FORGING PROCESS MODELS FOR USE WITH
GLOBAL OPTIMIZATION OF MANUFACTURING PROCESSES

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1. INTRODUCTION

Industrial manufacturing processes involve a series of operations that transform a workpiece into a useful finished product characterized by acceptable shape and mechanical properties. A typical sequence of thermomechanical operations consists of multiple-stage hot and cold deformation processes interspersed with suitable heat treatment, material removal, and inspection processes. The shape of the product is achieved through the deformation and material removal processes, while the properties of the product are, in general, dependent upon the entire thermomechanical processing history. The challenge in the design of a manufacturing process is to optimize the entire processing sequence in order to achieve the best balance of manufacturing and material costs, delivery schedules, and shape and mechanical properties of the final product.

Trial and error methods have long been used to select process parameters, including processing temperatures, machine speeds, and die geometries. These methods generally result in, at best, a working design, with no attempt at optimization. Furthermore, they result in high tooling costs and long lead times before production, which, in turn, adversely affects the manufacturing enterprise.
Recently, the long lead times characteristic of build and test methods have been overcome by the use of computerized process models based on the finite element method[1] or other numerical simulation techniques. However, these simulation techniques frequently serve only to replace trial and error on the shop floor with trial and error on the computer, still offering little or no optimization of costs or properties.

Very recently, research has moved to the application of conventional optimization techniques to individual manufacturing processes, such as forging [2,3,5], casting [6], and machining [7,8]. A major concern from the systems engineering point of view is that individual manufacturing operations could be over-optimized at the expense of the cost or performance of the entire enterprise. As shown in Figure 1, these process design approaches yield a series of concatenated local optima, which may be far from the global optimum for a given part. A new approach has been proposed in which the entire manufacturing enterprise is optimized in a single sequence.
1.1 Organization of this Dissertation

The research presented in this dissertation is drawn from a synthesis of two distinct fields of research: namely those of metal forming and operations research. Since advanced researchers in one field frequently will have only a cursory knowledge of the other field, a reasonably detailed introduction to both fields is presented. However, the focus of the research reported herein is the analysis of metal forming. Thus the fundamental mechanics and analysis techniques of metal forming will be described in greater detail.
Chapter 1 presents a generalized overview of the goals of this research and the context of this research within a larger US Air Force Office of Scientific Research directed research project.

This dissertation is concerned primarily with the modeling of processes associated with the manufacture of aircraft engine compressor and turbine rotors, particularly deformation processes. Chapter 2 presents a general overview of the process, with special attention devoted to the fundamental mechanics of deformation processing of aerospace alloys.

The software developed under this research is designed specifically to be interfaced with optimization software. Chapter 3 provides an introduction to optimization theory, then presents some of the specifics of the optimization techniques developed by collaborative researchers.

Chapter 4 contains an extensive review of existing manufacturing process models. Particular note is made of prior application of optimization techniques to thermomechanical manufacturing processes.

Since the focus of the optimization is aircraft engine rotor disks, Chapter 5 examines issues and requirements in aircraft engine rotor manufacture, including manufacturing processes and factors influencing cost, performance, and safety.
Most computer process models, either commercially available or described in the literature, are intended for interactive use in analyzing a single manufacturing process. Using computer models in optimization places unique demands on computer process models, but also offers unique opportunities. Chapter 6 examines unique issues in the application of computer process models for optimization.

Chapter 7 details the complete theory underlying the process models developed through this research. Included are all assumptions, derivations, and equations which have been implemented in the computer code.

The process models were validated against the commercial finite element code DEFORM. Chapter 8 includes results of this validation, including comparison of values and statistical analysis of the error.

Chapter 9 contains conclusions drawn from this research, and recommendations for future work.

The objective function used in association with these models was developed by US Air Force engineers. Appendix 1 contains complete details of the objective function.
1.2 Global Optimization

Global optimization of an entire manufacturing enterprise involves searching an extremely large candidate design space. In a typical example of a 5 step manufacturing process, with 5 independent design parameters at each step, and assuming each design parameter is discretized to 10 levels, the design space approaches $10^{35}$ possible combinations of process parameters. The probability of finding the true optimum by any search method is nearly zero. However, if the optimization goal is softened from finding the true optimum to finding a solution ranking in the top, say, 5%, the task becomes feasible[9].

In the manufacturing problem, by using “soft” optimization techniques developed in the field of operations research for Discrete Event Dynamic Systems (DEDS), a large number (typically thousands) of alternative processing methods are evaluated, and various search techniques [10,11,12,13,14] are used to find a solution set which has a high probability of being in the top 5% of all solution sets. In the early stages of this search, it is desirable to identify feasible or promising processing methods. At this stage, it necessary to quickly evaluate a large number of designs which span the search space. As the search progresses, the search space is reduced to better designs, and more time can be devoted to analyzing each design.
1.3 Process Model Requirements

The state of the art in computerized manufacturing process modeling is finite element analysis. Several commercial codes are available for casting and metal forming simulation which offer detailed, high fidelity prediction of workpiece behavior under processing conditions. However, even simple finite element simulations are very computationally expensive, with simple 2 dimensional simulations requiring $\frac{1}{2}$ hour to 2 hours, and a 3D simulation requiring anywhere from several hours to several weeks, depending on complexity and computational speed. Given that in global optimization a single candidate design sequence may require two or three simulations, and that thousands of candidate designs are evaluated during an optimization search, it is immediately clear that use of finite element analysis for analysis of a large search space is computationally prohibitive. Hence, a different form of numerical process model must be used which can provide reasonable answers without the computational overhead of finite element analysis.

Finite element analysis codes are by design general purpose codes which are suited to complex boundary conditions, geometry, and material behavior. As such, they provide a large amount of information, much of which is not necessary at the initial stages of an optimization search. By focusing on a few key process parameters which provide essential information about the suitability of the process, it is possible to develop a simulation method which captures the physics of the process and provides necessary information quickly and simply. Often, by representing a workpiece as an assembly of geometric
primitives, such as disks and rings, it is possible to apply existing analytic solutions to these geometric primitives to develop a solution for the entire part.

A further requirement of the process models is that they be compatible with the multiple optimization software codes with which they must be interfaced. This requires a standard input-output format among the various process models, and a standardized method for representing geometry and field data in the workpiece.

1.4 Example Problem: Aircraft Engine Compressor / Turbine Rotor

In order to demonstrate the global optimization methods, a class of parts which produced an interesting, yet reasonable, solution were necessary. Aircraft engine compressor and turbine rotor disks offer such a challenge. The geometry exhibits axial symmetry, allowing the use of 2D models. The titanium and nickel based superalloys from which they are manufactured must be processed in narrow windows of strain, strain rate, and temperature to obtain acceptable, defect-free microstructure and mechanical properties.

The parts operate in a harsh environment and are critical to safety. There are also a large number of parts with similar geometry, making the development of part-specific software a feasible option.
The manufacture of aircraft engine rotor disks involves several typical steps (Figure 1):

1) Billet formation—the metal is either cast into a billet from the molten state or consolidated from powder

2) Breakdown/ Microstructure refinement: The billet undergoes uniform hot working, either through upset forging or extrusion, to refine the microstructure

3) Impression die forging: the workpiece undergoes one or more impression die forging operations to achieve further microstructure refinement and to create a near net shape geometry,

4) Heat treatment: The workpiece is exposed to one or more cycles of controlled heating and cooling to achieve the final desired microstructure

5) Finishing: The workpiece is machined to the final desired shape.

There are frequently one or more intermediate inspection operations. The requirements of inspection frequently require a particular intermediate workpiece shape suitable for ultrasonic imaging, surface eddy current inspection, or other techniques.

The mechanical properties of the part are a function of the strain, strain rate, and temperature encountered at every phase of the manufacturing processes. These values, in turn, are functions of environment conditions and changes in geometry. Thus, models which give reasonable predictions of geometry and temperature evolution are necessary for giving reasonable predictions of mechanical properties.
1.5 Scope of Research

The research presented herein focuses on the development of theory for simplified methods of predicting geometry, strain, strain rate, and temperature during thermomechanical processing, specifically extrusion and forging of aircraft engine compressor and turbine rotor disks. The theory will be demonstrated through computer programs suitable for interfacing with global optimization simulation programs. Another researcher [15] has developed a simplified machining model, and the casting process models have yet to be implemented.

1.6 Associated Research

The research presented herein was supported by the United States Air Force Office Of Scientific Research (AFOSR). Two other groups of researchers, led by Dr. Sheldon Jacobson at Virginia Polytechnic Institute and State University, and Dr. Larry Ho at Harvard University, worked in conjunction with this effort, developing optimization techniques and software. The software discussed herein has been interfaced with software developed by both of these research groups, and with software developed by the Materials Research Lab (WL/MLIM) at Wright Patterson Air Force Base.
2. MECHANICS OF DEFORMATION PROCESSING

The response of a metal workpiece to deformation processing is governed by the forces exerted on it by tooling, and the workpiece materials responses to those forces. Before this behavior can be modeled, there must be an understanding of the fundamental behavior of the material, and of its interaction with the tooling and its environment.

2.1 Fundamental Definitions

The deformation of material can be characterized by the degree of deformation (strain), the rate of deformation (strain rate), and the force per unit area of the material required to produce an increment of deformation (stress).

Strain and Strain Rate

Incremental strain at a point can be defined by considering two arbitrary neighboring points $P_0$ at coordinates $(x_0,y_0,z_0)$ and $P$ at coordinates $(x,y,z)$, in a deforming body as illustrated in Figure 2. After the body has undergone some finite deformation, the points move to $P'_0$ and $P'$ with coordinates $(x'_0,y'_0,z'_0)$ and $(x',y',z')$ respectively.
The components of displacement of $P_0$ are

\begin{align}
  u_0 &= x'_0 - x_0 \\
  v_0 &= y'_0 - y_0 \\
  w_0 &= y'_0 - y_0
\end{align}

and the components of displacement of $P$ are

\begin{align}
  u &= x' - x \\
  v &= y' - y \\
  w &= y' - y
\end{align}

Through a derivation available in any common plasticity text (for example, [16]), three components of normal strain can be defined in a Cartesian coordinate system:
where \( \varepsilon_x, \varepsilon_y, \) and \( \varepsilon_z \) are the components of strain in the x, y, and z directions.

Six components of shear strain can be defined:

\[
\begin{align*}
\varepsilon_{xy} &= \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\
\varepsilon_{yz} &= \frac{1}{2} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\
\varepsilon_{zx} &= \frac{1}{2} \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)
\end{align*}
\]

where \( \varepsilon_{xy}, \varepsilon_{yx}, \varepsilon_{yz}, \varepsilon_{zy}, \varepsilon_{zx}, \) and \( \varepsilon_{xz} \) are the components of shear strain in the xy, yz, and zx planes, respectively. The complete 3x3 symmetric strain tensor can be written

\[
\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).
\]

These definitions of strain are applicable only to incremental deformations. Large deformation strains can be calculated by summing a series of incremental strains.

By writing the strain definition in a rate form

\[
\dot{\varepsilon}_{ij} = \frac{1}{2} \left( \frac{\partial \dot{u}_i}{\partial x_j} + \frac{\partial \dot{u}_j}{\partial x_i} \right)
\]

and integrating with respect to time, an expression for total strain is obtained:
\[ \varepsilon_{ij} = \int_0^t \dot{\varepsilon}_{ij} dt. \]  

where \( \dot{\varepsilon}_{ij} \) is the strain rate tensor, representing the time rate of deformation.

A quantity known as the effective, generalized, or equivalent plastic strain \( \bar{\varepsilon} \) is defined as

\[ \bar{\varepsilon} = \sqrt{\frac{2}{3}} \left( \varepsilon_{ij} \varepsilon_{ij} \right)^{1/2} \]  

This provides an effective measure of the total plastic distortion.

### 2.1.2 Stress and Stress Invariants

Stress is defined as the force acting on a unit area of a body. Consider an infinitesimal parallelepiped of dimensions \( dx, dy, dz \) in Cartesian coordinate system arbitrarily oriented within a body. Given forces (not necessarily aligned with the coordinate system) acting on each face of the element, the stresses acting on any face can be resolved into 3 components along the axes of the coordinate system.

The stress components on the positive \( x \) face are

\[ \sigma_x = \frac{P_{xx}}{dy \, dz} \]

\[ \sigma_{xy} = \frac{P_{xy}}{dy \, dx} \]

\[ \sigma_{xz} = \frac{P_{xz}}{dy \, dz} \]
Where $P$ is the force acting on the positive $x$ face, and $P_{xx}, P_{yy},$ and $P_{zz}$ are the components in the $x$, $y$, and $z$ directions respectively. $\sigma_x$, $\sigma_{xy}$, $\sigma_{xz}$ are the resolved normal and two shear components of stress acting on the face.

In the limit as the edge lengths of the parallelepiped approach a point, enforcing equilibrium conditions the stress at that point results in a stress state that can be described by the second order tensor

\[
\begin{bmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{xz} & \sigma_{yz} & \sigma_{zz}
\end{bmatrix}
\]

(10)

It can be shown [16] that the stress tensor can be uniquely represented by three invariant or “principal” stresses $(\sigma_1, \sigma_2, \sigma_3)$, which act in corresponding orthogonal principal directions defined by the eigenvectors of the stress tensor. The principal stresses can similarly be shown to be the roots of the cubic equation

\[
\sigma^3 - I_1 \sigma^2 - I_2 \sigma - I_3 = 0
\]

(11)

where $I_1, I_2$ and $I_3$ are termed the first, second, and third invariants of the stress tensor.

\[
I_1 = \sigma_1 + \sigma_2 + \sigma_3
\]

\[
I_2 = -(\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_3 \sigma_1)
\]

\[
I_3 = \sigma_1 \sigma_2 \sigma_3
\]
The first and second invariants have particular significance for plastic flow of materials.

In particular, the mean or hydrostatic component of stress is

\[ \sigma_m = \frac{1}{3} I_1. \]  

(12)

For non-porous metals, this hydrostatic stress has been shown to have negligible effect on plastic deformation. Thus, it is convenient to define a “deviatoric” stress tensor \( S_{ij} \) by subtracting the mean stress components from the actual stress state:

\[ S_{ij} = \sigma_{ij} - \sigma_m \delta_{ij} \]  

(13)

\[
\begin{bmatrix}
\sigma_{xx} - \sigma_m & \sigma_{xy} & \sigma_{xz} \\
\sigma_{xy} & \sigma_{yy} - \sigma_m & \sigma_{yz} \\
\sigma_{xz} & \sigma_{yz} & \sigma_{zz} - \sigma_m \\
\end{bmatrix}
\]

The invariants of the deviatoric stress tensor \((J_1, J_2, J_3)\) are related to the invariants of the complete stress tensor as follows:

\[ J_1 = 0 \]  

\[ J_2 = \frac{1}{3} \left( I_1^2 + 3I_2 \right) \]  

\[ J_3 = \frac{1}{27} \left( 2I_1^3 + 9I_1I_2 + 27I_3 \right) \]
2.1 Material Behavior

2.2.1 Fundamental Deformation Behavior

The fundamental behavior of metals under deformation conditions can be described by considering a cylinder of constant initial diameter \( D_0 \) and height \( h_0 \) placed between two perfectly flat, frictionless platens. Assume one of the platens is moved towards the other, compressing the specimen, and assume the force \( F \) exerted by the cylinder on the platens is recorded. If \( F \) is plotted against the change in height of the specimen \( \Delta h \), a curve similar to Figure 3 will result. This test is commonly referred to as the uniaxial compression test, and is a common way of characterizing material behavior under deformation conditions.

If the load does not exceed the value corresponding to point A, the cylinder will return to its original shape upon removal of the load. However, if the load exceeds the value at A, the workpiece will recover some of its original shape, but there will be some permanent deformation. Point A is referred to as the yield point.
Applying the definitions of stress from the previous section, the component of stress along the x axis $\sigma_{xx}$ at any point during the deformation is the instantaneous force $F$ divided by the instantaneous cross sectional area of the workpiece. Assuming the workpiece is incompressible (a reasonable assumption for fully dense metals [17]), the instantaneous area is given by

$$A = \pi \frac{D_0^2}{4} \frac{h_0}{(h_0 - \Delta h)}$$

and the stress is given by

$$\sigma_{xx} = \frac{F}{A}.$$
Following the definition of incremental strain from the previous section, at an instantaneous cylinder height $h_l$, an increment of strain is given as

$$d\varepsilon = \frac{dh}{h_l}. \tag{17}$$

The total accumulated strain is given by integrating over the strain path

$$\varepsilon_{xx} = \int_{h_0}^{h_l} \frac{dh}{h} = \ln \left( \frac{h_l}{h_0} \right). \tag{18}$$

### 2.2.2 Mathematical Plasticity

The fundamental purpose of a mathematical description of plasticity is to provide a numerical description of the permanent deformation of materials, particularly metals. Essential to this goal are two requirements:

1. A numerical description of the stress state required for permanent deformation to occur in a given metal with a given microstructure at a given temperature with a given amount of prior deformation, known as a *Yield Criterion*.

2. A numerical description of deformation behavior of the material for a given stress state, known as a *Flow Rule*.
2.2.2.1 Yield Criteria

A criteria for yielding is a numerical description of the stress state under which permanent deformation will occur. For the most general case, a yield criteria can be described by a function $f$:

$$f = f(\sigma_{ij}, \varepsilon_{ij}, \dot{\varepsilon}_{ij}, T, \mu)$$

(19)

where $\sigma$, $\varepsilon$, and $\dot{\varepsilon}$ are the stress, strain, and strain rate tensors, respectively, $T$ is the temperature, and $\mu$ is a microstructural parameter. $f$ is termed the yield function, which has the characteristic:

\begin{align*}
  \text{if } f < 0, & \text{ yielding will not occur} \\
  \text{if } f = 0, & \text{ yielding will occur}
\end{align*}

(20)

It can be shown that the value of $f$ can never exceed the value required for yielding. Thus, $f = 0$ describes a surface in 3 dimensional principal stress space, such that if the point defined by the principal stress vector is on this surface, yielding will occur. If the point lies inside the surface, yielding will not occur.

Based on experimental data, it is a reasonable assumption that most metals undergoing large bulk deformations behave isotropically[1]. Thus, plastic yielding can depend only on the magnitude of the tensor values in equation (19), not on their directions. Plastic yielding of non-porous materials is also, to a first approximation, unaffected by moderate hydrostatic pressure [1]. Thus, for a given strain rate, strain, temperature, and
microstructure, the yield criterion reduces to a function of the second and third invariant of the deviatoric stress tensor:

\[ f = f(J_2, J_3) \quad (21) \]

A criteria attributed variously to von Mises, Maxwell, and Huber states that yielding will occur for any stress state with the value of \( J_2 \) equal to the value of \( J_2 \) at yielding in uniaxial tension. Written in terms of principal stresses, yielding will occur if:

\[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2 \quad (22) \]

where \( Y \) is the stress in uniaxial tension. The criteria is also referred to as the distortion energy theory, for the assumption that yielding begins when the distortion energy at yield equals the distortion energy at yield in simple tension.

### 2.2.2.2 Flow Rules

Once a criteria for yielding is met, a flow rule is required to describe the relationship between the components of the stress tensor and the components of the strain tensor. In general, plastic strain behavior is non-linear and path dependent. That is, deformation behavior is dependent not only on the current stress state, but also on the entire loading and deformation history. By necessity, therefore, infinitesimal or incremental flow behavior is defined for a particular stress and deformation history.

For large deformation problems, it is acceptable to neglect the elastic component of strain. For isotropic behavior, the general equation is known as the Levy-Mises equation:
where \( d\lambda \) is a non-negative constant which varies with loading history.

Given the assumption that this flow rule is applicable to any general case, then the solution of a specific case for \( d\lambda \) will apply to a general case. From consideration of the uniaxial tension test, it may be shown that:

\[
d\lambda = \frac{2}{3} \frac{d\bar{\varepsilon}}{\bar{\sigma}}
\]  

where \( \frac{d\bar{\varepsilon}}{\bar{\sigma}} \) is a function of deformation history, rate of deformation, temperature, and microstructure. [16]. The relationship described in equation (24) is commonly referred to as a constitutive equation or constitutive rule, and in practice is determined experimentally for a given material.

### 2.2.2.3 Work of Deformation

The work done by an increment of plastic deformation on a unit volume element is given by:

\[
dJ_{plas} = \sigma_{ij} \varepsilon_{ij}.
\]
Hence, neglecting elasticity, the total work is given by

\[ J_{\text{plast}} = \int_{\varepsilon_i}^{\varepsilon_2} \sigma \, d\varepsilon . \]  

(26)

\section*{2.2.3 Friction Behavior}

Under conditions of bulk plastic deformation of materials, the shear stress exerted on the surface of the workpiece due to friction is generally limited by the shear yielding behavior of the material:

\[ \tau_f = k \]  

(27)

where \( \tau_f \) is the friction stress, and \( k \) is the shear yield stress of the material. Due to the presence of lubricants and surface aspirates, the apparent friction stress is generally substantially lower than the shear yield stress. Thus, a correction factor is applied:

\[ \tau_f = mk \]  

(28)

where \( m \) is referred to as the shear friction factor: \( 0 \leq m \leq 1 \).

\section*{2.2.4 Microstructural Behavior}

The microstructure of a material can be described by two microscopic structural properties: the atomic structure, and the crystal structure.
2.2.4.1 **Atomic Structure**

The atoms in metals are arranged in regular geometric arrays known as lattices. The arrangement of atoms in the lattice can be described by a unit cell, which is a minimal description of the lattice structure, and is repeated in a periodic manner throughout a crystal.

The lattice structure of a metal has significant influence on the properties of the metal under deformation processing conditions, particularly the ability to undergo large permanent deformations without fracture. Most metals assume a Face Centered Cubic (FCC), Body Centered Cubic (BCC), or Hexagonal Close Packed (HCP) structure.

Deformation tends to occur preferentially on certain planes within a lattice structure. Deformation occurs more readily when these preferential slip planes are correctly oriented with respect to the applied stress field. The number of favorable slip planes, and the interplanar spacing has a significant effect on the strength and ductility of materials. BCC metals tend to have relatively high strength with moderate ductility. FCC metals tend to have low strength and good ductility. HCP metals have only one preferential slip plane, giving them particularly low ductility.

Titanium, an important metal for aircraft engine rotor applications, is allotropic. That is, it has two stable lattice forms: HCP α is stable in unalloyed titanium below a temperature of 1625°F (885°C); BCC β is stable above that temperature. By the addition of various alloying
elements which dissolve preferentially in one or the other phase and stabilize that phase, it is possible to obtain stable $\alpha$, stable $\beta$, or a stable mixture of $\alpha$ and $\beta$ over a wide temperature range.

2.2.4.2 Crystal Structure

Within almost all metal parts in engineering applications, multiple, typically microscopic regions of lattice structures known as crystals or grains exist. The grains are formed during initial metal solidification, and may reform during thermomechanical processing. The lattice structure is essentially uniform within a grain, but discontinuous at the grain boundary.

Grain sizes generally vary, and have a significant influence on mechanical properties. Metals with a smaller grain structure tend to exhibit higher strength, impact resistance, and ductility. Metals with a coarser grain structure tend to exhibit better creep resistance.

As polycrystalline metals are deformed, their constituent grains are distorted. This distortion tends to place the grain in a higher energy state, due both to increased free surface energy and elastic strain energy. If the temperature is raised high enough, the energy due to distortion will seek to lower itself through the formation of new, undistorted, equiaxed crystals. This process is known as recrystallization, and is influenced by the temperature and amount of deformation. The size of the new grains is controlled by the prior grain size and the amount of deformation.
If sufficient deformation occurs below the temperature required for recrystallization, so called “static” recrystallization can be triggered by raising the temperature above the recrystallization temperature. If deformation occurs above the recrystallization temperature, so called “dynamic” recrystallization will occur during or shortly after deformation.

At sufficiently high temperatures, continued grain growth will occur over time, converting the small, equiaxed grains to larger equiaxed grains.

By careful control of thermomechanical processing, the grain size in a part can be controlled, with a substantial influence on the properties.

### 2.2.5 Thermal Behavior

As noted, the constitutive behavior of metals is influenced by the temperature of the metal. The thermal behavior is governed by Fourier’s law:

\[
\rho C_p \frac{dT}{dt} = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + q^* \tag{29}
\]

where \( \rho \) is the density, \( C_p \) is the specific heat, \( k \) is the thermal conductivity, \( T \) is temperature, \( t \) is time, and \( q^* \) is the change in internal heat energy (generation or absorption).
The heat term is due to:

- Plastic deformation work
- Friction
- Latent heat of phase transformation.

Heat generation due to plastic work is given by

\[ q_{\text{plas}} = \eta \int_{\epsilon_f}^{\epsilon_o} \sigma d\epsilon \]

where \( \eta \) is an efficiency term, \( 0 < \eta < 1 \).

The heat generation over a unit area due to friction is given by

\[ q_{\text{frict}} = \int_{u_i}^{u_f} m k \| du \| \]

where \( \| du \| \) is the norm of the relative surface sliding distance of the unit area.

The heat generation due to phase transformation is

\[ q_{\text{trans}} = \int_{t_i}^{t_f} \sum_{\xi_i} \phi_i \ d\xi_i \]

where \( \phi_i \) is the latent heat of the \( i \)th phase, \( \xi_i \) is the volume fraction of each phase, and \( t \) is time.

Heat is lost to the environment through convection and radiation, and is lost to tooling through interface conduction. These losses are modeled as boundary conditions on
equation (29). Specifically, for convective losses,

\[
\frac{\partial T}{\partial n} = \frac{h_{\text{conv}}}{k} (T - T_\infty)
\]

where \(h_{\text{conv}}\) is the convection coefficient, \(k\) is the thermal conductivity of the material, \(n\) denotes a direction normal to the boundary surface at the location under consideration, \(T\) is the surface temperature, and \(T_\infty\) is the ambient temperature[18].

Radiant losses are given by

\[
\frac{\partial T}{\partial n} = \frac{\sigma \varepsilon}{k} (T^4 - T_\infty^4)
\]

where in this equation, \(\sigma\) is the Stephan-Boltzmann radiation constant, and \(\varepsilon\) is the emissivity.

Losses to dies and other tooling is given by

\[
\frac{\partial T}{\partial n} = \frac{h_{\text{contact}}}{k} (T - T_{\text{die}})
\]

where \(h_{\text{contact}}\) is the contact conduction coefficient, and \(T_{\text{die}}\) is the die temperature.
3. CURRENT PRACTICE AND RESEARCH IN OPTIMIZATION

While the primary focus of this thesis is modeling of metal forming processes, the original developments are driven by the requirements of the optimization techniques with which they will be used. Thus a discussion of optimization, starting with fundamentals, and leading to the novel approaches introduced by collaborating researchers in this project, is necessary to provide a context for this research.

3.1 Optimization

Optimization of any problem is in essence an attempt to find the values of a set of independent variables or parameters, possibly subject to certain conditions or restrictions, for which some measure of "goodness" attains an optimum value[19]. The restrictions are termed "constraints," and the measure of goodness is termed an "objective" or "cost" function.

The essential elements of an optimization problem are a set of $n$ independent parameters $x_i$, subject to constraints $C_1 < x_i < C_2$, and a single valued function $F(x)$. The solution to the problem is found using some optimization technique to select the vector $x$ such that $F(x)$ is minimized. (Note that without loss of generality, a maximum or a target value may be achieved by restating $F(x)$ as a negative, or as an error from the target value). The effort
required in finding $x$ varies with form of $F(x)$. For any but the simplest optimization problems, $F(x)$ is stated numerically. The larger problem of optimization then becomes the selection of a technique by which the optimum value of $x$ can be found efficiently.

The selection of an efficient optimization technique requires an understanding of the behavior of $F$. Let us define a *feasible* value of $x$ as any value that satisfies the constraints $C_i < x_i < C_j$. Let us further define the response surface of $F$ as an $n+1$ dimensional surface defined by the values of $F(x)$ for feasible values of $x$. For a scalar value of $x$, a feasible response curve is shown in Figure 4.

![Response curve for a typical objective function, showing local (A, B) and global (C) minima.](image)

The curve illustrates several minimum values. Points A and B are both *local* minimum values. That is, they represent the minimum value in some neighborhood about the point. Point C is a *global* minimum value. That is, it represents the minimum value of $F$ for any feasible value of $x$. 

**Figure 4:** Response curve for a typical objective function, showing local (A, B) and global (C) minima.
The concept of maxima and minima in a response curve can be extended into an arbitrary number of dimensions. The likelihood of encountering local minima in a surface is one factor governing the selection of an optimization technique. The other major factor is the mathematical classification of the problem.

A few objective function forms lend themselves to solution using differential calculus techniques. Functions which do not lend themselves to such techniques must be solved using one of many search techniques. The applicability of search techniques can be judged on two principles

1. **Accuracy**: How closely does the predicted optimum value match the actual optimal value. Is the technique susceptible to finding local minima instead of global minima.

2. **Convergence**: How quickly or efficiently does the search technique find the optimal value.

Several techniques will be examined in the following sections. Differential calculus solutions provide the most straightforward approach (although not always the most easily implemented), and as such are introduced as a reference for other techniques. Two well-established search techniques will be introduced. Downhill search techniques tend to converge relatively quickly, and can be very accurate with certain problems, but are also very susceptible to finding local, rather than global, minima. At the other extreme, Monte Carlo searches are completely immune to local minima, but converge very slowly. Lying between these two extremes are other techniques such as threshold acceptance or simulated
annealing, which offer a blend between the convergence of the local downhill search and the global solution of the Monte Carlo technique. This class of techniques will not be discussed in detail, but will be alluded to in the discussion of the generalized hill climbing technique.

The downhill and Monte Carlo search techniques provide a basis for the introduction of novel techniques developed by research teams collaborating with this current effort. Generalized hill climbing, a synthesis of several search methods, was developed by S.H Jacobson and his coworkers at Virginia Technical Institute and State University. Ordinal Optimization is a novel approach proposed by L.C. Ho of Harvard University. It is specifically developed to work with mathematical models with some finite inaccuracies.

3.1.1 Differential Calculus Solutions

If a cost function $F$ can be written as an analytical equation, an optimum solution vector $x$ will satisfy the conditions

$$\frac{\partial F}{\partial x_i} = 0 \quad \text{and} \quad \frac{\partial^2 F}{\partial x_i^2} > 0$$

for all $x_i$. If multiple solution vectors exist, the global minimum is the one which evaluates to the minimum value of $F(x)$. [20] Solution techniques vary depending on the form of the equation.

This solution technique will show little applicability in the current research. However, it illustrates two important points:
1. Identifying a vector which represents a stationary point in $F$, with a positive second derivative is a sufficient condition for identifying a local minimum.

2. If multiple local minima are identified, the only means of comparison is evaluation of the objective function $F$ at each vector $x$.

The discussion of search methods in the following section will rely on these basic premises for many of their assumptions.

### 3.1.2 Search Methods

Differential calculus solutions are ideal where a differentiable objective function is available, and where a direct solution can be found. However, many optimization problems do not lend themselves to well-behaved objective functions. In these cases, if a solution is to be obtained, it must be found using a search method.

The general principle of search methods is illustrated in Figure 5. An initial candidate vector $x_1$ is chosen, either at random, or based on some a priori knowledge about the system being optimized. The objective function $F$ is evaluated to find $F(x_1)$. If derivative information is available, $F'(x_1)$ may also be evaluated.
The value of $F(x_1)$ is stored, and a new candidate optimum vector $x_2$ is selected. $F(x_2)$ is evaluated and stored, and the next value of $x$ is selected. The process is repeated until a predefined stopping criteria is met. Depending on the nature of the search space and the behavior of the search algorithm, the vector corresponding to the minimum value of $F(x_i)$ may be regarded as a global minimum, as a local minimum, or as a reasonable approximation of a global or local minimum.

The method by which new solution vectors $x_i$ are selected, and the stopping criteria are the features which separate different search techniques.
3.1.2.1 Downhill Search

Downhill search, or gradient descent methods, represent a class of relatively straightforward search methods. The objective function value is evaluated at the candidate vector \( x \), and the gradient of the surface is evaluated, either by direct calculations of the derivative equation, or by testing small perturbations about the value \( x \). The next candidate vector \( x \) is chosen by following a downhill gradient, and is continuously updated until the difference between iterations \( F(x_i) \) and \( F(x_{i+1}) \) is suitably small.

There are several variations on this method. For problems where the derivative of the objective function cannot be evaluated directly, the simplex method\[19,21\] is a well established, reasonably efficient method. More advanced methods, such as Powell’s method\[21\] are more complex, but offer improvements in efficiency.

For problems where the derivative of the function is available in addition to the objective function value, this information can be used to determine the best downhill direction. Search methods using this technique are referred to as Conjugate Gradient Methods.

Regardless of how derivative information is obtained, downhill search methods are, by design, local searches. If the cost function response surface is known to be strictly concave, then this does not pose a problem. However, if there are local minima in a response surface, or if the shape of the response surface is unknown, then these local search methods are prone to converging at a local minimum, rather than a global minimum.
Convergence to a global minimum is strongly dependent on the selection of the initial vector $x_1$.

In the next section, a method that is not subject to local minima will be discussed.

### 3.1.2.2 Monte Carlo Search

While downhill search methods converge quickly, they are subject to convergence to local minima which are not the global minimum. The Monte Carlo search offers exactly the opposite characteristics. It is nearly, if not totally, unaffected by local minima. However, an extremely large number of objective function evaluations must be performed to have a reasonable chance of finding the minimum value.

The Monte Carlo method uses unrestricted random sampling over the search space. That is, random values of each component of $x$ are generated, and the function $F$ is evaluated at this value. Then, a new, completely random $x$ is generated, and the process is repeated. A single simulation consists of a predetermined large number of evaluations of $F$ at random values of $x$. To quantify the accuracy of the solution, it is common practice to perform many Monte Carlo simulations, and perform statistical analysis to estimate the probability that a true global minimum has been identified.
3.1.2.3 Hill Climbing Techniques

Downhill search techniques and Monte Carlo search techniques represent two extremes of traversing a search space and selecting candidate solution vectors. The primary difference is the algorithms' method of selection of a new candidate vector. The downhill search bases the selection of a new candidate vector strictly on the history of prior solutions, and is constantly driven towards a lower objective function value. As such it is very efficient, but tends to be forced into local optima. The Monte Carlo search completely disregards any information from prior solutions, and chooses solution vectors randomly. Thus, the relative objective function value of each successive solution is also completely random, and the solution very inefficient. However, this randomness also effectively avoids entrapment in local minima.

Hill climbing techniques combine the random searching and immunity from local minima of the Monte Carlo techniques with the directed searching and relatively faster convergence of downhill techniques.

The generalized flow chart of Figure 5 can be modified as shown in Figure 6. A candidate solution $x$ is evaluated and returns a value $F(x)$. Some random perturbation of $x$ is made, resulting in $x_{new}$. If $F(x_{new})$ is less than $F(x)$, the new value of $x$ is taken as the current solution, and future perturbations are made on that value. Hence, downhill searching is enforced. However, if $F(x_{new})$ is larger than $F(x)$, $x_{new}$ may still be accepted, based on some acceptance criteria, and future perturbations are made from $x_{new}$. This acceptance of a
potential solution vector corresponding to a higher objective function value facilitates *uphill* searching or hill climbing. The rational is that, to avoid getting trapped in a local minimum, a solution must first get worse before it has the opportunity to get better.

Perhaps the best known hill climbing technique is "Simulated Annealing," based loosely on the physics of annealing of metals, which, if slowly cooled will achieve a low energy state microstructure. The analogy holds that early on in the optimization search, the "temperature" and available "energy" is high, and portions of the system can momentarily pass from an initial energy state through a higher state to ultimately achieve a low state. As the system "cools," the available energy is lower, and the system tends to settle into its lowest, or optimal, energy state. The practical implementation of simulated annealing is well documented in many resources, such as [21].

### 3.1.2.4 Generalized Hill Climbing

Many optimization algorithms based on hill climbing techniques have been proposed. As a parallel effort to the research documented in this thesis, S.H. Jacobson and his coworkers have developed a generalized theory of hill climbing algorithms, (GHC, or Generalized Hill Climbing) and is testing these algorithms using the metal forming analysis software developed in the present research [22].
Jacobson has formulated several hill climbing optimization techniques, including simulated annealing, threshold accepting, tabu search, and Weibull search, as well as local search and Monte Carlo search, in terms of the GHC algorithm. The algorithm follows the flow chart in Figure 6. The only differentiating parameter is the criteria on which the new $x$ vector is accepted or rejected if $F(x_{new}) < F(x_{old})$. Local search will never accept $x_{new}$ corresponding
to a higher value of $F(x)$. Monte Carlo will always accept the latest permutation of $x$.

Simulated annealing, and other similar techniques accept $x_{\text{new}}$ based on the magnitude of $F(x_{\text{new}}) - F(x_{\text{old}})$, some "cooling" or "temperature" parameter, and a random (0,1) probability variable.

### 3.1.2.5 Ordinal Optimization and Genetic Algorithms

All of the optimization techniques discussed to this point have sought to identify a single optimum value which is the single best solution of a stated problem. Achieving this goal using any of these methods relies on two implicit assumptions:

1. The value of the objective function can be evaluated precisely for any value of $x_i$
2. A single evaluation of the objective function can be calculated quickly enough that the number of evaluations necessary to obtain a convergent solution can be performed in a reasonable amount of time.

For many complex real world optimizations, including manufacturing process optimization, the time required to accurately evaluate the objective function for a single vector $x_i$ makes a direct solution by any of the hill climbing techniques time prohibitive.

Practical real world problems suggest a second question: How much optimization is enough? It seems reasonable to suppose that for complex problems, no optimization approach can guarantee with 100% certainty that the solution selected represents the single global minimum value of the objective function. Given this premise, the problem now can
be restated "What represents a good enough solution," and, more importantly, what incremental cost is justified in obtaining an incremental improvement in the solution.

If a solution can be guaranteed to be within 5% of the global optimum with an investment of computational resources $T$, is it worth investing $2T$ resources to achieve a solution within 2% of the global optimum? Is it worth investing $4T$ resources to achieve a solution within 1% of the global optimum? Clearly at some point, the benefits of additional incremental optimization will not be worth the cost of achieving that optimization.

Yang and Ho have addressed many of these issues in their research on Ordinal Optimization, particularly as applied to the manufacturing problem. Yang and Ho’s approach is based on three primary features:

1. Goal softening: Accepting a solution with a specified probability of being in the top $n\%$ of all designs, as opposed searching for the single best design.

2. Ordinal ranking: Simply ranking solutions from best to worst, instead of maintaining relative or absolute values of the objective function.

3. Compatibility with imprecise evaluations of the objective function: Accepting the idea that a precise evaluation of the objective function is time prohibitive, the ordinal optimization approach accounts for probable inaccuracy in the objective function value for any given vector $x_i$. 
These characteristics make ordinal optimization particularly adept for filtering a large search space and providing a set of potential solutions, with a high probability that at least one of the designs is optimal or near optimal.
4. REVIEW OF EXISTING MANUFACTURING PROCESS MODELS AND OPTIMIZATION TECHNIQUES

4.1 Deformation Models

As discussed previously, optimization of a manufacturing process requires evaluating the effect of varied parameters on some objective function which may measure the cost of producing a product within certain constraints, the likely quality of the product, the likelihood of producing non-conforming products, or other parameters related to the end product.

While it is in theory possible to perform experiments on actual parts and measure the results, in practice this is cost prohibitive for most manufacturing processes. Thus, before optimization can be attempted, process models which predict the response of the objective function to potential input parameters are required.

In thermomechanical processing typical for aircraft engine rotor disks, parameters influencing the objective function value may include material cost, final part geometry, final part microstructure, processing time, processing cost, tool life, and likelihood of producing a non-conforming part, among others. Influencing these factors are the strain, strain rate, temperature, and stress distributions in the workpiece, and the loads, temperatures, and stress distributions in the tooling throughout the manufacturing process.
Several analytical methods for predicting various parameters in the thermomechanical processing have seen wide use. Before the advent of digital computers, various solution techniques which relaxed static or dynamic boundary conditions and field equations were employed to estimate total forming loads, stress distributions, flow patterns, and temperature distributions. Two methods which were easily applied to practical metal forming processes were the slab method and the upper bound method. Both methods made the following simplifying assumptions about the process[23]:

- Deforming material is isotropic and incompressible
- The elastic component of deformation is neglected
- Inertial forces are small relative to the flow stress, and are neglected
- The magnitude of the friction stress is constant, and is described by $\tau = m\bar{\sigma} / \sqrt{3}$
- The deforming material can be discretized into zones in which the flow stress is constant

The basic approach for the practical use of both methods is:

- Assume a velocity field in the deforming material, or in each discretized zone.
- Estimate the average strains, strain rates, and temperatures within each zone,
- Knowing the flow stress, apply appropriate boundary conditions to estimate the forming load, and in the case of slab analysis, the average stress distribution.
4.1.1 Slab Analysis

The slab method of analysis relaxes velocity field behavior, and seeks a statically admissible stress field. The solution provides a stress distribution within the workpiece. The flow pattern is assumed to be 2 dimensional – either axisymmetric or plane strain. (Figure 1). Furthermore, the velocity in the x (plane strain) or r (axisymmetric) direction is assumed to be constant with respect to z, and vary only with x (or r). (Plane sections remain plain, or cylindrical sections remain cylindrical). There is no flow in the third dimension.

\[ \sum F_x = \sigma_x h - (\sigma_z + d\sigma_x) h - 2\pi dx = 0 \]  

Figure 7: Geometry and notation for slab analysis. (after [23])

For a plane strain section, a section of unit thickness, width l/2, and height h is considered to be in quasi-static equilibrium. Consider a slab of infinitesimal thickness dx as illustrated in figure 7. For equilibrium, the sum of the forces in the x direction is zero:
Applying von Mises yield criterion gives the stress in the $z$ direction as a function of $x$:

$$\sigma_z = -\frac{2\tau}{h}\left(\frac{1}{2} - x\right) - \frac{2}{\sqrt{3}}\bar{\sigma}$$  \hspace{1cm} (37)

Applying a similar analysis to axisymmetric geometry gives the $z$ stress distribution in a cylindrical disk as a function of $r$ as

$$\sigma_z = \frac{2\tau}{h}(r-R) - \bar{\sigma}$$  \hspace{1cm} (38)

The total forming load for either geometric configuration can be easily obtained by integrating $\sigma_z$ with respect to the independent variable ($x$ or $r$). Complete derivations of the basic cases are available in any text on metal forming, such as [23], and will not be reproduced here. Original derivations for complex geometries with varying height and incomplete contact are presented later in this dissertation.

4.1.2 Upper Bound Method

4.1.2.1 The Upper Bound Theorem

The upper bound theorem states that for a rigid-plastic material undergoing plastic deformation, the work rate associated with any kinematically admissible velocity field defines an upper bound to the actual work rate associated with deformation. Hence, for a class of kinematically admissible velocity fields, the velocity field associated with the lowest work rate is the lowest upper bound, and therefore is nearest the actual solution. This method can be used to estimate the deformation load and the average forming pressure.
The method is based on the assumptions stated above, and is implemented through the following steps[23]:

- For a given geometry, describe a family of admissible velocity fields which satisfy
  - Incompressibility
  - Continuity, and
  - Velocity boundary conditions
- Calculate the energy rate for deformation, internal shear, and friction shear. Sum these to obtain the total energy rate
- Minimize the total energy rate with respect to the unknown parameters of the velocity field formulation.

The total forming load is found from the minimized energy rate divided by the velocity of the tool-workpiece interface.

The total energy rate is given by

\[ \dot{E}_T = \dot{E}_D + \dot{E}_S + \dot{E}_F \]  

Where the \( T \) subscript refers to total energy rate, \( D \) refers to deformation, \( S \) refers to internal sliding discontinuities, and \( F \) refers to friction between the die and the workpiece.

Restated:

\[ \dot{E}_T = \int_V \bar{\sigma} \dot{\varepsilon} \, dV + \int_{SS} \tau |\Delta \dot{v}| \, ds + \int_{SF} \tau_i \dot{v}_i \, ds \]  

(40)
where $SS$ represents an internal sliding surface, $\tau$ is the shear stress on the sliding boundary, generally equal to the shear flow stress of the material, $\Delta v$ is the relative internal sliding velocity, $SF$ represents the sliding friction surface, $\tau_i$ is the friction stress, generally equal to $m\bar{\sigma}/\sqrt{3}$ for shear friction, $v_i$ is the interface sliding velocity.

Given a family of velocity fields, where the specific velocity is determined by a series of independent parameters $\alpha_i$. Then the value of the vector $\alpha$ which satisfies the equation

$$\frac{\partial \dot{E}_r}{\partial \alpha_i} = 0$$

for all values of $i$ describes the velocity field corresponding to the lowest upper bound. Derivations of standard shapes are presented in any standard text on metal forming theory such as [23], and will not be reproduced here. Original derivations of velocity field families, and minimization of energy rates are presented later in this dissertation.

### 4.1.2.2 Upper Bound Elemental Technique

In 1960 Kudo [24] proposed the concept of a “unit rectangular deforming region” to ease the upper bound solution of complicated plane strain problems. By dividing the deforming material into several unit regions, Kudo reported that it was possible to calculate good upper bounds for many working problems. Kobayashi [25] extended the solution to axisymmetric shapes. Since that time several researchers, notably Bramley and his coworkers [26-31], have applied various extensions and modifications of UBET to 2D and 3D analysis of forging. Wang and his coworkers extended the geometric flexibility of
UBET from simple rectangles and right triangles with an "arbitrarily inclined triangular UBET element" [32]. By using stream functions to improve the assumed elemental velocity fields, Lin and Wang [32,33,34] have succeeded in obtaining lower upper bounds than those obtained with traditional parallel velocity fields.

Recent work has focused on the application of backward tracing schemes and reverse simulation for preform design. Kim and Kim [35], Osman and Bramley [30], Mamalis and his coworkers [36] and Choi and his coworkers [37] have all applied various methods for arriving at a suitable preform design based on one or more forward simulations of the forging process.

4.1.3 Hill's General Method

In 1963, Hill introduced "A General Method of Analysis for Metal-Working Processes" [38]. Where upper bound methods and lower bound methods such as slab relax static or kinematic criteria to create an easier solution, Hill's method adheres strictly to well defined general principles. Nevertheless, it remains flexible enough to cover all normal metal forming situations, including arbitrary material properties, varying frictional conditions, and any tool shape. It is capable of reproducing and predicting main phenomena, and of delivering reliable information about loads and principal dimensional changes. Such fidelity to the actual solution requires extensive calculations.
A class of approximating velocity fields is assumed. This class of velocity fields must satisfy all kinematic conditions. From the class of velocity fields, the single velocity field which best satisfies stress equilibrium and continuity requirements is selected.

Hill’s method for selecting the single velocity field satisfying these conditions relies on the virtual work rate principle: the virtual work rate within a control volume is equal to the work rate acting on the surface of the control volume (neglecting body forces), or

\[ \int \left( \sigma_{ij} \frac{\partial w_j}{\partial x_i} \right) dV = \int t_j w_j dS \]  

where \( \sigma_{ij} \) is the stress tensor, \( w_j \) is the velocity field, \( x_i \) is the orthogonal coordinate system, \( V \) is the control volume, \( t_j \) is the surface traction, and \( S \) is the control surface. By applying the Gauss divergence theorem, Hill shows that

\[ \int_V \sigma_{ij} \frac{\partial w_j}{\partial x_i} dV = \int_{S_c} \left[ (n_i t_i) n_j + m k l_j \right] w_j dS : \text{for const shear} \]

\[ + \int_{S_c} \left[ (n_i t_i) (n_j - \mu l_j) \right] w_j dS : \text{for Coulomb} \]

where \( \mu \) is the Coulomb friction coefficient, \( m \) is the shear friction factor, and \( n_i \) is the surface normal.

Given a suitable family of virtual velocities, the calculus of variations is applied to find the particular velocity field \( w \) providing the best solution.
4.1.4 Finite Element Method

Hill’s observation that his method of solution is computationally intensive suggests that digital computer solutions are appropriate. The finite element method offers the capability of analyzing a continuous, piecewise smooth velocity function over an arbitrarily shaped domain [39]. A boundary value problem, such as a metal forming problem, is stated in terms of a function and its derivatives. An exact solution to the function is found at an arbitrary number of points (nodes), and the function value is approximated between these points by interpolating functions (shape functions).

Kobayashi and Lee [40] restated Hill’s general solution as a functional:

\[
\pi = \int_V E(\dot{\epsilon}) dV - \int_{S_r} F_i u_i dS
\]  

(44)

where \( \pi \) is a variational solution parameter to be minimized, \( E \) is the work function for rigid-viscoplastic materials

\[
\sigma_i^* = \frac{\partial E}{\partial \dot{\epsilon}_i}
\]  

(45)

\( F \) represents surface tractions, and \( u_i \) is the sliding velocity. The specific velocity field solution is obtained by allowing the variation of \( \pi \) to vanish:

\[
\delta \pi = \int_V \sigma \delta \dot{\epsilon} \ dV - \int_{S_r} F_i \delta u_i \ dS = 0
\]  

(46)

where \( \sigma = \sigma(\bar{\epsilon}, \dot{\bar{\epsilon}}, T) \) is the flow stress, as a function of effective strain, effective strain rate, and temperature.
For an incompressible solid, the hydrostatic part of the flow stress is not uniquely determined by the constitutive law for a flow field. Furthermore, the incompressibility constraint on the velocity field solution is not enforced in equation (46). Zienkiewicz [41] introduced a penalty on dilatational strains to the functional:

$$\delta \pi = \int_V \sigma \delta \varepsilon \, dV + K \int_V \dot{\varepsilon} \delta \varepsilon \, dV - \int_{S_p} F_i \delta u_i \, dS = 0$$

(47)

where $\dot{\varepsilon}$ is the dilatational strain rate, and $K$ is a large positive penalty constant. The hydrostatic stress is then given by

$$\frac{\sigma}{3} = K \dot{\varepsilon}.$$

The solution of equation (47) is in the form of a velocity $w_i$ at each node in a finite element mesh. For non-steady state problems, the shape of the domain may change continuously in time. To obtain discrete time solutions, the deformation is subdivided into a sequence of time increments. If the finite element mesh is considered to be attached to the deforming material (Lagrangian description), then the new geometry is determined by updating the coordinates of the nodes through:

$$x_i^I (t_0 + \Delta t) = x_i^I (t_0) + w_i^I \Delta t$$

(49)

where $x_i^I$ is the coordinate vector of the $Ith$ node, $w_i^I$ is the velocity vector, $t_0$ is the time at the current solution, and $\Delta t$ is the time increment.
In practical metal forming problems, updating the mesh with the workpiece geometry causes large distortions in the mesh, and frequently results in unacceptable element shapes. It is therefore necessary to periodically “remesh” the workpiece. Remeshing consists of two procedures: the definition of a new mesh, and the transfer of history information from the old mesh.

This approach was first successfully implemented in the finite element code ALPID[42]. In 1991, DEFORM [43] was developed based on experience gained from ALPID. The approach has been thoroughly validated for a wide variety of metal forming problems[1]. Of particular interest to the present research is the “Application of FEM Modeling to Simulate Metal Flow in Forging a Titanium Alloy Engine Disk.” [44] simulated the isothermal forging of a dual property Ti-6242 alloy disk, in which controlled thermomechanical processing is used to obtain good creep and stress rupture properties in the rim, and high fatigue strength in the bore.

### 4.2 Heat Transfer

Aerospace alloys are typically processed at elevated temperature. During the forming process, heat is generated from plastic work and friction work. Heat is also lost through contact with dies, and convection and radiation with the environment. Since material properties vary considerably with temperature [1], accurate temperature prediction is required.
The governing equation for heat transfer within an object is

$$\rho \ C_p \ \frac{dT}{dt} = k \ \frac{\partial^2 T}{\partial x_i^2} + q^*$$  \hspace{1cm} (50)

Where \( \rho \) is the density, \( C_p \) is the specific heat, \( T \) is the temperature, \( t \) is time, \( k \) is the thermal conductivity, and \( q^* \) is a heat generation term. Heat generation in metal forming is due to work of plastic deformation and friction. Heat due to plastic deformation is given by

$$q_{pw}^* = \kappa \int_V \sigma \dot{\varepsilon} \, dV$$  \hspace{1cm} (51)

where \( \kappa \) is a deformation efficiency term, representing the fraction of the work of deformation converted to heat. The heat generation due to friction at the surface is given by

$$q_f^* = \int_S f_x |u_s| \, dS$$  \hspace{1cm} (52)

where \( u_s \) is the sliding velocity, and \( f_x \) is the friction stress.

Convection and interface conduction boundary conditions are given by

$$\frac{\partial T}{\partial n} = \frac{h}{k} \Delta T$$  \hspace{1cm} (53)

where \( n \) is the surface normal, \( h \) is the convection or interface conduction coefficient, and \( \Delta T \) is the difference in temperature between the surface temperature and the environment or tool temperature. Radiation boundary conditions are given by
where in this equation, $\sigma$ is the Stefan-Boltzmann radiation constant, equal to $5.67 \times 10^{-8} \text{W/m}^2\text{K}^4$, and $\varepsilon$ is the emissivity of the surface.

4.2.1 Numerical Methods in Heat Transfer

While solutions exist for temperature distribution in simple to moderately complex shapes, the solutions are frequently quite involved. For mathematically inconvenient shapes, it may be difficult or impossible to obtain an analytical solution, especially for transient problems with heat generation terms included. By simplifying the problem to obtain temperature distribution at discrete points instead of over the continuum, significantly more complex problems can be solved.

4.2.1.1 Finite Difference Method

The finite difference method approximates equation (50) at a discrete set of points, or nodes throughout the object of interest. The temperature and other thermal properties in a finite (usually rectangular) region around the node are assumed to be uniform, and all heat generation is assumed to be lumped at the node. The unit thermal resistance between two nodes is given by

$$R = \frac{\Delta x}{k} \quad (55)$$
where $R$ is the unit resistance, $\Delta x$ is the distance between adjacent nodes, and $k$ is the thermal conductivity of the material. The total thermal resistance between nodes, which governs the total heat flow between nodes, is given by

$$ \mathcal{R} = \frac{\Delta x}{k A_x} $$

such that

$$ q = \frac{T_1 - T_2}{\mathcal{R}_{12}} $$

where $\mathcal{R}$ is the total thermal resistance, $A$ is the cross sectional area considered to be the area of conduction, defined normal to the line between nodes 1 and 2, and $q$ is the total heat flux.

The thermal capacity of a node is the thermal capacity of the volume of material assumed to be lumped at the node, given by

$$ C_a = V_a \rho C_p $$

where $C_a$ is the total thermal capacity, $V_a$ is the total volume assumed to be lumped at the node, $\rho$ is the density, and $C_p$ is the specific heat of the material.

For a network of interconnected nodes, the temperature at a node $i$ at some time $t + \Delta t$ can be determined if the temperature of all nodes at time $t$ is known. The implicit formulation gives
where $T_{i}^{t'}$ is the temperature at node $i$ at time $t$, and the nodes $j$ are the nodes connected to node $i$ through the network.

Boundary conditions are included in the network, with ambient or adjacent surface temperatures assigned to nodes. Resistances are defined for conduction:

$$\mathcal{R}_{ij} = \frac{1}{kA_{ij}}$$

and for convection

$$\mathcal{R}_{ij} = \frac{1}{hA_{ij}}$$

where $k$ is the conduction heat transfer coefficient, and $h$ is the radiation heat transfer coefficient.

The finite difference method formulation is reasonably straightforward, and implementation is straightforward for relatively uniform shapes. However, defining a suitable grid on an arbitrary shape can become difficult, especially if it requires fitting irregular boundaries, as is frequently the case with metal forming.

In this dissertation, the geometrically simplified forgings lend themselves well towards finite difference solutions. However, the more complex, arbitrary shapes accompanying
finite element analysis of deformation also tend to lend themselves better to finite element solution of the temperature distribution.

4.2.1.2 Finite Element Method for Heat Transfer

Equation (50) can be expressed in tensor notation as

$$k_i \, T_{,ii} + \dot{r} - \rho C \dot{T} = 0$$

where $k_i$ is the thermal conductivity, the comma $ii$ denotes the sum of the second partial derivatives with respect to the orthogonal coordinates, $\dot{r}$ is the heat generation rate, and $\dot{T}$ is the time rate of temperature change.

Assuming that all heat generation is due to plastic deformation, the rate of heat generation can be obtained from equation (51), as

$$\dot{r} = \kappa \sigma_{ij} \dot{\varepsilon}_{ij}$$

By substituting equation (63) into equation (62) and introducing a small, arbitrary temperature variation $\delta T$, and applying the divergence theorem, equation (62) can be written in the form

$$\int_V k_i \, T_{,j} \, \delta T_{,j} \, dV + \int_V \rho \, C \dot{T} \, \delta T \, dV + \int_V \kappa \sigma_{ij} \delta T \, dV - \int_{S_b} q_n \, \delta T \, dS = 0$$

where $q_n$ is the heat flux across the boundary. The temperature distribution function can be expressed through nodal temperatures and shape functions [1,39,41]. After discretization, Equation (64) can be expressed in matrix form as
\[ C \dot{T} + K_c T = Q \]  

(65)

where \( C \) is the heat capacity matrix, \( K_c \) is the heat conduction matrix, \( T \) is the node point temperature vector, and \( \dot{T} \) is a vector containing the time rate of change of temperature of node points. \( Q \) is the heat flux vector, which for metal forming simulation considers plastic work of deformation, heat generation due to sliding contact friction, and heat flux due to lubricant conduction, convection, and radiation.

This heat transfer calculation method is implemented in the code ALPID, and the commercial finite element code DEFORM\(^{TM}\), which was used for validation of the simplified models developed in this research.

### 4.2 Research in Deformation Process Optimization

The introduction, validation, and development of extremely accurate computerized FEM process models through the 1980s made practical the study of metal forming processes through the systematic numerical evaluation and modification of process design parameters.

In 1993, Grandhi, et al[45], combined state-space optimum control schemes with the ALPID finite element package to design an optimal ram velocity profile to maintain a desired strain rate at a limited number of control elements within the workpiece. Due to the impracticality of measuring state variables during the actual manufacturing process, an
open loop control system was envisioned for press control, based on the velocity profile
developed using ALPID as a state variable predictor in an off-line closed loop model.

Berg, et al[46], extended the work of Grandhi to control microstructure as opposed to only
strain rate. The work assumes a predetermined die geometry and preform geometry. They
further simplified the optimization by selecting a preferred operating temperature, and
optimizing only for die ram velocity at a discretized sequence of points. The process is
optimized using Brent’s method, a local search technique. A quadratic cost function to
maintain the average strain rate near 0.06:

\[ J = \sum_{n} (\dot{\varepsilon}_i - 0.06)^2 \]  

(66)

where \( n \) is the number of elements, and \( \dot{\varepsilon}_i \) is the effective strain rate in element \( i \).

Frazier and his coworkers extended these rate control techniques to extrusion, and refined
the control of forging systems by placing an emphasis on the dynamic behavior of
hydraulic vertical forge presses[47-50].

Fourment, et al[3], proposed a method to optimize the shape of a forging preform to obtain
the desired final part geometry using a given set of dies. The shape of the preform is
defined by a finite number of control points \( p \) which define a cubic spline curve. The
forging operation is simulated using the finite element method described above. The
objective function to be minimized is
where \( \pi(x) \) is the target profile, and \( x \) is the actual profile of surface \( s \). The objective function is minimized using multiple variable gradient descent techniques.

Zhao, Wright, and Grandhi[51] reported similar research in 1996, but focused on the optimal design of dies instead of the workpiece. The dies are represented using cubic B-spline curves. The objective is to minimize the difference between actual and desired final forging shape. The difference in shapes is measured by discretizing the actual and desired final shaped, then calculating the area of the regions where the shapes do not coincide. The design variables are the locations of control points of the B-splines defining the die surface. The function is minimized using the BFGS [52] algorithm.

He, et al[53] reported the optimization of the volume of an aircraft engine turbine disk. The design variables were 6 key dimensions on the cross section. The design was constrained by a minimum part cross section and a maximum allowable press load. The commercial optimization software iSIGHT was used to perform the optimization. Geometry was generated through non-interactive sessions in the CAD system Unigraphics, controlled by iSIGHT. The designs were evaluated using the DEFORM finite element analysis software. The research is part of a larger project in which the forging, heat treatment, and machining operations will all be individually optimized. No mention is made in this report of any attempt to optimize the overall process.
5. ISSUES IN AIRCRAFT ENGINE ROTOR DISK MANUFACTURE

This dissertation will focus on the analysis of particular manufacturing processes for aircraft engine compressor and turbine rotors. This class of parts was selected for focus by the air force because they pose challenging, yet resolvable, issues in manufacturing and analysis. They also represent a large class of geometrically similar parts, and as such lend themselves well to the type of simplified analysis which will be utilized in this project. Most importantly, due to the extreme cost of materials and processing, even minor incremental improvements in processing techniques offer the opportunity for large cost savings [53].

5.1 Service Requirements

Aircraft engine compressor and turbine rotors are rotating structural components to which airfoils are attached. The rotating parts are illustrated in black in Figure 8. The components are exposed to extreme conditions of dynamic rotating stress, fatigue loading, temperature and thermal stress.
Operating temperatures for compressor rotors approach 1000°F, and turbine rotors may exceed 2000°F. Rotational speeds in the engine core reach 15,000RPM, causing accelerations at the outside of a 12” diameter rotor in excess of 75,000G’s. The radial stresses due to these rotational speeds are extreme. Compounding these effects are cyclic loading during a typical flight cycle and over the numerous flight cycles in a typical disk lifetime. Also associated with these cyclic stresses are non-uniform thermal loads and associated thermal stresses.

5.2 Mechanical Requirements

These conditions require excellent fatigue life, fracture toughness, high strength at operating temperatures, and excellent creep resistance at operating temperatures. Frequently, these represent competing requirements. High strength often comes at the
expense of fracture toughness. Good low cycle fatigue behavior requires energy
dissipation mechanisms which inhibit crack growth, but are contrary to creep resistance[54].

A large group of titanium alloys and so-called “superalloys” - nickel, cobalt, or iron based
alloys with excellent high temperature properties - have been developed to address these
requirements. With proper thermomechanical processing, a well controlled microstructure
can be developed which provides a good compromise between desired properties. This
microstructure might include fine grained regions near the center of the disk where strength
and high cycle fatigue are a concern, and/or a coarse grain structure near the rim for
improved creep properties. The microstructure might also include multiple phases,
precipitates, or other strengthening components.

Titanium, for example, is characterized by a dual phase structure. For unalloyed titanium,
the $\alpha$ phase is stable from room temperature to the so-called $\beta$ transus temperature of
1620°F. Above this temperature, $\alpha$ is unstable and $\beta$ becomes stable. The $\alpha$ phase exhibits
better creep resistance, while the $\beta$ phase exhibits better heat treatability, hardenability, and
strength. By addition of various alloying elements which offer preferential solubility, the
range of stability of both phases can be extended so that they overlap each other, thus
facilitating a dual phase structure. Rapid cooling, or quenching, can preserve the structure
at low temperatures. For many alloys, overheating during processing must be avoided to prevent undesirable transformations to $\beta$.

Frequently, the very properties that make these materials ideal for high temperature aircraft applications also make them very difficult to process. Superalloys possess excellent high temperature strength, and as such are extremely difficult to process by deformation methods such as forging or extrusion. Titanium, which is extremely light and also relatively strong at high temperatures, tends to loose heat quickly to tooling. It also dissipates heat generated during deformation processing very poorly, and as such is susceptible to localized thermal softening and resultant flow localizations or shear banding.

### 5.3 Manufacturing Processes

#### 5.3.1 Primary Metallurgy

Primary metallurgy is the process of creating or consolidating the alloy in a useable form suitable for further processing. The two most widely used processes for providing primary stock for aircraft engine rotor components are casting (also referred to as ingot metallurgy) and powder metallurgy.
5.3.1.1 Casting

Casting is the process of melting a solid material, then allowing it to resolidify in a particular shape. While the material is in the molten state, modifications may be made to the chemical composition. Casting is frequently used as a net shape, or near net shape, process. However, for aerospace applications, the as-cast grain structure is generally unacceptable for final applications. Therefore, applications of casting are generally limited to simple shaped billets which will be subjected to considerable further processing.

For aerospace alloys, extremely careful control of composition is required. For titanium, in particular, contamination with even small amounts of nitrogen may lead to the formation of an extremely brittle phase known as hard $\alpha$. If left undetected, a small hard $\alpha$ inclusion may create a stress riser and an initiation site for a fatigue crack. In recent years, such hard $\alpha$ induced cracks have caused catastrophic failures of aircraft engines and have resulted in numerous deaths. For this reason, careful, complete inspection of aircraft disks is required.

5.3.1.2 Powder Metallurgy

In the powder metallurgy process, fine powdered materials are blended, pressed into a desired shape, then heated (sintered), possibly under pressure, to bond the particles. Powders are manufactured under carefully controlled conditions, and offer advantages in blending and alloying that are not available under ingot metallurgy processes. The particle size also forms a practical limit on grain sizes.
Adequate compaction of powders and complete void closure is a concern in aerospace alloys. Densification may be enhanced by any of several processes. Hot Isostatic Pressing (HIPing) is a process under which the powder formed part is placed in a high temperature, high pressure environment for a period of time. The combination of temperature and pressure tends to promote void closures.

As with ingot metallurgy, powder preforms are generally produced in simple shapes which are suitable for further deformation processing. Deformation processes such as extrusion or upset forging can also be used to close voids and promote full densification.

5.3.2 Deformation Processing

For aerospace alloys, deformation processing serves two, frequently equally important roles. One is to provide a final, or at least modified, shape of the workpiece. The other is to promote recrystallization and other similar microstructural enhancements. Although there are many variations on deformation processing, the two fundamental deformation methods for refining billet microstructure and forming turbine rotor disks are extrusion and forging.

When metals solidify, they tend to form large, coarse grain structures. These structures frequently possess geometric distributions based on cooling patterns, and may exhibit substantial segregation of alloying elements. Reheating these cast structures with no deformation will likely cause grain growth, with little other alteration of the structure. However, if sufficiently high temperatures are reached during or after the material receives
sufficient distortion, the coarse, distorted grain structure will be replaced by finer, distortion-free grains. This so-called recrystallization process can occur at sufficiently high temperatures after deformation, in which case it is referred to as "static recrystallization." It can also occur if sufficiently high energy is available due to temperature and strain energy during deformation, in which case it is referred to as "dynamic recrystallization." For sufficiently large degrees of deformation, dynamic recrystallization can occur several times throughout the deformation process. By careful control of deformation and temperature, a well controlled, uniform grain structure can be obtained.

5.3.2.1 Extrusion

In extrusion, a usually cylindrical billet is forced through a suitably shaped opening under extreme pressure. The product has a uniform cross section. Several variations on the extrusion process exist, including solid or hollow parts, hot or cold processes, and variations on the movement to the die relative to the container.

For aircraft engine disk forgings, extrusion is used strictly as a breakdown process. That is, the sole purpose of the extrusion is to impart a sufficient amount of deformation to promote dynamic recrystallization. The extruded shape is generally round, and is used as starting stock for the forging process. For extrusion of high strength materials such as titanium or superalloys, conic or streamlined dies are generally used.
5.3.2.2 Forging

Forging describes a number of deformation processes, where deformation is caused by compression of the deforming workpiece between two or more tools which move towards each other during the deformation. The tools may deform the entire workpiece at once, or they may be smaller than the workpiece, and deform the workpiece one small section at a time. The process may be performed hot or cold. Frequently, several forging steps are required to progress from the initial stock to the finished shape.

Descriptive names of forging operations vary from source to source. Some terms, such as "open die forging" may be used in different sources to describe distinctly different processes. The following discussion will seek to provide a basic explanation of processes typically used in the manufacture of aircraft engine rotor disks.

5.3.2.2.1 Open Die Forging

Open die forging generally describes the compression of the workpiece between dies which provide limited constraint for the workpiece. The dies may deform a small section of the workpiece at a time, in a process which is sometimes referred to as "cogging," or may be flat, and cover the entire workpiece, in a process referred to as "flat die" or "pancake" forging.
Open die forging is used as a breakdown process to induce recrystallization, and as a preforming process, used to form the material into a geometry appropriate for further deformation operations.

5.3.2.2 Impression Die Forging

Impression die forging describes a process in which the workpiece is compressed between two or more dies which have cavities designed to form the material to a particular geometry. The material may be partially or completely constrained, or may be relatively unconstrained. The dies may be designed such that as they close, a thin section remains unclosed. Controlled metal flow through this section can improve the filling of the impression. This thin section is referred to as “flash.”

In general, some finishing operations, such as machining, are required to produce the desired finish shape. Forgings which closely approximate the desired shape in order to minimize the required machining are referred to as “near net shape.” For certain shapes, forging geometry may match the desired finished shape. These forgings are referred to as “net shape.”

In general, higher degrees of precision in forging require greater constraints on material flow. Larger constraints on metal flow in turn tend to increase the loads required to form the part, which tends to lead to a decrease in tool life.
Forging processes may be carried out hot or cold. Forging at elevated temperatures offers increased material ductility, permitting significantly larger deformations in a single blow. However, cooling causes thermal distortions, and limits the degree of precision which can be attained in a hot forging. Exposure to the environment at elevated temperatures also causes development of surface scale, which typically must be removed before the part can be placed in service.

Cold forgings offer improved material flow, and frequently offer improved strength through work hardening. However, the limited ductility inherent in most structural materials at ambient temperature may require repeated heat treatments if large deformations are to be carried out without cracking.

5.3.3 Heat Treatment

Heat treatment is the process of heating a workpiece to a particular temperature, maintaining that temperature for a predetermined amount of time, then cooling the material at a controlled rate, for the purpose of modifying the microstructure, and hence the mechanical properties, of the material. The heat treating processes may induce any of several changes, including phase transformations, recrystallization, forcing secondary phases into solution, or precipitating secondary phases. By proper selection of hold temperatures, hold times, and cooling rates, heat treatment may be used to either strengthen or soften materials.
The common strengthening mechanisms used with aerospace alloys are

- Solid-solution strengthening
- Grain size refinement
- Precipitation hardening
- Dispersion hardening
- Phase transformations.

Many of the strengthening mechanisms rely on establishing equilibrium or slowly transitioning microstructures at high temperature, then rapidly reducing the workpiece temperature to produce non-equilibrium structures at service temperatures. These rapid temperature changes frequently produce non-uniform thermal strains, which can result in severe distortion and residual stresses. Thus, heat treatment processes must be carefully designed to obtain desired mechanical properties without inducing excessive stresses.

5.3.4 Finishing

Regardless of the thermomechanical processing used in disk manufacture, certain finishing processes are required to make the disk suitable for service. These processes may include material removal and surface treatments. The finishing process establishes the final shape of the part, and can have a substantial effect on fatigue behavior and safe service life.

5.3.4.1 Machining

Machining is the process of removing excess material from a workpiece. Traditionally, machining involves material removal by repeatedly cutting away layers with a hard, sharp
tool until the final shape is achieved. Non-traditional processes use chemical, electro-chemical, or electro-discharge techniques to remove bulk quantities of material. Other processes can be used for drilling and similar processes, but are not relevant to this discussion.

For round parts such as aircraft engine disks, turning on a lathe is the most common method of material removal. Several issues related to the material removal process affect the cost and performance of the finished part. Ideally, material removal rate would be optimized to minimize cost. However, distortion and surface finish both limit how material can be removed from a workpiece.

Distortion during turning, especially of thin parts, is a major concern. The tool tends to exert substantial forces on the workpiece, and elastic deformation of the workpiece may result in an unintended final shape. Of greater concern in quenched parts is residual stress. Residual stresses developed as a result of thermal gradients during cooling may or may not cause distortion in the workpiece. In any case, these stresses are distributed throughout the workpiece such that they are in equilibrium. Removal of a layer on one side of the workpiece will disrupt the equilibrium and cause elastic distortion. If material removal is non-uniform, the distortion may be permanent, and may lead to unacceptable final shapes.

Surface finish is a second concern in turning. Surface irregularities which result from the machining process can adversely affect fatigue life. Slower tool feed rates on the finishing
pass of a turning operation improve the surface finish. However, excessively slow feeds will increase production time, which in turn increases manufacturing costs. Improper tool selection, insufficient stiffness in the support, worn tools or other factors can cause chatter or other surface disturbances which can also adversely affect fatigue life.

Machining also tends to induce tensile residual stresses and microcracks. Localized heating due to plastic deformation and tool-workpiece friction can induce localized microstructural changes if it is not controlled.

5.3.4.2 Surface Treatments

Fatigue life of a component can be enhanced by inducing compressive residual stresses on the surface of the component, thus lessening the surface tensile stress under surface load and reducing the tendency towards crack initiation and propagation. Two common processes used to induce compressive residual stresses are shot peening and burnishing. Shot peening induces compression in the surface by repeated impacts of shot impelled at a high speed. Burnishing involves rubbing a smooth, relatively small, hard object over the surface of the component under very high pressure to smooth surface irregularities and induce compressive residual stresses.

5.3.5 Inspections

Undetected flaws in an aircraft engine disk act as stress risers and fatigue crack initiation sites. If left undetected, the resulting cracks can lead to catastrophic failure. Such failures can and have caused airplane crashes [55] and fatalities.
Titanium, in particular, is prone to melt-related anomalies resulting from impurities introduced into the furnace through inadequate atmospheric controls or inadequate screening and cleaning of the melt material. These anomalies will frequently lead to flaws inside the part which cannot be detected by any form of surface inspection.

Ultrasonic inspection is a technique which attempts to identify internal flaws by directing ultrasonic waves through the part and measuring the reflected sound waves. Defects reflect differently than normal material. Ultrasonic inspection is limited by the shape and size of the workpiece. Furthermore, smoother surface finishes lend themselves better to inspection than do rougher surfaces. Thus it is common practice to machine a workpiece to a “sonic shape” and inspect the part before machining to the final shape.

Of particular interest to the current research on global design and optimization is the effect of billet size on inspectability and flaw evolution in forging. Smaller diameter billets allow more sensitive inspection before forging. The larger degree of deformation also increases the likelihood of cracking or voiding around defects. This increases the chance of detection in post forge inspection, but also increases the risk of fracture if the defect remains undetected. Extremely small billet diameters also pose greater challenges in forging, due to the tendency towards buckling.
6. USING COMPUTER PROCESS MODELS IN OPTIMIZATION

6.1 Preliminaries

The search space for global optimization of a manufacturing process approaches unbounded. For a typical aircraft engine rotor disk manufacturing sequence with (conservatively) 6 operations, each operation having 5 controllable parameters discretized at 10 levels, the search space exceeds $10^{40}$ possible combinations. Hence, the optimization of a manufacturing process or sequence of processes requires a large number of objective function evaluations. He and his coworkers have reported 30 objective function evaluations for a simple optimization of a single value disk forging problem with six control variables [53]. For global optimization of a rotor disk manufacturing process, estimates range from the order of 1000 to 10,000 evaluations for the Ordinal Optimization methods proposed by Ho and Yang, and detailed in Chapter 3 of this dissertation, to the order of 300,000 evaluations for the generalized hill climbing methods proposed by Jacobson and his coworkers, and also detailed in Chapter 3.

As noted previously, the exact determination of objective function value is essentially impossible for all but the most simple optimizations. Thus it is necessary to rely on computer based process models to provide a function mapping candidate design parameters to an objective function value. By definition, any process model provides an estimate of
the parameters of interest. The accepted standard for modeling deformation processes is
the finite element method. The accuracy of this method has been thoroughly
demonstrated[1,43,44,56]. However, solution time for a single finite element run ranges
from 10 to 20 minutes for extremely simple 2D problems to weeks for complex 3D
problems. He and his coworkers[53] reported that a relatively simple optimization with 30
evaluations required 35 CPU Hours using DEFORM with a 1000 element mesh.

Finite element analysis codes are by design general purpose codes which are suited to
complex boundary conditions, geometry, and material behavior. As such, they provide a
large amount of information, much of which is not necessary at the initial stages of the
optimization search.

Even assuming that a single forming simulation could be completed in 10 minutes using
finite element analysis, a 1000 iteration global optimization of an aircraft engine rotor disk
manufacturing process with 3 deformation operations would take 6 weeks. Hence, if global
optimization is to be performed in a reasonable amount of time, the need exists for an
extremely fast computer model which will provide reasonable results required for
evaluation of the cost function.

The contributions of deformation processes to the objective function are geometry,
microstructural, and mechanical properties. Inspectability, and hence safe operating life, is
also influenced by the deformation process. Press loads and pressure on tooling affect
press selection and tooling life. The entire objective function implemented with the present research is detailed in the Appendix. The final part geometry depends on forging die geometry and metal flow patterns during deformation. Microstructure and mechanical properties depend on the complete thermomechanical history of the part, specifically, the strain, strain rate, and temperature distributions in the part with time. Thus, computer models for use with global optimization must be capable of predicting geometry, strain, strain rate, and temperature evolution during forming operations.

In order to attain the speed necessary to make global optimization practical, simplifying assumptions must be made regarding the behavior workpiece during deformation. These simplifying assumptions will invariably introduce inaccuracies in the results. While abundant literature exists detailing simplified approximate methods of analysis of metal forming problems, there has been little or no effort to quantify the uncertainty or to address its effect on the overall results of the analysis.

6.2 Uncertainty and Soft Optimization

In research parallel to the work reported in this dissertation, Yang and Ho have analyzed and quantified the effects of uncertainty in the context of global optimization of a manufacturing problem, and provided a methodology for effectively utilizing simplified approximate analysis models.
Their method relies on the principle of goal softening combined with ordinal optimization. The underlying assumption is that the search will be a preliminary search, providing a small set of candidate designs which can be further evaluated with more accurate models.

Global optimization of a manufacturing process is known in numerical complexity theory as a non-polynomial time decidable problem [57]. That is, the effort required to guarantee an absolute minimum has been reached grows exponentially with problem size. Goal softening recognizes that in a huge search space, such as the one presented by global optimization of a manufacturing process, identifying a single optimum process variable vector may be an extremely expensive proposition as compared to finding a set of process variables which is very good, but not necessarily optimal.

Suppose for example, that a 95\textsuperscript{th} percentile design can be obtained in one cpu-day. A 98\textsuperscript{th} percentile design might be obtained in 1 cpu week. A 99\textsuperscript{th} percentile design might require 1 cpu-month. At some point, the cost of an additional increment of optimality or certainty exceeds the value gained by attaining that optimality.

The principal of ordinal optimization is that candidate solutions are simply ranked in order of objective function value. Thus, screening is performed by evaluating a number of candidate solutions, then selecting the top n\% of the solutions for further evaluation. Given that there is an acknowledged uncertainty in the evaluation of the objective function, the
percentage of solutions selected must be sufficiently large to insure that some of the selected solutions based on the objective function values are in fact within the top n% of actual solutions.

Yang [57] defines the alignment probability as follows: Assume a search space Θ. Define the subset G to be the "Good Enough Subset," that is, the top n% of all actual designs. Define S to be the "Selected Subset," that is, the designs corresponding to the top n% of objective function values as calculated by the process model. Then \( G \cap S \) is the set of solutions which are both in the top n% of the objective function values and the "Good Enough Subset."

The alignment probability is defined as

\[
P(k,s,g) = P(\left| G \cap S \right| \geq k)
\]

(68)

where \( k \leq g \) and \( k \leq s \).

Given that \( G = S + \eta \), where \( \eta \) is some normally distributed noise with variance \( \sigma^2 \), the probability of the alignment being null is given by [58]

\[
P(S \cap G = \emptyset) \leq e^{\frac{-k|G|+k|S|}{\sigma^2}}
\]

(69)

Thus, a reasonable screening method has been presented, via which simplified approximate solutions with quantifiable uncertainty can be used as to screen a non-polynomial search
space and select a reasonable number of candidate solutions to be further evaluated by a more accurate method such as finite element analysis.
7. DEFORMATION MODELS

7.1 Requirements

The performance requirements for deformation models have been established by the requirements of the optimization routines. The models must produce reasonable predictions of geometry, strain, strain rate, and temperature for a given set of processing conditions during deformation processing. If the entire optimization is to take place in a reasonable amount of time, each analysis of a single deformation operation should take place extremely quickly. Execution times of less than 0.1 seconds would allow a 100,000 iterations to be performed within a day.

By focusing on a few key process parameters which provide essential information about the suitability of the process, it is possible to develop a simulation method which captures the physics of the process and provides necessary information quickly and simply.

A further requirement of the process models is that they be compatible with the multiple optimization software codes with which they must be interfaced. This requires a standard input-output format among the various process models, and a standardized method for representing geometry and field data in the workpiece.
7.2 Approach of this Research

The requirements of the objective function are:

- Strain
- Strain Rate
- Temperature
- Peak die pressure
- Total press load
- Final workpiece geometry

Strain and strain rate within a workpiece are driven by geometry changes in the workpiece. Temperature changes are due to heat lost to the environment, and heat generated due to the work of deformation and friction. Thus, a deformation model which produces a reasonable prediction of geometry evolution can also be used to predict strain, strain rate, and temperature in the workpiece. By representing a workpiece as an assembly of geometric primitives, such as cylinders, disks and rings, it is possible to apply existing analytic solutions to these geometric primitives to develop a solution for the entire part.

The overriding principle in the development of models is maintaining execution speed. The models are used for preliminary analysis of deformation processes, with detailed analysis left to finite element analysis, for which it is well suited. Hence, detail is sacrificed in favor of execution speed.
7.3 Program Structure

The ultimate result of this research is a series of computer programs written in C which will interface with an objective function evaluation routine and an optimization routine. The programs developed under this research model extrusion and forging. Another researcher [15] has developed a simplified machining model. The optimization routine selects a sequence of operations for a candidate process, then assigns parameters for each process. These values are passed to the C program cost.fit, which calls the appropriate extrusion, forging, or machining process models, then evaluates the values returned and calculates a single cost value. (The exact cost function is detailed in Appendix 1).

7.4 Geometry Representation

In the computer process model, the workpiece is an object consisting of geometry and field data. This object is passed between the optimization routine and each of the process models, as described above. A consistent format is necessary to avoid reassigning values for extrusion, forging, and machining. The geometry selected is disk or cylinder (depending on aspect ratio) surrounded by a concentric sequence of rings. The complete geometry can be defined, to a reasonable degree of approximation, by the diameter and height of each of these disks and rings.
For the various manufacturing processes, simplifying assumptions can be made about the geometric relationship between the rings. These assumptions are explored in the following sections.

7.4.1 Extrusion

As discussed in Chapter 5, the sole role of extrusion in the manufacture of aircraft engine rotor disks is to condition the billet by imparting a strain sufficient to produce recrystallization. As such, there is no need for extrusion of any shape other than round to round. This can be accomplished by either a conic or streamlined die. In either case, the geometry can be defined by a single cylinder with specified radius and height. This geometry will evolve through the extrusion process. The concentric rings around the cylinder are maintained for geometric consistency, but are assumed to have the same radius as the cylinder, and to have zero thickness.

7.4.2 Forging

The workpiece is axisymmetric, and as such can be modeled in 2 dimensions, with the rotational dimension understood to be unvarying. The part is also assumed to be top/bottom symmetric, so that only the upper portion of the part need be modeled (Figure 9).

- ring 1 has rectangular cross section. Inside radius is 0. Outside radius is \( w_{r1} \). Height is constant \( h_{l1} \). Thus, the volume is given by \( \text{vol}_1 = 2\pi(w_{r1})^2(h_{l1}) \).
- ring 2 has trapezoidal cross section. The inside radius is the outside radius of ring 1 \((wr_1)\). The outside radius is \(wr_2\). The height varies from an arbitrary height of \(wh_1\) to \(wh_2\) to a height \(wh_3\) at the outside. The volume is given by:

\[
vol_2 = 2\pi wh_2(wr_2^2 - wr_1^2) + \frac{2\pi (wh_3 - wh_1)}{3}(2wr_2 + wr_1)(wr_2 - wr_1)
\]

- ring 3 has rectangular cross section. The inside radius is the outside radius of ring 2 \((wr_2)\) and the outside radius is \(wr_3\). The height is \(wh_3\). The volume is given by \(vol_3 = 2\pi (wh_2)(wr_3^2 - wr_2^2)\).

- ring 4 (flash) is modeled as having rectangular cross section. The primary role of flash in the model is to account for material volume extruded from the die cavity.
The shape of the impression die is defined similarly using only 3 rectangular cross section rings (Figure 10). Ring 1 has constant height $dh_1$ and constant radius $dr_1$. Ring 2 has constant height $dh_2$. The inside radius of ring 2 is the outside radius of ring 1 ($dr_1$). The outside radius of ring 2 is $dr_2$. The flash gutter has height $dh_3$ and radius $dr_3$. 

**Figure 10: Die Geometry Nomenclature**
7.5 Deformation Models

7.5.1 Extrusion

The input parameters for extrusion are given in Table 1.

Table 1: Input Parameters for Extrusion

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Variable</th>
<th>Corresponding C Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Billet Half Length</td>
<td>wh₀</td>
<td>wh[0]</td>
</tr>
<tr>
<td>Initial Billet Radius</td>
<td>wr₀</td>
<td>wr[0]</td>
</tr>
<tr>
<td>Initial Billet Temperature</td>
<td>wT₀</td>
<td>wkpc-&gt;gridpt[1].T</td>
</tr>
<tr>
<td>Initial Billet Strain</td>
<td>ε₀</td>
<td>wkpc-&gt;gridpt[1].eps</td>
</tr>
<tr>
<td>Die Shape</td>
<td>Conic or Streamlined</td>
<td></td>
</tr>
<tr>
<td>Die Length</td>
<td>dh₁</td>
<td>dh[1]</td>
</tr>
<tr>
<td>Die Velocity</td>
<td>V_die</td>
<td>forset-&gt;die_speed</td>
</tr>
<tr>
<td>Die Exit Diameter</td>
<td>dr₁</td>
<td>dr[1]</td>
</tr>
<tr>
<td>Lubrication Conditions</td>
<td>fm</td>
<td>forset-&gt;m</td>
</tr>
<tr>
<td>Extrusion Container temp</td>
<td>dT</td>
<td>forset-&gt;die_temp</td>
</tr>
</tbody>
</table>

The output parameters for extrusion are given in Table 2.

Table 2: Output Parameters for Extrusion

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Variable</th>
<th>Corresponding C Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final Billet Half Length</td>
<td>wh₁</td>
<td>wh[1]</td>
</tr>
<tr>
<td>Final Billet Radius</td>
<td>wr₁</td>
<td>wr[1]</td>
</tr>
<tr>
<td>Final Billet Temperature</td>
<td>wT₁</td>
<td>wkpc-&gt;gridpt[1].T</td>
</tr>
<tr>
<td>Total Strain</td>
<td>ε</td>
<td>wkpc-&gt;gridpt[1].eps</td>
</tr>
<tr>
<td>Peak Strain Rate</td>
<td>˙e</td>
<td>wpkc-&gt;gridpt[1].epsdot</td>
</tr>
<tr>
<td>Peak Extrusion Force</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>Peak Pressure on Die</td>
<td>P</td>
<td>P</td>
</tr>
</tbody>
</table>

Assuming negligible swell or elastic recovery, the final radius of the billet is determined by
the radius of the extrusion die. Since the material is incompressible, volume is maintained
during the extrusion. For the degree of approximation in these calculations, we can assume that deformation is homogenous. The final dimensions of the workpiece are given by

$$w_{r_1} = r_1$$  \hspace{1cm} (70)

$$w_{h_1} = \frac{r_0^2 h_0}{r_1^2}.$$  

The homogenous effective strain is given by

$$\bar{\varepsilon} = \ln \left( \frac{w_{h_1}}{w_{h_0}} \right)$$   \hspace{1cm} (71)

For a conic die the strain rate at any distance along the die is given in the following derivation.

![Figure 11: Dimensions used for conic die extrusion analysis](image)

Referring to Figure 11, the constant slope of the conic surface of the die is given by

$$\frac{dr}{dx} = \frac{w_{r_0} - dr_1}{dh_1} = \tan \alpha.$$  \hspace{1cm} (72)

Where \(\alpha\) is the half angle of the die. The strain rate in a slab of material \(\Delta h\) thick at distance \(x\) from the inlet of the die is given by
\[ \dot{\epsilon} = \frac{dV_x}{dx} \]  

(73)

where \( V_x \) is the velocity of the workpiece material at some distance \( x \) from the entrance of the die. Let \( A_x \) be the cross sectional area at the same distance \( x \). Then

\[ V_x A_x = \left( V_x + \frac{dV}{dx} \delta x \right) \left( A_x + \frac{dA}{dx} \delta x \right) \]  

(74)

where \( \delta x \) is an incremental distance. Expanding this equation, and neglecting the \( \delta x^2 \) terms gives

\[ V_x \frac{dA}{dx} + \frac{dV}{dx} A_x = 0. \]  

(75)

\( V_x \) is related to the extrusion die velocity by

\[ V_x = V_{die} \frac{A_0}{A_x} \]  

(76)

where \( A_0 \) is the cross sectional area at the entrance to the die.

The radius \( r \) at any distance \( x \) is given by

\[ r = r_0 - x \tan \alpha \]  

(77)

where \( r_0 \) is the radius at the die entrance. Thus

\[ A = \pi \left( r_0^2 - 2 r_0 x \tan \alpha + x^2 \tan^2 \alpha \right) \]  

(78)

and
\[
\frac{dA}{dx} = 2\pi (r_0 \tan \alpha - x \tan^2 \alpha).
\] 

(79)

Substituting equations (76), (78), and (79) into (75) and rearranging gives

\[
\dot{\varepsilon} = \frac{dV}{dx} = 2V_{\text{die}} A_0 \frac{2 wr_0 \tan \alpha + x \tan^2 \alpha}{\pi \left(wr_0^2 - 2 wr_0 x \tan \alpha + x^2 \tan^2 \alpha\right)^2}. 
\]

(80)

The maximum strain rate occurs at \(x = dh_1\).

### 7.5.2 Forging

While extrusion is characterized by relatively straightforward geometry behavior, at least to a first order of approximation, forging, particularly closed die forging, presents significantly more challenges. Specifically, certain phases of forging with flash involves multiple free surfaces whose relative movement is controlled by die geometry and friction.

Die fill is controlled by the formation of flash. Ideally the flash land in the die is designed so that as the dies close and metal is forced between the dies, the pressure in the part cavity is sufficient to fill the cavity without breaking or deforming the die[4]. Figure 12 illustrates flash formation concurrent with underfill in the die cavity. The material flow pattern which will actually develop is the pattern which requires the least energy at any point in time. A common method, aside from the finite element method, used to predict die fill is the Upper Bound Elemental Technique (UBET) [2,26-34]. This method is less detailed, and hence less computationally intensive than the finite element method, but is still an incremental technique requiring numerous solutions at small time steps throughout the process. For this research, a hybrid technique based partly on UBET and partly on volume constancy
calculations has been developed. This method minimizes computational effort while still producing reasonable predictions of die fill.

![Figure 12: Schematic of typical forging showing underfill and flash formation](image)

While precise geometric details are not necessary given the applications of this research, a reasonable description of geometric behavior during deformation is necessary for accurate determinations of strain and associated properties (plastic work, recrystallization, etc). A study of typical velocity field patterns during die fill reveals four phases of flow (Figure 13). In the first two phases (top image in Figure 13), as the workpiece is upset from the center, the free surface expands until it contacts and fills the outside geometry of the die. The third phase (middle image in Figure 13) is characterized by flash formation and backfilling, wherein the formation of flash increases the hydrostatic stress in the die cavity and assists in backfilling unfilled cavities. After the die is completely filled, material flows exclusively through the flash gutter (bottom image in Figure 13). Predicting the behavior of the first, second, and fourth phases of flow is relatively straightforward, and can be
predicted to a reasonable degree of accuracy by volume constancy methods.

Figure 13: Velocity field flow phases in rotor forging showing flow reversals at the end of the second and third phases
The velocity fields behave in an easily predictable manner. However, according to the upper bound theorem, the relative velocities of the free surfaces in the flash gutter and in the backfilling region during the third phase will correspond to the velocity field with the lowest energy dissipation rate. Hence, an upper bound solution is necessary to determine these relative velocities. This hybrid volume constancy / UBET approach to die fill prediction is described in detail below.

7.5.3 Forging Parameters

Three general forging configurations can be analyzed. Flat die or “pancake” forging uses two flat dies to upset the workpiece to a uniform height. Blocker forging is assumed to indent the center of the workpiece, but produce no constraint on the outer diameter. Closed die forging, described above, produces a part which completely fills the die and forms flash.
7.5.3.1 Flat Die Forging

The input parameters for flat die forging are listed in Table 3.

Table 3: Input parameters for flat die forging

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Variable</th>
<th>Corresponding C Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Billet Half Length</td>
<td>$w_{h0}$</td>
<td>$wh[0]$</td>
</tr>
<tr>
<td>Initial Billet Radius</td>
<td>$w_{r0}$</td>
<td>$wr[0]$</td>
</tr>
<tr>
<td>Initial Billet Temperature</td>
<td>$wT_0$</td>
<td>wkpc-&gt;gridpt[1].T</td>
</tr>
<tr>
<td>Initial Billet Strain</td>
<td>$\varepsilon_0$</td>
<td>wkpc-&gt;gridpt[1].eps</td>
</tr>
<tr>
<td>Final die height</td>
<td></td>
<td>$dh[1] + offset$</td>
</tr>
<tr>
<td>Die Velocity</td>
<td>$V_{die}$</td>
<td>forset-&gt;die_speed</td>
</tr>
<tr>
<td>Die Exit Diameter</td>
<td>$dr_1$</td>
<td>dr[1]</td>
</tr>
<tr>
<td>Ambient Temperature</td>
<td>$T_{amb}$</td>
<td>forset-&gt;</td>
</tr>
<tr>
<td>Die Temperature</td>
<td>$T_{die}$</td>
<td>forset-&gt;die_temp</td>
</tr>
</tbody>
</table>

Table 4: Forging Output Parameters

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Variable</th>
<th>Corresponding C Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final Billet Half Height</td>
<td>$w_{h1}$</td>
<td>$wh[1]$</td>
</tr>
<tr>
<td>Final Billet Radius</td>
<td>$w_{r1}$</td>
<td>$wr[1]$</td>
</tr>
<tr>
<td>Final Billet Temperature</td>
<td>$wT_f$</td>
<td>wkpc-&gt;gridpt[1].T</td>
</tr>
<tr>
<td>Total Strain</td>
<td>$\varepsilon$</td>
<td>wkpc-&gt;gridpt[1].eps</td>
</tr>
<tr>
<td>Peak Strain Rate</td>
<td>$\dot{\varepsilon}$</td>
<td>wpkc-&gt;gridpt[1].epsdot</td>
</tr>
<tr>
<td>Peak Forging Force</td>
<td>$F$</td>
<td>$F$</td>
</tr>
<tr>
<td>Peak Pressure on Die</td>
<td>$P$</td>
<td>$P$</td>
</tr>
</tbody>
</table>

Flat die output parameters are listed in Table 4.

The final height of the workpiece is simply determined by the final height of the dies. The final radius can be determined from volume constancy. Assuming barreling is negligible (a reasonable first order approximation), the radius does not vary along the height of the workpiece. The radius is given by
The uniform strain is given by

\[ \varepsilon = \varepsilon_0 + \ln \left( \frac{w h_1}{w h_0} \right) . \]  \hspace{1cm} (82)

The peak strain rate is given by

\[ \dot{\varepsilon} = \frac{\dot{v}}{w h_1} . \]  \hspace{1cm} (83)

### 7.5.3.2 Open Die Forging

Open die forging assumes an indentation of the center of the disk, without constraint on the outside of the disk. The initial and final configuration for a quarter section of an open die forging are illustrated schematically in Figure 14 and Figure 15 respectively.
Figure 14: Initial configuration for open die forging

Figure 15: Final configuration for open die forging
The input parameters for open die forging are listed in Table 5.

**Table 5**: Input parameters for open die forging

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Variable</th>
<th>Corresponding C Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Billet Half Length</td>
<td>wh₀</td>
<td>wh[0]</td>
</tr>
<tr>
<td>Initial Billet Radius</td>
<td>wr₀</td>
<td>wr[0]</td>
</tr>
<tr>
<td>Initial Billet Temperature</td>
<td>wT₀</td>
<td>wkpc-&gt;gridpt[1].T</td>
</tr>
<tr>
<td>Initial Billet Strain</td>
<td>ε₀</td>
<td>wkpc-&gt;gridpt[1].eps</td>
</tr>
<tr>
<td>Half thickness of die at web</td>
<td>dh₁</td>
<td>dh[1]</td>
</tr>
<tr>
<td>Half thickness of die at flange</td>
<td>dh₂</td>
<td>dh[2]</td>
</tr>
<tr>
<td>Radius of die at web</td>
<td>dr₁</td>
<td>dr[1]</td>
</tr>
<tr>
<td>Outer radius of die</td>
<td>dr₂</td>
<td>dr[2]</td>
</tr>
<tr>
<td>Die Velocity</td>
<td>V_die</td>
<td>forset-&gt;die_speed</td>
</tr>
<tr>
<td>Die Exit Diameter</td>
<td>dr₁</td>
<td>dr[1]</td>
</tr>
<tr>
<td>Ambient Temperature</td>
<td>T_amb</td>
<td>forset-&gt;</td>
</tr>
<tr>
<td>Die Temperature</td>
<td>T_die</td>
<td>forset-&gt;die_temp</td>
</tr>
</tbody>
</table>

To model the process, flow patterns corresponding to a single flow phase will be modeled as a single step. In the initial condition, the workpiece is defined as two concentric rings of equal height wh₀ as illustrated in Figure 14. The radius of ring 1 corresponds to the shoulder in the die. Volume lost from ring 1 forms ring 2. The volume of ring 3 is assumed to remain constant. Through upper bound analysis, Lee [60] has shown that if ring 3 is assumed to maintain its rectangular cross section, then it is reasonable to assume that wh₃ remains constant if ring 3 is not in contact with a die. Thus from volume constancy, Δvol₁ = -Δvol₂ and Δvol₃ = 0. The radius of ring 1 (wr₁) remains constant, and equal to the die shoulder radius (dr₁). The inner height of ring 2 (wh₂) is equal to the height of ring 1 (wh₁) and the outer height, as always, is equal to wh₃.
This flow phase ends when the top of ring 3 contacts the die. By establishing the die height at this position, there is adequate information to uniquely define the heights, inside, and outside diameters of each of the 3 rings in the workpiece, given the constant volume considerations detailed above.

Define the value $offset$ as the difference between the current position of the die and the position of the die at full closure. Thus, when the center of the die first contacts the workpiece, the following equality holds:

$$wh_i = dh_i + offset .$$  \hfill (84)

The first phase of flow stops when the die contacts the top of ring 3. The value of $offset$ at that point is given by:

$$offset = wh_3 - dh_2 .$$  \hfill (85)

The new height of ring 1 is given by equation (84). The radius of ring 1 ($wr_1$) is still equal to the radius of the shoulder in the die ($dr_1$). Thus, a new, smaller, volume of ring 1 can be calculated, and the displaced volume is assigned to ring 2:

$$vol_{2_{new}} = vol_{1_{old}} - vol_{1_{new}} .$$  \hfill (86)
The volume of ring 2 is given by

\[ \text{vol}_2 = 2\pi \left[ \frac{(h_3 - h_1)r_2^3}{(r_2 - r_1)^6} + \frac{h_3}{2} \frac{r_2^2}{(r_2 - r_1)^3} - \frac{(h_3 - h_1) r_1^3}{(r_2 - r_1)^3} + \frac{(h_3 - h_1) r_2^2}{2} - \frac{h_3}{2} \frac{r_1^2}{(r_2 - r_1)^2} \right] . \] (87)

The combined volume of rings 1 and 2 is given by

\[ \text{vol}_1 + \text{vol}_2 = \pi r_1^2 h_1 + 2\pi \left[ \frac{h_3}{6(r_2 - r_1)} + \frac{h_3 r_2^3}{3(r_2 - r_1)} - \frac{h_3 r_1^3}{2(r_2 - r_1)} \right] + 2\pi \left[ \frac{r_2^3}{6(r_2 - r_1)} + \frac{r_1^3}{3(r_2 - r_1)} - \frac{r_2 r_1^2}{2(r_2 - r_1)} \right] h_1 \] (88)

where \(wr_1, \ wr_2, \ wh_1,\) and \(wh_3\) have been abbreviated \(r_1, r_2, h_1,\) and \(h_3\) for convenience, and \(wh_3\) is the height of ring 3, which is equal to the height of ring 2 at the outside of ring 2.

The outer radius of ring 2 (\(wr_2\)) can be found iteratively by bisection. Given \(wr_2\) and \(wh_3\), the outside radius of ring 3 can be determined from

\[ wr_3 = \sqrt{\left( \frac{\text{vol}_3}{\pi wh_2} \right) + wr_2^2} . \] (89)

In the second phase of flow, the die continues downward to the end of the stroke. Volume continues to be displaced from ring 1 into ring 2. By assumption, the volume of ring 3 remains constant. Since the height of the die is known, the height of ring 1 is known. The radius of ring 1 is known, hence the volume is known. The volume of ring 2 is therefore also known. The height of the outside of ring 2 is established by the die. Given the volume, height, and inside radius, equation (88) can be solved for \(wr(2)\) as described above. Again, as above, the outer radius of ring 3 is determined by equation (89).
Strain is calculated for each phase in the circumferential, radial, and thickness directions. The strain of each region is given by

\[
\varepsilon_c = \ln \left( \frac{w_f - w_i}{w_f - w_i} \right) \quad (90)
\]

\[
\varepsilon_r = \ln \left( \frac{w_f - w_i}{w_f - w_i} \right) \quad (91)
\]

\[
\varepsilon_h = \ln \left( \frac{w_h - w_i}{w_h - w_i} \right) \quad (92)
\]

The effective strain is

\[
\varepsilon = \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_c - \varepsilon_r)^2 + (\varepsilon_r - \varepsilon_h)^2 + (\varepsilon_h - \varepsilon_c)^2} \quad (93)
\]

The temperature rise is given by

\[
\Delta T = C_T \frac{\int \varepsilon \sigma d\varepsilon}{\rho C_p} \quad (94)
\]

where \( C_T \) is a correction factor for redundant work \( (C_T \geq 1) \), and \( \sigma \) is the flow stress, modeled by the equation

\[
\sigma = K \dot{\varepsilon}^n \exp(C_r/T) \quad (95)
\]

Where \( T \) is the absolute Temperature, and \( K, m, n, \) and \( C_T \) are constants related to the initial state of the material.
7.5.3.3 Closed Die Forging

7.5.3.3.1 Initial Phases: Volume Constancy

The fundamental phases of closed die forging are illustrated in Figure 13. For the simplified model, there are 4 assumed flow phases. The first is similar to the first phase of open die forging described above, where the web region of the die indents the center of the workpiece (ring 1), and ring 3 is unconstrained. This single free surface flow phase ends when ring 3 contacts the die. This contact can occur in one of two configurations (Figure 16 and Figure 17).

Figure 16: First possible configuration at end of closed die flow phase 1. Die contacts top of ring 3.
For the first configuration, (Figure 16) the dimensions of rings 1, 2, and 3 are the same as the end of phase 1 in open die forging, and equations (85) through (89) apply. For the second configuration (Figure 17), the outside of ring 3 ($wr_3$) is equal to the die rim diameter ($dr_2$). By assumption, the height of ring 3 ($wh_3$) is unchanged. Thus, the inside diameter of ring 3 is established by volume constancy:

$$wr_2 = \sqrt{wr_3^2 - \frac{vol_1}{\pi wh_2}}. \quad (96)$$

Equation (88) can be rearranged to find the height of ring 1 ($wh_1$).
\[
    h_1 = \frac{vol_1 + vol_2 - 2\pi \left[ -\frac{h_2 r_3^3}{6(r_2 - r_1)} + \frac{h_2^2 r_3^2}{2} - \frac{h_2 r_1^3}{3(r_2 - r_1)} + \frac{h_2 r_2 r_1^2}{2(r_2 - r_1)} - \frac{h_2}{2} r_1^2 \right]}{\pi r_1^2 + 2\pi \left[ \frac{r_2^3}{6(r_2 - r_1)} + \frac{r_1^3}{3(r_2 - r_1)} - \frac{r_2 r_1^2}{2(r_2 - r_1)} \right]}
\]

where, as with equation (88), \(wr_1, r_2, wh_1, \) and \(wh_2\) have been abbreviated \(r_1, r_2, h_1, \) and \(h_2\) for convenience.

The end of the second phase is illustrated in Figure 18. Ring 3 fills the outside corner of the rim region. The outside radius of ring 3 is established by the die radius (\(wr_3 = dr_3\)). The difference in heights between ring 1 and ring 3 is established by the difference in heights between the rim and web region of the die (\(wh_3 - wh_1 = dh_3 - dh_1\)).
All heights are unknown at the instant of contact. However, since the height of ring 3 can be written as a function of the height of ring 1, the heights can be reduced to a single variable. Define $\Delta h = dh_2 - dh_1$, then

$$vol_1 + vol_2 = \frac{\pi}{3}(wr_2 - wr_1) \left[ \frac{vol_3}{\pi (wr_3^2 - wr_2^2)} \right] 3(wr_3 - wr_2) - \Delta h(2wr_1 + wr_2)$$

and $wr_2$ can be found using an iterative technique. Given $wr_2$

$$wh_3 = \frac{vol_3}{\pi (wr_3^2 - wr_2^2)}$$
7.5.3.3.2 Backfilling and Flash Formation: Upper Bound Elemental Technique

In flow phase 4, flash formation increases the hydrostatic stress in the die cavity and causes the workpiece to completely fill the cavity. Flow is characterized by two free surfaces, and the relative motion of these two free surfaces is dictated by geometric, frictional and material considerations. If the die geometry is not properly designed, the part may not fill completely before the die is fully closed.

Since die fill is influenced by more than geometric consideration, constant volume analysis alone cannot be used to predict behavior. The upper bound elemental technique is an alternative, energy based method which can predict die fill.

7.5.3.3.2.1 Theory

The Upper Bound Elemental Technique is based on the upper bound theorem. A plastically deforming region is subdivided into simple shape elements linked together by shear surfaces. Each element has a general solution of a simple (usually parallel) velocity field, and the elements are linked together by shear surfaces. Hence, the kinematically admissible velocity field for the deforming region is simply the assembly of the velocity fields of the elements. According to the upper bound theorem, the specific velocity field

\[
wh_i = wh_j - \Delta h.
\] (100)
for the region corresponding to the lowest deformation work rate is the best estimate of
the actual velocity field.

The deformation work rate is given by

\[ W = \dot{W}_i + \dot{W}_s + \dot{W}_f \]  

(101)

where \( W_i \) is the internal power dissipation due to plastic deformation of the elements, \( W_s \) is
the power dissipation due to inter-element shearing, and \( W_f \) is the power dissipation due to
friction shearing between the die and workpiece:

\[ \dot{W}_s = k \int_{r_s} |\Delta U_s| dS \]  

(102)

\[ \dot{W}_i = \int_{vol} \sigma \dot{\varepsilon} dV \]  

(103)

\[ \dot{W}_f = m k \int_{r_f} |\Delta U_f| dS \]  

(104)

Where \( \Delta U_s \) and \( \Delta U_f \) are the inter-element and element-die sliding velocities, respectively,
\( r_s \) and \( r_f \) are the inter-element and element-die surface areas, \( k \) is the plastic flow stress in
shear, and \( m \) is the interface shear friction factor.

Previous researchers have addressed geometry evolution by updating the element geometry
by extrapolating the velocity field through a finite time step [35]. The deformed geometry
is then rezoned and used as the starting point for the next time step. Since only two free surfaces are present in the current research, and their location is well constrained, a fixed mesh will be used, and only the position of the free surfaces will be updated. The proposed mesh is shown in Figure 19.

Generalized velocity fields are required for rectangular and triangular elements. A parallel velocity field is assumed for both cases. That is \( u = u(r) \) and \( w = w(z) \), where \( u \) and \( w \) are the \( r \) and \( z \) components of velocity, respectively. For incompressibility in a cylindrical coordinate system:

\[
\varepsilon_r + \varepsilon_\theta + \varepsilon_z = 0
\]

(105)

or

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0.
\]

(106)

Figure 19: Upper bound element discretization mesh for rotor forging
For a typical rectangular element shown in Figure 20, the generalized velocity fields are given by

\[
    u(r) = -\frac{(w_{i,j+1} - w_{i,j})}{2(z_{j+1} - z_j)} r + \left[\frac{u_{i,j} r_i + (w_{i,j+1} - w_{i,j})}{2(z_{j+1} - z_j)} r_i^2\right] \frac{1}{r} \tag{107}
\]

and

\[
    w(r) = \frac{(w_{i,j+1} - w_{i,j})}{(z_{j+1} - z_j)} z + \frac{(w_{i,j+1} - w_{i,j})}{(z_{j+1} - z_j)}. \tag{108}
\]
7.5.3.3.2.3 Triangular Element

Previous researchers have derived velocity fields for arbitrarily inclined triangular elements [33]. However, for the present research, such a velocity field introduces an unnecessary element of computational complexity. Therefore, a computationally simpler velocity field has been derived, and is illustrated in Figure 21 and the following text.

![Figure 21: Dimensions for Eulerian-Lagrangian triangular upper bound element](image)

In order to derive a parallel velocity field for the triangular element, it is necessary to find the velocity of the Lagrangian boundary in Figure 21 based on the volumetric flow rate of material across the Eulerian boundaries. For simplicity, we can assume that the diagonal (Lagrangian) boundary moves parallel to the original boundary, thus
\[
\frac{dz_2}{dr_i} = \frac{z_2 - z_1}{r_2 - r_i}
\]  

(109)

The volumetric flow rate across the boundary \( z = z_1 \) is given by

\[
\dot{V}_z = \pi(r_2^2 - r_1^2)w(z_1)
\]

(110)

and the volumetric flow rate across the boundary \( r = r_2 \) is given by

\[
\dot{V}_r = 2\pi(h_2 - h_1)r_2u(r_2)
\]

(111)

where \( u(r) \) and \( w(z) \) are the velocity components in the \( r \) and \( z \) direction, respectively.

The velocity of the Lagrangian boundary is defined by the (virtual) velocity of its endpoints, given by

\[
u(r_i) = 3\frac{(r_2^2 - r_1^2)w(z_1) + 2(z_2 - z_1)r_2u(r_2)}{\left(-r_2h_2 - 2r_1h_2 + r_2h_1 + 2r_1h_1 + \frac{(h_2 - h_1)}{(r_2 - r_i)}(2r_2^2 - r_1r_2 - r_1^2)\right)}
\]

(112)

and

\[
w(z_2) = \frac{(h_2 - h_1)}{(r_2 - r_i)}u(r_i).
\]

(113)

### 7.5.3.3.2.4 Implementation

Each element edge velocity constitutes a degree of freedom in the energy minimization. The complete system, as defined, has 46 degrees of freedom (4 degrees of freedom for 11 quadrilateral elements, 2 degrees of freedom for 1 triangular element). Imposed boundary
conditions, volume constancy, and inter-element velocity continuity requirements reduce the number of unknown degrees of freedom to 6. The analysis of constraints, known, and unknown parameters is detailed in Table 6.

Table 6: Analysis of known and unknown degrees of freedom in UBET energy minimization.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Element</th>
<th>Component</th>
<th>Value/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>u1</td>
<td>0: symmetry</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>u2</td>
<td>f(eq. 1,3,4): volume constancy</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>w1</td>
<td>0: symmetry</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>w2</td>
<td>die velocity</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>u1</td>
<td>elem. 1 u2 : continuity</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>u2</td>
<td>f(5,7,8): volume constancy</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>w1</td>
<td>0: symmetry</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>w2</td>
<td>unknown</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>u1</td>
<td>elem. 1 u2 : continuity</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>u2</td>
<td>f(9,11,12): volume constancy</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>w1</td>
<td>elem. 2 w2 : continuity</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>w2</td>
<td>unknown</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>u1</td>
<td>0: die contact</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>u2</td>
<td>f(13,15,16): volume constancy</td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td>w1</td>
<td>elem. 3 w2 : continuity</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>w2</td>
<td>unknown</td>
</tr>
<tr>
<td>17</td>
<td>5</td>
<td>u1</td>
<td>elem. 4 u2 : continuity</td>
</tr>
<tr>
<td>18</td>
<td>5</td>
<td>u2</td>
<td>f(17,19,20): volume constancy</td>
</tr>
<tr>
<td>19</td>
<td>5</td>
<td>w1</td>
<td>elem. 3 w2 : continuity</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>w2</td>
<td>unknown</td>
</tr>
<tr>
<td>21</td>
<td>6</td>
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</tr>
<tr>
<td>22</td>
<td>6</td>
<td>w2</td>
<td>elem. 7 u1 : continuity</td>
</tr>
<tr>
<td>23</td>
<td>7</td>
<td>u1</td>
<td>f(24,25,26): volume constancy</td>
</tr>
<tr>
<td>24</td>
<td>7</td>
<td>u2</td>
<td>elem. 5 u2 : continuity</td>
</tr>
<tr>
<td>25</td>
<td>7</td>
<td>w1</td>
<td>elem. 5 w2 : continuity</td>
</tr>
<tr>
<td>26</td>
<td>7</td>
<td>w2</td>
<td>die velocity</td>
</tr>
<tr>
<td>27</td>
<td>8</td>
<td>u1</td>
<td>elem. 2 u2 : continuity</td>
</tr>
<tr>
<td>28</td>
<td>8</td>
<td>u2</td>
<td>f(27,29,30): volume constancy</td>
</tr>
<tr>
<td>29</td>
<td>8</td>
<td>w1</td>
<td>0: symmetry</td>
</tr>
<tr>
<td>30</td>
<td>8</td>
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</tr>
<tr>
<td>31</td>
<td>9</td>
<td>u1</td>
<td>elem. 3 u2 : continuity</td>
</tr>
<tr>
<td>32</td>
<td>9</td>
<td>u2</td>
<td>0: die contact</td>
</tr>
<tr>
<td>33</td>
<td>9</td>
<td>w1</td>
<td>elem. 8 w2 : continuity</td>
</tr>
</tbody>
</table>
The table shows that there are 6 unknown velocity values. The values of these 6 velocities which give the minimum value for total energy is the lowest upper bound for this assumed velocity field. The velocity field is minimized using Powel's method [21]. Given the velocity field for a given geometric configuration, a new workpiece geometry can be obtained by updating the free surfaces through an incremental distance \( u(\Delta t) \) where \( u \) is the velocity of the free surface, and \( \Delta t \) is a small time step.

In the software implementation \( \Delta t \) is selected such that 20 time steps will be required for the die to move from its position at the beginning of phase 3 to the fully closed position. After each step, the height of the inside of ring 2 (\( wr_2 \)) is checked against the height of the rim section of the die (\( dh_2 + \text{offset} \)) to see if fill has completed. If the fill has completed, control is passed to the phase 4 (flash formation) calculation. If fill has not occurred when the die reaches the fully closed position, the program returns an error code.

<table>
<thead>
<tr>
<th>34</th>
<th>9</th>
<th>w2</th>
<th>unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>10</td>
<td>u1</td>
<td>elem. 5 u2 : continuity</td>
</tr>
<tr>
<td>36</td>
<td>10</td>
<td>u2</td>
<td>0 : die contact</td>
</tr>
<tr>
<td>37</td>
<td>10</td>
<td>w1</td>
<td>elem. 9 w2 : contact</td>
</tr>
<tr>
<td>38</td>
<td>10</td>
<td>w2</td>
<td>die velocity</td>
</tr>
<tr>
<td>39</td>
<td>11</td>
<td>u1</td>
<td>elem. 8 u2 : continuity</td>
</tr>
<tr>
<td>40</td>
<td>11</td>
<td>u2</td>
<td>f(39,41,42) : volume constancy</td>
</tr>
<tr>
<td>41</td>
<td>11</td>
<td>w1</td>
<td>0 : symmetry</td>
</tr>
<tr>
<td>42</td>
<td>11</td>
<td>w2</td>
<td>die velocity</td>
</tr>
<tr>
<td>43</td>
<td>12</td>
<td>u1</td>
<td>elem. 11 u2 : continuity</td>
</tr>
<tr>
<td>44</td>
<td>12</td>
<td>u2</td>
<td>f(43,45,46) : volume constancy</td>
</tr>
<tr>
<td>45</td>
<td>12</td>
<td>w1</td>
<td>0 : symmetry</td>
</tr>
<tr>
<td>46</td>
<td>12</td>
<td>w2</td>
<td>unknown</td>
</tr>
</tbody>
</table>
7.5.3.3 Flash Formation

After the die has been determined to fill through the phase 3 UBET analysis, the die continues to move down to its final position. At this time, the only significant geometry change is uniform height reduction in the workpiece and formation of flash. All flash is assumed to originate from ring 3, thus reducing its volume. The radius of ring 2 is assumed to expand, as material from ring 1 fills ring 2, and ring 2 forces ring 3 into flash.

7.5.4 Die Load and Pressure Predictions: Modified Slab Analysis

Traditionally, slab analysis assumes a flat or smoothly varying workpiece in continuous contact with the die. In the general case of open or closed die forging, the workpiece may exhibit either abrupt changes in height or discontinuous contact with the die. Hence modifications to traditional slab analysis are necessary to address both of these issues.

7.5.4.1 Treatment of Rings

The die contact condition and surface geometry is consistent within a particular ring. Hence, slab analysis can be performed piecewise over each ring, with radial boundary conditions at the outside of each ring determined by the boundary conditions of the inside of the next outer ring.

The pressure distribution on a ring with inside diameter $r_1$ and outside diameter $r_2$ subject to uniform stress $\sigma(r_2)$ at the outside diameter can be derived using the slab method as illustrated in Figure 22.
Applying equilibrium equations in the radial direction yields

\[-hr\sigma_r + h(r + dr)(\sigma_r + d\sigma_r) - h\sigma_\theta - 2\tau dr = 0\]  \hspace{1cm} (114)

or

\[\frac{d\sigma_r}{dr} + \frac{\sigma_r}{r} - \frac{\sigma_\theta}{r} - \frac{2\tau}{h} = 0.\]  \hspace{1cm} (115)

where \(\sigma_\theta\) is the hoop stress acting on the slab. It may be shown [23] that \(\sigma_r = \sigma_\theta\). Thus, equation (115) may be rewritten

\[\frac{d\sigma_r}{dr} - \frac{2\tau}{h} = 0\]  \hspace{1cm} (116)

For sticking friction conditions which dominate in bulk forming processes, \(\tau = mk\), where \(m\) is the shear friction factor and \(k\) is the shear flow stress of the deforming material. By the Tresca yield criterion,
Thus, the pressure variation with radius is given by

\[ \frac{dp}{dr} = -\frac{2mk}{h}. \]  

(119)

Separating the variables and integrating gives

\[ \int_{p_1}^{p_2} dp = -\frac{2mk}{h} \int_1^r dr. \]  

(120)

or

\[ p_r = p_2 + \frac{2mk}{h} (r_2 - r) \]  

(121)

where \( p_r \) is the pressure at radius \( r \). \( p_2 \) is given by equation (119) and the boundary condition \( \sigma \) at \( r_2 \).
7.5.4.2 Changes in Height

Since the only known boundary condition is on the outside of the outermost ring, equation (121) must be solved ring by ring working from the outside in. Given a step change in height between adjacent rings, the boundary condition $\sigma_r$ at the outside of the inner ring is equal to the radial stress at the inside of the outer ring, where

$$\sigma_r = p - k$$

(122)

where $p$ is the die pressure at the inside radius of the outer ring.

7.5.4.3 Free Surfaces

The surface of ring 2 in the unfilled region (see for example Figure 15) is not in contact with the die. Hence, the boundary conditions are not established in the same manner as in step contact.

The governing equation for the stress field is given by [60]

$$\frac{\partial (h\sigma_x)}{\partial r} + \frac{h}{r} (\sigma_r - \sigma_\theta) = 0.$$  

(123)

where

$$h(r) = h_i + (r - r_i)T$$  

(124)

where $T = tan(\alpha) = \frac{h_2 - h_1}{r_2 - r_1}$ and, by the Tresca yield criterion
Integrating equation (123) gives

\[ h\sigma_r = -\bar{\sigma} \int_{r_1}^{r_2} \frac{h_1 - r_1T}{r} + T \, dr \]  

(126)

or

\[
\frac{\sigma_{\theta}}{\bar{\sigma}} = \left( \frac{h_1 - r_1T}{h} \right) \ln \left( \frac{r_1}{r_2} \right) + \bar{\sigma} \left[ \frac{h_2 - h_1}{r_2 - r_1} \right] + \sigma_{\theta}.
\]  

(127)

This equation can be combined piecewise with equation (121), either using (121) to provide outer boundary conditions for (127), or by using (127) to provide outer boundary conditions for (121).

### 7.5.5 Temperature Predictions: Simplified Finite Difference Approach

The temperature rise per unit volume due to plastic work is given by equation (94). Temperature rise can be calculated independently for each ring at each phase of deformation.

The temperature in each ring can be found using finite difference analysis, with each ring corresponding to a node. The thermal resistance network is illustrated in Figure 23.
The finite difference equations for the nodes are given by

\[
\frac{T_{1}^{\text{new}} - T_{1}^{\text{old}}}{\Delta t} = \frac{1}{\rho V_1 C_p} \left[ (T_2^{\text{old}} - T_1^{\text{old}}) k A_{12} \Delta x_{12} + (T_1^{\text{old}} - T_{\text{die}}) h_{\text{cond}} A_{\text{cond}} + (T_1^{\text{old}} - T_{\text{amb}}) h_{\text{amb}} A_{\text{amb}} \right] \tag{128}
\]

\[
\frac{T_{2}^{\text{new}} - T_{2}^{\text{old}}}{\Delta t} = \frac{1}{\rho V_2 C_p} \left[ (T_1^{\text{old}} - T_2^{\text{old}}) k A_{12} \Delta x_{12} + (T_3^{\text{old}} - T_2^{\text{old}}) k A_{23} \Delta x_{23} + (T_2^{\text{old}} - T_{\text{die}}) h_{\text{cond}} A_{\text{cond}} + (T_2^{\text{old}} - T_{\text{amb}}) h_{\text{amb}} A_{\text{amb}} \right] \tag{129}
\]

\[
\frac{T_{3}^{\text{new}} - T_{3}^{\text{old}}}{\Delta t} = \frac{1}{\rho V_3 C_p} \left[ (T_2^{\text{new}} - T_3^{\text{old}}) k A_{23} \Delta x_{23} + (T_3^{\text{old}} - T_{\text{die}}) h_{\text{cond}} A_{\text{cond}} + (T_3^{\text{old}} - T_{\text{amb}}) h_{\text{amb}} A_{\text{amb}} \right] \tag{130}
\]
Where $A_{ij}$ is the average cross sectional area between nodes $i$ and $j$, and $x_{ij}$ is the distance between the centers of rings $i$ and $j$. The temperatures can be solved numerically over several steps.
8. VALIDATION OF PROCESS MODELS

8.1 Comparison with an Accepted Standard

While the new deformation models are based on first principles and fundamental physics of the deformation process, several simplifying assumption were made to facilitate fast computation of results. While these extremely fast computations are necessary to make practical the global optimization of multiple operations, the results of these computations must be accurate enough that the calculated objective function value aligns well with the objective function value of the actual process. Furthermore, the error or variability in the process model relative to the actual process variables should be quantifiable.

8.1.1 Standard for Comparison

The rigid-viscoplastic finite element formulation utilizing a flow formulation has been implemented in the finite element codes such DEFORM [43], and its predecessor ALPID, among others. These codes have been widely validated and shown to give excellent agreement with experimental values for non-isothermal bulk deformation problems [41-44]. Because of its availability, DEFORM was chosen as a reference for comparison and validation of the new deformation models.
8.1.2 Basis of Comparison

As noted previously, the goal of the deformation models is to provide a reliable estimate of objective function value for a given set of input parameters, with a minimal computation time. The primary thermomechanical components of the objective function are geometry, strain, temperature, and forming pressures and loads. Other components of the objective function, including production costs and material costs, are substantially easier to predict, and less sensitive to the accuracy of the material model. Therefore, the primary validation of the deformation models as an objective function predictor will be based on the accuracy of geometry, strain, strain rate, and temperature predictions.

8.1.3 Measures of Error

A series of forging simulations were run using both DEFORM and the newly developed deformation model. The input parameters were generated randomly within a given range of acceptable values. The models were evaluated for accuracy of geometry behavior. Those parameter sets which produced an acceptable geometry were also evaluated for field variable and load predictions.

8.1.3.1 Geometry

The geometry of a fully filled impression die forging is determined by the shape of the die. Thus, geometry comparisons cannot be based simply on shape, except in cases of unfill. A second, scaled (not binary) parameter which describes the accuracy of geometry predictions both in cases of fill and unfill is desirable. An obvious measure of fill is the position of the die at the instant that the workpiece completely fills the die cavity.
For comparison of the new models to finite element simulations, define a fill parameter with value from -1 to 1, where -1 indicates that absolutely no fill has occurred, and 1 indicates that the cavity fills instantly. A value of 0 indicates that the cavity fills precisely as the die reaches the end of its stroke. A typical workpiece geometry showing partial fill in ring 2 is illustrated schematically in Figure 24.

Figure 24: Workpiece showing partial fill in ring 2.

The fill parameter $\phi$ can be defined numerically as

$$\phi = \frac{\text{offset at fill}}{\text{offset at end of phase 3}}$$

when fill occurs, and

$$\phi = \frac{(dh2 - dh1) - \text{error at closure}}{dh2 - dh1} - 1$$

(131)
when fill does not occur, where the *error at closure* is given by $\text{wh}_3 - \text{wh}_2$ in Figure 24. 

The value of $\phi$ is $-1$ for no filling, and $1$ for "instant" filling. Both are theoretical limits. Practical values will range between these two extremes. These values are easily obtained from both finite element and UBET simulations.

### 8.1.3.2 Field Variables

Titanium and other aerospace alloys must be processed within narrow windows of strain, strain rate, and temperature to produce acceptable microstructures. Thus, the maximum, minimum and average value of each of these variables at any location within the workpiece is important to the ultimate microstructural properties of the part.

The newly developed deformation model predicts only average field variable values within each ring. Finite element simulations predict strain and strain rate at the center of each element, and temperature values at each node. A short computer program was written to scan through the output data of each simulation, and identify all elements and nodes corresponding to each ring in the new deformation model. Once these nodes and elements have been identified, maximum and minimum values can be extracted directly. Average values can be calculated for a relatively uniform mesh density simply by taking the sum of all the values for each ring, then dividing by the number of nodes or elements.
8.1.3.3 Loads and Pressures

Die Load-Stroke curves are available directly in the DEFORM post-processing utility. A step tracing line allows reasonably precise extraction of load data, as illustrated in Figure 25. The peak load values are reported directly in the new deformation model.

Figure 25: Typical Load-Stroke curve from DEFORM showing step tracer.
8.2 Results of Comparison

8.2.1 Extrusion

Table 7: Workpiece and Die Geometry for Extrusion Model Comparisons

<table>
<thead>
<tr>
<th>Case Number</th>
<th>Workpiece Diameter</th>
<th>Workpiece Height</th>
<th>Product Diameter</th>
<th>Die Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
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<td>5</td>
<td>12</td>
<td>10</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

Five extrusion simulations were performed in both FEM and the simplified models. Initial workpiece diameters of 10” and 12” were used. Product diameters of 6”, 7” and 8” were produced. A summary of the comparison cases is listed in Table 7.

8.2.1.1 Geometry

Geometry calculations are based on constant volume assumptions. The extruded product diameter is established by the die diameter. The product length is thus established within minor variations for end effects.

8.2.1.2 Strain

Table 8 contains a comparison of strain values from FEM simulation and simplified model simulation of various extrusions.
Table 8: Comparison of Strain values from FEM and Simplified Models

<table>
<thead>
<tr>
<th>Case</th>
<th>FEM Average Strain</th>
<th>Simplified Model Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.09</td>
<td>1.02</td>
</tr>
<tr>
<td>2</td>
<td>0.765</td>
<td>0.71</td>
</tr>
<tr>
<td>3</td>
<td>0.793</td>
<td>0.71</td>
</tr>
<tr>
<td>4</td>
<td>0.885</td>
<td>0.81</td>
</tr>
<tr>
<td>5</td>
<td>1.47</td>
<td>1.38</td>
</tr>
</tbody>
</table>

On average, the simplified model underestimates strain in the range of geometries studied by 6%. A linear regression fit of the data yields the following fit:

$$
\varepsilon_F = 1.03 \varepsilon_S + 0.04737.
$$

Where $\varepsilon_F$ is the average strain predicted by finite element analysis, and $\varepsilon_S$ is the strain predicted by simplified models. The $r^2$ fit value is .9986.

8.2.1.3 Extrusion Loads

The simplified model consistently underestimates extrusion loads as compared to Finite Element Analysis. The FEM and Simplified Model predicted peak loads for extrusion are listed in Table 9. The underestimate is most likely due to the failure of the slab analysis technique to account for redundant deformation the extrusion die.

Table 9: Comparison of maximum extrusion force predicted by FEM and simplified models.

<table>
<thead>
<tr>
<th>Case</th>
<th>FEM Extrusion Load (kib)</th>
<th>Simplified Model Extrusion Load (kib)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5450</td>
<td>3100</td>
</tr>
<tr>
<td>2</td>
<td>4400</td>
<td>2600</td>
</tr>
<tr>
<td>3</td>
<td>3700</td>
<td>3650</td>
</tr>
<tr>
<td>4</td>
<td>6000</td>
<td>4260</td>
</tr>
<tr>
<td>5</td>
<td>8700</td>
<td>5080</td>
</tr>
</tbody>
</table>

Using linear regression, the following correction equation was developed:
Where $L_F$ is the load predicted by FEM, and $L_s$ is the load predicted by the simplified model. The $r^2$ fit value is .641.

### 8.2.2 Forging

A series of 23 forging simulations were performed using both FEM and simplified forging models. Dimensions for the die and workpiece were selected randomly within guidelines. Reasonable aspect ratios were required for the die. The workpiece volume was selected to exceed the die volume by 5 to 20%. Complete details of each workpiece and die geometry is given in Table 10.

#### Table 10: Geometry of test cases for simplified forging model validation.

<table>
<thead>
<tr>
<th>Case</th>
<th>wh1</th>
<th>wr1</th>
<th>dr1</th>
<th>dr2</th>
<th>dr3</th>
<th>dh1</th>
<th>dh2</th>
<th>dh3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.59</td>
<td>6.82</td>
<td>5.29</td>
<td>7.24</td>
<td>7.49</td>
<td>1.43</td>
<td>2.45</td>
<td>.26</td>
</tr>
<tr>
<td>2</td>
<td>2.01</td>
<td>5.52</td>
<td>4.29</td>
<td>5.83</td>
<td>6.08</td>
<td>.81</td>
<td>2.34</td>
<td>.16</td>
</tr>
<tr>
<td>3</td>
<td>1.62</td>
<td>4.91</td>
<td>4.67</td>
<td>5.6</td>
<td>5.85</td>
<td>.52</td>
<td>2.2</td>
<td>.11</td>
</tr>
<tr>
<td>4</td>
<td>3.73</td>
<td>6.01</td>
<td>4.94</td>
<td>6.49</td>
<td>6.74</td>
<td>1.62</td>
<td>4.12</td>
<td>.15</td>
</tr>
<tr>
<td>5</td>
<td>2.28</td>
<td>4.91</td>
<td>4.68</td>
<td>5.6</td>
<td>5.85</td>
<td>.52</td>
<td>2.2</td>
<td>.11</td>
</tr>
<tr>
<td>6</td>
<td>2.83</td>
<td>5.08</td>
<td>4.05</td>
<td>5.76</td>
<td>6.01</td>
<td>.71</td>
<td>3.07</td>
<td>.31</td>
</tr>
<tr>
<td>7</td>
<td>1.97</td>
<td>4.45</td>
<td>3.91</td>
<td>5.22</td>
<td>5.47</td>
<td>.51</td>
<td>2.26</td>
<td>.42</td>
</tr>
<tr>
<td>8</td>
<td>2.79</td>
<td>5.54</td>
<td>4.88</td>
<td>6.62</td>
<td>6.87</td>
<td>.72</td>
<td>3.13</td>
<td>.24</td>
</tr>
<tr>
<td>9</td>
<td>1.51</td>
<td>5.17</td>
<td>4.65</td>
<td>5.5</td>
<td>5.75</td>
<td>.92</td>
<td>1.64</td>
<td>.23</td>
</tr>
<tr>
<td>10</td>
<td>2.9</td>
<td>4.66</td>
<td>3.94</td>
<td>5.01</td>
<td>5.26</td>
<td>1.5</td>
<td>3.82</td>
<td>.48</td>
</tr>
<tr>
<td>11</td>
<td>2.47</td>
<td>4.6</td>
<td>4.32</td>
<td>5.46</td>
<td>5.71</td>
<td>1.3</td>
<td>2.08</td>
<td>.18</td>
</tr>
<tr>
<td>12</td>
<td>2.41</td>
<td>4.78</td>
<td>4.39</td>
<td>5.05</td>
<td>5.3</td>
<td>1.61</td>
<td>2.76</td>
<td>.18</td>
</tr>
<tr>
<td>13</td>
<td>1.7</td>
<td>5.26</td>
<td>4.54</td>
<td>6.15</td>
<td>6.4</td>
<td>.52</td>
<td>1.89</td>
<td>.35</td>
</tr>
<tr>
<td>14</td>
<td>2.3</td>
<td>5.87</td>
<td>5.29</td>
<td>6.04</td>
<td>6.29</td>
<td>1.52</td>
<td>3.29</td>
<td>.35</td>
</tr>
<tr>
<td>15</td>
<td>1.72</td>
<td>5.17</td>
<td>4.74</td>
<td>6.21</td>
<td>6.46</td>
<td>.63</td>
<td>1.67</td>
<td>.28</td>
</tr>
<tr>
<td>16</td>
<td>2.36</td>
<td>3.83</td>
<td>3.38</td>
<td>4.3</td>
<td>4.55</td>
<td>1.22</td>
<td>2.71</td>
<td>.31</td>
</tr>
<tr>
<td>17</td>
<td>1.86</td>
<td>5.47</td>
<td>3.18</td>
<td>5.09</td>
<td>5.34</td>
<td>1.14</td>
<td>2.43</td>
<td>.53</td>
</tr>
</tbody>
</table>
The forging simulations were evaluated for fill. The field variable values of those simulations which did fill was compared to the FEM solutions.

**8.2.2.1 Geometry**

The fill parameter $\phi$ defined in equations (131) and (132) was calculated for case for both the FEM and simplified forging calculation. The values and absolute error are given in Table 11. The average error is .334 (out of a scale of $-1$ to $1$), and the standard deviation is .25. Case 17 exhibited a fold during filling and is disregarded.
Table 11: Fill parameter $\phi$ calculated from FEM and simplified forging simulations

<table>
<thead>
<tr>
<th>fem $\phi$</th>
<th>simp $\phi$</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3805</td>
<td>0.1844</td>
</tr>
<tr>
<td>2</td>
<td>0.068</td>
<td>0.2604</td>
</tr>
<tr>
<td>3</td>
<td>-0.2976</td>
<td>0.0595</td>
</tr>
<tr>
<td>4</td>
<td>0.0896</td>
<td>0.185</td>
</tr>
<tr>
<td>5</td>
<td>-0.5144</td>
<td>0.5144</td>
</tr>
<tr>
<td>6</td>
<td>-0.7076</td>
<td>0.7876</td>
</tr>
<tr>
<td>7</td>
<td>-1</td>
<td>0.5</td>
</tr>
<tr>
<td>8</td>
<td>-0.7178</td>
<td>0.4207</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0.4036</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0.03</td>
</tr>
<tr>
<td>12</td>
<td>-0.2261</td>
<td>0.1652</td>
</tr>
<tr>
<td>13</td>
<td>-0.8686</td>
<td>0.6789</td>
</tr>
<tr>
<td>14</td>
<td>-0.1921</td>
<td>0.3077</td>
</tr>
<tr>
<td>15</td>
<td>-0.6923</td>
<td>0.7616</td>
</tr>
<tr>
<td>16</td>
<td>-0.438</td>
<td>0.0743</td>
</tr>
<tr>
<td>17 fold</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>-0.5478</td>
<td>0.7142</td>
</tr>
<tr>
<td>19</td>
<td>0.136</td>
<td>0.5783</td>
</tr>
<tr>
<td>20</td>
<td>0.159</td>
<td>0.1193</td>
</tr>
<tr>
<td>21</td>
<td>-0.325</td>
<td>0.3038</td>
</tr>
<tr>
<td>22</td>
<td>0.0566</td>
<td>0.1981</td>
</tr>
<tr>
<td>23</td>
<td>0.1896</td>
<td>0.4396</td>
</tr>
</tbody>
</table>

Detailed analysis indicates that the four largest error values occurred in cases where the thickness of the flash was more than 40% of the thickness of the web ($\frac{dh_3}{dh_1} > 0.4$). This represents an unusually large flash thickness. In all of these cases (6, 7, 13, 18), the simplified forging models overpredicted fill.
If the restriction is imposed that the flash thickness be less than 40% of the web thickness for fill predictions to be valid, then the average error is reduced to 0.25 with a standard deviation of 0.15.

### 8.2.2.2 Strain

Of the 23 cases studied, 7 were judged to have reasonable enough geometric similarity to make comparisons of strain calculations. Comparisons are given in Table 12, Table 13, and Table 14. The average percentage error in strain value is 9% for ring 1, 34% for ring 2, and 33% for ring 3. The standard deviations of the errors are 0.04, 0.13, and 0.16 for rings 1, 2, and 3 respectively. Ring 1 showed excellent agreement between the simplified forging models and FEM. Rings 2 and 3 showed improved agreement with the application of empirically derived correction factors.

**Table 12: Comparison of Strain predictions from FEM and simplified forging models for ring 1.**

<table>
<thead>
<tr>
<th>Case</th>
<th>fem ring 1</th>
<th>simp ring 1</th>
<th>abs error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6639</td>
<td>0.6712</td>
<td>0.0109956</td>
</tr>
<tr>
<td>2</td>
<td>0.928</td>
<td>1.035</td>
<td>0.1153017</td>
</tr>
<tr>
<td>4</td>
<td>0.8599</td>
<td>0.9485</td>
<td>0.1030352</td>
</tr>
<tr>
<td>9</td>
<td>0.4997</td>
<td>0.5685</td>
<td>0.1376826</td>
</tr>
<tr>
<td>10</td>
<td>0.6605</td>
<td>0.7066</td>
<td>0.0697956</td>
</tr>
<tr>
<td>11</td>
<td>0.6037</td>
<td>0.6833</td>
<td>0.1318356</td>
</tr>
<tr>
<td>19</td>
<td>0.49</td>
<td>0.5362</td>
<td>0.0942857</td>
</tr>
</tbody>
</table>
Table 13: Comparison of Strain predictions from FEM and simplified forging models for ring 2. *Corr ring 2* reflects an empirically determined .513 correction factor

<table>
<thead>
<tr>
<th>Case</th>
<th>fem ring 2</th>
<th>simp ring 2</th>
<th>Corr ring 2</th>
<th>abs error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8154</td>
<td>1.05</td>
<td>0.5390144</td>
<td>0.3389571</td>
</tr>
<tr>
<td>2</td>
<td>0.9302</td>
<td>1.6</td>
<td>0.8213552</td>
<td>0.1170122</td>
</tr>
<tr>
<td>4</td>
<td>0.8221</td>
<td>2.3</td>
<td>1.1806982</td>
<td>0.4361977</td>
</tr>
<tr>
<td>9</td>
<td>0.7669</td>
<td>1.78</td>
<td>0.9137577</td>
<td>0.1914952</td>
</tr>
<tr>
<td>10</td>
<td>0.7856</td>
<td>2.15</td>
<td>1.1036961</td>
<td>0.4049085</td>
</tr>
<tr>
<td>11</td>
<td>0.9323</td>
<td>0.85</td>
<td>0.436345</td>
<td>0.5319694</td>
</tr>
<tr>
<td>19</td>
<td>0.61</td>
<td>1.61</td>
<td>0.8264887</td>
<td>0.3548995</td>
</tr>
</tbody>
</table>

Table 14: Comparison of Strain predictions from FEM and simplified forging model for ring 3. *Corr ring 3* reflects an empirically determined .86 correction factor

<table>
<thead>
<tr>
<th>Case</th>
<th>fem ring 3</th>
<th>simp ring 3</th>
<th>Corr ring 3</th>
<th>abs error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8935</td>
<td>0.59</td>
<td>0.513</td>
<td>0.426</td>
</tr>
<tr>
<td>2</td>
<td>0.9432</td>
<td>0.84</td>
<td>0.730</td>
<td>0.226</td>
</tr>
<tr>
<td>4</td>
<td>0.9641</td>
<td>0.85</td>
<td>0.739</td>
<td>0.233</td>
</tr>
<tr>
<td>9</td>
<td>0.8027</td>
<td>1</td>
<td>0.870</td>
<td>0.083</td>
</tr>
<tr>
<td>10</td>
<td>0.7462</td>
<td>1.23</td>
<td>1.070</td>
<td>0.433</td>
</tr>
<tr>
<td>11</td>
<td>0.7625</td>
<td>1.44</td>
<td>1.252</td>
<td>0.642</td>
</tr>
<tr>
<td>19</td>
<td>0.55</td>
<td>0.84</td>
<td>0.730</td>
<td>0.328</td>
</tr>
</tbody>
</table>

8.2.2.3 Forging Loads

Forging load predictions are based on slab analysis, which is a lower bound analysis. A comparison of loads predicted by finite element analysis and the slab model used in the simplified forging model analysis is given in Table 15. Improved agreement was obtained by use of an empirically derived correction factor of 1.83. The average error is 20%, and the standard deviation is .15.
Table 15: Comparison of FEM and simplified forging model load predictions for forgings. Corrected loads reflect an empirical correction factor of 1.83

<table>
<thead>
<tr>
<th></th>
<th>FEM Load</th>
<th>slab load</th>
<th>Corr slab load</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25132</td>
<td>14216</td>
<td>26057.928</td>
<td>0.0294891</td>
</tr>
<tr>
<td>2</td>
<td>17850</td>
<td>9503</td>
<td>17418.999</td>
<td>0.0310667</td>
</tr>
<tr>
<td>4</td>
<td>25254</td>
<td>10897</td>
<td>19974.201</td>
<td>0.2146773</td>
</tr>
<tr>
<td>9</td>
<td>13000</td>
<td>8664</td>
<td>15881.112</td>
<td>0.21296</td>
</tr>
<tr>
<td>10</td>
<td>8910</td>
<td>6433</td>
<td>11791.689</td>
<td>0.3140359</td>
</tr>
<tr>
<td>11</td>
<td>12788</td>
<td>8005</td>
<td>14673.165</td>
<td>0.139279</td>
</tr>
<tr>
<td>19</td>
<td>8991</td>
<td>7427</td>
<td>13613.691</td>
<td>0.5034079</td>
</tr>
</tbody>
</table>

8.3 Objective Function Value

The ultimate goal of the simplified forging process model is to provide an accurate estimate of the objective function value. Yang[57] ran 80 randomly generated cases on both the simplified model and on the finite element program ANTARES. The average error of the estimated objective function value relative to the value predicted by finite element analysis is 6.3% The standard deviation is .087. Figure 26 shows the results of the comparison.
Figure 26: Comparison of objective function values predicted by FEM and Simplified models. Straight line is the $x=y$ line representing perfect predictions (after [57]).
9. CONCLUSIONS AND FUTURE WORK

9.1 Results and Conclusions

In this research, the viability of using simplified process models for optimization has been demonstrated. The new hybrid UBET models provide reasonable results with quantifiable variability and extremely low computation time requirements relative to alternative process models. This combination of characteristics makes possible the application of global optimization search techniques developed for optimization of discrete event dynamic systems to be applied to the complete sequence of manufacturing processes required to produce an aircraft engine rotor disk.

9.2 Future Work

The present research is limited to deformation models. A complete model requires prediction of behavior of all processes, including casting, heat treatment, and machining. Furthermore, the range of shapes is limited to parts that are top/bottom symmetric. The deformation models should be extended to consider non-symmetric parts.
9.2.1 Other Models

9.2.1.1 Casting

Solidification rates and conditions influence the microstructure of the billet, including grain size, alloy segregation, and porosity. A casting model should consider distributions of microstructure characteristics from the surface to the center of the workpiece.

9.2.1.2 Heat Treatment

Heat treatment is performed to produce certain non-equilibrium phases and precipitates in rotor disks. Achievement of these phases at reduced temperatures requires rapid cooling from solution heat treatment temperatures. This rapid cooling tends to introduce residual stresses. The residual stresses can introduce distortion in the disk directly. Residual stresses also tend to be released in machining, resulting in distortions and unacceptable geometry.

A heat treatment model should be capable of predicting microstructure, including both volume composition of phases as well as second phase precipitates. It should also be capable of predicting the distribution of quench-induced residual stresses. Residual stresses introduced during forging are essentially completely relaxed during solution heat treatment, and as such do not generally warrant consideration in the final part.
9.2.1.3 Machining

A simplistic machining model calculating volume of material removal and approximate time required to remove the material was developed by Eberhart[15]. This model does not account for the effects of residual stress release, which can have a substantial effect on the final geometry. An understanding of residual stress distributions, combined with proper tool path planning can minimize the effects of the release of residual stresses during machining.

A proper machining model should consider the thickness of each cut, and should contain an elasto-plastic deformation model which estimates equilibrium shape after each pass, given the fixturing conditions of the workpiece.

9.2.2 Refinement of Present Models

As noted, the present deformation process models assume top/bottom symmetry of the rotor disk forging. While this assumption is adequate for a large class of rotor disks, a more complete model should also consider non-symmetric rotors. An arbitrary number of rings, instead of the current limit of 3, will also increase the versatility of these deformation process models.

9.2.3 Improvement of Objective Function

The present objective function is limited by the results available from the models. More detailed consideration of the effect of microstructure, machining, and residual stress will be available with improved process models. Moreover, aircraft engine component design is
moving away from process design for a single set of design requirements, and towards life cycle design over the entire life of the product. True optimization will consider not only manufacturing costs, but the effects of manufacturing process parameters on part life, and tradeoffs between service life and manufacturing costs.
10. Bibliography


54. R. Tricot, Thermomechanical Processing of Titanium Alloys for the Aerospace Industry, Materiaux et Techniques, v76,n1,Jan-Feb 1988, p47-64.


11. APPENDIX: OBJECTIVE FUNCTION

These models were developed for use with optimization codes developed by research teams at Harvard University and Virginia Polytechnic Institute and State University. For research purposes, a simplified process was assumed. This appendix describes the objective function that was used to assign costs to each result set from the process models. The process was assumed to start with a cast billet. The processing options are illustrated in Figure 27.

![Diagram of process models]

**Figure 27**: Possible paths to producing a finish machined disks in simplified manufacturing process.
Each possible process has specified inputs and outputs, with both inputs and outputs influencing the objective function contribution of the process. The total value of the objective function is the sum of the contributions of each completed process.

The objective function described below was developed by Mullins (W. Mullins: C program cost_fn.c, 1996). It assumes isothermal forging of Titanium-6Al-4V rotors on a 45MN press. Initial microstructure beta quenched. Alpha-beta forging desired. Production run of 3500 disks.

### 11.1 Initial Billet

#### 11.1.1 Inputs

- Diameter of billet
- Height of billet

#### 11.1.2 Outputs

- None.

#### 11.1.3 Objective Function Contributions

- Cost of Material, based on cost per volume
11.2 Upset

11.2.1 Inputs

- Output workpiece height
- Friction factor
- Die speed
- Isothermal temperature

11.2.2 Outputs

- Output temperature
- Output workpiece diameter
- Output strain
- Time
- Die pressure
- Die force
- Output height
- Output strain rate

11.2.3 Objective Function Contributions

- Set up costs (fixed per operation)
- Processing costs (based on operation time)
- Die Damage costs (function of temperature and pressure)
- Penalty for probable wedge cracking * 1500, based on strain rate, temperature, and microstructure.

- Penalty for height/diameter aspect ratio > 2.5. based on Taguchi (quadratic) cost function. Penalty = 1e6 for aspect ratio > 3.0

- Penalty = 1e6 for exceeding press capacity.

11.3 Blocker and Closed Die Forge

11.3.1 Inputs

- Die speed
- Friction factor
- Die geometry
- Output workpiece height

11.3.2 Outputs

- Workpiece geometry
- Output temperature (each ring)
- Output strain (each ring)
- Peak strain rate (each ring)
- Peak die pressure
- Press load
11.3.3 Objective Function Contributions

- Set up costs (fixed per operation)
- Processing costs (based on operation time)
- Die Damage costs (function of temperature and pressure)
- Penalty for probable wedge cracking * 1500, based on strain rate, temperature, and microstructure.
- Penalty for height/diameter aspect ratio > 2.5. based on Taguchi (quadratic) cost function. Penalty = 1e6 for aspect ratio > 3.0
- Penalty = 1e6 for exceeding press capacity.

11.4 Round to Round Extrusion

11.4.1 Inputs

- Output billet diameter
- Die length
- Die shape (Conic or Streamlined)
- Friction Factor
- Extrusion ram velocity
- Billet temperature
11.4.2 Outputs

- Workpiece length
- Strain
- Peak strain rate
- Extrusion force
- Peak die pressure
- Temperature

11.4.3 Objective Function Contributions

- Set up costs (fixed per operation)
- Processing costs (based on operation time)
- Die Damage costs (function of temperature and pressure)
- Penalty for probable wedge cracking * 1500, based on strain rate, temperature, and microstructure.
- Penalty for height/diameter aspect ratio > 2.5. based on Taguchi (quadratic) cost function. Penalty = 1e6 for aspect ratio > 3.0
- Penalty = 1e6 for exceeding press capacity.
11.5 Rough and Finish Machining

11.5.1 Inputs

- Output Geometry

11.5.2 Outputs

- Processing Time
- Volume Removed

11.5.3 Objective Function Contributions

- Set up costs (constant)
- Machining Costs (tool wear, machine time)
Industrial manufacturing processes involve a series of operations that transform a workpiece into a useful finished product characterized by acceptable shape and mechanical properties. A typical sequence of thermomechanical operations consists of multiple-stage hot and/or cold deformation processes interspersed with suitable heat treatment, material removal, and inspection processes. The shape of the product is achieved through the deformation and material removal processes, while the properties of the product are, in general, dependent upon the entire thermomechanical processing history. The challenge in the design of a manufacturing process is to optimize the entire processing sequence in order to achieve the best balance of manufacturing and material costs, delivery schedules, and shape and mechanical properties of the final product.

Trial and error methods have long been used to select process parameters, including processing temperatures, machine speeds, and die geometries. These methods generally result in, at best, a working design, with no attempt at optimization. Recently, research has moved to the application of conventional optimization techniques to individual manufacturing processes. A major concern from the systems engineering point of view is