A COMPARATIVE STUDY OF
THE ALGEBRAIC RECONSTRUCTION TECHNIQUE AND
THE CONSTRAINED CONJUGATE GRADIENT METHOD AS
APPLIED TO
CROSS BOREHOLE GEOPHYSICAL TOMOGRAPHY/

A Thesis presented to
the Faculty of The College of Engineering and Technology
Ohio University

In Partial Fulfillment of
the Requirements for the Degree
Master of Science

OHIO UNIVERSITY
LIBRARY

by

Ryuichi Masuda

June, 1989
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<td>Reconstruction of Profile 6 by C.C.G., N.F.=2</td>
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I. Introduction

Civilization today heavily depends upon the natural resources found under the ground, which includes fossil fuels and various metals. In recent years, interest in the underground regions has become greater than before. There are needs for possible sites for nuclear wastes and underground facilities. In Japan, the government has begun to explore the possibilities of using "far underground regions" in order to alleviate the problem of overcrowded areas.

Technology for detecting underground structures has been developed and is called geophysical tomography (geotomography). This detection method uses seismic waves or electromagnetic (EM) waves. The main point of detection is to identify the structure without destroying the region. Therefore, the transmission of these waves through the region to be investigated must be analyzed.

Seismic waves are usually used in detecting petroleum, which is buried hundreds of meters beneath the surface, because they are the most powerful waves artificially available at this time. Electromagnetic waves are usually used for shorter distances (between ten and one hundred meters) because of its rapid attenuation under the ground. The distance depends on the conductivity of the soil. The advantage of EM geotomography is that it is possible to obtain high resolution and do continuous time scanning as long as desired.
This study is devoted to EM geotomography with two boreholes (Fig.1). This technology is related to computerized tomography (CT) which has the same mathematical foundation, the Radon inversion formula [1], [2]. The CT scanner is used in the medical field, and this technology is able to display an image of the two-dimensional density distribution of a cross-section of the human body.

Radon is an Austrian mathematician who proved that the inner structure of an object can be determined by the information in the infinite set of projections [2]. A projection is an estimate of a set of line integrals of a parameter, such as a signal attenuation. The process of producing an image of a two-dimensional distribution from projections is called image reconstruction from projections.

In real world applications, an infinite set of projections cannot be achieved; therefore, Radon's rigorous formula must be approximated by some other methods. There are two kinds of methods: transform methods and series expansion methods. With CT scanners, it is possible to make large number of measurements from many directions. The scanning machine can rotate around a patient, yielding a 360-degree viewing angle. This condition is well suited for transform methods.

The measurements in geotomography usually can only be done between two boreholes. Geotomography has an inherent disadvantage with viewing angles, in that there is usually no possible way to scan the region vertically between the ground
Fig. 1 EM Geotomography
surface and the bottom of the cross section of the area (Fig.1). This geometrical condition makes the measurement data insufficient to determine the cross section by transform methods. Therefore, geotomography must rely on the more generalized formulas to account for this problem. These methods are called series expansion methods. Series expansion methods begin with the "digitization" of an image that approximates the real image to be reconstructed.

In order to do the reconstruction, transmitting antennas are placed in one borehole, and receiving antennas in the other. The attenuation of transmitted waves or time-delay of transmitted waves to the receiver is measured. The scanning frequency of an electromagnetic wave is assumed to be ten megahertz to one hundred megahertz depending on the geometrical conditions. The electromagnetic wave is attenuated while it propagates through the area to be investigated. The amount of the attenuation varies when the measurement is taken in different positions. The time-delay also varies when the measuring position is changed. This variation is due to the signal attenuation or time-delay of the electromagnetic wave due to the anomalous part of the area or the distance between a transmitting antenna and a receiving antenna.

The measured data forms a set of projection data that is used to reconstruct the spatial distribution of an electrical property of the cross section of the investigated area. The
signal attenuation is simulated in this study in order to compare the results with prior research.

There is a well known series expansion method called A.R.T. (Algebraic Reconstruction Technique) which is well documented and its behavior is well understood. A.R.T. is easy to code into computer programs; however, there have been many efforts to find a better algorithm than A.R.T.. The C.C.G. algorithm was introduced into geotomography by Balanis [4] which is a class of the conjugate gradient methods (cg-method). The cg-method is known to have fast convergence behavior, but this method has not been analyzed for geophysical applications where the synthetic measurement data is contaminated with noise. Balanis showed superior reconstructions with the C.C.G. algorithm compared to the A.R.T. algorithm with noiseless cases [4].

However, C.C.G. was not analyzed for noisy data conditions. Noisy conditions means data that is contaminated by the imperfect measurement system and theoretical approximations in the interpretation of the data. Boreholes, which are assumed to be drilled straight, may not be perfectly straight, and data can be contaminated by background noise. Also in this study, the simulation was performed with the assumption that the electromagnetic wave propagates straight through the medium. This is a straight-line geometrical optics, which neglects reflection, refraction and diffraction of the electromagnetic wave. The electromagnetic wave could be reflected at the air-ground
boundary. Also the propagation path of the electromagnetic wave could be bent when the wave propagates through the refractive medium. The received power of transmitted wave is simulated according to the straight-line geometrical optics. Thus reflection, refraction and diffraction effects are considered as noise to these simulated data. Random noise is injected to the simulated data in order to account for this "noise". The refraction of the electromagnetic wave can be taken into account in series expansion methods [5]; however, diffraction is not accounted for in the series expansion methods. To alleviate this problem in some cases, diffraction tomography has been developed and examined [6]. But the diffraction tomography is limited in that the diffraction of the electromagnetic wave has to be well modeled by either the Born or Rytov approximations. This approximation is questionable [5].

In this study, the performance of C.C.G. algorithm is compared to A.R.T. algorithm with the data contaminated by the random noise. C.C.G. showed faster convergence over A.R.T. with noisy data, and produced better results in many cases. Then a new method was established to improve this advantage. This method was applied to C.C.G., and improved the performance of C.C.G. to noisy data.
II. Reconstruction Algorithms

Radon developed a mathematical formula which determines a picture (two-dimensional density distribution) from all of its line integrals. In the formula, the picture is represented by a function of two variables, whose domain is from minus infinity to plus infinity. Also, the formula requires infinite set of line integrals in order to calculate the values of the function.

In the application of the formula, only a finite set of measurements can be done, which can only be used to estimate the line integrals of the picture.

Radon's rigorous formula is very sensitive to such an imperfect condition [1]. The finite set of line integrals would not be enough to determine the desired image accurately, even if these exact estimates were available. The inaccuracy of estimates also adds to the problem.

In order to evaluate Radon's formula, an efficient algorithm is required which is able to produce an adequate estimate of the picture from finite and estimated data. The following assumptions are made to set up the background for Radon's reconstruction algorithm.

i) The picture is within a square region, which is called reconstruction region.

ii) The density distribution of a parameter is represented by a function of two variables. This
function is called picture function whose value is zero outside the reconstruction region.

iii) The picture function is continuous inside the reconstruction region.

iv) The center of the reconstruction region is the origin of the coordinate system.

If the cartesian coordinate system is chosen, the density at a point \((x,y)\) can be determined by the value of the picture function at that point \((x,y)\) (Fig.2).

In Fig.2, let \(L\) be a straight line which is the path of the wave from the transmitter to the receiver. Assume \(\alpha\) be a corresponding integrated signal attenuation and \(f(x,y)\) be a picture function. Based on the Radon's proof, \(\alpha\) can be defined approximately equal to the line integral of \(f\) along the line \(L\).

\[
\alpha \equiv \int_{0}^{D} f(x,y) \, dz \quad \text{(1)}
\]

The variable \(z\) is the distance of the point \((x,y)\) on the line \(L\). In practice, the estimate \(\alpha\), such as an integrated signal attenuation, can be obtained by the physical measurement. The problem is to calculate values of the function \(f(x,y)\) from estimates of \(\alpha\) along a number of lines. Equation (1) can be regarded a simplified formula for the image reconstruction problem.
Fig. 2 Reconstruction Region
1) Radon Transform

A mathematically more rigorous form of equation (1) is defined by the Radon transform of the function \( f \) [1]. The Radon transform is the basis for all reconstruction algorithms. The domain of the picture function is a polar coordinate system \((r, \phi)\), where

\[
\begin{align*}
  r &= \sqrt{x^2 + y^2}, & x &= r \cos \phi \\
  \phi &= \tan^{-1}(y/x), & y &= r \sin \phi
\end{align*}
\]

---

The Radon transform of a function \( f(r, \phi) \) is defined in the following formulas:

\[
[Rf](\ell, \theta) = \int_{-\infty}^{\infty} f(\sqrt{\ell^2 + z^2}, \theta + \tan^{-1}(z/\ell)) \, dz \quad \text{if } \ell \neq 0 \quad (3)
\]

\[
[Rf](0, \theta) = \int_{-\infty}^{\infty} f(z, \theta + \pi/2) \, dz \quad \text{if } \ell = 0 \quad (4)
\]

\( R \) is a mathematical operator denoting the Radon transform, acting on a picture function \( f(r, \phi) \) and producing another function \( [Rf](\ell, \theta) \). By observing Fig.2, \( [Rf](\ell, \theta) \) can be considered as a line integral of \( f \) along the line \( L \). It is proven that the values of \( f(r, \phi) \) can be determined by all of its integrals \( [Rf](\ell, \theta) \).
In equation (3) and (4), z=0 corresponds to the point where the perpendicular merges at the line L from the origin, instead of the point of z=0 in Fig. 2.

There are important differences between the domain of the function f and \( \mathcal{R}f \) in following points,

i) Point \((0,\phi)\) represents the origin of the reconstruction region for all values \(\phi\), while the value of the \( \mathcal{R}f \)(0,\(\phi\)) represents the line integral of f along a line through the origin making a angle \(\theta\) with positive y axis.

ii) A point in the \((\ell,\theta)\) space can be thought as a line L (at a distance \(\ell\) from the origin making an angle \(\theta\) with the positive y axis) in the \((r,\phi)\) space.

The relationship between these two variables of two space is shown in Fig.3 and,

\[
\ell = r \cos(\theta-\phi) \hspace{1cm} \hspace{1cm} \text{(3)}.
\]

In the reconstruction region, the values of \( \mathcal{R}f \) are:

\[
[\mathcal{R}f](\ell,\theta) = [\mathcal{R}f](\ell,-\theta+\pi) = [\mathcal{R}f](\ell,\theta+2\pi)
\]

and if the size of the reconstruction region is \( \sqrt{2}E \times \sqrt{2}E \)

\[
[\mathcal{R}f](\ell,\theta) = 0 \text{ if } |\ell| \geq E
\]
Fig. 3 Coordinate System in Radon Transform
If the values of $\mathcal{R}f(l, \theta)$ are known for the $I$ pair of points: $(l_i, \theta_i)$ $1 \leq i \leq I$, the mathematical operator $\mathcal{R}_i$ is defined as

$$\mathcal{R}_i f = [\mathcal{R}f](l_i, \theta_i)$$

$\mathcal{R}_i$ is an example of a functional which acts on $f$ and produces a real number. The input data to a reconstruction algorithm are estimates of values of $\mathcal{R}_i f$ for finite points.

The estimate of $\mathcal{R}_i f$ is referred as $y_i$ which forms a $I$-dimensional column vector, that is called a measurement vector $y$ [1]. Thus the problem is summarized.

i) Measure the $y_i$.

ii) Estimate the $f$.

There are two ways of estimating the picture function from $y$, transform methods and series expansion methods.

2) Transform Methods

The reconstruction methods which express $f$ in terms of its Radon transform are called transform methods. Transform methods define an estimate of the picture function $f$ in terms of $r$, $\Phi$, $y_1$, $y_2$, ..., $y_I$. Mathematically this approach is to find the operator $\mathcal{R}^{-1}$. 
In practice, the formula which defines the operator $\mathcal{R}^{-1}$ is approximated by numerical procedures. One method of approximation is called the convolution method [7], [8]. The convolution method is used in CT scanners, because of ease of implementation and good accuracy [1].

Transform methods generally require following:

i) A precise scanning geometry.

ii) The region of interest is scanned over a 360 degree view.

iii) A large number of measurements.

It is possible to fulfill these requirements in CT scanners. The scanning machine in CT can measure a large number of data in short time while rotating around the patient. Cross borehole tomography cannot provide such a convenient scanning geometry.
3) Series Expansion Methods

The reconstruction process using series expansion methods begins with the discretization of the problem. This method translates the problem of estimating a picture function into finding a finite set of numbers [1].

In order to discretize the problem, basis pictures, whose linear combinations are assumed to give an adequate approximation to any picture $f$, are defined. One example of such a "digitization" is the partition of the picture (of the cross section of the underground region) into $N \times N$ rectangular cells (Fig.4 and 5, N=8). Cells are numbered from 1 to $J$, where $J = N^2$. The basis picture is defined as:

$$b_j(r, \phi) = \begin{cases} 
1 & \text{if } (r, \phi) \text{ is inside the } j \text{th cell} \\
0 & \text{otherwise} 
\end{cases}$$

\[ \tag{1} \]

Then, the approximated picture of $f$ is defined by $f^*$.

$$f^*(r, \phi) = \sum_{j=1}^{J} x_j b_j(r, \phi) \tag{2},$$

where $x_j$ is the average value of $f(r, \phi)$ inside the $j$th cell. The image vector $x$, whose $j$th component is $x_j$, is defined. Thus, the problem is discretized to finding an image vector $x$ instead of estimating a picture $f$. 
Fig. 4 Picture of the Region
Fig. 5 Digitized Picture of the Region
In order to find an estimate of $x$ from the measurement vector $y$, two properties of the functional $R_i$ are required.

i) For all pictures $f_1$ and $f_2$, for all real numbers $c_1$ and $c_2$ and for $1 \leq i \leq I$:

$$R_i(c_1f_1 + c_2f_2) = c_1R_if_1 + c_2R_if_2.$$  

ii) If $f_1$ and $f_2$ are approximately equal to each other, so are $R_if_1$ and $R_if_2$.

Property i) can be proven by the definitions of $R_i$ and $R$. However, property ii) cannot be proven rigorously. Even though it is reasonable to agree with the assumption of ii), sometimes this assumption is badly violated. This is a basic weakness of series expansion methods.

Based on these properties, equation (3) is defined. For $1 \leq i \leq I$:

$$R_if \cong R_if^* = \sum_{i=1,I}^{j=1,J} x_j R_i b_j. \quad \text{(3)}$$

If $b_i$ is defined by equation (2), $R_i b_j$ can be calculated analytically by the length of intersection with the $j$th cell of the line of $i$th position of the transmitter-receiver pair. This line is called a ray path.

The calculated value of $R_i b_j$ is denoted by $D_{i,j}$. The estimate of $R_if$ is $y_i$. Then, equation (3) becomes
$y_i \cong \sum_{j=1}^{J} x_j D_{i,j}$ \hspace{1cm} (4).

The distance matrix $D$, whose $(i,j)$ th component is $D_{i,j}$, is defined and the $I$-dimensional error vector $e$ whose $i$ th component is $e_i$, is defined. The $e_i$ is the difference between the left hand side and right hand side of equation (4). Then, (4) is:

$$y = Dx + e \hspace{1cm} (5).$$

Based on the equation (5) the problem is summarized as:

i) Measure the data $y$.

ii) Estimate the image vector $x$.

If the estimate of the image vector is obtained as $x^*$, the estimate of the picture is defined by

$$f^* = \sum_{j=1}^{J} x^*_j b_j \hspace{1cm} (6).$$

The advantage of series expansion methods is that these methods can be applied to more general means of data collection systems [1]. This condition is well suited to cross borehole tomography.
The algebraic reconstruction technique is one of the algorithms in the class of series expansion methods. This algorithm is an iterative procedure.

The objective of A.R.T. is to produce the estimate of the image vector, \( x^* \), that minimizes the error vector \( e \). The error vector \( e \) is:

\[
e = y - Dx^* \quad (1),
\]

or for \( 1 \leq i \leq I \) and \( 1 \leq j \leq J \):

\[
e_i = y_i - \sum_{i=1,I}^{I} D_{i,j} x_j \quad (2),
\]

where,

\( I \) : Number of ray paths.

\( J \) : Number of cells.

In order to implement the A.R.T., initial estimates for \( x \) are fixed. These estimates are usually zeros. Then, an estimate \( y^q_i \) for \( i \) th path in \( q \) th iteration is calculated by:

\[
y^q_i = \sum_{i=1,I}^{I} D_{i,j} x^q_j \quad (3).
\]
Unless the values of \( x_{qj} \) are exactly correct, the values of \( y_{qi} \) are different from \( y_i \). The correction of the \( x_{qj} \) is done by using this information. The correction factor \( c_{qj} \) is defined by

\[
c_{qj} = (y_i - y_{qi}) D_{i,j} / \left( \sum_{i=1}^{I} \sum_{j=1}^{J} (D_{i,j})^2 \right) \quad (4).
\]

After all correction factors for all cells are calculated, they are added to existing estimate \( x_{qj} \). Then, the algorithm works on the next path. One iteration is considered to be complete when all paths are operated by this process. These procedures are repeated until, ideally, convergence is achieved.

The procedures are summarized as follows.

i) Set all \( x_j \) equal to zero; \( x_{i,j} = 0 \), \((1 \leq j \leq J)\).

ii) Calculate \( y^q \) by

\[
y_{qi} = \sum_{i,j} D_{i,j} x_{qj}.
\]

iii) Calculate correction factors for each cell by

\[
c_{qj} = (y_i - y_{qi}) D_{i,j} / \left( \sum_{i=1}^{I} \sum_{j=1}^{J} (D_{i,j})^2 \right).
\]

iv) Add correction factors \( c^q \) to existing estimates \( x^q \).

v) Go to ii) for the next ray path.
In practical application of the algorithms, the output is usually a map of some physical value. If the lower limit and the higher limit of the value are known, the output of the algorithms may be constrained. This avoids an unrealistic solution such as a negative attenuation constant. In this study, the lower limit is set to 0.001 and the higher limit is set to 2000.0:

\[
x^{q+1}_j = 0.001 \quad \text{for} \quad x^q_j + c^q_j < 0.001
\]

\[
= x^q_j + c^q_j \quad \text{for} \quad 0.001 \leq x^q_j + c^q_j \leq 2000.0
\]

\[
= 2000.0 \quad \text{for} \quad x^q_j + c^q_j > 2000.0
\]

3-2) Constrained Conjugate Gradient Method

(C.C.G.-method)

The C.C.G. method, previously investigated for geophysical applications by Balanis and Frank, is an iterative algorithm which produces the estimate of image vector \(x\) [4]. C.C.G. is based on the method of conjugate gradient (cg-method) developed by Hestenes and Stiefel [21].

The cg-method was originally developed for solving a system of linear equations:

\[
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1
\]

\[
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2
\]

\[\vdots \quad \vdots \quad \vdots \quad \vdots \]

\[
a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n
\]
Equation (1) can be written in vector form:

\[ Ax = b \]  \hspace{1cm} (2),

where, \( A \) is the matrix whose coefficient is \( a_{i,j} \), \( x \) and \( b \) are vectors \((x_1, \ldots, x_n)\) and \((b_1, \ldots, b_n)\). The matrix \( A \) is assumed to be symmetric. If \( A \) is not symmetric, equation (2) is normalized. The solution for equation (2) is denoted by \( h \) \((Ah = b)\). If \( x \) is an estimate of \( h \), the difference \( r = b - Ax \) is called the residual vector.

In each iteration \( q \), the residual vector \( r^q \) is calculated by

\[ r^q = b - Ax \]  \hspace{1cm} (3).

The \( r^q \) can be used as a measure of "goodness" of the estimate \( x_1 \).

The cg-method generates \( n \) mutually conjugate vectors whose linear combinations give solution \( h \):

\[ h = \alpha_1 p_1 + \alpha_2 p_2 + \ldots + \alpha_n p_n \]  \hspace{1cm} (4),

where, \( \alpha_i \)'s are scalar to be found and \( p_i \)'s are called search direction vectors.
Not only can the solution found in this way, but also, the \( h \) can be generated by an iterative fashion as

\[
x^{q+1} = x^q + \gamma^q p^q \quad q = 0, 1, 2, \ldots \ldots \ldots \quad n-1 \tag{5}
\]

with \( h = x^n \) at the \( n \)th step. Finding the scalar \( \gamma^q \) in equation (4) is the objective of this algorithm. The \( \text{cg-} \) method produces a solution in \( n \) steps if the \( n \) dimensional vector is unknown.

The formulas of \( \text{cg-} \) method are defined as follows.

Set \( x_0 = 0 \) (\( x_0 \), initial estimates)

\[
r_0 = p_0 = b \tag{6}
\]

for \( q = 0, 1, 2, \ldots n-1 \)

\[
\gamma^q = \sum_{i=1,n}^{i=1,n} (p^q_i \cdot r^q_i) / \sum_{i=1,n}^{i=1,n} (p^q_i \cdot A_{i,j} p^q_j) \tag{7}
\]

\[
x^{q+1}_i = x^q_i + \gamma^q p^q_i \tag{8}
\]

\[
r^{q+1}_i = x^q_i - (A_{i,j} p^q_j)_i \tag{9}
\]

\[
\beta^q = \sum_{i=1,n}^{i=1,n} (r^{q+1}_i \cdot A_{i,j} p^q_j) / \sum_{i=1,n}^{i=1,n} (p^q_i \cdot A_{i,j} p^q_j) \tag{10}
\]

\[
p^{q+1}_i = r^{q+1}_i + \beta^q p^q_i \tag{11}
\]
The procedures are summarized in two steps.

i) Initial step

Select an estimate \( x_0 = 0 \) and set the residual vector \( r_0 \) and the search direction vector \( p_0 \) equal to \( b \).

ii) Subsequent steps

Based on \( x_q, r_q \) and \( p_q \), calculate the \( y^{q+1}, x^{q+1}, r^{q+1} \) and \( p^{q+1} \) by formulas (6), . . . . . , (11) successively.

The cg-method can be modified to constrain its output. The process of constraining \( x_\chi \) removes the guarantee of generating \( n \) mutually conjugate direction vectors. Thus, C.C.G. may not converge to the solution \( h \) in \( n \) steps. The algorithm is restarted once the convergence of \( x_\chi \) has been reached. After the restart, the solution is assumed to be closer to the final solution \( h \). The procedures for C.C.G. algorithm are defined as follows:

Set \( x_0 = 0 \) (\( x_0 \); initial estimate).

\[
 r_0 = p_0 = b
\]

For \( q = 0, 1, 2, \ldots, n_{iter} \)

i) Perform cg-method calculations by formulas (6), (7), . . . . . , (11) successively.
ii) Constrain the output:

\[ 0.001 \leq x^q \leq 2000.0, \]

iii) If \( \text{ABS}(||r_k|| - ||r_{k-1}||) < \varepsilon, \)

set \( p_k = r_k \).

iv) Go to i) for next iteration.

One iteration is considered to be complete after step iii) is performed. This step was considered by Hestenes and Stiefel to minimize the accumulation of rounding off errors of cg-method [4], [21]. The \( \varepsilon \) is the order of the machine precision and \( n_{\text{iter}} \) is the predetermined maximum number of iterations.
The objective of EM geophysical tomography (EM geotomography) is to determine the distribution of an electrical property in a cross section of an underground region. The output of the process is either the attenuation constant distribution or refractive index distribution in the region of interest. Power is measured to obtain the attenuation constant map while time delay is measured to obtain the refractive index. It is possible to determine the content of the medium from these distributions.

In order to make measurements, two boreholes are drilled on either side of the region of interest. In one hole, transmitters are lowered to predetermined places and in the other hole receivers are lowered in the same manner (Fig. 6). The distance between these boreholes depends on conditions which include the scanning geometries and conductivity of the soil [9]. The location for the boreholes may be restricted by practical considerations.

The electromagnetic wave is assumed to propagate straight through the medium without reflection, refraction or diffraction effects. This is referred to as straight-line geometrical optics. In realistic cases, the transmitted wave could be refracted when the wave propagates through a refractive medium. Thus, the propagation path of the wave could be bent.
Fig. 6 Geometry of Geotomography
When the wave is refracted, the propagation speed of the wave changes. If variations in wave speed are sixteen percent or less, straight-line geometrical optics is assumed to be an adequate approximation [6]. Three criteria are proposed to justify straight-line propagation in [11].

i) Separation distance between the two boreholes should be much larger than $\lambda/2\pi$ ($\lambda$; wavelength).

ii) The refractive index of the medium should vary slowly with distance.

iii) $\lambda<\pi\delta$, where $\delta$ is the skin depth of the medium.

1) Friis Transmission Equation

The straight-line propagation of the electromagnetic wave between two antennas is expressed by Friis transmission equation [12].

If two antennas are assumed to be isotropic and are separated by a distance $R: R > 2D^2/\lambda$, where $D$ is the largest dimension of either antennas, Friis Transmission Equation is:

$$P_r/P_t = e_{te} \frac{(\lambda/4\pi R)^2 D_g D_{gr}}{\left| \hat{P}_t \cdot \hat{P}^*_r \right|^2}$$  \hspace{1cm} (1).$$

For lossy media, equation (1) becomes:

$$P_r/P_t = e_{te} \frac{(\lambda/4\pi R)^2 D_g D_{gr}}{\left| \hat{P}_t \cdot \hat{P}^*_r \right|^2 e^{-2\alpha R}}$$ \hspace{1cm} (2).$$

where,
$P_r, P_t$: power received and transmitted (W)
$\eta_{tt}, \eta_{tr}$: efficiency of transmitting antennas and receiving antennas
$\lambda$: wavelength (m)
$R$: separation distance between transmitting and receiving antennas (m)
$D_{gt}, D_{gr}$: directive gain of transmitting antennas and receiving antennas
$|\hat{\mathbf{p}}_t \cdot \hat{\mathbf{p}}_r^*|^2$: polarization loss factor
$\alpha$: attenuation constant (nepers/m)

The objective is to solve for the attenuation constant of the medium, which is related to its electrical parameters as follows [13].

$$\alpha = \omega \sqrt{\mu \varepsilon} \left( \frac{1}{2} \left[ 1 + (\sigma / \omega \varepsilon)^2 \right]^{\frac{1}{2}} \right)^{\frac{1}{2}} \text{-------(3)}$$

where,
$\omega$: angular frequency (rad/sec)
$\mu$: permeability of the medium (H/m)
$\varepsilon$: permittivity of the medium (F/m)
$\sigma$: conductivity of the medium (S/m).

If the antennas are aligned vertically (to prevent polarization loss), equation (2) becomes:

$$P_r / P_t = \eta_{tt} \eta_{tr} (\lambda / 4\pi R)^2 D_{gt} D_{gr} e^{-2\alpha R} \text{---------(4)},$$

the received power is:
\[ P_r = P_t e_t e_{tr} (\lambda/4\pi R)^2 D_{gt} D_{gr} e^{-2\alpha R} \]  \[ \text{(5)} \]

thus,

\[ \alpha_R = \frac{1}{2} \ln \left( \frac{K}{R^2 P_r} \right) \]  \[ \text{------------------- (6)} \]

where,

\[ K = P_t e_t e_{tr} D_{gt} D_{gr} (\lambda/4\pi)^2 \]

R: separation distance between transmitter and receiver

2) D Matrix and y Vector

The \( \alpha_R \) can be considered as the integrated signal attenuation of the transmitted wave. Equation (6) can be "digitized" into each ray path,

\[ \alpha_R = \frac{1}{2} \ln \left( \frac{K}{L_i^2 P_{rec_i}} \right) = y_i \]  \[ \text{------------------- (7)} \]

\( L_i \) is the total length of the \( i \) th ray path and \( P_{rec_i} \) is the received power of \( i \) th measurement. This digitization produces the measurement vector \( y \) whose \( i \) th component is \( y_i \).

In order to reconstruct an attenuation constant distribution by series expansion methods, the reconstruction region is partitioned into rectangular cells (Fig.7). The
attenuation constant of each cell is assumed constant. Thus, Friis Transmission Equation can be "digitized":

$$\alpha R = y_i = \sum_{\text{one path}}^{\text{one cell}} D_i,j x_j$$

or, a linear system of equations:

$$[y_i] = [D_i,j] [x_j]$$

The D matrix can be determined by the antenna placement and cell partitioning of the region (Fig.8) and $y_i$ can be calculated from the measurement data. In actual cases, the measurement data are the received powers. The $y$ vector is then calculated by equation (7). However, the unknown attenuation constant $x_j$ is available as a solution in the computer simulation. This fact enable us to calculate the $y$ vector without knowing the received powers. The values of $y_i$ will be calculated by equation (8) instead of equation (7).

The D matrix usually becomes very large in size, because many measurements are taken. The D matrix often consists of a nearly singular or singular matrix, which makes the system equation impossible to be solved accurately by direct matrix inversion methods. Besides this problem, errors in measurement affect the system equation. The system equation often becomes an inconsistent (i.e. no solution).
Transmitting Borehole

Receiving Borehole

10x7 partition of the region

Digitized region

Fig. 7 Digitization of the Region
Fig. 8 D Matrix and y Vector
The A.R.T. and C.C.G. are methods which can be applied in such a case.

3) Random Noise

The straight-line geometrical optics formulation neglects the effects of reflection, refraction, and diffraction of the electromagnetic wave. Random noise is injected into the simulated measurements to approximate these effects. The noise factors of 1, 2, 5, and 10 are selected and multiplied to the simulated measurement data in the following manner:

1) Noise factor is selected before the simulation begins.
   ( noise factor 1 is the noiseless condition )
2) Each simulated received power is multiplied by a random number between \( \frac{1}{\text{number selected}} \) and \( \frac{\text{number selected}}{\text{number selected}} \).

The received power contaminated by the random noise is defined by

\[
P_{\text{rec}}^* = P_{\text{rec}}(1 + xnf^*x)^{\text{R}}
\]

where,
P_{rec} : simulated received power by the straight-line geometrical optics
P_{rec}^* : simulated received power after noise is injected
xnf : noise factor (variable: 1, 2, 5, and 10)
x : the random number between 0 and 1
m : (-1)^N
N : an integer x*10

4) Euclidean Distance

The purpose of this work is to examine the performance of the A.R.T. and C.C.G. algorithms under noisy conditions. In the computer simulations, the solutions for the reconstruction are available. This fact enables us to evaluate the performance of algorithms.

The Euclidean Distance [5], [13] is defined by

$$\delta = \sqrt{\frac{1}{n} \sum (x_j - x_j^c)^2} \quad (1)$$

where,

n : total number of cells
x_j : reconstructed parameter
x_j^c : solution parameter.

The measure $\delta$ has shown to be more sensitive to large changes in fine detail than to small changes over a large area [13]. This measure is well suited to examine the quality of the
reconstruction in terms of the identification of the distribution of a parameter.

5) C.C.G. Residual Vector

The C.C.G. residual vector can be used to measure the "goodness" of the estimation of the solution. Advantages of using residual vectors for monitoring the performance of the algorithm are [4]:

i) The residual vector is available in real world measurement. The Euclidean distance works where the answer exists (in the simulation).

ii) The residual vector gives information concerning convergence behavior. This is useful in determining when to stop the execution.

In this study, Euclidean distance is employed to evaluate the performance of each algorithm (A.R.T. and C.C.G.). The performance of C.C.G. algorithm is also monitored by the residual vector.
In order to evaluate the performance of the A.R.T. algorithm and the C.C.G. algorithm, various analyses were done. It is important to consider performances, in terms of stability to noisy data, shape and location of the anomaly, sensitivity to the contrast between the background and the anomaly, resolution and accuracy of reconstructed images, convergence of the algorithm, and number of measurements. The Euclidean distance and the residual vector are employed to evaluate these factors quantitatively.

The random noise was injected into the simulated power, defined to be noise factors of 1, 2, 5, and 10. There are three kinds of shapes of anomalies, with low and high contrast. These profile conditions should indicate the sensitivity of the algorithm to the shape of anomaly and the contrast between the anomaly and background. Digitizations of test profiles are chosen to be thirty cells and one hundred cells, for two different resolutions. If a lesser number of cell partitions is chosen, the system equation becomes simpler, so that less error is expected in the output. But a larger number of cells could give us higher resolution. Therefore, there is a trade-off between accuracy of reconstruction and resolution. These factors and the convergence behavior are examined by Euclidean distance and examining residual vectors.
1) Test Profiles.

Test profiles in this simulation can be sorted into two groups in terms of shapes of anomalies and contrast. There are horizontal and vertical anomalies which are chosen to examine the effect of anomaly orientation on reconstructed images. More accurate results are expected for the horizontally oriented anomaly profile. This means that when the longer sides of cells are parallel to ray paths, one can expect better reconstructions. Whether or not this phenomenon occurs with the C.C.G. algorithm is also examined. The third test profile is the combination of these two, which is L-shaped and placed at the bottom of the reconstruction region. The results of this profile showed that not many ray paths go through the anomaly due to its complicated shape and location, thus a more difficult reconstruction is expected.

In this study, the attenuation profile is reconstructed. The lower contrast profile is 0.1 to 0.2 (neper/m), the higher contrast profile is 0.1 to 0.7 (neper/m). Performance of the algorithms, in terms of the contrast, is tested to examine the sensitivity of algorithms to the range of attenuation constant distribution.

There are two sizes of reconstruction regions-five meters in width and six meters in depth, and ten meters by ten meters. However, the FORTRAN program accepts any size test profiles.
2) Digitization of the Test Profiles.

In series expansion methods such as A.R.T. and C.C.G., reconstruction regions are digitized first. In this study, test profiles are partitioned into cells in two ways: thirty cells and one hundred cells. This digitization affects the resolution and accuracy of the reconstructed image. In particular, with the limited number of measurements, too many cell partitions may result in less accurate reconstructions. Accuracy and resolution of reconstructed images are examined for the same shape of anomaly with different cell partitioning. Digitized test profiles are shown in Fig.9, 10, 11, 12, 13, and 14. Instead of showing numbers, shading is used to display contrast. There are three shapes of anomalies with different cell partitionings, of thirty cells and one hundred cells. The entire region is partitioned into a group of rectangular cells and the size of cells are defined by the number of rows and columns in the FORTRAN program.

3) D Matrix.

After the digitization of the test profile is done, the D matrix is calculated. In order to calculate the D matrix, positions of antennas are given to the test profile. Ten pairs of antennas are assigned to the thirty cell profile, fourteen pairs of antennas are assigned to the one hundred
Fig. 9 Test Profile 1
Fig. 10 Test Profile 2
Fig. 11 Test Profile 3
Fig. 12 Test Profile4

Lower contrast

0.1–0.2 (neper/m)

Higher contrast

0.1–0.7 (neper/m)
Fig. 13 Test Profile 5

- Lower contrast
  - 0.1-0.2 (neper/m)

- Higher contrast
  - 0.1-0.7 (neper/m)
Fig. 14 Test Profile6
fourteen pairs of antennas are assigned to the one hundred cell profile. The number of ray paths are one hundred and one hundred ninety six respectively for each. The position of each antenna is shown in Fig.15.

4) Power Transmission under the Ground.

The electromagnetic waves used in the measurements are assumed to be governed by Friis transmission equation [12], [13]. (i.e. assuming geometrical optics).

4-1) Antennas

The power transmitted is assumed to be one thousand watts. The directive gain of each antenna is one, and no polarization loss is assumed. The frequency is one hundred megahertz and a continuous wave is assumed. The efficiency of each antenna is one.

\[
P_t = 1000 \text{ (W)}
\]
\[
f = 100 \text{(Mhz)}
\]
\[
D_{gt}, D_{gr} = 1 : \text{directive gain}
\]
\[
| \hat{\rho}_t \cdot \hat{\rho}_r^* |^2 = 1 : \text{polarization loss factor}
\]
\[
e_{tt}, e_{rr} = 1 : \text{efficiency of transmitting antenna and receiving antenna}
\]
Fig. 15 Scanning Geometry
4-2) Soil Conditions

The electrical conditions of the test profile are assumed to be

\[ \mu = 4\pi \times 10^{-7} \text{ (H/m)} \] permeability

\[ \varepsilon = 9 \times 8.85 \times 10^{-12} \text{ (F/m)} \] permittivity.

4-3) Received Power \((P_r)\) and \(y\) vector

The received power of \(i\) th measurement can be calculated by

\[ P_{\text{rec}_i} = P_t \ e_{t_{\text{t}}} \ e_{t_{\text{r}}} (\lambda/4\pi L_i)^2 \ D_{gt} D_{gr} \ |\ \hat{\rho}_{t} \ \hat{\rho}_{r}^* \ |^2 \ \ e^{-2\alpha L_i} \quad \text{---(1)} \]

where,

\[ P_t : 1000 \text{ (W)} \]

\[ \alpha = \omega \sqrt{\mu \varepsilon} \left( \frac{1}{2} \sqrt{1 + \left( \frac{\sigma}{ \omega \varepsilon} \right)^2} - 1 \right)^{1/2} \]

The separation distance between two antennas, \(L_i\), can be calculated by the positions of the antenna pair.

In the computer simulation, \(y_i\) can be calculated without knowing the received power \(P_{\text{rec}_i}\).

\[ y_i = \sum_{\text{one path}} D_{i,,j} x_j \quad \text{------------------(2)}, \]
and $y_i$ is related to $P_{rec_i}$ as:

$$y_i = \frac{1}{2} \ln\left(\frac{K}{(L_i^2 P_{rec_i})}\right)$$

--- (3),

where,

$$K = P_t e_t e_r D_{gt} D_{gr} (\lambda/4\pi)^2$$

$L_i$ : separation distance between $i$th transmitter-receiver pair

Thus, the received power can be obtained by equation (4) instead of equation (1):

$$
\frac{K}{L_i^2 e^{2y_i}}
$$

--- (4).

5) Noise injected to the Received Power

After the received power for each measurement is obtained by equation (4), random noise is injected. Noise factors are 1, 2, 5, and 10, where 1 is the noiseless condition. These noises are injected to received powers in order to make the simulation realistic, and also to account for reflection, refraction, and diffraction effects, and to examine the stability of the reconstruction algorithm.
Then, the $y$ vector is produced again by equation (3) with the received power contaminated by the random noise. This $y$ vector is put into equation (2) which will be solved by the algorithm to obtain the desired (attenuation constant distribution) $x_j$.

6) Computations

Computations are first done by A.R.T. and C.C.G. with one hundred iterations. The stopping criterion is developed for the C.C.G. algorithm in a later section. The simulation results, in terms of Euclidean distance, are shown in Tables 1 and 2. Some important points are implied in these results. Performances of C.C.G. and A.R.T. are compared in terms of noise stability, anomaly shape and location, sensitivity to the contrast of anomalies, and the convergence behavior of the algorithm.

6-1) Stability to Noisy Data

In all cases, with one hundred iterations, A.R.T. produces a better reconstruction than C.C.G. when there is no noise injected to the data. A.R.T. works better than C.C.G. in ideal cases, while the former takes more time for computations. An ideal case assumes that the straight-line theory is valid. However, once noise is injected into simulated data, C.C.G. performs better than A.R.T. in many
## Euclidean Distance

<table>
<thead>
<tr>
<th>Profile1</th>
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<th>Tig. CCG</th>
<th>ART</th>
<th>Tig. ART</th>
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<td>No.</td>
<td>(IT=100)</td>
</tr>
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<td>0.1-0.2</td>
<td>1</td>
<td>0.022481 (20)</td>
<td>0.017043</td>
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<td>(neper/m)</td>
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<td>0.033958 (21)</td>
<td>0.085015</td>
<td>0.077517 (25)</td>
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<td></td>
<td>5</td>
<td>0.049524 (22)</td>
<td>0.128732</td>
<td>0.160455 (26)</td>
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<td>0.147150 (31)</td>
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<td>(2%)</td>
<td>CCG</td>
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<td>0.040314 (37)</td>
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<td>(neper/m)</td>
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<td>0.199240 (61)</td>
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<td>0.200574 (62)</td>
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<td></td>
<td>10</td>
<td>0.196234 (63)</td>
<td>0.182551</td>
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Table 1. Convergence of C.C.G. and A.R.T.
### Table 2. Convergence of C.C.G. and A.R.T.

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<th>Fig. ART</th>
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<tr>
<td>5</td>
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<td>0.117001</td>
<td>0.121105</td>
<td>(70)</td>
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<td>0.155077</td>
<td>0.131763</td>
<td>(71)</td>
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<td>(neper/m)</td>
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<td>10</td>
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<th>ART(IT=100)</th>
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<td>0.1-0.2</td>
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<td>0.1-0.7</td>
<td>1</td>
<td>0.145230</td>
<td>0.106698</td>
</tr>
<tr>
<td>(neper/m)</td>
<td>2</td>
<td>0.147477</td>
<td>0.126639</td>
</tr>
<tr>
<td>5</td>
<td>0.153101</td>
<td>0.162412</td>
<td>0.209884</td>
</tr>
<tr>
<td>10</td>
<td>0.158738</td>
<td>0.193148</td>
<td>0.226155</td>
</tr>
</tbody>
</table>
cases. Balanis showed superior reconstruction of the C.C.G. algorithm to the A.R.T. algorithm in noiseless cases [4], where the simulated data are generated by the volume current method (V.C.M.), and not the straight-line theory [10], [19]. V.C.M. produces a more realistic y vector than the straight-line theory. If the y vector produced by the V.C.M. is comparable to the y vector produced by the straight-line theory with a little noise, this result seems to be feasible. Because V.C.M. takes into account reflection, refraction, and diffraction effects, which can be considered as noise for perfect data, C.C.G. has better stability to noisy data than A.R.T..

C.C.G. performs better than A.R.T. in many cases, (except noiseless case) with much faster convergence.

6-2) Anomaly's Shape and Location

The shape of the anomalous region affects the reconstructed image which is, also affected by the length of the ray path through that anomaly. Since the system equation often becomes ill-conditioned, the result is very sensitive to noise and to the anomaly's shape. If the anomaly has a vertically oriented shape, the path length of each ray path through the anomaly does not vary much from the horizontally oriented anomaly. The vertical shape results in components of the y vector with small variations which seem to make it harder to pick up the anomalous region [13].
The location of the anomaly also affects the reconstructed image. In order to reconstruct a quality image, the entire region of interest should be scanned thoroughly from all directions. But, it is very hard to do that because of the geometry of geotomography. If the anomalous region lies in the top, bottom, or corner of the area, detection of that part may become difficult, because the density of rays become small there and there is not a wide range of projection angles. Also, there may be certain places where no ray paths pass through, since ray paths may be bent by the refraction effect. Therefore, it seems to be important to draw ray path lines and see if there are any spots with no ray passes. This phenomenon may not be detected if straight-line theory is used, therefore a combination with a ray tracing method should be considered [5], [17].

In terms of the anomaly's shape, better reconstruction is obtained with horizontally oriented anomalies. The Euclidean distance shows this clearly. Both A.R.T. and C.C.G. algorithms showed same kind of reaction to the difference in the shape of the anomaly.

There is an L-shaped anomaly on the left bottom corner in the third profile, which is expected to be the most difficult reconstruction of the three kinds of profiles. The Euclidean distance showed the poorest reconstructed results for this type.
6-3) Sensitivity to the Contrast between Background Medium and Anomaly

Two kinds of contrasts are chosen to compare reconstructed images. Accuracy and the resolution of reconstructed images are important factors. When the contrast is lower, 0.1 to 0.2 (neper/m), the accuracy of the reconstructed attenuation constant is better than for higher contrasts, 0.1 to 0.7 (neper/m), for both algorithms. C.C.G. performed better when the injected noise level was lower in lower contrast cases. For higher contrast cases, C.C.G. showed better reconstruction when the noise level was higher.

C.C.G. tends to perform better for lower contrast cases with lower noise levels, and for higher contrast cases with higher noise levels. This fact should be noted in terms of refraction and diffraction effects, however further research should be done to examine whether or not this is important. When the contrast of a profile is small, those effects should be small. And for higher contrast case, effects should be larger. If refraction effects can be considered as noise induced in straight-line data, this characteristic of C.C.G. is well suited for geotomography.
6-4) Resolution and Accuracy

When the number of cells which partition the region become larger, the number of ray paths passing through each cell decreases. As a result, the accuracy of the reconstructed image may be affected. When the results of the one hundred cell profile are compared to the thirty cell profile, this becomes clear. The accuracy of reconstructed images is often better for thirty cell profiles. The number of measurements is also different. For thirty cells, one hundred measurements were taken, while, for one hundred cells, one hundred ninety six measurements were taken. When the region is partitioned into a larger number of cells, a larger number of measurements are required to maintain the accuracy of the reconstruction.

6-5) Convergence Behavior

Both A.R.T. and C.C.G. algorithm often showed output diverging from realistic values after a few iterations when the data was contaminated with noise. In order to achieve the best performance of the C.C.G. algorithm, a stopping criterion was developed.

When noise was injected into synthetic data, many C.C.G. outputs diverged from the correct answer, while its residual vector still converged to a smaller number or did not change its value much (Fig.16, 17, 18, and 19). If execution of the
Fig. 16 Convergence of C.C.G., N.F. = 1

Magnitude

0 100 200

Iteration no.

E. D.

||r||
Fig. 17 Convergence of C.C.G., N.F. = 2

![Graph showing convergence of C.C.G., N.F. = 2.](image-url)
Fig. 18 Convergence of C.C.G., N.F.=5
Fig. 19 Convergence of C.C.G., N.F. = 10
algorithm is stopped at the bottom of the curve (Euclidean distance), the best result can be expected. The problem with this kind of stopping criterion is that the Euclidean distance is not available in actual applications, so it is impossible to know whether the output is diverging from correct values or not.

Balanis suggested the enhancement of C.C.G. algorithms in rounding off errors, by taking the norm of residual vectors [4], [21]. This idea is employed to produce the stopping criterion for the C.C.G. algorithm.

In all cases, the norm of the residual vectors converges quickly to a certain value and after that it does not change its value much. The stopping criterion is defined as follows.

i) Store the first change in the norm:

\[ \text{ABS}( ||r^1|| - ||r^0||) \]

as a measure.

ii) Compare the change in the norm of following iterations:

\[ \text{ABS}(||r^i|| - ||r^{i-1}||) \]

with the first one.

iii) Calculate the percent change in the norm when best convergence is achieved.

As a result, 1.5 % for thirty cell profiles, and 2.5 % for one hundred cell profiles were obtained. The stopping criterion is put into the C.C.G. algorithm which stops the execution when a change in the norm of residual vectors
becomes less than 2.0% compared to the first one. Reconstructed images of all profiles are shown in (Fig.20 to 115).

Thus, the final step in the C.C.G. algorithm with stopping criterion is:

iv) Stop the execution of algorithm when the percentage changes in the norm of residual vectors becomes less than 2.0%.

6-6) C.C.G. with Stopping Criterion

The results showed useful enhancement of the C.C.G. algorithm for noisy data, particularly in the reconstruction of noise factor two for lower contrast profiles. The reconstruction became acceptable where it was not before. And also, many of reconstructed images of higher contrast with higher noise level were improved. Thus, this stopping criterion improved the advantage of the C.C.G. algorithm over the A.R.T. algorithm. However, the results of the higher contrast profile became poorer in many cases when injected noise level was low. This was because the convergence of the C.C.G. algorithm for higher contrast anomalies often did not diverge. Therefore, the stopping criterion stopped the execution of the algorithm too early. This is especially true for the thirty cell profile, which is better conditioned than
Fig. 20 Reconstruction of Profile 1 by C.C.G., N.F. = 1
Euclidean distance = 0.033958

Lower contrast 0.1-0.2 (neper/m)

Fig. 21 Reconstruction of Profile1 by C.C.G., N.F. = 2
Euclidean distance = 0.049524

Lower contrast 0.1 - 0.2 (neper/m)

Fig. 22 Reconstruction of Profile 1 by C.C.G., N.F. = 5
Euclidean distance = 0.065623

Lower contrast 0.1-0.2 (neper/m)

Fig. 23 Reconstruction of Profile 1 by C.C.G., N.F. = 10
Fig. 24 Reconstruction of Profile1 by A.R.T., N.F. = 1
Euclidean distance = 0.077517  

Lower contrast 0.1-0.2 (neper/m)

Fig. 25 Reconstruction of Profile1 by A.R.T., N.F. = 2
Euclidean distance = 0.160455

Lower contrast
0.1 - 0.2 (neper/m)

Fig. 26 Reconstruction of Profile 1 by A.R.T., N.F. = 5
Euclidean distance = 0.210904

Lower contrast 0.1 - 0.2 (neper/m)

Fig. 27 Reconstruction of Profile 1 by A.R.T., N.F. = 10
Fig. 28 Reconstruction of Profile1 by C.C.G., N.F. = 1
Fig. 29 Reconstruction of Profile 1 by C.C.G., N.F. = 2

Euclidean distance = 0.127986

Higher contrast
0.1-0.7 (neper/m)
Euclidean distance = 0.136586

<table>
<thead>
<tr>
<th>Euclidean distance = 0.136586</th>
<th>Higher contrast</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1-0.7 (neper/m)</td>
</tr>
</tbody>
</table>

Fig. 30 Reconstruction of Profile 1 by C.C.G., N.F. = 5
Euclidean distance = 0.147150

Higher contrast
0.1-0.7 (neper/m)

Fig. 31 Reconstruction of Profile 1 by C.C.G., N.F. = 10
Fig. 32 Reconstruction of Profile1 by A.R.T., N.F. = 1
Euclidean distance = 0.108033

Higher contrast
0.1-0.7 (neper/m)

Fig. 33 Reconstruction of Profile 1 by A.R.T., N.F. = 2
Euclidean distance = 0.213092

Higher contrast 0.1 - 0.7 (neper/m)

Fig. 34 Reconstruction of Profile 1 by A.R.T., N.F. = 5
Euclidean distance = 0.292944

Higher contrast
0.1-0.7 (neper/m)

Fig. 35 Reconstruction of Profile 1 by A.R.T., N.F. = 10
Euclidean distance = 0.030265

0.10 0.11 0.12 0.13 0.14 0.15 0.16 0.17 0.18 0.19 0.20

Lower contrast 0.1-0.2 (neper/m)

Fig. 36 Reconstruction of Profile2 by C.C.G., N.F. = 1
Euclidean distance = 0.040314

Lower contrast 0.1-0.2 (neper/m)

Fig. 37 Reconstruction of Profile2 by C.C.G., N.F. = 2
Euclidean distance = 0.054872

Lower contrast
0.1 - 0.2 (neper/m)

Fig. 38 Reconstruction of Profile2 by C.C.G., N.F. = 5
Euclidean distance = 0.067556

Lower contrast
0.1 - 0.2 (neper/m)

Fig. 39 Reconstruction of Profile2 by C.C.G., N.F. = 10
Euclidean distance = 0.024012

Lower contrast

0.1 - 0.2 (neper/m)

Fig. 40 Reconstruction of Profile2 by A.R.T., N.F. = 1
Euclidean distance=0.082030

Lower contrast 0.1-0.2(neper/m)

Fig. 41 Reconstruction of Profile2 by A.R.T., N.F.=2
Euclidean distance = 0.147347

Lower contrast 0.1 - 0.2 (neper/m)

Fig. 42 Reconstruction of Profile2 by A.R.T., N.F. = 5
Euclidean distance = 0.207806

Lower contrast
0.1-0.2 (neper/m)

Fig. 43 Reconstruction of Profile2 by A.R.T., N.F. = 10
Euclidean distance = 0.176646

Higher contrast

Fig. 44 Reconstruction of Profile 2 by C. C. G., N.F. = 1
Euclidean distance = 0.176899

Higher contrast
0.1–0.7 (neper/m)

Fig. 45 Reconstruction of Profile2 by C.C.G., N.F. = 2
Fig. 46 Reconstruction of Profile2 by C.C.G., N.F. = 5
Euclidean distance = 0.186586

Higher contrast
0.1 - 0.7 (neper/m)

Fig. 47 Reconstruction of Profile2 by C.C.G., N.F. = 10
Fig. 48 Reconstruction of Profile2 by A.R.T., N.F. = 1

<table>
<thead>
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<th>Euclidean distance = 0.080584</th>
<th>Higher contrast 0.1-0.7 (neper/m)</th>
</tr>
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<tbody>
<tr>
<td>0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6 0.65</td>
<td></td>
</tr>
</tbody>
</table>
Euclidean distance = 0.092579

Higher contrast
0.1 - 0.7 (neper/m)

Fig. 49 Reconstruction of Profile2 by A.R.T., N.F. = 2
Euclidean distance = 0.194674

Higher contrast
0.1-0.7 (neper/m)

Fig. 50 Reconstruction of Profile2 by A.R.T., N.F. = 5
Fig. 51 Reconstruction of Profile2 by A.R.T., N.F. = 10

Euclidean distance = 0.266959

Higher contrast
0.1-0.7 (neper/m)
Fig. 52 Reconstruction of Profile 3 by C.C.G., N.F. = 1
Euclidean distance = 0.043266

Lower contrast
0.1 - 0.2 (neper/m)

Fig. 53 Reconstruction of Profile 3 by C.C.G., N.F. = 2
Fig. 54 Reconstruction of Profile3 by C.C.G., N.F. = 5
Fig. 55 Reconstruction of Profile 3 by C.C.G., N.F. = 10
Fig. 56 Reconstruction of Profile 3 by A.R.T., N.F. = 1
Euclidean distance = 0.075614

Lower contrast
0.1 - 0.2 (neper/m)

Fig. 57 Reconstruction of Profile3 by A.R.T., N.F. = 2
Euclidean distance = 0.161385

Lower contrast
0.1-0.2 (neper/m)

Fig. 58 Reconstruction of Profile3 by A.R.T., N.F. = 5
Euclidean distance = 0.211460

Lower contrast 0.1 - 0.2 (neper/m)

Fig. 59 Reconstruction of Profile3 by A.R.T., N.F. = 10
Higher contrast 0.1-0.7 (neper/m)

Euclidean distance = 0.199967

Fig. 60 Reconstruction of Profile 3 by C.C.G., N.F. = 1
Fig. 61 Reconstruction of Profile3 by C.C.G., N.F. = 2
Euclidean distance = 0.200574

Higher contrast
0.1 - 0.7 (neper/m)

Fig. 62 Reconstruction of Profile3 by C.C.G., N.F. = 5
Euclidean distance = 0.196234

Higher contrast 0.1-0.7 (neper/m)

Fig. 63 Reconstruction of Profile3 by C.C.G., N.F. = 10
Fig. 64 Reconstruction of Profile3 by A.R.T., N.F. = 1

Higher contrast

0.7 0.75
Euclidean distance = 0.069779
0.1-0.7 (neper/m)
Euclidean distance = 0.082359

Higher contrast
0.1 - 0.7 (neper/m)

Fig. 65 Reconstruction of Profile3 by A.R.T., N.F. = 2
Euclidean distance = 0.242074  
Higher contrast  
0.1-0.7 (neper/m)  

Fig. 66 Reconstruction of Profile 3 by A.R.T., N.F.=5
Euclidean distance = 0.307855

Higher contrast
0.1-0.7 (neper/m)

Fig. 67 Reconstruction of Profile3 by A.R.T., N.F. = 10
Fig. 68 Reconstruction of Profile 4 by C. C. G., N. F. = 1
Fig. 69 Reconstruction of Profile 4 by C.C.G., N.F. = 2

Euclidean distance = 0.036141

Lower contrast
0.1 - 0.2 (neper/m)
Euclidean distance = 0.052731

Lower contrast 0.1-0.2 (neper/m)

Fig. 70 Reconstruction of Profile 4 by C. C. G., N. F. = 5
Fig. 71 Reconstruction of Profile 4 by C.C.G., N.F. = 10

Euclidean distance = 0.077825

Lower contrast 0.1 - 0.2 (neper/m)
Fig. 72 Reconstruction of Profile 4 by A.R.T., N.F. = 1

Lower contrast
Euclidean distance = 0.017173
0.1-0.2 (neper/m)
Euclidean distance = 0.117266

Lower contrast 0.1 - 0.2 (neper/m)

Fig. 73 Reconstruction of Profile 4 by A.R.T., N.F. = 2
Euclidean distance = 0.121105

Lower contrast
0.1 - 0.2 (neper/m)

Fig. 74 Reconstruction of Profile 4 by A.R.T., N.F. = 5
Euclidean distance = 0.131763

Fig. 75 Reconstruction of Profile 4 by A.R.T., N.F. = 10

Lower contrast
0.1 - 0.2 (neper/m)
Fig. 76 Reconstruction of Profile 4 by C.C.G., N.F. = 1

Higher contrast
Euclidean distance = 0.132623
0.1-0.7 (neper/m)
Euclidean distance = 0.133861

Higher contrast

0.1 - 0.7 (neper/m)

Fig. 77 Reconstruction of Profile4 by C.C.G., N.F. = 2
Fig. 78 Reconstruction of Profile4 by C.C.G., N.F.=5

Euclidean distance=0.137371

Higher contrast
0.1-0.7(neper/m)
Fig. 79 Reconstruction of Profile 4 by C.C.G., N.F. = 10
Fig. 80 Reconstruction of Profile by A.R.T., N.F. = 1
Fig. 81 Reconstruction of Profile4 by A.R.T., N.F. = 2
Euclidean distance = 0.230805
Higher contrast 0.1-0.7 (neper/m)

Fig. 82 Reconstruction of Profile4 by A.R.T., N.F. = 5
Euclidean distance = 0.270511

Higher contrast
0.1–0.7 (neper/m)

Fig. 83 Reconstruction of Profile 4 by A.R.T., N.F. = 10
Euclidean distance = 0.037188

Fig. 84 Reconstruction of Profile 5 by C.C.G., N.F. = 1
Euclidean distance = 0.041439  

Lower contrast  
0.1 – 0.2 (neper/m) 

Fig. 85 Reconstruction of Profile 5 by C.C.G., N.F. = 2
Euclidean distance = 0.060957

Lower contrast

0.1 - 0.2 (neper/m)

Fig. 86 Reconstruction of Profile 5 by C.C.G., N.F. = 5
Euclidean distance = 0.061924

Lower contrast

0.1 - 0.2 (neper/m)

Fig. 87 Reconstruction of Profile 5 by C.C.G., N.F. = 10
Fig. 88 Reconstruction of Profile5 by A.R.T., N.F.=1
Fig. 89 Reconstruction of Profile 5 by C.C.G., N.F. = 2
Fig. 90 Reconstruction of Profile5 by A.R.T., N.F. = 5

Euclidean distance = 0.118796

Lower contrast 0.1 – 0.2 (neper/m)
Euclidean distance = 0.131795

Lower contrast
0.1 - 0.2 (neper/m)

Fig. 91 Reconstruction of Profile5 by A.R.T., N.F. = 10
Fig. 92 Reconstruction of Profile 5 by C.C.G., N.F. = 1
Euclidean distance = 0.179680

Higher contrast 0.1-0.7 (neper/m)

Fig. 93 Reconstruction of Profile5 by C.C.G., N.F.=2
Euclidean distance = 0.182492

Higher contrast
0.1-0.7 (neper/m)

Fig. 94 Reconstruction of Profile 5 by C.C.G., N.F. = 5
Fig. 95 Reconstruction of Profile5 by C.C.G., N.F. = 10
Fig. 96 Reconstruction of Profile5 by A.R.T., N.F. = 1
Fig. 97 Reconstruction of Profile5 by A.R.T., N.F. = 2
<table>
<thead>
<tr>
<th>Euclidean distance</th>
<th>Higher contrast</th>
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<td>0.235391</td>
<td>0.1-0.7 (neper/m)</td>
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</table>

Fig. 98 Reconstruction of Profile5 by A.R.T., N.F.=5
Fig. 99 Reconstruction of Profile5 by A.R.T., N.F. = 10
Fig. 100 Reconstruction of Profile6 by C.C.G., N.F. = 1
Euclidean distance = 0.037026

Lower contrast
0.1 - 0.2 (neper/m)

Fig. 101 Reconstruction of Profile6 by C.C.G., N.F. = 2
Euclidean distance = 0.057883

Lower contrast 0.1-0.2 (neper/m)

Fig. 102 Reconstruction of Profile 6 by C.C.G., N.F. = 5
Euclidean distance = 0.057827

Lower contrast
0.1-0.2 (neper/m)

Fig. 103 Reconstruction of Profile 6 by C.C.G., N.F. = 10
Fig. 104 Reconstruction of Profile 6 by A.R.T., N.F.=1
Euclidean distance = 0.107698

Lower contrast
0.1 - 0.2 (neper/m)

Fig. 105 Reconstruction of Profile 6 by A.R.T., N.F. = 2
Euclidean distance = 0.111794

Lower contrast
0.1 - 0.2 (neper/m)

Fig. 106 Reconstruction of Profile6 by A.R.T, N.F. = 5
Fig. 107 Reconstruction of Profile6 by A.R.T., N.F. = 10
Fig. 108 Reconstruction of Profile6 by C.C.G., N.F. = 1
Euclidean distance = 0.147477

Higher contrast
0.1-0.7 (neper/m)

Fig. 109 Reconstruction of Profile6 by C.C.G., N.F. = 2
Euclidean distance = 0.153101

Higher contrast

0.1 - 0.7 (neper/m)

Fig. 110 Reconstruction of Profile 6 by C.C.G., N.F. = 5
Euclidean distance = 0.158738

Higher contrast
0.1 - 0.7 (neper/m)

Fig. 111 Reconstruction of Profile6 by C.C.G., N.F. = 10
Higher contrast
Euclidean distance = 0.058082
0.1-0.7 (neper/m)

Fig. 112 Reconstruction of Profile 6 by A.R.T., N.F. = 1
<table>
<thead>
<tr>
<th>Euclidean distance = 0.155441</th>
<th>Higher contrast 0.1–0.7 (neper/m)</th>
</tr>
</thead>
</table>

Fig. 113 Reconstruction of Profile 6 by A.R.T., N.F. = 2
Euclidean distance = 0.209884

Higher contrast 0.1-0.7 (neper/m)

Fig. 114 Reconstruction of Profile6 by A.R.T., N.F. = 5
Euclidean distance = 0.226155

Higher contrast
0.1 - 0.7 (neper/m)

Fig. 115 Reconstruction of Profile 6 by A.R.T., N.F. = 10
the one hundred cell profile. But in many cases, the anomalous area was detected to a certain degree.

Therefore, the following points should be noted for the C.C.G. algorithm with stopping criterion:

For overdetermined cases,

(number of measurement > number of cells).

i) When the contrast is known to be high, execution of the C.C.G. algorithm may be continued until the residual vector converges.

ii) When the contrast is known to be low, execution of the C.C.G. algorithm should be stopped when the percentage change in the norm of residual vector becomes less than 2.0 %.

Lytle suggested a scanning system which is different from this study [24]. One pair of transmitter and receiver is lowered into the borehole in unison, to scan the entire region. This method is more oriented toward detecting the horizontal position of the anomalous region rather than the lateral position. But the "signature" of the received power can tell us the contrast between the background and anomalous region. This method may be combined with C.C.G. to estimate the contrast of the region before C.C.G. is applied.
V. Conclusions

The main purpose of this study was to examine the noise stability of the C.C.G. algorithm and compared with A.R.T.. The stopping criterion was developed in this study, and applied to this algorithm. Useful enhancement was obtained, which further improved the stability of C.C.G.. It is important to note that less than five iterations produced acceptable reconstructions. This stopping criterion also contributed to reducing the computational time.

C.C.G. requires a larger core memory than A.R.T., but its strong noise stability and its quick convergence characteristics should outweigh this disadvantage. Since C.C.G. converges quickly, future enhancement of this algorithm, such as new stopping criterion, should be applied at an early step. This study was simulated under the straight-line geometrical optics, but a more realistic analysis can be done with sophisticated methods such as the ray tracing method. However, this study definitely showed stable behavior of the C.C.G. algorithm to the noisy data. Noise simulates the effect of reflection, refraction, and diffraction of EM waves and the non-perfect geometry of the boreholes. According to engineers involved in field measurements, drilling boreholes straight down is difficult and often results in spiral shaped boreholes, which can contaminate the data.
The advantages of the C.C.G. algorithm over the A.R.T. algorithm are:

i) Its quick convergence.

ii) Strong noise stability.

a) In high contrast cases:

C.C.G. produces superior reconstructions under noisy conditions, with no stopping criterion. With the stopping criterion, the performance of the algorithm was improved for high noise conditions.

b) In low contrast cases:

C.C.G. performed superior reconstructions under noisy conditions with no stopping criterion. With the stopping criterion, the performance of the algorithm was improved for low noise conditions.

In order to improve the stopping criterion which was developed in this study, additional computer simulations with test profiles of various contrasts may be performed. The Euclidean distance and the residual vector can be used to monitor the convergence of C.C.G.
Also, during this study, several pertinent issues were identified:

i) Polarization of the EM waves.

King considered the polarization of the transmitted waves, which affects the field intensity in far field under the ground [26]. He proved that the EM waves with horizontal polarization propagate most efficiently.

ii) Lateral wave

There are also lateral waves which propagate along the surface of the ground if the depth of transmitting antennas is relatively shallow [26]. In order to scan the shallow region for EM cross-borehole tomography, this effect should either be eliminated, or antennas should be lowered deep enough to avoid the lateral wave and reflection effect from the air-ground boundary.

King also suggested the development of array antennas, which may be used to enlarge the scanning distance of EM geotomography.

In conclusion, this study has shown that the C.C.G. algorithm is more stable in the presence of noisy data, produces a superior reconstruction, and requires shorter
computation time. However, further research, particularly in field measurements, is required to determine if the results from these computer simulations are valid for actual application.
VI. Bibliography


VII. Appendices

A: FORTRAN Listing of A.R.T.

C234567890
C THIS PROGRAM RECONSTRUCTS A GIVEN ATTENUATION PROFILE BY THE ALGEBRAIC RECONSTRUCTION TECHNIQUE.
C
C ARRAY ASSIGNMENT
C
C D : D MATRIX
C Y : Y VECTOR
C XE : ESTIMATION OF THE ATTENUATION PROFILE
C TRANS : TRANSMITTER COORDINATE
C REC : RECEIVER COORDINATE
C
C INPUT VARIABLES
C
C DEPTH : DEPTH OF THE RECONSTRUCTION REGION
C WIDTH : WIDTH OF THE RECONSTRUCTION REGION
C NROW : NUMBER OF ROWS
C NCOL : NUMBER OF COLUMNS
C NTRANS : NUMBER OF TRANSMITTER POSITIONS
C NREC : NUMBER OF RECEIVER POSITIONS
C ITER : MAXIMUM NUMBER OF ITERATIONS
C
C***************************************************
C DIMENSION TRANS(14,2),REC(14,2),SLOPE(196)
& XX(11),YY(11),REGION(100,4)
& ,B(196),XL(196),D(196,100),THETA(196)
& ,XA(100),Y(196),XE(100),YQ(196),SD(400)
& ,PREC(196),SED(400),XIT(400)
& ,ENO(200)
REAL PI
READ (5,401) DEPTH,WIDTH
READ (5,400) NROW,NCOL
READ (5,400) NTRANS,NREC
READ (5,404) ITER
404 FORMAT(I3)
NPATH=NTRANS*NREC
NCELL=NROW*NCOL
WRITE (6,400) NROW,NCOL
DO 112 I=1,NCELL
WRITE (6,113) I
C
113 FORMAT(3X,I3)
112 READ (5,502) XA(I)
111 DO 1 I=1,NTRANS
1 READ (5,401) TRANS(I,1),TRANS(I,2)
DO 8 I=1,NREC
8 READ (5,401) REC(I,1),REC(I,2)
400 FORMAT(212)
401 FORMAT (2F6.3)  
502 FORMAT (F6.3)  
       I = 1  
       DO 2 K = 1, NTRANS  
       DO 2 KK = 1, NREC  
       SLOPE (I) = (REC (KK, 2) - TRANS (K, 2)) / (REC (KK, 1) - TRANS (K, 1))  
       B (I) = TRANS (K, 2) - SLOPE (I) * TRANS (K, 1)  
       XL (I) = SQRT ((REC (KK, 2) - TRANS (K, 2))**2 + (REC (KK, 1) - TRANS (K, 1))**2)  
       THETA (I) = ATAN (ABS (SLOPE (I)))  
     2 I = I + 1  
     M = NCOL + 1  
     DO 5 I = 1, M  
       XX (I - 1) = WIDTH * FLOAT (I - 1) / FLOAT (NCOL)  
     5 CONTINUE  
     M = NROW + 1  
     DO 6 J = 1, M  
       YY (J - 1) = DEPTH * FLOAT (J - 1) / FLOAT (NROW)  
     6 CONTINUE  
     C DO 4 I = 0, NCOL  
     C WRITE (6, *) XX (I), YY (I)  
     4 CONTINUE  
     C DO 10 J = 1, NCOL  
       M = J  
       DO 10 I = M, NCELL, NCOL  
       REGION (I, 1) = XX (J - 1)  
       REGION (I, 2) = XX (J)  
     10 CONTINUE  
     C K = 0  
     DO 20 J = 1, NROW  
       L = I + (J - 1) * NCOL  
       M = J * NCOL  
       DO 15 I = L, M  
       REGION (I, 3) = YY (K)  
       REGION (I, 4) = YY (K + 1)  
     15 CONTINUE  
     K = K + 1  
     20 CONTINUE  
     C DO 11 I = 1, NCELL  
     C WRITE (6, *) REGION (I, 1), REGION (I, 2), REGION (I, 3), REGION (I, 4)  
     11 CONTINUE  
     C DO 30 I = 1, NPATH  
     C DO 30 L = 1, NCELL  
       JJ = 0  
       XP = 0.0  
       ZP = 0.0  
       N = 1  
       DO 40 LL = 1, 2  
       Z = SLOPE (I) * REGION (L, LL) + B (I)  
       IF (Z .LT. REGION (L, 3) .OR. Z .GT. REGION (L, 4)) GO TO 40  
       DIF1 = Z - REGION (L, 3)  
       DIF2 = Z - REGION (L, 4)  
     40 CONTINUE  
     C WRITE (6, 22) L, DIF1, DIF2  
     22 FORMAT (3X, 'CELL=', I2, 3X, 'DIF1=', F7.3, 3X, 'DIF2=', F7.3)  
     NTRA = IFIX (FLOAT (I) / FLOAT (NREC) - 0.01) + 1
C  IF(Z.EQ.REGION(L,3).OR.Z.EQ.REGION(L,4)) GO TO 110
DF1=ABS(DIF1)
DF2=ABS(DIF2)
IF(DF1.LE.1.0E-6.0R.DF2.LE.1.0E-4) GO TO 110
GO TO 130
110 WRITE(6,120) NTRA,I
120 FORMAT(3X,'PLEASE MOVE TRANSMITTER NO.',I2,'NPATH=',I4)
C GO TO 111
130 CONTINUE
XP=XP+REGION(L,LL)*(-1.0)**(N+1)
ZP=ZP+Z*(-1.0)**(N+1)
JJ=JJ+1
N=N+1
40 CONTINUE
IF (SLOPE(I).EQ.0.0) GO TO 30
DO 50 LL=3,4
X=(REGION(L,LL)-B(I))/SLOPE(I)
IF (X.LE.REGION(L,1).OR.X.GE.REGION(L,2)) GO TO 50
DIF3=X-REGION(L,1)
DIF4=X-REGION(L,2)
C WRITE(6,23) L,DIF3,DIF4
23 FORMAT(3X,CELL=',I2,3X,DIF3=',F7.3,DIF4=',F7.3)
XP=XP+X*(-1.0)**(N+1)
ZP=ZP+REGION(L,LL)*(-1.0)**(N+1)
JJ=JJ+1
N=N+1
50 CONTINUE
IF (JJ.EQ.2) GO TO 30
XP=0.0
ZP=0.0
30 D(I,L)=SQRT(XP*XP+ZP*ZP)
YA=0.0
DO 31 I=1,NPATH
DO 32 L=1,NCELL
Y(I)=D(I,L)*XA(L)+YA
YA=Y(I)
32 CONTINUE
YA=0.0
31 CONTINUE
WRITE(6,51)
51 FORMAT(3X,'SIMULATE WITH NOISE ? YES-1,NO-0')
READ(5,52) NOISE
52 FORMAT(I2)
IF(NOISE.EQ.0) GO TO 55
WRITE(6,53)
53 FORMAT(3X,'SELECT NOISE FACTOR')
READ(5,54) NF
54 FORMAT(I2)
XNF=FLOAT(NF)-1.0
C ADD NOISE TO RECEIVED POWER
PI=3.14159
FREQ=100.0
GAIN=1.0
BETAC=2.0*PI*FREQ*1.0E6*SQRT(4.0E-7*PI*9.0*8.85E-12)
&SQRT((1.0+SQRT(1.0+((1.0E-3/(2.0*PI*FREQ*1.0E6*9.0*8.85E-12)))**2)
&)))/2.0)

C

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C

WL=2.0*PI/BETAC

CONST=(WL/(4.0*PI))**2)*1000.0*(GAIN**2)

C

DO 56 I=1,NPATH

PREC(I)=CONST/((XL(I)**2)*EXP(2.0*Y(I)))

CALL ARAND(EX)

N=EX*10

PREC(I)=PREC(I)*((1.0+XNF*EX)**((-1.0)**N))

56 Y(I)=0.5**ALOG(CONST/(PREC(I)*(XL(I)**2)))

55 CONTINUE

DO 41 J=1,NCELL

41 XE(J)=0.0

DO 42 I=1,NPATH

SD(I)=0.0

DO 43 J=1,NCELL

43 SD(I)=SD(I)+D(I,J)**2

DO 44 IT=1,ITER

XIT(IT)=FLOAT(IT)

XIT(IT+200)=FLOAT(IT)

DO 45 I=1,NPATH

45 YQ(I)=0.0

EN=0.0

DO 46 J=1,NCELL

46 IF(XE(J).LT.0.001) XE(J)=0.001

IF(XE(J).GT.2000.0) XE(J)=2000.0

C

56 CONTINUE

EN0(IT)=EN

EDS=0.0

DO 75 J=1,NCELL

75 EDS=EDS+((XA(J)-XE(J))**2)

SED(IT)=SQRT(EDS/FLOAT(NCELL))

C

WRITE(6,450) IT, SED(IT)

C

CALL MPLOT(XIT, SED, NPTSPP, NPLTS, XL, YL)

C

CALCULATE EUCLIDEAN DISTANCE
C
ED=0.0
DO 49 J=1,NCELL
ED=ED+(XA(J)-XE(J))**2
ED=SQRT(ED/FLOAT(NCELL))
DO 35 I=1,NTRANS
35  WRITE(6,403) TRANS(I,1),TRANS(I,2)
DO 36 I=1,NREC
36  WRITE(6,403) REC(I,1),REC(I,2)
WRITE(6,402) WIDTH,DEPTH,NCELL,NPATH
DO 70 I=1,NPATH
DO 70 L=1,NCELL
C
70  CONTINUE
WRITE(6,500) I,L,D(I,L)
70 CONTINUE
DO 80 I=1,NPATH
C
80  CONTINUE
WRITE(6,600) I,Y(I)
80 CONTINUE
DO 90 J=1,NCELL
WRITE(6,700) J,XE(J)
90 CONTINUE
WRITE(6,800) ED,NF,ITER
C
402 FORMAT(1X,'WIDTH=',F6.2,3X,'DEPTH=',F6.2,/,&10X,'NCELL=',I3,3X,'NPATH=',I4)
403 FORMAT(3X,'X=',F7.3,3X,'Y=',F7.3)
500 FORMAT(3X,'D(',I2,',',I2,')=',F12.3)
600 FORMAT(3X,'Y(',I3,')=',F12.3)
700 FORMAT(3X,'XE(',I3,')=',F12.3)
800 FORMAT(3X,'EUCLIDEAN DISTANCE=',F8.6
&',/3X,'NOISE FACTOR =',I2,3X,'ITERATION=',I3)
STOP
END
C234567890
C THIS PROGRAM RECONSTRUCTS A GIVEN ATTENUATION PROFILE BY
C THE CONSTRAINED CONJUGATE GRADIENT METHOD
C
C ARRAY ASSIGNMENT
C
D : D MATRIX
Y : Y VECTOR
XE : ESTIMATION OF THE ATTENUATION PROFILE
R : RESIDUAL VECTOR
P : SEARCH DIRECTION VECTOR
TRANS : TRANSMITTER COORDINATE
REC : RECEIVER COORDINATE

C INPUT VARIABLES

DEPTH : DEPTH OF THE RECONSTRUCTION REGION
WIDTH : WIDTH OF THE RECONSTRUCTION REGION
NROW : NUMBER OF ROWS
NCOLS : NUMBER OF COLUMNS
NTRANS : NUMBER OF TRANSMITTER POSITIONS
NREC : NUMBER OF RECEIVER POSITIONS
ITER : MAXIMUM NUMBER OF ITERATIONS

C
C***************************************************************
C
DIMENSION TRANS(14,2),REC(14,2),SLOPE(196)
& XX(11),YY(11),REGION(100,4)
& B(196),XL(196),D(196,100)
& XA(196),Y(100),XE(100),DT(100,196),YN(100)
& PREC(196),R(100),P(100),DNP(100),DN(100,100)
& SED(200),XIT(200)
& GAMMA(200),ANR(200)
REAL PI

READ (5,401) DEPTH,WIDTH
READ (5,400) NROW,NCOL
READ (5,400) NTRANS,NREC
READ (5,404) ITER

404 FORMAT(I3)
NPATH=NTRANS*NREC
NCELL=NROW*NCOL
WRITE (6,400) NROW,NCOL
DO 112 I=1,NCELL
WRITE (6,113) I

C113 FORMAT(3X,I3)
112 READ(5,502) XA(I)
111 DO 1 I=1,NTRANS
1 READ(5,401) TRANS(I,1),TRANS(I,2)
DO 8 I=1,NREC
8 READ(5,401) REC(I,1),REC(I,2)

400 FORMAT(2I2)
401 FORMAT(2F6.3)
502 FORMAT(F6.3)
I=1
DO 2 K=1,NTRANS
DO 2 KK=1,NREC
SLOPE(I)=(REC(KK,2)-TRANS(K,2))/(REC(KK,1)-TRANS(K,1))
B(I)=TRANS(K,2)-SLOPE(I)*TRANS(K,1)
XL(I)=SQRT((REC(KK,2)-TRANS(K,2))^2)
&+(REC(KK,1)-TRANS(K,1))^2
C
THETA(I)=ATAN(ABS(SLOPE(I)))
2
I=I+1
M=NCOL+1
DO 5 I=1,M
XX(I-1)=WIDTH*FLOAT(I-1)/FLOAT(NCOL)
5 CONTINUE
M=NROW+1
DO 6 J=1,M
YY(J-1)=DEPTH*FLOAT(J-1)/FLOAT(NROW)
6 CONTINUE
C
DO 4 I=0,NCOL
WRITE(6,*) XX(I),YY(I)
4 CONTINUE
C
DO 10 J=1,NCOL
M=J
DO 10 I=M,NCELL,NCOL
REGION(I,1)=XX(J-1)
REGION(I,2)=XX(J)
10 CONTINUE
C
K=0
DO 20 J=1,NROW
L=1+(J-1)*NCOL
M=J*NCOL
DO 15 I=L,M
REGION(I,3)=YY(K)
REGION(I,4)=YY(K+1)
15 CONTINUE
K=K+1
20 CONTINUE
C
WRITE(6,*) REGION(I,1),REGION(I,2),REGION(I,3),REGION(I,4)
C
DO 30 I=1,NPATH
DO 30 L=1,NCELL
JJ=0
XP=0.0
ZP=0.0
N=1
DO 40 LL=1,2
Z=SLOPE(I)*REGION(L,LL)+B(I)
IF (Z.LT.LEVEL(L,3).OR.Z.GT.LEVEL(L,4)) GO TO 40
DIF1=Z-REGION(L,3)
DIF2=Z-REGION(L,4)
40 CONTINUE
C
WRITE(6,22) L,DIF1,DIF2
22 FORMAT(3X,'CELL=',I2,3X,'DIF1=',F7.3,3X,'DIF2=',F7.3)
NTRA=IFIX(FLOAT(I)/FLOAT(NREC))-0.01)
C
IF (Z.EQ.LEVEL(L,3).OR.Z.EQ.LEVEL(L,4)) GO TO 110
DF1=ABS(DIF1)
110 CONTINUE
DF2=ABS(DIF2)
IF(DF1.LE.1.0E-6.OR.DF2.LE.1.0E-6) GO TO 110
GO TO 130
110 WRITE(6,120) NTRA,I
120 FORMAT(3X,'PLEASE MOVE TRANSMITTER NO.','I2','NPATH='I4)
C
130 CONTINUE
XP=XP+REGION(L,LL)*(-1.0)**(N+1)
ZP=ZP+Z*(-1.0)**(N+1)
JJ=JJ+1
N=N+1
40 CONTINUE
IF (SLOPE(I).EQ.0.0) GO TO 30
DO 50 LL=3,4
X=(REGION(L,LL)-B(I))/SLOPE(I)
IF (X.LE.REGION(L,1).OR.X.GE.REGION(L,2)) GO TO 50
DIF3=X-REGION(L,1)
DIF4=X-REGION(L,2)
C
WRITE(6,23) L,DIF3,DIF4
23 FORMAT(3X,'CELL=','I2','DIF3=','F7.3','DIF4=','F7.3')
XP=XP+X*(-1.0)**(N+1)
ZP=REGION(L,LL)*(-1.0)**(N+1)
JJ=JJ+1
N=N+1
50 CONTINUE
IF (JJ.EQ.2) GO TO 30
XP=0.0
ZP=0.0
30 D(I,L)=SQRT(XP*XP+ZP*ZP)
YA=0.0
DO 31 I=1,NPATH
DO 32 L=1,NCELL
Y(I)=D(I,L)*XA(L)+YA
YA=Y(I)
32 CONTINUE
YA=0.0
31 CONTINUE
WRITE(6,51)
51 FORMAT(3X,'SIMULATE WITH NOISE ? YES-1,NO-0')
READ(5,52) NOISE
52 FORMAT(I2)
NF=0
IF (NOISE.EQ.0) GO TO 55
WRITE(6,53)
53 FORMAT(3X,'SELECT NOISE FACTOR')
READ(5,54) NF
54 FORMAT(I2)
XNF=FLOAT(NF)-1.0
C
C ADD NOISE TO RECEIVED POWER
C
PI=3.14159
FREQ=100.0
GAIN=1.0
BETAC=2.0*PI*FREQ*1.0E6*SQRT(4.0E-7*PI*9.0*8.85E-12)
&*SQRT((1.0+SQRT(1.0+((1.0E-3/(2.0*PI*FREQ*1.0E6*9.0*8.85E-12))**2))**2))
&)))/2.0)

**C**

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**C**

WL=2.0*PI/BETAC
CONST=((WL/(4.0*PI))*2)*1000.0*(GAIN**2)

**C**

DO 56 I=1,NPATH
PREC(I)=CONST/((XL(I)**2)*EXP(2.0*Y(I))
CALL ARAND(EX)
NN=EX*10
PREX(I)=PREC(I)*((1.0+XNFXEX)**((-1.0)**NN))
Y(I)=0.5*ALOG(CONST/(PREC(I)**(XL(I)**2)))
56 CONTINUE
55 CONTINUE

**C**

CCG CALCULATION BEGINS

**C**

DO 41 I=1,NPATH
DO 41 J=1,NCELL
41 DT(J,I)=D(I,J)

**C**

ADN=0.0
DO 42 M=1,NCELL
DO 42 N=1,NCELL
DO 43 I=1,NPATH
43 ADN=DT(M,I)*D(I,N)+ADN
DN(M,N)=ADN
42 ADN=0.0

**C**

AYN=0.0
DO 44 J=1,NCELL
44 AYN=DT(J,I)*Y(I)+AYN
YN(J)=AYN

**C**

DO 46 J=1,NCELL
XE(J)=0.0
R(J)=YN(J)
46 P(J)=R(J)
ENO=0.0

**C**

ITERATION BEGINS

**C**

72 DO 71 IT=1,ITER
XIT(IT)=FLOAT(IT)

**C**

SDNP=0.0
DO 47 M=1,NCELL
DO 48 N=1,NCELL
48 SDNP=DN(M,N)*P(N)+SDNP
DNP(M)=SDNP
47 SDNP=0.0

**C**

PR=0.0
DO 64 J=1,NCELL

**C**

NCC01700
NCC01710
NCC01720
NCC01730
NCC01740
NCC01750
NCC01760
NCC01770
NCC01780
NCC01790
NCC01800
NCC01810
NCC01820
NCC01830
NCC01840
NCC01850
NCC01860
NCC01870
NCC01880
NCC01890
NCC01900
NCC01910
NCC01920
NCC01930
NCC01940
NCC01950
NCC01960
NCC01970
NCC01980
NCC01990
NCC02000
NCC02010
NCC02020
NCC02030
NCC02040
NCC02050
NCC02060
NCC02070
NCC02080
NCC02090
NCC02100
NCC02110
NCC02120
NCC02130
NCC02140
NCC02150
NCC02160
NCC02170
NCC02180
NCC02190
NCC02200
NCC02210
NCC02220
NCC02230
NCC02240
NCC02250
NCC02260
64 PR=P(J)*R(J)+PR
C
60 DNPP=0.0
DO 60 J=1,NCELL
60 DNPP=P(J)*DNPP(J)+DNPP
IF(DNPP.LT.1.0E-7) DNPP=1.0E-7
GAMMA=PR/DNPP
GAMA(IT)=GAMMA
C
WRITE(6,610) IT,GAMMA,DNPP,ENO
610 FORMAT(3X,'IT=',I3,3X,'GAMMA=',F15.8,'DNPP=',F15.8,&,'ENO=',F15.8)
C
61 ENRO=0.0
DO 61 J=1,NCELL
ENRO=ABS(R(J))+ENRO
61 XE(J)=XE(J)+GAMMA*P(J)
C
62 DNX=0.0
DO 62 M=1,NCELL
DO 63 N=1,NCELL
63 DNX=DNX+DN(M,N)*XE(N)
R(M)=YN(M)-DNX
62 DNX=0.0
C
RDNP=0.0
DO 65 J=1,NCELL
RDNP=R(J)*DNP(J)+RDNP
BETA=RDNP/DNPP
C
66 P(J)=R(J)+BETA*P(J)
C
67 CONTINUE
EDS=0.0
DO 76 J=1,NCELL
76 EDS=EDS+(XA(J)-XE(J))**2
SED(IT)=SQRT(EDS/REAL(NCELL))
IF(IT.EQ.2) EN01=ENO
QUIT=ENO/ENO1
ENR=0.0
DO 68 J=1,NCELL
68 ENR=ENR+ABS(R(J))
ANR(IT)=ENRO
C
IF(QUIT.LT.0.02.AND.QUIT.NE.1.0.AND.QUIT.NE.0.0) GO TO 140
C
69 DO 74 J=1,NCELL
74 P(J)=R(J)
71 CONTINUE
140 ITE=IT
C
WRITE(1,1000)
1000 FORMAT(' INPUT DATA.')
C DO 1200 I=1,2
C DO 1200 J=1,200
C IF(I.EQ.2) GO TO 1100
C SED(J)=SED(J)*1000.0
C WRITE(1,1300) J, SED(J)
C GO TO 1200
C1100 WRITE(1,1300) J,ANR(J)
C1300 FORMAT(3X,I3,3X,F20.6)
C1200 CONTINUE
C WRITE(1,1400)
C1400 FORMAT(' END OF DATA. ')
DO 2000 I=1,100
SED(I)=SED(I)*1000.0
WRITE(1,3000) I, SED(I), ANR(I)
3000 FORMAT(3X,I3,2F8.3)
2000 CONTINUE
NPTSS=200
NPLOTS=1
C CALL MPLLOT(XIT,SED,NPTSS,NPLOTS,XL,YL)
C CALL MPLLOT(XIT,GAMA,NPTSS,NPLOTS,XL,YL)
C CALL MPLLOT(XIT,ANR,NPTSS,NPLOTS,XL,YL)
C C DO 440 IT=1,ITE
C 440 WRITE(6,450) IT, SED(IT), ANR(IT)
450 FORMAT(3X,'IT=',I3,3X,'ED=',F8.6,3X,'NR=',F12.6)
C CALCULATE EUCLIDEAN DISTANCE
C ED=0.0
DO 49 J=1,NCELL
49 ED=ED+(XA(J)-XE(J))**2
ED=SQR(T(ED/FLOAT(NCELL))
DO 35 I=1,NTRANS
35 WRITE(6,403) TRANS(I,1), TRANS(I,2)
DO 36 I=1,NREC
36 WRITE(6,403) REC(I,1), REC(I,2)
WRITE(6,402) WIDTH, DEPTH, NCELL, NPATH
C DO 70 I=1,NPATH
C DO 70 L=1,NCELL
C WRITE(6,500) I, LID(I,L)
70 CONTINUE
C DO 75 M=1,NCELL
C DO 75 N=1,NCELL
C 75 WRITE(6,550) M,N,DN(M,N)
C DO 80 I=1,NPATH
C 80 WRITE(6,600) I, Y(I)
C 88 WRITE(6,650) J, YN(J)
C 85 WRITE(6,650) J, YN(J)
C 86 WRITE(6,700) J, XE(J)
90 CONTINUE
WRITE(6,800) ED,NF, ITER
402 FORMAT(1H,'WIDTH=',F6.2,3X,'DEPTH=',F6.2,3X,'ED=',F8.6,3X,'NR=',F12.6)
403 FORMAT(3X,'X=',F7.3,3X,'Y=',F7.3)
500 FORMAT(3X,'D(',I2,',',I2,')=',F12.3)
550 FORMAT(3X,'DN(',I2,',',I2,')=',F12.3)
600 FORMAT(3X,'Y(',I3,')='F15.7)
650 FORMAT(3X,'YN(',I2,')='F15.7)
700 FORMAT(3X,'XE(',I3,')='F12.3)
800 FORMAT(3X,'EUCLIDEAN DISTANCE=',F8.6
&//,3X,'NOISE FACTOR =',I2,3X,'ITERATION=',I5)
STOP
END