CURRENT DEVELOPMENTS IN SIGNAL MODELING
OF THE PRECISION DISTANCE
MEASURING EQUIPMENT

A Thesis Presented to
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In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by
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<tr>
<td>ADS</td>
<td>automatic delay stabilization</td>
</tr>
<tr>
<td>AGC</td>
<td>automatic gain control</td>
</tr>
<tr>
<td>AFCS</td>
<td>aircraft flight control system</td>
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<tr>
<td>AIC</td>
<td>Akaike's Information theoretic Criterion</td>
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<tr>
<td>AR</td>
<td>autoregressive</td>
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<td>ARMA</td>
<td>autoregressive moving average</td>
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<td>AWOP</td>
<td>All Weather Operations Panel</td>
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<td>CMN</td>
<td>control motion noise</td>
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<td>DAC</td>
<td>delay-attenuate-compare</td>
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<td>DME</td>
<td>Distance Measuring Equipment</td>
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<td>DME/N</td>
<td>narrow spectrum DME</td>
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<td>DME/P</td>
<td>Precision DME</td>
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<tr>
<td>ERP</td>
<td>effective radiated power</td>
</tr>
<tr>
<td>FA</td>
<td>final approach</td>
</tr>
<tr>
<td>FAA</td>
<td>Federal Aviation Administration</td>
</tr>
<tr>
<td>FPE</td>
<td>Final Prediction Error</td>
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<tr>
<td>IA</td>
<td>initial approach</td>
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<tr>
<td>ICAO</td>
<td>International Civil Aviation Organization</td>
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<td>IF</td>
<td>intermediate frequency</td>
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<tr>
<td>ILS</td>
<td>Instrument Landing System</td>
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<tr>
<td>log</td>
<td>logarithm</td>
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<td>MA</td>
<td>moving average</td>
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<td>M/D</td>
<td>multipath-to-direct</td>
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LIST OF ABBREVIATIONS (Continued)

ML  maximum likelihood
MLS  Microwave Landing System
NAS  National Airspace System
ns   nanosecond
PAF  peak-amplitude-find
PFE  path following error
RF   radio frequency
RSS  root-sum-square
SARPs Standards and Recommended Practices
SNR  signal-to-noise ratio
TOA  time-of-arrival
ACKNOWLEDGEMENTS

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Final thanks is given to the heavenly Father. Without Him, nothing is possible.
CHAPTER I
INTRODUCTION

The Microwave Landing System (MLS) is an all-weather precision approach and landing guidance system (Figure 1.1). Its adoption by the International Civil Aviation Organization (ICAO) in 1978 made it the new world standard (ICAO, 1985). The transition from the Instrument Landing System (ILS) to the MLS is scheduled for completion by the year 2000 and several nations are already in the process of completing their transition plans.

With the imminent widespread implementation of the MLS comes the need for various support activities. Among these are the development of siting criteria, collision risk models, approach procedures, and signal models. Many of these activities are either well under way or are approaching completion.

The task of signal modeling stems from the need of airframe manufacturers to develop and certify aircraft flight control systems (AFCS's). Before aircraft can be used in the National Airspace System (NAS), they must be certified by the Federal Aviation Administration (FAA). Aircraft manufacturers use models of the navigation signal to design, develop and certify AFCS's. The signal modeling work performed thus far has been concerned only with the angle functions of the MLS (Vickers, 1988). A complete
Figure 1.1
The Microwave Landing System
(from Redlien, 1981)
signal model for the MLS must include its range function which is provided by the Precision Distance Measuring Equipment (DME/P) (Figure 1.2).

The purpose of this thesis is to describe the work performed by the author on the development of the DME/P signal model. This will involve, (1) an explanation of the need for signal modeling and the requirements for a signal model, (2) a description of previous signal modeling efforts and their deficiencies, (3) an overview of two types of distance measuring equipment (DME/N and DME/P), (4) the characterization of the deterministic error sources in the DME/P and a description of the deterministic error generator developed by the author, (5) an introduction to System Identification Theory, (6) the synthesis of the non-deterministic (random) error generator using System Identification Theory, and (7) a functional description, developed by the author, of the signal model.

Since the characterization of the deterministic errors is a fairly straightforward process, the signal modeling challenge lies in the formulation of a random error generation module. System Identification Theory is shown to be highly useful both in the determination of a compact module as well as in the analysis of collected error data. The compact random error module leads to a portable signal model which the airframe manufacturer may use in the design and certification of aircraft flight control systems.
Figure 1.2
DME/P Transponder
(from Bencivenga, 1989)
CHAPTER II
SIGNAL MODELING AND ITS ROLE IN THE DEVELOPMENT OF AIRCRAFT FLIGHT CONTROL SYSTEMS

In the process of developing and optimizing an AFCS, the designer must account for a multitude of factors. Among these are the aircraft dynamics, envisioned flight profiles, wind, turbulence, and navigation system error (Kelly, 1988a). The designer must quantify these effects and use them to determine the required AFCS. It is at this point where the need for a simulation becomes apparent. If the designer can be provided with a simulation which accurately represents the above stated factors, this simulation may then be used in the AFCS development. One important factor in the simulation is the quantification and reproduction of navigation system error.

The importance of environment simulation is seen again at the certification stage. Once an AFCS has been designed, optimized, and implemented, regulatory agencies require proof that the system will adhere to established specifications. This proof is given in terms of documentation which demonstrates the AFCS performance. Since the cost of actual flight testing is prohibitive, the majority of the documentation is produced through simulation. Again, generation of realistic navigation
system error is required as part of the simulation (Kelly, 1988a).

This chapter discusses the requirements for a signal model along with previous signal modeling methodologies. Signal noise modeling schemes have been devised both for the ILS as well as the MLS. The inability of these schemes to produce accurate MLS error signals is described.

2.1 Signal Model Requirements

Navigation system error is defined as indicated aircraft position minus actual aircraft position. It is this quantity that must be accurately reproduced by the signal model.

Accurate reproduction focuses primarily on the frequency domain characteristics of the signal (Kelly, 1977, Kelly, and Cusick, 1986). This frequency domain representation (error spectrum) must be considered together with the AFCS frequency response. The result is a model which accurately reproduces the navigation system error in the frequency range to which the AFCS is responsive (Vickers, 1988). Figure 2.1 shows the power spectral response of a typical autocoupled transport aircraft. For the purpose of analysis, the response is typically divided into two regions. The first region consists of the lower
Figure 2.1
Power Spectral Response of Typical Transport Aircraft
(from Kelly, 1988a)
part of the band, 0 to 0.5 radians/second. Errors in this region will cause the aircraft to deviate from its desired path and therefore are called Path Following Error (PFE). The second region consists of the upper part of the band, 0.3 to 10 radians/second. Errors in this region do not cause a path deviation but do cause unnecessary control surface activity. Hence these errors are called Control Motion Noise (CMN). CMN affects pilot and passenger acceptance (Kelly, 1977). It is important to note that any frequency component of error above 10 radians/second has a negligible effect on the AFCS.

A model which accurately reproduces the error spectrum, however, is not complete. In an operational environment, only a portion of the error spectrum will be observed at one instant in time. The portion observed will depend upon the position, orientation, and velocity of the aircraft with respect to the ground environment (Vickers, 1988). This is especially true in the presence of specular multipath disturbance of a landing-system guidance signal. Depending upon the orientation of the aircraft with respect to the reflector, the observed error will be either oscillatory or bias-like.
2.2 ILS Signal Modeling

Signal modeling performed for the ILS has consisted of several approaches. One attempt at fault simulation was used in Glide Slope modeling (Hofmann, et al, 1975). An ideal signal (no error) is input to the AFCS and then at a specified time, the input signal is corrupted by a step, sinusoid, or some specially designed waveform. The purpose was to determine the response of an AFCS to a sudden system fault.

Augmentation of the fault simulation was accomplished by simply cataloging actual flight error traces (Hofmann, et al, 1975). A variety of traces were collected from glide slope sites around the United States. These traces could be input to the AFCS with the advantage being an accurate representation of all error components.

A standardized approach to signal modeling for both the glide slope and localizer consists of a white noise source operating on a low pass filter (reference: Joint Aeronautical Regulations). The noise source and filter are specified to produce an output with a certain two sigma level. A bias term may then be added to the noise term. The advantage of this method is the simplicity. A very compact model is obtained.
2.3 Early MLS Signal Model Development

Early simulations of MLS errors were based on the federally specified maximum error limits (Thompson, et al, 1981). The PFE and CMN error budgets were used to define a complex error spectrum. An inverse Fast Fourier Transform was applied to the error spectrum with the result being a sample time series. It is important to note that the error components were taken from zero mean Gaussian distributions with standard deviations set according to the PFE and CMN error budgets (Walen, et al, 1981).

Later modeling was performed much in the same way as the standardized ILS modeling. Error data were collected from a Bendix MLS facility located at Wallops Flight Center. An error model was derived from this data and consisted of a white noise source operating on a low pass filter with a given correlation time. Recommendations were made to perform further simulation work after more flight data had been collected (Ho, 1986).

2.4 Limitations Of Previous Signal Modeling Methodologies

The signal modeling methodologies discussed thus far have some significant attractions. In the case of the catalog of flight error traces, confidence in accurate error
representation is assured. The models of white noise operating on low pass filters have the chief advantages of simplicity and compactness.

Several difficulties arise, however, in applying these models to current MLS facilities. Cataloging flight error traces is not practical due to the kinds of operations possible with the MLS. The ILS only allows for centerline approaches and therefore a catalog of several error traces can be manageable. Operations with the MLS, on the other hand, are not limited to centerline approaches. Radial, curved, and segmented approach procedures can be supported giving rise to virtually an infinite amount of possible operations. A catalog of error traces for all operations is therefore not feasible.

As will be shown later, the concept of white noise operating on a filter is a valid model type. It must be recognized, however, that the validity of the exact model depends upon how accurately it reproduces actual error. Specifying a two sigma error limit does not ensure an accurate representation throughout the spectrum. Finally, the model derived from actual MLS data was based upon a preproduction system which did not meet ICAO Standards and Recommended Practices (SARPs). The final signal model must be based on data taken from SARPs systems.
CHAPTER III
DME PRINCIPLE OF OPERATION

Distance measuring equipment (DME) is an outgrowth of the radar beacon concept (Kelly, 1986). The history of the beacon and DME concepts is beyond the scope of this thesis but may be found in the literature (Kelly, 1986 and Dodington, 1980).

The range function of the MLS is performed by the precision distance measuring equipment (DME/P). DME/P is an improved version of the DME/N (narrow spectrum). DME/N has been used extensively throughout the world as an enroute and terminal area navigation aid. For the MLS, a ranging system is required as a replacement for the marker beacons used in ILS, and to provide full three-dimensional guidance. It must provide the accuracy and data rates necessary for the support of precision approaches. This is accomplished through the enhancement of the DME/N into the present DME/P.

3.1 The DME/N System

The DME/N system is an internationally defined measurement scheme which determines the line-of-sight distance between an aircraft interrogator and a ground transponder (Kelly, 1986). Development of the DME/N has
continued over the last forty years with the most recent revisions in technical standards having been adopted by ICAO in 1985.

3.1.1 DME/N System Concept

The basic principle governing the operation of the DME/N is a timing measurement. The measurement involves determination of elapsed time between the signal transmission from the airborne interrogator and the reply signal received from a ground transponder. This time measurement may be combined with the knowledge of the propagation velocity of the signal to determine the line-of-sight distance (slant range) between the aircraft and ground transponder.

More specifically this is accomplished as follows. The airborne interrogator transmits a signal (pulse pair) and starts its counter. The ground transponder receives the pulse pair, holds for a known fixed delay, then replies by transmitting a pulse pair. The airborne interrogator receives the transponder reply and stops its counter. The known fixed delay is subtracted from the measured time and the result is divided by two times the propagation velocity to determine the slant range:
\[ R = \frac{t - r}{2c} \]

where: 
- \( R \) is the one-way range
- \( t \) is the total round-trip time
- \( r \) is the fixed delay
- \( c \) is the propagation velocity of the pulse pairs

Actual implementation of this scheme is complicated by several factors. First is the need for identification of the ground transponder. In the context of the NAS, many navigation systems are in operation. An aircraft attempting to use a specific facility must know that it is tuned to the correct one. DME identification is accomplished through the periodic transmission of pulse pairs by the ground transponder which are decoded in the airborne equipment as Morse code dots and dashes (ICAO, 1985).

A second complication stems from the need for one ground transponder to service many airborne interrogators. The problem which arises is the need to identify the replies to one's own interrogations amidst the replies to other aircraft. This is done by a process known as Search and Track and involves finding the time slot in which replies correlate well with one's own interrogations. Old receiver architectures performed this correlation process mechanically resulting in typical search times of twenty to
thirty seconds. Modern receivers incorporate digital computers and have reduced the search time to approximately two seconds. Once the proper time slot has been identified, the interrogator may then track the transponder replies for the duration of usage of the DME facility (Kelly, 1986).

A further complication is encountered when considering system bandwidth. In order to reduce the cost of airborne equipment, interrogators are allowed to transmit basically square pulses. Since these pulses have a fast rise time, wide-band signal processing is required in the ground transponder. The result is that off-frequency signals are processed. This problem of frequency rejection is addressed through the use of a Ferris discriminator. The Ferris discriminator employs two filters, one narrow-band and one wide-band. The narrow-band filter determines if the incoming signal is on frequency. The wide-band filter preserves the shape of the incoming pulse which is required for timing purposes. The output from the wide-band filter then is used only when there is a valid output from the narrow-band filter (Kelly, 1986).

Another spectrum issue is concerned with the channel plan. The spectrum available for DME is not sufficient for each facility to be assigned a unique frequency. A channel plan must then be established which allows various facilities to operate at the same frequency. Confusion
between two transponders operating at the same frequency is eliminated by two methods. One is to locate the two facilities far enough apart such that the airborne receiver may distinguish "desired" replies from "undesired" ones on the basis of power level. The other method involves the specific spacing of the pulse pairs known as pulse coding. Multiple channels may operate at the same frequency by dictating the separation of the pulse pairs. In the DME/N channel plan, the X codes have twelve microsecond separation between the two pulses in a pulse pair of both the interrogation and reply. The Y codes have thirty-six microsecond separation for the interrogation and thirty microsecond separation for the reply. Using these two codes, 126 interrogation/reply frequency pairs make up 252 operating channels (ICAO, 1985).

Given the fact that the ground transponder must identify valid interrogations and then reply after a fixed delay, a certain "dead time" is produced during which other valid interrogations are ignored. This is seen in the interrogator as a missed reply. The ratio of replies to interrogations is known as reply efficiency. The ICAO SARPS for the DME/N require reply efficiency to be at least 70% when 100 aircraft are using a particular ground transponder. This requirement is met through the limitation of transponder dead time and interrogation rate. The SARPS
specify 60 microseconds nominal dead time and 30 interrogations per second per aircraft (Kelly, 1986).

Another consideration lies in the dynamic range of the receivers in the interrogator and transponder. The dynamic range requirement for the interrogator receiver is easily satisfied. Since the range of the aircraft to the transponder does not vary rapidly, the received signal level is therefore fairly stable. This permits the employment of automatic gain control (AGC) circuitry to amplify the received signal to proper levels.

Unfortunately, received signal level at the transponder varies from interrogation to interrogation and as a result, normal AGC cannot be used. Transponder hardware requirements are therefore more complex with the need for logarithmic type amplifiers (Kelly, 1986).

3.1.2 DME/N Measurement Methodology

As mentioned earlier, the basic principle governing the operation of the DME/N is the measurement of elapsed round-trip time between the airborne interrogation and the ground transponder reply. In order to do this, a calibration point is defined on a pulse. This calibration point is the half amplitude point on the rising edge of the pulse. The ranging time measurement is made at the calibration point on
the first pulse in the pair. A time measurement is also made at the second pulse in the pair for code determination. Since the reference is the calibration point, the measurement is not altered by differences in pulse amplitude as long as both pulses have the same amplitude and pulse shape.

The specification of the calibration point requires consideration of the pulse shape. Pulse shape is governed by coverage, spectrum, and accuracy requirements. The transmitter power of the ground transponder must be large enough to ensure adequate signal levels throughout the service volume. Simultaneously, however, the transponder must minimize its radiation in adjacent channels. The SARPS limit effective radiated power (ERP) to 200 milliwatts in the first adjacent channel and 20 milliwatts in the second adjacent channel. In order to meet these conflicting requirements, the pulse shape must be strictly defined. The pulse shape used in the DME/N is essentially Gaussian (Figure 3.1) and meets all requirements.

With the standards set for pulse shape and timing measurement, interoperability among equipments from various manufacturers is ensured. A summary of these measurement principles is given in Figure 3.2.
Figure 3.1
DME/N Pulse Shape
(from ICAO, 1985)
**Range (nmi) = \frac{\text{Total Round Trip Delay} - \text{Transponder Delay}}{12.36 \mu s/\text{nmi}}**

- The \(\frac{1}{2}\) amplitude point (i.e., 6dB below the pulse peak) on the pulse leading edge is the reference point for all time measurements: range, transponder delay, and code. Hence, range measurements are independent of pulse shape.

- Range measurements are based on first pulse timing.

- Transmitted RF interrogation pulse pair is processed by interrogator receiver to begin range timing. Bias errors due to delays in the processing path are minimized since the received reply pulse pair is processed by the same receiver path.

- The transponder begins transmission of the reply pair such that the desired delay results between the \(\frac{1}{2}\) amplitude point on the leading edge of the first interrogation pulse received and the same point on the first pulse of the reply pair.

**Figure 3.2**
DME/N Measurement Principles
(from Kelly, 1986)
3.1.3 DME/N Error Sources and System Accuracy

Error sources in the DME/N system are segmented into three categories:

- airborne instrumentation
- ground instrumentation
- site-dependent

The instrumentation errors are mainly a function of imperfect hardware. Timing measurements in a receiver include the following error sources:

- clock drift
- quantization noise
- receiver noise

Site-dependent errors are composed of multipath and garble. Multipath error is the corruption of the direct signal by one or more reflected signals. Garble error is the corruption of the desired signal by signals from other DME equipments or other radio frequency (RF) sources. Since each of these components is assumed to be a zero mean random variable, the combined effect of all components may be determined by the root-sum-square (RSS) method:

\[
\text{Total error} = \left[ (\text{arbn})^2 + (\text{grnd})^2 + (\text{site})^2 \right]^{0.5}
\]
where: arbn = airborne instrumentation error
grnd = ground instrumentation error
site = site-dependent errors

In order to reduce airborne equipment cost, the majority of the system error budget is allocated to airborne instrumentation errors as shown in Table 3.1 (Kelly, 1986). The next largest component is ground instrumentation error and the remainder of the budget covers site dependent errors such as multipath and garble.

Adherence of ground and airborne equipment to this error budget is accomplished through proper design. Ground transponder errors are checked by a smart interrogator, known as a monitor, mounted near the transponder itself. The monitor checks such things as average reply delay, radiated power, pulse code, and frequency stability. If the monitor detects an out-of-tolerance condition, the ground facility is shut down or redundant equipment, if it is available, is turned on in its place (Kelly, 1986).

The remainder of the error budget is sufficient to cover the site-dependent errors. Multipath errors are inherently small (relative to the system error budget) due to the primary use of DME/N for enroute applications. First, aircraft are generally above the multipath region. Secondly, for multipath which is encountered, the delay is quite long and as a result, interference is not seen until
Error Source | 95% Error Value (nautical miles)
--- | ---
Instrumentation: | 
Ground | 0.08
Airborne | 0.17
Remainder | 0.07
System Error (total) | 0.20

Table 3.1
DME/N System Error Budget
(from Kelly, 1986)
after the ranging time measurement has been made. Finally, the strength of the multipath signal is typically far below that of the direct. Garble errors are also small due to the use of frequency rejection techniques and the proper separation of co-channel ground facilities (Kelly, 1986).

As noted in Table 3.1, the RSS total of all error components is not to exceed 0.20 nautical miles (nm) on a 95% probability basis. Ground equipment error limits are tighter if the DME/N facility is being used in combination with an ILS. In this case, the DME/N is being used to determine distance from runway threshold (Kelly, 1986).

Even with the strict error limits, however, DME/N lacks the accuracy necessary for advanced approach procedures. These approaches include three dimensional curved approaches, flare initiation, and height computation, and require range accuracy on the order of 100 feet (ICAO). This is an order of magnitude better than DME/N and therefore must be provided by a different system.

3.2 The DME/P System

As with the DME/N, the DME/P is an internationally defined system which determines the line-of-sight distance between an aircraft and a ground transmitter. However, higher data rates and better accuracy are required since the
DME/P supports complex precision approach and landing procedures.

3.2.1 DME/P System Requirements

During the development process of the DME/P, two issues were set forth as primary concerns regarding system performance. The first was the need for high accuracy data (100 feet, 95% probability) provided at a rate suitable for autocoupled approaches. The second was the decision that the new system should be fully compatible and interoperable with the existing DME/N. This second requirement will be discussed in the next section.

The requirement for high accuracy data is a natural consequence of the complex approach procedures to be supported. In order to perform procedures such as curved and segmented approaches, the airborne equipment must be able to calculate the aircraft's three dimensional position in space. This is made possible by combining the DME/P range function with the MLS Azimuth and Elevation angle functions. Other procedures requiring high accuracy range data are listed in Table 3.2 (ICAO, 1985).
Table 3.2
Terminal Area Procedures Requiring High Accuracy Ranging
(from ICAO, 1985)
3.2.2 Interoperability of DME/P and DME/N

As will be shown in the next section, in order to achieve the required accuracy, a new pulse shape was introduced for DME/P operation. The point at which timing measurements are made on this pulse is significantly below the half amplitude calibration point used in the DME/N. Since a significant time difference exists between the pulse calibration points of the two systems, an interoperability error exists. Specifically, a DME/N interrogator obtaining service from a DME/P transponder would see a bias error due to the calibration point difference. Several methods of combating this problem were proposed to ICAO with the two-pulse/two-mode system finally being accepted (Kelly, 1984).

The two-pulse/two-mode system divides the DME/P service into two modes of operation corresponding to two coverage regions. An initial approach (IA) region was defined from 7 to 22 nautical miles from the transponder. A final approach (FA) region was defined from the transponder out to 7 nautical miles. In the IA region, DME/P interrogators operating on an X channel send pulse pairs with the standard DME/N 12 microseconds pulse separation. The transponder replies with pulse pairs also with 12 microsecond spacing. The reply pulses are Gaussian in shape and are calibrated at the half amplitude point. Thus, a conventional DME/N
interrogator may use the DME/P transponder without a bias error. In the FA region, however, the DME/P interrogator changes to an 18 microsecond pulse spacing which is interpreted in the transponder as an FA mode request. The transponder replies with the same pulse spacing, twelve microseconds, as in the IA mode but with a different pulse shape and calibration point (ICAO, 1985). This process is illustrated in Figure 3.3. As will be discussed next, the use of the IA and FA modes helped to solve the spectrum management problem associated with the new pulse shape.

3.2.3 DME/P Pulse Shapes

As noted previously, a new pulse shape was required to improve ranging accuracy in the DME/P. Since range information is required through touchdown and rollout, multipath becomes a key issue. However, since the multipath signal always arrives after the direct signal, its effect can be minimized by making the timing measurement at a very early point on the pulse leading edge. In this way the measurement is made before the medium to long delay multipath can corrupt the signal.

Difficulties arise, however, in trying to realize this benefit in a practical equipment configuration. Fast pulse rise times are required to make time measurements ahead of
Figure 3.3
IA and FA Modes; X-channel codes
(from Kelly, 1986)
multipath. Additionally, pulse time-of-arrival (TOA) estimation techniques require the pulse to be linear during the rise time. A major consequence of this linear leading edge and fast rise time is wide bandwidth. This is a problem in spectrum management since radiation of fast rise time pulses causes spurious emissions in adjacent frequencies/channels. Wide bandwidth is also a problem at the receiver. TOA estimation requires a linear leading edge; the receiver front end must have a wide bandwidth to preserve the shape of the pulse. This wide bandwidth allows nearby RF emissions to interfere with the desired signal resulting in the possibility of error.

The bandwidth problem is addressed in the same manner as it was in the case of the DME/N transponder. In that case a Ferris discriminator was employed to maintain the shape of the interrogation pulse yet reject off frequency transmissions. For the DME/P, pulse shape preservation is required both in the transponder and the interrogator. As a result, Ferris discriminators are used in both.

The spectrum management issue was solved in two parts. Introduction of the IA and FA modes of operation afforded a lower radiated power for the precision pulse. The advantage here lies in the fact that the FA mode region occupies only a fraction of the total DME/P service volume. An adequate signal-to-noise ratio (SNR) may be obtained for a lower ERP
than what would be required to cover the entire service volume. Pulse shape considerations constituted the second part of the spectrum management solution. Since the time measurement was to be made very early on the leading edge, it was determined that linearity was required only between the 5% and 30% points (percentages referenced to pulse peak). With this partial rise time linearity defined, the rest of the pulse need only be designed to minimize ERP in adjacent channels (Kelly, 1986).

The pulse shape chosen is referred to as the COS/COS2 (Figure 3.4). The pulse consists of a sinusoid during the rise time and a squared sinusoid during the fall time. This waveform has been shown to meet both the linearity and the ERP requirements (Kelly, 1984).

In order to standardize the timing measurements made in the FA mode, a pulse reference point must be established. This reference point is known as the virtual origin. The virtual origin is defined to be the point where a line passing through the 5% and 30% pulse amplitude points crosses a time line corresponding to 0% pulse amplitude. The term "virtual" is used since actual pulses will vary slightly from the desired waveform and in the presence of noise the determination of exact start of the pulse is not possible. Pulse variations during the linear partial rise time are strictly controlled (Figure 3.5) (ICAO, 1985).
DME/P Pulse

Pulse shape defined by cos/cos2

Figure 3.4
FA Mode Pulse Shape
Figure 3.5
Allowable FA Mode Pulse Variations
(from Kelly, 1986)
3.2.4 DME/P Pulse Time-of-Arrival Estimation

Pulse TOA estimation in the IA mode is identical to the method used in the DME/N. The 50\% point on the pulse is determined using a peak-amplitude-find (PAF) circuit. The calibration point is determined by finding the peak amplitude and then comparing a delayed version of the pulse to it.

TOA estimation in the FA mode must be performed relative to the virtual origin of the pulse. This is accomplished by what is known as a delay-attenuate-compare (DAC) circuit (Figure 3.6). As the name implies, the incoming pulse is passed simultaneously through an attenuator and a delay line. Outputs from these two are compared and the decode time is declared when the delayed pulse is equal in amplitude to the attenuated one. As a result of the linearity of the partial rise time and the DAC characteristics, the decode time is always a fixed offset from the virtual origin. This is true regardless of pulse rise time or peak amplitude. Since specular ground reflection causes signal enhancement or attenuation (Kelly, 1980), it is important that the TOA estimation technique be independent of pulse peak amplitude (Kelly, 1986).
**Table 3.6**

<table>
<thead>
<tr>
<th>DECAY TIME ($T_D$) FOR ARBITRARY PULSE</th>
<th>$T_D \cdot p(t-T_D) \cdot A(t)$</th>
</tr>
</thead>
</table>

**DAC OUTPUT (LOGIC SIGNAL)**

- $0. T(t-T) < A(t)$
- $1. T(t-T) > A(t)$

**DEFINITION OF LINEAR LEADING EDGE PULSE**

$p(t) = \begin{cases} 1, & t < T_R, \\ 0, & t \geq T_R \end{cases}$

**DECAY TIME FOR LINEAR LEADING EDGE PULSE**

$T_D = \frac{T}{1-A}$

**RELATIVE THRESHOLD OF ATTENUATED LINEAR LEADING EDGE PULSE**

$T = \frac{T}{(1-A) T_R}$

**RELATIVE THRESHOLD OF DELAYED LINEAR LEADING EDGE PULSE**

$AT = \frac{T}{(1-A) T_R}$

**Definitions:**

- $p(t)$ = RECEIVED PULSE
- $T$ = DELAY
- $A$ = ATTENUATION
- $T_R$ = 100% RISE TIME OF LINEAR LEADING EDGE PULSE

---

**Figure 3.6**

DAC Processing

(from Kelly, 1986)
3.2.5 DME/P Error Sources and System Accuracy

Three primary error sources exist in the DME/P system. These are: instrumentation errors, garble, and multipath. Instrumentation errors can be large. However, major sources of instrumentation error are readily identified and can be constrained to a specified level (Kelly, 1986). The two other sources, unfortunately, are not so easily constrained and must be examined closely.

3.2.5.1 Instrumentation Errors

In the transponder, the critical parameter which must be maintained is reply delay. This parameter is held within tolerance by using automatic delay stabilization (ADS). ADS is performed at the transponder and involves measuring reply delay in both the IA and FA modes. Any slowly varying errors which occur are then eliminated by adjusting the transponder delay (Kelly, 1986).

It must be noted, however, that ADS does not eliminate the effects of rapidly varying errors. Such errors are due to:

- receiver noise
- clock quantization
-logarithmic (log) amplifiers

-intermediate frequency (IF) filters.

Clock quantization effects are minimized by employing a high rate clock. Log amplifiers usually have cyclical deviations around the desired response curve. These errors can be reduced by proper design or storage of calibration information. Narrow-band IF filter delay is dependent upon pulse rise time and can be significant. Proper choice of filter type can reduce this effect. Note that this is only a factor in the IA mode since wide bandwidth signal processing is employed in the FA mode (Kelly, 1986).

As mentioned above, receiver noise is a rapidly varying error component not compensated for by the ADS. The main concern regarding receiver noise is the determination of the signal-to-noise ratio (SNR) required for adequate noise error reduction. Extensive computer simulations have shown that an SNR of approximately 30 dB in combination with proper signal processing will reduce receiver noise errors to approximately 2 meters (Kelly, 1984).

Instrumentation errors in the interrogator are virtually the same as in the transponder. Likewise, control of these errors is accomplished in the same fashion. One exception to this is the ADS. Slowly varying circuit parameters are compensated for by processing the interrogation pulses with the same circuitry as that used
for the reply pulses. In this way, the effect due to circuit parameter variations cancels itself out (Kelly, 1986).

3.2.5.2 Garble

As defined earlier, garble error is the corruption of the desired signal by signals from other DME equipments or other RF sources. Although Ferris discriminators are employed to reject off-frequency pulses, garble pulses coincident with desired pulses will produce error. This is due to the wide bandwidth processing used to preserve the shape of the FA mode pulse leading edge. These errors can be on the order of tens of meters (Kelly, 1986). Proper filtering, however, will significantly reduce this. Computer simulations have estimated garble errors to be on the order of 2 to 3 meters (Kelly, 1984).

3.2.5.3 Multipath

Due to the nature of approach and landing profiles, multipath is the most significant threat to DME/P accuracy. Since aircraft on these profiles will inevitably fly through multipath regions, some method must be determined to counteract its effects.
Multipath is typically categorized into two types: diffuse and specular. Diffuse multipath arises from reflections from groups of small objects. The term "small" here refers to the electrical size of the object (i.e. less than one-tenth of a Fresnel zone). Reflections from individual objects will be of low intensity but the combination of reflections from all the objects is significant. Diffuse multipath is considered a random effect and indeed does appear as such in a flight error trace. Environment studies have indicated that diffuse multipath will be of low intensity (Evans, 1982). Actual flight test results have shown diffuse multipath error to be less than 3 meters (Kelly, 1986).

Specular multipath arises from reflections from an electrically large object. The strength of individual reflections of this nature can be close to that of the direct signal. The parameters which characterize DME/P multipath are multipath-to-direct (M/D) signal strength ratio, multipath delay, multipath phase (relative to the direct), and multipath phase rate of change (Kelly, 1980 and Kelly, 1976).

DAC processing virtually eliminates multipath with path delays greater than 300 nanoseconds (ns). RF environmental studies indicate that most runways will not have high level multipath with less than 300 ns delay (Evans, 1982). For
path delays less than 300 ns and realistic M/D ratios, it has also been shown that multipath errors will be less than 10 meters (Kelly, 1980).

3.2.5.4 System Accuracy

Having identified and quantified all major error sources, it is now possible to synthesize an error budget. Table 3.3 shows the error budget recommended by ICAO (1985). The figures given are for 95% probability. Since individual components are assumed to be zero mean, independent random variables, they may be combined on a RSS basis. Note: The RSS total of the individual components will not always equal the system specification. This number is a maximum which the combined components are not allowed to exceed on a 95% probability basis. As can be seen from the table, PFE is not allowed to exceed 30 meters (100 feet) and CMN may not exceed 18 meters.

3.2.6 DME/P Error Distributions

As noted in the previous section, the components in the error budget are to be satisfied on a 95% probability basis. Total system error, then, is also to be satisfied on the same basis. The specified PFE value of 30 meters reveals
<table>
<thead>
<tr>
<th>Error Source</th>
<th>FA Mode</th>
<th>IA Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PFE</td>
<td>CMN</td>
</tr>
<tr>
<td>Instrumentation:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transponder</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>Interrogator</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Site Related:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Downlink Specular Multipath</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>Uplink Specular Multipath</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>Diffuse Multipath</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>DME Garble</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Non-DME Garble</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total System:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Combined effects of all error sources must not exceed:</td>
<td>30</td>
<td>18</td>
</tr>
</tbody>
</table>

(95% probability values in meters)

Table 3.3
DME/P System Error Budget
(from ICAO, 1985)
little about the magnitude of low probability error. Obstacle clearance groups throughout the world, however, are interested in knowing this information so that they may establish approach procedures. Approach procedures are based upon the assumption that collision risk probability is low. Quantitatively, the probability of having a collision with any obstacle along the approach path must be less than $10^{-7}$ for each approach.

In the process of defining an approach, then, it is necessary to know the distribution of navigation system error. Once a distribution of total navigation system error is obtained, error bounds may be obtained by examining the tails of the distribution. The bound of error must be determined such that the probability of having an error greater than the error bound is less than $10^{-7}$.

The problem of determining the error distribution involves the assignment of probability distributions to each of the error components. Recent work performed in this area involved a combination of uniform and Gaussian distributions (Kelly, 1989a). The results presented here are a confirmation of that work.

As discussed previously, the major DME/P error components are instrumentation error, garble, and multipath. Their distributions are given in Tables 3.4 and 3.5. The airborne and ground instrumentation errors are assigned zero
<table>
<thead>
<tr>
<th>Error Source</th>
<th>System Specification</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airborne (PFE)</td>
<td>30 m (maximum)</td>
<td>Uniform</td>
</tr>
<tr>
<td>Ground (PFE)</td>
<td>15 m (maximum)</td>
<td>Uniform</td>
</tr>
<tr>
<td>Specular Multipath:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>uplink</td>
<td>75 m (maximum)</td>
<td>Uniform</td>
</tr>
<tr>
<td>downlink</td>
<td>75 m (maximum)</td>
<td>Uniform</td>
</tr>
<tr>
<td>Diffuse Multipath</td>
<td>3 m (2σ)</td>
<td>Gaussian</td>
</tr>
<tr>
<td>Garble</td>
<td>6 m (maximum)</td>
<td>Uniform</td>
</tr>
<tr>
<td>Receiver noise</td>
<td>2 m (2σ)</td>
<td>Gaussian</td>
</tr>
</tbody>
</table>

Table 3.4
DME/P Error Sources - IA Mode
(from Kelly, 1989a)
<table>
<thead>
<tr>
<th>Error Source</th>
<th>System Specification</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airborne (PFE)</td>
<td>15 m (maximum)</td>
<td>Uniform</td>
</tr>
<tr>
<td>Ground (PFE)</td>
<td>10 m (maximum)</td>
<td>Uniform</td>
</tr>
<tr>
<td>Specular Multipath: uplink</td>
<td>18.3 m (maximum)</td>
<td>Uniform</td>
</tr>
<tr>
<td>Specular Multipath: downlink</td>
<td>18.3 m (maximum)</td>
<td>Uniform</td>
</tr>
<tr>
<td>Diffuse Multipath</td>
<td>3 m (2σ)</td>
<td>Gaussian</td>
</tr>
<tr>
<td>Garble</td>
<td>6 m (maximum)</td>
<td>Uniform</td>
</tr>
<tr>
<td>Receiver noise</td>
<td>2 m (2σ)</td>
<td>Gaussian</td>
</tr>
</tbody>
</table>

Table 3.5
DME/P Error Sources - FA Mode
(from Kelly, 1989a)
mean, uniform distributions. Although this assumption needs verification through the collection of additional flight data, the reasoning stems from the way in which the errors appear in a flight error trace. If a bias error consistently occurred in the equipment, the manufacturer would eliminate it. As a result, any instrumentation error having a PFE component will be slowly varying yet not compensated by the ADS. The variations are slow enough that they will remain constant for any given approach. The magnitude of the error for each approach is random. The maximum values of these errors (i.e. the limits of the uniform distribution), are specified in the SARPs.

Since receiver noise error is a Gaussian process, it is not included as a part of the instrumentation error but is assigned its own distribution. Diffuse multipath is also a noise-like process and is assigned a Gaussian distribution.

Specular multipath must be divided into downlink and uplink distributions since these are independent. This independence stems from the fact that the uplink and downlink occur at distinctly different frequencies. Note that the ICAO error budget (Table 3.3) assigns 37 meters in the IA mode and 10 meters in the FA mode to each of these components on a 95% probability basis. Reasonable worst case assumptions are for an M/D ratio of -3dB in the IA mode and 0db in the FA mode. This results in maximum errors of
75 meters in the IA mode and 18.3 meters in the FA mode.

Due to the way in which it is seen by the user, garble is assumed to be uniformly distributed (Kelly, 1989a). Maximum values for garble error are set at ± 6 meters for both the IA and FA modes. Although computer simulations indicate that 2-3 meters is the largest garble error to be expected, the issue in dealing with collision risk modeling is low probability events. The figure of 6 meters was agreed upon by ICAO as being the absolute maximum value for garble error (ICAO, 1985).

Since all error components are viewed as zero mean, statistically independent random variables, their combination is performed through convolution. Formally, the probability density function of the sum of two independent random variables is given by (Helstrom, 1984):

\[ f_z(z) = \int_{-\infty}^{\infty} f_x(z-y) f_y(y) \, dy \]

where: \( z = x + y \)

\( x \) and \( y \) are independent random variables

This convolution may be generalized to find the probability density function of the sum of any number of independent random variables. Convolved distributions for the DME/P IA and FA modes are shown in Figures 3.7 through 3.10. As would be expected from the Central Limit Theorem,
Figure 3.7

IA Mode Error Distribution

Probability density

meters

$10^{-4}$
Figure 3.8
IA Mode Error Distribution (Expanded View)

Cumulative Probability
= 9.77 x 10^-8

Probability density

1.5 x 10^-8
1.4
1.2
1
0.8
0.6
0.4
0.2

196.25
197
197.5
198
196.5
196
195.5
195
Figure 3.9
FA Mode Error Distribution
Figure 3.10
FA Mode Error Distribution
(Expanded View)

Cumulative Probability
= 9.16 \times 10^{-8}
the convolved distributions approach a Gaussian distribution. Determination of error bounds involves finding the error beyond which the cumulative probability density is less than $10^{-7}$. Since the error distributions were discretized to facilitate convolution, an exact bound for $10^{-7}$ cumulative probability was not found. In the IA mode, a cumulative probability of $9.77 \times 10^{-8}$ was calculated for a bound at 196.25 meters. If the bound were moved closer to the origin, the cumulative probability would go above $10^{-7}$. Thus, 196.25 meters was chosen as the bound in the IA case. The bound for the FA case was found to be at 66.75 meters. These figures compare well with the results presented by Kelly (1989a).
CHAPTER IV
DME/P DETERMINISTIC ERROR SIMULATOR

The material presented in the previous chapter outlines the theory which governs DME errors. From this theory it is possible to simulate the effects of the deterministic error sources. A software simulator has been developed by the author which models the FA mode deterministic errors. Simulation of non-deterministic errors will be discussed in the next two chapters.

Errors categorized as deterministic are: instrumentation (not including receiver noise), specular multipath, and garble. Although garble errors are truly random, they may be modeled simply by a Poisson process and are included in the deterministic model. For any given run, instrumentation errors are modeled as a bias. For simulation purposes this bias is chosen randomly within the limits of the instrumentation error budget. Multipath is characterized by M/D ratio, delay, relative phase, and phase rate-of-change.

4.1 Software Simulator Description

The deterministic error simulator combines all of these effects on a per pulse basis. Figure 4.1 diagrams the flow
Figure 4.1
Flow Diagram for Deterministic Simulator
of the simulator and the full code listing is given in Appendix A. The program starts by reading a data file containing environment information. This information includes the number of pulse windows to be processed, garble arrival rate, the random instrumentation bias, and multipath source information.

For each multipath source, the environment data file specifies: the period over which the multipath exists, the fixed delay, the maximum M/D ratio, the relative phase, and a flag specifying whether the relative phase is fixed or varies. In an operational environment, the M/D ratio starts at zero, rises to a maximum, and then falls to zero as the specular point migrates over the reflecting object. The exact amplitude at a given instant depends upon the portion of the Fresnel zone lying on the object at that instant. As a result, over the period in which the multipath exists, the instantaneous M/D ratio is varied as one-half of a sine wave. The peak amplitude, then, is the maximum M/D ratio specified in the environment data file.

Having specified the environment, the model proceeds to process each pulse window. The "window" is the time period during which the desired and extraneous pulses are received. During each window the model reads in the pulse and phase information for the desired, garble (if any), and multipath (if any) pulses. The combination of the three pulses is the
total signal which is sent to the DAC subroutine. The DAC subroutine performs the TOA estimation described earlier and returns the TOA value to the main routine. The main routine subtracts the true TOA from the estimated and adds the instrumentation bias to form the final error value. This process continues until the model has processed all the windows specified in the environment data file.

4.2 Simulation Results

Figure 4.2 shows the results from a sample run. The environment file for this run specified an instrumentation bias of 25 nanoseconds, an average garble arrival rate of 3000 pulses per second, and two multipath sources. The aircraft was assumed to be in the FA region for the duration of the run. The total run consisted of 1000 windows with the desired pulse arriving at 6500 nanoseconds relative to the start of the window. A 40 Hertz interrogation rate was used with 100% reply efficiency. This resulted in a reply every 1/40 (0.025) seconds. The first multipath source interfered with the desired pulse between windows 200 and 400. It had a maximum M/D ratio of 0.50 and a fixed delay of 50 nanoseconds. The relative phase flag was set to variable. The second multipath source interfered between windows 600 and 800, had a maximum M/D ratio of 0.50, and a
Figure 4.2
Simulator Output
fixed delay of 90 nanoseconds. Its relative phase flag was also set to variable.

Upon first glance one immediately notices the multitude of garble induced spikes. Many of them have the same amplitude because of simulated outlier rejection. Their actual values were greater than those shown in the figure. The effects of the two multipath sources also can be clearly seen. The typical scalloping effect stems from the variable relative phase. In practice, this effect is the result of a relative phase rate-of-change as the receiver moves through an interference field.

The large magnitude of the errors results from the fact that the normal signal processing, such as alpha-beta filters, was not simulated. Therefore, the simulator models the raw effects of the error sources only. The typical signal processing will attenuate the magnitudes of these errors. As mentioned earlier in the case of garble, the largest error which is to be expected normally is approximately 2.5 meters in the FA mode.

Even without the simulation of signal processing, first order effects can be estimated and some studies may be initiated. For instance, the claim of DAC rejection of long delay multipath can be tested. As stated previously, the DAC technique in combination with the FA mode pulse shape can reject multipath with path delays of more than 300 ns.
To test this, the simulator modeled the effects of fixed phase and amplitude multipath as a function of multipath delay. Figure 4.3 shows the results. The two curves represent the effects of two fixed relative phases: 0 degrees and 180 degrees. It is interesting to note that no error is incurred if the delay is greater than 200 ns.

Examining the figure closely also reveals that the maximum error is 50 ns. These two numbers are in conflict with those stated earlier, namely: multipath immunity with delay greater than 300 ns and maximum error of 33 feet (10 meters). The answer to this problem again is the raw-data mode of this simulation. In reality, IF filters modify the multipath effects. The use of the IF filter reduces the peak error considerably but still permits some error even with delays up to 300 ns. Kelly (1986) gives results for both IF filtering and no filtering. The results with no filtering match extremely well with those presented here.

If the model were extended to simulate the usual signal processing, operationally encountered error could be estimated. Such a model could also be expanded to simulate actual airport environments. Such an effort would be monumental but could prove to be useful in the siting of DME/P transponders.
Figure 4.3
Time-of-Arrival Error Versus Multipath Delay
CHAPTER V
SYSTEM IDENTIFICATION THEORY

After examination of the various signal modeling methodologies which have been used for the ILS and the MLS, it was concluded that these methods had shortcomings in either accuracy or generality. This chapter addresses a candidate method which can overcome the shortcomings of previous techniques. It is known as System Identification Theory and can be an aid in signal analysis as well as signal generation. Ljung (1987) covers the breadth of the topic. In the DME/P signal model application, System Identification Theory will be used to reproduce the combination of all non-deterministic errors.

5.1 Time Series

The first step in system identification, as it applies to the DME/P signal, is to understand the concept of time series. A time series is a set of values having a definite order, and arising from some observation process. It is implied that the observer has no a priori knowledge regarding the process (model) by which the series was created. The observer or analyst has no information about
the underlying model, its input, or any external noise. Only the data are available for scrutiny.

In the case of the DME/P, the time series is the flight error trace. It consists of both deterministic (bias, garble, multipath) and non-deterministic errors. Assuming all deterministic errors have been subtracted out of the trace, what remains is the output of some stochastic process. A stochastic process is a random process which is controlled by a set of probabilistic laws (Kelly, 1989b). Indeed, it is the assumption that the non-deterministic errors are the output of some stochastic process which makes signal identification and modeling possible.

In attempting to identify a random process, one needs to decide what characteristics of the process are to be identified. Obviously, exact prediction of instantaneous values is not possible. If this were the case, the process would be deterministic rather than stochastic. As a result, the information quantity in the signal is not readily available. Individual realizations of a stochastic process can vary considerably from each other. The information contained in each realization then, must be some quantity which does not change. While the details of probability theory are beyond the scope of this work, a quick review will remind one that it is the moments of the process which do not change. Practically speaking, the first and second
moments of a process are seen in the serial correlation of any given realization (Kelly, 1989). Serial correlation describes the relationship of a given data point to the points immediately preceding or following it. This correlation, and the power spectrum it gives rise to, is the information which must be extracted. These are also the characteristics which affect the AFCS response and therefore are the characteristics to be modeled (Kelly, 1977). Without serial correlation, no information exists and consequently there is nothing to identify or explain.

A random process without serial correlation is known as white noise. One value has absolutely no relationship to the prior or succeeding value. The process is completely described by its mean and variance as is shown in its probability density function (also known as a normal or Gaussian distribution):

$$f_X(x) = (2\pi \sigma^2)^{-\frac{1}{2}} \exp\left(-\frac{(x - m)^2}{2\sigma^2}\right)$$

where: $m$ is the mean

$\sigma^2$ is the variance

Before the process of identification is to begin, however, a restriction must be placed on the time series.
That restriction is known as stationarity. A stationary random process is one whose statistical properties do not change with time (Priestley, 1981). The need for this may be understood very quickly. All that one has available to do the identification is the data. Consequently, it would be impossible to determine the underlying statistics of the data if they changed while the data were being collected. This will be seen later when the process of model identification first involves estimation of ensemble autocorrelation or autocovariance.

5.2 Model Identification and Signal Generation

Following Kelly (1988b, 1989b) and Braasch (1989a), the essence of system identification will be described as it applies to model identification and signal generation. As mentioned earlier, it is assumed that the time series to be identified is the output of some stochastic model. More specifically, since neither the model nor its input are known, it is assumed to be stationary and have white noise as its input. In general, the model may be described by an Autoregressive Moving Average (ARMA) filter of a given order. The characteristics of the filter will be discussed later.
Model identification is accomplished in the following manner. The collected data are passed through a filter whose coefficients are adjusted until its output (also known as residuals) is white noise (with zero mean, and variance $\sigma_r^2$). Once the filter outputs white noise, identification is finished. Since white noise has no serial correlation (structure), it has no information. The filter has removed all the structure from the data. As a result, the final settings of the filter coefficients embody the structure, and therefore the information, contained in the collected data.

Signal generation is simply the inverse process. The filter obtained through model identification must now be inverted. White noise (with zero mean, and variance $\sigma_r^2$) passed through the inverse filter generates data which is statistically equivalent to the collected data. As implied before, statistical equivalence means that the autocorrelation and power spectrum of the collected and generated data are the same. Figure 5.1 depicts the process of model identification and signal generation. This technique has its origins in the field of statistics and dates back to 1927 (Yule, 1927). Although the overall concept of model identification and signal generation is straightforward, the details can become quite involved.
The determination of the optimized filter has three parts. First, the type of model (filter) must be chosen. Should it be all poles, all zeros, or a combination of both? Once the type is chosen, the model order must be determined. Should the model have three coefficients or four, et cetera? Finally, having specified a certain type and order, the coefficients must be estimated. As noted in Kelly (1989b), choosing a wrong order for the model (too many or too few coefficients) results in noise being brought into the model and consequently variance inflation. Choosing the wrong type can result in an optimized model which requires an infinite number of coefficients (Kelly, 1989b). Finally, once the model type, order, and its coefficients have been optimized, validation must be performed. These four processes are described in the following sections.

5.3 Model Types

Time series identification involves the optimization of three types of stationary models. They are known as the Autoregressive (AR), the Moving Average (MA), and their combination, known as the Autoregressive Moving Average (ARMA). As will be shown, these models are general enough to identify and reproduce any stationary time series.
5.3.1 The Autoregressive Model

In an autoregressive process, the output is a weighted sum of previous outputs plus a noise term:

$$ Y_t = -a_1 Y_{t-1} - a_2 Y_{t-2} - \ldots - a_p Y_{t-p} + e_t $$ \hspace{1cm} (1)

where: $Y_t$ is the output at time $t$

$a$ is the coefficient vector

$e_t$ is the noise term

The process is generally described as an autoregressive process of order $p$ or $AR(p)$. The term autoregressive stems from the output being a function of previous values of itself.

For the sake of convenience in time domain descriptions, a backward shift operator, $q$, is defined as follows (Ljung, 1987):

$$ q^{-k} Y_t = Y_{t-k} $$ \hspace{1cm} (2)

Equation (1) can now be rewritten as:
\[ A(q)Y_t = e_t \]  \hspace{1cm} (3)

where \( A(q) = 1 + a_1q^{-1} + a_2q^{-2} + \ldots + a_pq^{-p} \).

5.3.2 The Moving Average Model

In a moving average process, the output is a weighted sum of noise inputs:

\[ Y_t = c_0e_t + c_1e_{t-1} + c_2e_{t-2} + \ldots + c_re_{t-r} \]  \hspace{1cm} (4)

The process is generally described as a Moving Average of order \( r \) or MA(\( r \)). Using the backward shift operator, (4) becomes:

\[ Y_t = C(q)e_t \]  \hspace{1cm} (5)

5.3.3 The Autoregressive Moving Average Model

The natural extension of the AR(\( p \)) and the MA(\( r \)) is the combination of the two. A moving average is used instead of the simple noise term in the AR process:

\[ A(q)Y_t = C(q)e_t \]  \hspace{1cm} (6)

generally referred to as an ARMA(\( p \),\( r \)).
Use of the z-transform allows the expression of the transfer function as:

\[ H(z) = \frac{\text{C}(z)}{\text{A}(z)} \]  

(7)

which confirms the earlier claim that the three processes are general enough to model any stationary time series. The process is a general network function capable of implementing any difference equation.

The next section shows how the process of coefficient estimation is done for an AR model.

5.4 Estimation of AR Models

Having defined the model types, the issues of model order and coefficient estimation must be addressed. Since coefficient estimation precedes final choice of model order, it will be discussed first.

Equation (3) describes the AR(p) process:

\[ A(q)Y_t = e_t \]  

(3)

Estimation of the coefficient vector:

\[ a = [a_1, a_2, \ldots, a_p] \]  

(8)
is achieved by minimizing

\[ \Phi = E[e_t^T e_t] = E[e_t^2] \]  \hspace{1cm} (9)

where \( E[\ ] \) designates expected value. \( \Phi \) is minimized by differentiating it with respect to \( a \) and setting the derivative equal to zero:

\[ \frac{\partial \Phi}{\partial a_1} = 2E[e_t \frac{\partial e_t}{\partial a_1}] = 0 \]  \hspace{1cm} (10)

Substitute (11) into (10):

\[ E[e_t^T Y_{t-j}] = 0 ; \text{ for } j = 1, 2, \ldots, p \]  \hspace{1cm} (12)

which is the orthogonality condition. Substituting (3) into (12) for \( e_t \), yields:

\[ \sum_{i=0}^{p} E[a_i Y_{t-1} Y_{t-j}] = 0 ; \text{ for } j = 1, \ldots, p \]  \hspace{1cm} (13)
or,

\[
\sum_{i=0}^{p} a_i R_{yy}(i-j) = 0; \quad \text{for } j = 1, \ldots, p
\]

(14)

where: \( R_{yy}(i-j) \) (autocovariance) = \( E[Y_{t-i} Y_{t-j}] \)

Equations (14) are known as the Yule-Walker equations.

They may be rewritten in matrix form:

\[
\begin{bmatrix}
R_{yy}(-1) & R_{yy}(0) & R_{yy}(1) & \ldots & R_{yy}(p-1)
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
a_2 \\
\vdots \\
a_p
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

By setting \( a_0 = 1 \), the equations become:

\[
\begin{bmatrix}
R_{yy}(0) & R_{yy}(1) & \ldots & R_{yy}(p-1)
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
a_3 \\
\vdots \\
a_p
\end{bmatrix}
+ 
\begin{bmatrix}
R_{yy}(-1) \\
R_{yy}(-2) \\
R_{yy}(-3) \\
\vdots \\
R_{yy}(-p)
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]
By rearranging terms and remembering $R_{yy}(-j) = R_{yy}(j)$

one obtains:

$$
\begin{bmatrix}
R_{yy}(0) & R_{yy}(1) & \ldots & R_{yy}(p-1) \\
R_{yy}(1) & \ldots \\
R_{yy}(2) & \ldots \\
\vdots & \ddots \\
R_{yy}(p-1) & R_{yy}(p-2) & \ldots & R_{yy}(0)
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
a_3 \\
\vdots \\
a_p
\end{bmatrix}
= 
\begin{bmatrix}
R_{yy}(1) \\
R_{yy}(2) \\
R_{yy}(3) \\
\vdots \\
R_{yy}(p)
\end{bmatrix}
$$

which can be written more concisely as:

$$
R_{yy} a = -R_{yy}(p) 
$$

Equations (15) are the "normal equations" for the AR process. For a true appreciation of the terms "normal equations" and "orthogonality condition," one needs to perform the derivation using the projection theorem in vector space. Kelly (1989b) does this along with the conventional derivation presented here.

Having obtained the normal equations, the solution of $a$ is found simply through multiplying both sides by the inverse of the autocovariance matrix:
At this point, estimation of the coefficient vector appears to be a fairly trivial matter of matrix mathematics. However, this is far from the truth. The autocovariance matrix given in (19) is an ensemble quantity. Since it is impossible to determine, it must be approximated. This can be done in a variety of ways and gives rise to the multitude of AR estimation algorithms available. The most straightforward method is simply to use the sample autocovariance function of the N samples of collected data:

\[ a = - [R_{yy}]^{-1} R_{yy}(p) \] (19)

An entirely different approach to AR estimation can be done using Maximum Likelihood (ML). This approach involves inserting the data vector \( \mathbf{y} \) into the probability density function of the model and then varying the model coefficients until a maximum value is obtained (Kelly, 1989b). The derivations for MA and ARMA coefficient estimates are performed using ML.

5.5 Model Validation

Once the coefficients have been optimized for a certain model type and order, the model must be validated. This is
done in two ways. Referring back to Figure 5.1, the identification process is complete if the output of the filter (the residuals) is white noise. One test of whiteness involves computation of the sample autocorrelation function. The autocorrelation function is a normalized version of the autocovariance function. The equation for the autocorrelation function for a stationary random process is given by:

\[ r(\tau) = \frac{E[(Y(t) - \mu)(Y(t+\tau) - \mu)]}{E[(Y(t) - \mu)^2]} \]  \hspace{1cm} (21)

for all \( \tau \); where: \( \mu \) is the process mean.

Thus, the autocorrelation function is normalized since \( r(0) = 1 \). \( r \) is known as the "lag." The autocorrelation function of a finite-length white noise sequence has a spike at lag zero and negligible values at all other lags. Another whiteness test is to examine the power spectrum. White noise is characterized by a flat spectrum since it contains equal power at all frequencies.

The second validation procedure is the comparison of the power spectra of the collected data and the generated data. If model identification and signal generation have been successfully completed, their power spectra will match well. As discussed earlier, the autocorrelation (serial correlation) and the power spectrum of a stochastic process
represent the information contained in any given realization. Actually, by assuring power spectral similarity, correlation similarity is implied. This stems from the relationship of the autocovariance function and power spectral density. The Weiner-Khinchin theorem shows the two to be related through the Fourier transform (Helstrom, 1984):

\[ S_x(\omega) = \int R_{xx}(\nu) \, e^{-j\omega\nu} \, d\nu \]  

(22)

Therefore, by specifying the power spectrum, one has also specified the autocovariance function. Since the autocorrelation function is a normalized autocovariance function, it too is uniquely specified by a given power spectrum.

5.6 Model Type and Order Determination

The process of determining the proper model type and order is a comparative process. Various criteria have been developed which rate the quality of a certain model type and order in relation to the collected data.

As described in Kelly (1989b), the natural approach to type and order determination contains a pitfall. One may start with an AR(1). Its filter coefficient is optimized, the residuals are checked for whiteness, and the power
spectrum of a generated signal is compared against that of the collected data. One then moves on to do the same process for an AR(2), AR(3), and so on. The same process could be done for MA and ARMA models. The problem with doing this is that as the order of the model increases, the model will continue to improve in matching the collected data. At first this sounds like exactly what is needed. It is necessary to remember, however, that the goal is to model the process which created the data, and not the particular data set. If the order is too high, the model begins to fit the given process realization rather than the process.

Clearly, some criteria must be used to determine the point at which the model is fitting the data rather than the process. Two such criteria are the Final Prediction Error (FPE) and the Akaike Information Criterion (AIC) proposed by Akaike in 1970 and 1974 respectively (Akaike, 1970, and 1974). The two criteria are formed as follows (Ljung, 1987):

\[
FPE = \frac{1 + n/N}{1 - n/N} \times V
\]

(23)

\[
AIC \approx \log[(1 + 2n/N) \times V]
\]

(24)

where: 
N is the total number of collected data samples 
n is the number of estimated parameters 
V is the variance of the residuals
Choice of criterion depends upon model type and the size of the collected data set. Priestley (1981) shows that for AR model estimation with large data samples, the two criteria yield the same results. The criteria are used as follows. For a given model and collected data, a value for FPE and AIC can be calculated. As model order increases, FPE and AIC will decrease. Eventually, the two criteria will reach a minimum and then start to increase. The model order with the lowest FPE or AIC is the highest order allowable. Beyond that, one is starting to fit the data.

The FPE and AIC define an upper bound on model order. The order of one's model should not go above that specified by the criteria. In some instances, however, it is not necessary to go as high as the criteria will allow. The overall goal in using System Identification Theory is to determine a model which can reproduce the stochastic (non-deterministic) component of DME/P error. In addition, the model must be as compact as possible. This affects the model identification process as follows. As one optimizes various models, the residuals are checked for whiteness and a generated signal is compared to the collected data. Once this is accomplished on a 95% basis, the job is finished. As will be shown in the next chapter, the optimized model will be of order three even though the upper bound was four. This idea of halting the identification process when the
model obtained is adequate, is known as the Principle of Parsimony (Tukey, 1961).

Combination of the Principle of Parsimony with the information criteria allows comparison of model types. Again, the simplest adequate model is the goal.
CHAPTER VI
SYNTHESIS OF THE NON-DETERMINISTIC MODULE

The process of model identification begins with data collection. Typical DME/P flight error traces will contain both deterministic and non-deterministic components. The components must be separated before identification can begin. Once the non-deterministic component is separated from the collected data, further processing may be required to obtain a stationary waveform. This stationary waveform, then, is identified through the process described in the previous chapter with the result being an optimized filter and computation of the variance of the white noise input.

This chapter discusses model identification as applied to DME/P flight error traces. The steps outlined above are detailed and the identification process is demonstrated for an actual flight error trace.

6.1 Separation of Deterministic and Non-Deterministic Components

In general, a DME/P flight error trace will contain equipment bias, multipath, and garble components in addition to the non-deterministic component. The deterministic
components must be removed from the error trace to obtain the non-deterministic component.

Removal of bias and garble effects is straightforward. The bias is removed by subtracting the mean of the trace from the trace itself. Garble errors appear as spikes and therefore may easily be subtracted or clipped.

Errors due to specular multipath are not so easily removed. They have distinct characteristics, however, which can be exploited in an effort to identify their existence and then to subtract them from the trace. The four parameters which characterize multipath errors are M/D ratio, delay, RF phase, and phase rate-of-change. Given these four parameters, the effect of multipath in an error trace can be determined. Conversely, an error trace which is observed to have multipath effects may be analyzed to determine the four critical parameters.

It is important to note that although multipath effects can be observed in an error trace, it is generally not possible to identify the effect precisely enough to allow subtraction. It is possible, though, to determine the four parameters. Once these parameters have been obtained, the multipath effect can be generated and then subtracted from the trace. Appendix B derives the relationships which allow parameter determination from an error trace.
DME/P error traces collected to date have not exhibited the effects of specular multipath. This is due to a combination of benign sites, proper transponder siting, and centerline approaches. As a result, the above technique has not been tested. It is expected, however, that multipath effects will be recorded when data is taken from complex approaches. At that time, the technique proposed above may be tested and implemented.

6.2 Non-Deterministic Component Identification

The identification process will now be demonstrated for an actual flight error trace. Figure 6.1 shows the FA mode section of raw DME/P data taken at Lebanon, New Hampshire. Both the ground transponder and the airborne interrogator were produced according to ICAO SARPs. The error trace shows no garble spikes and specular multipath does not seem to be present. A very small equipment bias of approximately 1.25 feet was removed (Figure 6.2).

The waveform which remains is quite obviously non-stationary. This is exhibited primarily by the non-zero mean which varies along the flight path. Kelly (1989d) refers to this as a low frequency "random trend." The nature of this random trend is uncertain. It could be due to thermal effects in the transponder or diffuse multipath.
Figure 6.2
DME/P Error - Bias Removed

Distance from transponder in nautical miles

Error in feet
For the purpose of identifying the high frequency component, the random trend was removed. Kelly (1989g) states that the random trend may be removed by passing the waveform through a nonrecursive orthogonal polynomial filter. For this case, however, a simpler method will suffice. The random trend was approximated by taking a 120-sample "moving average." This is done by determining the average of the first 120 samples and then letting the random trend equal that average over that period. The next 120 samples are then averaged and the process continues for the whole waveform. Figure 6.3 shows the approximated random trend and Figure 6.4 shows the error waveform with the trend removed. The detrended waveform now approximates a stationary waveform.

The identification process continues by performing coefficient estimation for a variety of models. The results are shown in Table 6.1. The table clearly shows the AR(4) to have the lowest FPE value. The Principle of Parsimony, however, dictates that the simplest model be used. Comparison of the residuals from the AR(3) and AR(4) models shows both to be white with very little difference between the two (Figures 6.5 and 6.6). The dotted lines indicate the 95% confidence limits for white noise. The sidelobes of the autocorrelation should lie within these limits for the residuals to be considered white noise. The residuals from the AR(2) model are also white (Figure 6.7) but the AR(3) is
Figure 6.3
Approximated Random Trend
Table 6.1  
Model Estimation Results

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<th>Zeros</th>
<th>FPE</th>
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</tr>
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</tr>
<tr>
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<td>49.32</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>49.46</td>
</tr>
</tbody>
</table>
Figure 6.5

Autocorrelation Function of AR(4) Residuals

Correlation function of residuals

AR(4) model
Correlation function of residuals

AR(3) model

Figure 6.6
Autocorrelation Function of AR(3) Residuals
Figure 6.7
Autocorrelation Function of AR(2) Residuals
a compromise between model simplicity and complete identification.

Having identified the model, the variance of the residuals may be calculated. Referring back to Figure 5.1, signal generation requires finding the inverse of the filter obtained in the identification process. The inverted filter is given as:

\[ H(z) = \frac{1}{1 - 0.6486z^{-1} - 0.0446z^{-2} + 0.0266z^{-3}} \]

with the variance of the white noise input (residuals) having been calculated to be \( \sigma^2_r = 46.43 \). Simulated data is obtained by passing white noise (with variance \( \sigma^2_r \)) through \( H(z) \). Figure 6.8 shows the result. Since the generated data is supposed to be statistically equivalent to the collected data, their power spectra may be compared (Figure 6.9). The similarity is close enough for the identification to be considered complete.

At this point it is important to note the large data reduction which has been achieved. The error trace submitted for identification consisted of 1970 samples. The AR(3) process along with the white noise variance are characterized by 4 parameters. The number of samples needing to be stored was reduced from 1970 to 4.
Figure 6.8
Simulated DME/P Error Data
Figure 6.9
Power Spectra of Simulated and Collected Error Data
6.3 Model Identification Issues

The previous section demonstrated the ability of System Identification Theory to identify and reproduce collected error traces accurately. Some issues need to be resolved, however, before a final model for non-deterministic error components may be produced. The first issue deals with the low frequency random trend. As of this time, enough flight data has not been collected to characterize the trend. Variations in trend amplitude and frequency need to be determined. Kelly (1989a) claims that slowly varying instrumentation errors will appear as a bias over any given run. If this random trend is actually due to instrumentation errors, this assumption is not valid. Another possible cause of the trend is diffuse multipath. However, more data must be collected before the origins and characteristics of the trend may be known. Once this has been completed, it may be incorporated into the model.

This lack of data is also a problem for the IA mode. Since the IA mode works with different processing methods, it will require its own non-deterministic model. A recent suggestion has been to use DME/N data for the initial identification of an IA model (Cassell, 1989). This could be used until more flight data has been collected.
Finally, DME/P data available thus far has been taken using equipment from one transponder manufacturer and one interrogator manufacturer. It is possible that the non-deterministic error could vary among equipment types. This must be investigated before a final signal model is released to the international community.
CHAPTER VII

FUNCTIONAL DESCRIPTION OF THE DME/P SIGNAL MODEL

A task for the development of a DME/P signal model has been included in the work program of the ICAO All Weather Operations Panel (AWOP). The model is required as one input to those large scale simulations used to support the development and certification of aircraft flight control systems.

Up to this point, research presented to the AWOP has concentrated on the analysis and reproduction of the non-deterministic components of DME/P error (Kelly, 1988b, 1989d, 1989e, and Braasch, 1989a). During the most recent meeting of AWOP Working Group A, it was concluded that the complete signal model should be advanced to the point of a functional description (AWOP, 1989). This chapter reviews the major DME/P error sources described earlier and details how they may be synthesized in the model. Upon review by Working Group A (Braasch, 1989b), the model will be coded and made available to interested users by the time of the next AWOP full panel meeting in March of 1990.
7.1 Error Sources Modeled

As with the MLS angle model, the DME/P signal model is designed to reproduce the typical errors which would be encountered in an operational environment (Kelly, 1989f, and Vickers, 1986). It is expected that the ground transponder will be sited to minimize the effects of lateral multipath at least along the extended runway centerline. These errors are from instrumentation, garble, multipath, and non-deterministic sources.

7.1.1 Instrumentation Errors

Instrumentation errors result from:
- receiver thermal noise
- intermediate frequency (IF) filter delays
- logarithmic amplifier distortions
- calibration loop errors
- clock inaccuracies
- errors dependent upon pulse characteristics

As demonstrated in the previous chapter, the high frequency random components of these errors can be modeled with a white noise source operating on an optimized filter. The error components remaining are random but are slowly varying. For the purpose of a first order approximation, it
will be assumed that over the course of an approach these errors will be seen as a bias (Kelly, 1989a). The amplitude of the bias from run to run will be random. In the model, this amplitude is chosen as a random number, uniformly distributed between the limits of the instrumentation error budget (ICAO, 1985).

7.1.2 Garble

Garble errors result from corruption of the desired signal by coincident signals from other DME equipments or other RF sources. The probability of a garble error occurring during a given interrogation is determined from a Poisson distribution:

\[ P(\text{garble event}) = 1 - e^{-ft} \]

where: \( f \) = average garble arrival rate
\( T \) = interaction interval

The interaction interval indicates the window of time around the arrival of the desired pulse when undesired RF energy may derogate the desired pulse. Thus, to determine if a garble error has occurred:
\( \text{rnd} = \text{random number between 0 and 1 chosen from a uniform distribution} \)

\[ t = -\frac{\ln(1 - \text{rnd})}{f} \]

If \( t < T \), then a garble error has occurred.

Once a garble error has occurred, the magnitude of the error must be determined. The magnitude is chosen randomly from a uniform distribution bounded at ± 2.5 meters in the FA mode and ± 1 meter in the IA mode. These values were derived from worst case traffic loading scenarios (Kelly, 1984).

7.1.3 Multipath Sources

Multipath error in the DME/P may be classified into two categories. Diffuse multipath errors appear noise-like in a flight error trace. As such they may be classified as non-deterministic errors and will be modeled with time-series identification techniques (see next section). Specular multipath is characterized by multipath-to-direct (M/D) ratio, delay, relative phase, and phase rate-of-change. It is important to note this derivation assumes geometrical optics to be a good approximation of the total signal.
Geometrical optics gives the total signal to be the sum of the direct and reflected signals and neglects diffraction. Neglecting diffraction in the case of the DME/P is reasonable since the transmissions are at L-band. The model input for multipath will consist of specification of a reflecting plate (dimensions, position, and orientation), and the reflection coefficient of that plate. The reflecting plates will be used to model large structures in the vicinity of the ground transponder.

7.1.4 Non-deterministic Sources

The final error component to be synthesized is categorized as non-deterministic. As described in the previous chapter, System Identification Theory can be used to model the non-deterministic error. The intent then, is to determine the optimal transfer function (filter) for both the IA and FA modes. The non-deterministic errors will be generated by passing white noise (of specified variance) through these filters.

7.2 Description of Model Operation

A block diagram of the signal model is given in Figure 7.1. The model takes samples of aircraft position and
Figure 7.1  DME/P Signal Model Block Diagram
computes true range. This true range is input to the error generation blocks since they are dependent upon mode of operation (IA or FA). Each error block computes its value and the sum of the errors is combined with the true range. The perturbed range values are passed through the receiver output filter and then to the user via three routes. MLS range is output directly from the filter and is available for the aircraft simulation input. True range is subtracted from MLS range resulting in raw range error. Filtered error, for analysis purposes, is obtained by passing the raw range error through Control Motion Noise (CMN) and Path Following Error (PFE) filters.
CHAPTER VIII
CONCLUSIONS AND RECOMMENDATIONS

This thesis has addressed current developments for the DME/P component of the MLS signal model. The model is required to reproduce the desired signal plus errors which would typically be encountered in an operational environment. The error components of the DME/P may be divided into two categories: deterministic and random (non-deterministic).

- The reproduction of DME/P deterministic errors is shown to be straightforward and a first order deterministic error generator is presented. The accuracy of the generator has been verified.

- For the generation of non-deterministic errors, the idea of passing white noise through a filter has the advantage of simplicity but, as mentioned, lacks accuracy. System Identification Theory has been shown to overcome this problem by providing the means to optimize the model chosen and to verify its accuracy. The simplicity of the optimized model is demonstrated through large data compression. The information
contained in several thousand data points may be stored with just a few parameters.

In order to confirm the model, more flight data is required. Characterization of the IA mode and the low frequency trends in the FA mode can only be done conclusively with more operational data. It is expected that these data will become available as more MLS systems are implemented.

Since only an empirical noise generator exists in the MLS angle signal model, it is suggested that System Identification Theory be used to synthesize a more accurate non-deterministic module. For the DME/P, receiver signal processing should be included as a part of the deterministic error module. Upon validation of both the angle and DME/P models, efforts should be focused upon integration of the two. The result will be a complete MLS signal model available for use by the airframe manufacturers.
REFERENCES


Cassell, R., (1989), Personal Conversation with the author.


Kelly, R., (1989c), Private correspondence with the author.


Yule, G., (1927), "On a Method of Investigating Periodicities in Disturbed Series, with Special Reference to Wolfer's Sunspot Number," Philosophical Transactions, Series A.
APPENDIX A

Deterministic Simulator Code Listing

dmesim.for

Program to generate a DME/P error trace by simulating multipath, garble, and equipment bias effects.

coded by Michael S. Braasch
October 1989

external environ,dac,winpul

complex sigtot(8000),sigdes(8000)
real sigamp(8000),sigtim(8000)
integer npupw(5000)
real amplmp(5000,3),deLymp(5000,3),phamp(5000,3)

Set previous error (feet); pulse duration c (nanoseconds);
Set pulse rise time (nanoseconds); define pi
prverr = 0.
pdur = 5800.
rtime = 800.
pi = acos(-1.)

Generate environment file
call environ(numwin,bias,npupw,amplmp,deLymp,
& phamp,arrate)

Open random number file for garble arrival calculation
open(3,file='c:rnd.dat',status='old')

Open output data file
open(10,file='c:error.dat',status='unknown')

Zero out the desired signal data vector
do 150 i = 1,8000
   sigdes(i) = (0.,0.)
150 continue
write(*,*)'finished zeroing out the data vector'
write(*,*)

Generate desired pulse window:
write(*,*)'call winpul2'
call winpul2(pdur,rtime,1.,6500.,0.,sigdes)
Loop through the windows:

```plaintext
do 500 ndum = 1,numwin
write(*,*)' WINDOW number ',ndum
```

Read the desired signal into the total signal vector

```plaintext
do 163 i = 1,8000
sigtot(i) = sigdes(i)
continue
```

Calculate garble arrival time. If it lies in the window, then add the garble pulse to the window:

```plaintext
read(3,*),rndnum
write(*,*)'rndnum = ',rndnum
pat = (-1.*log(1.-rndnum))/arrate
write(*,*)'pat = ',pat
if(pat.le.(6800.))then
  read(3,*),rnd1
  read(3,*),grbamp
  grbamp = grbamp*0.5
  phase = rnd1*2.*pi
  call winpul2(pdur,rtime,grbamp,pat,phase,sigtot)
endif
```

If multipath is present at this time, add the multipath to the window:

```plaintext
if(npupw(ndum).gt.0)then
  write(*,*)
  write(*,*)'multipath being processed'
  write(*,*)
  do 180 ms = 1,npupw(ndum)
    call winpul2(pdur,rtime,amplmp(ndum,ms),
      & 6500.+delymp(ndum,ms),phamp(ndum,ms),sigtot)
  continue
endif
```

```plaintext
do 200 j = 1,8000
  sigtim(j) = float(j)
sigamp(j) = cabs(sigtot(j))
continue
```

Perform DAC calculations

```plaintext
write(*,*)'Perform DAC calculations'
call dac(sigamp,sigtim,8000,100.,0.5,yndac,timdac)
```
Compute error, add bias
error = timdac - 6700.
if(abs(error).gt.(300.)) error = 300.*error/abs(error)
error = error + bias
write(*,*)'yndac = ',yndac
if(yndac.lt.(l.)) error = prverr

Save time and error to file
write(10,510) float(ndum)/40.,error
write(*,*)float(ndum)/40.,error
prverr = error

goto top of loop
500 continue
510 format(1x,2(f15.8,4x))
close(3)
close(10)
stop
dmend

Subroutine winpu12(puldur,ristn,amp,pstart,phi,s)

Routine to generate a window of sampled data points for a given set of pulse characteristics
coded by Michael S. Braasch
9/20/89

Real amp,pstart,puldup,tinc,ristn,omegml,omegml,phi
Complex s(8000)

In-only parameter descriptions:
amp: pulse peak amplitude
pstart: pulse start time relative to the beginning of the window, in nanoseconds
puldup: pulse duration in nanoseconds
tinc: time sample increment in nanoseconds
ristn: zero level to peak pulse rise time in nanoseconds
omegml: angular frequency of the COS part of the cos pulse
omegm2: angular frequency of the COS2 part of the pulse
phi: phase angle in radians (0 for the desired pulse)

Out-only parameter description:
s: vector containing the sum of the previous plus the new sampled pulse data points

data tinc /1.0472e06/
data omegml /1.0472e06/
data omegm2 /0.31416e06/
cosst = pstart
cosssp = pstart + ristn
cos2st = cosssp + tinc
cos2sp = pstart + puldur
sigst = pstart

skip = 0.
if(cosssp.gt.(8000.))then
cosssp = 8000.
skip = 1.
elseif(cos2sp.gt.(8000.))then
cos2sp = 8000.
endif

c
riset = ristn/1.0e09
n = int(sigst)
psts = pstart/1.0e09

c
Sample and store the "COS" part of the pulse
do 50 t = cosst,cossptinc
time = t/1.0e09
ak = amp*\sin(omegml*(time-psts))
s(n) = s(n) + cmplx(cos(phi)*ak,\sin(phi)*ak)
n = n + 1
50 continue
c
if(skip.lt.(1.))then
Sample and store the "COS2" part of the pulse
do 60 t = cos2st,cos2sp,tinc
time = t/1.0e09
ak = amp*((cos(omegm2*(time-(psts+riset))))**2)
s(n) = s(n) + cmplx(cos(phi)*ak,\sin(phi)*ak)
n = n + 1
60 continue
endif
n = n - 1
return
end

cccccccccccccccccccccccccccccccccccccccccc

subroutine dac(sigvid,vidtim,npts,delay,att, &
   dacyn,dactim)

Routine to determine the DAC detect time

coded by Michael S. Braasch

real sigvid(8000),vidtim(8000)
real delay,att,dacyn,dactim
integer npts

description of in-only parameters:
sigvid: vector containing the envelope of sampled
data points of the total signal
   (desired+multipath+garble)
delay: DAC delay time in nanoseconds
att: DAC attenuation constant

description of out-only parameters:
dacyn: contains a 0. if no decode, a 1. if otherwise
dactim: if dacyn = 1., then dactim = DAC decode time
   in nanoseconds relative to the start of the window

ynold = 1.
do 999 nt = 1,npts
   if((nt-delay).lt.1)then
      D = 0.
   else
      D = sigvid(nt-delay)
   endif
   A = att*sigvid(nt+1)
   if((D.ge.A).and.(ynold.lt.(1.)))then
      dacyn = 1.
      dactim = vidtim(nt)
      goto 1000
   endif
   if(D.lt.A)then
      ynold = 0.
      dacyn = 0.
      dactim = 0.
   endif
   if((nt-delay).lt.1)then
      D = 0.
   else
      D = sigvid(nt-delay)
   endif
   A = att*sigvid(nt+1)
   if((D.ge.A).and.(ynold.lt.(1.)))then
      dacyn = 1.
      dactim = vidtim(nt)
      goto 1000
   endif
   if(D.lt.A)then
      ynold = 0.
      dacyn = 0.
      dactim = 0.
   endif
if((dactime.gt.(8000.)).or.(dactim.lt.(5000.))) then
  dacyn = 0.
dactim = 0.
endif
return
end

subroutine environ(nwin,b,nppw,ampmp,dlymp,phmp,arate)

Routine generates environment file. The environment file contains information regarding pulse characteristics for multipath interference. Maximum number of windows allowed is 5000.

Dimensioning and initialization

integer nppw(5000)
real ampmp(5000,3), dlymp(5000,3), phmp(5000,3)

pi = acos(-1.)
twopi = 2.*pi

do 5 i = 1,5000
  nppw(i) = 0
continue

5 continue

do 8 j = 1,3
  do 7 i = 1,5000
    ampmp(i,j) = 0.
dlymp(i,j) = 0.
  phmp(i,j) = 0.
continue

8 continue

open(4,file='c:dmein.dat',status='old')

Read in number of windows
  read(4,10)nwin
10 format(///,1x,i4)
write(*,*),'Number of windows = ',nwin
write(*,*)

Read in equipment bias error
  read(4,15)b
15 format(///,1x,f5.1)
write(*,*)'Equipment bias error = ',b
write(*,*)

Read in garble arrival rate
read(4,18) arate
format(/,1x,f8.1)
write(*,*)'Garble arrival rate = ',arate

Read in number of multipath sources
read(4,20) nummlt
format(/,1x,i1,//////////)
write(*,*)'Number of multipath sources = ',nummlt
write(*,*)

Loop through sources and read in start window, end window, amplitude, delay, and phase

do 42 i = 1, nummlt
read in start window, end window, M/D ratio, delay, phase flag (1 if fixed, 0 if stepped), and phase start point
read(4,25) nstart, nend, ratmd, delay, nphfl, phstrt
format(1x,4x,i4,9x,i4,7x,f6.4,4x,f5.1,
& 6x,i1,9x,f6.4)
write(*,*)
write(*,*)'For multipath source : ',i
write(*,*)
write(*,*)' nstart = ',nstart
write(*,*)' nend = ',nend
write(*,*)' M/D ratio = ',ratmd
write(*,*)' fixed delay = ',delay
write(*,*)' phase flag = ',nphfl
write(*,*)' phase start = ',phstrt
write(*,*)

Assign parameters to arrays

do 30 j = nstart, nend
npww(j) = npww(j) + 1
k = npww(j)
ampmp(j,k) = ratmd*sin(pi*(float(j-nstart)/
& float(nend-nstart)))
dlymp(j,k) = delay
if(nphfl.gt.0) then
  phmp(j,k) = phstrt
else
\[ \text{phmp}(j, k) = \text{mod}(\text{float}(j) \times \text{phstr}, \text{twopi}) \]

\text{endif}

\begin{verbatim}
30     continue
C
42     continue
C
     close(4)
C
     return
end
\end{verbatim}
APPENDIX B
DME/P Multipath Relations

Consider the general DME/P multipath scenario given in Figure B.1. The transponder has an offset siting and the approaching aircraft is on the extended runway centerline. Geometrical Optics gives the total received signal to be the sum of the direct and reflected signals. The relationships defining the four critical multipath parameters (M/D ratio, delay, relative phase, and phase rate-of-change) may now be derived.

Multipath delay is given by the difference of multipath and direct signal propagation time from the transmitter to the receiver. This is a function of siting geometry. As noted in chapter three, multipath delays greater than 300 nanoseconds do not cause time-of-arrival errors.

Assuming plane wave propagation, the relationships defining the remaining three parameters may be obtained as follows:

Direct Signal, \( E_D = E_0 \ e^{j(k_D \cdot r_D - \omega_c t)} \)

where: \( k_D = \frac{(2\pi/\lambda)u_D}{\lambda} \) and is the wave number vector.
\( \lambda \) is the carrier wavelength
\( r_D \) is the position vector
\( \omega_c \) is the carrier frequency
\( t \) is time
Figure B.1
DME/P Multipath Scenario
assume: $E_0 = 1$, and let $\phi_D = k_D \cdot r_D - \omega_c t$,

then, $E_D = e^{j\phi_D}$

Similarly for the reflected signal,

$$E_R = E_0 \cdot e^{j(k_R \cdot r_R - \omega_c t - \theta_0)} = \rho e^{j\phi_R}$$

where: $\rho$ is the M/D ratio
$\theta_0$ is the phase of the reflection coefficient

Having defined the direct and reflected signals, the total received signal is:

received signal = direct + reflected

$$= E_D + E_R$$

$$= e^{j\phi_D} + \rho e^{j\phi_R}$$

$$= A(t)e^{j\phi(t)} \quad (1)$$

Now, change the phase terms to be relative to the direct:

$$A(t)e^{j\phi(t)} = e^{j(\phi_D - \phi_D)} + \rho e^{j(\phi_R - \phi_D)}$$

$$= 1 + \rho e^{j(\phi_R - \phi_D)} \quad (2)$$

$$= 1 + \rho \cos(\phi_R - \phi_D) + j[\rho \sin(\phi_R - \phi_D)]$$

where: $(\phi_R - \phi_D)$ is the relative phase

$A(t) = \text{Received signal magnitude}$

$$= \sqrt{(\text{Re}[\text{received signal}])^2 + (\text{Im}[\text{received signal}])^2}$$

$$= \sqrt{[1 + \rho \cos(\phi_R - \phi_D)]^2 + [\rho \sin(\phi_R - \phi_D)]^2}$$

$$= \sqrt{1 + 2\rho \cos(\phi_R - \phi_D) + \rho^2} \quad (3)$$
\[ \phi(t) = \text{Received signal phase} \]
\[ = \tan^{-1} \left[ \frac{\text{Im[received signal]}}{\text{Re[received signal]}} \right] \]
\[ = \tan^{-1} \left[ \frac{\rho \sin(\phi_R - \phi_D)}{1 + \rho \cos(\phi_R - \phi_D)} \right] \quad (4) \]
\[ = \Delta \phi \]

Now add \( \phi_D \) to convert from relative phase back to absolute phase:
\[ \phi(t) = \Delta \phi + \phi_D = \Delta \phi + k_D r_D - \omega_c t \quad (5) \]

The relative phase rate-of-change appears in a flight error trace as oscillatory behavior. This behavior is known as scalloping and the phase rate-of-change is known as the scalloping rate (or scalloping frequency).

The scalloping rate may be obtained starting with the phase term in (2):
\[ f_s = \frac{1}{2\pi} \frac{d}{dt} (\phi_R - \phi_D) \quad (6) \]
\[ = \frac{1}{2\pi} \left[ \frac{d}{dt} (\phi_R) - \frac{d}{dt} (\phi_D) \right] \quad (7) \]
\[ \frac{d\phi_D}{dt} = \frac{d}{dt} [k_D \cdot r_D - \omega_c t] \]

\[ = \frac{d}{dt} \left[ 2\pi r_D/\lambda \right] - \frac{d}{dt} [\omega_c t] \]

\[ = \frac{2\pi}{\lambda} \frac{d}{dt} (r_D) - \omega_c \]  \hspace{1cm} (8)

\[ \frac{d}{dt} (r_D) = -V \cos(\alpha) u_D, \]

hence,

\[ \frac{d\phi_D}{dt} = \frac{-V(2\pi)}{\lambda} \cos(\alpha) - \omega_c \]  \hspace{1cm} (9)

Similarly, for the reflected phase term:

\[ \frac{d\phi_R}{dt} = \frac{-V(2\pi)}{\lambda} \cos(\beta) - \omega_c \]  \hspace{1cm} (10)

Substituting (9) and (10) into (7), after minor algebraic manipulation, results in:

\[ f_s = \frac{(V/\lambda)[\cos(\alpha) - \cos(\beta)]}{cos(\beta)} \]  \hspace{1cm} (11)

For the case of a transponder sited on the extended runway centerline, (11) reduces to:

\[ f_s = \frac{(V/\lambda)[1 - \cos(\beta)]}{cos(\beta)} \]  \hspace{1cm} (12)