Sensitivity Analysis of
Cam-and-Follower Mechanism at High Speeds

A Thesis Presented to
The Faculty of the College of Engineering and Technology
Ohio University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by
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I. Introduction

Sensitivity analysis has been widely used in control systems. Tomović(1)* introduced this theory to analyze general dynamic systems. The method is concerned with how sensitive a system's output to the variation of system parameters. In design analysis, if one knows in advance the effect of system parameters to system's performance by sensitivity analysis, expensive hardware construction and lengthy laboratory tests can be avoided.

Recently, Huang(2) has applied sensitivity analysis to the design of two and three dimensional linkages. In a linkage containing n parameters, this technique yields 2(n+1) first-order coupled differential equations. By integrating this set of differential equations simultaneously, the sensitivity coefficients can be obtained and the system parameters can be automatically readjusted by iterations to minimize the performance index of the given system. Vukobratović and Gligorić(3) applied sensitivity theory to study the characteristics of relaxation oscillations of the machine tool slide. Watari and Iwamoto(4) applied sensitivity analysis to study the variation of parameters in automobile dynamics. Very recently, Young and Shoup(5) proposed a technique of sensitivity analysis, utilizing the eigenvalue and eigenvector derivatives of a system with respect to the system parameters.

* Numbers in bracket designate Bibliography at the end of the thesis.
thereby the designer can determine rapidly the most efficient design modifications needed to minimize the vibrational amplitude of follower output in a cam mechanism.

Generally speaking, parameters in a dynamical system may vary because of such things as aging, manufacturing tolerance, and changes of environmental conditions. In addition, changes in system parameters such as masses, the location of the gravity, and inertia might also occur during the operating process. In a feedback control system, the effect of changes in system parameters in the open loop results in a change in the system's output, which in turn is corrected by the feedback and error signals. Therefore, the effect of parameter changes on the output in a feedback system is reduced. In this paper, after a brief description of the method of sensitivity analysis, two dynamic systems representing cam-activated mechanisms are investigated to show the application of the method.
II. Sensitivity Function

In general, the equations of motion in a dynamic system may be expressed as

$$ F(\ddot{x}, \dot{x}, x, t, q) = 0 $$

(1)

where $x$ is the dependent variable, $t$ is the independent variable, and $q$ is the system parameter. Dots represent the differentiation with respect to $t$.

If the parameters are subjected to change, Eq.(1) may be written in the form

$$ F(\ddot{x}, \dot{x}, x, t, q+\Delta q) = 0 $$

(2)

where $\Delta q$ represents the variation of parameter.

The solutions of Eqs.(1) and (2) are

$$ x = x(t, q) $$

$$ x = x(t, q+\Delta q) $$

respectively.

Comparing these two solutions, the expression of the structural stability of the system can be immediately obtained, which is expressed by

$$ \Delta u = \frac{x(t, q+\Delta q) - x(t, q)}{\Delta q} $$

(3)

The sensitivity coefficient $u(t, q)$ is defined as follows

$$ u(t, q) = \lim_{\Delta q \to 0} \frac{x(t, q+\Delta q) - x(t, q)}{\Delta q} = \frac{d(t, q)}{dq} $$
If the dynamic system is in a steady state, that is, when it does not depend on \( t \), the sensitivity coefficient becomes a function of the parameter only. Thus

\[
u(t, q) = u(q)\]

When the dynamic system contains several parameters \( q_1, q_2, \ldots, q_n \), the equation of motion becomes

\[
F(\dot{x}, x, t, q_1, q_2, \ldots, q_n) = 0
\]  

(4)

Then the sensitivity depends on the values of all those parameters. In general, the sensitivity coefficient is the function \( u(t, q_1, q_2, \ldots, q_n) \); in other words, the sensitivity coefficient is a quantity associated with every point of the parametric space of the system, and it changes with the position of the point.

By differentiating Eq.(4) partially with respect to \( q_j \), the sensitivity equation is given by

\[
\frac{\partial F}{\partial \dot{x}} \frac{\partial \dot{x}}{\partial q_j} + \frac{\partial F}{\partial x} \frac{\partial x}{\partial q_j} + \frac{\partial F}{\partial x} \frac{\partial x}{\partial q_j} + \frac{\partial F}{\partial q_j} = 0
\]

\[
\frac{\partial F}{\partial \ddot{x}} \ddot{u}_j + \frac{\partial F}{\partial \dot{x}} \dot{u}_j + \frac{\partial F}{\partial x} u_j = -\frac{\partial F}{\partial q_j}
\]

(5)

Eq.(5) is called the sensitivity equation. By solving this equation, the sensitivity coefficient is obtained. When Eq.(5) and Eq.(4) are studied closely, it can be seen that Eq.(5) is always a linear differential equation with constant coefficients or with time-dependent coefficients,
while the original system Eq.(4) is generally non-linear. Furthermore, these coefficients and forced terms are generally determined by the trajectory of solution satisfying the non-linear equation. Consequently, the computer is generally an indispensable tool for obtaining solution.

What has been described above is a process of sensitivity analysis in the time domain starting from a given system equation. In a similar manner, by the concept of transfer function, the sensitivity analysis in frequency domain can also be derived. Under this situation, the dynamical characteristics expressed by a linear differential equation with constant coefficients can be written in state variable form as follows:

\[ \dot{\bar{X}}(t, q_j) = \bar{A} \bar{X}(t, q_j) + \bar{B} \bar{W}(t) \]

\[ \bar{Y}(t, q_j) = \bar{C} \bar{X}(t, q_j) \]

\[ \bar{X}(0, q_j) = \bar{X}_0 \]

where \( \bar{X} \) = n dimensional column vector representing the state variables of the system

\( \bar{X}_0 \) = initial value of \( \bar{X} \)

\( \bar{A} \) = n x n dimensional matrix

\( \bar{B} \) = n x r dimensional matrix

\( \bar{W} \) = the input, r dimensional column vector

\( \bar{Y} \) = the output, m dimensional column vector

\( \bar{C} \) = m x n dimensional matrix

Taking Laplace transformations using zero initial condi-
tions, we obtain

\[ s \bar{X}(s, q_j) = \bar{A} \bar{X}(s, q_j) + \bar{B} \bar{W}(s) \]  \hspace{1cm} (6)

\[ \bar{Y}(s, q_j) = \bar{C} \bar{X}(s, q_j) \]  \hspace{1cm} (7)

From Eq.(6)

\[ (s \bar{I} - \bar{A}) \bar{X}(s, q_j) = \bar{B} \bar{W}(s) \]

\[ \bar{X}(s, q_j) = (s \bar{I} - \bar{A})^{-1} \bar{B} \bar{W}(s) \]

\[ \bar{Y}(s, q_j) = \bar{C} (s \bar{I} - \bar{A})^{-1} \bar{B} \bar{W}(s) \]

\[ = \bar{G}(s, q_j) \bar{W}(s) \]

where

\[ \bar{G}(s, q_j) = \frac{\bar{Y}(s, q_j)}{\bar{W}(s)} = \bar{C}(s \bar{I} - \bar{A})^{-1} \bar{B} \]

is called the transfer function. Then the complex semi-logarithmic sensitivity coefficient in the s-domain is given as

\[ \bar{U}(\bar{Y}/q_j, s) = \frac{\partial \bar{Y}(s, q_j)}{\partial \ln q_j} \]

\[ = \bar{W}(s) \frac{\partial \bar{G}(s, q_j)}{\partial \ln q_j} \]

\[ = \frac{\bar{Y}(s, q_j)}{\bar{G}(s, q_j)} \frac{\partial \bar{G}(s, q_j)}{\partial \ln q_j} \]
\[ \frac{\partial \bar{G}(s, q_j)}{\partial G(s, q_j)} \]

or

\[ \bar{U}(\bar{Y}/q_j, s) = \bar{Y}(s, q_j) \frac{\partial \ln \bar{G}(s, q_j)}{\partial \ln q_j} \quad (8) \]

Eq. (8) is referred to as the complex sensitivity equation.
Fig. 1
III. Cam-and-Follower system with a Backlash

Fig. 1 (a) shows the equivalent dynamical system of a cam and follower system with backlash. \( K_1 \) is the equivalent spring constant. \( C_1 \) is the internal damping. \( K_2 \) is external spring constant. \( M \) is the equivalent mass of follower linkage. \( X_0 \) is the backlash. Fig. 1 (b) shows the free-body diagram of the system before the follower is in contact with the cam. Fig. 1 (c) shows the free-body diagram when the follower is in contact with the cam. The displacement equation for the simple harmonic motion is

\[
y = \frac{H}{2} \left(1 - \cos \frac{\pi t}{T}\right)
\]

where \( H \) is the total rise of follower. \( T \) is the period of cam rotation.

The displacement equation for the cycloidal motion is

\[
y = \frac{H}{2} t - \frac{H}{2\pi} \sin \frac{2\pi t}{T}
\]

The excitation is applied to the system as a pulse.

The data used here are

\[
\begin{align*}
M &= 0.00329 \text{ lb-sec}^2/\text{inch} \\
C_1 &= 2.45 \text{ lb-sec/inch} \\
H &= 0.5 \text{ inch} \\
K_1 &= 32100 \text{ lb/inch} \\
K_2 &= 850 \text{ lb/inch} \\
X_0 &= 0.0005, 0.0055, 0.0105, 0.015 \text{ inch}
\end{align*}
\]
From Fig. 1 (c), the equation of motion after the cam and follower is in contact may be expressed as

\[ M\ddot{Z} + C_1 \dot{Z} + (K_1 + K_2)Z = C_1 \dot{Y} + K_4 Y \]

where \( Z \) is the absolute displacement response of the mass.

The sensitivity coefficient of parameter \( q_j \) in time domain is

\[ U_j = \frac{\partial Z}{\partial \ln q_j} \]

Differentiating the above equation with respect to \( \ln C_1, \ln K_1, \ln M, \) and \( \ln K_2 \), we obtain the following sensitivity equations:

1. For parameter \( C_1 \)

\[ \ddot{U}_1 + \frac{C_1}{M} \dot{U}_1 + \frac{K_1 + K_2}{M} U_1 = \frac{C_1}{M} (\dot{Y} - \dot{Z}) \] (10)

2. For parameter \( K_1 \)

\[ \ddot{U}_2 + \frac{C_1}{M} \dot{U}_2 + \frac{K_1 + K_2}{M} U_2 = \frac{K_2}{M} (Y - Z) \] (11)

3. For parameter \( M \)

\[ \ddot{U}_3 + \frac{C_1}{M} \dot{U}_3 + \frac{K_1 + K_2}{M} U_3 = \frac{K_1}{M} Y + \frac{C_1}{M} (\dot{Y} - \dot{Z}) - \frac{K_1 + K_2}{M} Z \] (12)
4. For parameter $K_2$

$$\ddot{U}_4 + \frac{C_1}{M} \dot{U}_4 + \frac{K_1 + K_2}{M} U_4 = -\frac{K_2}{M} Z$$

(13)

The equation of motion before the cam and follower is in contact can be written as

$$M \ddot{Z} + K_2 Z = 0$$

(14)

The sensitivity equations for parameters are

$$\ddot{U}_1 + \frac{K_2}{M} U_1 = 0$$

(15)

$$\ddot{U}_2 + \frac{K_2}{M} U_2 = 0$$

(16)

$$\ddot{U}_3 + \frac{K_2}{M} U_3 = \frac{K_2}{M} Z$$

(17)

$$\ddot{U}_4 + \frac{K_2}{M} U_4 = -\frac{K_2}{M} Z$$

(18)

Solving these equations by digital computer programming with Runge-Kutta numerical scheme, the results may be plotted in graphical forms.

Fig. 2 through 5 and Fig. 6 through 9 show the response $Z$ and the sensitivity coefficients ($U_2$, $U_3$, $U_4$) of $Z$ to parameters ($K_1$, $M$, $K_2$) for a simple harmonic motion excitation and for a cycloidal motion excitation with different values of clearance $X_0$. From these plots, it
is obvious that the spring stiffness parameters $K_1$ and $K_2$ have a dominant effect upon the displacement output of the follower mass when the clearance $X_0$ is 0.0005 inch. Under this situation, change in $K_1$ or $K_2$ causes an appreciable position error in the response and consequently, causes variations in the acceleration pattern. Other system parameters such as $C_1$ and $M$ are having very small effect so far as the response in the follower displacement during the period of excitation is concerned. While in the residual part ($t > T$), $M$ has a dominant effect in a simple harmonic motion excitation. This implies that when an excessive output displacement error occurs, it is the parameters $K_1$ and/or $K_2$ rather than $C_1$ and/or $M$ that should be examined. When the clearance $X_0$ increases, the parameter of follower mass or the sensitivity $U_3$ becomes more significant. As shown in Fig. 5, when the clearance $X_0$ is equal to 0.015 inch, $U_3$ plays a more dominant role than other sensitivity coefficients. Observations show that the trend of sensitivity analysis is the same for a system with backlash subjected to both harmonic and cycloidal excitations. We can further plot the ratio of $U_2$ (or $U_3$) with finite backlash to $U_2$ (or $U_3$) with zero backlash versus time for the two cases of excitation—harmonic and cycloidal. Fig. 10 and Fig. 11 show such plots for the case of $X_0 = 0.0055$ inch, and $X_0 = 0.0105$ inch. It is apparent that the variation for harmonic motion is bigger than that for cycloidal motion with
the same backlash value $X_0$. This implies that a small variation in parameters for harmonic motion will result in more violent response than for cycloidal motion. In other words, it is better to use cycloidal cam in order to get more accurate displacement response of the follower at high speeds.
$X_0 = 0.0005 \text{ in.}$
$x_0 = 0.0055 \text{ in.}$

![Graph showing DEHIL, (INCHES) vs. TIME (x 10^{-2} \text{SEC}) for different values of $U_2$, $U_3$, and $U_4$.]
Fig. 4

$x_0 = 0.0105$ in.
$x_0 = 0.015 \text{ in.}$
$X_0 = 0.0005$ in.

Fig. 6
$X_0 = 0.0055 \text{ in.}$

Fig. 7
$x_0 = 0.0105 \text{ in.}$

Fig. 8
$X_0 = 0.015$ in.

Fig. 9
Backlash  Harmonic  Cycloidal
$X_0 = 0.0055 \text{ in.}$  
$X_0 = 0.0105 \text{ in.}$

Fig. 11
Fig. 12
IV. Cam-and-Follower System With Coulomb Friction

Fig. 12 (a) is the equivalent system of a cam-follower mechanism containing viscous damping and Coulomb friction. C is the coefficient of viscous damping. \( \alpha \) is the coefficient of Coulomb friction. Fig. 12 (b) shows the free-body diagram of the system. The numerical data assumed are

\[
M = 0.00329 \text{ lb-sec}^2/\text{in} \\
C = 2.45 \text{ lb-sec/in} \\
K = 32100 \text{ lb/in}
\]

\( \alpha \) is to vary

The equation of motion (when \( Y > Z \)) is

\[
M \ddot{Z} = K(Y - Z) + C(\dot{Y} - \dot{Z}) + \alpha \frac{|\dot{Y} - \dot{Z}|}{\dot{Y} - \dot{Z}} \tag{19}
\]

Using the nondimensional parameters

\[
\omega_n = \sqrt{\frac{K}{M}} \\
\zeta = \frac{C}{2M\omega_n}
\]

Eq. (19) becomes

\[
\ddot{Z} + 2\zeta_1 \omega_n \dot{Z} + \omega_n^2 Z = 2\zeta_1 \omega_n \dot{Y} + \omega_n^2 Y + \alpha \frac{\omega_n^2}{K} \frac{|\dot{Y} - \dot{Z}|}{\dot{Y} - \dot{Z}} \tag{20}
\]

The sensitivity equations are

1. For parameter \( \zeta \)

\[
\ddot{U}_1 + 2\zeta_1 \omega_n \dot{U}_1 + \omega_n^2 U_1 = 2\zeta_1 \omega_n \dot{Y} - 2\zeta_1 \omega_n \dot{Z} \tag{21}
\]
2. For parameter $\omega_n$

$$\ddot{U}_2 + 2\zeta \omega_n \dot{U}_2 + \omega_n^2 U_2 = 2\zeta \omega_n \dot{Y} + 2\omega_n^2 \ddot{Y} - 2\zeta \omega_n \ddot{Z}$$

$$- 2\omega_n^2 Z + \alpha \frac{2\omega_n^2}{k} \frac{\dot{Y} - \dot{Z}}{\ddot{Y} - \ddot{Z}} \tag{22}$$

Solving equation (20), (21), and (22) by the same method we did for the previous problem, we can plot the results in graphical form.

Fig. 13 through 18 and Fig. 19 through 24 show the curves of response $Z$ and the sensitivity coefficients $U_1$, $U_2$ of $Z$ to parameters $\zeta$, $\omega_n$ for a simple harmonic motion excitation and for a cycloidal motion excitation with different values of $\alpha$. From these plots, it is apparent that $\omega_n$ is to have a dominant effect on the displacement response of the system. It tends to decrease the displacement response of the follower during the period ($0 < t < T$) for both the harmonic and cycloidal excitations. However, it tends to increase the residual vibratory response of the follower for cycloidal excitation so long as value $\alpha$ is less than 17,000. When $\alpha$ is getting larger, say, when $\alpha$ reaches 40,000, there are no marked difference in the pattern of system displacement response between the case of the harmonic motion input and the cycloidal motion input. This is depicted in Fig. 18 and Fig. 24. When $\alpha$ is large, the sensitivity coefficients and the displacement response become more or less a constant during the
period $t > T$. Furthermore, in order to investigate the variations of the parameter of natural frequency $\omega_n(U_2)$, we again plot the ratio of $U_2$ with a finite value of $\alpha$ to $U_2$ with $\alpha = 1000$ versus time, as shown in Fig. 25 and Fig. 26. It can be seen from these plots that the only difference in the ratio of $U_2$ between the types of input motions — the harmonic and the cycloidal, is during the period of $t > T$. That means the variation of Coulomb damping has more influence on the cycloidal motion. Also, as $\alpha$ increase to 8,000, there is no residual oscillation for harmonic motion although there still is residual oscillation for cycloidal motion. Therefore, it is quicker to reach steady state for harmonic motion at small value of $\alpha$. That is to say, it is better to use harmonic motion to avoid residual oscillation for small Coulomb damping, other conditions being equal.
$\alpha = 1000$

Fig. 13
\( \alpha = 2000 \)

**Fig. 14**
Fig. 15
$\alpha = 8000$

Fig. 16
Fig. 17

α = 9000

DISPL. (INCHES)

0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5

0.4 0.8 1.2 1.6 2.0 2.4 2.8

TIME (×10^{-2} SEC.)

u_1

u_2
Fig. 18
Fig. 19
Fig. 20

\( \alpha = 2000 \)

Displ. (Inches)

Time \((\times 10^{-2}\text{ Sec.})\)
\( \alpha = 3000 \)

**Fig. 21**
Fig. 22

\[ \alpha = 3000 \]
Fig. 23

\[ \alpha = 9000 \]

Displ. (inches)

TIME (x10^{-2} SEC.)

-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5

0
0.8
1.2
1.6
2.0
2.4
2.8

U_1

U_2
Fig. 24
Fig. 26

- $\alpha = 2000$
- $\alpha = 3000$
- $\alpha = 4000$
- $\alpha = 8000$
- $\alpha = 10000$
- $\alpha = 40000$

$U_2(\alpha \neq 1000)$

Time (x 10^-2 sec.)
Bibliography


Appendix

Computer Program Listing

This program is to calculate the response $Z$ and sensitivity coefficients $u_1$, $u_2$, $u_3$, $u_4$ in a cam-and-follower system with three different values of backlash $x_0$. Each second-order differential equation can be expressed by two function subprograms, then calculate the values through one subroutine RK which is a Runge-Kutta numerical integration subroutine. These differential Eqs to be solved are as follows:

Before the cam and follower is in contact

\[ \ddot{z} + \frac{K_2}{M} z = 0 \]

As FUN1, FUN2

\[ \ddot{u}_1 + \frac{K_2}{M} u_1 = 0 \]

As FUN3, FUN4

\[ \ddot{u}_2 + \frac{K_2}{M} u_2 = 0 \]

As FUN5, FUN6

\[ \ddot{u}_3 + \frac{K_2}{M} u_3 = \frac{K_2}{M} z \]

As FUN7, FUN8

\[ \ddot{u}_4 + \frac{K_2}{M} u_4 = -\frac{K_2}{M} z \]

As FUN9, FUN10

After the cam and follower is in contact

\[ \ddot{z} + \frac{K_1 + K_2}{M} z = \frac{C_1}{M} \ddot{y} + \frac{K_1}{M} y \]

As FUN1, FUN22

\[ \ddot{u}_1 + \frac{C_1}{M} \ddot{u}_1 + \frac{K_1 + K_2}{M} u_1 = \frac{C_1}{M} (\ddot{y} - \dot{z}) \]

As FUN3, FUN44

\[ \ddot{u}_2 + \frac{C_1}{M} \ddot{u}_2 + \frac{K_1 + K_2}{M} u_2 = \frac{K_1}{M} (y - z) \]

As FUN5, FUN66

\[ \ddot{u}_3 + \frac{C_1}{M} \ddot{u}_3 + \frac{K_1 + K_2}{M} u_3 = \frac{K_1}{M} y + \frac{C_1}{M} (\ddot{y} - \dot{z}) \]

As FUN7, FUN88

\[ \ddot{u}_4 + \frac{C_1}{M} \ddot{u}_4 + \frac{K_1 + K_2}{M} u_4 = -\frac{K_2}{M} z \]

As FUN9, FUN100
REAL K1, K2, M
EXTERNAL FUN1, FUN2, FUN3, FUN4, FUN5, FUN6, FUN7, FUN8,
FUN9, FUN10, FUN22, FUN44, FUN66, FUN88, FUN100
DIMENSION T(600), Z(600), ZDT(600), ZDDT(600), U1(600),
U1DT(600), U1DDT(600), U2(600), U2DT(600), U2DDT(600),
U3(600), U3DT(600), U3DDT(600), U4(600), U4DT(600),
U4DDT(600), Y(600), YD(600), X(600), VET(3000), VEC(3000),
VED(3000)
DIMENSION XL(12), YL(12), YLD(12), YLZ(12), YLZD(12),
YL1(12), YL1D(12), YL2(12), YL2D(12), YL3(12), YL3D(12),
YL4(12), YL4D(12)
COMMON A, B, H, C1, K1, K2, M, PERI, Z1, Z1D

ENTER GIVEN DATA

READ(5,4) M, K1, C1, K2, H, RPM
 FORMAT(6F11.5)

ENTER THE LABELS FOR PLOTS

READ(5,5) XL, YL, YLD, YLZ, YLZD, YL1, YL1D, YL2, YL2D, YL3,
YL3D, YL4, YL4D
 FORMAT(6(12A1)/6(12A1)/(12A1))

SET INITIAL BACKLASH XO

X0=0.0005

SET THE NUMBER OF POINTS FOR CALCULATION AND
INCREMENT FOR RUNGE-KUTTA NUMERICAL SCHEME

N=600
D=0.00005
PERI=60.0/RPM
A=C1/M
B=(K1+K2)/M

SET INITIAL VALUES

T(1)=0.0
Z(1)=0.0
ZDT(1)=0.0
U1(1)=0.0
U1DT(1)=0.0
U2(1)=0.0
U2DT(1)=0.0
U3(1)=0.0
FOR HARMONIC MOTION PULSE

DO 100 I=1,J

Y(I)=((1.0-COS(3.14159*T(I)/PERI)))*(H/2.0)
X(I)=Y(I)-Z(I)

IF(X(I) .LT. XO) GO TO 10

CALL RK(FUN1,FUN2,T(I),YI,UI,D,Z1,Z1D)
CALL RK(FUN3,FUN4,T(I),Y1I,U1I,D,UU1,UU1D)
CALL RK(FUN5,FUN6,T(I),Y2I,U2I,D,UU2,UU2D)
CALL RK(FUN7,FUN8,T(I),Y3I,U3I,D,UU3,UU3D)
CALL RK(FUN9,FUN10,T(I),Y4I,U4I,D,UU4,UU4D)

GO TO 20

10 CALL RK(FUN1,FUN2,T(I),YI,UI,D,Z1,Z1D)
CALL RK(FUN3,FUN4,T(I),Y1I,U1I,D,UU1,UU1D)
CALL RK(FUN5,FUN6,T(I),Y2I,U2I,D,UU2,UU2D)
CALL RK(FUN7,FUN8,T(I),Y3I,U3I,D,UU3,UU3D)
CALL RK(FUN9,FUN10,T(I),Y4I,U4I,D,UU4,UU4D)

GO TO 20

20 T(I+1)=T(I)+D
Z(I+1)=Z1
ZDT(I+1)=Z1D
U1(I+1)=UU1
U1DT(I+1)=UU1D
U2(I+1)=UU2
U2DT(I+1)=UU2D
U3(I+1)=UU3
U3DT(I+1)=UU3D
U4(I+1)=UU4
U4DT(I+1)=UU4D
T(I+1)=T(I)+D
YI=Z1
UI=Z1D
Y1I=UU1
U1I=UU1D
Y2I=UU2
U2I=UU2D
Y3I=UU3
U3I=UU3D
Y4I=UU4
U4I=U4D

CONTINUE

CALCULATE THE ACCELERATIONS

DO 200 I=1,N
YD(I)=(H*3.14159/(2.0*PERI))*SIN(3.14159*T(I)/PERI)
X(I)=Y(I)-Z(I)
IF(X(I).LT.XO) GO TO 30
IF(T(I).GT.PERI) GO TO 40

\( \Delta DDT(I) = \frac{(K1/M)*YD(I)+(K1/M)*Y(I)-B*Z(I)-A*ZDT(I)}{M} \)

\( U1DDT(I) = \frac{(K1/M)*(YD(I)-2*ZDT(I))-B*U1(I)-A*U1DT(I)}{M} \)

\( U2DDT(I) = \frac{(K1/M)*(Y(I)-Z(I))-B*U2(I)-A*U2DT(I)}{M} \)

\( U3DDT(I) = \frac{(K1*Y(I)+C1*YD(I)-(K1+K2)*Z(I)-C1*ZDT(I))/M}{1-B*U3(I)} \)

\( U4DDT(I) = -(K2/M)*Z(I)-B*U4(I)-A*U4DT(I) \)

GO TO 200

ZDDT(I)=(K1/M)*H-B*Z(I)-A*ZDT(I)

U1DDT(I)=-(K1/M)*ZDT(I)-B*U1(I)-A*U1DT(I)

U2DDT(I)=(K1/M)*(H-Z(I))-B*U2(I)-A*U2DT(I)

U3DDT(I)=(K1*H-(K1+K2)*Z(I)-C1*ZDT(I))/M-B*U3(I)-

A*U3DT(I)

U4DDT(I)=-(K2/M)*Z(I)-B*U4(I)-A*U4DT(I)

GO TO 200

ZDDT(I)=-(K2/M)*Z(I)

U1DDT(I)=-(K2/M)*U1(I)

U2DDT(I)=-(K2/M)*U2(I)

U3DDT(I)=(K2/M)*Z(I)-(K2/M)*U3(I)

U4DDT(I)=-(K2/M)*Z(I)-(K2/M)*U4(I)

CONTINUE

PLOT THE CALCULATED VALUES

DO 300 I=1,N

VET(I)=T(I)

VET(I+N)=T(I)

VET(I+2*N)=T(I)

VET(I+3*N)=T(I)

VET(I+4*N)=T(I)

VEC(I)=Z(I)

VEC(I+N)=U1(I)

VEC(I+2*N)=U2(I)

VEC(I+3*N)=U3(I)

VEC(I+4*N)=U4(I)

VED(I)=ZDDT(I)

VED(I+N)=U1DDT(I)

VED(I+2*N)=U2DDT(I)

VED(I+3*N)=U3DDT(I)

VED(I+4*N)=U4DDT(I)

CONTINUE
WRITE(6,6)
FORMAT(1X,'TIME',15X,'Z',20X,'U1',20X,'U2',
120X,'U3',20X,'U4')
WRITE(6,7) (T(I),Z(I),U1(I),U2(I),U3(I),U4(I),I=1,N)
WRITE(6,8)
FORMAT(1X,'TIME',15X,'ZDDT',15X,'U1DDT',15X,
1'U2DDT',15X,'U3DDT',15X,'U4DDT')
WRITE(6,7) (T(I),ZDDT(I),U1DDT(I),U2DDT(I),U3DDT(I),
1U4DDT(I),I=1,N)
CALL MMPLLOT(T,Z,N,1,XL,YLZ)
CALL MMPLLOT(T,U1,N,1,XL,YL1)
CALL MMPLLOT(T,U2,N,1,XL,YL2)
CALL MMPLLOT(T,U3,N,1,XL,YL3)
CALL MMPLLOT(T,U4,N,1,XL,YL4)
CALL MMPLLOT(VET,VEC,N,5,XL,YL)
IF(XO .GE. 0.01) GO TO 90
XO=XO+0.005
GO TO 999
STOP
END

SUBROUTINE RK(FUNA,FUNB,X,Y,U,H,VEY,VEU)
REAL K1,K2,K3,K4
H2=H/2.0
K1=H*FUNA(X,Y,U)
T1=H*FUNB(X,Y,U)
K2=H*FUNA(X+H2,Y+K1/2.0,U+T1/2.0)
T2=H*FUNB(X+H2,Y+K1/2.0,U+T1/2.0)
K3=H*FUNA(X+H2,Y+K2/2.0,U+T2/2.0)
T3=H*FUNB(X+H2,Y+K2/2.0,U+T2/2.0)
K4=H*FUNA(X+H,Y+K3,U+T3)
T4=H*FUNB(X+H,Y+K3,U+T3)
VEY=Y+(K1+2.0*K2+2.0*K3+K4)/6.0
VEU=U+(T1+2.0*T2+2.0*T3+T4)/6.0
RETURN
END

FUNCTION FUN1(T,Z,ZDT)
REAL K1,K2,M
FUN1=ZDT
RETURN

BEFORE THE CAM AND THE FOLLOWER IS IN CONTACT
FUNCTION FUN2(T,Z,ZDT)
REAL K1,K2,M
COMMON A,B,H,C1,K1,K2,M,PERI,Z1,Z1D
FUN2=-K2*Z/M
RETURN
END

FOR PARAMETER C1 WITHOUT CONTACT

FUNCTION FUN3(T,U1,U1DT)
REAL K1,K2,M
FUN3=U1DT
RETURN
END

FUNCTION FUN4(T,U1,U1DT)
REAL K1,K2,M
COMMON A,B,H,C1,K1,K2,M,PERI,Z1,Z1D
FUN4=-(K2/M)*U1
RETURN
END

FOR PARAMETER K1 WITHOUT CONTACT

FUNCTION FUN5(T,U2,U2DT)
REAL K1,K2,M
FUN5=U2DT
RETURN
END

FUNCTION FUN6(T,U2,U2DT)
REAL K1,K2,M
COMMON A,B,H,C1,K1,K2,M,PERI,Z1,Z1D
FUN6=-(K2/M)*U2
RETURN
END

FOR PARAMETER M WITHOUT CONTACT

FUNCTION FUN7(T,U3,U3DT)
REAL K1,K2,M
COMMON A,B,H,C1,K1,K2,M,PERI,Z1,Z1D
FUN7=U3DT
RETURN
END

FUNCTION FUN8(T,U3,U3DT)
REAL K1,K2,M
COMMON A,B,H,C1,K1,K2,M,PERI,Z1,Z1D
FUN8=K2*Z1/M-(K2/M)*U3
RETURN
END
FOR PARAMETER K2 WITHOUT CONTACT

FUNCTION FUN9(T,U4,U4DT)
REAL K1,K2,M
FUN9=U4DT
RETURN
END

FUNCTION FUN10(T,U4,U4DT)
REAL K1,K2,M
COMMON A,B,H,C1,K1,K2,M,PERI,Z1,Z1D
FUN10=-(K2/M)*Z1-(K2/M)*U4
RETURN
END

AFTER THE CAM AND THE FOLLOWER IS IN CONTACT

FUNCTION FUN22(T,Z,ZDT)
REAL K1,K2,M
COMMON A,B,H,C1,K1,K2,M,PERI,Z1,Z1D

Y=(H/2.0)*(1.0-COS(3.14159*T/PERI))
YD=(H*3.14159/(2.0*PERI))*SIN(3.14159*T/PERI)

F4=(C1/M)*YD+(K1/M)*Y
IF(T.GE.PERI) F4=(K1/M)*H
FUN22=F4-B*Z-A*ZDT
RETURN
END

FOR PARAMETER C1 AFTER THE CAM AND THE FOLLOWER IS IN CONTACT

FUNCTION FUN44(T,U1,U1DT)
REAL K1,K2,M
COMMON A,B,H,C1,K1,K2,M,PERI,Z1,Z1D

YD=(H*3.14159/(2.0*PERI))*SIN(3.14159*T/PERI)
F4=(C1/M)*(YD-Z1D)
IF(T.GE.PERI) F4=-C1/M)*Z1D
FUN44=F4-B*U1-A*U1DT
RETURN
END

FOR PARAMETER K1 AFTER THE CAM AND THE FOLLOWER IS IN CONTACT

FUNCTION FUN66(T,U2,U2DT)
REAL K1,K2,M
COMMON A,B,H,C1,K1,K2,M,PERI,Z1,Z1D

Y=(H/2.0)*(1.0-COS(3.14159*T/PERI))
F6=(K1/M)*(Y-Z1)
IF(T .GE. PERI) F6=(K1/M)*(H-Z1)
FUN66=F6-B*U2-A*U2DT
RETURN
END

FOR PARAMETER M AFTER THE CAM AND THE FOLLOWER IS IN CONTACT

FUNCTION FUN88(T,U3,U3DT)
REAL K1,K2,M
COMMON A,B,H,C1,K1,K2,M,PERI,Z1,Z1D
Y=(H/2.0)*(1.0-COS(3.14159*T/PERI))
YD=(H*3.14159/(2.0*PERI))*SIN(3.14159*T/PERI)
F8=K1*Y+C1*YD
IF(T .GE. PERI) F8=K1*H
FUN88=(F8-(K1+K2)*Z1-C1*Z1D)/M-B*U3-A*U3DT
RETURN
END

FOR PARAMETER K2 AFTER THE CAM AND THE FOLLOWER IS IN CONTACT

FUNCTION FUN100(T,U4,U4DT)
REAL K1,K2,M
COMMON A,B,H,C1,K1,K2,M,PERI,Z1,Z1D
FUN100=-(K2/M)*Z1-B*U4-A*U4DT
RETURN
END
Sensitivity Analysis of Cam-and-Follower Mechanism at high speeds

In order to improve the dynamic performance of a cam mechanism at high speeds, it is desired to have a method that does not require a trial-and-error procedure based on total system re-analysis or resynthesis. In this paper, sensitivity coefficients utilizing the response of the system output derivatives with respect to the system design parameters are used to investigate the cam mechanism dynamics.