ANALYSIS, SIMULATION, AND EXPERIMENTS FOR ADDITIVE
NARROWBAND COMMUNICATION SYSTEMS

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by

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Abstract

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An expanding demand for wireless communication systems requires researchers to find novel ways to consume less bandwidth while providing connectivity for multiple users. This thesis proposes an additive narrowband communication system which allows multiple users to occupy the same bandwidth without the use of a spreading code. Amplitude and bandwidth (data rate) discrepancies among users provide a means of extracting each individual user’s signal from the aggregate received signal. Bit error rate expressions are derived for a two user additive narrowband system operating over an additive white Gaussian noise channel. The uplink and downlink systems are analyzed separately and bit error expressions are provided for each case. Custom built hardware used to model an additive narrowband system is discussed. Experimental and simulation results are provided and are in excellent agreement with the analytical results.

Approved: 

David W. Matolak
Associate Professor of Electrical Engineering
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Chapter 1: Introduction

1.1 Background

Over the past decade the wireless community has seen an exponential demand for wireless applications. High speed wireless internet connections, embedded GPS navigation systems, personal wireless communication devices, and satellite radios are just a few examples of systems that have had success penetrating the wireless market over the past decade. The wireless system boom has created a public demand for new wireless applications. To satisfy the public demand as well as make a profit, companies that supply wireless systems and services will undoubtedly provide higher data rate systems, increase user capacity, and design more reliable connections. Companies will achieve these goals through a combination of technological advances and innovative research.

New services will certainly stress an already congested frequency spectrum by demanding more bandwidth per user and/or per application. One way to accommodate the burgeoning requirements on wireless systems is to use portions of the spectrum that are currently free. These frequencies are typically at much higher carrier frequency (i.e., multi giga-hertz and beyond) and the electromagnetic wave propagation at these frequencies tend to be non-ideal for general terrestrial use due to the large free space path loss, atmospheric absorption, obstruction by elements in the environment, and poor rain and foliage penetration [1]. For these reasons, researchers are motivated to find new methods to achieve higher spectral efficiency – which essentially means increasing the amount of information that can be transmitted over a fixed bandwidth. One such method is code division multiple access (CDMA) which allows multiple users to occupy the same
bandwidth by assigning a unique pseudo-random spreading code to each user [2]; this can be thought of as “stacking” each user’s spectral content within a fixed bandwidth.

CDMA is an example of a wideband spectral overlay system, where spectral overlay refers to the technique of “stacking” spectral content, or simultaneously sharing spectrum among multiple users.

A direct consequence of the spreading code used in CDMA is that each user signal occupies more bandwidth than required to transmit their underlying information sequence [2]. If a system or service has a fixed bandwidth that does not support the spectral spreading of each user’s information sequence, can a form of spectral overlay still be used to increase spectral efficiency? This thesis considers the feasibility and performance of just such an additive narrowband (ANB) spectral overlay system.

The ANB spectral overlay system differs from the wideband spectral overlay system in that it does not use a spreading sequence to differentiate users. Instead each user’s waveform is a conventional narrowband signal, with known data rate, selectable amplitude, and independent information sequence. These waveforms are then added together, either in a common transmitter, or via wireless transmission, and this creates a spectral overlay system. The receiver is then tasked with the demodulation of each user’s signals. To do this reliably, the receiver performs successive interference cancellation, which removes each user’s waveform from the received signal for further, individual processing. One condition that allows the receiver to perform successive interference cancellation is having the amplitude of each user’s waveform unique with respect to that of the other users. A similar method was proposed by Janssen and Slimane in [3]. However, unlike Janssen and Slimane, we propose varying not only the amplitude of
each user but the bandwidth as well, thus enabling discrimination via both amplitude and signal bandwidth. Note that our use of multiple bandwidths does not constitute spectral spreading—each signal is still a narrowband, unspread, digitally modulated single-carrier signal.

1.2 Scope

In this thesis we provide and examine the transmitter and receiver structures for both the downlink and uplink. Here, and in connection with terrestrial cellular mobile radio terminology, the downlink is defined as the communication link from the base station to the mobile users, and the uplink is defined as the communication link from the mobile user to the base station [4]. For the purpose of this thesis we consider each of the signals in the downlink to be bit synchronous, whereas in the uplink each signal is bit asynchronous. The uplink consists of several mobile users each transmitting an independent narrowband signal from their communication device to the base station. Each user can not physically be located in the same location and as such the path/paths over which the electromagnetic waves propagate will vary from one user to another. Because the individual mobile users’ signals are summed in the atmosphere (i.e., channel) and propagate along unique paths, without sophisticated timing control, the bit alignment between users will inherently be asynchronous. On the other hand, when we consider the downlink we are considering a single transmitter which is used to broadcast to several mobile users. The individual narrowband signals of the mobile users can be considered encapsulated by the RF (radio frequency) signal because the transmitter at the base station sums them prior to transmission. The RF signal will propagate along a
unique path to a given user, at which time the carrier signal will be removed, leaving the combined narrowband signals of the participating mobile users. Therefore, the matter of bit alignment in the downlink is determined by the base station due to the “encapsulation” of the narrowband signals within the RF signal. Thus, we are compelled to investigate the effect of bit synchronism in the downlink whereas bit asynchronism will be investigated in the uplink.

When analyzing complex problems it is paramount to simplify the complexity by breaking the problem down into smaller and more manageable constituents. The complexity associated with analyzing a multi-user ANB system operating with arbitrary mobile user data rates requires just such an approach. For the purpose of this thesis, which explores the initial step in the feasibility of this type of ANB system, we simplified the multi-user ANB system by restricting the number of mobile users to two while setting their data rates to $R_{b_1}$ and $2R_{b_1}$ (user one and user two, respectively). We also forced the system to operate over an additive white Gaussian noise (AWGN) channel using either synchronous (downlink, Chapter 2) or asynchronous (uplink, Chapter 3) bit alignment between users. The primary focus of this thesis is on the bit error rate (BER) performance of the ANB system. For this, analytical expressions are derived for the BER performance of each user participating in the restricted ANB system described above. BER approximations are also derived for a two user ANB system with arbitrary data rates to illustrate the effect that bandwidth differences between users has on BER performance. These BER derivations for the restricted two user ANB system are an invaluable step in deriving a more general BER expression for a multi-user ANB system that supports arbitrary data rates.
One convincing way to support analytical and approximated results is by computer simulation. Therefore, a computer based model of an ANB baseband transmitter and receiver pair was developed in Matlab® to support our findings. The software model allowed an ANB transmitter to be simulated with arbitrary parameters, including the number of mobile users, individual data rates, individual amplitudes, bit alignment among users, and the signal-to-noise ratios. The corresponding software model for the ANB receiver demodulates the signal using serial interference cancellation and then computes the bit error rate (BER) for each mobile user. The results obtained through simulation are provided alongside the analytical and approximated expressions for comparison purposes.

Finally, a hardware based baseband ANB transmitter was designed and developed specifically for this project to provide a physical model of an ANB system. The baseband ANB transmitter provided us with the ability to store known information sequences onboard, and output the corresponding ANB baseband waveform. Using the baseband ANB transmitter we were able to experimentally verify both the simulated and analytical results.

1.3 Thesis Contents

In Chapter 2 we provide an overview of the ANB downlink (bit synchronous) system and derive bit error rate equations for a two user ANB system with fixed data rates set at \(R_{b1}\) and \(2R_{b1}\). Approximations are then derived and provided for a two user system with arbitrary data rates. Simulation and analytical results are provided, along with a discussion of behavior as a function of parameters.
Chapter 3 provides an overview of the ANB uplink (bit asynchronous) system and derives bit error rates for a two user ANB system with fixed data rates set at \( R_{b1} \) and \( 2R_{b1} \). The relationship between the asynchronous and synchronous systems is discussed. Simulated bit error rate results are then compared to the derived theoretical expressions for corroboration.

A description of the design and development procedure; which includes component identification, printed circuit board layout, fabrication, printed circuit board population, and programming is provided in Chapter 4. Testing procedures, data collection, data processing, and a comparison of experimental results with simulated and analytical results are also provided in Chapter 4. A summary of the work, conclusions, and future work are all provided in Chapter 5.
Chapter 2: System Definition and Cancellation Analysis

2.1 Additive Narrow Band Downlink System Model

An additive narrow band (ANB) downlink system, as defined in this thesis, is a system in which the baseband waveforms of multiple users are summed together and then modulated onto a specific carrier frequency. Figure 2.1 depicts a generic multiple phase shift keying (MPSK) baseband ANB downlink system where \( b_i, s_i(t), \) and \( \alpha_i \) are the \( i^{th} \) user’s data stream, complex baseband waveform, and gain respectively. The complex baseband output of the generic MPSK ANB downlink system can be written as a sum of the complex baseband waveforms (2.1), where \( N \) is the number of users.

\[
s(t) = \sum_{i=1}^{N} s_i(t) \quad (2.1)
\]

When we specify the MPSK modulators in Figure 2.1 as binary phase shift keying (BPSK) modulators, then the \( i^{th} \) user’s baseband waveform is real. Assuming rectangular pulses are used the \( i^{th} \) user’s baseband waveform can be written as (2.2), where
$b_{i,k} \in \{ \pm 1 \}$ is the $i^{th}$ users $k^{th}$ bit and the rectangular pulse, $p_x(t)$, is defined by (2.3) [2]. In communication systems we are concerned with the amount of energy per bit because “…the received energy (is what) does the work” [2]. The energy dissipated during the $i^{th}$ users bit duration is defined as the energy per bit for that user and is given in (2.4).

$$s_i(t) = \alpha_i \sum_k b_{i,k} p_{x}(t-kT_{bi})$$

$$p_x(t) = \begin{cases} 1 & 0 \leq t < x \\ 0 & \text{elsewhere} \end{cases}$$

$$E_{bi} = \int_{-T_{bi}/2}^{T_{bi}/2} s^2(t) \, dt$$

To aid in demodulation a restriction will be placed on the gain constants of each user. The restriction allows successive interference cancellation (SIC) to be performed on the received signal without loss of information (in the absence of noise). Equation (2.5) specifies the restriction imposed on the amplitudes, which, simply stated, ensures that the $i^{th}$ amplitude is greater than the maximum value of the potential destructive interference of the remaining $i-1$ amplitudes.

$$\alpha_i > \sum_{k=1}^{i-1} \alpha_k$$

The remaining constraint, given by (2.6), is placed on the data rate of each user.

$$R_{b1} \leq R_{b2} \leq \cdots \leq R_{bN}$$

2.2 Additive Narrow Band Downlink System Demodulation

As mentioned, successive interference cancellation (SIC) can be used to demodulate the received signal without loss of information (in the absence of noise).
Figure 2.2 depicts a SIC demodulator for the BPSK ANB downlink system. The SIC demodulator works by detecting the strongest signal, then subtracting the estimate of the strongest signal from the received signal, and repeating until all the signals have been detected. The input to the $i^{th}$ detector can be written as (2.7) where detector $N$ is the $N^{th}$ detector, $N$ is the number of users, and $\hat{s}_k(t)$ is the $k^{th}$ user’s estimated signal. User $N$’s signal is the first one detected, as its amplitude is the largest.

$$r(t) = s(t) + n_0(t)$$

To illustrate the basic operation of the SIC BPSK ANB downlink demodulator, an example will be given for a three user system where $\alpha_1 = 0.5, \alpha_2 = 1, \alpha_3 = 2$ and $R_{b1} = R_{b2} = R_{b3}$. If we say that user one, two, and three transmit bits equal to $+1$, $-1$, and $+1$, respectively, then the received signal pulse value, with no noise, is given by (2.2) and has an amplitude $r = \alpha_1 - \alpha_2 + \alpha_3 = 1.5$ over the bit duration (i.e. $r(t) = 1.5 p_r(t-kT)$).
We assume a unity gain, distortionless channel for illustration, and for the rectangular pulse case, abbreviate the waveforms over the bit duration of interest by their amplitudes. Using the SIC demodulator we first detect user three’s signal. Because a conventional BPSK demodulator is being used, the decision threshold is equal to zero (i.e., anything greater than zero is detected as a +1 and anything less than zero is detected as a -1) [2]. Thus, user three’s transmitted data bit is estimated to be a +1. Using (2.7) and assuming perfect amplitude estimates for each user we obtain the input to user two’s detector, which is \( r_2 = r - \alpha_3 = -0.5 \). User two’s data bit is then estimated to be a -1 and the input signal to user one’s detector is \( r_1 = r + \alpha_2 - \alpha_1 = 0.5 \). Finally user one’s signal is estimated to be a +1 and the demodulation of the BPSK ANB is complete.

2.3 Synchronous Two User Downlink Analysis

The analysis of the two user synchronous downlink will consist of deriving probability of bit error (BER) expressions for an additive white Gaussian noise (AWGN) channel for each user using the SIC demodulator. We will first consider a two user synchronous downlink system in which the data rate ratio, given by (2.8), is set to two. From this analysis we will then generalize and formulate approximate BER expressions for any integer valued data rate ratio, \( M \).

\[
M = \frac{R_{b2}}{R_{b1}} \quad (2.8)
\]

Setting the data rate ratio (2.8) to two indicates that user two will transmit two bits for every one bit that user one transmits. In other words \( R_{b1} = 1/T_b \) and \( R_{b2} = 2R_{b1} \), where \( T_b \) is the bit duration for user one. Because the system is synchronous, the bit
edges of each user’s signal align with one another as depicted in Figure 2.3. Figure 2.3 also defines the bit notation that will be used for each user: $b_1$ is the $k^{th}$ bit of user one, $b_{2,i}$ is the $(2k-1)^{th}$ bit of user two, and $b_{2,2}$ is the $(2k)^{th}$ bit of user two. Our analysis can also easily be generalized to apply to a slow, flat fading channel, in which each bit is affected by a constant multiplicative factor [7]. Thus, it is sufficient to consider an arbitrary bit in the analysis and we can drop the user-one bit index (i.e., $k$). Note that user two’s bits, $b_{2,i}$ and $b_{2,2}$, are superimposed onto user one’s bit, $b_1$.

User two’s signal has the largest amplitude and as such will be the first signal detected by the SIC demodulator and may be referred to as the strong user. Because we are only considering an AWGN channel it is sufficient to derive the probability of a bit error for bit $b_{2,1}$, as it will be the same for all of user two’s bits. Because the waveforms of user one and user two are summed together, the amount of energy contained within user two’s bit interval is not constant. The possible transmitted signals over the interval $kT_b < t < (k + 1/2)T_b$ are $s(t) = (\pm \alpha_1 \pm \alpha_2) p_{\phi/2} (t - kT_b/2)$.
and \( s(t) = (\pm \alpha_1 \mp \alpha_2) P_{b_{1,2}} (t - kT_b / 2) \). These signals have equal probability of occurrence.

The derivation for the probability of a bit error for a single user BPSK signal using a matched filter demodulator is straightforward and the result is given by (2.9) as a function of the bit energy, \( E_b \), and the two-sided power spectral density of the noise at the filter input, \( N_0/2 \) [2]. The \( Q \)-function, denoted as \( Q(x) \), is given in (2.10) and is defined as the integral of a normalized Gaussian pdf over the range \( x \) to \( \infty \) [5].

\[
P_b = Q\left( \sqrt{\frac{2E_b}{N_0}} \right) \tag{2.9}
\]

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left( -\frac{\xi^2}{2} \right) d\xi \tag{2.10}
\]

To determine the stronger signal’s (user two’s) probability of a bit error we compute the energy of a bit for each possible transmitted signal and substitute the result for \( E_b \) in (2.9). The final expression for the probability of a bit error for the stronger signal will have the form given by (2.11).

\[
P_{b_2} = \frac{1}{2} P\{ e \mid b_1 = b_{2,1} \} + \frac{1}{2} P\{ e \mid b_1 = -b_{2,1} \} \tag{2.11}
\]

The energy per bit for each user can be computed using (2.4) and these expressions are given in (2.12) and (2.13) for the weak and strong users respectively. The amplitude ratio constant, \( \zeta \), is defined by (2.14) and will be used to simplify the BER expressions.

\[
E_{b_1} = \alpha_1^2 T_b \tag{2.12}
\]
\[ E_{b2} = \alpha_2^2 \frac{T_b}{2} \]  

\[ \zeta = \frac{\alpha_2}{\alpha_1} \]  

For the case when the transmitted signal is \( s(t) = (\pm \alpha_1 \pm \alpha_2) p_{T_b/2} (t - kT_b/2) \) over the interval \( kT_b \leq t < (k + 1/2)T_b \), the received bit energy is computed using (2.4) and is given in (2.15). Likewise for the case when the transmitted signal is \( s(t) = (\pm \alpha_1 \mp \alpha_2) p_{T_b/2} (t - kT_b/2) \) over the interval \( kT_b \leq t < (k + 1/2)T_b \), the received bit energy is given in (2.16).

\[ \alpha_2^2 \left(1 + \frac{1}{\zeta}\right)^2 \frac{T_b}{2} = E_{b2} \left(1 + \frac{1}{\zeta}\right)^2 \]  

\[ \alpha_2^2 \left(1 - \frac{1}{\zeta}\right)^2 \frac{T_b}{2} = E_{b2} \left(1 - \frac{1}{\zeta}\right)^2 \]  

Substituting the above results for \( E_b \) in (2.9) yields the strong user’s probability of bit error expression for each of the possible transmitted signals, and these are given by equations (2.17) and (2.18). The final expression for the probability of a bit error for the stronger signal is obtained by substituting the results of (2.17) and (2.18) into (2.11), which is given in (2.19).

\[ P\{E \mid b_1 = b_{2,1}\} = Q\left(\sqrt{\frac{2E_{b2} (1+1/\zeta)^2}{N_0}}\right) \]  

\[ P\{E \mid b_1 = -b_{2,1}\} = Q\left(\sqrt{\frac{2E_{b2} (1-1/\zeta)^2}{N_0}}\right) \]
Equation (2.19) helps to illustrate two important facts about the SIC demodulator depicted in Figure 2.2, the first being a reinforcement of the restriction placed on the amplitudes by (2.5). As the amplitude ratio constant, $\zeta$, approaches one (i.e. each user’s signal has the same amplitude) the probability of an error in the detection of user two’s data stream has an error probability floor of 0.25 due to the second term in (2.19). This intuitively makes sense because when the amplitude of each user’s signal is equal then whenever they transmit opposing bits (i.e. $b_i = -b_{2,i}$) they completely cancel one another and the bit decision is based purely on noise. Because there is a fifty percent chance of an error and fifty percent chance of occurrence for this particular bit combination then user two will have an error floor of 0.25. The second fact we can observe from (2.19) is that as the amplitude ratio constant approaches infinity, the error performance approaches that of a single user BPSK signal, which is expected since user one’s signal amplitude becomes negligibly small.

To compute the probability of a bit error for user one, a multi-step approach must be taken due to the SIC. When analyzing a single bit of the weaker user there are two bits from the stronger user that may or may not have been estimated and subtracted out correctly from the received signal. Therefore, there are several distinct cases that need to be analyzed in the final probability of bit error expression for the weaker user (user one). Each possible case is listed in Table 2.1, where $\hat{b}_{2,i}$ and $\hat{b}_{2,2}$ are the estimates of $b_{2,i}$ and $b_{2,2}$ respectively.
$b_{2,2}$, respectively. The derivation for the expressions under the column heading “Probability of Event” in Table 2.1 will be discussed later.

The weak signal error probability for one case will be derived to illustrate the necessary steps in the analysis for each individual case. The case that will be analyzed is when both users transmit a plus one pulse value while no errors are made in the detection of the strong user’s bits. Therefore, $b_1 = +1$, $b_{2,1} = +1$, $b_{2,2} = +1$, $\hat{b}_{2,1} = +1$, and $\hat{b}_{2,2} = +1$.

Using (2.2) the waveforms for the weak and strong users’ are computed and given in (2.20) and (2.21) respectively, where we are looking at the $k^{th}$ bit of the weak user. The received baseband signal is given in (2.22) where $n_0(t)$ is the AWGN random process.

<table>
<thead>
<tr>
<th>Case</th>
<th>Variable</th>
<th>$b_1$</th>
<th>$b_{2,1}$</th>
<th>$b_{2,2}$</th>
<th>$\hat{b}_{2,1}$</th>
<th>$\hat{b}_{2,2}$</th>
<th>Probability of Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$q_1$</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>$\frac{1}{4} \left( 1 - P{e</td>
</tr>
<tr>
<td>2</td>
<td>$q_2$</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>$\frac{1}{2} \left( 1 - P{e</td>
</tr>
<tr>
<td>3</td>
<td>$q_3$</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>$\frac{1}{4} \left( P{e</td>
</tr>
<tr>
<td>4</td>
<td>$q_4$</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>$\frac{1}{2} \left( 1 - P{e</td>
</tr>
<tr>
<td>5</td>
<td>$q_5$</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>$\frac{1}{2} \left( 1 - P{e</td>
</tr>
<tr>
<td>6</td>
<td>$q_6$</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>$\frac{1}{2} P{e</td>
</tr>
<tr>
<td>7</td>
<td>$q_7$</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>$\frac{1}{2} \left( 1 - P{e</td>
</tr>
<tr>
<td>8</td>
<td>$q_8$</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>$\frac{1}{4} \left( P{e</td>
</tr>
<tr>
<td>9</td>
<td>$q_9$</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>$\frac{1}{2} \left( 1 - P{e</td>
</tr>
<tr>
<td>10</td>
<td>$q_{10}$</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>$\frac{1}{4} \left( 1 - P{e</td>
</tr>
</tbody>
</table>
\begin{align*}
    s_1(t) &= \alpha_1 p_{T_b} (t - kT_b), \quad kT_b \leq t < (k + 1)T_b \\
    s_2(t) &= \alpha_2 p_{T_b} (t - kT_b), \quad kT_b \leq t < (k + 1)T_b \\
    r(t) &= (\alpha_1 + \alpha_2) p_{T_b} (t - kT_b) + n_0(t), \quad kT_b \leq t < (k + 1)T_b 
\end{align*}

The received signal is first processed by the strong user’s detector where decisions will be made on bits \(b_{2,1}\) and \(b_{2,2}\). A typical BPSK receiver uses a matched filter or a correlator to obtain the optimal error performance in AWGN [2]. For rectangular pulses the BPSK correlator receiver is equivalent to the structure shown in Figure 2.4, where \(T_b\) is the bit duration and \(z((k+1)T_b)\) is the decision statistic [2].

The received signal contains two terms, namely the transmitted signal, \(s(t)\), and the noise, \(n_0(t)\). We will assume that the noise is white Gaussian, zero-mean, and has a two-sided power spectral density of \(N_0/2\) W/Hz. Integrating \(n_0(t)\) over a bit interval will yield a Gaussian random variable that can be completely described by its mean and variance [2]. Let

\[
    n_{0,1} = \int_{kT_b}^{(k+1)T_b} n(t) \, dt \quad \text{and} \quad n_{0,2} = \int_{(k+1)T_b}^{(k+2)T_b} n(t) \, dt
\]

be the Gaussian random variables associated with bits \(b_{2,1}\) and \(b_{2,2}\) respectively. Since the mean of the random noise processes is \(E\{n_0(t)\} = 0\), where \(E\{\cdot\}\) denotes the expected value operation, this implies (2.23) and (2.24) [2].

\[
    \bar{n}_{0,1} = E\{n_{0,1}\} = 0 \quad \text{(2.23)}
\]
\( \overline{n}_{0,2} = E \{ n_{0,2} \} = 0 \)  \hspace{1cm} (2.24)

The variance of each of the Gaussian random variables can be computed using (2.25), where \( \overline{n}_{0,i}^2 \) is the second moment of the \( i^{th} \) random noise variable [5]. Because the mean of each random variable is equal to zero the variance is equal to the second moment.

\[ \sigma_{0,i}^2 = \overline{n}_{0,i}^2 - \overline{n}_{0,i} \]  \hspace{1cm} (2.25)

Like the means, the second moments of each of the random variables are equal. The derivation of the second moment for \( n_{0,t} \) is given below.

The second moment for \( n_{0,t} \) is

\[ \overline{n}_{0,1}^2 = E \left\{ \int_{\tau}^{\frac{1}{2}} n(t) dt \int_{\tau}^{\frac{1}{2}} n(s) ds \right\} \]

\[ = \int_{\tau}^{\frac{1}{2}} \int_{\tau}^{\frac{1}{2}} E \{ n(t)n(s) \} dt ds \]

Expression (2.26) can be simplified by employing the autocorrelation function of the processes \( n_0(t) \), defined as \( R_{n_0,n_0}(t,s) = E \{ n_0(t)n_0(s) \} \) [2]. For thermal noise, we assume that \( n_0(t) \) is a stationary random processes, so that the autocorrelation function is dependent on only the time difference \( \tau = t - s \) [5]. Therefore, the autocorrelation for the process \( n_0(t) \) can be rewritten as \( R_{n_0,n_0}(\tau) = E \{ n_0(t)n_0(\tau) \} \) [2]. Since the Gaussian random processes are white, we have \( R_{n_0,n_0}(\tau) = \left( \frac{N_0}{2} \right) \delta(\tau) \), where \( \delta(\tau) \) is the unit impulse function [2]. Substituting these results in (2.26) yields (2.27) [2].

\[ \overline{n}_{0,1}^2 = \int_{\tau}^{\frac{1}{2}} \int_{\tau}^{\frac{1}{2}} \frac{N_0}{2} \delta(t-s) dt ds \]

\[ = \frac{N_0 T_b}{2} \]  \hspace{1cm} (2.27)
The variance for each of the random variables can be found by substituting (2.27) into (2.25). For completeness, equations (2.28) and (2.29) explicitly define the variance of the random noise variables.

\[
\sigma^2_{0,1} = \frac{N_0}{2} \frac{T_b}{2} \tag{2.28}
\]

\[
\sigma^2_{0,2} = \frac{N_0}{2} \frac{T_b}{2} \tag{2.29}
\]

Bit decisions for \( b_{2,1} \) and \( b_{2,2} \) will be made for the strong user based on the decision statistics given in (2.30) and (2.31) respectively, where \( s(t) \) was computed by substituting (2.20) and (2.21) into (2.1).

\[
z_{2,1} \left( \left( k + \frac{1}{2} \right) T_b \right) = \int_{kT_b}^{(k+1)T_b} s(t) \, dt + n_{0,1} = (\alpha_1 + \alpha_2) \frac{T_b}{2} + n_{0,1} \tag{2.30}
\]

\[
z_{2,2} \left( (k+1)T_b \right) = \int_{(k+1)T_b}^{(k+2)T_b} s(t) \, dt + n_{0,2} = (\alpha_1 + \alpha_2) \frac{T_b}{2} + n_{0,2} \tag{2.31}
\]

Knowing there were no errors in the detection of the strong user’s bits imposes a lower bound on both random noise variables, namely, \( n_{0,1} > -\left( \alpha_1 + \alpha_2 \right) T_b / 2 \) and \( n_{0,2} > -\left( \alpha_1 + \alpha_2 \right) T_b / 2 \). The shaded region in Figure 2.5 illustrates the possible range of values that the decision statistic can take on while producing no errors in the detection of the strong user’s bits (i.e. \( b_{2,1} \) and \( b_{2,2} \)) for this particular case.
Each of the constrained Gaussian random variables has a pdf given by (2.32) where the index \( i \in \{1, 2\} \), \( \sigma_{0,i}^2 \) is the variance given by (2.28) and (2.29), \( \beta_i \) is given by (2.33), and \( Q(x) \) is defined in (2.10). Equation (2.33) was expressed as a function of bit energy by using the definition of user one’s energy per bit (2.12), the amplitude ratio constant given in (2.14), and the noise variance (2.28).

\[
f_{n_{0,i}}(x) = \begin{cases} 
\frac{\beta_i}{\sqrt{2\pi\sigma_{0,i}^2}} \exp\left(-\frac{x^2}{2\sigma_{0,i}^2}\right) & x > -\left(\alpha_i + \alpha_z\right)\frac{T_b}{2} \\
0 & \text{elsewhere}
\end{cases}
\]  

(2.32)

\[
\beta_i = \frac{1}{\sqrt{1 - Q\left(\frac{\left(\alpha_i + \alpha_z\right)\frac{T_b}{2}}{\sigma_{0,i}}\right)}}
\]

(2.33)
The input to the weaker user’s detector is found via (2.7). Assuming perfect amplitude estimation the strong user’s signal is completely removed, leaving only the weaker signal plus noise. The decision statistic for the weaker user’s bit, \( b_1 \), is given in (2.34). Note that in (2.34) the constrained Gaussian random variables are summed together producing a new random variable. If we let \( Y = n_{0,1} + n_{0,2} \) then an error will occur in the detection of the weak user’s bit (i.e. \( b_1 \)) when \( Y < -\alpha_i T_b \).

\[
\begin{aligned}
z_1((k+1)T_b) &= \int_{kT_b}^{(k+1)T_b} s_i(t) dt + \sum_{j=1}^{2} n_{0,j} = \alpha_i T_b + n_{0,1} + n_{0,2} \\
(2.34)
\end{aligned}
\]

Calculating the probability of the event \( Y < -\alpha_i T_b \) requires that the pdf of the new random variable \( Y \) be derived. The sum of two statistically independent random variables has a pdf given by a convolution integral of their individual pdfs [5]. Therefore, the pdf of \( Y \) can be computed via (2.35).

\[
\begin{aligned}
f_Y(y) &= \int_{-\infty}^{\infty} f_{n,1}(x) f_{n,2}(y-x) dx \\
(2.35)
\end{aligned}
\]

The bounds imposed on the individual pdfs of the noise variables creates a bound on the pdf of \( Y \), which is shown in (2.36).

\[
\begin{aligned}
f_Y(y) &= \begin{cases} 
\int_{-(\alpha_i+\alpha_2)T_b/2}^{\alpha_i+\alpha_2 T_b/2} f_{n,1}(x) f_{n,2}(y-x) dx & y > -2(\alpha_i + \alpha_2)T_b/2 \\
0 & \text{elsewhere}
\end{cases} \\
(2.36)
\end{aligned}
\]

If we let \( \lambda = -(\alpha_i + \alpha_2)T_b/2 \), \( \sigma_n^2 = \sigma_{0,1}^2 = \sigma_{0,2}^2 \) and substitute the individual pdfs given in (2.32) into (2.36) then the expression for the joint pdf of \( Y \) for \( y > 2\lambda \) is given in (2.37).
$f_y(y) = \frac{\beta_1 \beta_2}{2 \pi (2 \sigma_n^2)} \exp \left( -\frac{y^2}{2 \sigma_n^2} \right) \exp \left( -\frac{(y-x)^2}{2 \sigma_n^2} \right) \int_{-\infty}^{y} \frac{1}{2 \pi (\sigma_n^2/2)} \exp \left( -\frac{(x-y/2)^2}{2 (\sigma_n^2/2)} \right) dx$  \hspace{1cm} (2.37)

Equation (2.37) can be written as (2.38), which contains a single exponential function in the integral that resembles a Gaussian pdf with a mean equal to $y/2$ and a variance equal to $\sigma_n^2/2$. This allows the integral in (2.38) to be replaced with a difference of $Q$-functions, which is given in (2.39).

$$f_y(y) = \frac{\beta_1 \beta_2}{2 \pi (2 \sigma_n^2)} \exp \left( -\frac{y^2}{2 \sigma_n^2} \right) \left[ Q \left( \frac{y-2\lambda}{\sqrt{2\sigma_n^2}} \right) - Q \left( \frac{y+2\lambda}{\sqrt{2\sigma_n^2}} \right) \right]$$

$$f_y(y) = \frac{\beta_1 \beta_2}{2 \pi (2 \sigma_n^2)} \exp \left( -\frac{y^2}{2 \sigma_n^2} \right) \left[ 1 - 2Q \left( \frac{y-2\lambda}{\sqrt{2\sigma_n^2}} \right) \right]$$  \hspace{1cm} (2.39)

Substituting (2.39) into (2.36) provides the pdf of $Y$. The probability of an error in the detection of the weak signal’s bit can now be calculated for this case. An error will occur when the random variable $Y < -\alpha_i T_b$. Therefore, the probability of this event is calculated by $P \{ Y < -\alpha_i T_b \} = \int_{-\infty}^{-\alpha_i T_b} f_y(y) dy$ [5].

$$P \{ Y < -\alpha_i T_b \} = \int_{-2(\alpha_1 + \alpha_2) T_b}^{-\alpha_i T_b} \frac{\beta_1 \beta_2}{2 \pi (2 \sigma_n^2)} \exp \left( -\frac{y^2}{2 (2 \sigma_n^2)} \right) \left[ 1 - 2Q \left( \frac{y+(\alpha_1 + \alpha_2) T_b}{\sqrt{2\sigma_n^2}} \right) \right] dy$$  \hspace{1cm} (2.40)
Equation (2.40) can be written as a function of the bit energies of each user. If we let $\xi = (E_1 + 2E_2 + 2\sqrt{2E_1E_2})$ and use (2.28), then equation (2.40) becomes (2.41).

$$P_Y < -\sqrt{E_1T_b} = \ldots$$

$$\int_{-\sqrt{E_1T_b}}^{\sqrt{E_1T_b}} \frac{\beta_1\beta_2}{\sqrt{2\pi(2\sigma_n^2)}} \exp \left( -\frac{y^2}{2(2\sigma_n^2)} \right) \left( 1 - 2Q \left( \frac{\sqrt{2(y^2 + 2y(\sqrt{E_1T_b + \sqrt{2E_2T_b}} + \xi)}}{N_0 T_b} \right) \right) dy$$

(2.41)

The probability of a bit error in (2.41) only represents the probability of an error given the predefined event that $b_1 = +1$, $b_{2,1} = +1$, $b_{2,2} = +1$, $\hat{b}_{2,1} = +1$, and $\hat{b}_{2,2} = +1$. To complete the expression for the probability of a bit error, the probability of this event must also be found. First we note that due to symmetry we only need to consider the cases when $b_1 = +1$ or $b_1 = -1$ as each will yield identical results. Now we can analyze bit $b_{2,1}$, which was detected correctly while $b_1 = +1$. The probability of $b_{2,1}$ being a $+1$ is $1/2$ and the probability of no error in the detection of this bit is $1 - P \{ \varepsilon | b_1 = b_{2,1} \}$; where $P \{ \varepsilon | b_1 = b_{2,1} \}$ was derived in equation (2.17) and is the probability of an error given that $b_1 = b_{2,1}$. Analyzing bit $b_{2,2}$ yields the same result as $b_{2,1}$ because $b_{2,2} = b_{2,1}$ and no error was made in the detection of this bit either. Therefore, the probability of this event is also $1/2 \left( 1 - P \{ \varepsilon | b_1 = b_{2,1} \} \right)$. These two events are independent of one another because $b_{2,1}$ and $b_{2,2}$ are independent. Thus, the probability of the event $b_{2,1} = +1$, $b_{2,2} = +1$, $\hat{b}_{2,1} = +1$, and $\hat{b}_{2,2} = +1$ given that $b_1 = +1$ can be written as a product of the underlying events, which is given in (2.42) [5]. To find the probability of the event for the remaining
nine cases an identical method is used, however, their derivation has been omitted for
brevity and the final results are provided in Table 2.1.

\[ P\{b_{2,1} = 1, b_{2,2} = 1, \hat{b}_{2,1} = 1, \hat{b}_{2,2} = 1 | b_1 = 1\} = \frac{1}{4} \left(1 - P\{e | b_1 = b_{2,1}\}\right)^2 \quad (2.42) \]

Table 2.2 provides the necessary pdf scaling factors used in determining the
probability of bit error expression for user one, where \(Q_1\) and \(Q_2\) are given in (2.43) and
(2.44) respectively. Table 2.3 contains the probability of an error for each of the ten
distinct cases; where \(\beta_1\) and \(\beta_2\) are given in Table 2.2 for each unique case and

\[ g(y, x) = \frac{\beta_1 \beta_2}{\sqrt{2\pi x}} \exp\left(-\frac{y^2}{2x}\right). \]

Each of the expressions under the heading “Probability of
Error” in Table 2.3 was derived in the same fashion as the case given above.

<table>
<thead>
<tr>
<th>Case</th>
<th>Variable</th>
<th>Probability of Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(P_1)</td>
<td>(\int_{-(\alpha_1 + \alpha_2)T_b}^{\alpha_1 T_b} g\left(y, 2\sigma_n^2\right) \left[1 - 2Q\left(\frac{y + (\alpha_1 + \alpha_2)T_b}{\sqrt{2\sigma_n^2}}\right)\right] dy)</td>
</tr>
<tr>
<td>2</td>
<td>(P_2)</td>
<td>(\int_{\infty}^{-(\alpha_1 + \alpha_2)T_b} g\left(y, 2\sigma_n^2\right) \left[1 - Q\left(\frac{y + (\alpha_1 + \alpha_2)T_b}{\sqrt{2\sigma_n^2}}\right)\right] dy)</td>
</tr>
<tr>
<td>3</td>
<td>(P_3)</td>
<td>(\int_{\infty}^{-(\alpha_1 + 2\alpha_2)T_b} g\left(y, 2\sigma_n^2\right) \left[2Q\left(\frac{y + (\alpha_1 + \alpha_2)T_b}{\sqrt{2\sigma_n^2}}\right) - 1\right] dy)</td>
</tr>
</tbody>
</table>
The final expression for the average probability of a bit error for the weak user (user one) is given in (2.45), where \( q_i \) is given in Table 2.1 and \( P_i \) is given in Table 2.3.

\[
P_{b1} = \sum_{i=1}^{10} q_i P_i \tag{2.45}
\]
2.4 Synchronous Two User Downlink Analytical Approximations

It was shown in the previous section how to derive the analytical BER expressions for each user in a synchronous downlink environment for the case when the data rate ratio of the strong to weak user was set to two. In this section we will derive BER expressions for a two user synchronous system with an arbitrary integer data rate ratio of $M$.

The BER expression for the strong user needs only a minor modification to include data rate ratios greater than two. The data rate ratio was defined in (2.8) as a ratio of user two’s to user one’s data rate. If we allow the data rate ratio to be a positive integer then we can compute the BER expression in an identical manner as before. First we re-evaluate the bit energies for each user by using (2.4) and the data rate ratio. The weak user’s bit energy is not affected by the increase in data rate and is given by (2.12); however, the strong user’s bit energy is inversely related to the data rate ratio as seen in (2.46).

$$E_{b2} = \alpha^2 \frac{T_b}{M}$$  \hspace{1cm} (2.46)

In (2.15) and (2.16) we found the respective bit energies, as seen at the correlator input, for the cases when the strong and weak user’s signal add constructively and destructively. Now we are interested in the bit interval $kT_b \leq t < (k + 1/M)T_b$. Therefore, (2.15) and (2.16) become (2.47) and (2.48) respectively. The respective bit energies of (2.47) and (2.48) are the same as (2.17) and (2.18). Therefore, when the data rate ratio is $M$ the final BER expression for the strong user will be equivalent to (2.19). This result makes sense because the strong user’s decision statistic does not change as a function $M$. 

\[ \alpha_2^2 \left( 1 + \frac{1}{\zeta} \right)^2 \frac{T_b}{M} = E_{b2} \left( 1 + \frac{1}{\zeta} \right)^2 \] (2.47)

\[ \alpha_2^2 \left( 1 - \frac{1}{\zeta} \right)^2 \frac{T_b}{M} = E_{b2} \left( 1 - \frac{1}{\zeta} \right)^2 \] (2.48)

Unfortunately a similar modification can not be made to the weak user’s BER expression, derived above, to include the data rate ratio. This is due to the fact that increasing the data rate ratio increases the number of unique cases that need to be analyzed. For the case when the data rate ratio was two there were ten unique cases as can be seen in Table 2.1. A Matlab® script was written to aid in determining the number of unique cases for any given date rate ratio. Equation (2.49) was found by inspection and was verified for values of \( M \) up to fifteen and is believed to hold true for values of \( M \) greater than fifteen.

\[ \sum_{k=0}^{M} (M + 1 - k)(k + 1) \] (2.49)

Table 2.4 shows the results of computing the number of unique cases given a fixed data rate ratio. It can be seen from this table that for the case when the data rate ratio is two there are a total of ten unique cases. The values next to the “Number of Unique Cases” in Table 2.4 show the number of unique cases per the number of errors made in the detection of the strong user’s signal. Because the strong user transmits \( M \) bits for every bit the weak user sends, there can be up to \( M \) errors made per weak user’s bit interval. To illustrate, when \( M = 2 \) there are three unique cases when there are no errors made in the detection of the strong user these cases are listed in Table 2.1 as 1, 5, and 10. Cases 3, 6, and 8 in Table 2.1 are the three unique cases when two errors are made in the
estimation of the strong user; the remaining cases in Table 2.1 are the four unique cases when there is a single error made in user two’s bit estimation.

<table>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<td>16</td>
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<td>35</td>
<td>32</td>
<td>27</td>
<td>20</td>
<td>11</td>
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</tbody>
</table>

Table 2.4 Data Rate Ratio Based Unique Cases

Equation (2.49) helps to illustrate how infeasible it is to produce exact BER expressions for the weak user as the data rate ratio $M$ gets large. For example, when $M=4$ there are 35 unique cases, which means the BER expression for the weak user would contain 35 weighted integrals each of which contain one or more $Q$-functions. Even with numerical approximation and a fast computer, evaluating this becomes a daunting task. Note that it is not impossible to compute the exact BER expression for any given data rate ratio; it requires only time. Finding the BER for an arbitrary value of $M$ can be accomplished using the same method shown above for the case when the data rate ratio was fixed at two.

One way to find an approximation for the weak user’s BER expression is to alter the block diagram of the receiver. Figure 2.6 is a modified, albeit fictional, realization of
the SIC ANB demodulator for a two user system. The only difference between Figure 2.6 and 2.2 is that the noise process on each branch is uncorrelated with that on the other.

![Diagram of SIC Demodulation](image)

**Figure 2.6 Approximate SIC Demodulation of BPSK ANB Downlink**

The AWGN in Figure 2.6 is denoted as $n_{1,0}(t)$ and $n_{2,0}(t)$ on branches one and two, respectively. Thus, the signal entering the weak user’s detector (i.e., Detector 1) can be written as (2.50) assuming ideal timing and estimation of the strong user’s amplitude.

The advantage of this model is that the noise, $n_{1,0}(t)$, is uncorrelated with the interfering noise generated by incorrect decisions made in the estimation of the strong user’s data.

The key now is to derive an expression that represents, statistically, the interference caused by the subtraction of the strong user’s signal with its corresponding estimate.

\[
r_1(t) = \alpha_1 \sum_k b_{1,k} p_{T_b} \left( t - kT_b \right) + \alpha_2 \sum_j p_{T_b/n} \left( t - j \frac{T_b}{N} \right) \left( b_{2,j} - \hat{b}_{2,j} \right) + n_{1,0}(t) \quad (2.50)
\]

The decision statistic produced at the correlator output for the weak user is given in (2.51) where the second term represents the sum of the interference caused by the strong user’s signal and $n_0$ represents the noise value at the output of the correlator.
\[ z((k+1)T_b) = \int_{kT_b}^{(k+1)T_b} r_1(t) \, dt = \left( \alpha_1 b_{1,k} + \alpha_2 \sum_{j=1}^{M} (b_{2,j} - \hat{b}_{2,j}) \right) T_b + n_0 \] (2.51)

Any one of the strong user’s \( M \) bits subtracted from its corresponding estimate can produce three values. This is because each bit is defined as \( b_i \in \{\pm 1\} \) which means that the subtraction of a bit from its estimate is defined as \( (b_i - \hat{b}_i) \in \{0, \pm 2\} \). Let 

\[ I_{i,j} = (b_{2,j} - \hat{b}_{2,j}) \]

be a random variable that represents a single interfering term caused by the \( j^{th} \) bit of the strong user’s signal. The corresponding probability mass function will thus be discrete, with non-zero probabilities at the allowed values (i.e., \( \{0, \pm 2\} \)). If it is assumed that the weak user has transmitted a positive pulse value then the discrete pdf for the random variable \( I_1 \) will equal (2.52) and will resemble Figure 2.7, where 

\[ P\{\varepsilon \mid b_i = b_{2,i}\} \text{ and } P\{\varepsilon \mid b_i = -b_{2,i}\} \]

are given in (2.17) and (2.18) respectively. Note that if the weak user transmits a negative pulse value then the discrete pdf will be the mirror image of Figure 2.7 about the origin. The pdf of the aggregate interference caused by the subtraction of the strong user’s \( M \) bits from their estimates is found by computing the convolution of the \( M \) independent, but identical, pdfs given in Figure 2.7. This is because it is assumed that each of the strong user’s \( M \) bits are independent of one another, hence the pdf of the sum of the \( M \) interfering terms is equal to the convolution of their individual pdfs [5].
Let $I_M = \sum_{k=1}^{N} I_{1,k}$ be a random variable that represents the sum of the $M$ interfering terms, where $I_{1,k}$ is the $k^{th}$ of $N$ random interfering terms defined above. As stated above, the pdf of $I_M$ is equal to the convolution of the individual pdfs of the random variable $I_{1,k}$. From basic signals and systems we know that the discrete convolution of two functions is equal to the multiplication of their individual Z-transforms, which is shown in (2.53) [6]. Therefore, the Z-transform of the pdf for the random variable $I_M$ is given in (2.54); where $A$, $B$, and $C$ are the amplitudes specified in (2.52) at indices -2, 0, and +2 respectively and $M$ is the data rate ratio. Taking the inverse Z-transform of (2.54) will yield the pdf for the aggregate interference caused by the subtraction of the strong user’s $M$ bits from their corresponding estimates. Figure 2.8 shows an example plot of the pdf for the aggregate interference, for $M = 32$, $\alpha_1 = 0.25$, and $\alpha_2 = 0.5$. 

$$f_{I_1}[k] = \begin{cases} \frac{1}{2} P\{\varepsilon | b_1 = -b_2\}, & k = -2 \\ 1 - \frac{1}{2} \left( P\{\varepsilon | b_1 = -b_2\} + P\{\varepsilon | b_1 = b_2\} \right), & k = 0 \\ \frac{1}{2} P\{\varepsilon | b_1 = b_2\}, & k = +2 \\ 0, & \text{elsewhere} \end{cases}$$

(2.52)
\[ \sum_{k=-\infty}^{\infty} f_1[k] f_2[n-k] \rightarrow F_1(z)F_2(z) \]  

(2.53)

\[ F_{IM}(z) = \left(Az^{-2} + B + Cz^{-2}\right)^M \]  

(2.54)

The weak user’s correlator output given in (2.51) can now be written as (2.55), which includes the random variable \( I_M \). A Gaussian approximation for the pdf of \( I_M \) can be found by computing the mean and variance of \( f_{IM} [k] \). Once the interference is characterized by a Gaussian approximation the decision statistic for the weak user is equal to the signal plus two statistically independent Gaussian random variables.

\[ z((k+1)T_b) = \int_{kT_b}^{(k+1)T_b} r_i(t) dt = (\alpha_1 b_{1,k} + \alpha_2 I_M) T_b + n_0 \]  

(2.55)

The sum of two statistically independent Gaussian random variables is itself a Gaussian random variable with a mean and variance completely described by the mean and variance of the individual Gaussian random variables [5]. The noise sample \( n_0 \) in
(2.55) has zero-mean and a variance of $\sigma_0^2 = \frac{N_0}{2}T_b$, where $N_0/2$ is the two-sided power spectral density and $T_b$ is the weak user’s bit duration (see 2.23 – 2.29 for derivation).

Let $\bar{T}_M$ and $\sigma_{i_M}^2$ equal the mean and variance of the random variable $I_M$, which will equal the mean and variance of our Gaussian approximation for the aggregate interference. The Gaussian random variable resulting from the addition of the noise and the interference will have a mean given by (2.56) and a variance given by (2.57).

\[
E\{n_0 + I_M\} = E\{n_0\} + E\{I_M\} = \bar{T}_M \tag{2.56}
\]

\[
E\{(n_0 + I_M)^2\} - E\{n_0 + I_M\}^2 = \sigma_0^2 + \sigma_{i_M}^2 \tag{2.57}
\]

The probability of a bit error in the detection of the weak user’s signal can be written as a single $Q$-function (2.58) once the mean and variance are found for the Gaussian approximation; which represents the interference caused by the incorrect estimation of the strong user’s data stream [6].

\[
P_{\text{bl}} = Q\left(\frac{\alpha T_b + \bar{T}_M}{\sqrt{\sigma_0^2 + \sigma_{i_M}^2}}\right) \tag{2.58}
\]

2.5 Simulation and Analysis of Two User Synchronous System

To verify the analytical results for the BER expressions derived above for the two-user synchronous ANB system a simulation was developed using Matlab®. This simulation allows multiple users with varying data rates to be simulated over an AWGN channel. In this section simulation results will be compared to both the analytical and approximate BER expressions.
Figure 2.9 is a standard BER vs. SNR curve that provides both the analytical and simulated results. A total of nine curves are plotted in Figure 2.6; four of them are simulation results, another four are analytical BER expressions derived above, and the ninth is a standard single user BPSK BER curve included for comparison. To accurately interpret Figure 2.9 it is to be understood that when user one has an $E_b/N_0$ of $K$ dB then user two will have an $E_b/N_0$ of $K + 20\log_{10}(\zeta) - 10\log_{10}(2)$ dB, where $\zeta$ is defined by (2.14). For example when $\zeta = 2$ and user one has an $E_b/N_0$ of 2 dB, the BER can be read directly off Figure 2.9 and is approximately 0.1. However, user two has an $E_b/N_0$ of 5 dB for the same condition and thus has a BER of approximately 0.05.
Plotting the BER of a particular user versus the near-far-ratio (NFR) is another way to show how the two signals interact. Figure 2.10 is a graph of user one’s BER as a function of the changing NFR. For each of the NFR graphs in this section the noise variance was kept constant at 0.1, which gives the noise a two-sided power spectral density of 0.05 W/Hz. The NFR can be defined as a ratio of powers or of relative bit energies. Figure 2.10 defines the NFR as a ratio of power, whereas Figure 2.8 defines the NFR as a ratio of bit energies.

Figure 2.10 Two User Synchronous Multi-Rate NFR Power Curves

When the NFR is defined as a ratio of power then (2.59) is used, where power is equal to the bit energy multiplied by the data rate (i.e. \( P_x = E_b \cdot R_{\text{data}} \)) [2]. Substituting the equations for bit energy and data rate into (2.59) allow the following reduction to be
made: \( NFR_{dB} = 20 \log_{10} \left( \frac{\alpha_2}{\alpha_1} \right) \). This indicates that the NFR is simply a function of the relative amplitudes of the individual signals. Figure 2.10 was obtained by maintaining user one’s amplitude and varying user two’s amplitude to achieve the desired NFR. The multiple curves in Figure 2.10 illustrate how varying user two’s data rate affects the achieved BER of user one’s signal.

\[
NFR_{dB} = 10 \log_{10} \left( \frac{P_2}{P_1} \right) \tag{2.59}
\]

One might ask, why does increasing user two’s data rate reduce user one’s BER at lower NFRs, whereas increasing user two’s data rate increases user one’s BER at higher NFRs? When user two’s data rate is increased, the number of user two’s bits that overlap a single bit of user one also increases. At lower NFRs the probability of an error in the detection of user two’s signal is high because the two users have comparable amplitudes. However, increasing the number of user two bits yields a higher likelihood of “self cancellation” at the output of user one’s correlator. In essence, user one’s correlator integrates over several of user two’s bits subtracted from their estimates, which yields several opposite signed components that in effect cancel each other out. Thus as user two’s data rate increases, despite the higher error probability of user two at lower NFRs, the effect of incorrect user two bit estimates is dominated by the “self cancellation” in user one’s correlator output. Thus user one’s error probability decreases as user two’s data rate increases at lower NFRs.

At higher NFRs, the probability of an error in the detection of user two’s signal is low and therefore, very few of user two’s estimates are incorrect. However, as user two’s data rate increases, for a particular NFR value, the corresponding energy per bit decreases
because the amplitude remains the same. The decision statistic used to estimate a single bit is proportional to the energy in that bit; therefore, a decrease in the energy per bit increases the chance of an error in one of the overlapping bits. Because user one makes use of subtractive interference cancellation, increasing the number of errors that are made in user two’s data will increase BER for user one. Analyzing Figure 2.10 it can be seen that regardless of user two’s data rate, as NFR increases, user one’s BER will ultimately achieve single user BPSK performance; however, the required NFR for this increases proportionally with the increase in user two’s data rate.

In Figure 2.11 we have defined the NFR as a ratio of respective bit energies, as in (2.60). Substituting the expressions for bit energies into (2.60) the following simplification is found: \( NFR_{dB} = 20 \log_{10} \left( \frac{\alpha_2}{M \alpha_1} \right) \), where \( M \) is defined by (2.8). Figure 2.11 was created by holding user one’s energy per bit constant while varying user two’s energy per bit to achieve the desired NFR.

\[
NFR_{dB} = 10 \log_{10} \left( \frac{E_{b2}}{E_{b1}} \right)
\]  

(2.60)
When the NFR is defined as a ratio of respective energies, an increase in user two’s data rate causes a decrease in BER for user one. The reason for this is that as user two’s data rate increases so does the corresponding amplitude (this ensures the desired NFR). Therefore, increasing user two’s data rate, for identical NFR values, will increase the amplitude such that the energy per bit remains the same; which in turn ensures that fewer errors are made in the estimation of user two’s data. When fewer errors are made in the estimation of user two’s data the less likely an error will be made in user one’s data.

Figure 2.12 plots user two’s BER vs. NFR for the case when NFR is defined as (2.60). As can be seen in Figure 2.12 the simulation and analytical results are in good
agreement with one another for each data rate ratio. For each of the curves in Figure 2.11 the bit energy of user two was held constant while the bit energy of user one was varied to achieve the desired NFR. Looking at (2.47) and (2.48) it is clear that in order for user two’s bit energy to remain constant as the data rate ratio $M$ increases, user two’s amplitude must increase by a factor of $\sqrt{M}$. If user two’s amplitude increases by the multiplicative term $\sqrt{M}$ then so does the amplitude ratio constant $\zeta$ (2.14). Looking at the BER expression for user two (2.19) it is evident that when $E_{b2}$ is held constant and the amplitude ratio constant, $\zeta$, increases as a result of an increase in the data rate ratio, $M$, then the BER will be reduced. This result is seen in Figure 2.12 as the data rate ratio gets larger user two’s BER reduces.

Figure 2.12 Strong Synchronous User Multi-Rate NFR Energy BER Curves
The simulation results shown in Figures 2.9, 2.10, 2.11, and 2.12 help to illustrate that the analytical results for user one (2.45) and user two (2.19) are accurate. Figure 2.13 shows the comparison of the simulated results with the approximations for user one’s BER (2.58) when NFR is defined by (2.60). The approximated BER curves follow the general trend of the simulated results. However, there are a few deviations between the approximation and the expected values. The primary mechanism that is underlining this behavior is the correlation between the random noise process and the incorrect decisions made in the estimation of the strong user (user two). Also, as user two’s data rate increases, agreement between the BER approximation and simulation is better because the Gaussian approximation for the interference pdf gets better via the central limit theorem.

Figure 2.13 Weak Synchronous User BER Simulation and Approximation Comparison
Chapter 3: System Uplink Definition and Analysis

3.1 Additive Narrow Band Uplink System Model

The ANB uplink system represents a scenario in which there are multiple users transmitting independent of one another using the same carrier frequency. Figure 3.1 illustrates the ANB uplink system model, where \( N \) is the number of users.

For \( M \)-PSK, assuming rectangular pulse shaping, each user’s complex base band waveform can be written as (3.1), where \( \alpha_i, T_{bi}, \theta_i, b_{i,k} \), and \( p_i(t) \) are the \( i^{th} \) user’s amplitude, bit duration, phase offset, \( k^{th} \) symbol (\( b_{k,i} \in \{0,1,\cdots,(M-1)\} \)), and rectangular pulse defined by (2.3) respectively [2].
\[ s_i(t) = \alpha_i \sum_k \exp \left( j \left( \theta_i + b_{i,k} \frac{2\pi}{M} \right) \right) p_{r_i}(t - kT_{bi}) \] (3.1)

If we define \( M=2 \) then each user is transmitting a BPSK signal. Thus, equation (3.1) becomes (3.2), where \( b_{i,k} \in \{ \pm 1 \} \) is the \( i^{th} \) users \( k^{th} \) bit. Note that the BPSK waveform in (3.2) is still complex due to the phase \( \theta_i \). The phase on the \( i^{th} \) user’s BPSK waveform emphasizes that each user operates independent of one another. As such each transmitter’s local oscillator will have a random phase; which is assumed to be uniformly distributed over \([0, 2\pi)\) [3].

\[ s_i(t) = \alpha_i \sum_k b_{i,k} e^{j\theta_i} p_{r_i}(t - kT_{bi}) \] (3.2)

### 3.2 Additive Narrow Band Uplink System Demodulation

Assuming an AWGN channel the received baseband ANB uplink signal can be written as (3.3) where \( s_i(t) \) is given in (3.2) and \( \tilde{n}(t) \) is the complex random Gaussian noise process.

\[ r(t) = \sum_{i=1}^{N} s_i(t) + \tilde{n}(t) \] (3.3)

The signal on the \( i^{th} \) branch of the SIC demodulator can be written as (3.4) where \( \hat{\theta}_i \) is the estimate of the phase rotation of the \( i^{th} \) signal and \( \hat{s}_k(t) \) is the \( k^{th} \) user’s estimated signal. The \( i^{th} \) detector phase rotates the received signal such that the \( i^{th} \) BPSK signal is completely on the real axis of the complex plane.

\[ r_i(t) = \left( r(t) - \sum_{k=i+1}^{N} \hat{s}_k(t) \right) e^{-j\hat{\theta}_i} + \tilde{n}(t) \] (3.4)
3.3 Asynchronous Two User Uplink Analysis

We will begin the analysis of the asynchronous uplink system by consider two users operating over an AWGN channel. For this analysis we will assume bit synchronicity between users (i.e. Figure 2.3 depicts the bit alignment for each user). As in the synchronous two user downlink analysis we will begin by deriving the BER expression for each user given that \( R_{b1} = 1/T_b \) and \( R_{b2} = 2R_{b1} \), where \( T_b \) is the bit duration for user one.

The SIC receiver will lock onto the strongest carrier first (i.e. user two) and will phase rotate the signal to ensure that the BPSK signal is on the real access of the complex plane. Therefore, assuming ideal phase and amplitude estimations, the signal present at user two’s detector can be written as (3.5), where \( \phi_{a,b} = \theta_a - \theta_b \) is the total phase rotation of signal \( a \) as seen from \( b \).

\[
r_2(t) = \alpha_2 \sum_k b_{2,k} p_{T_b/2} \left( t - k \frac{T_b}{2} \right) + \alpha_2 e^{j\phi_{a,b}} \sum_k b_{1,k} p_{T_b} \left( t - kT_b \right) + \bar{n}(t) \quad (3.5)
\]

Because we are considering an AWGN channel we only need to derive the BER expression for a single bit of user two, as it will be the same for all of user two’s bits. The two possible received signals (ignoring noise) at the input of user two’s detector during the interval \( kT_b \leq t < (k + 1/2)T_b \) are given in (3.6) and (3.7); which have equal probability of occurrence.

\[
r_2(t) = \left( \pm \alpha_2 \pm \alpha_1 e^{j\phi_{a,b}} \right) p_{T_b/2} \left( t - kT_b/2 \right) \quad (3.6)
\]
\[
r_2(t) = \left( \pm \alpha_2 \mp \alpha_1 e^{j\phi_{a,b}} \right) p_{T_b/2} \left( t - kT_b/2 \right) \quad (3.7)
\]
The received energy per bit for each of the signals given in (3.6) and (3.7) can be found using (2.4) and are given in (3.8) and (3.9) respectively.

\[
\alpha_2 \left(1 + \frac{\cos(\phi_{1,2})}{\zeta}\right) \frac{T_b}{2} = E_{b2} \left(1 + \frac{\cos(\phi_{1,2})}{\zeta}\right)^2
\]

(3.8)

\[
\alpha_2 \left(1 - \frac{\cos(\phi_{1,2})}{\zeta}\right) \frac{T_b}{2} = E_{b2} \left(1 - \frac{\cos(\phi_{1,2})}{\zeta}\right)^2
\]

(3.9)

Equations (3.10) and (3.11) are user two’s BER expressions for each of the possible transmitted signals given in (3.6) and (3.7) respectively. The final BER expression for the strong user can be found by substituting (3.10) and (3.11) into (2.11); which is given in (3.12).

\[
P\{e \mid b_1 = b_{2,1}\} = Q\left(\sqrt{\frac{2E_{b2} \left(1 + \cos(\phi_{1,2})/\zeta\right)^2}{N_0}}\right)
\]

(3.10)

\[
P\{e \mid b_1 = -b_{2,1}\} = Q\left(\sqrt{\frac{2E_{b2} \left(1 - \cos(\phi_{1,2})/\zeta\right)^2}{N_0}}\right)
\]

(3.11)

\[
P_{b2} = \frac{1}{2} Q\left(\sqrt{\frac{2E_{b2} \left(1 + \cos(\phi_{1,2})/\zeta\right)^2}{N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{2E_{b2} \left(1 - \cos(\phi_{1,2})/\zeta\right)^2}{N_0}}\right)
\]

(3.12)

As would be expected when the phase difference, \(\phi_{1,2}\), between the strong and weak user is zero the system becomes synchronous and (3.12) becomes (2.19). Note also that when the strong and weak users are orthogonal then (3.12) becomes the typical single user BPSK BER expression.
To construct the BER expression for the weak user we will build upon the derivation from the previous chapter. From the synchronous case we know that there is a correlation between the errors made in the estimation of the strong and weak users’ data bits. This is seen in Chapter 2 as a bound placed on the individual noise samples that are summed together to create the noise sample present at the output of the weak user’s correlator. The BER expression of the weak user was then found by constructing the pdf of the sum of the two noise samples and then integrating over a known interval for each of the possible signals present at the input of the weak user’s correlator, which are given in Table 2.1. We now have to consider that the noise is complex with independent Gaussian random variables on each the real and imaginary parts of the noise samples.

We will reference the phase difference between the two users by assuming that user two (strong user) has a phase of zero. Therefore, the strong user will completely exist on the real axis and will only be affected by the real part of the complex noise. If we let \( \tilde{n}_{0,1} = \int_{kT_0}^{(k+\frac{1}{2})T_0} \tilde{n}(t) \, dt \) and \( \tilde{n}_{0,2} = \int_{kT_0}^{(k+\frac{1}{2})T_0} \tilde{n}(t) \, dt \) be the complex Gaussian random variables associated with bits \( b_{2,1} \) and \( b_{2,2} \) respectively then the decision statistic for bits \( b_{2,1} \) and \( b_{2,2} \) are given in (3.13) and (3.14) respectively.

\[
\begin{align*}
\tilde{z}_{2,1} \left( \left( k + \frac{1}{2} \right) T_0 \right) &= \int_{kT_0}^{(k+\frac{1}{2})T_0} r(t) \, dt = \left( \alpha_1 b_1 \cos(\phi_{1,2}) + \alpha_2 b_{2,1} \right) \frac{T_0}{2} + \text{Re} \left[ \tilde{n}_{0,1} \right] \quad (3.13) \\
\tilde{z}_{2,2} \left( (k+1)T_0 \right) &= \int_{kT_0}^{(k+\frac{1}{2})T_0} r(t) \, dt = \left( \alpha_1 b_1 \cos(\phi_{1,2}) + \alpha_2 b_{2,2} \right) \frac{T_0}{2} + \text{Re} \left[ \tilde{n}_{0,2} \right] \quad (3.14)
\end{align*}
\]

Using (3.13) and (3.14) we can find the constraints placed on the real part of the complex noise for each of the ten distinct cases tabulated in Table 2.1. These results are
tabulated in Table A.2 in Appendix A, where we are considering only the real part of the complex noise sample, \( \Omega = (\alpha_1 \cos(\phi_{1,2}) + \alpha_2) T_b / 2 \), and \( \Psi = (\alpha_1 \cos(\phi_{1,2}) - \alpha_2) T_b / 2 \).

Appendix A, equations (A.9) through (A.18), provide the ten unique probability density functions that result from the sum of the real parts of the complex noise samples \( \tilde{n}_{0,1} \) and \( \tilde{n}_{0,2} \). The pdf for the \( i^{th} \) case is denoted as \( f_{Y_i}(y) \). The imaginary parts of the complex noise samples \( \tilde{n}_{0,1} \) and \( \tilde{n}_{0,2} \) are both Gaussian and therefore their sum will also be Gaussian [5], which we will define as \( f_z(z) \) and is given in (3.15). Let \( \Lambda \) be defined by (3.16), which will be a random variable that represents the noise that influences the decision of the weak user’s bit. In (3.16) the random variables \( \text{Re}[\tilde{n}_{0,1} + \tilde{n}_{0,2}] \) and \( \text{Im}[\tilde{n}_{0,1} + \tilde{n}_{0,2}] \) have pdfs given by \( f_{Y_i}(y) \) and \( f_z(z) \) respectively.

\[
f_z(z) = \frac{1}{\sqrt{2\pi(2\sigma_n^2)}} \exp\left(-\frac{z^2}{2(2\sigma_n^2)}\right) \quad (3.15)
\]

\[
\Lambda = \text{Re}[\tilde{n}_{0,1} + \tilde{n}_{0,2}] \cos(\phi_{1,2}) + \text{Im}[\tilde{n}_{0,1} + \tilde{n}_{0,2}] \sin(\phi_{1,2}) \quad (3.16)
\]

We are interested in finding the probability that \( \Lambda < \lambda_i \), where \( \lambda_i \) is a real value. We could accomplish this by finding the pdf of the random variable given by (3.16) and then integrating the pdf from \(-\infty\) to \( \lambda_i \). However, when we multiply a random variable by a constant, as we do in (3.16), we alter the variance of that random variable [5].

Therefore the pdfs associated with the random variables \( \text{Re}[\tilde{n}_{0,1} + \tilde{n}_{0,2}] \) and \( \text{Im}[\tilde{n}_{0,1} + \tilde{n}_{0,2}] \) in (3.15) would have to be solved for the change in their respective variances. Modifying the expression that represents the Gaussian pdf associated with the
random variable $\text{Im}[\tilde{n}_{0,1} + \tilde{n}_{0,2}]$ would simply be a matter of changing the variance of (3.15) from $2\sigma_n^2$ to $2\sin^2(\phi_{1,2})\sigma_n^2$. However, modifying the analytical expression for the pdf associated with the random variable $\text{Re}[\tilde{n}_{0,1} + \tilde{n}_{0,2}]$ would require more effort than its worth. Instead we can find the probability that $\Lambda < \lambda_i$ by realizing that for every single value that the random variable $\text{Re}[\tilde{n}_{0,1} + \tilde{n}_{0,2}\cos(\phi_{1,2})]$ takes on there is a probability associated with all the values that the random variable $\text{Im}[\tilde{n}_{0,1} + \tilde{n}_{0,2}\sin(\phi_{1,2})]$ can take on that ensure that $\Lambda < \lambda_i$. Therefore, we can use a double integral to compute the probability that $\Lambda < \lambda_i$, which is given in (3.17), where we constrain the phase difference between the two users signals by $0 < \phi_{1,2} \leq \pi/2$. Because $f_z(z)$ is Gaussian the inner integral of (3.17) can be replaced with a $Q$-function and (3.17) becomes (3.18) where $\sigma_n^2$ is equal to equation (2.28).

$$P\{\Lambda < \lambda_i\} = \int_{y=-\infty}^{\infty} \int_{z=-\infty}^{\infty} f_z(z) f_{y_i}(y) \, dz \, dy$$ (3.17)

$$P\{\Lambda < \lambda_i\} = \int_{y=-\infty}^{\infty} f_{y_i}(y) Q\left(\frac{y \cos(\phi_{1,2}) - \lambda_i}{\sin(\phi_{1,2}) \sqrt{2\sigma_n^2}}\right) \, dy$$ (3.18)

Table 3.1 provides the value associated with the condition which ensures an error in the detection of the weak user’s signal for each of the ten distinct cases.
Table 3.1 Two User Asynchronous Carrier Error Conditions

<table>
<thead>
<tr>
<th>Case</th>
<th>Variable</th>
<th>Error Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\lambda_1$</td>
<td>$-\alpha_i T_b$</td>
</tr>
<tr>
<td>2</td>
<td>$\lambda_2$</td>
<td>$-(\alpha_1 + \alpha_2 \cos(-\phi_{i,2})) T_b$</td>
</tr>
<tr>
<td>3</td>
<td>$\lambda_3$</td>
<td>$-(\alpha_1 + 2\alpha_2 \cos(-\phi_{i,2})) T_b$</td>
</tr>
<tr>
<td>4</td>
<td>$\lambda_4$</td>
<td>$-(\alpha_1 - \alpha_2 \cos(-\phi_{i,2})) T_b$</td>
</tr>
<tr>
<td>5</td>
<td>$\lambda_5$</td>
<td>$-\alpha_i T_b$</td>
</tr>
<tr>
<td>6</td>
<td>$\lambda_6$</td>
<td>$-\alpha_i T_b$</td>
</tr>
<tr>
<td>7</td>
<td>$\lambda_7$</td>
<td>$-(\alpha_1 + \alpha_2 \cos(-\phi_{i,2})) T_b$</td>
</tr>
<tr>
<td>8</td>
<td>$\lambda_8$</td>
<td>$-(\alpha_1 - 2\alpha_2 \cos(-\phi_{i,2})) T_b$</td>
</tr>
<tr>
<td>9</td>
<td>$\lambda_9$</td>
<td>$-(\alpha_1 - \alpha_2 \cos(-\phi_{i,2})) T_b$</td>
</tr>
<tr>
<td>10</td>
<td>$\lambda_{10}$</td>
<td>$-\alpha_i T_b$</td>
</tr>
</tbody>
</table>

The final BER expression can be computed using (3.19) where $f_Y(y)$ is the pdf of the sum of the constrained Gaussian noise samples for the $i^{th}$ case which are given in equations (A.9) through (A.18) in Appendix A, $\lambda_i$ is the $i^{th}$ constraint given in Table 3.1, and $q_i$ is the probability of the $i^{th}$ event and is given in Table 2.1 where $P\{e | b_i = b_{2,i}\}$ and $P\{e | b_i = -b_{2,i}\}$ are given in (3.10) and (3.11) respectively.

$$P_{ni} = \sum_{i=1}^{10} q_i \int_{y=-\infty}^{\infty} f_Y(y) Q\left(\frac{\lambda_i - y \cos(\phi_{i,2})}{\sin(\phi_{i,2}) \sqrt{2 \sigma_n^2}}\right) dy$$

(3.19)

3.4 Simulation and Analysis of Two User Asynchronous System

As in the previous chapter to verify the analytical results for the BER expressions derived for the two-user asynchronous ANB system a simulation was developed using Matlab®. This simulation allows multiple users with varying data rates and carrier
phases to be simulated over an AWGN channel. In this section the analytical BER
distributions will be compared against the results obtained from simulation.

Near-far ratio (NFR) plots are adequate in illustrating the agreement between the
analytical and simulated results. This is because of the dependence on the BER results of
each user with respect to the difference in amplitudes between each user. For each NFR
plot shown in this section the noise variance was kept constant at 0.1, which gives the
noise a two-sided power spectral density of 0.05 W/Hz. We are also using the definition
of NFR as defined by the ratio of user two’s to user one’s energy per bit. Mathematically
this was defined in Chapter 2 as equation (2.60).

Figure 3.2 is a plot of the weak user’s BER versus NFR. As expected when the
phase difference between the two users is zero we obtain the synchronous case. This can
be seen by comparing the simulated and analytical results for $\phi_{1,2} = 0$ in Figure 3.2 with
the analytical and simulated results for $R_{b2} = 2R_{b1}$ in Figure 2.11. The results obtained in
Figure 3.2 are intuitive and can be justified by the mere fact that as the phase difference
between the two user approaches orthogonal the percentage of interference caused by the
other user is reduced, which in turn allows better decisions to be made on each individual
bit. Figure 3.2 also servers to illustrate that the analytical BER expression derived for the
weak user (user one) is in agreement with the simulated results.
Figure 3.2 Weak Asynchronous User Multi-Rate NFR Energy Curves

Figure 3.3 is a plot of the BER for the strong user versus NFR. Again if we compare the results obtained for a phase difference of zero in Figure 3.3 we see that we in fact have a synchronous system and the results are identical to the plot of the synchronous case \((R_{b2} = 2R_{b1})\) in Figure 2.12. The steady increase in BER performance for the strong user in Figure 3.3 as the phase difference approaches orthogonal is contributed to only a fraction of the weak user’s signal being projected onto the strong user’s signal, thus less total interference and better BER performance.
Figure 3.3 Strong Asynchronous User Multi-Rate NFR Energy Curves

3.5 Synchronous Carrier, Asynchronous Bit, Two User Analysis

In this section we deviate slightly from the asynchronous carrier to consider the affects of bit asynchronicity in a synchronous ANB system. We will begin the analysis of the bit asynchronous system by consider two users operating over an AWGN channel with data rates set such that $R_{b1} = 1/T_b$ and $R_{b2} = 2R_{b1}$. Figure 3.4 illustrates the alignment of the two users’ waveforms independently from one another where Figure 3.5 shows the alignment before superposition. Because the system is bit asynchronous there
is a time delay, $\tau$, associated with the misalignment of the two signals. It should be evident that a delay value of zero or an integer multiple of $T_b/2$ produces a synchronous system and as such should return identical BER expressions for each user. For this reason we will only consider delay value constrained such that $0 \leq \tau < T_b/2$ holds true.

Figure 3.4 Two User Asynchronous Bit Definition

Figure 3.5 clearly shows the alignment of the two users waveforms prior to superposition. Every other bit that the strong user transmits completely overlaps a portion of a single bit transmitted by the weak user; in Figure 3.5 this bit is labeled $b_{2,1}(m)$. The other bit, which is labeled $b_{2,2}(m)$ in Figure 3.5, overlaps two of the weaker user’s bits (i.e. $b_1(k)$ and $b_1(k+1)$ in Figure 3.5).
If we were to, at random, select one of the strong user’s bits we would have a fifty
percent chance of selecting a bit that either resembles $b_{2,1}(m)$ or $b_{2,2}(m)$. These two
bits, however, do not have the same probability of an error because the interference
caused by the weak user is not equivalent for each case. Equation (3.20) provides the
form of the BER expression for the strong user. We must now find the individual
probabilities of an error for each case.

$$P_{b_2} = \frac{1}{2} P\{\in b_{2,1} \text{ was sent}\} + \frac{1}{2} P\{\in b_{2,2} \text{ was sent}\}$$  \hspace{1cm} (3.20)

If we select a bit that resembles $b_{2,1}(m)$ then the probability of a bit error is
equivalent to the expression derived in (2.13) and is given in (3.21). Computing the BER
expression for the case when the strong user’s bit resembles $b_{2,2}(m)$ in Figure 3.5
requires that the output of the correlator (Figure 2.4) be determined for each of the
possible bit patterns. If we assume that the strong user has transmitted a positive pulse
value then there will be four possible combinations that result in the overlapping of the
two adjacent weak user’s bits. Table 3.2 lists the four possible correlator outputs along
with their associated probability of error. Equation (3.22) gives the probability of an error given that a bit resembling $b_{2,3} (m)$ in Figure 3.5 was transmitted. The final BER expression for the strong user is found by substituting (3.21) and (3.22) into (3.20) and is given in (3.23).

$$P\{e | b_{2,3} \text{ was sent} \} = \frac{1}{2} \Phi \left( \frac{2E_2 (1+1/\xi)^2}{N_0} \right) + \frac{1}{2} \Phi \left( \frac{2E_2 (1-1/\xi)^2}{N_0} \right)$$ \hspace{1cm} (3.21)$$

<table>
<thead>
<tr>
<th>Probability of Occurrence</th>
<th>Correlator Output</th>
<th>Probability of Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{4}$</td>
<td>$-\alpha_1 \frac{T_b}{2} + \alpha_2 \frac{T_b}{2}$</td>
<td>$\Phi \left( \frac{2E_2 (1-1/\xi)^2}{N_0} \right)$</td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
<td>$-\alpha_1 \left( \frac{T_b}{2} - 2\tau \right) + \alpha_2 \frac{T_b}{2}$</td>
<td>$\Phi \left( \frac{2E_2 \left(1 - (T_b - 4\tau)/\xi T_b \right)^2}{N_0} \right)$</td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
<td>$\alpha_1 \left( \frac{T_b}{2} - 2\tau \right) + \alpha_2 \frac{T_b}{2}$</td>
<td>$\Phi \left( \frac{2E_2 \left(1 + (T_b - 4\tau)/\xi T_b \right)^2}{N_0} \right)$</td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
<td>$\alpha_1 \frac{T_b}{2} + \alpha_2 \frac{T_b}{2}$</td>
<td>$\Phi \left( \frac{2E_2 (1+1/\xi)^2}{N_0} \right)$</td>
</tr>
</tbody>
</table>

$$P\{e | b_{2,3} \text{ was sent} \} = \frac{1}{4} \Phi \left( \frac{2E_2 (1-1/\xi)^2}{N_0} \right) + \frac{1}{4} \Phi \left( \frac{2E_2 \left(1 - (T_b - 4\tau)/\xi T_b \right)^2}{N_0} \right) + ...$$ \hspace{1cm} (3.22)
\[ P_{b2} = \frac{3}{8} Q\left( \sqrt{\frac{2E_s (1-1/\zeta)^2}{N_0}} \right) + \frac{1}{8} Q\left( \sqrt{\frac{2E_s (1-(T_b - 4\tau)/\zeta T_b)^2}{N_0}} \right) + \cdots \] (3.23)

\[ \frac{1}{8} Q\left( \sqrt{\frac{2E_s (1+(T_b - 4\tau)/\zeta T_b)^2}{N_0}} \right) + \frac{3}{8} Q\left( \sqrt{\frac{2E_s (1+1/\zeta)^2}{N_0}} \right) \]

It was stated above that for a delay of zero we should obtain identical BER expressions for each user. If we set \( \tau = 0 \) in (3.23) we will obtain the BER expression given in (3.24) which is the BER expression for the strong user for the synchronous case.

\[ P_{b2} \{ e \mid \tau = 0 \} = \frac{1}{2} Q\left( \sqrt{\frac{2E_s (1-1/\zeta)^2}{N_0}} \right) + \frac{1}{2} Q\left( \sqrt{\frac{2E_s (1+1/\zeta)^2}{N_0}} \right) \] (3.24)

If we want to maximize BER performance for the strong user (user two) in equation (3.23) we need to jointly maximize the quantities \( (1-(T_b - 4\tau)) \) and \( (1+(T_b - 4\tau)) \), where \( 0 \leq \tau < T_b/2 \). The solution is to find the intersection of the two expressions, which occurs at \( \tau = T_b/4 \) indicating that each strong user bit overlaps a bit transition of the weak user. Thus, there is a potential for “self cancellation” of the weak users signal when integrating over the strong user’s bit duration. Solving equation (3.23) for the case when \( \tau = T_b/4 \) yields equation (3.25).

\[ P_{b2} = \frac{3}{8} Q\left( \sqrt{\frac{2E_s (1-1/\zeta)^2}{N_0}} \right) + \frac{3}{8} Q\left( \sqrt{\frac{2E_s (1+1/\zeta)^2}{N_0}} \right) + \frac{1}{4} Q\left( \sqrt{\frac{2E_s}{N_0}} \right) \] (3.25)

Figure 3.6 compares equations (3.24) and (3.25) to illustrate that the BER improvement between the bit synchronous system and the bit asynchronous system with \( \tau = T_b/4 \) is small. The largest gains in BER performance will be achieved when the amplitude ratio constant, \( \zeta \), is small. From Figure 3.6 we can see that for the case when...
\( \zeta = 2 \) we increase our BER performance by approximately 1 dB and as the amplitude ratio constant gets larger the increase in BER performance becomes negligible.

As the data rate difference between the two users increases the benefit, albeit small, of asynchronous bit alignment on BER performance is reduced even further. Figure 3.7 illustrates why this occurs. In Figure 3.7 the data rates are set such that \( R_{b2} = 8R_{b1} \) which means the strong user transmits eight bits for every one bit the weak user sends. The seven strong user bits that are highlighted will each have a probability of error given by (2.19), where the bit energies used in the expression are given by (2.47)
and (2.48), whereas the remaining bit will have a probability of error given by (3.23). Thus, only one bit has a slightly better BER performance as compared to the synchronous bits. Therefore, it is acceptable to upper bound the BER performance of a synchronous ANB system that has asynchronous bit alignment with the BER performance of a synchronous bit aligned system.

Figure 3.7 Asynchronous Bit Alignment Example
Chapter 4: Hardware Implementation

4.1 Introduction

Engineering, as a whole, can be viewed as the practical implementation of theoretical ideas. One goal, of course, is to experimentally show agreement between theory and measurement as a means of corroboration. That is the objective of this chapter, to provide a comparison between experimental results obtained with a physical implementation of an ANB system and the analytical expressions derived in the previous chapters.

We begin by introducing a digital hardware design that provides a versatile platform for which an ANB system can be physically implemented and easily tested. Hardware specifications are described, as are budget constraints on component selection and their impact on overall system performance. A brief overview of the layout and fabrication of the printed circuit board (PCB) will be given to provide insight. We then provide a synopsis of the software hierarchy implemented on the digital microcontroller and its role in overall system control. The section describing the hardware ends by providing a few key hardware tests and a broad system overview.

After the hardware is sufficiently characterized a detailed test procedure is given for both collecting and analyzing experimental data. Post processing techniques are then discussed, as are the supporting computational algorithms and their effect on experimental results.

The last section of this chapter provides a comparison of the derived analytical BER expressions found in the previous chapters with the experimental results obtained
from our digital hardware. In closing we comment on our findings and provide arguments that support our results.

4.2 Hardware Specifications

The initial concept behind our design was a general purpose baseband transmitter with independently programmable in-phase and quadrature channels. The high level design was broken down into functional blocks; which are illustrated in Figure 4.1. When designing any type of hardware, care must be taken in developing the initial specifications that outline the purpose and functionality of the device.

Because the primary purpose of our device was to obtain experimental BER data for an ANB system, we required that approximately $10^6$ bytes of data be stored onboard the device. The required size was intended to easily enable collection of statistically accurate results without a large number of repeated trials. We also required that the device be programmable so that the data stored onboard could easily be modified via a personal computer (PC).
We also required that our system be able to achieve a date rate of 300 kHz at minimum; which in turn means bit durations of no more than approximately 3 microseconds. This requirement was a key contributor in component identification because it imposes restrictions on all but the PC interface block. For example, during the decision process regarding storage devices we required that the read time be less than 1.5 microseconds, which is half the maximum bit duration; of course the faster the better.

The microcontroller is the heart of the device. It handles data I/O (input and output), I/O manipulation, PC communications, and the DAC interface. We decided on the PIC18F8585 made by Microchip. The reason we chose the PIC18F8585 is familiarity with the Microchip family as well as cost restrictions. The primary cost in choosing a processor, for a single use application, is incurred not from the integrated circuit (IC) itself but in the development hardware required to program and debug the processor. The School of Electrical Engineering and Computer Science in the Russ College of Engineering at Ohio University already had the Microchip programming interface, which ultimately reduced the total cost of our hardware. A few of the key features of the PIC18F8585 are that it has 68 I/O lines, 48 kbytes of program memory, onboard AUSART (used for RS-232 communication), a RISC architecture, and a 100 ns instruction time (based on a 40 MHz clock) [8].

Once the microcontroller was selected, we began looking for an EEPROM (electrically erasable programmable read-only memory) that would satisfy the $10^6$ bytes requirement. There are two primary types of interfaces supported by memory ICs: serial and parallel. Because we wanted to maximize the bandwidth of our device we opted for a parallel interface. The drawback to a parallel interface is the large number of I/O lines
that are required for control, addressing, and data I/O. Because we required $10^6$ bytes for each of the in-phase and quadrature channels, and taking into account cost restrictions, we decided upon a single one megabyte IC for each channel. We could have chosen a four megabyte EEPROM; however, the cost would have been multiplied by more than one hundred, and reduced access time (access times for the 4 MB devices are typically double that of the one megabyte ICs). The final restriction was that we have a fast (less than 200 ns) access time. The Atmel AT49F008AT EEPROM met all our requirements. It’s relatively inexpensive, has a fast access time of 70 ns and bus contention controls to allow the addressing and I/O lines to be shared on a common bus [9]. The bus contention controls were crucial, considering it takes nineteen lines to address the one megabyte EEPROM and another eight I/O lines for read/write operations, totaling twenty seven dedicated microcontroller I/O lines. Without the ability to use a shared bus we could not have used the PIC18F8585 because there would not have been enough I/O pins to support the remaining hardware. The quick access time of only 70 ns ensures that our EEPROMs do not reduce our bandwidth given that each microcontroller instruction cycle is 100 ns.

An RS-232 communication link was chosen as the interface between our device and a personal computer. This choice was based solely on our microcontrollers onboard AUSART and the ease of writing PC applications to read the RS-232 port. A higher data rate interface such as USB would be preferred. As is, we achieve a data rate of 56-kbits/s. With the overhead required to ensure data integrity, it takes approximately five minutes to program a single EEPROM. The only hardware required to interface the microcontroller to the RS-232 port is a line driver which converts TTL voltages to the line levels required by the RS-232 standard. Maxim produces the MAX233ACPP which
is a multi-channel RS-232 line driver [10]. The MAX233ACPP IC in conjunction with a few resistors and capacitors makeup the entire PC interface circuit.

The digital-to-analog converters and output amplifiers were the last components to be selected. We required that the DACs have a parallel interface for the same reason that we chose a parallel interface for the EEPROMs. The characteristics that we were looking for were an 8-bit parallel interface, a fast write time, and a step response with a fast settling time (less than 5% of maximum bit duration). We decided upon Analog Devices AD5424 which has an 8-bit parallel interface, fast 17 ns write time, adjustable reference voltage, bus contention control lines, $\pm 0.25$ LSB relative accuracy, and maximum settling time of 30 ns [11]. In [11] Analog Devices provide a circuit to produce a bipolar signal as an output; they also specify the AD8066 op-amp as the output amplifier. The AD8066 op-amp provides a reasonable slew rate of $180V/\mu s$ and operates over a large voltage range [12]. The outputs of the AD8066 op-amps are then fed to their respective BNC connectors which are terminated such that there is a $50\Omega$ input impedance to the device.

Once component identification and selection was finished the hardware schematic was designed using Altium’s Protel DXP board layout software. Portions of the schematic were taken directly from the manufacturer’s suggested application layout, which can be found in [8] - [12]. Appendix B provides the complete circuit diagram, PCB layout, and parts list for our hardware.
4.3 PCB Layout and Fabrication

Altium’s Protel DXP, which was used to create the schematic of our hardware, was also used to create the PCB layout. Once a schematic has been created a net list of all the nodes and connections between ICs is generated. This file is then imported into the PCB layout software which first looks up the physical footprint of each component in the schematic (e.g., ICs, resistors, capacitors, voltage regulators, etc.) and then visually displays the required connections for each component. The board dimensions are specified, and then each component must be placed inside the PCB area. After the location of each component is set, the board can be routed. Figure 4.2 is a snapshot of the baseband transmitter after the board dimensions had been specified and the components arranged. The connections that must be routed are indicated by lines extending from one pad to another.

Figure 4.2 Pre Routed Baseband Transmitter PCB
Routing a PCB is the process of defining the location of the tracks (transmission lines) that define the physical connection between each electrical node (DXP refers to these as nets) and their supporting components. Due to the number of connections in our design we required a double layer board; this means both the top and bottom of our PCB has tracks. Figure 4.3 is a snapshot of the baseband transmitter after routing. The center component is the microcontroller and the bottom portion (left to right) consists of the two EEPROMs, oscillator, and RS-232 line driver circuit. The digital and analog portions were intentionally separated to reduce switching noise on the analog outputs. The analog grounding plane is also evident in Figure 4.3 and helps minimize noise on the analog portion of the circuit [11].

Figure 4.3 Routed PCB for the Baseband Transmitter
Once a board has been completely routed a DRC (design rule check) is performed to verify that all nets have been established, no broken nets exist, spatial requirements between electrical components have been meet, and a host of other compliances fulfilled. After correcting all DRC contentions the PCB is ready for fabrication. Typically after the PCB layout has been finished a set of Gerber files would be generated and sent to a PCB production company for fabrication of the actual PCB. However, because cost and total turn around time were concerns, the PCB was fabricated “in house.” The processes involved printing the PCB mask onto a sheet of thermal transfer paper: “Press-n-Peel” made by Techniks Inc., was the thermal transfer paper we used. Once a mask for each layer is printed they can be transferred to a double sided copper board, one side at a time, using a standard house hold iron.

Figure 4.4 shows the double sided copper board that we used for our baseband transmitter next to the top layer of our design printed on a sheet of thermal transfer paper. Figure 4.5 is a picture of the top layer of our PCB after completing the thermal transfer process (i.e. ironing the mask onto the copper board).
After each layer’s mask had been transferred the board was soaked in a bath of ferric chloride (PCB etchant). The ferric chloride removed all exposed copper in approximately five minutes. The PCB was then washed and visually inspected. All vias and thru holes were then drilled to prepare the board for component population. Before components were soldered to the PCB all vias were populated and the board was tested for continuity to ensure that there were no broken traces and that isolation existed between traces in close proximity to one another. All surface mount and thru hole components were then soldered onto the PCB while periodic testing ensured proper functionality of the hardware (e.g., the microcontroller was soldered and tested, then the memory modules were soldered and tested, etc.).

Figure 4.6 was taken shortly after our PCB was etched and drilled. The components that had been populated when Figure 4.6 was taken were the microcontroller, microcontroller programming circuit, EEPROMs, oscillator, and the RS-232 line driver (socket only, IC not shown). Figure 4.7 is a picture of the final baseband transmitter after all the components had been populated.
4.4 **Microcontroller Software Hierarchy**

Figure 4.8 depicts the software hierarchy that is implemented on our baseband transmitter. The PC sends out commands to our device via the RS-232 port. These commands are received and processed by the onboard microcontroller of our hardware. We will provide a brief overview of each of the sub blocks underneath the baseband transmitter block in Figure 4.8.
Starting with the EEPROM sub block there are three commands that interact with the in-phase and quadrature storage devices. These commands are the erase, load, and read commands. The erase command sends out the required data sequence to force the desired chip to be cleared and prepped for programming. Note that in [9] Atmel specifies that before a sector of the AT49F008AT can be written to, it must first be erased. The load command allows the end user to program each of the EEPROMs entire one megabyte storage capacity via the PC. Reading the contents of the EEPROM is accomplished using the read command. The start address is sent along with the number of bytes that are to be read from memory. Then the device transmits the selected contents of the EEPROM back to the PC. Because we are using non-volatile memory, the
EEPROMs only need to be loaded once, that is of course unless the output signal is to be altered.

There are two commands that interact directly with the DACs. The reset command is internal to the microcontroller and cannot be issued directly by the PC. However, the reset command is executed during boot up and also after a continuous transmission is halted. The reset command sends the appropriate commands to the DACs to force each of them to output zero volts. The set command allows the end user to set either of the DACs to a specific value. This is particularly useful when the output voltage of any particular code word needs to be experimentally measured. Table 4.1 was taken from [11] and is used to compute the theoretical value of any code word, where $V_{ref}$ is the reference voltage and should be measured experimentally. Note that we are using the DACs in a bi-polar mode of operation (i.e., $-V_{ref}$ to $+V_{ref}$).

<table>
<thead>
<tr>
<th>Digital Input</th>
<th>Analog Output (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1111 1111</td>
<td>$+V_{ref}(127/128)$</td>
</tr>
<tr>
<td>1000 0000</td>
<td>0</td>
</tr>
<tr>
<td>0000 0001</td>
<td>$-V_{ref}(127/128)$</td>
</tr>
<tr>
<td>0000 0000</td>
<td>$-V_{ref}(128/128)$</td>
</tr>
</tbody>
</table>

The final set of commands handles the execution and termination of the baseband transmission of stored data. Under the continuous transmit command in Figure 4.8 we find the passive, active, and halt commands.

Executing a halt command terminates any continuous transmission in progress and places the microcontroller in an idle mode. After the halt command is processed a
DAC reset command is issued which, as mentioned, causes both the in-phase and quadrature DACs to output zero volts.

Passive mode transmission streams the contents of an EEPROM directly to its respective DAC. There are two modes of operation which support passive mode transmission: single channel output and dual channel output. Single channel output transfers the EEPROM contents of only the in-phase or quadrature channel’s data to its respective DAC while maintaining an output of zero volts on the other channel. Passive mode transmission on a single channel requires 15 microcontroller instructions to be performed. Therefore, with a 100 ns microcontroller instruction time the bit duration is equal to 1500 ns.

In the passive dual channel output mode both the in-phase and quadrature channels output simultaneously. However, because the EEPROMs are on a common bus, as are the DACs, the two channels cannot change states at the same time. Therefore, in the passive dual channel output mode the in-phase channel output leads the quadrature channel output by half a bit duration (see Figure 4.9). The microcontroller requires 21 instructions to perform all the tasks associated with the passive dual channel output mode. Therefore, each channel has a bit duration of 2100 ns. Note that the increase in bit duration from the passive single channel output to the passive dual channel output is due to the processing overhead of managing the second channel.

Active transmissions allow the end user to modify the contents of an EEPROM prior to outputting the value to its respective DAC. When the end user initiates an active transmission they specify the output channel as well as an array of byte pairs, where each byte pair represents the desired output value and its corresponding value to be replaced.
The active transmission is accomplished by constructing a lookup table that stores the desired output value in memory at a computed offset based on the value that is to be replaced. For example if we wanted to replace 0x08, 0x14, 0x2B, and 0x3E with 0x0C, 0x1A, 0x2C, and 0x62 respectively then the user would initiate an active transmission command and upload the four sets of byte pairs. The microcontroller would in turn construct Table 4.2 in memory using 0x0400 as the starting address for the lookup table. Every time a value is read from an EEPROM it uses this value as a computed offset from 0x0400 and sets the respective DAC’s output to the value stored at that location (e.g., if 0x2B is read from an EEPROM then the value stored at memory location 0x042B, which is 0x2C, would be returned and sent to the respective DAC). Active mode transmission was implemented in such a way that ensured its timing characteristics were the same as the single channel passive mode (i.e., the bit duration is 1500 ns). As will be seen in the succeeding sections, active transmission is particularly useful when performing experiments in which the underlying data stored on an EEPROM must remain constant while the parameters that define the output signal fluctuate from trial to trial.

<table>
<thead>
<tr>
<th></th>
<th>00</th>
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<th>0A</th>
<th>0B</th>
<th>0C</th>
<th>0D</th>
<th>0E</th>
<th>0F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0400</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>62</td>
</tr>
</tbody>
</table>

Table 4.2 Example Augmented Transmission Lookup Table

Both the passive and active transmission modes continually generate an output by successively incrementing an address that indexes into one or both the EEPROMs. The value stored at that address is then returned, processed, and sent to the respective DAC.
After the entire contents of one or both EEPROMs have been read, the address rolls back to zero and the process continues until a halt command is issued or the device is powered off.

4.5 Hardware Testing

Numerous hardware tests were conducted to verify proper functionality of our device. A majority of these tests were performed during the population of the digital portion of our hardware to ensure timing specifications were met, bus contentions were avoided, and to verify the general integrity of the PCB. Once the analog side of the PCB was populated, the main concern was the output waveforms generated by our baseband transmitter. The goal of the analog hardware test was to obtain characteristics for the output response of a full range transition (i.e., DAC transitions from $-V_{ref}$ to $+V_{ref}$); namely the settling time, rise time, and peak overshoot. We also wanted to verify the bit duration of the continuous single channel output. Table 4.3 provides a list of equipment that was used during both the hardware tests and final experiments.

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Model</th>
<th>Serial Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agilent Technologies</td>
<td>MSO6034A</td>
<td>MY44002665</td>
<td>300 MHz MegaZoom Oscilloscope</td>
</tr>
<tr>
<td>Agilent Technologies</td>
<td>34420A</td>
<td>MY42001814</td>
<td>7½ Digit Nano Volt Meter</td>
</tr>
<tr>
<td>Agilent Technologies</td>
<td>E3630A</td>
<td>MY40007810</td>
<td>Triple Output Power Supply</td>
</tr>
<tr>
<td>Fluke</td>
<td>87 III</td>
<td>74851458</td>
<td>True RMS Multimeter</td>
</tr>
</tbody>
</table>

Figure 4.9 is a screenshot of the 300 MHz MegaZoom oscilloscope during the testing of a full range transition of the quadrature DAC and the supporting output amplifier. From Figure 4.9 we can see that the bit duration is indeed 1500 ns as calculated and the full dynamic range is $\pm 5$ volts.
Since we are particularly interested in the step response characteristics we imported the oscilloscope’s measurements into Matlab® and processed the underlying data. Figure 4.10 is a plot of the quadrature output in response to a ten volt step input. It is evident from Figure 4.10 that the output response to the step input is overdamped. An overdamped response is characterized by an output that does not exceed the steady state value of the system [13]. The required time for a system’s output to settle within a percentage of the steady state value is referred to as the settling time [13]. This measurement is useful when sampling the signal because we want to avoid sampling during transitions and would like to capture as many samples during a steady state condition as possible. The rise time is another useful measurement because it indicates the “…swiftness of the response” [13]. For an overdamped system rise time is typically a measure of the time required to transition from 10% to 90% of the steady state output value. The rise time for our hardware was empirically found to be approximately 145 ns. Therefore, when the system is used for communication experiments any sample taken
within the first 145 ns of a bit transition has the potential of corrupting the average steady state value for that bit and could ultimately result in a bit error.

Figure 4.10 Ten Volt, Single Channel, Step Response

Figure 4.11 is a screenshot of the 300 MHz Megazoom oscilloscope during a dual channel output test. The primary focus of this test was to verify that the in-phase leads the quadrature channel by half a bit duration. As stated previously, each channel has a bit duration of 2400 ns during a continuous dual channel output. Therefore, the in-phase should lead the quadrature channel by 1200 ns; which can be verified by Figure 4.11.
The final test was to characterize an abnormality found in the output response during a transition. Figure 4.12 depicts a single bit during a continuous single channel transmission. The abnormality can be seen prior to each transition. At approximately 200 ns prior to transition there is a 475 mV peak-to-peak ringing. During this 200 ns abnormality the microcontroller pulls the DAC’s chip select line low, loads the code word to the I/O port, and then takes the DAC’s chip select line high, which forces the DAC to latch the code word present on the I/O lines. From [11] it is believed that this abnormality is caused by the absence of an additional $0.1 \mu F$ ceramic capacitor at the power terminals of the DACs. The purpose of the additional ceramic capacitor is to “…provide a low impedance path to ground at high frequencies” which would “…handle the transient currents due to internal logic switching” of the DACs [11]. For our purposes it is important to remember that when we sample the output any sample that falls within the last 200 ns of a bit could potentially corrupt the average steady state value for that bit and therefore should be discarded.
4.6 Hardware Summary

Figure 4.13 in conjunction with Table 4.4 provides a system overview of our baseband transmitter hardware. In summary we have designed and fabricated a device that is a versatile platform for which any number of experiments can be performed. With two megabytes of non-volatile memory and the ability to easily modify the contents via a PC, our device provides both the designer and end user with the storage required to implement large non-repeating sequences of data. Our 8-bit digital-to-analog converters provide the flexibility to create numerous pulse amplitude signals on each channel, with the ability to add baseband pulse shape filtering on each channel by connecting the filters to the output terminals of the device. An adjustable reference voltage from zero to five volts provides a maximum peak-to-peak voltage of ten volts and a means of controlling the energy of the output signal. Finally, the on board microcontroller allows another designer to modify the core code which would alter/enhance the functionality of the hardware. Overall our baseband transmitter is a multi-purpose device that can be used as a test bed for any number of digital communication experiments.
4.7 Experimental Test Procedures

The goal of our experimentation was to corroborate the analytical expressions derived in the previous chapters. We primarily focused on collecting BER vs. NFR curves for the two-user synchronous ANB system. The aim of this section is to provide a
detailed explanation of the equipment and methods that were used to collect and analyze the experimental data.

Figure 4.14 illustrates the general hardware setup that was used for collecting experimental data. First a sampled two user synchronous ANB system was generated on a PC using Matlab®. Once the known ANB system had been created the output signal was quantized and uploaded to the baseband transmitter’s EEPROM. With a two user ANB system we know that there are four possible output levels. Therefore, we took advantage of the active single channel continuous transmission of our baseband transmitter to alter the four output states so that we could quickly change the NFR of the two user ANB system. An attenuator was connected to the output terminal of the baseband transmitter to provide a means of adjusting the SNR of the output signal. We then used an oscilloscope to collect the raw baseband data. Because the oscilloscope operates with its own local oscillator and could not be easily synchronized with our baseband transmitter, an external timing circuit was required to provide bit transition information. The raw oscilloscope data (baseband and timing waveforms) was then uploaded to a PC and processed in Matlab® to obtain BER results for any given trial.

Figure 4.14 Hardware Signaling Flow Diagram
The BER analysis performed in the previous chapters was based on the transmission of the ANB system over an AWGN channel. We used thermal noise generated by the electronics in the power supply, baseband transmitter, and oscilloscope themselves to mimic an AWGN channel. Figure 4.15 is the average power spectral density (PSD) plot of the random noise measured at the output of our baseband transmitter. The PSD plot of Figure 4.15 was generated by taking the FFT (post-process) of the time domain noise samples collected by the oscilloscope. As can be seen, the PSD is flat over the \(2 \text{ MHz}\) bandwidth shown and has a two-sided PSD of approximately \(2.06 \times 10^{-4} \text{ W/Hz}\). The three noticeable spikes in the PSD are from RF interference coupled to the output of our device. However, the actual influence of the RF interference is considered negligible due to the bandwidth of each of the interfering RF signals relative to that of the bandwidth of the baseband transmitter (\(~666 \text{ kHz}\)). Therefore, we can reasonably assume that we operated over an AWGN channel.

![Figure 4.15 PSD of Thermal Noise](image)

The external timing circuit mentioned in Figure 4.14 uses a \(T\) flip-flop with the \(T\) input pulled to a logic level high while the clock input is connected to the chip select line
of the quadrature DAC of our baseband transmitter. In the described configuration, the \( T \) flip-flop will toggle the output from a logic level high to a logic level low whenever it detects a rising edge on the clock input (assuming positive edge triggering) [14].

Because the clock input is connected to the chip select line of the quadrature DAC, every time the microcontroller takes the chip select line from a logic level low to a logic level high to latch data into the DAC, the output of the external timing circuit will toggle states. Therefore, when the oscilloscope captures both the external timing waveform and the baseband waveform, the two can be aligned during post processing to define bit edges and bit duration. Figure 4.16 is the schematic used to construct the external timing circuit. Note that the SN74HC163N is a programmable 4-bit synchronous binary counter [13]. However, it internally uses four \( D/T \) flip-flops of which we make use of only one.

Figure 4.16 External Timing Circuit

Figure 4.17 clearly shows the external timing waveform aligned with the baseband transmitter waveform. During post processing the external waveform undergoes threshold detection and is transformed into an ideal rectangular pulse train. Obtaining a sample that represents the integration over a single bit simply becomes a
matter of summing the baseband transmitter waveform samples that fall between transitions of the ideal timing waveform.

To acquire enough data to obtain meaningful statistics we collected one second’s worth of data on the 300 MHz MegaZoom oscilloscope per trial. Collecting a second of data requires that the time per division be set at 100 ms/division; this in turn forces the oscilloscope to sample each channel at 4 MSa/s. The minimum bit duration that our hardware supports is 1500 ns. Therefore, if the baseband transmitter and oscilloscope were synchronized, each bit would contain precisely six samples. However, because the two devices are not synchronized the location of the oscilloscope samples within a bit will fluctuate as time progresses. This indicates that certain samples must be rejected in order to ensure that the samples we use represent the steady state value of a bit. Above we computed the settling time of our DAC to a step response to be approximately 302 ns and the measured abnormality near the end of a bit to be approximately 200 ns. The oscilloscope records a measurement every 250 ns and as such, the first two and last two
samples of every bit will be discarded to decrease the probability of corrupting the measured steady state value of a bit. Note that in actuality only the last sample of every bit needs to be discarded because the abnormality near the end of the bit is only approximately 200 ns and therefore does not have the potential to span two samples.

Once a trial was performed the raw oscilloscope data representing the ANB waveform and the external timing waveform were saved as two separate CSV (comma-separated values) files, each being approximately 40 MB in size. These files were then imported into Matlab® for post processing. The first step in post processing was threshold detection of the timing waveform. This produced the ideal rectangular pulse train mentioned above. Once the timing waveform had been formatted, the bit edges and bit durations of each bit in the ANB waveform were accurately known. The raw oscilloscope data which represented the ANB waveform plus AWGN noise was sent through a correlator that used the formatted timing waveform to determine bit alignment and duration. The output of the correlator was a sequence of samples that was cross-correlated with the known sequence of data such that BER analysis could be performed.

Before the ANB waveform could be sent to the detector, each user’s amplitude had to be estimated. For the two user ANB system we accomplished amplitude estimation by noting that there are only four possible output values; namely $\pm \alpha_i \pm \alpha_2$.

To find the amplitude estimate of each user we first defined $z[k]$ and $\hat{z}[k]$ to be the $k^{th}$ sample of the known and estimated ANB signal respectively. We then defined a set of indices $\{I_j\}, j=1, 2$, such that for $j=1$, $\forall i_1, |z[i_1]| = \alpha_2 + \alpha_1$ where $i_1 \in I_1$ and for $j=2$, $\forall i_2, |z[i_2]| = \alpha_2 - \alpha_1$ where $i_2 \in I_2$. The amplitude estimate for the second user was then
computed as $1/2 \left( E \left[ \hat{z} \left[ I_1 \right] \right] + E \left[ \hat{z} \left[ I_2 \right] \right] \right) = \hat{\alpha}_2$. Likewise the amplitude estimate for the first user was computed as $1/2 \left( E \left[ \hat{z} \left[ I_1 \right] \right] - E \left[ \hat{z} \left[ I_2 \right] \right] \right) = \hat{\alpha}_1$.

After amplitude estimation, the sampled ANB waveform was sent to the detector to estimate each user’s data sequence. The estimated data sequences were then compared to the known data sequences and the experimental BER for each user was then computed. Before a comparison between experimental and analytical results could be performed we needed to compute the average $E_b/N_0$ of the sampled ANB waveform. The average energy-per-bit for each user was readily available once amplitude estimation had been performed, leaving an estimate of the single-sided PSD (i.e., $N_0$) to be computed from the sampled ANB waveform. We obtained the noise density estimate by reconstructing a sampled ANB waveform using the estimated amplitudes of each user. The reconstructed ANB waveform was then subtracted from the experimental sampled ANB waveform to produce a sequence of samples that ideally represented the AWGN noise process. An estimate of the single-sided PSD was then found by computing the variance of this newly created sequence of samples. Once the average $E_b/N_0$ had been computed, the analytical BER for each user was calculated and compared with the experimental results.

### 4.8 Experimental Results

In the previous section we described the procedure for collecting and processing the experimental data. Here we show and discuss the results. Three sets of experiments were performed which collaborate the analytical analysis of the ANB BER performance. We focused our experiments on the two user synchronous ANB system because it is easiest to obtain results. Because we were using true thermal noise to generate the
AWGN channel the noise variance was considered constant (assuming the ambient temperature was constant during a trial) and could not be easily changed. Therefore, collecting NFR data proved to be the easiest means of direct comparison between experimental and analytical results. Each experiment consisted of several trials in which we varied the NFR of one user with respect to the other. As mentioned we collected 1 second of data for each trial. This provided approximately 667 kbits of data for the strong user and 333 kbits of data for the weak user (assuming a 1:2 ratio between the weak and strong users’ data rates). We also collected four trials per data point (i.e., at the same NFR) and averaged the data to compute the final experimental result.

The first experiment that we performed was to collect the BER performance of the weak user in a two user ANB synchronous system versus NFR where the data rate ratio was set at 2. Figure 4.18 shows the results of the experiment (red dashed line with diamond markers). We used the computed noise variance obtained during the experiment along with the estimated amplitude of each user and the achieved NFR to calculate the expected analytical result and plotted that in Figure 4.18 as well (blue solid line with circle markers). As can be seen in the results plotted in Figure 4.18, the experimental and analytical results are in very good agreement.
Our next experiment was to collect BER performance for the strong user in a two user ANB system and plot it against NFR where the data rate ratio was set at 2. Figure 4.19 illustrates the results that were obtained from the experimentation (red dashed line with diamond markers). As before, the analytical results are also plotted in the figure and were obtained by using the amplitude estimate of each user signal, estimated noise variance, and achieved NFR. It is seen in Figure 4.19 that the experimental and analytical results are again in good agreement and follow the same trend (i.e., towards single user BPSK performance).
The final experiment that was performed collected BER performance of the weak user in a two user ANB system where the data rate ratio was set at 8. Figure 4.20 plots the experimental results (red dashed line with diamond markers). Approximated results are also provided using the approximation models provided in Chapter 2. The difference between the simulation and experimental results at lower NFR in Figure 4.20 was unexpected. However, on closer examination it was clear that the difference was caused by the inaccuracy in the computation of the noise variance. The DACs that were used in the baseband transmitter have a $\pm 1/2$ LSB error in any transmitted code word. During amplitude estimation we assume that the user’s amplitude is constant over the entire sequence of samples and consider the rest of the samples AWGN. However, each user’s amplitude fluctuates within $\pm 1/2$ LSB of the set value. Recall that during the estimation
of the noise variance, the underlying ANB signal is reconstructed using the amplitude estimates of each user and then subtracted from the received signal. This, in theory, leaves only the AWGN signal. However, it also leaves behind a signal that consists of the fluctuations between the estimated amplitude and the $\pm 1/2$ LSB error caused by the DAC. This signal has the affect of increasing the overall estimated noise variance.

When the estimated noise variance is increased and used to approximate the results of the system you will see degraded BER performance (i.e., the curve moves upwards). This is the reason why the experimental results appear to achieve better BER performance than the approximated results in Figure 4.20.

![Figure 4.20 Experimental Weak Synchronous User NFR Energy BER Curves (M=8)](image)
The incorrect estimation of the noise variance was more noticeable in the experiment that used a data rate ratio of 8 than in the experiments that used a data rate ratio of 2, because there were four times as many samples per bit for the higher data rate ratio. Note that the $\pm 1/2$ LSB error in the DAC output does not necessarily have a zero mean. As such the affects of the $\pm 1/2$ LSB error are not averaged out with more samples but instead exacerbated. This is why there is a noticeable difference between the approximated and experimental results when the data rate ratio was set at 8.

In conclusion we have shown that the experimental and analytical results are in good agreement. We clearly conveyed the discrepancy between the approximation and experimental results obtained when the data rate ratio was set at 8, and showed that the experimental results corroborate the analytical models for the two user ANB system.
Chapter 5: Summary and Conclusions

5.1 Summary

In this thesis we proposed an additive narrowband communication system that allows both amplitude and bandwidth discrimination among multiple mobile users. We discussed the transmitter and receiver structures for both the uplink and downlink operating in a multi-user environment. Restrictions were imposed on the multi-user ANB system with arbitrary data rates to reduce the complexity of analysis: namely we reduced the system to a two user ANB system with fixed data rates of $R_{b,1}$ and $2R_{b,1}$ operating over an AWGN channel. The restrictions were imposed so that we could explore the preliminary feasibility of this type of ANB system.

Analytical BER expressions were derived for the restricted ANB system for both the uplink and downlink. Computer based simulation results were provided along with the analytical results for direct comparison, and showed excellent agreement. Analytical BER expressions for a two user ANB system with arbitrary data rates were not derived due to the exponential increase in the number of terms in the BER expressions as the data rate ratio between users increased. However, BER approximations were derived for a two user ANB system with arbitrary data rates. These approximations were compared to simulation results and illustrated two fundamental aspects of an ANB system. The first of these is the effect that an increased difference in bandwidth among user signals has on BER performance: specifically, BER performance tends to that of a single BPSK user as the bandwidth differences among users increases (the same is true for amplitude differences). The second aspect is the effect that correlated noise, which is fed back
during successive interference cancellation, has on BER performance. Correlated noise accounts for the differences seen between the approximation results and the results obtained through simulations—the approximations employ the uncorrelated noise model, whereas the simulations incorporate this correlation.

Finally, hardware was designed and developed to implement a physical baseband two user ANB system. This hardware was used to conduct a series of BER versus near-far ratio experiments in the laboratory. The experimental results obtained were compared to both the analytical and simulated results and showed excellent agreement.

5.2 Applications

One possible application of an ANB system is in conjunction with a preexisting narrowband system. It may be possible to overlay weaker and/or narrower-bandwidth users on top of a legacy system without significantly degrading its performance. Another application for which an ANB system could be used is in a point-to-point full duplex link. For this application a single terminal’s receive and transmit functions could be broken into two separate “users” each being assigned a unique amplitude and bandwidth. In a point-to-point full duplex link each terminal could also feedback the transmission signal into the successive interference cancellation receiver to aid in demodulation. A final proposed application of an ANB system is in a multi-sensor network where each type of sensor in the network has a different sampling rate requirement and therefore a different data rate requirement. For example if a small sensor network was measuring wind speed, ambient temperature, and humidity then it would not be unrealistic to expect each sensor to require a different sampling rate based on the expected rate of change of the measured...
quantity. We could consider each sensor as a user and implement an ANB system in which the base station could distinguish between the multiple sensors using bandwidth and amplitude discrimination.

5.3 Conclusions

The purpose of this thesis was to take the initial step in exploring the feasibility of an additive narrowband system which utilizes both amplitude and bandwidth discrimination among users. The successive interference cancellation incorporated into the receiver structure provided a BER performance improvement (over conventional demodulation) at the cost of receiver complexity. As expected, the BER performance of each user tends to that of a single BPSK user as the difference between amplitudes and/or bandwidths among user signals increases. The analytical BER derivations for the simplified two-user ANB system provided insight into how BER derivations for a multi-user, arbitrary data rate ANB system might be derived. However, the analytical BER derivations also illustrated the complexity (i.e., nested $Q$-function integrations) of computing BER data points. Therefore, we believe that good approximations for BER expressions will be required to analyze the more general multi-user arbitrary data rate ANB system. Both the simulated and experimental results corroborate the derived analytical BER expressions.

5.4 Future Work

We provide here a brief list of suggested work that would elaborate and extend the work described in this thesis.
The BER performance for a multi-user arbitrary data rate ANB system needs to be fully characterized. Either analytical BER expressions or good approximation for the BER performance for the multi-user arbitrary data rate ANB system would provide further insight into the overall system performance. The results obtained from such work would indicate how much degradation is induced by the increase in users as well as the importance that amplitude and bandwidth discrepancies among users has on BER performance.

Mobile communication systems rarely operate over a line-of-sight stationary Gaussian channel as modeled in this thesis. Therefore, it would be appropriate to characterize the performance of the ANB system when distortion like inter-symbol-interference (ISI) and fading multi-path propagation contribute to the received signal. Research into the performance of the ANB system over such real world channels would provide invaluable information regarding the appropriate applications for which the ANB system would be suited.

The serial interference cancellation receiver proposed in this thesis is only one way in which the received ANB signal could be detected. Another method might be to implement a parallel interference cancellation (PIC) receiver. Unlike the SIC, the PIC would decode each user signal at the same time. If only a single detection iteration was performed, the performance of all but the strongest user would likely suffer. However, if iterative detection was incorporated into the parallel structure the overall BER performance of each user would likely improve. The move towards both iterative decoding and parallel processing would increase system complexity, however, the gain in BER performance could warrant the extra effort. Other possible receiver structures for
detection in an ANB system exist, and researching alternative decoding methods would prove useful in maximizing overall system performance.

The BER performance gains that are achieved via forward error correcting coding have also been neglected in this thesis. However, if coding were incorporated into an ANB system it would likely allow both amplitude and bandwidth discrepancies among users to be relaxed while lowering the BER per user. As illustrated in the body of this thesis, the BER performance of each user tends to that of a single user BPSK signal when either the amplitude or bandwidth discrepancy among users increases. This is akin to saying that when the probability of an error decreases for a given user signal, then each subsequent user will achieve BER performance closer to that of a single BPSK user. Researching the achievable gains in BER performance of an ANB system via the use of forward error correcting coding would provide a more realistic system characterization, since virtually all modern communication systems employ error correction coding.


Appendix A

The primary use of this appendix is in the construction of the BER expression for the weak user in a two user ANB system with data rates set such that \( R_{b1} = 1/T_b \) and \( R_{b2} = 2R_{b1} \). As illustrated in Chapter 2, there are ten unique cases that need to be analyzed when computing the BER expression of the weak user (user one). Each case produces a random noise variable that is formed by the sum of two constrained Gaussian random variables. The probability density function of this random noise variable is required to compute the exact analytical BER expression for the weak user.

We start by describing a general method for finding the pdf of the sum of two constrained random variables. Because the two constrained Gaussian random variables are independent of one another we can find the pdf of their sum by using the convolution integral of their individual pdf’s [5]. The constraints placed on the individual Gaussian random variables create a discontinuity in the pdf resulting from their sum. For now we are only concerned with illustrating how to find a continuous, non-zero, portion of the pdf. In general we will have an equation that looks like (A.1), where \( f_{n_{0,1}}(x) \) and \( f_{n_{0,2}}(x) \) are the pdfs of the two constrained Gaussian random variables and \( a, b, \) and \( c \) are real values that define a continuous, non-zero, segment of the pdf denoted by \( f_{Y}(y) \).

Note that we are defining the random variable \( Y \) as \( Y = n_{0,1} + n_{0,2} \), where \( n_{0,1} \) and \( n_{0,2} \) are two constrained Gaussian random variables. When we substitute the constrained Gaussian density equations in for \( f_{n_{0,1}}(x) \) and \( f_{n_{0,2}}(x) \) in (A.1) we obtain (A.2), where \( \beta \)
and $\beta_2$ are scaling factors used to ensure that $f_{n_{0,1}}(x)$ and $f_{n_{0,2}}(x)$ fulfill the required pdf property of unity area, i.e., $\int_{-\infty}^{\infty} f_x(x) \, dx = 1$ [5].

$$f_Y(y) = \int_{a}^{b} f_{n_{0,1}}(x) f_{n_{0,2}}(y-x) \, dx, \quad y > c$$  \hspace{1cm} (A.1)

$$f_Y(y) = \int_{a}^{b} \frac{\beta_1}{\sqrt{2\pi\sigma_0^2}} \exp \left( -\frac{x^2}{2\sigma_0^2} \right) \frac{\beta_2}{\sqrt{2\pi\sigma_0^2}} \exp \left( -\frac{(y-x)^2}{2\sigma_0^2} \right) \, dx, \quad y > c$$ \hspace{1cm} (A.2)

Rearranging the terms in (A.2) we can create equation (A.3). The integrand in (A.3) is a Gaussian pdf with mean $y/2$ and variance $\sigma_0^2/2$. Therefore, the integral in (A.3) can be replaced with a difference of $Q$-functions, which is given in (A.4).

$$f_Y(y) = \frac{\beta_1 \beta_2}{\sqrt{2\pi(2\sigma_0^2)}} \exp \left( -\frac{y^2}{2(2\sigma_0^2)} \right) \int_{a}^{b} \frac{1}{\sqrt{2\pi(\sigma_0^2/2)}} \exp \left( -\frac{(x-y/2)^2}{2(\sigma_0^2/2)} \right) \, dx, \quad y > c$$ \hspace{1cm} (A.3)

$$f_Y(y) = \frac{\beta_1 \beta_2}{\sqrt{2\pi(2\sigma_0^2)}} \exp \left( -\frac{y^2}{2(2\sigma_0^2)} \right) \left( Q \left( \frac{2a-y}{\sqrt{2\sigma_0^2}} \right) - Q \left( \frac{2b-y}{\sqrt{2\sigma_0^2}} \right) \right), \quad y > c$$ \hspace{1cm} (A.4)

Equation (A.4) represents the generalized expression for a continuous, non-zero, segment of the pdf of the random variable resulting from the sum of two constrained Gaussian random variables. When deriving the pdf for each case we need only know the constraint imposed on each Gaussian noise sample. Then by using (A.4) and selecting the appropriate values for $a$, $b$, and $c$ we can quickly generate the pdf for each unique case.

Because these results are used for both the synchronous and asynchronous case we will generalize the pdfs for use in both systems. Let $\Omega = \left( \alpha_1 \cos(\phi_{1,2}) + \alpha_2 \right) T_s/2$ and
\[ \Psi = (\alpha_1 \cos(\phi_{1,2}) - \alpha_2)T_b/2, \] where \( \alpha_1 \) and \( \alpha_2 \) are the amplitudes of the weak and strong user signals, respectively, \( \phi_{1,2} \) is the carrier phase difference between the strong and weak user, and \( T_b \) is the bit duration for the weak user.

Table A.1 provides the scaling factors used for each unique case where \( Q_1 \) and \( Q_2 \) are given in (A.5) and (A.6) respectively. For clarity equation (A.5) and (A.6) use the terms \( E_{b_2} \) and \( \zeta \) where \( E_{b_2} \) is the bit energy for the strong user, and the amplitude ratio constant \( \zeta \) is defined by (2.14).

Table A.1 Probability Density Function Scaling Factors

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<th>Case</th>
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<th>( \beta_2 )</th>
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<td>1</td>
<td>1/( 1-Q_1 )</td>
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<td>1/( 1-Q_2 )</td>
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<td>1/( Q_1 )</td>
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<tr>
<td>10</td>
<td>1/( Q_1 )</td>
<td>1/( 1-Q_2 )</td>
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</table>

\[ Q_1 = Q \left( \sqrt{\frac{2E_{b_2}(1+\cos(\phi_{1,2})/\zeta)^2}{N_0}} \right) \quad (A.5) \]

\[ Q_2 = Q \left( \sqrt{\frac{2E_{b_2}(1-\cos(\phi_{1,2})/\zeta)^2}{N_0}} \right) \quad (A.6) \]

Table A.2 lists the constraints imposed on the Gaussian random noise variables for each case. These constraints define the range over which the pdf for the Gaussian random variable are non-zero. For clarity equations (A.7) and (A.8) explicitly define the pdf for the Gaussian random variables for case one where the scaling factors \( \beta_1 \) and \( \beta_2 \) are defined in Table A.1 under the column heading “Case 1”.
Finally the pdf resulting from the sum of the two constrained Gaussian random noise variables for each case are provided in equations (A.9) through (A.18).

\[
f_{n,1}(x) = \begin{cases} \frac{\beta_1}{\sqrt{2\pi}\sigma_0^2} \exp\left(\frac{-x^2}{2\sigma_0^2}\right) & x > -\Omega \\ 0 & \text{elsewhere} \end{cases}
\]  

(A.7)

\[
f_{n,2}(x) = \begin{cases} \frac{\beta_2}{\sqrt{2\pi}\sigma_0^2} \exp\left(\frac{-x^2}{2\sigma_0^2}\right) & x > -\Omega \\ 0 & \text{elsewhere} \end{cases}
\]  

(A.8)

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<tr>
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<th>(b_{2,2})</th>
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<th>(\hat{b}_{2,2})</th>
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<td>(x &gt; -\Omega)</td>
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<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>(x &gt; -\Omega)</td>
<td>(x &lt; -\Omega)</td>
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<td>(x &gt; -\Psi)</td>
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<td>-1</td>
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<td>(x &gt; -\Psi)</td>
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<td>+1</td>
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<td>(x &lt; -\Omega)</td>
<td>(x &lt; -\Psi)</td>
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<td>-1</td>
<td>-1</td>
<td>(x &lt; -\Psi)</td>
<td>(x &lt; -\Psi)</td>
</tr>
</tbody>
</table>

Finally the pdf resulting from the sum of the two constrained Gaussian random noise variables for each case are provided in equations (A.9) through (A.18).

\[
f_{n}(y) = \begin{cases} \frac{\beta_1\beta_2}{\sqrt{2\pi}(2\sigma_0^2)} \exp\left(\frac{-y^2}{2(2\sigma_0^2)}\right) \left[1 - 2Q\left(\frac{y + 2\Omega}{\sqrt{2\sigma_0^2}}\right)\right] & y > -2\Omega \\ 0 & \text{elsewhere} \end{cases}
\]  

(A.9)
\[ f_{y_1}(y) = \begin{cases} \frac{\beta_1 \beta_3}{\sqrt{2\pi (2\sigma_0^2)}} \exp \left(-\frac{y^2}{2(2\sigma_0^2)}\right) \left(1 - Q\left(\frac{y + 2\Omega}{\sqrt{2\sigma_0^2}}\right)\right) & y < -2\Omega \\ \frac{\beta_1 \beta_3}{\sqrt{2\pi (2\sigma_0^2)}} \exp \left(-\frac{y^2}{2(2\sigma_0^2)}\right) Q\left(\frac{y + 2\Omega}{\sqrt{2\sigma_0^2}}\right) & \text{otherwise} \end{cases} \tag{A.10} \]

\[ f_{y_2}(y) = \begin{cases} \frac{\beta_1 \beta_3}{\sqrt{2\pi (2\sigma_0^2)}} \exp \left(-\frac{y^2}{2(2\sigma_0^2)}\right) \left(1 - Q\left(\frac{y + 2\Omega}{\sqrt{2\sigma_0^2}}\right)\right) - Q\left(\frac{y + 2\Psi}{\sqrt{2\sigma_0^2}}\right) & y > -\Omega - \Psi \\ 0 & \text{otherwise} \end{cases} \tag{A.11} \]

\[ f_{y_3}(y) = \begin{cases} \frac{\beta_1 \beta_3}{\sqrt{2\pi (2\sigma_0^2)}} \exp \left(-\frac{y^2}{2(2\sigma_0^2)}\right) \left(1 - Q\left(\frac{y + 2\Omega}{\sqrt{2\sigma_0^2}}\right)\right) & y < -\Omega - \Psi \\ \frac{\beta_1 \beta_3}{\sqrt{2\pi (2\sigma_0^2)}} \exp \left(-\frac{y^2}{2(2\sigma_0^2)}\right) Q\left(\frac{y + 2\Psi}{\sqrt{2\sigma_0^2}}\right) & \text{otherwise} \end{cases} \tag{A.12} \]

\[ f_{y_4}(y) = \begin{cases} \frac{\beta_1 \beta_3}{\sqrt{2\pi (2\sigma_0^2)}} \exp \left(-\frac{y^2}{2(2\sigma_0^2)}\right) \left(1 - Q\left(\frac{y + 2\Psi}{\sqrt{2\sigma_0^2}}\right)\right) & y < -\Omega - \Psi \\ \frac{\beta_1 \beta_3}{\sqrt{2\pi (2\sigma_0^2)}} \exp \left(-\frac{y^2}{2(2\sigma_0^2)}\right) Q\left(\frac{y + 2\Omega}{\sqrt{2\sigma_0^2}}\right) & \text{otherwise} \end{cases} \tag{A.13} \]

\[ f_{y_5}(y) = \begin{cases} \frac{\beta_1 \beta_3}{\sqrt{2\pi (2\sigma_0^2)}} \exp \left(-\frac{y^2}{2(2\sigma_0^2)}\right) \left(1 - Q\left(\frac{y + 2\Omega}{\sqrt{2\sigma_0^2}}\right)\right) + Q\left(\frac{y + 2\Psi}{\sqrt{2\sigma_0^2}}\right) - 1 & y < -\Omega - \Psi \\ 0 & \text{otherwise} \end{cases} \tag{A.14} \]
\[ f_{x_i}(y) = \begin{cases} 
\frac{\beta_i \beta_z}{\sqrt{2(2\sigma_0^2)}} \exp\left(-\frac{y^2}{2(2\sigma_0^2)}\right) \left(1 - 2Q\left(\frac{y + 2\Psi}{\sqrt{2}\sigma_0^2}\right)\right) & y > -2\Psi \\
0 & \text{elsewhere} 
\end{cases} \] (A.16)

\[ f_{y_i}(y) = \begin{cases} 
\frac{\beta_i \beta_z}{\sqrt{2(2\sigma_0^2)}} \exp\left(-\frac{y^2}{2(2\sigma_0^2)}\right) \left(1 - Q\left(\frac{y + 2\Psi}{\sqrt{2}\sigma_0^2}\right)\right) & y < -2\Psi \\
\frac{\beta_i \beta_z}{\sqrt{2(2\sigma_0^2)}} \exp\left(-\frac{y^2}{2(2\sigma_0^2)}\right) Q\left(\frac{y + 2\Psi}{\sqrt{2}\sigma_0^2}\right) & \text{elsewhere} 
\end{cases} \] (A.17)

\[ f_{y_o}(y) = \begin{cases} 
\frac{\beta_i \beta_z}{\sqrt{2(2\sigma_0^2)}} \exp\left(-\frac{y^2}{2(2\sigma_0^2)}\right) \left(2Q\left(\frac{y + 2\Psi}{\sqrt{2}\sigma_0^2}\right) - 1\right) & y < -2\Psi \\
0 & \text{elsewhere} 
\end{cases} \] (A.18)
Appendix B

The following pages contain information pertaining to the baseband ANB transmitter that was designed for this thesis. The circuit schematic is provided in Figure B.1. PCB masks for the top layer, bottom layer, and overlay are shown to scale in Figures B.2, B.3, and B.4 respectively. Finally a list of parts used to construct the hardware is provided in Table B.1.
Figure B.1 ANB Baseband Transmitter Schematic
Figure B.2 PCB Mask: Top Layer
Figure B.3 PCB Mask: Bottom Layer
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