EXTRUSION OF AXISYMMETRIC SECTIONS
THROUGH STREAMLINED AND CONICAL DIES

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CHAPTER I
INTRODUCTION

Although the basic extrusion process for lead pipe appeared in the late 1700's, it does not have a long history of development. Extrusion was industrialized only after World War II. Since then, the hot extrusion technique generally has been used with square dies (or flat-faced dies) which normally need a higher extrusion force, and which produce a less uniformly deformed product than a shaped die. Shaped dies (or converging dies) ordinarily make the billet deform to the product shape with a gradual change of cross section. A lubricant is required for more effective work of the shaped die. Shaped dies have both disadvantages and advantages over square dies. One disadvantage of shaped dies is that they are more difficult to design and manufacture, but if designed properly, they offer the following advantages over square dies:

(a) better quality of product
(b) less extrusion force
(c) greater extrusion speed and productivity.

Most of the products in the metal forming industry are non-circular in cross section, but a circular section is advantageous for extrusion billet shape. Therefore, this research deals with a round-to-round shaped die which is also axisymmetric.
In the theoretical work, upper bound solutions have been introduced for extrusion through a streamlined die and two conical dies which are axisymmetric. Also, ALPID (Analysis of Large Plastic Increment Deformation) simulations have been performed to validate the experimental results as a part of theoretical work.

For the experiment, a streamlined die and two conical dies which are essentially straightly converging dies were manufactured by EDM (Electrical Discharge Machining). Commercially pure aluminum was used as the billet material and grease as the lubricant. Extrusion was conducted at room temperature through shaped dies by a vertical press machine and the measurements of load were in agreement with the predicted results from the theoretical works. The Rockwell hardness test was performed to check the strength properties of the material.

The objective of this study, therefore, is to examine both theoretically and experimentally the performance of axisymmetrically shaped dies, using the direct extrusion method.
CHAPTER II
THEORY OF EXTRUSION

2.1 Extrusion Process

Extrusion is a process in which material is forced to flow, under high pressure, through a shaped die, thereby producing a length of product with the required constant cross section. The metal flow pattern is important to investigate in an extrusion process. This technique will be further discussed in Chapter IV.

The material, which is called billet, is placed in a container. The billet is normally in cylindrical form. A die is placed at one end of the container. By pressing a ram along the container against the billet, the billet is forced to flow through the die openings.

2.2 Types of Extrusion Process

The four common methods of extrusion processes, known as direct, indirect, hydrostatic, and impact extrusion, are shown in Fig. 2-1 (from Alexander [1] and Kalpakjian [2]).

Direct extrusion is much like squeezing toothpaste through its tube. The billet and the ram travel in the same direction, so that
Fig. 2-1. Types of Extrusion (Alexander [1]).
Fig. 2-1. Types of Extrusion (Kalpakjian [2]).
the billet is moved forward relative to the container wall. As a result, the ram force is increased by the wall friction considerably. Generally, this process is convenient mechanically.

In indirect extrusion, the billet and the ram travel in opposite directions. In this method, there is almost no relative movement between the billet and the container walls. Consequently, there is only negligible frictional force at that interface.

Hydrostatic extrusion is a form of direct extrusion, but there is no friction between the container and the wall. The chamber is filled with a fluid which transmits the pressure to the billet. Despite producing no friction, this method has been limited in applications in real industry, because of the complex nature of tooling.

Impact extrusion is similar to indirect extrusion, and this method is useful for hollow shapes such as a tube.

2.3 Extrusion Dies

There are two common types of extrusion dies. The first type is called the square die which is also called the flat-faced die. The second type is the shaped die or converging die. The shape and the deforming regions for both square and shaped dies are illustrated
The square die has one or more openings, each of which is similar to the section of the desired product. It is not necessary for the openings to be identical, because the die may deform under load, and the extruded section itself may exhibit some post-extrusion recovery. The shaped die has a smooth entry with a circular cross section and changes gradually towards the final extruded shape.

It is clear that an unlubricated square die produces a severely deforming shear surface which separates the deforming zone from the dead metal zone. If a new and oxide-free surface of the product is required, the square die is specifically useful. In general, however, the square die requires a higher extrusion force than the shaped die, and its extrusion speed may be limited due to the occurrence of hot shortness in the material. To overcome those defects, the shaped extrusion dies are used. However, there are also some limitations in the application of shaped extrusion dies, because they are difficult to design and manufacture.

2.4 Types of Die Surfaces

For this research, two types of die surfaces were studied. They are shown in Fig. 2-3. In Fig. 2-3(a) a straight line connects the corresponding exit and entry points, and in Fig. 2-3(b)
Fig. 2-2. Extrusion Dies (Gunasekera [3]).
Fig. 2-3. Types of Die Surfaces (Gunasekera [3]).
the corresponding exit and entry points are connected by third-order polynomials with zero gradients at the ends.

2.4.1 Die Type (a) in Fig. 2-3

This is probably the simplest form of the shaped die. The die surface is connected by a straight line drawn from points on the entry shape to corresponding points on the exit shape.

2.4.2 Die Type (b) in Fig 2-3

In this die, points on the entry section are connected with corresponding points on the exit section by a third order polynomial, the coefficients of which are determined from the known boundary conditions. When r is the radial distances, r is expressed as a function of z. Hence, the expression of the third order polynomial is the following:

\[ r = f(z) = a_0 + a_1z + a_2z^2 + a_3z^3 \]  \hspace{1cm} (2-1)

where \( a_0, a_1, a_2, a_3 \) are all constants to be determined.

If the length of the die is L, the following boundary conditions are derived.

At entry section: \( z = 0, \ r = r_0, \ dr/dz = 0 \)
hence,
\[ r = a_0 = r_0 \]  \hspace{1cm} (2-2)

and
\[ \frac{dr}{dz} = a_1 = 0 \]  \hspace{1cm} (2-3)

At exit section: \( z = L, r = r_1, \frac{dr}{dz} = 0 \)

hence,
\[ r_1 - r_0 = a_2 L^2 + a_3 L^3 \]  \hspace{1cm} (2-4)

and
\[ \frac{dr}{dz} = 2a_2 L + 3a_3 L^2 = 0 \]  \hspace{1cm} (2-5)

By eq. (2.5) \* L - eq (2-4) \* 2, it was obtained that
\[ a_3 L^3 = 2r_0 - 2r_1 \]

\[ a_3 = 2(r_0 - r_1)/L^3 \]  \hspace{1cm} (2-6)

By substituting eq. (2-6) into eq. (2-5), \( a_2 \) was achieved as follows:

\[ a_2 = -3(r_0 - r_1)/L^2 \]  \hspace{1cm} (2-7)

Now, all the necessary values of constants \( a_0, a_1, a_2, \) and \( a_3 \) were determined. By substituting eqs. (2-2), (2-3), (2-6), and (2-7) into eq. (2-1), the following result can be made:

\[ r(z) = 2(r_0 - r_1)z^2/L^3 - 3(r_0 - r_1)z^2/L^2 + r_0 \]  \hspace{1cm} (2-8)
If the values of \( L \), \( r_0 \), and \( r_1 \) in eq. (2-8) are specified, an equation of a streamlined die can be derived. This will be discussed in Chapter IV.

2.5 Upper Bound Solutions

Limit analysis has been developed to give a more realistic idea of the collapse loads of structures. The science of limit analysis applies generally to the plastic-rigid non-work-hardening material only. There are two different forms of limit analysis theorem. One is a lower bound technique which generalizes stresses, such as force, torques, or bending moments, and the other is an upper bound technique which is specifically concerned with kinematically admissible velocity fields such as strains, angular rotation, or linear displacements. If another kinematically admissible velocity field which requires a lower energy consumption exists, it will always come into play first as the load is increased. Therefore, the postulated field can never require less force than the real one.

A lower bound solution underestimates the load. An upper bound solution, on the other hand, overestimates the load, and it is of practical interest in metal forming processes. For this project, therefore, only the upper bound method will be considered.

In the upper bound technique, the whole deformation zone is
divided into several small zones in which the velocity of a particle is continuous. In adjacent zones, the particle velocities may be different. At the boundaries, however, between the zones, or between a zone and the die surface, all movement must be such that velocity discontinuities occur only in the tangential direction. In this method, the total power consumed in an operation is the sum of the ideal power of deformation, shearing force, and friction force.

2.5.1 Upper Bound Solution for Conical Die

The upper bound solution to axisymmetric extrusion through a conical die has been developed by so many outstanding reports. Kudo [1960, 1961] applied velocity fields similar to those of plane strain by introducing conical surfaces as surfaces of velocity discontinuity in cylindrical regions of deformation. Kobayashi [1963] established a new method to find the admissible velocity field which may contain curved surfaces of velocity discontinuity instead of conical surfaces. The method which was proposed by Halling and Mitchell [1964] has greater flexibility than Kudo's method and has been extended to incorporate both frictional effects and strain hardening characteristics. Their principles are based on the energy balance between the work input and the plastic-work dissipation due to a kinematically admissible solution. The method which was suggested by Kobayashi [4] was introduced in this research. Once again, the material is assumed to be perfectly plastic rigid and to follow the
Von Mises yield criterion.

In Fig. 2-4, it is easy to obtain the velocity components in region 1 and region 3.

At region 1: \( u_1 = 0, v_1 = 0 \)
At region 3: \( u_3 = 0, v_3 = 1/b^2 \)

In region 2, on the other hand, the following velocity components are suggested by an inspection of the incompressibility equation and velocity boundary conditions.

\[
\begin{align*}
    u_2 &= -v_0 \left( 1 + (z - \alpha a) \tan \delta / r \right) \tan \delta \\
    v_2 &= v_0 \left( 1 + (z - \alpha a) \tan \delta / r \right)
\end{align*}
\]

For here, \( \delta \) is a semidie angle, and a constant \( v_0 \) is to be determined by a requirement for the velocity-discontinuity curve. The equations for the curves of velocity discontinuity 13 and 23 can be obtained by integrating the continuity condition across these curves. Now the constant \( v_0 \) is found to be:

\[
v_0 = 1/(1 - \alpha a \tan \delta)^*\]

in order to satisfy the conditions for the curves 13 and 23. Then, the equations for 13 and 23 are given by:
Fig. 2-4. Assumed Velocity Field for the Flow through Conical Dies (Kobayashi [4]).
\[ z_{1a} = a_a(1 - r) \]

and

\[ z_{2a} = \frac{a}{b}(1 - \alpha)r + a_a \]

where \( a = (1 - b)cot\delta \)

It is possible in an axisymmetric case, as shown in Fig. 2-5(a), to assume a series of deformation patterns as shown in Fig. 2-5(b). The velocity field of the ith deforming zone, by the use of the notation in Fig. 2-5(b), is defined as following. In this study, the \( n \) will be 1, although the upper bound solution for general case will be developed.

\[ u_{a1} = -v_{o1}(1 + (z_1 - a_{a1})tan\delta/r)\tan\delta \]
\[ v_{a1} = -v_{o1}(1 + (z_1 - a_{a1})tan\delta/r) \]

(2-9)

where

\[ z_1 = z - z_{o1} = z - \sum_{j=1}^{i-1} a_j \]

and

\[ v_{o1} = 1/(b_1 - a_{a1}tan\delta)^2 \]

The equations for the surfaces of velocity discontinuity including the die surface are given by

\[ z_{(12)1} = (b_1 - r)cot\delta \]
\[ z_{(13)1} = a_{a1}(1 - r/b_1) \]
Fig. 2-5. Assumed Velocity Field for the Flow through Conical Dies (n=3). (Kobayashi [4]).
Now, it is ready to obtain the upper bound to the extrusion pressure. It can be determined by calculating the total energy dissipation rate based on the assumed velocity field.

The upper bound solution of the extrusion is given as,

\[
P_{\text{ave}} / \bar{\sigma} = \frac{E}{\pi \bar{\sigma}}
\]  

(2.10)

where \( E \) is the total energy dissipation rate which is based on an assumed velocity field with the unit entrance velocity.

In Fig. 2-5(b), the energy dissipation rate due to shear along the line \( l_{13} \) can be found, and it is given by

\[
\dot{E}_{(13)} = \int_{-\infty}^{b_4} \left( \frac{\bar{\sigma}}{\sqrt{3}} \right) \left| \dot{S}_{13} \right| 2\pi r \cdot dr / \sin \theta
\]

(2.11)

where \( \theta \) is the angle between the z-axis and the line \( l_{13} \) as shown in Fig. 2-5(b), and \( \left| \dot{S}_{13} \right| \) is the amount of velocity discontinuity along the line \( l_{13} \).
For here,

\[
\dot{\mathbf{S}}_{13} = \frac{1}{\sin \theta} \cdot b_1^2 (\cot \delta - \cot \theta) \quad (2-12)
\]

Substituting eq. (2-12) into eq. (2-11), and by integration, the result obtained is

\[
\dot{E}_{(13)1} = \left( \frac{\pi \sigma}{43} \right) \left\{ \frac{\sin \delta}{\sin \theta} \cdot \sin(\theta - \delta) \right\} \quad (2-13)
\]

With the same method, the energy dissipation rate along with the line 2_3_4 is given by

\[
\dot{E}_{(23)4} = \int_0^{b_{4+1}} \left( \frac{\sigma}{43} \right) [\dot{\mathbf{S}}_{23}] \cdot 2\pi r \cdot dr \cdot \sin \psi
\]

\[
\dot{E}_{(23)4} = \left( \frac{\pi \sigma}{43} \right) \left\{ \frac{\sin \delta}{\sin \psi} \cdot \sin(\psi + \delta) \right\} \quad (2-14)
\]

where \( \psi \) is the angle between the line 2_3_4 and the z-axis, as shown in Fig. 2-5(b). For the continuously deforming region 1_2_4_3_4, the energy rate can be calculated according to

\[
\dot{E}_{(123)4} = \int \int \sigma \cdot \dot{\mathbf{e}} \cdot 2\pi r \cdot dr \cdot dz \quad (2-15)
\]

Here the integration must be performed over the total volume of deformation. In eq. (2-15), the effective strain rate \( \dot{\mathbf{e}} \) is defined by

\[
\dot{\mathbf{e}} = [(2/3) (\dot{E}_r^2 + \dot{E}_\theta^2 + \dot{E}_z^2 + \frac{1}{2} \dot{\mathbf{r}} \cdot \dot{\mathbf{r}})^{1/2} \]

(2-16)
It can be expressed as:

\[ \dot{E} = (2/3)^{1/2} \left( v_{01} \tan \delta/r \right) \left( Mx^2 + Nx + L \right)^{1/2} \]  

(2-17)

where

\[ x = (z_1 - a_{a1})/r \]
\[ M = \frac{1}{2} + 2\tan^2 \delta \]
\[ N = \tan \delta \]

and

\[ L = 2 + \frac{1}{2}\tan^2 \delta \]

because, by definition, the strain rate components in eq. (2-16) are given by:

\[ \dot{e}_r = \frac{\partial u}{\partial r} = v_{01}\tan \delta \left\{ (z_1 - a_{a1})/r^2 \right\} \]
\[ \dot{e}_\theta = \frac{\partial u}{\partial r} = (-v_{01}\tan \delta/r) \left\{ 1 + (z_1 - a_{a1})\tan \delta/r \right\} \]
\[ \dot{e}_z = \frac{\partial v}{\partial z} = v_{01}\tan \delta/r \]
\[ \dot{e}_{rz} = \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} = (v_{01}\tan \delta/r)\left\{ \tan \delta - (z_1 - a_{a1})/r \right\} \]  

(2-18)

The polar coordinate system \((\rho, \theta)\) is introduced in Fig. 2-6 to integrate eq. (2-15), and it can be written as:

\[ \dot{E}_{(12\theta)} = \iint_\sigma \dot{E} \, 2\pi \rho^2 \cos \theta \, d\theta d\rho \]  

(2-19)

where the limits of integration are

\[ \theta = -(\pi/2 - \varphi) \sim (\pi/2 - \psi) \]
Fig. 2-6. Polar Coordinate System on a Meridian Plane (Kobayashi [4]).
\[ \rho = 0 \quad \Gamma \]

for here,

\[ \Gamma = \frac{b_4 - \alpha a_1 \tan \delta}{(\cos \theta + \sin \theta \tan \delta)} \]

for the die surface.

By substituting the effective strain rate \( \dot{\varepsilon} \), which was given by eq. (2-17) into eq. (2-19), it is certain that

\[ \dot{E}_{(123)} = 2\pi \Omega (2/3)^{1/2} v_{o1} \tan \delta \]

\[ \int_{-\pi/2}^{\pi/2} \int_0^R \left( Mx^2 + Nx + L \right)^{1/2} \rho \cdot d\rho \] (2-20)

Using eq. (2-9) again for \( v_{o1} \), and noting that \( x = \tan \theta \), the energy dissipation rate due to plastic deformation, eq. (2-20) can be rewritten as:

\[ \dot{E}_{(123)} = \pi \Omega (2/3)^{1/2} \tan \delta \int_{-\cot \psi}^{\cot \psi} \left( \frac{Mx^2 + Nx + L}{(Nx + L)^2} \right) dx \] (2-21)

Finally, the total energy dissipation rate of the ith deforming region is obtained by adding eq. (2-13), eq. (2-14) and eq. (2-21). That is,

\[ \dot{E}_i = \frac{\pi \Omega}{3} \sin \delta \left\{ \frac{1}{\sin \theta \sin (\theta - \delta)} \right\} + \frac{1}{\sin \psi \sin (\psi + \delta)} \]

\[ + (J2/\cos \delta) I \] (2-22)

where
Therefore, the total energy dissipation rate for the velocity field of triangular patterns is given by:

\[ I = \int_{-\infty}^{\infty} \{(Mx^2 + Nx + L)^{1/2}/(Nx + 1)^2\}dx \]

\[ \dot{E} = (n\pi\delta/4\sqrt{3})\sin\left[\left\{1/\sin\theta \sin(\theta - \delta)\right\} + \left\{1/\sin\psi \sin(\psi + \delta)\right\} \right] \\
+ (4\sqrt{2}/\cos\delta)I \]  

(2-23)

In Fig. 2-5(a), the reduction in area \(A\) defined by \(A = (1-b^2)/1\) is related to \(n\), \(\theta\), and \(\psi\) according to the equation

\[ (1 - A)^{1/3n} = \{\sin\psi \sin(\theta - \delta)\}/\{\sin\theta \sin(\psi + \delta)\} \]

Green, who developed an upper bound solution for the plane strain condition, assumed \(\theta = \psi + \delta\). Since the first two terms of eq. (2-23) are exactly the same as Green's solution, the same assumption can be made. Then, the upper bound to the bar-extruding can be written as follows:

\[ \dot{P}_{\text{ave}} / \sigma = \dot{E}/\pi\delta = (n \sin\delta/3J3)[\{2/\sin\theta \sin(\theta - \delta)\} + (4\sqrt{2}/\cos\delta)I] \]  

(2-24)

where

\[ (1 - A)^{1/3n} = \sin(\theta - \delta)/\sin\theta \]

and

\[ \psi = \theta - \delta \]
The constants x, M, N, and L are already given by eq. (2-17), and I is defined by eq. (2-22). The number n can be selected in such a way that the upper bound solution becomes minimum. Therefore, the upper bound solution for conical die to axisymmetric extrusion can be obtained.

2.5.2 Upper Bound Solution for Streamlined Die

An upper bound solution developed by Hoshino [5] will be modified for the streamlined die in this thesis.

An equation of cubic die can be formulated as,

\[ r = a_0 + a_1 z + a_2 z^2 + a_3 z^3 \]  \hspace{1cm} (2-25)

For this equation, the boundary conditions are given as follows:
At entry section: \[ z = 0, \quad r = r_o, \quad \frac{dr}{dz} = 0 \]

At exit section: \[ z = L, \quad r = r_1, \quad \frac{dr}{dz} = 0 \]

By substituting those boundary conditions in eq. (2-25), the equation of \( r \) can be obtained.

\[
r = r_o - 3(r_o - r_1)z^2/L^2 + 2(r_o - r_1)z^3L^3 \tag{2-26}
\]

The velocity component along the extrusion axis \([V_z]\) is expressed by

\[
V_z = V_o \left( \frac{r_o}{r} \right)^2
\]

This equation can be rewritten as:

\[
V_z = V_o r_o^2/[r_o - 3(r_o - r_1)z^2/L^2 + 2(r_o - r_1)z^3/L^3]^2
\]

The incompressibility condition for this study is found by

\[
\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_z = 0
\]

Therefore,

\[
\frac{\partial V_z}{\partial r} + \frac{V_z}{r} + \frac{\partial V_z}{\partial z} = 0 \tag{2-27}
\]

If eq. (2-27) is multiplied by \( r \), it can be written as
By the integration of eq. (2-28), an equation for $V_r$ can be obtained as follows:

$$V_r = -(1/r) \int r V_x/\partial z \, dr$$

$$= -(1/r) \int r V_o(-2) r_o^2/r^3 \cdot (\partial r/\partial z) \cdot dr$$

For here:

$$\partial r/\partial z = 6(r_o - r_1)z^2/L^3 - 6(r_o - r_1) z/L^2$$

For the streamlined die, the total power consumption $J^*$ is represented as the sum of the following terms.

$W_1 = \text{work dissipated by the plastic deformation}$

$W_m = \text{work dissipated by the die surface friction}$.

Hence,

$$J^* = W_1 + W_m$$

For here,

$$W_1 = 2/J^3*Y_m \int \left[ \frac{1}{2} \frac{\partial \epsilon_{ij} \partial \epsilon_{ij}}{\partial z} \right]^{1/2} \, dv$$

$$= 2/J^3 * Y_m * 2\pi \int_0^r \int_0^L \left[ \frac{1}{2} \frac{\partial \epsilon_{ij} \partial \epsilon_{ij}}{\partial z} \right]^{1/2} \, rdr \, dL$$  \hspace{1cm} (2-29)
where

\[ Y_m = \text{yield stress} \]

\[ \dot{\varepsilon}_{ij} = \frac{1}{2}(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}) = \text{strain rate component expressed by the tensor form} \]

and

\[ W_m = \frac{mY_m}{J3} \int_{V} |\Delta V| \, ds \]

\[ = \frac{mY_m}{J3} \cdot 2\pi \int_{0}^{2\pi} \left[ V_r^2 + V_\theta^2 \right]^{1/2} r \, dz/\cos \alpha \quad (2-30) \]

where

\[ m = \text{frictional factor} \]

\[ \alpha = \text{maximum angle of inclination of the die surface element} \]

A more detailed explanation for eq. (2-29) and eq. (2-30) is included in Appendix B.

The power consumption can be converted to the average pressure.

\[ P_{ave} = J^*/\pi r^2 o V_o \]

The method which has been introduced here can also be applied for the conical dies, by changing eq. (2-26) for a straight line.
From the two different approaches in Sections 2.5.1 and 2.5.2, the basic idea of an upper bound technique has been achieved. The finite element method usually gives a more precise value than the upper bound method. Hence, ALPID was chosen for the theoretical value because it is based on the finite element method (FEM).
CHAPTER III
ALPID SIMULATION

3.1 ALPID Simulation

ALPID (Analysis of Large Plastic Incremental Deformation) developed by Battelle Columbus division [6] is a finite element method (FEM) code which is suitable for the simulation of plastic deformation in metal forming processes. ALPID 2.0 is composed of four independent FORTRAN packages. They are:

(i) the FEM code ALPID
(ii) the post-processor FEMGRA
(iii) the rezoning/interpolation program
(iv) the data base management program.

The FEM code ALPID program is based on the coupled thermo-viscoplastic formulation. Hence, it is remarkably useful to simulate a wide class of metal forming processes which are under either isothermal or non-isothermal conditions. It is applicable to rigid-plastic and rigid-viscoplastic materials, that is, the program can be applied not only to hot and cold forming of metals, but also to hot forming of glasses and polymers. The FEM code ALPID program also has a capability of performing automatic initial guess. Therefore, an excellent computational efficiency is achieved and data requirements
are reduced to those of a linear elastic FEM program.

The following input data are necessary for ALPID 2.0.

(i) Die data: die geometry, movement, and friction at die surface
(ii) Workpiece geometry: elements and nodal points
(iii) Material properties: flow stresses of workpiece, temperature, and thermal properties of die and workpiece

As a result of the simulation, with the above input data, ALPID 2.0 provides load requirement, metal flow, stress distribution, strain distribution, and die-workpiece interface traction at any stage of the deformation.

In this research, ALPID simulations were performed for four different cases of extrusion. In case study 1, a streamlined die was used. In case study 2, the same die was used, but with a new mesh system for grid enhancement as shown in Fig. 3-12. A conical die was used in case study 3 and case study 4, each with a different die geometry. For all those simulations, 1100-0 aluminum was used as the billet material, and all of those 4 cases were performed under a condition of lubricated cold extrusion.
From Fig. 3-10, Fig. 3-30, and Fig. 3-40, which are the effective strain rate contours, the following results were observed. As it was expected from the experiment, the contour of the streamlined die is more uniform than both conical dies. Also, the 14° conical die has a better result than the 26.6° conical die. In those figures, there is one more important fact. For the extruded product, the effective strain rate at center in cross section has a smaller value than the outside for all the cases. It is the same result as the hardness test, so that those contours look similar to the graph of hardness in Fig. 4-10.

The other comparisons of the results obtained by ALPID simulation will be discussed in Chapter V.

3.2 Case Study 1: Streamlined Die

<table>
<thead>
<tr>
<th>Die Shape</th>
<th>Streamlined Die</th>
</tr>
</thead>
<tbody>
<tr>
<td>Billet Diameter</td>
<td>1 inch</td>
</tr>
<tr>
<td>Product Diameter</td>
<td>0.5 inch</td>
</tr>
<tr>
<td>Extrusion Ratio</td>
<td>4</td>
</tr>
<tr>
<td>Die Length</td>
<td>1 inch</td>
</tr>
<tr>
<td>Friction Coefficient</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Using the above input parameters, the following output results were obtained from the simulation.
### 3.3 Case Study 2: Streamlined Die with a New Mesh

<table>
<thead>
<tr>
<th>Die Shape</th>
<th>Streamlined Die</th>
</tr>
</thead>
<tbody>
<tr>
<td>Billet Diameter</td>
<td>1 inch</td>
</tr>
<tr>
<td>Product Diameter</td>
<td>0.5 inch</td>
</tr>
<tr>
<td>Extrusion Ratio</td>
<td>4</td>
</tr>
<tr>
<td>Die Length</td>
<td>1 inch</td>
</tr>
<tr>
<td>Friction Coefficient</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Using the above input parameters, the following output results were obtained from the simulation.

<table>
<thead>
<tr>
<th>Total Stroke (Die II)</th>
<th>0.73869 inch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Load (Die I)</td>
<td>0.5013 E+05 pounds</td>
</tr>
<tr>
<td>Step Number</td>
<td>from 1 to 70</td>
</tr>
</tbody>
</table>

### 3.4 Case Study 3: Conical Die (14 degrees)

<table>
<thead>
<tr>
<th>Die Shape</th>
<th>Conical Die</th>
</tr>
</thead>
<tbody>
<tr>
<td>Billet Diameter</td>
<td>1 inch</td>
</tr>
<tr>
<td>Product Diameter</td>
<td>0.5 inch</td>
</tr>
</tbody>
</table>
Extrusion Ratio 4
Die Length 1 inch
Semidie Angle 14 degrees
Friction Coefficient 0.3

Using the above input parameters, the following output results were obtained from the simulation.

Total Stroke (Die II) 0.66570 inch
Maximum Load (Die I) 0.5416 E+05 pounds
Step Number from 1 to 70

3.5 Case Study 4: Conical Die (26.6 degrees)

Die Shape Conical Die
Billet Diameter 1 inch
Product Diameter 0.5 inch
Extrusion Ratio 4
Die Length 0.5 inch
Semidie Angle 26.6 degrees
Friction Coefficient 0.3
Using the above input parameters, the following output results were obtained from the simulation.

Total Stroke (Die II) 0.37365 inch
Maximum Load (Die I) 0.5489 E+05 pounds
Step Number from 1 to 60
Fig. 3-1. Case 1 - Streamlined Die Load-Stroke.
Fig. 3-2. Case 1 - Streamlined Die Initial Grid (Step 0).
Fig. 3-3. Case 1 - Streamlined Die Grid Distortion (Step 35).
Fig. 3-4. Case 1 - Streamlined Die
Grid Distortion (Step 70).
Fig. 3-5. Case 1 - Streamlined Die
Eff. Stress (PSI) (Step 35).
Fig. 3-6. Case 1 - Streamlined Die
Eff. Stress (PSI) (Step 70).
Fig. 3-7. Case 1 - Streamlined Die Effective Strain (Step 35).

\[ A = 4.23 \times 10^{-1} \]
\[ B = 1.12 \times 10^{0} \]
\[ C = 3.19 \times 10^{0} \]
\[ D = 3.27 \times 10^{0} \]
\[ E = 3.35 \times 10^{0} \]
\[ F = 6.43 \times 10^{0} \]
\[ G = 5.10 \times 10^{0} \]
Fig. 3-8. Case 1 - Streamlined Die
Effective Strain (Step 70).
Fig. 3-9. Case 1 - Streamlined Die
Eff. Strain Rate ( /sec) (Step 35).
Fig. 3-10. Case 1 - Streamlined Die
Eff. Strain Rate (/sec) (Step 70).
Fig. 3-11. Case 2 - Streamlined Die Load-Stroke.
Fig. 3-12. Case 2 - Streamlined Die Initial Grid (Step 0).
Fig. 3-13. Case 2 - Streamlined Die Grid Distortion (Step 35).
Fig. 3-14. Case 2 - Streamlined Die
Grid Distortion (Step 70).
Fig. 3-15. Case 2 - Streamlined Die
Eff. Stress (PSI) (Step 35).
Fig. 3-16. Case 2 - Streamlined Die
Eff. Stress (PSI) (Step 70).
Fig. 3-17. Case 2 - Streamlined Die Effective Strain (Step 35).
Fig. 3-18. Case 2 - Streamlined Die Effective Strain (Step 70).
Fig. 3-19. Case 2 - Streamlined Die
Eff. Strain Rate ( /sec) (Step 35).
Fig. 3-20. Case 2 - Streamlined Die
Eff. Strain Rate ( /sec) (Step 70).
Fig. 3-21. Case 3 - Conical Die (14 degrees)
Load-Stroke.
Fig. 3-22. Case 3 - Conical Die (14 degrees) Initial Grid (Step 0).
Fig. 3-23. Case 3 - Conical Die (14 degrees) Grid Distortion (Step 35).
Fig. 3-24. Case 3 - Conical Die (14 degrees) Grid Distortion (Step 70).
Fig. 3-25. Case 3 - Conical Die (14 degrees) Eff. Stress (PSI) (Step 35).
Fig. 3-26. Case 3 - Conical Die (14 degrees)  
Eff. Stress (PSI) (Step 70).
Fig. 3-27. Case 3 - Conical Die (14 degrees) Effective Strain (Step 35).
Fig. 3-28. Case 3 - Conical Die (14 degrees)
Effective Strain (Step 70).
Fig. 3-29. Case 3 - Conical Die (14 degrees)
Eff. Strain Rate ( /sec) (Step 35).
Fig. 3-30. Case 3 - Conical Die (14 degrees)  
Eff. Strain Rate ( /sec) (Step 70).
Fig. 3-31. Case 4 - Conical Die (26.6 degrees) Load-Stroke.
Fig. 3-32. Case 4 - Conical Die (26.6 degrees)
Initial Grid (Step 0).
Fig. 3-33. Case 4 - Conical Die (26.6 degrees)
Grid Distortion (Step 30).
Fig. 3-34. Case 4 - Conical Die (26.6 degrees) Grid Distortion (Step 60).
Fig. 3-35. Case 4 - Conical Die (26.6 degrees)
Eff. Stress (PSI) (Step 30).
Fig. 3-36. Case 4 - Conical Die (26.6 degrees)
Eff. Stress (PSI) (Step 60).
Fig. 3-37. Case 4 - Conical Die (26.6 degrees) Effective Strain (Step 30).
Fig. 3-38. Case 4 - Conical Die (26.6 degrees) Effective Strain (Step 60).
Fig. 3-39. Case 4 - Conical Die (26.6 degrees)
Eff. Strain Rate ( /sec) (Step 30).
Fig. 3-40. Case 4 - Conical Die (26.6 degrees)
Eff. Strain Rate (1/sec) (Step 60).
4.1 Preparation of the Experiment

For this experiment, soft aluminum 1100-0 which was annealed was chosen as the billet material. Among the basic types of extrusion explained in Chapter II, the direct extrusion method was selected by examination of a press machine and a die set for the press machine. With the above basic concepts, this experiment was performed by the following procedure.

(a) Designing and manufacturing of the die
(b) Preparing of the billet
(c) Performing of the experiment
(d) Hardness test

Prior to all those procedures, however, the approximate ram force was determined to ensure that the press machine had adequate power for the experiment. According to Kalpakjian [2], the constitutive equation for 1100-0 aluminum is obtained as:

\[ \sigma = K\varepsilon^n = 26,000\ \dot{\varepsilon}^{0.2} \]

where

\[ K = \text{strength coefficient} \]
n = strain hardening exponent
\bar{\sigma} = effective stress
\bar{\varepsilon} = effective strain

The billet used for this experiment had a diameter of 1 inch and the product diameter was 0.5 inch. Hence, the extrusion ratio R is obtained as,

\[ R = \frac{A_o}{A_f} = \frac{1}{(0.5)^2} = 4 \]

where

\( A_o \) = the billet cross-sectional area
\( A_f \) = the area of the extruded product

Now, the value of the effective strain is,

\[ \bar{\varepsilon} = \ln 4 = 1.3863 \]

Therefore,

\[ \bar{\sigma} = 26000 \ast (1.3863)^{0.2} = 27755.23 \text{ (psi)} \]

and

\[ Y_m = \frac{1}{\bar{\varepsilon}} \int \bar{\sigma} \, d\bar{\varepsilon} = \frac{26000}{\bar{\varepsilon}} \int \bar{\varepsilon}^n \, d\bar{\varepsilon} \]

\[ = \frac{\bar{\sigma}}{n+1} = 23129.36 \text{ (psi)} \]

where

\( Y_m \) = mean yield stress

Hirst and Ursell [9] suggested the basic equation of aluminum alloy for the extrusion pressure required to extrude a billet of
length 1 as,

\[ P/Y_m = (0.47 + 1.2 \ln R) \] (4-1)

Hence, the required extrusion pressure is

\[ P = 49347.72 \text{ (psi)} \]

and

\[ \text{Force} = P \times \pi/4 = 38757.61 \text{ (lb.)} \]

The above calculation was for the frictionless and shear die. However, rough estimation of the required power is valuable because the press machine has a maximum capacity of 400,000 lbs. and the eq. (4-1) is the (well known) basic equation of the extrusion pressure.

4.2 Design and Manufacturing of the Die

The die for this project is designed to fit into the container of the die set. From the measurement of the container, the following dimensions were obtained.

Diameter of entrance = 1 inch
Diameter of exit = 0.5 inch
Length of the die = 1 inch
By recalling eqs. (2-1), (2-2), (2-3), (2-6) and (2-7), the equation of a streamlined die for this thesis can be found. That is,

\[ r = f(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 \]

where

- \( a_0, a_1, a_2, \) and \( a_3 \) are all constants
- \( r_0 \) = radius of entrance
- \( r_1 \) = radius of exit
- \( L \) = length of the die

By substituting all the necessary values, the equation of a streamline was determined as,

\[ r(z) = 0.5z^3 - 0.75z^2 + 0.5 \]

Using this equation, the streamlined die was designed as shown in Fig. 4-1. By connecting the die surface between the entry point and the exit point with a straight line instead of the streamline, a comparable conical die is formed. A diagrammatic comparison between the streamlined die and the conical die is shown in Fig. 4-2. The angle \( \alpha \) of the conical die is 14 degrees.
Fig. 4-1. Design of the Streamlined Die.
1: Streamlined Die
2: Conical Die
\[ \alpha = 14^\circ \]

*Fig. 4-2. Diagrammatic Comparison between the Streamlined and Conical Die.*
A second conical die was designed for comparison with the above conical die and the dimensions are as follows.

Diameter of entrance = 1 inch
Diameter of exit = 0.5 inch
Length of the die = 0.5 inch
Die angle $\alpha = 26.6$ degrees

A simple design of the die is shown in Fig. 4-3.

Those three different dies based on the designs were manufactured by EDM (Electrical Discharge Machining). For this purpose, first of all, an electro graphite must be prepared. This was accomplished with a MAZAK (Mazatrol T-2) CNC turning center which is shown in Fig. A-1 in Appendix A. The graphite was shaped exactly the same as the design, and was placed on the EDM (Easco Sparcatron) machine. A photograph of this procedure is included in Appendix A (Fig. A-2). The EDM was operating at 100V in voltage for about 4 hours and 7.5V in voltage for about 1 hour for finishing. When using the EDM, a lower voltage gives better surface finish, but takes a much longer time.

4.3 Preparation of the Billet

To make a billet from raw material, first the length of the
Fig. 4-3. Design of a Conical Die (26.6°).
billet should be decided. To do so, the following simple diagram was used. In this diagram, the length of region (a) was decided as 0.5 inch and 0.25 inch for region (c). To determine region (b), a simple calculation is performed.

At the point of 0.5 inch from the entrance, the reduction \( R \) is given as,

\[
R = \frac{1}{(0.75)^2} \approx 1.778
\]

Hence, the billet length of region (b) is determined as,

\[
\frac{1}{1.778} \approx 0.562
\]

Hence, the required length of billet is achieved.

\[
0.5 + 0.562 + 0.25 = 1.312 \text{ (inches)}
\]

Since 1.312 inches are enough for the billet, the billet length was decided as 1.5 inches.

The metal flow pattern in extrusion is an important factor, and a common technique for investigating the flow pattern is explained below.
(a) mill off a half of the round billet lengthwise
(b) draw on remaining half a grid pattern as shown in
Fig. 4-5 (Kalpakjian [2])
(c) cut as a billet length, and two pieces are placed
    together in the container

For the purpose of the procedure (b), a Bridgeport R2E4 CNC milling
machine which is shown in Fig. A-3 of Appendix A was used and
the depth and the width of the grid was determined as 0.0025 inch and
0.003 inch, respectively.

4.4 Performing of the Experiment

A Forney vertical press machine (Fig. A-4) with 400,000 lbs.
capacity load was used for this experiment. A schematic diagram of
the die set for the press machine is shown in Fig. 4-6. Some grease
was applied at the interface of the container wall and the billet.

From the experiment, the maximum loads were achieved, as well
as the metal flow patterns shown in Fig. 4-7, Fig. 4-8 and Fig. 4-9.

Max. Load of Streamlined Die = 56676.27 (lbs.)
Max. Load of Conical Die (14°) = 59137.46 (lbs.)
Max. Load of Conical Die (26.6°) = 61775.64 (lbs.)
Fig. 4-4. Illustration of the Billet Length.

Fig. 4-5. Pattern of Grid.  
(Kalpakjian [2])
Fig. 4-6. Schematic Diagram of the Die Set.
Fig. 4-7. The Metal Flow Pattern (Streamlined Die).
Fig. 4-8. The Metal Flow Pattern (Conical Die 14°).
Fig. 4-9. The Metal Flow Pattern (Conical Die 26.6°).
4.5 **Hardness Test**

The hardness of a material is defined as resistance to permanent indentation. It has been found that the resistance to indentation depends on the shape of the indenter and the load which is applied. Hence, the hardness is not a fundamental property, but it is often important to check the strength properties of a material without destroying it.

Even though it is not possible to use the hardness test to determine the stress-strain curve of a metal, the hardness test is still useful as a guide to the overall strength of a material. The deformation of the material can be determined by the hardness test, because there is a correlation between hardness and yield stress. According to Alexander [7], for example, the correlation between Brinell Hardness $H_B$ and ultimate tensile strength of a material is found to be approximately,

$$\text{U.T.S.} \approx 0.22 H_B$$

The dimensions of U.T.S. and $H_B$ must be the same. Among the most common hardness tests which include Brinell, Rockwell, Vickers, and Scleroscope tests, the Rockwell hardness test was chosen for this project. In this test, the depth of the indentation is measured as a hardness. For this experiment, a $1/8$ inch diameter steel ball was
used as an indenter with a load of 60 kg and an H scale. Thus, when a hardness number is 80, it is written as 80 HRH.

The following data were obtained from this experiment.

<table>
<thead>
<tr>
<th>At point 1</th>
<th>Streamlined Die</th>
<th>Conical Die 14°</th>
<th>Conical Die 26.6°</th>
</tr>
</thead>
<tbody>
<tr>
<td>103</td>
<td>106</td>
<td>117</td>
<td></td>
</tr>
<tr>
<td>At point 2</td>
<td>102</td>
<td>105</td>
<td>114</td>
</tr>
<tr>
<td>At point 3</td>
<td>101</td>
<td>103</td>
<td>111</td>
</tr>
<tr>
<td>At point 4</td>
<td>101</td>
<td>104</td>
<td>114</td>
</tr>
<tr>
<td>At point 5</td>
<td>102</td>
<td>106</td>
<td>118</td>
</tr>
</tbody>
</table>

Table 4-1. Rockwell Hardness (HRH).

The values obtained from different hardness tests can be easily converted to different scales in which a correlation with a yield stress can be found by using a converting chart. Unfortunately, no converting chart is available for the HRH scale.
which indicating it to be very soft at the present time. In fact, the following comparison must be plotted: \( r \) (radius) vs. \( \bar{\varepsilon} \) (effective strain). However, it is plotted as \( r \) vs. HRH for this experiment and it is still useful at this time since we know that the hardness is related to the strength of the material. In this plot, higher value means more deformation. Analysis of the data will be discussed in the next chapter.
Fig. 4-10. Graphs of Hardness.
CHAPTER V
COMPARISON AND CONCLUSION

In this work, 4 different types of dies were studied for ALPID simulation, and 3 different dies for the experiment. By comparing these different types, the extrusion process can be analyzed.

5.1 Comparison Between Old Mesh and New Mesh for ALPID

The new mesh system was applied for an ALPID simulation. Mesh is a very important factor and a basic conception of the ALPID work, since ALPID analyzes the metal forming process with all the elements created by a mesh.

After several steps of execution, remeshing is expected for some of the elements because they may be distorted too much. Fig. 5-1 explains the movement of an element. At the region II, the angle 123 is bigger than 135° which is not within ALPID standards. Hence, it is necessary to remesh the element or it will shift to region III causing some error in analysis. A shape of the element at region III should be avoided for effective use of ALPID.

To be compared with this, a new mesh method shown as follows was attempted. This element will never touch a die with three nodal points on a line and it will not be distorted so greatly as to
Fig. 5-1. Movement of an Element within Old Mesh Method.
require remeshing.

This mesh was suggested by Mr. Vijay Shende in U.E.S. through an informal communication.

ALPID simulations have been performed with the above mesh systems. The following results were obtained from those meshes.

<table>
<thead>
<tr>
<th></th>
<th>Old Mesh</th>
<th>New Mesh</th>
<th>Diff. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Die Stroke (Die II)</td>
<td>0.71750</td>
<td>0.73869(in)</td>
<td>3</td>
</tr>
<tr>
<td>Max. Load (Die I) (lbs)</td>
<td>0.5044E+05</td>
<td>0.5013E+05</td>
<td>0.6</td>
</tr>
<tr>
<td>Eff. Stress (Contour G) (lbs)</td>
<td>0.271E+05</td>
<td>0.270E+05</td>
<td>0.37</td>
</tr>
<tr>
<td>Eff. Strain (Contour G)</td>
<td>0.153E+01</td>
<td>0.147E+01</td>
<td>3.9</td>
</tr>
<tr>
<td>Eff. Strain Rate (Contour G)</td>
<td>0.604E+01</td>
<td>0.601E+01</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 5-1. Comparison of ALPID Result Between Old Mesh and New Mesh.

Table 5-1 shows that every result is in very good agreement. There is also the similarity of the grid distortion in ALPID simulation in Chapter III. Therefore, the new mesh can be used for the streamlined die.
5.2 Comparison of Grid Distortion Between ALPID and the Experiment

In Fig. 5-2, Fig. 5-3, and Fig. 5-4, the similarity of grid distortions between ALPID and the experiment was observed. Thus, the assumed friction coefficient 0.3 for ALPID simulation is suitable.

5.3 Comparison Between a Streamlined Die and a Conical Die

Using the same reduction (4) and die length (1 inch), a streamlined die which connects two points between the exit and the entry point with a third order polynomial curve and a conical die with a straight line were considered. Maximum load was selected as a comparing factor. From both experiment and ALPID simulation, Table 5-2 was obtained.

<table>
<thead>
<tr>
<th></th>
<th>Streamlined Die</th>
<th>Conical Die (14°)</th>
<th>Diff. %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Load (experiment) (lbs)</td>
<td>56676.27</td>
<td>59137.46</td>
<td>4.3</td>
</tr>
<tr>
<td>Max. Load (ALPID) (lbs)</td>
<td>50440.00</td>
<td>54160.00</td>
<td>7.4</td>
</tr>
</tbody>
</table>

Table 5-2. Max. Load of Streamlined Die and Conical Die.

According to the table, the streamlined die requires less power than the conical die, as expected. There is, however, only a small difference. When comparing the experimental result and ALPID result, the experimental result for the streamlined die was 11%
Fig. 5-2. Comparison of Grid Distortion (Streamlined Die).
Fig. 5-3. Comparison of Grid Distortion (Conical Die 14°).
Fig. 5-4. Comparison of Grid Distortion (Conical Die 26.6°).
higher and 8.4% higher for the conical die. That may be caused by a
greater loss of energy in the experimental work. During the
experiment, a flashing was found. Hence, the hold down ring in Fig.
4-6 was manufactured to avoid that problem. Also, expansion of the
container was observed. All those ineffective modifications require
power consumption.

5.4 Comparison Between Two Conical Dies

Two different conical dies are compared in this section. They
have different lengths and different angles. As they are shown in
Fig. 4-2 and Fig. 4-3, the conical die with a 14° die angle is 1
inch in die length, and the other die with 26.6° die angle has a 0.5
inch die length. The result, for those conical dies, from the
experiment and ALPID is shown in Table 5-3.

<table>
<thead>
<tr>
<th></th>
<th>Conical</th>
<th>Conical</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(14°)</td>
<td>(26.6°)</td>
<td>(%)</td>
</tr>
<tr>
<td>Max. Load (experiment) (lbs)</td>
<td>59137.46</td>
<td>61775.64</td>
<td>4.46</td>
</tr>
<tr>
<td>Max. Load (ALPID) (lbs)</td>
<td>54160.00</td>
<td>54890.00</td>
<td>1.35</td>
</tr>
</tbody>
</table>

Table 5-3. Max. Load of Two Conical Dies.

The smaller angle (longer length) conical die requires less
power than the other. Because of the same reason mentioned in the
previous Section 5.2, the experimental result is 8.4% and 11.1%
higher than ALPID result respectively.

5.5 Analysis of the Result of the Hardness Test

Using the Rockwell hardness test, the results in Table 4-1 and Fig. 4-10 were measured. From the results, it is obvious that the center of the material is less deformed than both sides since a higher hardness number represents more deformation. It is also obvious that the conical die with 26.6° angle is deformed more severely and the streamlined die is the least deformed. Fig. 4-10 indicates a very important phenomenon. In the case of the streamlined die, the hardness was changed only slightly along the cross section. However, the conical die (26.6°) was changed greatly. The streamlined die, thus, gives a uniform deformation along the cross section of the product.

5.6 Conclusion

Several remarkable aspects have been observed in Sections 5.1 through 5.4. By arranging those comparisons and analyses, the following conclusions are expected.

1. A streamlined die is more efficient than a conical die.
2. A streamlined die gives a uniform deformation along the cross section indicating that the product through a
streamlined die is better than a conical die.

3. Since they have the same reduction, in the case of conical dies, a smaller angle which is identical as longer length is better than a bigger angle (shorter length).

4. The new method of mesh, for ALPID simulation, can remove the inconvenience of ALPID execution such as remeshing.
REFERENCES


APPENDIX A

PICTURES OF THE MACHINES

FOR

THE EXPERIMENT
Fig. A-1. CNC Turning Center (Mazatrol T-2) Manufacturing Electro Graphite.
Fig. A-2. EDM (EASCO Sparcatron) Manufacturing Die.
Fig. A-3. CNC Milling Machine (R2E4)  
Mark Grid on the Billet.
Fig. A-4. Press Machine (QC-410-D).
APPENDIX B

DETAILED EXPLANATION OF

THE POWER CONSUMPTION $w_1$ AND $w_2$. 
By recalling eq. (2-29),

\[ W_1 = \frac{2}{J3} \cdot Y_m \int \left[ \frac{\dot{E}_{x1} \dot{E}_{y1}}{J3} \right]^{1/2} \, dv \]

\[ = \frac{2}{J3} \cdot Y_m \int_0^L \int_0^{2\pi} \left[ \frac{\dot{E}_{x1} \dot{E}_{y1}}{J3} \right]^{1/2} \, r \, d\theta \cdot dr \cdot dL \]

\[ = \frac{2}{J3} \cdot Y_m \cdot 2\pi \int_0^L \int_0^{2\pi} \left[ \frac{\dot{E}_{x1} \dot{E}_{y1}}{J3} \right]^{1/2} \, rdr \cdot dL \]

For the above equations, \( dv \) can easily be converted to \( r \, d\theta \cdot dr \cdot dL \) using the following diagram.

In this diagram, \( dv = r \, d\theta \cdot dr \cdot dL \). Therefore, eq. (2-29) is clear.

On the other hand, in eq. (2-30),

\[ W_m = \frac{mY_m}{J3} \int s \mid \Delta v \mid ds \]

\[ = \frac{mY_m}{J3} \cdot 2\pi \int_0^L \left[ v_r^2 + v_z^2 \right]^{1/2} \, dz / \cos\alpha \]

For here, \( \cos\alpha \) can be determined from the below diagram.
By enlargement of the broken line section, it is obvious that

\[ ds = \frac{dz}{\cos \alpha}. \]