Investigation of The Benefits of Multi-Chain Loran-C and Hybrid GPS/Loran-C Positioning

A Thesis Presented to

The Faculty of the

Fritz J. and Dolores H. Russ
College of Engineering and Technology

Ohio University

In Partial Fulfillment

of the Requirement for the Degree

Master of Science

By

Wen-Jye Huang
March 1997
Acknowledgments

My deepest appreciation belongs to my thesis advisor, Dr. Michael S. Braasch for his unending support, advice and encouragement throughout this work. I have really enjoyed the pleasant association that I have had with him over the last year.

I would like to thank my thesis committee members, Dr. Robert W. Lilley, Dr. Frank van Graas and Dr. Gene Kaufman for their critical review of my work.

Sincere thanks goes to the full support from the entire staff of the Avionics Engineering Center at Ohio University in flight testing and other areas. In particular, the constant support as a pilot and encouragement given by Dr. Lilley is greatly appreciated. I also thank Dr. Frank Van Graas for his full support of the software source code.

I would also like to thank Locus Inc. and NovAtel Inc. for the provision of the Loran-C and GPS receivers. In addition, Dr. G. Linn Roth, Mr. Paul Schick and Mr. Tom Blandino of Locus Inc. and Mr. Bryan Townsend of NovAtel Inc. are thanked for their technical help and guidance.

My parents have my greatest love and admiration, and I deeply appreciate the sacrifices they have made for my studies. Simply saying thanks is not enough.
Table of Contents

Chapter 1  Introduction.................................................................................... 1

Chapter 2  Loran-C System Overview.............................................................. 4

2.1 Transmitted Pulse................................................................ 4

2.2 Group of Transmission Signal....................................................... 6

2.3 Chain Configuration.................................................................. 7

2.4 Control Console.......................................................................... 8

2.5 Travel Path................................................................................. 9

2.6 Envelope-to-Cycle Difference (ECD)........................................... 9

Chapter 3  GPS System Overview................................................................. 11

3.1 Space Segment....................................................................... 11

3.2 Control Segment....................................................................... 13

3.3 User Segment........................................................................ 14

3.4 GPS Signal Characteristics.......................................................... 15

3.5 GPS System Errors..................................................................... 16
Chapter 4  Loran-C Pseudorange Positioning................................................... 18
  4.1  Loran-C Pseudorange Measurement............................................. 20
    4.1.1  Primary Phase Factor (PF)................................................... 21
    4.1.2  Secondary Phase Factor (SF).................................................. 22
    4.1.3  Additional Secondary Phase Factor (ASF)............................... 22
  4.2  Calculation Method for Pseudorange Model.................................... 22
  4.3  Geometry Effect........................................................................... 25
    4.3.1  2-D Compensation Method.................................................... 26
    4.3.2  3-D Compensation Method.................................................... 27
    4.3.3  Comparison of the Compensation Methods................................ 29
  4.4  Static Performance for Pseudorange Positioning Method.................. 30
    4.4.1  Local Chain Performance.................................................... 30
    4.4.2  Non-Local Chain Performance.............................................. 33
  4.5  Analysis....................................................................................... 39

Chapter 5  Loran-C Multi-Chain Pseudorange Positioning............................. 41
  5.1  Multi-Chain Pseudorange Model Calculations.................................... 42
  5.2  Two Chain Combination.................................................................... 44
    5.2.1  Local Chain Combination...................................................... 44
    5.2.2  Two Non-Local Chain Combination......................................... 47
Chapter 6  Loran-C Flight Test................................................................. 59
  6.1  Hardware Configuration........................................................ 59
  6.2  Single Chain Result.............................................................. 61
  6.3  Two Chain Combination Result.......................................... 63
  6.4  Three Chain Combination Result........................................ 64
  6.5  Conclusion........................................................................... 65

Chapter 7  New Coverage Diagram for Loran-C.................................. 67
  7.1  Loran-C Coverage Prediction for NPA Navigation................. 67
  7.2  Loran-C Coverage Prediction for Remote Navigation.......... 70
  7.3  New Loran-C Receiver Algorithm....................................... 74
Chapter 8
Hybrid GPS/Loran-C Pseudorange Positioning

8.1 Pseudorange Measurement for GPS

8.1.2 Propagation Delay for GPS Signal Transmission

8.1.2.1 Ionospheric Propagation Delay

8.1.2.2 Tropospheric Propagation Delay

8.2 Calculation Method for Hybrid Pseudorange Model

8.3 Hardware Configuration for Hybrid GPS/Loran-C System

8.4 Static Performance for Hybrid System

8.4.1 Hybrid GPS/Local Chain Loran-C Pseudorange Positioning

8.4.2 Hybrid GPS/Non-Local Chain Loran-C Pseudorange Positioning

8.4.3 Hybrid GPS/Multi-Chain Loran-C Pseudorange Positioning

8.5 Conclusion

Chapter 9
Flight Test II

9.1 Local Chain (GRI 9960) Positioning Result

9.2 Two Chain Combination (with local chain) Positioning Result
9.3 Two Chain Combination (without local chain) Positioning Result

9.4 Three Chain Combination Positioning Result

9.5 Hybrid GPS/Loran-C Positioning Results

9.6 Analysis

Chapter 10 Conclusion

Recommendations

References

vi
List of Figures

Figure

2-1 Loran-C pulse structure ................................................................. 5
2-2 The group of pulse timing for Loran-C ........................................... 6
2-3 The chain timing for Loran-C .......................................................... 8
3-1 GPS satellite orbit arrangement ..................................................... 12
3-2 GPS constellation .......................................................................... 13
3-3 GPS control segment locations ........................................................ 14
3-4 GPS signal modulation ..................................................................... 16
4-1 Graphical depiction of time axis for computing Loran-C TD .......... 18
4-2 Loran-C time-difference (TD) positioning ....................................... 19
4-3 Loran-C pseudorange (PR) positioning ............................................. 21
4-4 2-D pseudorange model adjustment for Loran-C transmitter .......... 26
4-5 3-D pseudorange model adjustment for Loran-C transmitter .......... 28
4-6 Coverage diagram for Northeast U.S. chain (GRI 9960) ................. 31
4-7 The single chain GRI 9960 (M X Y Z) horizontal positioning error .... 32
4-8 The single chain GRI 9960 (M W X Y Z) horizontal positioning error .... 33
4-9 Coverage diagram for Great Lakes chain (GRI 8970) ..................... 34
4-10 Coverage diagram for Southeast U.S. chain (GRI 7980) ............... 35
4-11 Coverage diagram for South Central U.S. chain (GRI 9610) .......... 36
4-12 The single chain GRI 8970 (M W X Y Z) horizontal positioning error .... 37
4-13 The single chain GRI 7980 (M W X Y Z) horizontal positioning error .... 38
4-14 The single chain GRI 9610 (M V Y Z) horizontal positioning error .... 38
4-15 Single chain coverage distance versus horizontal positioning error .... 39
4-16 Single chain coverage distance versus standard deviation ............ 40

5-1 The horizontal positioning error of a two chain combination (GRI 9960 M Y Z plus GRI 7980 M W Y) .................................................. 45
5-2 The horizontal positioning error of a two chain combination (GRI 9960 M Y Z plus GRI 9610 M V Y) .................................................. 46
5-3 The horizontal positioning error of a two chain combination (GRI 9960 M Y Z plus GRI 8290 W X) .................................................. 47
5-4 The horizontal positioning error of a two chain combination (GRI 7980 W Y Z plus GRI 8290 M X) .................................................. 48
5-5 The horizontal positioning error of a two chain combination (GRI 9610 M V Y plus GRI 5930 M X) .................................................. 48
5-6 The horizontal positioning error of a three chain combination (GRI 9610 M V, GRI 7980 W X and GRI 8290 W X) .................................. 49
5-7 The horizontal positioning error of a three chain combination (GRI 9610 M V Y, GRI 7980 M Y Z and GRI 5930 M X) .............................. 50
5-8 The horizontal positioning error of a multi-chain combination (GRI 9610 M V W X, GRI 7980 M W X Y, GRI 8290 M X and GRI 5930 M X) ...... 51
5-9 Coverage diagram for West Coast U.S. chain (GRI 9940) ................. 54
5-10  The horizontal positioning error of a two chain combination (GRI 9960 W
X Y plus GRI 9940 W X) with 2 skywaves............................................. 55
5-11  Multi-chain coverage distance versus horizontal positioning error ........ 57
5-12  Multi-chain coverage distance versus standard deviation .................. 58
6-1   Hardware configuration of Loran-C flight test ................................. 60
6-2   Loran-C flight test reference trajectory (GPS card tracking result) ........ 62
6-3   Loran-C flight test : single chain (GRI 8970 M W X Y Z) pseudorange
positioning error .................................................................................. 62
6-4   Loran-C flight test : two chain combination (GRI 7980 M W X plus GRI
9960 M Y Z ) pseudorange positioning error ........................................ 63
6-5   Loran-C flight test : three chain combination (GRI 7980 M W X plus GRI
9960 M Y Z ) pseudorange positioning error ........................................ 64
6-6   Loran-C flight test coverage distance versus horizontal positioning error...
6-7   Loran-C flight test coverage distance versus standard deviation .......... 66
7-1   Loran-C master station coverage with 1400 km in U.S. and Canada ...... 68
7-2   Loran-C coverage prediction for NPA navigation in U.S. with 1400 km
radius .......................................................... 69
7-3   The relationship between coverage distance and SNR ....................... 71
7-4   New Loran-C coverage prediction for remote navigation in the vicinity of the
U.S. ( 2500 km radius coverage from four stations)......................... 72

ix
7-5 New Loran-C coverage prediction for oceanic navigation in the Atlantic Ocean area with 2500 km radius .......................................................... 73
7-6 New Loran-C coverage prediction for oceanic navigation in the Pacific Ocean area with 2500 km radius ................................................ 74
7-7 Multi-chain pseudorange positioning Algorithm for Loran-C receiver................................................ 75
8-1 Block diagram for hybrid GPS/Loran synchronization ................................................. 81
8-2 Synchronization circuit for hybrid GPS/Loran-C system ....................................... 82
8-3 The block diagram of hybrid GPS/Loran-C system .............................................. 83
8-4 The hybrid system (2 GPS plus 3 Loran-C GRI 9960 M Y Z) horizontal positioning error .......................................................... 85
8-5 The relationship between crossing angles and area of fix uncertainty ................ 86
8-6 The hybrid system (3 GPS plus 3 Loran-C GRI 9960 M Y Z) horizontal positioning error .......................................................... 87
8-7 The hybrid system (2 GPS plus 3 Loran-C GRI 7980 X Y Z) horizontal positioning error .......................................................... 88
8-8 The hybrid system (3 GPS plus 2 Loran-C GRI 9610 V Y) horizontal positioning error .......................................................... 89
8-9 The hybrid system (3 GPS plus two Loran chain combination (GRI 9610 M V and GRI 7980 X Y) ) horizontal positioning error ....................... 90
9-1 Flight test II reference trajectory (GPS card tracking result)............................. 91
9-2 The horizontal error for local Loran-C chain (GRI 9960 M W X Y Z) pseudorange positioning ................................................................. 92
9-3 The horizontal error for two chain combination (GRI 9960 M X Z+GRI 7980 M Y Z) positioning result ......................................................... 94
9-4 The horizontal error for two chain combination (GRI 9960 M X Z+GRI 9610 M Z) positioning result ............................................................ 95
9-5 The horizontal error for two chain combination (GRI 7980 M Y Z+GRI 5930 M X) positioning result ............................................................ 96
9-6 The horizontal error for three chain combination (GRI 5930 M X + GRI 7980 M Y Z+GRI 9610 M Z) positioning result ................................. 97
9-7 The horizontal error for hybrid system (2 GPS +GRI 9960 M Y Z) positioning result .......................................................... 98
9-8 The horizontal error for hybrid system (3 GPS + GRI 9610 MYZ + GRI 7980 MYZ) positioning result ......................................................... 99
9-9 The horizontal error for hybrid system (3 GPS +GRI 9610 MYZ + GRI 7980 MYZ + GRI 9960 MX) positioning result ................................. 100
9-10 Coverage distance versus horizontal error ........................................ 101
Chapter 1 Introduction

Today the use of Global Navigation Satellite Systems (GNSS) is increasing rapidly throughout the world. Although many users are satisfied with the accuracy which the Global Positioning System (GPS) provides, satellite-only solutions such as GPS still cannot provide the integrity or availability necessary to be certified as a sole means of navigation [1]. To solve the GPS integrity problem, modern techniques such as receiver autonomous fault detection and isolation and ground/space-based augmentations (DGPS, WAAS) have been developed. Another way to fulfill the integrity and availability requirement is through the use of hybrid or integrated navigation systems [4]. The idea behind a hybrid navigation system is to provide a solution which is based on all data available from various position sensors. Currently, hybrid navigation systems are being used to provide robust operations. Examples include the integration of GPS with barometric altimeter and integration of GPS with inertial measurement units [2].

Owing to the development of advanced digital signal processing, modern Loran-C receivers have been developed and demonstrate a remarkable improvement in performance. New Loran-C receivers can track up to 40 signals at a range of 2500 Km from transmitters 24 hours/day simultaneously [3]. Using these available data, the Loran-C system coverage is enhanced significantly. This thesis will present an
algorithm to utilize the abundant signals to enhance the Loran-C accuracy and coverage for different kinds of navigation applications. It will also provide test results to show the new coverage of the Loran-C system extending beyond the continental U.S and into selected overseas terrestrial areas. Using modern Loran-C receivers, the coverage of the Loran-C transmitters covers an approximately circular area with 1400 km radius for non-precision approach (NPA) navigation, 2000 km for terminal navigation and 2500+ km for enroute navigation. Moreover, it is possible to use Loran-C in transoceanic navigation.

The Loran-C system's characteristics including such features as low frequency, high signal level, and being ground-based are dissimilar with GNSS being high frequency, very low signal level, and satellite-based. Because of the dissimilar redundancy, the Loran-C system could be the complement system of choice to back up GNSS where GNSS might be temporarily unavailable [3]. The hybrid use of the Loran-C with GPS has been shown to be capable of meeting very stringent availability and integrity criteria related to the provision of primary or sole-means enroute air radio navigation [4]. It means the Loran-C system not only could play the role of back up to GNSS, but also could be a partner with GNSS. This thesis identifies a new algorithm that hybridizes the GNSS/Loran-C system in range mode. It also provides results of testing which indicate that the hybrid GNSS/Loran-C not only can improve the system integrity; but also addresses the availability problem when GPS/GNSS and
Loran-C are both independently unavailable. The results of testing presented in this thesis demonstrate the strength of the GNSS/Loran-C partnership.

The hybrid use of GNSS/Loran-C can provide a powerful and robust navigation capability which is available worldwide for the next century.
Chapter 2  Loran-C System Overview

The Loran-C is a low frequency, land-based radio navigational aid which provides two-dimensional (horizontal) navigation capability. The carrier frequency of the Loran-C system is 100 kHz and it operates in the protected 90 to 110 kHz frequency band. The basic applications of Loran-C system are navigation, communication, precise timing and frequency reference [13].

The Loran-C system can be described as follows:

- Transmitter Segment : Generate and monitor the Loran-C signals.

2.1 Transmitted Pulse

To save on prime power required for transmission and to facilitate signal identification and precise timing, the Loran-C system is designed for pulse transmission. Each station transmits signals which have standard pulse leading-edge characteristics [9]. The standard Loran-C pulse waveform [20], is defined as $e(t)$:

\[ e(t) = 0; \quad \text{for } t < \tau \tag{Eq.2.1} \]

\[ e(t) = A \left( \frac{t-\tau}{65} \right)^2 \exp \left( -2 \frac{t-\tau}{65} \right) \sin (\omega t + \phi) \quad ; \quad \text{for } \tau \leq t \leq 65 + \tau \tag{Eq.2.2} \]
where:

\[ A \] : Amplitude constant

\[ t \] : Time (\( \mu \) sec)

\[ \tau \] : Envelope-to-cycle difference (ECD \( \mu \) sec)

\[ \omega \] : Carrier frequency (\( 2\pi \times 100 \) kHz)

\[ \phi \] : Phase coding (radians)

The standard Loran-C signal structure is illustrated in Figure 2-1. The third positive-going zero crossing of the pulse is used to track the signal. The peak value of the pulse occurs at 65 \( \mu \) seconds.

![The Standard Loran-C Pulse Waveform](image)

**Figure 2-1** Loran-C pulse structure
2.2 Group of Transmission Signal

To facilitate identification of master and secondary stations, a Loran-C transmitted signal consists of a group of pulses. For the master signal, a series or group of pulses is transmitted (eight spaced 1,000 \( \mu \text{sec} \) apart, followed by a ninth 2,000 \( \mu \text{sec} \) later). Secondary stations transmit a series of only eight pulses, each spaced 1,000 \( \mu \text{sec} \) apart [5]. Figure 2-2 shows the group timing for Loran-C pulses.

![Figure 2-2 The group pulse timing for Loran-C](image)
Within each pulse group, each pulse may be transmitted with a carrier phase of either 0° or 180°, referred to respectively as positive (+) or negative (-) phase code. Standard Loran-C signals are transmitted with a fixed phase code sequence which extends over two successive groups of pulses and then repeats. Master stations use one phase code sequence, secondary stations use another. Phase coding serves two purposes. First, it makes the master station's transmission distinguishable from secondary station transmissions. Second, it reduces the effect on system accuracy of the unstable skywave interference [10].

2.3 Chain Configuration

Loran-C transmitters are grouped into chains. Each chain consists of one master station and two to five secondary stations. To facilitate identification of the chain, the length of time between successive transmissions of the master’s pulse groups is defined and termed the group repetition interval (GRI). The GRI is different for each chain identification. GRI’s range from 40 to 99.99 milliseconds. All Loran-C chains transmit at the same 100 kHz frequency; they are distinguished by the use of different GRIs.

Each GRI begins with the first pulse of the master group. Then, relative to this start time (time reference), the emission of each of each secondary transmitter is delayed by a time called the Emission Delay (ED) for that Secondary. The ED for each secondary in a chain is unique to ensure that the pulse groups from the various
transmitters of the chain will not overlap anywhere in the coverage area. The idea of Loran-C chain timing is depicted in Figure 2-3.

![Diagram of Loran-C chain timing](image)

**Figure 2-3** The chain timing for Loran-C.

2.4 **Control Console**

The Loran-C chain is controlled by Chain Control Center which provides centralized monitoring and control of the transmitters in the chain and record keeping.
of chain performance. The Control Center also processes data from the System Area Monitor Unit (SAM) to determine whether each transmitter is operating within specification. The Control Center is often located with the master transmitter [10].

2.5 Travel Path

Loran-C signals mainly travel by groundwave propagation. The signal from each transmitter is attenuated as it travels over land and sea paths. The major reason for attenuation is the conductivity of the paths. Generally speaking, the lower the ground conductivity, the more rapidly the signals are attenuated.

2.6 Envelope-to-Cycle Difference (ECD)

As the Loran-C groundwave propagates, the pulse envelope travels at a slightly different speed than the carrier. The reason for the change of ECD is that the velocity of propagation of groundwave signals in the region of 100 kHz is a weakly non-linear function of frequency [11]. The velocities of the spectral components of the Loran-C signal thus vary non-linearly across the band from 90 to 110 kHz. The group velocity of the signal is slightly different from the phase velocity. The receiver measures the difference between the third zero-crossing tracking point and 30 \( \mu \) seconds to obtain the ECD [5]. The rate of change of ECD with distance is such
that the shift builds up to approximately \(-2.5 \, \mu\text{sec}\) by the edge of the coverage area
(near to the station in the near far-field) [11]. In order to facilitate the receiver zero-
tracking, some transmitters are designed to transmit a pulse with a non-zero ECD
\((+2.5 \, \mu\text{sec})\) so that the ECD falls toward zero at the edge of coverage [20].
Chapter 3 GPS System Overview

The Global Positioning System (GPS) is a satellite radio navigation system capable of providing a highly accurate service. GPS provides 24-hour, all weather, worldwide coverage with three dimensional position, velocity and timing information. The system uses the NAVSTAR (NAVigation Satellite Timing And Ranging) satellite constellation which consists of twenty four satellites to provide a GPS receiver with a six to twelve satellite coverage at all times [12].

The GPS system design consists of three parts:

- The Space Segment : The orbiting satellite configuration.
- The Control Segment : The ground stations which monitor and control the satellite orbits and the broadcast signals.
- The User Segment : The different application receivers which track the GPS signals.

3.1 Space Segment

The space segment is composed of the NAVSTAR GPS satellites. The constellation of the system consists of 24 satellites in six 55° orbital planes, with four satellites in each plane. Figure 3-1 shows the NAVSTAR satellites orbit arrangement. The GPS constellation is shown in Figure 3-2. The orbit period of each satellite is 11
hours and 58 minutes at an altitude of 20162.61 km above the Earth's equatorial radius of 6378.137 km. This provides a GPS receiver with six to twelve satellites in view from any point on earth, at any particular time.

Figure 3-1 GPS satellite orbit arrangement

The GPS satellites transmit on two L-band frequencies; one centered at 1575.42 MHz (L1) and the other at 1227.60 MHz (L2). The L1 carrier is modulated by the C/A (Coarse/Acquisition) code and P (Precision) code. The L2 carrier is modulated with the P code [15].

3.2 Control Segment

The control segment consists of a master control station, monitor stations and data up-loading stations. The major objective of the GPS control segment is to track
the GPS satellites and generate and upload the navigation data to each of the GPS satellites [12]. Figure 3-3 shows the GPS control segment locations.

![GPS control segment locations](image)

**Figure 3-3 GPS control segment locations**


**3.3 User Segment**

The user segment consists of equipment which track and receive the satellite signals. There are many applications for GPS such as aircraft, marine vessel and land mobile navigation and survey. For a civil user, GPS provides the Standard Positioning Service (SPS). SPS is based on the C/A code. The horizontal position accuracy for SPS is 100m (95 %). For authorized (military) users, GPS provides the Precise
Positioning Service (PPS). PPS is based on the P code. The horizontal position accuracy for PPS is 20 m (95 %) [21].

3.4 GPS Signal Characteristics

The basic signal structure of GPS is the carrier (L1/L2) modulated with the pseudo random noise (PRN) code and navigation information to produce the modulated carrier. The signal transmitted by the satellite is written as follows [14]:

\[
S_{L1}(t) = \sqrt{2P_c} XG(t)D(t)\cos(\omega_1 t + \Phi_1) + \sqrt{2P_p} XP(t)D(t)\sin(\omega_1 t + \Phi_1) \quad \text{(Eq.3.1)}
\]

\[
S_{L2}(t) = \sqrt{2P_p} XP(t)D(t)\cos(\omega_2 t + \Phi_2) \quad \text{(Eq.3.2)}
\]

where

\[
\omega_1, \omega_2 : \quad \omega_1 \text{ is the } L_1 \text{ frequency (1575.42MHz).}
\]

\[
\omega_2 \text{ is the } L_2 \text{ frequency (1227.6MHz).}
\]

\[
\Phi_1, \Phi_2 : \quad \Phi_1, \Phi_2 \text{ are a small phase noise and oscillator drift components.}
\]

\[
P_{C/A}, P_p : \quad \text{C/A code and P code signal powers.}
\]

\[
XG(t) : \quad \text{The C/A code is a } \pm 1 \text{ pseudorandom sequence (Gold code) of a period 1023 bits with a clock rate of 1.023 Mbps and period } 1 \text{ m sec.}
\]

\[
XP(t) : \quad \text{The P code is a } \pm 1 \text{ pseudorandom sequence with a clock rate of 10.23 Mbps and period 1 week.}
\]
$D(t)$: Navigation data at a rate of 50 bits per second. This data contains the satellite ephemeris (orbit) information, satellite clock correction data, satellite health and status information.

An modulate example is given in Figure 3-4.

![Figure 3-4 GPS signal modulation](image)

### 3.5 GPS System Errors

Because of propagation effects on the GPS signal, there are numerous factors which influence the positioning accuracy of GPS.

The earth's ionospheric layers cause varying degrees of GPS signal
propagation delay. The troposphere is the lower portion of the earth’s atmosphere consisting of dry gases and water vapor. It also causes propagation delays.

The satellites contribute three significant error sources. The ephemeris error is the difference between the predicted satellite position (as given by the broadcast parameters) and the actual satellite position [12]. The satellite clock error is the difference between the actual clock offset (from GPS time) and that predicted by the broadcast parameters. Finally, selective availability (SA) is an intentional degradation of the satellite clock and ephemeris parameters. SA is designed to deny the full accuracy of the C/A-code.

Multipath is the phenomenon whereby signals are received via multiple paths due to reflection and diffraction. The additional signals distort the desired line-of-sight signal and cause ranging error.
Chapter 4 Loran-C Pseudorange Positioning

The Loran-C system was initially designed to be used in the hyperbolic mode. In this mode, the basic position determination method implemented in a traditional Loran-C receiver is accomplished via the time difference (TD) between the reception time of master and secondary pulses. Each master-secondary pair defines a hyperbolic line-of-position (LOP). Figure 4-1 shows the graphical depiction of time axis for computing the Loran-C TD. The idea of hyperbolic positioning is depicted in Figure 4-2. Reception time measurements from one master and two secondary stations are necessary to solve for latitude and longitude [4].

Loran-C Time Difference

![Loran-C Time Difference Diagram]

Figure 4-1 Graphical depiction of time axis for computing Loran-C TD
Figure 4-2 Loran-C time-difference (TD) positioning.
4.1 Loran-C Pseudorange Measurement

An alternative positioning method via pseudorange measurements is possible if the receiver makes time-of-arrival (TOA) measurements available. Each transmitter defines a circular line-of-position (LOP) on which the receiver is located. The idea of pseudorange positioning is shown in Figure 4-3.

The ideal Loran-C pseudorange equation is expressed as follows [15]:

\[ PR_i = TOA(i) \times c \]  \hspace{1cm} (Eq.4.1)

where \( PR_i \) : The ith pseudorange measurement.
\( TOA(i) \) : The ith Loran-C time of arrival.
\( c \) : The speed of light.

For the actual Loran-C pseudorange measurement, each \( PR_i \) includes three components and can be written as follows:

\[ PR_i = R_i + b + SF(R_i) + ASF(R_i) \]  \hspace{1cm} (Eq.4.2)

where \( R_i \) : The true range between ith transmitter and user.
\( b \) : The clock offset between receiver and transmitter.
\( SF(R_i) \) : The ith secondary correction factor.
\( ASF(R_i) \) : The ith additional secondary correction factor.
Loran-C Pseudorange Positioning

4.1.1 Primary Phase Factor (PF)

According to Eq.4.1, the Loran-C pseudorange is basically the product of $TOA(i)$ multiplied by the speed of signal propagation $c$. Because the signal propagation speed in the atmosphere is slightly slower than in free space, the primary factor accounts for this. According to Bowditch, the speed at which the Loran-C signal propagates in the atmosphere is 161,829 NM/sec ($\approx 299707308$ m/sec) [5].
4.1.2 Secondary Phase Factor (SF)

The secondary phase factor (SF) reflects the fact that the Loran-C groundwave is further retarded when traveling over different media such as sea water. The amount of time required for travel over a specified distance will exceed that calculated using the PF. A Harris polynomial correction equation for SF is shown below which relates the SF to the distance traveled $R_i$ (in statute miles) [5].

$$SF = \begin{cases} 
-0.01142 + 0.00176 \times R_i + 0.510483 / R_i; & R_i \leq 100SM \\
-0.40758 + 0.00346776 \times R_i + 24.0305 / R_i; & R_i \geq 100SM 
\end{cases}$$

(Eq. 4.3)

4.1.3 Additional Secondary Phase Factor (ASF)

In practice, Loran-C signals travel over a mixed path. The correction arising from the additional retardation of the signal is termed the additional secondary factor (ASF) [5]. ASF is calculated by considering the overall path as separate segments, each segment having a uniform conductivity value.

4.2 Calculation Method for Pseudorange Model

According to the section 4.1 discussion, the pseudorange equations for Loran-C can be written as follows:

$$PR_i = \sqrt{(X_i - U_X)^2 + (Y_i - U_Y)^2 + (Z_i - U_Z)^2} + b + SF(\hat{R}_i) + ASF(\hat{R}_i) ;$$

$$\Rightarrow PR_i - SF(\hat{R}_i) - ASF(\hat{R}_i) = \sqrt{(X_i - U_X)^2 + (Y_i - U_Y)^2 + (Z_i - U_Z)^2} + b ;$$
where: \( PR_i \) : The ith pseudorange measurement.

\( (\text{without SF and ASF}) \)

\( X_i, Y_i, Z_i \) : The position of ith Loran-C transmitter.

\( U_x, U_y, U_z \) : The user position.

\( b \) : The receiver clock offset.

\( SF \) : The secondary correction factor.

\( ASF \) : The additional secondary correction factor.

\( \hat{R}_i \) : The estimated range between transmitter and user.

Solving for the three dimensional user position \((U_x, U_y, U_z)\) and clock offset \((b)\), Newton's method for nonlinear systems is used for a generic navigation solution [16].

Assume that

\[
\begin{bmatrix}
U_x \\
U_y \\
U_z \\
b
\end{bmatrix}, \quad \begin{bmatrix}
PR_1 \\
PR_2 \\
PR_3 \\
PR_4
\end{bmatrix}, \quad \begin{bmatrix}
PR_i \\
PR_2 \\
PR_3 \\
PR_4
\end{bmatrix}
\]

In equation (4.4), the user position \((U_x, U_y, U_z, b)\) is not known. In order to linearize the equation (4.4) and use the Taylor series expansion, a priori estimation
\((\hat{U}_X, \hat{U}_Y, \hat{U}_Z, \hat{b})\) is necessary. The \textit{a priori} position estimate is used to calculate the estimate of the distances to the stations \(\hat{R}_i\). Then the pseudorange measurement (Eq.4.4) can be expressed by the \textit{a priori} estimate plus first order term expansion.

\[
PR_i' = \hat{R}_i + \frac{\partial PR_i}{\partial \hat{U}_X}(\hat{U}_X, \hat{U}_Y, \hat{U}_Z, \hat{b}) + \frac{\partial PR_i}{\partial \hat{U}_Y}(\hat{U}_X, \hat{U}_Y, \hat{U}_Z, \hat{b}) + \frac{\partial PR_i}{\partial \hat{U}_Z}(\hat{U}_X, \hat{U}_Y, \hat{U}_Z, \hat{b}) + \frac{\partial PR_i}{\partial \hat{b}}(\hat{U}_X, \hat{U}_Y, \hat{U}_Z, \hat{b})
\]

(Eq.4.5)

Let

\[
\frac{\partial PR_i}{\partial \hat{U}_X}(\hat{U}_X, \hat{U}_Y, \hat{U}_Z, \hat{b}) = \alpha_{11} ;
\]

\[
\frac{\partial PR_i}{\partial \hat{U}_Y}(\hat{U}_X, \hat{U}_Y, \hat{U}_Z, \hat{b}) = \alpha_{12} ;
\]

\[
\frac{\partial PR_i}{\partial \hat{U}_Z}(\hat{U}_X, \hat{U}_Y, \hat{U}_Z, \hat{b}) = \alpha_{13} ;
\]

\[
\frac{\partial PR_i}{\partial \hat{b}}(\hat{U}_X, \hat{U}_Y, \hat{U}_Z, \hat{b}) = \alpha_{14} ;
\]

(Eq.4.6)

\[
PR_i' - \hat{R}_i = \alpha_{11}\delta U_X + \alpha_{12}\delta U_Y + \alpha_{13}\delta U_Z + \alpha_{14}\delta \hat{b}
\]

(Eq.4.7)

To solve for the user state \((U_X, U_Y, U_Z, b)\), at least four measurement equations (4.7) are necessary and can be written in matrix form:

\[
\begin{bmatrix}
\delta PR_1' \\
\delta PR_2' \\
\delta PR_3' \\
\delta PR_4'
\end{bmatrix} =
\begin{bmatrix}
\alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\
\alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\
\alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\
\alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44}
\end{bmatrix}
\begin{bmatrix}
\delta U_X \\
\delta U_Y \\
\delta U_Z \\
\delta \hat{b}
\end{bmatrix}
\]

(Eq.4.8)
\[ \delta x = H \delta \hat{x} \quad \text{(Eq. 4.9)} \]

Equation (4.9) can be rewritten as follows:

\[ H^T \delta x = H^T H \delta \hat{x} \]

\[ \therefore \delta x = (H^T H)^{-1} H^T \delta \hat{x} \quad \text{(Eq. 4.10)} \]

Equation 4.10 can be used iteratively to solve for the user state vector \( x \).

The iteration algorithm (Newton’s method) proceeds as follows:

1. Obtain an a priori user estimate \((\hat{U}_x, \hat{U}_y, \hat{U}_z, \hat{b})\).
2. Calculate \( \delta x \) use the pseudorange measurement and calculate \( H \).
3. Calculate the change in the user position \( \delta x \) from (Eq. 4.10).
4. Update the user position: \( x_{\text{NEW}} = x_{\text{OLD}} + \delta x \).
5. If the change in the user position is \( |\delta x(1)^2 + \delta x(2)^2| \leq \varepsilon \) the procedure is completed, otherwise repeat steps b-e. \( \varepsilon \) is a selected tolerance.

4.3 Geometry Effect

One of the problems related to the Loran-C pseudorange method is that Loran-C groundwaves travel by following the earth’s curvature, the so-called great-circle path [22]. It is not a line-of-sight propagation. For local chain distances (hundreds of K\text{m level}), this effect is small and can be neglected. For a long distance (thousands K\text{m level}) or for a hybrid system, the geometric effect is one of the major causes of position inaccuracies. It is necessary to reconsider this curvature problem and take it into account before using the Loran-C pseudorange positioning method.
4.3.1 2-D Compensation Method

In order to eliminate the geometry effect, one method straightens the groundwave travel path and projects it onto the local-level-tangent-plane. Secondly, it converts the Loran-C transmitter coordinates to the East-North-Up (ENU) coordinates with respect to the estimated user position [4]. Figure 4-4 illustrates the idea of the adjustment.

**Loran-C 2-D Pseudorange Model**

![Diagram showing 2-D pseudorange model adjustment for Loran-C transmitter](image)

**Figure 4-4** 2-D pseudorange model adjustment for Loran-C transmitter
4.3.2 3-D Compensation Method

Another method (derived by the author) is to straighten the groundwave propagation path into a line and project it onto a plane which is determined by the earth’s center, the earth’s radius, the Loran-C transmitter location and the user position. This method treats the Loran-C transmitters, after adjustment, in the same manner as the GPS constellation. Figure 4-5 shows the idea of how to adjust the geometry curvature factor. The Loran-C transmitter equivalent position equation is derived as follows:

**D**: Distance between the Loran-C Transmitter and the estimated user position (m).

**Re**: Radius of earth (m).

\[ \theta = 2 \cdot \arcsin \left( \frac{D}{2 \cdot Re} \right) \]

\[ S = Re \cdot \theta \]

Assume \( X = h + Re; \)

According to the Cosine Law:

\[ S^2 = X^2 + Re^2 - 2 \cdot X \cdot Re \cdot \cos \theta \]

\[ X^2 - 2 \cdot X \cdot Re \cdot \cos \theta + Re^2 - S^2 = 0 \]

The reasonable solution for \( X \) is:

\[ X = Re \cdot \cos \theta + \sqrt{Re^2 \cdot \cos^2 \theta - (Re^2 - S^2)} \]

\[ X = Re \cdot \cos \theta + \sqrt{(S^2 - Re^2 \sin^2 \theta)} \]

\[ h = X - Re = Re \cdot \cos \theta + \sqrt{(S^2 - Re^2 \sin^2 \theta)} - Re \quad \text{(Eq. 4.11)} \]
By translating the Loran-C transmitter position into the Earth-Centered-Earth-Fixed (ECEF) coordinate system and adding the geometric compensation adjustment $h$, the user state $x = \begin{bmatrix} U_x & U_y & U_z & b \end{bmatrix}^T$ can be determined in exactly the same manner as Newton’s solution for the pseudorange equation we discussed in section 4.2. A minimum of four pseudorange measurements are required to solve for the user three dimensional position $(U_x, U_y, U_z)$ and clock offset $b$.

**Loran-C 3-D Pseudorange Model**

![Loran-C 3-D Pseudorange Model](image)

**Figure 4-5** 3-D pseudorange model adjustment for Loran-C transmitter
4.3.3 Comparison of The Two Compensation Methods

Theoretically, the Loran-C position results for the pseudorange model plus the 2-D or 3-D compensation methods should be the same. The only difference for both methods is the coordinate conversion. In practice, this 3-D compensation model neglects two effects.

- The earth is not a perfect sphere.
- The signal propagation for a receiver at altitude is no longer described by a pure groundwave.

The advantage of the 2-D compensation is that only three pseudorange measurements are required for solving user state $x = [U_x, U_y, b]^T$. For remote Loran-C chain positioning, the 2-D compensation mode should take the additional secondary phase factor (ASF) into account. Instead of the 2-D method, the 3-D method is an easy and direct approach to implement remote Loran-C chains’ data without concerning ASF correction. The disadvantage of 3-D compensation is that at least four measurements are required for solving user state $x = [U_x, U_y, U_z, b]^T$. Furthermore, the 3-D method suffers an accuracy degradation due to the fact it solves for user height. For easy modeling, later discussions focus on 3-D compensation method.
4.4 Static Performance for Pseudorange Positioning Model

This section documents field test results for a stationary receiver.

4.4.1 Local Chain Performance

To qualify the basic performance of the Loran-C pseudorange positioning model, single chain measurements were taken on the Northeast U.S. Loran-C Chain (GRI 9960) on June 26, 1996 for a 6 hour period (sampling period: 10 seconds). This chain consists of five transmitters and provides basic coverage for southeast Ohio. Figure 4-6 shows the coverage diagram for Northeast U.S. Chain (GRI 9960) [5]. The east and north position errors as given by measurements from GRI 9960 station M (Seneca, NY), X (Nantucket, ME), Y (Carolina Beach, NC) and Z (Dana, IN) are depicted in Figure 4-7. The mean of the horizontal error is 148.7215 (m); the standard deviation of the horizontal error is 4.4077 (m). The horizontal dilution of position (HDOP) is 2.0641. The result shows that the Loran-C pseudorange method is accurate and meets the requirement of Loran-C absolute accuracy (1/4 NM ≈ 463 m) [5]. The small variance of the scatter points in Figure 4-7 is also a direct result of the stability of the Loran-C signals.
Northeast U.S. Loran-C Chain GRI 9960

SNR 1:3
Fix Accuracy 1/4 NM (95% 2 dRM(S)
Atmospheric Noise 58.1 dB above 1uV/m
M Seneca, NY
W Caribou, ME
X Nantucket, MA
Y Carolina Beach, NC
Z Dana, IN

Note: Estimated Groundwave coverage, actual coverage will vary

Figure 4-6 Coverage diagram for Northeast U.S. chain (GRI 9960).

To double check the accuracy of the Loran-C pseudorange scheme, a redundant measurement GRI 9960 W (Caribou, ME) is added for the last calculation. The north and east errors from GRI 9960 M, W, X, Y and Z are shown in Figure 4-8. The mean of the horizontal error is 30.5075 (m); the standard deviation of the horizontal error is 7.8251 (m). The HDOP is 1.5771. Compared to the last result, more pseudorange measurements provide better performance than the minimum of four. It also shows that in some cases the accuracy of the Loran-C pseudorange measurements meet the requirement of GPS Standard Position Service with Selective Availability (100 m).
Figure 4-8 The single chain GRI 9960 (M W X Y Z) horizontal positioning error

4.4.2 Non-Local Chain Performance

Thanks to advanced digital signal processing, new Loran-C receivers can also track signals from multiple of chains simultaneously (such as the Great Lakes Chain (GRI 8970), the South Central U.S. Chain (GRI 9610) and the Southeast U.S. Chain (GRI 7980)). Figure 4-9, Figure 4-10 and Figure 4-11 are the coverage diagrams for the three chains [5].
Great Lakes Loran-C Chain GRI 8970

SNR 1:3
Fix Accuracy 1/4 NM (95% 2 dRMS)
Atmospheric Noise 58.1 dB above 1 uV/m
M Dana, IN
W Malone, FL
X Seneca, NY
Y Baudette, MN
Z Boise City, OK

Note: Estimated Groundwave coverage, actual coverage will vary

Figure 4-9 Coverage diagram for Great Lakes chain (GRI 8970).

Southeast U.S. Loran-C Chain GRI 7980

SNR
Fix Accuracy
Atmospheric Noise

1:3
1/4 NM (95% 2 dRMS)
60.5dB above 1uV/m

M Malone, FL
W Grangeville, LA
X Raymondville, TX
Y Jupiter, FL
Z Carolina Beach, FL

Note: Estimated Groundwave coverage. Actual coverage will vary.

Figure 4-10 Coverage diagram for Southeast U.S. chain (GRI 7980).

South Central U.S. Loran-C Chain GRI 9610

SNR 1:3
Fix Accuracy 1/4 NM (95% 2 dRMS)
Atmospheric Noise 57.8 dB above 1 uV/m

M  Boise City, OK
V  Gillette, WY
W  Searchlight, NV
X  Las Cruces, NM
Y  Raymondville, TX
Z  Grangeville, LA

Note: Estimated Groundwave coverage, actual coverage will vary

Figure 4-11  Coverage diagram for South Central U.S. chain (GRI 9610).

Figure 4-12, Figure 4-13 and Figure 4-14 show positioning errors resulting from use of GRI 8970 (M, W, X, Y, Z), GRI 7980 (M, W, X, Y, Z) and GRI 9610 (M, V, Y, Z). The means of the horizontal errors are 500.4056 (m), 681.7433 (m) and 789.3971 (m); the standard deviations of the horizontal errors are 6.2412(m), 43.9534(m) and 189.4188 (m) respectively. The HDOPs are 1.3846, 6.1273 and 8.3497 respectively.

Figure 4-12 The single chain GRI 8970 (M W X Y Z) horizontal positioning error
Figure 4-13 The single chain GRI 7980 (M W X Y Z) horizontal positioning error

Figure 4-14 The single chain GRI 9610 (M V Y Z) horizontal error
4.5 Analysis

Based on the previous results, the mean of horizontal errors versus the average distance from the user to the transmitters is depicted in Figure 4-15. Figure 4-16 shows the relationship of the standard deviation versus the average distance from the user to the transmitters.

Figure 4-15 Single chain coverage distance versus horizontal position error
Obviously, the results presented indicate that using the Loran-C pseudorange model can meet the absolute accuracy of the Loran-C system (local chain). In addition, the modern Loran-C receiver easily enlarges the coverage of the Loran-C system, if some additional error can be accommodated.

**Figure 4-16** Single chain coverage distance versus standard deviation
Chapter 5 Loran-C Multi-Chain Pseudorange Positioning

Due to the increased sensitivity of the modern digital Loran-C receiver, today a single receiver can receive groundwaves and skywaves from up to 40 Loran-C transmitters (8 chains) simultaneously. In a practical sense, these measurements can easily improve the performance of the Loran-C system, but for traditional positioning methods such as the TD model, the majority of these measurements are redundant and cannot improve the system performance. In order to utilize these redundant measurements, a multi-chain pseudorange positioning model will be derived. The Loran-C multi-chain pseudorange positioning method can convert these redundant measurements into useful information and can easily improve the performance of the Loran-C system.

In this case study, the modern receiver can track 27 signals (15 stations) at all times in southeast Ohio (Ohio University Airport, Albany, OH). For chain GRI 8290, the receiver only receives three signals: M (Havre, MT), W (Baudette, MN), X (Gillette, WY). The same situation happened on chain GRI 5930 where only M (Caribou, ME) and X (Nantucket, MA) have been tracked at all times. These measurements are insufficient to solve for a user position (whether using TD's or the single chain pseudorange method plus 3-D compensation). However, it is possible to
combine two or more Loran-C chain measurements to solve for a user position. That means multi-chain pseudorange positioning can increase the Loran-C coverage and help address system integrity and continuity. The following results also show that the multi-chain pseudorange positioning method can improve the absolute accuracy for the Loran-C system.

One thing should be pointed out here. This results presented in this chapter ignore the chain synchronization and station dual-rating which are currently in practice. For North America Loran-C master stations have been synchronized with UTC time. The European Loran-C master stations are also synchronized. In addition, dual-rating is used in most of the North American and European stations. For worst case estimates, we neglect these two effects and focus on the general case discussion.

5.1 Multi-Chain Pseudorange Model Calculations

Applying the same method used in section 4.2, the pseudorange equations can be written in general form as follows:

\[ \delta z = H^* \delta \epsilon \]  \hspace{1cm} (Eq.5.1)

\[ \therefore \delta \epsilon = (H^T H)^{-1} H^T \delta z \]  \hspace{1cm} (Eq.5.2)

where 

\[ x : \text{ User state vector.} \]

\[ x = \begin{bmatrix} U_x & U_y & U_z & b_1 & \ldots & b_{j-3} \end{bmatrix}^T \]

\[ z : \text{ Measurement vector.} \]
\[ z = [PR_1 \ PR_2 \ PR_3 \ PR_4 \ ... \ PR_n]^T \]

\( H \): A matrix containing data related to the geometry of the transmitting stations with respect to the user.

\[
H = \begin{bmatrix}
\alpha_{11} & \alpha_{12} & \alpha_{13} & \ldots & \alpha_{1j} \\
\alpha_{21} & \alpha_{22} & \alpha_{23} & \ldots & \alpha_{2j} \\
\alpha_{31} & \alpha_{32} & \alpha_{33} & \ldots & \alpha_{3j} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\alpha_{i1} & \alpha_{i2} & \alpha_{i3} & \ldots & \alpha_{ij}
\end{bmatrix}
\]

\[ \alpha_{ij} = \frac{\partial PR_j}{\partial \hat{U}_j} \bigg|_{(\hat{\varphi}_x, \hat{\varphi}_y, \hat{\varphi}_z, \hat{b}_1, \hat{b}_2, \ldots, \hat{b}_{j-1})} \quad \text{if} \quad j \leq 3 ; \]

or \[ \alpha_{ij} = \frac{\partial PR_j}{\partial \hat{b}_j} \bigg|_{(\hat{\varphi}_x, \hat{\varphi}_y, \hat{\varphi}_z, \hat{b}_1, \hat{b}_2, \ldots, \hat{b}_{j-1})} \quad \text{if} \quad j > 3 ; \]

Note that for the Loran-C system, if transmitter synchronization is established within each chain only, j clock offsets should be added to the user state vector \( x \) in Eq.5.1 for j chain combinations (Using multi-chain pseudorange positioning).

Secondly, using the iteration algorithm (Newton’s method) as follows:

1. **Obtain an a priori user estimate** \( (\hat{U}_x, \hat{U}_y, \hat{U}_z, \hat{b}_1, \hat{b}_2, \ldots, \hat{b}_{j-1}) \).

2. **Calculate & use the pseudorange measurement and calculate** \( H \).

3. **Calculate the change in the user position** & from (Eq.5.2).
4. **Update the user position**: \( x_{\text{NEW}} = x_{\text{OLD}} + \Delta x \).

5. **If the change in the user position** \( |\Delta x(1)^2 + \Delta x(2)^2| \leq \varepsilon \) **the procedure is completed,** otherwise repeat steps b-e. \((\varepsilon \text{ is a selected tolerance})\)

The user state \( x = [U_x, U_y, U_z, b_1, \ldots, b_{j-3}]^T \) can be obtained.

In practice, the U.S and Canada Loran-C master stations have been synchronized to within 100 ns sec with UTC time.

### 5.2 Two Chain Combination

For the two chain combination, at least 5 pseudorange measurements are required to solve for 3 dimensions \((U_x, U_y, U_z)\), clock offset for chain 1 \((b_1)\) and clock offset for chain 2 \((b_2)\).

#### 5.2.1 Local Chain Combination

To make sure the Loran-C multi-chain pseudorange method works correctly, a two chain combination with one local chain (GRI 9960) and one remote chain (GRI 7980) was selected. The horizontal errors of the two chain combination (GRI 9960 M, Y, Z plus GRI 7980 M, W, Y) is depicted in Figure 5-1. The mean of the horizontal error is 88.4041 (m); the standard deviation of the horizontal error is 4.7952 (m) and
HDOP is 2.3703. Compared to the single chain results (Figure 4-13 the result of GRI 7980 M, W, X, Y, Z), the outcome of the two chain combination shows significant improvement in positioning compared to "remote" chain accuracy. It implies that multi-chain pseudorange positioning is better than single chain positioning under conditions of poor geometry.

Figure 5-1 The horizontal positioning error of a two chain combination (GRI 9960 M,Y,Z plus GRI 7980 M,W,Y)
Figure 5-2 and Figure 5-3 present the result of the two chain combination from (GRI 9960 M, Y, Z plus GRI 9610 M, V, Z) and (GRI 9960 M, Y, Z plus 8290 W,X). The mean of the horizontal errors are 455.3121 (m) and 191.4680 (m); the standard deviation of the horizontal errors are 7.0723 (m) and 78.3563 (m). The HDOPs are 1.6620 and 2.8969.

Figure 5-2 The horizontal positioning error of a two chain combination (GRI 9960 M,Y,Z plus GRI 9610 M,V,Z)
5.2.2 Two Non-Local Chain Combination

Figure 5-4 and Figure 5-5 show the results of a two non-local chain combination (GRI 7980 W, Y, Z plus GRI 8290 M, X) and (GRI 9610 M, V, Y plus GRI 5930 M, X). The mean of the horizontal errors are 1004.40 (m) and 2517.40 (m); the standard deviation of the horizontal errors are 270.4441 (m) and 282.7033 (m), respectively. The HDOPs are 8.0039 and 8.1662. Although the results cannot meet the requirement for the Loran-C system (1/4 NM), it is still acceptable for some applications such as oceanic positioning. The results present direct evidence of increasing the coverage of the Loran-C system.
Loran-C Multi-Chain Positioning: (GRI 7980 XYZ + GRI 8290 MX) Result

Figure 5-4 The horizontal positioning error of a two chain combination (GRI 7980 W,Y,Z plus GRI 8290 M,X)

Loran-C Multi-Chain Positioning: (GRI 9610 M,V,Y plus GRI 5930 M,X) Result

Figure 5-5 The horizontal positioning error of a two chain combination (GRI 9610 M,V,Y plus GRI 5930 M,X)
5.3 Three Chain Combination

For a three-chain combination, a minimum of 6 pseudorange measurements are required to solve for 3 dimensions \((U_x, U_y, U_z)\), clock offset for chain 1 \((h_1)\), clock offset for chain 2 \((h_2)\) and clock offset for chain 3 \((h_3)\). The result of a three chain combination (GRI 9610 MV, GRI 7980 WX and GRI 8290 MX) is illustrated in Figure 5-6. The mean of the horizontal error is 1076.20 (m); the standard deviation of the horizontal error is 143.6010 (m) and the HDOP is 5.3959. Comparing the results of the two chain combinations in Figure 5-4 and Figure 5-5, the three chain combination provides better accuracy.

![Figure 5-6 The horizontal positioning error of a three chain combination (GRI 9610 MV, GRI 7980 WX and GRI 8290 MX)](image-url)
Figure 5-7 shows the result of an 8 station combination selected from (GRI 9610 M V Y, GRI 7980 M Y Z and GRI 5930 M X). The mean of the horizontal errors is 585.6652 (m); the standard deviation of the horizontal errors is 67.0015 (m). The HDOP is 2.8433.

![Figure 5-7 The horizontal positioning error of three chain combination (GRI 9610 M V Y, GRI 7980 M Y Z and GRI 5930 M X)](image)

Obviously, chain combination provides good geometry and increases the accuracy of positioning.
5.3.1 k-Chain Combination

A multi-chain (k chain) combination is also possible to solve for user position. To solve for the user position \((U_x, U_y, U_z)\) and \(k\) clock offsets \((b_1, \ldots, b_k)\), at least \(k+3\) pseudo-range measurements are required. Figure 5-8 shows the result of all possible chain combinations except the local chain (GRI 9960) in southeast Ohio. The mean of the horizontal error is 840.6016 (m); the standard deviation of the horizontal error is 40.0925 (m). The HDOP is 2.1146.

Figure 5-8 The horizontal positioning error of a multi-chain combination (GRI 9610 MVWX, GRI 7980 MWXY, GRI 8290 MX and GRI 5930 MX)
5.4 Skywave Navigation

The modern Loran-C receiver is more powerful than previously thought. The modern receiver can track skywaves traveling 5,000 miles and reaching levels of 28 $dB\mu V / M$ [3]. Traditionally, skywave tracking did not yield enough measurements to allow for solving for position. Moreover, skywaves were viewed as interference, so numerous receiver architectures try to reject skywaves. This situation could be changed in some applications such as oceanic navigation. Using the multi-chain pseudorange positioning method, skywave navigation can increase the coverage of the Loran-C system.

5.4.1 Skywave Propagation

Loran-C signal travel paths are grouped into two major categories:

(i) groundwave.

(ii) skywave.

The groundwave signal propagates in the atmospheric medium below the ionosphere; however, the signal strength of the groundwave is attenuated as it follows the contour of the earth. At great distances from the transmitter, the groundwave signal is substantially attenuated.

Skywave signals propagate via reflection from the ionosphere. For the 100 kHz carrier frequency of the Loran-C signal, this reflection will take place in the lower E or D region of the ionosphere [5]. Because the intensity of the ionosphere varies seasonally and diurnally, the average reflection height will vary from approximately 60
km during the day to approximately 90 km at night [6]. From the geometry of the reflection, it is obvious that the skywave signal must travel a longer distance than the groundwave to reach an observer and will arrive after the corresponding groundwave - generally 35 \( \mu \text{sec} \) to 1000 \( \mu \text{sec} \) after the groundwave [5]. Because skywaves do not travel over the surface of the earth and hence are not to significantly attenuated, they can travel longer distances than groundwaves.

5.4.2 Skywave Propagation Correction

In order to use skywaves, it is necessary to apply a skywave propagation correction to the pseudorange measurement. One method [6] presents skywave delay as follows:

\[
t = \frac{2}{c} \left( \left( h^2 + 4R(R + h)\sin^2 \frac{\beta}{2} \right)^{\frac{1}{2}} - R\beta \right)
\]

(Eq. 5.3)

where

- \( c \): The Velocity of EM waves.
- \( h \): The effective height of the ionosphere.
- \( R \): The earth’s radius.
- \( \beta \): \( D / 2R \).
- \( D \): The range of the receiver from the transmitter.

To test the skywave correction factor, the first step is to make sure which signals are skywaves. By carefully checking the tracking data, GRI 9940 W and X
should be skywaves. Figure 5-9 shows the coverage diagram for GRI 9940. Apparently, the coverage range is far from southeast Ohio. Note that a typical coverage limit for groundwave SNR is approximately -10 dB. The range from southeast Ohio to both transmitters is over 3,000 km and the SNR is -2 dB and -8 dB respectively. It follows that the received signals are skywaves.

West Coast U.S. Loran-C Chain GRI 9940

![Coverage diagram for West Coast U.S. chain (GRI 9940).](image)

**Figure 5-9** Coverage diagram for West Coast U.S. chain (GRI 9940).

Secondly, a two chain combination is selected from (GRI 9960 W, X, Y plus GRI 9940 W, X) and is calculated for a one and half hour period (sampling period 10 seconds) on July 26, 1996. The resulting horizontal error is plotted in Figure 5-10. The mean of the horizontal error is 1836.2 (m); the standard deviation of the horizontal error is 435.1730 (m). The HDOP is 9.5041.

![Loran-C Skywave Positioning](image)

**Figure 5-10** The horizontal positioning error of two chain combination (GRI 9960 W,X,Y plus GRI 9940 W,X) with 2 skywaves
One item that should be pointed out is that in Eq.5.3, the effective height of the ionosphere \( h \) depends on time [5]; but for easy calculation, the Figure 5-10 results assume the height is fixed at \( h = 65 \) km. The scatter of the result in Figure 5-10 is caused by the deviation between actual and estimated (65km) ionosphere height.

Thus, for oceanic navigation, Loran-C skywave navigation shows promise as a backup system.

5.5 Conclusion

Quantitative analysis of the results presented in this chapter are summarized in Figure 5-11. It should be noted that there is a strong correlation between the horizontal errors and the average range from the user location to the transmitters. Figure 5-12 shows the relationship of the standard deviation versus average range from the user location to the transmitters.

Apparently, multi-chain pseudorange positioning has the following merits.

- Increases the coverage distance of the Loran-C system.
- Improves the performance of remote single chain positioning.
- Improves the Loran-C system integrity and continuity.
Horizontal Position Errors for Multi-Chain

Coverage Range (Km) vs Horizontal Positioning Error (m)

Figure 5-1: GRI9960 MYZ + GRI 7980 MWZ
Figure 5-2: GRI9960 MYZ + GRI 9610 MVZ
Figure 5-3: GRI9960 MYZ + GRI 8290 MWX
Figure 5-4: GRI 7980 XYZ + GRI 8290 MX
Figure 5-5: GRI 9610 MVY + GRI 5930 MX
Figure 5-6: GRI 9610 MV + GRI 7980 WX + GRI 8290 MX
Figure 5-7: GRI 9610 MVY + GRI 7980 MYZ + GRI 8290 WX
Figure 5-8: GRI 9610 MVWX + GRI 7980 MWXY + GRI 8290 MX + GRI 5930 MX

Figure 5-11 Multi-chain coverage distance versus horizontal position errors
Standard Deviation for Multi-Chain

Figure 5-12 Multi-chain coverage distance versus standard deviation
Chapter 6 Loran-C Flight Test

According to the static results, Loran-C multi-chain pseudorange positioning seems to have numerous advantages. Among these are increased coverage distance and improved the accuracy. To determine the feasibility and robustness of the Loran-C multi-chain pseudorange positioning model, actual flight test data is required. The flight data employed here were collected during a trip from Athens, Ohio to Madison, Wisconsin on July 1, 1996.

6.1 Hardware Configuration

The block diagram of the hardware configuration for the flight test experiment is shown in Figure 6-1. A ten-channel GPS receiver (NovAtel GPS PC card) and a modern digital Loran-C receiver (LOCUS LAD Loran-C LRS receiver), both with continuous tracking (sample period: 5 seconds), were used to collect GPS and Loran-C pseudorange data. The two receivers were interfaced with a microcomputer (IBM 486 PC). For this flight test, the receivers were not fully synchronized. The method of synchronization for this test uses software to synchronize the GPS clock with the PC clock at the beginning of the flight, and record the PC time in the first Loran-C receiver data.
Block Diagram of the GPS/Loran-C Hardware

Figure 6-1 Hardware configuration of Loran-C flight test
6.2 Single Chain Result

Figure 6-2 shows the GPS tracking results. Using GPS as "truth" position, the Loran-C single chain (GRI 8970 M W X Y Z) pseudorange positioning result is depicted in Figure 6-3. The mean of the horizontal error is 965.4800 (m); the standard deviation of the horizontal error is 442.1231 (m). The HDOP is 1.3026. Compared to the static test results in chapter 4, this dynamic result is far from the tolerance of the Loran-C accuracy (1/4 NM). Moreover, the positioning errors become large and unstable during aircraft turns. It is important to point out, however, that the receiver was designed for use in timing and not for aviation. These large errors and disturbances occur because the Loran-C receiver tracking loop is optimized for stationary users. This receiver's tracking loop has a long integration time which enables it to receive weak signals. For high dynamic applications such as aircraft flight, this tracking loop cannot fully respond to the change of position within a short period of time. In addition, there is a slight synchronization problem since the PC clock and the GPS card clock have different accuracies.
Figure 6-2 Loran-C flight test reference trajectory (GPS card tracking result)

Figure 6-3 Loran-C flight test: single chain (GRI 8970 M W X Y Z) pseudorange positioning errors
6.3 Two Chain Combination Result

The horizontal positioning error for the two chain combination (GRI 9960 M Y Z plus GRI 7980 M W X) is plotted in Figure 6-4. The mean of the horizontal error is 1332.4800 (m); the standard deviation of the horizontal error is 702.0347 (m). The HDOP is 2.1906.
6.4 Three Chain Combination Result

The horizontal position error for the three chain combination (GRI 9960 M Y Z + GRI 7980 M W X + GRI 8290 W X) is shown in Figure 6-5. The mean of the horizontal error is 1198.03 (m); the standard deviation of the horizontal error is 568.9566 (m). The mean of the HDOP is 2.6310. Compared to the results of Figure 6-3 and Figure 6-4, the disturbance during the turn decreases in size. This shows that the multi-chain pseudorange positioning method is useful for integrity and increases absolute accuracy.

![Flight Test I: Loran-C Pseudorange Positioning (GRI 9980 M W X + GRI 9960 M Y Z + GRI 8290 W X) Result](image)

**Figure 6-5** Loran-C flight test: three chain combination (GRI 7980 M W X, GRI 9960 M Y Z and GRI 8290 W X) pseudorange positioning error
6.5 Analysis

The mean of the horizontal errors versus the coverage range from the user location to the transmitter is depicted in Figure 6-6. Figure 6-7 shows the relationship of the standard deviation versus the coverage range from user to transmitter. Although the results of the flight test are worse than the static test, the multi-chain pseudorange positioning method can significantly increase the coverage distance of the Loran-C system and eliminate some of the disturbances.

Generally speaking, the multi-chain pseudorange positioning method not only extends the coverage of the Loran-C system; but also addresses system integrity.

Figure 6-6 Loran-C flight test coverage distance versus horizontal position error
Figure 6-7 Loran-C flight test coverage distance versus standard deviation

As will be discussed later, the advantages of modern digital signal processing will be received in aviation when adaptive tracking loops are utilized.
Chapter 7 New Coverage Diagram for Loran-C

Due to the performance of the Loran-C multi-chain pseudorange positioning method and modern receiver technique, it is reasonable to increase the coverage range without losing accuracy. Therefore, it is prudent to reconsider the coverage of Loran-C for several navigation applications.

The maximum range of the Loran-C system is defined as that range which satisfies both accuracy and SNR criteria [5]. For the USCG (United States Coast Guard) method, the minimum acceptable SNR (Signal to Noise Ratio) is -10 dB [5]. In fact this SNR limit is vastly conservative, since modern Loran-C receiver can track signals adequately with SNR of -20 dB or less [7]. In addition, signal strength as measured at a receiver location depends upon the transmitter power, antenna type and conductivity of the wave's travel path. For easy modeling, the coverage prediction criteria ignores the transmitter power and wave travel path. Using the same standard as the USCG method, i.e. SNR and accuracy, the following discussion focuses on the ideal relation between accuracy and maximum coverage range.

7.1 Loran-C Coverage Prediction for NPA Navigation

The accuracy criteria for non-precision approach navigation is 0.3 NM absolute accuracy [4]. In addition, the SNR criteria for NPA is SNR > -6 dB.
First, based on the results presented in Figure 5-11, the maximum distance to fit the NPA rule is about 1400 km.

Second, after checking the receiver SNR measurements (Figure 7-3), the SNR level is greater than -6 dB within a range of 1400 km.

Using a 1400 km coverage radius, the Loran-C master stations' coverage diagram in North America is shown in Figure 7-1 [3].

\[Figure\ 7-1\ Loran-C\ master\ station\ coverage\ with\ 1400km\ in\ U.S.\ and\ Canada]
With a minimum requirement of coverage from at least 4 stations for each point, Figure 7-2 shows the useful coverage prediction diagram for Loran-C NPA in North America. Compared to the USCG coverage diagram, the new Loran-C coverage for NPA navigation is substantially increased in the fringe areas.

**Figure 7-2** Loran-C coverage prediction for NPA navigation in U.S. with 1400 km radius
7.2 Loran-C Coverage Prediction for Remote Navigation

The criteria for terminal and enroute navigation are 1.1 NM (2037m) and 1.5 NM (2778m) [4]. In addition, the tolerance for oceanic navigation is 2 NM (3704m) [5]. According to the results in chapter 5 and chapter 6, even though there are large errors for the non-local multi-chain position solution, the horizontal errors are still acceptable and fit the tolerance needs for remote or oceanic navigation. Extrapolating the result from Figure 5-11, the maximum distance to fix the oceanic or remote navigation requirement is about 2400+ km.

Secondly, a 48 hour data collection effort in southeastern Ohio (June, 1996) found that at least 27 signals (15 stations) are available 24 hours/day. The station data and distances between the stations and southeastern Ohio (Ohio University Airport, Albany, OH) are shown in Table 6-1. This indicates that the coverage range for Loran-C is more than 2400+ km, 24 hr/day.

Thirdly, the relationship between the coverage range and receiver SNR measurement is plotted in Figure 7-3. If the minimum acceptable SNR (Signal to Noise Ratio) is set at -15 dB for modern Loran-C receivers, the new Loran-C coverage for oceanic or remote navigation is about 2500 km, 24 hours/day.
<table>
<thead>
<tr>
<th>Station</th>
<th>Distance (Km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seneca, NY (9960 M, 8970 X)</td>
<td>598</td>
</tr>
<tr>
<td>Caribou, ME (9960 W, 5930 M)</td>
<td>1436</td>
</tr>
<tr>
<td>Nantucket, MA (9960 X, 5930 X)</td>
<td>1066</td>
</tr>
<tr>
<td>Carolina Beach, NC (9960 Y, 7980 Z)</td>
<td>689</td>
</tr>
<tr>
<td>Dana, IN (9960 Z, 8970 M)</td>
<td>458</td>
</tr>
<tr>
<td>Malone, FL (7980 M, 8970 W)</td>
<td>951</td>
</tr>
<tr>
<td>Grangeville, LA (7980 W, 9610 Z)</td>
<td>1225</td>
</tr>
<tr>
<td>Raymondville, TX (7980 X, 9610 Y)</td>
<td>2022</td>
</tr>
<tr>
<td>Jupiter, FL (7980 Y)</td>
<td>1365</td>
</tr>
<tr>
<td>Baudette, MN (8970 Y, 8290 W)</td>
<td>1436</td>
</tr>
<tr>
<td>Boise City, OK (8970 Z, 9610 M)</td>
<td>1840</td>
</tr>
<tr>
<td>Havre, MT (8290 M)</td>
<td>2447</td>
</tr>
<tr>
<td>Las Cruces, NM (9610 X)</td>
<td>2359</td>
</tr>
<tr>
<td>Gillette, WY (9610 V, 8290 X)</td>
<td>2014</td>
</tr>
<tr>
<td>Searchlight, NV (9610 W, 9940 Y)</td>
<td>2906</td>
</tr>
</tbody>
</table>

Table 1 The station data and distances from the stations to southeastern Ohio

Figure 7-3 The relationship between coverage distance and SNR
At least 4 Loran-C stations across within the coverage diagram, Figure 7-4 illustrates the predict Loran-C coverage diagram for remote navigation in the U.S and Canada with 2500 km circles.

**Figure 7-4** New Loran-C coverage prediction for remote navigation in the vicinity of the U.S. (2500km radius coverage from four stations)

Figure 7-5 shows the Loran-C oceanic navigation prediction coverage diagram for the Atlantic Ocean. Note that in Figure 7-5 the coverage for European part are made by Bo Chain (GRI 7001), Sylt Chain (GRI 7499), Ejde Chain (GRI 9007) chains and Lessay Chain (GRI 6731) [18]. Figure 7-6 depicts the Loran-C oceanic navigation
prediction coverage diagram for the Pacific Ocean. The coverage for Asian part in Figure 7-6 are made by Russia Chain (GRI 7950), Korea Chain (GRI 7499), North West Pacific Chain (GRI 8930) and China North Sea Chain (GRI 9007) [19].

Clearly, new Loran-C prediction coverage illustrations above indicate that transoceanic navigation is possible for aviation or marine applications in some of the Atlantic ocean and the Pacific ocean areas.

**Figure 7-5** New Loran-C coverage prediction for oceanic navigation in the Atlantic ocean area with 2500 km radius
7.3 New Loran-C Receiver Algorithm

From the previous results, a new Loran-C navigation software algorithm for multi-chain pseudorange positioning method can be derived. Figure 7-6 shows the flowchart for Loran-C multi-chain positioning.

**Figure 7-6** New Loran-C coverage prediction for oceanic navigation in the Pacific Ocean area with 2500 km radius
Figure 7-6 Multi-chain pseudorange positioning Algorithm for Loran-C receiver

Data presented here indicate the performance of the Loran-C system can be significantly enhanced if contemporary technology and the new algorithm of Loran-C multi-chain pseudorange positioning method is applied to receivers.
Chapter 8 Hybrid GPS/Loran-C Pseudorange Positioning

Many methods can be implemented to combine the data from Loran-C and GPS. For example, a Kalman filter could be developed that processes the position solution for Loran-C and GPS. Even though this method can obtain an optimized solution, it is not an easy task. The navigation solution should be based on a generic design that emphasizes effective and transparent rather than optimal processing [4]. Using pseudorange positioning, Loran-C measurements could be integrated with other pseudorange systems such as GPS, easily and effectively. In addition, the results presented in this chapter will show the hybrid GPS/Loran-C pseudorange positioning method can address the availability problem when GPS and Loran-C are both independently unavailable simultaneously.

8.1 Pseudorange Measurement for GPS

GPS pseudorange measurements are calculated by taking the time difference between the signal time of arrival (TOA) and the corresponding signal time of transmission (TOT). Note that the time of transmission and the time of reception (arrival) must be expressed with respect to the same time reference. The equation for the GPS pseudorange measurement is shown as follows:
\[ TOT_{GPS} = TOT_{sv} + \Delta t_{sv} \]

\[ TOA_{Re} = TOA_{UE} + \Delta t_{UE} \]

\[ PR_i = c \times (TOA_{Re} - TOT_{GPS}) \]

\[ PR_i = c \times (TOA_{UE} - TOT_{sv}) + c \times (\Delta t_{UE} - \Delta t_{sv}) \]  

(Eq.8.1)

where  

- \( PR_i \) : The ith GPS satellite pseudorange measurement.
- \( TOT_{GPS} \) : Time of transmission with respect to UTC time.
- \( TOT_{sv} \) : Time of transmission with respect to satellite clock time.
- \( TOA_{UE} \) : Time of reception with respect to UTC time.
- \( TOA_{Re} \) : Time of reception with respect to receive clock time.
- \( \Delta t_{sv} \) : Clock offset between GPS satellite clock and UTC time.
- \( \Delta t_{UE} \) : Clock offset between receiver clock and UTC time.
- \( c \) : The GPS signal propagation speed (299792458 m/s)

### 8.1.2 Propagation Delay For GPS Signal Transmission

Because the signal propagation speed \( c \) varies in the earth's atmosphere, we usually subdivide earth's atmosphere into several regions based on common physical properties. The majority of the propagation delays for GPS are ionospheric and tropospheric delay.
8.1.2.1 Ionospheric Propagation Delay

The ionospheric error is a function of the integrated electron count over the ray path after one has accounted for the ray bending effects of the ionosphere [14]. Thus the effect is dependent on both the character of the ionosphere at zenith and the elevation angle to the satellite. To neutralize the effect of the ionosphere on signal propagation, GPS is designed to operate dual frequencies $f_{L1}$ and $f_{L2}$. Civil aviation receivers do not have access to L2, however, and a model is used to estimate ionospheric delay. This model accounts for about 50% of the error on a worldwide average. The mean ionospheric delay at nighttime is on the order of 10 nsec. During daytime the delay increases to as much as 50 nsec.

8.1.2.2 Tropospheric Propagation Delay

The tropospheric error is caused by the lower portion of the atmosphere (less than 40 km above the surface) is non-disperse medium [21].

The tropospheric refraction, does not depend on the frequency and thus affects both the code modulation and the carrier phase in the same way.

Tropospheric propagation delays are modeled using the following equation [4]:

\[
\text{delay}_{\text{TROPO}} = \frac{2.4224 \times e^{-0.13345h}}{0.026 + \sin(E)} \quad \text{(m)}
\]

(Eq.8.2)

where:

- $E$ is the satellite elevation angle
- $h$ is the altitude of the receiver (km)
8.2 Calculation Method for Hybrid Pseudorange Model

Taking ionospheric and tropospheric delay into account, the GPS pseudorange measurement is expressed as follows:

\[ PR_i = c \cdot (TOA_{UE} - TOT_{SV}) + c \cdot (\Delta t_{UE} - \Delta t_{SV}) - c \cdot (t_{ION} + t_{Tropo}) \]  
(Eq.8.3)

where \( t_{ION} \) : The ionosphere delay for GPS.
\( t_{Tropo} \) : The troposphere delay for GPS.

According to the previous discussion in section 4.1, the Loran-C pseudorange measurement can be expressed as follows:

\[ PR_i = c \cdot TOA(i) - SF(R_i) \]  
(Eq.8.4)

Combining both GPS and Loran-C pseudorange together, the pseudorange equations can be written in the same manner as 4.2.

\[ \Delta \xi = H \cdot \Delta \xi \]  
(Eq.8.5)

\[ \therefore \Delta \xi = (H^T H)^{-1} H^T \Delta \xi \]  
(Eq.8.6)

where \( \Delta \xi \) : User state vector.
\[ x = \begin{bmatrix} U_x & U_y & U_z & b_{\text{GPS}}^\text{Loran} & b_{\text{Chain}-1}^\text{Loran} & \ldots & b_{\text{Chain}-m-1}^\text{Loran} \end{bmatrix}^T \]

\( z \) : Measurement vector.
\[ z = \begin{bmatrix} PR_1^\text{GPS} & PR_2^\text{GPS} & \ldots & PR_j^\text{Loran} \end{bmatrix}^T \]

\( H \) : A matrix containing data related to the geometry of the transmitting stations with respect to the user.
Using hybrid GPS/Loran pseudorange positioning, j clock offsets should be added to the user state vector $x$ in Eq.8.5. for m-1 Loran chains and GPS combinations.

Secondly, using the iteration algorithm (Newton’s method) as follows:

i) Obtain an a priori user estimate $(\hat{U}_x, \hat{U}_y, \hat{U}_z, \hat{b}_{\text{GPS}}, \hat{b}_{\text{Loran}}, ..., \hat{b}_{\text{Loran}})$.

ii) Calculate $\Delta$ and use the pseudorange measurement and calculate $H$.

iii) Calculate the change in the user position $\Delta x$ from (Eq.8.6).

iv) Update the user position: $x_{\text{NEW}} = x_{\text{OLD}} + \Delta x$.

v) If the change in the user position $|\Delta x(1)^2 + \Delta x(2)^2| \leq \varepsilon$ the procedure is completed, otherwise repeat steps b-e. ($\varepsilon$ is a selected tolerance)

The user state $x = [U_x, U_y, U_z, b_{\text{GPS}}, b_{\text{Loran}_{\text{Ch1-l}}, ..., b_{\text{Loran}_{\text{Ch1-m-1}}}]}^T$ can be obtained and the position solution is equal $[U_x, U_y, U_z]$. 
8.3 Hardware Configuration for Hybrid GPS/Loran-C System

The time reference in the transmitter and receiver is the primary concern to obtain accurate pseudorange measurements in GPS and Loran-C. Different systems usually have no correlated time reference. Traditional Loran-C systems use the master station to be the time reference for each chain. For a hybrid system, it is important to correlate both time references together to acquire useful pseudorange measurements.

In our experiments, we use two receivers to measure GPS and Loran-C pseudoranges. In order to get useful pseudorange measurements, synchronization is a major concern. Figure 8-1 illustrates the block diagram of how to synchronize the GPS and Loran-C receivers.

Figure 8-1 Block diagram for hybrid GPS/Loran synchronization
Figure 8-2 is a diagram of the synchronization circuit designed for the hybrid tests. Using Loran-C receiver output signals as a trigger point, the GPS pseudorange measurement is correlated with the Loran-C pseudorange measurement. Although the method is not perfect and still has an offset between Loran-C pseudorange and GPS pseudorange, the offset should be constant and thus can be eliminated.

Figure 8-2 Synchronization circuit for hybrid Loran-C/GPS system
Applying this synchronization circuit to both receivers, the block diagram of the hardware configuration is given in Figure 8-3. A modern digital Loran-C receiver (LOCUS LAD Loran-C LRS receiver) is used to collect Loran-C pseudorange data continuously. As the Loran-C starts the data collection, a signal triggers a ten-channel GPS receiver (NovAtel GPS PC card) to collect GPS pseudorange measurements at the same time.

Block Diagram of Hybrid GPS/Loran-C Hardware

![Block Diagram of Hybrid GPS/Loran-C Hardware](image)

**Figure 8-3** The block diagram of hybrid GPS/Loran-C system
8.4 Static Performance for Hybrid System

To verify the hybrid use of the Loran-C pseudorange with GPS is capable of providing a user position, static measurement were taken on December 1, 1996 for a two hour period (sampling period: 5 seconds).

8.4.1 Hybrid GPS/Local Chain Loran-C Pseudorange Positioning

First, a hybrid solution with 2 GPS and 3 Loran-C (GRI 9960 M Y Z) measurements is formed. The horizontal error is plotted in Figure 8-4. The mean horizontal error is 172.6048 (m); the standard deviation of the horizontal error is 82.5009 (m). The HDOP is 4.38. Note that the GPS SV’s selection for this result picks up the higher elevation angles for all the GPS observations.

Compared to previous results, the mean and standard deviation of the horizontal error are larger than the result of single chain positioning (GRI 9960 M X Y Z , Figure 4-7). There are several sources contributing to the errors.

- The distance between the Loran-C transmitter and the user is smaller than the range between GPS satellite and user. The satellite range (≈ 21425 km) is more than 40 times of the range of GRI 9960 M (≈ 598 km). If there are small errors, such as ephemeris error for GPS and ASF error for Loran-C, added to the GPS and Loran-C pseudorange measurements, the crossing
angle of the LOP will change by large amount. Figure 8-5 shows the idea of how the crossing angle affects the accuracy [5].

- The relative geometry of the selected GPS satellites and the Loran-C transmitters will affect the position result. The relative discrete groups of points in Figure 8-4 is caused by a satellite switch during the data collection.

- The GPS Selective Availability (SA) is the major error source for the hybrid GPS/Loran-C method.

Figure 8-4 The hybrid system (2 GPS plus 3 Loran-C GRI 9960 M Y Z) horizontal positioning error
Figure 8-5 The relationship between crossing angles and area of fix uncertainty

Positioning results with 3 GPS plus 3 Loran-C (GRI 9960 M Y Z) pseudorange measurements is illustrated in Figure 8-6. The mean of the horizontal
error is 167.2096 (m); the standard deviation of the horizontal error is 31.3849 (m).

The HDOP is 1.71.

![Hybrid Pseudorange Positioning: (3 SV + GRI 9960 M Y Z) Result](image)

**Figure 8-6** The hybrid system (3 GPS plus 3 Loran-C GRI 9960 M Y Z) horizontal positioning error

### 8.4.2 Hybrid GPS/Non-Local Chain Loran-C Pseudorange Positioning

Figure 8-7 and Figure 8-8 show the positioning results with a 2 GPS plus 3 Loran-C (GRI 7980 M Y Z) combination and a 3 GPS plus 2 Loran-C (GRI 9610 V Y) combination. The mean of horizontal errors are 502.3757 (m) and 539.1546 (m);
the standard deviation of horizontal errors are 76.1276 (m) and 356.7115 (m) respectively. The HDOPs are 3.8830 and 21.0419.

Figure 8-7 The hybrid system (2 GPS plus 3 Loran-C GRI 7980 X Y Z) horizontal positioning error
Figure 8-8 The hybrid system (3 GPS plus 2 Loran-C GRI 9610 V Y) horizontal positioning error

8.4.3 Hybrid GPS/Multi-Chain Loran-C Pseudorange Positioning

A hybrid pseudorange measurements solution with two GPS and two different Loran-C chain (GRI 9610 M V, GRI 7980 X Y) is plotted in Figure 8-9. The mean of horizontal error is 354.4370(m); the standard deviation of horizontal error is 193.5952(m). The HDOP is 5.1348.
8.5 Conclusion

It is important to note that the aforementioned results represent worst case conditions. Note that neither GPS nor Loran-C were sufficient for navigation given the limited number of measurements. In practice there are usually three to four times the number of measurements used here and the accuracy improves accordingly.

Obviously, the hybrid GPS/Loran-C pseudorange method provides an effective backup for both systems. Using hybrid GPS/Loran-C positioning can help address availability and continuity when GPS and Loran-C are both independently unavailable.

Figure 8-9 The hybrid system (3 GPS plus two Loran chain combination (GRI 9610 MV and GRI 7980 XY) ) horizontal positioning error
Chapter 9 Flight Test II

To verify that the Loran-C multi-chain pseudorange positioning method and hybrid GPS/Loran-C pseudorange positioning scheme are useful in the real navigation applications, actual flight test data is required.

The hybrid systems were flown on December 10, 1996 for a period of 52 minutes in the vicinity of the Ohio University Airport (Albany, OH). The hybrid GPS/Loran-C equipment was installed in a Piper Saratoga PA-32-301, N8238C. Figure 9-1 shows the reference trajectory generated by the GPS-PC-card.

![Flight Test II: GPS Card Tracking Trajectory](image)

**Figure 9-1** Flight test II reference trajectory (GPS card tracking result)
9.1 **Local Chain (GRI 9960) Positioning Result**

Using the GPS tracking trajectory as a reference, the Loran-C single chain positioning result (GRI 9960 MWXYZ) is depicted in Figure 9-2. The horizontal bias is 1852.2 (m). Note that the data latency for Loran-C receiver is about 30sec [17].

![Figure 9-2](image-url)
Obviously, the positioning bias before take-off fits the previous static result; the horizontal bias during this period is approximately 258.0736 (m). However, when the airplane departed, the horizontal error increased substantially. This large offset is due to the delay in the Loran-C receiver tracking loop. Note again that this receiver is optimized for stationary users. With the addition of tracking loop logic to accommodate dynamics, these bias errors could be eliminated.

9.2 Two Chain Combination (with local chain) Positioning Result

Owing to the aforementioned Loran-C receiver problem, the following multi-chain results use the Loran-C single chain (GRI 9960 MWXYZ) positioning result as a reference or baseline. The positioning result of a two chain combination (GRI 9960 MXZ +GRI 7980 MWZ) is depicted in Figure 9-3. The horizontal bias is 241.7374 (m); the standard deviation of horizontal bias is 99.7543 (m). The HDOP is 2.4798.
Flight Test II: (9960 MWXYZ) Position Result vs 2 Chain Combination (9960 MXZ + 7980 MWZ) Result

Assuming the single chain positioning (GRI 9960 MWXYZ) works correctly (i.e. the receiver problem has been eliminated), this result indicates that the two chain combination result drifts slightly more (about 241m) than the local chain does. Roughly speaking, the two chain combination (with local chain) appears to meet the criteria of NPA navigation. Although it is not a direct evidence; it is an indirect result to show the Loran-C multi-chain pseudorange positioning method is feasible in actual flight.
The positioning result of a two chain combination (GRI 9960 MYZ + GRI 9610 MZ) is plotted in Figure 9-4. The horizontal bias is 596.1597 (m); the standard deviation of horizontal bias is 60.8523 (m). The HDOP is 2.0307.

Figure 9-4 The horizontal error for two chain combination (GRI 9960 M X Z + GRI 9610 M Z) positioning result

9.3 Two Chain Combination (without local chain) Positioning Result

Figure 9-5 illustrates the positioning result of two chain combination (GRI 7980 MYZ + GRI 5930 MX). The horizontal bias is 1480.8 (m); the standard deviation of horizontal bias is 123.639 (m). The HDOP is 5.5743.
Figure 9-5 The horizontal error for two chain combination (GRI 7980 MYZ + GRI 5930 MX) positioning result

9.4 Three Chain Combination Positioning Result

Figure 9-6 shows the result from a three-chain combination (GRI 9610 MZ + GRI 7980 MYZ + GRI 5930 MX). The horizontal error is 816.3497 (m); the standard deviation for horizontal error is 222.5622 (m). The HDOP is 3.3057.
Figure 9-6 The horizontal error for three chain combination (GRI 5930 M X + GRI 7980 M Y Z + GRI 9610 M Z) positioning result

9.5 Hybrid GPS/Loran-C Positioning Results

Using the GPS PC card tracking result as a reference, the positioning error from hybrid use of two GPS pseudorange measurements plus 3 Loran-C pseudorange measurements (GRI 9960 MYZ) is depicted in Figure 9-7.
Figure 9-7 The horizontal error for hybrid system (2 GPS + GRI 9960 MYZ) positioning result

Figure 9-8 shows the hybrid result of three GPS pseudorange measurements and six Loran-C measurements, (GRI 9610 MYZ) and (GRI 7980 MYZ). Compared to previous results, the large bias during the turn decreases. It indicates that the hybrid GPS/Loran-C method is applicable for integrity purposes.
The hybrid result of three GPS pseudorange measurements and eight Loran-C measurements, (GRI 9610 MYZ), (GRI 7980 MYZ) and (GRI 9960 MX) is illustrated in Figure 9-9. The result is getting close to the reference. It means the hybrid GPS/Loran-C scheme not only solves the availability problem when both systems are independently unavailable; but also can address integrity issues.
9.6 Analysis

Assume the single chain GRI 9960 MWXYZ works correctly and the horizontal bias is 200 (m). Quantitative analysis of the previous result in this chapter, the horizontal errors of Loran-C multi-chain pseudorange positioning versus the coverage range from the user to the transmitter is depicted in Figure 9-10. Although this result is based on the assumption, it still shows that the multi-chain positioning method can reduce the poor geometric combination errors and increase the coverage simultaneously.
The Multi-Chain Horizontal Errors for Flight Test II

![Graph showing the relationship between Coverage Range (Km) and Horizontal Positioning Bias (m).]

Figure 9-10 Flight test II coverage distance versus horizontal error
Chapter 10 Conclusion

Based on the research described in this thesis, the current Loran-C system can be significantly enhanced by using the multi-chain pseudorange positioning. This method not only increases the coverage of the Loran-C system; but also addresses accuracy and integrity. In addition, the multi-chain scheme can improve areas plagued by poor geometry. When countries from Ireland to India and China are installing Loran-C transmitters and advanced digital signal processing is improving receiver performance, it is quite feasible to provide worldwide coverage for navigation users. Moreover; if the Russian Chayka system is added to Loran-C to form the hybrid Loran-C/Chayka system, it is clear that the coverage of the composite is also worldwide. In general, for non-precision approach navigation or enroute applications, the hybrid Loran-C/Chayka could provide a powerful and robust service worldwide for the next century.

The hybrid use of Loran-C/GPS(GNSS) provides a powerful and robust navigation capability. It addresses the availability problem when GPS and Loran-C are both independently unavailable. In addition, the hybrid system can help to address the integrity problem. The Loran-C is an ideal complement to GPS(GNSS).
In a word, the Loran-C is a global partner of GPS(GNSS) for the next century. The integration of the GPS/Loran-C system provides complete dissimilar redundancy to guarantee availability and integrity in addition to accuracy.

Further research should be conducted to verify the absolute accuracy of the hybrid GPS/Loran-C system in actual flight. In addition, advanced DSP-based Loran-C receivers should be outfitted with adaptive tracking loops to provide robust performance under varying dynamics. A fully digital hybrid GPS/Loran-C receiver would exploit the benefits of each system.
Recommendations

The Loran-C pseudorange positioning and hybrid GPS/Loran-C positioning results presented in this paper show that the Loran-C is an ideal complement to GNSS. It is recommended that future efforts should continue to develop the following topics for Loran-C.

- Add additional secondary phase factor (ASF) calibration in Loran-C pseudorange positioning method plus 2-D compensation model for remote chains. Theoretically, it can provide a wider coverage range for Loran-C than the results which were shown in this thesis.
- Improve receiver tracking loop design and fix dynamic averaging by using heading to provide Loran-C or a hybrid system with robust performance under dynamics.
- Advanced Loran-C antenna designs such as the H-field Loran-C antenna should be used in conjunction with advanced receivers to enhance the availability and continuity in urban applications.
- Investigate new algorithms to calibrate the effect of receiver altitude on propagation of the Loran-C signal.
In addition, for the hybrid GPS/Loran-C system, it is recommended that efforts should be continued to work on weighted solutions. Moreover, fault detection and isolation could be added to provide robust performance.
References


