ANALYSIS OF A TEST TECHNIQUE FOR HARDNESS SURVEILLANCE,

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Frank Marcum
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INTRODUCTION

A sensor designed by C. C. Herrmann of TRW for the determination of shielding degradation around apertures is analyzed. The sensor is based on techniques developed by Yang and Baum [1].

The principal components of the system are a parallel plate transmission line antenna with a large slit on the bottom plate along the center of its length (Figure 1), a field sensor (in this case, a length of wire bonded across the aperture in the form of a loop), and a network analyzer for driving the transmission line and pickup of the signal on the shielded side of the aperture. The transmission line is slit so that it may be placed over a region containing an aperture and illuminate it uniformly. It is well known that a uniform field produces maximum penetration through an aperture antenna at low frequency (hence, a maximization of the signal to noise ratio).

\[
\begin{align*}
\frac{d\Phi}{dn} & = 0 \\
v_1 & = 0 \\
v_2 & = W_2 \\
\frac{d\Phi}{dn} & = 0
\end{align*}
\]

Figure 1: Geometry of Transmission line.

The transmission line is analyzed to determine the fields illuminating the aperture region and also to determine its characteristic
impedance. The conductors were made of a resistive material because practical considerations require the transmission line to have a small characteristic impedance. Network analyzers typically have source and load impedances of 50 Ω; hence a severe impedance mismatch occurs. The losses of the conductors quenched standing waves generated by the improperly terminated transmission line.

The coupling of the field through the aperture onto the sense wire is analyzed. The open circuit voltage and short circuit current are found so that the Thevenin equivalent can be obtained. Comparisons of computed sense wire currents to measured currents are made to verify the analysis. With this knowledge, the voltage (or current) driving the transmission line can be compared with the current and/or voltage induced in the sense wire over a wide range of frequencies. This comparison provides a measure of the electromagnetic field penetration through the aperture.

Finally, simple formula are derived for computation of per unit length capacitances of the multiconductor transmission line.

I. Comparison of TEM Field to the Static Field

This chapter shows that the electric and magnetic field vectors have the same spatial distribution as the static field solution in the plane transverse to the direction of propagation [2].

Solutions to the electric and magnetic field intensity vectors may then be obtained in the steady state with

\[ E(x, y, z, t) = E(x, y, z) e^{j\omega t} \quad \text{and} \quad H(x, y, z, t) = H(x, y, z) e^{j\omega t} \]
respectively, where

\[
E(x, y, z) = E_y(x, y, z) \hat{y} + E_z(x, y, z) \hat{z} \quad (1.1a)
\]

\[
H(x, y, z) = H_y(z, y, z) \hat{y} + H_z(x, y, z) \hat{z} \quad (1.1b)
\]

for a transverse electromagnetic (TEM) wave propagating in the \( \hat{x} \) direction. The dielectric is assumed lossless, and the conductors only slightly lossy, so that losses may be added to the TEM field solution as a perturbation on the lossless case. There are then no field components in the direction of propagation: \( E_x \equiv H_x \equiv 0 \). Maxwell's equations are written as

\[
\nabla \times E = -\mu \frac{dH}{dt} \quad (1.2a)
\]

\[
\nabla \times H = J = \sigma E + \epsilon \frac{dE}{dt} \quad (1.2b)
\]

\[
\nabla \cdot D = \rho \quad (1.2c)
\]

\[
\nabla \cdot B = 0 \quad (1.2d)
\]

The \( \nabla \) operator can be split up into two parts: one representing differentiation with respect to the transverse plane and one representing differentiation with respect to the longitudinal or axial coordinate \( x \)

\[
\nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} = \nabla_T + \hat{x} \frac{\partial}{\partial x} \quad (1.3)
\]

Maxwell equations in the transverse plane are then written as

\[
\nabla_T \times E = 0 \quad (1.4a)
\]
Equation (1.4) are identified as the equations for a static field distribution. Except for the sinusoidal time dependence, the TEM field has exactly the same spatial distribution as the static field.

II. Formulation of the Electric Scalar Potential

Finite differencing can be used to iteratively solve for the electric and magnetic scalar potentials in a bounded region whose properties are at least piecewise homogeneous [3]. The scalar potentials are obtained by solving Laplace's equation as a boundary value problem.

The transverse electromagnetic (TEM) spatial field distribution is exactly the same as the static field solution. Hence, the TEM field may be referred to as the quasi-static field. Although higher order modes will be present, it is assumed that the TEM field is the dominant mode of wave propagation.

Since the curl of a static electric field is always zero, it is advantageous to express the electric field as the gradient of a scalar potential

$$ E = - \nabla \phi_e $$

Substituting (2.1) into (1.4d) reveals that the electric scalar potential satisfies from Poisson's equation,
\[- \nabla \cdot D = \nabla \cdot (\varepsilon \nabla \phi_e) = \varepsilon \nabla^2 \phi_e + \nabla \phi_e \cdot \nabla \varepsilon = - \rho_e. \tag{2.2}\]

\(\varepsilon\) is locally homogeneous except at the interface. Therefore, \(\nabla \varepsilon = 0\) except at the interface and at the conductor surface where bound electric surface charges exist. For a charge free region Poisson's equation simplifies to Laplace's equation. Integrating and applying the divergence theorem yields

\[\iiint V \cdot [\varepsilon \nabla \phi_e] \, dv = \iint_S \varepsilon \nabla \phi_e \cdot \hat{n} \, ds = 0. \tag{2.3}\]

The line is initially assumed lossless and terminated in its characteristic impedance. The solution is then uniform in the \(x\) direction and its solution may be generalized as a two dimensional problem (i.e., \(\phi = f(y, z)\)). In the two dimensional solution,

\[\oint_c \varepsilon \nabla \phi_e \cdot \hat{n} \, dl = 0, \tag{2.4}\]

where units are charge/meter.

Divide the \(y, z\) plane into many small cells. Apply (2.4) over each cell and keep in mind that the potentials must be continuous across each cell boundary. Assume that the cell size is so small that the potential distribution varies slowly over the surface (or cubic volume in 3 dimensions) so that only terms of second order or less need be retained. This represents an approximation of the potential by the first three terms of its Taylor series expansion. The two dimensional gradient can be further approximated as

\[\nabla \phi_e \approx \frac{\Delta \phi_e}{\Delta y} \hat{y} + \frac{\Delta \phi_e}{\Delta z} \hat{z} \tag{2.5}\]
The gradient of the potential is constant along the differential length (or the differential area in 3 dimensions) and can be brought outside the integral.

The electric scalar potential in a charge free region is then obtained from (2.4) by integrating over the cell centered at "0",

\[ \nabla \Phi_e \cdot \oint_C \hat{n} \varepsilon \, dl = \sum_{n=1}^{N} \frac{\Phi_n - \Phi_0}{d_{no}} < \varepsilon >_{no} = 0 \]  

where \( N \) is the number of neighboring cells. \( N = 4 \) for the case considered, \( N = 6 \) in 3 dimensions. Here \( < \varepsilon >_{no} \) is the integration of the permittivity times its differential length along the cell boundary between cells 0 and \( n = 1, 2, \ldots, N \) (its differential area along the cell boundary surface in the 3 dimensional solution), and \( d_{no} \) is the distance between the centers of cells 0 and \( n = 1, 2, \ldots, N \). The geometry considered around \( \Phi_0 \) is illustrated in Figure 2.

![Figure 2: Contour Integral Around Cell \( \Phi_0 \).](image-url)
Solving for \( \Phi_0 \) from (2.6) yields

\[
\Phi_0 = \frac{\sum_{n=1}^{4} \frac{\Phi_n}{d_{no}}}{\sum_{n=1}^{4} \frac{1}{d_{no}}} < \epsilon >_{no}
\]  

(2.7)

Now that the relationship between the potentials around each cell is known, the potential at coordinates \( y \) and \( z \) for \( 0 \leq y \leq W_2 \) and \( 0 \leq z \leq H_2 \), can be determined iteratively. Define an array of size \( M \times N \), say \( V(I, J) \), where \( 1 \leq I \leq M \) and \( 1 \leq J \leq N \). Then,

\[
V(y, z) = V \left[ y_0 + \sum_{p=1}^{J} \Delta Y_p, z_0 + \sum_{q=1}^{I} \Delta Z_q \right]
\]  

(2.8)

where \((y_0, z_0)\) is the coordinate origin, and \( \Delta Y \) and \( \Delta Z \) refer to the distance between individual cell points in the \( \hat{y} \) and \( \hat{z} \) directions respectively. Generally \( V(y, z) \) will be referred to as \( V(I, J) \). Figure 3 identifies the cell geometry for points along the dielectric interface shown in Figure 1.

Several simplifications to the solution procedure can be made. For example, the cell spacing can be uniform in the \( \hat{y} \) direction and uniform up to the dielectric interface. There is then only one discontinuity in the \( \hat{z} \) direction. At the point where \( z = H_1 \) \((I = IP)\), the spacing between points will change from \( \Delta Z_1 \) to \( \Delta Z_2 \) and the relative dielectric constant from \( \epsilon_1 \) to \( \epsilon_2 \). This row will also demarcate the two return plates at potential \( V_1 \). The other plates are identified as:
the signal plate with potential $V_2$ at $z = H_2$ ($I = M$); the grounded plate with zero potential at $z = 0$ ($I = 1$). Note that $y = 0$ corresponds to $J = 1$; $y = W_2$ to $J = N$. One can then write the differential lengths as

\[
\Delta Y = \frac{W_2}{N - 1} \\
\Delta Z_1 = \frac{H_1}{IP - 1} \\
\Delta Z_2 = \frac{(H_2 - H_1)}{(M - IP)}
\]

(2.9a)  
(2.9b)  
(2.9c)

An initial guess for the potentials is needed and then (2.7) is used to compute the potential at each point in the array iteratively until a specified convergence is reached. The potentials can be initially set to any arbitrary set of numbers [typically $\phi^{(0)} = 0$] except for those points corresponding to the boundaries of each region. It is not uncommon to iterate each point thousands of times to meet a strict convergence criterion and/or large array size.

Figure 3: Contour Integral Around Cell (I, J)
For those points not on the conductors along the interface (I = IP), the potentials may be evaluated as

\[
V(I, J) = \frac{e_1 \Delta Z_1 + e_2 \Delta Z_2}{\Delta Y} + \frac{e_2 \Delta Y}{\Delta Z_2} + \frac{e_1 \Delta Y}{\Delta Z_1}
\]

(2.10)

\[
= \frac{[V(I, J + 1) + V(I, J - 1)]}{\Delta Y} + \frac{e_1 V(I - 1, J)}{\Delta Z_1} + \frac{e_2 V(I + 1, J)}{\Delta Z_2} + \frac{\Delta Y}{\Delta Z_1}
\]

This is the most general form of the finite difference equation to be considered here.

For points not lying on the interface (I ≠ IP), the medium is locally homogeneous. The dielectric and variations in \( \Delta Z \) cancel out of the above equation since they are uniform throughout the region.

\[
V(I, J) = \frac{[V(I, J + 1) + V(I, J - 1)] \Delta Y + [V(I + 1, J) + V(I - 1, J)] \Delta Z}{2 \left[ \frac{\Delta Z}{\Delta Y} + \frac{\Delta Y}{\Delta Z} \right]}
\]

(2.11)

where it is understood that \( \Delta Z = \begin{cases} \Delta Z_1, & I < IP \\ \Delta Z_2, & I > IP \end{cases} \) For points that are locally homogeneous, the potential does not depend on the electrical properties of the medium, only on the cell dimensions.
The exact solution to the potential distribution of any open ended transmission line has a non-zero and non-uniform potential in and around itself extending to infinity. The finite difference solution requires knowledge of the potential at the boundaries of the space to be considered: all of space in such a case. However, the potential within the transmission line is the only quantity of interest and is the dominant source of field interaction.

Mixed boundary conditions are applied.* The potentials are known at the upper and lower conductors and are fixed. These are taken as the horizontal boundaries (Dirichlet boundary conditions, or the forcing function) and are not iterated. The vertical boundaries are taken along an imaginary line that connects the upper and lower plate edges. Neumann boundary conditions are applied along this line. At the edge of the transmission line (that portion which does not contain conductors), we force the horizontal derivative of the potential to zero

\[
\frac{3\phi}{3n} = \frac{3\phi}{3y} = 0.
\]  

---

* Ibid. P. 129.
The gradient of a constant function is identically zero. Therefore, (2.12) sets the \( \hat{y} \) component of the field to zero at the vertical transmission line edge. The fields outside the transmission line can be considered negligible provided that there are no other sources or conducting surfaces within the vicinity of the transmission line and that the plate spacing is small. Note that the boundary conditions have been chosen so that the fields outside of the transmission line are zero.

The effect on the array is to force \( V(I, 1) = V(I, 2) \) and \( V(I, N - 1) = V(I, N) \). Set \( V(I, 1) = V(I, 2) \) and \( V(I, N - 1) = V(I, N) \) in (2.11) and solve for \( V \)

\[
V(I, J) = \frac{V(I, J + 1) \frac{\Delta Z}{\Delta Y} + [V(I + 1, J) + V(I - 1, J)] \frac{\Delta Y}{\Delta Z}}{\frac{\Delta Z}{\Delta Y} + 2 \frac{\Delta Y}{\Delta Z}}
\]  

for \( J = 2 \) and \( N - 1 \).

This condition yields accurate results when fringing fields are negligible, such as when the height of the transmission line is much less than its width (\( W_2 \ll H_2 \)). The error that is introduced will be examined subsequently.

III. Formulation of the Magnetic Scalar Potential

The magnetic scalar potential can be solved in an analogous manner.
from (1.2b) and (1.2d). Assume that the dielectric is not a magnetic material so that \( \mu \) is homogeneous everywhere. In a non-magnetic medium free from currents (a region containing no conductors so that \( \sigma = 0 \)), the magnetic field can be expressed,*

\[
H = \hat{x} \times \left[ -\frac{\nabla \Phi_m}{Z_0} \right] \tag{3.1}
\]

where \( Z_0 \) is the impedance of free space, 120 \( \pi \) \( \Omega \). \( \nabla \Phi_m \) is crossed in the \( \hat{x} \) direction because of the curl in (1.2b). This will become more apparent when boundary conditions are discussed. Dividing by \( Z_0 \) forces the units of \( \Phi_m \) to be in volts. Moreover, \( \Phi_m \) will satisfy the same general boundary conditions as \( \Phi_e \).

It is important to note that this is not the usual magnetic scalar potential. It is a pseudo-quantity that is simpler to work with.

Substituting (3.1) into (1.2d) yields

\[
\nabla \cdot B = \mu_0 \nabla \cdot \left[ \hat{x} \times -\frac{\nabla \Phi_m}{Z_0} \right] = \sqrt{\mu_0 \varepsilon_0} \nabla \cdot (\nabla \Phi_m \times \hat{x}) = 0 \tag{3.2}
\]

This reduces to Laplace's equation for \( \Phi_m \); \( \nabla^2 \Phi_m = 0 \). Derivation of the difference equation is accomplished as before.

* This definition utilizes the property \( H = \hat{x} \times E/Z_0 \), for TEM fields, where \( \hat{x} \) is the direction of propagation.
\[ \iiint \nabla^2 \Phi_m \, dv = \oint_S \nabla \Phi_m \cdot ds = 0 \]

In 2 dimensions,
\[ \oint_c \nabla \Phi_m \cdot dl = 0, \]

where units are coulomb/meter. Using the same procedure as for $\Phi_e$ yields

\[ \nabla \Phi_m \cdot \oint_c dl \approx \sum_{n=1}^{N} \frac{\Phi_n - \Phi_o}{d_{no}} \langle l \rangle_{no} = 0 \]

Equation (2.11) can be used to solve for the magnetic scalar potential at points interior to the boundaries.

A few words need to be said in regard to boundary conditions. Neumann boundary conditions are simple to apply since they require no knowledge of the potential. Equation (2.13) is used at the vertical boundaries.

The Dirichlet boundary conditions are in general, more difficult to determine. We must obtain the equivalent conductor potentials, designed as $V'_i$. The current density at the conductor must be found using (1.2b). Note that the magnetic field computation requires

\[ \hat{x} \times \nabla \Phi_m \approx \frac{\Delta \Phi_m}{\Delta Z} \hat{y} - \frac{\Delta \Phi_m}{\Delta Y} \hat{z} \]

due to the cross product with $\hat{x}$. A discussion of the procedure used to obtain the Dirichlet boundary conditions is presented in detail in Chapter 5b after a discussion of transmission line parameters.
IV. Determination of Transmission Line Parameters [4]

It is necessary to determine transmission line parameters with respect to the conductors at terminals $V_1$ and $V_2$. In a system of $n + 1$ conductors, there are at most $n$ linearly independent equations that can be written to describe the system. The transmission line considered has two conductors that are always at the same potential so that $n$ can be reduced to two. The distributed capacitances for the multiconductor configuration are defined in (4.1). Refer to Figure 4 for the schematic.

$$
\begin{bmatrix}
Q_1 \\
Q_2 \\
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} \\
c_{21} & c_{22} \\
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\end{bmatrix} =
\begin{bmatrix}
c_{11} + c_{12} & -c_{12} \\
-c_{21} & c_{21} + c_{22} \\
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\end{bmatrix} \quad (4.1)
$$

Here $V_1$ and $V_2$ are the conductor voltages relative to the metal plate ground, the lowercase $c$ designates the capacitive coefficients, and the uppercase $C$ the capacitances between conductors.

Since the elements are assumed lossless, the matrix must be symmetric [5]. Losses will be added later. The losses will be small so that the matrix will be approximately symmetric. Therefore, $c_{12} = c_{21}$ and $c_{12} = c_{21}$. Once the electric potential is known, the electric charge per unit length can be solved by integrating the electric flux normal to the surface of each conductor

$$
Q_k = \oint_C \nabla(\varepsilon \Phi) \cdot \hat{n} \, dl_k \quad (4.2)
$$

where $\hat{n}$ is directed normal to the conductor surface ($\pm \hat{z}$ direction) and $l_k$ is the differential length along the $k^{th}$ conductor.
Figure 4: Equivalent Transmission Line Capacitances.

Numerically the charge per unit length of each conductor is obtained by the sums,

\[ Q_2 \approx - \varepsilon_2 \varepsilon_0 \sum_{J=1}^{N} (V(M, J) - V(M - 1, J)) \frac{\Delta Y}{\Delta Z_2} \]  

(4.3)

\[ Q_1 = Q_{1a} + Q_{1b} \]  

(4.4)

where

\[ Q_{1a} \approx - \varepsilon_1 \varepsilon_0 \sum_{J=1}^{N} (V(IP, J) - V(IP - 1, J)) \frac{\Delta Y}{\Delta Z_1} \]  

(4.5a)

\[ Q_{1b} \approx - \varepsilon_2 \varepsilon_0 \sum_{J=1}^{N} (V(IP + 1, J) - V(IP, J)) \frac{\Delta Y}{\Delta Z_2} \]  

(4.5b)

The conductors are assumed to be infinitesimally thin so that end contributions are negligible. The vertical field component is the only contributor to this summation. The bound charge on the dielectric is typically neglected.
The charge induced on each conductor determines the capacitive coefficients. Let \( V_1 = 1, V_2 = 0 \). Solve using finite difference to obtain \( C_{11} = Q_1, C_{21} = Q_2 \).

Next let \( V_1 = 0, V_2 = 1 \). Solve the potential to obtain \( C_{21} = Q_1, C_{22} = Q_2 \). Due to the finite word length and the truncation errors of successive iterations, the matrix will not be exactly symmetric (\( C_{12} \neq C_{21} \)). The capacitances are calculated using

\[
C_{12} = C_{21} = -\frac{1}{2} (C_{12} + C_{21}),
\]

(4.6)

\[
C_{11} = C_{11} + C_{12},
\]

(4.7)

\[
C_{22} = C_{22} + C_{12},
\]

(4.8)

The total capacitance between conductors 1 and 2 is

\[
C_T = C_{12} + \frac{C_{11}C_{22}}{C_{11} + C_{22}}
\]

(4.9)

The transmission line inductance per unit length must be determined for characteristic impedance and magnetic field calculations. The dual of the above problem must be solved also. It requires the "magnetic" capacitances from \( \Phi_m \) using unit "magnetic" voltages as above in (4.3) - (4.9), substituting primed quantities for unprimed ones. It is important to note that \( \varepsilon_1 = \varepsilon_2 = 1 \) for this part of the solution.

The charge induced on the conductors from the magnetic scalar potential is obtained from

\[
Q'_k = \varepsilon_0 \oint \nabla \Phi_m \cdot \hat{n} \, dA_k
\]

(4.10)
The transmission line inductance is related to the magnetic capacitance as

$$L'_T = L_T = \frac{\mu_0 \varepsilon_0}{C'_T}$$  \hspace{1cm} (4.11)

and the characteristic impedance is for the lossless case

$$Z_C = \left[ \frac{L_T}{C_T} \right]^{1/2}$$  \hspace{1cm} (4.12)

V. Calculation of Field Quantities

**Electric Field**

The electric field depends only upon the voltages, the permit-tivities, and the conductor configuration. It is obtained by the following steps:

1. Determine the capacitance \((C_{ij})\) of the inhomogeneous transmission line from the electric scalar potential.

2. Determine the transmission line voltages for the inhomogeneous transmission line.

$$\sum_i Q_i = Q_1 + Q_2$$  \hspace{1cm} (5.1)

There is conductive isolation between the transmission line and the reference conductor (grounded plate). In this case, called the balanced-mode operation by Frankel, \(- Q_1 = Q_2 \) \cite{6}. Therefore,

$$- C_{11} V_1 + C_{12} (V_2 - V_1) = C_{12} (V_2 - V_1) + C_{22} V_2.$$  \hspace{1cm} (5.2)

Then

$$V_1 = - V_2 \frac{C_{22}}{C_{11}}.$$

\hspace{1cm} (5.3)
One is free to set $V_2$ to any arbitrary potential. It may be convenient to normalize $V_2$ to 1 V.

These voltages are referenced to the grounded test object. $V_2 - V_1$ is the voltage of the driver.

3. The finite differencing solution is then applied to the transmission line with the voltages from (5.3). The electric field is determined by solving $E = - \nabla \phi$, which by (2.5) yields

$$E_y(I, J + \frac{1}{2}) = - \frac{V(I, J + 1) - V(I, J)}{\Delta y} \hat{y} \quad (5.4)$$

$$E_z(I + \frac{1}{2}, J) = - \frac{V(I + 1, J) - V(I, J)}{\Delta z} \hat{z} \quad (5.5)$$

The indices with $\frac{1}{2}$ added to them indicate that the exact location of the field point is at the cell boundary, not the cell center. $\Delta y$ and $\Delta z$ are assumed so small that the location of the field point is approximately that of the location of the potential ($I = I + \frac{1}{2}, J = J + \frac{1}{2}$).

**Magnetic Field**

The magnetic field depends only upon the currents, the permeabilities, and the conductor configuration. If the capacitances and voltages are already known, the magnetic field can be determined as
follows.* Note that the regions of different dielectric permittivities do not affect the magnetic field directly.

4. Determine the transmission line current from the inhomogeneous transmission line as

\[
I = \frac{V_2 - V_1}{Z_c} = \frac{V_2}{Z_c} \frac{C_{11} + C_{22}}{C_{11}},
\]

(5.6)

5. Determine the capacitances obtained from the magnetic scalar potential \((C'_{ij})\) of the transmission line using \(\mu_1 = \mu_2\) in the same physical configuration as in 1.

6. Obtain the equivalent voltages from the magnetic scalar potential \((V'_{i})\) for the transmission line that causes the same current to flow as in 2.

\[
I = \frac{V_2 - V_1}{Z_c} = \frac{V'_{2} - V'_{1}}{Z'_c}
\]

(5.7)

\[
I = \frac{V_2}{Z_c} \frac{C_{11} + C_{22}}{C_{11}} = \frac{V'_{2}}{Z'_c} \frac{C'_{11} + C'_{22}}{C'_{11}}
\]

(5.8)

Solve (5.8) for \(V'_{2}\)

\[
V'_{2} = V_2 \frac{C_{11} + C_{22}}{C'_{11} + C'_{22}} \frac{C'_{11}}{C_{11}} \left[ \frac{C'_T}{C_T} \right]^{1/2}
\]

(5.9)

The continuity equation states that whatever current flows in must flow out. There is no net magnetic charge, so

* C. D. Taylor, Internal Memo.
\[ V'_{1} = - V'_{2} \frac{C'_{22}}{C'_{11}}. \] (5.10)

7. Determine the magnetic field tangential to the conductors as in 3. Note that (2.5) means the tangential \( H \) means the difference in magnetic potential in the direction normal to the conductors.

Solve (3.1) as

\[ H_{y}(I + \frac{1}{2}, J) = + \frac{V'(I + 1, J) - V'(I, J)}{Z_{0} \Delta Z} \dot{y} \] (5.11)

\[ H_{z}(I, J + \frac{1}{2}) = - \frac{V'(I, J + 1) - V'(I, J)}{Z_{0} \Delta Y} \dot{z} \] (5.12)

where \( Z_{0} \) is the impedance of free space.

VI. Considerations for Lossy Conductors

Transmission line parameters are determined so that small losses may be added to the solution. Mechanical and practical considerations forced the line to be small in height and wide in width, reducing its characteristic impedance well below that of the 50 \( \Omega \) source and load terminations. The transmission line was deliberately made of a lossy material so that impedance mismatches would not cause significant standing wave interference. Transfer functions were then generated to compare with measured data.

Once \( L \) and \( C \) have been determined for the transmission line, losses can be added to the solution as a perturbation. Distributed parameters are used to characterize the transmission line along its
length. A series resistance $R$ can be added to the differential length to account for small losses in the conductor and a shunt conductance $G$ to account for small losses in the dielectric as shown in Figure 5.

The propagation constant $\gamma$ and characteristic impedance $Z_c$ of the lossy transmission line is then given as [7],

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (6.1)$$

$$Z_c \left[ \frac{R + j\omega L}{G + j\omega C} \right]^{1/2} \quad (6.2)$$

The frequency domain forcing function $F(\omega)$ is 1 A/Hz for $I(x, \omega)$ for the current transfer function of 1 V/Hz for the voltage transfer function. The source impedance is designated as $Z_G$, the terminating impedance as $Z_T$, and the line length as $D$ meters as shown in Figure 6. The voltage and current along the line are then

$$E(x, \omega) = \frac{Z_c}{Z_c + Z_G} \frac{e^{-\gamma x} + \Gamma_T e^{-\gamma(2D - x)}}{1 - \Gamma_G \Gamma_T e^{-2\gamma D}} \quad (6.3)$$

$$I(x, \omega) = \frac{1}{Z_c + Z_G} \frac{e^{-\gamma x} - \Gamma_T e^{-\gamma(2D - x)}}{1 - \Gamma_G \Gamma_T e^{-2\gamma D}} \quad (6.4)$$

where

$$\Gamma_G = \frac{Z_G - Z_c}{Z_G + Z_c}, \quad \text{and} \quad \Gamma_T = \frac{Z_T - Z_c}{Z_T + Z_c} \quad (6.5)$$

are the reflection coefficients at the source and load ends, respectively.
Figure 5: Transmission Line Parameters.

Vance and others have shown that at low frequencies, the attenuation constant $\alpha$ [Re $\gamma$] will equal the wave number $\beta$ [Im $\gamma$] and both will increase as the square root of frequency [3]. At higher frequencies, $\beta$ rises linearly and $\alpha$ approaches a constant value.

The characteristic impedance of the line has a low frequency phase of $45^\circ$ which decreases as the square root of frequency. At higher frequencies, the impedance approaches a constant real value.

Figure 6: Equivalent Transmission Line for Loss Calculation.
VII. Considerations for Speedy Convergence of Solution

A plane of symmetry passes vertically through the center of the transmission line as illustrated in Figure 1. It can be shown (and was demonstrated in trial runs) that the solution in the left-half space is exactly the same (within machine computational tolerance) as that in the right half-space. The finite difference solution is obtained by forcing the solution immediately to the right of vertical center to be equal to that immediately left of vertical center. This yields symmetry along vertical center.

Utilizing the symmetry of the potential, the matrix size has been reduced from $M \times N$ to

$\begin{align*}
M \times \frac{N}{2} + 1, & \text{ for } N \text{ even;} \\
M \times \frac{N + 1}{2}, & \text{ for } N \text{ odd.}
\end{align*}$

(7.1a, 7.1b)

The number of computations and thus the computation time has been reduced proportionately (≈ ½). Note also that memory allocations have also been proportionately reduced. To force the symmetry on the array, set

$\begin{align*}
V(I, \frac{N}{2}) &= V(I, \frac{N}{2} + 1), \text{ for } N \text{ even } ;
\end{align*}$

(7.2a)

$\begin{align*}
V(I, \frac{N + 1}{2} - 1) &= V(I, \frac{N + 1}{2} + 1), \text{ for } N \text{ odd.}
\end{align*}$

(7.2b)
This technique should be considered more accurate since the symmetry is forced upon the solution. Without this technique, symmetry is only approximately approached due to the nature of the iterative algorithm. The array is not solved simultaneously. It is scanned from left to right, top to bottom for each iteration.

VIII. Results of Computer Simulation on Driver

One source of error inherent in the solution is the approximate boundary conditions at the vertical boundaries. The exact solution should include fringing fields throughout all space. Another source of error results from successive truncation errors associated with finite computer word lengths.

The finite differencing procedure was verified by obtaining results for the capacitance per unit length of a single plate oriented over a ground plane with a paint layer. The configuration was analyzed by considering it to be a series combination of two capacitors, one having an air dielectric and one having a paint dielectric. For the air dielectric, the Morse and Feshbach result (which includes fringing fields) is used [9]. It is

\[ C_{\text{air}} = \varepsilon_0 \left[ \frac{W_2}{H_2 - H_1} + \frac{2}{\pi} \left( 1 + \ln \frac{\pi W_2}{2(H_2 - H_1)} \right) \right] \]  

(8.1)

and for a thin layer of paint,

\[ C_{\text{paint}} = \varepsilon_1 \frac{W_2}{H_1} \]  

(8.2)

Therefore, the total capacitance per unit length is
\[ C = \frac{C_{\text{air}} C_{\text{paint}}}{C_{\text{air}} + C_{\text{paint}}} \]  

(8.3)

For \( H_2 - H_1 = 10.6" \), \( W_2 = 28" \), \( H_1 = 0.001" \) and \( \varepsilon_1 = 3\varepsilon_0 \), (8.3) yields \( C = 370 \) nF/m, whereas, the numerical technique yields 240 nF/m. This significant difference (\( \approx 35\% \)) occurs as a result of the neglect of fringing effects by the numerical solution. For \( H_2 - H_1 = 0.19685" \) (5mm), (8.3) yields \( C = 1.298 \) nF/m, the same as the numerical solution. Lastly for \( H_2 - H_1 = 0.19685" \), \( H_1 = 0.04" \), (8.3) yields \( C = 1.486 \) nF/m, whereas the numerical solution yields 1.537 nF/m. These results confirm the numerical procedure for structures where \( H_2/W_2 \) is small.

Aperture illumination is determined by the incident fields developed on the conducting surface placed over the aperture. The aperture is shorted for field calculations. As an example, the electric field distribution is shown in Figure 7 for \( W_1 = 7" \), \( W_2 = 28" \), \( H_2 = 3/16" \), \( H_1 = 0.01" \), \( \varepsilon_1 = 3 \varepsilon_0 \), and \( V_2 = 1 \) volt. A significant reduction of the electric field in the gap region is observed. This reduction is primarily due to the permittivity of the paint. There is only a small reduction of the electric flux density. This is verified by the small reduction of magnetic field in the gap region as shown in Figure 8.

Lumped multiconductor transmission line parameters such as the propagation constant and the characteristic impedance must also be determined. These are shown in Figures 9 and 10 respectively for the above configuration. The data was obtained with the assumption that the plate material (mylar with a conductive backing) had a resistance of 3 \( \Omega/\text{square} \).
Figure 7: Electric Field Distribution for Example.

Figure 8: Magnetic Field Distribution for Example.
A second example was also considered. The parameters involved were $W_1 = 7''$, $W_2 = 96''$, $H_2 = 3/8''$, $H_1 = 0.01''$, $\varepsilon_1 = 3\varepsilon_0$, and $V_2 = 1$ volt. The data showed different numbers but the same trends. Since the data does not illustrate anything new, it is included as Appendix A.

Table 1 lists quantities derived from the finite difference method for field and transmission line calculations.

Table 1: Transmission Line Parameters.

<table>
<thead>
<tr>
<th>E FIELD CAPACITANCES</th>
<th>H FIELD CAPACITANCES</th>
<th>INDUCTANCE</th>
<th>E FIELD VOLTAGES</th>
<th>H FIELD VOLTAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$</td>
<td>$C_{11}$</td>
<td>$C_{22}$</td>
<td>$C_{12}$</td>
<td>$C_T$</td>
</tr>
<tr>
<td>7.00</td>
<td>3.9834E-08</td>
<td>6.8547E-10</td>
<td>7.4785E-10</td>
<td>1.4217E-09</td>
</tr>
</tbody>
</table>

| $W$                  | $C'_{11}$            | $C'_{22}$  | $C'_{12}$        | $C'T$            |
| 7.00                 | 1.3278E-08           | 6.6102E-10 | 7.4794E-10       | 1.3776E-09       |

INDUCTANCE

8.0665 nH

E FIELD VOLTAGES

<table>
<thead>
<tr>
<th>$W$</th>
<th>$V_1$</th>
<th>$V_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.00</td>
<td>-1.72081646E-02</td>
<td>1.00000000</td>
</tr>
</tbody>
</table>

H FIELD VOLTAGES

<table>
<thead>
<tr>
<th>$W$</th>
<th>$V'_1$</th>
<th>$V'_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.00</td>
<td>-4.90043461E-02</td>
<td>0.98435706</td>
</tr>
</tbody>
</table>

DATA: $W_2 = 28.00''$, $W_1 = 7.00''$, $\varepsilon_r = 3.00$

ACCURACY - 6 DECIMAL PLACES

$H_2 = 0.1875''$, $H_1 = 0.010''$, $48 \times 56$ MATRIX
Figure 9: Propagation Constant of Transmission Line.

Figure 10: Transmission Line Characteristic Impedance.
Figure 11: Effect of Losses on Current at Source.

Figure 12: Effect of Losses at Center of Line.
Mismatches cause standing waves on a transmission line. Losses in the conductor can minimize standing waves at a cost of some loss of signal. Figures 11 and 12 show that current with lossy conductors and with perfect conductors at the source and at the center of the transmission line. There is not much difference observed except for some slight destructive interference at or above the normal operating frequency of the transmission line driver in the lossless case. It is observed that losses attenuated the current by 14%.

It is postulated that if the current distribution is uniform on the transmission line, the field distribution is uniform in the aperture region. This simplifies the solution of the aperture coupling problem. Figure 13 compares the current at the driving end to that at the center of the transmission line. Below 1 MHz, the current distribution is seen to be nearly equal at these two points.

This led to an examination of the current distribution along the line to observe any simplifying trends. Figure 14 shows the current distribution along the transmission line for several frequencies. At low frequencies the current distribution is uniform along the line.

Apertures that are small compared to the length of the transmission line should only be significantly affected by the field in proximity to the aperture. This is not necessarily true for an electrically large aperture.
Figure 13: Effect of Standing Wave Current.

Figure 14: Current Distribution of Transmission Line.
IX. Aperture Calculations [10]

A sense wire circuit is considered to be located below a circular aperture and the induced voltage measured in order to quantitate the electromagnetic field penetration through the aperture.* The measurement configuration under consideration is shown in Figure 15. The resistor $R_s$ is selected to be sufficiently large so only a small current flows through it (i.e., the aperture is not loaded) yet it must be small enough to allow a measurable current. The magnetic flux penetrating the aperture is measured by the voltage induced in the circuit, the voltage across $R_s$.

![Figure 15: Sense Wire Circuit.](image)

The sense wire circuit configuration is easily implemented for apertures in flat surfaces with open regions behind the aperture. However, for apertures in curved surfaces and for apertures with restricted interior regions, some spacing between the aperture and the

---

* Yang and Baum.
sense wire may be required.* An analytical study of the voltage and the current induced in the sense wire circuit as the spacing between the aperture surface and the wire is varied shall now be considered.

A circular aperture in a perfectly conducting sheet is used to model an aperture perforated shield. The penetrating field is derived using the approach presented by Jackson [11]. A quasi-static solution technique is used, but the results can be generalized to higher frequencies provided the aperture dimensions are small in terms of wavelength.

**Calculation of Voltage Induced on Sense Wire**

Faraday's law of induction is used to obtain the induced voltage in the sense wire circuit. The equivalent circuit for the sense wire is derived, and an expression is obtained for determining the sense wire current.

From Chapter 8, it is known that the magnetic field is very nearly uniform and directed parallel to the sheet, \( (i.e., H_y) \). A magnetostatic solution can be obtained by solving Laplace's equation for the magnetic scalar potential while imposing the appropriate boundary conditions. Details of the procedure are given by Jackson. The resulting magnetic field distribution is essentially the same that would exist if the aperture were immersed in a low frequency field where the maximum linear dimension of the aperture is small compared to the wavelength.

* Taylor, Marcum, et al.
In this case \( H_0 \) is simply the surface magnetic field that would exist at the aperture location if the aperture were absent.

Define the magnetic scalar potential, \( \Phi_M \), as

\[
H = -\nabla \Phi_M
\]  

(9.1)

and use the solenoidal property of the magnetic field. This yields

\[
\nabla^2 \Phi_M = 0.
\]  

(9.2)

It is convenient to express the potential

\[
\Phi_M = -H_0 y + \Phi^{(1)}, \text{ for } z > 0
\]  

(9.3a)

\[
= -\Phi^{(1)}, \text{ for } z < 0.
\]  

(9.3b)

According to Jackson, the solution for \( \Phi^{(1)} \) subject to the appropriate boundary conditions for a circular aperture in the sheet is

\[
\Phi^{(1)} = \frac{2H_0 a^2}{\pi} \int_0^\infty dk \ j_1(ka) \ e^{-k|z|} \ J_1(kp) \sin \phi
\]  

(9.4)

where

\[
j_1(ka) = \frac{\sin (ka)}{ka^2} - \frac{\cos (ka)}{ka}
\]  

(9.5)

\( j_1 \) is the spherical Bessel function of the first kind, order 1, and \( J_1 \) is the cylindrical Bessel function of the first kind, order 1.

The magnetic field is determined by substituting (9.3) and (9.4) into (9.1). An analytical evaluation of (9.4) is not possible in
general, but a numerical method yields satisfactory results. The gradient of (9.4) is

\[
\mathbf{H}^{(1)} = - \left[ \hat{\rho} \frac{\partial \Phi}{\partial \rho} + \hat{\phi} \frac{1}{\rho} \frac{\partial \Phi}{\partial \phi} + \hat{z} \frac{\partial \Phi}{\partial z} \right]
\]  

(9.6)

\[
\frac{\partial \Phi}{\partial \rho} = \frac{2H_0a^2}{\pi} \int_0^\infty dk \ e^{-kz} j_1(ka) \left[ \frac{k J_0(k\rho) - \frac{J_1(k\rho)}{\rho}}{\sin \phi} \right] (\sin \phi)
\]  

(9.7)

\[
\frac{1}{\rho} \frac{\partial \Phi}{\partial \phi} = \frac{2H_0a^2}{\pi} \int_0^\infty dk \ e^{-kz} j_1(ka) \frac{J_1(k\rho)}{\rho} \cos \phi
\]  

(9.8)

\[
- \frac{\partial \Phi}{\partial z} = \frac{2H_0a^2}{\pi} \int_0^\infty dk \ e^{-kz} j_1(ka) \frac{k}{1 - \frac{J_1(k\rho)}{\rho}} \sin \phi
\]  

(9.9)

The H field components can be written in cartesian coordinates as

\[
\hat{x} = \cos \phi \hat{\rho} - \sin \phi \hat{\phi}, \quad \hat{y} = \sin \phi \hat{\rho} + \cos \phi \hat{\phi}.
\]

\[
H_z = \frac{2H_0a^2}{\pi} \int_0^\infty dk \ e^{-kz} j_1(ka) k J_1(k\rho) \sin \phi
\]  

(9.10a)

\[
H_y = - \frac{2H_0a^2}{\pi} \int_0^\infty dk \ e^{-kz} j_1(ka) \left[ k J_0(k\rho) \sin^2 \phi - \frac{J_1(k\rho)}{\rho} \cos 2\phi \right]
\]

\[
H_x = - \frac{2H_0a^2}{\pi} \int_0^\infty dk \ e^{-kz} j_1(ka) \left[ \frac{k}{2} J_0(k\rho) - \frac{J_1(k\rho)}{\rho} \right] \sin 2\phi
\]

Jackson indicates that the total field in the aperture opening is
The determination of the open circuit voltage on a sense wire crossing the aperture requires the determination of the total flux linking the aperture. The maximum flux linkage occurs when the sense wire is located in the plane of the aperture so that it bisects the aperture at an angle \( \alpha \) from the \( x \) axis. The maximum magnetic flux linking the sense wire circuit is

\[
\Psi_M|_{\text{max}} = \mu_0 H_0 a^2 \cos \alpha
\]  

Substituting (9.11b) in (9.12) yields

\[
\Psi_M|_{\text{max}} = \mu_0 H_0 a^2 \cos \alpha
\]  

The voltage induced on the sense wire is then

\[
V_{oc} = j\omega \Psi_M
\]  

As the sense wire moves away from the aperture, it links fewer lines of magnetic flux. Hence,

\[
\Psi_M = \Psi_M|_{\text{max, } \alpha} - 2 \mu_0 \int_{-h}^{0} \int_0^{\lambda/2} H\cdot\hat{\alpha} \, d\rho \, dz
\]  

where \( \hat{\alpha} \) is a unit vector in the direction of the wire segment of length \( \lambda \). For maximum pickup \( \alpha = 0 \).
The expression may be simplified by evaluating the integral over z and applying a change of variables.

\[
\Psi_M = \mu_0 H_0 a^2 \left[ 1 - \frac{4}{\pi} \int_0^{\infty} du \, j_1(u) \frac{1 - e^{-uh/a}}{u} \right] \int_0^{ul/2a} dv \, \frac{J_1(v)}{v}
\]

The following table lists flux calculations from the above equation.* Observe that the flux linkage and hence the voltage falls off as the wire moves away from the aperture as expected.

Table 2: Voltage Induced by Magnetic Flux Linking a Sense Wire Behind a Circular Aperture.

<table>
<thead>
<tr>
<th>h/a</th>
<th>( \frac{V_{oc}}{j\omega\mu_0 H_0 a^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( l/2a = 1 )</td>
</tr>
<tr>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9088</td>
</tr>
<tr>
<td>0.25</td>
<td>0.8005</td>
</tr>
<tr>
<td>0.50</td>
<td>0.6761</td>
</tr>
<tr>
<td>0.8</td>
<td>0.5869</td>
</tr>
<tr>
<td>1.0</td>
<td>0.5493</td>
</tr>
<tr>
<td>2.0</td>
<td>0.4689</td>
</tr>
</tbody>
</table>

*Taylor, Marcum, et al.
Figure 16: Equivalent circuit for the excitation of the sense wire behind an aperture.
Calculation of Sense Wire Current

The reduction in the wire current is studied for an open \((Z_L = \infty)\) aperture and the sense wire resistance \(R_w = 0\). A hardened aperture can be modeled as an equivalent sheet impedance \(Z_L\) as detailed in Casey [12]. For maximum wire current, the sense wire should extend across the aperture opening \((h = 0, \ell = 2a)\). The resulting maximum wire current is

\[
I_w(h = 0) = \frac{V_{oc}}{j\omega(L_a + L_w(h = 0))} = \frac{\Psi_M(h = 0)}{L_a + L_w(h = 0)} \tag{9.18}
\]

where the inductance for a straight wire is,

\[
L_w(h = 0) = \frac{\mu_0 \ell}{2\pi} \left[ \ln \frac{2\ell}{a_w} - 0.75 \right] \tag{9.19}
\]

where \(a_w\) is the sense wire radius. The inductance for a small circular aperture is approximately,

\[
L_a \approx \frac{1}{2} \mu_0 a \tag{9.20}
\]

where \(a\) is the aperture radius. Note that in deriving (9.18), the image contributions of the equivalent circuit in Figure 16 are not required.

For \(h \neq 0\) the equivalent circuit yields the sense wire current

\[
I_w = \frac{2\Psi_M}{L_a + 0.5 L_w} \tag{9.21}
\]

where \(L_w\) is the inductance of the sense wire circuit including the image, i.e., the inductance of a loop.
\[ L_w = \frac{\mu_0}{\pi} \left[ \ell \ln \left( \frac{4h\ell}{a_w(1 + d)} \right) + 2h \ln \left( \frac{4h\ell}{a_w(2h + d)} \right) + 2d - \frac{7}{4} (\ell + 2h) \right] \]

where

\[ d = \sqrt{(2h)^2 + \ell^2} \]  

(9.22)

The sense wire current reduction can be expressed as

\[ \frac{I_w}{I_w(h = 0)} = \frac{\Psi_M}{\Psi_M(h = 0)} \frac{L_a + L_w(h = 0)}{L_a + 0.5 L_w} \]  

(9.23)

X. Sensor Results

Equation (9.15) can not be evaluated in closed form. However, a numerical evaluation can be accomplished by brute force numerical integration. A high degree of accuracy can be obtained by using a very fine \( \Delta k \) resolution (\( \Delta k \) on the order of 0.001). Evaluating \( 0 \leq k \leq 100 \) yields truncation errors on the order of \( 10^{-4} \).

Sensitivity to angle of incidence can be obtained by substituting (9.10b) and (9.10c) into (9.15). It changes the absolute magnitude of the final results but not the shape of the frequency curve.

One aspect that was not studied was sensitivity to alignment of the sense wire to the center of the aperture. One of the assumptions made is that the wire is symmetric about the aperture opening. If it were asymmetric, it would couple differing amounts of flux along both vertical and horizontal segments. This is even more important at higher frequencies where the field illumination is not exactly uniform.
Asymmetries might increase the electric flux coupling. They are negligible in part due to symmetry and in part due to the magnetic coupling through the aperture.

Measured data for the sense wire were obtained at 3 frequencies - 10, 50, and 90 MHz [13]. Table 3 presents the average and standard deviation from the three measurements. The calculated data are obtained by considering the sense wire separation to be \( h \leq 0.4 \, a \), the reduction in flux linking the sense wire.

Inductances of the loop and aperture were determined and the circuit analyzed. The results were in good agreement.

Table 3: Reduction in Sense Wire Current for a Circular Aperture, \( \ell = 2a = 10'' \).

<table>
<thead>
<tr>
<th>( h/a )</th>
<th>( \Psi_M^w/\Psi_M^w(h = 0) )</th>
<th>( I_w/I_w(h = 0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CALCULATED *</td>
<td>CALCULATED *</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>- 0.83</td>
<td>- 3.8</td>
</tr>
<tr>
<td>0.25</td>
<td>- 1.93</td>
<td>- 6.7</td>
</tr>
<tr>
<td>0.40</td>
<td>- 2.87</td>
<td>- 13.59</td>
</tr>
<tr>
<td>0.50</td>
<td>- 3.40</td>
<td></td>
</tr>
<tr>
<td>0.80</td>
<td>- 4.63</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>- 6.58</td>
<td></td>
</tr>
</tbody>
</table>

* In dB

However, the measured results were obtained for a square aperture. In order to make meaningful comparisons the inductance for a square aperture is used in (9.23). Since \( L_w >> L_a \) generally occurs, using the square aperture inductance rather the circular aperture inductance provides...
only a small change in the results. For the 10" x 10" square aperture the inductance is estimated to be less than 300 nH.

The aperture may be of arbitrary shape with a minimum error for voltage calculations. The answer is approximate but of the same order of magnitude. The current is primarily dependent on the inductance of the aperture. The inductance is dependent on the shape of and orientation of the field to the aperture. Inductance calculations are difficult to make for arbitrary geometries although a method is currently being sought by Taylor and Marcum [14].

XI. Conclusions

A lossy multiconductor transmission line with severely mismatched terminations was analyzed and found to be in good agreement with measured data. Aperture coupling was also determined so that the overall shielding degradation due to the aperture could be characterized by frequency.

It was found that finite differencing could be used to iteratively solve for the electric and magnetic scalar potentials in a bounded region whose properties are at least piece-wise homogeneous. The static field distributions, and therefore the TEM field, could then be calculated from the scalar potentials using the appropriate boundary conditions.

The finite differencing approach yielded accurate solutions for structures where fringing fields were negligible. Magnetic fields
coupled very efficiently to the aperture with the transmission line analyzed. Electric field coupling was determined to be dependent on the ratio of the permittivity over the aperture to that of the transmission line antenna.

Apertures small compared to the gap region of the antenna were observed to be illuminated by a nearly uniform magnetic field. The techniques derived in Chapter 9 yielded very good results on the voltage induced on a wire behind the aperture. For larger apertures at higher frequencies, it was decided that the field distribution was only approximately uniform in the direction of transmission line current flow. Solutions using Chapter 9 only approximate that of the actual result.

Use of a lossy material for the conductors of the antenna increased the effective operating frequency of the device. At low frequencies, losses had little effect on the performance of the antenna. Signals were attenuated by 1.41 dB by the use of lossy material estimated at 3 Ω/square of resistance.
APPENDIX A - Example 2 Data

As a second example, a larger plate configuration is considered. The electric field distribution is shown in Figure A1 for $W_1 = 7\"$, $W_2 = 96\", H_2 = 3/8\", H_1 = 0.01\", \varepsilon_1 = 3 \varepsilon_0$, and $V_2 = 1$ volt. It is similar to that shown in Figure 7. In Figure A2 the corresponding reduction of the magnetic field is shown.

Table A1: Transmission Line Parameters - Example 2

<table>
<thead>
<tr>
<th>E FIELD CAPACITANCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
</tr>
<tr>
<td>7.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>H FIELD CAPACITANCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
</tr>
<tr>
<td>7.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>INDUCTANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.313525hN</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E FIELD PLATE VOLTAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
</tr>
<tr>
<td>7.00000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>H FIELD PLATE VOLTAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
</tr>
<tr>
<td>7.00000000</td>
</tr>
</tbody>
</table>

DATA: $W_2 = 96.00\", W_1 = 7.00\", \varepsilon_r = 3.00$

ACCURACY - 6 DECIMAL PLACES

$H_2 = 0.37\", H_1 = 0.010\", 48 \times 96$ MATRIX
Figure A1: Electric Field Distribution for Example 2.

Figure A2: Magnetic Field Distribution for Example 2.
Figure A3: Propagation Constant of Example 2.

Figure A4: Characteristic Impedance of Example 2.
Figure A5: Effect of Losses on Source Current - Example 2.

Figure A6: Effect of Losses at Center of Line - Example 2.
APPENDIX B

Approximate Formulas for the Transmission Line Parameters

The distributed capacitances for the three conductor configuration are defined:

\[ Q_1 = C_{11} V_1 + C_{12} (V_1 - V_2) \]
\[ Q_2 = C_{21} (V_2 - V_1) + C_{22} V_2 \]  

(B1)

Accordingly

\[ C_{11} = \frac{Q_1}{V_1} \quad V_2 = V_1 \]  

(B2)

In this case the flux lines extend from conductor(s) # 1 to the ground plane. Then

\[ C_{11} \approx \varepsilon_2 \varepsilon_0 \frac{2 W_1}{H_1} \]  

(B3)

Correspondingly,

\[ C_{22} = \frac{Q_2}{V_2} \quad V_1 = V_2 \]  

(B4)

For this situation the flux lines extend from conductor # 2 to the ground plane through the opening in conductor # 1. Considering that the flux must penetrate the dielectric interface \( C_{22} \) can be represented by a series combination of the air dielectric capacitor with the paint dielectric capacitor. Therefore

\[ C_{22} \approx \varepsilon_2 \varepsilon_0 \frac{W_2 - 2 W_1}{H_1 + \varepsilon_2 (H_2 - H_1)} \]  

(B5)

Finally the mutual capacitance is
\[ C_{12} = C_{21} = -\frac{Q_1}{V_2} \quad \text{with} \quad V_1 = 0 \quad (B6) \]

Here the flux lines extend from conductor \# 1 to conductor \# 2.

Therefore,

\[ C_{12} \approx \varepsilon_0 \frac{2 W_1}{H_2 - H_1} \quad (B7) \]

The total distributed line capacitance for the balanced line configuration is

\[ C_T = C_{12} + \frac{C_{11} C_{22}}{C_{11} + C_{22}} \quad (B8) \]

Since \( C_{11} \gg C_{22} \), (B7) is approximately

\[ C_T \approx C_{12} + C_{22} \quad (B9) \]

This result can be combined with the general transmission line formulas to approximate the characteristic impedance

\[ Z_c \approx \left[ \frac{R(C_{12} + C'_{22}) + j\omega \mu_0 \varepsilon_0}{(C_{12} + C'_{22})(C_{12} + C_{22})} \right]^{\frac{1}{2}} \quad (B10) \]

and the propagation constant

\[ \gamma^2 = [ R(C_{12} + C'_{22}) + j\omega \mu_0 \varepsilon_0 ] j\omega \frac{C_{12} + C_{22}}{C_{12} + C'_{22}} \quad (B11) \]

Here

\[ C'_{22} = C_{22} \quad \left| \varepsilon_2 = 1 \right. \quad (B12) \]

Results obtained from the foregoing approximations are presented in
Table B1. A comparison is made with the numerically obtained values and excellent agreement is obtained.

Table B1: Comparison of Capacitance Formulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Approx.</th>
<th>Finite Difference</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Example 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C11</td>
<td>37.17 nF/m</td>
<td>39.84 nF/m</td>
<td>6.69</td>
</tr>
<tr>
<td>C12</td>
<td>696.0 pF/m</td>
<td>747.8 pF/m</td>
<td>6.66</td>
</tr>
<tr>
<td>C22</td>
<td>660.8 pF/m</td>
<td>685.5 pF/m</td>
<td>3.60</td>
</tr>
<tr>
<td><strong>Example 2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C11</td>
<td>37.17 nF/m</td>
<td>42.49 nF/m</td>
<td>12.52</td>
</tr>
<tr>
<td>C12</td>
<td>339.4 pF/m</td>
<td>387.9 pF/m</td>
<td>12.49</td>
</tr>
<tr>
<td>C22</td>
<td>1.935 nF/m</td>
<td>1.971 nF/m</td>
<td>1.81</td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY


