MULTIPATH ERRORS INDUCED BY ELECTRONIC COMPONENTS IN RECEIVER HARDWARE

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1. Introduction

The satellite-based Global Positioning System (GPS) has been demonstrated to be a navigation system capable of meeting the high accuracy demands of many applications. High accuracy techniques such as carrier phase differential are capable of removing many of the error sources found in stand-alone receiver solutions, through the use of a reference receiver in a known location. Multipath errors still remain as a major error source for high precision applications. Multipath is the result of a signal arriving at a receiver via multiple paths due to reflection or diffraction. Multipath distorts the signal modulation and degrades the accuracy of the system. Typical sources of multipath include large reflective surfaces such as buildings, airframes, lakes, and the ground.

Electronic components used to condition GPS signals can also be a source of reflections. Impedance mismatch between components causes reflections within a circuit. These reflections could result in multipath tracking errors, which can be both similar and dissimilar for different satellites. As a result, an examination of the impact of hardware induced multipath is required. Specifically, this thesis discusses the effects of multipath on code tracking receivers by examining the impact of multipath on the coherent Delay Lock Loop (DLL) discriminator function. This thesis also briefly touches on the theory behind impedance mismatch and reflections created within electronic circuits. This leads into a discussion of hardware-induced multipath, whose characteristics are determined by the difference in frequency of the input signals as well as the overall delay of the reflected signals with respect to the direct signals.
Measurement of the delay and amplitude of the multipath signal is required to calculate
the magnitude of the multipath error. However, the nature of most electronic devices
makes it difficult to determine the amplitude and the path traveled by the reflected
signals. Fortunately, bounding of the multipath tracking error is feasible through
indirect measurements and the modeling of various aspects of the input signals such as
difference in input frequencies, amplitude of the reflected signals and the delay of the
reflected signals. This thesis explores the process of bounding hardware-induced
multipath errors through two case studies that examine the impact of frequency
separation between two input signals.

Finally, this thesis discusses data collected in a laboratory environment using live
satellites. The laboratory data is compared against the theoretical data presented earlier
in the thesis.
2. Multipath Error

2.1. Background

A GPS receiver can receive multiple copies of a signal broadcast by a GPS satellite. These copies are created when the signal reflects or diffracts off surrounding structures and terrain. These copies are referred to as multipath signals since the signals arrive at the receiver via multiple paths. A pseudorandom noise (PRN) ranging receiver that is subjected to multipath may incorrectly calculate a position solution. This problem has been studied for over 25 years with the basic relationship between PRN code tracking errors and multipath parameters derived by Hagerman in the early 1970’s (Hagerman, 1973).

Multipath is a localized phenomenon, as the sources of multipath signals for one receiver do not necessarily serve as sources of reflected signals for other receivers. For this reason, advanced, high accuracy techniques such as differential GPS do not necessarily remove multipath errors. Multipath distorts the signal modulation resulting in degraded accuracy in both stand-alone and differential systems. Multipath represents the dominant error source in satellite-based precision guidance systems (Braasch, 1994-95).

For the analysis of multipath error, the composite received GPS signal can be expressed as:

\[ s(t) = p(t)\sin(\omega_o t) + ap(t - \delta)\sin(\omega_o t + \theta_m) \]  

(1)
where

\[ p(t) = \text{pseudorandom noise (PRN) code} \]
\[ \alpha = \text{ratio of the multipath signal strength to the direct signal strength} \]
\[ \omega_o = \text{carrier frequency including Doppler shift} \]
\[ \delta = \text{delay of the multipath signal relative to the direct signal} \]
\[ \theta_m = \text{phase of the multipath signal relative to the direct signal} \]

In general, \( \alpha, \omega_o, \delta, \theta_m \) are time varying quantities. The PRN code can either be the 1.023 Mchips per second Course-Acquisition (C/A) code or the 10.23 Mchips per second Precision (P) code. This thesis only addresses the performance of the C/A code. All other sources of errors such as those introduced by the ionosphere and troposphere have been ignored in this equation. The 50 Hz data rate has been omitted, as it does not impact multipath performance (Braasch, 1997).

In normal operation, the GPS signal spectral density is well below the thermal white noise level of the receiver (Spilker, 1996). The performance of the receiver is dependent upon its ability to pull the signal out of the noise and accurately track the PRN code sequence. A delay-lock loop (DLL) receiver multiplies the received PRN code sequence by a reference PRN code. The multiplier output is filtered with a low pass filter. The result is one point on the autocorrelation function (Spilker, 1980).

2.2. Autocorrelation Function

Autocorrelation is the process of matching a signal with a delayed version of itself. The autocorrelation function of an infinite length truly random code is given by (Sklar, 1988)
\[ R(\tau) = \frac{1-|\tau|}{T} \quad \text{for } |\tau| \leq T \]
\[ R(\tau) = 0 \quad \text{for } |\tau| > T \quad (2) \]

The variable T is the pulse duration and \( \tau \) is the time lag between the two signals. The autocorrelation function of a satellite’s PRN code can be approximated by equation (2). For this approximation T is 1 chip and \( \tau \) is the time lag between the locally generated reference PRN code and the PRN code received from the GPS satellite. Since the GPS codes are not truly random, the autocorrelation function has sidelobes. The approximation for the autocorrelation function made in equation (2) assumes the correlation sidelobes to be zero. In addition, this approximation assumes an infinite bandwidth. The finite bandwidths of band pass filters (BPFs) used in receivers distort the shape of the PRN code chips and results in a smoothing of the correlation function. The smoothing slightly reduces the maximum range error due to multipath. Therefore, use of the approximated correlation function yields slightly conservative results (Braasch, 1995). The approximated autocorrelation function is shown in figure 1.

Since the received GPS signal power (approximately -160 dBW) is below the noise floor, the PRN code received from the satellite must be extracted by correlating it with a copy of the PRN code generated by the receiver. The received PRN code is tracked by advancing or delaying the locally-generated code sequence until maximum correlation is obtained. The correlation function is symmetrical about the zero axis which results in the same correlation value when the locally-generated code is advanced or delayed by the same amount. (Braasch, 1992).
Figure 1. The autocorrelation function.

2.3. Discriminator Function

The delay lock loop (DLL) solves the tracking ambiguity problem due to the symmetrical correlation function. Three versions of the PRN code sequence are generated by the receiver. One copy is advanced by a certain amount, one is delayed by the same amount and the third is “on-time”. The early and the late versions of the autocorrelation functions are subtracted, yielding the discriminator function. This function changes sign depending on whether the on-time code needs to be advanced or delayed. The discriminator function with an early-late separation of one chip is shown in figure 2.
The equation for the coherent DLL discriminator function in the presence of multipath is (Braasch, 1996):

\[ D(\tau) = \left[ R\left(\tau + \frac{d}{2}\right) - R\left(\tau - \frac{d}{2}\right) \right] \cos(\theta_c) + \alpha \left[ R\left(\tau + \frac{d}{2} - \delta\right) - R\left(\tau - \frac{d}{2} - \delta\right) \right] \cos(\theta_m - \theta_c) \]  

(3)

where \( d \) is the early-late correlator spacing. \( \theta_c \) is the carrier-tracking loop estimate of the phase of the received carrier and is given by (Braasch, 1996):

\[ \theta_c = \arctan\left(\frac{\alpha R(\tau - \delta) \sin(\theta_m)}{R(\tau) + \alpha R(\tau - \delta) \cos(\theta_m)}\right) \]  

(4)

2.4. Multipath Tracking Error

The multipath tracking error for the coherent DLL is determined by setting equation (3) to zero and solving for \( \tau \). Multipath distorts the discriminator function such that the discriminator function does not cross the zero axis at \( \tau = 0 \). The distortion is a
function of the amplitude of the multipath signal \((\alpha)\) with respect to the direct signal, delay of the signal \((\delta)\) with respect to the direct signal, as well as the phase of the multipath signal \((\theta_m)\) with respect to the direct signal. Figures 3 through 7 demonstrate the impact of the delay and phase of multipath on the discriminator function. As shown in figure 3, if the amplitude of the multipath signal is held constant and the delay is set to zero, changes in the multipath phase affects the amplitude of the discriminator function. However, the discriminator function crosses the zero axis at \(\tau = 0\) for every shift in phase. There is no multipath tracking error introduced in this case.

![Figure 3](image1.png)

Figure 3. The discriminator function in the presence of multipath. Relative multipath amplitude is -4.4 dB, multipath delay is set at zero chips, multipath phase is varied from 0 to 180 degrees.
In figure 4, the amplitude of the multipath signal is held constant and the multipath phase is set to zero while the multipath delay is varied. The distortion caused by the multipath delay results in the discriminator function crossing through zero at non-zero values of $\tau$. The magnitude at which the discriminator function crosses the zero axis is the multipath tracking error. Note that the zero crossing of the discriminator function does not continually increase as the delay is increased. In fact, for this set of multipath parameters the maximum tracking error occurs with a delay of 0.9 chips. When the delay is further increased to 1.2 chips, the multipath tracking error decreases to near zero.

Figure 4. The discriminator function in the presence of multipath. Relative multipath amplitude is -4.4 dB, multipath phase is set at zero degrees, multipath delay is varied.
Combining the effects of multipath delay and phase yields the plots shown in figure 5. The same data used to generate the plots in figure 4 is used again in figure 5 with the exception of multipath phase. By setting the phase equal to 180 degrees, the zero crossings all have negative values. Again, notice that the magnitude of the multipath tracking error does not necessarily increase with additional multipath delay.

Figure 5. The discriminator function in the presence of multipath. Relative multipath amplitude is -4.4 dB, multipath phase is held at 180 degrees, multipath delay is varied.

As a final example of a multipath signal distortion of the discriminator function, a multipath signal with an arbitrary delay of 0.9 chips was selected with the multipath phase varying from 0 to 180 degrees. The resulting curves are plotted in figures 6 and 7 for different values of $\alpha$. In these figures, the multipath tracking error is both positive and negative with a phase shift of 90 degrees yielding a very small multipath tracking error.
error. The impact of changing the magnitude of the multipath signal can be seen in the magnitude of the multipath tracking error. The stronger of the two signals produces larger multipath tracking errors.

Figure 6. The discriminator function in the presence of multipath. Relative multipath amplitude is -4.4 dB, multipath delay is 0.9 chips, varied multipath phase.
Figure 7. The discriminator function in the presence of multipath. Relative multipath amplitude is -10 dB, multipath delay is 0.9 chips, varied multipath phase.

2.5. Multipath Tracking Error Curves

As shown in figures 3 through 7, the multipath tracking error can vary widely in both magnitude and sign as a function of magnitude, delay, and phase of the multipath signal. Figures 6 and 7 show that maximum error occurs when the multipath signal is either in phase or completely out of phase with respect to the direct signal. This property of multipath error results in the error envelope shown in figure 8. The inner curve (dashed lines) represents the maximum multipath errors for an autocorrelation function with zero sidelobes (equation (2)) and infinite bandwidth. For this figure, the separation between the early and late correlators is one chip with the relative multipath amplitude equal to -20 dB. The outer curves represent the multipath error envelope for an autocorrelation
function with non-zero sidelobes. To generate the outer curves, non-zero sidelobe levels of $\Gamma = -1/1023$, $\Gamma_{\text{max}} = 63/1023$, and $\Gamma_{\text{min}} = -65/1023$ are used. These values represent the C/A code sequence with the largest correlation sidelobes (Braasch, 1997).

![Figure 8. Multipath error envelopes, zero and nonzero sidelobes.](image)

The phase of the multipath signal is determined by a large number of factors including delay of the reflected signal and the properties of the object that is the source of the reflected signal. Assume that the multipath phase is solely a function of delay:

$$\theta_m = \delta \cdot f \cdot 2\pi$$

(5)

where $\delta$ is the delay in seconds and $f$ is the carrier frequency (1575.42 MHz). Using this assumption for multipath phase, the multipath tracking error over a range of delays for a correlator spacing of 1 chip and relative multipath amplitude of $-20\,\text{dB}$ is shown in figure 9. Figure 10 shows the impact of increasing the multipath amplitude to $-4.4\,\text{dB}$. The amplitude of the reflected signal determines the shape of the multipath error curve.
Figure 9. C/A-code multipath error for relative multipath amplitude of −20 dB.

Figure 10. C/A-code multipath error for relative multipath amplitude of −4.4 dB.

The next chapter discusses how electronic devices can also cause reflections.
3. Impedance Mismatch

3.1. Terminology

Impedance mismatch between electronic components such as cables, amplifiers, filters, and mixers create reflections of a signal within a circuit. The impedance mismatch is characterized in terms of voltage reflection coefficient ($\Gamma$) and voltage standing wave ratio (VSWR). The voltage reflection coefficient is the ratio of the reflected voltage to incident voltage at the input of a device. The reflection coefficient can range from 0 for a perfect match (no reflection) to ±1 for a total mismatch (total reflection).

VSWR is a measure of how much power is reflected back from a device when an external signal is applied. VSWR can vary from 1 for a perfect match to infinity for a total mismatch. The higher the VSWR, the stronger the signal strength of the reflected signal. VSWR and the reflection coefficient are related by (Cheng, 1990):

$$|\Gamma| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1}$$

Figure 11 shows how components can be a potential source of multipath signals. Reflected signals may be produced at the input and output ports of one or more devices in a circuit. One or more of these reflected signals may eventually make it to the output of the circuit as a delayed and diminished copy of the direct signal. For the example system shown in figure 11, the reflection coefficients of devices 1 and 2 not only control the magnitude of the direct signal but the magnitude of the reflected signal as well.
3.2. Relative Multipath Amplitude

Figure 11 is used to analyze the amplitude of a multipath signal relative to the amplitude of the direct signal. For this analysis, the boundaries are assumed to be symmetric in terms of their reflection coefficients, and the signals are not attenuated when traveling between the two boundaries. Boundary 1 attenuates the direct and any multipath signals equally, such that the relative amplitude is not changed. For boundary 2, the amplitude of the reflected signal $V_1$ is the product of the input signal, $V_{in}$ and the reflection coefficient of the second device, $\Gamma_2$ (Marshall and Skitek, 1987):

$$V_1 = V_{in} \cdot \Gamma_2$$

(7)

The magnitude of the output signal is the difference of the input signal $V_{in}$ and the reflected signal $V_1$. This can be written in terms of the input signal:

$$V_o = V_{in} \cdot (1 - \Gamma_2)$$

(8)

The amplitude of the multipath signal can be similarly written in terms of $V_{in}$ and the reflection coefficients of devices 1 and 2, $\Gamma_1$ and $\Gamma_2$ respectively:

$$V_{mp} = V_{in} \cdot \Gamma_2 \Gamma_1 (1 - \Gamma_2)$$

(9)
The relative amplitude in decibels of the multipath signal can be written in terms of \( V_{mp} \) and \( V_o \):

\[
\alpha = 10 \log_{10} \left( \frac{V_{mp}^2}{V_o^2} \right)
\]  

Equation (10)

This can be reduced to contain only the reflection coefficients.

\[
\alpha = 20 \log_{10} \left( \Gamma_1 \Gamma_2 \right)
\]  

Equation (11)

Equation (11) can be rewritten in terms of the VSWR of the devices.

\[
\alpha = 20 \log_{10} \left( \frac{(VSWR_1 - 1)(VSWR_2 - 1)}{(VSWR_1 + 1)(VSWR_2 + 1)} \right)
\]  

Equation (12)

Many devices also attenuate the signal between the two boundaries, which reduces the relative multipath amplitude. The multipath signal travels the path between the two boundaries three times, versus only once for the direct signal. If the attenuation between the two boundaries is equal to \( \beta \) dB, then the relative multipath amplitude is further reduced by \( 2\beta \) dB.

If the VSWRs of the devices upstream of the correlators in a GPS receiver are high enough, the reflected signals created by these devices will have sufficient amplitude to impact the performance of the correlators. For this reason hardware is a potential source of multipath errors. The relative multipath amplitude of a reflected signal can be computed given the VSWRs of all the components in a system. Once determined, the amplitude can be applied to computing the multipath tracking error introduced by the reflected signal. However, the nature of multipath created within a system is significantly different than multipath created outside the receiver’s antenna. The
techniques used to calculate the multipath tracking error using relative multipath amplitude and delay do not necessarily apply to multipath created inside the receiver. The next section discusses what the differences are and how they affect the magnitude of the multipath error.
4. Hardware Multipath

4.1. Assumptions

With the knowledge that impedance mismatch can be a source of reflected signals and given that hardware typically does not have perfect impedance matches, one must conclude that multipath is not limited to sources outside the antenna. For multipath signals created outside the antenna, factors such as the electromagnetic properties of the reflective surface, location, and the incident angle of the direct signal influence the characteristics of the reflected signal. These characteristics include amplitude, phase, delay, and polarization that determine how harmful the multipath signal is to the discriminator function. Suffice it to say that multipath tracking errors caused by reflections outside of the antenna cannot be simply modeled by assuming that the multipath phase is a function of delay as stated in equation (5).

Inside the electronic circuit, however, the environment is significantly different. All GPS signals travel the same physical path and encounter the same reflection points. For this study, it is assumed that the characteristics of the electronic devices do not vary with frequency. All reflected signals will have the same relative multipath amplitude and all signals will experience the same phase and group delays. It is also assumed that the multipath phase can be modeled as a function of delay (equation (5)).

4.2. Common Multipath

Hardware creates two types of multipath signals: common and relative. Given multiple input signals to a circuit, common multipath signals are those reflected signals that
create the same or common multipath error on each input signal. If the input signals shown in figure 11 have the same frequency and the paths are identical for all the input signals, the signals that are reflected off common boundaries will travel the same path. As a result, all of the reflected signals will have the same amplitude, delay, and phase relative to their respective input signals. These reflections will distort the discriminator function for each input signal the same, creating a common multipath error. This common error will be attributed to the clock by the receiver and will be removed from the position solution.

Although the common multipath error does not affect the position solution, it does affect other receiver operations, including:

1. Timing receivers will calculate a clock error that is proportional to the common multipath error. Even a relatively small common multipath error of 1 m results in a timing error of approximately 3 nanoseconds, which is unacceptable for many time transfer operations.

2. Advanced receivers that perform Signal Quality Monitoring (SQM) to determine the quality of the shape of the correlation function (Akos, 2000) must take the common multipath error into account.

3. Receivers that process different codes at the same frequency, such as both the C/A and the P codes may observe different biases due to different multipath error envelopes (Braasch, 1992). This is of particular concern for applications where a mix of C/A and P codes is used.
4.3. Relative Multipath

Each input signal can have an associated multipath signal created by impedance mismatches. If the input signals are not at the same frequency, then the multipath errors are not necessarily the same even though the signals traveled the same physical path. For example, each input signal will experience a different Doppler frequency shift due to satellite velocity and the motion of the receiver antenna. As shown in figure 12, two signals differing in frequency contain a different number of wavelengths over a given time period. If the two signals represent reflected signals traveling identical paths within a circuit, it can be seen that the phases of the reflected signals will differ at the input of the correlator. This difference in phase impacts the discriminator function given by equations (3) and (4).

Figure 12. Phase delay introduced by two signals with different frequencies traveling common paths.
The phase of the multipath signal inside hardware is a function of the delay and frequency of the signal. A change in the frequency of the input signal will result in a change in the multipath tracking error curve. Examples of large frequency separation include off-frequency pseudolites and Russian GLONASS satellites that are separated by multiples of 0.5625 MHz (Kayton and Fried, 1997). Figures 13 and 14 show the multipath tracking error for a 10 MHz separation in frequency at two values of relative multipath amplitude. The two tracking errors create two distinct error curves giving arise to a non-zero relative multipath error. Smaller separations in frequency will also produce significant errors. As will be discussed in the next section, the frequency separation between input signals affect the magnitude of the relative multipath tracking error.

![Multipath Tracking Error Graph](image)

Figure 13. Multipath tracking error of two signals separated by 10 MHz. Relative multipath amplitude is -4.4 dB.
4.4. Relative Multipath Tracking Error

The relative multipath tracking error is the difference between the multipath tracking error of one satellite signal compared to another. The relative multipath tracking errors of two signals are calculated by subtracting the multipath tracking errors of each signal. The relative multipath error for two signals separated by 8 kHz is shown in figures 15 and 16 for two values of relative multipath amplitude. The plots show a potential source of error for high-accuracy position or timing receivers. The two multipath tracking errors curves for satellite signals separated by 10 MHz shown in figures 13 and 14 are subtracted and shown in figures 17 and 18. Designers of receivers with large frequency differences such as off frequency pseudolites need to be aware of this potential error source. Since we are examining multipath created within hardware and all input signals
encounter the same reflection points, we can assume the relative multipath amplitude ($\alpha$) of multipath signals created within the circuit are the same for both signals.

Figure 15. The relative multipath tracking error of two signals separated by 8 kHz. Relative multipath amplitude is -4.4 dB.

Figure 16. The relative multipath tracking error of two signals separated by 8 kHz. Relative multipath amplitude is -20 dB.
Figure 17. The relative multipath tracking error of two signals separated by 10 MHz. Relative multipath amplitude is -4.4 dB.

Figure 18. The relative multipath tracking error between two signals separated by 10 MHz. Relative multipath amplitude is -20 dB.
The results plotted in figures 15 through 18 show the amplitude of the relative multipath error is dependent upon the difference in the frequencies of the input signals, their relative multipath amplitudes, and the overall multipath delay with respect to the direct signal. The magnitude of the peaks of the relative multipath tracking error curve is shown to increase as the multipath delay increases. In fact, figures 19 and 20 show that the shape of the relative multipath tracking error curve is similar to the multipath tracking error curve shown in figure 8. The jagged edge in figure 19 is due to undersampling.

Figure 19. The relative multipath tracking error of two signals plotted over 2 chips. Input frequency separation is 8 kHz, relative multipath amplitude is -4.4 dB.
Figure 20. The relative multipath tracking error of two signals plotted over 2 chips. Input frequency separation is 8 kHz, relative multipath amplitude is -20 dB.

For comparison, the relative multipath tracking error curve for a correlator spacing of 0.1 chips is presented in figures 21 and 22.

Figure 21. The relative multipath tracking error of two input signals separated by 8 kHz. Relative multipath amplitude is -4.4 dB. Correlator spacing is 0.1 chips.
Figure 22. The relative multipath tracking error of two input signals separated by 8 kHz. Relative multipath amplitude is -20 dB. Correlator spacing is 0.1 chips.

Having explored the theoretical aspects of hardware multipath, the next section begins the discussion of the various characteristics of a circuit that affects the magnitude of the hardware induced multipath error and the methodology to determine the bounds of the error.
5. Bounding Relative Multipath Errors

5.1. Overview

The amplitude of the relative multipath error is dependent upon the difference in the frequencies of the input signals, their relative multipath amplitudes, and the overall multipath delay with respect to the direct signal. The first two dependencies, frequency difference and multipath amplitude can be measured or calculated from the characteristics of the devices within the circuit. However, the nature of some devices such as filters makes it very difficult to calculate the delay of reflected signals. It is possible, however, to bound the relative multipath tracking error based upon the shape of the multipath tracking error curve. Two cases will be examined to demonstrate the bounding of relative multipath errors. The first case investigates the relative multipath error created by a small difference in the input signal frequencies such as that due to Doppler shifts. The second case investigates bounding relative multipath error when the input signal frequencies are separated by a larger amount. Examples of large frequency separation include off-frequency pseudolites and Russian GLONASS satellites that are separated by multiples of 0.5625 MHz (Kayton and Fried, 1997).

The parameters that control the relative multipath tracking error are delay, multipath amplitude, frequency separation, and correlator width. It is assumed that delay is not measurable or easily calculated for some devices in a circuit. The correlator width is considered to be constant. The frequency separation of the input signals is assumed to be fixed at the worse-case value. That leaves the relative multipath amplitude as the only design parameter that can be varied. The relative multipath amplitude is controlled
through the selection of circuit devices. Devices can be selected with appropriate VSWRs to keep the relative multipath amplitude at or below a desired level.

Figures 15 through 18 show that as the multipath amplitude is changed, the peaks in the relative multipath tracking error curve shift slightly in delay. For example, in figure 15 there is a peak at a delay of 5.5e-10 chips when the multipath amplitude is -4.4 dB. Figure 16 shows that when the multipath amplitude is decreased to -20 dB, the peak shifts forward to approximately 5.4e-10 chips. The relationship between the relative multipath tracking errors plotted over a range of relative multipath amplitudes for a fixed delay is shown in figure 23. For the delay of 5.5e-3 chips, the relative multipath error exceeds .001 meters when the multipath amplitude rises above -4.4 dB.

Unfortunately, not every delay produces the well-defined curves as those shown in figure 23. The same plot is generated for a multipath delay of 0.6 chips in figure 24. Inspection of the relative multipath tracking curve reveals in figure 25 that a delay of 0.6 chips corresponds with a zero crossing.
Figure 23. Relative multipath tracking error vs. relative multipath amplitude where multipath delay $= 5.5\text{e{-3}}$ chips. Input signal frequency separation $= 8$ kHz.

Figure 24. Relative multipath tracking error vs. relative multipath amplitude where multipath delay $= 0.6$ chips. Input signal frequency separation $= 8$ kHz.
In bounding the relative multipath tracking error, the relative multipath amplitude is varied until the relative multipath tracking error is less than the desired error limit. This process is repeated over the desired range of multipath delays. To maximize the bounds, the values at the peaks (both max and min) of the relative multipath tracking error curve are used in the calculations.

5.2. Bounding Methodology

The method to determine the bounding of the relative multipath tracking error is as follows:

1. Set the frequency separation of the input signals to the desired value.
2. Select the desired limit for the relative multipath tracking error.
3. Set the initial multipath delay to 0 chips.
4. Choose an initial value for alpha.
5. Find the nearest peak in the relative multipath tracking error curve, whether it is positive or negative amplitude.

6. Increase or decrease alpha to keep the absolute value of that peak below the set limit.

7. Increment the multipath delay.

8. Go to step 4 until the entire range of the multipath tracking error curve has been covered.

Assuming the system under consideration is similar to the one depicted in figure 11, the multipath signal $V_{mp}$ is likely to be the multipath signal with the largest amplitude as it contains the fewest number of reflections. To simplify the analysis, the reflection coefficients ($\Gamma$) of the two boundaries that caused the reflections are assumed to be the same. Rewriting equation (11) in terms of relative multipath amplitude yields:

$$\Gamma = \sqrt{\frac{\alpha}{20}}$$

(13)

Once the reflection coefficient for the two devices has been determined, their VSWR can be calculated using equation (6) rewritten as:

$$VSWR = \left( \frac{-1-|\Gamma|}{|\Gamma|-1} \right)$$

(14)

5.3. Case Study I

The largest source of frequency shift between GPS signals is the Doppler shift created by the motion of the user and the GPS satellites. Using typical Doppler shifts, the L1 carrier frequency of the GPS signal ranges over 1575.42 MHz ±4 kHz for a stationary
receiver. The largest relative multipath tracking error due to Doppler shift occurs when the carrier frequencies are 8kHz apart.

For this first case study, the input signals are separated by 8 kHz, as is found with typical maximum Doppler shifts due to satellite motion. The desired limit for the relative multipath error is set to 1 mm and the correlator spacing is assumed to be 1 chip. The size of the relative multipath error is consistent with today’s high-quality receivers, which have inter-channel, or relative, tracking errors below 100 picoseconds (3 cm) (Miranian and Powers, 2000). Figure 26 shows that if the relative multipath error introduced by hardware is to be below 1 mm, the relative multipath amplitude will have to remain below −72 dB.

![Figure 26](image)

Figure 26. The relative multipath amplitude required to keep the relative multipath tracking error below 1 mm. Input frequency separation is 8 kHz.
Repeating the calculations with the relative multipath tracking error limit set to 1 cm yields the graph shown in 27. In order to keep the relative multipath tracking error below the set limit, the relative multipath amplitude must remain below $-55$ dB.

![Graph showing relative multipath amplitude vs. multipath delay in chips]

Figure 27. The relative multipath amplitude required to keep the relative multipath tracking error below 1 cm. Input frequency separation is 8 kHz.

Solving for VSWR using equations (13) and (14), the first example of a 1 mm error limit and a $-72$ dB value for $\alpha$ yields a required VSWR better than 1.032. For the second example where the relative multipath error should not exceed $-53$ dB in order to keep the relative multipath tracking error below 1 cm, the VSWR of the two devices needs to be better than 1.099. High-quality components typically have VSWRs ranging from 1.15 to 1.05 with the lowest VSWRs approaching 1.01. For this case study it is likely that components to keep the relative multipath tracking error below the 1-cm limit could be found but it may be difficult to find components for the 1-mm error limit. The example does represent a worst-case scenario, since in practice, the multipath signal will
experience additional attenuation due to the signal loss incurred while traveling between the two boundaries.

5.4. Case Study II

Larger frequency separations of the input signals are possible. They may be due to the utilization of an off-frequency pseudolite that implements a 10MHz offset from L₁ or the result of the GLONASS frequency division design. Since the relative multipath tracking error increases with the increased separation of input frequencies, the required VSWRs of the devices will need to improve.

Following the methodology established in the previous case study, the frequency separation of the input signals is assumed to be 10 MHz. The relative multipath amplitudes required to keep the relative multipath tracking error below a set limit of 1 mm is shown in figure 28. The increase in input frequency separation decreases the maximum allowable relative multipath amplitude from −72 dB to −109 dB.

If the limit for the relative multipath tracking error is increased to 1 cm, the maximum relative multipath amplitude allowed is −89 dB as shown in figure 29. As expected, when the limit for the relative multipath tracking error is increased, the allowable relative multipath amplitude increases as well.
Figure 28. The relative multipath amplitude required to keep the relative multipath tracking error below 1 mm. Input frequency separation is 10 MHz.

Figure 29. The relative multipath amplitude required to keep the relative multipath tracking error below 1 cm. Input frequency separation is 10 MHz.
Calculating the VSWRs of the two devices in the sample circuit, as was done for Case
Study 1, the larger frequency separation is expected to yield maximum allowable
VSWRs closer to 1. For the case where the relative multipath error limit was set to 1
mm, the VSWRs of the two devices need to be better 1.004. For the second case where
the limit is set to 1 cm, the maximum relative multipath amplitude of –89 dB translates
to a VSWR for each device that cannot exceed 1.012.

As expected, the larger frequency separation of the input signals produces larger relative
multipath errors requiring much lower VSWRs to meet the set error limit. For this case
study it may not be possible to acquire components with such low VSWRs. If the circuit
under study represented a system, the reflected signal will experience twice the
attenuation of the direct signal as it travels through the cable connecting the devices.
The target relative multipath tracking error can be achieved by increasing the length of
cable, which decreases the relative multipath amplitude and in turn lowers the relative
multipath error. If the circuit under study is a component such as a multistage filter
where there may be little or no attenuation between boundaries, other avenues of
lowering the relative multipath amplitude will have to be explored.
6. Impact on Receiver Performance

6.1. Producing Hardware Multipath

Following the analysis of hardware-induced multipath error, data were collected with a test setup specifically designed to produce this type of error in such a way that it can be seen in receiver data.

To measure hardware-induced multipath, a special test setup was designed at Ohio University so that a tracking loop receiving a minimal amount of multipath could have its raw measurements compared against an identical tracking loop receiving a significant amount of hardware multipath. The NovAtel Beeline receiver used in this test setup implements a modified non-coherent DLL. The multipath tracking performance of this tracking loop can be approximated by the performance of a coherent DLL. The setup is shown in figure 30. Live GPS signals were amplified and split into two paths, where one path only experienced signal attenuation, while the second path contained a source of hardware-induced multipath. A long cable was inserted between two BNC T connectors, which represented the two boundaries depicted in figure 11. The cable has an attenuation of 11 dB and a delay of 132 nanoseconds for GPS signals. The third leg of the BNC T connectors prior to the long cable was shorted, while the third leg of the BNC T connector after the long cable was either shorted or properly terminated with a 50-Ohm impedance. The shorted terminators create reflections with a relative amplitude of -5 dB, while the 50-Ohm impedances create reflections with a relative amplitude of -20 dB. Any differences in the raw pseudorange measurements between the two paths
will be primarily due to induced multipath from the two BNC T connectors in combination with the long cable. To help ensure that this is the case, isolators are included in the design to minimize reflections from other portions of the system.

Figure 30. Multipath test setup.

The test procedure was as follows:

1. Allow the receivers to track for 150 seconds.
2. Change the terminator on the receiver side from 0-Ohms to 50-Ohms.
3. After 300 seconds, change the terminator back to 0-Ohms.

The relative multipath amplitude is calculated from the reflection coefficients of the BNC-Ts and the receivers, and the cable loss.

\[
\alpha_{\text{short}} = 10^{-\frac{11-11-5-5}{20}} = 0.025 \quad (15)
\]

\[
\alpha_{50\Omega} = 10^{-\frac{11-11-5-20}{20}} = 0.0045 \quad (16)
\]
To determine if multipath is introduced, the raw pseudorange measurements of each receiver for a particular satellite are compared. Not only should a difference be visible when the short is exchanged for a 50-Ohm termination, but the error will be less than or equal to the multipath error envelope. The differences in raw pseudorange measurements are shown in figures 31 and 32 for two satellites. The change in multipath error introduced by switching the BNC-T termination is calculated by averaging the values of the initial interval and the final interval and subtracting it from the average of the second interval. The mean change in raw pseudorange measurements for SV 2 and SV 8 are 0.33 m and .34 m respectively with an average standard deviation of .07 m and .06 m each. The C/No plots are taken from the port where the terminators are switched.

The multipath error envelope for a correlator with 0.1 chip spacing and a delay of 304 nanoseconds gives the maximum multipath error for $\alpha = 0.025$ to be no greater than 0.37 m and 0.068 m for $\alpha = 0.0045$. In the worst-case, multipath would decrease from 0.37 m with the short in place to 0.068 m with the 50-Ohm termination in place. The maximum predicted change in the multipath error is therefore 0.302 m. The multipath error seen in the data in figures 31 and 32 is consistent with the predicted maximums.
Figure 31. Pseudorange difference for SV2 and the carrier to noise ratio.

Figure 32. Pseudorange difference for SV8 and the carrier to noise ratio.
A short piece of cable measured to add a 220 degree phase shift to the multipath is inserted into the setup and the test is repeated. The cable changes the delay of the multipath such that it falls on a different location on the multipath error curve resulting in a change in the amplitude of the multipath error. The cable length was selected to be close to 180 degrees to maximize the change in multipath amplitude if the original cable length resulted in the multipath delay to occur at a maximum or a minimum. If the first test was indeed close to a maximum of 0.3 m or $0.3\sin(90^\circ)$, then the additional delay should result in a minimum error of $0.3\sin(310^\circ) = -0.23$ m. The results from the second test, shown in figures 33 and 34, show a negative change of approximately 0.26 m for SV 2 and SV 8 with a standard deviation of .07 m and .05 m respectively, which is again consistent with the predicted value of 0.23 m.

Figure 33. Pseudorange difference for SV2 with 220 degree phase shift added to multipath and the carrier to noise ratio.
Figure 34. Pseudorange difference for SV8 with 220 degree phase shift added to the multipath and the carrier to noise ratio.

It is clear from the data analyzed in this section that hardware multipath does exist and that it can be accurately predicted if the characteristics of the hardware are known.

Relative multipath errors between multiple satellites cannot be evaluated with the limited set of data provided in this section, since relative errors for the above scenario would be below 1 cm. However, the change in path delay due to different Doppler frequency shifts for each of the satellites is a relatively straightforward extension of the results provided in this section.
7. Conclusions and Recommendations

7.1. Summary

The focus of this effort has been to demonstrate that the sources of multipath errors are not limited to objects outside the antenna. Electronic components in the front-end of a GPS receiver can introduce multipath errors. The work presented in this thesis primarily addressed hardware-induced multipath error on the code measurements.

The nature of some electronic components such as filters makes it very difficult to measure the delay of a reflected signal. For this reason, this thesis is focused on bounding the multipath error. The maximum expected multipath error is calculated based on the VSWR of the components within the circuit. Through modeling and laboratory tests, this thesis has shown that hardware multipath can significantly affect the performance of a GPS receiver in a high accuracy positioning or timing application.

7.2. Conclusions

Electronic components in the front-end of a GPS receiver can introduce hardware-induced multipath error. It is possible to design an electronic circuit that minimizes the impact of multipath errors through the careful selection of electronic components. The relative multipath amplitude of the multipath signal is determined by the VSWRs, the attenuation, and the propagation delay through the electronic device. By choosing a desired relative multipath tracking error threshold and the frequency separation between input signals, the VSWR of a device can be selected to keep the relative multipath tracking error below the desired threshold.
The results obtained from the laboratory tests show multipath errors that are consistent with those predicted by the mathematical model presented in this thesis.

Engineers and designers who are using GPS receivers in high-accuracy position or timing applications need to be aware of this potential source of errors. Receiver test setups could be the source of multipath errors, which may then be incorrectly attributed to the receiver.

7.3 Recommendations

This work focused on the coherent DLL and its susceptibility to multipath signals created within a circuit. Further work is needed to examine the non-coherent DLL and its vulnerability to hardware-induced multipath error. The research presented in this thesis made many assumptions to simplify the analysis. Additional work is recommended to improve modeling of the components used in the test setup in order to accurately predict the magnitude of hardware-induced multipath signals.
8. References


