ANALYSIS OF RIB-PLATE RESPONSE 
TO EXTERNAL LOADING

A Thesis Presented To 
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In Partial Fulfillment 
Of The Requirement For The Degree 
Master Of Science

By
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Laboratory results were obtained under a concentrated load and under a constant bending moment for a corrugation and rib configuration. There were three different configurations of rib-plate combination - no rib, one rib, and two ribs. The electric strain gages were installed on rib and plate to measure strain. Dial gages were mounted to measure deflection along the longitudinal direction. Load was applied by means of 40 kip, GILMORE loading machine.

The composite and non-composite response were studied thoroughly. The section response to the external load varies with the loading. The degree of performance is a ideal measurement to differentiate each individual response. In addition, both normal stress and shear stress transfer has a relation with composite response. Experimental data analysis also discussed about principal stress direction and the stress distribution around the bolt connection.
The theoretical analysis of the corrugation is based on the orthotropic plate theory. Deflections are in agreement with the experimental displacements. The theoretical stresses are close to experimental data in no-rib case. Simply-supported end condition and assumption of noncomposite response is a good approach to one-rib and two-ribs configurations.
DEVELOPMENT OF THE CORRUGATION

1.1 THE HISTORY AND APPLICATION OF THE CORRUGATION

The invention of the corrugated structure occurred in seventeen century. As a result of the development of metal products and fabrication techniques, the corrugation has gained corrosion resistance and can be designed to conform to safety objectives. In recent years, there has been successful investigations of and development of site-assembly techniques. The application of corrugated structure has rapidly increased in highway accessory construction and building construction.

Corrugated metal culvert pipe was the first important application to highway accessory construction, introduced by Watson (1) in 1896.
Culverts and subdrains are highway accessory structures that use corrugation extensively. Disposal of water in roadside ditches is largely accomplished by means of culverts, which are transverse conduits used under road to pass through road embankments; see Figures 1.1 and 1.2.

Figure 1.1 Corrugated Steel Pipe Culvert Conveying Runoff From Winter's Snow (1)

Figure 1.2 An Aluminum Box Culvert is Used to Pass Water Through Under the Roadway (20)
A comparison is drawn between two types culvert: a corrugated pipe culvert and a box culvert. Both are made to order and stocked in standard gages, diameters and lengths for ready delivery to site and assembled on site. Sections may be welded or bolt together; see Figures 1.3 and 1.4. The first corrugated culverts were small, seldom exceeding 3 ft or 4 ft in diameter. Today, corrugated steel pipe is furnished in diameters up to 10 ft, and the box culvert can span more than 20 ft. They are frequently reinforce with ribs. The box is be formed to a trapezoidal or plate culvert.

Figure 1.3 Spot Welding Corrugated Steel Pipe

Figure 1.4 Spot Bolt Connection in Box Culvert
Corrugated sheets have been used in building construction since about 1784. It is one of the oldest type of formed metal products. Today, there are many other types of corrugated plates with different coatings being produced by manufacturers. Several standard corrugated sheets are generally available for building construction. Corrugated sheets are frequently used for roofing and siding in buildings because the sheets are strong, lightweight, and easy to erect. There were over 100 such roof forms built for a variety of applications to school buildings, churches and halls which are ideally suited for such architecturally pleasing construction. In many cases, corrugated sheets are also used as shear diaphragms to replace conventional bracing and to stabilize entire structures or individual members such as columns and beams; see Figure 1.5. Corrugated sheets are also used in flooring systems for building and bridge construction. These products have also been used as web elements for built-up girders in order to increase web stiffness instead of using a relatively thicker plate or to use a thin web with stiffeners. The Macomber Panlweb girder shown in Figure 1.6 consists of 14 gage to 9 gage corrugated web for depths from 20 to 40 in.

![Figure 1.5 Shear Diaphragms (2)]
1.2 THE STANDARD CORRUGATION

Several types corrugation are common used in engineering industry, varying from trapezoidal-type corrugated sheets to arc-and-tangent type corrugated sheets; see Figures 1.7 and 1.8.
The standard corrugated sheets have arc and tangent-type corrugation. The corrugation profiles are represented by circle arcs connected by tangents; referring to Figure 1.7. Strength satisfied to meet loading requirements and ductility necessary for forming requirements are important metal parameters. Chemical composition of steels and aluminum is specified by AASHO (3) for culverts and under drains. Structural components must correspond to ASTM designation. Certain simplified formulas for computing the sectional properties of standard corrugated sheets can be used in design. Following an investigation conducted by American Iron and Steel Institute during 1955-1957, a publication entitled "Sectional Properties of Corrugated Steel Sheets" was issued by the Institute in 1964 to provide necessary design information for standard corrugated sheets.

1.3 RECENT RESEARCH

Since the invention of corrugated structure, it has been applied to a variety of industry situations. Engineers and researchers have done many investigations on projects which were related to the corrugated configuration. These results are published in three different ways based on each researcher's emphasis: determination of sectional properties, laboratory and field investigation, numerical analysis, and theoretical analysis.
1.4 OBJECTIVES OF STUDY

The Galerkin method was developed for a corrugated plate. Corrugated plate was instrumented and the behavior of the plate was monitored under a static incremental load. The comparison was made between experimental data and theoretical solutions. Composite and non-composite study of rib and plate interaction was also presented. The following points are mainly concerned in this study.

1). Discussion about the reaction of rib and plate, and their connections during the loading

2). Discussion about the distribution on the transverse cross section and principal stress direction.

3). Present the comparison of theoretical and experimental results.
2.1 SECTION PROPERTIES OF CORRUGATED SHEETS

Based on H. B. Blodgett's investigation (11) for the standard corrugation (arc-and-tangent-type), D.S. Wolford (15) simplified the formula of the moment of inertia by extensive calculations giving numerical values for the complex factors contained in the formulas. The factors are shown as curves. In computation, design and tables can be used to determine factors $C_5$ and $C_6$ in Equations (2.1) and (2.2):

$$I = C_5bt^3 + C_6bd^2t \quad (2.1)$$

$$S = \frac{2I}{b+t} \quad (2.2)$$
For a given standard corrugated sheet, the cross sectional properties can be determined from a given set of dimensions. It is always necessary to know $t$, $p$, and $d$, and one other of the quantities $\alpha, \gamma, m$, or $m$ to determine the cross sectional properties of the arc-and-tangent corrugation. The following steps are suggested by the writer when using Wolfrord's charts.

a) Compute the mid-thickness radius

$$r = R + \frac{t}{2}$$

b) Calculate value $q$ and $k$

$$q = \frac{r}{d} \quad \text{and} \quad k = \frac{p}{d}$$

which $p$=pitch of the corrugation

c) Find web angle $\alpha$ from $q$ and $d$ in Figure 2.2, $q$ and $k$ is given in step(b)

d) Determine $C_5$ in Figure 2.3 and $C_6$ in Figure 2.4 by using $\alpha$ and computed value $k$

e) Determine $\lambda$ in Figure 2.5 and the ratio $u = m/d$ in Figure 2.6 from $k$
and $\alpha$

f) Calculate value $I$ and $S$ in Equations (2.1) and (2.2)

g) Find the radius of gyration $\rho$

$$\rho = \sqrt{\frac{I}{A}}$$

where, $A = \lambda bt$

h) Determine the length of tangent

$L = d \cdot (m/d) = d \cdot u$

![Figure 2.2](image)

**Figure 2.2** Radius-Depth Ratio is Plotted Against Pitch-depth Ratio at Various Web Angles (5)

![Figure 2.3](image)

**Figure 2.3** Factor $C_5$ is Plotted Against Pitch-Depth Ratio at Various Web Angles (5)
Let the corrugation shown in Figure 2.1 is as same as the experimental one in laboratory test, which

- pitch \( p = 9 \text{ in.} \),
- width \( b = 27 \text{ in.} \) (projecting width not the developed width)
- depth \( d = 2.5 \text{ in.} \),
- inside radius \( R = 2.31 \text{ in.} \),
- thickness \( t = 0.2 \text{ in.} \).
Following the above procedures, we have
\( r=2.41, q=0.96 \) and \( k=3.6 \)

By using Wolford's Charts, it is easy to obtain,
\( \alpha=38^\circ, C_5=0.121, C_6=0.138, \lambda=1.17 \) and \( u=1.1 \)

Equations (2.1) and (2.2) give,
\[
I=0.121 \times 27 \times 0.2^3 + 0.138 \times 27 \times 2.5^2 \times 0.2 = 4.51
\]
\[
S=3.34, \quad A=6.3, \quad L=2.75
\]

Based on the method outlined above, some of the sectional properties for galvanized and uncoated corrugated sheets are reproduced in standard table forms. These forms have been verified by AISI or supplied by manufacturers and are more convenient for engineers. Table 2.1 is one of these typical formats.

<table>
<thead>
<tr>
<th>Sheet thickness, p=6 in., D=2 in.</th>
<th>Uncoated</th>
<th>A, in²</th>
<th>I, in⁴</th>
<th>S, in³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.2758</td>
<td>4.119</td>
<td>1.990</td>
<td>1.749</td>
</tr>
<tr>
<td>3</td>
<td>0.2451</td>
<td>3.658</td>
<td>1.754</td>
<td>1.562</td>
</tr>
<tr>
<td>5</td>
<td>0.2145</td>
<td>3.199</td>
<td>1.523</td>
<td>1.376</td>
</tr>
<tr>
<td>7</td>
<td>0.1838</td>
<td>2.739</td>
<td>1.296</td>
<td>1.187</td>
</tr>
<tr>
<td>8</td>
<td>0.1644</td>
<td>2.449</td>
<td>1.154</td>
<td>1.066</td>
</tr>
<tr>
<td>10</td>
<td>0.1345</td>
<td>2.003</td>
<td>0.938</td>
<td>0.879</td>
</tr>
<tr>
<td>12</td>
<td>0.1046</td>
<td>1.556</td>
<td>0.725</td>
<td>0.689</td>
</tr>
</tbody>
</table>

Noted: \( p=\)Corrugation Pitch, \( D=\)Corrugation Depth
2.2 LABORATORY AND FIELD INVESTIGATION

Recent laboratory test and field investigation were performed by Raymond (4) and Tan (20) respectively.

LABORATORY INVESTIGATION

Raymond related critical corrugation dimensioned limits to critical bucking strain, minimum curving radius, and ultimate moment capacity. He conducted twenty-four flexural tests for arc-and-tangent corrugated profiles in laboratory. The specimens were measured 36 in. long and three or five corrugated wide. The properties of corrugation profile were measured by the following parameters: arc inside radius-to-thickness ratios (R/t), tangent length-to-thickness ratio (m/t), and material yield strengths (F_y); see Figure 2.7.

Figure 2.7 A Sample of Standard Corrugated Profile with Dimension

Since the number of the laboratory test was limited, the following conclusion was only for the parameter, 0.45 \leq m \leq 1.7R
(1) Nominal critical flexural strain; see Figure 2.8.

for $7.85R \leq m \leq 1.1R$

$$\varepsilon_{cr} = 7.85/(R/t_n)^2$$

(2.3)

for $0.45 \leq m \leq 1.7R$

$$\varepsilon_{cr} = 5.80/(R_i/t_n)^2$$

(2.4)

where, $\varepsilon_{cr}$ = nominal critical flexural strain,

$R/t_n = R/t$ normal by multiplying by $(F_y/33)^{1/2}$, and

$F_y$ = material yield strength in ksi.

Figure 2.8 Critical Bucking Strain for Standard Corrugations (4)

(2) Minimum curving radius

$$R_c = (d+t)/2\varepsilon_{cr}$$

(2.5)

where, $R_c$ = mean radius of curvature of the corrugation,

d = corrugation depth, and

t = material thickness.

(3) Ultimate moment capacity; see Figure 2.9.
$M_{\mu_c} = [1.429 - 0.156 \ln (m/t_n)] M_p \leq M_p$

Where,

$M_{\mu_c}$ = ultimate moment capacity,

$M_p$ = calculated plastic moment, and

$m/t_n$ = $m/t$ normalized by multiplying by $(F_y/33)^{1/2}$

---

**FIELD INVESTIGATION**

Civil Engineering Department, Ohio University, Athens conducted several box culvert projects in the field for ODOT (Ohio Department of Transportation). Tan's thesis (5), was mainly concerned with composite, non-composite in box culverts and interaction between soil and culverts. At begin of backfilling the experimental data were not in agreement between composite and non-composite action. However, the assumption of composite action showed improvement with an increase in load and appears to be very good for a 42 kip traffic load; see Figure 2.11. Moment
compares favorably with theoretical calculations with maximum moments at the center and haunches of the culvert. The analytical thrusts compare poorly with experimental values by underestimating at the crown but overestimating at the haunch. With regard of bolt connection, it is clear the bolt disrupts the stress distribution since the circumferential stresses are compressive and decrease in magnitude away from the bolt. Large stresses close to the bolt represent stress concentrations and possibly local forces.

Figure 2.10  Moment Comparisons with 42 kip Live Load at Center Position (20)
There is always another subject in the field investigation—the interaction of culvert and the surrounding soil. The common and successful tool for the analysis and design of buried culvert is Culvert Analysis And Design (CANDE), which is a FORTRAN program. This program, developed for the Federal Highway Administration, is based on finite element method. In addition to the various solution levels and option of design or analysis, this program has another executive routine that provides for the selection of common types of pipe culverts and five different constitutive soil models. In Tan (20), the analytical predictions tend to overestimate the haunch moments in the way that they underestimate crown moments; see Table 2.2.

Figure 2.11 Thrust Comparisons with 42 kip Live Load at Center Position (20)
Table 2.2 Comparison Between Analytical and Experimental Moments for Live Load at Center Position (20)

<table>
<thead>
<tr>
<th>Position</th>
<th>Live Load</th>
<th>F. E. moments (k-ft/ft)</th>
<th>Expt. mom. (k-ft/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CROWN</td>
<td>16 kip</td>
<td>1.63</td>
<td>1.82</td>
</tr>
<tr>
<td></td>
<td>32 kip</td>
<td>1.99</td>
<td>2.07</td>
</tr>
<tr>
<td></td>
<td>16 kip</td>
<td>-1.48</td>
<td>-1.13</td>
</tr>
<tr>
<td>HAUNCH</td>
<td>32 kip</td>
<td>-2.04</td>
<td>-1.20</td>
</tr>
</tbody>
</table>

2.3 NUMERICAL ANALYSIS

There has not been many publications on the topics of corrugated plate by numerical methods, which include Finite Element, Finite Difference and Numerical Series Solution. but, The following are a few related to that subject.

Ruzickova, H.(6) developed a finite element model for analyzing static problems of three-dimensional thin panel constructions. The modification was based on Herrmann and Campbell's model for shells. The triangular element is obtained by combining the well known constant-strain plane stress element and constant-moment plate bending element. The results for moments are satisfactory, the results for deflections are comparatively good. Table 2.3 showed the results for a clamped and uniformly loaded square plate.

Table 2.3 The Maximum Deflections and Extreme Moments in Case of a Uniformly loaded cube

<table>
<thead>
<tr>
<th>Division of plates into squares</th>
<th>4*4</th>
<th>6*6</th>
<th>8*8</th>
<th>Exact values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflections at centers of plates</td>
<td>0.871</td>
<td>0.692</td>
<td>0.627</td>
<td>0.535</td>
</tr>
<tr>
<td>Moments at centers of plates</td>
<td>0.1874</td>
<td>0.1997</td>
<td>0.2032</td>
<td>0.2079</td>
</tr>
<tr>
<td>Moments at centers of edges</td>
<td>-0.362</td>
<td>-0.4092</td>
<td>-0.4305</td>
<td>-0.4653</td>
</tr>
</tbody>
</table>
Craig (7) started from the governing equation and applied finite difference technique for moments and shears. He also presented the different boundary conditions (simply-supported, fixed, free and specific combinations) to be applied along with the computational approach that is to be used by the computer. Table 2.4 showed the result for simply-supported square plate.

<table>
<thead>
<tr>
<th>Thickness ratio h/a</th>
<th>Concentrated Central Load, P</th>
<th>Finite Elements</th>
<th>Finite Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.01170</td>
<td>0.011991</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>0.01219</td>
<td>0.012377</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.01353</td>
<td>0.013660</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>0.01551</td>
<td>0.015646</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>0.01801</td>
<td>0.017974</td>
<td></td>
</tr>
</tbody>
</table>

The main problem involving in corrugated plate is solving the governing differential equation. Foil (8) introduced a matrix series method. He has solved a variety of structural problems involving sandwich plates and shells. The basic idea was to write the differential equation in the first derivative of the matrix form, referred to the initial boundary restraint matrix and solved for a solution. For example, the equation for an orthotropic plate can be solved by this method.

\[
D_1 \frac{\partial^4 W}{\partial X^4} + 2D_3 \frac{\partial^4 W}{\partial X^2 \partial Y^2} + D_2 \frac{\partial^4 W}{\partial Y^4} = q
\]  

(2.7)

suppose \( w = w_m \sin \alpha \)

we can rewrite Equation (1.5) as
\[
\begin{bmatrix}
\frac{d}{dx} \left[ Z_1 \right] \\
\frac{d}{dx} \left[ Z_2 \right] \\
\frac{d}{dx} \left[ Z_3 \right] \\
\frac{d}{dx} \left[ Z_4 \right]
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
D_2'/D_1 & 0 & 2D_3'/D_1 & 0
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
q/D_1
\end{bmatrix}
\]

in which  
\( Z_1 = w_m \)  
\( Z_2 = \frac{dw_m}{dx} \)  
\( Z_3 = \frac{d^2w_m}{dx^2} \)  
\( Z_4 = \frac{d^3w_m}{dx^3} \)

### 2.4 THEORETICAL ANALYSIS

To apply orthotropic plate theory in the corrugated plate, we can always avoid the consideration of the irregular geometrical properties. Corrugation may be considered approximately homogeneous and orthotropic if number of corrugations is sufficiently large, or if the chord of the corrugation is small in comparison with the length of the side of the plate. Seydel (9) was the first person to do research on the rigidities as early as 1931. The typical corrugated sheet is a sinusoidal wave corrugation.
The principal direction $x$ and $y$ are perpendicular to and along the corrugations. Let $E$ and $\nu$ be the elastic constants of the material of the sheet, $h$ be its thickness,

$$Z = H \cdot \sin \frac{\pi x}{S}$$

the form of the corrugation; and $S$ the length of the arc of one-half a wave; see Figure 2.12. Then the following rigidities were given by Seydel:

$$D_1 = \frac{1}{S} \frac{Eh^3}{12(1-\nu^2)}$$

$$D_2 = EI$$

where,

$$S = 1 + \frac{\pi^2 H^2}{4 l^2}$$
\[ I = \frac{H^2 h}{2} \left[ 1 - \frac{0.81}{1 + 2.5(\frac{H}{2l})^2} \right] \]

While the bucking limit of isotropic curved plates subject to shear loading \((10,11,12)\), limit design \((13)\) and four-boundary simply-supported rectangular plates with Navier type solution \((14)\) have been extensively investigated, the writer is not aware of any publication dealing with the application of Galerkin method in corrugated plate.

### 2.5 DESIGN OF CORRUGATION

In culvert design, after investigation failures of these structures, many engineers believe that the improper construction procedures or backfill properties are the main reason to cause to fail. These two failures can be avoided by checking design handbook \((16)\) or manufacturer manual. In most common case, some reinforced features may be added to the basic shape of culvert in order to protect the structure from excessive change in the shape due to assembling and the increasing backfill. Reinforced features can be varied from ribs, thrust beams, slabs, compacting wings to certain stiffness reinforced concrete foundation; see Figure 2.13. Overall, the main considerations in design of culvert should be emphasized on, (1) seam strength, (2) component connection strength, (3) buckling strength, (4) installation rigidity, and (5) deflection.
For corrugated sheets, the determination of load/carrying capacity can be expressed as the allowable bending moments (16):

\[ M = SF_b \]  \hspace{1cm} (2.8)

where, 
- \( M \) = allowable bending moment
- \( S \) = section modulus obtained from Equation (2.2)
- \( F_b \) = allowable bending stress, take as 0.6\( F_y \)
To prevent local bucking, corrugations should be designed with R/t and m/t nearly equal and as small as possible. In addition, material should be close to the minimum yield strength of 33 ksi.
CHAPTER THREE

THEORETICAL DISCUSSION OF ORTHOTROPIC PLATES

3.1 INTRODUCTION

Usually, elastic plates have been analyzed as composed of a single homogeneous and isotropic material. However, plates of structural anisotropic properties have important applications, owing to their exceptionally high bending stiffness. A nonisotropic or anisotropic material displays direction dependent properties. Simplest among them are those in which the material properties differ in two mutually perpendicular directions. A material so described is orthotropic, and which can be divided into two ways; 1) natural orthotropic, such as wood, plywood and
fiber-reinforced plastics. 2) structural orthotropic, such as gridwork and corrugation. Consequently, in such cases, to obtain reasonable agreement between the analysis and the actual structural behavior, it is necessary to consider the orthotropic of such plates in our calculation.

![Wood Material Orthotropic](image)

![Corrugated Plate](image)

**Figure 3.1 Typical Orthotropic Plate**

When using the orthotropic plate theory to analyzed the practical applications of orthotropic plates in engineering field, although the actual structural behavior of a corrugation cannot be completely replaced by that of an orthotropic plate, experimental data indicate good agreement with this idealization, provided that the flexural rigidities are uniformly distributed in the X and Y directions, respectively.

### 3.2 STRAIN ENERGY

The expressions of stress and strain in plate are shown as;

\[ \varepsilon_x = \frac{\partial U}{\partial x} - \gamma \frac{\partial^2 w}{\partial x^2} \]
Strain energy expressed as,

\[ U = \frac{1}{2} \int_V (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy}) \, dV \]

\[ = U_m + U_b \]

where, \( U_m \) is membrane strain energy (in plane)

\( U_b \) is bending strain energy

Neglecting surface force and only considering bending strain energy.
3.3 MINIMUM POTENTIAL ENERGY AND BOUNDARY CONDITIONS

Since the equilibrium configuration is represented by the admissible functions which make the total potential energy of the system a minimum, we can write

\[ U = \frac{1}{2} \int \int \left[ D_1 \left( \frac{\partial^2 W}{\partial x^2} \right)^2 + 2D_1^{1/2} \frac{\partial W}{\partial y} \frac{\partial W}{\partial x} + D_2 \left( \frac{\partial^2 W}{\partial y^2} \right)^2 + 4D_k \left( \frac{\partial^2 W}{\partial y \partial x} \right)^2 \right] \, dx \, dy \]

\[ S \Pi = S(U - W) = S(U - W) = 0 \]

where, \( W \) is external work. The details expression for \( S \Pi \) will be;

\[ S \Pi = \int \int \left[ D_1 \frac{\partial^2 W}{\partial x^2} \frac{\partial^2 W}{\partial x^2} + D_1^{1/2} \frac{\partial W}{\partial y} \frac{\partial W}{\partial x} + D_2^{1/2} \frac{\partial^2 W}{\partial y^2} \frac{\partial^2 W}{\partial y^2} \right. \]
\[ + \left. D_2 \frac{\partial^2 W}{\partial y^2} \frac{\partial^2 W}{\partial y^2} + 4D_k \frac{\partial W}{\partial y} \frac{\partial W}{\partial x} \right] \, dx \, dy - \int \int S \omega \, dx \, dy \]
\[
=- \iint [M_x \frac{\partial^2 w}{\partial x^2} + M_y \frac{\partial^2 w}{\partial y^2} + 2M_{xy} \frac{\partial^2 w}{\partial x \partial y}] \, dx \, dy - \iint P \delta w \, dx \, dy
\]

\[
= - \left[ M_x \frac{\partial^2 w}{\partial x^2} + M_{xy} \frac{\partial^2 w}{\partial x \partial y} \right]_0^a \, dy - \left[ M_y \frac{\partial^2 w}{\partial y^2} + M_{xy} \frac{\partial^2 w}{\partial y \partial x} \right]_0^b \, dx
\]

\[
+ \iint [ - \frac{\partial M_x}{\partial x} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial M_{xy}}{\partial x} \frac{\partial^2 w}{\partial x^2} + \frac{\partial M_y}{\partial y} \frac{\partial^2 w}{\partial y \partial x} + \frac{\partial M_{xy}}{\partial y} \frac{\partial^2 w}{\partial y^2}] \, dx \, dy
\]

\[
- \iint P \delta w \, dx \, dy
\]

Figure 3.2 Rectangular Plate with Different Boundary Condition on Two Ends

Let $BC_y$ as the boundary condition in $y$ direction.

\[
BC_y = - \left[ M_x \frac{\partial^2 w}{\partial x^2} + M_{xy} \frac{\partial^2 w}{\partial x \partial y} \right]_0^a \, dy
\]

and $BC_y$ as the boundary condition in $x$ direction

\[
BC_x = - \left[ M_y \frac{\partial^2 w}{\partial y^2} + M_{xy} \frac{\partial^2 w}{\partial y \partial x} \right]_0^b \, dx
\]

So,

\[
\delta \Pi = BC_x + BC_y + \iint [ Q_x \frac{\partial^2 w}{\partial x} + Q_y \frac{\partial^2 w}{\partial y}] \, dx \, dy - \iint P \delta w \, dx \, dy \quad (3.1)
\]
\[ \delta \Pi = - \int_0^b \left[ M_x \frac{\partial^2 w}{\partial x^2} + M_{xy} \frac{\partial^2 w}{\partial x \partial y} - Q_x \delta w \right] \, dy - \int_0^a \left[ M_y \frac{\partial^2 w}{\partial y^2} + Q_y \delta w \right] \, dx - \int_0^a \left[ \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} \right] \, dy \, dx \]

From the boundary condition shown on the Figure 3.7 and ignored the corner reaction, we have,

Since, \( SW \neq 0 \)

\[ \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + P = 0 \] (3.2)

After substituting \( Q_x \) and \( Q_y \) expression into Equation (3.2), this turns out to be the governing differential equation for orthotropic plate.

\[ D_1 \frac{\partial^4 W}{\partial x^4} + 2D_3 \frac{\partial^4 W}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 W}{\partial y^4} - P = 0 \]

This demonstrates another method to obtain the governing differential equation.

Processing of the variation of strain energy, will result the following several forms boundary condition.

\[ M_x \frac{\partial w}{\partial x} = 0 \quad \text{or} \quad M_y \frac{\partial w}{\partial y} = 0 \] (3.3)
3.4 CLASSICAL METHODS

Within classical plate theory—thin plate and small deflection, there are several methods which are common used by most engineers. These methods, only valid on certain boundary condition or can be classified by the boundary condition.

1) For simply-supported ends on four sides.
   Suggestive method: Navier's Method.

2) For one side is simply-supported, the other side is arbitrary boundary condition.
   Suggestive method: Levy Method.

3) For arbitrary boundary condition on four sides.
   Suggestive method: (a) Energy Methods (Ritz Method).
   (b) Virtual Work Principle (Galerkin Method)
3.5 GALERKIN METHOD

3.5.1 INTRODUCTION

Galerkin method is based on virtual work principle, and this method has been applied successfully to such diverse types of problems as small deflection and large deflection theories. In this study, we applied Galerkin method on two typical cases, partially fixed edge and simply-supported edge; see Figure 3.2. The more detail results are presented in Chapter 4.

Assume the displacement function takes the following form:

\[ W = W_m(x) \psi_m(y) \]

where, \( W_m(x) \): unknown function,

\( \psi_m(y) \): known function which satisfies boundary condition

along \( y=0 \) and \( y=b \).

Since \( \delta \psi_m = 0 \), the variation of displacement function,
Substituting Equation (3.4) into Equation (3.1), we have

\[ \sum_{k=1}^{m} \psi_k \]

After rewrite above equation, \( (3.5) \)

The general form of the rigorous of governing differential equation can be written as,

\[ W = W_h + W_m \]

where, \( W_h \) represents the homogeneous solution, and

\( W_p \) is particular solution.
3.5.2 HOMOGENEOUS SOLUTION

In order to solve Equation (3.5), let us select the following solution,

\[ W_{1h} = C e^{\lambda x} \]  (3.6)

substituting Equation (3.6) into Equation (3.5) yields homogeneous solution as,

\[ W_{mh} = (A_m \cos \beta_m x + B_m \sin \beta_m x) C \alpha_m x + (C_m \cos \beta_m x + D_m \sin \beta_m x) S \alpha x \]  (3.7)

Which,

\[ \alpha_m = \frac{1}{\sqrt{2A_{11}D_1}} \sqrt{-D_3 B_{11} + \sqrt{A_{11}D_1 D_2 C_{11}}} \]

\[ \beta_m = \frac{1}{\sqrt{2A_{11}D_1}} \sqrt{D_3 B_{11} + \sqrt{A_{11}D_1 D_2 C_{11}}} \]

and \( A_m, B_m, C_m, D_m \) and constants.

3.5.3 BOUNDARY CONDITION

Only takes one term in Equation (3.5),

\[ [A_{11} D_1 \frac{d^4 W_1}{dx^4} + 2D_3 B_{11} \frac{d^2 W_1}{dx^2} + D_2 C_{11} W_1] - P_1 = 0 \]  (3.8)
Consider the case of the boundary conditions in Y axis which parallels to the free edges; referring to Figure 3.3. This implies that the bending moment and the Kirchhoff type shear must vanish at this edge.

a) At the edge,

\[ M_x = -D_1 \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right] = 0 \]

From Equation (3.3), we have,

\[ \int_{0}^{l} M_x \frac{\partial w}{\partial x} \bigg|_0^b = 0 \]

\[ \int_{0}^{l} \left[ \psi_1 \frac{d^2 w}{dx^2} + \nu_2 \psi_1 W_1 \right] \psi_1 \frac{\partial w}{\partial x} \bigg|_0^b dx = 0 \]

\[ A_{11} \frac{d^2 w_1}{dx^2} + \nu_2 B_{11} W_1 = 0 \]  
(3.9)

b) At the edge,

\[ V_x = Q_x + \frac{\partial M_{xy}}{\partial y} = 0 \]

From Equation (3.17), we have,

\[ \int_{0}^{l} V_x \delta W_1 \bigg|_0^b dx = 0 \]

\[ \int_{0}^{l} \left[ D_1 A_{11} \frac{d^3 w_1}{dx^3} + (2D_2 + D_3) B_{11} \frac{dw_1}{dx} \right] \delta W_1 dx = 0 \]

\[ D_1 A_{11} \frac{d^3 w_1}{dx^3} + (2D_2 + D_3) B_{11} \frac{dw_1}{dx} = 0 \]  
(3.10)
3.5.4 THEORETICAL SOLUTIONS

The following procedures have been suggested by the writer while solving this typical problem.

a) Find homogeneous solution; see Equation (3.7)

b) Find particular solution for different configuration.

c) Find constant of differential equation from boundary condition.

d) Combine particular solution and homogeneous solution to have the expression for displacement.

e) Find stress or strain expression from displacement function.

By means of above procedures, we first investigated simply-supported edges and took the coordinates as shown in Figure 3.4.

![Coordinate System in One-Rib with Center Load for Simply-Supported Edge](image)

Figure 3.4 Coordinate System in One-Rib with Center Load for Simply-Supported Edge
Theoretical solution is,

\[ W = [A_m \cos \beta_m x \text{Ch} \alpha_m x + D_m \sin \beta_m x \text{Sh} \alpha_m x + \frac{2}{\pi^4} \frac{q_0 b^3}{D_2} ] \sin \frac{\pi y}{b} \]  \hspace{1cm} (3.11)

For elastic restrained ends; see Figure 3.5, we have,

\[ W = [A_m \cos \beta_m x \text{Ch} \alpha_m x + D_m \sin \beta_m x \text{Sh} \alpha_m x + Q_m] [A + B B y^2 + y^4] \]  \hspace{1cm} (3.12)

Figure 3.5  Coordinate System for Elastic Restrained Edges

In Equation (3.11) and (3.12), \( A_m, D_m, Q_m, \alpha_m, \beta_m, A A \) and \( B B \) are constant.

The components of the stress were given,

\[ \sigma_x = - \frac{E_x Z}{1 - \nu^2} [S_x + \nu S_y] \]  \hspace{1cm} (3.13)

\[ \sigma_y = - \frac{E_y Z}{1 - \nu^2} [S_y + \nu S_x] \]
CHAPTER
FOUR

LABORATORY TEST INSTRUMENTATION AND DESCRIPTION

4.1 TEST SPECIMEN

In laboratory study, an aluminum corrugated plate, 6 ft, 10 in. span by 2 ft, 3 in. width, was loaded with two sides free and the other two sides bolted to I-beams; see Figure 4.1.

Figure 4.1 Laboratory Test Setup for One-Rib Configuration
The corrugation measured a 9 in. pitch and 2-1/2 in. depth. Two aluminum ribs of type VI with 67 in. long were used. These ribs were bolted to the plate when needed. There were seven bolt holes on the rib which corresponded to the plate in corrugation direction, the diameter of the bolt hole was 7/8 in. A torque wrench was used to apply 150 ft-lb to prevent any mechanical slips and model field installation.

4.2 INSTRUMENTATION

The whole experimental setup was supported in Gilmore machine frame. Load applied to the plate was distributed with granular fill. The applied load was calibrated before the test; see Figures 4.2 and 4.3.
Two loading cases were tested: center line load and constant moment; See Figure 4.4. Five dial gages were installed to measure deflection of the convex surface of the plate; Figure 4.5.
Figure 4.4  Experiment Setup and Load Configuration
Three types electrical strain gages were cemented on rib and plate: uniaxial strain gages on rib, biaxial strain gages on the crown and valley of the plate, and rosettes around the end bolt connection on the plate; Figure 4.6.
Figure 4.6 Laboratory Plate-Rib Combinations
The rosette group is located at the bolt connection between section 3 and section 4. A total of twelve rosettes were mounted at $0^\circ$, $45^\circ$, $90^\circ$, and $180^\circ$ planes with respect to the corrugation direction of the plate, three on each plane; Figure 4.7. All strain gages data were read by an HP computer and stored on a disk for analyzing.
Figure 4.7 Laboratory Rosette Group
4.3 TEST PROCEDURE

There were three configurations of the corrugation and two cases loading;

No-rib and center load,
No-rib and constant moment,

One-rib and center load,
One-rib and constant moment,

Two-ribs and center load,
Two-ribs and constant moment.

Totally six cases were involved in the test. The one-rib with center load was used as the final test when loaded to failure. The specimen were loaded in increments. Deflections in plate increased while load cell readings decreased at failure. The maximum load at 15 kip which was ten percent less than the calculated value.

4.4 LABORATORY RESULTS

Within elastic load range, predicted displacement which is calculated from beam theory agrees with experimental deflection; see Figure 4.8.
As shown in Figures 4.9 and 4.10, the results indicated that two kinds of load configuration, center load and constant moment, were accomplished.
Figure 4.10  Crown stresses of Corrugation Direction in Different Section for No-Rib, Constant Moment (section number referring to Figure 4.6)

Figure 4.11  Load VS. Stress for No-Rib, Center Load at Section 1
Figure 4.12  Load VS. Stress for One-Rib with Center Load at Section 1

Figure 4.13  Load VS. Stress for Two-Ribs with Center Load at Section 1
Figures 4.11, 4.12 and 4.13 presented three types of rib and plate combination. By examining these figures, the following observations are drawn;

1) The experimental results shown the expected load configurations were accomplished; see Figures 4.9 and 4.10.

2) Composite degree develops as the number of stiffeners increase, as shown by Figure 4.13. Because experimental stresses in the connection area measured at rib and at plate are close.

3) The stress near the bolt connection is disrupted by the ribs, the greater number of ribs are placed, the more complicated stress distribution becomes; see Figures 4.12 and 4.13. Rib only affects stresses within the connection area, or the relationship between load and stress is linear out of the bolt connection.

4) Increasing the number of stiffeners resulted in reducing stresses in stiffeners and but did not affect the maximum stresses in corrugated plate. So stiffeners only improve culvert stability to retain the original configuration.

As load increased, principal stresses and shear stresses increased in magnitude. Principal axes coincide with corrugation direction and transverse direction as the location is away from the bolt connection. Change of principal stress direction was mainly induced by shear stress and local bending. The one-rib case represented such a typical pattern, or, we concluded that noncomposite reaction was fully developed in one-rib case other than the cases of two-ribs and no-rib.

Figure 4.14 indicates that the principal stress direction is disrupted, and the bolt connection is the main factor. Another point should be noticed, that the transverse stresses were almost tension stresses as long as
they were far from the bolt.

Figure 4.14  Principal Stresses in Rosette Group Located Between Section 3 and 4 with Concentrated Load, One-Rib, and at 7.43 kip
5.1 DISCUSSION OF THRUST IN FULLY COMPOSITE REACTION

The value of thrust as calculated by Equation (5.2) in Figure 5.1, was too large to be directly acceptable in Equation (5.2). Beal (18) suggested these equations be used in composite design for moment and thrust based on data from the outside two gages, whereas noncomposite design be based on data from all four gages.

\[
M = \frac{I}{d} (\sigma_1 - \sigma_2) \tag{5.1}
\]

\[
T = \frac{A}{d} (C_2 \sigma_1 - C_1 \sigma_2) \tag{5.2}
\]

which, I is moment of inertia of the cross section,
A is the area of the cross section,
d is the distance between two gages,
C₁, C₂ is the distance from neutral axis to each gage, respectively,
\(\sigma_1, \sigma_2\) is the crown and valley gage readings.
For composite action the no-rib case is the ideal condition to be analyzed, however large thrusts were recorded. The cause of excessive thrusts may result from three considerations: end supports, loading and stress distribution.

The consideration of end support can be removed from examining of thrust as shown by experimental data, since the ratio of (tension stress)/(compression stress) is independent of load increases; see Table 5.1.
Table 5.1  Ratio of Crown Stress and Valley Stress In Corrugation Direction

<table>
<thead>
<tr>
<th>Load (kip)</th>
<th>section 1</th>
<th>section 2</th>
<th>section 3</th>
<th>Load (kipfs)</th>
<th>section 1</th>
<th>section 2</th>
<th>section 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{no-rib, center load}}$</td>
<td>$\sigma_3/\sigma_1$</td>
<td>$\sigma_7/\sigma_5$</td>
<td>$\sigma_{11}/\sigma_9$</td>
<td>$M_{\text{no-rib, constant moment}}$</td>
<td>$\sigma_3/\sigma_1$</td>
<td>$\sigma_7/\sigma_5$</td>
<td>$\sigma_{11}/\sigma_9$</td>
</tr>
<tr>
<td>-0.53</td>
<td>1.36</td>
<td>1.37</td>
<td>1.34</td>
<td>0.68</td>
<td>1.37</td>
<td>1.46</td>
<td>1.68</td>
</tr>
<tr>
<td>-0.90</td>
<td>1.38</td>
<td>1.39</td>
<td>1.31</td>
<td>1.10</td>
<td>1.39</td>
<td>1.49</td>
<td>1.68</td>
</tr>
<tr>
<td>-1.52</td>
<td>1.36</td>
<td>1.36</td>
<td>1.29</td>
<td>2.18</td>
<td>1.39</td>
<td>1.50</td>
<td>1.85</td>
</tr>
<tr>
<td>-2.12</td>
<td>1.36</td>
<td>1.37</td>
<td>1.29</td>
<td>2.64</td>
<td>1.37</td>
<td>1.50</td>
<td>1.85</td>
</tr>
<tr>
<td>-2.86</td>
<td>1.36</td>
<td>1.37</td>
<td>1.32</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-3.01</td>
<td>1.36</td>
<td>1.37</td>
<td>1.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: $\sigma_3, \sigma_7, \sigma_{11}$ are stresses at crown of corrugation, $\sigma'_1, \sigma'_5, \sigma'_9$ are stresses at valley of corrugation. $\sigma'_1, \sigma'_5, \sigma'_9$ are different from $\sigma_1, \sigma_5, \sigma_9$ by one corrugation thickness.

The stress distribution in Equation (5.2) is different from the experimental situation, and this difference may result the exceptional thrust. Figures 5.2 and 5.3 present the stress distribution in corrugation direction of the transverse cross section.
Figure 5.2 Stresses in Corrugation Direction of the Cross Section are Plotted for No-Rib Configuration and Center Load
Figure 5.3 Stresses in Corrugation Direction of the Cross Section are Plotted for No-Rib Configuration and Constant Moment
Thrust should vanish in this experimental setup as the ends were not fixed. That (moment)/(unit width) varies with position may provide an explanation of this behavior. This variation is calculated by Equation (5.3). The ratio is a constant with respect to different loading level; as shown in Table 5.2.

Table 5.2 The Value of Ratio \( M_2/M_1 \) for No-Rib with Center Load Configuration

<table>
<thead>
<tr>
<th>Load Configuration</th>
<th>Center Load (kip)</th>
<th>Constant Moment (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section 1</td>
<td>0.53 0.902 1.52 2.12 2.86 3.01</td>
<td>0.68 1.10 2.18 2.64</td>
</tr>
<tr>
<td>Section 2</td>
<td>1.36 1.37 1.36 1.36 1.36 1.36</td>
<td>1.38 1.39 1.39 1.38</td>
</tr>
<tr>
<td>Section 3</td>
<td>1.37 1.38 1.36 1.36 1.37 1.37</td>
<td>1.46 1.49 1.50 1.50</td>
</tr>
<tr>
<td></td>
<td>1.33 1.31 1.28 1.30 1.33 1.33</td>
<td>1.68 1.72 1.80 1.82</td>
</tr>
</tbody>
</table>

Note, \( M_1 \) : Moment at the symmetrical plane of the corrugation.

\( M_2 \) : Moment at the plane which is a half corrugation away from \( M_1 \).

\[
\frac{M_2}{M_1} = \frac{\sigma_2 C_1}{\sigma_1 C_2}
\]  \hspace{1cm} (5.3)

Using Equation (5.2) we have,
Theoretically, $M_1$ is aligned with $M_2$ in Equation (5.4). Whereas in experimental setup $M_1$ is located a half corrugation away from $M_2$. This difference results a significant thrust value. In order to adjust thrust value, we take $M_1' = kM_1$ and make $M_1'$ as $M_1$ substituting in Equation (5.4). In here, $M_1'$ is aligned with $M_2$, $K$ is a coefficient which accounts for the moment as it varies with position. Referring to Table 5.2, $K$ can be taken as 1.36 for center load case. The result of thrust after considering that the moment varies with position is presented in Figure 5.4.

![Figure 5.4 Thrust Values at Different Sections for No-Rib, Center Load, P=0.53 kip](image)

Some conclusions may be drawn from the above discussion.

1) Equation (5.2) appears to be only good when $\sigma_1$ is aligned with $\sigma_2$, but in corrugation case, $\sigma_2'$ is always located off $\sigma_1$ by a half corrugation; see Figure 5.5. In this experimental setup, half corrugation
length is 4.5". If using $\sigma_2'$ instead of $\sigma_2$, this difference in position will induce 10% difference for the correct $\sigma_2$ in case of a plate with four sides simply-supported and uniformly loaded. Comparison is available in instrumented corrugated plate for the positions which are different with one corrugation. The experimental data shows that the difference is 26% for no-rib, center load configuration and 33% for no-rib, constant moment configuration.

![Figure 5.5 Comparison is Drawn Between Corrugation and Solid Section](image)

2) The calculation of thrust should consider the tensile area and compression area, and stress distribution on the transverse cross section. This could be expressed as,

\[ \text{Thrust} = \text{Tensile stress} \times \text{Tensile area} + \text{Comp. stress} \times \text{Comp. area} \]

\[ = \Sigma A_i \times S_i \]

By introducing the integration method, as suggested above, the calculation may be simplified by adopting the idea presented in reinforce concrete (R.C.) design; see Figure 5.6.
5.2 DEGREE OF PERFORMANCE

We have two considerations about the rib and plate combination; composite design (using data from the outside two gages) and noncomposite design (using data from all four gages). For composite reaction, we assume that rib and plate works as a unit, moment and thrust can be calculated by Equations (5.1) and (5.2). On the other hand, in noncomposite reaction, most engineers consider the reaction of rib and plate separately by using composite section; see Beal (18). In the field investigation (20), these two considerations were compared in Figures 5.7 and 5.8.
Two conclusions are drawn from the above observations,

1) Comparison between moment is consistent.
2) Comparison between thrust is in disagreement.

The incorrect assumption of the sectional response may result in the discrepancy between the expected thrust and the measured thrust. As we assume here, the section changes the range (virtual noncomposite, noncomposite, and composite) with respect to the loading. The calculation of thrust is totally different for each range. To differentiate each range, we use the degree of performance ($\alpha$) as a measurement; see Figure 5.9.

![Figure 5.9 Degree of Performance VS. Load](image)

Since the development of these range is dependent on the rib response, plate response and rib-plate connection, so that $\alpha$ may be a function of load, load configuration, location of bolt and degree of bolt slip. We also notice that $\alpha$ is meaningful in noncomposite stage (point A to point B). When $\alpha > \alpha_{cr}$ and $\alpha < \alpha_A$, $\alpha$ has no meaning, the combination section tends to be
reacted as composite and virtual noncomposite respectively. In noncomposite range, compared to Beal's noncomposite formula (17), we have added the considerations of moment and thrust in connection area. These considerations are expressed by introducing degree of performance; see Figure 5.10.

![Figure 5.10 Noncomposite Reaction in Plate-Rib Combination](image)

After considered local bending, moment and thrust are developed as;

\[ T = T_C + T_R + \alpha T_B \]
\[ M = M_C + M_B + M_R + T_C D_1 + T_R D_2 + \alpha T_B D_3 \]

Which,

C = Corrugation,
B = Bolt connection part,
R = Rib,
\( \alpha \) = Degree of performance,
and \( D_1, D_2, D_3 \) is the distance from neutral axis to each section.

Referring to Figure 5.9, at the initial stage up to point A, very slight slip may occur between the plate and the stiffening ribs as loading begins.
The reaction in the combination section can be referred as virtual noncomposite, in this stage, there is no relation to the degree of performance $\alpha$. As the load increases, between point A and B, local bending and shear transfer create thrust within the connection area. Since there is an interface here, we quantify this thrust by introducing one more consideration, the degree of performance ($\alpha$). At this stage $\alpha$ has certain value. Within the elastic range from Equation (5.4), we know that the $\alpha$ should have a limit to avoid the domination of the other two terms. This limit is defined as critical degree of performance $\alpha_{cr}$. After $\alpha_{cr}$ the mechanics of reaction will appear as composite. When bolt reaches its maximum load capacity, or $\alpha > \alpha_{cr}$, composite response begins. The composite response will be maintained with increasing load until the structure fails. For one-rib, center load and one-rib, constant moment, the degree of performance is presented in Figures 5.11 and 5.12. The comparisons between predicted moment and analyzed moment are shown in Figures 5.13 and 5.14.

![Figure 5.11](image)

Figure 5.11 Degree of Performance as Plotted Against Load with One-Rib, Center Load Configuration.
Figure 5.12 Degree of Performance as Plotted Against Load with One-Rib, Constant Moment Configuration.

Figure 5.13 Comparisons Among Predicted, Composite and Noncomposite Moments in Section 1 for One-Rib, Center Load Configuration.
5.3 EFFECTS OF BOLT HOLE

The existence of a bolt hole disturbs stress distribution in all three configurations: no-rib, one-rib and two-ribs. Effects of bolt depend on boundary conditions, load types and rib configurations. The experimental data show that in the center load configuration the laboratory setup could be modelled as a center loaded plate with simply-supported end. In the above case, there are only two stress patterns (either tension or compression) that can be applied; see Figure 5.15. In Figure 5.15, point A is close to bolt hole (point B), E is close to the end support. When drawing the disturbed stress
line, four principles need to be considered; 1) stress line is continuous, 2) first derivative continues smoothly, 3) stress in bolt hole is zero, and 4) stress line should be compatible with original moment diagram for each load configuration. In no-rib, center load case, experimental stress distribution is presented in Figure 5.16, and similar to pattern A of Figure 5.15. On the other hand, the one-rib case behaves as shown in pattern B; see Figure 5.17, (after subtracting the constant thrust through the each section).

Figure 5.15  Disturbed Stress Line Along Corrugation Direction for Center Load Configuration
Figure 5.16 Rosette Stresses Along Corrugation Direction in Line with Bolt Hole for No-Rib, Center Load Configuration.

Figure 5.17 Rosette Stresses Along Corrugation Direction in Line with Bolt Hole for One-Rib, Center Load

For two-ribs, center load configuration as shown by Figure 5.18, the
distribution line tended to be pattern A of Figure 5.15.

![Graph showing rosette stresses along corrugation direction in line with bolt hole for two-ribs, center load configuration.]

Figure 5.18 Rosette Stresses Along Corrugation Direction in Line with Bolt Hole for Two-Ribs, Center Load Configuration.

Similarly, we can analyze constant moment in the same way. Two patterns A and B are shown in Figure 5.19
Figure 5.19 Disturbed Stress Line Along Corrugation Direction for Constant Moment Configuration

Figure 5.20, 5.21 and 5.22, shows stress distribution line for no-rib, one-rib and two-ribs configurations under constant moment. All three cases are close to pattern B.

Figure 5.20 Rosette Stresses Along Corrugation Direction in Line with Bolt Hole for No-Rib, Constant Moment
Figure 5.21 Rosette Stresses Along Corrugation Direction in Line with Bolt Hole for One-Rib, Constant Moment Configuration.

Position 4 is bolt hole

M=1.23kip-ft
M=2.68kip-ft
M=4.34kip-ft
M=5.68kip-ft
M=6.55kip-ft

Figure 5.22 Rosette Stresses Along Corrugation Direction in Line with Bolt Hole for Two-Ribs, Constant Moment Configuration.

Position 4 is bolt hole

M=1.92kip-ft
M=2.38kip-ft
M=4.30kip-ft
M=5.36kip-ft
5.4 DISCUSSION OF SHEAR STRESS

5.4.1 FUNCTION OF BOLT

As we know, to make rib to be more responsive to external loading in the structure, the load must be carried out through the connection between rib and corrugated plate. From the basic principles, an individual bolt can resist shear stress. On the other hand, a set of bolts may form a mechanism to resist both shear force and normal bending. Many researchers focus on the transfer of shear stress (18). The effect of transfer of bending stress may prominence in many cases and local bending will then be induced by this kind of transfer. Because effect of local bending, we can consider the combination of these structures. Whereas considering shear stress transfer can tell how strong coherent of their interaction; see Figures 5.9 and 5.10. So, physically, some loads are transferred from plate to rib; logically, it has to be divided out some parts between shear stress transfer and normal stress transfer. The percentage of those transfers are dependent on load configuration, bolt type, bolt combinations and bolted joints. There are two types of bolted joints, either standard (unslotted) bolted joints and slotted joints. Slotted joints are applied in box culvert and hole is slotted in circumferential direction in order to reduce stiffeners in that direction. In performance, this shifts large thrust stress to compacted surrounding soil.

The degree of performance in Figure 5.9 may be explained in the
other way under the case of only shear stress transfer or this term becomes projecting in the whole process. Let \( R_1 = (\sigma_1 - \sigma_2) / 2 \), \( R = \sqrt{(R_1)^2 + (\tau)^2} \)

which, \( \sigma_1, \sigma_2 \) is principal stresses respectively.

\( \tau \) is shear stress.

we know, the ratio \( R_1 / R \) equals unity, indicates that the principal stress is virtually directed in the vertical and horizontal directions signifying the absence of shear stress. This would indicate slippage or noncomposite reaction occurs under the situation \( \tau = 0 \). Vice verse, if this ratio reaches minimum value (0.707), it would indicate composite response or finish of slippage.

In center load configuration, there were two forms of transfer: shear stress and normal stress transfer, but in constant moment configuration transfer of normal stress is prominent, since shear transfer diminished in this case. Therefore, this observation may explain why the degree of performance in Figure 5.11 (center load configuration) is greater than the the degree of performance in Figure 5.12 (constant moment configuration).

5.4.2 PRINCIPAL STRESS DIRECTION AND AFFECTED SHEAR STRESS RANGE

The experimental results show that the assumption of orthotropic plate theory for corrugated plate is acceptable. Since the ratio of \( R_1 / R \) approximately equals to unity, as long as the gage is located away from the bolt connection, (like gages 24,15 etc in Figure 4.7). From the previous investigations (18) and (20), the following conclusion is reached: shear stresses only exists around the bolt hole, in other words the direction of
principle stresses coincides with the horizontal and vertical axes beyond the affected range. The following discussions of principal stress direction and affected shear stress range are an extension of the above conclusion.

In the center line of the corrugated plate along the corrugation direction, the shear stress would vanish in the no-rib or two-ribs configurations, since that axis is the symmetrical plane. This anticipation is also verified by experimental results shown by Figure 5.23, where the value of $R_1/R$ approximately equals to unity. It is not surprising that rosette gage 48 was off in the graph, because that gage was apparently affected by the end support.

Table 5.3 represents the ratio $R_1/R$ for one-rib configuration. The value of $R_1/R$ increases as the load increases. Also the value of principal stress direction increases at the same time. Even so in the failure stage, this relationship is also consistent. But the data in the failure stage must be
disregarded since Hook's law cannot be applied beyond the yield point.

Table 5.3 Value of R1/R for One-Rib, Center Load Configuration in Corrugation Direction

<table>
<thead>
<tr>
<th>Load Gage position</th>
<th>Center Load P (kip)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.51</td>
</tr>
<tr>
<td>15</td>
<td>.767</td>
</tr>
<tr>
<td>18</td>
<td>.709</td>
</tr>
<tr>
<td>21</td>
<td>.709</td>
</tr>
<tr>
<td>33</td>
<td>.868</td>
</tr>
<tr>
<td>42</td>
<td>.884</td>
</tr>
<tr>
<td>48</td>
<td>.968</td>
</tr>
</tbody>
</table>

Along the perpendicular direction, the experimental data is represented in the Tables 5.4 and 5.5. In either one-rib and two-ribs constant moment configuration, the principal stress direction does not change as load increases, since the ratio of individual gage stayed unchanged.

Table 5.4 Value of R1/R for One-Rib Configuration in Perpendicular Direction

<table>
<thead>
<tr>
<th>Load Gage</th>
<th>Center load P (kip)</th>
<th>Constant moment M (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.51</td>
<td>2.74</td>
</tr>
<tr>
<td>24</td>
<td>.91</td>
<td>.94</td>
</tr>
<tr>
<td>27</td>
<td>.96</td>
<td>.97</td>
</tr>
<tr>
<td>30</td>
<td>.98</td>
<td>.92</td>
</tr>
</tbody>
</table>

Table 5.5 Value of R1/R for Two-Ribs Configuration in Perpendicular Direction

<table>
<thead>
<tr>
<th>Load Gage</th>
<th>Center load P (kip)</th>
<th>Constant moment M (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.6</td>
<td>3.8</td>
</tr>
<tr>
<td>24</td>
<td>.72</td>
<td>.75</td>
</tr>
<tr>
<td>27</td>
<td>.78</td>
<td>.80</td>
</tr>
<tr>
<td>30</td>
<td>.72</td>
<td>.72</td>
</tr>
</tbody>
</table>
Along the 45° plane, gage 36, 39, and 45, the ratio of $R_1/R$ stayed unchanged; see Tables 5.6 and 5.7. This indicates the increasing of load has a less effect for the principal stress direction along this plane.

### Table 5.6 Values of $R_1/R$ for One-Rib Configuration Along 45° Plane

<table>
<thead>
<tr>
<th>Load</th>
<th>one-rib center load $P$ (kip)</th>
<th>one-rib constant moment $M$ (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gage</td>
<td>Position 1.51 2.74 4.01 5.57 7.43</td>
<td>1.23 2.68 4.34 5.68 6.55</td>
</tr>
<tr>
<td>36</td>
<td>0.71 0.71 0.80 0.99 0.96</td>
<td>0.85 0.85 0.85 0.85 0.85</td>
</tr>
<tr>
<td>39</td>
<td>0.98 0.97 0.98 0.99 0.99</td>
<td>1.0 1.0 1.0 1.0 1.00</td>
</tr>
<tr>
<td>45</td>
<td>0.75 0.75 0.75 0.75 0.75</td>
<td>0.73 0.72 0.72 0.72 0.72</td>
</tr>
</tbody>
</table>

### Table 5.7 Values of $R_1/R$ for Two-Ribs Configuration Along 45° Plane

<table>
<thead>
<tr>
<th>Load</th>
<th>two-ribs center load $P$ (kip)</th>
<th>two-ribs constant moment $M$ (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gage</td>
<td>Position 1.60 3.18 3.97 4.71</td>
<td>1.90 2.38 4.30 5.36</td>
</tr>
<tr>
<td>36</td>
<td>0.75 0.75 0.74 0.73</td>
<td>0.72 0.72 0.71 0.71</td>
</tr>
<tr>
<td>39</td>
<td>0.78 0.81 0.82 0.81</td>
<td>0.96 0.95 0.96 0.97</td>
</tr>
<tr>
<td>45</td>
<td>0.97 0.99 0.99 0.98</td>
<td>0.87 0.85 0.80 0.78</td>
</tr>
</tbody>
</table>

The following discussion of affected shear stress range is based on one-rib and two-ribs configurations.
Figure 5.24 Illustration of Possible Affected Shear Stress Range in One-Rib Configuration

One-rib center load
$P = 1.51, 7.43$ kip

One-rib constant moment
$M = 1.23, 6.55$ kip-ft

Figure 5.25 Illustration of Possible Affected Shear Stress Range in Two-Ribs Configuration

Two-ribs center load
$P = 1.6, 4.71$ kip

Two-ribs constant moment
$M = 1.92, 5.36$ kip-ft
In Figures 5.24 and 5.25, the points where the semi-circles intersect the axes indicate both the gage position and where $R_1/R$ equals unity. The pair of numbers presented on the graph show change of ratio for initial load and final load. A single number indicates that the ratio is constant and is independent of load. One may conclude from the above figures that the affected shear stress range varies with configuration, and has a greater range in the corrugation direction than in either of the other two directions. In Figure 5.25 (two-ribs configuration) the line in corrugation direction represents the plane of symmetry so that the ratios are approximately equal to unity.

The examination of experimental results of rosette group may be summarized in the following statements.

1) Away from the rib, principal stress direction along the 45° plane stays unchanged with respect to load level. For the positions around the bolt connection, the principal stress direction changes as load increases.

2) Ratio of $R_1/R$ approaches to unity would indicate that shear stress vanishes and noncomposite response occurs.

3) In the data shown above only shear is affected in the perpendicular direction, while both normal and shear stress is affected in the corrugation direction.

4) The affected shear stress range varied with configuration and loading.
5.5 SHEAR FLOWS FOR THE DIFFERENT CONFIGURATION

It is of interest in composite response studies to investigate shear flows in corrugated plate and plate-rib combinations. The experimental shear flows are represented in Figure 5.26. The corresponding shear flows' values are shown in Tables 5.8 and 5.9.
Figure 5.26 Shear Flows in Corrugated Plate for Different Configuration

No-Rib

One-Rib

Two-Ribs

Center Load Configuration
Table 5.8  The Value of Shear Flow $q_c$ (ksi/in) for Center Load Configuration

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Gage Load P (kip) Position</th>
<th>24</th>
<th>27</th>
<th>30</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-Rib</td>
<td>0.529</td>
<td>.15</td>
<td>-.3</td>
<td>-.9</td>
<td>-.45</td>
</tr>
<tr>
<td></td>
<td>0.902</td>
<td>.20</td>
<td>-.5</td>
<td>-1.5</td>
<td>-.66</td>
</tr>
<tr>
<td></td>
<td>1.52</td>
<td>.04</td>
<td>-.9</td>
<td>-2.6</td>
<td>-1.10</td>
</tr>
<tr>
<td></td>
<td>2.12</td>
<td>.65</td>
<td>-1.2</td>
<td>-3.7</td>
<td>-1.53</td>
</tr>
<tr>
<td></td>
<td>2.86</td>
<td>.95</td>
<td>-1.5</td>
<td>-4.8</td>
<td>-1.92</td>
</tr>
<tr>
<td></td>
<td>3.01</td>
<td>1.0</td>
<td>-1.6</td>
<td>-5.0</td>
<td>-2.01</td>
</tr>
<tr>
<td>One-Rib</td>
<td>1.51</td>
<td>1.1</td>
<td>1.85</td>
<td>-.15</td>
<td>6.99</td>
</tr>
<tr>
<td></td>
<td>2.74</td>
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<td>2.7</td>
<td>.03</td>
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<td>1.8</td>
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<td></td>
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<td>2.1</td>
<td>9.15</td>
<td>5.50</td>
<td>17.55</td>
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<td>Two-Ribs</td>
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<td>-4.0</td>
<td>3.35</td>
<td>.095</td>
</tr>
<tr>
<td></td>
<td>3.18</td>
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<td>-7.6</td>
<td>6.5</td>
<td>.085</td>
</tr>
<tr>
<td></td>
<td>3.97</td>
<td>-4.65</td>
<td>-8.7</td>
<td>8.0</td>
<td>.069</td>
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<tr>
<td></td>
<td>4.71</td>
<td>-5.90</td>
<td>-10.3</td>
<td>9.75</td>
<td>-.01</td>
</tr>
</tbody>
</table>

Table 5.9  The Value of Shear Flow $q_c$ (ksi/in) for Constant Moment Configuration

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Gage M (kip-ft) Position</th>
<th>24</th>
<th>27</th>
<th>30</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-Rib</td>
<td>0.68</td>
<td>-1.05</td>
<td>-2.3</td>
<td>-3.5</td>
<td>-1.03</td>
</tr>
<tr>
<td></td>
<td>1.10</td>
<td>-1.75</td>
<td>-3.7</td>
<td>-5.4</td>
<td>-1.72</td>
</tr>
<tr>
<td></td>
<td>2.18</td>
<td>-3.4</td>
<td>-6.6</td>
<td>-8.6</td>
<td>-3.96</td>
</tr>
<tr>
<td></td>
<td>2.64</td>
<td>-4.1</td>
<td>-7.5</td>
<td>-9.3</td>
<td>-5.16</td>
</tr>
<tr>
<td>One-Rib</td>
<td>1.23</td>
<td>.35</td>
<td>1.92</td>
<td>1.85</td>
<td>2.82</td>
</tr>
<tr>
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<tr>
<td></td>
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<td>.95</td>
<td>5.95</td>
<td>7.7</td>
<td>.49</td>
</tr>
<tr>
<td></td>
<td>5.68</td>
<td>.85</td>
<td>7.35</td>
<td>10.1</td>
<td>-2.38</td>
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<tr>
<td></td>
<td>6.55</td>
<td>.80</td>
<td>8.35</td>
<td>11.9</td>
<td>-5.10</td>
</tr>
<tr>
<td>Two-Ribs</td>
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<td>-4.3</td>
<td>1.4</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>2.38</td>
<td>-5.3</td>
<td>-5.3</td>
<td>1.6</td>
<td>4.22</td>
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<td>4.30</td>
<td>-9.7</td>
<td>-9.2</td>
<td>2.6</td>
<td>6.9</td>
</tr>
<tr>
<td></td>
<td>5.36</td>
<td>-12.0</td>
<td>-11.9</td>
<td>3.6</td>
<td>8.85</td>
</tr>
</tbody>
</table>
In the center load configuration, the external load is resisted by the middle part of corrugation for the no-rib case. This conclusion could also be verified by examining of shear flow on I-beam. Two points need to be noted in rib-plate combination. 1) Shear force is carried mainly by the rib; the shear stresses in corrugated plate are considerably small. 2) Shear stresses are increasingly shifted to the rib as load increases. These two points may be justified by either experimental data or theoretical analysis. Experimental data show the progression of shifting shear stress is evident in one-rib, center load configuration. At the initial loading stage, plate resists some load (shear flows in gage position 24 and 30 have same direction). As load increases, shear flow changes direction in these two gages. Theoretically, for plate-rib combination the plate is more flexible and more likely to distort than the rib. The absence of shear force in the constant moment configuration will explain why shear flows consistently through loading. Two conclusions are reached by examining of Figure 5.26. 1) Direction of shear flow points to the loading position in no-rib case. 2) Direction of shear flow points to the rib attachment.

5.6 MODELS FOR THE DIFFERENT RIB-PLATE CONFIGURATION

If $\tau /\sigma_y$ reaches a maximum value ( $\tau$ and $\sigma_y$ is shear stress and normal stress which are defined in beam theory), this shows the location of the neutral axis. In addition, if this ratio remains unchanged during the loading could result no shear stress transfer. Since $\tau$ and $\sigma_y$ are
proportional to the loading as the section reacts fully composite. Experimental data justified these two points evidently in no-rib case. First, from engineering principles, we know the neutral axis is located close to gage position 24 and 30. So the ratio is greater in these gage positions than any other gage positions. Then, since no shear stress transfer occurs in no-rib case (fully composite response), so the $T/\sigma_y$ stays unchanged.

Table 5.10 The Value of $T/\sigma_y$ for Center Load Configuration

<table>
<thead>
<tr>
<th>Case</th>
<th>Load (kip)</th>
<th>Gage position</th>
<th>24</th>
<th>30</th>
<th>27</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\tau/\sigma_y$</td>
<td>$\tau/\sigma_y$</td>
<td>$\tau/\sigma_y$</td>
<td>$\tau/\sigma_y$</td>
<td>$\tau/\sigma_y$</td>
</tr>
<tr>
<td></td>
<td>0.529</td>
<td>.03  .007  4.4</td>
<td>-18  .18  .10</td>
<td>-06  .49  .12</td>
<td>-09  .2  .45</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.902</td>
<td>.04  .025  1.7</td>
<td>-3    .3  .10</td>
<td>-1    .81  .12</td>
<td>-1.3  .31  .42</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.52</td>
<td>.08  .04   21</td>
<td>-52   .56  .93</td>
<td>-1.8  1.5  .12</td>
<td>-2.2  .56  .39</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.12</td>
<td>.13  .09   1.3</td>
<td>-74   .80  .92</td>
<td>-2.4  2.2  .11</td>
<td>-3.1  .79  .39</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.86</td>
<td>.19  .22   .87</td>
<td>-95   .10  .93</td>
<td>-3.0  2.8  .11</td>
<td>-3.8  .98  .39</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.01</td>
<td>.20  .25   .83</td>
<td>-1.0  1.1  .95</td>
<td>-3.1  3.0  .95</td>
<td>-4.0  1.0  .39</td>
<td></td>
</tr>
<tr>
<td>One-Rib</td>
<td>1.51</td>
<td>.22  .86   .26</td>
<td>-0.3  .35  .09</td>
<td>.37   1.9  .19</td>
<td>1.40  1.5  .92</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.74</td>
<td>.27  1.4   .19</td>
<td>.01   .45  .01</td>
<td>.54   3.3  .16</td>
<td>2.05  3.0  .68</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.01</td>
<td>.30  2.09  .14</td>
<td>.25   .37  .68</td>
<td>.82   4.8  .17</td>
<td>2.36  4.1  .58</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.57</td>
<td>.36  2.88  .13</td>
<td>.60   .45  1.3</td>
<td>1.3   6.5  .2</td>
<td>2.81  4.7  .60</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.43</td>
<td>.42  3.07  .11</td>
<td>1.1   .92  1.2</td>
<td>1.8   8.4  .22</td>
<td>3.51  4.6  .76</td>
<td></td>
</tr>
<tr>
<td>Two-Ribs</td>
<td>1.6</td>
<td>-.47 -.76  .62</td>
<td>-.67  1.1  .61</td>
<td>-.8   1.4  .56</td>
<td>.02   .8  .03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.18</td>
<td>-.85 -.17  .51</td>
<td>-.13  2.1  .63</td>
<td>-.15  3.2  .47</td>
<td>.02   1.3  .01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.97</td>
<td>-.93 -.19  .48</td>
<td>-.16  2.4  .68</td>
<td>-.17  4.0  .43</td>
<td>.01   1.4  .01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.71</td>
<td>-.12 -.15  .76</td>
<td>-.20  2.7  .72</td>
<td>-.21  5.0  .41</td>
<td>.002  1.5  0.0</td>
<td></td>
</tr>
</tbody>
</table>

In one-rib and two-ribs configurations since this ratio varies with the loading would indicate change of the neutral axis and shear stress transfer. This effect results some degree of composite response taking place in rib-plate combination, or composite reaction is developed in one-rib and two-ribs configuration. So the degree of affected composite response may be
measured by shear stresses transfer and the change of normal stresses. Shear flow value in gage position 24 is greater than the one in gage position 30 in one-rib case and vice versa in two-ribs case; see Table 5.8. This indicates the large degree shear transfer for the area which is close to the rib. Normal stress in gage position 24 is greater than the one in gage position 30 in one-rib configuration and vice versa in two-ribs configuration. These indicate that the section becomes more rigidity as it moves to the rib attachment. When composite action increases, shear transfer and sectional rigidity also increase. The result of above discussion can be expressed by Figure 5.28.

![Physical view](image)

**Physical view**

- **No-Rib**
- **One-Rib**
- **Two-Ribs**

**Models**

- $\tau \sigma$ diminishes as away from center
- Combination section expands until composite response finish, or, $\alpha$ reaches $\alpha_{cr}$

Figure 5.27 Models for the Different Rib-Plate Configuration
5.7 EXPERIMENTAL RESULTS COMPARE WITH GALERKIN SOLUTION

Theoretical stress is calculated in only one direction by Culvert Analysis And Design (CANDE). In box culvert design, the circumferential stress is presented and compared with experimental data (20). When using Galerkin method, however, we can calculate stresses in two directions and vertical deflection. Comparisons for no-rib, one-rib and two-ribs configuration are presented.

Taking the end supports as either elastic restrained or simply-supported and sides free for all three configurations, the comparisons for no-rib configuration are presented in Tables 5.11,5.12 and Figures 5.28,5.29.

<table>
<thead>
<tr>
<th>Stress</th>
<th>$\sigma_x$ (Perpendicular Direction) ksi</th>
<th>$\sigma_y$ (Corrugation Direction) ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gage</td>
<td>2 4 6 8 10 12 14 39</td>
<td>1 3 5 7 9 11 13 41</td>
</tr>
<tr>
<td>The.</td>
<td>1.7 2.31 1.5 1.95 0.90 1.2 0.04 0.09</td>
<td>-9.07 10.9 -7.6 9.18 -4.78 5.74 -0.5 0.57</td>
</tr>
<tr>
<td>Exp.</td>
<td>1.4 1.09 1.2 1.49 1.14 0.8 0.97 -0.16</td>
<td>-10.7 17.2 -7.10 11.6 -3.71 5.85 -0.5 0.58</td>
</tr>
<tr>
<td>Abs. %</td>
<td>19 112 21 31 21 53 96 153</td>
<td>15 37 8 21 29 2 4 5</td>
</tr>
</tbody>
</table>

| Differ. | 15 37 8 21 29 2 4 5 |

Exp.= Experimental results.
Abs. Differ.= Absolute difference between experimental and theoretical values.
No-rib, center load
P=2.86kip, K=15200

No-rib, constant moment
M=2.18kip-ft, K=0

Figure 5.28 Illustration of Comparisons for Displacements
Figure 5.29  Comparisons Among the Rosette Gages for No-Rib, Constant Moment, $M=2.18\text{kip-ft}$

Table 5.12 Stress Comparisons Among Biaxial Gage for No-Rib with Constant Moment, $M=2.18\text{kip-ft}$ and Elastic Constant $k=0$

<table>
<thead>
<tr>
<th>Stress</th>
<th>$\sigma_x$ (Perpendicular Direction) ksi</th>
<th>$\sigma_y$ (Corrugation Direction) ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gage</td>
<td>2 4 6 8 10 12 14 39</td>
<td>1 3 5 7 9 11 13 41</td>
</tr>
<tr>
<td>The.</td>
<td>0.5 1.3 0.5 1.1 0.3 0.71 0.09 0.21</td>
<td>-7.37 9.03 -6.19 7.58 -4.02 4.9 -1.21 1.48</td>
</tr>
<tr>
<td>Exp.</td>
<td>-0.39 1.12 -0.16 0.81 0.5 0.76 0.65 -1.3</td>
<td>-6.0 9.92 -5.38 9.59 -4.52 9.7 -2.07 3.91</td>
</tr>
<tr>
<td>Abs. %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Differ</td>
<td>234 16 375 35 41 7 87 117</td>
<td>23 9 15 21 11 49 42 62</td>
</tr>
</tbody>
</table>

Exp.= Experimental results.
Abs. Differ.= Absolute difference between experimental and theoretical values.
Either in experimental analysis or in theoretical analysis, it is necessary to discuss composite and noncomposite reaction when ribs are assembled to the corrugated plate. In composite response, we can calculate the moment of inertia and centroid for the section. On the other hand, in noncomposite response, we cannot calculate the moment of inertia and centroid for the whole section (the combination of individual section). As in the combination section (rib-plate combination), centroid and moment of inertia for the whole section cannot be calculated because of the noncomposite reaction. In the noncomposite reaction, the individual structural behavior during the performance is dependent on their flexural rigidity. Also compatible of displacement must be satisfied for whole structure. From this point, we should not use Beal's formula (18) as the way to describe noncomposite reaction, since a composite section was used to develop a noncomposite formulation. A better approach is to use Equation (5.5) and it describes the progression between noncomposite and composite performance.

The fixed end conditions are modelled in the laboratory for one-rib with center load configuration, theoretical results show that the simply supported end condition compares well with deflection, with $\sigma_y$ near the end support and with $\sigma_y$ at the crown of the corrugation, see Tables 5.13 and 5.14. The result of elastic restrained end conditions compares well for $\sigma_y$ in the center at the valley. The following detail discussion is mainly based on the simply-supported end condition.
Table 5.13  Stress Comparisons Among Biaxial Gage for One-Rib with Center Load, $P=7.42$ kip and Elastic Constant $k=0$

<table>
<thead>
<tr>
<th>Stress</th>
<th>$\sigma_x$ (Perpendicular Direction) ksi</th>
<th>$\sigma_y$ (Corrugation Direction) ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gage</td>
<td>2 4 6 8 10 12 14 39</td>
<td>1 3 5 7 9 11 13 41</td>
</tr>
<tr>
<td>The.</td>
<td>1.1 2.61 0.9 2.20 0.6 1.43 .17 0.43</td>
<td>-14.8 18.1 -12.4 15.2 -8.1 9.9 -2.4 2.97</td>
</tr>
<tr>
<td>Exp.</td>
<td>7.2 2.81 4.4 4.16 2.3 2.07 2.7 -4.79</td>
<td>-11.9 23.0 -6.3 15.7 -2.5 10.9 2.3 4.93</td>
</tr>
<tr>
<td>Abs.%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Differ.</td>
<td>85 7 80 47 74 31 94 109</td>
<td>25 21 99 4 227 9 206 40</td>
</tr>
</tbody>
</table>

Note: Gage position is same as in Figure 4.6.

Table 5.14  Deflection Comparison for One-Rib with Center Load, $P=7.42$ kip and $k=0$

<table>
<thead>
<tr>
<th>Section</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>The.(in)</td>
<td>0.70</td>
<td>0.59</td>
<td>0.38</td>
<td>0.11</td>
</tr>
<tr>
<td>Exp.(in)</td>
<td>0.80</td>
<td>0.69</td>
<td>0.48</td>
<td>0.24</td>
</tr>
<tr>
<td>Abs.%</td>
<td>12</td>
<td>14</td>
<td>20</td>
<td>52</td>
</tr>
</tbody>
</table>

Note: Section number is same as in Figure 4.6.

Table 5.15  Stress Comparisons Among Biaxial Gage for Two-Ribs with Center Load, $P=3.18$ kip and Elastic Constant $k=0$

<table>
<thead>
<tr>
<th>Stress</th>
<th>$\sigma_x$ (Perpendicular Direction) ksi</th>
<th>$\sigma_y$ (Corrugation Direction) ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gage</td>
<td>2 4 6 8 10 12 14 39</td>
<td>1 3 5 7 9 11 13 41</td>
</tr>
<tr>
<td>The.</td>
<td>0.3 0.77 0.3 0.7 0.2 0.42 0.05 0.13</td>
<td>-4.38 5.36 -3.68 4.50 -2.39 2.9 -0.72 0.88</td>
</tr>
<tr>
<td>Exp.</td>
<td>0.2 -2.2 0.1 -6.42 -.12 -.89 0.03</td>
<td>0.08 8.8 0.2 5.46 -2.05 3.7 -3.85 1.79</td>
</tr>
<tr>
<td>Abs.%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Differ.</td>
<td>94 135 190 214 140 451 106 58</td>
<td>5570 39 1938 18 16 20 81 51</td>
</tr>
</tbody>
</table>

In one-rib center load configuration, theoretical $\sigma_x$ is less than experimental values. But theoretical $\sigma_y$ is greater than the experimental values at the valley and is close to the experimental values at the crown. Theoretical deflection is greater than the experimental values for small load.
This difference between two values diminishes as load increases. Also it is shown that the stresses at some part of corrugation are out of elastic range, otherwise the differences should remain constant during the whole performance. Figure 5.30 represents a typical pattern for one-rib, center load and simply-supported end condition. We are not surprising that the theoretical stress at the middle plane of corrugated plate was zero, since Galerkin method is based on the noncomposite assumption. In constant moment configuration, however the theoretical values overestimate the experimental values, the difference between these two values are constant during the whole procedure. Comparing these two configurations, the bending case had a better agreement than the center load case. This indicates since the rib did not join the end could affect result in the center load configuration when compared to the constant moment configuration.
Figure 5.30  Stress Comparisons Among the Rosette Gage for One-Rib, Center Load, P=7.43 kip

Table 5.16  Deflection Comparison for Two-Ribs with Center Load, P=3.18 kip and k=0

<table>
<thead>
<tr>
<th>Section</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>The.(in)</td>
<td>0.21</td>
<td>0.17</td>
<td>0.11</td>
<td>0.03</td>
</tr>
<tr>
<td>Exp.(in)</td>
<td>0.34</td>
<td>0.28</td>
<td>0.19</td>
<td>0.08</td>
</tr>
<tr>
<td>Abs. Differ.</td>
<td>39</td>
<td>38</td>
<td>39</td>
<td>56</td>
</tr>
</tbody>
</table>
For two-ribs case and simply-supported end condition, theoretical deflection and $\sigma_y$ are less than the experimental values, whereas theoretical $\sigma_x$ was greater than the experimental value, see Tables 5.13 and 5.16. From these two tables, the following observation can be made: In two-ribs case, the reaction of whole plate is divided into two parts in the corrugation direction. 1) Near the center in the corrugation direction, the section reacted as composite section, since the experimental stress was close to zero at the valley and this indicates the section behaved as composite reaction. Because the sectional centroid was near the valley under the composite reaction, and normal stress must vanish along the centroid. 2) In the plan off the center in corrugation direction, the section reacted as a virtual noncomposite, because stress was prominent at the valley and the centroid of the plate. Having virtual in front of noncomposite, we just want to note that this is different from total noncomposite. Since in the total noncomposite reaction, the stress should vanish at the individual centroid in the section of rib and plate.

The above detail discussion may be summarized as follows;

1. It is difficult to model the end condition exactly.

2. For no-rib configuration, Galerkin solution is recommended as a good approach. Experimental deflection in corrugated plate agrees with the assumption of simply supported end condition, and experimental stress agrees with the assumption of elastic restrain end condition.

3. The result of theoretical analysis underestimates the experimental stress in perpendicular direction.

4. Simply supported end condition and the assumption of noncomposite reaction in theoretical analysis is a good approach to one-rib and two-ribs configurations.
5. Greater the number of ribs in the corrugated plate, the more complicated the situation will be.
The study of rib-plate structure is presented. After instrumentation, several rib-plate configurations were tested, a detailed review of the data analysis procedure followed. The experimental analysis emphasized the topics of composite response and the effects of bolt connection. Several findings of the former researchers were extended. Stresses in corrugation direction were studied completely. Experimental data analysis began with the normal stresses measured at the crown and valley of corrugated plate. Then shear stresses and normal stresses in corrugation direction at different plane around the bolt hole were analyzed. Principal stress direction was presented. Three models were developed for three kinds of rib-plate response. Analysis ended with the comparison between experimental results and theoretical results. Theoretical results were calculated from Galerkin method applied to an orthotropic plate, which is based on two sides free and ends as simply-supported or elastic restrained. The following conclusions were reached:

(1) Increasing the number of stiffeners resulted in reducing stresses in stiffeners and but did not affect the maximum stresses in the plate. So, stiffeners only improve culvert stability to retain original configuration.
(2) The normal stresses in the cross section varies with position with the result that \(\text{(moment)/(unit width)}\) at the crown is different from \(\text{(moment)/(unit width)}\) at the valley of corrugation.

(3) For rib-plate combination, the section does not perform as either composite or noncomposite. As the load increases, structural response changes in the ranges of virtual noncomposite, noncomposite and composite. The degree of performance \(\alpha\) is an ideal measurement to differentiate each individual response.

(4) Bolt hole disrupts normal stresses in corrugation direction and shear stresses in all directions.

(5) For plate-rib combination, shear stresses are transferred in the perpendicular direction, while both normal and shear stresses are transferred in corrugation direction.

(6) The fundamental model for plate and rib-plate combination is established on the fact that section becomes more rigid as it moves to the rib-location.

(7) Away from rib, principal stress direction stays unchanged along the \(45^\circ\) plane. For the positions around the bolt hole, the principal stress direction will change as the load increases.

(9) In Galerkin method; theoretical results show that simply-supported end conditions compares well only for deflection at the center, while elastic restrained end conditions give the best description for stresses distribution at the ends. Constant moment loading compares well to theory than center load loading. For no-rib configuration, Galerkin method is proved to be good approach.

(10) Simply-supported end conditions and assumption of noncomposite response in theoretical analysis is a good approach to one-rib and two-ribs configuration.
REFERENCES


