RANGING AIRPORT PSEUDOLITE FOR
LOCAL AREA AUGMENTATION
USING THE GLOBAL POSITIONING SYSTEM

A Dissertation Presented to

The Faculty of the

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Doctor of Philosophy

by

Chris Gregory Bartone

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<th>Description</th>
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<tbody>
<tr>
<td>AGC</td>
<td>Automatic Gain Control</td>
</tr>
<tr>
<td>APL</td>
<td>Airport Pseudolite</td>
</tr>
<tr>
<td>B-values</td>
<td>Bias-value</td>
</tr>
<tr>
<td>bps</td>
<td>bits per second</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary-Phase Shift Keyed</td>
</tr>
<tr>
<td>CAT</td>
<td>Category (CAT I/II/III for Aircraft Precision Approach)</td>
</tr>
<tr>
<td>C/A</td>
<td>Coarse Acquisition</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>C/N₀</td>
<td>Carrier-to-Noise Ratio in units of dB-Hz</td>
</tr>
<tr>
<td>DAPL</td>
<td>Differential Airport Pseudolite</td>
</tr>
<tr>
<td>DD</td>
<td>Double Difference</td>
</tr>
<tr>
<td>DGPS</td>
<td>Differential Global Positioning System</td>
</tr>
<tr>
<td>DME</td>
<td>Distance Measuring Equipment</td>
</tr>
<tr>
<td>DOD</td>
<td>Department of Defense</td>
</tr>
<tr>
<td>DOP</td>
<td>Dilution of Precision</td>
</tr>
<tr>
<td>D/U</td>
<td>Desired-to-Undesired</td>
</tr>
<tr>
<td>ECM</td>
<td>Electronic Counter Measures</td>
</tr>
<tr>
<td>EMC</td>
<td>Electromagnetic Compatibility</td>
</tr>
<tr>
<td>EMI</td>
<td>Electromagnetic Interference</td>
</tr>
<tr>
<td>ENU</td>
<td>East, North, Up</td>
</tr>
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</table>
FTE  Flight Technical Error
GPS  Global Positioning System
GNSS  Global Navigation Satellite System
GS  Ground Station
HZA  High-Zenith Antenna
i  Index Referring to a GPS Satellite or APL, 1 ≤ i ≤ 37
ICD  Interface Control Document
iid  independent-identically-distributed
j  Index Referring to a Reference Receiver
L1  The center frequency of “Link 1” for GPS = 1575.42 MHz
L2  The center frequency of “Link 2” for GPS = 1227.60 MHz
LAAS  Local Area Augmentation System
λ  Carrier Wavelength in units of meters
LLT  Local, Level, Tangent
LPF  Low Pass Filter
M  Number of Ground Station sites (a.k.a. Reference Receivers)
m  A Particular Reference Receiver Number
MASPS  Minimum Aviation System Performance Standards
MGC  Manual Gain Control
MLA  Multipath Limiting Antenna
N  The Number of GNSS Observables to the Differential GNSS
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>A Particular GNSS Observable</td>
</tr>
<tr>
<td>NORTEL</td>
<td>Northern Telecommunications</td>
</tr>
<tr>
<td>NSE</td>
<td>Navigation System Error</td>
</tr>
<tr>
<td>OCXO</td>
<td>Ovenized Crystal Controlled Oscillator</td>
</tr>
<tr>
<td>OU</td>
<td>Ohio University</td>
</tr>
<tr>
<td>$\phi_m$</td>
<td>Accumulated Doppler Measurement</td>
</tr>
<tr>
<td>P/Y</td>
<td>Precise/Encrypted</td>
</tr>
<tr>
<td>PPS</td>
<td>Pulse Per Second</td>
</tr>
<tr>
<td>PR</td>
<td>Pseudorange</td>
</tr>
<tr>
<td>$PR_d$</td>
<td>Difference Between the Measured and Smoothed Pseudoranges</td>
</tr>
<tr>
<td>$PR_m$</td>
<td>Pseudorange Measurement</td>
</tr>
<tr>
<td>$PR_p$</td>
<td>Pseudorange Measurement Propagation Term</td>
</tr>
<tr>
<td>$PR_s$</td>
<td>Smoothed Pseudorange Measurement</td>
</tr>
<tr>
<td>PRc</td>
<td>Pseudorange correction</td>
</tr>
<tr>
<td>PRN</td>
<td>Pseudorandom Noise</td>
</tr>
<tr>
<td>q</td>
<td>Index Referring to a GPS Satellite, $1 \leq q \leq 32$</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>RHCP</td>
<td>Right Hand Circular Polarization</td>
</tr>
<tr>
<td>RR</td>
<td>Reference Receiver, Maximum Number is M</td>
</tr>
<tr>
<td>r.s.s.</td>
<td>Root Sum Squared</td>
</tr>
<tr>
<td>RTCA</td>
<td>RTCA Incorporated</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Definition</td>
</tr>
<tr>
<td>--------------</td>
<td>------------</td>
</tr>
<tr>
<td>SBAS</td>
<td>Space-Based Augmentation System</td>
</tr>
<tr>
<td>SD</td>
<td>Single Difference</td>
</tr>
<tr>
<td>SPS</td>
<td>Standard Positioning Service</td>
</tr>
<tr>
<td>SU</td>
<td>Stanford University</td>
</tr>
<tr>
<td>STel</td>
<td>Stanford Telecommunications Incorporated</td>
</tr>
<tr>
<td>SV</td>
<td>Space Vehicle</td>
</tr>
<tr>
<td>SVID</td>
<td>Space Vehicle Identification</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>Smoothing Time Constant $1 &lt; \tau &lt; 100$ seconds</td>
</tr>
<tr>
<td>TSE</td>
<td>Total System Error</td>
</tr>
<tr>
<td>U.S.</td>
<td>United States</td>
</tr>
<tr>
<td>VDB</td>
<td>Very High Frequency Data Broadcast</td>
</tr>
<tr>
<td>VDOP</td>
<td>Vertical Dilution of Precision</td>
</tr>
<tr>
<td>VHF</td>
<td>Very High Frequency</td>
</tr>
<tr>
<td>WAAS</td>
<td>Wide Area Augmentation System</td>
</tr>
<tr>
<td>WB</td>
<td>Wideband (Code for APL Applications)</td>
</tr>
<tr>
<td>w.r.t.</td>
<td>With Respect To</td>
</tr>
<tr>
<td>Z</td>
<td>Total Number of Observables to all M Reference Receivers</td>
</tr>
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</table>
1.0 INTRODUCTION

The Local Area Augmentation System (LAAS) is being developed to support precision approach and landing operations in and about the local area surrounding an airport. The LAAS Program is currently under development by the Federal Aviation Administration (FAA) with Minimum Aviation System Performance Standards for the LAAS being developed by RTCA, Incorporated [1, 2]. The LAAS is to provide service to the United States (U.S.) National Airspace System (NAS) and will likely be fielded as a worldwide Ground-Based Augmentation System (GBAS) for the Global Navigation Satellite System (GNSS) in cooperation with the International Civil Aviation Organization (ICAO) and the European Organization for Civil Aviation Equipment (EUROCAE). Furthermore, the LAAS is likely to be included, in some form, within the U.S. Department of Defense (DOD) Joint Precision Approach and Landing System (JPALS), currently in the early stages of concept development. One reason for this is to allow DOD aircraft to land at civilian airfields when required. The LAAS acquisition plan calls for procurement of 143 systems, scheduled for installation throughout the U.S. beginning in the year 2003 [3]. The LAAS is comprised of three elements: 1) The Satellite Subsystem which includes the GPS space vehicles (SVs) and any potential Space-Based Augmentation System (SBAS) satellites; 2) The Ground Subsystem that includes the differential measurements and associated very high frequency (VHF) data broadcast (VDB), and Airport Pseudolite(s) (APL(s)) to increase availability; and 3) The Airborne Subsystem. Figure 1.1 illustrates these three LAAS Subsystems.
The LAAS uses differential Global Positioning System (DGPS) and includes APL(s) to increase the availability for certain installations. Using the current satellite constellation of the Global Positioning System (GPS) for the LAAS, not all Category II/III precision approach airports would have a sufficient level of availability without APL augmentation. Out of the 143 LAAS installations scheduled for acquisition, 112 are CAT II/III systems. With the addition of APL(s) the availability of the CAT II/III installations is expected to be increased to meet the requirements.

This dissertation addresses the addition of a differentially corrected, ranging APL into a LAAS. The ranging APL enables accurate code and carrier measurement information to be incorporated into the differentially corrected position solution for increased availability. Prior to this work, no ranging APL has been integrated into a
prototype LAAS and demonstrated in a real-time flight environment showing that an increase in LAAS availability is feasible.

To investigate the feasibility of integrating the ranging APL in a real-time fashion into a prototype LAAS, the APL requirements for receiver integration, frequency allocation, APL tracking performance, and electromagnetic interference (EMI) with nominal GPS performance were considered. This resulted in a prototype APL transmitting and receiving subsystem with a coarse-acquisition (C/A) code format that could be operated at any frequency within the L1 ± 10.0 MHz band.

Two major error components exist that are unique for the APL link. First, the ground multipath from the APL transmitting antenna to the DGPS Ground Stations (GS) reception antenna is more severe than for nominal DGPS operations. To minimize ground multipath for the APL geometry, a multipath limiting antenna (MLA) was designed, fabricated, and tested within a 4-month period. The implementation of this MLA concept was a first for APL applications and also contributed to the successful multipath limiting of ground multipath at the DGPS LAAS GS. This effort successfully demonstrated that ground multipath can be limited for the APL geometry and that suitable performance for precision approach applications can be achieved. The second major error component is the large power level variation the APL receiver observes over a typical mission profile. The APL power level variation is much greater than that for nominal GPS reception. This, in part, sets the peak power level of the APL transmitter, which affects the requirement for the APL not to produce EMI to nominal GPS performance.
To investigate the two major APL error components, the developmental approach was performed in two phases. Phase I concentrated on an APL operating at a center frequency off-L1. An automatic gain control (AGC) was implemented to control the received APL signal power such that optimum APL tracking performance could be achieved. For this first phase, EMI to GPS was not a major issue because additional isolation between the APL and GPS spectrums was achieved by means of frequency separation. Phase II of the development concentrated on a pulsed on-L1 APL that achieved electromagnetic compatibility (EMC) with nominal DGPS performance by means of pulsing the APL transmission and controlling the APL power-level in the GPS receiver paths. The on-L1 APL architecture implemented a unique AGC and GPS Blanker technique for APL power-level control in the common reception path to maximize APL signal tracking while minimizing EMI to nominal DGPS performance.

For this effort a total of 11 flight tests with three test aircraft (Piper Saratoga, FAA Boeing 727, and Ohio University DC-3) and 14 distinct laboratory tests were conducted to produce the APL subsystem architecture, data, and system performance documented in this research. This final APL subsystem architecture was fully integrated with a prototype LAAS and demonstrated with the Ohio University DC-3.
2.0 BACKGROUND

2.1 The GPS Constellation

The GPS is based on the premise of trilateration using signals received from the GPS SVs. With the GPS SV positions, time-of-transmissions, and time-of-receptions known (calculated and measured), the user can determine its position and time. The quality, quantity, and availability of these GPS SV signals directly affect the user’s solution for position and time. The current GPS Space Segment consists of 24 satellites. Each SV travels around the earth in a near-circular orbit with a radius of approximately 26,600 km [4]. Figure 2.1 depicts the GPS SV constellation. These 24 SVs orbit the earth in six orbital planes with 4 SVs in each plane. Each SV orbit has an inclination of

Figure 2.1 GPS Space Vehicle Constellation Diagrams: a) View from Space, b) Planar Projection View
approximately 55° and each SV completes an orbit in approximately ½ sidereal day (11 hours 58 minutes). This orbital period results in repeating ground tracks with a period of approximately 23 hours and 56 minutes.

Several factors can affect the number of GPS SVs which are observable to a user at a given time, and position. Global factors such are SV maintenance (scheduled and unscheduled), SV failures and replenishment actions, as well as, other actions taken by the GPS Ground Control Segment can affect the availability of GPS SVs. Local factors that can affect the number of GPS SVs that a user can utilize could be obstructions, antenna mask angles, significant multipath in a particular direction, local EMI, environmental effects (e.g., heavy rain, tree cover), receiver hardware/software limitations, and the user’s location on the surface of the earth.

2.2 The General GPS Position Solution Concept

The GPS utilizes an earth-centered-earth-fixed (ECEF) coordinate frame and geoid defined by the World Geodetic Survey of 1984 (WGS-84); this coordinate frame rotates as the earth rotates. Data source-encoded onto the GPS signal provides information on the precise position of the satellite in terms of Kepler orbital parameters, which is often called broadcast ephemeris. The GPS receiver translates the GPS SV orbital position information into an ECEF position of the SV. With only a rough estimate of the user’s initial position, a minimum of four GPS SVs are required to calculate the user’s position and clock bias as follows:
\[(x_i-x_u)^2 + (y_i-y_u)^2 + (z_i-z_u)^2 = (\rho_i - c t_u - \varepsilon_{\rho i})^2\]  \hspace{1cm} (2.1)

where:

\((x_u, y_u, z_u)\) = users position (to be calculated),

\((x_i, y_i, z_i)\) = the \(i\) th satellite position,

\(\rho_i\) = pseudorange measurement to the \(i\) th satellite,

\(c\) = GPS propagation constant = 299,792,458 m/s,

\(t_u\) = users receiver clock bias,

\(\varepsilon_{\rho i}\) = errors on the \(i\) th pseudorange measurement.

The GPS code observable is called a pseudorange since it contains the true geometric range to the satellite, the user’s clock bias, and errors. Note that the raw pseudorange measurement to a particular SV made by the user’s receiver “corrects for” known error terms estimated by the GPS Ground Control Segment. Information contained within the broadcast ephemeris about these error sources (i.e., satellite clock offset, relativistic effects, etc.) is used to correct the raw pseudorange measurement prior to inclusion of equation (2.1). Appendix A describes the linearization of equation (2.1), and the determination of the user’s state (position and time) and its statistics when the number of GNSS observables is greater than four.

2.3 Precision Approach

Based on the user’s requirements the GPS may require augmentation. Within the FAA and DOD, developmental programs have been established to enhance the GPS
capabilities to meet precision approach requirements. For precision approach applications, the user’s requirements for its position solution are often quantified in terms of accuracy, continuity, integrity, and availability. The Wide Area Augmentation System (WAAS) is being developed to enhance en-route positioning and to provide a CAT I precision approach capability using GPS [5]. The WAAS, still under development, uses ground stations located over a wide geographic area. Pseudorange errors are separated into several components which are broadcast to the user over a wide area using geostationary satellites. The LAAS is being developed by the FAA for CAT I, II, and III requirements. The LAAS is intended to be used for CAT II and III at most installations, and for CAT I at airports where WAAS is not available.

Figure 2.2 illustrates a typical precision approach profile for an aircraft on final approach. While the exact requirements for the LAAS are still being refined, Table 2.1

![Figure 2.2. Typical Precision Approach Profile](image)
Table 2.1 Performance Parameters for the Local Area Augmentation System

<table>
<thead>
<tr>
<th>Measure of Performance</th>
<th>CAT I</th>
<th>CAT II</th>
<th>CAT III</th>
</tr>
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<tbody>
<tr>
<td>Decision Height Above</td>
<td>200+ feet</td>
<td>100+ feet</td>
<td>0 - 100 feet</td>
</tr>
<tr>
<td>Threshold</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical Accuracy (95%)</td>
<td>4.4 meters</td>
<td>2.0 meters</td>
<td>2.0 meters</td>
</tr>
<tr>
<td>NSE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continuity</td>
<td>$10^{-5}$ per approach</td>
<td>$10^{-5}$ per approach</td>
<td>$10^{-7}$ per 30 sec.</td>
</tr>
<tr>
<td>Integrity</td>
<td>$10^{-7}$ per approach</td>
<td>$10^{-9}$ per approach</td>
<td>$10^{-9}$ per approach</td>
</tr>
<tr>
<td>Availability</td>
<td>Modular; Service Availability is 0.999 to 0.99999 per day (Depends Upon Specific Airport Requirements)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

illustrates qualitative measures of the performance parameters for the LAAS [1]. The accuracy of the position solution refers to the measured (user determined) position as compared to the user’s true position. This accuracy is usually quantified statistically for the navigation system error (NSE) in a 95% probabilistic sense. The level of accuracy required depends upon the category of service supported, with CAT III being the most stringent. Availability is a qualitative indication of the navigation systems’ ability to provide the required level of performance at the beginning of the intended operation. This level of performance for availability refers to the required accuracy, integrity, and continuity at the beginning of the approach. The availability of the LAAS is specified from 0.999 up to 0.99999 per day, dependent upon the specific airport requirement for Service Availability. For a particular airport site, the availability is affected by the configuration and status of the LAAS GS, the desired level of performance required, the accuracy of the airborne and GS receivers, the number and location of GPS SVs and APLs, as well as, the approach path [6]. LAAS availability is broken down in terms of 1) Service Availability, and 2) Operational Availability. The Service Availability (in
the long-term) is the probability that the LAAS service will be available to a user at an arbitrary time. The Operational Availability refers to the operational impact of different types of long-term failures in a LAAS. The integrity applies to the trust the user has in its position solution. The LAAS integrity includes provisions for a timely warning when the integrity exceeds the specified limits. The continuity of function is the ability of the system to perform adequately over the intended operation without an unscheduled interruption of service. Thus, it is the systems' ability to complete an approach after the system has been declared to be available and the decision has been made to begin the approach. Availability is only defined at one point (the beginning of the approach) while continuity, integrity, and accuracy have meaning throughout the approach.

The accuracy requirements for the LAAS drive it to be based on a DGPS architecture. The FAA conducted a LAAS CAT III feasibility demonstration with four prototype DGPS systems using the Standard Positioning Service (SPS) [8]. Of these four systems, two were code-based systems (carrier-smoothed code), and two were carrier based systems (carrier resolved ambiguity resolution using the code and carrier). One of these carrier based systems also used the L2 carrier, which was not part of the SPS at the time of testing but is now [9]. The results of this demonstration illustrate that a code-based system with moderate flight technical error (FTE) can provide the accuracy and continuity to support CAT III precision approach operations. Figure 2.3 illustrates a DGPS LAAS with a VHF data broadcast. Numerous studies have been performed analyzing the availability requirement to be met by the LAAS with the conclusion that some level of augmentation is required for CAT II/III operations [6].
2.4 The GPS Signal Structure

The GPS signal structure is based on a spread-spectrum format where navigation data at a low rate of 50 bits per second (bps) is multiplied by high rate pseudorandom noise (PRN) codes [10]. These high-rate codes are the C/A code (1.023 MHz chip rate) and precision (P) (10.23 MHz chip rate) code. During times when the GPS anti-spoofing capability is “on”, the P code is multiplied by an encrypted code and the “Y” code is produced and transmitted. The modulation of each signal is binary-phase shift keyed (BPSK). These GPS signals are transmitted by the GPS SVs at center frequencies...
of L1 (1575.42 MHz) and L2 (1227.60 MHz). Currently the GPS SVs transmit signals on-L1 with both the C/A and P/Y-codes in quadrature and only the P/Y code format on-L2, although the GPS SVs have the capability to transmit C/A code on-L2. Equation (2.2) represents a general signal description of a GPS signal at L1 or L2.

\[ s_i(t) = C_i(t)d_i(t)A_c\cos(\omega_c t + \phi_i) + P_i(t)d_i(t)A_p\sin(\omega_c t + \phi_i) \]  

(2.2)

where:

\[ s_i(t) = \text{GPS signal from } i^{th} \text{ GPS SV}, \]

\[ C_i(t) = \text{High rate C/A binary PRN spreading code from the } i^{th} \text{ GPS SV}, \]

(Equals zero for L2),

\[ P_i(t) = \text{High rate P/Y binary PRN spreading code from the } i^{th} \text{ GPS SV}, \]

\[ d_i(t) = \text{Low rate navigation data at 50 bps from the } i^{th} \text{ GPS SV}, \]

\[ A_c = \text{Power in the carrier for the C/A BPSK modulation from the } i^{th} \text{ GPS SV}, \]

\[ A_p = \text{Power in the carrier for the P/Y BPSK modulation from the } i^{th} \text{ GPS SV}, \]

\[ \omega_c = \text{Carrier center frequency in radians per second} = 2\pi f_c, \text{ with } f_c = \text{L1 or L2}, \]

\[ \phi_i = \text{Arbitrary carrier phase term from the } i^{th} \text{ GPS SV}, \]

\[ i = \text{SV index, where } 1 < i < 32. \]

The total signal transmitted from a particular GPS SV will be a composite of the C/A and P/Y code signals BPSK modulated in quadrature, onto the carrier frequencies.

Original design features of the C/A code allow for independent codes from each SV (hence Gold codes vice ideal m-sequences) and of moderate length to allow for rapid synchronization (1023 chips repeating every 1.0 millisecond). The selection of the C/A code does limit the amount of cross-correlation between codes in the Gold family
selected for GPS. This C/A cross-correlation performance for in-band (at approximately the same center frequency) codes have worst-case correlation of about -21 dB, and a 95% probability of about -25 dB [11]. Explicitly, when one C/A code is approximately 25 dB stronger than the other, it will interfere with it 95% of the time. Only on rare occasions is this C/A code-to-code cross-correlation observed when signals are received from SVs. However, when a mobile user receives a C/A signal from a ground-based transmitter, then the power variation exceeds the C/A code-to-code cross-correlation performance. This effect will be discussed in the next section.

2.5 Introduction to Pseudolites

Although not part of the permanent GPS constellation in space, the GPS Interface Control Document (ICD) does provide for ground-based transmitters with PRN Gold Codes similar to that transmitted by the GPS SVs [12]. These ground-based transmitters or pseudo-satellites are called pseudolites. Pseudolites were used in the early 1970's by the DOD to develop the GPS by augmenting the constellation for position determination when only a few SVs were operational [13]. Other military applications for pseudolites are to strategically place a transmitter for augmentation of a platform’s position or time solution in an area where electronic countermeasures (ECM) are being conducted. Because the pseudolite power can be made very strong relative to the nominal GPS signal power received and relative to a potential jamming source, additional anti-jam margin (AJM) can be achieved for a specific scenario. This additional AJM is obtained
over a limited window of power levels due to the fact that at some point, the APL may
start to produce EMI to nominal GPS performance.

The C/A Gold Code cross-correlation capability is a severe limitation for ground-
based transmitters. With a C/A code format pseudolite, the limiting factor is the amount
of cross-correlation between the pseudolite PRN and the GPS PRNs and measures need
to be taken to ensure the pseudolite does not inadvertently produce EMI with nominal
GPS performance. These measures can be in the form of space, frequency, code, or time
separation of the pseudolite signal from the GPS signals. All of these measures will
decrease the relative power that is correlated between the pseudolite and GPS signals
and hence reduce the amount of cross-correlation between them.

2.6 The Pseudolite Signal Structure

To minimize receiver complexity and integration of the pseudolite into a GPS
receiver, the pseudolite signal structure is kept similar to that of the GPS signal structure.
The pseudolite signal structure design involves the optimization of the spatial,
frequency, code, and time separation of the pseudolite signal from the nominal GPS
signals. Equation (2.3) represents a general signal description of an APL signal at L1 or
L2.

\[ s_i(t) = p_i(t) C_i(t) d_i(t) A_e \cos(\omega_e t + \Phi_i) + p_i(t) W B_i(t) d_i(t) A_{wb} \sin(\omega_c t + \Phi_i) \]  (2.3)

where:

\[ s_i(t) = \text{Signal from the } i^{th} \text{ pseudolite}, \]

\[ p_i(t) = \text{Pulsing format (On Off Keyed Modulation) from the } i^{th} \text{ pseudolite}, \]
\( C_i(t) \) = High rate C/A binary PRN spreading code from the \( i \) \textsuperscript{th} pseudolite,

\( d_i(t) \) = Low rate navigation data at 50 bps from the \( i \) \textsuperscript{th} pseudolite,

\( WB_i(t) \) = High rate Wideband binary PRN spreading code from the \( i \) \textsuperscript{th} pseudolite, (Equals zero for a C/A only APL),

\( A_c \) = Power in the carrier used for C/A BPSK modulation from the \( i \) \textsuperscript{th} pseudolite,

\( A_{wb} \) = Power in the carrier used for WB BPSK modulation from the \( i \) \textsuperscript{th} pseudolite,

\( \omega_c \) = Carrier frequency in radians per second = 2\( \pi f_c \), with \( f_c = L1 \pm 10\alpha \) MHz or \( L2 \pm 10\alpha \) MHz, where \( 0 \leq \alpha \leq 1 \),

\( \phi_i \) = Arbitrary carrier phase term from the \( i \) \textsuperscript{th} pseudolite,

\( i \) = Pseudolite identification index, where \( 33 \leq i \leq 37 \).

If the pseudolite is a “Wideband” (WB) code pseudolite then the total pseudolite signal transmitted will be a composite of the C/A and WB code signals BPSK modulated in quadrature, at a particular the carrier frequency. An additional factor of \( p_i(t) \) is included in the pseudolite signal structure and represents the pulsing format from the \( i \) \textsuperscript{th} pseudolite. This additional pulsing (or On Off Keyed Modulation) is performed as part of the pseudolite signal structure design such that EMI to nominal GPS performance can be minimized. Studies have shown that most GPS receivers are fairly tolerant to pulsed EMI as long as the duty cycle is less than approximately 30 % [14]. A non-cooperative GPS receiver (a user that does not know about the pseudolite signal) will see the pseudolite signal as a pulsed interference source. This pulsing is a good way to decrease the pseudolite C/A code to GPS C/A code cross-correlation. Also note that the pseudolite center frequency is defined to be anywhere within the L1 \( \pm 10 \) MHz or
L2 ± 10 MHz bands. Operating the pseudolite at a frequency other than directly on-L1 or on-L2 increases the frequency separation and pseudolite to GPS C/A cross-correlation performance. However, as the center frequency of the APL moves further away from the GPS center frequency, additional factors need to be considered such as "code - carrier" divergence and the receiver clock bias solution.

2.7 Pseudolites for Precision Approach

Civil applications of pseudolites have mostly been in support of precision approach requirements for the FAA. Research over the past eight years has concentrated on the addition of pseudolites to increase the availability of a LAAS. When a pseudolite is located on the airport property it is referred to as an APL. APLs are the preferred method of augmentation for LAAS to increase its availability. Studies have indicated that pseudolites can effectively increase the LAAS availability [6]. In order to realize an increase in availability with the addition of an APL to a LAAS, the APL must be of high quality and able to be used as a ranging source.

As with the prototype DGPS precision approach systems, both code (code only or carrier-smoothed code) and carrier phase (carrier resolved ambiguity or differential carrier phase) based pseudolite systems have been developed.

Research by Stanford University (SU) on pseudolites for civil applications began in the early 1990's and have been based on a carrier phase approach that can be divided into two categories. Initial pseudolite development placed two pseudolites off the airport property directly underneath (straddling) the aircraft flight approach path. These
pseudolites were called GPS Marker/Integrity Beacons [15]. The CAT III landing system tested was based on a kinematic GPS solution (resolved carrier cycle ambiguities) and performed well. This system required a kinematic position solution, a bottom-mounted aircraft pseudolite antenna, and the pseudolites to be located off the airport property. The logistic restriction to locate, operate, and maintain these off-airport pseudolites made the system unpractical for an operational LAAS.

More current research by SU (1995-1997) on pseudolites has been based on an “in-track” APL approach [16]. This architecture places two APLs, one at each end of a supporting runway. Each APL is at a center frequency of L1 and pulsed at a 9.0% duty cycle. The in-track APL geometry obtains a geometric advantage in tracking the carrier phase from each APL transmission as the aircraft approaches on a 3° glidepath. As the aircraft flies in for an approach, the differential carrier phase from each APL is received and processed in a differential solution. The SU LAAS architecture required both in-track APL’s to obtain the desired increase in system availability. Only the carrier phase information is used and no attempt is made to perform autonomous ranging with the APL C/A code. This architecture does perform well but requires both APLs to be in an ‘in-track” configuration. The system was flight tested with a separate nose-mounted antenna for APL reception.

Pseudolite research was performed by Stanford Telecommunications (STel), Incorporated in 1993-1995 [17]. This system was a C/A code-based system. Parameters of this design were to offset the APL by 1.023 MHz and pulse the RF output at a 9.0% duty cycle with an integrated 250 bps data broadcast modulated onto the C/A code. The
data link supported the DGPS position solution for this prototype precision approach system. STel tried various antenna configurations on the ground and in the air to optimize performance. Results from this effort were non-conclusive with respect to its applicability to the LAAS.
3.0 OU LAAS APL DESIGN CONSIDERATIONS

During the FAA LAAS CAT III feasibility demonstration, the OU LAAS architecture demonstrated that a code-based system, utilizing carrier-smoothed code, with differential corrections broadcast by a VHF transmitter can provide the accuracy and continuity suitable for CAT III precision approach [7, 18]. The OU system does not require carrier cycle ambiguity resolution. The overall design objective of this research effort was to add an APL in the L1 ± 10 MHz band utilizing the C/A code format in a differential APL (DAPL) fashion such that the APL signal could be treated just like an additional SV signal. A carrier-smoothed code solution was desirable such that autonomous DAPL ranging could be obtained from the added APL observable. As a result, the APL would increase the overall availability of the LAAS. This DGPS/DAPL augmented architecture is depicted in Figure 3.1. To achieve a differential APL suitable for CAT III precision approach, several system level design challenges must be considered [19]. These challenges are described in the next section.
Figure 3.1 DGPS-based LAAS with a VHF Data Broadcast and an APL

3.1 APL System Level Design Challenges in a LAAS

The goal with respect to accuracy is that the APL should be “as good as a high-elevation GPS SV” otherwise the APL ranging source may not add value to the position solution, and hence no addition in availability will be realized. Although the APL measurements could be deweighted to a certain level, they cannot be deweighted too much, otherwise they will be of no added value. Despite the goal to make the APL measurements as accurate as GPS SV measurements, it is more difficult. This fact is recognized by the FAA and RTCA as specified in [20]. Allocations of the APL pseudorange measurements are about two to three times as large as for a GPS SV pseudorange measurements. The APL signal is not needed to increase the accuracy of the basic
LAAS, but it must be accurate enough to allow for an increase in availability of the LAAS. These accuracy requirements drive the APL architecture to be differential in some fashion. Therefore, **accurate APL code and carrier measurements need to be performed at the DGPS GS (performing reference, monitor, and integrity functions), and at the aircraft on final approach, over the APL operational range (80 m to 20 nmi), with no EMI to nominal DGPS operation.**

At the LAAS GS, measurements of the APL code and carrier phase must be made at the same location as the DGPS measurements are being made (without added complexity and risk). This means that the APL signal is received in a severe multipath signal environment, worse than the DGPS GS case. The DGPS GS must receive the APL signal at approximately 0° in elevation, while the GPS SVs are at elevation angles of 5° and higher. Additionally, the transmitted APL signal cannot interfere or degrade the DGPS LAAS performance. This degradation must not only be assessed in terms of carrier-to-noise (C/N₀) degradation on a GPS PRN basis, but more importantly on a code - carrier (code “minus” carrier) tracking error basis. In other words, the C/N₀ degradation is not as important as increased GPS tracking errors.

*At the Aircraft, the APL signal “must” be received by the top-mounted GPS Antenna.* This “must” comes from the reality of using a single LAAS antenna on the aircraft to:

- Minimize platform integration costs,
- Eliminate use of lever-arm corrections if separate APL and GPS antennas are used,
• Use existing GPS antenna installations in fleet aircraft.

Of concern for the APL subsystem is the antenna gain at low elevation angles which is 3° below the local horizon for the reception of the APL on a 3° approach path. The details of this gain will be discussed later in this document but the amount of power coupled by the antenna and provided to the APL code and carrier tracking loops is important. Additionally, at the aircraft, the multipath is of some concern. While not as severe as the DGPS GS case, the aircraft will receive ground-based transmissions at an elevation angle of +3° from the APL transmitting antenna where multipath can be present; again this is more severe than the GPS case because the GPS SVs do not have a large reflector (earth) close to their transmission antennas.

The APL subsystem must be able to handle the additional spatial loss factor for this non-space or terrestrial application. To date, all GPS receivers that have been used for APL applications were originally designed for GPS reception. These receivers process nominal GPS signals at -130 dBm with some power variation due to SV elevation angle and antenna pattern variation as a function of the user's platform orientation. Thus, most GPS receivers have a relatively small dynamic range (30 dB or so) as compared to typical terrestrial-based communication systems. Furthermore, a limiting factor with most code-division multiple access (CDMA) systems is the cross-correlation between different codes used in the systems. Thus most GPS receivers were not designed for a wide dynamic range. A type of "near-far" problem occurs in the applications of APL utilizing a typical GPS receiver because the cross-correlation between the APL PRN code (C/A or WB) and GPS SV C/A PRN code becomes the
limiting factor. However, for APL applications a wider dynamic range is required simply due to free space \((\lambda/4\pi R)^2\) losses. The variation in power from a minimum operational range of 80 meters to 20 nmi at L1 is 53.3 dB, not including antenna pattern variations, installed cable loss variations, multipath losses, link margin, etc. Thus, the dynamic range required for an APL channel (about 53.3 dB + 30 dB = 83 dB) far exceeds the typical dynamic range available in a GPS receiver operating at L1. Different approaches in APL signal structure exist on how to handle this large dynamic range and will be discussed later.

*One of the most important challenges for the APL subsystem is not to interfere with nominal DGPS performance that is supporting the precision approach system.*

The limiting factor of this interference to the DGPS LAAS is based on its use of the L1 C/A code and close proximity of the DGPS GS relative to the APL transmitting antenna; it will be discussed later why the APL transmitter should be close to the DGPS GS receiving antenna. This isolation from the APL transmitter to the DGPS GS detection circuitry (in time, frequency, and position) is on the order of the dynamic range of the APL subsystem needed to support the operational range requirements. There are several ways to attain this isolation by APL signal structure design. The signal structure design could include one or more of the following:

- Frequency separation of the APL and GPS spectrums,
- APL code selection to minimize cross-correlation,
- Pulsing the APL transmission to minimize EMI to GPS,
• APL code rate (outside the GPS code rate tracking bandwidth) to minimize cross-correlation.

The requirement not to interfere with DGPS performance is the primary factor that drives the APL signal structure with APL subsystem tradeoffs (increasing APL errors, added cost, complexity, etc.). The RTCA and FAA are currently in the process of defining what constitutes insignificant EMI to nominal GPS performance caused by an APL. Some have proposed "not to degrade GPS tracking by 1 dB"; however, this parameter does not directly indicate potential increased tracking errors in the code and carrier loops while in the presence of the APL signal.

The APL subsystem should be modular to the precision approach system it supports, so that it can be added only at sites where availability augmentation is required.

The APL subsystem should be easily integrated into a DGPS receiver system. For practical implementation into a GPS receiver, it is obviously desirable to make the APL signal-in-space as similar to the GPS waveform received from the SVs as possible. Making the APL PRN one of the original 37 codes and placing the center frequency on-L1 is an obvious choice but difficult due to interference with nominal GPS performance and tradeoffs associated with the APL signal structure must be considered. If the APL center frequency is placed at the center frequency of GPS then measures need to be taken to ensure EMC with nominal GPS performance.
Lastly, the all encompassing challenge to the APL subsystem is to meet all the above challenges simultaneously. It is relatively easy to accomplish each requirement individually, but when combined, the final combined challenge must be met as well.

3.2 APL Power Link Budget Calculations

This section addresses the link calculations of the APL for the desired function of operation. EMI of this link to the DGPS GS reception operation is not considered here. A tailored Friis transmission equation, equation (3.1), is used to calculate operating ranges for the APL subsystem [21].

\[ P_t = \frac{P_r(L)M}{G_r G_t} \left( \frac{4\pi R}{\lambda} \right)^2 \]

where:

\( P_t \) = Transmitted Power,

\( P_r \) = Received Power,

\( L \) = Losses in Link,

\( M \) = Margin Allocated for the Link,

\( G_r \) = Gain of Reception Antenna,

\( G_t \) = Gain of Transmission Antenna,

\( R \) = Range of Link in units of meters,

\( \lambda \) = Carrier Wavelength in units of meters.

Several key assumptions for the power link calculation are that the GPS and APL signals are received by the same reception antenna at the LAAS GS and at the top-mounted right-handed circular polarization (RHCP) GPS antenna on the aircraft, and
that the Global Navigation Satellite System (GNSS) Sensor ARINC Characteristic 743A-1, 1993 (either 2MCU or Alternate Receiver Configuration) will be used as a basis for the APL subsystem with airborne component variations noted [22]. This same specification will be used as a guideline for the LAAS ground component until the appropriate LAAS document is complete. The GNSS Sensor ARINC Characteristics document makes allocations for GPS antenna reception gain (loss) not to exceed -4.5 dB and expected cable loss to be 3 to 13 dB. APL antenna gains on the ground were selected based on the eventual implementation of a MLA designed in this effort for the APL link. Several trade-offs with respect to antenna size, gain, and multipath will be discussed in Section 3.4 of this document. Key parameters for the APL link are defined in Table 3.1.

Table 3.1. Key APL Link Budget Parameters

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{t,\text{peak}}$</td>
<td>Peak power transmitted by the APL transmitter (does not include $L_{\text{cab,xtm}}$)</td>
</tr>
<tr>
<td>$P_{t,\text{ave}}$</td>
<td>Average power transmitted by the APL transmitter (does not include $L_{\text{cab,xtm}}$)</td>
</tr>
<tr>
<td>$d.c.\text{avg}$</td>
<td>Average transmitted duty cycle averaged over the receiver’s integration window</td>
</tr>
<tr>
<td>$L_{\text{d,c.}}$</td>
<td>Average loss in received signal average power for an APL Receiver as a result of pulsing $= 10 \log (d.c.)$</td>
</tr>
<tr>
<td>$L_{\text{cab,xtm}}$</td>
<td>Loss of APL transmission cable from APL Transmitter to the APL transmission antenna $= 2$ dB</td>
</tr>
<tr>
<td>$G_t$</td>
<td>Gain of APL transmitting antenna in the desired direction of propagation</td>
</tr>
<tr>
<td>$G_t,\text{APLG-to-APL}\text{A}$</td>
<td>Gain of APL transmitting antenna in the direction of an aircraft approaching at $3^\circ$ for vertical polarization</td>
</tr>
<tr>
<td>$G_t,\text{APLG-to-APLG}$</td>
<td>Gain of APL transmitting antenna in the direction of LAAS Ground Station for vertical polarization</td>
</tr>
</tbody>
</table>
| $dBi_l$           | dB relative to an isotropic radiator for linear polarization, i.e.,
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{\text{spatial}}$</td>
<td>Spatial free space signal loss factor $= (\lambda/4\pi R)^2$</td>
</tr>
<tr>
<td>$R_{\text{max}}$</td>
<td>Maximum operating range of APL subsystem $= 20.0$ nmi. At this range the APL received power level shall be $P_{r,\text{nom}}$ &quot;at the skin&quot; of the aircraft prior to the reception antenna</td>
</tr>
<tr>
<td>$P_{r,\text{nom}}$</td>
<td>The nominal APL power level received at the airborne APL receiver for an APL transmission at $R_{\text{max}}$, which does not include link margin. This level is defined to be $-130$ dBm</td>
</tr>
<tr>
<td>$G_r$</td>
<td>Gain of the receiving antenna in the desired direction of propagation</td>
</tr>
<tr>
<td>$G_{r,\text{APLG-to-APLA}}$</td>
<td>Gain of the APL receiving antenna (Airborne LAAS Participant) in the desired direction of propagation $= -14.5$ dBi for vertical polarization on a $3^\circ$ approach path ($-3^\circ$ elevation angle)</td>
</tr>
<tr>
<td>$G_{r,\text{APLG-to-APLG}}$</td>
<td>Gain of the APL receiving antenna (GS LAAS Participant) in the desired direction of propagation for vertical polarization at $0^\circ$ elevation $= -17.5$ dBi</td>
</tr>
<tr>
<td>$dG_r$</td>
<td>The difference between the actual reception antenna gain for pseudolite applications and the amount allocated per [22]</td>
</tr>
<tr>
<td>$L_{\text{cab,air}}$</td>
<td>Cable loss in the aircraft per [22]</td>
</tr>
<tr>
<td>$dL_{\text{cab,air}}$</td>
<td>The difference between the actual cable loss for pseudolite applications in the aircraft and the amount allocated per [22]</td>
</tr>
<tr>
<td>$L_{\text{cab,gs}}$</td>
<td>Cable loss in the LAAS Reference Ground Station $= 12$ dB for this effort</td>
</tr>
<tr>
<td>$dL_{\text{cab,gs}}$</td>
<td>The difference between the actual cable loss in the LAAS Reference Ground Station and the amount allocated per [22]</td>
</tr>
<tr>
<td>$dG_{\text{preamp}}$</td>
<td>The difference in the preamplifier gain in the OU LAAS ($G = 40$ dB) as compared to [22]; 2MCU Receiver Configuration ($G = 26.5$ dB). The number used here is $13.5$ dB</td>
</tr>
<tr>
<td>$L_{\text{sig fad}}$</td>
<td>Signal fading component due to multipath when the $D/U &lt; 1$</td>
</tr>
<tr>
<td>$L_{\text{margin(+)}}$</td>
<td>Link Margin (positive) for the APL$_G$-to-APL$_A$ Link in and above all other losses $= +2$ dB. (This is sometime referred to as M for Margin implied as a loss quantity.)</td>
</tr>
<tr>
<td>$L_{\text{margin(-)}}$</td>
<td>Link Margin (negative) for the APL$_G$-to-APL$_G$ Link in and above all other losses $= -2$ dB</td>
</tr>
</tbody>
</table>

These data as well as the following information will be used for the power link calculation.
3.2.1 APL Maximum Range Link Calculation

The maximum range (desired link) from the ground-based APL transmitter to the airborne APL receiver link is analyzed to calculate $P_{t,\text{avg}}$ and $P_{t,\text{peak}}$ in Table 3.2.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_t$</td>
<td>(-) - 6 dBil</td>
<td>(@ +3° elevation w.r.t. LLT with MLA)</td>
</tr>
<tr>
<td>$dG_r$</td>
<td>(-) - 10 dBil</td>
<td>(@ - 3° elevation w.r.t. LLT)</td>
</tr>
<tr>
<td>$\alpha_{\text{spatial}}$</td>
<td>(-) - 127.8 dB</td>
<td>(@ R = 20 nmi, L1)</td>
</tr>
<tr>
<td>$L_{\text{cab,xtm}}$</td>
<td>(+)</td>
<td>2 dB</td>
</tr>
<tr>
<td>$P_{t,\text{nom}}$</td>
<td>(+) - 130 dBm</td>
<td></td>
</tr>
<tr>
<td>$dL_{\text{cab,air}}$</td>
<td>(+)</td>
<td>0 dB</td>
</tr>
<tr>
<td>$L_{\text{sig fad}}$</td>
<td>(+)</td>
<td>1 dB</td>
</tr>
<tr>
<td>$L_{\text{margin},(+)}$</td>
<td>(+)</td>
<td>2 dB</td>
</tr>
<tr>
<td>$P_{t,\text{avg}}$</td>
<td>+</td>
<td>18.8 dBm</td>
</tr>
<tr>
<td>$L_{\text{d.c.}}$</td>
<td>+</td>
<td>6.0 dB</td>
</tr>
<tr>
<td>$P_{t,\text{peak}}$</td>
<td>+</td>
<td>24.8 dBm</td>
</tr>
</tbody>
</table>

Thus an average value of +18.8 dBm in APL transmission power at 100 % duty cycle is needed for the APL link, using the parameters defined in Table 3.2. If the duty cycle of the APL transmission is 25 % then an additional 6.0 dB of peak power will be needed for the APL link.

3.2.2 APL Minimum Range Link Calculation to the DGPS GS

The power received at a minimum operational range of approximately 80 meters can be calculated for the APL link. This link is similar for the APL transmitter-to-DGPS GS (ground-to-ground) link or the APL transmitter-to-aircraft on taxiway link with the exception of $G_r$. The link to the DGPS GS is calculated here. Transmitter antenna gain
will be reduced because of the transmission angle at 0° elevation. The concern is now having too much power for the APL channel hence a negative link margin is introduced.

The minimum range of the APL link from the ground APL transmitter to the GS APL receiver is analyzed for calculation of $P_{r,\text{avg}}$ and $P_{r,\text{peak}}$ in Table 3.3.

Table 3.3 APL Minimum Range Link Calculation

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{t,\text{avg}}$</td>
<td>(+)</td>
<td>18.8 dBi</td>
</tr>
<tr>
<td>$G_t$</td>
<td>(+)</td>
<td>-17.5 dBi</td>
</tr>
<tr>
<td>$\alpha_{\text{spatial}}$</td>
<td>(+)</td>
<td>-74.5 dB</td>
</tr>
<tr>
<td>$dG_T$</td>
<td>(+)</td>
<td>-13.0 dBi</td>
</tr>
<tr>
<td>$dG_{\text{preamp,gs}}$</td>
<td>(+)</td>
<td>13.5 dB</td>
</tr>
<tr>
<td>$dL_{\text{cab,gs}}$</td>
<td>(-)</td>
<td>0 dB</td>
</tr>
<tr>
<td>$L_{\text{cab,xtm}}$</td>
<td>(-)</td>
<td>2 dB</td>
</tr>
<tr>
<td>$L_{\text{sig fad}}$</td>
<td>(-)</td>
<td>1 dB</td>
</tr>
<tr>
<td>$L_{\text{margin fad}}$</td>
<td>(-)</td>
<td>-2 dB</td>
</tr>
<tr>
<td>$P_{r,\text{avg}}$</td>
<td>(-)</td>
<td>-73.7 dBm</td>
</tr>
<tr>
<td>$L_{\text{d.c.}}$</td>
<td>(-)</td>
<td>+6.0 dB</td>
</tr>
<tr>
<td>$P_{r,\text{peak}}$</td>
<td>(-)</td>
<td>-67.7 dBm</td>
</tr>
</tbody>
</table>

Thus, with the DGPS GS reception antenna 80 meters away from the APL transmission antenna, an APL peak power level of -73.7 dBi ( @ 100% d.c.) or -67.7 dBm ( @ 25% d.c. pulsed APL) is received. It should be noted that this is not the actual power level received at the APL antenna port because of allocations made in the GNSS Sensor ARINC Characteristic 743A-1 [22]. Depending upon the particular APL signal structure, this received signal power level can be used to assess the desired APL signal tracking performance or the undesired EMI to the DGPS GS performance. To assess the
APL signal power level as compared to the nominal GPS power level received, an analogous GPS calculation is performed. This is shown in Table 3.4.

Table 3.4 Nominal GPS Power Calculation

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{r,nom}$</td>
<td>(+) 130.0 dBm</td>
<td></td>
</tr>
<tr>
<td>$dG_r$</td>
<td>(+) 0.0 dBil</td>
<td>@ 5° elevation w.r.t. LLT</td>
</tr>
<tr>
<td>$dG_{preamp,gs}$</td>
<td>(+) 13.5 dB</td>
<td></td>
</tr>
<tr>
<td>$P_{r,avg}$</td>
<td>- 116.5 dBm</td>
<td>For GPS @ 100 % d.c.</td>
</tr>
</tbody>
</table>

Thus we see that the APL signal power level at 100% d.c. is 42.8 dB stronger than the nominal GPS power level which does require consideration in the APL system design.

3.3 Error Budget for the APL Subsystem

In this section, a generic error budget for the APL subsystem is introduced with key error terms identified in Table 3.5. Depending upon the APL signal structure employed, these terms may or may not be present, however, a budget for each, and a total error budget is established. Key assumptions are stated for these APL error budget parameters. It is assumed that the GPS and APL signals are received by the same reception antenna at the LAAS GS and a top-mounted RHCP GPS antenna at the aircraft. It is also assumed that the APL differential correction between the GS APL receiver and the aircraft APL receiver is taken to remove all common errors in the APL transmission. As with the APL power link calculation, a set of GPS and APL parameters are defined.
Table 3.5 Key APL Error Budget Parameters

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>DESCRIPTION</th>
<th>ERROR TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>APL Transmission Segment:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{\text{clk,xmt}}$</td>
<td>APL clock stability error</td>
<td>Differentially Corrected</td>
</tr>
<tr>
<td>$\varepsilon_{\text{surv}}$</td>
<td>APL survey location error</td>
<td>Bias if Survey is off</td>
</tr>
<tr>
<td>$\varepsilon_{\text{temp}}$</td>
<td>APL thermal errors</td>
<td>Noise Like</td>
</tr>
<tr>
<td><strong>APL Airport Control Segment:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{\text{cs}}$</td>
<td>APL control errors</td>
<td>Zero for Fault-Free Operations</td>
</tr>
<tr>
<td><strong>APL User Segment:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{\text{rec,code}}$</td>
<td>APL receiver code jitter error</td>
<td>Noise Like</td>
</tr>
<tr>
<td>$\varepsilon_{\text{trop}}$</td>
<td>APL error from troposphere delay</td>
<td>Slowly varying, can be modeled</td>
</tr>
<tr>
<td>$\varepsilon_{\text{mp,env}}$</td>
<td>APL multipath error</td>
<td>Bias Like</td>
</tr>
<tr>
<td>$\varepsilon_{\text{mp,sf}}$</td>
<td>APL multipath error (signal fading component)</td>
<td>High Frequency</td>
</tr>
<tr>
<td>$\varepsilon_{\text{freq}}$</td>
<td>APL error as a result of APL operating off-L1</td>
<td>Slowly Varying Bias</td>
</tr>
<tr>
<td>$\varepsilon_{\text{pul}}$</td>
<td>APL error as a result of pulsing APL transmission</td>
<td>High Frequency Noise; May increase $\varepsilon_{\text{pow}}$</td>
</tr>
<tr>
<td>$\varepsilon_{\text{pow}}$</td>
<td>APL error as a result of power variation in APL reception</td>
<td>Slowly Varying</td>
</tr>
<tr>
<td>$\varepsilon_{\text{clk,rec}}$</td>
<td>APL error as a result of APL clock difference w.r.t. GPS clock</td>
<td>Bias Like</td>
</tr>
<tr>
<td>$\varepsilon_{\text{APL,ss}}$</td>
<td>Total APL signal structure induced errors $= \varepsilon_{\text{mp,sf}} + \varepsilon_{\text{mp,sf}} + \varepsilon_{\text{freq}} + \varepsilon_{\text{pul}} + \varepsilon_{\text{pow}} + \varepsilon_{\text{clk,rec}}$</td>
<td>Total Signal Structure Error (r.s.s.), Code Jitter is Considered Separately, Assuming Independence</td>
</tr>
<tr>
<td><strong>APL Subsystem:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{\text{tot,goal}}$</td>
<td>Desired error to be as good as a high elevation SV $= 0.3$ meters (95%)</td>
<td>After smoothing</td>
</tr>
<tr>
<td>$\varepsilon_{\text{tot,design}}$</td>
<td>Total Designed Error with MLA APL Transmission and Reception Antennas with 95% Confidence</td>
<td>After smoothing</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>Time constant used to smooth code $= 100$ seconds</td>
<td></td>
</tr>
<tr>
<td>$T$</td>
<td>APL receiver’s integration window</td>
<td></td>
</tr>
</tbody>
</table>
These error budget parameters are generic in nature and are analogous to a GPS error budget with a few alterations. No ionospheric propagation errors are present in the APL link for the LAAS and the total APL signal structure induced errors ($\varepsilon_{\text{APL,ss}}$), can be written as a composite of the APL signal structure parameter effects introduced. This total error is composed of a root sum square (r.s.s.) of the individual error terms by assuming that they are independent. Estimates of these error values are presented for the ground-to-air and ground-to-ground links. It should be emphasized that these budgets may or may not be attainable based on a particular APL subsystem architecture.

### 3.3.1 APL Subsystem Error Budget Parameters, Desired Ground-to-Air Link

The error budget for the desired $\text{APL}_G$-to-$\text{APL}_A$ link is analyzed to investigate the system design parameters as illustrated in Table 3.6.

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Error Value (m), (2$\sigma$, 95%)</th>
<th>Error Reduction Issue</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{\text{clk,xmt}}$</td>
<td>0.00</td>
<td>Differentially Corrected</td>
</tr>
<tr>
<td>$\varepsilon_{\text{surv}}$</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{\text{temp}}$</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_{\text{trop}}$</td>
<td>0.05</td>
<td>Mostly Removed by Modeling</td>
</tr>
<tr>
<td>$\varepsilon_{\text{code}}$</td>
<td>0.13</td>
<td>With C/A Code @ Normal C/N$_0$</td>
</tr>
<tr>
<td>$\varepsilon_{\text{mp,env}}$</td>
<td>0.15</td>
<td>Goal with Antenna Design</td>
</tr>
<tr>
<td>$\varepsilon_{\text{APL,ss}}$</td>
<td>0.15</td>
<td>Total After Smoothing</td>
</tr>
<tr>
<td>$\varepsilon_{\text{tot,design}}$</td>
<td>0.254</td>
<td>r.s.s., Assuming Independent Errors</td>
</tr>
<tr>
<td>$\varepsilon_{\text{tot,goal}}$</td>
<td>0.300</td>
<td></td>
</tr>
</tbody>
</table>
Thus a total designed error of 0.254 meters is obtained using the parameters identified in Table 3.6, which is less than the goal of 0.3 meters.

### 3.3.2 APL Subsystem Error Budget Parameters, Desired Ground-to-Ground Link

The error budget for the desired APL\(_G\)-to-APL\(_G\) link is analyzed to investigate the system design parameters as illustrated in Table 3.7.

#### Table 3.7 APL Subsystem Error Budget Parameters, Desired APL\(_G\)-to-APL\(_G\) Link

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Error Value (m), (2(\sigma), 95%)</th>
<th>Error Reduction Issue</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varepsilon_{\text{clk,xmt}})</td>
<td>0.00</td>
<td>Differentially Corrected</td>
</tr>
<tr>
<td>(\varepsilon_{\text{surv}})</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>(\varepsilon_{\text{temp}})</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>(\varepsilon_{\text{trop}})</td>
<td>0.00</td>
<td>Same Height Assumed for GS</td>
</tr>
<tr>
<td>(\varepsilon_{\text{code}})</td>
<td>0.10</td>
<td>With C/A Code @ High C/N(_o)</td>
</tr>
<tr>
<td>(\varepsilon_{\text{mp,env}})</td>
<td>0.15</td>
<td>Goal with Antenna Design</td>
</tr>
<tr>
<td>(\varepsilon_{\text{APL,tot}})</td>
<td>0.15</td>
<td>Total After Smoothing</td>
</tr>
<tr>
<td>(\varepsilon_{\text{tot}})</td>
<td>0.235</td>
<td>r.s.s., Assuming Independent Errors</td>
</tr>
<tr>
<td>(\varepsilon_{\text{tot,goal}})</td>
<td>0.300</td>
<td></td>
</tr>
</tbody>
</table>

Thus a total designed error of 0.235 meters is obtained using the parameters identified in Table 3.6, which is less than the goal of 0.3 meters.

As stated previously, the above budgets are allowable errors for the APL subsystem. An assessment of these parameters will be discussed later based on data collected with the prototype APL subsystem developed in this research.
3.4 Multipath Design Considerations for the APL Subsystem

3.4.1 APL Multipath and APL Link Polarization Investigation

Early in 1996, the need to assess the severity of the multipath for the APL subsystem link was recognized. In July 1996, characterization of the multipath from a ground-based APL was performed at the OU UNI Airport where data were collected with a L1 Northern Telecommunications (NORTEL) Satellite Simulator, a standard 12 Channel NovAtel 3951R GPS Receiver, and the OU Piper Saratoga Aircraft [23]. Code and carrier measurements were made for all observable GPS and the APL PRN signals. The code and carrier measurements were subtracted on a PRN by PRN basis to illustrate the remaining residual error, of which the multipath error is typically the major contributor. Data for the final seconds of flight (approach profile) indicate that the code - carrier (i.e., code minus carrier) residual remaining from a ground-based pseudolite is approximately ± 2 meters, as illustrated in Figure 3.2. These code - carrier residuals do include the predominant multipath error as well as receiver noise and APL noise added for the APL channel. Of importance is that the magnitude of the multipath error from the APL is bounded by ± 2 meters. While this error is greater than that received from a GPS SV, which is approximately 0.1 m (one σ), it appears to be manageable with proper APL system design.
The design approach to minimize multipath interference for the APL subsystem was to limit the multipath at the antenna before it was applied to the receiver. The outcome of this design approach was the MLA, which is discussed in Section 5.2.

3.4.2 Aircraft Top-Mounted RHCP GPS Antenna Performance for the APL Subsystem

Use of the same GPS antenna for APL and GPS reception on the aircraft is a design goal such that system implementation and integration costs can be minimized. Of primary consideration are the antenna pattern shading/blockage and nulls that may occur in the pattern for ground-to-aircraft operation. The APL links (ground-to-air and
ground-to-ground) are not power-limited channels, however the signal loss through the RHCP GPS antenna for the APL link at low elevation angles needs to be considered for various polarizations. Figure 3.3 is a measured outdoor ground range radiation pattern.

![Ground Model Range S68-1575-14 GPS Antenna Radiation Pattern (At L1)](image)

Figure 3.3 Typical L1 Measured GPS Patch Antenna Elevation Radiation Pattern

of a typical L1, L2 GPS patch antenna (Sensor Systems S68-1575-14) for various polarizations on-L1.

The patterns in Figure 3.3 were obtained by placing the antenna on a 4'x7' curved surface with a 96” radius, in accordance with ARINC Characteristic 743A [24]. They were obtained from the Naval Air Warfare Center, Aircraft Division, Antenna Test
Laboratory Automated System (ATLAS) Data Base and is typical for an L1 ± 10 MHz passive patch antenna [25]. These patterns were cut from “wing-to-wing” but can be projected for “tail-to-nose” performance where the antenna would be mounted at a more forward, slightly downward sloping part of the airframe. These radiation patterns give an indication of the performance for horizontal polarization as compared to vertical polarization and RHCP for the APL channel. These data indicate that power coupling for horizontal polarization into the top-mounted RHCP GPS antenna would be very difficult at low grazing angles. Theoretically, the vertical component should never couple more energy than the composite RHCP, but in practice, the antenna pattern is not RHCP in all directions. Thus, the gain of vertical polarization is about the same or greater than that of RHCP at elevation angles below the horizon.

3.4.3 Multipath Limiting Antenna Design Considerations for the APL Subsystem

The multipath from an APL is highly site dependent and requires consideration in the APL transmission and GS APL reception design [23]. Since multipath is the dominant error source, the APL ground-based transmitter antenna design needs to minimize ground multipath, but still maintain good coverage. To analyze the APL geometry, consider the plane-of-incidence to be spanned by the unit vector normal to the reflecting surface and the unit vector in the direction of incidence [26]. By comparing the perpendicular (geometric horizontal) polarization component to the parallel (geometric vertical) polarization component, it is evident that the magnitude of the reflection coefficient for parallel polarization is slightly smaller than that for
perpendicular polarization at low grazing angles incident on a lossy dielectric (earth) [27]. Thus, it can be concluded that the multipath will be minimized for vertical polarization on the APL channel.

Unlike the GPS SV signals, the APL transmissions do not pass through a plasma medium such as the ionosphere. Therefore, Faraday rotation of the signal is not a consideration, and we have more freedom to choose the polarization of the APL-transmitted signal. Vertical polarization was selected for the APL transmission antenna for the following reasons:

1) The aircraft receiving antenna has a strong vertical polarization component at low elevation angles;

2) The ground reflection coefficient at low grazing angles is smaller than that for horizontal polarization;

3) The ease of vertical antenna construction of a suitable APL transmission multipath limiting antenna.

The driving requirement for the APL transmission multipath limiting antenna is a sharp roll-off in the antenna radiation characteristics as a function of elevations angle between +3° to -3° for the ground-to-air link and between 0° to -3° for the ground-to-ground link, assuming only single bounce reflections off the surface of the earth. This design is similar to that used in existing land-based navigation systems such as Tactical Air Navigation (TACAN) and Distance Measuring Equipment (DME), where the elevation patterns roll-off very sharply to minimize ground reflections. As the requirement for the APL transmission multipath limiting antenna became more defined,
combined with the requirement to limit ground multipath for the APL and GPS signals for the LAAS reception, it became clear that these requirements have common characteristics. These common characteristics are for omni-directional coverage in azimuth, a very sharp roll-off about the horizon, coverage at moderate elevation angles, and high desired-to-undesired (D/U) ratios. Furthermore, previous analysis performed on multipath mitigation at a DGPS GS illustrate that a vertically polarized antenna array could achieve good multipath rejection [28]. These factors resulted in a common APL transmission and LAAS GS reception multipath limiting antenna that was designed, built, and tested.

Because the APL link is not a power-limited channel, the APL transmission power level and consequently the APL transmitter antenna gain at +3° was not a primary requirement. The sharpness of roll-off between +3° to -3° and the MLA size will primarily determine the gain (G_t) at +3°.
4.0 APL CALCULATIONS IN LAAS

4.1 Difference Calculations

Code and carrier measurements are taken from the GPS and APL receivers and made available to the navigation processor. Since the APL 50 bps navigation data is not synchronous with GPS timing, the APL pseudorange measurements are first divided down by a C/A code epoch (i.e., modulus 1 ms) at each receiver. This is a valid operation since 1 millisecond corresponds to a range of 299,792 meters, which is larger than the operational range of the APL. Next, the APL pseudorange measurements are smoothed by the carrier, consistent with the LAAS smoothing, using equations (4.1) through (4.3), [29].

\[
PR_p(k) = PR_s(k-1) + [\phi_m(k) - \phi_m(k-1)] \frac{\lambda}{4} \tag{4.1}
\]

\[
PR_d(k) = PR_m(k) - PR_p(k) \tag{4.2}
\]

\[
PR_s(k) = PR_p(k) + \alpha(k) [PR_d(k)] \tag{4.3}
\]

where:

- \( PR_p \) = Propagated pseudorange measurement in units of meters,
- \( \phi_m \) = Accumulated Doppler measurement in units of wavelengths,
- \( \lambda \) = Carrier wavelength in units of meters,
- \( PR_m \) = Pseudorange measurement in units of meters,
- \( PR_d \) = Difference between measured and propagated pseudoranges,
\( \alpha(k) = \text{Inverse of the filter time constant} = \frac{1}{\tau_s}, \)

\( \tau_s = \text{smoothing time constant}, 1 < \tau_s < 100 \text{ seconds}, \)

\( \text{PR}_s(k) = \text{Smoothed Pseudorange Measurement}. \)

APL integration into the LAAS can be analyzed in the pseudorange domain or in the position domain. For pseudorange domain analysis, an APL single difference (SD) is taken between the ground receiver (subscript 1) and user receiver (subscript 2) in a similar fashion as is done for DGPS [30]. This removes all common errors in the APL signal generation, such as the APL transmission clock error. The SD calculation is as follows for \( i = 1 \) to 37:

\[
\text{SD}^1_{\text{PR},2-1} = \text{PR}^1_{2} - \text{PR}^1_{1} + c\Delta t_{2-1}
\]  

(4.4)

where:

\( \Delta t_{2-1} = \text{clock offset between the two receivers}. \)

Next, the APL double difference (DD) between the APL SD and a GPS SD is calculated to remove the receiver clock bias term \( (\Delta t_{2-1}) \) as:

\[
\text{DD}^{\text{q}}_{\text{PR},2-1} = (\text{PR}^1_{2} - \text{PR}^1_{1}) - (\text{PR}^q_{2} - \text{PR}^q_{1})
\]  

(4.5)

where the superscript \( q \) is for GPS SVs only: \( 1 \leq q \leq 32. \)

The ground ranging accuracy of the APL DD measurements is calculated to investigate the APL ground multipath. A “Truth APL DD” \( (\text{True DD}^{\text{q}}_{\text{PR},2-1}) \) is calculated and then subtracted from the measured APL DD of equation (4.5). Note that for this analysis, the user receiver is the ground-based monitor receiver. The Truth APL DD is calculated from the GPS broadcast ephemeris data, the surveyed APL
transmission antenna location, and the surveyed GS reception antenna locations. The APL DD Error can then be calculated as:

\[ \text{DD}_{iQ}^{\text{PR,2-1 \ Error}} = \text{DD}_{iQ}^{\text{PR,2-1}} - \text{True DD}_{iQ}^{\text{PR,2-1}} \]  

(4.6)

This ground DD Error will be used to assess error in the pseudorange domain.

4.2 B-value Calculations and Associated Statistics

For the LAAS, the concept of multiple GSs is utilized for signal monitoring, integrity, and enhanced accuracy [1]. A LAAS configuration with multiple GSs or Reference Receivers (RRs) and APLs is illustrated in Figure 3.4. Differential corrections based on multiple, averaged measurements can be used to decrease multipath errors. Once differential corrections are formed, they can be compared to assess signal integrity. This is implemented by comparing a particular differential correction with the average of the differential corrections for that particular satellite or APL. The resulting test statistics are referred to as B-values, since they allow for the detection of bias errors. The implementation of B-value calculations for the APL provides a direct indication of the quality of the APL signal.

The B-values and their associated statistics are calculated in the remainder of this section and in Appendix B, consistent with the LAAS integrity methodology as described in reference [31]. These calculations closely follows the equations of reference [31] but with APLs included. With a maximum number of Z observables available (N satellites plus A APLs, \( Z = N + A \)), a total of N SV signals, as well as, A APL signals will be received by all M RRs. The M RRs for the LAAS Ground
Subsystem will be spatially separated by a distance great enough to decorrelate multipath error. The APL transmission will be omni-directional in azimuth and can be received by all Ground Subsystem RRs.

For a particular RR $m$ and a particular SV $n$, the pseudorange measurement ($\rho_m^n$) can be written as follows:

$$\rho_m^n = R_m^n + \text{errors} = R_m^n + S\alpha^n + t_m + \text{iono}^n + \text{tropo}^n + n_m^n + \epsilon^n \quad (4.7)$$
where:

\[ R^n_m = \text{True Distance from RR m to SV n}, \]
\[ SA^n = \text{Selective Availability Error for SV n}, \]
\[ t^n = \text{Clock Error of SV n}, \]
\[ iono^n = \text{Ionospheric Delay for SV n}, \]
\[ tropo^n = \text{Troposheric Delay for SV n}, \]
\[ n^n_m = \text{Noise Component (i.e., Receiver Noise, Multipath) with RR m for SV n}, \]
\[ \varepsilon^n = \text{Ephemeris Error from SV n}. \]

With the true pseudorange known (calculated from the ephemeris and surveyed receiving antenna phase center locations) a particular RR m can calculate a pseudorange correction to a particular SV n.

\[ PRe^n_m(t_m) = \rho^n_m - R^n_m = \left\{ SA^n + t^n + iono^n + tropo^n + \varepsilon^n \right\} + \left\{ n^n_m \right\} + t_m \quad (4.8) \]

In equation (4.8), the terms in the first set of brackets represent errors in the pseudorange corrections that are considered common between the LAAS GS and the airborne user. These error terms will mostly cancel in the DGPS/DAPL solution as long as the separation between the GS and airborne user is not too large. The term in the second bracket of equation (4.8) is not common between the LAAS GS and the airborne user. This error term will not cancel in the DGPS/DAPL corrections and should be considered in the B-value calculations. Finally, the clock bias term must also be removed to limit the magnitude of the differential corrections and to allow for a comparison between different RRs.
Since the clock bias will apply across all pseudorange measurements for a particular RR m, an estimate of the clock bias is obtained by averaging the pseudorange corrections for all SVs that are common to all M RRs. This is expressed as:

\[
\hat{\Delta m} = \frac{1}{N} \sum_{i=1}^{N} PRc_{m}^{i}(t_{m}) - t_{m} - \Delta t_{m} \quad (4.9)
\]

where the estimate of the clock bias for RR m is represented as the true clock bias for RR m minus a difference. The clock bias estimate is then subtracted from the pseudorange correction:

\[
PRc_{m}^{n} = PRc_{m}^{n}(t_{m}) - \hat{\Delta m} = \{SA^{n} + t^{n} + iono^{n} + tropo^{n} + \epsilon^{n}\} + \{n_{m}^{n}\} + \Delta t_{m} \quad (4.10)
\]

After the removal of the estimated clock biases from each of the RRs, an averaged pseudorange correction is made as follows:

\[
PRc^{n} = \frac{1}{M} \sum_{j=1}^{M} PRc_{j}^{n} \quad (4.11)
\]

An indication of the error of the PRc for a particular RR m can be calculated by taking the difference between \(PRc_{m}^{n}\), and \(PRc_{m}^{n}\) with the particular RR m removed. This is represented as:

\[
B_{m}^{n} = PRc^{n} - \frac{1}{M - 1} \sum_{j \neq m}^{M} PRc_{j}^{n} \quad (4.12)
\]

Equation (4.12) can also be written as:

\[
B_{m}^{n} = \frac{1}{M - 1} \left( PRc_{m}^{n} - PRc^{n} \right) \quad (4.13)
\]

Appendix B calculates the expected value of the B-value to be zero and the variance of the B-values as:
\[
\text{VAR}[B_m^n] = \left(\sigma_m^n\right)^2 \left(\frac{N + 1}{MN(M - 1)}\right)
\]

The above expression for the variance of the B-values illustrates that it is directly related to the variance of the pseudorange measurements, the number of RRs used, and the number of satellites used. The \(\left(\sigma_m^n\right)^2\) term in equation (4.14) is the variance of the pseudorange measurement. It is noted that the above derivation also holds for the APL as long as its measurement variance is the same as that of the GPS satellites. If the variances are different, then the clock removal process will generate additional terms in equation (B-1). This significantly complicates the variance calculation for the B-values. This complication is beyond the scope of this document. For the purpose of this research, the APL signal was not used in the GPS receiver clock bias solution because the value of the noise on the APL pseudorange measurement was expected to be greater than the noise on the GPS pseudorange measurements.

4.3 DAPL Position Integration and Associated Statistics

DGPS and DAPL position integration is accomplished in a least-squares sense as outlined in equations (A-1) through (A-14). For the integrated APL, the direction cosines of equation (A-7) will point in the direction of the APL transmission, such that \(\hat{g}_i = (g_{xi}, g_{yi}, g_{zi})\) will be the unit vector in the direction of the APL transmitting MLA phase center location with \(i = 34\). The APL integrated position analysis will be with respect to an ENU coordinate system where VDOP with and without the APL integrated into the DGPS/DAPL solution will be assessed using equation (A-25).
5.0 PHASE I APL ARCHITECTURE WITH MLA DEVELOPMENT

5.1 Off-L1 APL Subsystem - Isolation of Major APL Error Sources

As stated in Section 2.6, the amount of cross-correlation between the APL PRN and the GPS PRNs can be decreased by spatial, frequency, code, or time separation. The two major error sources for APLs in the precision approach environment are ground multipath and the large power level variation over the final approach path [19]. To assess these major error sources, a prototype APL subsystem was developed such that it could operate at any frequency in the L1 band (L1 ± 10 MHz). Figure 5.1 illustrates the APL transmitter used in this first phase. Both single and dual receiver configurations were used. In the single receiver configuration, both the GPS and APL signals are

![Figure 5.1 Top-Level APL Transmitter Block Diagram](image-url)
received with the same receiver. A second APL-only receiver is added if the APL operating frequency is outside the passband of the GPS receiver. Figures 5.2 illustrate a general block diagram of the APL receiver configurations.

Figure 5.2 Top-Level GPS/APL Receiver Block Diagram

The first phase of the OU APL architecture was based primarily on a frequency separation approach. This allowed the investigation of the APL multipath and power level variation factors, independent from APL signal structure factors. The center frequency was tuned off L1 far enough such that the transmitted, filtered APL spectrum is "out-of-band" from typical GPS receiver. Placement of the APL center frequency at multiples of the C/A code rate further minimizes cross-correlation between the APL
PRN code and any GPS PRN code. To compensate for the APL transmitter clock bias, the APL signal was differentially corrected. With this off-L1 approach, the only expected residual is caused by the difference in the two APL receiver channels, one at the GS and the other in the airborne unit, similar to the DGPS case. All APL transmissions maintained a constant code rate of 1.023 MHz such that accurate ranging could be accomplished. The local oscillator within the receiver was tuned to enable off-L1 operation. Internal receiver code smoothing by the carrier was kept to a minimum such that a potential code-to-carrier divergence could be observed; a 2-second smoothing time was used.

The large power level variations that are needed in an APL subsystem places design constraints on the APL signal structure such that EMI is not induced on the nominal DGPS performance of the LAAS. When the APL frequency of operation is within the passband of the GPS receiver, several modifications must be made to the APL signal structure. To isolate the effects from error sources induced by ground multipath and error sources induced by the APL signal structure, an APL with a frequency offset of \( L_1 + 8 \times (1.023 \text{ MHz}) = 1583.6 \text{ MHz} \) was tested. This prototype was extremely useful in terms of assessing APL subsystem performance, error components, system parameter trade-offs, and their sensitivity to design parameters. The MLA was designed with APL requirements included and then tested with this APL configuration to assess its effectiveness to limit ground multipath for APL integration in precision approach applications [19].
5.2 Multipath Limiting Antenna for the APL Subsystem

A photograph of the MLA that was designed, built, and tested for APL applications is shown in Figure 5.3. This MLA has an aperture (size) of approximately 2.2 meters and a sharp roll-off radiation characteristic in elevation from +3° to -3°. Figure 5.4 is a plot of the elevation radiation characteristics of this antenna at L1 for various aspect angles in azimuth. This MLA has an “omni-directional” radiation characteristic in azimuth. The primary design factor for the APL application was the sharpness of the roll-off in gain with a reasonable size aperture. Apertures that are significantly larger than 2 meters would pose siting, installation, and structural limitations. Figure 5.5 is a plot of the Desired/Undesired (D/U) ratio and indicates how much ground multipath rejection is attained as a function of elevation angle. For the APL ground-to-air link the D/U can be read directly from Figure 5.5 and is at least 25 dB for an undesired multipath signal reflecting off the surface of the earth transmitted at -3° elevation angle. The APL ground-to-ground multipath rejection is determined from Figure 5.4, and is approximately 11 dB (Difference between the gain at 0° and -3°) for each antenna. Hence, at least 22 dB of D/U protection is provided for a first bounce undesired multipath signal received at -3°, using the MLA for both APL transmission and APL reception.
Figure 5.3 MLA used for APL Transmission
Figure 5.4 MLA Elevation Radiation Plot for S/N003 where 0 dB ≡ 7.5 dBi
Figure 5.5 MLA Desired/Undesired Ratio Plot for S/N003

Error performance data utilizing the MLA for APL transmission and reception will be presented in the next section.

5.3 APL Multipath Error Analysis

In May 1997, flight tests were performed with the FAA Boeing 727 Aircraft and the OU/FAA LAAS at the William J. Hughes Technical Center. During these tests, data were collected at the GS and at the aircraft to assess the APL subsystem multipath performance using the OU Prototype APL subsystem. An APL signal structure format off-L1 (@1583.6 MHz) was used where the APL transmission was not pulsed (100% duty cycle). This allowed for multipath error analysis independent from APL signal
structure design factors. Data were collected using the MLA for APL transmission while APL reception at the GS was performed simultaneously with a MLA and a choke ring antenna mounted on top of the receiving MLA. The FAA Boeing 727 Aircraft received the APL signal through a single top-mounted Sensor Systems S67-1575-14 passive L1, L2 antenna at flight station 282.0. Figure 5.6 shows the APL transmitter MLA-to-LAAS GS MLA code - carrier residual data and sampled standard deviation (typical) with mean and rate divergence removed. Figure 5.7 shows the APL transmitter MLA-to-LAAS GS (choke ring antenna) code - carrier residual data and sampled standard deviation (typical) with mean and rate divergence removed. Figure 5.8 shows the typical APL transmitter.

Figure 5.6 APL Transmitter MLA-to-LAAS Reference Station (MLA) Code - Carrier Residual Variations; and Received Carrier-to-Noise Ratios
MLA-to-Boeing 727 code - carrier residual data and sampled standard deviation (typical) with mean and rate divergence removed. The data in Figure 5.8 starts at a pseudorange

![Normalized Code-Carrier Residual Variation, Std Dev = 0.1976 m](image)

![Carrier-to-Noise Ratio](image)

Figure 5.7 APL Transmitter MLA-to-LAAS Reference Station (Choke Ring Antenna) Code - Carrier Residual Variations; and Received Carrier-to-Noise Ratio

(PR) of approximately 10,050 meters (t = 0 seconds), stops at a PR of 250 meters (t=130 seconds), and shows low code - carrier variations (σ = 0.11 m). There was no additional smoothing performed external to the receiver (NovAtel 3951R), where a default time constant of 20 seconds was used on the internal code smoothing.

Figures 5.6 and 5.7 indicate that the multipath variations for APL reception are much better with the receiving MLA, limited to ± 0.4 meters (σ = 0.12 m) than with the receiving choke ring antenna, where the multipath residual is limited to ± 0.7 meters
(\( \sigma = 0.20 \) m). External smoothing would further reduce these errors. Figure 5.8 shows that the transmitting APL MLA works effectively to limit the code-carrier residual to approximately \( \pm 0.4 \) meters with no external receiver smoothing for the ground-to-air APL link. It should be emphasized that these results are for a non-

![Normalized Code-Carrier Residual Variation, Std Dev = 0.1053 m](image)

![Carrier-to-Noise Ratio](image)

Figure 5.8 APL Transmitter MLA-to-Boeing 727 Code-Carrier Residual Variations; and Received Carrier-to-Noise Ratio

saturating APL receiver and they indicate that after smoothing, the proposed error budget of 0.15 meters can be met for the ground-to-air and probably also for the ground-to-ground link using the C/A code format. It should also be noted that the mean has been removed in these data. However, for the dual-receiver APL configuration, a random start-up bias remained in the differential calculation. This random start-up bias error
precluded the observation of a possible ground-to-ground standing multipath error component. In other words, successful multipath limiting was achieved on the APL transmitter-to-aircraft path but not yet on the ground-to-ground link. This was the major shortfall of the off-L1 dual receiver configuration. The random startup bias was anywhere in the interval of -49 ns to +49 ns. The limits of the random bias correspond to ± 1 clock cycle of the internal 20.473 MHz clock. This bias exists even though both receivers were connected to a common Rubidium frequency standard, the same phase-locked loop was used in the receiver chassis frequency translator card, and GPS time was transferred from the GPS receiver to the APL receiver for the APL pseudorange calculations. It was unclear from the manufacturer if the bias results from an inaccurate time transfer from the GPS receiver to the second APL receiver or if it is an internal clock synchronization issue. The need to eliminate this clock bias error was the major reason why a single GPS and APL receiver configuration was used for the remainder of the APL research.
6.0 PROTOTYPE ON-L1 C/A APL IN LAAS ARCHITECTURE (PHASE II)

All on-L1 APL development was performed at the OU UNI Airport with one or two LAAS GSs, and the OU DC-3 Aircraft. Ten developmental flight tests and twelve laboratory tests were conducted to arrive at the following on-L1 configuration.

6.1 APL Transmitter

The APL transmitter consists of a modified GPS signal generator with power amplifiers and a pulser circuit. The APL transmitter block diagram is shown in Figure 6.1. The APL signal generation is performed with a modified NORTEL GPS Simulator with a 10.23 MHz oven-controlled crystal oscillator (OCXO) frequency reference. PRN code 34, in compliance with the GPS-ICD-200, is used with an arbitrary GPS Week

Figure 6.1 On-L1 C/A Code Prototype APL Transmitter Block Diagram
number of 800 [12]. A power level of -37 dBm is generated directly by the modified GPS simulator. Figure 6.2 illustrates a detailed block diagram of the APL transmitter.

Pulsing is accomplished with two high-isolation, high-speed RF switches. The isolation of the RF switches was measured to be 118 dB of on/off isolation “on the cable”. A final stage of power amplification follows the pulsing to produce a peak power output of +12 dBm. The RF amplifier and pulser RF network is enclosed within a RF-shielded enclosure so that a truly pulsed APL signal is seen at the LAAS receivers. A photograph of the shielded RF APL transmitter is shown in Figure 6.3. The pulser circuitry within the RF shielded enclosure was controlled from the LAAS GS location via a 100-meter
Figure 6.3 Shielded APL Transmitter Photograph

cable and low pass filter (LPF). The pulse on/off timing was driven from the GPS 1 pulse-per-second (PPS) at the LAAS GS GPS and APL receivers. The pulsing format selected for the APL transmission was 310 C/A chips of duration “on” and 806 C/A chips of duration “off” at a 1/11th code-cycle rate. This slides the APL transmission through each C/A code epoch of the receiver’s integration window. The transmitted duty cycle was 27.8%. Figure 6.4 illustrates this APL pulsing technique with respect to the GPS 1 PPS and the C/A code epoch timing. Figure 6.4 also illustrates the “GPS Blanking” of the APL signal for GPS reception, which will be discussed in the next section. Each C/A code epoch is divided into 11 sub-epochs because 11 is exactly divisible into 1023 (the number of C/A chips per C/A code epoch). The selection of the
pulsing format was primarily driven by the requirement to operate in a linear region (non-saturating) of the receiver. The APL equipment is housed in a ruggedized enclosure and located in close proximity to the transmitting MLA.

Figure 6.4 APL Transmitter Pulsing and APL Blanking Timing Diagram Illustration

6.2 APL Receiving Subsystem in the Prototype LAAS

6.2.1 APL Receiving Subsystem at the Ground Station

The distance between the APL transmitting antenna and the GS reception antenna was chosen to be approximately 80 meters based on the height of each antenna and the first null ( @ 3° down) in the elevation pattern of the MLA to minimize ground
multipath. Due to siting and logistic constraints at the OU UNI Airport, the LAAS GS was placed approximately 1000 feet prior to the runway threshold in line with Runway 25 in close proximity to a line of trees.

Figure 6.5 illustrates a two GS configuration for prototype APL signal reception used in developmental tests. The APL subsystem was integrated into the OU LAAS described in [29]. The transmitted APL signal can be received by either the high-zenith antenna (HZA) used for high-elevation GPS SV reception or by the MLA (for low-elevation GPS SV reception).

One hundred meters of low-loss heli-axial cable connected the field antennas to the LAAS GS located in a mobile van. Data have indicated that satisfactory APL performance can be obtained with either APL reception on the top HZA or the lower MLA based on antenna siting constraints. With good antenna siting (as is the case at the FAA William J. Hughes Technical Center), APL reception with the MLA is preferred. With poor antenna siting (as is the case at the OU UNI Airport) the HZA is preferred. When the HZA is used for the desired APL reception, this path is called the “Desired
GPS/APL Path”, and the MLA is called the “Desired GPS-Only Path”. The opposite configuration is also possible with good antenna siting.

Figure 6.6 is a photograph of the APL Gain Control and GPS Blanking System for a M=2 GS Configuration. This system performs APL MGC and GPS Blanking for the Desired GPS/APL Path, and GPS Blanking for the Desired GPS-Only Path. Figure 6.7 is a block diagram of the APL Gain Control and the GPS Blanker Subsystem for the “Desired GPS/APL Path”. This subsystem simultaneously performs APL signal power level control in the “Desired APL Sub-Path” and blanks the APL signal,
during the times it is “on” in the “Desired GPS Sub-Path”. (The blanking of the APL signal in the Desired GPS Sub-Path is referred to as “GPS Blanking”.) Figure 6.8 illustrates a composite GPS and APL signal in the Desired GPS/APL Path where the APL signal peak power is greater than the GPS peak power by a ratio of A/G (APL-to-GPS signal ratio). This figure also illustrates the modifications to the signals in each sub-path to produce a power-controlled APL signal with minimal effect on the GPS signal.

Within the APL manual gain control (MGC) and GPS Blanking Subsystem, the GPS and APL composite signal (Desired GPS/APL Sub-Path) is split into a Desired
GPS Sub-Path and a Desired APL Sub-Path, as shown in Figure 6.7. When the signal is split and recombined a 3 dB loss will occur in each operation; this is designated by dividing the signal in half as illustrated in Figure 6.8. In the Desired GPS Sub-Path

![Diagram of APL MGC and GPS Blanking Subsystem]

Figure 6.7 Ground Station APL Gain Control and GPS Blanker Subsystem Block Diagram

the APL signal was blanked “totally” with two series combination TTL switches; the two TTL switches provided an on/off isolation of approximately 70 dB. The isolation of this switch is designated as $I_s$ in Figure 6.8. In the Desired APL Sub-Path a programmable attenuator was used to manually control the amplitude of the APL signal presented to the receiver. The APL signal was usually attenuated by approximately
Figure 6.8 GS APL Gain Control and GPS Blanker Subsystem Time Domain Diagram
25 dB at the GS. The level of attenuation is designated as $\alpha_a$ in Figure 6.8. Since the APL signal was pulsed and blanking of the APL signal occurred in the other sub-path (Desired GPS Sub-Path), the attenuator operated in a continuous time fashion (i.e., it was not pulsed). The power-level-controlled APL signal in the Desired APL Sub-Path, and the desired GPS blanked signal in the Desired GPS Sub-Path were then recombined. To minimize signal fading, both paths were calibrated with a network analyzer. All cable lengths were minimized to mitigate temperature variation effects.

Within the GPS Blanking Subsystem ("Desired GPS-Only Path", see Figure 6.5) a single TTL switch is placed in series to "blank" the APL signal during the time when it is present. The timing control for all blanking is via a GPS 1 PPS input and a microcontroller. In the ground station a guard time of approximately 2 $\mu$ seconds is added to ensure complete blanking overlap, which is illustrated in Figure 6.4. For this Desired GPS-Only Path, the level of blanking was not complete, but only to a comparable power level as is done in the Desired GPS/APL Signal Path. The APL power level was turned down/blanked by approximately 35 dB. This was done to ensure that the $C/N_0$ calculations in the receivers were valid.

All GPS and APL receivers were NovAtel 3951R Receivers with standard or modified firmware for APL reception. A Rubidium frequency standard was used to tie measurements from the HZA and MLA antennas such that the phase variation as a function of GPS SV elevation angle could be observed. The Rubidium standard would not be required for an operational system.
For differential corrections the VDB used a horizontally-polarized 8-ary differentially phase shift keyed modulation-formatted signal to support DGPS operation [29].

6.2.2 APL Receiving Subsystem at the Aircraft

Figure 6.9 illustrates the GPS and APL reception in the aircraft. All signal reception is with a single top-mounted RHCP passive L1/L2 patch antenna. Flight data presented here were collected with the OU DC-3 Aircraft. The antenna location is top-mounted, on-centerline at flight station 143.0. The RF signal was split and fed into an Ashtech Z-12 (used for truth), an unmodified NovAtel 3951R GPS-Only Receiver (used...
for APL interference assessment and DGPS reference), and the combined GPS and APL NovAtel Receiver via the airborne APL AGC and GPS Blanker Subsystem.

For APL applications, a wide dynamic range is required due to free space \((\lambda/4\pi R)^2\) losses. The variation in power from a minimum operational range of 80 meters to a maximum range of 10 nmi at L1 is 47.3 dB, not including antenna pattern and other variations. Thus, the dynamic range required for an APL channel far exceeds the dynamic range available in a GPS receiver.

Early laboratory and flight tests revealed an error source that was present as the receiver saturated over a 63 dB dynamic range. Due to these nonlinear effects in the receiver, an external AGC was added to the receiver. The RF power level was controlled such that the receiver never saturated.

The APL AGC and GPS Blanker Subsystem, prior to the GPS and APL receiver is similar to the GS APL MGC and GPS Blanking Subsystem described in Section 6.2.1 with MGC replaced by AGC and increased guard time. Again, GPS 1 PPS timing was used from the GPS and APL receiver for blanker timing control. The GPS and APL signal was split into a Desired GPS Sub-Path and a Desired APL Sub-Path, see Figure 6.7. In the Desired GPS Sub-Path the APL signal was blanked “totally” with two series combination high-isolation switches; each switch provided an on/off isolation of 63 dB and the total on/off isolation was measured to be 118 dB. A guard time of 62 C/A chips was used to support a guard distance of approximately 10.0 nmi. The airborne blanking sequence was 372 chips “on” and 744 chips “off” at a 1/11 th code-cycle rate, again sliding through each epoch integrated in the receiver. The timing of the airborne
blanking is illustrated in Figure 6.4. This blanking could produce a maximum loss in GPS C/N₀ of approximately 2 dB, which was not an issue for the feasibility demonstration. An operational system would have a shorter duty-cycle such that the loss in C/N₀ is negligible. A programmable attenuator was used in the Desired APL Sub-Path to automatically control the gain of the APL signal presented to the receiver. This programmable attenuator was driven by the APL (PRN 34) C/N₀ measurement via the Airborne Processor to maintain a C/N₀ between 42 to 43 dB-Hz. Since the APL signal was pulsed and the blanking of the APL signal occurred in the other sub-path (Desired GPS Sub-Path), the attenuator operated in a continuous time fashion. This RF AGC power-controlled APL signal in the Desired APL Sub-Path, and the desired GPS signal in the Desired GPS Sub-Path were then recombined. To minimize signal fading, both paths were calibrated with a network analyzer. All cable lengths were minimized to mitigate temperature variation effects.

For an on-L1 APL, pulsing (with high on/off isolation) is required to mitigate EMI. Laboratory tests indicate that the amount of EMI protection is directly proportional to the duty cycle [14]. Laboratory tests also indicate that the C/N₀ calculations for the receivers tested were inaccurate for pulsed APL signals. The C/N₀ calculation became more non-linear as the duty cycle decreased. Hence, at low duty cycles the linear operating region ("comfort zone") became extremely small. Since the desired operation region was a linear portion of C/N₀ indication in a non-saturation region, the duty cycle had to be high enough so that successful C/N₀ tracking could occur to drive the RF AGC. These were the primary factors to operate the APL at a duty cycle
of 27.8 %. With C/N_o calculations for pulsed signals, lower duty cycles are likely supported in a linear region of operation within the GPS/APL receiver.
7.0 PROTOTYPE ON-L1 C/A APL IN LAAS PERFORMANCE

7.1 DAPL Ground Performance Assessment

The location of the APL transmission antenna, and the GS reception antennas were surveyed with an Ashtech Z-12 GPS Receiver. A peak transmitted power of +12 dBm was used, which is limited primarily by the components in the current APL RF transmitter. This power level supports an APL operational range of approximately 6-8 nmi. At this power level and duty cycle, EMI was not recorded by an Ashtech Z-12 GPS Receiver located on top of the OU Hanger, 578 meters from the APL transmitting MLA. Additionally, using the unmodified NovAtel 3951R GPS Receiver and a Trimble LTN-2000 GPS Receiver aboard the aircraft, EMI was not observed from the APL transmission during a typical approach path. It should be noted however that EMI was observed in the GS receivers if they were not blanked. In the absence of blanking, the pulsed APL signal level would be approximately 25 dB stronger than the GPS signal levels. This causes the receiver to lose lock on the weaker GPS SV signals. An operational APL augmentation would use lower pulsing duty cycles which would mitigate this EMI effect. Furthermore, the proposed WB APL signal structure would render this EMI effect insignificant [32].

To investigate the possible existence of “standing wave multipath” from the APL transmission MLA to the LAAS GS reception antenna due to the static APL geometry, APL DD error analysis was performed [33]. For this GS APL DD error analysis the
measured APL DD was formed and then subtracted from the truth APL DD, using equation (4.6). This True APL DD was formed using surveyed antenna positions and the SV identification (SVID) 19 orbit data from the broadcast ephemeris. SVID 19 was used in the APL DD because it had the lowest B-value and hence presumed to be of the best quality. Data indicates that the criteria for the GPS DD should be based on B-value and not simply SV elevation angle. Figure 7.1 shows the computed APL DD error from equation (4.6). The large spikes in the APL DD are caused when the APL receiver loses lock and the code filter is reset. This loss of lock results when the DC-3 over-flies directly between the APL transmitting antenna and the APL reception antenna. In this

![Figure 7.1 Ground APL DD Error Using SVID 19 (20 February 1998)](image-url)

Figure 7.1 Ground APL DD Error Using SVID 19 (20 February 1998)
data analysis no PR limit checks were performed (i.e., APL exclusion) but this is incorporated in the real-time system. With better antenna siting, these events would likely not occur. While these data still contain some ground multipath, the multipath from the GPS SV used in the double difference is mixed in. The APL transmitting MLA with some multipath limiting protection from the reception antenna limits the APL multipath and no substantial long-term bias exists.

To monitor and detect anomalies in the pseudorange measurements at the LAAS GS, the B-values are computed between the two ground reference sites (i.e., RRs). Figure 7.2 shows the B-values for the APL (PRN 34) and a GPS SVID 19 used in the APL DD calculation for the HZA calculated in accordance with equation (4.13). The

![Figure 7.2 B-values at the LAAS GS for APL DD Error Calculation](image)

Figure 7.2 B-values at the LAAS GS for APL DD Error Calculation
variations in ground multipath can be observed in real-time from these B-values. The calculated variances and standard deviations of the B-values are also included on Figure 7.2. (Note that for $M=2$ the B-value for a particular RR $m$ will be the complement of the other; this will not be the case for $M > 2$.)

With reference to equation (4.13), the measured variances of the B-values and the corresponding standard deviations are given by:

$$\text{VAR}[B_m^{GPS(SVID19)}] = 0.012 \text{ meters, } \sigma[B_m^{GPS(SVID19)}] = 0.11 \text{ meters}$$

$$\text{VAR}[B_m^{APL(PRN34)}] = 0.034 \text{ meters, } \sigma[B_m^{APL(PRN34)}] = 0.19 \text{ meters}$$

Using the measured standard deviations of the B-values, the pseudorange standard deviations for the GPS satellite and the APL can be approximated using equation (4.14):

$$\sigma^n = \sigma[B_m^n] \sqrt{\frac{MN(M-1)}{(N+1)}}$$

Therefore,

$$\sigma_m^{GPS(SVID19)} = \sigma[B_m^{GPS(SVID19)}] \sqrt{\frac{MN(M-1)}{(N+1)}} = 0.11 \sqrt{\frac{(2)(8)(2-1)}{(8+1)}} = (0.11) \frac{4}{3} = 0.15 \text{ meters}$$

$$\sigma_m^{APL(PRN34)} = \sigma[B_m^{APL(PRN34)}] \sqrt{\frac{MN(M-1)}{(N+1)}} = 0.19 \sqrt{\frac{(2)(8)(2-1)}{(8+1)}} = (0.19) \frac{4}{3} = 0.25 \text{ meters}$$

where $N$ equals 8, which was the average number of common SVs used in the receiver’s clock solutions. Table 7.1 summarizes the measured values and the proposed requirements for the pseudorange standard deviations.
Table 7.1 Measured and Proposed Requirements for the GPS Satellite and APL Pseudorange Standard Deviations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measured</th>
<th>RTCS MASPS [20]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{GPS}$</td>
<td>0.15 m</td>
<td>0.15 m (^{(1)})</td>
</tr>
<tr>
<td>$\sigma_{APL}$</td>
<td>0.25 m</td>
<td>0.5 m (^{(2)})</td>
</tr>
</tbody>
</table>

Note 1. This is the most stringent proposed requirement for a high-elevation satellite.

Note 2. Reference [20] notes that this accuracy specification requires validation. The achieved APL accuracy is expected to be in the range of 0.25 to 1.5 meters, RMS.

Based on Table 7.1, that shows the ground APL pseudorange measurement errors are approximately 0.25 meters, and Figure 7.1, that shows that data collected on the ground exhibits APL bias errors of less than 0.1 meters, it is concluded that the APL feasibility of accuracy has been demonstrated.

7.2 DGPS/DAPL Airborne Performance Assessment

Once GPS and APL measurement quality was established, GPS and APL PR corrections were broadcast in real-time via the VDB, received by the Airborne Processor, and then applied to the airborne GPS and APL PR measurements. The DAPL measurements were then incorporated into the DGPS position solution in real-time. The APL was excluded from the position solution if the DAPL PR measurement exceeded a 5 meter spheroid around the DGPS position solution, the APL signal was lost, or invalid ground data were received.

At the beginning of a typical approach, acquisition was fairly rapid where the APL channel was assigned to search over a window of \(\pm 700\) Hz. This window is large...
enough to cover the Doppler frequency shift uncertainty and temperature dependent variations of the APL frequency reference OCXO. Acquisition of the APL signal typically occurred at a C/N₀ of approximately 35 dB-Hz. The C/N₀ was allowed to increase to 43 dB-Hz, where the RF AGC then activated to maintain the C/N₀ between 42 to 43 dB-Hz until its dynamic range of 63 dB was expended. At +12 dBm peak transmitted power, power control over a 55 dB dynamic range was performed. Since all approaches were of short range and of low altitude, tropospheric delay corrections were not applied. Not modeling this effect does produce a small amount of error at the maximum range of the APL (6-8 nmi) but is insignificant by the time the decision height is reached. Further investigation into the tropospheric delays at these low elevation angle/low heights for APL applications will likely be needed.

Figure 7.3 illustrates a ground track of a typical DC-3 flight approach and shows when the APL was in or out of the DGPS solution. In general, the APL was included in the DGPS/DAPL position solution most of the time, except for periods outside the antenna patterns or dynamic range of the APL link.
The VDOP for a typical DC-3 approach loop is illustrated in Figure 7.4 as calculated from equation (A-25). The VDOP is related to the satellite geometry, which is illustrated in the sky plot in Figure 7.5 during the same time (501300 < GPS Time < 504000 on 20 February 1998). In Figure 7.5 the beginning of this approach corresponds approximately to the beginning of the sky plot traces, where the GPS SVID number is located to the right of the beginning for each of the traces. This good SV geometry corresponds to a good VDOP even without the APL. The benefit of the APL can be seen in Figure 7.4 for the GPS-Only Receiver and GPS/APL Receivers when the APL is incorporated into the DGPS/DAPL solution. The benefit of the APL with respect to VDOP reduction is clearly illustrated. The VDOP reduction ratio is not “flat” due to the
siting of the APL at the OU UNI Airport prior to the runway threshold where a constant 3° approach path is not maintained. It should be noted that the small jump in VDOP for the GPS-only receiver is caused by the blockage of SVID 7 by the OU UNI Hanger as the aircraft was close to the ground.

Figure 7.4 VDOP of Typical Approach Path With and Without APL in the Position Solution
Figure 7.5 Sky Plot of SVID and APL at Ground Station

Figure 7.6 shows typical position differences between the position solutions without the APL from the DGPS receiver (solid trace) and with the APL from the DGPS/APL receiver (circle trace) over the final phase of an approach. (Previous flight tests showed that the absolute DGPS errors with respect to the post-processed kinematic Ashtech Z-12 solution are sub-meter in each ENU coordinate.) It should be noted that no significant difference in the North solutions was observed due to the aircraft's approach to Runway 25 at the OU UNI Airport; the North coordinate axis is approximately perpendicular (within 20°) to the line-of-sight to the pseudolite transmitter.
This on-L1 prototype system supported an operational range of approximately 6 - 8 nmi with accuracies consistent with precision approach requirements. The operational range for testing was primarily limited by the 1 dB compression operating points of the final stage APL transmitting amplifier. This operational range can likely be increased easily with higher-powered APL transmitting amplifiers and additional blanking in the GPS/APL signal paths, which is within the capabilities of the AGL AGC and GPS Blanker Subsystem. This real-time DGPS/APL position integration without accuracy degradation beyond precision approach limits has increased the availability of the prototype LAAS. This illustrates the successful integration of a ranging APL in a real-time fashion into a prototype LAAS and demonstrates that an increase in availability can be achieved.
8.0 SUMMARY AND CONCLUSIONS

The LAAS is being developed by the FAA to support precision approach and landing operations in and about the local area surrounding an airport. It is planned to provide service within the U.S. NAS and will likely be fielded as a worldwide GBAS for the GNSS and be interoperable with the DOD JPALS. The LAAS is based on a DGPS and includes one or more APLs at select installations where availability augmentation is required.

This dissertation documents the addition of a ranging APL into a prototype LAAS to assess the feasibility of ranging APLs in LAAS. Prior to this work, no ranging APL has been integrated into a prototype LAAS. This effort successfully integrated a ranging APL in a real-time fashion into a prototype LAAS and demonstrated that an increase in availability can be achieved. Data collected on the ground show that APL bias errors are less than 0.1 meters, and that the ground APL pseudorange measurement errors are approximately 0.25 meters (one $\sigma$). Thus, it is concluded that the APL feasibility of accuracy has been demonstrated. Real-time DGPS/DAPL position integration without accuracy degradation beyond precision approach limits has increased the availability of the prototype LAAS.

For this effort, the APL requirements for receiver integration, frequency allocation, APL tracking performance, and EMI with nominal GPS performance were considered. This led to a prototype APL transmitting and receiving subsystem with a C/A code format that could operate within the $L1 \pm 10.0$ MHz band. This APL test tool
was very useful in isolating the two unique major error components that exist for the APL link. First, the ground multipath from the APL transmitting antenna site to the DGPS GS reception antenna site, which is more severe than that for the nominal DGPS operations, was successfully limited. To limit this ground multipath for the APL geometry, a MLA was designed, fabricated, and tested within a 4-month period. The implementation of this MLA concept was a first for APL applications and also contributed to the successful multipath limiting of ground multipath at the DGPS LAAS GS. The second major error component investigated with this APL test tool was the large power level variation the APL receiver observed over a typical mission profile, which is much greater than for nominal GPS reception. This sets the power level of the APL transmitter which affects the requirement for the APL not to produce EMI to nominal GPS performance. Data indicates that this power level variation can be managed effectively by the addition of a RF AGC loop.

Investigation of the two major error components for the APL link, led to a phase developmental approach. Phase I concentrated on an APL operating at a center frequency off-L1 in a continuous wave fashion where the MLA was implemented to minimize ground multipath for APL applications, and an additional RF AGC loop was implemented to control the APL signal power such that optimum APL tracking performance could be achieved. For Phase II, an APL system architecture was used based on a C/A code operating on-L1, pulsed at a 27.8 % duty cycle, transmitted with a MLA where APL signal gain control and blanking was incorporated in the GPS/APL receiving path. This is the first time this type of technique has been designed, built,
integrated, tested, and successfully demonstrated to maintain linear APL tracking (to optimize APL tracking performance) and minimize EMI with nominal DGPS performance.

For this effort a total of 11 flight tests with three test aircraft (Piper Saratoga, FAA Boeing 727, and Ohio University DC-3) and 14 distinct laboratory tests were conducted to produce the APL Subsystem Architecture, data, and system performance documented in this dissertation. This APL subsystem was fully integrated with a prototype LAAS and demonstrated with the Ohio University DC-3.
REFERENCES


APPENDIX A: Position Solution with Linearization and Associated Statistics

With more than four observables, an enhanced or “overdetermined” position solution can be obtained to reduce errors [1-A]. For N observables where N > 4, the SV index i will be constrained by 1 ≤ i ≤ N, and the subscript n will represent a particular SV. The non-linear equations of (2.1) can be solved simultaneously in a number of ways. One approach is linearization using the Taylor Series expansion followed by an iterative least-squares position and time solution. The linearization process begins with writing the measured pseudorange in the absence of errors.

\[ \rho_i(x) = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2 + c_t u} \]  

(A-1)

Next, the user state, \( \mathbf{x} = [x_u, y_u, z_u, t_u]^T \), can be expressed as the estimated state, \( \hat{\mathbf{x}} = [\hat{x}_u, \hat{y}_u, \hat{z}_u, \hat{t}_u]^T \) plus an offset term \( \Delta \mathbf{x} = [\Delta x_u, \Delta y_u, \Delta z_u, \Delta t_u]^T \).

\[ \mathbf{x} = \begin{bmatrix} x_u \\ y_u \\ z_u \\ t_u \end{bmatrix} = \begin{bmatrix} \hat{x}_u + \Delta x_u \\ \hat{y}_u + \Delta y_u \\ \hat{z}_u + \Delta z_u \\ \hat{t}_u + \Delta t_u \end{bmatrix} = \hat{\mathbf{x}} + \Delta \mathbf{x} \quad (A-2) \]

The Taylor Series expansion of \( \rho_i(x) \) is given by equation (A-3) where the measured pseudorange is expanded about the estimated user state and higher-order non-linear terms are ignored.

\[ \rho_i(x) \equiv \rho_i(\hat{x}) + \frac{\partial \rho_i(x)}{\partial x_u} \bigg|_{\hat{x}} \Delta x_u + \frac{\partial \rho_i(x)}{\partial y_u} \bigg|_{\hat{x}} \Delta y_u + \frac{\partial \rho_i(x)}{\partial z_u} \bigg|_{\hat{x}} \Delta z_u + \frac{\partial \rho_i(x)}{\partial t_u} \bigg|_{\hat{x}} \Delta t_u \]  

(A-3)
The partial differentiations are performed to establish the following relationships:

\[
\frac{\partial \rho_i(x)}{\partial x_u} \bigg|_{\hat{x}} = -\frac{x_i - \hat{x}_u}{\hat{r}_i}
\]

\[
\frac{\partial \rho_i(x)}{\partial y_u} \bigg|_{\hat{x}} = -\frac{y_i - \hat{y}_u}{\hat{r}_i}
\]

\[
\frac{\partial \rho_i(x)}{\partial z_u} \bigg|_{\hat{x}} = -\frac{z_i - \hat{z}_u}{\hat{r}_i}
\]

\[
\frac{\partial \rho_i(x)}{\partial t_u} \bigg|_{\hat{x}} = c
\]

(A-4)

where:

\[
\hat{r}_i = \sqrt{(x_i - \hat{x}_u)^2 + (y_i - \hat{y}_u)^2 + (z_i - \hat{z}_u)^2}
\]

When the equations of (A-4) are substituted back into (A-3) the unknown elements of \( \Delta \mathbf{x} \) can be written in terms of all the known parameters (measured and estimated pseudoranges) as follows.

\[
\rho_i(\mathbf{x}) = \rho_i(\hat{\mathbf{x}}) - \left[ \frac{x_i - \hat{x}_u}{\hat{r}_i} \Delta x_u + \frac{y_i - \hat{y}_u}{\hat{r}_i} \Delta y_u + \frac{z_i - \hat{z}_u}{\hat{r}_i} \Delta z_u \right] + c\Delta t_u
\]

(A-5)

For the least-squares solution, the differences in the measured pseudoranges and estimated pseudoranges can be expressed from (A-5), as follows [1-A].

\[
\Delta \rho_i = \rho_i(\hat{\mathbf{x}}) - \rho_i(\mathbf{x}) = \left[ g_{x_i} \Delta x_u + g_{y_i} \Delta y_u + g_{z_i} \Delta z_u \right] - c\Delta t_u
\]

(A-6)

where:
Thus the term $g_{xi}$ is the difference between the $i^{th}$ SV $x_i$ position and the estimated user $x_u$ position scaled by the estimated range between the user and the $i^{th}$ SV. Similar interpretations can be made for the $y$ and $z$ coordinates, such that the elements of the unit vector $\hat{g}_i = (g_{xi}, g_{yi}, g_{zi})$ represent the direction cosines from the estimated user position to the $i^{th}$ SV position. The pseudorange differences, the direction cosines, and position differences can be expressed in matrix notation:

$$\Delta \rho = \begin{bmatrix} \Delta \rho_1 \\ \Delta \rho_2 \\ \vdots \\ \Delta \rho_n \\ \Delta \rho_N \end{bmatrix} = \begin{bmatrix} g_{x1} & g_{y1} & g_{z1} & -1 \\ g_{x2} & g_{y2} & g_{z2} & -1 \\ \vdots & \vdots & \vdots & \vdots \\ g_{xn} & g_{yn} & g_{zn} & -1 \\ g_{xN} & g_{yN} & g_{zN} & -1 \end{bmatrix} \begin{bmatrix} \Delta x_u \\ \Delta y_u \\ \Delta z_u \\ c \Delta t_u \end{bmatrix} = G \Delta x$$

(A-7)

where the clock bias term is written in terms of meters. With the $g_i$'s placed in matrix form, it is clear that the $G$ matrix provides information on the geometric relationship between changes in the GPS SV positions and changes in the estimated user state. The $G$ matrix provides a direct relationship between the pseudorange differences and the position and clock differences based on the GPS SV geometry. ($G$ will be a function of time but a point-by-point solution is assumed such that when the state estimate is updated so will the $G$ matrix.) With $N = 4$, the solution for the unknown user state
difference $\Delta x$ is simply $\Delta x = G^{-1} \Delta \rho$, as long as $G$ is nonsingular. For $N > 4$, the goal will be to minimize the error in $\Delta x$. To do this, a solution residual vector ($r$) is defined such as:

$$r = G\Delta x - \Delta \rho$$  \hspace{1cm} (A-8)

The ordinary least squares solution is the particular solution that minimizes the sum of the elements of the solution residual vector squared. Therefore, the following expression is minimized.

$$r^2 = (G\Delta x - \Delta \rho)^2$$  \hspace{1cm} (A-9)

To minimize this expression, the gradient is taken with respect to $\Delta x$ and set equal to zero; the maximum could also be obtained but it can be shown that this is not the solution we seek [2-A]. Expanding (A-9), taking the gradient, and setting it equal to zero, leads to the following:

$$r^2 = (G\Delta x - \Delta \rho)^2 = (\Delta x)^T G^T G \Delta x - 2(\Delta x)^T G^T \Delta \rho + (\Delta \rho)^2$$  \hspace{1cm} (A-10)

$$\nabla \{r^2\} = \nabla \{(\Delta x)^T G^T G \Delta x\} - \nabla \{2(\Delta x)^T G^T \Delta \rho\} + 0 = 0$$  \hspace{1cm} (A-11)

$$\nabla \{r^2\} = 2(\Delta x)^T G^T G - 2(\Delta \rho)^T G = 0$$  \hspace{1cm} (A-12)

The solution of the unknown user state differences $\Delta x$ is what we seek so the transpose of both side of (A-12) is taken, and then solved for $\Delta x$, as long as $G^T G$ is nonsingular, and the pseudorange error is assumed to have a zero mean. The solution of $\Delta x$ is:

$$\Delta x = (G^T G)^{-1} G^T \Delta \rho$$  \hspace{1cm} (A-13)
Once these differences in the estimated user state have been determined, a new estimate of the users state can be computed:

\[ \hat{x}_k = \hat{x}_{k-1} + \Delta x \]  

(A-14)

where \( k \) is an integer index of update. This process is iterated until desired convergence of the user state is obtained.

Statistics of the user state error can now be determined from the statistics of the pseudorange errors. The pseudoranges contain several error sources, including multipath, receiver noise, and propagation delays. Also, at some point in the GPS position solution, a coordinate transformation usually occurs and produces the user position in the desired coordinate frame. For the purpose of position and pseudorange error statistics, an East, North, Up (ENU) coordinate frame will be used. The state error statistics will have more meaning in an ENU coordinate frame compared to an ECEF coordinate frame. Assuming that the error sources on the pseudorange measurements are jointly independent-identically-distributed (iid) Gaussian random variables with zero mean, then the state errors will also be jointly iid Gaussian random variables with zero mean since \((G^T G)^{-1} G^T\) is linear, see equation (A-13). (If the variances are not equal, an additional weighting matrix can be introduced.) The covariance of the user state and the covariance of the pseudorange errors can be calculated, by taking the covariance of both sides of equation (A-13), [4-A].

\[
\text{cov}(x_{\text{ENU}}) = \mathbb{E}\left\{ (\Delta x_{\text{ENU}})(\Delta x_{\text{ENU}})^T \right\} = \mathbb{E}\left\{ (G^T G)^{-1} G^T \Delta \rho (G^T G)^{-1} G^T \Delta \rho)^T \right\}
\]
where:

\( x_{ENU} \) = state vector expressed in ENU coordinates

or,

\[
\text{cov}(x_{ENU}) = \mathbb{E}\left\{(G^T G)^{-1} G^T \Delta \rho (\Delta \rho)^T G (G^T G)^{-1}\right\}
\]

\[
= (G^T G)^{-1} G^T \mathbb{E}\{\Delta \rho (\Delta \rho)^T\} G (G^T G)^{-1}
\]

where the covariance of the pseudoranges can be identified,

\[
\text{cov}(x_{ENU}) = (G^T G)^{-1} G^T \text{cov}(\rho) G (G^T G)^{-1}
\]  \hspace{1cm} (A-15)

With the assumption of jointly iid Gaussian pseudorange measurements, the pseudorange covariance matrix will only have diagonal components representing the errors of each pseudorange component \([4-A]\). Thus the covariance matrix of the pseudoranges can be represented as a diagonal identity matrix multiplied by the variance of the noise as:

\[
\text{cov}(\rho) = I \sigma^2_p
\]  \hspace{1cm} (A-16)

Substituting equation (A-16) into (A-15), simplifying, and then expanding the \((G^T G)^{-1}\) term into its positional error components leads to:

\[
\text{cov}(x_{ENU}) = (G^T G)^{-1} G^T \sigma^2_p G (G^T G)^{-1} = \sigma^2_p (G^T G)^{-1}
\]  \hspace{1cm} (A-17)

\[
\text{cov}(x_{ENU}) = \begin{bmatrix}
\text{var(E)} & \text{cov(EN)} & \text{cov(EU)} & \text{cov(Ect)} \\
\text{cov(NE)} & \text{var(N)} & \text{cov(NU)} & \text{cov(Nct)} \\
\text{cov(UE)} & \text{cov(UN)} & \text{var(U)} & \text{cov(Uct)} \\
\text{cov(CE)} & \text{cov(CT)} & \text{cov(UCT)} & \text{var(CT)}
\end{bmatrix}
\]  \hspace{1cm} (A-18)
which can be expressed as:

\[
\text{cov}(\mathbf{x}_{\text{ENU}}) = \begin{bmatrix}
\sigma_E^2 & . & . & . \\
. & \sigma_N^2 & \text{cross terms} & . \\
. & \text{cross terms} & \sigma_U^2 & . \\
. & . & . & \sigma_{ct}^2
\end{bmatrix}
\] (A-19)

For a particular \( \mathbf{G} \) matrix, the dilution of precision (DOP) parameters are defined as ratios of the standard deviations of components from the position covariance matrix in the numerator and the standard deviation of the pseudorange error (\( \sigma_p \)) in the denominator. The matrix relationship between the pseudorange errors and the state covariance is given by:

\[
\text{cov}(\mathbf{x}_{\text{ENU}}) = \sigma_p^2 (\mathbf{G}^T \mathbf{G})^{-1} = \sigma_p^2 \mathbf{D}
\] (A-20)

where:

\[
\mathbf{D} = (\mathbf{G}^T \mathbf{G})^{-1} = \begin{bmatrix}
d_{EE} & d_{EN} & d_{EU} & d_{Ect} \\
d_{NE} & d_{NN} & d_{NU} & d_{Nct} \\
d_{UE} & d_{UN} & d_{UU} & d_{Uct} \\
d_{cE} & d_{cN} & d_{cU} & d_{ct}
\end{bmatrix}
\] (A-21)

The DOP parameters are defined for the specific coordinate variable(s) of interest as:

\[
\text{GDOP} = \sqrt{\text{Tr}(\mathbf{D})} = \sqrt{d_{EE} + d_{NN} + d_{UU} + d_{cct}} = \text{Geometrical DOP}
\] (A-22)

\[
\text{PDOP} = \sqrt{d_{EE} + d_{NN} + d_{UU}} = \text{Positional DOP}
\] (A-23)

\[
\text{HDOP} = \sqrt{d_{EE} + d_{NN}} = \text{Horizontal DOP}
\] (A-24)

\[
\text{VDOP} = \sqrt{d_{UU}} = \text{Vertical DOP}
\] (A-25)

\[
\text{TDOP} = \sqrt{d_{cct}} = \text{Time DOP}
\] (A-26)
The GDOP, PDOP, HDOP, VDOP, and TDOP provide a measure of the dilution from the pseudoranges domain to the solution domain for the coordinate variables of interest. These DOP equations relate the pseudorange errors in the pseudorange domain to the state variable errors in the position domain. With $N > 4$ the increased observables allow for better geometric trilateration from the GPS SVs in the sky. Poor dispersion of GPS SVs in the sky provide for high DOP. Good dispersion of the GPS SVs in the sky provide for a low DOP.

References for Appendix A


APPENDIX B: Expected Values and Variances of the B-Values

To calculate the expected values and variances of the B-values, equation (4.13) is rewritten using equations (4.10) and (4.11), [1-B]:

\[
B^n_m = \frac{1}{M - 1} \left\{ \left\{ SA^n + t^n + iono^n + tropo^n + \varepsilon^n \right\} + \left\{ n^n_m \right\} + \Delta t_m \right\} \\
- \frac{1}{M - 1} \left\{ \left\{ SA^n + t^n + iono^n + tropo^n + \varepsilon^n \right\} + \left\{ \frac{1}{M} \sum_{j=1}^{M} n^n_j \right\} + \frac{1}{M} \sum_{j=1}^{M} \Delta t_j \right\} \\
= \frac{1}{M - 1} \left\{ \left\{ n^n_m - \frac{1}{M} \sum_{j=1}^{M} n^n_j \right\} + \left\{ \Delta t_m - \frac{1}{M} \sum_{j=1}^{M} \Delta t_j \right\} \right\} \\
= \frac{1}{M - 1} \left\{ \left\{ \sum_{j=m}^{M} n^n_m \left( \frac{M - 1}{M} \right) - \frac{1}{M} \sum_{j=m}^{M} n^n_j \right\} + \left\{ \Delta t_m - \frac{1}{M} \sum_{j=m}^{M} \Delta t_j \right\} \right\} \\
= \left\{ \frac{n^n_m}{M} - \frac{1}{M(M - 1)} \sum_{j=m}^{M} n^n_j \right\} + \left\{ \frac{\Delta t_m}{M} - \frac{1}{M(M - 1)} \sum_{j=m}^{M} \Delta t_j \right\} \quad \text{(B-1)}
\]

Next, the clock bias term is rewritten using equations (4.9) and (4.10):

\[
\Delta t_m = -\frac{1}{N} \sum_{i=1}^{N} \left\{ SA^i + t^i + iono^i + tropo^i + \varepsilon^i \right\} - \frac{1}{N} \sum_{i=1}^{N} n^i_m = -\Delta T - \frac{1}{N} \sum_{i=1}^{N} n^i_m \quad \text{(B-2)}
\]

In equation (B-2) it is important to recognize that \( \Delta T \) is common between all RRs. Substituting equation (B-2) into (B-1) yields:
The cancellation of the $\Delta T$ terms results in the B-value expressed in terms of just the noise components. Statistics on the B-values can now be calculated. Earlier, the noise component $n_m^n$ was defined to include receiver noise and multipath. Under the assumption that the error sources comprising $n_m^n$ are jointly iid Gaussian with zero mean, then the B-values should also be jointly iid Gaussian random variables with zero mean, since there exists a linear relationship between the B-values and the differential correction errors, see equation (4.13). Hence, the expected values of the B-values are equal to zero as shown below.

\[
\begin{align*}
E[B_m^n] &= E \left[ \left( \frac{n_m^n}{M} - \frac{1}{M(M-1)} \sum_{j \neq m} n_j^n \right) + \left( -\Delta T - \frac{1}{N} \sum_{i=1}^{N} n_i^n \right) \right] \\
&= \left( \frac{n_m^n}{M} - \frac{1}{M(M-1)} \sum_{j \neq m} n_j^n \right) + \left( -\Delta T + \frac{1}{M} \sum_{j \neq m} \Delta T \right) + \frac{1}{MN(M-1)} \sum_{j \neq m} \sum_{i=1}^{N} n_i^n \\
&= \left( \frac{n_m^n}{M} - \frac{1}{M(M-1)} \sum_{j \neq m} n_j^n \right) + \left( -\frac{1}{MN} \sum_{i=1}^{N} n_i^n \right) + \frac{1}{MN(M-1)} \sum_{j \neq m} \sum_{i=1}^{N} n_i^n \\
&= \left( \frac{n_m^n}{M} - \frac{1}{M(M-1)} \sum_{j \neq m} n_j^n \right) + \left( -\frac{1}{MN} \sum_{i=1}^{N} n_i^n \right) + \frac{1}{MN(M-1)} \sum_{j \neq m} \sum_{i=1}^{N} n_i^n \\
&= E \left[ \frac{n_m^n}{M} \right] + E \left[ -\frac{1}{M(M-1)} \sum_{j \neq m} n_j^n \right] + E \left[ -\frac{1}{MN} \sum_{i=1}^{N} n_i^n \right] + E \left[ \frac{1}{MN(M-1)} \sum_{j \neq m} \sum_{i=1}^{N} n_i^n \right] = 0
\end{align*}
\]
The variance of the B-value can be calculated using equations (B-3) and (B-4) as follows:

\[
\text{VAR}[B_m^2] = E[(B_m^2)^2] - (E[B_m^2])^2 = E[(B_m^2)^2] - 0 = E[(B_m^2)^2]
\]

\[
= E \left[ \left\{ \left( \frac{n_m^a}{M} - \frac{1}{M(M-1)} \sum_{j=1}^{M} n_j^a \right)^2 \right\} + \left\{ - \frac{1}{MN} \sum_{j=1}^{N} n_j^a \right\} \right] \left[ \left\{ \frac{1}{M(M-1)} \sum_{j=1}^{M} n_j^a \right\} + \left( \frac{1}{MN} \sum_{j=1}^{N} n_j^a \right) \right]^2
\]

(B-5)

Since the error sources comprising \( n_m^a \) are jointly iid Gaussian random variables, all the cross components will equal zero and the individual terms of equation (B-5) can be separated as follows:

\[
\text{VAR}[B_m^2] = E \left[ \left( \frac{n_m^a}{M} \right)^2 \right] + E \left[ \left( \frac{-1}{M(M-1)} \sum_{j=1}^{M} n_j^a \right)^2 \right] + E \left[ \left( \frac{-1}{MN} \sum_{j=1}^{N} n_j^a \right)^2 \right] + E \left[ \left( \frac{1}{MN(M-1)} \sum_{j=1}^{M} n_j^a \right)^2 \right]
\]

\[
= \left( \frac{1}{M^2} \right) E\left[ (n_m^a)^2 \right] + \left( \frac{(M-1)}{M^2(M-1)^2} \right) E\left[ (n_m^a)^2 \right] + \left( \frac{N}{M^2 N^2} \right) E\left[ (n_m^a)^2 \right] + \left( \frac{(M-1)N}{M^2 N^2(M-1)} \right) E\left[ (n_m^a)^2 \right]
\]

\[
= E\left[ (n_m^a)^2 \right] \left( \frac{(M-1)N + N + (M-1) + 1}{M^2 N(M-1)} \right) = E\left[ (n_m^a)^2 \right] \left( \frac{M(N+1)}{M^2 N(M-1)} \right) = E\left[ (n_m^a)^2 \right] \left( \frac{(N+1)}{MN(M-1)} \right)
\]

(B-6)

Reference for Appendix B

ABSTRACT

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RANGING AIRPORT PSEUDOLITE FOR LOCAL AREA AUGMENTATION

USING THE GLOBAL POSITIONING SYSTEM (111 pp.)

Director of Dissertation: Dr. Frank van Graas

The Local Area Augmentation System (LAAS) is being developed to support precision approach and landing operations in and about the local area surrounding an airport. The LAAS Program is currently under development by the Federal Aviation Administration (FAA) with Minimum Aviation System Performance Standards for the LAAS being developed by RTCA, Incorporated. The LAAS uses differential Global Positioning System (DGPS) and includes one or more airport pseudolites (APL) to increase the availability for certain installations.

This dissertation addresses the addition of a differentially corrected, ranging APL into a LAAS. Prior to this work, no ranging APL has been integrated into a prototype LAAS and demonstrated in a real-time flight environment showing that an increase in LAAS availability is feasible.

The APL requirements resulted in a prototype APL transmitting and receiving subsystem with a coarse-acquisition (C/A) code format that could be operated at any frequency within the L1 ± 10.0 MHz band.

To investigate the major APL error the developmental approach was performed in two phases. Phase I implemented an APL operating at a center frequency off-L1 and