EXPERIMENTAL VALIDATION OF
ROLL FORCE AND PROFILE FILL FOR A PROFILE
RING ROLLING COMPUTER MODEL

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# TABLE OF CONTENTS

INTRODUCTION........................................................................................................ 1
1.1 The Aircraft Manufacturing Industry............................................................... 1
1.2 The Profile Ring............................................................................................... 2
1.3 Background....................................................................................................... 3
1.4 Development of a Computer Model................................................................. 6
1.5 Purpose of The Study...................................................................................... 7

LITERATURE REVIEW......................................................................................... 8

THEORY..................................................................................................................... 16
3.1 Mathematical Models....................................................................................... 16
3.2 The Slab Method.............................................................................................. 17
3.3 The Finite Element Method............................................................................ 19
3.4 Material Flow.................................................................................................. 25

EXPERIMENTAL EQUIPMENT AND PROCEDURES........................................28
4.1 Rolling Mill...................................................................................................... 28
4.2 Roll Specifications.......................................................................................... 28
4.3 Design of Experiments.................................................................................. 29
4.4 Experimental Billets .................................................. 31
4.5 Material Characterization ............................................ 31
4.6 Roll Force Data Compilation .......................................... 35
4.7 Percent Roll Fill ........................................................... 40

RESULTS ........................................................................... 43
5.1 Roll Force and Profile Fill Results ................................. 43
5.2 Experiment One ............................................................ 43
5.3 Experiment Two ........................................................... 46
5.4 Experiments Three and Eight ......................................... 48
5.5 Experiment Four .......................................................... 48
5.6 Experiments Five and Six ............................................... 52
5.7 Experiment Seven ......................................................... 52
5.8 Effects of Varying Parameters on Roll Force ................. 54
5.9 Percentage Difference ................................................ 62
5.10 Profile Filling .............................................................. 62

DISCUSSION ................................................................ 69
6.1 Project Summary .......................................................... 69
6.2 ROLLCAD ................................................................. 70
6.3 Experimental Results .................................................. 71
6.4 Problems Encountered ............................................... 72
CHAPTER 1

INTRODUCTION

1.1 The Aircraft Manufacturing Industry

The improvements of aircraft engines and the materials from which they are constructed has been an ongoing challenge in the manufacture of metals and alloys. Alloys, solid solutions containing two or more pure metals, have been available for centuries. However, within the last fifty years the focus has been to develop metals for high strength, high temperature, and high corrosion resistant applications (Shramko, 1994). The search for “super” alloys has increased competitiveness in the manufacturing industry and has forced a higher awareness of research and design, cost and performance.

One result of this has been the development of profile ring rolling for the manufacture of air seals and vane stator shrouds such as at Pratt and Whitney’s East Hartford, Connecticut facility. Due to its high material utilization, short production times, and preferred grain growth, profile ring rolling has generated a great deal of attention. Because the profile ring is considered to be near net shape, very little machining is required to finish a part and minimal material is wasted. Furthermore because profile rings are used in high temperature, high strength applications, there is the added benefits of preferred grain growth inherent in the cold rolling process. Although
the manufacturing industry will continue to search for more efficient solutions, at the present time profile ring rolling is a solution that shows promise for the near future.

1.2 The Profile Ring

Ring rolling was first introduced in 1842 by Bodmer who designed the first ring rolling mill to produce seamless rings. One of the first practical application of the ring rolling process was the manufacture of railway tires. Since then ring rolling has generated a lot of interest in the automotive and aerospace industries. The capabilities of rolling flat seamless rings has been extended to producing rings with a cross sectional profile which has improved the yield of the workpiece and decreased the amount of machining cost.

Profile rings are generated by rolling flat or preformed bar stock through top and bottom profile rolls with different diameters. The larger diameter roll, called the main roll, is usually powered while the smaller diameter roll, called the mandrel, remains idle. Once the bar stock or preform is rolled into a ring, the ends are butt welded. This initial stage produces the pre-formed cylinder which is further rolled until the final profile and ring size is achieved. A classical “T” section profile ring process is shown in Figure 1.2.1.
1.3 Background

Although ring rolling has been utilized for over one hundred years, until 1976 there was little research conducted in this area. The process is extremely complicated with sometimes unpredictable behavior and complex boundary conditions. As the ring grows, the profile fills, and the material becomes work hardened. It is difficult to predict the exact product of these events. Studies of ring rolling by Mamalis and Johnson in 1976 attempted to address the theoretical aspects of the process. Recently, the aid of Finite Element Analysis (FEA) has sparked a whole new interest in ring rolling. As
FEA’s capabilities have increased so has the understanding of the complex conditions of ring rolling. Simulations of ring rolling using a rigid-plastic finite element model have been presented by a number of researchers.

The manufacturing industry typically relied on proven empirical methods in developing the ring rolling process. The time and money consumed by this trial and error approach has been greatly reduced by utilizing an initial approximate mathematical model for predicting power requirements, pressure distribution, workpiece preform, speed of rolls, reduction per pass, and other roll parameters. The results of this mathematical analysis can ultimately be used as initial input to a more accurate three dimensional finite element program capable of predicting defect formation, profile filling, material flow, and microstructure. This two-stage design methodology (Fig. 1.3.1), which is a more systematic approach, saves time and money during development. With these more accurate tools it is now feasible to map the complex three dimensional flow characteristics of the workpiece. It is through the use of both the mathematical models developed for slab rolling and such techniques as finite element that the complexities of profile ring rolling are becoming better understood.
Figure 1.3.1 Two-Stage Design Methodology for Profile Ring Rolling.
1.4 Development of a Computer Model

The Ohio University Department of Mechanical Engineering in 1992 developed a computer model for Pratt & Whitney to aid in the development of a near net-shape ring rolling processes. The process can be used to produce ring and shroud details with simple and complex shapes. The rings, currently manufactured by Pratt & Whitney, are forgings requiring considerable machining before the part is finished. Ring rolling is an alternative manufacturing process aimed at reducing the raw material cost of the part by beginning with less material. The design and development of the profile rolls are crucial factors to the success of the rolling process development. The computer models developed by Ohio University help to reduce the expensive trial and error development approach used by Pratt & Whitney in the 1990’s.

Computer simulation consists of both approximate and accurate model. The approximate model program, executed on a personal computer using AutoCad software, can, by using mathematical and empirical models, predict machine parameters such as roll force, torque, power, heat transfer, microstructure, and preferred part preforms. These approximate parameters can then used to construct a more accurate model based on finite element analysis. The more accurate model is complex in that it considers the process in three dimensions, presenting a more exact idea of the process parameters. In addition, die fill and potential defects such as out of round can be predicted.
1.5 Purpose of The Study

Developing a computer program to predict manufacturing parameters requires the use of both mathematical and empirical models. Although these models are based on sound principles that have been validated throughout history, there still remains the question of accuracy. The ring rolling processes developed for Pratt & Whitney consisted of rings with complex shapes that could not be directly compared to past research. Without initial testing to validate the accuracy of the software, there could be no confidence in the results. The purpose of this research was to validate the roll profile and the roll force predicted by the computer models.
CHAPTER 2

LITERATURE REVIEW

In the manufacture of jet engines, profile ring rolling is very attractive because of the reduced production costs. However, research, development, and validation for ring rolling is extremely difficult and expensive due to the unpredictability of the complex material flow. Thus, relatively little research has taken place until recently when the aid of computers changed what had been, heretofore, largely trial and error.

The analyses developed for roll force and torque in flat rolling are not appropriate for ring rolling. There are major differences between flat and ring rolling. Ring rolling requires relatively small reductions, the diameters of the rolls are unequal, and one roll is undriven (Mamalis, 1976). As a result of the difference in diameters of the top and bottom rolls different velocities are imposed within the cross section of the roll gap. Therefore, the deformation patterns in the plastic region between the rolls differs from those in flat rolling.

A detailed analysis of profiled rings was published by Mamalis, Johnson, and Hawkyard in 1976. In their study, a T-shape is formed from an initially rectangular cross-section. The main roll is flat with the mandrel containing the column of the T-section. A mathematical relationship for the roll force was formulated and the driving
torque was obtained from the resultant forces and the lever arm (length of the perpendicular on the line of action of the force from the center of the main roll).

Flow patterns in the roll gap have been studied using various techniques. In 1883 Hollengberg inserted rods of the same material into a workpiece. Others have used such techniques as scribing a square network of lines on the outside surfaces of the workpiece. Orowan (1943) in carrying out rolling tests on Plastacine bars, found that the Plastacine laminae became curved in a direction opposite to that of rolling; thus indicating the inhomogeneity of the straining process. Mamalis, Hawkyard, and Johnson (1976) scribed a fine network of equidistant concentric circles and radii at equal angular positions on the top, bottom and side faces of an initial blank. By drilling holes axially and radially and inserting rods of similar material, the flow of the material inside the blank was examined. They found that the concentric circles remain concentric after the deformation but the distance between them changes due to the unequal stretching of the various regions of the ring. The edge closest to the main roll stretched the least. It was also found that the straight radii curved in the rolling direction, indicating that in the actual rolling process there is a combined bending and shearing effect which is most pronounced at fast feed-rates. Also, the peak of the curve or the point at which a change in curvature of the distorted radii occurs corresponds to the position of minimum spread in the central regions of the ring.

In commercial ring-rolling operations, the edge shape of the ring is generally controlled by the use of edge rolls or flanges on the main roll. When rolling rings are
unconstrained axially, spreading of the material in this axial direction occurs and an irregular and non-rectangular spread profile is developed. This is referred to in the industry as “fishtail”. In general, spread increases with increasing total reduction (Mamalis, 1976).

Mamalis, Hawkyard, and Johnson (1976) also found cavity formation while rolling T-shaped rings at large total reductions. They discovered that for certain values of feed-rate, groove and shape factor, a specific type of cavity formation appears. Separation of the material from the main roll occurred where earlier it was in contact. The cavity ran along the length of the billet on top of the T-shape directly across from the column of the “T”. This formation was compared to that which sometimes occurs on the rear end of an extruded product; i.e. the beginning of piping or cavity formation on the bottom of the slug during the final unsteady phase of extrusion.

For theoretical investigations of the rolling processes, the required force can be calculated from the stress applied over the region of contact between tool and workpiece. However, experimental validation must support the assumptions made. One of the most accepted methods for measuring contact pressure is the “pin technique”. A small diameter pin passes through the body of the tool making contact with the workpiece as it is deformed between the roll gap. The force is measured with a load cell equipped with electro-resistance strain gauges or with piezo-electric crystals. The first successful attempt at measuring the pressure in rolling was made by Siebel and Lueg (1933).
Since then, there have been numerous investigations into the theoretical nature of rolling. The slab, the slip-line, and the upper-bound methods have been widely used. However, due to the complexities involved, various degrees of simplifications and assumptions have been employed. Although these methods have provided useful information they are limited. The development of the finite element method has offered a powerful new tool to predict deformation behavior more accurately.

Over the years, several attempts have been made to solve the rolling problem by the use of the finite-element technique. Tamano (1973) used the elastic-plastic material model which is suitable only for small reduction rolling. Oh and Kobayashi (1974) developed a finite element model for the analysis of three-dimensional deformation in rolling. Their model was based on the extremum principle of rigid perfectly plastic materials. Theoretical solutions were obtained for single-pass rolling in terms of sideways spread, roll torque and the location of neutral points. The theoretical results for axial spread revealed that the amount of spread decreases as initial width-to-height ratio increases and as the initial thickness (with respect to roll diameter) increases. Zienkiewicz (1978) and Dawson (1978) individually addressed the rolling problem in terms of visco-plasticity in which temperature effect were incorporated into the computation. Kanazawa and Marcal (1978) used Oh’s and Kobayshi’s (1974) upper-bound method in finite-element form. Their model took into account the sideways spread in rolling which differs from the plain-strain assumed by the others. Kobayashi (1982) solved the plain-strain rolling problem by the rigid-plastic finite-element method, based
on the infinitesimal theory of plastic deformation. Solutions for both steady-state and nonsteady-state problems were solved. His solution calculated the velocity field in the deforming material and the external forces along the arc of contact. From these solutions it is possible to calculate roll separating force, roll torque, and roll pressure.

The already complex problem of rolling becomes increasingly complex to analyze with such added variables such as profiles and rings. Ryoo and Yang (1986) developed a simple method of analysis using force polygon diagrams for determining roll torque and pressing load in plain strain ring rolling. Pressing velocity together with driven roll rotation was incorporated in constructing a hodograph for upper-bound analysis in order to find an optimized tangential velocity pattern. Results from two types of tangential velocity discontinuity patterns and related force polygon diagrams were compared at optimized values. The ratio of resultant friction force and pressing load, or an equivalent coefficient of friction was determined from the force polygon diagrams.

Again in 1987, Ryoo and Yang tackled the plain-strain ring rolling problem. Using the concept of “equivalent” coefficient of friction, the ratio of frictional force of the driven roll to pressing load of the pressure roll was determined. Considering the ring geometry and the related kinematics, the relationship between equivalent coefficient of friction and process parameters was derived. A technique introduced by Tszeng and Altan (1991) separates the 3-D finite-element ring rolling problem into a 2-D in-plane flow and a 1-D out-plane flow (in rolling direction). The metal flow was studied by using a pseudo plane strain scheme which was incorporated into a finite element method-based
model. The average deformation of a ring during each rolling revolution was approximated by a kinematic steady state deformation process. The out-plane deformation was solved semi-analytically while the in-plane deformation was derived by the rigid-plastic finite element method.

The rigid-plastic finite element method (FEM) is now regarded as one of the most useful techniques for metal forming analysis. In profile ring rolling FEM most accurately predicts the complex material flow within the roll gap. With the development of more powerful computers, simulations have become more detailed. However, because of the increasing demands FEM places on the computer it is necessary to simplify the models. Kim and Yang (1989) analyzed only a segment spanning the roll gap, since the ring rolling process is incremental and, at any given time, the deforming region is restricted to the vicinity of the roll gap while the remainder of the workpiece is relatively rigid. Therefore, the boundary conditions were given by the kinematics of the ring rotation.

More recently Kim, Machida, and Kobayshi (1990) analyzed T-section profile ring rolling by a three-dimensional rigid-plastic fine element method using a different approach in which the entire ring was meshed using larger elements. Even with the use of larger incremental elements, there were problems with computer memory size and computation time when handling complicated profiles. The power of today’s computers will not allow for a full mesh of the ring when acquiring accurate results. The larger the incremental mesh becomes the less accurate the results. Yang, Kim, and Hawkyard (1990) used a three-dimensional rigid-plastic finite element method to simulate profile
ring rolling of a T-shaped section from an initially rectangular cross-section, without any axial constraints, the same truncated approach as Kim and Yang (1989). However, they used finer elements and pertinent remeshing for the deforming region. This allowed for very accurate results with the same computing capacity. As compared to meshing the entire ring this method assumes that once the material exits the roll gap, it follows rigid motion with the axis of the ring until it enters the roll gap at the next pass.

Although FEM provides a more accurate and complete picture of the profile ring rolling process, there is no substitute for experimental research. Hawkyard and Moussa (1984), experimenting with closed-pass rolling of a range of symmetrical and unsymmetrical profiles, found that the material flow associated with each profile was complex and each profile differed from the others. In addition, most of the profiles tested proved to be impossible to roll from initial rectangular blanks. Lubrication did not improve profile filling significantly and roll closure rate had only a small effect. The incorporation of entry radii and tapered faces to assist flow appear to have allowed for some improvement. However, the most useful tool in filling the profile was the use of pre-formed blanks which, in the roll gap, decreased the axial flow, allowing the material to fill the desired profiles.

Although research on ring rolling has not heretofore been abundant, FEM has sparked renewed interest due to its capability to accurately predict complex material flow. However, FEM is very taxing on today’s computers. Especially when the number of elements is increased for more accurate models. The research efforts at Ohio University
have focused on finding a more efficient way to develop the profile ring rolling process and to minimize the time consuming and costly finite element models. Ohio University has developed an approximate model on the personal computer. Using the approximate model in conjunction with the more accurate finite-element model, it is now possible to devise a solution for ring rolling profiles with minimal simulation effort. In this study, experimental work was performed to validate the accuracy of the computer models developed by Ohio University.
CHAPTER 3

THEORY

3.1 Mathematical Models

The development methodology for profile ring rolling begins with an approximate model. Using empirical formulas and the slab method (an approximate mathematical method for estimating pressure distribution in rolling), an initial ring forming strategy can be formed. The approximate mathematical method provides estimates on compression roll design, workpiece preform design, and such equipment parameters as power requirements, roll speed, and reduction per pass. Because these mathematical and empirical models can be run quickly on an IBM PC they are very cost effective. These initial approximated results provide a starting place for developing a more accurate finite element model.

Finite element analysis (FEA) gives a relatively accurate description of what can be expected during an actual ring rolling trial. However, FEA is very demanding for today's computers and the time and money consumed for the simulation can be costly. Therefore, the parameters presented in the approximate model will help to minimize the number of FEA simulations required.
3.2 The Slab Method

In this study actual lead billets were profile rolled and the experimental forces were compared to the predicted results in the approximate model using the slab method. The theory behind the slab method for ring rolling is subdividing the roll “bite” into a finite number of small equal intervals. Figure 3.2.1 shows the equational parameters. Here \( R_2 \) is the main roll radius, \( R_1 \) the radius of the mandrel, and \( R_0 \) the outer diameter of the ring before the next pass. The “bite” angles measured from the center of the main

Figure 3.2.1. Roll “Bite”.
roll, the center of the mandrel, and the center of the ring are represented by \( \psi, \Phi, \) and \( \theta \), respectively.

The slab model for rolling becomes more complex in ring rolling because of the differing diameters of the mandrel and the main roll. In ring rolling, the curvature of the ring must also be taken into account, making the slab analysis a bit more complicated.

The free body diagram (Figure 3.2.2) shows the force acting on any elemental section along the roll “bite” in strip rolling. Force equilibrium leads to the following two differential equations for the top and bottom sections of the workpiece, respectively.

\[
\frac{d(p.h.)_1}{d\theta} = \{S_1 \sin(\Phi - \theta) \pm \tau_1 \cos(\Phi - \theta)\} R \frac{d\Phi}{d\theta} + \\
\{S_2 \sin(\psi - \theta) \pm \tau_2 \cos(\psi - \theta)\} R_2 \frac{d\psi}{d\theta}
\]

\[
(3.2.1)
\]

\[
\frac{d(p.h.)_1}{d\theta} = \{S_1 \sin(\Phi - \theta) \pm \tau_1 \cos(\Phi - \theta)\} R \frac{d\Phi}{d\theta} + \\
\{S_2 \sin(\psi - \theta) \pm \tau_2 \cos(\psi - \theta)\} R_1 \frac{d\psi}{d\theta}
\]

\[
(3.2.2)
\]
where,

\[ S_1 \] is the roll pressure exerted by the mandrel

\[ S_2 \] is the roll pressure exerted by the main roll

\[ \tau_1 \] is the shear (friction) between the mandrel and the ring

\[ \tau_2 \] is the shear (friction) between the main roll and the ring

Equations 3.2.1 and 3.2.2 can be solved numerically using a fourth order Runge Kutta algorithm to obtain the roll pressures \( S_1 \) and \( S_2 \).

3.3 The Finite Element Method

Prior to the 1970’s, mathematical modeling of metal forming processes incorporated one of several approximate methods. These approximate methods include slab analysis for approximating stress, the slip-line method characterized by perfectly plastic material and plane strain conditions, and the upper-bound method utilizing energy principles. Although these methods have provided very useful information, a more accurate model has been developed. Under more realistic conditions, accurate determination of detailed metal flow in forming processes has recently been made.
possible with the introduction of the finite element method.

Finite element can accurately model the forming process by dividing the cross-section of deforming material, for the case of axisymmetric or plan-strain processes, into a two-dimensional network of discrete elements called “finite elements”. The deformation at selected points, “nodes”, in each element is determined by the application of some variational principle. By analyzing the mesh of elements one at a time, the deformation pattern in a complex shape can be determined. Furthermore, the accuracy of the model can be increased by dividing the finite elements into an even finer mesh. However, increasing the number of elements can have a profound effect on hardware and computational time. Thus, the application of the finite element method to more complex problems has been limited by the power of today’s computers.

Two of the most popular finite element methods presently in use are the rigid-plastic and the rigid-viscoplastic. Both of these mathematical models resulted from the infinitesimal theory of plasticity. The infinitesimal theory of plasticity is the result of ignoring the elastic part of deformation. This method has been proven to be adequate for most practical applications. A more complete approach to metal forming analysis involves a more complete elastic-plastic approach that requires a large deformation theory of plasticity. It is also a more complicated mathematical approach. The elastic-plastic finite element method has been developed to analyze a class of metal forming problems wherein controlling springback and residual stresses are very important.
The ANTARES finite element code used in this study is an updated Lagrangian form that uses an approximate time integration scheme. ANTARES is capable of analyzing elastic-plastic, rigid-plastic, rigid-viscoplastic and rigid-thermoviscoplastic metal forming problems for both two-dimensional (2D) and three-dimensional (3D) cases. Triangular elements are used in 2D analysis and are automatically generated using the Quadtree Algorithm. A tetrahedral formulation using four and ten-node elements is used for the 3D analysis.

The rigid-viscoplastic material is an idealization of an actual material. This is accomplished by neglecting the elastic response. The elastic response is considered negligible due to the short period at which the material is in the elastic state. The material shows the dependence of flow stress on strain-rate in addition to the total strain and temperature. This material can sustain a finite load without deformation. The rigid-viscoplastic material simplifies the solution process with a less demanding computational procedure.

Although three dimensional analysis was used in this study, for simplicity, the following theory which supports two dimensional analysis of ANTARES is presented. Consider a body of volume $V$ at a generic moment with the traction $F$ prescribed on a portion of the surface, $S_F$, and the velocity $U$ on $S_U$. Let $S_C$ be the remainder of the surface where the frictional stress acts. The deformation of the body $V$ is characterized by the following equations:
Equilibrium Conditions, Neglecting Body Forces

\[ \sigma_{i,j,j} = 0 \]  \hspace{1cm} (3.3.1)

Where \( \sigma_{ij} \) is the stress component and the comma, “,” denotes the differentiation.

Strain-Rate and Velocity Relation

\[ \varepsilon_{ij} = \frac{1}{2} (v_{i,j} + v_{j,i}) \]  \hspace{1cm} (3.3.2)

Where \( \varepsilon_{ij} \) and \( v_i \) are the strain-rate component and a velocity component, respectively.

Constitutive Relation

\[ \sigma' = \frac{2}{3} \bar{\varepsilon} \varepsilon_{ij} \]  \hspace{1cm} (3.3.3)

Where, \( \bar{\sigma} \) is the effective flow stress and \( \bar{\varepsilon} \) is the effective strain-rate. They are defined by \( \sqrt{\frac{3}{2} \sigma'_i \sigma'_j} \) and \( \sqrt{\frac{2}{3} \varepsilon_{ij} \varepsilon_{ij}} \), respectively. The effective flow stress is, in general, a function of temperature, total strain, strain-rate and temperature. It should be noted that
equation (3.3.3) relates the deviatoric stresses to strain-rates and implicitly states incompressibility $\varepsilon_{ii} = 0$.

**Boundary Conditions**

\[ \sigma_y n_i = F_j \text{ on } S_f \quad (3.3.4) \]

\[ v_i = U_i \text{ on } S_u \quad (3.3.5) \]

\[ |f_s| = \text{given, sign } (f_s) = \text{sign } (\Delta v_s) \text{ on } S_c \quad (3.3.6) \]

Where $n_i$ is a component of unit normal to the surface, and $\Delta v_s$ is the slipping velocity.

The above equations (3.3.1-3.3.6) can be put into the variational principle as:

\[ \delta \Phi = \delta \left[ \int_V E(\varepsilon) dV + \int_V \frac{1}{2} K \varepsilon_{kk} dV - \int_{S_c} \left\{ \int_{S_c} f_{sbr} \right\} dS - \int_{S_f} F_j v_j dS \right] = 0 \quad (3.3.7) \]
The first term in equation (3.3.7) is the workfunction $E(\vec{e})$, which can be expressed as follows:

$$E(\vec{e}) = \int_0^\infty \bar{\sigma} d\bar{e} \quad (3.3.8)$$

Here, $K$ is a large positive constant which penalizes the dilatational strain. The above functional $E(\vec{e})$ reduces to that of a rigid-plastic material if the effective flow stress $\bar{\sigma}$ is expressed as a function of $\bar{e}$ only. In this case, $E(\vec{e})$, the work function is not strictly convex, and the uniqueness argument follows that of a rigid-plastic material. The penalty constant $K$ has been substituted for the Lagrangian multiplier, $\lambda$, which has been proven to be equal to the hydrostatic stress component. The penalty constant $K$ is used because less computer storage space and computer time is required per iteration. The use of the Lagrangian multiplier variable increases the bandwidth and the number of equations. Also, because some of the main diagonal terms inadvertently reduce to zero, it is necessary to take additional precautions in the programming procedure. Thus, it can be shown that $\lambda = \sigma_m = K\varepsilon_{kk}$. 
3.4 Material Flow

Deformation patterns in a given cross-section, their relation to material type and pertinent process parameters, remain to be better understood as they are critical in the prediction of surface defects and residual stresses that lead to material loss and instability problems (Eruc 1991). In profile rolling, as well as flat rolling, the material can flow both in the rolling direction and across the work piece as transverse spread. The flow rate of the material is greater near the surface of the workpiece. The difference in material flow throughout the cross section produces a phenomenon commonly known as “fishtailing” (Fig. 3.4.1). The fishtail can be seen on the sides of the billet due to transverse spread and at the exit end of the billet due to flow in the rolling direction. The fishtail suggests that the material in the middle of the billet is resisting deformation and the material near the surface is promoting deformation. The difference in material expansion creates residual stress throughout the workpiece. The material in the middle is under tensile
stress due to resisting the outer material from flowing. Therefore, the outer material is under compressive stress.

The residual stresses inherent in rolling are more pronounced when a profile is introduced. The profile, with its peaks and valleys, creates an uneven growth throughout the cross section. The valleys are under a much greater total reduction because they flow faster than the peaks. For the same reasons as the fishtailing phenomenon, residual stresses are imposed. Figure 3.4.2 shows the compressive and tensile stresses in a simple cross section. As can be seen, residual stress can be the source of a common defect known as cracking. Areas of high velocities can experience tears in material in zones of low velocity.

Again, the material flow pattern changes when rolling profile slabs into rings.

Figure 3.4.2. Residual Stresses.
Since the inner roll of ring rolling is smaller than the outer it promotes slower radial growth. The surface material near the smaller roll flows faster than the material in the middle. However, the outer surface of the ring must travel a greater distance than the inner, causing a difference in deformation rate. Mamlis, Hawkyard, and Johnson (1976), noted these effects when rolling profile rings. They scribed a network of equidistant concentric circles and radii at equal angular intervals on the top, bottom, and sides of an initial blank to detect material flow on the outer surface. The inner flow was analyzed by drilling holes axially and radially and inserting rods of similar material. They found that the concentric circles remain concentric after the deformation but the distance between them varies due to the unequal stretching of the various regions of the ring. The material stretched least at the edge close to the inner roll.

The material flow characteristics in profile rolling can have a profound effect on both roll force and profile fill. The different velocities of the material in the roll gap have the greatest effect causing redundant work. Material shears between the two velocity zones increasing the energy required to develop the shape, as a result the rolling force is increased. Profile fill is also effected due to the stretching of the material. As the work piece is stretched between different velocity zones it is restricted from flowing into peaks and valleys of the profile. The velocity zones of profile ring rolling can be accurately modeled by the Finite Element Method to improve development lead-times.
CHAPTER 4

EXPERIMENTAL EQUIPMENT AND PROCEDURES

4.1 Rolling Mill

The experimental rolling for this research was performed at Ohio University on a Stanat rolling mill. The rolling mill has a maximum closing force of seventy five thousand pounds. The maximum experimental force was less than nine thousand pounds. The top and bottom rolls are equally powered by a variable speed gearbox. The speed of the rolls was set at thirty three revolutions per minute for all experiments.

The roll gap was controlled mechanically by manually rotating the crank wheel of a worm gear drive. One revolution of the crank corresponds to a change in roll gap of one six-thousandths of an inch. Micrometer dials on the wheel shaft allow for accurate positioning of the rolls. Photographs of the Stanat Rolling Mill can be seen in Appendix A.

4.2 Roll Specifications

The experimental rolls were designed to model a typical near-net shape roll formed compressor shroud found in jet engines produced at Pratt and Whitney’s air craft engine plant in East Hartford, Connecticut. The preferred grain growth in cold rolling is ideal for the extreme temperatures and pressures these parts experience. Pratt and Whitney previously machined these parts from forgings, but the profile generated by rolling greatly increases the yield of the material. The experimental rolls at Ohio University were machined from forged 4140 MHT steel at Ohio University. The profile rolls and their nominal dimensions were designed to prevent material from flowing in the
axial direction since, as noted by Eruc (1991), rolling is better performed in closed passes. [Figure 4.2.1]

Figure 4.2.1. Experimental Rolls.

4.3 Design of Experiments

Preparing the equipment and materials for rolling was an essential aspect in obtaining accurate results. The rolling was designed to gain the most information from a limited number of experiments, taking into account the impact of changing roll
parameters. In order to gain the optimum amount of information on the effects of roll parameters during the profile rolling, two-level fractional factorials were used. Fractional factorials are arrays that indicate the level and number of runs at which the experiments should be performed. These arrays represent only a fraction of all the possible experiments that could be conducted.

There are many factors that can be considered in profile rolling. However, this study focuses on three of the most controlling factors: initial billet thickness, frictional conditions (lubrication), and reduction percentage. The factorial case considered is that of $k$ parameters, each at two levels. Each parameter is varied between its respective high and low extremes. Therefore, the number of experiments conducted was $2^3$ or $2*2*2 = 8$. The experiments were set at either 20 or 30 percent, lubrication or no lubrication, and 0.4 or 0.45 initial thickness. [Table 4.3.1]

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Initial Thickness</th>
<th>Lubrication</th>
<th>Percent Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.45 inch</td>
<td>No</td>
<td>20%</td>
</tr>
<tr>
<td>2</td>
<td>0.45 inch</td>
<td>No</td>
<td>30%</td>
</tr>
<tr>
<td>3</td>
<td>0.40 inch</td>
<td>Yes</td>
<td>30%</td>
</tr>
<tr>
<td>4</td>
<td>0.40 inch</td>
<td>Yes</td>
<td>20%</td>
</tr>
<tr>
<td>5</td>
<td>0.45 inch</td>
<td>Yes</td>
<td>20%</td>
</tr>
<tr>
<td>6</td>
<td>0.40 inch</td>
<td>No</td>
<td>30%</td>
</tr>
<tr>
<td>7</td>
<td>0.40 inch</td>
<td>No</td>
<td>20%</td>
</tr>
<tr>
<td>8</td>
<td>0.45 inch</td>
<td>Yes</td>
<td>30%</td>
</tr>
</tbody>
</table>

Table 4.3.1. Experimental Array.
4.4 Experimental Billets

The test material used for the experiments was lead, obtained as bar stock with rough dimensions of 4" x 4" x 1". To insure accurate results, it was necessary to keep the thickness and width of each initial test billet consistent. The lead billets were carefully flat rolled to initial rolling thickness of 0.45 inches or 0.40 inches, depending on the experimental array mentioned above. Since the width of the billet can effect roll parameters such as roll force, frictional effects, and die fill, they were machined to 5.385 inches. The relative length of the billets varied between 5.0 and 6.5 inches. Since the length of the billet was not a factor in evaluating the results, there was no need to maintain a consistent billet length.

4.5 Material Characterization

When analyzing a process with an approximate model, it is necessary to determine the properties of the working material. The behavior of a material from initial yield to ultimate load is adequately described by an expression of the form

\[ \sigma = Ke^n \]  

(4.5.1)

where, for an induced strain \( \varepsilon \), the corresponding value of stress \( \sigma \) is the new yield strength caused by the degree of cold working that induced the strain (Hosford, 1983). If Eq. (4.5.1) is descriptive of the plastic behavior of the metal, the simplest approach is to plot the stress/strain data on logarithmic coordinates. A power-law expression plots as a straight line on a log/log scale.
A sample piece of the rolling material was tested on the UTM machine at Ohio University. The stress/strain results of the test were plotted on a log/log scale. The numerical values of $K$ and $n$ from Eq. (4.5.1) were found by linear regression on a spreadsheet and the analytical curve was then superimposed on the graph. The resulting analytical model is a good approximation of the material properties. [Figure 4.5.1]

![True Stress & Strain For Lead](image)

Figure 4.5.1. Stress Strain for Lead.

The range of data presented represents the "fully plastic" condition of the material. The slope of this line defines the strain-hardening exponent, $n$, and the intersection of this line with unit strain gives the stress value, defining the magnitude of $K$ in Equation (4.5.1).
Although the work-hardening constants have been defined, for some materials, such as lead, flow stress is also dependent upon strain rate. When the flow stress is dependent on strain rate, the flow stress is described by an expression of the form

\[ \sigma = K \varepsilon^n \beta^m \]  

(4.5.2)

where

\[ \beta = \text{Strain Rate} \]

\[ m = \text{Strain Rate Constant} \]

Since the rate of deformation changes throughout the arc of roll contact, it is necessary to define an average strain rate. The average strain rate for rolling is defined by Eq. (4.5.3) and is illustrated in Figure 4.5.2.

![Figure 4.5.2. Average Strain Rate.](image)
where \( r \) is the fractional reduction in the pass, namely

\[
\beta = V \left( \frac{r}{R h_l} \right)^{\frac{1}{2}}
\]

(4.5.3)

The strain rate constant \( m \), according to a materials handbook is \( m = 0.04 \), a typical value for lead at room temperature. Since the average strain rate is a constant, Eq. (4.5.3) can be used in the form

\[
\sigma = K' \dot{\varepsilon}^n
\]

(4.5.5)

where

\[
K' = K \beta^m
\]

(4.5.6)

Therefore, the work hardening constants used in the approximate model were the constants \( K' \) and \( n \).
4.6 Roll Force Data Compilation

Force measurements in the laboratory were collected using PC based data acquisition equipment. The force data for all experiments can be seen graphically in Appendix B. In this study experimental roll force was compared to a mathematical model known as the “slab” method. In addition, the effects of varying parameters were considered.

When mathematical models are used to predict forming parameters the accuracy of the results depends on the users assumptions. The assumptions made in this study will be presented as they occurred. The first assumption to be explained is actually a normalization procedure. One of the parameters varied in the experiments was percentage of reduction. For the case of profile rolls percent reduction can be subjective. As for the simpler case of closed pass flat rolling the percent reduction is defined as

\[
\% \text{REDUCTION} = \frac{(A_o - A_f)}{A_o} \tag{4.6.1}
\]

where:

- \(A_o\) is the initial cross sectional area of the workpiece.
- \(A_f\) is the final cross section after reduction.

However, the cross sectional area of profile rolling can not be defined until there is complete die fill. The profile rolls only make partial contact with the workpiece during
the initial rolling passes. However, during profile rolling, the roll gap can actually travel farther than the initial thickness of the workpiece. [Figure 4.6.1.]

The high and low values of reduction considered in this study were 30% and 20% respectfully. The reduction was calculated using the actual change in roll gap, $\Delta H$. Since the rolls were designed for closed pass rolling the width remained constant, $W=5.385$

Figure 4.6.1. Profile Roll Gap
(in). In this case, \( \Delta H \) can be substituted for \( (A_o - A_f) \) and \( H \) for \( A_o \) in Equation 4.6.1.

The slab method assumes there is complete die fill. In fact, the slab method is based on a rectangular cross section. Therefore, it was necessary to normalize the percent reduction used in the approximate model. The reduction value used in the model was based on the final cross sectional area, \( A_f = 1.1 \) (in\(^2\)), and the actual change in roll gap.

\[
A_o = A_f + (\Delta H)(W) \tag{4.6.2}
\]

Of course, due to the fixed reduction, \( \Delta H \) varies as the rolls become closer to each other.
The roll force was monitored by data acquisition using a 286 IBM PC equipped with an WB-800 12 bit analog to digital converter. [Figure 4.6.2] Pre-calibrated Sensotec load cells, each having a maximum capacity of 20,000 [lbf], were placed under each end of the bottom roll. The load cells contained strain gauges that varied in voltage according to the force. The analog voltage was converted into a digital signal and recorded by a Turbo

![Diagram](image-url)

Figure 4.6.2. Data Acquisition Set-Up.
Pascal program. The load cell voltage data was then imported into a spreadsheet and converted into force. See Appendix C. The left and right side force was plotted on a graph. [Figure 4.6.3] See Appendix D.

Figure 4.6.3. Experimental Roll Force.

In developing profile rolls the worst case or highest force must be assumed. The force recorded was an average across a maximum force period. The period varied from experiment to experiment depending on reasonable stability of the curve. The graph in Figure 4.6.3 has a maximum force of 4406 [lbf], a result of averaging the force curve between reading number 240 and 280.
4.7 Percent Roll Fill

The profile of each billet was measured with a dial indicator which was secured along the table of the vertical mill. The dial indicator was magnetically fastened to the chuck and positioned directly above the work piece. [Figure 4.7.1] The billet was translated horizontally with the mill allowing the dial indicator to travel up and down along the profile. The vertical and horizontal positions were recorded at each incremental horizontal displacement. Both the top and bottom profiles of the test billets were recorded.

Figure 4.7.1 Profile Measurement.
The more complex areas of the profile, namely the peaks and valleys, required shorter horizontal increments. The accuracy of the profile was also dependent on the choice of dial indicator pointers which had to be slender enough so as not to interfere with steep grades, yet blunt enough as so not to penetrate the soft lead material.

The corresponding horizontal and vertical data was recorded on a spreadsheet and exported to a DOS text file. An Auto-LISP program imported the data into AutoCAD where a poly-line was created. The poly-lines defining the top and bottom profiles were closed at each end with reference to the thickness of the test billet. The billet thickness was

![Diagram of profiles](image)

**Figure 4.7.2. Imported Profiles**
determined by a micrometer. The profiles and sides of the drawing were then joined into a closed loop poly-line. Figure 4.7.2 shows the imported profiles and their resultant poly-lines. As can be seen, the closed poly-line represent the cross section of the test billets.

The cross sectional areas of the test billets were determined by AutoCAD. These cross sectional areas were compared to the actual roll profile when calculating the percent of roll fill. Areas of poor fill were identified by superimposing the test billets on the desired roll gap profile.
CHAPTER 5

RESULTS

5.1 Roll Force and Profile Fill Results

Eight experiments were designed for the analysis of roll force and profile fill. Each experiment was conducted with different controllable factors. The controllable factors considered were; initial billet thickness, reduction, and lubrication condition. Each factor was varied between a high and low level as defined in Table 4.3.1. Results and conclusions were drawn from the comparison of these eight experiments and the mathematical models.

5.2 Experiment One

In Experiment number One, as in all of the experiments, the initial rolling failed to yield profile fill due to the peaks and valleys of the profile. Most of the deformation of the rolling billet was in bending, where there was very little radial and axial flow. For this reason, the forces during this initial deformation were much lower than would be expected from an approximate model. In fact, in most cases the forces associated with the first pass were not even great enough to trigger the data acquisition program.

The billet for Experiment One had an initial thickness of 0.45 inches. No lubrication was applied to the workpiece and the reductions were considered low at a
constant 20%. The graph in Figure 5.2.1. show some inconsistencies in the experimental data. Since the reductions of the roll gap were held constant, a more uniform change in force was expected. It was expected that run number four would follow the trend more closely. It could be that there was some human error in run number four or that the electronic equipment did not function properly. At first glance, run number eleven appears to have some inconsistency. However, the low force was to be expected on this run as a result of a lower gap reduction. Variance in reductions at initial or final forming was due to the experimental workpiece having different parameters but a common final area. This variance from the constant in one of the reductions was sometimes necessary to assure a consistent final cross-sectional area in all experiments.

The forces calculated by the approximate model appear to follow a constant trend. Although the theoretical trend demonstrated an even sloping curve it consistently predicted low forces which was expected considering the mathematical background of the approximate model. While the mathematics represented by the slab method considered most of the work involved in the deformation during rolling, not all deforming modes were considered. The slab method does not consider redundant work nor does it take into consideration the lateral flow inherent in profile rolling. Furthermore, the slab did not take into
Table 5.2.1. Experiment #1, Approximate and Experimental Force Data.

<table>
<thead>
<tr>
<th>RUN</th>
<th>ACTUAL PROFILE AREA (in²)</th>
<th>ROLL GAP THICKNESS (inches)</th>
<th>REDUCTION OF GAP TRAVEL</th>
<th>APPROXIMATE MODEL FORCE (lbf)</th>
<th>EXPERIMENTAL FORCE (lbf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RUN #1</td>
<td>0.288</td>
<td>0.090</td>
<td>20%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RUN #2</td>
<td>0.288</td>
<td>0.072</td>
<td>20%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RUN #3</td>
<td>2.117</td>
<td>0.230</td>
<td>20%</td>
<td>5738</td>
<td>6255</td>
</tr>
<tr>
<td>RUN #4</td>
<td>11.72%</td>
<td>1.869</td>
<td>20%</td>
<td>5200</td>
<td>5135</td>
</tr>
<tr>
<td>RUN #5</td>
<td>10.62%</td>
<td>1.671</td>
<td>20%</td>
<td>4629</td>
<td>5034</td>
</tr>
<tr>
<td>RUN #6</td>
<td>9.51%</td>
<td>1.512</td>
<td>20%</td>
<td>4166</td>
<td>4766</td>
</tr>
<tr>
<td>RUN #7</td>
<td>8.40%</td>
<td>1.385</td>
<td>20%</td>
<td>3743</td>
<td>0</td>
</tr>
<tr>
<td>RUN #8</td>
<td>7.34%</td>
<td>1.283</td>
<td>20%</td>
<td>3358</td>
<td>4025</td>
</tr>
<tr>
<td>RUN #9</td>
<td>6.34%</td>
<td>1.202</td>
<td>20%</td>
<td>3010</td>
<td>0</td>
</tr>
<tr>
<td>RUN #10</td>
<td>5.41%</td>
<td>1.137</td>
<td>20%</td>
<td>2694</td>
<td>3502</td>
</tr>
<tr>
<td>RUN #11</td>
<td>3.20%</td>
<td>1.100</td>
<td>20%</td>
<td>1982</td>
<td>2221</td>
</tr>
</tbody>
</table>

Figure 5.2.1. Experiment #1, Approximate and Experimental Force.
account the different diameters of the top and bottom roll. The different diameters promote different velocities of the material inside and outside of the ring. These deformations not considered added up to a significant amount of work which increased the rolling force. Therefore, the slab method must be considered a lower bound mathematical method which will always predict low forces for profile rolling.

5.3 Experiment Two

The experimental and approximate model data for Experiment Two are compared in Table 5.3.1. and Figure 5.3.1. The results of this experiment presented expected trends. Experiment Two started with an initial billet thickness of 0.45 inches, no lubrication, and a high percent reduction of 30% in the roll gap. The relative approximate and theoretical forces remained relatively consistent until the last pass where the theoretical and experimental forces were equal. As stated above, the slab method predicts low forces because of the deformation variables not considered in it’s calculation. Perhaps, as the billet conformed to the profile of the roll, there was less lateral flow; thus, the work associated with it becomes zero. The absence of lateral flow would result in a more realistic approximation from the model. The rolls modeled were considered closed. That is, they were designed to restrict material flow in the lateral direction. However, during initial rolling some material did escape from the sides of the rolls. This extra material, referred to as “flash”, was cut away in order to proceed with the next pass. Possibly there was more lateral flow in the initial rolling due to the “flash”.
### Table 5.3.1. Experiment #2, Approximate and Experimental Force Data.

<table>
<thead>
<tr>
<th>Run</th>
<th>Actual Profile Reduction (%)</th>
<th>Area (in²)</th>
<th>Roll Gap Thickness (inches)</th>
<th>Reduction Change in Thickness (inches)</th>
<th>Reduction of Gap Travel (%)</th>
<th>Approximate Model Force (bf)</th>
<th>Experimental Force (bf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run #1</td>
<td>21.2%</td>
<td>2.671</td>
<td>0.450</td>
<td>0.099</td>
<td>22%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Run #2</td>
<td>18.87%</td>
<td>2.104</td>
<td>0.246</td>
<td>0.105</td>
<td>30%</td>
<td>6641</td>
<td>6700</td>
</tr>
<tr>
<td>Run #3</td>
<td>16.28%</td>
<td>1.707</td>
<td>0.172</td>
<td>0.074</td>
<td>30%</td>
<td>5577</td>
<td>6837</td>
</tr>
<tr>
<td>Run #4</td>
<td>13.61%</td>
<td>1.429</td>
<td>0.120</td>
<td>0.052</td>
<td>30%</td>
<td>4726</td>
<td>5800</td>
</tr>
<tr>
<td>Run #5</td>
<td>11.03%</td>
<td>1.234</td>
<td>0.084</td>
<td>0.036</td>
<td>30%</td>
<td>3987</td>
<td>4915</td>
</tr>
<tr>
<td>Run #6</td>
<td>8.68%</td>
<td>1.098</td>
<td>0.059</td>
<td>0.025</td>
<td>30%</td>
<td>3416</td>
<td>3411</td>
</tr>
<tr>
<td>Run #7</td>
<td></td>
<td>1.003</td>
<td>0.041</td>
<td>0.018</td>
<td>30%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 5.3.1.** Experiment #2, Approximate and Experimental Force.
As the roll gap closed, there was increasingly less “flash” suggesting less lateral flow, resulting in lower forces.

5.4 Experiments Three and Eight

The experimental results for Experiment Three and Eight were indeterminate due to unreasonable process parameters. A combination of high reduction and lubrication prevented the rolls from gripping the work piece. As a result no data could be compiled for these experiments.

5.5 Experiment Four

The experimental and approximate model results for Experiment Four can be seen in Table 5.5.1 and Figure 5.5.1. Experiment Four had an initial billet thickness of 0.40 inches with lubrication applied and a low gap reduction of 20%. This experiment followed the same trends as previous trials, except for a slight deviation in run number eight. A closer look at the difference between the approximate and experimental forces reveals a more agreeable relationship as the roll gap closes. The close relationship again supports the idea of less lateral flow. Unfortunately run number ten was lost due to equipment error. It would have been interesting to see if this roll pass revealed an even closer relationship between the actual and approximated forces.
Table 5.5.1. Experiment #4, Approximate and Experimental Force Data.

<table>
<thead>
<tr>
<th>RUN</th>
<th>ACTUAL SECTION</th>
<th>REDUCTION</th>
<th>APPROXIMATE MODEL FORCE</th>
<th>EXPERIMENTAL FORCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>2.00</td>
<td>0.340</td>
<td>6000</td>
<td>2788</td>
</tr>
<tr>
<td>#2</td>
<td>2.00</td>
<td>0.340</td>
<td>6000</td>
<td>2788</td>
</tr>
<tr>
<td>#3</td>
<td>2.00</td>
<td>0.340</td>
<td>6000</td>
<td>2788</td>
</tr>
<tr>
<td>#4</td>
<td>2.00</td>
<td>0.340</td>
<td>6000</td>
<td>2788</td>
</tr>
<tr>
<td>#5</td>
<td>2.00</td>
<td>0.340</td>
<td>6000</td>
<td>2788</td>
</tr>
<tr>
<td>#6</td>
<td>2.00</td>
<td>0.340</td>
<td>6000</td>
<td>2788</td>
</tr>
<tr>
<td>#7</td>
<td>2.00</td>
<td>0.340</td>
<td>6000</td>
<td>2788</td>
</tr>
<tr>
<td>#8</td>
<td>2.00</td>
<td>0.340</td>
<td>6000</td>
<td>2788</td>
</tr>
<tr>
<td>#9</td>
<td>2.00</td>
<td>0.340</td>
<td>6000</td>
<td>2788</td>
</tr>
<tr>
<td>#10</td>
<td>2.00</td>
<td>0.340</td>
<td>6000</td>
<td>2788</td>
</tr>
</tbody>
</table>
Table 5.6.1. Experiment #5, Approximate and Experimental Force Data.

<table>
<thead>
<tr>
<th>RUN</th>
<th>PROFILE REDUCTION</th>
<th>AREA (in²)</th>
<th>ROLL GAP THICKNESS (inches)</th>
<th>CHANGE IN THICKNESS (inches)</th>
<th>REDUCTION %</th>
<th>APPROXIMATE MODEL FORCE (lbf)</th>
<th>EXPERIMENTAL FORCE (lbf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RUN #1</td>
<td>12.70%</td>
<td>2.133</td>
<td>0.230</td>
<td>0.068</td>
<td>20%</td>
<td>6151</td>
<td>6451</td>
</tr>
<tr>
<td>RUN #2</td>
<td>11.63%</td>
<td>1.885</td>
<td>0.184</td>
<td>0.006</td>
<td>20%</td>
<td>5370</td>
<td>5925</td>
</tr>
<tr>
<td>RUN #3</td>
<td>10.53%</td>
<td>1.666</td>
<td>0.147</td>
<td>0.006</td>
<td>20%</td>
<td>4628</td>
<td>5401</td>
</tr>
<tr>
<td>RUN #4</td>
<td>9.42%</td>
<td>1.527</td>
<td>0.118</td>
<td>0.004</td>
<td>20%</td>
<td>4164</td>
<td>4713</td>
</tr>
<tr>
<td>RUN #5</td>
<td>8.32%</td>
<td>1.400</td>
<td>0.094</td>
<td>0.002</td>
<td>20%</td>
<td>3743</td>
<td>4161</td>
</tr>
<tr>
<td>RUN #6</td>
<td>7.26%</td>
<td>1.299</td>
<td>0.075</td>
<td>0.001</td>
<td>20%</td>
<td>3357</td>
<td>3839</td>
</tr>
<tr>
<td>RUN #7</td>
<td>6.26%</td>
<td>1.217</td>
<td>0.060</td>
<td>0.001</td>
<td>20%</td>
<td>3007</td>
<td>3715</td>
</tr>
<tr>
<td>RUN #8</td>
<td>5.34%</td>
<td>1.152</td>
<td>0.048</td>
<td>0.001</td>
<td>20%</td>
<td>2693</td>
<td>3461</td>
</tr>
<tr>
<td>RUN #9</td>
<td>4.52%</td>
<td>1.100</td>
<td>0.039</td>
<td>0.001</td>
<td>20%</td>
<td>2412</td>
<td>2423</td>
</tr>
</tbody>
</table>

Figure 5.6.1. Experiment #5, Approximate and Experimental Force.
<table>
<thead>
<tr>
<th>RUN</th>
<th>INITIAL THICKNESS</th>
<th>PERCENT REDUCTION</th>
<th>ROLL GAP</th>
<th>REDUCTION</th>
<th>APPROXIMATE MODEL FORCE</th>
<th>EXPERIMENTAL FORCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.222</td>
<td>0.272</td>
<td>0.128</td>
<td>32%</td>
<td>1663</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.782</td>
<td>0.190</td>
<td>0.082</td>
<td>30%</td>
<td>6641</td>
<td>7443</td>
</tr>
<tr>
<td>3</td>
<td>1.475</td>
<td>0.133</td>
<td>0.057</td>
<td>30%</td>
<td>5843</td>
<td>7012</td>
</tr>
<tr>
<td>4</td>
<td>1.269</td>
<td>0.093</td>
<td>0.040</td>
<td>30%</td>
<td>4965</td>
<td>5889</td>
</tr>
<tr>
<td>5</td>
<td>1.109</td>
<td>0.065</td>
<td>0.028</td>
<td>30%</td>
<td>3957</td>
<td>5711</td>
</tr>
<tr>
<td>6</td>
<td>1.003</td>
<td>0.046</td>
<td>0.020</td>
<td>30%</td>
<td>3529</td>
<td>4031</td>
</tr>
</tbody>
</table>

Table 5.6.2. Experiment #6, Approximate and Experimental Force Data.

Figure 5.6.2. Experiment #6, Approximate and Experimental Force.
5.6 Experiment Five and Six

The experimental and approximate model results for Experiments Five and Six can be seen in Table 5.6.1; Figure 5.6.1, Table 5.6.2, and Figure 5.6.2. Experiment Five had an initial billet thickness of 0.45 inches with lubrication applied and a low reduction of 20%. Experiment Six had an initial billet thickness of 0.40 inches with no lubrication applied and a high reduction of 30%. The same trends were again observed. The approximate model predicted low forces until the last pass where the differences were minimal.

5.7 Experiment Seven

The experimental and approximate results for Experiment Seven can be seen in Table 5.7.1 and Figure 5.7.1. Experiment Seven had an initial billet thickness of 0.40 inches with no lubrication and a low gap reduction of 20%. Experiment Seven had a similar relationship, as did the others, with the approximate force on the low end and more agreeable results on the last pass. The approximate models force curve appeared smooth, as with the other experiments. However, the data from the earlier runs was not as consistent as in the previous experiments. There seems to have been a step function associated with their progression. Roll passes five and six seem to have had an irregular change in force with roll gap, wherein the differences were not as great as was expected. This same pattern can be seen again in comparing passes seven and eight and even passes nine and ten. These differences can not be explained at the present time. Perhaps there
### Table 5.7.1. Experiment #7, Approximate and Experimental Force Data.

<table>
<thead>
<tr>
<th>RUN</th>
<th>PROFILE</th>
<th>AREA (m²)</th>
<th>ROLL GAP</th>
<th>REDUCTION OF GAP (inches)</th>
<th>Reduction in Thickness (inches)</th>
<th>Reduction in Thickness (%)</th>
<th>APPROXIMATE MODEL FORCE (lbf)</th>
<th>EXPERIMENTAL FORCE (lbf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td></td>
<td>0.400</td>
<td>0.340</td>
<td>0.060</td>
<td>15%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#2</td>
<td></td>
<td>0.272</td>
<td>0.068</td>
<td>0.024</td>
<td>20%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#3</td>
<td></td>
<td>0.218</td>
<td>0.054</td>
<td>0.030</td>
<td>20%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#4</td>
<td>11.57%</td>
<td>1.792</td>
<td>0.174</td>
<td>0.044</td>
<td>20%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#5</td>
<td>10.46%</td>
<td>1.604</td>
<td>0.139</td>
<td>0.035</td>
<td>20%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#6</td>
<td>9.35%</td>
<td>1.455</td>
<td>0.111</td>
<td>0.028</td>
<td>20%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#7</td>
<td>8.25%</td>
<td>1.335</td>
<td>0.089</td>
<td>0.022</td>
<td>20%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#8</td>
<td>7.19%</td>
<td>1.239</td>
<td>0.071</td>
<td>0.018</td>
<td>20%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#9</td>
<td>6.20%</td>
<td>1.162</td>
<td>0.057</td>
<td>0.014</td>
<td>20%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>#10</td>
<td>5.29%</td>
<td>1.100</td>
<td>0.046</td>
<td>0.011</td>
<td>20%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Figure 5.7.1. Experiment #7, Approximate and Experimental Force.
was human error or the equipment was not functioning properly. It is curious and a little too coincidental that this relationship would repeat itself throughout the experiment. Still, there is no solid explanation for this behavior.

5.8 Effects of Varying Parameters on Roll Force

Each of the eight experiments had unique parameters. They differed by a high and low value of initial billet thickness, lubrication, and reduction as indicated in Chapter 4. [Table 4.3.1] Unfortunately, two of the eight proposed experiments were not completed due to their lubrication and reduction values. Experiments Three and Eight show lubrication and 30% reduction. [Table 4.3.1] It proved impossible to feed the workpiece through the rolls. Although the combined effects of lubrication and high reduction could not be analyzed, it was possible to study the direct effect of changing parameters on the remaining six experiments.

The effects on roll force due to high and low value of initial billet thickness can be seen in Figures 5.8.1 and 5.8.2. Two sets of experiments with varying initial billet thickness were compared to each other. These particular experiments were compared because they had common parameter combinations for lubrication and reduction. As can be seen in Experiments Two and Six, there appears to have been a consistent relationship, suggesting that the greater initial billet thickness of Experiment Two results in higher forces. However, in the comparison of Experiments One and Seven the opposite is true.
Figure 5.8.1. Initial Billet Thickness Comparison.

Figure 5.8.2. Initial Billet Thickness Comparison.
Although Experiment One had a high initial billet thickness the rolling forces were consistently lower than that of Experiment Seven with a low initial billet thickness. In this case there is no evidence that the initial billet thickness had any significant bearing on resultant roll force.

Lubricated and non-lubricated experiments were compared in two sets, shown in Figures 5.8.3 and 5.8.4, where the common parameters were initial billet thickness and percentage reduction. Again we see conflicting results. Experiments Four and Seven suggest that the application of lubrication may have slightly reduced the rolling force, since Experiment Four was lubricated and Experiment Seven was not. However, a comparison of Experiments One and Five reveal the opposite. Experiment One did not apply lubrication although it had lower or equivalent forces compared to Experiment Five which did apply lubrication. There is no substantive evidence that lubricating the workpiece had any effect on the rolling force.

Perhaps a more intuitive relationship with rolling force is percent reduction. Higher reductions require more material to flow. This is shown graphically when comparing Experiments One and Two as shown in Figure 5.8.5, where the common parameters were lubrication and initial billet thickness. It is evident that billet two induces the largest force having the high reduction with most of the lateral flow taking place in the first few roll passes. The maximum force for Experiment Two was 8128 (lbs) but only 6255 (lbs) for Experiment One. The rolling forces in the experiments became much more compatible in the final passes of Experiment Two. Most of the
Figure 5.8.3. Lubricated and Non-Lubricated Comparison.

Figure 5.8.4. Lubricated and Non-Lubricated Comparison.
Figure 5.8.5. High and Low Reduction Comparison.

Figure 5.8.6. High and Low Reduction Comparison.
lateral flow stopped in the final phases of Experiment Two. Experiment One, has three more rolling passes than Experiment Two. This explains the increasing similarity in the two curves. Figure 5.8.6 shows the same trend with Experiments Six and Seven with Experiment Six having the high reduction and Experiment Seven having the low reduction.

5.9 Percentage Difference

The average percentage difference between experimental and theoretical forces can be seen in Figure 5.9.1. Observation of the data suggests a relationship between experiments. The lowest percentage of difference presented was in Experiments Four and Seven. The common rolling parameters for these two experiments were a low initial billet thickness of 0.4 (in) and a low reduction of 20% roll gap. The difference can be attributed to the fact that the number four work piece was lubricated while work piece seven was not. It was earlier suggested that lubrication played a minor roll in the rolling process. This comparison now suggests that there is less redundant work involved in experiments with low initial thickness and low reductions. Such effects were not considered by the approximate model. In Experiments Two and Six the common parameters were lubrication and high reduction. Here the difference was the initial billet thickness. Perhaps it can be stated that higher reductions contribute to more redundant work and axial flow.
Figure 5.9.1. Average Percent Difference.
5.10 Profile Filling

As explained earlier in Chapters Two and Three, material deformation patterns in profile ring rolling are very complex. The profile fill is directly affected by the way the material flows in the roll gap. If there is too much axial flow, the material does not flow into the peaks and valleys of the profile because of the constant stretching of the material across the roll gap. Axial flow can be controlled with roll profiles of a closed cross section. The rolling profile in this study was considered semi-closed in that it allowed “flash” to escape the sides until the rolls were nearly 100% closed. Only the final rolling passes experienced no lateral flow.

Another deterrent of profile filling is the difference in radial flow throughout the profile. The peaks and valleys associated with the profiles in this study presented different velocities within the roll gap that not only imposed residual tensile and compressive stresses but also stretched the material, preventing it from filling the profile. Since the rolling profiles of these experiments do not appear to be extreme, it can be said that radial spread was the leading factor in insufficient profile fill.

The final cross sectional areas of the work pieces were digitized and are compared to a 100% cross section in Figure 5.10.1. Although the profiles show a high percent of fill, the sharp edges of the peaks and valleys are incomplete. The dimple on the far sides of the profile shows relatively good fill as compared to that on the inside. The thickness of the outside of the billets is relatively constant. Constant thickness
would produce a more uniform radial spread and minimize stretching of the material and associated residual stresses. However, the dimple in the middle of the billet is near a major “step” in the profile thickness. The thinner material in the middle deformed at a much greater rate than that of the thicker. This difference in velocities stretched the material, preventing it from filling the profile. Again, the same result can be seen in the abrupt incline adjacent to the dimple. The areas of incomplete profile on the workpiece actually show stretch marks and material tearing between the two velocity zones.
Figures 5.10.2, 5.10.3, and 5.10.4 show the ANTARES F.E.A. simulation results of the experimental profiles. The ANTARES model showed a remarkable similarity to the actual profile cross sections. Again, the same deficiencies in the profile fill can be seen. Figure 5.10.3 presents the strain in the cross section. The lighter colors indicate the areas of maximum strain. These “hot” areas are to be expected because of the difference in radial spread. It must be noted that the areas of incomplete spread are close to transition zones in strain. The material is simply being stretched across the profile.

Figure 5.10.2. Antares Model Profile Fill
Figure 5.10.3. Antares Strain Across the Profile.
Figure 5.10.4. Antares Model
CHAPTER 6

DISCUSSION

6.1 Project Summary

The high strength and high temperature applications of metal needed in the aircraft industry have prompted manufacturers to develop specialized forming processes. Often these forming processes are very expensive to execute. In order to achieve complex shapes with the desired microstructure, excessive machining and unacceptable material loss usually occurs. One alternative to machining complex shapes from bars of metal is to form the shape in the rolling process. Profile ring rolling is a near net shape roll forming process that does roll the form into the part; thus, requiring very little machining. This reduces the time and cost of manufacturing and, at the same time, provides the capability to develop complex parts with ideal microstructure.

Although a successful profile ring rolling process is very efficient, the development for such a process is very difficult. The profile ring presents complex material deformation within the roll gap. It is not intuitive, even for the most experienced personnel, to predict the rolling parameters for any given ring. In the past, trial and error was the method used by designers. However, with the demand for more intricate profiles from super alloys, trial and error has not proven to be reliable. It is the use of today’s powerful computers and increased understanding of forming processes which has
presented an acceptable alternative to trial and error. Hence, the subject for this study "The Development of a Computer Model to Design Compression Rolls" is relevant. Computer simulation allows for the early prediction of shape defects and process parameters. By applying process prediction with computer model input into the development equation, the cost of prototyping can be significantly reduced. Most of the trial and error can be performed in the imaginary world of the computer which is a much cheaper alternative than building excessive prototype rolling mills. However, before computer models can be used to predict parameters of metal forming processes they must be validated by experimentation. The scope of this research presents validation for rolling forces predicted by ROLLCAD and a three dimensional comparison of profile fill with the ANTARES package.

6.2 ROLLCAD

The computer model ROLLCAD for approximating rolling parameters, developed by Ohio University, runs on a personal computer requiring limited hardware cost and minimal computational time. The approximations calculated by ROLLCAD are then input into the more accurate finite element analysis package ANTARES. The two-part design methodology is an extremely efficient development tool. Because ANTARES uses a more complex finite element code and requires more advanced hardware and longer computational time, the most efficient and cost effective development is to run ROLLCAD time and time again until expectable parameters are found. Input into
ANTARES will further refine the approximates and produce a more accurate three-dimensional representation.

6.3 Experimental Results

The results of the profile ring rolling process was modeled in the laboratory on a small rolling mill using lead as the working material. The force results collected from data acquisition equipment strongly agreed with simulations by ROLLCAD. While ROLLCAD consistently calculated low forces when compared to the experiments, this was to be expected. The “slab” method fails to take into account either the redundant work in rolling, the axial or unequal radial flow; thus, the approximate forces are consistently low. Because, when using ROLLCAD to develop profile rolls, it is imperative to consider the material flow characteristics as they relate to the profile, a shape complexity factor should be considered in the development of profile rolling.

The profile billets rolled in the laboratory had an average die fill of 97%. The three percent loss was in material flow patterns. The profile rolls were considered to be closed to restrict axial flow, and although some material escaped the sides in the form of “flash” during the initial rolling passes, the final rolling produced flashless billets. Therefore, axial flow played but a minor part in deficient die fill. The majority of the deficient die fill was due to uneven radial flow across the cross section. Evidence of radial tearing of the work piece was observable on all billets. Tearing between velocity zones prevented the material from flowing into the profile. These exact events were
predicted by the finite element model ANTARES where the strain graph showed velocity transitions at low die fill areas. In fact, the areas of incomplete die fill on the ANTARES model exactly matched those of the experimental billets. The results of this study suggest that a high level of confidence can be placed on computer simulations when modeling the profile rolling process. It is imperative, however, that there be a sound understanding of the process being modeled.

6.4 Problems Encountered

Initial rolling trials in the study presented some obstacles. Since the rolls used were designed to model the ring rolling process, the mandrel was slightly smaller in diameter than the main roll. Because of the size difference, the lead billet curved upward as it exited the rolls. Since the mill was not equipped to control the billet, it was difficult to feed the curved billet for the next pass. As the billet lengthened, it ran into the top of the mill when exiting the rolls. To counter this problem, the top and bottom rolls were interchanged so the billet would curve down. The height of the exit table was then lowered and greased so that, as the billet curved down, it was restricted by the flat table which straightened the billet. This straightening was not previously mentioned when explaining the low prediction of the “slab” method. The slab method did not take into consideration the work required to straighten the billet since this amount of work was considered to be negligible. Another problem encountered was controlling the billet through the rolls. Side guides were constructed to prevent the workpiece from vying
from a straight path. The energy expanded in resisting the frictional force was another source of error considered to be negligible.

The largest problem encountered with certain combinations of process parameters did affect the results of the study. Rolling Experiments Three and Eight failed completely due to the fact that it was impossible to safely feed the billets through the rolls. Experiments Three and Eight had a high reduction and were lubricated. The absence of friction prevented the billet from being pulled into the rolls. For this reason, the combined effects of the three-process parameters considered could not be studied.

6.5 Recommendations for Future Work

The largest source of error in this study was the inability of the “slab” method to address all of the work present in the deformation process. In profile rolling, as in flat rolling, redundant work is a significant factor in the rolling force. In addition, radial and lateral deformations in the roll gap were not considered in the “slab” mathematical model. It is possible that these sources of error could be minimized by the addition and incorporation of other models. Possibly, the redundant work could be calculated separately and added to the result of the “slab” method. Another approach might be to use an upper bound mathematical model to determine a normalized median between the two results. Finally a shape complexity factor could be empirically formulated. Such a factor, would consider the work results in other deformations, to enhance the accuracy of the “slab” method result.
Thus while ROLLCAD proved to be very effective in predicting force in profile rolling, in future work, the mathematical models employed could be made more accurate with the additions presented while still retaining the speed and simplicity that ROLLCAD was initially designed for.

6.6 Conclusions

The purpose of this study was to validate the accuracy of the forces calculated by ROLLCAD and the profile fill predicted by the finite element model ANTARES. Rolling experiments were conducted in the laboratory and the resultant forces were compared to those calculated by ROLLCAD. The mathematical model used by ROLLCAD, referred to as the “slab” method, was successful in predicting the actual experimental forces. The average percentage of difference between ROLLCAD and experimental forces was 13% with the results from ROLLCAD being consistently lower than experimental results. This was to be expected when considering the deformation processes that were not included in the “slab” method.

The resultant profile fill and strain (velocity) zones were modeled by the finite element program ANTARES and compared to actual lead billets rolled in the laboratory. ANTARES predicted die fill with surprising accuracy and areas of incomplete die fill directly correlated with the experiments. ANTARES also predicted the torn material on the experimental billets at velocity transition zones.
The results of this study support the application of computer models in the development of profile ring rolling. Computer models provide more efficient and cost effective research and development for the near net shape rolling of high strength and high temperature metals. ROLLCAD and ANTARES, which have been experimentally validated and field tested are presently being used by Aerospace engineers. ROLLCAD is used to approximate separating force, rolling mill horsepower requirements, the maximum accumulated strain, material hardness, material temperature rise during rolling, ring growth rates, centering forces required to stabilize rolls, inner roll speed necessary to avoid slippage, and microstructure change during rolling.
BIBLIOGRAPHY


APPENDIX A

PHOTOGRAPHS

OF THE

EXPERIMENTAL

ROLLING MILL
Figure A.1. Experimental Rolling Mill.
APPENDIX B

EXPERIMENTAL FORCE RESULTS
Figure B.1. Experiment One (Billet #3).
Figure B.2. Experiment One (Billet #3).
Figure B.3. Experiment One (Billet #3).
Figure B.4. Experiment Two (Billet #4).
Figure B.5. Experiment Two (Billet #4).
Figure B.6. Experiment Four (Billet #6).
Figure B.7. Experiment Four (Billet #6).
Figure B.8. Experiment Four (Billet #6).
Figure B.9. Experiment Five (Billet #7).
Figure B.10. Experiment Five (Billet #7).
Figure B.11. Experiment Five (Billet #7).
Figure B.12. Experiment Five (Billet #7).
Figure B.13. Experiment Six (Billet #8).
Figure B.14. Experiment Six (Billet #8).
Figure B.15. Experiment Seven (Billet #9).
Figure B.16. Experiment Seven (Billet #9).
Figure B.17. Experiment Seven (Billet #9).
APPENDIX C

DATA ACQUISITION

TURBO PASCAL

COMPUTER PROGRAM
program expl1;
VAR
bfile: text;
x: real;
I, N, f: integer;
Y: array[1..1000] of integer; {channel 0 data}
P: array[1..1000] of integer; {channel 1 data}
Q: array[1..1000] of integer; {channel 2 data}
LABEL REP;
PROCEDURE DEL; {TIME DELAY ROUTINE}
VAR
D: integer;
BEGIN
FOR D := 1 TO 20 DO
BEGIN
D := D + 1
END;
END;
BEGIN
WRITELN('RUNNING...
');
I := 0;
assign(bfile, 'result.DAT'); {OPEN A DATA FILE AND INITIALIZE IT}
rewrite(bfile);
REP:
PORT[$311] := $40; {CHECK FOR DATA INPUT ABOVE THRESHOLD}
PORT[$312] := 0;
DEL;
f := portw[$313]
if F < 25 then GOTO REP
ELSE
BEGIN
WRITELN('collecting data...
');
REPEAT
I := I + 1;
PORT[$311] := $40;
PORT[$312] := 0;
DEL;
Y[I] := PORTW[$313];
PORT[$311] := $41;
PORT[$312] := 0;
DEL;
P[I] := PORTW[$313];
PORT[$311] := $02;
PORT[$312] := $0;
DEL;
Q[I] := PORTW[$313];
DELAY(5);
UNTIL KEYRESSED;
END;
FOR N := 1 TO I DO
BEGIN
WRITELN(BFILE, N, ', Y[N], ', P[N], ', Q[N]);
END;
CLOSE(BFILE);
END.

program exp;
VAR
x: real;
y: integer;
PROCEDURE DEL;
VAR
N: INTEGER;
BEGIN
FOR N := 1 TO 20 DO
BEGIN
N := N + 1;
END;
EDN;
BEGIN
PORT[$311] := $40;
PORT[$312] := $0;
DEL;
Y := PORTW[$313];
X := Y / 204.8;
WRITELN('X IS ', X);
WRITELN('Y IS ', Y);
PORT[$311] := $40;
PORT[$312] := $0;
DEL;
Y := PORTW[$313];
X := Y / 204.8;
WRITELN('X IS ', X, ' Y IS ', Y);
END.
APPENDIX D

CALLIBRATION

OF

LOAD CELLS
Calibration Equation

\[
\frac{\text{Calibration Constant} \left[ \frac{\text{mV}}{\text{V}} \right]}{\text{Max Cell Force} [\text{lbf}]} = \left( \frac{\text{Read Voltage} [\text{mV}]}{\text{Battery Voltage} [\text{V}]} \right) \frac{\text{Force} [\text{lbf}]}{}
\]

Figure D.1. Load Cell Calibration