DESIGN AND CONSTRUCTION OF A MULTI-SEGMENT
SNAKE-LIKE WHEELED VEHICLE

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1.1 Background Information

In this chapter background information concerning multi-degree of freedom systems is provided. In addition, the previous work on dexterous mechanisms and the organization of this thesis are presented.

Creatures such as worms and snakes have the ability to move around objects by using their bodies dexterity. Throughout the world various scientific institutions examined the use of mechanisms which can simulate the motion of these creatures. One type of these mechanisms are the snake-like robots. They can be used in a wide variety of tasks such as mine clearing, planet exploration, and inspection of hazardous environments. In addition, the use of such robots in the medical field has been explored recently. Miniature snake-like robots fitted with micro-diagnostic sensors can assist physicians in many areas, such as inspection of intestinal lining, and by taking temperature, pressure and acidity readings (Slatkin, 1995, abstract).

Mechanisms which have many degrees of freedom are important in environments where other legged or wheeled vehicles can not be used, as in chemical and nuclear powerplants. In order for a robot to perform surveillance or inspection in these environments it has to be able to move around obstacles, (Chiang, 1992, pp. 405). Hyper-redundant or multi-degree of freedom robots are used for this purpose. Such mechanisms consist of many independent segments connected by joints.
1.2 Literature Review

Many researchers and scientists have explored the use of hyper-redundant robots in a variety of tasks. Ridgeway and Crane, designed a multi-segment serially connected mobile robot for possible use in nuclear powerplants in two navigation modes; the horizontal and the vertical-bridge. Different joint actuation possibilities are examined for the parallel planar actuator.

Agrawal and Chang, have developed two motion algorithms for a multi-segment wheeled vehicle, one based on position kinematics and the other on rate kinematics. Both motion algorithms have been simulated on a Silicon Graphics workstation. The snake-like robot represents an experimental platform of the development of these motion algorithms.

Saha, has both kinematically and dynamically analyzed a two degree of freedom three wheeled vehicle for modeling, simulation and control purposes.

Hemani, designed a light-weight flexible string controlled robot arm used as a manipulator. The arm was analyzed kinematically in 3 D space. It can be used for obstacle avoidance and for motion through narrow passages.

Chiang, developed a 3D path planning of a serially connected 18 segment robot for possible use in a nuclear powerplant. Inspection, maintenance, and obstacle avoidance are some of the tasks the vehicle can perform. The vehicle is designed to execute both horizontal and vertical motions.
Barraquand and Latonde, developed path planning of mobile robots subjected to non-holonomic constraints. Using differential geometry and non-linear control theory the controllability of these robots is examined. Then the results obtained are applied to two types of vehicles the Car-Like Robot and the Trailer-Like Robot.

Wit and Roskam, studied the controllability of a two degree of freedom wheeled mobile robot subjected to both task related constraints, and actuator power constraints. The dynamic control of the vehicle was based on tangential navigation velocity and orientation, and on-line path profile generation. This method of dynamic control was tested on an experimental robot.

Shiller and Chen, studied the motion optimization of autonomous vehicles in 3D space by considering the vehicle’s dynamics, and the nature of the path. The path is optimized by the use of the motion time as the cost function. The path is described by cubic B patch.

Feng and Krogh, developed an algorithm to steer an autonomous mobile robot on an unknown obstacle filed path. It accomplishes this task by the use of on board sensors feeding the generated path trajectories to a servo-controller.

Novel and Campion, developed a general dynamical model of a three wheel vehicle subjected to non holonomic constraints. The dynamics of the system were reduced to an input-output linearized control by the use of a static state feedback.
1.3 Organization

In this thesis the design and construction of a snake-like wheeled vehicle consisting of four segments which are connected by ball and socket joints is described. It represents the means of implementing two motion algorithms developed, and simulated on a Silicon Graphics workstation.

In chapter 2 the assumptions made in the development of the equations of motion, and a description of the model used for simulation purposes are presented. In addition, the design requirements and the design criteria for the snake-like robot are discussed. During the preliminary design the vehicle was divided to four major subassemblies, namely the upper base plate, the lower base plate, the coupling between the two base plates, and the variable length ball and socket joint.

In chapter 3 the detailed design of the vehicle’s components is presented. The design of the vehicle’s components is divided in three categories. The first one involves the design of the gear drive shaft, and a subsequent force analysis of the worm-worm gear drive used for pitch control. The second one involves the force analysis of the spur gear drive used for roll control. The design of other vehicle components such as base plates, pillow blocks is also presented. Finally the process of evaluating three techniques to balance the multi-segment wheeled vehicle is described.

In chapter 4 the machining techniques employed in the fabrication of four segments of the snake-like vehicle are discussed.
In chapter 5 an experimental determination of the torque requirements of the vehicle’s servomotors and the closed loop with position feedback servocontrol are described.

In chapter 6 the results obtained are analyzed.

In chapter 7 the derived conclusions are stated.
2.1 Simulation of a multi-segment wheeled vehicle

A mathematical model of the snake robot was derived and simulation on the Silicon Graphics workstation was performed. In this model the vehicle is composed of identical segments connected via ball and socket joints. Each segment has the same path.

Fig 2.1 Model of a multi-segment wheeled robot.
while the end-effector follows a different trajectory. Each segment is composed of four parts, a wheel, a steering rod, a right part and a variable length left part, Fig. 2.1. It was assumed that during motion there is pure rolling between the wheel of each segment and the ground. The position and orientation of the end effector as a function of the joint variables and the path the vehicle is derived. Furthermore knowing the position and orientation of the end effector the joint variables were found.

2.2 Design requirements and design criteria

Following the computer simulation, the implementation of the motion algorithm on an experimental platform was to be performed. In the beginning, both the design requirements and the design criteria had to be established. These design requirements for the snake robot are the following:

. The motions of each segment can be achieved using two active control actuators and one passive control actuator.

. The snake robot should be able to execute planar motions, as well as motions in 3-D space with some design modifications.

. The ability to carry various end-effector assemblies for manipulation, inspection and surveillance tasks.

. The motion algorithm requires that pure rolling conditions exist between the vehicle's wheels and the ground, meaning that there is no slipping.
The compatibility of vehicle’s electronics with existing systems in the Mechanical Systems Laboratory is required.

- The use of one wheel per segment.
- The fabrication of four identical segments connected together by variable length ball and socket joints.

The various design concepts were evaluated based on certain design criteria. These criteria are:

- Cost of the design
- Availability of the vehicle’s components.
- Manufacturability of the snake-like vehicle.
- Compactness of the design.
- The number of parts needed for the vehicle fabrication.
- The weight of each segment should not exceed 6 lbs (2.72 Kg).
- The length of each segment should not exceed 12 in (30.4 cm), its width no more than 8 in (20.3 cm), and its height no more that 12 in.
- The minimum distance between two segments to be more than 1.5 in (3.8 cm), while the maximum distance no more than 5 in (12.7 cm).
Mobility Analysis

According to one of the design requirements each segment of the snake-like robot had to be supported on a single wheel. This requirement was analyzed from two perspectives, static stability, and dynamic stability.

Static stability

If each segment had one wheel then the robot will have to start from the sagging position. As a result of it the vehicle’s motors will need additional power to bring it first in the horizontal position and then start moving it. This requires the use of additional servomotor control inputs.

Dynamic stability

During motion there is a need for continuous servomotor control inputs which will ensure that the vehicle remains in the horizontal position. The design requirement for relative translation between two adjacent segments will further complicate this task. Failure to maintain balance during the snake-like robot’s motion might damage the vehicle’s power transmission components such as servomotors, gearing, e.t.c.

After reaching to the conclusion that the use of one wheel per segment will cause both static and dynamic instability, it was decided that two wheels per segment are needed.
2.3 Preliminary design

In this phase of the design process a general approach is taken concerning the way the snake robot will be designed. To be determined is the type of motors used for roll, yaw and pitch control of the vehicle. In addition, the type of the gear drives which will transmit power from the motors to the steering rod, and the wheels is also determined. In the developed mathematical model there is a relative translation and rotation of one segment with respect to the other. The mechanism which will accomplish these motions is considered in this phase of the design. The vehicle is divided in four major assemblies; the upper base plate, the lower base plate the variable length ball and socket joint, and the coupling between the two base plates.

i) Upper base plate assembly

Gear drive

The first step is to choose the type of the gear drive to be used for turning the vehicle. A worm-worm gear drive was selected for this purpose. Although worm gears are less efficient than other gears they have certain advantages. The main advantages are the shaft orientation, and the speed ratio.

Shaft orientation

The orientation of the worm - worm gear drive has the input shaft to be perpendicular
to the output shaft, and non-intersecting. This orientation considerably simplifies the design, and allows various arrangements.

**Speed ratio**

The worm-worm gear drive offers high speed ratios and thus reducing the need for multi-stage gearing. Such gearing will both increase the weight of the vehicle, it will add to its complexity, and increase the overall cost. Worm gears can produce as high as 60 to 1 speed ratios in a single reduction.

The arrangement of the power transmission components is determined in this stage of the design process. A representation of the relative position of these components in a form of a diagram assists in visualizing the design concept. In Fig. 2.2 such an

![Diagram of the upper base plate assembly](image)

**Fig. 2.2** Schematic of the upper base plate assembly.
arrangement is presented. Since in the worm - worm gear drive radial and thrust loads are
developed both radial and thrust bearings are needed. The motor is connected with the
worm shaft with a coupling. Both bearings are mounted in pillow blocks and they are
held in place with pressure collars. The whole mechanism in fastened on the upper base
plate. The position of the worm gear shaft is adjusted with a precision collar. On the
lower surface of the base plate a collar holding a radial bearing is rigidly attached.

**ii) Lower Base Plate Assembly**

Power transmission to the vehicles wheels is done with a spur gear drive. Spur

![Diagram of major components of the lower base plate assembly.](image-url)

Fig. 2.3 Major components of the lower base plate assembly.
gears are the least expensive and the most commonly used in industrial applications. Since power transmission is done through rolling action, efficiencies can be as high as 99%. These gears are mostly used for drives with parallel shafts. The lower part of the multi-segment snake robot is turned by a steering rod as shown in Fig.2.3. It is composed of a base plate in where the vehicle’s steering rod is attached to. Each segment is supported on two wheels which are turned by a spur gear drive coupled to a motor. Two shock absorbers provide vibration dampening for the base plate.

**iii) Variable length ball and socket joint**

For two adjacent segments to translate and rotate relative to each other the

![Diagram of variable length ball and socket joint](image)

Fig. 2.4 Variable length ball and socket joint mechanism.
mechanism shown in Fig. 2.4 is used. It consists of a linear guide attached to one of the segments. A linear guide slides on a V shape rail attached to the upper base plate. The guide is attached to a ball and socket joint which is connected with the other segment via a threaded fitting.

**iv) Coupling between the base plates**

In order to couple the upper and the lower base plate assemblies the mechanism shown in Fig. 2.5 is employed. Since the worm gear shaft is put in compression a thrust bearing is

![Diagram of coupling assembly of the upper and the lower base plates.](image-url)

Fig. 2.5 Coupling assembly of the upper and the lower base plates.
used. It provides the means of supporting the upper plate on the lower one. A spacer prevents any interference between the ball and socket mechanism and the base plates. A complete assembly of one segment of the snake robot is shown in fig. 2.6 It includes the upper base plate sub-assembly the lower base plate and the coupling mechanism between the two base plates. The variable length ball and socket joint is not shown.

Fig. 2.6 Cross sectional view of a single segment of the snake vehicle.
3.1 Shaft design

In order to determine the diameter of the steering rod a shaft element shown in fig. 3.1 is taken. The diameter of both the worm shaft and the worm gear shaft were found by relying on the fatigue strength analysis and the Sodenberg criterion\(^1\). Both shafts of the worm - worm gear drive are subjected to both bending stresses and torsional stresses.

![Diagram](image)

Fig. 3.1 An element subjected to both fluctuating normal and shear stresses at an angle $\phi$

---

\(^1\) The shaft design drawings and equations are taken from Burr, 1995, pp. 342-357.
Where \(\tau_e\) and \(\sigma_e\) are the combined shear and bending stresses respectively. The factors \(k_f\) and \(k_{fs}\) are depending on stress concentration in bending and shear respectively.

The angle \(\phi\) is formed between any plane where the combined stresses are larger than the bending and shear stresses, and the horizontal plane. The subscripts a and m represent the alternate and mean components of the bending stresses and the torsional stresses respectively.

By summing the forces in the x direction we get:

\[
\tau_c \cdot d_c + (\sigma_m + k_f \cdot \sigma_a) \cdot dy \cdot \cos\phi + (\tau_m + k_{fs} \cdot \tau_a) \cdot dx \cdot \cos\phi - (\tau_m + k_{fs} \cdot \tau_a) \cdot dy \cdot \sin\phi = 0
\]

Dividing both sides by \(dc\), using trigonometric identities and solving for the combining shear stress

\[
\tau_c = \frac{(\sigma_m + k_f \cdot \sigma_a)}{2} \cdot \sin 2\phi + (\tau_m + k_{fs} \cdot \tau_a) \cdot \cos 2\phi
\]

Separating the mean and the alternating terms of the stresses

\[
\tau_c = (\frac{\sigma_m}{2} \cdot \sin 2\phi + \tau_m \cdot \cos 2\phi) + (\frac{k_f \cdot \sigma_a}{2} \cdot \sin 2\phi + k_{fs} \cdot \tau_a \cdot \cos 2\phi)
\]

(3.1)

The Soderberg criterion provides the relationship between the endurance limit of a material and its yield strength. Figure 3.2 shows that this relationship is simply represented by a straight line. From the similar triangles ABE and CDE we get:
Sodenberg criterion.

Solving for the reciprocal of the safety factor we get

\[
\frac{1}{n} = \frac{\tau_{cm}}{\frac{S_y}{2n}} + \frac{\tau_{ca}}{\frac{S_e}{2n}}
\]  

(3.2)
Where \( \tau_{cm} \) and \( \tau_{ca} \) are the mean and the alternating components of of the combined shear stress. \( S_y \) is the yield strength of a material and \( S_e \) is the endurance limit of the material. The endurance limit of a material is given by:

\[
S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot S_e'
\]  
(3.3)

where \( k_a \) is a surface dependent factor, \( k_b \) is a size factor, \( k_c \) is a load factor, \( k_d \) is a temperature dependent factor, \( k_e \) depends on factors such stress concentration, residual stresses, corrosion, and \( S_e' \) is the endurance limit of a test specimen.

Substituting equation 3.2 in to equation 3.1 and separating the bending stress from the torsional stress

\[
\frac{1}{n} = \left( \frac{\sigma_m}{S_y} + \frac{k_f \cdot \sigma_a}{S_e} \right) \cdot \sin 2\phi + 2 \cdot \left( \frac{\tau_m}{S_y} + \frac{k_f \cdot \tau_a}{S_e} \right) \cdot \cos 2\phi
\]  
(3.4)

By setting

\[
\Gamma = \left( \frac{\sigma_m}{S_y} + \frac{k_f \cdot \sigma_a}{S_e} \right) \quad \text{and} \quad \Delta = \left( \frac{\tau_m}{S_y} + \frac{k_f \cdot \tau_a}{S_e} \right)
\]

and by differentiating eqn. 3.4 w.r to \( \phi \) and set it equal to zero to find the maximum stresses

\[
\frac{\sin 2\phi}{\cos 2\phi} = \tan 2\phi = \frac{\Gamma}{2 \cdot \Delta}
\]

But the \( \sin 2\phi \) and \( \cos 2\phi \) are given as
\[ \sin 2\phi = \frac{\Gamma}{\sqrt{\Gamma^2 + 4\Delta^2}} \quad \text{and} \quad \cos 2\phi = \frac{2\Delta}{\sqrt{\Gamma^2 + 4\Delta^2}} \]  

(3.5)

Putting eqn 3.5 in to eqn 3.4 the reciprocal of \( n \) becomes

\[ \frac{1}{n} = \sqrt{\Gamma^2 + 4\Delta^2} \]

or

\[ \frac{1}{n} = \sqrt{\left(\frac{\sigma_m}{S_y} + k_f \cdot \sigma a\right)^2 + 4 \cdot \left(\frac{\tau_m}{S_y} + k_f \cdot \tau a\right)^2} \]  

(3.6)

By factoring out the yield strength, \( n \) becomes:

\[ n = \frac{S_y}{\sqrt{\left(\sigma_m + \frac{S_y \cdot k_f \cdot \sigma a}{S_e}\right)^2 + 4 \cdot \left(\frac{\tau_m}{S_y} \cdot \frac{1}{S_e} \cdot \tau a\right)^2}} \]  

(3.7)

For a solid shaft with diameter \( d \) the bending stress and torsional stress are given by:

\[ \sigma = \frac{32M}{\pi d^3} \quad \text{and} \quad \tau = \frac{16T}{\pi d^3} \]

Where \( T \) and \( M \) are the applied torque and the applied moment respectively. By

substituting the values of the bending stress and torsional stress in to equation 3.7 and

designating by \( m \) and \( a \) the mean and alternating components of the stresses \( n \) becomes

\[ n = \frac{S_y}{\sqrt{\left(\frac{32 \cdot M_m}{\pi d^3} + \frac{S_y \cdot k_f \cdot 32 \cdot M a}{S_e \cdot \pi d^3}\right)^2 + 4 \cdot \left(\frac{16 \cdot T_a}{\pi d^3} + \frac{1}{S_y} \cdot \frac{1}{S_e} \cdot k_f \cdot \frac{16 \cdot T_m}{\pi d^3}\right)^2}} \]  

(3.8)

Solving for the diameter \( d \) in eqn 3.8 we get:
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For steady state rotation the mean component of the bending moment and the alternating component of the torque are zero therefore \( d \) becomes

\[
d = \left[ \frac{32 \cdot n}{\pi \cdot S_y} \cdot \sqrt{(M_m + \frac{k f^2 S_y^2 M_a}{S_e})^2 + (T_m + \frac{k f^2 S_y^2 T_a}{S_e})^2} \right]^{\frac{1}{3}}
\]

For steady state rotation the mean component of the bending moment and the alternating component of the torque are zero therefore \( d \) becomes

\[
d = \left[ \frac{32 \cdot n}{\pi \cdot S_y} \cdot \sqrt{(\frac{k f^2 S_y^2 M_a}{S_e})^2 + (T_m)^2} \right]^{\frac{1}{3}}
\]

(3.9)
3.2 Worm-worm gear drive

A worm gear set consists of a screw-like worm which is meshing with the worm gear, see Fig. 3.3. Several revolutions of the worm produce a single revolution of the worm gear. In this way large speed ratios can be achieved through a single reduction. Most worms have uniform pitch diameter but more teeth can be engaged if a variable pitch diameter worm is used. During power transmission the worm actually slides rather than rolls as it drives the worm gear. As a result of it high level of friction is developed between the worm’s threads and the worm gear’s teeth. Therefore the use of lubrication is essential to avoid metal to metal contact which produces wear.

Fig. 3.3. Uniform pitch diameter worm-worm gear drive.
The selection of the materials the worm and the worm gear are made of plays a role in reducing friction. For example, a bronze made worm gear and a hardened steel made worm have low coefficient of friction when they mesh together.

**Force analysis**

In the worm worm gear interface equal and opposite in direction forces\(^2\) are acting on the

![Diagram of worm and worm gear forces](image)

Fig. 3.4 Forces acting on both the worm and worm gear.

\(^2\) The force analysis for the worm-worm gear drive is taken from Shigley, 1989, pp. 565-570.
worm-worm gear interface as shown in figs. 3.4 (a) and 3.4 (b).

From Figure 3.4 (c) the x, y, and z components of force \( W \) are:

\[
W_x = W \cdot \cos \phi_n \cdot \sin \lambda \tag{3.11 (a)}
\]

\[
W_y = W \cdot \sin \phi_n \tag{3.11 (b)}
\]

\[
W_z = W \cdot \cos \phi_n \cdot \cos \lambda \tag{3.11 (c)}
\]

Where \( \phi_n \) is the pressure angle and \( \lambda \) is the lead angle.

As seen from Figure 3.4 (a), and 3.4 (b) the forces acting on the worm and the worm gear are:

\[
w_{wt} = -w_{Ga} = W_x
\]

\[
w_{wr} = -w_{Gr} = W_y
\]

\[
w_{wa} = -w_{Gt} = W_z
\]

If friction is introduced then

\[
W_f = \mu \cdot W \tag{3.11 (g)}
\]

or

\[
w_x = W \cdot (\cos \phi_n \cdot \sin \lambda + \mu \cdot \cos \lambda ) \tag{3.12 (a)}
\]

\[
w_y = W \cdot \sin \phi_n \tag{3.12 (b)}
\]

\[
w_z = W \cdot (\cos \phi_n \cdot \cos \lambda - \mu \cdot \sin \lambda) \tag{3.12 (c)}
\]

Solving for \( W \)
\[ W = \frac{W_z}{(\cos \phi \cdot \cos \lambda - \mu \cdot \sin \lambda)} \]

\[ W = -\frac{W_{Gl}}{(\cos \phi \cdot \cos \lambda - \mu \cdot \sin \lambda)} \quad \text{(3.12 (d))} \]

Substituting eqn. 3.12 (d) into eqn. 3.11 (g) the frictional force becomes

\[ W_f = \frac{\mu \cdot W_{Gl}}{(\mu \cdot \cos \lambda - \cos \phi \cdot \cos \lambda)} \quad \text{(3.13)} \]

From eqns 3.11 (d) and 3.12 (d) the force acting on the worm in the tangential direction is

\[ W_{wt} = W \cdot (\cos \phi \cdot \sin \lambda + \mu \cdot \cos \lambda) \]

or

\[ W_{wt} = \frac{W_{Gl} \cdot (\cos \phi \cdot \sin \lambda + \mu \cdot \cos \lambda)}{(\mu \cdot \cos \lambda - \cos \phi \cdot \cos \lambda)} \]

The force on the worm in the tangential direction when friction is not present is:

\[ W_{wt} = \frac{-W_{Gl} \cdot (\cos \phi \cdot \sin \lambda)}{\cos \phi \cdot \cos \lambda} \quad \text{(3.14)} \]

The efficiency of the worm gear drive is by definition the ratio of frictionless \( W_w \) to the frictional \( W_w \). By performing some simplifications the efficiency of the drive is given by

\[ \eta = \frac{\cos \phi \cdot \mu \cdot \tan \lambda}{\cos \phi + \mu \cdot \cot \lambda} \quad \text{(3.15)} \]
To determine the sliding velocity in the worm-worm gear interface, the sliding velocities of both the worm and the gear are represented in the way shown in Fig. 3.5. By adding all the sliding velocities vectorially, the worm velocity is:

\[ \vec{V}_W = \vec{V}_G + \vec{V}_S \]  

The sliding velocity in the worm-worm gear interface is:

\[ V_s = \frac{V_w}{\cos \lambda} \]

Where \( \lambda \) is the lead angle.

Fig. 3.5 Sliding velocity in the worm-worm gear interface.

The sliding velocity in the worm-worm gear interface is:

\[ V_s = \frac{V_w}{\cos \lambda} \]
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The center distance between the worm and the worm gear is given by:

\[ C = \frac{d_w + d_G}{2} \]  \hspace{1cm} (3.17)

The pitch velocity of the gear is

\[ V_G = \frac{\pi \cdot d_G \cdot n_G}{12} \]  \hspace{1cm} (3.18)

Where \( d_G \) and \( d_w \) are the pitch diameters of the worm gear and the worm respectively and \( n_G \) is the rotational speed of the gear in rpms.

The lead of the worm is:

\[ L = P_x \cdot n_w \]  \hspace{1cm} (3.19)

Where \( P_x \) is the axial pitch and \( n_w \) is the rotational speed of the worm in rpms.

The lead angle of the worm is giving by

\[ \lambda = a \tan\left(\frac{L}{\pi \cdot d_w}\right) \]  \hspace{1cm} (3.20)

To determine the forces acting on the bearings and the output torque moments are taken as well as all the forces are summed about the x, y and z axis.

\[ \sum M_x = 0 \]  \hspace{1cm} 3.21 (a)

\[ \sum M_y = 0 \]  \hspace{1cm} 3.21 (b)

\[ \sum M_z = 0 \]  \hspace{1cm} 3.21 (c)

\[ \sum F_x = 0 \]  \hspace{1cm} 3.22 (a)
To find the bearing forces as well as the gear forces and the output torque the isometric diagram shown in figure 3.6 is used, where $x_2$ is the radius of the worm gear.

Fig. 3.6 Diagrammatic representation of the forces acting on both the worm gear and the bearings.

\[ \sum F_y = 0 \quad \text{3.22 (b)} \]
\[ \sum F_z = 0 \quad \text{3.22 (c)} \]
3.3 Spur gear drive

Force analysis

The forces acting in the interface between two spur gears are shown in fig. 3.7.

![Diagram of forces](image)

Fig. 3.7 Forces acting in the interface of a spur gear drive.

The only force which transmits load is the tangential component while the radial component serves no purpose. Therefore

\[ W_t = F_{32}^x \]

---

3 The spur gear force analysis is taken from Shigley, 1989, pp. 556-558.
The torque transmitted in gear #2 is given by

\[ T = \frac{d \cdot W_1}{2} \]  

(3.24)

The pitch velocity is

\[ V = \frac{\pi \cdot n_2 \cdot d}{12} \]  

(3.25)

Where \( n_2 \) is the rotational speed of the driver in rpms, and \( d \) is its diameter.

### 3.4 Design of other components

#### Bearing selection

Knowing the forces acting on both the worm- worm gear drive and the spur gear drive the selection of the proper type of bearings follows. The purpose of the bearings is to allow smooth rotary and linear motion between two surfaces. In some cases bearings have to accommodate both motions.

#### Forces

One of these cases involves the worm - worm gear drive. As seen in Fig. 2.2 four bearings are needed for the support of both the worm shaft and the worm gear shaft. There are two types of loads applied to bearings; radial and thrust, Fig. 3.8. Radial loads are acting in a direction normal to the bearing’s axis of rotation while thrust loads are applied parallel to the bearing’s axis of rotation. In order to determine the bearing’s capacity the PV factor is used. It is the product of the pressure in psi exerted on the bearing area and the shaft’s rotational speed in ft/min.
Bearing life

Lubrication of bearings is important because friction is developed during both rotary and linear motion. Bearings with low friction generate low heat such prolonging the life of the bearing.

![Diagram showing forces acting on the bearing's axis of rotation]

Fig. 3.8 Forces acting on the bearing's axis of rotation.

Housing

The bearing housing or pillow block construction requires tolerance of the order of one to two thousandths of an inch. Although there is a variety of pillow blocks in the market it was chosen to be built because of weight and size considerations.

During the preliminary design of the upper base plate assembly two pillow blocks were needed to house two radial and two thrust bearings mounted on the worm shaft. The design of mounted bearing blocks is based on shaft size, radial and thrust load, load characteristics, shaft speed, mounting restrictions, and the environment they are used to.
**Shaft coupling**

Power transmission from one shaft to the other requires that both shafts turn about the same axis. Shaft couplings belong to two categories; rigid and flexible couplings. Flexible couplings are widely used in power transmission applications because they can allow up to 3 degrees of angular misalignment as well as small parallel offset between the two shafts, Fig. 3.9.

![Diagram of shaft coupling](image)

*Fig. 3.9 Parallel offset (a), and angular misalignment (b), between two shafts coupled together by a flexible coupling.*

Flexible couplings can handle, to a certain degree, lateral loads caused by lateral displacement, (parallel offset or angular misalignment, or both) thus relieving the bearings from such loads. In addition they can act as vibration cushions allowing smoother power transmission between the two shafts.
CHAPTER 3: DETAILED DESIGN

Pressure collars

As seen in Fig.2.2 there is a need to keep both the radial and the thrust bearings inside the pillow blocks by using collars which exert pressure on the bearings. They are selected based on the shaft size, weight considerations, and mounted bearing block size.

Base plates

The selection of the material of the bases for both the upper and lower assembly, Figs. 2.2 and 2.3 was based on material strength, weight, machinability, and cost. In Table 1 there is a comparison between materials considered for the base plates based on the criteria listed above.

Table 1. Comparison of Aluminum, Delrin, and acrylic plates.

<table>
<thead>
<tr>
<th>Machinability</th>
<th>Cost</th>
<th>Density (g/cm³)</th>
<th>Tensile Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>Good</td>
<td>Medium</td>
<td>2.70</td>
</tr>
<tr>
<td>Delrin</td>
<td>Very Good</td>
<td>High</td>
<td>1.42</td>
</tr>
<tr>
<td>Acrylic</td>
<td>Excellent</td>
<td>High</td>
<td>1.19</td>
</tr>
</tbody>
</table>

Taking into account Table 1 cast acrylic was chosen the material for both the upper and the lower base assemblies.

Coupling between the base plates

In order to couple the upper and the lower base plates, and provide better lateral support for the steering rod a bushing is needed. The selection criteria for the material are resistance to oils and greases, impact strength, coefficient of friction, and density. In
Table 2, Delrin, Aluminum, and Cast Acrylic are compared based on the properties mentioned above.

Table 2. Comparison of various materials based on mechanical, physical and chemical properties.

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficient of friction (g/cm^3)</th>
<th>Ozod impact strength (J/m)</th>
<th>Resistance to oils and greases</th>
<th>Density (g / cm^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delrin</td>
<td>0.1 - 0.3</td>
<td>64 - 123</td>
<td>Good</td>
<td>1.42</td>
</tr>
<tr>
<td>Aluminum</td>
<td>0.2 - 0.4</td>
<td>-</td>
<td>Very Good</td>
<td>2.70</td>
</tr>
<tr>
<td>Acrylic</td>
<td>-</td>
<td>16 - 32</td>
<td>Poor</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Based on the properties mentioned in Table 2 Delrin was chosen for this component.

**Motor support**

In order to mount the selected motors in both the upper and lower base plates, a motor support block has to be designed. The design criteria are low weight, size, and high tensile strength. For this purpose 6061 Aluminum plate with rectangular cross sectional area was selected.

**Ball and socket joint**

In the computer simulation of the snake robot the rotation of one segment with respect to the other was performed via a ball and socket joint. B ∈ S joints allow three angular motions as seeing in Fig. 3.10.
Fig. 3.10 Ball and Socket joint’s three angular motions.

These motions are angular displacements $\alpha$ and $\beta$ and angular rotation $\gamma$, therefore having three degrees of freedom. The only disadvantage of the B$\in$S joints is that their turning radius is limited, in most cases to about 30°.
3.5 Balancing the vehicle

To balance the inherently unstable snake vehicle three different methods were considered. That is, the installment of an active sensor, the static balance of the system and balance by pairs.

Active sensor

This method involves the introduction to the system of a sensor which will not permit the shift of the center of mass of vehicle while it moves. This can be achieved by the use of a counterbalance mechanism which will keep the center of mass unchanged.

Static balance

In this approach both the upper and the power base plate assemblies will be statically balanced by the use of counterweights. Since the vehicle is rotating around its wheel axis, static balance about this axis is necessary. Therefore the moments around this axis of rotation should be zero. That is

\[ \sum M_x = 0 \]

Balance by pairs

The center of mass of the motor assemblies for both the upper and lower base plates is offset by the same distance from the center of each plate. If the segments are to be assembled in the orientation shown in Fig. 3.11 then each pair is balanced.
Fig. 3.11 Balancing by pairs of the snake-like vehicle.

The advantages of this method of balance are:

. There is no increase in the vehicle’s weight since counterweights are not needed.

. No additional expense is added to the project’s overall cost.

. Fewer parts are needed and the design time is shorter.
4.1 Machining operations

In order to construct the vehicle, most of the parts were purchased from various manufacturers. When there was a need to make a part like the bearing pillow block of the worm-worm gear drive various machining techniques were employed. Such methods include: drilling, milling, grinding, cutting, turning, and facing operations. When precision drilling and milling operations were needed CNC, Computer Numerical Control, machines were used.

The various machining operations were used to provide the part the final dimensions, the desired surface finish, and to provide the desired part shape when this is not economical by the use of manufacturing processes. In the case of the construction of the vehicle machining operation were extensively used due to the limited number of similar parts required for it.

4.2 Machinability

An important element considered in the construction of the vehicle is the machinability of the material. It is defined in terms of the surface finish of the part, tool life, force and power requirements. Although it is difficult to have machinability ratings of a particular material the following principle was used. The desired dimensions and surface finish of a part should be obtained by limiting tool wear, and by minimizing force requirements, (Kalpakjian, 1991, pp. 515-519).
5.1 Torque requirements

In order to determine the torque required to turn the snake robot, an experimental method was used. In order to estimate the torque needed to turn the vehicle a pulley was affixed to the worm gear shaft, Fig. 5.1. One end of a cord was secured to the outer surface of the pulley and the rest of it was wrapped around it. The other end of the cord was tied to a spring scale. A manual force was exerted on the rope until the shaft started turning. The product of the force indicated on the spring scale and the radius of the pulley gives the torque. Subsequently, the force needed to turn the worm a few of the revolutions was found in the manner described above. This time the approximate value...
CHAPTER 5: TORQUE AND SERVOCONTROL

running torque is found.

**Continuous torque**

Based on the required load torque a servomotor is chosen. Based on motor operating conditions the continuous torque\(^4\) is given by:

\[
T_{\text{cont}} = \sqrt{\frac{(155-T_{\text{amb}})}{\text{TRP}}} - \frac{T_m S}{C} \cdot k \cdot PKO - T_m
\]

(5.1)

Where:

\(T_{\text{cont}}\) = Continuous load torque

155 = Maximum winding temperature (155 °C)

\(T_{\text{amb}}\) = Ambient temperature (°C)

\(S\) = Motor speed (rev/min)

\(\text{TRP}\) = Motor thermal impedance (°C / W)

\(C = 1352\) for \(T_m = \text{oz} \cdot \text{in}\)

\(9.549\) for \(T_m = \text{N} \cdot \text{m}\)

\(\text{PKO}\) = Motor constant

\(T_m\) = Motor friction torque

\(K = 0.71\) for brush commutated motors, ferrite magnet motors

0.78 for brush commutated rare earth magnet motors

---

\(^4\) Equation 5.1 is taken from Pittman, servomotor manufacturer.
0.79 for brushless, rare earth magnet motors

0.80 for brushless ferrite magnet motors

Equation 5.1 is given by the servomotor manufacturer in order to obtain an accurate estimation of the continuous motor torque. This value is calculated under the worst possible operating conditions. For gearmotors the speed of the motor in rpms is

\[ S_{motor} = S_{output} \cdot GR \]  \hspace{1cm} (5.2)

Where GR is the motor gear ratio

The relationship between the output torque and the motor torque is given by:

\[ T_{motor} = \frac{T_{output}}{GR \cdot \eta} \]  \hspace{1cm} (5.3)

Where \( \eta \) is the motor efficiency

### 5.2 Servocontrol

Servomotors are commonly used in closed loop control systems in where they are connected to a load. As seen in Fig. 5.2 the controller sends velocity signals to the amplifier which controls the motor. The system includes integral feedback components such as optical encoders and tachometers. These devices are either an integral part of the motor itself or they are mounted to the load. The signal sent to the controller by these feedback devices is compared with the desired motion profile and is used to change its
velocity signal. The desired motion profile consists of time, position, and velocity signals which are controlling the operation. The position and the velocity feedback signal is sent back to the PC's Data Acquisition Board Card. The velocity and acceleration of the gearmotors is controlled by the SCC, Servo Control Card.

Fig. 5.2 Diagram of a typical closed loop with position feedback servosystem.
Results

To determine the forces acting on the bearings and the output torque for both the worm-worm gear and spur gear drives the following plots were created. In Fig. 6.1 the range of the torque output of the worm-worm gear drive was found as a function of the operating rotational speed of the worm. The forces acting on the bearings of the same gear drive in the radial as well as in the tangential direction for bearing A, Figs. 6.2, 6.3, and for the tangential direction for bearing B, Fig 6.4, were determined. In addition, the output torque of the spur gear drive as a function of the driving tangential force, Fig. 6.5, was estimated.

![Graph](image)

Fig. 6.1 Effect of the rotational speed of the worm on the output torque of the worm-worm gear drive.

The values of both the bearing forces and output torque at a specified speed range
CHAPTER 6: RESULTS

determine the type of the bearings and the type of the servomotors needed.

Fig. 6.2 Forces developed in the radial direction of bearing A at different rotational speeds of the worm.

To determine the type and the dimensions of the shafts the equations of chapter 3.1 were used. An example of these calculations is described in the Appendix B. The snake-like vehicle was designed to be modular, to easily incorporate end-effectors for surveillance, maintenance, and inspection purposes. It was originally designed for motion in 2D space, but with minor modifications, use of shock absorbers, for use in a 3D environment. Due to stability problems the use of one wheel per segment was not adapted. This led to the adjustment of the already developed motion algorithm. The equations of motion are being adjusted to two wheels per segment.
CHAPTER 6: RESULTS

Fig. 6.3 Forces acting on the tangential direction of bearing A at different worm speeds.

Fig. 6.4 Forces acting on the tangential direction of bearing B at different rotational speeds of the worm.
CHAPTER 6: RESULTS

Fig. 6.5 Resulting output torque of the spur gear drive at different forces in the tangential direction.

During testing both the worm-worm gear and spur gear drives powered by gearmotors were able to move and turn the vehicle. The new motion algorithm will be tested on the already developed snake robot. The specifications of the snake-like vehicle are shown in Table 3.

Table 3. Technical specification for each segment of the snake robot.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>10.0 in (25.4 cm)</td>
</tr>
<tr>
<td>Width</td>
<td>6.0 in (12.2 cm)</td>
</tr>
<tr>
<td>Height</td>
<td>10.0 in (25.4 cm)</td>
</tr>
<tr>
<td>Weight with BES</td>
<td>5.5 lbs (2.5 Kg)</td>
</tr>
<tr>
<td>Weight without BES</td>
<td>4.5 lbs (2.0 Kg)</td>
</tr>
<tr>
<td>Max distance between two segments</td>
<td>5.0 in (12.7 cm)</td>
</tr>
<tr>
<td>Min distance between two segments</td>
<td>2.3 in (5.8 cm)</td>
</tr>
</tbody>
</table>
Conclusions

Following the simulation of two motion algorithms on a Silicon Graphics workstation, the multi-segment snake-like wheeled vehicle was designed and constructed based on established design requirements and design criteria.

Each segment of the robot is powered by two high torque, high gear ratio servomotors with closed loop position feedback. These actuators can turn and move each segment of the vehicle individually. Power is transmitted from the motors to the wheels, and to the steering rod by two gear drives. A worm-worm gear drive turns each segment independently. The movement of each segment is accomplished by the use of a spur gear drive. Translation between two adjacent segments is achieved by the use of a linear sliding mechanism, incorporating ball bearing guides and induction hardened ground steel rails. A York steel ball and socket joint provides relative rotation between two adjacent segments.

The robot is designed to perform planar motions, but it has the capability to execute motions in three dimensional space, with minor design modifications performed. In order to ensure that only pure rolling conditions exist between the vehicle’s wheels and the ground, the vehicle was designed in such a way that slipping of the wheels is avoided.

During the design of the snake-like vehicle, emphasis was placed on incorporating various end-effector assemblies. The design of these assemblies will enable the vehicle to be used for manipulation, surveillance, and inspection tasks. The vehicle’s Servocontroller Cards, power supplies, Data Acquisition Cards are compatible
with existing electronic components in the lab. This interchangability reduces the cost of designing and operating various robotic systems in the Mechanical Systems lab.

In the computer simulation and in the mathematical model, the use of one wheel per segment was required. Physical implementation of this design requirement will have required additional control inputs during the vehicle’s movement. Any failure to maintain static and dynamic balance will have caused damage of the robot’s components, such as gears, steering rods, servomotors, e.t.c. For these reasons the one wheel per segment design requirement was not incorporated.

A four segment snake-like vehicle was fabricated in house within the design specifications by using various machining methods. During testing the motion of the vehicle in unprescribed paths was achieved.

The design of the vehicle is compact, minimizing the number of parts needed, providing a low cost experimental platform implementing the two motion algorithms already developed.

The development of motion algorithms based on kinematics is the subject of research being conducted in the Mechanical Engineering Department of the University of Delaware. The future of snake-like vehicles in the medical field, in nuclear powerplants, and in search and rescue operations following natural disasters is promising.
REFERENCES


REFERENCES


REFERENCES

Saha, K., S., Angeles, J. (1989). Kinematics and Dynamics of a Three-Wheeled 2-DOF AGV, Department of Mechanical Engineering & McGill Centre of Intelligent Machines, McGill University, Montreal, Canada, 1572, 1573.


REFERENCES


APPENDIX A

Shaft design

By summing all the forces in the direction of the combined shear stress and by setting it equal to zero, fig. 3.2, we get

\[
\begin{align*}
\int -\tau_c \cdot dc + \left( \sigma_m + k_f \sigma_a \right) dy \cdot \cos \phi + \left( \tau_m + k_f \tau_a \right) dx \cdot \cos \phi &= \left( \tau_m + k_f \tau_a \right) dy \cdot \sin \phi \\text{[3.2]} \\
\tau_c &= \left( \sigma_m + k_f \sigma_a \right) \frac{dy}{dc} \cdot \cos \phi + \left( \tau_m + k_f \tau_a \right) \frac{dx}{dc} \cdot \cos \phi - \left( \tau_m + k_f \tau_a \right) \frac{dy}{dc} \cdot \sin \phi \\
\tau_c &= \left( \sigma_m + k_f \sigma_a \right) \sin \phi \cdot \cos \phi + \left( \tau_m + k_f \tau_a \right) \cos \phi^2 - \left( \tau_m + k_f \tau_a \right) \sin \phi^2 \\
\tau_c &= \frac{\sigma_m + k_f \sigma_a}{2} \cdot \sin 2\phi + \left( \tau_m + k_f \tau_a \right) \cos 2\phi \\
\end{align*}
\]

Combining the mean and alternating stress terms

\[
\tau_c = \left( \sigma_m \sin 2\phi + \tau_m \cos 2\phi \right) + \left( k_f \sigma_a \sin 2\phi + k_f \tau_a \cos 2\phi \right) \quad (3.1)
\]

Using the Sodenberg criterion and from the similar triangles ABE and CDE we get:

\[
\frac{DE}{BE} = \frac{CD}{AB}
\]

or

\[
\frac{S_y}{2} - \frac{\tau_{cm}}{2n} = \frac{\tau_{ca}}{S_e} \quad \frac{S_y}{2n} = \frac{S_e}{2n}
\]

Solving for the safety factor n
APPENDIX A: DERIVATIONS

\[ \frac{1}{n} = \frac{\tau_{cm}}{S_y} + \frac{\tau_{ca}}{S_e} \]  \hspace{1cm} (3.2)

Putting eqn. 3.2 in to eqn. 3.1

\[ \frac{1}{n} = \frac{\sigma_m \sin (2 \phi) + \tau_m \cos (2 \phi)}{2 \left( \frac{S_y}{2} \right)} + \frac{\sigma_a - k_f \sin (2 \phi) + \tau_a \cos (2 \phi)}{2 \left( \frac{S_e}{2} \right)} \]

\[ \frac{1}{n} = \left( \frac{\sigma_m \sin (2 \phi) + \tau_m \cos (2 \phi)}{S_y + \frac{k_f \sigma_a}{S_e}} \right) + \left( \frac{\tau_a \cos (2 \phi)}{S_y + \frac{k_f \sigma_a}{S_e}} \right) \]

By making the following simplification eqn. 3.4 becomes

\[ \Gamma = \left( \frac{\sigma_m \sigma_a}{S_y} + \frac{k_f \sigma_a}{S_e} \right) \Delta = \left( \frac{\tau_m \tau_a}{S_y} + \frac{k_f \tau_a}{S_e} \right) \]

\[ \frac{1}{n} = \Gamma \cdot \sin (2 \phi) + 2 \cdot \Delta \cdot \cos (2 \phi) \]  \hspace{1cm} 3.4 (a)

By setting the derivative w.r to the angle phi equal to zero

\[ \frac{d}{d \phi} \left( \frac{1}{n} \right) = 0 \]

\[ 2 \cdot \Gamma \cdot \cos (2 \phi) - 4 \cdot \Delta \cdot \sin (2 \phi) = 0 \]

\[ \sin (2 \phi) = \frac{\Gamma}{\sqrt{\Gamma^2 + 4 \cdot \Delta^2}} \]

\[ \cos (2 \phi) = \frac{2 \cdot \Delta}{\sqrt{\Gamma^2 + 4 \cdot \Delta^2}} \]  \hspace{1cm} (3.5)
APPENDIX A: DERIVATIONS

For a solid shaft with diameter \(d\) the bending stress and shear stress are given by

\[
\sigma = \frac{M \cdot c}{I} = \frac{M \cdot d}{\pi \cdot d^4} = \frac{32 \cdot M}{\pi \cdot d^3}
\]
\[
\tau = \frac{T \cdot c}{J} = \frac{T \cdot d}{2 \cdot \pi \cdot d^4} = \frac{16 \cdot T}{\pi \cdot d^3}
\]

Then the alternating, mean shear and bending stresses are:

\[
\sigma_a = \frac{32 \cdot M_a}{\pi \cdot d^3}, \quad \sigma_m = \frac{32 \cdot M_m}{\pi \cdot d^3}, \quad \tau_a = \frac{16 \cdot T_a}{\pi \cdot d^3}, \quad \tau_m = \frac{16 \cdot T_m}{\pi \cdot d^3}
\]

\[
n = \frac{S_y}{\sqrt{\left(\frac{32 \cdot M_m + S_y \cdot k_{f_b} \cdot 32 \cdot M_a}{\pi \cdot d^3}\right)^2 + 4 \cdot \left(\frac{16 \cdot T_a + S_y \cdot k_{f_b} \cdot 16 \cdot T_m}{\pi \cdot d^3}\right)^2}}
\]

\[
n = \frac{S_y}{\left[\frac{32}{\pi \cdot d^3} \left(\frac{M_m + \frac{k_{f_b} \cdot S_y \cdot M_a}{S_e}}{S_e}\right)^2 + 4 \cdot \left(\frac{16 \cdot T_a + \frac{k_{f_b} \cdot S_y \cdot T_m}{S_e}}{S_e}\right)^2\right]^{1/3}}
\]

Solving for the shaft diameter we get

\[
d = \frac{32 \cdot n}{\pi \cdot S_y} \sqrt{\frac{M_m + \frac{k_{f_b} \cdot S_y \cdot M_a}{S_e}}{S_e} + \left(\frac{T_a + \frac{k_{f_b} \cdot S_y \cdot T_m}{S_e}}{S_e}\right)^2}^{1/3}
\]

For steady state shaft rotation the mean moment and the alternating torque are zero

\[
d = \frac{32 \cdot n}{\pi \cdot S_y} \sqrt{\frac{k_{f_b} \cdot S_y \cdot M_a}{S_e} + \left(\frac{T_m}{S_e}\right)^2}^{1/3}
\]
Worm - worm gear force analysis

The lead of the worm is given by

\[ L = P_x N_w \]

The lead angle is:

\[ \tan \lambda = \frac{L}{\pi \cdot d_w} \]

The frictional forces acting in the x, y, and z directions

\[ W_x = W \cdot \cos \phi_n \cdot \sin \lambda \]
3.11 (a)

\[ W_y = W \cdot \sin \phi_n \]
3.11 (b)

\[ W_z = W \cdot \cos \phi_n \cdot \cos \lambda \]
3.11 (c)

The forces acting on the gear and the worm are obtained by

\[ W_{(w_x)} = W_{(g_x)} = W_x \]
3.11 (d)

\[ W_{(w_y)} = W_{(g_y)} = W_y \]
3.11 (e)

\[ W_{(w_z)} = W_{(g_z)} = W_z \]
3.11 (f)

When friction is introduced then:

\[ W_f = \mu \cdot W \]

\[ W_x = W \cdot (\cos \phi_n \cdot \sin \lambda + \mu \cdot \cos \lambda) \]
3.12 (a)

\[ W_y = W \cdot \sin \phi_n \]
3.12 (b)

\[ W_z = W \cdot (\cos \phi_n \cdot \sin \lambda - \mu \cdot \sin \lambda) \]
3.12 (c)
APPENDIX A: DERIVATIONS

Since

\[ W_f = \mu \cdot W \]

From equation 3.12 (c) we get

\[ W = \frac{W_Z}{\left( \cos \phi_n \cdot \cos \lambda - \mu \cdot \sin \lambda \right)} = \frac{W(U)}{\left( \cos \phi_n \cdot \cos \lambda - \mu \cdot \sin \lambda \right)} \]

Therefore the frictional force is:

\[ W_f = \frac{\mu \cdot W(U)}{\left( \mu \cdot \sin \lambda - \cos \phi_n \cdot \cos \lambda \right)} \]

(3.13)

\[ W_{w_t} = W(U_n) = W_x = W \cdot \left( \cos \phi_n \cdot \sin \lambda + \mu \cdot \cos \lambda \right) \]

\[ W_{w_t} = \frac{W(U) \cdot \left( \cos \phi_n \cdot \sin \lambda + \mu \cdot \cos \lambda \right)}{\left( \mu \cdot \sin \lambda - \cos \phi_n \cdot \cos \lambda \right)} \]

The efficiency can be defined as the ratio of the frictionless force in the tangential direction to the same force but with the friction not included.

\[ \eta = \frac{W_{w_t}}{W_{w_{tf}}} \]
Therefore the efficiency of the worm gear drive is:

\[
\eta = \frac{W (g_1) \cdot (\cos \phi_n \cdot \sin \lambda)}{\cos \phi_n \cdot \cos \lambda}
\]

\[
\eta = \frac{W (g_1) \cdot (\cos \phi_n \cdot \sin \lambda + \mu \cdot \cos \lambda)}{(\mu \cdot \sin \lambda - \cos \phi_n \cdot \cos \lambda)}
\]

With further simplifying the expression above the efficiency is given by:

\[
\eta = \frac{\sin \lambda \cdot (-\mu \cdot \sin \lambda + \cos \phi_n \cdot \cos \lambda)}{\cos \lambda \cdot (\cos \phi_n \cdot \sin \lambda + \mu \cdot \cos \lambda)}
\]

(3.15)
Worm - worm gear drive

The pressure angle is:  \( \phi_n := \frac{14.5 \cdot \pi}{180} \text{ rads} \)

The diameter of the worm gear is:  \( d_g := 1.875 \text{ in} \)

The diameter of the worm is:  \( d_w := 0.465 \text{ in} \)

The number of threads of the worm  \( N_w := 6 \)

The number of teeth of the worm gear  \( N_g := 60 \)

The rotational speed of the worm  \( n_w := 150, 160.. 400 \)

From fig. 3.6 the distances \( x_1, x_2, \) and \( x_3 \) are:

\[ x_1 = 1.0 \text{ in} \quad x_2 = \frac{d_g}{2} \text{ in} \quad x_3 = 2.5 \text{ in} \]

The circular pitch of the gear \( P \) is given by:

\[ P = \frac{N_g}{d_g} \]

The axial pitch of the worm is the same as the circular pitch of the gear which is:

\[ P_t = \frac{\pi}{P} \]

The lead then is

\[ L = P_t \cdot N_w \]

The transmitted horsepower for an input torque of 40 oz.in is:

\[ H_p(n_w) = \frac{40}{63000} \cdot n_w \]

The center distance between the gears is given by eqn 3.17

\[ C := \frac{d_w + d_g}{2} \]
The lead angle is obtained from eqn. 3.20

\[ \lambda := \text{atan} \left( \frac{L}{\pi \cdot \text{dw}} \right) \]

The pitch line velocity of the worm is

\[ V_w(nw) := \frac{\pi \cdot \text{dw} \cdot nw}{12} \]

The rotational speed of the gear is

\[ n_g(nw) := \frac{N_w \cdot nw}{N_g} \]

The pitch line velocity of the worm gear is calculated via eqn. 3.18

\[ V_g(nw) := \frac{\pi \cdot \text{dg} \cdot n_g(nw)}{12} \]

The sliding velocity is

\[ V_s(nw) := \frac{V_w(nw)}{\cos(\lambda)} \]

The coefficient of friction is given by:

\[ \mu(nw) = 0.103 \cdot e^{0.11 \cdot V_s(nw)^{0.450}} + 0.012 \]

The force on the gear in the tangential direction is

\[ W_{wt}(nw) := \frac{33000 \cdot H_p(nw)}{V_w(nw)} \]

Combining eqns 3.12 (b), 3.11 (d) the force W is:

\[ W(nw) := \frac{W_{wt}(nw)}{\cos(\phi n) \cdot \sin(\lambda) + \mu(nw) \cdot \cos(\lambda)} \]

The force Wy is given by eqn 3.12 (b)

\[ W_y(nw) = W(nw) \cdot \sin(\phi n) \]
The force in the z direction is obtained from eqn 3.12 (c)

\[ W(z) = W(nw) \cdot (\cos(\phi) \cdot \cos(\lambda) - \mu(nw) \cdot \sin(\lambda)) \]

From eqns 3.11 (d), 3.11 (e), 3.11 (f)

\[ W_{ga}(nw) = W_{wt}(nw) \]
\[ W_{gr}(nw) = W_{y}(nw) \]
\[ W_{gt}(nw) = W_{z}(nw) \]

Summing the forces in the x direction, eqn 3.22 (a)

\[ F_{bx}(nw) = W_{ga}(nw) \]

Taking moments about the z axis, eqn 3.21 (c)

\[ F_{by}(nw) = \frac{W_{ga}(nw) \cdot x_2 - W_{gr}(nw) \cdot x_1}{x_3} \]

Taking moments about the y axis, eqn 3.21 (b)

\[ F_{bz}(nw) = \frac{W_{gt}(nw) \cdot x_1}{x_3} \]

Summing forces in the y direction, eqn 3.22 (b)

\[ F_{ay}(nw) = W_{gr}(nw) - F_{by}(nw) \]

Summing the forces in the z direction, eqn 3.22 (c)

\[ F_{az}(nw) = W_{gt}(nw) - F_{bz}(nw) \]

Summing moments about x, eqn 3.21 (a)

\[ T_{out}(nw) = W_{gt}(nw) \cdot x_2 \]

The efficiency of the gear drive is given by eqn 3.15

\[ \eta(nw) = \left( \frac{\cos(\phi) - \mu(nw) \cdot \tan(\lambda)}{\cos(\phi) + \mu(nw) \cdot \cot(\lambda)} \right) \cdot 100 \]
### APPENDIX B

Table 1  Output torque and forces on the bearings of the worm-worm gear drive

<table>
<thead>
<tr>
<th>nw</th>
<th>Tout (nw)</th>
<th>( \eta ) (nw)</th>
<th>Fay (nw)</th>
<th>Faz (nw)</th>
<th>Fby (nw)</th>
<th>Fbz (nw)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>20.083</td>
<td>80.301</td>
<td>4.616</td>
<td>29.991</td>
<td>1.562</td>
<td>8.569</td>
</tr>
<tr>
<td>170</td>
<td>20.167</td>
<td>80.637</td>
<td>4.646</td>
<td>30.117</td>
<td>1.554</td>
<td>8.605</td>
</tr>
<tr>
<td>180</td>
<td>20.207</td>
<td>80.794</td>
<td>4.66</td>
<td>30.175</td>
<td>1.55</td>
<td>8.622</td>
</tr>
<tr>
<td>190</td>
<td>20.245</td>
<td>80.946</td>
<td>4.674</td>
<td>30.232</td>
<td>1.546</td>
<td>8.638</td>
</tr>
<tr>
<td>200</td>
<td>20.281</td>
<td>81.092</td>
<td>4.687</td>
<td>30.287</td>
<td>1.542</td>
<td>8.653</td>
</tr>
<tr>
<td>210</td>
<td>20.316</td>
<td>81.233</td>
<td>4.7</td>
<td>30.339</td>
<td>1.539</td>
<td>8.668</td>
</tr>
<tr>
<td>230</td>
<td>20.384</td>
<td>81.502</td>
<td>4.724</td>
<td>30.44</td>
<td>1.532</td>
<td>8.697</td>
</tr>
<tr>
<td>240</td>
<td>20.416</td>
<td>81.63</td>
<td>4.735</td>
<td>30.487</td>
<td>1.528</td>
<td>8.711</td>
</tr>
<tr>
<td>250</td>
<td>20.447</td>
<td>81.754</td>
<td>4.746</td>
<td>30.534</td>
<td>1.525</td>
<td>8.724</td>
</tr>
<tr>
<td>270</td>
<td>20.506</td>
<td>81.992</td>
<td>4.768</td>
<td>30.623</td>
<td>1.519</td>
<td>8.749</td>
</tr>
<tr>
<td>280</td>
<td>20.535</td>
<td>82.106</td>
<td>4.778</td>
<td>30.665</td>
<td>1.516</td>
<td>8.761</td>
</tr>
<tr>
<td>290</td>
<td>20.562</td>
<td>82.217</td>
<td>4.788</td>
<td>30.707</td>
<td>1.513</td>
<td>8.773</td>
</tr>
<tr>
<td>300</td>
<td>20.59</td>
<td>82.325</td>
<td>4.797</td>
<td>30.747</td>
<td>1.511</td>
<td>8.785</td>
</tr>
<tr>
<td>310</td>
<td>20.616</td>
<td>82.431</td>
<td>4.807</td>
<td>30.787</td>
<td>1.508</td>
<td>8.796</td>
</tr>
<tr>
<td>320</td>
<td>20.642</td>
<td>82.534</td>
<td>4.816</td>
<td>30.825</td>
<td>1.505</td>
<td>8.807</td>
</tr>
<tr>
<td>330</td>
<td>20.667</td>
<td>82.635</td>
<td>4.825</td>
<td>30.863</td>
<td>1.503</td>
<td>8.818</td>
</tr>
<tr>
<td>340</td>
<td>20.692</td>
<td>82.733</td>
<td>4.834</td>
<td>30.92</td>
<td>1.5</td>
<td>8.828</td>
</tr>
<tr>
<td>350</td>
<td>20.716</td>
<td>82.83</td>
<td>4.843</td>
<td>30.926</td>
<td>1.498</td>
<td>8.839</td>
</tr>
<tr>
<td>360</td>
<td>20.739</td>
<td>82.924</td>
<td>4.851</td>
<td>30.971</td>
<td>1.495</td>
<td>8.849</td>
</tr>
<tr>
<td>370</td>
<td>20.762</td>
<td>83.016</td>
<td>4.859</td>
<td>31.005</td>
<td>1.493</td>
<td>8.859</td>
</tr>
<tr>
<td>380</td>
<td>20.785</td>
<td>83.107</td>
<td>4.867</td>
<td>31.039</td>
<td>1.491</td>
<td>8.868</td>
</tr>
<tr>
<td>390</td>
<td>20.807</td>
<td>83.195</td>
<td>4.875</td>
<td>31.072</td>
<td>1.488</td>
<td>8.878</td>
</tr>
<tr>
<td>400</td>
<td>20.829</td>
<td>83.282</td>
<td>4.883</td>
<td>31.105</td>
<td>1.486</td>
<td>8.887</td>
</tr>
</tbody>
</table>
Worm - worm gear drive

The pressure angle is:
\[ \phi_n = \frac{14.5 \pi}{180} \text{ rads} \]

The diameter of the worm gear is:
\[ d_g = 1.875 \text{ in} \]

The diameter of the worm is:
\[ d_w = 0.465 \text{ in} \]

The number of threads of the worm:
\[ N_w = 6 \]

The number of teeth of the worm gear:
\[ N_g = 60 \]

The rotational speed of the worm:
\[ n_w = 300 \text{ rpm} \]

From fig. 3.6 the distances \( x_1, x_2, \) and \( x_3 \) are:
\[ x_1 = 1.0 \text{ in} \quad x_2 = \frac{d_g}{2} \text{ in} \quad x_3 = 2.5 \text{ in} \]

The circular pitch of the gear \( P \) is given by:
\[ P = \frac{N_g}{d_g} \]

The axial pitch of the worm is the same as the circular pitch of the gear which is:
\[ P_t = \frac{\pi}{P} \]
\[ P_t = 0.098 \]

The lead then is
\[ L = P_t \cdot N_w \]
\[ L = 0.589 \text{ in} \]

The transmitted horsepower for an input torque of 40 oz.in is:
\[ H_p = \frac{40 \cdot n_w}{16 \cdot 63000} \]
\[ H_p = 0.012 \]
The center distance between the gears is given by eqn 3.17

\[ C = \frac{d_w + d_g}{2} \]

\[ C = 1.17 \text{ in} \]

The lead angle is obtained from eqn. 3.20

\[ \lambda = \tan \left( \frac{L}{\pi \cdot d_w} \right) \]

\[ \lambda = 0.383 \text{ rads} \]

The pitch line velocity of the worm is

\[ V_w = \frac{\pi \cdot d_w \cdot n_w}{12} \]

\[ V_w = 36.521 \text{ ft/min} \]

The rotational speed of the gear is

\[ n_g = \frac{N_w \cdot n_w}{N_g} \]

\[ n_g = 30 \text{ rpm} \]

The pitch line velocity of the worm gear is calculated via eqn. 3.18

\[ V_g = \frac{\pi \cdot d_g \cdot n_g}{12} \]

\[ V_g = 14.726 \text{ ft/min} \]

The sliding velocity is

\[ V_s = \frac{V_w}{\cos (\lambda)} \]

\[ V_s = 39.378 \text{ ft/min} \]
The coefficient of friction is given by:

\[ \mu = 0.103 e^{0.11 V_s^{0.450}} + 0.012 \]

\[ \mu = 0.07 \]

The force on the gear in the tangential direction is

\[ W_{wt} = \frac{33000 Hp}{V_w} \]

\[ W_{wt} = 10.757 \text{ lbs} \]

Combining eqns 3.12 (b), 3.11 (d) the force \( W \) is:

\[ W = \frac{W_{wt}}{\cos(\phi n) \sin(\lambda) + \mu \cos(\lambda)} \]

\[ W = 25.194 \text{ lbs} \]

The force \( W_y \) is given by eqn 3.12 (b)

\[ W_y = W \cdot \sin(\phi n) \]

\[ W_y = 6.308 \text{ lbs} \]

The force in the \( z \) direction is obtained from eqn 3.12 (c)

\[ W_z = W \cdot (\cos(\phi n) \cdot \cos(\lambda) - \mu \cdot \sin(\lambda)) \]

\[ W_z = 21.962 \text{ lbs} \]

From eqns 3.11 (d), 3.11 (e), 3.11 (f)

\[ W_{ga} = W_{wt} \]

\[ W_{gr} = W_y \]

\[ W_{gt} = W_z \]
APPENDIX B

Summing the forces in the x direction, eqn 3.22 (a)

\[ F_{bx} = Wga \]

Taking moments about the z axis, eqn 3.21 (c)

\[ F_{by} = \frac{Wga \cdot x2 - Wgr \cdot x1}{x3} \]

\[ F_{by} = 1.511 \text{ lbs} \]

Taking moments about the y axis, eqn 3.21 (b)

\[ F_{bz} = \frac{Wgt \cdot x1}{x3} \]

\[ F_{bz} = -8.785 \text{ lbs} \]

Summing forces in the y direction, eqn 3.22 (b)

\[ F_{ay} = Wgr - F_{by} \]

\[ F_{ay} = 4.797 \text{ lbs} \]

Summing the forces in the z direction, eqn 3.22 (c)

\[ F_{az} = Wgt - F_{bz} \]

\[ F_{az} = 30.747 \text{ lbs} \]

Summing moments about x, eqn 3.21 (a)

\[ T_{out} = Wgt \cdot x2 \]

\[ T_{out} = 20.59 \text{ lb.in} \]

The efficiency of the gear drive is given by eqn 3.15

\[ \eta = \left( \frac{\cos(\phi n) - \mu \cdot \tan(\lambda)}{\cos(\phi n) + \mu \cdot \cot(\lambda)} \right) \cdot 100 \]

\[ \eta = 82.325 \% \]
The output torque is less than the product of the input torque and gear ratio

\[ T_{\text{out}} < T_{\text{in}} \times \text{GR} \text{ due to friction, or} \]

\[ T_{\text{out}} < 25 \]

The bearing forces as well as the forces acting on the worm gear are combined in the shear force and bending moment diagram shown in fig B1. The moments in both the X, Y and X, Z planes are combined in order to find the maximum moment.

Therefore the maximum moment is:

\[ M_{\text{z}} = 10.09 \text{ lb-in and } M_{\text{y}} = 21.96 \text{ lb-in} \]

\[ M_{\text{max}} = \sqrt{M_{\text{z}}^2 + M_{\text{y}}^2} \]

\[ M_{\text{max}} = 24.167 \text{ lb-in} \]

The value of the maximum bending moment and the output torque are used in the finding the steering rod diameter.

**Shaft design**

The safety factor is:

\[ n = 1.8 \]

The yield strength of 303 SS is:

\[ S_y = 35000 \text{ psi} \]

The stress concentration factor

\[ K_f = 1.0 \]

The bending moment is:

\[ M_a = 24.16 \text{ lbs-in} \]

The mean torque is:

\[ T_m = 20.6 \text{ lbs-in} \]

Factor for ground finish

\[ a = 1.34 \]

Exponent for ground finish

\[ b = -0.085 \]
Min tensile strength  
$Su_1 = 87.3 \text{ Kpsi}$

Guessed shaft diameter  
$d_l = 0.2$

Min tensile strength  
$Su = 87300 \text{ Psi}$

In order to find the endurance limit of the shaft the factors $Ka$, $Kb$, $Kd$, $Ke$ and $Se'$ have to be calculated.

The surface finish factor $Ka$ for ground shaft is

$$Ka = a \cdot Su_1^b$$

$Ka = 0.916$

The size factor based on $d_l$ is given by

$$Kb = \frac{d_l}{0.3}^{0.1133}$$

$Kb = 1.047$

The endurance limit of the specimen is given by

$$Se_1 = 0.504 Su_1$$

$Se := Ka \cdot Kb \cdot Se_1$

Therefore the endurance limit of the shaft is

$$Se = 42.22 \text{ Kpsi}$$

$$Se_1 = 42220 \text{ Psi}$$

The shaft diameter is given by eqn 3.9

$$d = \left[ \frac{32 \cdot n}{\pi \cdot Sy} \left( \frac{Kf \cdot Sy \cdot Ma}{Se} \right)^2 + Tm^2 \right]^{1/3}$$
Fig. B1 Shear force and bending moment diagrams of the steering rod.
APPENDIX B

Continuous servomotor torque

Maximum winding temperature \( T_{\text{max}} = 155 \) °C
Ambient temperature \( T_{\text{amb}} = 25 \) °C
Motor speed \( S = 150 \) rpm
Motor thermal impedance \( R_{\text{th}} = 24.4 \) °C/W
Constant based on torque in oz.in or N.m \( C = 1352 \)
Motor constant \( K_m = 0.93 \) oz.in/(W)^0.5
Motor friction torque \( T_m = 0.35 \) oz.in
Constant based on motor type \( K = 0.71 \)
Gear ratio \( G_R = 60.5 \)
Motor efficiency \( \eta = 0.66 \)

The continuous torque is obtained by eqn 5.1

\[
T_{\text{cont}} := \frac{T_{\text{max}} - T_{\text{amb}} - \frac{T_m \cdot S}{R_{\text{th}}} \cdot K \cdot K_m - T_m}{C} \sqrt{\frac{K m}{C}}
\]

\( T_{\text{cont}} = 1.169 \) oz.in

The output motor torque is given by eqn 5.3

\[
T_{\text{out}} := T_{\text{cont}} \cdot G_R \cdot \eta
\]

\( T_{\text{out}} = 46.66 \) oz.in
### Upper base plate assembly

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Base plate</td>
<td>Cast acrylic plate, 3/8 in thick</td>
</tr>
<tr>
<td>2. Motor support</td>
<td>6061 Aluminum, 1/4 in thick plate</td>
</tr>
<tr>
<td>3. Worm shaft</td>
<td>303 Stainless Steel ground shaft, dia = 3/16&quot;.</td>
</tr>
</tbody>
</table>
| 4. Coupling | Miniature precision flexible coupling  
bore = 3/16", max torque = 120 oz-in  
max allowable parallel offset 0.008",  
max angular displacement at 1000 rpm  
0.5 degrees.                                                                 |
| 5. Collar | Three pressure collars  
Bore 3/16", ¼" thick                                                                                                                                 |
| 6. Pillow block | Two 6061 Aluminum bearing housings  
¼" thick.                                                                                     |
<p>| 7. Radial bearing | Two miniature precision Stainless Steel shielded bearings, max allowable radial load 75 lbs.                                                   |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8.</td>
<td>Thrust bearing</td>
</tr>
<tr>
<td></td>
<td>Two three piece 410 Stainless steel thrust bearings, load rating at 15 rpm 73 lbs.</td>
</tr>
<tr>
<td>9.</td>
<td>Worm gear shaft</td>
</tr>
<tr>
<td></td>
<td>303 Stainless Steel threaded shaft</td>
</tr>
<tr>
<td></td>
<td>dia = $\frac{1}{4}$&quot;, length 5.5&quot;</td>
</tr>
<tr>
<td>10.</td>
<td>Worm gear</td>
</tr>
<tr>
<td></td>
<td>Precision Bronze worm gear,</td>
</tr>
<tr>
<td></td>
<td>bore = $\frac{1}{4}$&quot;, 60 teeth, 32 pitch</td>
</tr>
<tr>
<td></td>
<td>pressure angle 14.5°.</td>
</tr>
<tr>
<td>11.</td>
<td>Worm</td>
</tr>
<tr>
<td></td>
<td>Precision 303 Stainless Steel worm, 32 pitch, bore 3/16&quot;</td>
</tr>
<tr>
<td></td>
<td>pressure angle 14.5°.</td>
</tr>
<tr>
<td>12.</td>
<td>Radial Bearing</td>
</tr>
<tr>
<td></td>
<td>Steel ball bearing, bore = $\frac{1}{4}$&quot;</td>
</tr>
<tr>
<td></td>
<td>OD = 1.0&quot;.</td>
</tr>
<tr>
<td>13.</td>
<td>Collar</td>
</tr>
<tr>
<td></td>
<td>Steel bearing collar, bore = 1.0&quot;</td>
</tr>
<tr>
<td></td>
<td>OD = 1 - $\frac{1}{4}$&quot;.</td>
</tr>
<tr>
<td>14.</td>
<td>Screws</td>
</tr>
<tr>
<td></td>
<td>Two number 12 screws fastening the bearing collar to the base plate.</td>
</tr>
<tr>
<td>15</td>
<td>Screws</td>
</tr>
<tr>
<td></td>
<td>Four number 10 screws fastening the blocks on the base plate.</td>
</tr>
</tbody>
</table>
### Lower base plate

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Base plate</td>
<td>Clear cast acrylic plate 3/8&quot; thick</td>
</tr>
<tr>
<td>2. Motor support</td>
<td>6061 Aluminum, 1/4&quot; thick</td>
</tr>
<tr>
<td>3. Spur gear</td>
<td>Precision Stainless Steel Spur gear, 32 pitch, bore 3/16&quot;</td>
</tr>
<tr>
<td></td>
<td>48 teeth.</td>
</tr>
<tr>
<td>4. Wheel and axle</td>
<td>Assembly of two 2.0&quot; diameter rubber wheels, Stainless Steel shaft</td>
</tr>
<tr>
<td></td>
<td>dia = 3/16&quot;, slip type differential.</td>
</tr>
<tr>
<td>5. Support</td>
<td>6061 Aluminum Bar with rectangular cross section, length = 2.5&quot;</td>
</tr>
<tr>
<td>6. Rod</td>
<td>Two L shape 1/4-20 threaded</td>
</tr>
<tr>
<td>7. Coupling</td>
<td>Two 6061 Aluminum squares</td>
</tr>
<tr>
<td></td>
<td>a=0.5&quot;.</td>
</tr>
<tr>
<td>8. Support</td>
<td>Two 8 -32 threaded rods</td>
</tr>
<tr>
<td>9. Nut</td>
<td>Two hexagonal flex</td>
</tr>
<tr>
<td></td>
<td>1/4 -20</td>
</tr>
<tr>
<td>10. Nut</td>
<td>Two hexagonal flex nuts</td>
</tr>
<tr>
<td>11. Fastener</td>
<td>Two 1/4 -20 bolts</td>
</tr>
</tbody>
</table>
**Plate coupling assembly**

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Bushing</td>
<td>Delrin rod OD=1.75 &quot;</td>
</tr>
<tr>
<td>2. Bearing</td>
<td>Antifriction ball thrust bearing OD=2.344 in, height =0.625 &quot;</td>
</tr>
<tr>
<td>3. Nut</td>
<td>Two flex nuts ¼ - 20</td>
</tr>
<tr>
<td>4. Nut</td>
<td>Two socket type, length = 1.75 &quot;</td>
</tr>
<tr>
<td>5. Washer</td>
<td>Two plain washers, ID = 0.300 &quot;</td>
</tr>
<tr>
<td>6. Washer</td>
<td>Two plain washers, ID = 0.200 &quot;</td>
</tr>
</tbody>
</table>
### Variable length ball and socket joint

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Joint</td>
<td>York steel universal ball and socket joint</td>
</tr>
<tr>
<td></td>
<td>max allowable torque 635 in-lbs, max angular displacement 30°.</td>
</tr>
<tr>
<td>2. Guide</td>
<td>Radial load linear motion precision ground guide.</td>
</tr>
<tr>
<td>3. Rail</td>
<td>Hardened precision ground Steel V shape rail.</td>
</tr>
<tr>
<td>4. Housing</td>
<td>6061 Aluminum guide plate</td>
</tr>
<tr>
<td>5. Fitting</td>
<td>Aluminum ball and socket housing</td>
</tr>
<tr>
<td>6. Rod</td>
<td>Two steel threaded rods, 3/8-16.</td>
</tr>
<tr>
<td>7. Bolt</td>
<td>Four M4 x 7 metric bolts.</td>
</tr>
<tr>
<td>8. Pin</td>
<td>Two safety pins</td>
</tr>
<tr>
<td>9. Plate</td>
<td>Thin rectangular plate</td>
</tr>
<tr>
<td>10. Bolt</td>
<td>One 6-32, 0.75 &quot; long bolt.</td>
</tr>
</tbody>
</table>
Abstract

This thesis presents the design and fabrication of a multi-degree of freedom snake-like wheeled vehicle. The vehicle represents the means of implementing two motion algorithms developed, and simulated on a Silicon Graphics workstation. Due to the snake-like robot’s hyper-redundancy and dexterity it can perform tasks which can not be performed by other legged or wheeled vehicles. Snake-like robots are used in hazardous environments, such as nuclear powerplants and chemical factories, for surveillance, manipulation and inspection purposes.