A CARRIER PHASE ONLY PROCESSING TECHNIQUE 
FOR 
DIFFERENTIAL SATELLITE-BASED POSITIONING SYSTEMS 

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<tr>
<td>3-D</td>
<td>3-Dimensional</td>
</tr>
<tr>
<td>ADR</td>
<td>Accumulated Delta Range</td>
</tr>
<tr>
<td>C/A Code</td>
<td>Coarse/Acquisition Code (of GPS)</td>
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<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
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<tr>
<td>DD</td>
<td>Double Difference</td>
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<tr>
<td>DLL</td>
<td>Delay Lock Loop</td>
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<tr>
<td>DoD</td>
<td>Department of Defense</td>
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<tr>
<td>DOP</td>
<td>Dilution of Precision</td>
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<td>EKF</td>
<td>Extended Kalman Filter</td>
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<tr>
<td>FDI</td>
<td>Fault Detection and Isolation</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
</tr>
<tr>
<td>GLONASS</td>
<td>Global Orbiting Navigation Satellite System (Russian)</td>
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<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System (U.S.)</td>
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<td>ID^3</td>
<td>Integrated Doppler Double Difference</td>
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<tr>
<td>LEO</td>
<td>Low-Earth Orbit</td>
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<td>LS</td>
<td>Least Squares</td>
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<td>MEKF</td>
<td>Modified Extended Kalman Filter</td>
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<td>MSS</td>
<td>Mobil Satellite Service</td>
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<tr>
<td>NCO</td>
<td>Numerical Controlled Oscillator</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Definition</td>
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<tr>
<td>P Code</td>
<td>Precision Code (of GPS)</td>
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<td>PLL</td>
<td>Phase Lock Loop</td>
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<tr>
<td>PPS</td>
<td>Precise Positioning Service (of GPS)</td>
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<tr>
<td>PR</td>
<td>Pesudo Range</td>
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<tr>
<td>PRN</td>
<td>Pseudo Random Noise</td>
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<tr>
<td>PZ-90</td>
<td>Geodetic Coordinate System used by GLONASS</td>
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<tr>
<td>RA</td>
<td>Radio Astronomy</td>
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<td>RSF</td>
<td>Russian Space Forces</td>
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<td>RTK</td>
<td>Real-Time Kinematic</td>
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<td>SA</td>
<td>Selective Availability (on GPS)</td>
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<td>SD</td>
<td>Single Difference</td>
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<td>SPS</td>
<td>Standard Positioning Service (of GPS)</td>
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<td>SU</td>
<td>Soviet Union</td>
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<tr>
<td>SV</td>
<td>Space Vehicle</td>
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<tr>
<td>TD</td>
<td>Triple Difference</td>
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<tr>
<td>TEC</td>
<td>Total Electron Content</td>
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<tr>
<td>T_R</td>
<td>Time of Reception</td>
</tr>
<tr>
<td>T_T</td>
<td>Time of Transmission</td>
</tr>
<tr>
<td>U(E)RE</td>
<td>User (Equivalent) Range Error</td>
</tr>
<tr>
<td>UTC</td>
<td>Universal Time Coordinated</td>
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<tr>
<td>USNO</td>
<td>United States Naval Observatory</td>
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<tr>
<td>WGS-84</td>
<td>Worldwide Geodetic System used by GPS</td>
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1. Introduction

This dissertation will present the concept and an algorithm for processing carrier phase measurements from satellite-based signal sources, to determine the three-dimensional (3-D) relative position between two receivers. Position accuracies on the order of 1 - 50 centimeters can be obtained depending on the number of measurements, satellite geometries, and the separation distance between the two users. The proposed algorithm is based on an extended Kalman filter, which uses satellite geometry variations, together with an accurate relative position propagation, to converge the user's 3-D relative position down to sub-meter level within a few minutes. This converged position can serve as the initial estimate to launch a small ambiguity search, and then refine the user position to centimeter level accuracy after the ambiguities are resolved. If one of the users is at a known location, then the absolute location of the second user is found by adding the relative position vector to the known location. This is also referred to as differential positioning.

The necessity of a carrier phase processing technique originates from the demand for centimeter level accuracy in positioning, which is required for many applications such as aircraft landings on aircraft carriers, platform attitude determination, spacecraft docking, photogrammetry, mining, dredging, and agricultural machine control. This dissertation focuses on determining a user's 3-D position using primarily satellite-based navigation systems such as the United States' Global Positioning System (GPS) or the Russian Global Orbiting Navigation Satellite System (GLONASS). The application of GPS satellites for
positioning and navigation has been broadly adopted in a variety of areas in land, sea, and air transportation over the last decade. The GPS signal provides two kinds of observables for positioning, one of which is the code phase or pseudorange measurement, the other is the carrier phase or integrated Doppler frequency shift measurement. Code phase position accuracy that can be achieved by an autonomous (stand-alone) GPS user with the standard positioning service (SPS) is approximately 100 meters (95%) horizontal and 156 meters (95%) vertical.[35] If a differential operation is employed to cancel the correlated errors between two receivers, the resultant code phase positioning error is approximately 1-5 meters. Therefore, the use of a carrier phase differential processing technique is indispensable for users who require sub-meter accuracies.

The purpose of this dissertation is to develop, simulate, and analyze the concept and a preliminary algorithm for processing the carrier phase measurements from multiple satellites to determine a user's position with respect to a stationary or dynamic reference receiver. Actual data will be used to demonstrate the feasibility of the algorithm by post processing.

The balance of this dissertation is divided into 9 chapters. In the second chapter, satellite positioning principles as well as usable signal sources will be presented. Chapter 3 will cover the theoretical foundation and equations associated with the processing algorithms. The application of integrating a Low-Earth-Orbit (LEO) satellite to rapidly converge the 3-D position will be studied in Chapter 4. A refinement of the measurement model will be investigated in Chapter 5 to accommodate longer distances between the two
receivers. In Chapter 6, the motivation and investigation of a statistical analysis as well as a demonstrative example will be provided. Chapter 7 will address the need for a system observability analysis. Chapter 8 will cover the concept of combining GPS and GLONASS processing, the potential problems of using GLONASS measurements, and the advantages of this integrated processing. Finally, a summary and conclusion will be provided in Chapter 9.
2. **Background**

In this chapter, the principles of satellite positioning and carrier phase differential processing will be discussed. The U.S. Global Positioning System (GPS) is mainly, but not exclusively, the satellite system on which this proposed technique is based. First, error sources and an error budget associated with the GPS will be presented in this chapter. Next, this chapter will introduce other usable signal sources, such as GLONASS or LEOs that can be integrated into the proposed algorithm. Finally, existing carrier phase processing techniques will be reviewed.

2.1 **Principle of Satellite Positioning**

In general, there are two modes of operation for satellite positioning, one is referred to as autonomous or stand-alone positioning and the other is called relative or differential positioning. Differential positioning can be conducted with either code phase or carrier phase measurements. Usually, carrier phase differential positioning is used for high-accuracy applications.

2.1.1 **Autonomous Positioning**

The principle of satellite positioning in the autonomous mode is based on trilateration of multiple ranging measurements from satellites to a single user. By applying the recursive
ordinary Least Squares solution, and knowing the ranges to a minimum of 4 satellites, the user's 3-D position and time can be estimated. Existing satellite positioning systems are designed as passive systems. This means that there is no transmission from the user to the satellite. Because of this, range measurements can only be made directly if the satellite clock and the user clock are synchronized. In general, this is not the case and the range measurement thus includes a clock offset. Therefore, it is referred to as a pseudorange measurement to reflect the presence of the clock offset, and the user must solve for both 3-D position and clock offset relative to the satellite system. Due to this reason, a minimum of four pseudorange measurements are required, for determining a user's 3-D position. Also, the orbit information of the four satellites must be obtained before applying the Least Squares estimation to determine the user's position. Pseudorange measurements are the major observables for autonomous positioning.

The Least Squares estimator is based on the expression of the navigation solution given by

\[ PR^i_k = \sqrt{(x^i - x_k)^2 + (y^i - y_k)^2 + (z^i - z_k)^2 + c \cdot b_k} \]  

(2-1)

where \( PR^i_k \) is the pseudorange from satellite \( i \) to receiver \( k \), and \( x^i, y^i, z^i \) are the coordinates of the satellite (obtained from the navigation message), \( c \) is the speed of light, and \( b_k \) is the clock offset. Since the expression for the pseudorange as given by Equation 2-1 is a non-linear equation, it has to be linearized about the user's position in order to apply the linear Least Squares estimator. A change in the estimated user position is calculated based on the differences between the measured and the estimated positions. The calculated change is then
added to the estimated position. This process is repeated until the user’s change in position is very small. At this point, the estimated user position has converged and the user position is determined.

The positioning accuracy is affected by ranging errors which will be addressed in Section 2.2. For GPS, the accuracy for autonomous positioning under SPS as stated in Chapter 1 is 100 m (95%) horizontal and 156 m (95%) vertical. To further improve the accuracy, differential positioning techniques must be used.

2.1.2 Differential or Relative Positioning

In the relative or differential mode of satellite positioning, the accuracy can be improved by correcting the correlated ranging errors. Differential operation is comprised of a local reference station, a data broadcast capability, and users which can receive and process the broadcast data. Depending on the application, the position of the reference station’s receiving antenna can be surveyed in advance. Pseudorange corrections are determined by comparing the measured pseudoranges with the predicted pseudoranges that are based on the pre-surveyed location of the reference station, the calculated satellite positions, and clock offsets. Next, the pseudorange corrections are broadcast to local users. The local user can then correct its pseudorange measurements using the received corrections from the reference station, thus obtaining absolute position accuracies on the order of 1-5 meters (95%). If the reference antenna is not surveyed with high accuracy, the stand-alone GPS position of the reference antenna can be used instead. Note that the accurate relative position is thus
determined with respect to the reference antenna. Due to the dependency on the spatial correlation of the error sources, the performance of a differential positioning technique degrades as the separation between the reference and user antennas increases. Usually, antenna separations of 10 - 20 km are tolerable for sub-meter accuracies.

2.1.3 Carrier Phase Differential Positioning

The differential operation introduced in Section 2.1.2 is referred to as code phase differential positioning, since pseudorange or code phase measurements are used. As stated in the previous section, the achievable accuracy of code phase differential positioning is approximately 1 - 5 meters. To further improve the level of accuracy, carrier phase measurements can be used. The wavelength of the GPS L1 carrier at 1575.42 MHz is 19 cm, which is approximately 1500 times smaller than that of the code phase (293.4 m). Therefore, carrier phase measurements have better resolution than code phase pseudorange measurements. As a result, centimeter level positioning accuracy can be achieved by using the carrier phase measurements. The accumulated carrier phase is what the receiver produces; it is also referred to as the integrated Doppler frequency shift measurement.

If a user's position is determined by measuring the carrier phases from the satellites, and comparing that to the carrier phases measured at the reference site, then this operation is called carrier phase differential positioning, sometimes also referred to as carrier phase interferometry. Although a carrier phase measurement yields better resolution, the measurement is ambiguous due to an unknown number of integer wavelengths, which is a
constant bias relative to the start of integration or accumulation, and it must be removed from the measurement. Figure 2-1 illustrates the carrier phase ambiguity problem for measurements taken at two receivers.

\[ \Delta \phi = \phi_1 - \phi_2 = \Delta \lambda + N \lambda \]

Figure 2-1. The Interferometry of Carrier Phase

The difference between the integrated carrier phase at the two receivers is referred to as a Single Difference (SD), which contains an unknown integer ambiguity. The ambiguity must be solved for in order to determine accurate relative or absolute user positions. Once the ambiguity is resolved, the 3-D position solution can be maintained at the centimeter level as long as the carrier phase measurements are continuously tracked without loss of cycles.
Under certain circumstances, it is possible for the carrier-phase measurement from the receiver to jump in either a positive or negative direction. This discontinuity in the carrier-phase measurement is called a cycle slip and is usually due to the SV signal being blocked temporarily. The blockage could be caused by surrounding obstacles, or, in the case of an aircraft, temporary shielding by the airframe during maneuvering. Other potential causes are low signal-to-noise ratio due to ionospheric effects, severe signal multipath, and low satellite elevation angle. If any of these events occurs, carrier-phase tracking can be interrupted (possibly terminated), and cycle slips may be present in the carrier-phase measurements.

In any type of interferometric-based or carrier phase differential GPS positioning system, the detection of cycle slips is crucial if high confidence in the accuracy of the results is to be maintained. For a detailed discussion of cycle-slip detection, the reader is referred to Hofmann-Wellenhof.[19]

2.2 Error Sources and Error Budget

Satellite-based positioning is accomplished by measuring the precise ranges from satellites to the user. Therefore, the ranging error will be reflected in the position accuracy. Ranging errors originate from different sources which can be classified into correlated errors, uncorrelated errors, and signal propagation delays. Errors can be correlated in either or both the time and space domains.
2.2.1 Correlated Errors

Correlated errors consist of satellite clock error, satellite orbit determination error, and Selective Availability (deliberate satellite clock and/or orbit errors induced by the U.S. Department of Defense). The satellite clock error is the deviation of the onboard atomic clock from the true GPS system time. This error is the same in all directions. In other words, all differential stations and users observe an identical satellite clock error. Differential operations can be used to eliminate the satellite clock error. The satellite clock error is typically on the order of 10 ns.[29]

The satellite orbit determination error is reflected in the satellite ephemeris. This error shows in radial, tangential, and cross-track directions with respect to the velocity of the satellite. The tangential and cross-track components are an order of magnitude larger than the radial component. However, the former do not affect ranging precision too much, since only the projection of the satellite position error on the line-of-sight creates a ranging error. The root-mean-square (rms) ranging errors due to the ephemeris are about 2.1 m. These errors are the same for both C/A code and P code. For relative or differential users, the tangential and cross-track errors can be significant since their projections are different for separated users.

The largest error source for GPS SPS is Selective Availability (SA). This error is deliberately induced by the DoD to degrade the user’s position accuracy. It is implemented through the manipulation of the broadcast ephemeris data and through the dithering of the
satellite clock. The resultant SA rms error is typically 20 m. Therefore, SA is dominant among the correlated errors for autonomous positioning.

2.2.2 Uncorrelated Errors

Uncorrelated errors include receiver clock error, receiver inter-channel biases, receiver noise (thermal noise and measurement noise), receiver tracking errors, receiver software errors, antenna phase center variations, and multipath. Like satellite clock errors, the clock inside the receiver also has an offset with respect to GPS system time, which is referred to as the receiver clock offset. It can cause the receiver position to be calculated at the wrong time, thus introducing position errors. Inter-channel biases arise from the hardware biases and phase delays associated with each channel. These biases can be minimized by digitally sampling the signal and using an all-digital design. Receiver measurement noise is caused by thermal noise, quantization noise, and oscillator noise. The receiver tracking error occurs due to the inability of the receiver tracking loops to precisely track the signal under dynamics. This error is a function of the types of filters used and their bandwidths. The software error mainly results from the number of bits used to express the range measurements in full precision.

All of the above mentioned errors are generated by the receiver, and vary from receiver to receiver. Modern receiver designs usually produce less than 0.5 m in biases and 0.2 m in noise.
Multipath is a phenomenon in which a direct signal reaches a receiving antenna via more than one propagation path. The additional paths are a result of reflection and/or diffraction of the direct ray due to surrounding objects. The composite signal differs in magnitude and phase from that of the direct signal. The signal modulation and the carrier phase will be distorted, which causes tracking errors. Multipath degrades the positioning accuracies of conventional and differential systems as well as interferometric systems. Multipath has a stronger influence on code phase than on carrier phase measurements by approximately two orders of magnitude. It has also been shown that code-phase multipath error is not zero mean and can have periods on the order of tens of minutes. For a moving user, the high frequency components of multipath will usually be averaged out. For a static user, a filter can be used to average the high frequency components of the multipath. The low frequency components of multipath, however, cannot be reduced by filtering. A variety of methods can be used to reduce the multipath error, such as antenna siting, antenna design, and receiver design. The proper antenna siting reduces multipath error because the multipath error is location-dependent. Special antenna design can ease the impact of multipath by shaping the antenna pattern to reject the multipath that enters the antenna from low elevation angles. Also state-of-the-art receiver designs apply different correlator techniques to alleviate the multipath error.

If the antenna is properly sited, the net multipath error for a moving user or for a static user with an averaging filter would be less than 1-2 m for code phase measurements, and less than 5 cm for carrier phase measurements. For the differential positioning mode, multipath turns out to be the dominant source for ranging error. It is noted that multipath
could be correlated for small separation distances between receiving antennas, but there is practically little correlation for users separated by more than a few hundred meters.[35]

2.2.3 Signal Propagation Delays

Signal propagation delays consist of the ionospheric delay and the tropospheric delay. The ionospheric delay is caused by free electrons in the ionosphere, due to which the signal does not travel at the speed of light. The modulation of the signal is delayed by an amount that is proportional to the Total Electron Content (TEC), and is also proportional to the inverse of the carrier frequency squared, $1/f^2$. However, the phase of the carrier frequency is advanced by the same amount, which phenomenon is called a dispersive property. The ionospheric effect is fairly stable and predictable for temperate zones, but fluctuates considerably near the equator or magnetic poles. The correction for the ionospheric delay can be calculated from a diurnal model that relies on the parameters broadcast in the navigation message. The obtained correction can achieve 2-5 m accuracy in ranging for users in the temperate zones. Also, a dual frequency technique (L1 and L2) can be applied to directly solve for the delay, which produces a ranging accuracy of 1-2 m.[19, 35]

The troposphere is another layer that will contribute to signal delay. Variations in temperature, pressure, and humidity all contribute to variations in the signal’s traveling velocity and bending in the troposphere. This delay is the same for both the code phase modulation and the carrier phase. Also, beside weather changes, the tropospheric effect is a function of altitude. A tropospheric model using temperature, pressure, and humidity can
be used to correct the delay, such that the corrected ranging accuracy is better that 1 m. For relative or differential applications, the propagation delays de-correlate as a function of the separation distance between the two receivers.

2.2.4 Geometric Dilution of Precision

Satellite-based position errors are functions of both the ranging errors and the satellite geometry. The satellite geometry can be expressed as the Position Dilution of Precision (PDOP), and its relation to the position error is given by: [12]

\[
\sigma_p = PDOP \cdot \sigma_m
\]  

(2-2)

where \(\sigma_p\) is the rms position error, \(\sigma_m\) is the rms measurement error, and PDOP is the geometric factor. The average PDOP value varies from as good as 1.7 to as poor as 6 or 7 during a 24-hour observation period. Details on the derivation of PDOP can be found in [12]. For the Kalman filter-based technique proposed in this dissertation, it is found that the PDOP factor does not directly affect the positioning error as it does in the case of general satellite-based positioning. The system’s observability must be analyzed to find the relationship between position error and measurement error, which will be covered in Chapter 7.
2.3 Usable Signal Sources for Carrier Phase Processing

There are several signal sources that can provide measurements to determine the user position. Currently, there are two different satellite systems available for civilian use: the Global Positioning System (GPS) operated by the U.S. Department of Defense, and the Global Orbiting Navigation Satellite System (GLONASS) operated by the Russian Space Forces. In addition, there are many other signals available from low-earth orbit (LEO) satellite systems. Although these satellites do not all transmit at protected navigation frequencies, the use of these satellites will also be discussed in this dissertation.

2.3.1 Global Positioning System (GPS)

The U.S. Global Positioning System (GPS) is an advanced satellite system that is being used by millions of users worldwide. The GPS system operated by the U.S. Department of Defense (DoD), comprises three segments -- space segment, control segment, and user segment. The space segment consists of 24 satellites, every four of which are evenly distributed in each of the six orbital planes, which are inclined 55° with respect to the equatorial plane. Every satellite has an orbiting period of approximately 11 hours and 58 minutes. The control segment contains 5 monitor stations, four ground antenna upload stations, and the Operational Control Center (OCC). The 5 monitor stations are to track and to provide information to the OCC for updates to the satellites' precise orbit and clock information. All GPS satellites transmit signals at the L1 frequency of 1.57542 GHz and
at the L2 frequency of 1.2276 GHz. The L1 frequency is modulated by the C/A code for civilian users and the P code for authorized users, L2 is only modulated by the P code, and both L1 and L2 are modulated by a 50 Hz navigation message. GPS system characteristics and signal structures are well documented in [35] and [40]. In this dissertation, only the observables directly related to position determination are of interest. The GPS satellites provide two major observables for positioning purposes:

1) Pseudorange (PR).

The pseudorange measurement is obtained by multiplying the time difference \((T_R - T_T)\) between the time of arrival and time of transmit by the speed of light. By finding the maximum correlation between the locally-generated Pseudo Random Noise (PRN) code and the PRN code modulated on the incoming signal, the time of transmit can be extracted from the tracking loop inside the receiver. Every epoch, the satellite PRN code is precisely aligned with the GPS time of week (TOW) as maintained by the satellite’s onboard clock. After the receiver successfully correlates the received signal with the locally-generated PRN code, the phase offset of the replica PRN code with respect to the beginning of the GPS week represents the transmit time of the satellite. Subtracting this time from the current local receiver time produces the time difference. The time difference, if scaled by the speed of light, results in the code phase observable, which is called the pseudorange.

2) Integrated Doppler frequency shift or Accumulated Delta Range (ADR).

Doppler frequency shift is generated by the relative motion between the satellite and receiver. In a GPS receiver, to successfully maintain carrier tracking, the local carrier
should exactly lock onto the carrier of the incoming signal. This is implemented by a signal processor Phase-Lock Loop (PLL). The input to the numerical controlled oscillator (NCO) is the Doppler shift required for the PLL to lock onto the signal carrier. The accumulation of this Doppler shift is called integrated Doppler or accumulated delta range (ADR) because it keeps track of the changes in the observed range to the satellite. In the GPS literature, this measurement is also called “carrier beat phase” or carrier phase for short. The integrated Doppler or accumulated carrier phase measurement is approximately 1000 times less noisy than the code phase (pseudorange) measurement. The noise level of integrated Doppler is approximately 1 mm while the pseudorange noise is approximately 1 meter. Although the integrated Doppler measurement has a very good resolution, it is ambiguous since the initial carrier phase between satellite and receiver is unknown when the receiver locks onto the satellite signal. Therefore, to benefit from the carrier phase positioning technique, the unknown carrier cycle ambiguity must be resolved.

2.3.2 Global Orbiting Navigation Satellite System (GLONASS)

Another satellite system similar to the GPS is the Global Orbiting Navigation Satellite System (GLONASS) owned and operated by the Russian Space Forces (RSF). GLONASS also has 24 satellites in the space segment, but it has only three orbital planes, inclined at an angle of 64.8° with respect to the equatorial plane. Each of the three orbital planes contains 6 evenly spaced satellites. GLONASS also broadcasts two frequencies, L1
and L2, with the C/A code and the P code modulated on the L1 frequency, and only the P code on the L2 frequency. However, each GLONASS satellite uses a different frequency, which is referred to as Frequency Division Multiple Access (FDMA). Differences between GPS and GLONASS are listed in Table 2-1.

Table 2-1. Parameters of GPS and GLONASS Systems

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>GLONASS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Satellites</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>Number of Orbit Planes</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Orbit Inclination</td>
<td>55°</td>
<td>64.8°</td>
</tr>
<tr>
<td>Orbit Radius</td>
<td>26,560 km</td>
<td>25,510 km</td>
</tr>
<tr>
<td>Orbit period (hour:minute)</td>
<td>11:58</td>
<td>11:16</td>
</tr>
<tr>
<td>Number of Sidereal Days for Ground Track Repeat</td>
<td>1</td>
<td>8</td>
</tr>
</tbody>
</table>
| Carrier Frequencies k = 1,2, ... 24 | L1 : 1575.42 MHz  
L2 : 1227.6 MHz | L1 : 1602 + 0.5625k MHz  
L2 : 1246 + 0.4375k MHz |
| Modulation Type and Modulated Codes | CDMA     
L1 : C/A , P  
L2 : P | FDMA     
L1 : C/A, P  
L2 : P |
| Code Frequencies      | C/A : 1.023 MHz  
P : 10.23 MHz | C/A : 0.511 MHz  
P : 5.11 MHz |
| Reference Coordinate System | WGS-84                          | PZ-90                                   |
| Reference Time System | UTC (USNO)                             | UTC (SU)                                |
Additional comments on differences between GPS and GLONASS are provided below.

1. Due to frequency interference in the radio astronomy (RA) band (1610.6 - 1613.8 MHZ) and frequency allocation for mobile satellite service (MSS) (1610 - 1626.5 MHZ), the frequency allocation of GLONASS is planned to be changed in three stages [30]:

<table>
<thead>
<tr>
<th>Period</th>
<th>Frequency Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present to 1998</td>
<td>1602.0 - 1608.8 MHZ</td>
</tr>
<tr>
<td></td>
<td>1614.4 - 1615.5 MHZ</td>
</tr>
<tr>
<td></td>
<td>(Channels 1-12, 22-24)</td>
</tr>
<tr>
<td>1998 - 2005</td>
<td>1602.0 - 1608.8 MHZ</td>
</tr>
<tr>
<td></td>
<td>(Channels 0-12, antipodal)</td>
</tr>
<tr>
<td>Beyond 2005</td>
<td>1598.1 - 1605.4 MHZ</td>
</tr>
<tr>
<td></td>
<td>(Channels -7 to +6)</td>
</tr>
</tbody>
</table>

2. The GLONASS broadcast navigation message does not contain ionospheric correction data for GLONASS measurements.

3. The GLONASS satellite position, velocity and acceleration is directly broadcast in the navigation message and changes every 15 minutes. In the case of GPS, the orbit ephemeris parameters are broadcast. The types of measurements available from GLONASS are the same as for GPS: Pseudoranges and Accumulated Delta Ranges. However, because of the difference in signal structure between GPS and GLONASS, the processing of these measurements is somewhat different.
2.3.3 GPS and GLONASS

GPS and GLONASS can be combined to provide redundant measurements for increasing the overall system availability and to improve the positioning accuracy. Since this research work concentrates on carrier phase processing, issues with the integration of GPS and GLONASS carrier phase measurements will be addressed. The main issue is inter-frequency biases among the measurements from the different GLONASS satellites, which originate from the FDMA design of GLONASS.

The ranging accuracy of GLONASS for stand-alone users is approximately 10 m (1σ), as compared to approximately 21 m (1σ) for GPS.[35] The difference is due to Selective Availability (SA) on the GPS system. In addition to the better ranging accuracy of GLONASS, the combination of GPS and GLONASS improves the overall spatial geometry and reduces the average position DOP values by providing more available satellites. The comparison of stand-alone system accuracy and a more detailed description for GPS, GLONASS, and GPS+GLONASS is provided in [21] and [35].

2.3.4 Low-Earth-Orbit (LEO) Satellites

A Low-Earth-Orbit (LEO) satellite is typically in an orbit at an altitude of approximately 600 km above the earth’s surface. LEOs are used for many purposes, including navigation, communication, broadcasting, remote sensing, and tracking. The first
satellite positioning system, TRANSIT, was a low-earth-orbit system.[12] Unfortunately, Transit was phased-out at the end of 1998.

A LEO has a small semi-major orbit axis, resulting in a relatively short orbit period. The main advantage of a LEO orbit is that the distance to the user is much smaller, thus signal propagation delays are short and transmission power can be small compared to geostationary satellites. Another important LEO characteristic is its high speed; typically a LEO is only visible for a period of time of 10 to 15 minutes. On the other hand, this also means that many more LEO satellites are required (on the order of hundreds) to cover the whole earth for positioning compared to only 24 satellites for GPS or GLONASS.

In the context of this dissertation, the integrated Doppler frequency shift measurement from the LEO is very useful. As will be shown in Chapter 4, GPS combined with the rapidly changing LEO geometry provides for a fast convergence of the position solution.

Several new LEO satellite systems are proposed for global wireless communications, e.g. IRIDIUM and QUALCOMM. These systems will be comprised of tens of LEO satellites. If the carrier phase measurements from these satellites are included in the proposed algorithm, then this will result in an extreme high availability of centimeter-level satellite positioning, even in cases where only a small section of the sky can be observed.

2.4 Historical Review of Carrier Phase Processing Techniques

The concept of precise satellite positioning using carrier phase dates back to 1981. Evans, et al.[11] describe the first relative GPS Doppler positioning experiment, which
resulted in sub-decimeter accuracies for antenna displacements of 2 to 3 meters. Also, [8] reports on the use of only the fractional portion of GPS carrier phase in combination with satellite orbit information to perform GPS surveying. Similar experiments with GPS carrier phase are also reported in [16].

Experimental results using integrated Doppler aiding of the code phase measurements for precise differential positioning using GPS are reported in [3]. The first evaluation of the use of redundant integrated Doppler measurements to achieve carrier phase ambiguity resolution are reported in [28]. This technique is extended in [18] to shorten the initialization time of the ambiguity-resolved position solution.

Previous research on the use of carrier phase measurements has not resulted in efficient and robust methods that allow for dynamic relative positioning of a user with respect to a reference receiver. Also, no methods have been published that make use of the carrier phase only for sub-meter relative or differential positioning. By efficient positioning, we mean that the following characteristics are met:

- Initialization time of less than 2 to 5 minutes;
- Centimeter accuracies for separation distances of less than 2 km, and sub-meter accuracies for separation distances of up to 50 km;
- The use of no more than 7 satellites.

This dissertation presents the research work to achieve efficient dynamic relative positioning using carrier phase measurements only.
3. The Carrier Phase Processing Technique

To achieve centimeter level accuracy, the use of carrier phase differential positioning is inevitable. In this chapter, a technique that combines a Least Squares ambiguity search and Kalman filter estimation will be presented. The Kalman filter produces the user's 3-D floating solution which can then serve as the initial estimate for the ambiguity search. The Double Difference is the metric used as the input observable of the Kalman filter.

3.1 Double Difference Interferometry

As stated in Section 2.1.1, pseudorange measurements from at least four satellites are needed to solve for the user position. Carrier phase measurements or integrated Doppler can also be used for the same purpose. As pseudorange is used to express the true range plus a clock bias. Accumulated carrier phase can also be used to express the true range, clock bias, and a constant bias existing between the satellite and receiver. This unknown constant bias is also referred to as integer ambiguity, and should be resolved to benefit from the precise carrier phase tracking. The double difference (DD) metric is used to cancel the correlated errors associated with satellites and the receivers. The derivation of the DD equation starts from the expression of the range measurement between satellite and receiver in terms of carrier phase given by:
\[ \rho^i_A(t) = (\phi^i_A(t) - \phi^i(t)) \cdot \lambda + N^i_A \cdot \lambda + S^i_A + \beta^i + \beta_A - d_{\text{iono}} + d_{\text{tropo}} \]
\[ = \phi^i_A \cdot \lambda + N^i_A \cdot \lambda + S^i_A + \beta^i + \beta_A - d_{\text{iono}} + d_{\text{tropo}} \]  \hspace{1cm} (3-1)

where \( \rho^i_A(t) \) is the true range from satellite \( i \) to receiver \( A \) at time \( t \), and

- \( \phi_A \) is the received signal phase at receiver \( A \).
- \( \phi^i \) is the transmitted signal phase from satellite \( i \),
- \( N^i_A \) is the integer ambiguity from satellite \( i \) to receiver \( A \).
- \( S^i_A \) is the noise (e.g. multipath) for satellite \( i \) at receiver \( A \).
- \( \beta \) is the clock bias of satellite \( i \) or receiver \( A \).
- \( d \) is the signal delay in the propagation path caused by the ionosphere or troposphere.

It should be noted here that due to the dispersive property of the ionosphere, the ionospheric delay is opposite that of the tropospheric delay and thus the difference in sign.\cite{2}

To eliminate the satellite-related errors, two carrier phase range measurements from receivers \( A \) and \( B \) with respect to the same satellite are subtracted from each other. The resulting quantity is called a single difference (SD):

\[ SD^i_{AB} = \Delta \phi^i_{AB} \cdot \lambda + N^i_{AB} \cdot \lambda + S^i_{AB} + \beta_{AB} \]  \hspace{1cm} (3-2a)

The integer ambiguity \( N^i_{AB} \), the signal noise, and the receiver clock bias represent combined terms from the two receivers involved. The satellite clock bias term, the ionospheric, and the tropospheric delays cancel, provided that the distance between the two receivers, or
baseline length, is within a certain limit \(< 1\) km. An alternate form of the SD is shown in Equation 3-2b to reflect the spatial geometric relationship.

\[
SD_{AB}^i = b \cdot e^i
\]  

(3-2b)

where \(b\) is the baseline vector pointing from the reference receiver to the user receiver, and \(e^i\) is the unit vector from the baseline midpoint to satellite \(i\).

In order to eliminate the receiver clock biases, two single-differences with respect to two different satellites are differenced to generate a double-difference (DD) as shown below:

\[
DD_{AB}^{ij} = \Delta \phi_{AB}^{ij} \lambda + N_{AB}^{ij} \lambda + S_{AB}^{ij} = b \cdot (e^i - e^j)
\]  

(3-3)

Here the integer ambiguity \(N\) is a combination of the SD ambiguities. It is clear from Equation 3-3 that besides the integer ambiguity, all the error sources except noise and multipath are canceled in the formation of the double difference. The DD spatial geometry is also illustrated in Figure 3-1.

The double difference carrier phase construction is also referred to as double difference interferometry. The advantage of this operation is two-fold. First, DD measurements no longer contain any correlated errors associated with the satellites and the receivers. Second, in the DD formulation, the integer ambiguity can be solved for. This is not possible in the SD, since the ambiguity cannot be distinguished from the clock offset.
Once the receiver locks on to the incoming signal, the carrier phase SD and DD metrics can be formed using Equations 3-1 through 3-3; However, one still has to resolve the initial carrier phase integer-cycle ambiguity in the DD equation. The receiver carrier phase tracking loop can maintain a count of whole carrier cycles received from the time of signal acquisition. Using the receiver clock, a fractional phase measurement can be made at any given epoch. The sum of these two measurements yields a phase measurement at any
given instant in time, but does not account for the initial integer ambiguity. Therefore, the integer ambiguity \( N \) in Equation 3-3 must be resolved to determine the baseline solution accurately.

In general, ambiguity resolution is performed in two steps -- (1) ambiguity searching and (2) validation testing. The ambiguity search is to cycle through the many combinations of potential ambiguity sets or positions representing the ambiguity sets inside a predetermined search space. The search space can be defined in a mathematical space, defined in the ambiguity domain, or defined in the position domain.[1] The ambiguity searching method used in this dissertation is based on a Least-Squares solution from the Double Difference (DD) carrier phase measurements, and the application of the QR-factorization to perform a search in the parity space residual domain rather than in the position or ambiguity domains. All ambiguity search techniques require redundant information, which can be in the form of additional satellites or multiple measurement sets.

Assuming that the same five satellites are continuously tracked by the receivers on both ends of the baseline, the following DD equation results:

\[
\begin{align*}
\text{DD}_{ij}^{AB} &= (\varepsilon^i - \varepsilon^j)^T b \\
\text{DD} &= \mathbf{H}b
\end{align*}
\]  

(3-4)  

where \( i = 1, 2, 3, 4 \), and \( j = 2, 3, 4, 5 \). In matrix form this can be expressed as:
where $H$ is a 4x3 design matrix composed of the differences between the unit vector of satellite 1 and the unit vectors of the other four satellites; $b$ is a 3x1 baseline vector relative to the reference station with components $x, y, z$; and $\mathbf{DD}$ is the 4x1 measurement vector.

As long as the $H$ matrix is over-determined, a QR-factorization can be applied to decompose the $H$ matrix into the product of an orthonormal matrix $Q$ with dimension 4x4 and an upper triangular matrix $R$ with dimension 4x3 as in Equation 3-6.

$$H = QR \quad (3-6)$$

where $Q$ is the orthonormal matrix such that $Q^T = Q^{-1}$, and $R$ is the upper triangular matrix. As long as the matrix $H$ has full rank (3 in this case), after the QR-factorization, the first 3 columns of the matrix $Q$ form an orthonormal basis for the space of $\text{range}(H)$ to which the space of double difference $\mathbf{DD}$ measurements belongs. The fourth column of the matrix $Q$ represents the parity space, and implies the inconsistency associated with the $\mathbf{DD}$ measurements. Therefore, two separate equations can be set up as a result of this decomposition. One is the Least-Squares baseline solution:

$$b = U^{-1}Q_x\mathbf{DD} \quad (3-7)$$
and the other is the parity equation showing the inconsistency residual:

\[ Q_{DD} = \gamma \quad (3-8) \]

where \( U \) is the upper 3x3 sub-matrix of the triangular matrix \( R \), \( Q \), in the upper 3x4 sub-matrix of \( Q^T \), and \( Q_p \) is the last row vector of \( Q^T \).

Equation 3-8 is indicative of the estimation error in the Least-Squares baseline vector solution. This estimation error is also called the parity space residual. The parity space residual is used to evaluate the potential ambiguity combinations. Initially, after all the error sources such as satellite and receiver clock offset, and ionospheric and tropospheric effects are eliminated through the formation of the double differences using pseudorange (code) measurements, the accuracy achieved with a dual-frequency C/A-code receiver in a differential configuration is approximately ± 3 m or approximately ±15 wavelengths. This accuracy can be further improved by smoothing the code-based DDs with DDs derived from the carrier-phase measurements of the receiver using a complementary Kalman filter. This technique was developed and implemented by van Graas & Braasch.[43] Termed carrier-smoothed code double-difference measurements (DDC), their accuracy is on the order of 1 - 2 m (95%), which is 5 - 10 wavelengths. This results in a positioning accuracy of 1 - 5 meters (95%), depending on the satellite geometry. In the case of 4 DD measurements, each having an uncertainty of for example 4 meters, a search space of approximately \((20 + 20 + 1)^4\)
= 2,825,761 integer ambiguity combinations should be searched. This search is clearly computationally intensive. That is why much research has been performed to improve the ambiguity searching strategy, and reduce the time to determine the correct ambiguity. An efficient method of performing the ambiguity search called Fast QR-Search was explored by Lee in 1994 as documented in [25].

The searched candidates of integer ambiguity sets are subject to a validation test to decide which one is the correct set. The criterion used in this research is referred to as the ratio test (or γ-test). The ratio of the second smallest parity residual to the smallest parity residual is evaluated over time. If this ratio is greater than a predefined value (typically this value is 3 or 4 determined empirically), the ambiguity set associated with the smallest residual is selected as the correct set, which completes the ambiguity resolution.

The search scheme adopted in this dissertation is called the Fast QR-search as compared to the traditional QR-search.[43] The details of these search methods and achieved performance can be found in reference [25]. Two examples showing a performance comparison between the Fast QR-search and the traditional QR-search methods are illustrated in Tables 3.1 and 3.2. In the tables, each time period consists of 100 seconds of data collection prior to the ambiguity searching process, and the solution status indicates whether the validation test is passed or not. A fixed solution status indicates that the correct ambiguity set has been found.
Table-3.1. The Comparison of QR and Fast QR Search Algorithm, Search Range ± 10λ.

<table>
<thead>
<tr>
<th>Period</th>
<th>QR-Search</th>
<th>Fast QR-Search</th>
<th>Improvement Factor</th>
<th>Solution Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.83 s</td>
<td>0.99 s</td>
<td>4.9</td>
<td>Floating</td>
</tr>
<tr>
<td>2</td>
<td>4.73 s</td>
<td>0.87 s</td>
<td>5.4</td>
<td>Floating</td>
</tr>
<tr>
<td>3</td>
<td>4.62 s</td>
<td>0.77 s</td>
<td>6.0</td>
<td>Fixed</td>
</tr>
</tbody>
</table>

Table-3.2. The Comparison of QR and Fast QR Search Algorithm, Search Range ± 20λ.

<table>
<thead>
<tr>
<th>Period</th>
<th>QR-Search</th>
<th>Fast QR-Search</th>
<th>Improvement Factor</th>
<th>Solution Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>63.77 s</td>
<td>5.61 s</td>
<td>11.4</td>
<td>Floating</td>
</tr>
<tr>
<td>2</td>
<td>63.33 s</td>
<td>5.16 s</td>
<td>12.3</td>
<td>Floating</td>
</tr>
<tr>
<td>3</td>
<td>62.84 s</td>
<td>4.67 s</td>
<td>13.5</td>
<td>Fixed</td>
</tr>
</tbody>
</table>

From the comparison, it can be seen that the time required for the regular search increases dramatically when the search range is doubled. That further explains the necessity for a better ambiguity search strategy. It is also shown that the improvement factor of the Fast-QR search method increases when the search range increases. The ambiguity resolution technique discussed above, has several important considerations. First of all, the search process needs to be activated by an initial position or DD vector estimate, which in turn
determines the search range of the ambiguity space. To reduce the ambiguity search time, a more accurate code phase solution is desired, which implies advanced receiving equipment. Furthermore, software complexity may be added to cope with the time-consuming searching process. Also, using additional time information can improve the ambiguity resolution for the following three reasons. First, if data is collected over a longer time period, the code phase solution can be smoothed to improve the accuracy, which in turn reduces the search range of the ambiguity space. Second, a longer data collection period provides more carrier phase data to average, and improves the observability of the correct set of ambiguities. Finally, as time elapses, satellite geometry variation results in redundant measurements to cross-check the potential ambiguity sets, which reduces the number of candidate ambiguities and thus minimizes the search time. The next section is focused on improving the accuracy of the floating baseline solution using a Kalman filtering technique.

3.3 The Extended Kalman Filter

To dynamically determine a user’s 3-D position with centimeter accuracy, resolving the integer ambiguities by searching a large space is not efficient. Therefore, an extended Kalman filter is used to estimate the user’s relative 3-D position with sub-meter accuracy without searching. This high-accuracy estimated floating solution can further serve as the initial estimate to launch a small ambiguity search, if needed.

An extended Kalman filter is linearized about the estimated state trajectory instead of the pre-computed nominal state trajectory, and is used to enable dynamic, real-time state
estimation. Unlike a linearized Kalman filter which provides only incremental quantities for the nominal states, the extended Kalman filter makes it possible to conveniently keep track of all parameters of interest, including position, velocity, and ambiguity estimates.

Starting with equations 3-1 through 3-3, and the carrier phase double difference geometry as shown in Figure 3-1, the measurement equation can be rearranged to relate the spatial geometry to the carrier phase observables. Each equation contains three unknown coordinates $X, Y,$ and $Z$ of the baseline vector $b$, and an additional unknown DD ambiguity term $N$. In the case of seven satellites, the measurement model will be [44]:

$$
\begin{bmatrix}
DD^{12} \\
DD^{13} \\
DD^{14} \\
DD^{15} \\
DD^{16} \\
DD^{17}
\end{bmatrix} =
\begin{bmatrix}
(e^1 - e^2)^T \\
(e^1 - e^3)^T \\
(e^1 - e^4)^T \\
(e^1 - e^5)^T \\
(e^1 - e^6)^T \\
(e^1 - e^7)^T
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
b \\
N^{12} \\
N^{13} \\
N^{14} \\
N^{15} \\
N^{16} \\
N^{17}
\end{bmatrix}
$$

(3-9)

where the superscripts denote the double difference of satellite $j$ ($j = 2, 3, \ldots, 7$) with respect to satellite 1, in this case satellite 1 is taken to be the common satellite. The discrete-time form can, in the presence of noise, be expressed as:
\[ DD_k = H_k X_k + v_k \]  

(3-10)

where the subscript \( k \) denotes the value obtained at time instant \( k \), \( v_k \) represents the uncorrelated errors, and can be reasonably assumed to be normally distributed with zero mean and standard deviation \( \sigma \) provided that the multipath effect is not severe. This results in the measurement error covariance matrix \( R_k = E[v_k v_k^T] = \sigma^2 \cdot I \). The system model for the Kalman filter is given by:

\[ X_{k+1} = \Phi_k X_k + w_k \]  

(3-11)

where \( \Phi_k \) is the state transition matrix, and \( w_k \) contains the systematic errors which can also be assumed to be normally distributed with zero mean and standard deviation \( \delta \), and uncorrelated with each other. This results in a system error covariance matrix \( Q_k = E[w_k w_k^T] = \delta^2 \cdot I \).

Measurement Equation 3-9 shows that the system is under-determined. No matter how many DD measurements are added, there are always three more unknowns in the state estimate than in the measurements. Therefore, it is only possible to directly solve for the baseline solution and unknown ambiguities if a precise initial position estimate is available. However, by applying the Kalman filter, taking advantage of new DD measurements
available for every time epoch, together with the statistical information that carries over from the previous time epochs, the state vector can be continuously estimated. Convergence is an important issue for Kalman filter estimation, which is a function of the exactness of the system model, and the observability of the system’s state vector. Since the measurement equation is linearized about the filter’s estimated trajectory rather than a (pre-computed) nominal trajectory, the extended Kalman filter is more vulnerable to divergence if the system is not correctly modeled. However, in the case of GPS, the linearization is not overly sensitive to the estimated trajectory, since the distances to the satellites are large compared to the baseline length. However, with other applications, the linearization may be a critical concern for the consistency of system models. One example of this is the integration of Low-Earth-Orbit (LEO) satellites with GPS satellites for positioning purposes. In Chapter 4, this will be further explained. The computed Kalman gain, estimated solution, and error covariance matrix for each time instant \( k \) are given by:

\[
K_k = P_k H_k^T (H_k P_k H_k^T + R_k)^{-1}
\]

(3-12)

\[
\hat{x}_k = \hat{x}_k^- + K_k (D_k - H_k \hat{x}_k^-)
\]

(3-13)

\[
P_k = (I - K_k H_k) P_k^-
\]

(3-14)
where the overhat symbol ^ means estimate, the superscript symbol - means initial guess or estimate before the measurement is included, and the subscript k indicates the time instant.

The state propagation proposed in this work is different from the ordinary extended Kalman filter. Using the state transition matrix to propagate the state vector from time epoch to time epoch results in a slow convergence, as is described in [20] and [28]. Therefore, an innovative approach of state propagation is used to accurately characterize the user dynamics, and speed up the estimation convergence. The ambiguity estimates are propagated using Equation 3-11 because the ambiguity states are not supposed to have a great variation once the estimation reaches steady-state, but the baseline vector is propagated based on a new technique. The baseline vector propagation is accomplished using a very accurate adjustment obtained from the triple difference (TD) carrier phases measurement between two consecutive time epochs. The triple difference is formed by subtracting the measured DD of the previous time epoch from the DD of the current time epoch

\[ TD_k = H_k^* b_k - H_{k-1}^* b_{k-1} \]  \hspace{1cm} (3-15)

where \( H^* \) is the sub-matrix of \( H \) which contains only the unit vectors. Triple difference measurements can be used to cancel the constant ambiguity. It is noted that the triple difference is also a common measure for monitoring carrier phase cycle slips. Equation 3-15 can be rewritten as
and then the three-dimensional adjustment in the baseline vector $\Delta b$ from time $k-1$ to time $k$ can be obtained using the ordinary Least-Squares solution

$$TD_k = H_k^*(b_{k-1} + \Delta b) - H_{k-1}^*b_{k-1}$$ (3-16)

Therefore, the previously estimated baseline vector is adjusted using Equation 3-17 to account for the relative dynamics between time $k-1$ and $k$.

$$\Delta b = (H_k^TH_k^*)^{-1}H_k^*TD_k - (H_k^* - H_{k-1}^*)b_{k-1}$$ (3-17)

Equation 3-17 is the key to propagate the baseline vector from one time epoch to the next, and obtain the centimeter level estimated trajectory using integrated Doppler measurements only. This accurate baseline vector propagation can be used to determine 3-D velocity with accuracies better than 1 centimeter/second as well. It should be noted that Equation 3-17 implies that the 3-dimensional position adjustment or the determined velocity is a function of the position estimate of the previous time epoch, geometry variation from previous time to the next time epoch, and triple difference measurements as measured between the consecutive time instants. Thus, not only the estimated baseline vector and triple difference
measurements can help propagate the accurate state vector, but any spatial relationship between the satellites and the receiver that is carried in the geometry variation will also help to speed up the estimation convergence. That is why the modified extended Kalman filter (MEKF) is effective for dynamic users as well.

3.3.1 Simulation of The Modified Extended Kalman Filter (MEKF)

Preliminary simulations of the modified extended Kalman filter were performed using seven GPS satellites, a simple flight path, and a short initial offset distance (\( \Delta t_0 = 15 \) meters). The flight path was generated for a 100-second period, with the starting point offset by 15 m with respect to a local origin. The local origin was taken as the approximate location of Stocker Center (Latitude = 39.33 degrees; Longitude = -82.11 degrees; and height = 168.29 m). The flight starts at a constant velocity of 100 m/s to the east for 10 sec, followed by a 1.5-g left turn for 21 sec, and continues to the west for the remainder of the 100-second period. The simple flight path is shown in Figure 3-2. The altitude was kept constant at 5 meters for the whole flight path.

The simulation was set up based on the generated flight path. For each time epoch, the satellite positions and elevation angles were calculated using the satellite almanac parameters, then based on the flight path, the true ranges from the satellites to the reference station and user were both calculated. In the first time epoch, the calculated true ranges were divided by the L1 carrier wavelength, the integral part was taken as the initial integer ambiguities, and the fractional part was added with error to serve as the carrier phase
measurement. The added error represents the noise of the double differences. The standard deviation for the phase error was chosen as 5 millimeters. Starting from the second time epoch, the fractional carrier phase measurement kept accumulating to serve as the input to the MEKF. Also, the single difference, double difference, and unit vectors were calculated to construct the measurement and system models for the MEKF. The estimation process was initialized with an uncertainty of 15 meters, and then the 3-D baseline adjustment model was applied before the Kalman filter update equations to yield an accurate estimate. A flow chart describing the simulation procedures is illustrated in Figure 3-3. The convergence results in three local coordinates are shown in Figures 3-4 through 3-6. The error shown is the comparison with the simulated reference flight path. The results show that the 3-D error
Figure 3-3 Flow Chart of The Simulation
Figure 3-4 The Estimated Position Error in East Coordinate, Simulation

East error std = 0.3173

Figure 3-5 The Estimated Position Error in North Coordinate, Simulation

North error std = 0.06943
of the baseline vector converges to within 50 cm in approximately one minute. Also, the estimated DD ambiguity errors are presented in Figures 3-7 through 3-12. Note that the errors of the estimated ambiguities show similar convergence as that of the estimated baseline solution.

3.3.2 Post-Processing of The MEKF

The modified extended Kalman filter-based algorithm was also tested by post-processing the data collected during a 1995 flight test using a Boeing 757 aircraft operated by the United Parcel Service (UPS).[44] The truth flight path is shown in Figure 3-13. The results of the application of the described technique to this flight test are shown in Figures
Figure 3-7 The Estimated DD Ambiguity Error SV 3 - 21

Figure 3-8 The Estimated DD Ambiguity Error SV 11 - 21
Figure 3-9 The Estimated DD Ambiguity Error SV 12 - 21 Simulation

Figure 3-10 The Estimated DD Ambiguity Error SV 15 - 21 Simulation
Figure 3-11 The Estimated DD Ambiguity Error SV 17 - 21

Simulation

Figure 3-12 The Estimated DD Ambiguity Error SV 18 - 21

Simulation
3-14 through 3-16. The figures show the comparison between the estimated solution and a truth reference. The truth reference was based on a commercial software package that utilizes dual-frequency measurements to resolve the ambiguities and correct all the ionospheric delays. This package also uses a high-quality tropospheric model to reduce the tropospheric measurement delays. For the flight test, the estimated baseline vector error converges to within 50 cm after 3 minutes of processing. The slower convergence in the vertical error as illustrated in Figure 3-16 is explained as follows. There was an altitude difference of about 450 meters between the aircraft antenna and the ground reference station at the time of initialization. The altitude difference results in different tropospheric delays between the reference station and the aircraft, which are not included in the MEKF post-processing. Even though many additional error sources are present in this flight test compared to the simulation, the modified extended Kalman filter performance was excellent. The initial offset distance from the reference station is approximately 4.6 km, and the initial estimate was obtained from a code phase differential solution with a 3-D error of approximately 10 meters. Therefore, from the post-processed results shown, the real-time estimator based on the modified extended Kalman filter (MEKF) can achieve sub-meter accuracy after approximately 3 minutes of convergence time.

3.4 Combined MEKF and Ambiguity Resolution

A longer estimation time can converge the baseline solution down to centimeter-level accuracy if the steady-state is reached. For applications that demand real-time tracking with
Figure 3-13 The Boeing 757 Flight Trajectory

Figure 3-14 The Estimated Position Error in East Coordinate Post-Processing
Figure 3-15 The Estimated Position Error in North Coordinate Post-Processing

Figure 3-16 The Estimated Position Error in Vertical Coordinate Post-Processing
centimeter accuracy, the initialization time should be as short as possible. Therefore, rather than relying on the Kalman filter itself to reach steady-state and provide centimeter level accuracy, the modified extended Kalman filter in combination with an ambiguity searching scheme is applicable to quickly resolve the ambiguities, and thus refine the 3-D baseline solution. As mentioned in Section 3-2, the ambiguity search requires an initial estimate or floating solution to start the search process. The modified extended Kalman filter is used to obtain this floating solution quickly and accurately. The error covariance matrix associated with the estimated solution reveals the uncertainty of the estimated floating solution. If the error covariance matrix is continuously monitored, then once the estimate converges to within a predefined value, a small ambiguity search can be launched to find the correct ambiguities and thus fix the position solution with centimeter accuracy. The search range could be as small as a few wavelengths depending on the predefined threshold value. As an example, simulated noise with statistical parameters as specified in Figure 3-18 was added to the true flight trajectory from the B-757 flight test to demonstrate the feasibility of the proposed algorithm. The noise was added to show a distortion between the MEKF and the truth solutions. The MEKF was initialized with the same code phase solution as used during post-processing. In this example, only the last 300 seconds of the approach were used. The ambiguity search process was launched at the 84th second time mark, with a search range of ±2 wavelengths. The ambiguities were resolved right away. The actual processing time for resolving the ambiguities using the C programming language is not available, since the process was implemented in MATLAB™. However, the required time can be estimated as follows. The total number of integer combination to be searched is \((5)^4 = 625\). Using a QR
search, this would require approximately \((5)^4 / (21)^4 \times 5 \text{ seconds} \approx 20 \text{ ms}\) (see Table 3.1).

The 3-D position error comparison between the MEKF-only and the ambiguity-resolved solution after the 84th second time mark is illustrated in Figure 3-17. Also, the 3-D error of the ambiguity resolved baseline solution as well as the statistical parameters are shown in Figure 3-18. It is clearly shown that after the ambiguities are resolved, the 3-D errors can be reduced to within 2 cm, whereas in case of the MEKF alone, the 3-D errors can only converge to 10 cm during the 300 seconds of run time.

3.5 Cycle-Slip Monitoring

As stated in Section 2.3.1, the receiver is capable of measuring the fractional phase difference between the incoming carrier signal and a locally generated reference. Each epoch (1 second for this research), a set of carrier-phase measurements is made in the receiver. In between epochs, a counter in the receiver is used to keep track of the advance or retreat of whole carrier cycles based upon integrating the SV Doppler frequencies. As the observation continues, the counter in the receiver is either incremented or decremented based upon whether the Doppler is positive (SV approaching) or negative (SV retreating). Each count represents a change in carrier phase of \(2\pi\) radians.
Figure 3-17 3-D Error Comparison Between MEKF and Ambiguity-Resolved Solution

Figure 3-18 3-D Position Errors of The Ambiguity-Resolved Least-Squares Solution
In any type of interferometric-based GPS positioning system, the detection of cycle slips is very important if high confidence in the accuracy of the results is to be maintained. In surveying applications, most of the processing is done after the data is collected and often it is possible to repair the cycle clip using redundant measurements from other SVs, for example. For real-time continuous interferometric GPS systems, detection and isolation of cycle slips is the governing factor. Once the cycle slip is detected, the offending SV can be ignored for a period of time until it is reacquired and its new carrier-cycle integer ambiguity resolved. During this period, the highly-accurate fixed-baseline solution may or may not be lost depending upon whether the isolated SV is a part of the position solution or is a redundant SV held in reserve for just such an occurrence. In the case of the former, should the fixed-baseline solution be jeopardized, the system is designed to switch to any of the redundant SVs (ones whose carrier-cycle integer ambiguity has been resolved) in order to maintain the solution. With the GPS constellation now available, redundant satellites are the norm, and all-in-view GPS receivers with up to twelve independent channels are common place. In the absence of suitable redundant SVs, the only recourse is to drop back to the less accurate floating-baseline solution until such time as sufficient SVs with resolved ambiguities are once again available.

Also, a cycle slip will directly affect the estimation performance of the modified extended Kalman filter. Any abnormal carrier phase measurement showing a bias will result in an abrupt inconsistency in the measurement vector, which will lead the filter to adjust all the parameters trying to accommodate the observed anomaly. This will deviate the estimation process from the correct solution. Furthermore, if any of the measurements is lost
or regained, then the Kalman filter faces the problem of a dimensionality change which should be carefully handled to avoid an interruption of the estimation convergence. Chapter 6 will address the details on dimensionality changes of the Kalman filter. For a detailed discussion of cycle-slip detection, the reader is referred to Hofmann-Wellenhof.[19]
4. Integration of a Low-Earth-Orbit (LEO) Satellite

The motivation for integrating a LEO satellite into the modified extended Kalman filter (MEKF) algorithm originates from the circumstances when the number of visible GPS satellites is less than seven. Since the MEKF algorithm requires carrier phase measurements from at least 7 GPS satellites in order to converge quickly in the beginning of the estimation process. However, GPS may not be able to provide at least 7 visible satellites at all times during a day. Therefore, a LEO satellite is used to augment the MEKF by improving the availability of measurements. In addition to providing an additional measurement source, the LEO satellite also provides a fast geometry change which benefits the MEKF. The LEO satellite with its lower altitude as compared to the higher altitude of a GPS satellite, produces greater geometry variation and thus helps the MEKF to converge faster.

LEO satellites orbit the earth at about 600 km above the earth surface. They are normally used for the purposes of communication, broadcasting, remote sensing, and tracking. Its lower altitude results in a smaller orbital radius, and consequently, a shorter period for the satellite orbit. Because of the LEO satellite’s smaller orbit radius and period, the LEO satellite is only visible for a short period of time. Therefore, in order to provide sufficient global coverage for navigation purposes, many LEO satellites have to be put into a constellation. The next section shows an example of coverage versus the number of satellites that need to be in the constellation. This example uses simulated LEO orbital parameters.
4.1 Simulation of LEO Satellites

In this section, a simulation example is given to show how the coverage and visibility is calculated based on the Keplerian parameters. The satellite’s orbit can be characterized by six Keplerian parameters, namely mean anomaly \( M \), right ascension \( \Omega \), inclination angle \( i \), semi-major axis \( A \), eccentricity \( e \), and reference time. The mean anomaly determines where the satellite is within a particular orbit. The right ascension is the angle in the equatorial plane between the periapsis direction and the point where the satellite ascends through the equatorial plane.[35] The inclination angle is the angle of the orbital plane with respect to the earth equatorial plane. The semi-major axis equals the orbit radius under the assumption that the eccentricity is zero, and is related to the orbit period \( T \) through Kepler’s third law:

\[
T = 2\pi \sqrt{\frac{A^3}{\mu}}
\] (4-1)

where \( \mu \) is the earth’s gravitational parameter \( (3986005 \times 10^8 \text{ m}^3/\text{sec}^2) \).

In the simulation of a LEO constellation, the reference location was chosen to be Stocker Center, Ohio University (N 39.33°, W 82.11°, Height = 168.29 m), and the orbit radius of the LEO satellites was set to 6,652,557 meters so as to yield an orbit period of 1.5 hours. This orbiting period is proposed for communication purposes to keep the power of hand-held transmitters as low as 2 Watts. By evenly distributing 384 LEO satellites over 24-evenly spaced orbits inclined 55° with respect to the equatorial plane, the coverages for a one day
period at Stocker Center is shown in Figure 4-1. Figure 4-1 indicates that even with 384 LEO satellites, there are not always seven satellites visible. This explains why it is unlikely that a constellation of LEO satellites will be designed for navigation purposes. However, LEO satellites do provide redundant measurements for a GPS-based navigation solution.

4.2 Simulation of MEKF Combining 6 GPS Satellites and 1 LEO Satellite

A simulation of integrating a LEO satellite with 6 GPS satellites was conducted to demonstrate how the MEKF performs under this scenario. First, the orbital parameters for the LEO satellite must be selected. In this simulation, all the orbital parameters associated with the added LEO satellite except for the semi-major axis were kept the same as those used
for GPS satellites. The value for the semi-major axis was changed to 6,652,557 meters, which corresponds to a orbital period of 1.5 hours (see Section 4.1). Due to the much shorter orbital period as compared to GPS, the LEO satellite is visible for only a small period of time. An example showing the visibility of a LEO satellite is shown in Figure 4-2, as compared with the visibility of a GPS satellite shown in Figure 4-3. In this example, identical orbital parameters except for the semi-major axis were used. Azimuth and elevation angles were calculated based on the local origin chosen at Stocker Center, Ohio University.

Since the required observables for running the MEKF are the integrated Doppler or carrier phase measurements, and the satellite's position, as long as the LEO satellite's orbit information is modulated in the broadcast signal, the LEO signal can be directly integrated into MEKF with GPS signals. Otherwise, a small network of ground stations may be constructed to track the satellite's position in order to provide the LEO satellites' orbit information.

The simulation of combining a LEO satellite with 6 GPS satellites was conducted using the same simulation model as in Section 3-4, and the estimation results of the MEKF are shown in Figures 4-4 through 4-6. Compared to the case of 7 GPS satellites which were shown in Figures 3-4 through 3-6, the speed of convergence of the baseline vector is much faster. Also, the estimated ambiguities show much faster convergence. Due to the large number of plots, only one example of the convergence errors is presented in Figure 4-7.
Figure 4-2 The Visibility of a LEO Satellite at Stocker Center

Figure 4-3 The Visibility of a GPS Satellite at Stocker Center
As the simulation results show, the integration of a LEO signal significantly improves the convergence speed of the MEKF.

![Figure 4-4 The Estimated Position Error in East Coordinate 6 GPS + 1 LEO Simulation](image)

4.3 **Investigation of Hardware Bias When Combining GPS/LEO**

In this section, the potential problem of hardware biases due to different tracking channels and/or different frequencies associated with different signal sources will be discussed. Each channel from the antenna to the measurement point in the receiver, consists of many filters, amplifiers, and other devices. The phase delay through these devices are a function of the frequency. Therefore, different carrier frequencies, when passing through different channels, will result in different carrier phase delays.
Figure 4-5 The Estimated Position Error in North Coordinate 6 GPS + 1 LEO Simulation

Figure 4-6 The Estimated Position Error in Vertical Coord. 6 GPS + 1 LEO Simulation
Modern GPS receivers solve this problem by using only one hardware channel for all the satellites. The inter-channel biases or hardware biases, for a modern GPS receiver are on the order of millimeters. Therefore, the hardware bias problem for tracking GPS signals is negligible. However, when it is desired to combine GPS and LEO satellites to speed up the convergence of MEKF, the hardware channel bias problem will become more evident since the LEO satellite utilizes a different frequency than the GPS.

In order to determine the effect of hardware biases, when applying the modified extended Kalman filter, on the GPS/LEO signals to estimate the user's position, a simulation of combining GPS/LEO signals with hardware channel bias added to the LEO channel was conducted. This simulation is similar to the example of integrating 6 GPS satellites and 1
LEO satellite as shown in Section 4.2. The estimated user's 3-D position errors are shown in Figure 4-8. The estimated double difference ambiguity (with respect to GPS) associated with the LEO satellite is illustrated in Figure 4-9. The other estimated DD ambiguities show similar convergence trends as in Section 4.2, and are not repeated here. It is clearly shown that the estimated 3-D position is not affected by the hardware bias associated with the added LEO satellite. Interestingly, the error of the estimated DD ambiguity associated with the added LEO satellite converges towards the hardware bias. In other words, the MEKF-estimated DD ambiguity for the LEO satellite is the sum of the true constant ambiguity offset and the hardware bias. Therefore, the MEKF can be used to estimate a

![Figure 4-8 The Estimated 3-D Position Error 6 GPS + 1 LEO Simulation + Bias](image-url)
user's 3-D relative position with a variety of different signal sources. Note that the DD ambiguities of GPS signals can be found exactly; however, the DD ambiguities associated with different signal sources will be in error by the amount of their channel hardware biases.

The concept of integrating LEO satellites with GPS can also be extended to pseudolites as the redundant signal source. The similarity between the LEO and the pseudolite is that they both provide a redundant measurement when the user moves into a certain area and/or during a certain time period. Furthermore, the signals from both systems provide rapid, relative geometry variation, which speeds up the convergence of the MEKF estimation process. The details of the pseudolite application and its operational concerns will not be covered in this dissertation, but can be found in [35].

Figure 4-9 The Estimated Ambiguity Error Associated with The Biased LEO Signal
5. **Modification of The Measurement Model**

Using the modified extended Kalman filter to continuously estimate the user’s 3-D relative position was discussed in Chapter 3. Sub-meter level positioning accuracies are achievable in approximately 2 - 3 minutes (in real-time). It was shown in Chapter 4 that the integration of a LEO satellite can improve the convergence speed of the algorithm. By integrating GPS and LEO carrier phase measurements, the estimated solution reaches steady-state faster, and thus the convergence accuracies are also improved during the estimation process. If a short period of run time (e.g. 100 seconds) is used for the GPS/LEO algorithm, the modified extended Kalman filter functions properly. When the simulation is carried out for longer run times (e.g. 200 seconds or more), the estimation errors may show a divergence-like behavior. Since the divergence behavior is usually related to an imperfect system model, the exactness of the system’s measurement model will be investigated next.

5.1 **Exactness of The Measurement Model**

As mentioned in Section 3.2, convergence is an important issue for any Kalman filter-based algorithm. In the case of an extended Kalman filter, the convergence problem is even more critical, due to its linearization about the estimated trajectory. For GPS signals alone, the linearization process usually does not pose too much of a problem. This is because the distance from the GPS satellites to users on the earth is large compared to the separation
distance between the users. However, this is not the case when introducing a LEO satellite's signal into the estimation algorithm.

Numerically, the LEO satellite is approximately 274,420 m above the earth surface compared to 20,182,487 m for a GPS satellite. The signal wave-fronts to both receivers of the baseline can not be assumed parallel, as was the case with the higher-orbit GPS satellites. This can be illustrated by the following example. For differential operation, when the antenna separation reaches 16,166 m, the angle between the two unit vectors to a GPS satellite is approximately 0.03 deg; however, the angle between the two unit vectors to a GPS satellite and a LEO satellite is approximately 1.6 deg. This difference significantly violates the parallel wave-front assumption. Thus, the unit vector to the satellite from the midpoint of the baseline is not accurate, and results in an inaccurate measurement model.

The divergence due to an inaccurate measurement model is illustrated using the simulation of Chapter 4. Figure 5-1 shows divergent behavior after time 8830 second. At the end of the run time in Figure 5-1, the antenna separation is 16,166 m.

Due to the properties of the Kalman filter, if the system and measurement models are constructed correctly and accurately, the innovation which is defined as:

$$ v = D D - H \hat{x} $$  \hspace{1cm} (5-1)

should be a white noise process. Thus, if the system model is designed correctly, the state vector should be estimated in an optimal sense, and the innovation is a sequence of white noise. In other words, there is no information carried in the sequence $v$ if $\hat{x}$ is an optimal
estimate. However, as can be seen in Figure 5-2, the innovation of the GPS/LEO estimation with the regular measurement model, used in the previous chapter, is not a white noise process. The system model does not cause a problem because the identity propagation is the only method used in the modified extended Kalman filter algorithm. Therefore, the measurement model used in this MEKF algorithm must be inexact.

5.2 The Approach for The Model Modification

Now that the measurement model is known to be inexact, it should be modified in order to eliminate the systematic errors. To precisely construct the measurement model, consider the geometric relations of the differential operation as shown in Figure 5-3.
The Innovations of Unmodified GPS/LEO MEKF Estimation

From the geometric relations, the geometric single difference $SD$ can be expressed as the projection of the baseline length onto the unit vector from the reference antenna to the satellite minus the length of vector $X$, as shown in Equation 5-2.

$$SD = b \cdot \hat{\Phi}_1 - \|X\|$$  \hspace{1cm} (5-2)

where $b$ is the baseline vector, $\hat{\Phi}_1$ is the unit vector from the reference receiver to the satellite, and the vector $X$ can characterized by the trigonometric relation among side $\|X\|$, angle $\Delta$, and side $\|b\|\sin\Theta_1$ as given by Equation 5-3.
Also, the relation between the geometric SD and the phase measurements can be expressed as in Equation 5-4.
where $N$, $b$, and $e$ are the single difference ambiguity, clock bias, and lumped noise in simplified form, respectively. $\Delta \phi$ is the single difference (SD) carrier phase measurement.

Therefore, using equations 5-3 and 5-4, Equation 5-2 can be rearranged to construct the SD phase measurement model in terms of the baseline vector, unit vector, integer ambiguity, and clock bias. It is shown in the Equation 5-5 that a correction term must be added to the SD phase measurement to exactly describe the spatial geometric relation.

$$SD = \Delta \phi + N\lambda + b + e$$

(5-4)

\[
\Delta \phi + \frac{\|b\| \sin \Theta_1}{\tan A} = b \cdot \hat{\Phi} + N\lambda + b + e
\]

(5-5)

In order to apply Equation 5-5, the angles $A$, and $\Theta_1$ must be determined. The angle $A$ can be calculated using the trigonometric property of an equal-sided triangle.

$$\triangle A = \frac{180 - \triangle \gamma}{2} = 90 - \frac{\triangle \gamma}{2}$$

(5-6)

Where the angle $r$ can be obtained from the range vectors $\Phi_1$, and $\Phi_2$, using the cosine theorem,

$$\triangle r = \cos^{-1}\left(\frac{\Phi_1 \cdot \Phi_2}{\|\Phi_1\| \|\Phi_2\|}\right)$$

(5-7)
Next, the angle $\Theta_1$ can be obtained from the baseline vector and range vector $\Phi_1$, using the cosine theorem, and is given by:

$$\Delta \Theta_1 = \cos^{-1}\left(\frac{\mathbf{b} \cdot \mathbf{\Phi}_1}{\|\mathbf{b}\| \|\mathbf{\Phi}_1\|}\right)$$  \hspace{1cm} (5-8)

Special attention should be paid to the tangent function in Equation 5-5. From Equation 5-5, it follows that if the tangent of the angle $A$ approaches zero, the modified measurement model will be undefined. This situation occurs when the angle $A$ approaches zero, or alternatively when the angle $\gamma$ approaches 180°. This is unlikely to happen in real application, since the satellite’s position would not fall in between the user and reference receivers. On the other hand, the tangent of the angle $A$ will approach infinity when angle $A$ approaches 90°, or equivalently when angle $\gamma$ is close to zero. Fortunately, this is also unlikely to happen, because a mask angle of at least 5 degrees is usually applied to avoid poor measurements due to low signal-to-noise ratios and large atmospheric delays.

5.3 Improvement of The Model Modification

To investigate the improvement of the measurement model modification, the simulation as described in Section 5.1 is used again. The single difference phase measurements are adjusted with the modified measurement model as is given by Equation
and the unit vectors to the satellites are calculated based on the position of the reference receiver. The convergence results of the 3-dimensional position errors, for 200 seconds of run-time, are illustrated in Figure-5-4. The innovation sequence of the MEKF is also examined, and the results are shown in Figure-5-5. After the measurement model adjustment, the innovation sequences obtained from Equation 5-1 demonstrate the white noise property.

The verification of the white-noise innovation property as well as the improvement of the convergence performance verify the correctness of the approach used for modifying the measurement model. Although the motivation for modifying the measurement model originated from the integration of a LEO satellite for positioning purposes, this model can also be used for GPS-only systems or integrated systems as long as the antenna separation for the carrier phase differential operation does not exceed the range that de-correlates the error sources. As mentioned in Section 4.3, the application of integrating a pseudolite is similar to integrating a LEO satellite; therefore, this modification approach for the carrier phase differential measurement model is also applicable for GPS/Pseudolite systems.
Figure 5-4 The 3-D Convergence Error of The Modified GPS/LEO MEKF Estimation

Figure 5-5 The Innovation of The Modified GPS/LEO MEKF Estimation
6. Statistical Calibration

In the previous chapters, the modified extended Kalman filter (MEKF) has been shown to provide 3-D relative position as well as DD ambiguities estimates in real-time. Another requirement for the MEKF is that it must be able to handle changes in the satellite tracking status; i.e. the MEKF must have the ability of self-adjusting its state dimensionality while tracking transitions occur during the estimation process. This is the motivation for a statistical calibration study.

6.1 Necessity of Statistical Calibration

When the MEKF is initialized, the system and measurement models are constructed based on the number of available measurements. The number of available measurement, as well as the number of desired states, establish the dimensionality of the filter. The filter uses the models with fixed dimensionality to continuously estimate the unknown states. However, during the estimation process, a certain satellite may move below the mask angle and become invisible, or signal interference may cause cycle slips, such that the receiver loses track on a certain satellite. This situation will force the Kalman filter to stop estimating, re-establish the models, and reinitialize the estimation process.

Another situation occurs when there is a new satellite available for positioning purposes, or when there is a LEO satellite (or Pseudolite signal) to be incorporated into the
MEKF models for accelerating the estimation process. Again, the filter models would have to be re-established and the estimation process has to be reinitialized.

The above mentioned two situations lead to a dimensionality change (either increase or decrease) of the Kalman filter models. In this chapter, the addition and deletion of a measurement, when the estimation process is in steady-state, will be investigated. This is followed by an investigation into dimensionality changes of the filter before it has reached the steady-state. An analytic formulation as well as simulation results will be provided for three different statistical calibration approaches.

6.2 What is Statistical Calibration

Calibration is a process used to determine and adjust the systematic errors or biases of a measuring instrument based on an informative or calibration experiment.[34] This type of calibration is sometimes referred to as scientific calibration. The calibration process can be classified into pre-mission calibration and real-time calibration. The pre-mission calibration is usually conducted prior to the operation of the equipment, to ensure that the equipment is under normal operating conditions. Real-time calibration is usually completed periodically, during the operation, by the aid of redundant systems.

Statistical calibration is to determine the parameters characterizing the calibration curve for later prediction, and is sometimes referred to as inverse prediction or inverse regression.[32, 34] It is easier to realize the definition of statistical calibration by considering an experiment: "For each of the N samples with known values, one or more measurements
to be calibrated are made with a less accurate instrument, the known values have been determined with an extremely accurate instrument. The resulting data constitute the calibration experiment and are used to estimate the calibration curve (parameters), and this calibration curve (parameters) is then ready for use to predict the values for new samples measured with the less accurate instruments".[34] Hence, the statistical calibration is to determine a set of parameters of the calibration curve for estimation or prediction purposes.

Understanding the definition of statistical calibration leads to the realization about its application to the MEKF for those situations where measurements are added or dropped from the solution.

6.3 Approaches of Statistical Calibration for MEKF

The Kalman filter's system statistical parameters are defined as P, Q, and R covariance matrices in the Kalman equations. The statistical information is used to calculate the Gain matrix which is used to weigh the differences between the new measurements and their predicted values. Once calibration is necessary during the course of the estimation, these system parameters must be adjusted based on some experiment. Also, because the statistical information is carried from previous time epochs, the calibration process is not necessarily a batch process.
6.3.1 A Satellite Dropped

When the situation occurs that a certain satellite is dropped by the receiver, the MEKF should be able to handle this by leaving out the associated measurements and the states to be estimated. Also, the corresponding elements in the P, Q, and R matrices have to be taken out carefully in order to decrease the system’s dimension correctly. Since this process involves the removal of rows and/or columns, it is expected that the statistical information carried from the initialization all the way to the calibration point can be preserved under the system dimension change. An example of this will be shown in Section 6.4.

6.3.2 A Satellite Added

When a satellite is added to the MEKF, the new measurement should be added and the associated state vector should be expanded by adding the new state variable. Also, the corresponding rows and/or columns of the covariance matrices (P, Q, and R) should be determined and inserted properly. Clearly, determining these values is not as straightforward as just leaving out the corresponding values.

First, the measurement covariance matrix, R, changes will be addressed. Since all the measurements are assumed independent, and the statistical error standard deviation (σ) of the measurement is available from empirical data, the measurement covariance matrix is expanded as follows.
Next, the system states are assumed to be independent, too. This is accurate, because
the state vector contains only the 3-D position coordinates (x, y, and z) and DD ambiguities,
and the state propagation for the MEKF is the identity propagation. The state error standard
deviation (δ) is actually related to the DD ambiguities only, since the position coordinates
are propagated with the carrier phase measurements. Therefore, the new state can be
assigned the same value as those of the other states. The expansion for the system
covariance matrix Q is thus expressed as

\[ Q = \delta^2 I_{(n+1) \times (n+1)} \] (6-2)

The P matrix is referred to as the error covariance matrix and is defined in Equation
6-3.

\[ P = E[(X-\hat{X})(X-\hat{X})^T] \]
\[ = \begin{bmatrix} \sigma_1^2 & C_{12} & C_{13} & C_{1n} \\ C_{21} & \sigma_2^2 & C_{23} & C_{2n} \\ C_{31} & C_{32} & \ldots & C_{3n} \\ C_{n1} & C_{n2} & C_{n3} & \sigma_n^2 \end{bmatrix} \] (6-3)
It characterizes the error variances and covariances of the states during the estimation process. The error covariance and variance describe the statistical differences between the estimated and the true states. Therefore, the expansion for the error covariance matrix becomes non-trivial, and requires further exploration. The following section presents the details and formulas of three approaches for expanding the P matrix.

6.3.3 Three Statistical Calibration Methods

For investigating three methods for calibrating the P matrix, the MEKF equations are revisited. The measurement model is expressed as (see Chapter 3 for an explanation of the variables used)

\[
\begin{bmatrix}
DD^{12} \\
DD^{13} \\
DD^{14} \\
DD^{15} \\
DD^{16} \\
DD^{17}
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_1^T & 0 & 0 & 0 & 0 & 0 \\
\varepsilon_2^T & 0 & 1 & 0 & 0 & 0 \\
\varepsilon_3^T & 0 & 0 & 1 & 0 & 0 \\
\varepsilon_4^T & 0 & 0 & 0 & 1 & 0 \\
\varepsilon_5^T & 0 & 0 & 0 & 0 & 1 \\
\varepsilon_6^T & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
b \\
N^{12} \\
N^{13} \\
N^{14} \\
N^{15} \\
N^{16} \\
N^{17}
\end{bmatrix}
\]

The discrete-time matrix form for measurement and system models are given by
\[ DD_k = H_k x_k + v_k \] \hspace{1cm} (6-5)

and

\[ x_{k+1} = \Phi_k x_k + w_k \] \hspace{1cm} (6-6)

The equations for Kalman gain, state estimate, and covariance propagation are given by

\[ K_k = P_k^{-1} H_k^T (H_k P_k^{-1} H_k^T + R_k)^{-1} \] \hspace{1cm} (6-7)

\[ \hat{x}_k = \hat{x}_k^{-} + K_k (D_k - H_k \hat{x}_k^{-}) \] \hspace{1cm} (6-8)

\[ P_k = (I - K_k H_k) P_k^{-} \] \hspace{1cm} (6-9)

6.3.3.1 High-Uncertainty Method

The first statistical calibration method investigated in this dissertation is referred to as the High-Uncertainty Method. Since the Kalman filter is an optimal, unbiased estimator using the minimum variance as the performance criterion, the optimal estimate is obtained by minimizing the trace of the error covariance matrix \( P \), which is the sum of the variances of all the elements of the state vector. If no prior information is available for a new state variable, the corresponding element in the main-diagonal of the \( P \) matrix can be assigned.
with a large value, indicating a high-level of uncertainty. Furthermore, the covariances in the row and column corresponding to the new measurements are initialized at zero, indicating no correlation between the new state and the existing state variables. This method, although ad-hoc, usually works well. The performance of this method will be illustrated in a 2-D example shown in Section 6.4.

6.3.3.2 Statistical Mapping Method

In this section, a systematic approach will be presented to perform the statistical calibration on the P matrix. The concept is to calculate values for the variance and covariances corresponding to the new state variable. To calculate these values, the initial DD ambiguity estimate of the new measurement must be obtained first. From Equation 6-4, the initial DD ambiguity estimate can be calculated from

\[ \hat{N}_A^j = DD^j - (\epsilon^j_1 - \epsilon_A^j)^T \hat{b} \] (6-10)

where \( j \) is the superscript for the new measurement, and the subscript \( A \) denotes the added quantity. Using Equation 6-10, the estimation error of the new ambiguity can be obtained as follows

\[ \hat{N}_A^j - N_A^j = (\epsilon^j_1 - \epsilon_A^j)^T (b - \hat{b}) \] (6-11)
This is the information used to derive the calibration formulas for the added variance and covariances of the P matrix. In general, the P matrix is calculated from

\[
P = \text{cov}[(\hat{X} - X), (\hat{X} - X)] = E[(\hat{X} - X)(\hat{X} - X)^T]
\]

\[
= E\left[ \begin{pmatrix} \hat{b} - b \\ \hat{N}^{12} - N^{12} \\ \vdots \\ \hat{N}^{1j} - N^{1j} \\ \vdots \\ \hat{N}^{18} - N^{18} \end{pmatrix} \begin{pmatrix} \hat{b} - b \\ \hat{N}^{12} - N^{12} \\ \vdots \\ \hat{N}^{1j} - N^{1j} \\ \vdots \\ \hat{N}^{18} - N^{18} \end{pmatrix}^T \right]
\]

It is noted that the error covariance matrix, P, is symmetric, therefore the corresponding added column is the same as the added row, but the variance term has to be determined separately.

The elements of the inserted column are \( P_{1A}, P_{2A}, \ldots, P_{nA} \), where \( n \) is the number of state estimates. The covariance elements \( P_{iA} \), where \( i = 1 : n, i \neq A \), can be found as follows

\[
P_{iA} = P_{Ai} = \text{cov}[(\hat{x}_i - x_i), (\hat{N}_A - N_A)] = E[(\hat{x}_i - x_i), (\epsilon^{1A}(b_x - \hat{b}_x) + \epsilon^{1A}(b_y - \hat{b}_y) + \epsilon^{1A}(b_z - \hat{b}_z)]
\]

where \( \epsilon^{1A} \) is the Double Difference unit vector between the added satellite and the common satellite number 1. Under the assumption that the unit vector associated with the added
satellite is true and deterministic, Equation 6-13 can be rewritten by applying the definition of the P matrix into the expression

\[ P_{iA} = P_{Ai} = -(\varepsilon^A_{i1}P_{i1} + \varepsilon^A_{i2}P_{i2} + \varepsilon^A_{i3}P_{i3}) \]  

(6-14)

Equation 6-14 is the formula used to calibrate the covariance terms of the inserted column as well as the inserted row associated with the added satellite. The assumption that the DD unit vector of the added satellite is a truth value is reasonable since the orbital parameters of the added satellite are available, as well as an accurate estimate of the user's position.

For the calibration of the variance element of the P matrix, the same covariance property is applied. The variance of the estimation error of the new ambiguity is

\[ P_{AA} = E[(\hat{N}_A - N_A)(\hat{N}_A - N_A)^T] \]
\[ = E[(\varepsilon^A_{x}(b_x - \hat{b}_x) + \varepsilon^A_{y}(b_y - \hat{b}_y) + \varepsilon^A_{z}(b_z - \hat{b}_z)) \cdot (\varepsilon^A_{x}(b_x - \hat{b}_x) + \varepsilon^A_{y}(b_y - \hat{b}_y) + \varepsilon^A_{z}(b_z - \hat{b}_z))] \]

(6-15)

after some manipulation and by applying the definition of the P matrix, the calibration formula for the error variance is obtained as

\[ P_{AA} = \varepsilon^A_{x}P_{11} + \varepsilon^A_{y}P_{22} + \varepsilon^A_{z}P_{33} + 2[\varepsilon^A_{x}\varepsilon^A_{y}P_{12} + \varepsilon^A_{y}\varepsilon^A_{z}P_{23} + \varepsilon^A_{z}\varepsilon^A_{x}P_{31}] \]

(6-16)
In summary, Equation 6-14 and Equation 6-16 constitute the calibration formulas of the statistical mapping method. When a satellite is added at time epoch $k$, before calculating the Kalman gain matrix, $K_k$, and the error covariance matrix, $P_k$, the error covariance matrix propagated from the previous time epoch, $P_{k-1}$, is calibrated using Equation 6-14 and Equation 6-16. From this point on, the Kalman equations are resumed by updating the error covariance matrix, calculating the Kalman gain matrix, followed by the state update equation.

6.3.3.3 Error Covariance Propagation

The calibration mapping method of Section 6.3.3.2 uses the previous error covariance matrix $P$ in combination with the satellite geometry to create the variance and covariance elements for the new state variable. This is a one-way method, which does not provide for interaction between the state variables. To create this interaction, the error covariance matrix, $P$, of Kalman filter is analyzed without the actual measurement input. This property of the Kalman filter is also referred to as covariance analysis.

Basically, when the new measurement is incorporated, the error covariance matrix will be initialized, and then, as the Kalman filter continues to estimate, the error covariance matrix should converge to the steady state. At this point, the steady state error covariance matrix can be used as the new error covariance matrix at the time epoch that the new measurement is incorporated.
In principle, the above method of calibrating the $P$ matrix consists of performing the error covariance analysis iteratively based on the following set of three equations:

\begin{align}
K &= P^{-1}H^T(HP^{-1}H^T + R)^{-1} \\
\bar{P} &= (I - KH)P^{-1} \\
P^{-1} &= P + Q
\end{align}

In the above equations, the subscript denoting the time epoch is dropped for simplicity. This iterative process can be performed based on a pre-set time span, or the process can be terminated if the error covariance matrix does not change much, which implies that the process has reached the steady-state. The resulting error covariance matrix can be treated as the calibrated $P$ matrix for the MEKF. Note that to initiate the iterative covariance analysis process, the elements of the $P$ matrix corresponding to the new measurement can be obtained by either the high-uncertainty or by the statistical mapping method. The following section presents a 2-D simulation example to compare the three calibration methods.
6.4 2-D Simulation Example to Compare the Three Calibration Methods

In this section, a 2-dimensional (2-D) simulation is performed to illustrate statistical calibration, and to compare the three approaches documented in Section 6.3.3.

In this simulation, 5 satellites are simulated with an orbital radius of 20,000 km and an orbit period of 12 hours. The 5 satellites are initially at [5°, 45°, 85°, 115°, 150°] angular location relative to the positive X-axis, and start to move in a circle with the direction of [1, -1, 1, -1, -1], where 1 indicates counterclockwise and -1 indicates clockwise directions of travel. The satellite trajectories for the simulation are shown in Figure 6-1.

Figure 6-1 Two-Dimensional Satellite Geometry for Statistical Calibration Analysis.
The reference receiver is assumed at the origin, and the user is located 100 m east and 100 m south of the reference receiver. In static mode, the user starts the MEKF estimation process by using a position uncertainty of 10 m east and 5 m south from the true location. Observables for this simulation are integrated Doppler Double Differences (DD) with respect to satellite number 1. The error standard deviation for the integrated Doppler DD measurement is set at 1 cm. All other parameters and equations can be found in Chapter 3.

For easier reference, the three different statistical calibration methods are referred to as:

(M1) High-Uncertainty Method
(M2) Statistical Mapping Method
(M3) Covariance Propagation Method

In addition to the three calibration methods, there are 3 different scenarios that will be explored with the 2-D simulation, which are described below.

<S1> Add one satellite during the converging period.
<S2> Drop one satellite and add the same one later.
<S3> Drop one satellite and add another new one at the same time.

In the S1 scenario, the integrated Doppler measurements from 5 satellites are used to start the convergence. A certain satellite is added at the 10th second during the initial phase of the convergence process of the MEKF. The simulation results showing the comparison among the three different methods are illustrated in figures 6-2 through 6-4. In
Figure 6-2 X-Component Errors for Three Statistical Calibration Methods, Scenario 1

Figure 6-3 DD Ambiguity 2 Errors for Three Statistical Calibration Methods, Scenario 1
these figures, the High-Uncertainty method is indicated with a dotted line, the Mapping method with a solid line, and the Covariance Propagation method with a dash-dotted line. Only plots of the X-component and two DD ambiguities are shown, since these were found to be representative for all the results. Based on the results in figures 6-2 through 6-4, it is clear that all three methods perform well. The Covariance Propagation method is somewhat biased in Figure 6-3, which is probably due to the fact that the error covariance matrix has been stabilized too early in the convergence process.

For the scenario 2 (S2), a certain satellite is dropped at the 10th second and added back at the 20th second of the time history. This scenario characterizes the situation of temporary signal blockage, which is not uncommon for real operation of satellite positioning using carrier phase. Again, only the X-component and 2 DD ambiguities of the simulation
Figure 6-5 X-Component Errors for Three Statistical Calibration Methods, Scenario 2

Figure 6-6 DD Ambiguity 3 Errors for Three Statistical Calibration Methods, Scenario 2
The 2-D error, DD ambiguity 4

- . High-Uncertainty Method
- Mapping Method
- Covariance Propagation Method

Time in seconds

Figure 6-7 DD Ambiguity 4 Errors for Three Statistical Calibration Methods, Scenario 2

results are presented in figures 6-5 through 6-7. All three methods handle scenario 2 very well. The more advanced Mapping method and Covariance methods converge faster, as expected, but the simple High-Uncertainty method reaches an accurate steady-state faster.

The third scenario (S3) of this simulation is to drop one satellite and add another different satellite at the same time. This scenario may occur when a backup measurement is applied immediately once the number of measurement drops below a certain minimum. The convergence results from the simulation are illustrated in figures 6-8 through 6-10. The convergence trends are similar to those of scenario 2.
6.5 3-D Simulation Example for the Mapping Method Based on Flight Data

In this section, the convergence of the 3-D error with the mapping method will be demonstrated. For this simulation, a portion of the B-757 flight path as shown in Chapter 3 is used. This portion is the final 120 seconds before the touchdown point. A LEO satellite with mean anomaly and right ascension of 43° and 285°, respectively, is simulated and added to the GPS constellation.

The integrated Doppler measurements are calculated based on the reference flight trajectory and the satellite positions determined from the orbital parameters of the satellites, plus 2 mm (1-σ) of carrier phase noise. The MEKF is initialized using the differential code phase solution with a 3-D position standard deviation of approximately 15 m, as described

Figure 6-8 X-Component Errors for Three Statistical Calibration Methods, Scenario 3
Figure 6-9 DD Ambiguity 3 Errors for Three Statistical Calibration Methods, Scenario 3

Figure 6-10 DD Ambiguity 4 Errors for Three Statistical Calibration Methods, Scenario 3
in Chapter 3. Both satellite add (number of satellite from 6 to 7) and drop (number of satellite from 7 to 6) scenarios are simulated to show that the proposed statistical calibration algorithm can be used for both cases. Figure 6-11 shows the convergence of the MEKF for the case where the number of satellites decreases from 7 to 6. Figure 6-12 shows the convergence of the MEKF for the case where the number of satellites increases from 6 to 7. In both demonstration cases, the time instant for the transition was intentionally chosen around 20 seconds from the start of the run time (GPS time 39353 seconds for Figure 6-11, GPS time 39239 seconds for Figure 6-12). The reason for this is to test the performance of the statistical calibration algorithm while the estimation is still in the transient phase.

Figure 6-11 shows that the position convergence starts with a fast response and becomes sluggish after the tracked number of satellites drops from 7 to 6. The MEKF filter was not interrupted while estimating. On the other hand, Figure 6-12 demonstrates that the position convergence starts with a slow response, but improves significantly after the tracked number of satellites increases from 6 to 7. Not surprisingly, more satellites with better geometry tend to improve the convergence of the MEKF. The non-interrupted estimation while integrating another measurement to improve convergence is the objective of statistical calibration for the MEKF. The Mapping method was found to be most efficient, since it does not introduce a transient in the solution and its steady-state performance is similar to that of the High-Uncertainty method.
Figure 6-11 Convergence results with the Mapping Method While Dropping One SV

Figure 6-12 Convergence results with the Mapping Method While Adding One SV
7. **Observability Analysis**

The reason to investigate the observability of the MEKF algorithm is due to its occasional lack of convergence. It was observed that in the case of an added LEO, for some spatial geometries the filter converges well, while for other, similar geometries, the filter would converge slowly or even diverge. Figures 7-1 through 7-4 show examples of good and poor convergence as well as the associated DOP history for each case.

![3-D position error with added LEO M0:43, OM0:285](image)

Figure 7-1 Good Convergence Performance for 5 GPS + 1 LEO

Both examples show results for 120 seconds of simulation time with 5 GPS satellites and one LEO satellite. Note that this is a case of marginal availability, since the MEKF normally
requires 7 satellites for rapid convergence. Evidently, the geometry of the satellite constellation plays a significant role in the estimation performance (convergence speed) of the MEKF algorithm.

![3-D position error with added LEO M0:11, O0:46](image.png)

Figure 7-2 Poor Convergence Performance for 5 GPS + 1 LEO

7.1 Dilution of Precision (DOP) Analysis

To investigate the influence of the position DOP (PDOP) on the convergence performance, figures 7-3 and 7-4 show the PDOP histories that correspond to figures 7-1 and 7-2, respectively. As can be observed from these two PDOP figures, the one (Figure 7-4) associated with the poor convergence (Figure 7-2) actually has smaller PDOP values, which further decrease while the simulation runs. On the contrary, the one (Figure 7-3) associated
Figure 7-3 The Position DOP History of Figure 7-1.

Figure 7-4 The Position DOP History of Figure 7-2.
with the good convergence (Figure 7-1) has larger PDOP values, which further increase during the first half of the simulation period. Therefore, the position DOP is not the only factor that determines the convergence behavior of the MEKF.

This becomes evident by re-examining Equation 2.2, which shows that PDOP provides an instantaneous relationship between the measurement errors and the position errors. The MEKF, however, uses tens of seconds of observations to achieve convergence. It seems therefore appropriate to investigate the contributions of geometry changes, rather than just the instantaneous geometries.

7.2 Observability Analysis

Given that the convergence performance of the MEKF is a function of time, and by the inherent properties of the Kalman filter, a system can be characterized by its controllability and observability. However, since the MEKF uses only the integrated Doppler measurements from the outside and provides no control over any input of the system, it is manifest that the system's observability should determine the convergence speed of the MEKF filter. To derive a metric for observability, consider the Kalman filter estimation system as a linear time-varying model,

\[
X_{k+1} = \Phi_k X_k + w_k \\
Y_k = H_k X_k + v_k
\]  

(7.1)

Since the system is time-varying, the observability grammian could be used.[41]
The observability grammian is a measure of how observable a linear state-space system is. By definition, a linear state-space system as described in Equation 7.1 "is called observable on \([k_0, k_f]\) if any initial state \(x(k_0) = x_0\) is uniquely determined by the corresponding zero-input response \(y(k)\) for \(k = k_0, ..., k_{f-1}\)." [39] Here, the output response \(y(k)\) is the measurement of the MEKF, and the initial state \(x(k_0)\) can be the state estimates at any initial time instant of the estimation process. Derived from Equation 7.1, the measurement vector \(Y_k\) relates to the initial state vector \(X_0\) by:

\[
Y = \begin{bmatrix}
H_0 \\
H_1 \Phi_0 \\
H_2 \Phi_1 \\
\vdots \\
H_f \Phi_{k-1}
\end{bmatrix} X_0 = OX_0
\] (7.2)

The matrix \(O\) is called the observability test matrix. Also from reference [39], the linear state-space system is observable on the prescribed time period if and only if the observability test matrix \(O\) has full rank, such that the initial state vector can be uniquely determined by:

\[
X_0 = (O^T O)^{-1} O^T Y
\] (7.3)

For the observability test matrix \(O\) to have full rank, the if and only if condition is that a new formed matrix \(N = (O^T O)\) should be invertible, and satisfy the Lyapunov equation. This
symmetric and positive semi-definite matrix $N$ is also called the observability grammian. The advantage of analyzing observability grammian is because it can be computed by numerically solving certain matrix difference equations.

For a discrete-time, linear time-varying system, the observability grammian can be defined as: [41]

$$N(k_f, k_0) = \sum_{k=k_0}^{k_f} \Phi^T(k, k_0)H^T(k)H(k)\Phi(k, k_0)$$  \hspace{1cm} (7-4)

For the MEKF, the state transition matrix $\Phi(k)$ is set to the identity matrix, and the measurement matrix $H(k)$ is updated every epoch (typically every second). The system's observability grammian should be invertible to make the system observable over the time span. Moreover, for an observable linear system, it is implied that every state of the system can be uniquely determined (estimated) from the measurement inputs.

Therefore, the observability grammian should have full rank over the time span. However, the rank test of the observability grammian usually provides full rank results for the cases under test, which does not help much in arriving at a metric for convergence performance. Therefore, in addition to the rank of the grammian, a rank-deficiency or numerical rank test will be performed as well, which will indicate how close the system is to non-observability.

The Singular Value Decomposition (SVD) is a linear transformation to decompose a matrix into a set of orthogonal transformations along with the associated singular values.
of the matrix. The smallest singular value indicates how close the matrix is to being rank-
deficient. The SVD is applied to the observability grammian and the smallest eigenvalue
will be used to assess the convergence of the MEKF.

7.3 Observability Simulation Results

The observability grammian was calculated for the same two cases as shown in
Figure 7-1 and Figure 7-2. The first one is the good convergence example and the second
is the poor convergence example. In addition, the smallest singular values of observability
grammian are calculated and recorded. The smallest singular values of the observability
grammian were calculated every second and are shown in Figure 7-5 for both cases. As
indicated in Figure 7-5, for the entire 120-second run time period, the smallest singular value
of the observability grammian for the good convergence example has larger values and
increases faster than those of the poor convergence example.

To gain additional insight into the satellite geometries for the two simulation cases,
Figure 7-6 and 7-7 show the polar plots for the satellites used. It is illustrated in figures 7-6
and 7-7 that the added LEO satellite in the good convergence case provides more geometry
change than in the poor convergence case.

In summary, both the relative spatial geometry change as well as the system
observability analysis yield the same and consistent results for the two cases presenting the
good and poor convergence performance of the MEKF algorithm. It is concluded that the
smallest singular value of the observability grammian is an effective tool for the assessment
Figure 7-5 The Smallest Singular Values of the Observability Grammian.

Figure 7-6 Polar Plot of the Satellites-In-View for The Good Convergence Example.
of the convergence performance.

Figure 7-7 Polar Plot of the Satellites-In-View for The Poor Convergence Example.

This tool enables the prediction of the convergence performance, which allows for the scheduling of an operation using the MEKF. It is noted, that sufficient observability is not the only criterion for successful high-accuracy positioning; the measurement errors must also be within acceptable bounds. However, this dissertation provides all the tools necessary for predicting the positioning performance.
8. Combined GPS/GLONASS Processing

Although GPS, through carrier phase differential operation, can achieve centimeter-level accuracy after the integer ambiguities are resolved, the high availability requirements for some applications, such as aircraft automatic landing, may not be met by the GPS alone. This necessitates the use of additional measurements. As discussed previously, carrier phase measurements from any other satellite can be added to the MEKF algorithm. Since the existing GLONASS is a satellite system similar to GPS and its measurements are currently available, this chapter considers the addition of GLONASS to the MEKF algorithm.

8.1 Background for GPS/GLONASS

Due to the similarity to the GPS, the Russian GLONASS satellites will be considered as additional signal sources to augment the GPS for integrated processing. GLONASS satellites not only provide additional ranging signals to increase the positioning accuracy, they also serve as the augmented system to provide the required availability.

Background information on GLONASS is provided in Section 2.3.2. To date, many research activities have been conducted to investigate GPS/GLONASS positioning. Due to the fact that GLONASS does not contain SA degradation error, the stand-alone positioning accuracy tends to be better than that of GPS.[30] Furthermore, the increased number of measurements and the improved geometry benefit the ambiguity resolution of the carrier
phase differential operation. The GLONASS can also be used to cross check the systematic errors.

Although GLONASS can provide many advantages, it has some issues that need to be resolved in order to implement the combined GPS/GLONASS processing.

### 8.2 GLONASS Issues

There are three items that must be investigated in order to consider GLONASS as an additional measurement source for the combined process with GPS.

1. Time differences between GLONASS and GPS;
2. Coordinate differences between GLONASS and GPS;
3. Inter-frequency biases among the GLONASS satellites.

The first issue is that the time reference system of GLONASS is different from that of GPS. GPS time is referenced to Universal Time, Coordinated (UTC) as maintained by the United States Naval Observatory (USNO), while the GLONASS time is referenced to UTC as maintained by the National Time and Frequency Service (NTFS) of the Russian Federation, which has a three-hour offset relative to Greenwich Mean Time (GMT). In addition, unlike GPS, GLONASS considers leap seconds, which results in another time difference between GPS and GLONASS of several seconds. Besides these differences, there is also another time difference on the order of several microseconds between the GPS and
GLONASS time frames.[30,31] However, since a time difference, which is common to all satellites, contributes mainly to the clock bias, the time difference problem can actually be mitigated by using one more measurement to solve for the time difference.

The second issue is the geodetic coordinate frame difference between GPS and GLONASS. The GPS uses the WGS-84 coordinate system, while the GLONASS uses the PZ-90 coordinate system. The transformation between WGS-84 and PZ-90 is characterized by the following equation [31]:

\[
\begin{align*}
\begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 0 \\ \mu \\ 0 \end{pmatrix} + \begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} \\
\end{align*}
\]

(8-1)

where \([x,y,z]^T\) denotes the WGS-84 coordinate vector, and \([u,v,w]^T\) represents the PZ-90 coordinate vector. The parameter \(\mu\) is 2.5 meters, and \(\alpha\) is 0.4 seconds (0.4\(^\circ\)). With this transformation, the GLONASS position can be converted from the PZ-90 coordinate frame to the WGS-84 coordinate frame.

The third issue are the GLONASS interchannel biases which stem from the different frequencies of the GLONASS satellites. Frequency-dependent delays are introduced by the front-end of the receiver. This problem is more severe for the pseudorange measurement than for the integrated Doppler measurements. A portion of the interchannel biases result from the clock of the receiver. The common clock offset in the receiver will generate different errors associated with different channels due to different wavelengths being scaled.
These issues are critical and need to be considered especially when constructing the Double Difference (DD) carrier phase formulation.

Interchannel biases are significant; up to 15 nanoseconds for pseudorange measurements and 0.2 cycles for carrier phase measurements, without calibration. With calibration, the interchannel biases of the receiver can be controlled to within sub-meter level for pseudorange, and of course with better front-end filter design, the interchannel biases can be further reduced. However, without calibration, a 0.2 cycle error will pose a significant problem for carrier phase ambiguity resolution. Furthermore, interchannel biases tend to change with temperature. Typical drift rates are on the order of 0.7 ns/7 degrees Centigrade. Therefore, one of the best ways to alleviate interchannel biases is through real-time calibration. One such approach will be presented in Section 8.3.4.

8.3 The GPS/GLONASS MEKF algorithm

In this section, the combined GPS/GLONASS processing with the MEKF will be discussed. The primary focus will be on integrating GLONASS integrated Doppler measurements to dynamically estimate the user position with greater convergence speed. This will be followed by a search scheme to determine the ambiguities for GPS. Finally, the Least Squares (LS) solution with ambiguities resolved is used to calibrate the interchannel biases of GLONASS in real-time. The flowchart of the GPS/GLONASS MEKF processing algorithm is depicted in Figure 8-1.
Figure 8-1 Flow Chart of GPS/GLONASS Algorithm.
8.3.1 GPS/GLONASS Double Difference Formulation

To derive the GPS/GLONASS Integrated Doppler Double Difference equation for the MEKF, consider the phase equation for a GLONASS satellite:

\[
\rho_A^i(t) = \phi_A^i \lambda_i + N_A^i \lambda_i + S_A^i \beta f_i + \beta A f_i - d_{\text{iono}} + d_{\text{tropo}} + b_A^i
\]  

(8-2)

where \( \rho_A^i(t) \) is the true range from satellite \( i \) to receiver \( A \) at time \( t \), and

- \( \phi_A^i \) is the measured signal phase from satellite \( i \) to receiver \( A \).
- \( N_A^i \) is the integer ambiguity from satellite \( i \) to receiver \( A \).
- \( S_A^i \) is the lumped noise (e.g. multipath) from satellite \( i \) to receiver \( A \).
- \( \beta \) is the clock bias of satellite \( i \) or receiver \( A \).
- \( d \) is the signal delay in the propagation path caused by the ionosphere or troposphere.
- \( b_A^i \) is the receiver phase delay for satellite \( i \).

Note that the channel bias which is a function of satellite frequency is added, and the clock offsets of satellite and receiver are scaled by the specific frequency.

The single difference (SD) is formulated by differencing two phase observables from two receivers:
The satellite clock bias term, the ionospheric, and the tropospheric delays cancel, provided that the baseline length is within a certain limit (5 - 10 km). Also note that the channel phase delay is common between the two receivers. An alternate form of the SD is shown in Equation 8-3b to reflect the spatial geometric relationship.

\[ SD_{AB}^i = b \cdot \xi^i \]  

Equation 8-3b

\( b \) is the baseline vector pointing from the reference receiver to the user receiver, and \( \xi^i \) is the unit vector from the baseline midpoint to satellite \( i \).

Two single-differences with respect to two different satellites are themselves differenced to generate a double-difference (DD) as shown below:

\[ DD_{AB}^{ij} = (\Delta \phi_{AB}^i \cdot \lambda_i - \Delta \phi_{AB}^j \cdot \lambda_j) + (N_{AB}^i \cdot \lambda_i - N_{AB}^j \cdot \lambda_j) + S_{AB}^{ij} \]
\[ + \beta_{AB}(f_i^j - f_j^i) + (b_{AB}^j - b_{AB}^i) \]  

(8-4)

Due to the different frequencies of the GLONASS satellites, there are several differences with respect to GPS. First, the DD phase observable contains different scaling factors associated with each satellite frequency. Therefore, the user should be careful when converting the DD phase observable from carrier cycles to a distance. Second, the DD integer ambiguity can not be directly distinguished from the biases. Thus the DD ambiguity for GLONASS carrier phase processing is considered undefined. Third, the receiver
common clock bias is not eliminated because it is scaled by the different frequencies. Finally, there is an additional term; the interchannel bias.

8.3.2 Dynamic Floating Estimation With MEKF

When integrating the GLONASS integrated Doppler measurements into the MEKF processing, the DD phase equation (Equation 8-4) needs to be combined with Equation 3-3, which results in the following measurement model

\[
DD_{ij}^{AB} = \Delta \phi_{ij}^{AB} + F N_{AB}^{ij} + S_{AB}^{ij} B_{AB}^{ij} = b \cdot (\epsilon^i - \epsilon^j)
\]  

(8-5)

where, FN stands for the DD floating ambiguity, and B represents the overall DD bias error. Denote the DD phase observable \(\Delta \phi_{AB}^{ij}\) as DD\(^{ij}\), such that the DD measurement model can be written as follows

\[
\begin{bmatrix}
DD^{12} \\
\vdots \\
DD_{\text{GPS}n}^{1} \\
DD_{\text{GLN}1}^{1} \\
\vdots \\
DD_{\text{GLNm}}^{1}
\end{bmatrix}
= 
\begin{bmatrix}
(\epsilon^1 - \epsilon^2)^T & 1 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
(\epsilon^1 - \epsilon^{\text{GPS}n})^T & 0 & 1 & \cdots & 0 \ \\
(\epsilon^1 - \epsilon^{\text{GLN}1})^T & 0 & \cdots & 1 & 0 \ \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
(\epsilon^1 - \epsilon^{\text{GLNm}})^T & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
b \\
N^{12} \\
\vdots \\
FN^{\text{GLN}1} + B^{\text{GLN}1} \\
\vdots \\
FN^{\text{GLNm} + B^{\text{GLNm}}}
\end{bmatrix}
\]  

(8-6)

where GPS\(n\) in the maximum number of the visible GPS satellites, and GLN\(m\) is the maximum number of visible GLONASS satellites.
Note that the common satellite, referred to as 1, should be one of the GPS satellites. The reason for this is that the GPS satellites do not have the channel bias and the GPS DD ambiguities can be observed directly. Usually, the GPS satellite with the highest elevation angle is selected as the common satellite for the formulation. Also, the GLONASS associated floating ambiguities plus the lumped bias (receiver clock offset and channel bias) are to be estimated together. Thus, Equation 8-6 is the measurement model for the MEKF to dynamically estimate the user’s 3-dimensional relative position and the associated floating DD ambiguities. The remaining equations of the MEKF, such as the system model as well as the state propagation and update equations, are the same as equations 3-11 through 3-18.

8.3.3 Resolving Ambiguities with GPS/GLONASS MEKF

One purpose of carrier phase differential processing is to achieve centimeter-level position accuracy through carrier cycle ambiguity resolution. For the combined GPS/GLONASS MEKF processing, the ambiguity resolution is only successful for the GPS ambiguities. The strategy is as follows: While estimating the floating solution with MEKF using the combined GPS/GLONASS integrated Doppler measurements, the process covariance matrix $P$ is monitored. Once there are more than or equal to 5 GPS DD ambiguities whose variance (or standard deviation) converge within a certain limit, for example, 1 cycle (0.2 meter) for the standard deviation, then a ambiguity search is launched to determine the DD ambiguities for GPS.
From the selected 5 double-difference measurements and corresponding floating ambiguities, it is calculated that (see also Section 3-2)

$$DD_{GPS} = H^*b + N_{\text{float}}\hat{\lambda}_{GPS}$$  \hspace{1cm} (8-7)

where $H^*$ is the measurement sub-matrix containing only GPS-related components, and $N_{\text{float}}$ is the floating DD ambiguity vector obtained from the MEKF estimation.

By applying the QR factorization, the parity ambiguity search equation can be formulated:

$$Q_p[DD_{GPS} - (N^* + N_{\text{offset}})\hat{\lambda}_{GPS}] = \gamma$$  \hspace{1cm} (8-8)

where $\gamma$ is the parity space ambiguity search residual, $N^*$ is the rounded DD ambiguity from $N_{\text{float}}$, and $Q_p$ is the parity vector.

8.3.4 Calibration of the GLONASS Channel Biases

Once the integer ambiguities are found, the position accuracy should be at the centimeter-level. For the GPS/GLONASS combined process, the position solution can be used to calibrate the interchannel biases for GLONASS. The interchannel biases are defined in both pseudorange and carrier phase domains. For pseudorange measurements, the only unknowns are the channel bias and measurement noise, and can be determined from

$$b_{AB}^{ij} + S_{AB}^{ij} = b^i (e^i - e^j) - \Delta PR_{AB}^{ij}$$  \hspace{1cm} (8-9)
where the noise can be reduced through averaging over time.

For carrier phase measurements, see Equation 8-5, the floating DD ambiguity, common receiver clock offset, and channel bias can be considered as one bias term. The reason is that the DD GLONASS ambiguity is undefined and it is not possible to separate the receiver common clock offset from the channel bias. Thus it is treated altogether and determined from

\[
BB_{AB}^{ij} + S_{AB}^{ij} = h \cdot (e_i - e_j) - \Delta \phi_{AB}^{ij}
\]  

(8-10)

Again, the lumped noise term can be averaged out and is negligible. Please note that this bias term, although a lumped quantity, is computed in real-time after the GPS ambiguities are resolved. Therefore, the “calibrated” lumped GLONASS bias can be used to compute the user position for future time epochs. Thus, high-accuracy GLONASS data would be available, after calibration, during GPS outages.

8.4 Simulation Results of the GPS/GLONASS MEKF

The performance of the GPS/GLONASS MEKF algorithm is investigated through a computer simulation. The simulation model is expanded from the one used in Chapter 3, with more channels simulated to process the GLONASS integrated Doppler measurements. Also, the interchannel biases are simulated by random constants with a standard deviation of 0.4 meters for carrier phase measurements. The 0.4 meter is much larger than that
achievable with current GPS/GLONASS receivers. The enlargement of the channel bias level is to show that the GPS/GLONASS MEKF algorithm is not affected by the channel bias.

The flight path and the associated carrier phase data are generated based on the truth flight path of the B-757 flight test. The noise on the carrier phase measurements is set to 5 millimeters (1 \sigma) for both GPS and GLONASS. The simulation time is 200 seconds. Five GPS and 8 GLONASS satellites are simulated for the entire simulation period. The MEKF is initialized with the code phase solution which was logged from the flight test.

The convergence of the 3-D relative position coordinates are shown in Figure 8-2.

Figure 8-2 The 3-D Position Convergence of The GPS/GLONASS MEKF
It is observed that the errors converge to within 1 meter after less than 30 seconds. At the 83rd second, the ambiguity-associated process variances converge to within the pre-defined threshold value, such that the ambiguity search is launched. After the DD ambiguities are fixed, the Least Squares solution as compared to the MEKF estimated solution are shown in Figure 8-3. As the plot shows, the Least Squares (LS) solution with the ambiguities fixed stays well within +/- 3 centimeter for the remainder of the run time. The MEKF solution remains at the sub-meter level as well.

![Comparison between MEKF and ambiguity resolved LS solution](image)

Figure 8-3 Error Comparison of LS and MEKF Solutions

Figure 8-4 shows the detailed 3-D position errors for the ambiguity-fixed Least Squares solution. The channel bias calibration results for 4 ambiguities are shown in figures 8-5.
through 8-8. These figures show that the error between the computed lumped bias and the true bias are in close agreement.

Figure 8-4 Error Statistics of the LS Solution

This completes the demonstration through simulation of the combined GPS/GLONASS MEKF algorithm.
Figure 8-5 Comparison of Ambiguity 4 Channel Biases Between MEKF and LS

Figure 8-6 Comparison of Ambiguity 5 Channel Biases Between MEKF and LS
Figure 8-7  Comparison of Ambiguity 7 Channel Biases Between MEKF and LS

Figure 8-8  Comparison of Ambiguity 8 Channel Biases Between MEKF and LS
9. Summary and Conclusions

The research documented in this dissertation provides the concept and establishes the framework of a robust carrier phase processing technique for high-accuracy, dynamic, satellite-based differential positioning systems.

A novel modified extended Kalman filter (MEKF) is introduced to converge the position of a differential or relative user within a time period of two minutes using carrier phase measurements from seven or more satellites. The MEKF was successfully applied to post-processed flight data from a Boeing 757 aircraft on final approach. Position accuracies were found to be at the sub-meter level. Following the convergence of the MEKF, a small ambiguity search was implemented to increase the position accuracy to the centimeter level.

Next, a low-earth-orbit (LEO) satellite was integrated into the MEKF to speed-up the convergence process from 2 minutes to approximately 30 seconds. To reduce systematic modeling errors of the MEKF, an improved measurement models was developed and successfully tested through a computer simulation.

Three statistical calibration methods were developed and evaluated to accommodate new measurements into the MEKF. It was found that the statistical mapping method is the most effective way to initialize the variance and covariances associated with the new measurement in the system error covariance matrix.
Convergence properties of the MEKF were found to be characterized effectively by the smallest singular value of the observability grammian.

Finally, a combined GPS/GLONASS processing algorithm is presented, which uses the GLONASS measurements to speed-up the MEKF convergence. This is followed by an ambiguity resolution for the GPS measurements. Next, the ambiguity-resolved position solution is used to calibrate the GLONASS interchannel biases. The resulting position solution accuracy is at the centimeter level and can be maintained as long as at least four satellites with good geometry continue to be available.
10. References


Appendix A. QR Factorization

The QR factorization consists of finding a set of orthogonal basis vectors for the range space of the mapping \( H : \mathbb{R}^n \rightarrow \mathbb{R}^m \), where \( n \) is number of unknowns and \( m \) is the number of measurements. Consider the general linear measurement model:

\[
y = Hx + \epsilon
\]

where \( x \) is a \( nx1 \) state vector, \( y \) is a \( mx1 \) measurement vector, and \( \epsilon \) is the measurement error vector. Usually, \( m > n \) to form an over-determined system. The \( H \) matrix represents the mapping from the state space to the measurement space, and is also referred to as measurement matrix or design matrix.

The general application associated with the linear measurement model is to provide the optimal estimate for the unknown states by minimizing the sum-square errors of the estimates. This results in the Least-Squares estimate

\[
x_{LS} = (H^TH)^{-1}H^Ty
\]

with a residual vector which is the difference between the true measurement vector and the calculated measurement vector

\[
w = y - y_{LS} = y - Hx_{LS} = [I - H(H^TH)^{-1}H^T]y = [I - H(H^TH)^{-1}H^T] \epsilon
\]
Generally, the measurement residual vector is used as an indication of consistency among the measurements. However, it is not ideal because there are \( n \) constraints associated with the unknowns among the \( m \) elements of \( w \), which obscure some of the aspects of the inconsistency that are of interest. Therefore, a transformation that transforms the information in the measurement domain into another domain is needed. The QR factorization is one of the methods to transform the information into the Parity Space for consistency evaluation.

By performing the QR factorization on the \( H \) matrix, as long as the \( H \) matrix has full rank \( = n \), the following is obtained

\[
H = QR
\]  
(A-4)

where \( Q \) is an orthonormal matrix \( (Q^T = Q^1) \), and \( R \) is an upper triangular matrix. The first \( n \) columns of the \( Q \) matrix form the orthogonal basis for \( \text{range}(H) \).

For the true linear measurement model, this can be visualized as

\[
y_{m \times I} = \begin{bmatrix} Q_{x_{mn}} & Q_{p_{(m-n)n}} \end{bmatrix} \begin{bmatrix} U_{nxn} \\ O_{(m-n)xn} \end{bmatrix} x_{n \times I} + \epsilon 
\]  
(A-5)

By applying the orthonormal property of the \( Q \) matrix,
The upper part of the equation represents the Least-Squares solution, while the lower part represents the parity equation. The parity vector is obtained as follows

$$y = \begin{bmatrix} Q^T_{x_{nxm}} \\ Q^T_{p_{(m-n)xm}} \end{bmatrix} x + \begin{bmatrix} U_{nxn} \\ 0_{(m-n)xn} \end{bmatrix} x + \begin{bmatrix} Q^T_{x_{nxm}} \\ Q^T_{p_{(m-n)xm}} \end{bmatrix} \epsilon$$  \hspace{1cm} (A-6)

The evaluation of the parity residuals provides more efficiency than the evaluation of the residuals given by Equation (A-3).

$$w_p = Q^T_{p_{(m-n)xm}} y = Q^T_{p_{(m-n)xm}} \epsilon$$  \hspace{1cm} (A-7)