DEVELOPMENT OF BASIC SYSTEMS ANALYSIS AND CONTROLS
EXPERIMENTS TO DEMONSTRATE SPECIFIC THEORIES

A Thesis Presented to
The Faculty of the College of Engineering and Technology
Ohio University

In Partial Fulfillment
of the Requirements for the Degree
Masters of Science

By
William J. Triska
June, 1993
TABLE OF CONTENTS

LIST OF FIGURES

vi

LIST OF TABLES

viii

LIST OF PROGRAMS

ix

CHAPTER 1

INTRODUCTION

1.1 Thesis Objective 1
1.2 Experiments 2
1.3 Conclusions 7

CHAPTER 2

INITIAL CONDITION

SPRING–MASS SYSTEM

2.1 Theoretical Development 8
2.2 Experimental Setup 15
2.3 Experimental Procedure 17
2.4 Student Questions 19
2.5 Results 20
CHAPTER 3
FREQUENCY RESPONSE SYSTEM

3.1 Theoretical Development
3.2 Experimental Setup
3.3 Experimental Procedure
3.4 Student Questions
3.5 Results

CHAPTER 4
TACHOMETER CALIBRATION

4.1 Theoretical Development
4.2 Experimental Setup
4.3 Experimental Procedure
4.4 Student Questions
4.5 Results

CHAPTER 5
D.C. MOTOR PARAMETERS

5.1 Theoretical Development
5.2 Experimental Setup
5.3 Experimental Procedure
5.4 Student Questions
5.5 Results
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2-1</td>
<td>Initial Condition Spring-Mass System Diagram.</td>
<td>15</td>
</tr>
<tr>
<td>2.5-1</td>
<td>LVDT Calibration Curve.</td>
<td>21</td>
</tr>
<tr>
<td>2.5-2</td>
<td>Nonlinear Spring.</td>
<td>22</td>
</tr>
<tr>
<td>2.5-3</td>
<td>Linear Spring Constant.</td>
<td>23</td>
</tr>
<tr>
<td>2.5-4</td>
<td>Experimental Response $x_1(0) = 0.03937$ in.</td>
<td>24</td>
</tr>
<tr>
<td>2.5-5</td>
<td>Experimental Response $x_2(0) = 0.05906$ in.</td>
<td>25</td>
</tr>
<tr>
<td>2.5-6</td>
<td>Experimental Response $x_3(0) = 0.07874$ in.</td>
<td>26</td>
</tr>
<tr>
<td>2.5-7</td>
<td>Theoretical Response $x_1(0) = 0.03937$ in.</td>
<td>28</td>
</tr>
<tr>
<td>2.5-8</td>
<td>Theoretical Response $x_2(0) = 0.05906$ in.</td>
<td>29</td>
</tr>
<tr>
<td>2.5-9</td>
<td>Theoretical Response $x_3(0) = 0.07874$ in.</td>
<td>30</td>
</tr>
<tr>
<td>2.5-10</td>
<td>Comparison Plot $x_1(0) = 0.03937$ in.</td>
<td>32</td>
</tr>
<tr>
<td>2.5-11</td>
<td>Comparison Plot $x_2(0) = 0.05906$ in.</td>
<td>33</td>
</tr>
<tr>
<td>2.5-12</td>
<td>Comparison Plot $x_3(0) = 0.07874$ in.</td>
<td>34</td>
</tr>
<tr>
<td>3.2-1</td>
<td>Frequency Response System Diagram.</td>
<td>48</td>
</tr>
<tr>
<td>3.2-2</td>
<td>Box *1 (Integrating Circuit).</td>
<td>49</td>
</tr>
<tr>
<td>3.2-3</td>
<td>Box *2 (Differentiating Circuit).</td>
<td>49</td>
</tr>
<tr>
<td>3.2-4</td>
<td>Box *3 (Differentiating Circuit).</td>
<td>50</td>
</tr>
<tr>
<td>3.2-5</td>
<td>Box *4 (Lead-Lag Filter Circuit).</td>
<td>50</td>
</tr>
<tr>
<td>3.5-1</td>
<td>Experimental Magnitude and Phase Angle Bode Diagrams, Box 1.</td>
<td>57</td>
</tr>
<tr>
<td>3.5-2</td>
<td>Experimental Magnitude and Phase Angle Bode Diagrams, Box 2.</td>
<td>58</td>
</tr>
</tbody>
</table>
3.5-3 Experimental Magnitude and Phase Angle
   Bode Diagrams, Box 3.

3.5-4 Experimental Magnitude and Phase Angle
   Bode Diagrams, Box 4.

3.5-5 Theoretical Magnitude and Phase Angle
   Bode Diagrams, Box 1.

3.5-6 Theoretical Magnitude and Phase Angle
   Bode Diagrams, Box 2.

3.5-7 Theoretical Magnitude and Phase Angle
   Bode Diagrams, Box 3.

3.5-8 Theoretical Magnitude and Phase Angle
   Bode Diagrams, Box 4.

4.2-1 Tachometer Calibration System Diagram.

4.5-1 Tachometer Calibration Curve.

4.5-2 Tachometer Constant

5.2-1 D.C. Motor Parameters System Diagram.

5.5-1 Step Response Curve.

5.5-2 Step Response Time Constant Estimation.

5.5-3 Time Constant Estimation.

5.5-4 Motor Constant Curve.

6.1-1 Position Control System Schematic.

6.1-2 Position Control System Block Diagram.

6.1-3 Position Control System Closed-Loop Transfer Function.

6.2-1 Position Control Diagram.

6.5-1 Potentiometer Constant Curve.

6.5-2 Sample Root-Locus Plot.
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5-1</td>
<td>No Load Error Gain and Position Control Results.</td>
<td>114</td>
</tr>
<tr>
<td>6.5-2</td>
<td>Under Load Error Gain and Position Control Results.</td>
<td>115</td>
</tr>
<tr>
<td></td>
<td>Link *1 6&quot;x1/2&quot;x1/4&quot; @ 33.8g.</td>
<td></td>
</tr>
<tr>
<td>6.5-3</td>
<td>Under Load Error Gain and Position Control Results.</td>
<td>115</td>
</tr>
<tr>
<td></td>
<td>Link *2 12&quot;x1/2&quot;x1/4&quot; @ 62.8g.</td>
<td></td>
</tr>
<tr>
<td>6.5-4</td>
<td>Under Load Error Gain and Position Control Results.</td>
<td>116</td>
</tr>
<tr>
<td></td>
<td>Link *3 14&quot;x1/2&quot;x1/4&quot; @ 79.6g.</td>
<td></td>
</tr>
</tbody>
</table>
# LIST OF PROGRAMS

<table>
<thead>
<tr>
<th>Program</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position Control Program (\rightarrow) used to Control the Single Link Manipulator Position Control System.</td>
<td>106</td>
</tr>
<tr>
<td>Root-Locus Program (\rightarrow) used to simultaneously determine the position and velocity error gain values need to stabilize the system at 90° and construct the Root-Locus Diagram.</td>
<td>111</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1 THESIS OBJECTIVE

The objective of this thesis is to develop basic "Systems Analysis and Controls Experiments" for a new Mechanical Engineering Controls Laboratory. This laboratory will initially contain three Initial Condition Spring-Mass Systems, one Frequency Response System, and one Single Link Manipulator Position Control System. These systems have been broken down into five different experiments that will emphasize some theoretical component of each system. These experiments are designed to be used by undergraduate students in conjunction with M.E. 401 over a ten week quarter. This allows the students ample time (two weeks) to preform each experiment, analyze the experimental and theoretical components of the system, and write a report comparing the experimental and theoretical results and other findings. These laboratories will provide a unique opportunity for the students to see
the actual differences between physical and theoretical engineering. This will ultimately enhance their ability to distinguish between the two in the future when they are in the working world.

1.2 EXPERIMENTS

The first experiment is the Initial Condition Spring-Mass system. The three spring-mass systems are developed using different spring strengths and masses in order to achieve a variety of responses and damping constants. Only one of the three experiments will be evaluated here. The other two are expected to behave similarly.

The Initial Condition Spring-Mass experimental setup can be seen in Figure 2.2-1. This setup utilizes a Macintosh IIx computer with LABVIEW software, a Phillips PM 3335 60 MHz. 20 Ms/s oscilloscope, a Data Instruments 1/16 amp. amplifier, and a Linear-Variable Differential Transformer (LVDT). Prior to performing the experiment, all equipment is setup and properly calibrated as described in the Experimental Setup Section 2.2. The experiment is then performed and the responses plotted utilizing the initial conditions governed by the system. These experimental responses can be seen in figures 2.5-4, 2.5-5, and 2.5-6. In order for the theoretical responses to be developed the damping factor must be known. To accomplished this, the logarithm
of the decreasing amplitude of vibration is taken from the experimental responses over a set number of cycles. The value produced is the logarithmic decrement and thus the damping factors for each initial condition are determined to be $\xi_1 = 0.1125$, $\xi_2 = 0.1330$, and $\xi_3 = 0.1015$. The analytical process for each initial condition can be seen in the Theoretical Development Section 2.1. The corresponding theoretical responses are illustrated in figures 2.5-7, 2.5-8, and 2.5-9. Experimental and theoretical comparison plots are also plotted in figures 2.5-10, 2.5-11, and 2.5-12. The results show the responses to be in phase with a slight difference in the magnitude and response time to equilibrium. These are the expected results and thus constitutes a successful experiment.

The second experiment is the Frequency Response System and can be seen in Figure 3.2-1. This setup utilizes a Global Specialties Corporation 2001 Function Generator, a Leybold 30 watt amplifier, a Phillips PM 3335 60 MHz. 20Ms/s oscilloscope, and four different circuit networks: one integrating circuit, two differentiating circuits, and one lead-lag filter circuit. These circuits and their transfer functions are presented in figures 3.2-2, 3.2-3, 3.2-4, and 3.2-5.

Using the Experimental Setup Procedure located in section 3.2, the frequency response equipment is prepared for testing. Once prepared, the function generator is set to generate a sine wave with the frequency varying from 1 to 100,000 hertz in a logarithmic manner according to Experimental Procedure located in section 3.3. At each frequency, the input and output amplitudes and phase angles are recorded for each
network. These amplitudes and phase angles are then used to generate Bode Diagrams for the magnitude and phase angle of each network. These responses are located in figures 3.5-1, 3.5-2, 3.5-3, and 3.5-4. The theoretical response are developed in section 3.1. This is done by first determining the transfer function for each network and then using MATLAB on the macintosh computer to plot the bode diagrams. The theoretical responses for the circuit networks are illustrated in figures 3.5-5, 3.5-6, 3.5-7, and 3.5-8. The experimental and theoretical results closely approximate each other, thus depicting a successful experiment.

The Single Link Manipulator Position Control system is divided up into three different experiments: Tachometer Calibration and Constant, Time and Motor Constants, and Position Control. The experimental setups for these experiments can be seen in figures 4.2-1, 5.2-1, and 6.2-1 respectfully. The Tachometer Calibration and Constant experiment utilizes a Macintosh IIx computer with LABVIEW software, a Leybold 30 watt amplifier, a Phillips PM 3335 60 MHz. 20 Ms/s Oscilloscope, and a tachometer. The Time and Motor Constants experiment utilizes the same equipment with the addition of a Global Specialties Corporation 2001 Function Generator and a hand held Extech Instruments Digital Photo/Contact Tachometer. The Position Control experiment utilizes the Macintosh IIx computer with Labview software, a Leybold 30 watt amplifier, a Fluke multimeter voltmeter, a tachometer, and a Giannini 1500 ohm 10 turn potentiometer. Each experiment is designed to emphasizes a different theoretical component of the system.
Following the Experimental Setup and Experimental Procedure, sections 4.2 and 4.3 respectfully, for the Tachometer Calibration experiment, the variable power supply (VAR.POWERSUPPLY) program in LABVIEW is selected. This program allows the motor input voltage to be varied from -10.0 volts to +10.0 volts with an increment of 0.01 volts while monitoring the tachometer output voltage. This tachometer calibration curve can be seen in figure 4.5-1. Next, the power supply (POWERSUPPLY) program in LABVIEW is activated. This program allows any specific voltage between -10.0 volts and +9.99 volts to be input to the motor. The voltage is selected in 1 volt intervals and the speed is recorded using the hand held RPM indicator. Assuming $V_{\text{tachometer}} = k_v w$, the tachometer output voltage, $V_{\text{tachometer}}$, versus the speed, $w$, in radians/second is plotted yielding the tachometer constant, $k_v$. Figure 4.5-2 shows the tachometer constant curve.

The D.C. Motor Parameters experiment uses a function generator to generate a one hertz square wave illustrating the system's step response. Looking at the step response, the time constant, $t_\tau$ is equal to the time during which the free response increased by 63.2% of its initial value. Plots for the step response and time constant can be seen in figures 5.5-1 and 5.5-2 respectfully. The D.C. motor is then analyzed in section 5.1 to show the connection between the analytical and experimental system models. These calculations also lead to an expression for the motor constant $k_m$. The motor constant plot can be seen in figure 5.5-4.

The Position Control experiment uses a potentiometer to determine the position of the motor. Assuming $V_{\text{potentiometer}} = k_p w$, the motor
shaft is rotated from 0 to 360 degrees at 10 degree intervals on the
degree wheel and the corresponding voltages are taken on the voltmeter.
The potentiometer voltage versus the motor displacement in radians is
then plotted in figure 6.5-1 yielding the potentiometer constant, k_e.
This constant is used in the position control program.

The system block diagram is then reduced to the closed-loop
transfer function in section 6.1. Using MATLAB on the macintosh
computer, values for the position error gain, k_p, and the velocity
error gain, k_v, that made the system stable at 90° are determined
utilizing the Root-Locus Method. A sample root-locus plot for this
experiment is illustrated in figure 6.5-2. The results of this analysis
are then entered back into the computer and the "no load" position
control observed and recorded in table 6.5-1. Load is then applied in
the form of three single links with different inertias and the best
error gain values are reentered. The results of these tests are
recorded in tables 6.5-2, 6.5-3, and 6.5-4. The final results indicate
system control.

NOTE: These experiments do not emphasize error by statistical
analysis since these topics are covered in Junior and Senior Laboratory
classes, M.E. 398 and M.E. 498.
1.3 CONCLUSIONS

These systems analysis and controls experiments are designed to be used by undergraduates in conjunction with M.E. 401 over a ten week quarter. The theories covered in these experiments effectively complement the courses’ curriculum and will become an invaluable learning tool in the future.

This laboratory has great potential and will not be limited to these few experiments. Other potential experiments being developed at this time include: forced response on circuits, analytical tachometer modeling and analysis, analytical potentiometer modeling and analysis, students programming the controller used in the Single Link Manipulator Position Control System, and Inverted Pendulum Balance Control. These experiments will be broken down into nine or ten individual experiments (five of them coming from the Inverted Pendulum Balance Control Setup) providing enough quantity and diversity to compliment an Advanced Controls course for another ten week quarter.
CHAPTER 2

INITIAL CONDITION
SPRING–MASS SYSTEM

2.1 THEORETICAL DEVELOPMENT

Estimated Theoretical Spring Constant:

\[ k = \frac{Gd^4}{8nD^3} \]  \hspace{1cm} (2.1-1)

Where:
\[ d = \text{wire diameter} \]
\[ D = \text{helix diameter} \]
G = Modulus of Rigidity
\( k = \) spring constant
\( n = \) number of active coils

The theoretical spring constant values are:

\[ d = 2.02 \text{ mm} = 0.07953 \text{ in} \]
\[ D = 20.18 \text{ mm} = 0.79449 \text{ in} \]
\[ G = 11.5 \times 10^6 \text{ lb/in}^2 \]
\[ n = 10 \]

Thus:

\[ k = (11.5 \times 10^6)(0.07953)^4/(8)(10)(0.79449)^3 \]
\[ k = 11.47 \text{ lb/in} \]

SYSTEM ANALYSIS

System Sketch

Free-body diagram

Theoretical Known Values:

Mass = 1.4 lbs
Spring Constant:
\[ k = 10.699 \text{ lb/in} \quad \rightarrow \quad \text{From figure 2.5-3.} \]

Natural Angular Frequency:
\[ \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10.699}{3.6232 \times 10^{-3}}} \]
\[ = 54.34 \text{ rad/sec} \]

Frequency:
\[ f = \frac{\omega_n}{2\pi} = \frac{54.34}{2\pi} \]
\[ = 8.6486 \text{ cyc/sec} \]

Period:
\[ T = \frac{1}{f} = \frac{1}{8.6486} \]
\[ = 0.1156 \text{ sec/cyc} \]

Unknown Values:

Damped Angular Frequency:
\[ \omega_d = \omega_n \sqrt{1 - \xi^2} \]
\[ \omega_d = ? \]

Logarithmic Decrement:
\[ \delta = \ln\left(\frac{x_t}{x_n}\right) \]
\[ \delta = ? \]

Damping Factor:
\[ \delta = 2\pi \xi / \sqrt{1 - \xi^2} \]
\[ \xi = ? \]

Damping Constant:
\[ c = 2\xi \sqrt{km} \]
\[ c = ? \]
Equation of Motion:

\[ m\ddot{x} = -kx - cx \quad (2.1-11) \]
\[ m\ddot{x} + c\dot{x} + kx = 0 \quad (2.1-12) \]
\[ \ddot{x} + (c/m)\dot{x} + (k/m)x = 0 \quad (2.1-13) \]
\[ \ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = 0 \quad (2.1-14) \]

Solving:

\[ x_p = x_h + x_n \quad (2.1-15) \]

where \( x_n \) is the homogeneous solution and \( x_p \) is the particular solution. In this case:

\[ x_n = 0 \quad (2.1-16) \]

Therefore:

\[ x_p = x_n \quad (2.1-17) \]

The Homogeneous solution:

\[ x_h = e^{-\gamma \omega_n t}(A \cos \omega_0 t + B \sin \omega_0 t) \quad (2.1-18) \]

Substituting the initial conditions:

\[ x(0) = 1 (A(1) + B(0)) = x_n \]
\[ A = x_n, \quad \text{The 2nd I.C. yield:} \quad B = \frac{x_n}{\sqrt{1 - \zeta^2}} \quad (2.1-19) \]

Thus the General Solution is:

\[ x_n = x_ne^{-\gamma \omega_n t}(\cos \omega_0 t + \frac{\phi}{\sqrt{1 - \zeta^2}} \sin \omega_0 t) \quad (2.1-20) \]

By utilizing the experimental responses shown in figures 2.5-4, 2.5-5, and 2.5-6, the logarithmic decrement, damping factor, damping constant, and damped angular frequency can be found. Using these values, the theoretical responses can be calculated and plotted.
Test *1

Initial Conditions:

\[ x(0) = 1 \text{ mm} = 0.03937 \text{ in} \]  \hspace{1cm} (2.1-21)
\[ \dot{x}(0) = 0 \]  \hspace{1cm} (2.1-22)

Amplitudes:

The subscripts correspond to the peak being evaluated.

\[ x_1 = 0.03248 \text{ in} \]  \hspace{1cm} (2.1-23)
\[ x_2 = 0.01595 \text{ in} \]  \hspace{1cm} (2.1-24)

Logarithmic Decrement:

Subscript corresponds to test number.

From equation (2.1-8):

\[ \psi_i = \ln\left(\frac{0.03248}{0.01595}\right) \]  \hspace{1cm} (2.1-25)
\[ \psi_i = 0.7112 \]

Damping Factor:

From equation (2.1-9):

\[ 0.7112 = \frac{2 \pi \psi_i}{\sqrt{1 - \psi_i^2}} \]  \hspace{1cm} (2.1-26)
\[ \zeta_i = 0.1125 \]

Damping Constant:

From equation (2.1-10):

\[ c_i = 2(0.1125)(10.699)(3.6232 \times 10^{-3}) \]  \hspace{1cm} (2.1-27)
\[ c_i = 0.0442 \text{ (lb sec)/in} \]

Damped Angular Frequency:

From equation (2.1-7):

\[ w_{d,1} = 54.34 \sqrt{1 - 0.1125^2} \]  \hspace{1cm} (2.1-28)
\[ w_{d,1} = 53.99 \text{ rad/sec} \]

Utilizing equation (2.1-20), the theoretical response for
test *1 is:

\[ x_1(t) = 0.03937e^{-0.113t}(\cos 53.99t + 0.1132\sin 53.99t) \]  
(2.1-29)

This response can be seen in figure 2.5-7.

Test *2

Initial Conditions:

\[ x(0) = 1.5 \text{ mm} = 0.05906 \text{ in} \]  
(2.1-30)

\[ \dot{x}(0) = 0 \]

Amplitudes:

\[ x_1 = 0.04415 \text{ in} \]  
(2.1-31)

\[ x_3 = 0.00817 \text{ in} \]  
(2.1-32)

Logarithmic Decrement:

\[ \delta_2 = \ln(0.04415/0.00817) \]  
(2.1-33)

\[ \delta_2 = 0.8433 \]

Damping Factor:

\[ 0.8433 = \frac{2\pi\xi_2}{\sqrt{1 - \xi_2^2}} \]  
(2.1-34)

\[ \xi_2 = 0.1330 \]

Damping Constant:

\[ c_2 = 2(0.1330)(10.699)(306232 \times 10^{-3}) \]  
(2.1-35)

\[ c_2 = 0.0524 \text{ (lb sec)/in} \]

Damped Angular Frequency:

\[ \omega_d = 54.34 \sqrt{1 - 0.1330^2} \]  
(2.1-36)

\[ \omega_d = 53.86 \text{ rad/sec} \]

The theoretical response for test *2 is:

\[ x_2(t) = 0.05906e^{-0.227t}(\cos 53.86t + 0.1342\sin 53.86t) \]
and can be seen in figure 2.5-8.

Test *3

Initial Conditions:
\[
\begin{align*}
    x(0) &= 2 \text{ mm} = 0.07874 \text{ in} \quad (2.1-38) \\
    \dot{x}(0) &= 0
\end{align*}
\]

Amplitudes:
\[
\begin{align*}
    x_1 &= 0.06263 \text{ in} \quad (2.1-39) \\
    x_4 &= 0.00915 \text{ in} \quad (2.1-40)
\end{align*}
\]

Logarithmic Decrement:
\[
\begin{align*}
    \delta_3 &= \ln(0.06263/0.00915) \quad (2.1-41) \\
    \delta_3 &= 0.6412
\end{align*}
\]

Damping Factor:
\[
0.6412 = \frac{2 \pi \delta}{\sqrt{1 - \delta^2}} \quad (2.1-42)
\]
\[
\xi_3 = 0.1015
\]

Damping Constant:
\[
\begin{align*}
    c_3 &= 2(0.1015)(10.699)(3.6232 \times 10^{-3}) \quad (2.1-43) \\
    c_3 &= 0.0400 \text{ (lb sec)/in}
\end{align*}
\]

Damped Angular Frequency:
\[
\begin{align*}
    \omega_{3d} &= 54.34 \sqrt{1 - 0.1015^2} \quad (2.1-44) \\
    \omega_{3d} &= 54.06 \text{ rad/sec}
\end{align*}
\]

The theoretical response for test *3 is:
\[
    x_3(t) = -0.07874e^{-0.1015t}(\cos 54.06t + 0.102\sin 54.06t) \quad (2.1-45)
\]

and can be seen in figure 2.5-9.
2.2 EXPERIMENTAL SETUP

Figure 2.2-1 Initial Condition Spring-Mass System Diagram
SETUP:

Apparatus:
1. Plug LVDT into the amplifier.
2. Connect the amplifier to channel zero of the computer circuit board.
3. Select a LVDT mount position that will allow a full range of motion for the LVDT through the desired responses. Be sure mount is securely fastened so there is no slippage. Slippage will produce faulty data.
4. Make sure LVDT insert is snug on screw.

Computer:
1. Turn on Computer
2. Select FINDER under the "apple" icon
3. Select FIND FILE under the "apple" icon
4. Enter MTR_POS_CONTROL
5. Double click on MTR_POS_CONTROL
6. Double click on Hard Drive
7. Select LABVIEW 2.2
8. Select ME401.VI's
9. Select ME401_lab
10. Select ME401-Spring-Mass
11. Switch save data to disk lever to no - click on it
12. Set Time Base to 2 (can be adjusted to extend response if necessary)
13. Set Sample Interval to 25
14. Set Number of Samples to 2048

15. Use the "Coax T" to simultaneously plug the amplifier into the oscilloscope. Check the computer response against the oscilloscope response to make sure it is correct. (check the period)

2.3 EXPERIMENTAL PROCEDURE

Calibrations:

1. Linear-Variable Differential Transformer (LVDT) Calibration - Volts to Displacement(in). Use vernier calipers to hold the mass at a constant displacement(in). While holding, activate computer analysis, click on arrow \( \rightarrow \), and wait for response before releasing. Then use the cursors to find the corresponding average value in volts. Record these numbers and repeat until a sufficient number of data points are obtained. Plot Displacement(in) versus Volts(V) and use a regression technique of your choice to determine the calibration constant.

2. Spring Constant Calibration - Zero a vernier caliper at the systems equilibrium position. Then using a standard set of
measuring weights, place a weight on top of the mass and measure the displacement using the caliper. Record the data and repeat until a sufficient number of data points are obtained. **Note:** The spring may not be linear; therefore, small increments should be considered. Plot Load(lbs) versus Displacement(in). If the curve looks linear use a regression technique to find the actual spring constant. If the curve is nonlinear, plot the first linear portion of the curve and use regression to find the spring constant. **Note:** Your initial condition displacements used later in the experiment should not exceed the range of your spring constant.

**Initial Conditions:**

1. Select three initial condition displacements within the linear range of the spring. Zero the vernier caliper at the system's equilibrium position. Next, set the caliper to the first initial condition displacement and snug the set nut so it will not move. Now using the caliper, displace and hold the system in this position until ready to record the response. When ready, click on the arrow -> to start recording data and release the system from the initial condition. **Note:** If your timing is off you might not get a usable curve. Repeat until a sufficient curve is obtained.

2. Once a suitable curve is obtained, use the cursors to obtain a
sufficient number of data points to replicate the graph.

3. Repeat this procedure for the remaining initial conditions.

2.4 STUDENT QUESTIONS

Find:
1. Spring constant, k  (both the actual and theoretical)
2. Mass in slugs, m
3. Natural angular frequency, Wn
4. Damped natural angular frequency, Wd
5. Frequency, f
6. Period, T
7. Damping constant, c
8. Logarithmic decrement, δ
9. Damping factor, ξ

Plots:
1. Computer calibration curve (Displacement(in) vs. Volts(V))
2. Spring constant calibration curve (Load(lbs) vs. Displacement(in))
3. A comparison plot of the Actual and Theoretical responses for each initial condition. (Response(in) vs. Time(sec))
Discussion:

1. Explain the differences between the experimental and theoretical values of the spring constant. Tell which one was selected and why.

2. Discuss the comparison plots.

3. Make suggestions to improve the results of the experiment.

2.5 RESULTS

Following the Experimental Procedure section 2.3, data is taken for the LVDT and spring constant calibrations. The resulting plots and coefficients can be seen in figures 2.5-1, 2.5-2, and 2.5-3. Figure 2.5-3 shows the linear displacement portion of the spring to range from 0.0 to 0.09 inches with a resulting spring constant of 10.699 lb./in.

It is from this graph that the spring constant $k = 10.699$ lb./in. and initial conditions: $x_1(0) = 0.03937$ in., $x_2(0) = 0.05906$ in., and $x_3(0) = 0.07874$ in. are chosen. Using these initial conditions, the remainder of the experiment is carried out producing the experimental responses found in figures 2.5-4, 2.5-5, and 2.5-6. These plots illustrate the presence of viscous damping rather than coulomb damping due to the arc created by the descending free response amplitude.
LVDT CALIBRATION CURVE

\[ y = 0.68025 - 9.7235e^{-2x} \quad R^2 = 0.998 \]

Figure 2.5-1 LVDT Calibration Curve
SPRING CONSTANT CALIBRATION CURVE
(NON-LINEAR SPRING)

Figure 2.5-2 Nonlinear Spring
SPRING CONSTANT CALIBRATION CURVE (LINEAR RANGE)

\[ y = -1.7733 \times 10^{-2} + 10.699x \quad R^2 = 0.994 \]

Figure 2.5-3 Linear Spring Constant
EXPERIMENTAL RESPONSE
\[ X_1(0) = 1.0 \text{ mm} = 0.03937 \text{ in} \]

Figure 2.5-4 Experimental Response \( x_1(0) = 0.03937 \text{ in} \).
EXPERIMENTAL RESPONSE
\[ X_2(0) = 1.5 \text{ mm} = 0.05906 \text{ in} \]

Figure 2.5-5  Experimental Response \( x_2(0) = 0.05906 \text{ in.} \)
EXPERIMENTAL RESPONSE

\[ x_3(0) = 2.0 \text{ mm} = 0.07874 \text{ in.} \]

Figure 2.5-6 Experimental Response \( x_3(0) = 0.07874 \text{ in.} \)
The Theoretical Development section 2.1 systematically goes through the viscous damped free response analytical procedure used to determine the theoretical responses of the system subjected to the different initial conditions. Using the experimental responses in figures 2.5-4, 2.5-5, and 2.5-6, the damping factors for each initial condition were found to be $\xi_1 = 0.1125$, $\xi_2 = 0.1330$, and $\xi_3 = 0.1015$. The theoretical responses are then solved for in equations (2.1-29), (2.1-37), and (2.1-45) and plotted in figures 2.5-7, 2.5-8, and 2.5-9.
THEORETICAL RESPONSE

$X_1(0) = 1.0 \text{mm} = 0.03937 \text{ in}$

Figure 2.5-7 Theoretical Response $x_1(0) = 0.03937$ in.
THEORETICAL RESPONSE
\[ X_2(0) = 1.5 \text{mm} = 0.05906 \text{ in} \]

Figure 2.5-8 Theoretical Response \( x_2(0) = 0.05906 \text{ in} \).
THEORETICAL RESPONSE
\[ x_3(0) = 2.0 \text{mm} = 0.07874 \text{ in} \]

Figure 2.5-9 Theoretical Response \( x_3(0) = 0.07874 \text{ in} \).
Comparison plots located in figures 2.5-10, 2.5-11, and 2.5-12 are created to evaluate the exactness between the experimental and theoretical responses for each initial condition. In each case, the responses are in phase with a slightly smaller theoretical amplitude and longer theoretical time to equilibrium.
COMPARISON PLOT

$X_1(0) = 1.0\text{mm} = 0.03937\text{ in}$

Figure 2.5-10 Comparison Plot $x_1(0) = 0.03937\text{ in.}$
Figure 2.5-11 Comparison Plot $x_2(0) = 1.5\text{mm} = 0.05906\text{ in.}$
COMPARISON PLOT

\[ x_3(0) = 2.0 \text{mm} = 0.07874 \text{ in} \]

Figure 2.5-12 Comparison Plot \( x_3(0) = 0.07874 \text{ in} \).
A reason for the lack of exactness in amplitude could be due to the lack of precision when releasing the system from the initial condition position. These inaccuracies produced a difference in amplitude of approximately 0.01 inches (about 0.25 mm). The releasing action was performed by hand and was thus subjected to a number of possible inaccuracies. By taking away this hand motion and replacing it with a mechanical device to more precisely release the system the problem could be minimized.

The second inaccuracy in the responses was the longer theoretical time to equilibrium. The physical system is subjected to changing friction forces during the response that are not accounted for in the theoretical analysis. The resulting difference between the physical and theoretical amplitudes at the physical responses time to equilibrium is approximately 0.005 inches (less than 0.2 mm) and can be assumed negligible for this case.

In fact, the actual system response is governed by a nonlinear differential equation. (Nonlinearity due to Columb friction) The model used here is linear since I assume linear viscous damping.

The system as it stands is close enough to demonstrate to the students that many variables affect the physical system that are not accounted for in the theoretical analysis and that the theoretical analysis really does approximate the actual response.
3.1 THEORETICAL DEVELOPMENT

EXPERIMENTAL ANALYSIS:

The experimental data is expressed using "Bode Diagrams". These diagrams, pioneered by H. W. Bode on feedback amplifier design, make use of logarithmic charts to portray the magnitude and phase characteristics used in describing the dynamic performance of linear systems. Because extensive work has been done using Bode Diagrams, it is possible to determine the governing equations for the system in question from its frequency response. The Bode Diagrams consist of two plots, magnitude and phase. The magnitude curve is the response ratio of the output amplitude over the input amplitude and is plotted as a function of the log of the frequency. The magnitude response or response ratio in decibels is:
where $A$ is the magnitude of the input signal (channel A) and $B$ is the magnitude of the output signal (channel B). On semi-log paper plot the magnitude response ($R$) in decibels versus the frequency ($f$) in rad/sec or Hz.

The phase angle curve ($\phi$) in degrees is plotted as a function of the log of the frequency and is:

$$\phi = t*f*360$$

where $t$ is the time offset in seconds between the input and output signals and $f$ is the frequency in Hertz. On semi-log paper plot the phase angle ($\phi$) in degrees versus the frequency ($f$) in rad/sec or Hz.

Amplitude and phase angle curves were plotted for each of the frequency response systems and can be seen in figures 3.5-1, 3.5-2, 3.5-3, and 3.5-4.

**THEORETICAL ANALYSIS:**

The procedure for preparing a set of electrical circuit Bode Diagrams is shown below:

1. Determine the system transfer function $T(s)$. This is done using Kirchoff’s Laws (current and voltage) and Laplace Transforms.
2. To obtain the frequency response, replace \( s \) with \( jw \) to get 
\[
T(jw) = T(s)
\]
3. Develop expressions for the magnitude, \( T(w) = T(jw) \), and phase angle \( \Phi_T(w) = \angle T(jw) \) of \( T(jw) \).
4. Amplitude plot in decibels — On semi-log paper plot 
\[
20\log T(w)
\]
versus frequency \( (w \text{ or } f) \). Use the same type of frequency used for the experimental analysis. Phase angle plot — on semi-log paper plot \( \Phi_T \) in degrees versus frequency \( (w \text{ or } f) \).

This procedure was used to determine the Bode Diagrams for each network.

**Box *1**

(Integrating Circuit)

![Integrating Circuit Diagram](image)

\[
R = 10 \text{ ohms} \quad C = 50E-6 \text{ farads}
\]

Find the transfer function of the network:

Kirchoff's Voltage Law (KVL):

\[
iR + \frac{1}{C} \int_0^t i \, dt = V_{in} \tag{3.1-3}
\]

The Laplace Transform is:
(R + 1/C_\text{s})I(s) = V_{\text{in}}(s) \quad (3.1-4)

The output voltage is:
\[ V_{\text{out}}(s) = (1/C_\text{s})I(s) \quad (3.1-5) \]

Thus the transfer function of the network is:
\[ V_{\text{out}}(s)/V_{\text{in}}(s) = ((1/C_\text{s})I(s))/(R + 1/C_\text{s})I(s) = 1/(RC_\text{s} + 1) \quad (3.1-6) \]

Where \( t_\text{s} \) is equal to RC.

The Sinusoidal Transfer Function is:
\[ T(\omega) = \frac{T(s)}{s \rightarrow \omega} = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} \bigg|_{s \rightarrow j\omega} = \frac{1}{j\omega t_\text{s} + 1} \quad (3.1-7) \]

Find expressions for the magnitude and phase angle of the network through a range of frequencies in order to plot the Bode Diagrams.

To simplify obtaining expressions for \( T(\omega) \) and \( \theta(\omega) \), note that \( T(\omega) \) can be presented as a ratio of complex functions \( N(\omega) \) and \( D(\omega) \).

\[ T(\omega) = \frac{N(\omega)}{D(\omega)} = \frac{N(\omega)e^{j\phi_\text{in}(\omega)}}{D(\omega)e^{j\phi_\text{out}(\omega)}} \quad (3.1-8) \]

Simplifying:
\[ T(\omega) = \frac{N(\omega)}{D(\omega)} e^{j(\phi_\text{in}(\omega) - \phi_\text{out}(\omega))} \quad (3.1-9) \]

Thus the magnitude, \( T(\omega) \), is given by:
\[ T(\omega) = \frac{N(\omega)}{D(\omega)}, \quad N(\omega) = 1, \quad D(\omega) = \left| j\omega t_\text{s} + 1 \right| \quad (3.1-10) \]

and
\[ T(\omega) = \frac{1}{\left| j\omega t_\text{s} + 1 \right|} = \frac{1}{\sqrt{1 + \omega^2 t_\text{s}^2}} \quad (3.1-11) \]

The phase angle is given by:
\[
\phi_T(w) = \phi_N(w) - \phi_D(w)
\]  
(3.1-12)

Where:
\[
\phi_N(w) = \tan^{-1}(0/k) = 0
\]  
(3.1-13)
\[
\phi_D(w) = \tan^{-1}(\omega t_a/1) = \tan^{-1}(\omega t_a)
\]  
(3.1-14)

Thus:
\[
\phi_T(w) = 0 - \tan^{-1}(\omega t_a) = -\tan^{-1}(\omega t_a)
\]  
(3.1-15)

The Bode diagrams for the magnitude and phase angle of box *1 have been plotted and can be seen in figure 3.5-5.

Box *2

(Differentiating Circuit)

\[
R = 10 \text{ ohms}
\]
\[
C = 100E-6 \text{ farads}
\]

Find the transfer function of the network:

Kirchoff’s Voltage Law (KVL):
\[
1/C \int_0^t dt + Ri = V_{in}
\]  
(3.1-16)

The Laplace Transform is:
\[
\left(1/C + R\right)I(s) = V_{in}(s)
\]  
(3.1-17)

The output voltage is:
\[
RI(s) = V_{out}(s)
\]  
(3.1-18)

Thus the transfer function of the network is:
\[ V_{out}(s)/V_{in}(s) = R(I_s)/(1/C_s + R)I_s \]  
\[ = R/(1/C_s + R) \]
\[ = RC_s/(RC_s + 1) \]
\[ = st_s/(st_s + 1) \]

Where \( T_s \) is equal to RC.

The Sinusoidal Transfer function is:

\[ T(j\omega) = T_s \mid_{s=j\omega} = V_{out}(s)/V_{in}(s) \mid_{s=j\omega} \]
\[ = j\omega t_s/(j\omega t_s + 1) \]

Find expressions for the magnitude and phase angle of the network through a range of frequencies in order to plot the Bode Diagrams.

Applying equations (3.1-8) and (3.1-9) yield equation (3.1-21) and thus the magnitude, \( T_s(j\omega) \).

\[ T(j\omega) = N(j\omega)/D(j\omega) \]
\[ N(j\omega) = |j\omega t_s|, \quad D(j\omega) = |j\omega t_s + 1| \]

and

\[ T(j\omega) = |j\omega t_s|/|(j\omega t_s + 1)| \]
\[ = \sqrt{\omega^2 t_s^2}/\sqrt{1 + \omega^2 t_s^2} \]

Applying equation (3.1-12) yields equations (3.1-23) and (3.1-24) and thus the phase angle:

\[ \phi_N(j\omega) = \tan^{-1}(\omega t_s/0) = \text{UND.} \]  
\[ \phi_D(j\omega) = \tan^{-1}(\omega t_s/1) \]  

Thus:

\[ \phi(j\omega) = -\tan^{-1}(\omega t_s) \]

The Bode Diagrams for the magnitude and phase angle of box 2 have been plotted and can be seen in figure 3.5-6.
Find the transfer function of the network:

Kirchoff’s Current Law (KCL):
\[ C\frac{d}{dt}(V_{in} - V_{out}) + \left(\frac{1}{R_1}\right)(V_{in} - V_{out}) = \left(\frac{1}{R_2}\right)V_{out} \]
(3.1-26)

The Laplace Transform is:
\[ C(s)(V_{in}(s) - V_{out}(s)) + \left(\frac{1}{R_1}\right)(V_{in}(s) - V_{out}(s)) = \left(\frac{1}{R_2}\right)V_{out}(s) \]

Thus the transfer function of the network is:
\[ \frac{V_{out}(s)}{V_{in}(s)} = \left(\frac{C + \left(\frac{1}{R_1}\right)}{C + \left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right)}\right) \]
\[ = \frac{(s + (1/R_1C))}{(s + (1/R_1C) + (1/R_2C))} \]
\[ = \frac{(s + a)}{(s + b)} \]
(3.1-28)

Where \( a = \frac{1}{R_1C} \) and \( B = \frac{1}{R_1C} + \frac{1}{R_2C} \)

The Sinusoidal Transfer Function is:
Find expressions for the magnitude and phase angle of the network through a range of frequencies in order to plot the Bode Diagrams.

Applying equations (3.1-8) and (3.1-9) yield equation (3.1-30) and thus the magnitude, $T(\omega)$.

$$T(\omega) = \frac{N(\omega)}{D(\omega)}$$

(3.1-30)

$$N(\omega) = |j\omega + a|, \quad D(\omega) = |j\omega + b|$$

and

$$T(\omega) = \frac{|(j\omega + a)|}{|(j\omega + b)|}$$

(3.1-31)

$$= \frac{\sqrt{\omega^2 + a^2}}{\sqrt{\omega^2 + b^2}}$$

Applying equation (3.1-21) yields equations (3.1-32) and (3.1-33) and thus the phase angle:

$$\Phi_N(\omega) = \tan^{-1}(\frac{\omega}{a})$$

(3.1-32)

$$\Phi_D(\omega) = \tan^{-1}(\frac{\omega}{b})$$

(3.1-33)

Thus:

$$\Phi(\omega) = \tan^{-1}(\frac{\omega}{a}) - \tan^{-1}(\frac{\omega}{b})$$

(3.1-34)

The Bode Diagrams for the magnitude and phase angle of box 3 have been plotted and can be seen in figure 3.5-7.
Box 4

(Lead-Lag Filter Circuit)

Vin

R1

C1

R2

C2

Vout

R1 = 10 ohms
R2 = 10 ohms
C1 = 47E-6 farads
C2 = 100E-6 farads

Find the transfer function of the network:

Kirchoff’s Current Law (KCL):

\[ C_1 \frac{d}{dt}(V_{in} - V_{out}) + \frac{1}{R_1}(V_{in} - V_{out}) = i \quad (3.1-35) \]

The Laplace Transform is:

\[ C_1(s)(V_{in}(s) - V_{out}(s)) + \frac{1}{R_1}(V_{in}(s) - V_{out}(s)) = \mathcal{L}\{i(t)\} \]

\[ = I(s) \quad (3.1-36) \]

The output voltage and the current are related by:

\[ R_2i + \frac{1}{C_2s}\int_0^t i \, dt = V_{out} \quad (3.1-37) \]

Taking the Laplace Transform and simplifying:

\[ I(s) = \frac{V_{out}(s)}{R_2 + (1/C_2s)} \quad (3.1-38) \]

Combining equations (3.1-36) and (3.1-38):

\[ (C_1s + \frac{1}{R_1})(V_{in}(s) - V_{out}(s)) = \frac{V_{out}(s)}{(\frac{1}{C_2s} + R_2)} \quad (3.1-39) \]
Simplifying:

\[
((1/R_1) + C_1)(V_{in}(s) - V_{out}(s)) = V_{out}(s)C_2s/(1 + R_2C_2s) \quad (3.1-40)
\]

\[
(1 + R_2C_2s)((1/R_1) + C_1)(V_{in}(s) - V_{out}(s)) = V_{out}(s)C_2s \quad (3.1-41)
\]

\[
(1 + R_2C_2s)(1 + R_1C_1)(V_{in}(s) - V_{out}(s)) = V_{out}(s)R_1C_2s \quad (3.1-42)
\]

Thus the transfer function of the network is:

\[
V_{out}(s)/V_{in}(s) = (1 + R_1C_1s)(1 + R_2C_2s)
\]

\[
/((R_1C_2s + (1 + R_1C_1s)(1 + R_2C_2s)) = (1 + R_1C_1s)(1 + R_2C_2s)
\]

\[
/(R_1C_1R_2C_2s^2 + (R_1C_1 + R_1C_2 + R_2C_2)s + 1) = (1 + st_*)(1 + st_*)
\]

\[
/(1 + st_*)(1 + st_2) \quad (3.1-45)
\]

Where \( t_* = R_1C_1 \), \( t_0 = R_2C_2 \), \( t_{ab} = R_1C_2 \), \( t_{12} = t_*t_0 \), and

\( t_1 + t_2 = t_* + t_0 + t_{ab} \)

The Sinusoidal Transfer Function is:

\[
T(j\omega) = T(s) \bigg|_{s=j\omega} = (1 + t_*j\omega)(1 + t_0j\omega)
\]

\[
/(1 + t_1j\omega)(1 + t_2j\omega) \quad (3.1-46)
\]

Find expressions for the magnitude and phase angle of the network through a range of frequencies in order to plot the Bode Diagrams.

Applying equations (3.1-8) and (3.1-9) yield equation (3.1-47) and thus the magnitude, \( T(j\omega) \).

\[
T(j\omega) = n(j\omega)/D(j\omega) \quad (3.1-47)
\]

\[
N(j\omega) = |1 + t_*j\omega| \ast |1 + t_0j\omega|
\]

\[
D(j\omega) = |1 + t_1j\omega| \ast |1 + t_2j\omega|
\]
Thus:

\[
T(\omega) = \sqrt{1 + t_w^2 \omega^2} \cdot \sqrt{1 + t_\ell^2 \omega^2} / \sqrt{1 + t_1^2 \omega^2} \cdot \sqrt{1 + t_\omega^2 \omega^2} \quad (3.1-48)
\]

Applying equation (3.1-21) yields equations (3.1-49) and (3.1-50) and thus the phase angle:

\[
\hat{T}(\omega) = \tan^{-1}\left(\frac{t_w}{l}\right) = \tan^{-1}\left(\frac{t_\ell}{t_w}\right) \quad (3.1-49)
\]

\[
\hat{T}(\omega) = \tan^{-1}\left(\frac{t_1}{l}\right) = \tan^{-1}\left(\frac{t_\omega}{t_1}\right) \quad (3.1-50)
\]

Thus:

\[
\hat{T}(\omega) = \tan^{-1}\left(\frac{t_w}{l}\right) - \tan^{-1}\left(\frac{t_\omega}{t_1}\right) \quad (3.1-51)
\]

The Bode Diagrams for the magnitude and phase angle of box *4 have been plotted and can be seen in figure 3.5-8.

Using MATLAB on the Macintosh, the Bode Diagrams can be plotted in a fraction of the time required using the above method. To utilize this software, the coefficients of the ordered system must be known. Utilizing the transfer function for box *1 and equation (3.1-6).

\[
T(s) = \frac{1}{(t_a S + 1)}, \quad t_a = RC = 0.0005
\]

thus equation (3.1-6) becomes:

\[
T(s) = \frac{1}{(0.0005S + 1)} \quad (3.1-52)
\]

Select MATLAB on the computer until the command window appears. Now type in the coefficients using this format.
num=[a2 a1 a0];
den=[b2 b1 b0];
>Bode(num,den)

Equation (3.1-52) would look like

num=[0 1];
den=[0.0005 1];
bode(num,den)

After the last return, the computer analyzes the coefficients and plots the Bode Diagrams. At this point, the graphs can be printed or screen copied into a word processor and saved to be printed later. The Bode Diagrams for the theoretical analysis have been plotted in figures 3.5-5, 3.5-6, 3.5-7, and 3.5-8.
3.2 EXPERIMENTAL SETUP

Figure 3.2-1 Frequency Response System Diagram
NETWORK SCHEMATICS:

Box 1 (Integrating Circuit)

\[ \frac{V_{out}}{V_{in}} = \frac{1}{R C (R C + 1)} \]

\[ R = 10 \text{ Ohms} \]
\[ C = 5 \times 10^{-6} \text{ farads} \]

![Integrating Circuit Diagram]

Figure 3.2-2 Box 1 (Integrating Circuit)

Box 2 (Differentiating Circuit)

\[ \frac{V_{out}}{V_{in}} = \frac{R C S}{R C S + 1} \]

\[ R = 10 \text{ Ohms} \]
\[ C = 1 \times 10^{-6} \text{ farads} \]

![Differentiating Circuit Diagram]

Figure 3.2-3 Box 2 (Differentiating Circuit)
Box 3 (Differentiating Circuit)

\[ V_{out}/V_{in} = \frac{S+1/R_1 C}{S+(R_1+R_2)/R_1 R_2 C} \]

\[ R_1 = 10 \text{ Ohms} \]
\[ R_2 = 4.7 \text{ Ohms} \]
\[ C = 47E-5 \text{ farads} \]

Figure 3.2-4 Box 3 (Differentiating Circuit)

Box 4 (Lead-lag Filter Circuit)

\[ V_{out}/V_{in} = \frac{1}{(1+S t_a)(1+S t_b)} \]
\[ = \frac{1}{(1+S t_a)(1+S t_b)} \]
\[ = \frac{1}{(1+S t_a)(1+S t_b)} \]

\[ R_1 = 10 \text{ Ohms}, R_2 = 10 \text{ Ohms} \]
\[ C_1 = 47E-6 \text{ f}, C_2 = 100E-6 \text{ f} \]
\[ t_a = R_1 C_1, \ t_b = R_2 C_2 \]
\[ t_{ab} = R_1 C_1, \ t_1 t_2 = t_{ab} \]
\[ t_1 + t_2 = t_a + t_b + t_{ab} \]

Figure 3.2-5 Box 4 (Lead-Lag Filter Circuit)
SETUP:

**Function Generator:**
1. Turn on generator.
2. Select the sine waveform.
3. Plug a black wire into the ground (GND) slot.
4. Plug a red wire into the HI slot.

**Amplifier:**
1. Turn on amplifier.
2. Select the DC gain.
3. Plug the black wire from the generator into the input ground slot of the amplifier.
4. Plug the red wire from the generator into the yellow input slot of the amplifier.
5. Set the gain dial to 1.
6. Plug a red wire into the yellow output slot.
7. Plug a black wire into the output ground slot.

**Oscilloscope:**
1. Turn on oscilloscope.
2. Set channels A and B to DC.
3. Plug cables into channels A and B.
4. Press any one of the buttons below the screen to get the display.
5. Press the cursors button.
6. Press the mode button.
7. Press the V-Curs ON button until ON is highlighted.

8. Press the V button until V is highlighted.

9. Press the T-Curs ON button until ON is highlighted.

10. Press the T/PH/RATIO until T is highlighted.

11. Press the return button.

**Apparatus:**

1. Make sure the + and − banana clips are screwed on tight to ensure good connection.

2. Plug the black wire from the amplifier into the black lead on the IN side of Box 1.

3. Plug the red wire from the amplifier into the red lead on the IN side of Box 1.

4. Clip the wire from channel A of the oscilloscope onto the leads on the IN side of Box 1. Be sure to clip the positive to the red lead and the negative to the black lead. Channel A will carry the input signal.

5. Clip the wire from channel B of the oscilloscope onto the leads on the OUT side of Box 1. Be sure to clip the positive to the red lead and the negative to the black lead. Channel B will carry the output or response signal.
3.3 EXPERIMENTAL PROCEDURE

PROCEDURE:

Calibrating Signals:

1. Switch the channel B leads from the OUT side of Box 1 to the IN side of Box 1 and clip them beside the channel A leads.

2. Select a frequency (100 Hz).

3. View the oscilloscope. Select a gain on the amplifier or a voltage/division on the oscilloscope to give the signal a large amplitude. A large amplitude will be easier to read and provide better accuracy at high frequencies than a small amplitude.

4. Use the seconds/division control on the oscilloscope to obtain only one cycle of the signal on the screen. This will allow for better measurement of the input frequency throughout the experiment.

5. Signals A and B should overlap each other exactly. It should appear as if only one signal is present. To do this, Set the A and B grounds to the same position. Next, release channel A and leave channel B grounded. Use Channel A to select the exact desired input amplitude. The A VAR. knob on the oscilloscope should be used to achieve this. When the desired amplitude is achieved, release channel B and match the signal on channel A using the B VAR. knob. Make sure the signals are identical before moving to the next step.

6. Switch the channel B leads back to the OUT side of Box 1. At
100 Hz., the forced response signal on channel B should be slightly smaller than the input signal on channel A.

**Experiment:**
1. Select 1 Hz. on the Function Generator.
2. Select seconds/division to allow at least one full cycle.
4. Move the cursors to measure the period of the signal on channel A. It might help to set channel B to ground (to avoid confusion) while the frequency is being set. Pressing the "lock" button on the oscilloscope will freeze the signal so the frequency can be measured.
5. Adjust the frequency on the function generator until the oscilloscope reads 1 Hz.
6. Release channel B and use the cursors to find the phase shift.
7. Record this data in a table labeled Box 1.
9. Measure the input signal, channel A. The signal should be measured from the highest point to the lowest point. (ie. the whole signal)
10. Measure the forced response signal, channel B. Measure the whole signal as was done with channel A. Record this data in the table labeled Box 1.
11. Switch all leads from Box 1 to Box 2. Make sure all leads are properly connected.
12. Proceed to take readings for the phase shift, input signal,
and forced response signal.

13. Record this data in a table labeled Box 2.

14. Repeat steps 11 through 13 for Boxes 3 and 4 and record the data appropriately.

15. Repeat steps 1 through 15 varying the frequency from 1 to 100,000 in a logarithmic manner. (i.e., 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, ..., 80,000, 90,000, 100,000)

3.4 STUDENT QUESTIONS

Find:

1. Using the analytical method discussed, find the transfer function \( T(s) \) for each of the forced systems.

2. Determine the coefficients to be used in the MATLAB software program.

3. Develop the expressions for the magnitude \( T(w) \), and the phase angle \( \phi_r(w) \).

Plots:

1. Experimental plots -- Amplitude plot -- On semi-log paper plot equation (3.1-1) versus the frequency. Phase angle plot -- On semi-log paper plot equation (3.1-2) versus the frequency.

   Make plots for each of the forced systems.

2. Theoretical plots -- Amplitude plot -- On semi-log paper plot
20 \log T(w) \text{ versus the frequency. Phase angle plot -- on semi-log paper plot } \phi_r(w) \text{ versus the frequency.}

3. If MATLAB is used, *2. can be ignored.

Discussion:

1. Compare the experimental plots to the theoretical plots.
2. Estimate the break frequency for each system.
3. What is the slope of each Amplitude plot in decibels/decade or decibels/octave.
4. By observing the plot characteristics of the Bode diagrams for each of the systems, determine the type or behavior of the system. (For example: simple resistor circuit, low pass filter, etc.)
5. Make suggestions to improve the results of the experiment.

3.5 RESULTS

Following the experimental analysis procedure outlined in the theoretical development section 3.1, the magnitude and phase angle Bode Diagrams are constructed for each network using equations (3.1-1) and (3.1-2) over a frequency ranging from 1 to 100 KHz. These diagrams are plotted in figures 3.5-1, 3.5-2, 3.5-3, and 3.5-4.
Figure 3.5-1 Experimental Magnitude and Phase Angle Bode Diagrams, Box 1
Figure 3.5-2 Experimental Magnitude and Phase Angle Bode Diagrams, Box 2
Figure 3.5-3 Experimental Magnitude and Phase Angle Bode Diagrams, Box 3
Figure 3.5-4 Experimental Magnitude and Phase Angle Bode Diagrams, Box 4
The theoretical analysis procedure in the Theoretical Development section calculates the transfer function and expressions for the magnitude and phase angle for each network. These final expressions could have been used to determine the theoretical Bode Diagrams but in this case are not. This process has been greatly simplified with the use of MATLAB on the Macintosh computer. MATLAB has a program within it capable of plotting Bode Diagrams when the transfer function coefficients are known. This is a great time saver and helps to eliminate calculating mistakes. The transfer functions for each network; equations: (3.1-6), (3.1-19), (3.1-28), and (3.1-45) have been plotted and can be seen in figures 3.5-5, 3.5-6, 3.5-7, and 3.5-8.
Figure 3.5-5  Theoretical Magnitude and Phase Angle Bode Diagrams, Box 1
Figure 3.5-6 Theoretical Magnitude and Phase Angle Bode Diagrams, Box 2
Figure 3.5-7 Theoretical Magnitude and Phase Angle Bode Diagrams, Box 3
Figure 3.5-8 Theoretical Magnitude and Phase Angle Bode Diagrams, Box 4
Comparison plots could not be constructed due to the software involved; however, viewing the experimental and theoretical responses for each individual network at the same time under close observation show great similarity between the two. At high frequencies, the experimental responses tend to be a little shaky possibly due to lack of resolution on the oscilloscope and the 10% tolerance of the circuit components. Overall the results look very good and demonstrate the accuracy of the theoretical analysis of the components on which the students are working.
4.1 THEORETICAL DEVELOPMENT

The objective of this experiment is not to theoretically analyze the tachometer, but to determine the tachometer parameters experimentally. The calibration curve and constant are then used to help manipulate the data to meet the system's requirements in the subsequent experiments.

For this experiment, it is assumed that the tachometer output voltage is equal to the tachometer constant multiplied by the angular velocity of the motor ($V_{\text{tachometer}} = k\omega$).

Other potential experiments to be developed on this position control system will analyze the theory behind the tachometer in greater depth.
4.2 EXPERIMENTAL SETUP

Figure 4.2-1 Tachometer Calibration System Diagram
SETUP:

Apparatus:

1. Plug the *1 red and gray wires with the red and black banana plugs into the amplifier's yellow output slot and black ground slot respectively. This wire supplies the input voltage to the motor. (On the connection plate, the odd wires are on top and the even wires are on bottom.)

2. Plug the *2 coax wire with the coax lead into channel A of the oscilloscope. This wire will allow you to track the input motor voltage on the scope.

3. Plug the *3 coax wire with the coax lead into channel zero (CHO) on the computer circuit board. This wire carries the tachometer output voltage to the computer.

4. Plug the *4 coax wire with the coax lead into channel B of the oscilloscope. This wire will allow you to track the tachometer output voltage on the scope.

5. Plug the *8 free coax wire with the red and black banana leads into channel DACO OUT on the computer circuit board and into the yellow input slot and the black ground slot on the amplifier. This wire supplies the input voltage to the motor.

6. Let the *5, *6 & *7 wires hang. They will be used in a later experiment.

7. Unscrew the potentiometer's lead screws A, B and C.

8. Disconnect the potentiometer's voltage in (red wire), voltage out (white wire), and ground (green wire). Screw the lead screws back in so they will not be lost.
9. Loosen the two set screws on the sides of the potentiometer’s mounting bracket and the set screw on the motor side of the flexible coupling.

10. Carefully remove the potentiometer with mounting bracket.

**Amplifier:**

1. Set the AC/DC switch to DC.
2. Set the Cal knob on the right Cal dot.
3. Set the gain to 1.
4. Turn the offset knob to the left until the green light just goes off.

**Computer:**

1. Turn on computer.
2. Select FINDER under the "apple" icon.
3. Select FIND FILE under the "apple" icon.
4. Enter MTR_POS_CONTROL
5. Double click on MTR_POS_CONTROL
6. Double click on the Hard Drive
7. Select LABVIEW 2.2
8. Select ME401.VI’s
9. Select ME401_lab
10. Select VAR.POWERSUPPLY.
11. Switch save data to disk lever to no. —Click on it.
12. Set Time Base to 2.
13. Set Sample Interval to 25.
14. Set Number of Samples to 2048.
15. Set the Upper Voltage Limit to 10.0.
16. Set the Lower Voltage Limit to -10.0.
17. Set the voltage increment to 0.01.

Oscilloscope:
1. Center channel A and channel B grounds.
2. Set A and B to 2 volts/division.
3. Set the time to 5 seconds/division.

4.3 EXPERIMENTAL PROCEDURE

Tachometer Calibration:
1. Click on the -> at the upper left hand corner of the Labview screen on the computer. This will activate the single link manipulator and plot the Tachometer Output Voltage versus Motor Input Voltage calibration curve. If the time on the oscilloscope is switched to 5 seconds/division at the same time the motor is activated, the complete response will be traced across the scope's screen.
2. The cursors, on the computer, can then be used to obtain the data needed to recreate the curve.
Tachometer Constant:

1. Close the file.
2. Open POWERSUPPLY.
3. Use the hand held rpm indicator with the rubber cone shaped adaptor.
4. Attach the coupling to the end of the transducer shaft.
5. Select a voltage ranging from 9.99 to -10.0 volts.
6. Once the desired voltage is selected, click on the enter button. Then activate motor by clicking on the continuous, revolving ->, button.
7. Place the rubber cone shaped end of the rpm indicator against the coupling with some pressure and hold down the button on the left side of the indicator. The instantaneous rpm will appear on the screen.
8. Record both the voltage and the rpm.
9. Change the voltage and click on the enter button. The change in the motor speed should be evident. Take readings ranging from 9.99 to -10.0 volts so that an acceptable curve can be plotted.
4.4 STUDENT QUESTIONS

Plots:

Tachometer Calibration Curve:

Plot Tachometer Output Voltage versus Motor Input Voltage.

Be sure to include equations for each segment of the curve. These equations will be utilized in the following experiments.

Tachometer Constant Curve:

Assuming that $V_{\text{tachometer}} = k \cdot W$.

Plot Tachometer Output Voltage versus Speed (rad/sec) of the motor. The motor speed is ten times greater than the transducer speed measured due to the transducer’s 10:1 ratio. Calculate the best fit curve. The slope of this line will be the tachometer constant $k_t$. This value will also be used in the following experiments.

4.5 RESULTS

The tachometer calibration and constant curves were constructed following the Experimental Procedure section 4.3. A simple curve fit was applied to each grouping of data to determine the calibration curves.
and constant. The tachometer calibration curves were: \( y = 1.1167x + 1.5384 \) \( \forall -10.0 < \text{motor input voltage} < -1.52 \), \( y = 0 \) \( \forall -1.52 < \text{motor input voltage} < 1.65 \), and \( y = 1.1641x - 1.6066 \) \( \forall 1.65 < \text{motor input voltage} < 10.0 \); and the tachometer constant \( k_t = 0.066144 \). These plots can be seen in figures 4.5-1 and 4.5-2 respectfully.
TACHOMETER CALIBRATION CURVE

Figure 4.5-1 Tachometer Calibration Curve
TACHOMETER CONSTANT
$K_t = 0.066139$

Figure 4.5-2 Tachometer Constant
5.1 THEORETICAL DEVELOPMENT

D.C. Motor:
D.C. Motor analysis to determine the analytical model and the motor constant, $k_m$.

**D.C. Motor Schematic**

```
\[ L \frac{di}{dt} + Ri = V_{in} - e \]  \hspace{1cm} (5.1-1)
```

Where:
Substituting equation (5.1-2) into (5.1-1) gives:

\[ L \frac{di}{dt} + Ri + k_m A = V_i \quad (5.1-3) \]

The torque is:

\[ T = k_t i \quad (5.1-4) \]

Where \( k_t \) is the torque constant.

The Laplace Transform of equation (5.1-3) is:

\[ (Ls + R)I(s) + k_m W(s) = V_i(s) \quad (5.1-5) \]

Where:

\[ I(s) = \mathcal{L}(i), \quad W(s) = \mathcal{L}(A), \quad V_i = \mathcal{L}(V_i) \quad (5.1-6) \]

Solving for current, \( I \):

\[ I(s) = \frac{(V_i(s) - k_m W(s))}{(Ls + R)} \quad (5.1-7) \]

The Laplace Transform of equation (5.1-4) is:

\[ T(s) = k_t I(s) \quad (5.1-8) \]

Where:

\[ T(s) = \mathcal{L}(T) \quad (5.1-9) \]

Substituting equation (5.1-7) into (5.1-8) gives:

\[ T(s) = \frac{(k_t V_i(s) - k_m W(s))}{(Ls + R)} \quad (5.1-10) \]

Dividing the top and bottom of equation (5.1-10) by \( R \) gives:

\[ T(s) = \frac{((k_t/R)V_i(s) - (k_t k_m/R)W(s))}{((L/R)s + 1)} \quad (5.1-11) \]

For most D.C. Motors \( L/R \ll 1 \); therefore:

\[ T(s) = \frac{(k_t V_i(s) - (k_t k_m/R)W(s))}{(L/R)s + 1} \quad (5.1-12) \]

and \( T = \text{Inertia}(\frac{d}{dt}) \)

\[ T(s) = (-\text{Inertia}sW(s)) \quad (5.1-13) \]

Substituting equation (5.1-13) into (5.1-12) and separating
the variables:

\[(\text{Inertia})\dot{\theta}(s) + (k_{\text{tor}}k_{\text{emf}}/R)\dot{\theta}(s) = (k_{\text{tor}}/R)V_{in}(s)\]

\[\text{(5.1-14)}\]

Taking the inverse Laplace Transform of equation (5.1-14):

\[\text{Inertia}(d\theta/dt) + (k_{\text{tor}}k_{\text{emf}}/R)n = (k_{\text{tor}}/R)V_{in}\]

\[\text{(5.1-15)}\]

Dividing through by \text{Inertia}:

\[d\theta/dt + (k_{\text{tor}}k_{\text{emf}}/R\text{Inertia})n = (k_{\text{tor}}/R\text{Inertia})V_{in}\]

\[\text{(5.1-16)}\]

Equation (5.1-16) is the analytical model for this D.C. Motor. The experimental model used in this experiment will be:

\[t_{c}(d\theta/dt) + kmV_{in}\]

\[\text{(5.1-17)}\]

Where \(t_{c}\) is the time constant.

Dividing equation (5.1-17) through by \(t_{c}\) gives:

\[d\theta/dt + (1/t_{c}) = (km/t_{c})V_{in}\]

\[\text{(5.1-18)}\]

Notice the similarity between equations (5.1-16) and (5.1-18).

Say:

\[1/t_{c} = k_{\text{tor}}k_{\text{emf}}/R\text{Inertia}, \quad km/t_{c} = k_{\text{tor}}/R\text{Inertia}\]

\[\text{(5.1-19)}\]

Solve the differential equation (5.1-18) at \(\Delta(0) = 0\) to find an expression for the motor constant, \(km\).

Homogeneous solution:

\[\Delta_h(t) = A_1 e^{-t/t_{c}}\]

\[\text{(5.1-20)}\]

Particular solution:

\[\Delta_p(t) \Rightarrow c/t_{c} = (km/k_{\text{tor}}t_{c})V_{in}\]

\[\text{(5.1-21)}\]
Thus:
\[ c = \left(\frac{k_m}{k_t}\right)V_i \quad \text{(5.1-22)} \]
The total solution is:
\[ \mathcal{N}(t) = A_1 e^{-t'/t_c} + \left(\frac{k_m}{k_t}\right)V_i \quad \text{(5.1-23)} \]
Substituting in the initial conditions:
\[ \mathcal{N}(0) = A_1 + (k_m/k_t)V_i \quad \text{(5.1-24)} \]
\[ A_1 = -(k_m/k_t)V_i \quad \text{(5.1-24)} \]
Thus the solution is:
\[ \mathcal{N}(t) = \left(\frac{k_m}{k_t}\right)V_i \left(1 - e^{-t'/t_c}\right) \quad \text{(5.1-25)} \]
As \( t \) approaches infinity:
\[ \mathcal{N}_{\pi} = \frac{k_m}{k_t}V_i \quad \text{(5.1-26)} \]
Solving for the motor constant, \( k_m \):
\[ K_m = k_t \mathcal{N}_{\pi}/V_i \quad \text{(5.1-27)} \]
A motor constant graph has been plotted and can be seen in figure 5.5-4.
5.2 EXPERIMENTAL SETUP

Figure 5.2-1 D.C. Motor Parameters System Diagram
**SETUP:**

**Apparatus:**

1. Plug the "1 red and gray wires with the red and black banana plugs into the amplifier’s yellow output slot and black ground slot respectively. This wire supplies the input voltage to the motor. (On the connection plate, the odd wires are on top and the even wires are on bottom.)

2. Plug the "2 coax wire with the coax lead into channel A of the oscilloscope. This wire will allow you to track the input motor voltage on the scope.

3. Plug the "3 coax wire with the coax lead into channel zero (CHO) on the computer circuit board. This wire carries the tachometer output voltage to the computer.

4. Plug the "4 coax wire with coax lead into channel B of the oscilloscope. This wire will allow you to track the tachometer output voltage on the scope.

5. Plug the "9 free red and gray wires with the red and black banana plugs into the red HI slot and black ground slot of the function generator respectively. Plug the other ends into the amplifier’s yellow input slot and black ground slot. This wire supplies the step function to the motor.

6. Let the "5, 6 & 7 wires hang. They will be used in the next experiment.

7. Unscrew the potentiometer’s lead screws A, B and C.

8. Disconnect the potentiometer’s voltage in (red wire), voltage out (white wire), and ground (green wire). Screw the lead
screws back in so they won’t be lost.
9. Loosen the two set screws on the sides of the potentiometer’s mounting bracket and the set screw on the motor side of the flexible coupling.
10. Carefully remove the potentiometer with mounting bracket.

Function Generator:
1. Turn on function generator.
2. Set to generate a 1 Hz. square wave.
3. The exact frequency can determined and achieved using the cursors on the oscilloscope to measure the signal.

Amplifier:
1. Set the AC/DC switch to DC.
2. Set the Cal knob on the right Cal dot.
3. Set the gain to 1.
4. Turn the offset knob to the left until the green light just goes off.

Computer:
1. Turn on computer.
2. Select FINDER under the "apple" icon.
3. Select FIND FILE under the "apple" icon.
4. Enter MTR_POS_CONTROL.
5. Double click on MTR_POS_CONTROL.
6. Select LABVIEW 2.2
7. Select ME401.VI’s
8. Select ME401_lab
9. Select STEPRESPONSE.
10. Switch save data to disk lever to no. —Click on it.
11. Set Time Base to 2.
12. Set Sample Interval to 25.
13. Set Number of Samples to 2048.

Oscilloscope:
1. Center channel A and B grounds.
2. Set Channel A and B to 1 volt/division.
3. Set time to .2 seconds/division.

5.3 EXPERIMENTAL PROCEDURE

Step Response:
1. Click on the revolving -> button at the top left corner of the computer screen. This will activate LabView’s data acquisition of the tachometer output voltage step response.
2. Position the mouse on the stop button. (looks like a stop sign, upper left hand corner)
3. Click on the stop button when a full step (1/2 period)
response is showing on the screen. This graph can be seen in figure 5.5-1.

4. Use the cursors to obtain the data needed to recreate the curve. This is Tachometer Output Voltage versus Time (msec).

**Time Constant:**

**First Method:**

1. On the recreated Tachometer Output Voltage versus Time (msec) graph, draw a horizontal line from the y-axis to the step response at 63.2% of the step. Now draw a vertical line from where the horizontal line intersects the step response to the x-axis. The time constant \( t_c \), is the change in time from the input of the step voltage to the drawn vertical line.

**Second Method:**

1. Plot \( y(t) \) versus Time (msec) for a time range between the input of the step voltage and steady state. Where \( y(t) \) is

\[
y(t) = -\ln \left( \frac{V_{tach} - V_{tach}(t)}{V_{tach} - V_{tach}(0)} \right)
\]

(5.3-1)

Where \( V_{tach} \) is the steady state value of the tachometer output voltage after the input of the step voltage to the motor and \( V_{tach}(0) \) is the steady state value before the input of the step voltage.
2. Calculate the best fit simple curve through these data points.

The slope of the line is another estimate of the time constant, \( t_c \).

**Motor Constant:**

The experimental system will be modeled using the following differential equation.

\[
t_c \ k_t \ (d\omega/dt) + k_m \omega = k_m \ V_{in}
\]  

(5.3-2)

Where \( t_c \) in the time constant, \( k_t \) is the tachometer constant, \( k_m \) is the motor constant, \( V_{in} \) is the motor input voltage, and \( \omega \) is the angular speed in rad/sec.

Solve this differential equation for initial condition \( \omega(0) = 0 \). Analyze the final solution as \( t \) goes to infinity and solve for the motor constant \( k_m \).

A graph can then be plotted to obtain a value for \( k_m \).
5.4 STUDENT QUESTIONS

PLOTS:

Step Response:
Plot Tachometer Output Voltage versus Time (msec).

Step Response Time Constant Estimation:
Plot tachometer Output Voltage versus Time (msec). On this graph include the time constant estimation at 63.2%.

Time Constant Estimation:
Plot y(t) versus Time (msec) utilizing equation (5.3-1). Include the equation of the line.

Motor Constant:
After analytically determining the equation for the motor constant, plot the appropriate graph including the equation of the line.

5.5 RESULTS

Following the Experimental Setup and Procedure sections 5.2 and 5.3 respectfully, the function generator is set to generate a 1 Hertz square wave producing a step response in the position control system. This step response shows the ramp up time and thus the time constant of the system. The time constant is equal to the time during which the
free response increases by 63.2% of its initial value. In this estimation, the time constant was approximately 27.5 msec. Figures 5.5-1 and 5.5-2 show the step response and step response time constant estimation plots.

A second time constant estimation is conducted using the method described in the Experimental Procedure section 5.3 and equation 5.3-1. A plot of this estimation is located in figure 5.5-3 and yields the time constant to be 25.5 msec.

Since the results of both estimations were close and neither one is much more exact than the other, the average of the two is used to perform necessary calculations. Time constant $t_c = 26.5$ msec.
STEP RESPONSE

Figure 5.5-1  Step Response Curve
Figure 5.5-2  Step Response Time Curve Constant Estimation
TIME CONSTANT ESTIMATION

\[ y = -0.41017 + 2.5566x \cdot 2^x \quad R^2 = 0.914 \]

Figure 5.5-3 Time Constant Estimation
The Theoretical Development section 5.1 performs an analysis on the D.C. motor used in this system. The derived theoretical model for the motor is equation (5.1-16). However, the experimental model used in this experiment is equation (5.1-18). A similarity between these two equations can be seen and thus they are related by equations (5.1-19). After solving the experimental model differential equation (5.1-18) subjected to the initial conditions of the system, the motor constant, $k_m$ is solved for as time approaches infinity. Thus, the motor constant is equal to the tachometer constant multiplied by the steady state speed divided by the motor input voltage ($k_m = k_t \omega_s / V_{in}$). Figure 5.5-4 illustrates this constant.
MOTOR CONSTANT

\[ K_m = 1.0854 \]

\[
y = -1.3275 + 1.0854x \quad R^2 = 0.999
\]

Figure 5.5-4  Motor Constant Curve
6.1 THEORETICAL DEVELOPMENT

In this experiment the remaining system parameters: potentiometer constant, the closed loop system transfer function, stable root-locus values, and values for the position and velocity error gains are determined. Once determined the system was successfully controlled.
Position Control System:

Figure 6.1-1 Position Control System Schematic

Figure 6.1-2 Position Control System Block Diagram
Determine the closed-loop transfer function for the position control system.

Reducing the inner loop:

\[
\begin{align*}
\frac{K_1 K_2 K_m'}{ts + K_1 K_2 K_m' K_{vel} + 1} + \frac{1}{s}.
\end{align*}
\]

Reducing the outer loop:

\[
\begin{align*}
\frac{K_1 K_2 K_m' K_{pos}}{ts + (K_1 K_2 K_m' K_{vel} + 1)s + K_1 K_2 K_m' K_{pos}}.
\end{align*}
\]

Figure 6.1-3  Position Control System Closed-Loop Transfer Function

Thus:

\[
\frac{\Theta_{out}(s)}{\Theta_{in}(s)} = \frac{k_1 k_2 k' k_{pos}}{ts^2 + (k_1 k_2 k' k_{vel} + 1) + k_1 k_2 k' k_{pos}} \quad (6.1-1)
\]
6.2 EXPERIMENTAL SETUP

Figure 6.2-1 Position Control System Diagram
SETUP:

Apparatus:

1. Plug the *1 red and gray wires with the red and black banana plug into the amplifier’s yellow output slot and black ground slot respectively. This wire supplies the input voltage to the motor. (On the connection plate, the odd wires are on top and the even wires are on bottom.)

2. Plug the *3 coax wire with the coax lead into channel 1 (CH1) on the computer circuit board. This is the tachometer feedback wire.

3. Plug the *5 coax wire with the coax lead into channel 0 (CHO) on the computer circuit board. This is the potentiometer feedback wire.

4. Plug the *7 red wire with the red banana plug into the red 5V slot on the amplifier. This wire carries the voltage input to the potentiometer.

5. Plug the *8 free coax wire with the red and black banana plugs into channel DACO OUT on the computer circuit board and into the yellow input slot and the black ground slot on the amplifier. This wire supplies the input voltage to the motor.

6. Let *2, *4 & *6 coax wires hang. They are not used in this experiment.

7. Slide the potentiometer’s mounting bracket and flexible coupling over the free end of the motor and motor shaft respectively. The red, white and green wires should be facing up. Slide the potentiometer on just enough to allow the set
screws in the mounting bracket to pinch the edge of the motor casing. Secure the potentiometer and flexible coupling by tightening the set screws. **Note:** If the potentiometer is pushed on too far, the flexible coupling exerts a force on the motor shaft that will alter the performance of the system. (ie. previous data and constants will no longer apply.)

8. Connect the potentiometer’s voltage in (red wire), voltage out (white wire), and ground (green wire) wires to lead screws A, B and C respectively.

9. Fasten degree wheel to transducer output shaft.

**Voltmeter:**

1. Clip the voltmeter wires to the apparatus’ connection plate leads where coax wires *5 and *6 are mounted. This will allow the potentiometer’s output voltage to be seen and the exact position calculated.

**Amplifier:**

1. Set the AC/DC switch to DC.
2. Set the Cal knob on the right Cal dot.
3. Set the gain to 1.
4. Turn the offset knob to the left until the green light just goes off.

**Computer:**

1. Turn on Computer
2. Select FINDER under the "apple" icon
3. Select FIND FILE under the "apple" icon
4. Enter MTR_POS_CONTROL
5. Double click on MTR_POS_CONTROL
6. Double click on the Hard Drive
7. Select LABVIEW 2.2
8. Select ME 401.VI's
9. Select ME 401_lab
10. Select MTR_POS_CONTROL

6.3 EXPERIMENTAL PROCEDURE

**Potentiometer Constant:**

1. Use the flexible coupling between the motor and the transducer to turn the motor shaft clockwise until it stops and the voltmeter reads approximately zero.

2. Make a line of reference on the transducer so that accurate degrees of measure can be taken.

3. Turn the flexible coupling counterclockwise until the degree wheel reads 10 degrees. Record the voltage on the voltmeter.

4. Repeat *3 until the potentiometer reaches the end. (ie. one complete turn of the degree wheel)
5. Assuming that $V_{potentiometer} = k_p W$. Plot Volts versus Radians of the motor. (ie. motor radians = 10 * degree wheel radians)

The potentiometer constant, $K_e$ is equal to the slope of the line.

Closed loop Transfer Function:
1. Find the transfer function for the closed-loop control system shown below.

Where $K_{pos}$ is the position gain error, $K_{vel}$ is the velocity gain error, $K_1$ is equal to one, $K_2$ is equal to one, and $K_m'$ is equal to $K_m/K_t$.

2. Make a root-locus plot for the system.

3. Find values for $K_{pos}$ and $K_{vel}$ that make the system stable at 90°. To do this, find the characteristic equation from the closed-loop transfer function. It will look something like this.
Characteristic Equation:

\[ s^2 + A_1 s + A_0 = 0 \]  \hspace{1cm} (6.3-2)

where \( A_1 \) and \( A_0 \) are functions of \( K_{\theta\theta} \) and \( K_{\theta\theta} \) and

\[(s - \lambda_1)(s - \lambda_2) = 0 \]  \hspace{1cm} (6.3-3)

where \( \lambda_1 \) and \( \lambda_2 \) are the desired roots of the characteristic equation and make the system stable at 90°. These roots can then be taken from the stable portion of the root-locus plot. From equation (6.3-3):

\[ s^2 + (\lambda_1 + \lambda_2)s + \lambda_1 \lambda_2 = 0 \]  \hspace{1cm} (6.3-4)

Comparing equations (6.3-2) and (6.3-4) gives:

\[ A_1 = (\lambda_1 + \lambda_2) \hspace{0.5cm} & \hspace{0.5cm} A_0 = \lambda_1 \lambda_2 \]  \hspace{1cm} (6.3-5)

Thus values for \( K_{\theta\theta} \) and \( K_{\theta\theta} \) can be found.

4. The values of \( \theta \) (90°), \( K_{\theta\theta} \) and \( K_{\theta\theta} \) can then be entered into the computer and the resulting position measured. Use the potentiometer constant, \( K_{\theta} \) to calculate the position. Be sure to take in account the 10:1 transducer ratio.
5. Repeat until values of $K_{o.o}$ and $K_{v.1}$ are found that make the system stable at 90°.

6. Once the values of $K_{o.o}$ and $K_{v.1}$ are found, detach the degree wheel and attach one of the links. Enter the same values into the system and measure the resulting position. It should be the same.

7. Repeat using the two remaining links.

6.4 STUDENT QUESTIONS

Plots:

Potentiometer Constant:

Assuming that $V_{potentiometer} = k_{p}W$.

Plot potentiometer voltage versus motor radians. The slope of the line is the potentiometer constant $K_{p}$.

Root-locus:

Sketch the typical root-locus plots for the closed-loop transfer function found in procedure "1."

Results:

Clearly explain the results obtained in the experiment.
Compare the different responses of the system and explain the system’s behavior.

6.5 RESULTS

The object of the first part of this experiment was not to theoretically analyze the potentiometer, but to determine the potentiometer constant experimentally. For this experiment it was assumed the potentiometer voltage was equal to the potentiometer constant times the speed of the motor \( V_{\text{potentiometer}} = k_w \). This constant was then used in the position control program to convert between the potentiometer voltage and the motor displacement in radians. The potentiometer constant was determined to be equal to 0.0055562 volts/radian. This is shown in figure 6.5-1. This constant was not used by the students in this experiment because the position control program was already written for them. A copy of this program has been inserted on pages 106 through 110.

Other potential experiments yet to be developed on this position control system will have the students theoretically analyze the potentiometer schematics and write their own position control program.
POTENTIOMETER CONSTANT
Kp = 0.0055562

\[ y = 1.9737 \times 10^{-3} + 5.5562 \times 10^{-3} x \quad R^2 = 1.000 \]

Figure 6.5-1 Potentiometer Constant Curve
Position Control Program

#include <stdio.h>
#include <console.h>
#include <math.h>
#include <stdlib.h>
#include <string.h>

#include "NI_DAQ_MAC.h"

#define COUNT 1000L
#define COUNT_2 3072
#define SWITCH 512
#define OK 1
#define PI 3.141592654

void pause( void );
double get_float( char *, double, double );

int16 brd;
int16 chan_out;
int16 *bin_v_out;
int16 out_mode, update_mode, update_signal, update_edge;

float volt_out;
float out_range;

int16 chan_in_0;
int16 chan_in_1;
int16 input_mode, input_range, polarity;
int16 gain_0, *bin_v_in_0;
int16 gain_1, *bin_v_in_1;
float *volt_in_0;
float *volt_in_1;

float pos_err;
float v_in_0, ang_vel, pos_err;
float pos_req, K_w, K_p;
float gain_err, gain_vel, theta_d, theta_0;

main()
{
    int16 i;
    long j;

    brd = 2;
    chan_out = 0;
    out_mode = 0;
    out_range = 10.0;
    update_mode = 0;
    update_signal = 0;
    update_edge = 0;
Position Control Program

chan_in_0 = 0;
chan_in_1 = 1;
input_mode = 0;
input_range = 10;
polarity = 0;
gain_0 = 1;
gain_1 = 1;

/* Initialize board and input - output */
AO_Setup( brd, chan_out, out_mode, out_range, update_mode, update_signal, update_edge);
AI_Config( brd, input_mode, input_range, polarity );
volt_out = 0.0;
AO_VScale( brd, chan_out, volt_out, bin_v_out );
AO_Write( brd, chan_out, *bin_v_out );

/* Read the potentiometer voltage to find the voltage at theta_0 */
AI_Read( brd, chan_in_0, gain_0, bin_v_in_0 );
AI_Scale( brd, gain_0, *bin_v_in_0, volt_in_0 );
theta_0 = (float)*volt_in_0;

pos_err = 0.0;
ang_vel = 0.0;
v_in_0 = 0.0;
K_p = 0.0055414; /* potentiometer constant: units (volts/rad) */
K_w = 0.06614; /* motor 2 tachometer constant: units (volt-sec/rad
*/

/* Get the desired angle, the error gain and the velocity gain */
theta_d = (float)get_float("the desired angle of rotation (degrees)",-135.0,135.0);
theta_d *= PI/180.0;
pos_req = theta_d*10.0;
gain_err = (float)get_float("the position error gain",-1.0e3,1.0e3);
gain_vel = (float)get_float("the velocity feedback gain",-1.0e3,1.0e3);

for( j = 0; j < COUNT; j++ ) {
/* Output the voltage command */
Position Control Program

AO_VScale( brd, chan_out, volt_out, bin_v_out );
AO_Write ( brd, chan_out, *bin_v_out );

/*
 * Read the potentiometer voltage
 */
AI_Read ( brd, chan_in_0, gain_0, bin_v_in_0 );
AI_Scale( brd, gain_0, *bin_v_in_0, volt_in_0 );
v_in_0 = (float)(*volt_in_0-theta_0)/K_p;

/*
 * Read the tachometer voltage
 */
AI_Read ( brd, chan_in_1, gain_1, bin_v_in_1 );
AI_Scale( brd, gain_1, *bin_v_in_1, volt_in_1 );
ang_vel = (float)*volt_in_1/K_w;

/*
 * Compute the error, i.e., error = position_required - position_output
 */
pos_err = pos_req-v_in_0;

/*
 * Output voltage
 */
volt_out = pos_err*gain_err - gain_vel*ang_vel;
}
volt_out = 0.0;

AO_VScale( brd, chan_out, volt_out, bin_v_out );
AO_Write ( brd, chan_out, *bin_v_out );

AO_Update(2);

get_string - get string from user with prompt

Return pointer to string of input text, prompts user with string
passed by caller. Indicates error if string space could not be
allocated. Limited to 80 char input.

char *get_string(char *prompt_string)

prompt_string string to prompt user for input

-----------------------------------------------
------
char *get_string(title_string)
char *title_string;
{
    char *alpha;                         /* result string pointer */

    alpha = (char *) malloc(80);
    if(!alpha) {
        printf("String allocation error in get_string\n");
        exit(1);
    }
    printf("Enter %s: ",title_string);
    gets(alpha);

    return(alpha);
} /* ------------------------------------------------------------------------ */

get_float - get float from user with prompt and range

Return double of input text, prompts user with prompt string and range of values (upper and lower limits) passed by caller.

double get_float(char *title_string,double low_limit,double up_limit)

    title_string string to prompt user for input
    low_limit lower limit of acceptable input (double)
    up_limit upper limit of acceptable input (double)

------------------------------------------------------------------------

double get_float(title_string,low_limit,up_limit)

    char *title_string;
    double low_limit,up_limit;
{
    double x;
    int error_flag;
    char *get_string( char *);                     /* get string routine */
    char *cp,*endcp;                                /* char pointer */
    char *stemp;                                    /* temp string */

    /* check for limit error, low may equal high but not greater */
    if(low_limit > up_limit) {
        printf("Limit error, lower > upper\n");
        exit(1);
    }
}
/* make prompt string */
   stemp = (char *) malloc(strlen(title_string) + 80);
   if(!stemp) {
       printf("String allocation error in get_float\n");
       exit(1);
   }

   sprintf(stemp,"%s [%1.2g...%1.2g]",title_string,low_limit,up_limit);

/* get the string and make sure x is in range */
do {
   cp = get_string(stemp);
   x = strtol(cp,&endcp);
   error_flag = (cp == endcp) || (*endcp != '\0'); /* detect errors */
   free(cp);
   free(string space */
} while(x < low_limit || x > up_limit || error_flag);

/* free temp string and return result */
free(stemp);
return(x);

void pause()
{
   char c;
   printf("Pause ...");
   c = getchar();
   c = c;
}
The Root-Locus plot for the system was then developed to find values for the position error gain, \( k_p \), and the velocity error gain, \( k_v \), that make the system stable at ninety degrees. A program was written in MATLAB to accomplish this. The root-locus program can be seen below and the resulting root-locus plot is illustrated in figure 6.5-2.

Root-Locus Program

clear;
kl=1.0;k2=1.0;km=1.0854/0.066144;kv=0.1;tc=0.0265
for i = 1:110
    kp = .1*(i-1);
    a = [0,1;kl*k2*km*kp/tc,-(kl*k2*km*kv+1)/tc];
    r(i,:) = eig(a).';
    fprintf('%g %g
',real(r(i,1)),imag(r(i,1)));
    fprintf('%g %g
',real(r(i,2)),imag(r(i,2)));
    fprintf('%g
',kp);
end;
plot(r,'x');
Figure 6.5-2 Sample Root-Locus Plot
Various values of the position and error gains are selected from the root-locus plots and entered into the position control system. These values and their results are recorded and are listed in table 6.5-1. These results indicate that the system is more dependent on the velocity error gain and therefore is the controlling feedback coefficient. It is also noticed that the displacements produced are within approximately 3.0% (approximate error of the potentiometer constant) of the desired position. Thus these error gains are considered acceptable. Other stable values from the root-locus plot may not make the system stable due to the speed inefficiencies in the Macintosh IIx computer and the tolerance constraint within the Leybold 30 watt amplifier.

The above results are produced on an unloaded system and needed to be tested under load. To determine if the system would perform similarly under load, three different aluminum links: sizes and weights; 6"x1/2"x1/4" @ 33.8g, 12"x1/2"x1/4" @ 62.8g, and 14"x1/2"x1/4" @ 79.6g are attached by their ends to the transducer output shaft to simulate different inertia loads on the system. Five of the previous error gain values used in the no load test were then used to determine if the system was stable under various loads. The under load results for each link can be seen in tables 6.5-2, 6.5-3, and 6.5-4. Comparing the no load and under load tests show the results to be the same. Thus the system has been successfully controlled.

In order to achieve better results, the calibration curves and constants used in this single link manipulator system would have to be measured using smaller increments and more precise equipment. However,
the results here are sufficient enough to demonstrate to the student the theory behind the physical control of a position control system.

Table 6.5-1  No Load Error Gain And Position Control Results

<table>
<thead>
<tr>
<th>Trial</th>
<th>Od (deg)</th>
<th>A1</th>
<th>A2</th>
<th>Kvel</th>
<th>Kpos</th>
<th>Vout</th>
<th>O (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
<td>38.1</td>
<td>310</td>
<td>0.001</td>
<td>0.50</td>
<td>0.078</td>
<td>80.43</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>43.9</td>
<td>434</td>
<td>0.010</td>
<td>0.70</td>
<td>0.083</td>
<td>85.50</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>50.1</td>
<td>619</td>
<td>0.020</td>
<td>1.00</td>
<td>0.088</td>
<td>90.74</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
<td>56.3</td>
<td>743</td>
<td>0.030</td>
<td>1.20</td>
<td>0.088</td>
<td>90.74</td>
</tr>
<tr>
<td>5</td>
<td>90</td>
<td>62.5</td>
<td>991</td>
<td>0.040</td>
<td>1.60</td>
<td>0.088</td>
<td>90.74</td>
</tr>
<tr>
<td>6</td>
<td>90</td>
<td>68.7</td>
<td>1177</td>
<td>0.050</td>
<td>1.90</td>
<td>0.088</td>
<td>90.74</td>
</tr>
<tr>
<td>7</td>
<td>90</td>
<td>74.9</td>
<td>1363</td>
<td>0.060</td>
<td>2.20</td>
<td>0.088</td>
<td>90.74</td>
</tr>
<tr>
<td>8</td>
<td>90</td>
<td>81.1</td>
<td>1610</td>
<td>0.070</td>
<td>2.60</td>
<td>0.091</td>
<td>93.84</td>
</tr>
<tr>
<td>9</td>
<td>90</td>
<td>87.2</td>
<td>1858</td>
<td>0.080</td>
<td>3.00</td>
<td>0.088</td>
<td>90.74</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>93.4</td>
<td>2168</td>
<td>0.090</td>
<td>3.50</td>
<td>0.091</td>
<td>93.84</td>
</tr>
<tr>
<td>11</td>
<td>90</td>
<td>99.6</td>
<td>2477</td>
<td>0.100</td>
<td>4.00</td>
<td>0.090</td>
<td>92.80</td>
</tr>
<tr>
<td>12</td>
<td>90</td>
<td>156.0</td>
<td>6069</td>
<td>0.191</td>
<td>9.80</td>
<td>0.090</td>
<td>92.80</td>
</tr>
<tr>
<td>13</td>
<td>90</td>
<td>223.5</td>
<td>12447</td>
<td>0.300</td>
<td>20.10</td>
<td>0.091</td>
<td>93.84</td>
</tr>
<tr>
<td>14</td>
<td>90</td>
<td>285.4</td>
<td>20065</td>
<td>0.400</td>
<td>32.40</td>
<td>0.088</td>
<td>90.74</td>
</tr>
<tr>
<td>15</td>
<td>90</td>
<td>347.4</td>
<td>1858</td>
<td>0.500</td>
<td>3.00</td>
<td>0.090</td>
<td>92.80</td>
</tr>
<tr>
<td>16</td>
<td>90</td>
<td>347.4</td>
<td>30160</td>
<td>0.500</td>
<td>48.70</td>
<td>0.090</td>
<td>92.80</td>
</tr>
</tbody>
</table>
Table 6.5-2  Under Load Error Gain And Position Control Results

Link "1  6"x1/2"x1/4"  @  33.8g

<table>
<thead>
<tr>
<th>Trial</th>
<th>θd (deg)</th>
<th>A1</th>
<th>A2</th>
<th>Kvel</th>
<th>Kpos</th>
<th>Vout</th>
<th>θ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>90</td>
<td>50.1</td>
<td>619</td>
<td>0.020</td>
<td>1.00</td>
<td>0.088</td>
<td>90.74</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>93.4</td>
<td>2168</td>
<td>0.090</td>
<td>3.50</td>
<td>0.091</td>
<td>93.84</td>
</tr>
<tr>
<td>11</td>
<td>90</td>
<td>99.6</td>
<td>2477</td>
<td>0.100</td>
<td>4.00</td>
<td>0.090</td>
<td>92.80</td>
</tr>
<tr>
<td>12</td>
<td>90</td>
<td>156.0</td>
<td>6069</td>
<td>0.191</td>
<td>9.80</td>
<td>0.090</td>
<td>92.80</td>
</tr>
<tr>
<td>15</td>
<td>90</td>
<td>347.4</td>
<td>1858</td>
<td>0.500</td>
<td>3.00</td>
<td>0.090</td>
<td>92.80</td>
</tr>
</tbody>
</table>

Table 6.5-3  Under Load Error Gain And Position Control Results

Link "2  12"x1/2"x1/4"  @  62.8g

<table>
<thead>
<tr>
<th>Trial</th>
<th>θd (deg)</th>
<th>A1</th>
<th>A2</th>
<th>Kvel</th>
<th>Kpos</th>
<th>Vout</th>
<th>θ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>90</td>
<td>50.1</td>
<td>619</td>
<td>0.020</td>
<td>1.00</td>
<td>0.088</td>
<td>90.74</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>93.4</td>
<td>2168</td>
<td>0.090</td>
<td>3.50</td>
<td>0.091</td>
<td>93.84</td>
</tr>
<tr>
<td>11</td>
<td>90</td>
<td>99.6</td>
<td>2477</td>
<td>0.100</td>
<td>4.00</td>
<td>0.090</td>
<td>92.80</td>
</tr>
<tr>
<td>12</td>
<td>90</td>
<td>156.0</td>
<td>6069</td>
<td>0.191</td>
<td>9.80</td>
<td>0.090</td>
<td>92.80</td>
</tr>
<tr>
<td>15</td>
<td>90</td>
<td>347.4</td>
<td>1858</td>
<td>0.500</td>
<td>3.00</td>
<td>0.090</td>
<td>92.80</td>
</tr>
</tbody>
</table>
Table 6.5-4 Under Load Error Gain And Position Control Results

Link *3 14"xl/2"xl/4" θ 79.6g

<table>
<thead>
<tr>
<th>Trial</th>
<th>Od (deg)</th>
<th>A1</th>
<th>A2</th>
<th>Kvel</th>
<th>Kpos</th>
<th>Vout</th>
<th>O (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>90</td>
<td>50.1</td>
<td>619</td>
<td>0.020</td>
<td>1.00</td>
<td>0.088</td>
<td>90.74</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>93.4</td>
<td>2168</td>
<td>0.090</td>
<td>3.50</td>
<td>0.091</td>
<td>93.84</td>
</tr>
<tr>
<td>11</td>
<td>90</td>
<td>99.6</td>
<td>2477</td>
<td>0.100</td>
<td>4.00</td>
<td>0.090</td>
<td>92.80</td>
</tr>
<tr>
<td>12</td>
<td>90</td>
<td>156</td>
<td>6069</td>
<td>0.191</td>
<td>9.80</td>
<td>0.090</td>
<td>92.80</td>
</tr>
<tr>
<td>15</td>
<td>90</td>
<td>347.4</td>
<td>1858</td>
<td>0.500</td>
<td>3.00</td>
<td>0.090</td>
<td>92.80</td>
</tr>
</tbody>
</table>
CONCLUSIONS

The object of this thesis is to design systems analysis and controls experiments that emphasize specific theories to be used as a teaching aide for undergraduate students in conjunction with M. E. 401 over a ten week quarter. The experiments chosen are a single degree of freedom initial condition free response spring-mass system, a frequency response system, a single link manipulator tachometer calibration and constant system, a single link manipulator time and motor constants system, and a single link manipulator position control system. Each of these experiments emphasizes specific theory coinciding with the student's course work and allows them to see the physical aspects involved as well as the theoretical. The students are expected to perform an experimental and theoretical analysis on each experiment and compare the results and other findings in a formal report.
This new controls laboratory has great potential and will not be limited to the three spring-mass systems, one frequency response system, and one single link manipulator system divided up into the five experiments as listed above. Using these same systems, four or five more experiments are going to be implemented emphasizing these theories: forced response on circuits, analytical modeling and analysis of a tachometer, analytical modeling and analysis of a potentiometer, and position control programming by the students. Another system being developed at this time is the Inverted Pendulum Balance Control System. It will be divided up into at least five different experiments and be used in conjunction with another controls course.

The experiments and their results discussed in this thesis demonstrate a unique opportunity available to the undergraduate students. The opportunity to see the physical responses from their theoretical work and the differences that occur between the two due to outstanding variables. It is hoped these experiments will bring the students insight into their course work and the working world in the future.
REFERENCES


