SELECTION OF CONTROLLER GAINS FOR AN ELECTROMAGNETIC SUSPENSION SYSTEM

A Thesis Presented to
The Faculty of College of Engineering and Technology
Ohio University

In partial Fulfillment
of the Requirements for the Degree in
Master of Science

by
Jong Teck Foo
March 1993
Abstract

This thesis consists of using different optimization methods to find a suitable linear feedback controller for a single axis electromagnetic suspension. This suspension is intended to be used as a vibration isolation device. The equations of motion for this single axis electromagnetic suspension system are derived using Lagrange's approach. These equations are highly nonlinear and are linearized via a Taylor series expansion in order to facilitate the controller design.

Two sets of feedback control signals are considered. The first set of feedback control signals considered are the absolute velocity of the support piece, the relative displacement of the support piece with respect to the base and the current. The second set includes the previous three control signals and two additional control signals, the absolute displacement and relative velocity of the support piece. Stability test is performed to find the ranges of controller gains that will make the system stable. Some stability boundary plots are presented. Given the stiffness constants of the electromagnet, these plots give direct intuition of what the ranges of controller gains should fall in.

The design of state feedback controllers is considered based on two criteria, minimizing the mean square absolute displacement of the suspension system and the energy dissipated by the suspension. The input disturbance is assumed to be ideally white in both cases.

First, several design procedures developed from frequency domain are proposed to find the optimum controller gains. Then Linear Quadratic (LQ) optimal theory developed for time domain is
used to find optimum controller gains. The results obtained from these design methods are compared with that from LQ method. It is shown that the design methods developed from frequency domain yield better controller gains selection than LQ method. The transmissibility of the electromagnetic suspension is compared with that of the passive isolation system. It can be concluded that the disturbance attenuation characteristic of the electromagnetic suspension system is overall superior as opposed to the passive isolation system.
Table of Contents

Abstract i
Table of Contents iii
Acknowledgement v
Chapter 1 Introduction
1.1 Definition 1
1.2 The Need of Vibration Isolation Device 1
1.3 Types of Vibration Isolation System 2
1.4 Passive Vibration Isolation System 3
1.5 Recent Research Reviews 5
1.6 Objectives of Study 6
Chapter 2 Control and Stability
2.1 Equation of Motion for Electromagnetic Suspension System 8
2.2 Feedback Control of Electromagnetic Suspension 11
2.3 Stability Test 14
Chapter 3 Optimization of Three Control Signals
3.1 Introduction 22
3.2 Transfer Functions for Electromagnetic Suspension System with Three Feedback Control Signals 23
3.3 Design Methods to Select the Controller Gains 27
3.4 Root Locus Analysis for the Design Methods 29
3.5 Controller Gains Selection Using Linear Quadratic Optimal Technique 38
3.6 Example 41
3.7 Dynamic Simulation 45
Acknowledgement

I would like to express my great appreciation to my advisor, Dr. Brian C. Fabien, for all his guidance, patience and encouragement made possible the completion of this work.

I would also like to give my gratitude to the members of my thesis committee: Dr. Sunil K. Agrawal and Dr. Larry Snyder, for their patience and time in reviewing my thesis and for their comments and suggestions.

Finally, my special great gratitude to my family, especially to my parents, Chee Tan Foo and Ah Mui Goh, and my grandparents, Ming Goh and Ah Mui Tan, for their continual support and encouragement during my study in Ohio University.
Chapter 1
Introduction

1.1 Definition

Vibration isolation device is used to reduce the transmission of vibrating displacements and forces from one mechanical or electrical system to its surrounding or vice versa.

1.2 The Need of Vibration Isolation Device

In the present, the demand of vibration isolation device has increased dramatically because almost all mechanical or electrical systems need to have vibration isolation device installed on them in order to completely utilize their functions. With the developments of lightweight materials and reduction in size of new mechanical or electrical systems, a lot of sensitive measuring equipments are easily affected by the presence of floor vibration and noise of the surrounding. As a result, the performances of these sensitive measuring equipments have been adversely affected. In order to obtain accurate measurement outputs, these sensitive measuring equipments need to be mounted on the vibration isolation devices. Many production machinery and engines also suffer from the vibration which is created from the high speed performance of these devices. This vibration will create dynamic stress which will lead to fatigue and failure of mechanical components. One way of reducing failure of mechanical components due to the vibration is to install vibration isolation devices on these mechanical systems.
1.3 Types of Vibration Isolation System

In general, vibration isolation system can be divided into two categories, passive and active systems. Passive isolation system is mostly considered by a designer when a mechanical or electrical system need to have vibration isolation system installed on it. There are two reasons which can explain why passive vibration isolation system is usually favored by a designer to use. First, it is a simple and reliable device, especially the simplicity of the passive isolation devices make them easy to manufacture comparing with the active isolation device. Second, it is usually less expensive than active isolation system.

Four types of most commonly used passive vibration isolation device are metal spring, rubber, cork and felt. These devices have disadvantages. The disadvantage of metal spring obviously is that it transmits high frequency very readily [1]. As a result, metal spring alone is not suitable to be used as a vibration isolation device on a system which vibrates at high frequency. Rubber cannot be used practically in an oily environment. Normally, cork and felt do not have good vibration isolation performance when vibrating frequency is below 40 Hz. Therefore, they are commonly used on multi-cylinder reciprocating machines which vibrate at a frequency higher than 40 Hz.

Passive vibration isolation systems can only dissipate energy. While active vibration isolation systems can dissipate energy as well as supply energy. Active vibration control system requires an external energy source and a sophisticated feedback control system with some sensors and some electronic components. As a result, the complexity of the active vibration isolation system makes it costly.
However, active vibration isolation system is used where the performance benefit of the mechanical or electrical system is higher than the cost of the active vibration isolation system. Active pneumatic (hydraulic) vibration isolator is one of the most commonly used active vibration isolation devices. One of the disadvantages of a pneumatic vibration isolation system is that it requires to have a large pressurized air supply system. The other weakness of hydraulic vibration isolation system is that it is not suitable to be used in an environment which has to be maintained clean all the time. Electromagnetic vibration isolation system is also an active vibration control system. Recent research has shown that electromagnetic vibration isolation systems have superior vibration isolation characteristic and the potential to eliminate the problems discussed above.

1.4 Passive Vibration Isolation System

The simplest passive vibration isolation system consists of a spring and a damper, as shown in Figure 1.1. The equation of motion of the passive vibration isolation system in Figure 1.1 can be derived using Newton’s second law

\[-k(x-y)-c(\dot{x}-\dot{y})=m\ddot{x}\]  

(1.1a)

or

\[F_p=m\ddot{x}\]  

(1.1b)

where

\[F_p=-k(x-y)-c(\dot{x}-\dot{y})\]  

(1.2)

Rearrange equation (1.1a), we get
Figure 1.1 Passive Vibration Isolation System
\[
\dot{x} + \frac{c}{m} \dot{x} + \frac{k}{m} x = \frac{c}{m} \dot{y} + \frac{k}{m} y
\]  
\[\text{(1.3a)}\]

or

\[
\dot{x} + 2\zeta \omega_n \dot{x} + \omega_n^2 x = 2\zeta \omega_n \dot{y} + \omega_n^2 y
\]  
\[\text{(1.3b)}\]

where \( \omega_n = (k/m)^{1/2} \) is the natural frequency of the passive isolation system, and \( \zeta = c/(2(mk)^{1/2}) \) is the damping ratio.

Taking Laplace transform for equation (1.3b) and setting all the initial conditions to be zero, the transfer function of absolute displacement for the passive vibration isolation system becomes

\[
\frac{X(s)}{Y(s)} = \frac{2\zeta \omega_n s + \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]  
\[\text{(1.4)}\]

where \( s \) is the Laplace transform variable, \( X(s) \) is the Laplace transform of \( x(t) \), \( Y(S) \) is the Laplace transform of \( y(t) \).

1.5 Recent Research Reviews

Recently, some research has been done on active vibration isolation control system and the use of electromagnetic force as an active force generator in an active vibration control system. An electrohydraulic suspension for isolating the low frequency roll and pitch in off-road vehicles has been developed by Young and Suggs [2]. Esmailzadeh [3] has proposed using a three-way servo-valve controlled pneumatic suspension system for ground vehicles. A fail-safe active vibration isolation control system with the reliability provided by a passive system has been suggested by Guntur and Sankar [4]. Hulme and Sinha [5] have used a real time adaptive
control algorithm to design the controller of electromagnetic suspension system for nonlinear case. Multi-variable frequency response techniques have been used to design an active anti-vibration platform [6]. An active anti-vibration system designed with using electromagnetic force as an active force generator is given by Sandercock [7]. Rakheja, Sankar and Su [8] have investigated the vibration-isolation performance of an active electromagnetic force generator. Fabien [9] has proposed two design procedures on how to select controller gains for an electromagnetic vibration control system.

1.6 Objectives of Study

In this study, the vibration-isolation characteristic of an active electromagnetic vibration control system is investigated and analyzed. The dynamic equations of the electromagnetic suspension system is derived in Appendix B [9]. Different optimization methods and criteria are used to find optimum controller gains for the electromagnetic suspension system. The vibration isolation performance of electromagnetic suspension system is compared with that of simple spring-damper passive system. Frequency Response and Root-Locus methods are used to analyze the electromagnetic suspension system.

In Chapter 1, a brief introduction of vibration isolation device is described, and some research on active vibration isolation system and electromagnetic vibration isolation system is reviewed. In Chapter 2, feedback control law is used to derive transfer functions for the electromagnetic suspension system. Five controller gains are used in this feedback control law, they are absolute displacement,
absolute velocity, relative displacement, relative velocity and current. A stability test of the suspension system is illustrated by using the Hurwitz stability criterion. Transfer functions with three feedback control signals which are relative displacement, absolute velocity and current are shown in Chapter 3. Two design procedures of selecting the controller gains proposed by Fabien [9] are introduced. The root locus method is applied to examine how the roots of the characteristics equation vary with the changes in the controller gains. Linear Quadratic Optimization method is applied to find the optimum control gains for the suspension system based on different performance measures. The frequency response technique is used to examine the characteristics of the suspension system. A comparison of dynamic response of the linear and nonlinear equation for the electromagnetic vibration isolation system is shown. A step function (input) is used in this dynamic simulation to see how the electromagnetic suspension system responds. The transfer functions with five controller gains derived in Chapter 2 are analyzed in Chapter 4. Several selection procedures of controller gains are proposed. Linear Quadratic Optimal theory is utilized to find the optimum controller gains. Root locus and frequency response techniques are employed to investigate the transient responses, the relative stability and the steady-state responses of the vibration isolation system. The results obtained in chapter 3 and 4 are discussed and concluded in Chapter 5.
Chapter 2
Control and Stability

2.1 Equation of Motion for Electromagnetic Suspension System

The schematic of the electromagnetic vibration isolation system is shown in Figure 2.1. The displacement of the support piece and the core measured with respect to a datum are separately denoted by x and y. R is the resistance of the circuit and i is the applied current. The applied voltage is given by v and the mass of the support piece is represented by m. The inductance of the circuit denoted by L, $\alpha_0/(\alpha_1+x-y)$, is derived and shown in Appendix C. The derivation of the equations of motion for the electromagnetic vibration isolation system is shown in Appendix B. The Lagrange’s approach is used to derive the equation of motion. Coordinate q being the charge and x being the absolute displacement of the support piece measured with respect to the datum are selected as the generalized coordinates. Equation (2.1) is the Lagrange equation for the electromagnetic suspension system.

\[
\begin{align*}
\ddot{m} + \frac{\alpha_0}{2[\alpha_1 + x - y]}\dot{q}^2 - mg &= F & (2.1a) \\
\frac{\alpha_0}{\alpha_1 + x - y}\ddot{q} - \frac{\alpha_0}{[\alpha_1 + x - y]^2}(x - y)\dot{q} + R\ddot{q} &= v & (2.1b)
\end{align*}
\]

The equation (2.1) is then rewritten into the state variable
Figure 2.1 Schematic of the Electromagnetic Vibration Isolation System
form by letting $x_1$, $x_2$ and $x_3$ be the deviation of airgap size, the absolute velocity and the current. Therefore, equation (2.1) becomes

$$
\dot{x}_1 = x_2 - \dot{y} 
$$

(2.2a)

$$
\dot{x}_2 = \frac{F}{m} + g - \frac{\alpha_0 (I + x_3)^2}{2 m (\alpha_1 + a + x_1)^2} 
$$

(2.2b)

$$
\dot{x}_3 = \left( \frac{\alpha_1 + a + x_1}{\alpha_0} \right) \left[ V + v + \frac{\alpha_0}{(\alpha_1 + a + x_1)^2} (x_2 - \dot{y}) - R \right] (I + x_3) 
$$

(2.2c)

However, equation (2.2) is the nonlinear state equation for the electromagnetic vibration isolation system. A linearization approach using Taylor series expansion is applied to linearize equation (2.2):

$$
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
\dot{z}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
\frac{k_z}{m} & 0 & -\frac{k_i}{m} \\
0 & \frac{k_z}{k_i} & \frac{R}{L_0}
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2 \\
z_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
\frac{1}{L_0}
\end{bmatrix} u_0 +
\begin{bmatrix}
-1 \\
0 \\
0 \\
0 \\
\frac{k_z}{k_i}
\end{bmatrix} \dot{e}_0
$$

(2.3)

where $L_0 = \frac{\alpha_0}{\alpha_1 + a}$ is the inductance of the magnet at equilibrium position (airgap size=a), $k_z = \frac{\alpha_0 I^2}{(\alpha_1 + a)^3}$ and $k_i = \frac{\alpha_0 I}{(\alpha_1 + a)^2}$ can be treated as the stiffness constants of the magnet.

Equation (2.3) is the linear equation for the electromagnetic suspension system. Equation (2.3) can be written in the dimensionless form by substituting $w_1 = \frac{z_1}{a}$, $w_2 = \frac{z_2 L_0}{a R}$, $w_3 = \frac{z_3}{I}$, $\dot{w}_1 = \frac{\dot{z}_1 L_0}{a R}$, $\dot{w}_2 = \frac{\dot{z}_2 L_0^2}{a R^2}$, $\dot{w}_3 = \frac{\dot{z}_3 L_0}{a R}$, $u = \frac{u_0}{IR}$, $\dot{e} = \frac{\dot{e}_0 L_0}{a R}$ and the new time variable $\tau = t R / L_0$. Equation (2.4) is the
nondimensional form linear equation for the electromagnetic suspension system.

\[
\begin{bmatrix}
\dot{w}_1 \\
\dot{w}_2 \\
\dot{w}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
-\frac{k_1}{k_2} & 0 & -k_2 \\
0 & \frac{k_1}{k_2} & -1
\end{bmatrix}
\begin{bmatrix}
w_1 \\
w_2 \\
w_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u +
\begin{bmatrix}
-1 \\
0 \\
-\frac{k_1}{k_2}
\end{bmatrix} \dot{e}
\]  

(2.4)

where \( k_1 = k_2 a L_0^2 / m a R^2 \) and \( k_2 = k_1 L_0^2 / m a R^2 \) are dimensionless constants.

2.2 Feedback Control of Electromagnetic Suspension System

In this section, feedback control law is applied and the transfer functions for the electromagnetic suspension system are obtained. Before consider the feedback control law, two additional variables will be introduced. They are \( w_0 \) and \( w_4 \); \( w_0 \) is defined as the absolute displacement of the support piece; \( w_4 \) represents the relative velocity of the support piece. They can be expressed by the following two equations,

\[
\dot{w}_0 = w_2 \quad \text{(2.5a)}
\]
\[
w_4 = \dot{w}_1 \quad \text{(2.5b)}
\]

Five feedback control signals are considered in the control law. They are absolute displacement \( (w_0) \), relative displacement \( (w_1) \), absolute velocity \( (w_2) \), current \( (w_3) \) and relative velocity \( (w_4) \). This
is a practical consideration of applying the control law because the deviations in all of the five feedback control signals can be measured physically. The absolute displacement and the relative displacement can be measured by using a displacement sensor. The absolute velocity and the relative velocity can be obtained by integrating the output of an accelerometer which is attached on the core and the support piece. The change in current can be easily measured by an ammeter. Feedback control law of the electromagnetic is expressed as follow,

\[ u = -g_0 w_0 - g_1 w_1 - g_2 w_2 - g_3 w_3 - g_4 w_4 \]  

(2.6)

where \( g_0, g_1, g_2, g_3 \) and \( g_4 \) are the controller gains.

Equation (2.4) can be rewritten into the following form:

\[ \dot{w}_1 = w_2 - \dot{e} \]  

(2.4a)

\[ \dot{w}_2 = k_1 w_1 - k_2 w_3 \]  

(2.4b)

\[ \dot{w}_3 = \frac{k_1}{k_2} w_2 - w_3 + u - \frac{k_1}{k_2} \dot{e} \]  

(2.4c).

Taking derivative with respect to time of equation (2.4b), we get

\[ \dot{w}_2 = k_1 \dot{w}_1 - k_2 \dot{w}_3 \]  

(2.7)

Substitute equations (2.4a) and (2.4c) into equation (2.7), it becomes

\[ \dot{w}_2 = k_2 w_3 - k_2 u \]  

(2.8)

Substitute equation (2.6) into equation (2.8), and then using
equations (2.5a), (2.5b), (2.4a) and (2.4b) into equation (2.8) yields

\[ w_2^{(3)} + (1 + g_3) \ddot{w}_2 - k_2(g_2 + g_4) \dot{w}_2 - (k_1 + k_2g_0 + k_2g_1 + k_1g_3)w_2 = -k_2g_4\ddot{e} - (k_1 + k_1g_3 + k_2g_1)\dot{e} \]

(2.9)

Taking Laplace transform for equation (2.9) and setting all the initial conditions to zero, the transfer function for the absolute velocity becomes

\[
\frac{W_2(s)}{E(s)} = T_2(s) = \frac{d_2s^2 + d_1s}{s^3 + \sigma_2s^2 + \sigma_1s + \sigma_0}
\]

(2.10)

where \( s \) is the Laplace transform variable, \( W_2(s) \) is the Laplace transform of \( w_2(\tau) \), \( E(s) \) is the Laplace transform of \( e(\tau) \) and

\[
\sigma_0 = -k_1(1 + g_3) - k_2(g_0 + g_1)
\]

(2.11a)

\[
\sigma_1 = -k_2(g_2 + g_4)
\]

(2.11b)

\[
\sigma_2 = 1 + g_3
\]

(2.11c)

\[
d_1 = -(k_1 + k_1g_3 + k_2g_1)
\]

(2.12a)

\[
d_2 = -k_2g_4
\]

(2.12b)

Applying the derivative property of the Laplace transform, \( W_2(s) = sW_0(s) \), the transfer function for the absolute displacement becomes

\[
\frac{W_0(s)}{E(s)} = T_0(s) = \frac{d_2s + d_1}{s^3 + \sigma_2s^2 + \sigma_1s + \sigma_0}
\]

(2.13)
where $W_0(s)$ is the Laplace transform of $w_0(\tau)$.

Also, the transfer function for the current is obtained in the similar way as the transfer function for the absolute velocity. As the result, the transfer function for the current is given by

$$
\frac{W_3(s)}{E(s)} = T_3(s) = \frac{h_3 s^3 + h_2 s^2 + h_1 s + h_0}{s^3 + \sigma_3 s^2 + \sigma_1 s + \sigma_0}
$$

where $W_3(s)$ is the Laplace transform of $w_3(\tau)$ and

$$
\begin{align}
    h_0 &= k_1 g_0 \\
    h_1 &= k_1 g_2 \\
    h_2 &= g_1 \\
    h_3 &= g_4 - \frac{k_1}{k_2}
\end{align}
$$

2.3 Stability Test

One of the objective in feedback controller design is to make the system to be stable. In physical sense, a stable system means a steady-state response will be resumed in the system as soon as the transient response of the system damps out. An unstable system means that the response of the system diverges from an initial value and oscillates with ever-increasing amplitude.

In the mathematical sense, the system is stable if and only if all the roots of the characteristic equation have negative real parts. In contrast, if any one of the roots in the characteristic equation has a positive real part, then the system is unstable.

In this section, Hurwitz stability criterion is used to determine the range of the controller gains which will ensure all the roots of the
characteristic equation of the electromagnetic suspension system to have negative real parts. A brief review of Hurwitz stability criterion is shown in Appendix D.

The denominator in the equation (2.10) (and so as (2.13) and (2.14)) is the characteristic equation of the electromagnetic suspension system. Thus the characteristic equation of electromagnetic vibration isolation is given as follow

\[ s^3 + \sigma_2 s^2 + \sigma_1 s + \sigma_0 = 0 \]  

(2.16)

Apply the Hurwitz stability test to the characteristic equation (2.16), the minor principal determinants and the determinant are

\[ \Delta_1 = |\sigma_2| \]  

(2.17a)

\[ \Delta_2 = \begin{vmatrix} \sigma_2 & \sigma_0 \\ 0 & \sigma_1 \end{vmatrix} \]  

(2.17b)

\[ \Delta_3 = \begin{vmatrix} \sigma_2 & \sigma_0 & 0 \\ 1 & \sigma_1 & 0 \\ 0 & \sigma_2 & \sigma_0 \end{vmatrix} \]  

(2.17c)

According to the Hurwitz stability criterion that in order to have a stable system, the determinant and all the principal minor determinants constructed from the coefficients of the characteristic equation must greater than zero. Therefore, equation (2.17) becomes

\[ \sigma_1 > 0 \]  

(2.18a)

\[ \sigma_2 \sigma_1 - \sigma_0 > 0 \]  

(2.18b)

\[ \sigma_2 \sigma_1 \sigma_0 - \sigma_0^2 > 0 \]  

(2.18c)
Substituting equation (2.11) into equation (2.18), it becomes

\[ 1 + g_3 > 0 \] \hspace{1cm} (2.19a)

\[ g_1 + g_0 - \left[ (g_2 + g_4) - \frac{k_1}{k_2} \right] (1 + g_3) > 0 \] \hspace{1cm} (2.19b)

\[-k_2^2 g_1^2 + \left[ k_2^2 (g_2 + g_4)(1 + g_3) - 2k_1 k_2 (1 + g_3) - 2k_2^2 g_0 \right] g_1 + \]

\[ \left[ k_1 k_2 (g_2 + g_4)(1 + g_3)^2 + k_2^2 g_0 (g_2 + g_4)(1 + g_3) \right. \]

\[-k_1^2 (1 + g_3)^2 - 2k_1 k_2 (1 + g_3) g_0 - k_2^2 g_0^2 ] > 0 \] \hspace{1cm} (2.19c)

The controller gains \( g_0, g_1, g_2, g_3 \) and \( g_4 \) that satisfy the equation (2.19) will ensure to result a stable electromagnetic suspension system. It is obvious that \( g_3 \) must be greater than the value of \(-1\). Stability boundary plot as a function of \( g_1 \) and \( g_2 \) will be done to show the bounds of the controller gains. Before doing the stability boundary plot, equations (2.19a) and (2.19b) can be rewritten as

\[ g_3 > -1 \] \hspace{1cm} (2.20a)

\[ g_1 > -g_0 + \left[ (g_2 + g_4) - \frac{k_1}{k_2} \right] (1 + g_3) \] \hspace{1cm} (2.20b)

Equation (2.19c) is a quadratic equation, and the solution of this equation is found to be

\[ g_1 > -g_0 + \left[ (g_2 + g_4) - \frac{k_1}{k_2} \right] (1 + g_3) \] \hspace{1cm} (2.20b)

\[ g_1 < -g_0 - \frac{k_1}{k_2}(1 + g_3) \] \hspace{1cm} (2.20c)
The stability boundary plot can be categorized into four kinds due to the different combinations of selecting the range of controller gains for \( g_0 \) and \( g_4 \). They are separately plotted as shown in Figures 2.2, 2.3, 2.4 and 2.5. The controller gains that fall into the shaded area in these plots will result in a stable electromagnetic suspension system. In contrast, the controller gains fall outside the shaded area will make the system to become unstable.

It is clear that \( k_1 \) and \( k_2 \) are the important constants other than controller gains to determine the region of stability for the electromagnetic suspension system. The dimensionless constants, \( k_1 \) and \( k_2 \), are expressed in terms of the stiffness constants of the electromagnet \( (k_i \text{ and } k_z) \), the steady-state current \( (I) \), the mass of the support piece \( (m) \), the inductance of the magnet at the equilibrium position \( (L_0) \), the airgap size \( (a) \) and the resistance \( (R) \). Therefore, the values of \( k_i, k_z, I, m, L_0, a \) and \( R \) play important roles of selecting the controller gains as well as determining the stability region for the suspension system. Figures 2.2, 2.3, 2.4 and 2.5 can be used as the design tools to select the controller gains for the electromagnetic suspension system if the values of \( k_1 \) and \( k_2 \) are known. For most of the electromagnets, the ratio of \( (k_1/k_2) \) is about unity [9]. It should be noted that similar approach of using relative displacement, absolute velocity and applied current as the feedback controllers for the electromagnetic suspension system has been considered by Fabien [9].
Given Conditions:

\( g_3 > -1 \)

\( g_2 < \frac{g_0}{1+g_3} \)

\( g_4 < 0 \)

\( g_2 < g_4 \)

\( g_0 \geq \frac{k_1}{k_2} (1+g_3) \)

\[ g_1 = -g_0 \frac{k_1}{k_2} (1+g_3) \]

\[ g_1 = -g_0 + \frac{[g_2 + g_4 - \frac{k_1}{k_2}] (1+g_3)}{1+g_3} \]

\( (0, \frac{g_0}{1+g_3} - g_4 + \frac{k_1}{k_2}) \)

\( (-g_0 - \frac{k_1}{k_2} (1+g_3), -g_4) \)

\( (-g_0 + (g_4 - \frac{k_1}{k_2}) (1+g_3), 0) \)

Figure 2.2 Stability Boundary Plot
Given Conditions:

\[ g_3 > -1 \]
\[ g_4 \geq 0 \]
\[ g_2 < g_4 \]
\[ g_0 \geq \frac{k_1}{k_2} (1 + g_3) \]

Figure 2.3 Stability Boundary Plot
Figure 2.4 Stability Boundary Plot
Given Conditions:

\[ g_3 > -1 \]

\[ g_4 \geq 0 \]

\[ g_2 < \frac{k_1}{k_2} \]

\[ (0, \frac{g_0}{1+g_3} - g_4 + \frac{k_1}{k_2}) \]

\[ ( -g_0 - \frac{k_1}{k_2} (1+g_3), -g_4) \]

**Figure 2.5 Stability Boundary Plot**
Chapter 3
Optimization of Three Control Signals

3.1 Introduction

Almost all the feedback control systems we are using today are characterized by the necessity of satisfying different performance requirements and constraints. Hence, the synthesis of the electromagnetic vibration isolation system is necessary to be done. It can be done by using either conventional trial and error method or optimization theory. However, the synthesis based on conventional trial and error method is time consuming. In contrast, the synthesis using optimization theory can be done in less time than the synthesis using trial and error method. Moreover, the synthesis by optimization theory could give the best solution to the problem.

Three feedback control signals are considered in this chapter. They are relative displacement, absolute velocity and current. The feedback control law considered in this chapter is expressed as follows,

\[ u = -g_1 w_1 - g_2 w_2 - g_3 w_3 \]  (3.1)

where \( g_1 \) is the controller gain of relative displacement; \( g_2 \) is the controller gain of absolute velocity; \( g_3 \) is the controller gain of current.

In this chapter, the transfer functions with three feedback control signals for the electromagnetic suspension system will be derived and used to analyze the characteristics of the system. Two
controller gains selection methods based on different requirements will be introduced [9]. The root locus method will be applied to examine how the roots of the characteristics equation change with respect to the changes in controller gains. Linear Quadratic optimal control theory will be applied to find the optimum controller gains for the electromagnetic vibration isolation system based on several different performance measure requirements. An example using frequency response method is carried out to see how the steady-state response of the electromagnetic suspension system corresponding to a changing frequency sinusoidal input signal. The vibration isolation characteristic of the suspension system is compared with that of the passive isolation system. Dynamic simulation for the linear and nonlinear equations of the electromagnetic vibration isolation system will be illustrated to examine the differences between the linear and nonlinear equations.

3.2 Transfer Functions for Electromagnetic Suspension System with Three Feedback Control Signals

The nondimensional form linear equation for the electromagnetic suspension system is given by

\[
\begin{bmatrix}
\dot{w}_1 \\
\dot{w}_2 \\
\dot{w}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
k_1 & 0 & -k_2 \\
0 & \frac{k_1}{k_2} & -1
\end{bmatrix}
\begin{bmatrix}
w_1 \\
w_2 \\
w_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u +
\begin{bmatrix}
-1 \\
0 \\
\frac{k_1}{k_2}
\end{bmatrix} \dot{e}
\]

where \( k_1 = \frac{k_2 a L_0^2}{m a R^2} \) and \( k_2 = \frac{k_1 I L_0^2}{m a R^2} \) are dimensionless.
constants.

The input voltage, \( u \), is given as a function of relative displacement, absolute velocity and current. It is expressed as follows,

\[
u = -g_1 \dot{w}_1 - g_2 \dot{w}_2 - g_3 \dot{w}_3 \tag{3.1}
\]

Substituting equation (3.1) into equation (3.2) yields

\[
\begin{bmatrix}
\dot{w}_1 \\
\dot{w}_2 \\
\dot{w}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
-k_2 & k_1 & 0 \\
-g_3 & k_2 & g_2
\end{bmatrix}
\begin{bmatrix}
w_1 \\
w_2 \\
w_3
\end{bmatrix}
+ \begin{bmatrix}
-1 \\
0 \\
0
\end{bmatrix} \dot{e}
\]

Equation (3.3) can be represented as follow

\[
\dot{w}(t) = Aw(t) + B \dot{e}(t) \tag{3.4}
\]

In order to obtain the transfer functions, we take the Laplace transform of the equation (3.4) and set all the initial conditions to be zero, we get

\[
sW(s) = AW(s) + BS(s)
\]

\[
[sI - A]W(s) = BS(s)
\]

\[
W(s) = (sI - A)^{-1}BS(s)
\]

\[
W(s) = \Phi(s)BS(s) \tag{3.5}
\]

where \( \Phi(s) = (sI - A)^{-1} \) is the state-transition matrix for the
electromagnetic suspension system. The \([sI-A]\) matrix is

\[
[sI-A] = \begin{bmatrix}
s & -1 & 0 \\
-k_1 & s & k_2 \\
g_1 & \frac{-k_1}{k_2} + g_2 & s + l + g_3
\end{bmatrix}
\]

The state-transition matrix is

\[
\Phi(s) = [sI-A]^{-1}
\]

\[
= \begin{bmatrix}
s^2 + s + sg_3 + k_2g_2 + k_1 & s + l + g_3 & -k_2 \\
k_1s + k_1g_1 + k_2g_1 & s^2 + s + sg_3 & -k_2s \\
-k_1g_2 + \frac{k_1}{k_2}g_1s & sg_2 + \frac{k_1g_1}{k_2} & s^2 - k_1
\end{bmatrix}
\]

\[
= \frac{s^3 + (1 + g_3)s^2 - k_2g_2s - k_2g_1(1 + g_3)}{s^3 + (1 + g_3)s^2 - k_2g_2s - k_2g_1(1 + g_3)}
\]

Application of equation (3.5) and rearrangement of the equation give the transfer functions for the absolute velocity, absolute displacement and the current. The transfer function for the absolute velocity is given by

\[
\frac{W_2(s)}{E(s)} = G_2(s) = \frac{\gamma_0s}{s^3 + \gamma_2s^2 + \gamma_1s + \gamma_0}
\]

(3.6)

where \(s\) is the Laplace transform variable, \(W_2(s)\) is the Laplace
The transform of \( w_2(\tau) \), \( E(s) \) is the Laplace transform of \( e(\tau) \) and

\[
\gamma_0 = -k_1(1+g_3) - k_2 g_1
\]

(3.7a)

\[
\gamma_1 = -k_2 g_2
\]

(3.7b)

\[
\gamma_2 = 1 + g_3
\]

(3.7c)

Using the derivative property of the Laplace transform, \( W_2(s) = sW_0(s) \), the transfer function for the absolute displacement becomes

\[
\frac{W_0(s)}{E(s)} = G_0(s) = \frac{\gamma_0}{s^3 + \gamma_2 s^2 + \gamma_1 s + \gamma_0}
\]

(3.8)

where \( W_0(s) \) is the Laplace transform of \( w_0(\tau) \).

In addition, the transfer function for the current is given by

\[
\frac{W_3(s)}{E(s)} = G_3(s) = \frac{c_2 s^3 + c_1 s^2 + c_0 s}{s^3 + \gamma_2 s^2 + \gamma_1 s + \gamma_0}
\]

(3.9)

where \( W_3(s) \) is the Laplace transform of \( w_3(\tau) \) and

\[
c_0 = k_1 g_2
\]

(3.10a)

\[
c_1 = g_1
\]

(3.10b)

\[
c_2 = \frac{k_1}{k_2}
\]

(3.10c)

The stability test of this electromagnetic suspension system with three feedback control signals is shown in Appendix E.
3.3 Design Methods to Select the Controller Gains

In order to make the electromagnetic vibration isolation system to be superior in isolation performance and economical to operate when compared to passive vibration isolation system, the best way is to minimize the absolute displacement and the power dissipated by the system. Fabien [9] has proposed two design procedures of selecting the controller gains to meet the requirements mentioned previously. The functions he has minimized are expressed as follow

\[ J_1 = \int_{-\infty}^{\infty} |G_0(w)|^2 S_{\text{disp}}(w) \, dw \]  

(3.11a)

and

\[ J_2 = I^2 R \int_{-\infty}^{\infty} |G_3^*(w)|^2 S_{\text{vel}}(w) \, dw \]  

(3.11b)

where \( J_1 \) is the mean square response of the absolute displacement; \( S_{\text{disp}}(w) \) is the power spectral density of the input displacement; \( J_2 \) is the power dissipated by the system; \( G_3^*(s) \) is the transfer function of the current which has velocity as the input disturbance instead of the displacement, it is given by

\[ \frac{W_3(s)}{sE(s)} = G_3^*(s) = \frac{c_2s^2 + c_1s + c_0}{s^3 + \gamma_2s^2 + \gamma_1s + \gamma_0} \]  

(3.12)

\( S_{\text{vel}}(w) \) is the power spectral density of the input velocity.
Minimizing equation (3.11a), \( J_1 \), will yield an electromagnetic suspension system which is superior in vibration isolation performance. On the other hand, minimizing equation (3.11b), \( J_2 \), will result in an electromagnetic vibration isolation suspension system which consumes less energy; in other words, the operating cost of the electromagnetic vibration isolation suspension system is less. Before finding the analytical results for equations (3.11a) and (3.11b) the input disturbance power spectrum densities \( S_{\text{disp}}(w) \) and \( S_{\text{vel}}(w) \) are assumed to be ideally white, i.e., \( S_{\text{disp}}(w) = S_{\text{disp}} \) (a constant) and \( S_{\text{vel}}(w) = S_{\text{vel}} \) (a constant) for all frequencies. Using the result obtained by Crandall and Mark [10] (see Appendix F), equations (3.11a) and (3.11b) become

\[
J_1 = -\pi S_{\text{disp}} \frac{k_1(1+g_3)+g_1}{k_2(g_1+\frac{k_1}{k_2}-g_2)(1+g_3)} \quad (3.13a)
\]

and

\[
J_2 = \pi l^2 R S_{\text{vel}} \frac{g_2^2 k_1^2 - g_2^2 k_1^2(1+g_3)}{(k_1-\frac{g_2}{k_2})(1+g_3)+\frac{g_1 g_2}{k_2}} \quad (3.13b)
\]

The controller gains selection procedure which will yield a minimal \( J_1 \), Design Procedure I (DPI), is described as follows,

1) Choose the absolute velocity controller gain, \( g_2 \), to be a negative number as large as possible that can be implemented in practice.
2) Choose the current controller gain, \( g_3 \), to be greater than -1.
3) Determine \( k_1 \) and \( k_2 \), then calculate the relative displacement
controller using equation (3.14) stated below,

$$g_1 = \left(\frac{\xi}{k_2} - \frac{k_1}{k_2}\right)(1+g_3)$$  \hspace{1cm} (3.14)

where $\xi$ is a constant, $g_2 < \xi < 0$.

Substituting equation (3.14) into equation (3.13a) yields,

$$J_1 = -\pi S_{disp} \frac{\xi(1+g_3)}{\xi-g_2}$$  \hspace{1cm} (3.15)

It is obvious that we can make $J_1$ as close to zero as possible by selecting a very small $\xi$.

The design procedure of selecting controller gains for $J_2$, Design Procedure II (DP II), is stated as follows,

1) Determine $k_1$ and $k_2$, then choose the current controller gain, $g_3$, to be greater than -1.

2) Solve the equations $\frac{\partial J_2^*}{\partial g_1} = 0$ and $\frac{\partial J_2^*}{\partial g_2} = 0$ to find the $g_1$ and $g_2$ that minimize $J_2^*$. $J_2^*$ is expressed below,

$$J_2^* = \frac{J_2}{\pi I^2 R S_{vel}}$$  \hspace{1cm} (3.16)

Minimizing $J_2^*$ is the same as minimizing $J_2$ because $R$, $\pi$, $I$ and $S_{vel}$ are all constants.

### 3.4 Root Locus Analysis for the Design Methods

Root Locus analysis is a plot of the roots of the characteristic equation for a feedback control system as a function of the controller gains or other physical parameters in the system. It is a graphical
approach which clearly shows a designer the effects of the changes in controller gains or other physical parameters in the system with relatively small effort compared with other methods. In addition, a designer can easily get the best controller gains from the root locus plot for the system he analyzed to meet the requirements.

Root locus analysis will be applied to the controller gains selections methods given in the previous section to see how the changes of the controller gains will alter the locations of the poles of the characteristic equation and the behavior of the electromagnetic suspension system. The data set for the electromagnetic suspension system which will be used in all the analysis is listed in Table 3.1.

The root locus analysis for the design procedure which will yield a suspension system superior in vibration isolation performance is carried out as follows,

1. Pick $g_2=-5$ and $\xi=-0.2$.
2. Vary $g_3$ from -0.8 to 25 and compute $g_1$.
3. Find the roots of the characteristic equation and plot the roots as a function of $g_3$.
4. Repeat steps 1-3 with using $g_2=-10$ and -15.
5. Repeat steps 1-4 with using $\xi=-0.5$ and -1.0.

All the root-locus plots are shown in Figure 3.1. It is interesting to see that all the plots are similar to each other except the one using the parameters $g_2=-5$ and $\xi=-1.0$. At the beginning, the poles are a pair of complex roots and a real root. With the increase of $g_3$, the pair of complex poles meet together at the real axis and become real numbers. One of them goes to the far left of the break-in point, and the other one goes to the right of the break-in point. The one which goes to the right of break-in point meets the
Table 3.1  Data Set for Electromagnetic Suspension Isolation System

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_0)</td>
<td>(4.5239893 \times 10^{-4} \text{kgm}^3/\text{s}^2\text{A}^2)</td>
</tr>
<tr>
<td>(a_1)</td>
<td>(6.45 \times 10^{-4} \text{m})</td>
</tr>
<tr>
<td>I</td>
<td>(1.3265638199 \text{A})</td>
</tr>
<tr>
<td>R</td>
<td>(8.7382728296 \text{ohms})</td>
</tr>
<tr>
<td>(m)</td>
<td>(4.0 \text{kg})</td>
</tr>
<tr>
<td>a</td>
<td>(0.00254 \text{m})</td>
</tr>
<tr>
<td>(k_i)</td>
<td>(59.16037 \text{N/A})</td>
</tr>
<tr>
<td>(k_z)</td>
<td>(2.46405024 \times 10^4 \text{N/m})</td>
</tr>
<tr>
<td>(L_0)</td>
<td>(0.14204048 \text{H})</td>
</tr>
<tr>
<td>(k_1)</td>
<td>(1.627656)</td>
</tr>
<tr>
<td>(k_2)</td>
<td>(2.0409781)</td>
</tr>
<tr>
<td>(k_1/k_2)</td>
<td>(0.7975)</td>
</tr>
</tbody>
</table>
Figure 3.1 Root-Locus Plots: (a) $g_2=-5, \xi=-0.2$;
(b) $g_2=-10, \xi=-0.2$;
(c) $g_2=-15, \xi=-0.2$
Figure 3.1 Root-Locus Plots: (d) $g_2 = -5, \xi = -0.5$; 
(e) $g_2 = -10, \xi = -0.5$; 
(f) $g_2 = -15, \xi = -0.5$
Figure 3.1 Root-Locus Plots:

- (g) $g_2 = -5$, $\xi = -1.0$
- (h) $g_2 = -10$, $\xi = -1.0$
- (i) $g_2 = -15$, $\xi = -1.0$
real number pole at the breakaway point, and both become a complex pair. While the root-locus plot for the parameters \( g_2 = -5 \) and \( \xi = -1.0 \) does not have any break-in and breakaway points.

Using Figure 3.1 and equations (3.15) and (3.16), the following observations can be made:

1. Increase \( g_3 \) up to a certain limit will improve the robustness of the closed-loop system.
2. It is obvious that the disturbance attenuation of the suspension system will be decreased with an increase in \( g_3 \) or an increase in \( \xi \).
3. \( J_2 \) can be made smaller by using larger value of \( g_3 \) or \( \xi \).
4. The isolation characteristic will be improved with a larger value (a negative number) of \( g_2 \). However, energy dissipated by the suspension system will be increased with a decrease in \( g_2 \).
5. A smaller value of \( g_2 \) will enhance the robustness of the closed-loop system. While a smaller value of \( \xi \) will reduce the robustness of the closed-loop system.

Clearly it can conclude that in DPI there is always a trade off between robustness and disturbance attenuation when selecting controller gains \( g_1, g_2 \) and \( g_3 \).

DPII is applied to find the optimum controller gains for the electromagnetic suspension system before performing root locus analysis. The results are shown in Figures 3.2, 3.3 and 3.4. It can be seen from Figure 3.2 that \( J_2^* \) is large at \( g_3 = 0 \). It is slowly decreasing with an increasing in \( g_3 \). Both \( g_1 \) and \( g_2 \) are separately plotted in Figure 3.3 and 3.4 as a function of \( g_3 \). It is interesting that both curves are straight lines. The root locus analysis is done by varying \( g_3 \) value from -0.8 to 25. The root locus plot is drawn in Figure 3.5.
Figure 3.2 Optimum $J_2^*$ vs. $g_3$

Figure 3.3 Optimum $g_1$ vs. $g_3$
Figure 3.4 Optimum $g_2$ vs. $g_3$

Figure 3.5 Root Locus Plot for DPII
From Figure 3.5, it can be seen that an increase in $g_3$ from -0.8 to 25 will enhance the robustness of the closed-loop system.

3.5 Controller Gains Selection Using Linear Quadratic Optimal Technique

A brief introduction of Linear Quadratic (LQ) control theory is given in Appendix G. In this section, we will utilize LQ to find the optimum controller gains for the suspension system based on two different performance measures, minimizing mean square response of the absolute displacement and the energy dissipation. The state equations for the electromagnetic suspension system can be written as

$$\tilde{\mathbf{w}} = A\tilde{\mathbf{w}} + B\tilde{\mathbf{u}} + E\tilde{\mathbf{e}}$$ \hspace{1cm} (3.17)

where $\tilde{\mathbf{w}}$ is the state variables, $\tilde{\mathbf{u}}$ is the control input, and $\tilde{\mathbf{e}}$ is the input disturbance. $A$, $B$, and $E$ are separately defined in equation (3.2).

We want to minimize

$$J_1 = \int_{-\infty}^{\infty} \left| G_0(w) \right|^2 S_{\text{disp}}(w) \, dw$$ \hspace{1cm} (3.11a)

and

$$J_2 = \int_{-\infty}^{\infty} \left| G_3^*(w) \right|^2 S_{\text{vel}}(w) \, dw$$ \hspace{1cm} (3.11b)

Minimize $J_1$ will result in an electromagnetic suspension system with superior performance in vibration isolation. Minimize $J_2$ will yield a
energy saver vibration isolation system. Equations (3.11a) and (3.11b) are expressed in the frequency domain. Nevertheless, LQ theory is developed to optimize the closed-loop system with an performance index in the time domain. In order to minimize $J_1$ using LQ theory, the performance index, $J_1$, is equivalent to,

$$J_1 = \lim_{t \to \infty} \frac{1}{t} \mathbb{E} \left[ \int_0^t (\bar{w}^T Q_1 \bar{w} + u^T R u) \, dt \right]$$

(3.18)

where

$$Q_1 = 2\pi a S_{\text{disp}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(3.19)

and

$$R = [0.00001]$$

(3.20)

In the LQ formulation $J_2$ is equivalent to

$$J_2 = \lim_{t \to \infty} \frac{1}{t} \mathbb{E} \left[ \int_0^t (\bar{w}^T Q_2 \bar{w} + u^T R u) \, dt \right]$$

(3.21)

where

$$Q_2 = 2\pi I^2 R S_{\text{vel}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3.22)

note that the $R$ in $Q_2$ matrix (equation (3.22)) is the resistance value.
The $R$-matrix is the same as equation (3.20).

Upon the examination of the performance measures in equations (3.18) and (3.21), we recognize that the magnitude of the feedback control signal is not accounted because the value chosen in $R$ matrix is very small. However, there are many cases where we concern about the expenditure of the control signal energy which is expressed as $[u]^2$, the second term in the equations (3.18) and (3.21). In electromagnetic suspension system, $[u]^2$ represents the expenditure of the total energy consumption. In order to lessen the total energy consumption, we will use larger value in $R$ matrix. To achieve the goal which an electromagnetic suspension will provide superior isolation characteristics and consume less energy, we want to minimize the following performance index

\[
J_{1u} = \lim_{t \to \infty} \frac{1}{t} \mathbb{E} \left[ \int_0^t (\overline{w}^T Q_1 \overline{w} + \overline{u}^T R_{1u} \overline{u}) dt \right] 
\]

(3.23)

where

\[
R_{1u} = [a] 
\]

(3.24)

Similarly, we want to lower energy dissipation and total energy consumption together by minimizing equation (3.25)

\[
J_{2u} = \lim_{t \to \infty} \frac{1}{t} \mathbb{E} \left[ \int_0^t (\overline{w}^T Q_2 \overline{w} + \overline{u}^T R_{2u} \overline{u}) dt \right] 
\]

(3.25)

where

\[
R_{2u} = \left[ ma^2 \left( \frac{R}{L_0} \right)^3 \right] 
\]

(3.26)
Thus equations (3.18), (3.21), (3.23) and (3.25) can be used to obtained optimum controller gains which will minimize $J_1$, $J_2$, $J_{1u}$ and $J_{2u}$ for any given set of electromagnetic suspension data. However, LQ is an indirect approach to be used to optimize the electromagnetic suspension system comparing with the design procedures given in section 3.3 because it cannot penetrate the limitation performance capabilities of the electromagnetic suspension system.

3.6 Example

Now, it is time to apply frequency response technique to find out the steady-state responses of the electromagnetic suspension system. The physical parameters of the electromagnetic suspension used in the analysis are listed on Table 3.1. The optimum controller gains obtained from minimizing $J_1$ using LQ are listed in the first row of Table 3.2. The values of $J_1$ and $J^*_2$ are calculated and shown. Utilizing LQ to minimize the performance measures $J_{1u}$, $J_2$ and $J_{2u}$ gives the optimum controller gains that are separately listed in the second, third and fourth rows of the Table 3.2. The values of $J_1$ and $J^*_2$ are found and shown respectively in Table 3.2. The controller gains listed in the third row are very large because LQ method does not yield an insight into the limitation performance capabilities of the electromagnetic suspension system. The results obtained from DPI and DPII are also tabulated in Table 3.2 respectively. Moreover, notice that the controller gain $g_3$ is chosen to be zero in both cases because eliminating current sensor will cut down the cost of implementing an electromagnetic suspension system. The values $g_2=-10.0$ and $\zeta=0.3403$ are picked because they yield the same value
Table 3.2 Controller Gains from Various Design Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>$g_1$</th>
<th>$g_2$</th>
<th>$g_3$</th>
<th>$J_1$</th>
<th>$J_2^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LQ($J_1$)</td>
<td>-8.5194</td>
<td>-22.7233</td>
<td>8.6827</td>
<td>0.0352</td>
<td>18.2821</td>
</tr>
<tr>
<td>LQ($J_{1u}$)</td>
<td>-3.3692</td>
<td>-2.3025</td>
<td>2.2247</td>
<td>0.3881</td>
<td>2.6753</td>
</tr>
<tr>
<td>LQ($J_{2}$)</td>
<td>-2.7991e3</td>
<td>-2.1932e3</td>
<td>1.7553e3</td>
<td>0.6379</td>
<td>1.9944</td>
</tr>
<tr>
<td>LQ($J_{2u}$)</td>
<td>-5.6462</td>
<td>-3.6994</td>
<td>3.6065</td>
<td>0.6030</td>
<td>2.2293</td>
</tr>
<tr>
<td>DPI</td>
<td>-1.1375</td>
<td>-10.0</td>
<td>0.0</td>
<td>0.0352</td>
<td>18.7705</td>
</tr>
<tr>
<td>DPII</td>
<td>-1.5951</td>
<td>-2.0468</td>
<td>0.0</td>
<td>0.6338</td>
<td>2.6300</td>
</tr>
</tbody>
</table>
\( J_1 = 0.00352 \) as in minimizing \( J_1 \) using LQ method. Notice that the controller gains in the first and fifth rows that give superior disturbance attenuation have poor energy dissipation characteristic.

Equations (3.8) and (1.4) represent the transmissibility of an electromagnetic suspension system and a passive isolation system [12]. In order to show that electromagnetic suspension system has superior disturbance attenuation property over a passive vibration isolation system, the transmissibility of the electromagnetic suspension system is compared with the later’s. The transmissibility characteristic of a passive vibration isolation system has been extensively studied, and it has been shown that damping factor \( \zeta = 0.7071 \) will yield an optimum passive vibration isolation system. Hence, the value \( \zeta \) is selected to be 0.7071. The natural frequency of the passive isolation system is 5 (rad/sec).

The transmissibility plots of the passive vibration isolation and the electromagnetic suspension system using the controller displayed in Table 3.2 are depicted in Figures 3.6 and 3.7. In Figure 3.6, the transmissibility is plotted in a linear scale; whereas the transmissibility in Figure 3.7 is plotted in decibel (db). From Figure 3.6, all the frequency responses are the same in the frequency range of \( 1 \times 10^{-3} \) (rad/sec) to \( 2.9 \times 10^{-3} \) (rad/sec). The transmissibility with using controller gains from DPI and LQ\((J_1)\) are very similar and closed to each other. The frequency response with controllers DPIII has a peak transmissibility at 2 (rad/sec); similarly, DPI has a resonant peak at 4.4 (rad/sec). LQ\((J_2)\) and LQ\((J_{2u})\) have the same transmissibility until 1.3 (rad/sec). The frequency response of the electromagnetic suspension system is overall much better than that of the passive vibration isolation system.
Figure 3.6 Frequency Response Plot for Displacement (Linear Scale)
Legend: --- Passive, DPII; •••• LQ(J1), LQ(J1 u), LQ(J2);
•••• LQ(J2 u); --- DPI

Figure 3.7 Frequency Response Plot for Displacement (Decibel Scale)
Legend: --- Passive, DPII; •••• LQ(J1), LQ(J1 u), LQ(J2);
•••• LQ(J2 u); --- DPI
In Figure 3.7, we can clearly see that the transmissibility with controllers DPI is slightly lower than that of the controllers \( LQ(J_1) \) in 9 (db) for frequencies > 7 (rad/sec). In the frequency range of 10 (rad/sec) to 100 (rad/sec), \( LQ(J_1) \), \( LQ(J_{1u}) \) and DPII have the same level of transmissibility.

The current frequency response plots using controller gains listed in Table 3.2 are presented in Figures 3.8 (linear scale) and 3.9 (db scale). For frequency < 0.45 (rad/sec), the current frequency responses with controllers \( LQ(J_{1u}), LQ(J_2), LQ(J_{2u}) \) and DPII are much lower compared with that of the \( LQ(J_1) \) and DPI. In the frequency range of 0.7 (rad/sec) to 3.4 (rad/sec), \( LQ(J_1) \) and DPI have the lowest energy dissipation. For frequencies > 25 (rad/sec), the current frequency responses of \( LQ(J_1), LQ(J_{1u}), LQ(J_{2u}) \), DPI and DPII are exactly the same.

From the results given above and the frequency response plots, the controller gains from DPI can be considered as the best solution to the electromagnetic suspension system. The reasons are

1. They yield a lower displacement frequency response overall. Especially for frequencies > 7 (rad/sec), the transmissibility is the lowest.

2. No current sensor is needed. It will directly reduce the cost of implementing the electromagnetic vibration isolation system.

3. For frequencies > 0.7, the power dissipated by the system is similar to that from other methods.

3.7 Dynamic Simulation

The equations of electromagnetic suspension system are originally nonlinear. The equations have been linearized because we
Figure 3.8  Frequency Response Plot for Current (Linear Scale)
Legend : --- DPII; •••• LQ(J₁), LQ(J₁₁), LQ(J₂); ——•• LQ(J₂₁);
        —— DPI

Figure 3.9  Frequency Response Plot for Current (Decibel Scale)
Legend : --- DPII; •••• LQ(J₁), LQ(J₁₁), LQ(J₂); ——•• LQ(J₂₁);
        —— DPI
want to simplify the analysis and find the transfer functions for the electromagnetic suspension system. The linear model is valid as long as that the results obtained from the linear case yield a good approximation to the results from the nonlinear model. In order to find out the differences between linear and nonlinear equations, simulation of the control system using numerical technique on a digital computer is needed. The Runge-Kutta-Fehlberg method is used. It uses a Runge-Kutta method with local truncation error of order five to estimate the local truncation error in a Runge-Kutta method of order four [13].

The feedback control law is expressed as

\[ u = -g_1w_1 - g_2w_2 - g_3w_3 \quad (3.1) \]

in the nondimensional form. It becomes

\[ u_0 = -\bar{g}_1z_1 - \bar{g}_2z_2 - \bar{g}_3z_3 \quad (3.27) \]

where

\[ \bar{g}_1 = \frac{g_1IR}{a} \quad (3.28a) \]
\[ \bar{g}_2 = \frac{g_2IL_0}{a} \quad (3.28b) \]
\[ \bar{g}_3 = g_3R \quad (3.28c) \]

in the dimensional linear case. In the nonlinear model, the control voltage is given by

\[ v = -\bar{g}_1x_1 - \bar{g}_2x_2 - \bar{g}_3x_3 \quad (3.29) \]

where \( \bar{g}_1, \bar{g}_2 \) and \( \bar{g}_3 \) are defined in equation (3.28) respectively.
A step input disturbance is used to examine the differences between linear and nonlinear models and the behavior of the electromagnetic suspension in time domain. The step input function has the following relations:

\[
\begin{align*}
\dot{y} &= \dot{e}_0 = -0.06 \text{ (m/sec)} \quad 0 \text{ (sec)} \leq t \leq 1 \text{(sec)} \\
\dot{y} &= \dot{e}_0 = 0 \text{ (m/sec)} \quad t > 1 \text{(sec)}
\end{align*}
\]  

(3.30)

The controller gains chosen from DPI are used. They are \( g_1 = -5.9849 \), \( g_2 = -5.0 \) and \( g_3 = 5.0 \). The state responses are shown in Figure 3.10. From Figure 3.10, the only difference in the responses of the system between linear and nonlinear is near the end of the transient response regions. The relative displacement response curve and the current response curve have the similar shape. It can be concluded that the linear model is a good approximation for the nonlinear model.

![Relative Displacement Response](image-url)
Figure 3.10b Absolute Velocity Response

Figure 3.10c Current Response
Chapter 4
Optimization of Five Controller Signals

4.1 Introduction

Optimization is desirable in most of the design of feedback control system. Many research has been done on solving various types of optimal control problems. However, the difficulty of the optimization increases with the number of variables considered in the optimization.

A performance index is usually used in optimal control problem. It can be defined as a quantitative measure of the performance of a control system. In the design of an optimum control system, performance index is always minimized in order to find the best parameters to meet the important system specifications. Two performance indices are used in optimizing the electromagnetic suspension system.

In Chapter 3, optimization on an electromagnetic suspension system of three feedback controller gains has been considered; similarly, optimization will be utilized to find an optimum electromagnetic suspension system of five control signals. These five control signals are absolute displacement, relative displacement, absolute velocity, relative velocity and current. They can be expressed in the mathematical form as follow,

\[ u = -g_0 w_0 - g_1 w_1 - g_2 w_2 - g_3 w_3 - g_4 w_4 \]  \hspace{1cm} (4.1)

where \( g_0 \) is the control signal of absolute displacement; \( g_1 \) is the
control signal of relative displacement; $g_2$ is the control signal of absolute velocity; $g_3$ is the control signal of current; $g_4$ is the control signal of relative velocity.

The transfer functions derived in Chapter 2 will be used to investigate the characteristic of the electromagnetic suspension system. Two performance measures will be studied and minimized. One emphasizes on minimizing the mean square response of the absolute displacement, and the other one highlight on minimizing the dissipation energy. Several design procedures on selecting controller gains are proposed based on these two performance indices. The design methods are analyzed using root locus technique to see how the changes in selecting different combinations of controller gains will influence the behave of the system. Linear Quadratic (LQ) optimal theory is then employed to find controller gains that will minimize the performance measures mentioned earlier. Via an example to find out which controller gains selection method lead the best solution is illustrated.

### 4.2 Controller Gains Selections

An active vibration isolation system that possesses these two features, superior in disturbance attenuation and economical to operate, is considered to be an optimum active isolator. To make the electromagnetic suspension system to be superior as opposed to other types of isolator, we want to minimize

$$J_0 = \int_{-\infty}^{\infty} |T_0(w)|^2 S_{\text{disp}}(w) \, dw$$

(4.2)
and

\[ J_3 = I^2 R \int_{-\infty}^{\infty} |T_3^*(w)|^2 S_{\text{vel}}(w) \, dw \]  (4.3)

where \( J_0 \) is the mean square response of the absolute displacement; \( T_0 \) is the transfer function of the absolute displacement as defined in equation (2.13); \( S_{\text{disp}}(w) \) is the power spectral density of the input displacement; \( J_3 \) is the mean square response of the current; \( S_{\text{vel}}(w) \) is the power spectral density of the input velocity; \( T_3^*(s) \) is the transfer function of the current which has velocity as the input disturbance instead of the displacement. It is given by the following relation:

\[ \frac{W_3(s)}{sE(s)} = T_3^*(s) = \frac{h_3 s^2 + h_2 s + h_1}{s^3 + \sigma_2 s^2 + \sigma_1 s + \sigma_0} \]  (4.4)

where \( h_3, h_2, h_1, \sigma_2, \sigma_1 \) and \( \sigma_0 \) are defined in equations (2.11) and (2.15) respectively.

The similar approaches used in optimizing an active vibration isolation system have been considered by Fabien and Karnopp [9, 14].

Minimizing \( J_0 \), equation (4.2), will result in an electromagnetic suspension system which is superior in disturbance attenuation feature. Whereas minimizing \( J_3 \), equation (4.3), will yield an electromagnetic suspension system that is economical to operate.

In order to use the result obtained by Crandall and Mark [10], the power spectrum densities \( S_{\text{disp}}(w) \) and \( S_{\text{vel}}(w) \) are assumed to be...
ideally white, i.e., $S_{\text{disp}}(w) = S_{\text{disp}}$ (a constant) and $S_{\text{vel}}(w) = S_{\text{vel}}$ (a constant) for all frequencies. Note that in equation (4.4), the absolute displacement controller gain is chosen as zero because we want to have velocity as the input disturbance instead of displacement and utilize the result given by Crandall and Mark [10]. In addition, letting the power spectrum densities to be ideally white is to simplify the analysis and make the analysis to be widely used in most of the situations.

Applying the result from Crandall and Mark [10] (see Appendix F), equation (4.3) becomes

$$J_0 = \pi s_{\text{disp}} \frac{[k_1(1+g_3) + k_2g_1]^2(1+g_3) + k_2^2 g_4^2[-k_1(1+g_3) - k_2(g_0+g_1)]}{[-k_2(g_2+g_4)(1+g_3) + k_1(1+g_3) + k_2(g_0+g_1)] - k_1(1+g_3) - k_2(g_0+g_1)}$$

(4.5)

The steps of choosing controller gains that will minimize $J_0$, Design Procedure 1 (DP1), is listed as follows,

1) Determine the magnet stiffness constants $k_1$ and $k_2$ and select a value (negative number) as large as possible that can be implementing physically for the absolute velocity controller $g_2$.

2) Select the current controller gain $g_3 > -1$ and the relative displacement controller $|g_4| < -1$.

3) Select the absolute controller gain $g_0$ such that $2(g_2+g_4)(1+g_3) < g_0 \leq 0$.

4) Select the relative displacement controller gain

$$g_1 = -g_0 \frac{k_1(1+g_3)}{k_2} \delta$$

(4.6)

where $\delta$ value is selected in the range of $-g_0/2 < \delta < -(g_2+g_4)(1+g_3)$. 


Using equation (4.6) in equation (4.5) results in

\[ J_0 = \frac{\left( \frac{g_0}{\delta} - 1 \right)^2 (1+g_3) + k_2 g_4^2}{\left[ -(g_2 + g_4 - \delta) \right]} \]  

(4.7)

The other way of minimizing \( J_0 \) is to make the numerator of equation (4.5) to approach zero. Examine equation (4.5) gives the following solutions

(i) \( J_0 \rightarrow 0 \) as \( g_2 \rightarrow -\infty \) and \( g_4 \rightarrow 0 \)
(ii) \( J_0 \rightarrow 0 \) as \( g_3 \rightarrow -1 \) and \( g_4 \rightarrow 0 \)
(iii) \( J_0 \rightarrow 0 \) as \( g_1 \rightarrow -k_1(1+g_3)/k_2 \) and \( g_0 \rightarrow 0 \)
(iv) \( J_0 \rightarrow 0 \) as \( g_1 \rightarrow -k_1(1+g_3)/k_2 \) and \( g_4 \rightarrow 0 \)

Solutions (i), (ii) and (iii) are impractical because solution (i) needs an infinite controller gain \( g_2 \) and solutions (ii) and (iii) will make an unstable electromagnetic suspension system. Solution (iv) is the only practical solution. Using the result from (iv), a design procedure which will yield an electromagnetic suspension system superior in vibration isolation characteristic is developed. It is listed as follows:

**Design Procedure 2 (DP2)**

1) Determine the magnet stiffness constants \( k_1 \) and \( k_2 \).
2) Set the controller \( g_4 = 0 \).
3) Select the controllers \( g_2 < 0 \) and \( g_3 > -1 \).
4) Select the absolute displacement controller \( g_0 \) such that

\[ g_2(1+g_3) < g_0 < 0. \]

5) Calculate the relative displacement controller gains from

\[ g_1 = \frac{k_1(1+g_3)}{k_2} \]  

(4.8)
Using the controller chosen from DP2 will make the mean square response of the absolute displacement close to zero.

Apply the result given by Crandall and Mark [10] (see Appendix F) to equation (4.3), we get

\[
J_3 = I^2 R \frac{\pi s_{vel}^2}{k_1 g_2^2} \left( \frac{1}{1+g_3} + g_1^2 - 2 k_1 g_2 \left( g_4 - \frac{k_1}{k_2} \right) k_2 + g_4^2 \left( g_4 - \frac{k_1}{k_2} \right)^2 \right)
\]

\[
\left[ - k_2 (g_2 + g_4) (1+g_3) + k_1 (1+g_3) + k_2 (g_0 + g_1) \right]
\]

(4.9)

The purpose here is to figure out a procedure in which is used to find the values of the controller gains that will minimize \( J_3 \). One such procedure, Design Procedure 3 (DP3), can be stated as follows:

1) Determine the values of \( k_1 \) and \( k_2 \).
2) Choose a value for the controller \( g_3 > -1 \).
3) Determine the values for \( g_1 \), \( g_2 \) and \( g_4 \) that will minimize \( J_3 \) by solving the following equations

\[
\frac{\partial J_3^*}{\partial g_1} = 0
\]

(4.10a)

\[
\frac{\partial J_3^*}{\partial g_2} = 0
\]

(4.10b)

\[
\frac{\partial J_3^*}{\partial g_4} = 0
\]

(4.10c)

where

\[
J_3 = \frac{J_3}{\pi I^2 R s_{vel}}
\]

(4.11)
The values of I and R are constant, so that minimizing $J_3^*$ is the same as minimizing $J_3$.

The DP3 is illustrated by finding the values of the controller for various $k_1$, $k_2$ and $g_3$. The plots of $J_3^*$ versus $g_3$ for different combinations of $k_1$ and $k_2$ are shown in Figure 4.1. We can clearly see that $J_3^*$ is largest at the stability boundary region, $g_3 \rightarrow -1$. $J_3^*$ decreases dramatically in the range of $g_3$ from -0.8 to 0. For $g_3 > 2$, $J_3^*$ is a constant for a given ratio $(k_1/k_2)$. For a given $g_3$, $J_3^*$ can be made smaller by decrease the ratio $(k_1/k_2)$. It can be seen from Figure 4.1, given a $g_3$ and a ratio $(k_1/k_2)$, $J_3^*$ can be made smaller by using larger values of $k_1$ and $k_2$.

Illustrated in Figure 4.2 are the plots of $g_1$ versus $g_3$ for various $k_1$ and $k_2$. The plots of $g_2$ and $g_4$ versus $g_3$ for various $k_1$ and $k_2$ are presented in Figures 4.3 and 4.4 respectively.

From Figures 4.2 and 4.3, it can concluded that the values of $g_1$ and $g_2$ depend on the ratio $(k_1/k_2)$ as well as the individual value of $k_1$ and $k_2$. For a given ratio $(k_1/k_2)$, $g_1$ and $g_2$ decrease with increasing $g_3$. Given $g_3 > 2$, $g_1$ and $g_2$ decrease with increasing the ratio $(k_1/k_2)$. In Figure 4.4, it can be seen that the value of $g_4$ equal to the given ratio $(k_1/k_2)$ when $g_3 > 3$. Hence, $g_4$ depends on the ratio $(k_1/k_2)$ only when $g_3 > 3$.

4.3 Root Locus Analysis for the Design Procedures

Root locus method is a popular graphic technique used in designing and analyzing a control system. Root locus plot can clearly show the trend movements of the poles. Changing parameters in a control system will directly influence the characteristics of the control system. Thus root locus technique is always favored by a
Figure 4.1a Optimum $J_3^*$ vs. $g_3$ for various $k_1$ and $k_2$ ($k_1/k_2 = 0.8$)

Figure 4.1b Optimum $J_3^*$ vs. $g_3$ for various $k_1$ and $k_2$ ($k_1/k_2 = 0.9$)
Figure 4.1c Optimum $J_3^*$ vs. $g_3$ for various $k_1$ and $k_2$ ($k_1/k_2=1.0$)

Figure 4.2a Optimum $g_1$ vs. $g_3$ for various $k_1$ and $k_2$ ($k_1/k_2=0.8$)
Figure 4.2b Optimum $g_1$ vs. $g_3$ for various $k_1$ and $k_2$ ($k_1/k_2=0.9$)

Figure 4.2c Optimum $g_1$ vs. $g_3$ for various $k_1$ and $k_2$ ($k_1/k_2=1.0$)
Figure 4.3a Optimum $g_2$ vs. $g_3$ for various $k_1$ and $k_2$ ($k_1/k_2=0.8$)

Figure 4.3b Optimum $g_2$ vs. $g_3$ for various $k_1$ and $k_2$ ($k_1/k_2=0.9$)
Figure 4.3c Optimum \( g_2 \) vs. \( g_3 \) for various \( k_1 \) and \( k_2 \) (\( k_1/k_2=1.0 \))

Figure 4.4a Optimum \( g_4 \) vs. \( g_3 \) for various \( k_1 \) and \( k_2 \) (\( k_1/k_2=0.8 \))
Figure 4.4b Optimum $g_4$ vs. $g_3$ for various $k_1$ and $k_2$ ($k_1/k_2=0.9$)

Figure 4.4c Optimum $g_4$ vs. $g_3$ for various $k_1$ and $k_2$ ($k_1/k_2=1.0$)
control engineer to apply in analyzing the changes of behavior in a control system due to the changes in the parameters of the control system.

In this section, root locus technique is used in analyzing how the different combinations of controller gains will influence the locations of the poles of the electromagnetic suspension system. The same set of data tabulated in Table 3.1 will be used in the analysis.

The root locus analysis for DP1 is accomplished as follows,

1. Pick $\delta=0.8$.
2. Select $g_2=-5.0$, $g_4=-0.5$ and $g_0=0.0$.
3. Vary $g_3$ from -0.8 to 25 and compute $g_1$.
4. Find the roots of the characteristic equation, then plot the roots on a s-plane.
5. Repeat doing steps 2-4 by using $g_4=0.0$ and 0.5.
6. Repeat steps 2-4 with using $g_0=-1$.
7. Redo steps 2-4 with using $g_2=-10.0$, $g_0=0.0$ and -1.0.

The root-locus plots are depicted in Figure 4.5. It can be seen from Figures 4.5a, 4.5b and 4.5c, increase $g_4$ from -0.5 to 0.5 does not change much in the locations of the poles. While increase $g_3$ will change the poles dramatically. All of these root locus plots have the similar shape; with an increase in $g_3$ up to a certain number, the complex pair meet each other at the break-in point. One of them goes to the far left of the break-in point, and the other goes to the right of the break-in point. The real axis pole meets the pole that goes to the right of the break-in point at the breakaway point becoming a complex pair. Notice that in Figure 4.5a, change $g_0$ will not change the locations of the poles of the system. The robustness of the system will be improved with an increase in $g_2$. 
Figure 4.5 Root-Locus Plots: (a) $g_0 = 0.0$ or -1.0, $g_2 = -5$, $g_4 = -0.5$, $\delta = 0.8$; (b) $g_0 = 0.0$, $g_2 = -5$, $g_4 = 0.0$, $\delta = 0.8$
Figure 4.5 Root-Locus Plots: (c) $g_0=0.0$, $g_2=-5$, $g_4=0.5$, $\delta=0.8$;
(d) $g_0=0.0$ or $-1.0$, $g_2=-10.0$,
$g_4=-0.5$, $\delta=0.8$
The steps of doing root-locus analysis for DP2 is listed below,

1. Select $g_2 = -10.0$.
2. Pick $g_0 = -0.5$.
3. Vary $g_3$ from -0.8 to 12 and calculate $g_1$ then find the roots of the characteristic equation and draw the locus of the roots in a s-plane as $g_3$ is varied.
4. Redo steps 2-3 with using $g_0 = -1.0$ and -1.5.
5. Repeat steps 2-4 by sing $g_2 = -15.0$.

Figure 4.6 shows the root-locus plots of DP2. Increase $g_3$ will enhance the robustness of the system. Whereas increase $g_0$ will reduce the robustness of the system. Decrease $g_2$ will weaken the robustness of the closed-loop system.

Before performing root-locus analysis on DP3, DP3 is employed to find optimum controller gains for the electromagnetic suspension. The results are presented in Figures 4.7, 4.8, 4.9 and 4.10. The root-locus analysis is carried out in the range of $g_3 = -0.8$ to 6.0. The plot is shown in Figure 4.11. From Figure 4.11, an increase in $g_3$ will strengthen the robustness of the closed-loop system.

4.4 Linear Quadratic Optimal Method on Selecting Controller Gains

In section 4.2, three design methods are proposed to find optimum controller gains to minimize the transmissibility, $J_0$, and the dissipation power, $J_3$. Those design procedures are developed using equation in the frequency domain.

In this section, Linear Quadratic (LQ) optimal control theory which is primary developed to solve problem in time domain is applied to find controller gains that will minimum $J_0$ and $J_3$. The
Figure 4.6 Root-Locus Plots: (a) $g_0 = -0.5$, $g_2 = -10.0$, $g_4 = 0.0$; (b) $g_0 = -1.0$, $g_2 = -10.0$, $g_4 = 0.0$; (c) $g_0 = -1.5$, $g_2 = -10.0$, $g_4 = 0.0$
Figure 4.6 Root-Locus Plots: (d) \( g_0 = -0.5, g_2 = -15.0, g_4 = 0.0 \);
(e) \( g_0 = -1.0, g_2 = -15.0, g_4 = 0.0 \);
(f) \( g_0 = -1.5, g_2 = -15.0, g_4 = 0.0 \)
Figure 4.7 Optimum $J_3^*$ vs. $g_3$ ($k_1/k_2=0.7975$)

Figure 4.8 Optimum $g_1$ vs. $g_3$ ($k_1/k_2=0.7975$)
Figure 4.9 Optimum $g_2$ vs. $g_3$ ($k_1/k_2=0.7975$)

Figure 4.10 Optimum $g_4$ vs. $g_3$ ($k_1/k_2=0.7975$)
Real Axis

Figure 4.11 Root Locus Plot for DP3
state equation of the electromagnetic suspension system is

\[
\ddot{\bar{w}} = A\bar{w} + Bu + E\dot{e}
\]

(4.12)

where \( \bar{w} \) is the state variables, \( \bar{u} \) and \( \dot{e} \) are the control voltage and the input velocity disturbance respectively. \( A, B \) and \( E \) are defined in equation (2.4). If we apply LQ method in equation (4.12) to find optimum controller gains for the electromagnetic suspension system in minimizing \( J_0 \) and \( J_3 \), then we will only get three controller gains, \( g_1, g_2 \) and \( g_3 \) because \( A \) matrix in equation (4.12) is a 3 by 3 matrix.

In order to consider five controller gains, \( g_0, g_1, g_2, g_3 \) and \( g_4 \), as shown in equation (4.1) when apply LQ optimal theory, \( A \) matrix has to be modified into a 5 by 5 matrix. Control of linear system by output proportional plus derivative feedback has been considered by Haraldsdattir, Kabamba and Ulsoy [15]. The similar approach shown in [15] is used to expand \( A \) matrix from a 3 by 3 matrix to a 5 by 5 matrix.

First, let us assume the input disturbance is a second order equation as shown below,

\[
\ddot{e} + m_1\dot{e} + m_2e = 0
\]

(4.13)

where \( m_1 \) and \( m_2 \) are constants. Then rewrite equation (4.13) into state variable form, we get

\[
\begin{bmatrix}
\dot{e}_1 \\
\dot{e}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-m_2 & -m_1
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix}
\]

(4.14)

where

\[
e_1 = e
\]

(4.15)
Substitute equation (4.14) into equation (2.4), then rearrange the equation to obtain

\[
\begin{bmatrix}
\dot{w}_1 \\
\dot{w}_2 \\
\dot{w}_3 \\
\dot{e}_1 \\
\dot{e}_2
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 & 0 & -1 \\
- \frac{k_1}{k_2} & 0 & -k_2 & 0 & 0 \\
0 & \frac{k_1}{k_2} & 1 & 0 & \frac{k_1}{k_2} \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -m_2 & -m_1
\end{bmatrix}
\begin{bmatrix}
w_1 \\
w_2 \\
w_3 \\
e_1 \\
e_2
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
0
\end{bmatrix} u
\]

where the control input \( u \) has the following relations:

\[ u = -(g_0 + g_1)w_1 - (g_2 + g_4)w_2 - g_3w_3 - g_0e_1 - (-g_4)e_2 \]  

(4.17)

Equation (4.17) can be rewritten as

\[ \dot{p} = Ap + Bu \]  

(4.19)

where \( p = [w^T, e^T]^T \).

A matrix in equation (4.19) is a 5 by 5 matrix. Therefore, equation (4.17) or (4.19) is used in LQ optimal method to compute the controller gains, \( g_0, g_1, g_2, g_3 \) and \( g_4 \), that minimize \( J_0 \) and \( J_3 \). Equation (4.20) in time domain is equivalent to equation (4.2) in frequency domain,

\[ J_0 = \lim_{t \to \infty} \mathbb{E} \left[ \int_0^t (p^T Q_0 p + u^T R u) dt \right] \]  

(4.20)
where

\[
Q_0 = 2\pi a S_{\text{disp}} \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]  \hspace{1cm} (4.21)

and

\[
R = \begin{bmatrix} 0.00001 \end{bmatrix}
\]  \hspace{1cm} (4.22)

Similarly, \( J_3 \) is equivalent to

\[
J_3 = \lim_{t \to \infty} \mathbb{E} \left[ \int_0^t (p^T Q_3 p + u^T R u) dt \right]
\]  \hspace{1cm} (4.23)

where

\[
Q_3 = 2\pi l^2 R S_{\text{vel}} \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]  \hspace{1cm} (4.24)

and \( R \) matrix is the same as equation (4.22).

The design procedures given in section 4.2 and the LQ methods proposed in this section will be demonstrated through an example in next section.

4.5 Example

The physical parameters of the electromagnetic suspension system used in the example are listed in Table 3.1. The design
methods and LQ optimal theory will be utilized to find the optimum controller gains for this electromagnetic suspension. Frequency response method is applied to analyze the steady state responses of the system.

The optimum controller gains found from using the design methods and LQ theory are listed in Table 4.1. The values of $J_0$ and $J_3^*$ are also determined and shown in Table 4.1. Notice that the controller gains obtained from LQ($J_3$) is very large because LQ theory does not give an insight to the limitation of the electromagnetic suspension.

The absolute displacement frequency response using the controller values listed in Table 4.1 are illustrated in Figures 4.12 (linear scale) and 4.13 (decibel scale). Also the transmissibility of the passive vibration isolation system is shown in Figures 4.12 and 4.13. The frequency responses with controllers from DP3, DP1 and passive system have resonant peak at 1 (rad/sec), 4 (rad/sec) and 4.5(rad/sec) respectively. For frequencies $< 0.24$ (rad/sec), the frequency response with controllers LQ($J_3$) is less than the frequency responses with controllers from DP1 and DP3. From Figure 4.12, the transmissibility with controllers DP2 is almost zero. The frequency response with LQ($J_0$) is also very low compared with that of LQ($J_3$), DP1 and DP3. We can clearly see that the frequency response with DP2 has a resonant peak at 4.3 (rad/sec) in Figure 4.13.

The current frequency response curves are shown in Figures 4.14 (linear scale) and 4.15 (decibel scale). For frequencies $< 0.7$ (rad/sec), the frequency responses with controllers from LQ($J_3$), DP1, DP2 and DP3 are much lower than that of LQ($J_0$). For frequencies $> 5.8$ (rad/sec), the controller gains from DP2 yield the lowest current
Table 4.1 Controller Gains from Various Design Methods

<table>
<thead>
<tr>
<th></th>
<th>DP1</th>
<th>DP2</th>
<th>DP3</th>
<th>LQ(J₀)</th>
<th>LQ(J₃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>g₀</td>
<td>0.0</td>
<td>-5.0</td>
<td>0.0</td>
<td>-0.7843</td>
<td>-1.963e3</td>
</tr>
<tr>
<td>g₁</td>
<td>-1.7975</td>
<td>-0.7975</td>
<td>-0.9737</td>
<td>-7.7351</td>
<td>-8.361e2</td>
</tr>
<tr>
<td>g₂</td>
<td>-10.0</td>
<td>-10.0</td>
<td>-0.3944</td>
<td>-22.675</td>
<td>-2.094e3</td>
</tr>
<tr>
<td>g₃</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>8.6827</td>
<td>1.7553e3</td>
</tr>
<tr>
<td>g₄</td>
<td>-0.5</td>
<td>0.0</td>
<td>-0.3261</td>
<td>-0.048</td>
<td>-9.94e1</td>
</tr>
<tr>
<td>J₀</td>
<td>0.1590</td>
<td>0.0</td>
<td>0.7266</td>
<td>3.1188e-5</td>
<td>0.1084</td>
</tr>
<tr>
<td>J₃*</td>
<td>6.5435</td>
<td>1.3343</td>
<td>2.2575</td>
<td>18.1774</td>
<td>1.0968</td>
</tr>
</tbody>
</table>
Figure 4.12 Frequency Response Plot for Displacement (Linear Scale)
Legend: --- Passive, LQ(J_0); • • • LQ(J_3); — — • DP1; ——— DP2, DP3

Figure 4.13 Frequency Response Plot for Displacement (Decibel Scale)
Legend: --- Passive, LQ(J_0); • • • LQ(J_3); — — • DP1; ——— DP2, DP3
Figure 4.14 Frequency Response Plot for Current (Linear Scale)  
Legend : --- LQ(J₀), DP3; •••• LQ(J₃); —— DP1; — DP2

Figure 4.15 Frequency Response Plot for Current (Decibel Scale)  
Legend : --- LQ(J₀), DP3; •••• LQ(J₃); —— DP1; — DP2
frequency response. The frequency response with controller DP3 has a resonant peak at 1.1 (rad/sec). Similarly, the current frequency responses with controllers DP1 and DP2 exhibit similar resonant peak at 5 (rad/sec) and 4.8 (rad/sec) respectively.

Based on the frequency response plots, DP2 gives the best selection of controller gains for the electromagnetic suspension. The reasons are:

1. The controller obtained from DP2 gives the lowest transmissibility overall compared with other design procedures.
2. For frequencies >5.8 (rad/sec), the energy dissipation is better than that from DP1, DP3 and LQ(J_o).
3. No current sensor and relative velocity transducer are needed.
Chapter 5
Conclusions

In this thesis, the equation of motion for a single axis electromagnetic suspension is shown. The state feedback and state plus derivative feedback are employed to control the suspension system. The original state variables of the system are the deviation of airgap size, absolute velocity and current. Two additional state variables used in the feedback control are absolute displacement and relative velocity.

The stability test is performed to determine the ranges of the controller gains that will make the suspension system become stable. The stability boundary plots are presented. They can be used as design tools to determine stabilizing controller gains for an electromagnetic suspension system.

Two performance indices, mean square responses of absolute displacement and current, are used as the design criteria to find optimum controller gains. Minimize the mean square response of the absolute displacement gives a suspension that is superior in vibration isolation performance. On the other hand, minimize the mean square response of the current yield an electromagnetic suspension that has low energy dissipation; in other wards, it is economical to operate. These two performance measures are minimized in both frequency and time domains. Direct approach on transfer function is used in frequency domain minimization. Linear Quadratic optimal theory (LQ) is used in time domain minimization. In both cases, the input disturbance is assumed to be ideally white.
In Chapter 3, an active control scheme, based upon relative displacement, absolute velocity and current, is considered. Two design procedures are given. Design Procedure I (DPI) yields an electromagnetic suspension that has superior isolation performance. Design Procedure II yields a suspension whose energy dissipation is the lowest. Linear quadratic method is applied to find optimal controller gains to minimize the performance indices mentioned previously. From the transmissibility plots and current frequency response plots, the best choice of controllers are found to be the ones obtained from DPI because they yield minimum transmissibility.

In Chapter 4, several design procedures are proposed. Design Procedure 1 (DP1) and 2 (DP2) are developed based on minimizing the mean square response of the absolute displacement. Design Procedure 3 (DP3) gives a suspension which has lowest energy dissipation when the suspension is subjected to random white noise vibration. It is illustrated for various combinations of \( k_1 \) and \( k_2 \). These plots can be used as design tools.

The results obtained from DPI are compared with the results obtained from DP2. The transmissibility plot and current frequency response plot are shown in Figures 5.1 and 5.2. From these two plots, it is obvious that DP2 yields a superior suspension system. However, control of absolute displacement is very costly.

Using the proposed design procedures, we can get many sets of controller gains for particular \( k_1 \) and \( k_2 \); while in LQ method to minimize \( J_0 \) or \( J_3 \), only one set of controller gains can be obtained. In addition, the controller gains obtained from the design procedures can always be implemented physically. Therefore, it can be concluded that using the proposed design procedures we can get
better selection of controllers than using LQ method.
Figure 5.1  Frequency Response Plot for Displacement (db Scale)
Legend : – DPI; --- DP2

Figure 5.2  Frequency Response Plot for Current (db Scale)
Legend : – DPI; --- DP2
Appendix A

Nomenclature

\( A_c \) = the cross section area of the core normal to the flux path
\( A_s \) = the cross section area of the support piece normal to the flux path
\( A_g \) = the cross section area of the airgap normal to the flux path
\( a \) = nominal airgap size
\( b \) = length of the core
\( c \) = damping constant
\( D \) = dissipation energy
\( E(S) \) = Laplace transform of \( e(\tau) \)
\( E(\cdot) \) = expectation operator
\( \dot{e} \) = nondimensional form of the input disturbance
\( \dot{e}_0 \) = the input disturbance
\( F \) = the supported load
\( f_1 \) = a factor used to account for fringing of the airgap
\( f_2 \) = the coefficient of leakage
\( G_0(s) \) = the transfer function for the absolute displacement with three control gains
\( G_2(s) \) = the transfer function for the absolute velocity with three control gains
\( G_3(s) \) = the transfer function for the current with three control gains
\( G_3^*(s) \) = the transfer function for the current, which has velocity as the input disturbance instead of displacement, with three control gains
\( g \) = gravity constant (9.81 m/s\(^2\))
\( g_0 \) = controller gain for absolute displacement
\( g_1 \) = controller gain for relative displacement
\( g_2 \) = controller gain for absolute velocity
\( g_3 \) = controller gain for current
\( g_4 \) = controller gain for relative velocity
\( I \) = the steady-state current of the circuit at the nominal equilibrium position
\( i \) = the current of the circuit
\( J_1 \) = the mean square response of the absolute displacement
\( J_2 \) = the power dissipated by the electromagnetic suspension system
\( k \) = spring constant
\( k_i \) = stiffness constant of the magnet
\( k_z \) = stiffness constant of the magnet
\( k_1 \) = dimensionless constant
\( k_2 \) = dimensionless constant
\( KE \) = kinetic energy
\( L \) = the inductance of the circuit
\( L_0 \) = inductance of the magnet at equilibrium position
\( L \) = difference of kinetic energy and potential energy
\( l_c \) = the mean flux path length in the core
\( l_{\text{max}} \) = maximum allowable airgap size
\( l_s \) = the mean flux path length in the support piece
\( m \) = mass of the support piece
\( N \) = the number of turns in the coil
\( P.E \) = potential energy
\( Q_x \) = the generalized force in the generalized x coordinate
\( Q_q \) = the generalized force in the generalized q coordinate
\( q \) = the charge
R = the resistance of the circuit

R_c = the reluctance due to the core

R_s = the reluctance due to the support piece

R_g = the reluctance due to the airgap

R_{leakage} = the reluctance due to the leakage

s = Laplace transform variable

S_{disp} = the power spectral density of the input displacement

S_{vel} = the power spectral density of the input velocity

T_0(s) = the transfer function for the absolute displacement with five control gains

T_2(s) = the transfer function for the absolute velocity with five control gains

T_3(s) = the transfer function for the current with five control gains

T_3^*(s) = the transfer function for the current which has velocity as the input disturbance instead of displacement with five controller gains

u = nondimensional form of the control voltage

u_0 = the control voltage

V = the steady-state applied voltage during the nominal equilibrium position

v = the control voltage

v = the applied voltage

W_0(s) = Laplace transform of w_0(\tau)

W_2(s) = Laplace transform of w_2(\tau)

W_3(s) = Laplace transform of w_3(\tau)

w = circular frequency

w_{core} = width of the core

w_n = natural frequency
\( w_0 = \) nondimensional form of the deviation of the absolute displacement
\( w_1 = \) nondimensional form of the deviation of the nominal airgap size
\( w_2 = \) nondimensional form of the deviation of the absolute velocity
\( w_3 = \) nondimensional form of the deviation of the steady-state nominal current
\( w_4 = \) nondimensional form of the deviation of the relative velocity
\( x = \) the displacement of the support piece to be measured with respect to a datum
\( x_0 = \) the deviation of the absolute displacement
\( x_1 = \) the deviation of the nominal airgap size
\( x_2 = \) the deviation of the absolute velocity
\( x_3 = \) the deviation of the steady-state nominal current
\( x_4 = \) the deviation of the relative velocity
\( y = \) the displacement of the core of the electromagnetic system to be measured with respect to a datum
\( \dot{y} = \) the input disturbance
\( z_0 = \) the deviation of the absolute displacement
\( z_1 = \) the deviation of the nominal airgap size
\( z_2 = \) the deviation of the absolute velocity
\( z_3 = \) the deviation of the steady-state nominal current
\( z_4 = \) the deviation of the relative velocity
\( \zeta = \) damping factor
\( \Phi = \) the total magnetic flux
\( \Phi_l = \) the magnetic flux loss due to leakage
\( \Phi_m = \) the magnetic flux remains in the magnet
\( \mu_0 \) = the permeability of free space
\( \mu_r \) = the relative permeability of the core and support piece material
\( \alpha_0 \) = constant that depends on the material properties and geometry of the magnet
\( \alpha_1 \) = constant that depends on the material properties and geometry of the magnet
\( (\cdot) \) = derivative with respect to time
Appendix B

The dynamic equation of the electromagnetic vibration isolation system is derived by using the Lagrange’s equation [9]. The schematic diagram of the electromagnetic vibration isolation system is shown in Figure 2.1. The base of the electromagnetic vibration isolation system is assumed to be rigidly attached to the ground as shown in Figure 2.1. In other word, the base of the electromagnetic vibration isolation system vibrates at the same frequency and amplitude as the ground does. Nomenclature of all the symbols used in the equations are listed in Appendix A. The variables m, g, R and F which are not functions of time are constants. The variables x, y, v and q are the functions of time. The inductance of the suspension system is a function of the size of airgap (x-y). Its derivation is given in Appendix C. The kinetic energy, potential energy and the dissipation energy of the suspension system are given below. The (’) represents the differentiate with respect to time, d/dt.

\[ K.E. = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} L \dot{q}^2 \]  \hspace{1cm} (B1a)

\[ P.E. = -mgx \]  \hspace{1cm} (B1b)

\[ D = \frac{1}{2} Rq^2 \]  \hspace{1cm} (B1c)

let \[ L = K.E. - P.E. \]

The Lagrange’s equations of the suspension system are given below. The x and q are chosen as the generalized coordinates.
The inductance of the suspension system is given by

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial x} \right) - \frac{\partial L}{\partial x} + \frac{\partial D}{\partial x} = Q_x
\]  
(B2a)

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial q} \right) - \frac{\partial L}{\partial q} + \frac{\partial D}{\partial q} = Q_q
\]  
(B2b)

The inductance of the suspension system is given by

\[
L = \frac{\alpha_0}{\alpha_1 + (x - y)}
\]  
(B3)

where \( \alpha_0 = \mu_0 A_c f_1 f_2 N^2 / 2 \) and \( \alpha_1 = (l_s + l_c) f_1 / 2 \mu \) are constants.

The \( Q_x \) and \( Q_q \) are generalized forces, \( Q_x = F \) and \( Q_q = v \). Using equations (B1) and (B3) in (B2), we get

\[
mx + \frac{\alpha_0}{2[\alpha_1 + x - y]^2} q^2 - mg = F
\]  
(B4a)

\[
\frac{\alpha_0}{\alpha_1 + x - y} q - \frac{\alpha_0}{[\alpha_1 + x - y]^2} (x - y) \dot{q} + R \dot{q} = v
\]  
(B4b)

In order to write the state equations for the suspension system, we assume the airgap size = a, the current = I and the applied voltage = V at the nominal equilibrium position. Moreover, we let

\[
x_1 + a = x - y \quad \text{(B5a)}
\]

\[
x_2 = \dot{x} \quad \text{(B5b)}
\]

\[
x_3 + I = \dot{q} \quad \text{(B5c)}
\]

\[
v = v + V \quad \text{(B5d)}
\]

where \( x_1, x_2 \) and \( x_3 \) are the state variables and \( v \) is the change of
applied voltage. The deviation of the airgap size, the absolute velocity and the current are separately represented by $x_1$, $x_2$ and $x_3$. The equations (B6a), (B6b) and (B6c) are obtained by differentiating the equations (B5a), (B5b) and (B5c) first and then substituting the equations (B4) into the differentiated equations of (B5a), (B5b) and (B5c).

\[ \dot{x}_1 = x_2 - \dot{y} \quad \text{(B6a)} \]
\[ \dot{x}_2 = \frac{F}{m} + g \frac{\alpha_0 (1 + x_3)^2}{2 m (\alpha_1 + a + x_1)^2} \quad \text{(B6b)} \]
\[ \dot{x}_3 = \left[ \frac{\alpha_1 + a + x_1}{\alpha_0} \right] \left[ V + v + \left( \frac{\alpha_0}{(\alpha_1 + a + x_1)^2} (x_2 - \dot{y}) - R \right) (1 + x_3) \right] \quad \text{(B6c)} \]

Equations (B6a), (B6b) and (B6c) are the nonlinear state equations for the electromagnetic suspension system. They are valid under the following three conditions,

1. The material used for the core and the support piece are unsaturated all the time.
2. The flux fringing between the airgap and the support piece can be accounted by a constant which is proportional to the increasing area of the airgap and support piece.
3. The useful magnetic flux remaining in the magnet is proportional to the magnetic flux lost through the leakage.

In order to linearize the equations (B6a), (B6b) and (B6c), we consider the following assumptions, $x_1 = \varepsilon z_1$, $x_2 = \varepsilon z_2$, $x_3 = \varepsilon z_3$, $v = \varepsilon u_0$ and $\dot{y} = \varepsilon \dot{e}_0$ are the small perturbations about the nominal positions $x_1 = x_2 = x_3 = v = \dot{y} = 0$ and $\varepsilon << 1$. Substituting these small
perturbations into equation (B6), then using Taylor series to expand the equations about the nominal positions and only keeping the term with the first order of $\varepsilon$, equations (B6a), (B6b) and (B6c) become

$$
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
\dot{z}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
\frac{k_z}{m} & 0 & -\frac{k_i}{m} \\
0 & \frac{k_z}{k_i} & -\frac{R}{L_0}
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2 \\
z_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
\frac{1}{L_0}
\end{bmatrix} u +
\begin{bmatrix}
0 \\
0 \\
\frac{k_z}{k_i}
\end{bmatrix} \varepsilon
$$

(B7)

where $L_0 = \alpha_0 / (\alpha_1 + a)$ can be interpreted as the inductance of the magnet at equilibrium position (airgap = a), $k_z = \alpha_0 I^2 / (\alpha_1 + a)^3$ and $k_i = \alpha_0 I / (\alpha_1 + a)^2$ are the stiffness constants of the magnet.

In order to make the analysis to be general and widely used, we are going to substitute the following expressions, $w_1 = z_1 / a$, $w_2 = z_2 L_0 / aR$, $w_3 = z_3 / I$, $\dot{w}_1 = \dot{z}_1 L_0 / aR$, $\dot{w}_2 = \dot{z}_2 L_0^2 / aR^2$, $\dot{w}_3 = \dot{z}_3 L_0 / IR$, $u = u_0 / IR$, $\dot{\varepsilon} = \dot{\varepsilon}_0 L_0 / aR$ and the new time variable $\tau = tR/L_0$ into equation (B7) to make equation (B7) to be dimensionless. Equation (B8) is the nondimensional form of the linear state equation for the vibration isolation suspension system.

$$
\begin{bmatrix}
\dot{w}_1 \\
\dot{w}_2 \\
\dot{w}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
k_1 & 0 & -k_2 \\
0 & \frac{k_1}{k_2} & -1
\end{bmatrix}
\begin{bmatrix}
w_1 \\
w_2 \\
w_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix} u +
\begin{bmatrix}
0 \\
0 \\
\frac{k_1}{k_2}
\end{bmatrix} \dot{\varepsilon}
$$

(B8)
where $k_1 = \frac{k_2 a L_0^2}{m a R^2}$ and $k_2 = \frac{k_1 L_0^2}{m a R^2}$ are dimensionless constants.

Equation (B6) is the nonlinear state equation for the suspension system. Equation (B7) and (B8) are the linear state equations of the suspension system. Most of the analysis will be carried out by using these three equations.
Appendix C

In this section, the inductance of the electromagnetic suspension system, equation (B3), will be derived. If the materials used for the core and the support piece are unsaturated, then the inductance of the electromagnet is given by equation (C1) [16].

\[ L = \frac{N \Phi}{i} \]  

(C1)

The total magnetic flux is composed by the flux that losses due to leakage and the flux that remains in the magnet. It is expressed by equation (C2).

\[ \Phi = \Phi_l + \Phi_m \]  

(C2)

A magnetic flux field analysis is needed for the flux leakage, \( \Phi_l \), if accurate estimation of the flux leakage is required. The flux leakage is measured to be proportional to the flux remaining in the magnet. It is given by

\[ \Phi_l = f_2 \Phi_m - 1 \]  

(C3)

where \( f_2 \) is the coefficient of leakage.

Consider the equivalent magnetic circuit shown in Figure C1. From the equivalent circuit, we get

\[ \Phi_m \left( R_c + R_s + R_g \right) = Ni \]  

(C4)

where \( R_c = \frac{l_c}{\mu_0 \mu_r A_c} \) is the reluctance due to the core; \( R_s = \frac{l_s}{\mu_0 \mu_r A_s} \)
is the reluctance due to the support piece; \( R_g = 2(x - y)/(\mu_0 A_g) \) is the reluctance due to the airgap.

For the electromagnet, we assume that \( A_c = A_s \) and \( A_g = f_1 A_c \), where \( f_1 \) is the factor used to account for the fringing of the airgap [17]. The fringing factor, \( f_1 \), can be estimated by equation (C5).

\[
f_1 = \frac{(w_{\text{core}} + l_{\text{max}})(b + l_{\text{max}})}{A_c}
\]

(C5)

where \( l_{\text{max}} \) is the maximum allowable airgap size and \( A_c = w_{\text{core}} b \).

Substituting equations (C2), (C3) and (C4) into equation (C1), the inductance of the electromagnet is obtained.

\[
L = \frac{\alpha_0}{\alpha_1 + (x - y)}
\]

(C6)

where \( \alpha_0 = \mu_0 A_c f_1 f_2 N^2 / 2 \) and \( \alpha_1 = (l_s + l_c) f_1 / 2 \mu_r \) are constants.
Figure C1 Equivalent Magnetic Circuit
Appendix D

Hurwitz Stability Criteria

Consider the following \( n^{th} \) order characteristic equation

\[
an s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0 = 0
\]

where \( a_i, i=0, 1, \ldots, n, \) are the constants, \( s \) is the Laplace transform variable. A matrix that can be formed from the coefficients of the characteristic equation is shown below

\[
\begin{bmatrix}
an_{-1} & a_{n-3} & a_{n-5} & \ldots & 0 & \ldots & 0 \\
a_n & a_{n-2} & a_{n-4} & \ldots & 0 & \ldots & 0 \\
0 & a_{n-1} & a_{n-3} & a_{n-5} & \ldots & 0 & \ldots & 0 \\
0 & a_n & a_{n-2} & a_{n-4} & \ldots & 0 & \ldots & 0 \\
0 & 0 & a_{n-1} & a_{n-3} & a_{n-5} & \ldots & 0 & \ldots & 0 \\
0 & 0 & a_n & a_{n-2} & a_{n-4} & \ldots & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_2 & a_0
\end{bmatrix}
\]

The principal minor determinants of the matrix, \( \Lambda_i, i=1,2,\ldots,n-1, \) are formed as follow

\[
\Lambda_1 = a_{n-1}
\]

\[
\Lambda_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} = a_{n-1}a_{n-2} - a_{n-3}a_n
\]

\[
\Lambda_3 = \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} \\ a_n & a_{n-2} & a_{n-4} \\ 0 & a_{n-1} & a_{n-3} \end{vmatrix} = a_{n-1}(a_{n-2}a_{n-3} - a_{n-4}a_{n-1}) - a_n(a_{n-3}^2 - a_n(a_{n-5}a_{n-1})
\]
and so on up to $A_{n-1}$.

Hurwitz stability criterion states that all the roots of a characteristic equation will have negative real parts if and only if the determinant of the matrix, which formed from the coefficients of the characteristic equation, and the principal minor determinants ($A_i$, $i=1,2,...,n-1$) are all greater than zero.
Appendix E

The characteristic equation of the electromagnetic suspension system with three control signals is the denominator of equation (3.6) (and so as (3.8) and (3.9)). It is given by

\[ s^3 + \gamma_2 s^2 + \gamma_1 s + \gamma_0 = 0 \]  

(E1)

After using the Hurwitz Stability test, the determinant and the minor principal determinants are obtained as

\[ \Delta_1 = |\gamma_2| > 0 \]  

(E2a)

\[ \Delta_2 = \begin{vmatrix} \gamma_2 & \gamma_0 \\ 0 & \gamma_1 \end{vmatrix} > 0 \]  

(E2b)

\[ \Delta_3 = \begin{vmatrix} \gamma_2 & \gamma_0 \\ 1 & \gamma_1 \\ 0 & \gamma_2 \gamma_0 \end{vmatrix} > 0 \]  

(E2c)

Substituting equation (3.7) from Chapter 3 into equation (E2), equation (E2) becomes

\[ 1 + g_3 > 0 \]  

(E3a)

\[ g_1 \left[ k_1 \frac{1}{k_2} - g_2 \right] (1 + g_3) > 0 \]  

(E3b)

\[-g_1^2 + g_1 (1 + g_3) \left[ g_2 - \frac{2k_1}{k_2} \right] + \frac{k_1 (1 + g_3)^2}{k_2} \left( g_2^2 - \frac{k_1}{k_2} \right) > 0 \]  

(E3c)

Equations (E3a) and (E3b) can be rewritten as
Equation (E4c) is a quadratic equation for $g_1$, thus solution of this equation is found to be

$$g_1 > \frac{[k_1 - g_2]}{[k_2]}(1 + g_3)$$  \hspace{1cm} (E4b)$$

and

$$g_1 < \frac{k_1}{k_2} (1 + g_3)$$  \hspace{1cm} (E4c)$$

It is clear that $g_3$ must be greater than -1 (equation (E4a)) in order to make the electromagnetic suspension to be stable. Using equation (E4), we can construct the stability plot as a function of $g_3$ for the electromagnetic vibration isolation system. The stability plot is shown in Figure E1. The controller gains $g_1$ and $g_2$ which fall into the shaded area portion in Figure E1 will ensure a stable electromagnetic suspension system and satisfy the Hurwitz criterion.
Figure E1 The Stability Plot

Given Conditions:
\[ g_3 \geq -1 \]
Appendix F

Given a transfer function as follow:

\[ T(s) = \frac{N_2 s^2 + N_1 s + N_0}{D_3 s^3 + D_2 s^2 + D_1 s + D_0} \]  

(F1)

The mean square response can be found by

\[
\int_{-\infty}^{\infty} |T(s)|^2 ds = \pi \frac{\frac{N_0^2 D_2 D_3 + D_3 (N_1^2 - 2N_0 N_2) + D_1 N_2^2}{D_0}}{D_1 D_2 D_3 - D_0 D_3^2}
\]

(F2)
Appendix G

Linear Quadratic (LQ) Optimal Control Theory

Let us consider a state variables equation

\[ \dot{x} = Ax + Bu \]  

where \( \bar{x} \) is a \( n \) by 1 vector, \( A \) is a \( n \) by \( n \) square matrix, \( B \) is a \( n \) by \( m \) matrix, and \( \bar{u} \) is a \( m \) by 1 vector.

we want to find the controller input, \( \bar{u} \), that minimizes the following cost function,

\[
J = \frac{1}{2} \int_0^\tau x^T Q \bar{x} + 2 \bar{x}^T S \bar{u} + \bar{u}^T R \bar{u} \, dt
\]

where \( Q \) is \( n \) by \( n \) positive semi-definite matrix, \( S \) is a \( n \) by \( m \) matrix, and \( R \) is a \( m \) by \( m \) positive definite matrix.

The controller input, \( \bar{u} \), is defined by the feedback control law as

\[
\bar{u} = -k \bar{x}
\]

It has been found to be

\[
\bar{u} = -R^{-1}(B^T P + S^T) \bar{x}
\]

where \( P \) can be calculated from Riccati equation. Riccati equation is given as follow

\[
-P(A - BR^{-1} S^T) - (A^T S R^{-1} B^T) P + PBR^{-1} B^T P + Q = 0
\]
The superscript "T" and "-1" denote transpose and inverse of a matrix respectively.

For the electromagnetic suspension system, we have the time-invariant regulator problem. Hence, the controller input, $\bar{u}$, has been simplified to

$$\bar{u} = -R^{-1}B^TP\bar{x}$$ (G5)

where $P$, a positive definite matrix, can be calculated from

$$0 = PA + A^TP - PB^{-1}B^TP + Q$$ (G6)

MATLAB [11] will be used to solve the $P$ matrix and find the controller input $\bar{u}$. 
References


