VISUALIZATION AND MATHEMATICAL MODELLING OF HORIZONTAL
MULTIPHASE SLUG FLOW

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CHAPTER 1
INTRODUCTION

Long distance multiphase flow pipelines have become a key technological frontier in oil and gas exploration and production. The future production of oil and gas is expected to be from natural sources located in water depths of 500 m or more (Fairhurst, 1988). Given that fixed-platform costs are roughly proportional to the square of water depth, their use in deep water is uneconomical. There is a considerable incentive to reduce platform size and cost by employing minimum processing or by dispensing with a fixed platform altogether. It is expected that increasing use of subsea technology will occur in the future, with a goal of maximum offshore production with onshore processing.

Multiphase transportation is central in these technological developments. A minimum of offshore processing implies no separation of the phases and only a control of undesirable effects such as corrosion, wax, or hydrate formation in the system. The term "full well-stream transportation" (FWST) has been developed to describe these future systems. In these cases, the accurate prediction of multiphase flow characteristics in the flowlines is essential for the design, and economical and safe operation of these pipelines.

An important problem in these long distance multiphase transportation systems is the internal corrosion of the pipeline due to the severe service environment. The common approach is to design corrosion inhibitor programs with the predominant use of carbon steel pipelines. The critical parameter in the selection of a corrosion inhibitor is its effectiveness in the service environment. Since all corrosion phenomena involve interaction between the metal
and the flowing fluids, the nature of this physicochemical interaction is the controlling factor in corrosion inhibitor performance in these pipelines. For a given chemistry, the flow characteristics determine the rate of metal reaction with the flowing fluids. Relative motion between the fluid and the metal surface will enhance the corrosion rate. Ellison and Wen (1981), proposed three different controlling mechanisms for flow related corrosion, convective mass transfer, phase transport, and erosion corrosion. All three types of corrosion effects have been reported in multiphase flow pipelines.

Given that flow related corrosion is of great importance in multiphase flow, the characteristics of the flow and the different regimes involved become central in the analysis of these systems. Multiphase flow is a complicated phenomenon that is still not completely understood. Considerable research has been focused on two-phase gas-liquid flow. However, it is to be noted that multiphase flow in long distance pipeline transportation in the oil and gas industry is expected to involve gases, liquid mixtures, and solids. There is currently little understanding of three phase flow. Recently, research has been initiated to extend the mechanistic knowledge of two-phase gas-liquid flow to three-phase gas-liquid-liquid flows (Jepson, 1990).

Cocurrent flow of liquid and gas in a pipeline results in several different flow patterns. These are dependent on the flow configuration and the flow rates of the gas and liquid. Different flow patterns arise in vertical and horizontal pipelines, and the distinction is noted at the very outset. Figure 1.1 shows a schematic of horizontal two-phase gas-liquid flow patterns in a 10 cm pipe. At low velocities of gas and liquid, a stratified flow regime exists. In this regime, the gas flows as a stratified layer over the liquid. At very low velocities, the
Figure 1.1: Schematic of horizontal two phase gas liquid flow patterns.
interface between the gas and the liquid is smooth and this regime is called smooth-stratified (SS) flow. With increasing gas velocity, regular two-dimensional waves begin to appear at the interface between the gas and liquid. This regime is called wavy-stratified (WS) flow. With further increase in velocity two different types of transitions can occur. At higher gas velocities, the two dimensional waves grow further in height and the front of the wave begins to roll over. This gives the wave a three dimensional feature. This regime is called roll wave.

If the liquid velocity is increased, the flow changes from stratified to an intermittent flow pattern. The waves formed on the liquid film now grow to bridge the pipe. At low gas velocities, these waves form lumps of liquid called plugs. The plugs flow over the liquid film intermittently between elongated gas bubbles with little turbulence. This regime is called plug flow. At higher gas velocities, the front of the plugs begins to overrun the liquid film and assimilate it into its structure. This results in an acceleration of the front resulting in a highly turbulent flow pattern called slug flow. The mechanisms involved in slug flow are very different from those in plug flow.

As the gas velocity is increased even further, large three dimensional roll waves begin to appear on the liquid film between slugs. The slugs become highly aerated at this point. This flow regime is called pseudo-slug flow. As the gas velocity is increased even more, the slugs are no longer able to hold the gas. A blow-through phenomenon occurs and results in gas flowing in a central core of the pipe, with liquid flowing in an annulus around it. This flow regime is called annular flow. With further increase in gas velocity, the liquid is now entrained in the form of small bubbles in the flowing gas and regime is called mist flow.
If the liquid flow rate is increased at low gas velocities, then starting from an initial stratified flow regime, a transition occurs to the intermittent flow patterns described above. If the liquid velocity is now increased even further, the gas becomes distributed in the continuous liquid phase in the form of discrete bubbles. This flow regime is called bubble flow.

The knowledge of two phase gas-liquid flow regimes has been advanced through the classification of the above flow regimes. Flow regime maps have been constructed that plot the various regimes as functions of gas and liquid velocities. Figure 1.2 shows a flow regime map for two-phase water carbon-dioxide system, in a 10 cm I.D. pipe. The map is a plot of liquid velocity vs gas velocity and plots the different flow regimes described above. It is to be noted that the flow regime transitions are dependent on the fluid properties, the pipe diameter, and the flow configuration. The effect of inclination is particularly important. An inclination of ±1° can produce significant changes in the flow regime transitions. Figure 1.3 shows a comparison between two-phase gas-liquid flow regimes and three-phase gas-liquid-liquid flow regimes. The effect of the second liquid phase is not known. As mentioned, there is no knowledge of the distribution of phases in the liquid and its dependence on the flow regimes. This is particularly true for slug flow, where all three phases can be intimately mixed with one another.

The production rates in oil and gas wells are of such proportion that, in most cases, the multiphase flow lines are expected to be in slug flow at some time in their lives. As mentioned, this is a highly turbulent flow regime, leading to increased pipe damage from
Figure 1.2: Flow regime map for two-phase water carbon-dioxide system, in a 10 cm pipe.
Figure 1.3: Comparison between two phase gas-liquid flow and three phase gas-liquid-liquid flow regimes.
internal corrosion and mechanical impacts. The presence of slug flow in pipelines can significantly reduce the effectiveness of inhibitors.

Figure 1.4 shows the profile of a slug. Waves form on the liquid film, that grow to bridge the pipe. This causes the liquid to be accelerated by the gas. As the slug front moves through the pipe, it overruns slow moving liquid film ahead of it and accelerates it to the velocity of the slug. A mixing vortex is created in this process. This leads to a scouring mechanism on the pipe wall with high rates of shear. Also, as the liquid is scooped up into the slug, the leading edge of the slug jumps to the top of the pipe and entrains considerable amounts of gas in its wake. This leads to the creation of a highly frothy turbulent region behind the slug front called the mixing zone.

The liquid and gas are assimilated into the slug and accelerated to the velocity of the slug in this region of the slug. The gas is released in the mixing zone in the form of pulses of bubbles (Jepson, 1987). These bubbles are trapped by the mixing vortex and shot to the bottom of the pipe, where they can impact and collapse. The resulting synergy between the high rates of shear and scouring due to the mixing vortex and the bubble impact and collapse can degrade the performance of any inhibitor film that might have otherwise formed on the pipe wall. Beyond the mixing region of the slug, the level of turbulence is reduced, and buoyancy forces move the gas towards the top of the pipe. The cross sectional area available for liquid flow increases and the liquid velocity decreases. This is the slug body. Eventually a point is reached where the liquid velocity is no longer sufficient to sustain the bridging of the pipe, and the slug falls off. This is called the slug tail. The liquid velocity decreases in the liquid film, its height rebuilds, with waves forming on its surface and the next slug is initiated.
It is of great importance to understand the detailed mechanisms involved in slug flow. Several studies have been conducted to gain a better understanding of the characteristics of this flow regime. These will be reviewed in detail in the next chapter. However, the essentials of the problem involve the determination of the slug frequency, slug length, and the distribution of the different phases as a function of time in the pipe. Knowledge about the detailed features of slug flow are still lacking. The highly frothy and turbulent nature of slug flow has deterred any simple analysis. Slug lengths cannot still be predicted due to a lack of knowledge of the slug flow characteristics.

Dukler and Hubbard (1975) showed that slug flow was characterised by sixteen variables, many of which were time dependent. Mathematical models have developed that describe the relationship between different variables as knowledge of slug flow features have increased over the last decade. Still, detailed understanding of the motion of gas within the slug, and the distribution of phases in the different zones of the slug is not known. These are essential information that may then be used to develop a complete mathematical model to predict slug length and related features of slug flow.

In this study, flow visualisation techniques are developed to conduct a detailed analysis of the local characteristics within the slugs. The flow is recorded on video and the images digitized on a computer. Digital image processing techniques are then used to perform quantitative analysis of the images to obtain data on slug translational velocity, liquid film height, velocity and void profiles in the slug and the lengths of the different regions of the slug. The data are then incorporated into a mathematical model that is used to predict slug length and phase distribution within the slug.
CHAPTER 2
LITERATURE REVIEW

Two phase slug flow has been studied extensively both experimentally and theoretically. Early investigations into slug flow were restricted to empirical studies involving a large number of data in small diameter pipes. It was soon realised that these models were restricted in their applicability and mechanistic models were needed.

2.1 Modelling Studies of Slug Flow

Kordyban and Ranov (1970) published the first mechanistic results on slug formation. Their experiments were performed in a rectangular channel 2.54 cm deep and 15.2 cm wide. Air and water at atmospheric conditions were used as the working fluids. They proposed that slugs were formed as a consequence of a Kelvin-Helmholtz type instability of the liquid waves. The waves were substantially enhanced by the presence of the upper wall. By considering the stability of finite amplitude waves on shallow water, they derived approximate conditions for instability and slug formation.

The Kelvin-Helmholtz mechanism cannot be applied to large diameter pipes. In their study, Kordyban and Ranov had to assume a limiting steepness for large waves in order to successfully apply the theory. In large diameter pipes, however, the growth of waves to form slugs requires much greater momentum than predicted by the Kelvin-Helmholtz theory.

Dukler and Hubbard (1975) published the first realistic mechanistic model for slug flow characteristics. Based on observations in a 3.75 cm horizontal pipe with air and water as the
working fluids, they were able to determine the physical mechanisms involved in slug formation and formulate a mathematical model to predict slug flow characteristics. Figure 2.1 shows the schematic of their slug unit. The model was illustrated as a fast moving slug which overruns a slow moving liquid film. This process creates a mixing vortex at the front of the slug. The liquid is accelerated to the full slug velocity in a mixing zone located at the front of the slug. A new liquid film is shed behind the slug which decelerates with time. The gas pocket flows in a stratified layer over the liquid film between slugs.

From a momentum balance over the film region and the slug, they established fundamental equations describing slug velocity. This was expressed as:

\[ v_s = (1 + C) v_i \]  

(2.1)

where, \( v_i \) = slug translational velocity, m/s

\( v_s \) = average velocity in slug, m/s

\( C \) = constant = 0.2

The model can predict slug velocity, translational velocity of the slug, liquid film velocity as a function of time and distance, length of the slug and mixing zone, film region behind the slug, and also the shape of the surface of the film region. The agreement with experimental data was good and a better understanding of the mechanisms was given. However, many parameters, such as slug frequency and the void fraction within the slug, were required to complete the calculation.
Figure 2.1: Schematic of slug unit
Jepson (1989) derived an expression for the slug velocity, $v_s$, in Equation (2.1), by considering the mass balance for gas and liquid in a coordinate system moving with the slug front. He showed that the slug velocity may be calculated as:

$$v_{SL} + v_{SG} = v_s \quad (2.2)$$

where, $v_{SL} = \text{superficial liquid velocity, m/s} = q_L / A$

and, $v_{SG} = \text{superficial gas velocity, m/s} = q_G / A$

$q_L$, $q_G$ = volumetric flow rate of liquid and gas, $m^3/s$

$A$ = area of pipe, $m^2$

The superficial velocity is defined as the volumetric flow rate divided by the cross sectional area of the pipe. Since the slug velocity is the sum of the gas and liquid superficial velocities, the slug velocity, $v_s$, is also referred to as the mixture velocity, $v_m$.

Nicholson et.al. (1978) found that Equation (2.1) was not adequate for prediction of the slug translational velocity, $v_t$. From visual and photographic observations of the flow of air-light oil mixtures in 2.58 cm and 5.12 cm I.D. pipes, they incorporated a drift velocity, $v_d$, for the calculation of $v_t$. They defined $v_d$ as the weighted mean drift velocity of the gas phase relative to the liquid. This is induced from the liquid static pressure head which exists simply as a result of the pipe diameter. The static head causes the liquid velocity to decrease, resulting in a shedding of the liquid from the slug. Its effect on the gas velocity is negligible and results in a slip between gas and liquid. Nicholson et.al. found a non zero gravity induced drift velocity, even for horizontal pipes. However, they gave no expressions for $v_d$. Using the
drift velocity, $v_d$, in their calculations, and an empirical estimation of the average liquid fraction (or holdup) in the slug, they modified and extended the model of Dukler and Hubbard (1975) to apply to the entire intermittent flow regime. They divided the overall intermittent flow regime into three distinct regions, elongated bubble flow (or plug flow), elongated bubble flow with dispersed bubbles, and, slug flow. In order to use the model, an initial estimate of either the slug frequency or the slug length corresponding to a given design condition must be known. However, they found that the calculated pressure gradient and in situ liquid holdup were relatively insensitive to these parameters, and in fact, good results were obtained assuming a constant slug length. It is to be noted that the model ignores any slip between gas and liquid within the slug. It has been found (Jepson, (1987)) that for large diameter pipes, an assumption of homogeneous flow is not true for higher slug velocities.

Maron et.al. (1982) derived a model for slug flow based on new concepts of periodic distortion of the hydrodynamic boundary layer within the slug front followed by a recovery process in the body of the slug. As the slug front overruns the liquid film, the boundary layer is destroyed by the mixing eddy. At the end of the mixing zone, this boundary layer begins to redevelop. The model considered two separate types of slug, one with aeration and the other without. In the first case, the entrained gas bubbles in the slug leave the boundary layer region due to buoyancy effects and tend to agglomerate in the upper portion of the pipe. The slug length is the distance required for complete separation of the gas from the liquid. In the second case, the slug length is given by the distance required for the boundary layer to fully develop and reach the center of the pipe. Maron et.al. developed their boundary layer analysis in a coordinate system moving with the slug front and introduced a one-seventh power law
model to describe the velocity profile within the boundary layer. They showed that the model could be applied to predict pressure drop for a wide quasi-steady frequency range of slugs. However, stability analysis indicated that the slug pattern stabilised over a narrow frequency range corresponding to a minimum pressure drop. It is to be noted that no information about the pipe diameter or working fluids were given.

Dukler et. al. (1983) applied the concepts developed by Maron et. al. (1982) and formulated a generalised model for the prediction of the minimum stable slug length for horizontal and vertical slug flow. The model utilized the velocity profile developed by Maron et. al. (1982) in the boundary layer, and combined this with a inviscid potential core. An assumption was made that a flat velocity profile resulted at the end of the mixing zone. The velocity at the center was allowed to decrease due to frictional effects predicted by a Blasius-type equation. The slug length was predicted by the distance required for the complete development of the boundary layer. The results of the model were applied to 5 cm I.D. vertical and horizontal, and 3.8 cm horizontal pipes, with air and water as the working fluids. It was found that the experimental results were bounded between the value predicted by the model and twice that value. It should be noted that the model ignored the contribution of gas to the mechanisms in the slug.

Kouba (1986) formulated a model to account for both liquid and gas phase distribution as well as velocity distribution within the slug. By considering a mass balance between the slug and the liquid film he was able to formulate a generalised expression for liquid film velocity ahead of the slug as follows:
\[ v_{LF} = v_t \left[ 1 - \frac{\bar{a}_{LS}}{\bar{a}_{LF}} \right] + \frac{v_{LS} a_{LS}}{a_{LS}} \]  \hspace{1cm} (2.3)

where,

- \( v_{LF} \) = average liquid film velocity ahead of the slug, m/s
- \( \bar{a}_{LS}, \bar{a}_{LF} \) = liquid phase fractions in the slug and the film
- \( v_{LS} \) = average liquid velocity in the slug, m/s

Equation (2.3) is the most generalised equation for the film velocity. It assumes stratified flow conditions between slugs and relates the film velocity to the velocity in the slug. At high slug velocities, the film between slugs is subject to large three-dimensional roll waves, which contribute to the liquid flow rate. With increasing slug frequency, the effect of these roll waves on the liquid film increases and Equation (2.3) overpredicts the liquid velocity. In such situations, the contribution of both slugs and the film to the liquid flow rate becomes important and a liquid film velocity that is dependent on the slug frequency must be developed.

From observations in a 7.5 cm I.D., 418 m long pipeline, using kerosene and air as the working fluids, Kouba concluded that the drift velocity, \( v_d \), was significant for horizontal slug flow. Utilising the theory of shearing flow over a wavy boundary developed by Benjamin (1968), he developed an involved expression for the drift velocity as a function of pipe diameter. The incorporation of the drift velocity improved slug length predictions by twenty percent.
Taitel and Dukler (1976) provided a rational model for transition from stratified flow to other flow regimes. Figure 2.2 shows their physical model for stratified flow. The gas flows in a stratified layer above the liquid. From a consideration of a momentum balance in the liquid and gas phase the following equation is obtained:

\[
\tau_{WG}\frac{S_G}{a_{GF}} - \tau_{WL}\frac{S_L}{a_{LF}} + \tau_i \frac{S_i}{a_{LF}} \left( \frac{1}{a_{LF}} + \frac{1}{a_{GF}} \right) + (\rho_L - \rho_G)g \sin \alpha = 0 \tag{2.4}
\]

In the above equations, the variables are defined as follows:

- \(a_{GF}\) = area occupied by the gas above the stratified layer of liquid, m\(^2\)
- \(a_{LF}\) = area occupied by the stratified layer of liquid, m\(^2\)
- \(S_i\) = perimeter of liquid contact with wall over which shear stress acts, m
- \(S_G\) = perimeter of gas contact with wall over which gas phase shear stress acts, m
- \(S_i\) = width of gas-liquid interface, m
- \(\tau_{WL}\), \(\tau_{WG}\) = wall shear stress for gas and liquid respectively, Pa
- \(\tau_i\) = shear stress at gas-liquid interface, Pa
- \(\rho_L\), \(\rho_G\) = density of liquid and gas respectively, kg/m\(^3\)
- \(\alpha\) = pipe inclination (small values, close to horizontal), degrees

They calculated the shear stresses by conventional expressions as follows:

\[
\tau_{WL} = f_L \frac{\rho_L v_{LF}^2}{2}, \quad \tau_{WG} = f_G \frac{\rho_G v_{GF}^2}{2}, \quad \tau_i = f_i \frac{\rho_G (v_{GF} - v_L)^2}{2} \tag{2.5}
\]

where, \(f_L, f_G, f_i\) = friction factors for liquid, gas, and interface respectively.
Figure 2.2: Physical model for stratified flow
\( v_{LF}, v_{GF}, v_i \) = average velocities for gas, liquid and interface respectively.

From a knowledge of the areas of gas and liquid flow, it is possible to calculate, the average velocities, \( v_{LF} \), and \( v_{GF} \).

\[
v_{LF} = \frac{A_{VL}}{a_{LF}} \quad \quad v_{GF} = \frac{A_{VG}}{a_{GF}}
\]  

(2.6)

The friction factors for the two phases are evaluated as follows:

\[
f_L = C_L \left( \frac{D_L v_{LF} \rho_L}{\mu_L} \right)^{-a} \quad \quad f_G = C_G \left( \frac{D_G v_{GF} \rho_G}{\mu_G} \right)^{-a}
\]

(2.7)

where,

\[
C_L = C_G = 0.046, \quad \text{turbulent flow}
\]
\[= 16, \quad \text{laminar flow}
\]
\[n = m = 0.2, \quad \text{turbulent flow}
\]
\[= 1.0, \quad \text{laminar flow}
\]
\[
\mu_L, \mu_G = \text{viscosities of liquid and gas},
\]

\( D_L, D_G \) are hydraulic diameters as defined by Agrawal (1973):

\[
D_L = \frac{4 a_{LF}}{S_L} \quad \quad D_G = \frac{4 a_{GF}}{S_G + S_l}
\]

(2.8)

In their model, they assumed that the interface between gas and liquid was always smooth and therefore:
The assumption of smooth interface between gas and liquid is valid only at very low velocities, and this is not applicable to the slug flow regime.

Andritsos and Hanratty (1987) using experimental data from 2.52 and 9.53 cm I.D. pipes, and liquid viscosities ranging from 1 cP to 70 cP, proposed a relationship between $f_l$ and $f_G$:

\[
\frac{f_l}{f_G} = 1 + 15 \left( \frac{h_{LF}}{D} \right)^{0.5} \left[ \frac{v_{SG}}{5 \cdot \rho_G} \left( \frac{\rho_G}{\rho_{Go}} \right)^{0.5} - 1 \right] \quad v_{SG} \geq 5 \text{m/s}
\]

\[
\frac{f_l}{f_G} = 1 \quad v_{SG} < 5 \text{m/s}
\]

where, $\rho_{Go} = \text{gas density at 1 atm., kg/m}^3$

$h_{LF} = \text{liquid film height as shown in Figure 2.2, m}$

With this modification the model can adequately predict film heights in stratified flow. In slug flow, as stated earlier, there are large three dimensional roll waves that are present on the liquid film between slugs. It should be noted that as the slug frequency increases the liquid flow rate becomes dependent on the liquid film characteristics and the slug length. However, Equation (2.4) is still used to predict the liquid film height in slug flow. Account must be made of the effect of slug frequency on the liquid film height.

Jepson (1989) presented a physical model for the prediction of transition to slug flow. The model assumes that the slug is formed as a result of a hydraulic jump propagating along the conduit. He defined a dimensionless Froude Number for the film ahead of the slug. The definition allowed a comparison between slugs and hydraulic jumps. Chow (1959) showed
that the strength of the hydraulic jump is governed by the Film Froude Number, as shown in Figure 2.3. Using equations of continuity and momentum conservation at a condition where the jump just touches the top of the wall of the conduit he provided the necessary conditions for the existence of slugs. Jepson (1989) pointed out that the motion of the slug tail is similar to that of a breaking dam, the theory of which gives the necessary condition for the formation of a stable slug. The model can calculate the minimum liquid film thickness ahead of the slug as a function of the degree of aeration in the slug. It also predicted the Film Froude Number and the slug translational velocity. At low superficial gas velocities, the model predicted that $v_t = 2v_s$. As the gas velocity is increased to a point where the slug is fully aerated, the model predicted a $v_t = 1.25v_r$. Using air-water mixtures in a 0.3 m I.D. pipe, he generated data that confirmed the model predictions. All the model predictions agreed well with those of previously mentioned workers also.

Lin and Hanratty (1986) have examined the stability of an infinitesimal disturbance of two-phase flows using a viscous Kelvin-Helmholtz instability theory. To develop this theory, they used continuity and momentum equations for a stratified flow and imposed small sinusoidal disturbances upon it and examined the conditions for instability. Unlike previous approaches using the inviscid Kelvin-Helmholtz instability, the instability criterion for wave growth in this model becomes a function of viscosity. Using experimental data from 5.12 and 15 cm pipes, they confirmed the results of the model.

All of the above models require extensions to fluids of different compositions of the phases. This involves density, viscosity and surface tension. It is therefore important to develop theoretical relationships relating these parameters to the variables in slug flow.
Figure 2.3: Various types of hydraulic jumps

- $F_t = 1.1 - 1.7$ Undular jump
- $F_t = 1.7 - 2.5$ Weak jump
- $F_t = 2.5 - 4.5$ Oscillating jump
- $F_t = 4.5 - 9.0$ Steady jump
- $F_t > 9.0$ Strong jump
2.2 Theoretical and Experimental Studies of Slug Flow Characteristics

Slug flow characteristics involve the phase holdup variation within the slug, velocity profiles, average velocity variations, slug lengths and slug frequencies, and slug velocities. The relationship between slug velocity and slug translational velocity has already been demonstrated. There is a significant influence upon the slug velocity of the local velocity and phase distribution within the slug. This in turn influences the total slug length. The influence of the slug frequency on the liquid film ahead of the slug was noted in the previous section in the discussion on liquid film height. The literature on these variables is discussed next.

2.2.1 Phase Holdup

The holdup of a phase in a multiphase system is defined as the in-situ volume fraction of that phase in the conduit. The terms holdup and void fraction are used synonymously in this context. Usually holdup refers to the liquid fraction.

Several studies have been conducted to determine holdup in slugs. Gregory et al. (1978) proposed an empirical correlation for liquid holdup in the slug. They conducted experiments in 2.58 cm, and 5.12 cm I.D. pipes, with a light refined oil in the liquid phase and air in the gas phase. The superficial liquid velocity ranged from 0.03 m/s to 2.3 m/s, and the superficial gas velocity ranged from 0.09 m/s to 15.4 m/s. The liquid holdup was experimentally measured using a capacitance type liquid volume fraction sensor. They proposed the following correlation:
where, \( \alpha_s \) = void fraction in the slug

This correlation is widely used in industry. It however does not predict the liquid holdup (void fraction) in the slug for high viscosity liquids. It was suggested that a modification was necessary to include a pipe diameter and fluid property effects into this correlation.

Bamea and Brauner (1985) proposed a physical model for the prediction of void fraction in slugs for horizontal and vertical pipes. They proposed that the gas was distributed in the form of dispersed bubbles in fully developed slugs. This void fraction was determined from a balance between breakage forces due to turbulence and coalescence forces due to buoyancy and surface tension. When coalescence dominates, agglomeration of small bubbles occurs, leading to the formation of elongated gas bubbles separated by liquid slugs. However, when turbulence dominates, the breakage forces will lead to a dispersed bubble flow pattern. They used conditions proposed by Taitel and Dukler (1976) for horizontal pipes and by Barnea et al. (1982) for vertical pipes, for transition between bubble flow and slug flow to predict the void fraction. The void fraction in bubble flow is given by:

\[
\alpha_s = \frac{v_{SG}}{v_{EL} + v_{SG}}
\]  

(2.12)
At a fixed mixture velocity in bubble flow, if the superficial gas velocity is increased, there is a transition to slug flow. They assumed that the level of turbulence in the slug was the same as that in the dispersed bubble flow, and therefore the void fraction in the slug would be given by Equation (2.12). The model agreed with data presented by Ferschneider et.al. (1983), and with the data from Gregory et.al. (1978) for low slug velocities. Actually, this model is seen to work for plug flow and low velocity slugs. At higher velocities, the assumption that the turbulence in the slug is the same as in the dispersed bubble flow is not correct. This causes the model to under predict the void fraction in the slug.

Andreussi and Bendiksen (1989) proposed a correlation for air-water slug flow in horizontal and near horizontal pipes using a conductance probe technique. They conducted experiments in 5 cm and 9 cm I.D. pipes, with inclinations ranging from -3° to +0.5° and used this data along with those of Gregory et.al. (1978) and Ferschneider (1983), to formulate the correlation. It had provisions to allow for the effect of pipe diameter, inclination and fluid physical properties. The model agreed with the data of Gregory et.al. (1978) and Ferschneider (1983). However, it involved a number of empirical coefficients and it was not clear how to calculate them.

Kouba (1986) conducted slug holdup experiments in his study also. The liquid holdup in the slug was a strong function of the slug velocity but was not uniquely dependent on it. The holdup decreased approximately linearly with increasing slug velocities.

Jepson and Taylor (1988) carried out liquid holdup measurements for air-water slugs in a 300 mm I.D. pipeline. They found a strong dependence of liquid holdup on pipe diameter. As the superficial gas velocity was increased up to 5m/s, the holdup decreased to 0.45. There
was a limiting holdup value of 0.38 at very high gas velocities. This value was much lower than those predicted by Gregory et al. (1978) and for other small diameter pipes. There was negligible effect of the pipe diameter on the liquid holdup in the slug body below a gas velocity of 3 m/s.

Jepson (1987) used a hydraulic jump to form a stationary slug in a 10 cm horizontal pipe for an air-water system. This method allowed the insertion of a pitot tube and sampling probe without seriously affecting the flow. Local liquid holdup profiles at different locations within the slug were obtained and good agreement with the average liquid holdup predicted by Gregory et al. (1978) were achieved. He found that the gas was reasonably well mixed at a distance of 19 cm from the slug front. However, at a distance of 30 cm from the slug front, there was no gas in the lower third of the pipe cross section, while the void fraction near the top of the pipe rose to about sixty percent.

Jepson and Kouba (1987) conducted experiments with stationary slugs in a 15 cm pipeline with air and water in the gas and liquid phases. They defined a Froude Number for the film ahead of the slug, and found that the liquid fraction in the slug decreased approximately linearly with increasing Froude Number.

### 2.2.2 Velocity Profiles

The models developed so far in slug flow make it possible to obtain average velocities in the slug, if input parameters, such as, gas and liquid superficial velocities, and empirical data, such as, liquid holdup, are known. However, there is insufficient information about the velocity profile across a section of pipe within the slug. Jepson (1987) made velocity profile
that at a distance of 19 cm from the slug front, the incoming liquid was still affecting the flow within the slug. At a distance of 31 cm from the front, the flow was more uniform, but the profile was influenced by the distribution of gas bubbles. Increasing the liquid velocity did not affect the profile near the slug front, but farther downstream of the jump, the velocity profile became flat. This study made it clear that there was significant interdependence between the local velocity and void profile within the slug, and that this was a function of the slug velocity.

2.2.3 Slug Frequency

Taitel and Dukler (1977) presented a model for calculating slug frequency based on their stratified flow model. They argued that slug formation was an entry level phenomenon, and therefore, used the criterion of rebuilding of a wavy film on the liquid surface after the passage of a slug to determine the slug frequency. The model provided satisfactory agreement with existing slug frequency data of Dukler and Hubbard (1975) and Gregory and Scott (1969). It should be noted that these data were generated in small diameter pipes for air-water systems. Hill and Wood (1990) note that existing methods for predicting slug frequency from small diameter pipes are unsatisfactory for large diameter pipes.

Cercignani and Battara (1983) criticized the Taitel and Dukler model on the grounds that it was unwarranted to assume that the gaseous phase was in steady state. The assumption of steady state in the gas phase allows the elimination of the interfacial pressure by substitution. Also, the model proposed by Taitel and Dukler (1977) contained a term called as the "Bernoulli effect". This term described the creation of unstable waves as a result of decreased pressure above the wave due to gas acceleration. This term was neglected by Taitel and
Dukler (1977) in their analysis. Cercignani and Battara (1983) argued that this term was at least as large as the other terms in the model. They proposed a modified model that considered a steady state problem with coordinates moving with the slug front. This resulted in the "Bernoulli term" being automatically eliminated. However, they made an assumption that there was a region in the pipe through which gas does not flow. This allowed the gas velocity term to be dropped from the analysis. No results were given to substantiate their model.

Tronconi (1990) presented a physical model for the prediction of the slug frequency. He expressed the condition for wave growth as a function of $h_0$, the undisturbed equilibrium depth of the gas, and $u_{GF}$, the average gas velocity in stratified flow, and then computed the wave number for the maximum growth of waves. The model used the original model of Taitel and Dukler (1976) given by Equations (2.4) to (2.8) to predict $h_0$ and $u_{GF}$, with $f_i = f_0$ for laminar flow, and $f_i = 2f_0$ for turbulent flow. By assuming that the slug frequency is one-half the frequency of the unstable waves over the liquid film which are the precursors for slugs, he was able to predict slug frequencies with satisfactory agreement with the experimental data from several different researchers. However, all the data were for pipe diameters between 1.27 cm and 5.12 cm. It is not known if this approach is applicable for larger diameters.

Hill and Wood (1990) provided a method correlating the slug frequency with the liquid holdup in the film ahead of the slug and the gas-liquid slip velocity. Starting from the original dimensionless numbers given by Taitel and Dukler (1976) for flow regime transition and accounting for the slip between gas and liquid, slug frequency in real field pipelines were successfully predicted. The correction for slip velocity was accomplished by setting the
difference between the average velocity of gas and liquid in the stratified region ahead of the slug equal to the ratio of the slug velocity and the liquid film holdup. This accounted for the usually greater gas velocities compared to the liquid velocities in the systems.

Crowley et. al. (1984) conducted slug flow studies in 17 cm pipes, with water and glycerine (viscosity 400 cP) for the liquid phase and Freon for the gas phase at densities one to twenty times that of air. It was found that slug frequency for glycerine were higher by a factor of two than water. It was suggested that liquid viscosity was one of the key parameters controlling slug frequency.

Jepson and Taylor (1988) compared slug frequency data from 15 cm and 30 cm pipes to smaller diameter pipes. They found that the slug frequency was reduced from 100 slugs/min to 10 slugs/min, when the pipe diameter was increased from 2.54 cm to 30 cm. In an attempt to incorporate the effect of pipe diameter, a nondimensional slug frequency was defined and correlated with the slug velocity.

2.2.4 Slug Lengths

Dukler and Hubbard (1975) presented slug length data for 3.75 cm pipe, for air water system, for slug velocities ranging from 1.5 m/s to 10 m/s. The slug lengths ranged from a minimum of 0.5 m to a maximum of 1 m. The model proposed by Dukler and Hubbard adequately predicted the slug lengths. This corresponds to a range of 12-25 pipe diameters in slug lengths.

Nicholson et.al. (1978) measured slug lengths in 2.58 and 5.12 cm I.D. pipes for a light oil and air slug flow system. The superficial liquid velocity ranged from 0.06 m/s to 1.83 m/s,
and the superficial gas velocity ranged from 0.05 m/s to 20 m/s. There was a wide range of
slug lengths in the two pipes, ranging from about 0.5 m to 3 m. Again, the modified model
developed by the authors was successful in predicting the slug lengths. This corresponds to
a range of ten to sixty pipe diameters in a 5.12 cm pipe.

Kouba (1986) measured slug lengths for kerosene-air system in a 7.6 cm I.D., 418 m long
pipe line. The slug velocity ranged from 0.9 m/s to 7.6 m/s. The slug lengths in this case
ranged from 1.5 m to 7 m, or 25 to 100 pipe diameters. Most of the slugs were about 3 m
long, or forty pipe diameters. There will be an effect of gas expansion over the length of the
pipeline in this study resulting in the acceleration of slugs, and thereby an increase in slug
lengths.

2.2.5 Large Diameter Pipe Studies

Several investigations on slug flow have been conducted for large diameter pipes.
Crowley et. al. (1986) conducted slug flow studies in 17 cm pipes, with water and glycerine
(viscosity 400 cP) for the liquid phase and Freon for the gas phase at densities one to twenty
times that of air. They found that the translational velocity of the slug was predicted for all
cases using the drift flux model. There was negligible effect of the gas density on the slug
velocity. However, there was a large effect of liquid viscosity. The slug velocities for
glycerine were fifty percent large than water for the same conditions. They found that the
Taitel and Dukler (1976) model for transition to slug flow did not correctly predict the
transition at higher gas densities. It was found that the interfacial friction factor was about ten
transition at higher gas densities. It was found that the interfacial friction factor was about ten times the gas phase friction factor in this case. The slug lengths ranged from 0.5 m to about 3.5 m.

Jepson and Taylor (1988) found that there was an increase in slug length with an increase in pipe diameter. Below a gas superficial velocity of 5 m/s, the slugs appeared to be growing. This indicates that there may be gas expansion in large diameter pipes, or a significant drift velocity causing the slug to grow.

Brill et al. (1981) have modelled the growth of slugs in large diameter pipes in oilfields. Using field data from 30 cm and 40 cm pipes they predicted the pressure drop in slug flow. They found that slug lengths were an order of magnitude higher than that expected for normal slug flow. It was concluded that topography had a significant effect on the flow characteristics, and even a small change in inclination could cause large changes in the flow mechanisms.

Scott et al. (1986, 1989) developed correlations for predicting slug length in large diameter pipes using data from 30, 40, 50, and 60 cm I.D. pipes in oil fields in Prudoe Bay, Alaska. They found that in these pipelines there was an additional factor in slug length analysis. They termed this factor the "long term growth". This was related to gas expansion within the pipeline due to pressure changes.
CHAPTER 3
EXPERIMENTAL SETUP AND PROCEDURE

3.1 Description of Flow System

Figure 3.1 shows a schematic view of the experimental setup. Liquid is stored in a 0.6 m$^3$ stainless steel tank and is pumped by a 2.3 kW stainless steel centrifugal pump into a 50 mm I.D. PVC pipe. The flow rate of the liquid is controlled by a by-pass system and is monitored by an orifice plate with an orifice to pipe diameter ratio of 0.75. The liquid flow rate was experimentally calibrated as a function of pressure drop across the orifice plate, using U-tube manometers with Meriam blue liquid (specific gravity 1.75), and mercury (specific gravity 13.6) as the manometric liquids. The blue liquid was used for a lower range of velocities, and the mercury was used for the higher range. The calibration chart is given in Appendix A.1.

Carbon dioxide from compressed cylinders is stored in a 1.67 m$^3$ carbon steel tank at a pressure of 1300 kPa. It is introduced into the system at an inlet pressure of 800 kPa. The gas flow rate is controlled by a flow regulating valve and is monitored by Omega FL 4000 Series variable area gas flow meters. The calibration of gas flow rate reading in the flow meter as a function of pressure and gas density was done according to the specifications provided by the manufacturer. This is shown in Appendix A.2.

It is possible to generate both moving as well as stationary slugs in this system. For moving slugs, the gas and liquid are introduced into a 0.015 m$^3$ stainless steel mixing tank, and the two-phase mixture is allowed to flow out into the 0.75 m, 10 m long Plexiglass
Figure 3.1: Schematic layout of experimental flow system
pipeline. The mixture flows back into the liquid storage tank and is separated by means of a specially designed de-entrainer table. The gas is vented to the atmosphere and the liquid is recirculated into the system.

For stationary slugs, the liquid is forced under a gate, to generate a fast moving liquid film. The gate is formed by cutting out a semi-circular section from a 0.625 cm thick, aluminium plate, with a height at the center equal to one-third the pipe diameter. The gas is injected into the system downstream of the gate and its flow controlled by needle valves and a slug is generated in the Plexiglass pipeline. By adjusting the needle valves, the slug can be moved to the test section and made stationary. The test section is located 5 m from the tank.

An Omega DP8000 series differential pressure transducer was used for pressure measurements. One terminal of this transducer was connected to the pressure tap and the other kept open to the atmosphere. In this manner, single point pressure could be recorded as a function of time in slugs. The output from the transducer cell was connected to a Omega DP87 programmable display unit. The display was then connected to a Gateway 2000 386 personal computer (PC), equipped with a DAS20 data acquisition board, which was accessed by the data acquisition software, Labtech Notebook. Notebook parameters were set in the software and pressure data collected on the PC.

3.2 Experimental Test Matrix

Slug flow experiments were conducted for two phase gas-liquid systems. In order to study the effect of fluid composition, liquids of two different viscosities were studied, deionised water and an oil of medium viscosity, ARCOPAC90\textsuperscript{TM}. Carbon dioxide (CO\textsubscript{2}) was
used for the gas. This is used for all other projects in the Corrosion in Multiphase Systems Center since carbon dioxide is of the main contributing factors to corrosion in oil and gas pipelines. Table 3.1 lists the liquids used, their properties, and the range of variables studied.

Table 3.1: Experimental Test Matrix

<table>
<thead>
<tr>
<th>Variable</th>
<th>Deionised Water</th>
<th>ARCOPAC90</th>
<th>Carbon dioxide</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>998</td>
<td>850</td>
<td>1.9</td>
</tr>
<tr>
<td>Viscosity (Pa.s)</td>
<td>0.0010</td>
<td>0.015</td>
<td>0.000015</td>
</tr>
<tr>
<td>Surface Tension (N/m)</td>
<td>0.070</td>
<td>0.040</td>
<td>-</td>
</tr>
<tr>
<td>Superficial Velocity (m/s)</td>
<td>0.2 - 1.3</td>
<td>0.16 - 0.88</td>
<td>1 - 5</td>
</tr>
</tbody>
</table>

The superficial velocities are calculated from the ratio of volumetric flow rate and the area of cross section of pipe as shown for Equation (2.2).

The above range of velocities covered the entire range of the slug flow regime. All of the above values are listed for 298 K and 101.3 kPa.

3.3 Experimental Procedure

The basic experimental procedure was as follows. First, the pump was turned on and the liquid flow rate was adjusted to a given value using valves A and B. Once the liquid had attained a steady flow, the inlet gas valve was opened and gas was introduced into the system. The gas-liquid mixture then flowed through the Plexiglass pipeline and slugs were generated.
As the slugs moved through the system they were recorded on video using the flow visualisation system described later in Section 3.4. This was then used to obtain data regarding their detailed characteristics. For moving slugs, the flow was recorded for an interval of three to four minutes for each data set. Approximately thirty to fifty slugs were recorded in this time period. Two sets of recording were taken to confirm the reproducibility of the data. For stationary slugs, the flow was recorded for ten minutes. The visualisation was concentrated at the slug front and mixing zone. This allowed a detailed examination of the extremely complicated mechanisms involved in these parts of the slug.

3.3.1 Pressure Measurements

Data was also taken for single point pressures in stationary slug flow. This was used for a correlation with Froude Numbers as described later in Section (6.5).

Pressure data was taken for a total of ten seconds at a sampling rate of 30 Hz. To obtain a pressure profile at specific points along the length of the slug, data was taken at every 15 cm beginning from the slug front up to 60 cm into the slug. This is a realistic distance for slug length. In case of highly turbulent slugs, characterised by a Froude Number of 12-14, it was not realistic to take data every 15 cm. It was therefore decided to take data in these cases at 30 cm and 60 cm into the slug.

The voltage data was converted to psi units using a linear calibration scale. A voltage range of 1-5 volts was converted to 0-2.5 psi. The offset was set at -6.03 volts.
3.4 Flow Visualization System

Figure 3.2 shows a schematic of the flow visualisation system. The flow is observed by two cameras at right angles to the flow. A Panasonic WV-D5100, super-VHS camera is used for a closeup view of slug flow features and to obtain detailed data regarding slug flow characteristics. For moving slugs, the film height of the liquid before the slug, slug void fraction distribution and velocity profiles, lengths of the mixing zone and total slug lengths were all obtained using this camera. To obtain values for the slug translational velocity, a second camera, a Panasonic AG190 industrial camcorder is used. This camera was positioned 3 m back from the pipeline. This allows the slug front to be tracked over a sufficient number of frames and from the change of position of the slug front with time, the translational velocity is calculated. It is also used to confirm slug length data.

To obtain a high resolution image, a shutter speed of 1/1000 s is used in both cameras with an image resolution of 640 by 480 pixels. In this study, a 600 watt Smith-Victor fluorescent lamp was used to illuminate the system.

As shown in Figure 3.2, the video outputs from both cameras were connected to a Panasonic WJ-MX12 digital color Audio-Visual (AV) mixer, with a built-in frame synchronizer. The use of this AV-mixer facilitates the production of high quality, high resolution video images on a single screen resulting in easy analysis of the images. The image from the S-VHS closeup camera occupied the lower two-thirds of the screen from the bottom with the VHS image in the upper section.

The video signal from the AV-mixer was then sent to a Panasonic AG-1960, Hi-Fidelity, industrial Super-VHS video cassette recorder (VCR). This VCR has an important
Figure 3.2: Schematic view of flow visualisation system
feature, the super-still playback mode. In this mode, it is possible to obtain sixty video images every second. This is accomplished by playing the VCR in the frame advance mode. The super-still playback feature freezes each frame and displays one field at a time resulting in sixty images every second.

The primary display element in the visualisation system is a SONY PVM-1341Q Trinitron S-VHS, Hi-Fidelity TV monitor. The video signal from the VCR is connected to the monitor and final adjustments to camera positions and image quality were accomplished by examining the picture on this monitor.

The video images were then digitised on a Gateway 2000 486 PC using an Analog-to-Digital (A-D) board and software installed in the computer. The digitizer board and software used in this study was the ComputerEyesRT made by Digital Vision Inc. of Massachussetts. The resolution of this digitizer was 640 by 480 pixels with 256 solid colors. The PC was a 50 MHz machine equipped with a 8Mb RAM.

The digitized images are finally transferred to a SGI™ Indigo Elan graphics workstation for analysis. This workstation has a speed of 33 Mflops, with a video memory of 24 Mb. It is equipped with a 20 inch Mitsubishi color monitor capable of displaying 16 million colors. The SGI™ has several hundred graphics modules (subroutines) as part of a Graphics Language (GL) software package. These modules can be customized for particular image processing and analysis applications.
CHAPTER 4
DATA ANALYSIS

4.1 Image Analysis Summary

Most of the data on slug characteristics were obtained by digital image analysis. Some essential features of this technique are now described.

A visual images on the computer is described by pixels. Each image is described by exactly 640 pixels by 480 pixels. This corresponds to an area which depends on the camera position and the magnification during recording. It is therefore, necessary, to specify a conversion factor from length dimensions to pixels. This is done by marking two points 1 cm apart on the pipeline and recording this. The marks on the image will then represent a given number of pixels.

A technique called "edge tracking" was used for much of the image analysis. An algorithm was developed to have the computer record the coordinates of any desired point in the image. The point was identified by the movement of a cursor which was controlled manually.

Several image processing programs are written to analyse the images for each type of data, e.g., slug translational velocity, liquid film height, velocity of bubbles in slugs, and slug void fraction distribution. Each program is written in "C" and is listed in Appendix B, and is discussed in the appropriate data analysis section. Figure 4.1 illustrates the general flow
Figure 4.1: General flow chart for image analysis programs
chart for the image analysis programs. Modules available as a part of the SGI graphics system were modified to tailor them to each individual data acquisition application.

Essentially, the image processing scheme described in Figure 4.1 involves the creation of a buffer in memory for the display of the image, setting the image size and other relevant parameters, and then drawing the image on the screen. A mesh is then drawn over the pipe cross section, dividing it into ten segments at equal intervals along the diameter of the pipe. Finally, the appropriate subroutine is used to accomplish the necessary edge tracking.

4.2 Mesh Generation

It was necessary to divide the pipe cross sectional area into different segments to obtain velocity and void distribution profiles. A concentric cylindrical element was created and the element divided into ten sections. This cylindrical element was then super-imposed on the pipe. Figure 4.2 shows a schematic of this procedure. For the sake of clarity, only four sections are shown in the figure. The length of the element was determined by the velocity in the slug. It was chosen to cover the particular distance travelled by the gas bubbles between consecutive images. Programs normalmesh.c, and mesh.c, listed in Appendix B, were written for this purpose.

Jepson (1987) in his experiments on stationary slugs, successfully used a division of ten sections across the cross area of the pipe for determining void and velocity profiles. Consequently this number was chosen as the appropriate number of sections.
Figure 4.2: Schematic view of mesh generation.
4.3 Slug Translational Velocity

Figure 4.3 shows the motion of the slug front from image to image. The leading edge of the slug at the top of the pipe does not move with constant velocity. It is seen that the shape of the slug front changes significantly in each image. As the slug front scoops up liquid, its leading boundary grows rapidly and touches the top of the pipe. This leads to a sudden acceleration of the leading edge. Hence, to obtain an accurate measurement of the slug translational velocity, it is necessary to track the slug front over a sufficient number of frames. The second camera, positioned further back from the flow, was used for this purpose. Typically, the slug front was tracked over twelve to eighteen frames. In each image, the coordinates of the point where the front touches the top of the pipe was recorded. Program vtp.c, listed in Appendix B, was written to do this.

Once the spatial coordinates of the slug front have been obtained, the slug translational velocity, \( v_p \), may be obtained by the following equation:

\[
v_t = \frac{\sum_{i=1}^{n} x_i - x_{i+1}}{n-1} \frac{1}{c_v \cdot 100}
\]  

(4.1)

where, 
- \( v_t \) = translational velocity of slug, m/s
- \( x_i \) = \( x \)-coordinate of slug front in image \( i \), pixels
- \( c_v \) = conversion factor from pixels to cm as defined in section 4.1
- \( n \) = number of frames over which the slug front is tracked
Figure 4.3: Schematic view of translational motion of the slug front.
4.4 Liquid Film Height

As Figure 4.3 shows, there is a stratified liquid film of height $h$, ahead of the slug. Program film.c was written to obtain the film height in this region. The coordinates of the film surface and the bottom of the pipe were recorded with distance and the film height was calculated. The conversion factor for the image was used to convert the film height value from pixels to cm.

The film height in cm is given by:

$$h = \frac{y_c - y_b}{c_v} \quad (4.2)$$

where,

- $h$ = film height, cm
- $y_c$ = y-coordinate of film surface, pixels
- $y_b$ = y-coordinate of pipe bottom, pixels

This data was obtained directly from image analysis using the program film.c listed in Appendix B.

4.5 Velocity Profiles

Figure 4.4 shows a schematic of the motion of gas bubbles within the slug. It is seen that the bubbles move a particular distance in each image. Tracking a single bubble over several images and calculating the distance it has moved, and since each image corresponds to 1/60-th of a second, the bubble velocity can be obtained. Program path.c listed in Appendix B was written for this purpose.
Figure 4.4: Schematic view of bubble motion in slug
In most cases, the bubble was in view for approximately four images. The program was run simultaneously for all four images, and the bubble coordinates in each image were recorded. By tracking bubbles at various heights from the bottom of the pipe, velocity profiles across the cross section of pipe were generated.

To understand the flow mechanisms at different points in the slug, velocity profiles were generated at the slug front, at the end of the mixing zone, in the slug body, and at the slug tail. These correspond to 0 cm, 20-30 cm, 45 cm, and 60-80 cm from the slug front, for the slug velocities studied. All velocity profiles were obtained from water-carbon dioxide studies.

There are two components to the bubble velocity, axial and transverse. The axial velocity of the bubbles can be equated to the local liquid velocity if there is no slip between the gas bubbles and the liquid locally. In this way, a velocity profile for the liquid can be generated from a corresponding profile for the bubbles. In addition, it is possible to obtain a transverse velocity component for the bubbles, since there is a buoyancy force acting on the bubbles.

From the coordinates of the bubbles in each image, the axial and transverse velocities may be calculated as follows:

$$v_x(h) = \sum_{i=1}^{n} \frac{x_i(h)-x_{i+1}(h)}{c_v (n-1) 100}$$  \hspace{1cm} (4.3)

$$v_y(h) = \sum_{i=1}^{n} \frac{y_{i+1}(h)-y_i(h)}{c_v (n-1) 100}$$  \hspace{1cm} (4.4)

where,$
\[ v_x(h), v_y(h) = \text{axial and transverse velocity of bubble at height } h, \text{ m/s} \]

\[ x_i(h), y_i(h) = \text{the x-y coordinates of the bubble at height } h \text{ from the bottom in image } i, \text{ pixels} \]

\[ x_{i+1}(h), y_{i+1}(h) = \text{the x-y coordinates of same bubble in image } (i+1) \]

\[ n = \text{Total number of images over which bubble is tracked} \]

\[ c_v = \text{conversion factor from pixels to cm as defined before} \]

Program vel.c was written to calculate \( v_x(h) \) and program vely.c was written to calculate \( v_y(h) \). Once the local transverse velocity was calculated, an average rise velocity for the bubbles, \( v_r \), was calculated. A Program vyavg.c was written to do this. All of the above programs are listed in Appendix B.

### 4.6 Void Fraction Data

No information regarding the void fraction distribution in slugs is currently available. Hence, a detailed void fraction profile across the cross section of the pipe throughout the slug was generated. Typically, this involved the generation of void fraction distribution profiles every 5-10 cm within the slug. Again, void profiles in slugs were obtained from water-carbon dioxide slug systems.

Figure 4.5 shows a schematic view of the two different types of void structures in slugs. In the mixing zone, there is a great deal of turbulence, and the gas and liquid are well mixed. In this region, the voids are large frothy structures covering the entire cross section of the pipe. This type of voids are shown as Type b in Figure 4.5.
Figure 4.5: Schematic of void structures in slug.
Figure 4.6a: Video image of voids in mixing zone of slug

Figure 4.6b: Video image of voids in slug body
At the end of the mixing zone, the gas is pushed towards the top of the pipe due to buoyancy forces. The degree of turbulence is reduced and the gas is now distributed in the form of bubbles in the liquid. These are the kinds of voids that occur mostly in the slug body. They are classified as Type a. Figure 4.6a and 4.6b show actual video images of the voids.

Figure 4.5 also shows the schematic of the four section mesh depicted in Figure 4.2. This provides an idea of how the void fraction distribution can be obtained using the mesh. A program voidn.c, listed in Appendix B, was written to track the edges of voids within the mesh. Each void was classified as Type b or Type a, and the volume of the void was calculated. This was then used to determine the void fraction.

For Type b void structures, the area of the face seen from the front was first calculated. Since the void is distributed throughout the depth of the cross section, the volume of this type of void structure may obtained by multiplying the face area by the depth of the pipe at that height. This is given by the length of the chord T, as shown in Figure 4.5, and is calculated at the middle point of a particular section. This height h, of the middle point of a section is given simply by, $h_2 - h_1$, where $h_1$ and $h_2$ are the lower and upper bounds of that section. Once h is known, the chord length, T, is calculated as:

$$T = D \sqrt{1 - (2 \bar{h} - 1)^2}$$

(4.5)

where, $T = \text{chord length at height } h, \text{ m}$

$D = \text{pipe diameter, m}$

$\bar{h} = \frac{h}{D} \text{ at mid. pt. of a particular section of the mesh}$
The front face area as seen by the camera is calculated from the coordinates \((x_v, y_v)\) of the boundary points of the void profile as follows:

\[
A_f = \sum_{i=0}^{n-1} (x_i - x_{i+1}) (y_{i+1} - y_i)
\]  

(4.6)

where, 
- \(A_f\) = area of front face of void as viewed by the camera, \(m^2\)
- \(n\) = total number of points describing the boundary edge of void

The volume of the type b void structure is then given by the product of equations (4.5) and (4.6).

Most of the type a voids are spherical bubbles, the volume of these bubbles were calculated using the formula for the volume of a sphere. The face area of each bubble was calculated using equation (4.6). The radius of the sphere was then calculated from this projected area and the volume of the sphere was generated.

Next, the volume of each section of the mesh was calculated. The area of any section of pipe at height \(h\) from the bottom can be calculated as follows:

\[
A = \frac{D^2}{4} \left[ \pi - \cos^{-1}(2\bar{h} - 1) + (2\bar{h} - 1) \sqrt{1 - (2\bar{h} - 1)^2} \right]
\]

(4.7)

where, 
- \(A\) = Area of pipe section with height \(h\), \(m^2\)
- \(\bar{h}\) = \(h/D\), the dimensionless height of section

Hence in Figure 4.5, the area section of section 1 at height \(h_1\) and the cumulative areas of sections 1 and 2 (height \(h_2\)) can be calculated using equation (4.7). The area of section 2 is then obtained by subtracting the area of section 1 from the cumulative area. The areas of all
ten sections of the mesh can be obtained in this manner. The volume of each section is then the product of its area and the length of the mesh.

Knowing the volume of each section of the mesh and the void volume in each section the void fraction profile can be calculated. The average void fraction is then simply the ratio of the total void volume to the total volume of the cylindrical mesh. Programs voidar.c and voidfr.c, listed in Appendix B were written to accomplish all of the above.

4.7 Slug length and mixing length

In this visualisation study, the number of images during which the slug remains in view, starting from the slug front to the slug tail, was used to determine the slug length. Knowing this, and the slug velocity, the slug length can be computed. Since each image corresponds to 1/60-th of a second, the number of images multiplied by the slug velocity yields the slug length. The slug length was confirmed using the second camera.

The mixing length is difficult to define. In this study, it is based on two criteria. It was found that beyond a certain distance into the slug, the void fraction distribution in the slug reaches a near steady-state profile. The minimum distance at which this occurred was defined as the mixing zone. Visually, this also corresponds to the end of the frothy, turbulent, highly aerated part of the slug. This distance was also calculated by the same manner as above. The number of images for which this turbulent part of the slug was in view multiplied by the slug velocity was taken as a measure of the mixing length.
4.8 Pressure Data

The pressure data obtained was reduced to an "equivalent head" of pressure as follows:

\[ P = h_L \cdot \rho_L \cdot g \]  \hspace{1cm} (4.8)

where,

- \( P \) = pressure in Pa
- \( h_L \) = equivalent head of pressure
- \( \rho_L \) = density of liquid
- \( g \) = acceleration due to gravity

This equivalent height of pressure is necessary to correlate with the Froude Number, as will be described later.
CHAPTER 5

RESULTS AND DISCUSSION

5.1 Slug Translational Velocity

Figures 5.1a and 5.1b show the variation of slug translational velocity, \( v_t \), with slug velocity, \( v_s \). In Figure 5.1a, results presented by Kouba (1986) are also given. It can be seen that the ratio of \( v_t \) with \( v_s \) decreases with increase in slug velocity. At low slug velocities, the maximum ratio of \( v_t \) to \( v_s \) is about 1.75 and this decreases to about 1.2 at higher slug velocities. These results are similar to those reported by Kouba (1984). Figure 5.1b indicates similar results for ARCOPAC90\textsuperscript{TM}-carbon dioxide slugs.

Figures 5.1c shows the variation of the average ratio of \( v_t \) to \( v_s \) for both water and ARCOPAC90\textsuperscript{TM}-carbon dioxide slug systems. These agree with those of Kouba (1986). A model for slug translational velocity similar to that of Kouba (1986) is also included in Figure 5.1c. The model is written as:

\[
  v_t = (1 + C) v_s + v_d
\]  

(5.1)

where,

- \( v_t \) = slug translational velocity, m/s
- \( v_s \) = slug velocity, m/s
- \( v_d \) = drift velocity, m/s
- \( C \) = constant, usually equal to 0.2

As shown by Jepson (1989), for flow conditions in the slug, the slug velocity \( v_s \) is equal to the mixture velocity, \( v_m \). By considering mass balances on both liquid and gas, the mixture velocity is found to be the sum of the liquid and gas superficial velocities:
Figure 5.1a: Variation of slug translational velocity with slug velocity
Water-CO₂ system

Figure 5.1b: Variation of slug translational velocity with slug velocity
ARCOPAC90™-CO₂ system
Figure 5.1c: Modelled variation of slug translational velocity with slug velocity

Figure 5.1d: Variation of the drift velocity, $v_d$, with slug velocity
\[ v_{sl} + v_{sg} = v_s \]  \hspace{1cm} (5.2)

where, \( v_{sl} \) = superficial liquid velocity, m/s = \( q_{L} / A \)
\( v_{sg} \) = superficial gas velocity, m/s = \( q_{G} / A \)
\( q_{L}, q_{G} \) = liquid and gas volumetric flow rate, m\(^3\)/s
\( A \) = cross sectional area of pipe, m\(^2\)

In this study, the drift velocity, \( v_d \), is estimated by a nonlinear regression fit applied to the data in Figure 5.1c. This gives:

\[ v_d = \exp(-c_1 v_s) \]  \hspace{1cm} (5.3)

where, \( c_1 = 0.7 \)

Equation (5.3) is preferred in this study due to its simplicity. The values computed by this equation are very similar to those developed by Kouba (1986). It also predicts slug translational velocity for large diameter pipes. As reported by Crowley et.al. (1988), using data from 17 cm pipe, the drift flux model correctly predicted ratios of \( v_t / v_s \) for their large diameter pipe.

5.2 Liquid Film Height

Figure 5.2a and 5.2b show the variations of liquid film height ahead of the slug as a function of the slug velocity for water-carbon dioxide (CO\(_2\)) and ARCOPAC90\(^{TM}\)-CO\(_2\) systems.
Figure 5.2a: Variation of liquid film height with slug velocity
Water-CO$_2$ system

Figure 5.2b: Variation of liquid film height with slug velocity
for ARCPAC90™-CO$_2$ system
Figure 5.2c: Variation of average liquid film height with slug velocity
The liquid film height measurements ahead of the slug vary substantially. The presence of large amplitude three dimensional roll waves on the liquid film between slugs give large spacial variations in the liquid film height. Similar variations are seen for ARCOPAC90™.

The non dimensional average liquid film heights in all cases are between 0.3 and 0.4. This is in agreement with the results shown by Kouba (1986) for a 7.6 cm pipe, and Jepson and Taylor (1988) for a 300 mm diameter pipe at similar velocities.

5.3 Velocity Profiles in Slugs

As mentioned earlier, velocity profiles were generated at various distances into the slug for water-carbon dioxide slug systems.

Figure 5.3a to 5.3d show the velocity profiles at various distances into the slug for a superficial liquid velocity of 0.2 m/s and a superficial gas velocity of 1.07 m/s. Figure 5.3a shows the profile at the slug front. It is seen that the high degree of turbulence there results in a virtually flat velocity profile with a range from about 1 m/s to 1.3 m/s all across the pipe cross sectional area.

Figures 5.3b and 5.3c show the velocity profile 20 cm and 45 cm into the slug. At 20 cm into the slug, it is seen that a boundary layer is developing with a maximum velocity of 1.3 m/s being at the center of the pipe. At 45 cm into the slug, Figure 5.3c shows that the boundary layer is now well established. The profile is similar to that at 20 cm, with a maximum velocity of about 1.2 m/s near the center.
Figure 5.3a: Velocity Profile at the Slug Front
Vsl = 0.2 m/s, Vsg = 1.07 m/s

Figure 5.3b: Velocity profile 20 cm into slug
Vsl = 0.2 m/s, Vsg = 1.07 m/s
Figure 5.3c: Velocity Profile 45 into slug
\( V_{sl} = 0.2 \) m/s, \( V_{sg} = 1.07 \) m/s

Figure 5.3d: Velocity profile at the slug tail
\( V_{sl} = 0.2 \) m/s, \( V_{sg} = 1.07 \) m/s
Figure 5.3d shows the velocity profile at the slug tail. In this case, the velocity profile has not changed substantially, indicating the maintenance of fully developed flow at the tail of the slug.

Figures 5.4a to 5.4d show the velocity profile for a liquid superficial velocity of 0.3 m/s, and a superficial gas velocity of 1.07 m/s. It is seen from Figure 5.4a that the high local turbulence level has distorted the flat velocity profile. Here the maximum velocity at the center is approximately 1.4 m/s.

It is seen from Figure 5.4b that at a distance of 20 cm into the slug, the boundary layer is again developing. The maximum velocity is at the center of the pipe and has a velocity of 1.3 m/s. Figure 5.4c shows that at a distance of 45 cm into the slug, the boundary layer is fully developed. At the slug tail however, the effect of the gas behind the slug is beginning to be felt. This is shown in Figure 5.4d. The gas pocket behind the slug is now accelerating the liquid near the top of the pipe and this effect is felt on the structure of the boundary layer.

Figures 5.5a to 5.5c show the velocity profiles for a superficial liquid velocity of 0.4 m/s and a superficial gas velocity of 1.43 m/s. The same effects as before are seen. In Figure 5.5a a boundary layer is seen to be developing at the slug front but is distorted by the extreme turbulence at the slug front. The maximum velocity at the center in this case is as high as 2.2 m/s. It is seen from Figure 5.5b that at 30 cm into the slug, the boundary layer has become fully developed with a center line velocity of about 1.8 m/s. Figure 5.5c indicates that the boundary layer is again distorted at the slug tail due to the effect of the gas pocket behind it.

From the above analysis, it may be said that a fully developed boundary layer profile is a reasonable assumption for the velocity profile in the slug body after the mixing zone.
Figure 5.4a: Velocity Profile at the Slug Front
\( V_{sl} = 0.3 \, \text{m/s}, \quad V_{sg} = 1.07 \, \text{m/s} \)

Figure 5.4b: Velocity profile 20 cm into slug
\( V_{sl} = 0.3 \, \text{m/s}, \quad V_{sg} = 1.07 \, \text{m/s} \)
Figure 5.4c: Velocity profile 45 cm into slug
\( V_{sl} = 0.3 \text{ m/s}, \ V_{sg} = 1.07 \text{ m/s} \)

Figure 5.4d: Velocity Profile at the Slug Tail
\( V_{sl} = 0.3 \text{ m/s}, \ V_{sg} = 1.07 \text{ m/s} \)
Figure 5.5a: Velocity Profile at the slug front
\( V_{sl} = 0.4 \text{ m/s}, \ V_{sg} = 1.43 \text{ m/s} \)

Figure 5.5b: Velocity Profile 30 cm into slug
\( V_{sl} = 0.4 \text{ m/s}, \ V_{sg} = 1.43 \text{ m/s} \)
Figure 5.5c: Velocity Profile at the slug tail
V_{sl} = 0.4 m/s, V_{sg} = 1.43 m/s
This profile begins to distort at the tail end of the slug due to the effects of the gas pocket behind the slug. Hence, a profile may be assumed for the velocity in the slug from the end of the mixing zone. Dukler et al. (1983) used a one-seventh power law profile to describe the velocity profile in the slug. The law is written as:

\[ v(y) = v_o \left( \frac{y}{\delta} \right)^{1/7} \]  

(5.4)

where, \( v(y) \) = velocity at any distance \( y \) from the pipe wall, m/s
\( v_o \) = center line velocity, m/s
\( \delta \) = thickness of the boundary layer, m

It is seen that at the end of the mixing zone, the profile can be assumed to be fully developed. It is noted that the centerline velocity, \( v_o \), in all the above cases is very close to the slug velocity \( v_s \), given by equation (5.2).

Consequently, following Dukler et al. (1983), a one-seventh power law profile, with the centerline velocity set equal to the slug velocity according to Equation (5.2) can be used to describe the liquid velocity profile from the end of the mixing zone in slugs.

5.4 Void Fraction Distribution in Slug Body

Figures 5.6a to 5.6d show the void fraction distribution across the cross section of the pipe in the slug body for superficial liquid velocities of 0.2 m/s, 0.3 m/s and 0.4 m/s, and superficial gas velocities of 1.07 m/s and 1.43 m/s respectively.
Figure 5.6a: Void fraction profile across cross section of pipe in slug body
Water-CO\textsubscript{2} slug systems
\( V_{sl} = 0.2 \, \text{m/s}, \, V_{sg} = 1.07 \, \text{m/s} \)

Figure 5.6b: Void fraction profile across cross section of pipe in slug body
Water-CO\textsubscript{2} slug system
\( V_{sl} = 0.3 \, \text{m/s} \, \) \( V_{sg} = 1.07 \, \text{m/s} \)
Figure 5.6c: Void fraction profile across cross section of pipe in slug body
Water-CO₂ slug system
V₁ = 0.3 m/s, V₂ = 1.43 m/s

Figure 5.6d: Void fraction profile across cross section of pipe in slug body
Water-CO₂ slug system
V₁ = 0.4 m/s, V₂ = 1.43 m/s
It is seen that the void fraction increases in a nonlinear manner from the bottom to the top of the pipe. There is little difference in void fraction in the lower half of the pipe. In the upper half of the pipe, however, there is a significant increase in void fraction. This is expected, since at the end of the mixing zone, the gas moves towards to the top of the pipe due to buoyancy forces. However, the turbulence in the slug holds some gas bubbles in the mainstream of the flow. As noted before, most of the gas in this region of the slug is now distributed in the form of discrete bubbles.

It is seen from Figure 5.6a, that for a superficial liquid velocity of 0.2 m/s, and a superficial gas velocity of 1.07 m/s, the void fraction in the lower half of the pipe is less than ten percent. It increases to about 20 percent at a h/D of 0.9, and then increases rapidly to values of 60 to 90 percent in the top tenths of the pipe.

Figure 5.6a shows that the void fraction at the top of the pipe increases with distance into the slug. At a distance of 10 cm into the slug the void fraction at the top of the pipe is about thirty percent. This increases to about fifty percent at a distance of about 40 cm into the slug. Near the tail of the slug, at a distance of 75 cm, this increases to more than eighty percent. This shows that a thin gas film is formed at the very top of the pipe and is one of the characteristics of slug flow.

Figure 5.6b shows the void fraction variation for a superficial liquid velocity of 0.3 m/s, and a superficial gas velocity of 1.07 m/s. In this case the void fraction is less than twenty percent in the lower part of the pipe. It increases to about forty percent at a h/D of 0.8, and then increases to about fifty to eighty percent near the top of the pipe. This again shows the formation of a thin film of gas at the top of the pipe.
Figure 5.6c shows the void fraction variation for a superficial liquid velocity of 0.3 m/s, and a superficial gas velocity of 1.43 m/s. The trend in this case is the same as before in the lower half of the pipe. However, the void fraction has increased in the upper section of the pipe. At a distance of about 40 cm into the slug, the void fraction at the top of the pipe is about sixty percent. At the tail of the slug, at a distance of about 70 cm, this has increased to a value of more than ninety percent.

Figure 5.6d shows the void fraction variation for a superficial liquid velocity of 0.4 m/s, and a superficial gas velocity of 1.43 m/s. The trend is similar to those in Figures 5.6a to 5.6c but higher void fraction are present in the lower section of the pipe.

It is noted that all of the over one hundred and fifty void profiles at various distances in the slug body that were generated had the same shape as shown in Figures 5.6a to 5.6e. It is, therefore, reasonable to assume that they may be described by a single equation as follows:

$$ a(\bar{h}) = \frac{a \bar{h}}{b (1 - \bar{h})} $$

(5.5)

where, \( \bar{h} \) = the nondimensional distance from the bottom of the pipe

a, b = nonlinear regression coefficients.

Figures 5.7a and 5.7b show that a nonlinear regression fit to the data gives the variation of coefficient 'a' and the ratio of coefficients 'b/a' in Equation (5.6). It is seen that the values of coefficient 'a' are less than 0.2, and the ratio of coefficients 'b/a' lie between 0.7 and 1.0. A mean value of 0.1 for coefficient 'a' and a value of 0.8 to 0.85 for the ratio of 'b/a' are reasonable values to use to approximate the void fraction profile in the slug body for all
Figure 5.7a: Regression coefficient 'a' as a function of distance for various slug velocities

Figure 5.7b: Ratio of regression coefficient 'b/a' as a function of distance for various slug velocities
cases. These are the recommended values to use with Equation (5.6) in void fraction calculations in the slug body.

Jepson (1986), Zhou et al. (1992) generated void fraction data for stationary slugs. The void fraction profiles given in Figures 5.6a to 5.6e and Equation (5.5) agree with the void profile given by these authors.

5.5 Liquid Holdup in Slugs

Liquid holdup in the slug is determined by subtracting the void fraction in the slug from unity. Figures 5.8a to 5.8d show the variations of liquid holdup along the length of the slug. As is expected, in all cases, the liquid holdup also increases in a nonlinear manner, from the front of the slug to the tail. The liquid holdup in the body of the slug tends towards a constant value.

Figure 5.8a shows the variation of the average liquid holdup across a cross section of pipe as a function of distance into the slug for a superficial liquid velocity of 0.2 m/s, and a superficial gas velocity of 1.07 m/s. It is seen that the average liquid holdup varies substantially near the slug front. Values from ninety percent to as low as 30 percent are noted within the first 15 cm of the slug. This corresponds to the flow in the mixing zone in the slug. As mentioned previously, Jepson (1987) showed that gas was entrained and released into the slug in form of pulses of bubbles at a definite frequency. This explains the variation in the liquid holdup in this region of the slug.
Figure 5.8a: Model for liquid holdup variation in slug Water-CO₂ slug system
Vsl = 0.2 m/s, Vsg = 1.07 m/s

Figure 5.8b: Model for liquid holdup variation in slug Water-CO₂ slug system
Vsl = 0.3 m/s, Vsg = 1.07 m/s
Figure 5.8c: Model for liquid holdup variation in slug
Water-CO₂ slug system
Vsl = 0.2 m/s, Vsg = 1.43 m/s

Figure 5.8d: Model for liquid holdup variation in slug
Water-CO₂ slug system
Vsl = 0.3 m/s, Vsg = 1.43 m/s
Figure 5.8e: Model for variation of liquid holdup in slug Water-CO$_2$ slug system

$V_{sl} = 0.4 \text{ m/s, } V_{sg} = 1.43 \text{ m/s}$
Kouba (1986) proposed that there is a maximum void fraction (or minimum liquid holdup) in the mixing zone. If it is assumed that bubbles assemble themselves in a manner similar to crystallization phenomena in metallic structures, then the concept of "packing efficiency" in metals may be applied to the gas bubble arrangement. The maximum packing efficiency of a body centered cubic structure is 0.68, while that of a face centered cubic structure and a hexagonal close packed structure is 0.74. Therefore, he postulated that a maximum void fraction exists in the mixing zone somewhere around seventy percent. This is borne out for slug #3 in Figure 5.8a and the video image in Figure 4.6a. Slug #3 provides void fraction data at the slug front within a pulse of bubbles.

Beyond 20 cm, it is seen from Figure 5.8a, that the liquid holdup increases to about ninety percent at 35 cm into the body of the slug. This also provides an insight into the reason for the change in velocity profile with distance into the slug body. With the void profile reaching a quasi-steady state, the liquid boundary layer has a chance to fully develop.

Figure 5.8b shows the variation of the average liquid holdup across a cross section of pipe as a function of distance for a superficial liquid velocity of 0.3 m/s, and a superficial gas velocity of 1.07 m/s. It is seen that the same trend as Figure 5.8a is followed. The liquid holdup oscillates from eight percent just behind the slug front to a minimum of about forty to fifty percent within the first 20 cm of the slug. Beyond this region it increases and reaches a terminal value of about ninety percent within 40 cm into the slug.

Figure 5.8c shows the variation of the average liquid holdup across a cross section of pipe as a function of distance for a superficial liquid velocity of 0.2 m/s, and a superficial gas velocity of 1.43 m/s. In this case, the liquid holdup drops from about ninety percent at the
slug front to a minimum of twenty five percent in the mixing zone. There is tremendous
turbulence in this case, and the frequency of the release of bubbles has become high enough
to generate a continuous frothy mixing zone with low values of liquid holdup. It is seen that
the liquid holdup continuously increases in this case. The quasi-steady state is not reached
until the very end of the slug.

Figure 5.8d shows the variation of the average liquid holdup across a cross section of pipe
as a function of distance for a superficial liquid velocity of 0.3 m/s, and a superficial gas
velocity of 1.43 m/s. In this case the oscillation of the liquid holdup near the slug front is
much more pronounced. The liquid holdup oscillates between forty five percent and seventy
five percent before rising asymptotically to a value of eighty percent.

Figure 5.8e shows the variation of the average liquid holdup across a cross section of pipe
as a function of distance for a superficial liquid velocity of 0.4 m/s, and a superficial gas
velocity of 1.43 m/s. The same trend as before is observed. The liquid holdup increases from
about forty percent near the slug front asymptotically to a value close to eighty percent near
the tail of the slug.

An inspection of the holdup variation within the slug for the various sets of velocities
show that in all cases, these variations resemble the response of a second order system to a
step input (Coughanour and Koppel, (1982)).

They show that a second order process system is one that may mathematically described
by the following differential equation:
\[
\tau^2 \frac{d^2 Y}{dt^2} + 2\zeta \tau \frac{dY}{dt} + Y = X(t) \tag{5.6}
\]

where, \( \tau = \) time constant of the second order system
\( \zeta = \) damping coefficient characterising the process
\( X(t) = \) step input to second order system

A second order process is usually characterised by a viscous damping force and an elastic spring constant. A classical example is the rheology of a viscoelastic fluid. \( \tau \) and \( \zeta \) are usually some ratio of these forces.

The response of the second order system is dependent on the value of \( \zeta \). Three cases arise. For \( \zeta < 1 \), the response is given by:

\[
Y(t) = A \left[ 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \tau} \sin \left( \sqrt{1-\zeta^2} \frac{t}{\tau} + \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \right) \right] \tag{5.7}
\]

For the case when \( \zeta = 1 \), the response is:

\[
Y(t) = A \left[ 1 - \left(1 + \frac{t}{\tau}\right) e^{-\zeta \tau} \right] \tag{5.8}
\]

and, for \( \zeta > 1 \), the response becomes:

\[
Y(t) = A \left[ 1 - e^{-\zeta \tau} \left( \cosh \sqrt{\zeta^2-1} \frac{t}{\tau} + \frac{\zeta}{\sqrt{\zeta^2-1}} \sinh \sqrt{\zeta^2-1} \frac{t}{\tau} \right) \right] \tag{5.9}
\]

where, \( A = \) The amplitude of the step change
Figure 5.9 shows an example of the response of a classical second order process system to a step input.

If Figures 5.8a to 5.8e are examined closely, it is found that the response curves defined by equations (5.8) to (5.10) fit the data for the liquid holdup in the slug very well. Jepson (1987) showed that the slug front is a hydraulic jump. It appears that the hydraulic jump at the slug front acts as a step input to the liquid film. The resultant variation of liquid film holdup in the slug then behaves as a second order system.

The values of $\zeta$ and $\tau$ were estimated by a best fit regression curve. In each case, an appropriate $\zeta$ value was used as an initial guess with the value of $\tau$ fixed. A program reg.f, listed in Appendix B was written for this purpose. The values of $\zeta$ and $\tau$ can then be equated to ratios of dimensionless numbers characterising the slug flow.

It is seen from Figure 5.8d that near the slug front, a sinusoidal wave pattern exists for the liquid holdup. The remnants of this wave can be seen in the other cases as well, and the initial data point is not predicted by the fitted curve.

It will be shown in the next chapter that $\zeta$ and $\tau$ both can be expressed by dimensionless parameters controlling the flow. These are the Reynolds Number, Eotvos Number, and Froude Number.
Figure 5.9: Response of a second order process to a step input.
5.6 Transverse Velocity in Slug Body

Figures 5.10a to 5.10c show the transverse velocity profiles at different distances in the slug body. Due to buoyancy forces the gas bubbles are pushed towards the top of the pipe. This gives rise to a transverse velocity profile for the gas bubbles across the section of the pipe.

It is seen that the transverse velocities are mostly in the range of 0-0.15 m/s. There are some transverse velocities that are slightly negative, indicating that the bubbles in these cases have moved in a slightly downward direction. This is especially true near the slug tail. This is explained by the presence of the gas pocket following the slug tail. This gas tends to accelerate the liquid near the top of the pipe, causing a small downward motion towards the bottom of the pipe.

Figures 5.11a to 5.11c show the variation of the average transverse or rise velocity as a function of distance into the slug. It is seen that there is very little variation in the average rise velocity with distance into the slug and this average lies around 0.07 m/s. This result is later used for the calculation of the drag coefficient in the slug body.

5.7 Mixing Length

Figure 5.12 shows the variation of the mixing length in the slug as a function of slug velocity for water-carbon dioxide slug systems. Table 5.1 shows the distribution of mixing lengths for Figure 5.12.

Mixing lengths are difficult to measure. In this study, end of the mixing zone was determined by the minimum distance into the slug where the void profile could be described
Figure 5.10a: Transverse velocity profile at different distances into slug
\( V_{sl} = 0.2 \, \text{m/s}, \, V_{sg} = 1.07 \, \text{m/s} \)

Figure 5.10b: Transverse velocity profile at different distances into slug
\( V_{sl} = 0.3 \, \text{m/s}, \, V_{sg} = 1.47 \, \text{m/s} \)
Figure 5.10c: Transverse velocity profile at different distances into slug

$V_{sl} = 0.4 \text{ m/s, } V_{sg} = 1.43 \text{ m/s}$
Figure 5.11a: Average transverse velocity at different distances into slug
\( V_{sl} = 0.2 \text{ m/s}, \ V_{sg} = 1.07 \text{ m/s} \)

Distance from the slug front, cm

Figure 5.11b: Average transverse velocity at different distances into slug
\( V_{sl} = 0.3 \text{ m/s}, \ V_{sg} = 1.07 \text{ m/s} \)
Figure 5.11c: Average transverse velocity at different distances into slug

\[ V_{sl} = 0.4 \text{ m/s}, \ V_{sg} = 1.43 \text{ m/s} \]
Figure 5.12: Variation of mixing length with slug velocity
Water-CO₂ slug system

Slug velocity, m/s

Mixing Length, m
by Equation (5.5) when the void fraction distribution reached a quasi-steady profile. Visually,
this also corresponds to the end of the frothy, highly turbulent region of the slug in the video
image.

Table 5.1: Variation of mixing length as a function of slug velocity for water-carbon dioxide
slugs

<table>
<thead>
<tr>
<th>Slug velocity, m/s</th>
<th>Mixing length, m</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
<td>Maximum</td>
<td>Mean</td>
<td>St. Deviation</td>
</tr>
<tr>
<td>1.27</td>
<td>0.11</td>
<td>0.32</td>
<td>0.21</td>
<td>0.067</td>
</tr>
<tr>
<td>1.37</td>
<td>0.11</td>
<td>0.28</td>
<td>0.18</td>
<td>0.067</td>
</tr>
<tr>
<td>1.63</td>
<td>0.27</td>
<td>0.44</td>
<td>0.3</td>
<td>0.058</td>
</tr>
<tr>
<td>1.73</td>
<td>0.11</td>
<td>0.41</td>
<td>0.26</td>
<td>0.076</td>
</tr>
<tr>
<td>1.83</td>
<td>0.18</td>
<td>0.45</td>
<td>0.28</td>
<td>0.069</td>
</tr>
<tr>
<td>2.7</td>
<td>0.23</td>
<td>0.50</td>
<td>0.36</td>
<td>0.069</td>
</tr>
<tr>
<td>3.2</td>
<td>0.43</td>
<td>0.85</td>
<td>0.70</td>
<td>0.13</td>
</tr>
</tbody>
</table>

It is seen from Table 5.1 that the variation in mixing lengths in all the cases are within
±1.5 standard deviations. In general, the mixing length increases with an increase in the slug
velocity. At a slug velocity of 1.27 m/s, the mixing length is 0.2 m. This increases to about
0.7 m at a slug velocity of 3.2 m/s.

The length of the mixing zone is determined by the release of pulses of bubbles behind the
slug front. As mentioned previously, the slug front overruns slow moving liquid film ahead
of it and assimilates it into the slug. This creates a mixing vortex behind the slug front. The
assimilation of the liquid film accelerates the slug front and it jumps to the top of the pipe, and
entrains large amounts of gas in the process. This gas is trapped in the mixing vortex and is
released into the mixing zone in the form of pulses of bubbles. The mixing of gas and liquid
in this region creates large amounts of turbulence resulting in the variation of the length of the mixing zone. Figure 5.12 also shows the mixing length predicted by Dukler and Hubbard (1975). It is seen that the model does not predict the length of the mixing zone. The model was developed in terms of velocity heads to explain the assimilation of the liquid film into the slug. It is seen that the length of the mixing zone is determined by the mixing of gas and liquid and not the liquid alone.

5.8 Total Slug Length

Figures 5.13a and 5.13b show the variations of total slug length as a function of slug velocity for water-carbon dioxide slug and ARCOPAC90™-carbon dioxide slug system. Tables 5.2 and 5.3 list the variations for the two systems.

Table 5.2: Variation of total slug length with slug velocity for water-carbon dioxide slugs

<table>
<thead>
<tr>
<th>Slug velocity, m/s</th>
<th>Total slug length, m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
</tr>
<tr>
<td>1.27</td>
<td>.44</td>
</tr>
<tr>
<td>1.37</td>
<td>.34</td>
</tr>
<tr>
<td>1.63</td>
<td>.51</td>
</tr>
<tr>
<td>1.73</td>
<td>.35</td>
</tr>
<tr>
<td>1.83</td>
<td>.43</td>
</tr>
<tr>
<td>2.7</td>
<td>.73</td>
</tr>
<tr>
<td>3.2</td>
<td>1.01</td>
</tr>
</tbody>
</table>
Figure 5.13a: Variation of slug length with slug velocity
Water-CO₂ slug system

Figure 5.13b: Variation of slug length with slug velocity
ARCOPAC90-CO₂ slug system
It is seen that the slug length is distributed about the mean value within ±2 standard deviations. In general, the total slug length increases with an increase in slug velocity, from 0.7 m at a slug velocity of 1.27 m/s to about 1.3 m at a slug velocity of 3.2 m/s.

Table 5.3 shows the variations of total slug length as a function of slug velocity for ARCOPAC90™-carbon dioxide slug system. Again, it is seen that the slug length is within ±2 standard deviations.

Table 5.3: Variation of total slug length with slug velocity for ARCOPAC90™-carbon dioxide slug systems

<table>
<thead>
<tr>
<th>Slug velocity, m/s</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>St. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>.40</td>
<td>.83</td>
<td>.61</td>
<td>.17</td>
</tr>
<tr>
<td>2.3</td>
<td>.46</td>
<td>.76</td>
<td>.67</td>
<td>.14</td>
</tr>
<tr>
<td>2.47</td>
<td>.50</td>
<td>1.07</td>
<td>.73</td>
<td>.17</td>
</tr>
<tr>
<td>3.3</td>
<td>.50</td>
<td>1.10</td>
<td>.78</td>
<td>.16</td>
</tr>
<tr>
<td>4</td>
<td>.60</td>
<td>1.60</td>
<td>.94</td>
<td>.25</td>
</tr>
<tr>
<td>5.88</td>
<td>.98</td>
<td>1.76</td>
<td>1.16</td>
<td>.34</td>
</tr>
</tbody>
</table>

Figure 5.14 shows the variation of slug lengths for different systems. Data from this study is shown along with those of Nicholson et.al. (1978), Kouba (1986), Crowley et.al. (1986), and Jepson (1988). The slug lengths have been represented as a number of pipe diameters, to compare slug lengths from systems of different pipe diameters. Nicholson et.al. perfomed experiments in 2.58 cm and 5.12 cm pipes.
Figure 5.14: Variation of slug lengths with slug velocity as a function of pipe diameter.

- ARCPAC90™
- Water
- Nicholson et.al. (2.58 cm)
- Nicholson et.al. (5.18 cm)
- Crowley et.al. (17 cm)
- Jepson and Taylor (30 cm)
- Kouba (7.6 cm)
It is seen that, in general, the slug length varies from about five to twenty pipe diameters.

The data from Kouba (1986) are twice as large as the rest of the data. As discussed in chapter 2, this is ascribed to gas expansion over the length (418 m) of the pipe. The data in this study for both water and ARCOPAC are very similar to those of others.
CHAPTER 6

MATHEMATICAL MODELS

6.1 Flow Scheme of Model

In this chapter, a mathematical model is developed to relate all the variables described in Chapter 5. Figure 6.1 shows a schematic flow chart of the model development.

Step 1 involves the calculation of the Froude Number in the liquid film ahead of the slug. Knowing the input superficial velocities of liquid and gas, the pipe diameter, and the fluid properties, the liquid film height, $h_{LF}$, and the average velocity of the liquid in the film, $v_{LF}$, are calculated. Using the superficial liquid and gas velocity, the slug translational velocity, $v_t$, is also calculated. These three variables, $v_t$, $v_{LF}$, and $h_{LF}$, are then used to calculate a Film Froude Number.

Step 2 involves the calculation of the Froude Number in the slug. Once the Film Froude Number is calculated, the length of the mixing zone in the slug, $l_m$, and the effective height of the slug, $h_s$, are calculated. Next, knowing the superficial gas and liquid velocity, the slug velocity, $v_s$, is computed. Then, using the void fraction distribution and the velocity profile equations developed in Chapter 5, the effective average liquid velocity in the slug, $v_{LS}$, is calculated. Finally, Using $v_t$, $v_{LS}$, and $h_p$, the Froude Number in the slug is found.

In step 3, the decrease of the liquid velocity in the slug is calculated using pressure drop relations. The decrease in the effective average liquid velocity in the slug is then related to the increase in Froude Number in the slug. The point where the Froude Number equals unity is found, and the slug length is calculated.

In this chapter, the equations involved in the flow chart are developed systematically.
Figure 6.1: Flow Chart for Model Development
6.2 Liquid Film Height

The film height is predicted by the model given by Taitel and Dukler (1976) using Equations (2.4) to (2.8). As mentioned in Chapter 2, the model, with the modification for the interfacial friction factor given by Equation (2.10) adequately predicts the height of the liquid film in stratified flow.

It should be noted that the model is strictly applicable for stratified flow. It yields reasonable results in slug flow if the slug frequency is low. In that case the liquid flow rate as part of the slug is small compared to that as part of the film, and the model predictions for the film height agree with experimental results. In slug flow, as discussed previously, there exist large amplitude roll waves between slugs. As the slug frequency increases, the contributions of these roll waves to the liquid film height increases, as was shown in Figures 5.2a and 5.2b. In such situations, Equations (2.4) to (2.8) do not generally predict the liquid film height, even with the correction factor given by Equation (2.10). It becomes necessary to increase the ratio of the interfacial friction factor, $f_i$, to the gas phase friction factor, $f_g$, to higher values than those estimated by Equation (2.10).

In this study, the interfacial friction factor is estimated as a constant times the gas phase friction factor.

$$f_i = k_1 f_g$$  \hfill (5.4)

where, \hspace{1cm} f_i \hspace{1cm} = \hspace{1cm} \text{interfacial friction factor as used in equation (2.5)} \\
\hspace{1cm} f_g \hspace{1cm} = \hspace{1cm} \text{gas phase friction factor as defined in equation (2.7)} \\
\hspace{1cm} k_1 \hspace{1cm} = \hspace{1cm} 50
6.3 Liquid Film Velocity

As slugs flow in the pipeline, the liquid film velocity between slugs is not constant. This is due to the drainage from the rear of the slug mixing with new incoming liquid. This rebuilds the liquid film to an equilibrium height on which the next slug can propagate. Further, at high gas velocities, waves are formed on the liquid film that affect the film velocity. Approximations are usually made to determine the liquid film velocity.

One model that can be used to predict the film velocity is the stratified model of Taitel and Dukler (1976). Equation (2.6) describes the film velocity for a stratified film of height $h$. This model was developed for stratified flow and neglects the contribution of slugs to the liquid flow rate. Hence, the liquid film velocity can be lower than the value predicted by this model.

Another situation arises when the liquid flow rate is negligible compared with the gas flow rate. In this case, the slug characteristics are controlled by the gas, and the contribution from the liquid film velocity may be neglected.

Using the generalised model of Kouba (1986), described by equation (2.3), we would expect a value somewhere in between the two extremes discussed above. Therefore, an assumption can be made, that on average, the liquid film velocity, $v_{LF}$, will be close to the superficial liquid velocity. Hence this model sets:

$$v_{LF} \approx v_{SL} \quad (6.1)$$

This is used in the model for slug length prediction.
6.4 Film Froude Number Definition

It has been shown that slugs are hydraulic jumps (Jepson, 1988). Their strengths can be determined by the Froude Number ahead of the slug. The Film Froude Number is defined as follows:

$$ Fr_f = \frac{V_t - V_{LF}}{\sqrt{g \cdot h_{EF}}} \quad (6.2) $$

where,

- $V_t$ = translational velocity of the slug, m/s
- $V_{LF}$ = liquid film velocity, m/s
- $h_{EF}$ = effective film height ahead of the slug, m

If $a_{LF}$ is the area occupied by the liquid film and the width of the gas-liquid interface is given by $S_i$, then $h_{EF}$ is given by:

$$ h_{EF} = \frac{a_{LF}}{S_i} \quad (6.3) $$

The film height, $h_{LF}$, is usually expressed as a fraction of the pipe diameter as follows:

$$ \bar{h} = \frac{h_{LF}}{D} \quad (6.4) $$

where,

- $h_{LF}$ = liquid film height fraction, m
- $D$ = pipe diameter, m

Knowing $\bar{h}$, $a_{LF}$ and $S_i$ are determined by the following equations:
\[ a_{LF} = 0.25D^2 \left[ \pi - \cos^{-1}(2\bar{h} - 1) + (2\bar{h} - 1) \sqrt{1 - (2\bar{h} - 1)^2} \right] \quad (6.5) \]

and,

\[ S_1 = D \sqrt{1 - (2\bar{h} - 1)^2} \quad (6.6) \]

where, \( \bar{h} \) is given by Equation (6.4).

The definition of the Film Froude Number as above allows a comparison between hydraulic jumps in open channel flow and slugs in pipelines. This results in slug characteristics being determined by the Film Froude Number.

6.5 Pressure Relationship in Slugs

In a coordinate system moving with the slug front, the momentum equation becomes the same as that for a hydraulic jump. This is given for channel flow by Stoker (1957) as:

\[ \frac{d}{dt} \int_{x_0}^{x_1} p_L a_L u_L \, dx = \int_{a_{LF}}^{a_{LS}} p_{LF} \, dA - \int_{a_L}^{a_{LS}} p_s \, dA \quad (6.7) \]

where,

\[ u_L = \text{average velocity of the liquid, m/s} \]
\[ p_{LF}, p_s = \text{pressures in the liquid film and the slug, Pa} \]
\[ a_{LF}, a_{LS} = \text{areas of the liquid film and the liquid in the slug, m}^2 \]
\[ x_0, x_1 = \text{points in the liquid film and slug respectively, m} \]
\[ a_L = \text{local area of liquid, m}^2 \]
Figure 6.2 describes the physical situation. In a coordinate system moving with the slug front, the relative velocities of the fluids in the slug and in the film are described by \((v_t - v_a)\) and \((v_i - v_{LF})\) respectively, and the slug front itself becomes stationary. Under such a condition, following Stoker (1957), equation (6.7) can be integrated to give:

\[
\rho_L \left[ a_{LS} (v_t - v_{LS})^2 - a_{LF} (v_i - v_{LF})^2 \right] = p_{LF} a_{LF} - p_{LS} a_{LS} + p_{GF} a_{GF} + F_D
\]  

(6.8)

where, \(\rho_L\) = density of liquid, kg/m\(^3\)
- \(a_{LF}, a_{LS}\) = fraction of area occupied by the liquid in film and slug
- \(v_{LF}, v_{LS}\) = average liquid velocity in film and slug, m/s
- \(a_{GF}\) = fraction of area occupied by the gas over the liquid film.
- \(p_{GF}\) = pressure in gas pocket over liquid film, Pa

and,
- \(F_D\) = drag force exerted by the gas bubbles on the liquid, N

A momentum equation for the gas phase can be similarly written:

\[
p_G \left[ a_{GS} (v_t - v_{GS})^2 - a_{GF} (v_i - v_{GF})^2 \right] = p'_o a_{GF} - p_{GS} a_{GS} - p_{GF} a_{GF} - F_D
\]  

(6.9)

where, \(p'_o\) = \(p_{LF} - \rho_O \zeta'_o\) and \(\zeta'_o\) is the depth from the surface of the center of pressure in the gas over which \(p'_o\) acts, m
- \(v_{GS}, v_{GF}\) = gas velocity in the slug and gas pocket over liquid film, m/s
- \(a_{GS}, a_{GF}\) = fraction of area occupied by gas in slug and by gas pocket over liquid film
Figure 6.2: Schematic view of stationary slug
Adding equations (6.8) and (6.9) and neglecting the terms involving \( \rho \), since \( \rho \ll \rho_L \) we get:

\[
\rho_L (a_{LS} (v_t - v_{LS})^2 - a_{LF} (v_t - v_{LF})^2) = p_{LF} - p_{LS}
\]  

(6.10)

Next, dividing throughput by \( \rho_L \), the following expression is obtained:

\[
\frac{(v_t - v_{LS})^2}{a_{LS}} - \frac{(v_t - v_{LF})^2}{a_{LF}} = \frac{p_{LF} - p_{LS}}{\rho_L} = \frac{h_f - h_j}{\rho_L}
\]  

(6.11)

where, \( h_f, h_j \) = hydrostatic head corresponding to the pressures \( p_o, p_1 \)

This can then be rearranged to give:

\[
\frac{(v_t - v_{LS})^2}{gh_j} \frac{h_j}{a_{LS}} - \frac{(v_t - v_{LF})^2}{gh_f} \frac{h_j}{a_{LF}} = 1 - \frac{h_j}{h_f}
\]  

(6.12)

The second term in equation (6.12) provides a basis for the definition of a Froude No. in the slug: \( \frac{(v_t - v_{LF})}{\sqrt{gh_f}} \), is similar to the Film Froude No. defined in Equation (6.2). In open channel flow, the Film Froude Number is defined as:

\[
Fr_f = \frac{v_t - v_{LF}}{\sqrt{gh_f}}
\]  

(6.13)

In pipe flow, the definition of the Film Froude Number must be modified to account for the geometry. In this case \( h_f \) is the height of the liquid film at the center of the pipe. To obtain an effective height over the entire area occupied by the liquid film, Equations (6.4) to (6.6) are
used. The definition of the Film Froude Number is then written as in Equation (6.3). Equation (6.12) is also based on a channel geometry. To use it in pipes, $h_f$ is replaced by $h_{EF}$. The term \((v_i - v_{1F}) / \sqrt{g(h_f)}\) would then be replaced by the term \((v_i - v_{1F}) / \sqrt{g(h_{EF})}\) with a geometric correction factor given by Equations (6.5) and (6.6). This would then be the Froude Number in the film, as defined in Equation (6.3). The term \((v_i - v_{LS}) / \sqrt{gh_j}\), then may be said to be the Froude Number in the slug.

### 6.6 Slug Froude Number Definition

As is indicated from equation (6.12), a Froude No. in the slug may be defined as:

$$Fr_s = \frac{v_i - v_{LS}}{\sqrt{gh_j}} \tag{6.14}$$

where, \(v_{LS}\) = effective average liquid velocity in the slug at any point, m/s

\(h_j\) = effective "height" of the jump, m

It should be remembered, that \(v_{LS}\) is not constant in the slug. As one proceeds into the body of the slug momentum losses occur. Further, the fraction of the total pipe cross sectional area available for liquid flow increases with distance into the slug body. Here the gas is pushed pushed towards the top of the pipe decreasing the overall void fraction. This leads to a decrease in the slug liquid velocity, \(v_{LS}\), and consequently, gives an increase in the slug Froude Number.
6.7 Ratio of Heights in Slugs

A relationship to predict \( h_j \) as function of the Film Froude Number is now needed. Chow (1959) gives a correlation between \( h_F \) and \( h_j \) for channel flow:

\[
\frac{h_j}{h_F} = \frac{1}{2} \sqrt{1+8F_r^2} + 1
\]  
(6.15)

where, \( F_r \) = Film Froude Number in open channel flow as given by Equation (6.13)

As mentioned in Section 6.3, this equation needs to be modified for the geometry of the pipe. Figures 6.3a and 6.3b show the variation of the ratio \( h_j/h_{EF} \) and \( h_j/h_F \) for a stationary slug. \( h_{EF} \) is the effective height of the liquid film, given by Equation (6.4) and \( h_F \) is the height of the film at the center. The data shown in Figures 6.3a and 6.3b were obtained from the single point pressure measurements described in Sections (3.3.1) and (4.8).

Table 6.1 shows the variations of \( h_j/h_{EF} \) as a function of distance into the slug at various Film Froude Numbers for water-carbon dioxide stationary slug systems.

<table>
<thead>
<tr>
<th>Froude Number</th>
<th>0 cm</th>
<th>15 cm</th>
<th>30 cm</th>
<th>45 cm</th>
<th>60 cm</th>
<th>Eq. (6.10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.6</td>
<td>5.2</td>
<td>7.2</td>
<td>7.8</td>
<td>7.8</td>
<td>7.9</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>3.6</td>
<td>9.3</td>
<td>11.4</td>
<td>11.9</td>
<td>11.9</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>5.7</td>
<td>11.1</td>
<td>14.6</td>
<td>14.0</td>
<td>16.4</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td>11.1</td>
<td>15.8</td>
<td>19.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1: Table for \( h_j/h_{EF} \) with distance for different Film Froude Numbers
Figure 6.3a: Variation of equivalent height ratio with distance into slug as a function of Film Froude No. for stationary slug.

Figure 6.3b: Variation of maximum height ratio with distance into slug as a function of Film Froude Number for stationary slug.
Table 6.1 shows that beyond a distance of 45 cm from the slug front, the ratio $h_j / h_{EF}$ varies very closely as the values predicted by Equation (6.15), with $h_{EF}$ used for estimating $h_j$ in the equation.

It is seen that Equation (6.10) provides a reasonable estimate for $h_j$ for slug flow in pipes, if the effective film height, $h_{EF}$ from Equation (6.3) is used. This is due to the curved geometry of pipes. In channels, the depth is uniform across the width. Hence, a single height is sufficient to describe the liquid film. Using Equation (6.3), the effective height of the liquid film is obtained as an effective average of the height over the area occupied by the liquid film. Using this value for the liquid film height in Equation (6.10) gives the correct estimate of the effective height of the slug, $h_j$.

Figure 6.3b shows the ratios of $h_j$ over $h_F$, if the height of the liquid film at the center is used. This is the height of the liquid film, $h_{LF}$. It is seen that at a distance greater than 30 cm into the slug, the ratio of $h_j$ to $h_F$ varies as 6.6, 9, 12, and 13.5. This is the same as the values for Film Froude Numbers. This can be explained as follows; for Film Froude Numbers much greater than unity, Equation (6.10) tends to give a ratio of $h_j / h_F$ close to $\sqrt{2}$ Fr. In slug flow, as was shown in Figures (5.2a), (5.2b) and (5.2c), the liquid film heights vary between 0.25 and 0.4. In such cases, Equation (6.5) and (6.6) give $h_{EF} = 1/\sqrt{2}$. Hence, when $h_F$ is used instead of $h_{EF}$ in slug flow, the ratio becomes equal to the Film Froude Number.

In this study, Equation (6.10) with $h_F = h_{EF}$ is used to estimate the Froude Number in the slug.
6.8 Average Liquid Velocity in Slug

The volumetric flow rate of liquid in the slug is given by:

\[ q_l = \int_A v (1 - \alpha) \, dA \quad (6.16) \]

where \( q_l \) = volumetric flow rate of liquid as part of slug, m\(^3\)/s

The local velocity, \( v \), and the void fraction, \( \alpha \), within the differential area are given by Equations (5.4) and (5.5) respectively. The integral in equation (6.11) is numerically integrated using the trapezoidal rule. The differential area is taken as the area of one section of the mesh as described in Section 4.6 on the analysis of void fraction distribution, and the velocity and liquid fraction

Once \( q_l \) is known the effective average velocity in the slug is calculated as:

\[ v_{LS} = \frac{q_l}{\alpha_{LS}} = \frac{q_l}{A (1 - \alpha_{avg})} \quad (6.17) \]

where, \( \alpha_{avg} \) = average void fraction over a cross sectional area

Once \( v_{LS} \) is known, the Froude Number at any distance in the slug can be calculated using Equation (6.14). The programs Froude.c and Frjump.c, listed in appendix B, were written to calculate the Froude Number at various distances into the slug.

6.9 Other Dimensionless Numbers

The analysis of average liquid holdup in the slug as a function of distance requires the definition of two other dimensionless numbers. They are defined as follows:
\[ \text{Re}_{SL} = \frac{D \, v_d \, \rho_L}{\mu_L} \quad (6.18) \]

and,

\[ Eo = \frac{g \, \rho_L \, D^2}{\sigma} \quad (6.19) \]

where, \( \text{Re}_{SL} \) = Reynolds Number for liquid

\( Eo \) = Eotvos Number for the liquid

\( D \) = pipe diameter, m

\( \sigma \) = surface tension of liquid, N/m

The values of \( \zeta \) are then estimated using the ratios of these dimensionless numbers and the Froude Number as mentioned in Section 5.5.

### 6.10 Slug Length Model

The slug front is a propagating hydraulic jump (Jepson, (1989)) and represents a transition from subcritical flow to supercritical flow. Figure 6.4 shows how the Froude Number varies throughout the film and the slug. The momentum equations for the slug have been written in a coordinate system moving with the front of the slug. This was done to facilitate the mathematical analogy with a hydraulic jump. In such a situation, the flow is supercritical in the film and subcritical in the slug. For a moving slug is observed in a stationary coordinate system, the reverse occurs and the flow is subcritical in the film ahead of the slug and it changes to supercritical in the slug.
The above analysis implies that in a coordinate system moving with the slug front, the Froude Number in the film is always greater than unity indicating supercritical flow, and that in the slug is always less than unity, showing subcritical flow. The Froude Number decreases rapidly at the slug front and then in the slug slowly rises back up to unity once more. The point where it tends to unity again corresponds to the end of the slug and the formation of a new liquid film. The increase in Froude Number according to Equation (6.14) may be used to find the point where the Froude Number in the slug equals unity, and hence the slug length. The slug is broadly composed of a mixing zone, and the slug body. The total slug length is then a sum of the individual lengths of each of these zones.

6.10.1 Length of Mixing Zone

The characteristics of the mixing zone are very difficult to analyse. In this region of the slug, the gas and liquid are being accelerated to the slug velocity, and there is a great deal of turbulence and shear force. Jepson (1987) has shown that this length is proportional to the Film Froude Number ahead of the slug. Hence knowing the Film Froude Number, the mixing length of the slug can be estimated.

The mixing length of the slug has been modelled as an empirical function of the Film Froude Number based on experimental data described in Section 5.9. It was discussed that the variation in the mixing length was due to the large scale turbulence and mixing in this region of the slug. It was seen that the mixing length increased with increasing slug velocities. This can then be correlated to an increase in the Film Froude Number and the length of the mixing zone expressed as a function of the Film Froude Number.
Figure 6.4: Froude No. variations in slug.
The dependence is given by:

\[ l_m = m \cdot Fr_f + c \]  \hspace{1cm} (6.20)

where, \( l_m \) = length of the mixing zone, m
\( m, c \) = linear regression coefficients

### 6.10.2 Slug Body Length

In the slug body, the void fraction distribution is obtained by using equation (5.6) and the velocity profile may be obtained by Equation (5.5). The effective average liquid velocity may be calculated using Equations (6.16) and (6.17). The liquid velocity decreases due to momentum losses. This is expressed by the following relation:

\[ u_o \frac{du_o}{dx} = - \frac{1}{\rho_L} \frac{dp}{dx} \]  \hspace{1cm} (6.21)

where, \( u_o \) = centerline velocity in a moving coordinate system, m/s
\( = v_t - v_o \)

It is noted that the velocity profile in the slug body is similar to that given by the one-seventh power law, with \( v_o \) as the center-line velocity. However, unlike a normal fully developed flow in pipes, the center-line velocity decreases in slugs due to momentum losses. The profile, however, has the same shape throughout the body of the slug, as was seen from Figures (5.3) to (5.5).
The pressure gradient, \( \frac{dp}{dx} \), in the slug body is given by the sum of two components, the pressure drop due to friction alone, \( \frac{dp}{dx_f} \), and, an excess pressure drop due to agitation by the gas bubbles in the slug \( \frac{dp}{dx_g} \).

The pressure drop due to friction is given by the well known equation:

\[
\frac{dp}{dx_f} = \frac{4f P_L v_{LS}^2}{D} \frac{2}{2}
\]  
(6.22)

where, \( f \) = the Fanning friction factor

\( f \) is calculated using a Blasius-type equation, given by Taitel and Dukler (1976):

\[
f = C \left( \frac{D v_{LS} \rho_L}{\mu_L} \right)^n
\]  
(6.23)

where, \( C = 0.046 \)

\( n = -0.2 \)

The term \( \{Dv_{LS} \rho_L / \mu_L\} \) is the Reynolds Number for the liquid \( (Re_{LS}) \) in the slug. Equation (6.23) is applicable for values of \( Re_{LS} \) greater than 2100.

The pressure drop due to agitation by bubbles is more difficult to evaluate. This is given by the expression:

\[
\frac{dp}{dx_g} = \frac{3C_D a_{avg} \rho_L V_f^2}{2d_b} \frac{2}{2}
\]  
(6.24)

where, \( C_D = \) drag coefficient
\[ d_b = \text{diameter of bubble, m} \]

\[ \alpha_{av} = \text{average void fraction across pipe cross section} \]

\[ v_r = \text{measured rise velocity of bubbles in the slug body, m/s} \]

The drag coefficient \( C_D \) is a complicated function of several dimensionless groups.

Wallis (1969) modified the derivation of Peebles and Garber (1953) and used the following:

\[ \text{Re}_b = \frac{\rho_L d_b v_r}{\mu_L} \quad (6.25) \]

where,

\[ \text{Re}_b = \text{bubble Reynolds Number} \]

Also, a dimensionless group \( G_1 \), is defined as follows:

\[ G_1 = \frac{g \mu_L^4}{\rho_L \sigma^3} \quad (6.26) \]

Then a table for the drag coefficient may be used as shown in Table 6.2.

<table>
<thead>
<tr>
<th>Region</th>
<th>Drag Coefficient</th>
<th>Range of Applicability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1</td>
<td>( C_D = 24 \text{Re}_b^{-1} )</td>
<td>( \text{Re}_b \leq 2 )</td>
</tr>
<tr>
<td>Region 2</td>
<td>( C_D = 18.7 \text{Re}_b^{-0.68} )</td>
<td>( 2 \leq \text{Re}_b \leq 4.02 \text{G}_1^{-0.214} )</td>
</tr>
<tr>
<td>Region 3</td>
<td>( C_D = 0.0275 \text{G}_1 \text{Re}_b^4 )</td>
<td>( 4.02 \text{G}_1^{-0.214} \leq \text{Re}_b \leq 3.10 \text{G}_1^{-0.25} )</td>
</tr>
<tr>
<td>Region 4</td>
<td>( C_D = 0.82 \text{G}_1^{0.25} \text{Re}_b )</td>
<td>( 3.10 \text{G}_1^{-0.25} \leq \text{Re}_b )</td>
</tr>
</tbody>
</table>

Hence, the total pressure gradient can be calculated. Starting at the end of the mixing zone, the centerline velocity, \( u_o \), can be updated using Euler's method in Equation (6.21). \( v_{LS} \) can
then be found as before. The Slug Froude Number can then be checked for by Equation (6.14) until it reaches unity and that would yield the length of the slug body. The sum of the mixing length and the slug body length would then yield a measure of total slug length.

The sequence of steps for the calculations of slug characteristics are as follows; knowing the superficial gas and liquid velocities, \( v_{SL} \) and \( v_{SG} \), the translational velocity for the slug \( v \) is calculated using Equations (5.1), (5.2), and (5.3). The liquid film velocity, \( v_{LF} \), is estimated from Equation (6.2). Using the liquid film height, \( h_{LF} \), from Equations (2.4) through (2.8) and (6.1), the Froude Number in the film is calculated by Equation (6.3) and (6.4). The liquid film area, \( a_{LF} \) and gas-liquid interface width, \( S \), in Equation (6.4) are calculated from Equations (6.5) and (6.6). This completes Step 1 in Figure 6.1.

Once the Film Froude Number is known, the slug characteristics may be calculated. The length of the mixing zone in the slug is calculated by Equation (6.20). The effective average liquid velocity, \( v_{LS} \), in the slug is then calculated as a function of the slug length, using Equations (5.5), (5.6), (6.16), and (6.17). The liquid holdup in the slug can be calculated by Equations (5.7) through (5.9). The liquid velocity is allowed to vary with distance by using Equation (6.14). Equation (6.16) is solved using Euler's method, and the liquid velocity is updated at each incremental distance. The total pressure gradient in Equation (6.16) is obtained using Equations (6.17) to (6.21) and Table 6.2. The Froude Number is then calculated at each point using Equations (6.9) and (6.10) with \( h_{F} \) in Equation (6.10) set equal to the \( h_{LF} \). The Froude Number is updated at each point and checked to see if it has reached unity. When the Froude Number in the slug reaches unity, the computation is stopped and the slug length is printed out as the sum of the lengths of the mixing zone and the slug body.
CHAPTER 7
MODEL RESULTS

7.1 Liquid Film Height

Figure 7.1a and 7.1b show the variation of the mean film height ahead of the slug as a function of slug velocity. It is seen that the mean height of the film in both cases lie between 0.3 and 0.4 pipe diameters. This mean height does not change significantly over the range of velocities studied in both cases. The modified Taitel and Dukler model gives a reasonable value of film height prediction. The slight overprediction of the model for the case of water is due to the neglect of liquid flow rate as part of slugs and large roll waves that occur in slug flow. These do not seem to have a major effect for the ARCOPAC90™ slug systems.

7.2 Film Froude Number

Figure 7.2 shows the variation of the Film Froude Number as a function of the slug velocity for both water-carbon dioxide and ARCOPAC90™-carbon dioxide slug systems. It is seen that the lowest Froude Number in the range studied is about 3.8. There is a linear increase in the Film Froude Number with an increase in slug velocity from 3.8 to a maximum of 14. The data for water as well as ARCOPAC90™ slugs fall in the same linear range of values. An increase of slug velocity from 1.5 m/s to 3 m/s results in an increase of Froude Number from 4 to 6. However an increase of slug velocity from 3 m/s to 6 m/s, causes the Froude Number to jump from around 6 to 14. This is explained by the fact that the translational velocity of the slug increases in proportion to the slug velocity but the height of the liquid film ahead of the slug does not change significantly. This causes the momentum
Figure 7.1a: Variation of mean liquid film height fraction with slug velocity
Water-CO₂ slug system

Figure 7.1b: Variation of mean liquid film height with slug velocity
ARCOPAC90™-CO₂ slug systems
Figure 7.2: Variation of Film Froude Number with slug velocity.
term in the Froude Number definition in Equation (6.2) to increase significantly while the gravity term in the denominator remains approximately the same.

It is seen that the effect of viscosity on the Film Froude Number is negligible in this range of slug velocity.

Figure 7.2 also shows the Froude Number calculations for the data of Kouba (1986) and Jepson and Taylor (1988). The data of Kouba falls into the same line as that for the data in the present study. This is expected since both studies were conducted in 7.6 cm pipes. The data for Jepson and Taylor (1988) falls in a lower range of velocities. This is also expected, since their studies were conducted in 30 cm pipe, and for the same slug velocity the Froude Number in a larger diameter pipe is expected to be lower.

It is seen that the Froude Number definition works well for all range of data.

7.3 Froude Number in Slug

Figures 7.3a to 7.3c show the variation of Froude Number with distance in the slug for water-carbon dioxide slug system. Figure 7.3a describes the Froude Number variations for a superficial liquid velocity of 0.2 m/s, and a superficial gas velocity of 1.07 m/s. The Film Froude Number in this case is 3.8. It is seen that the Froude Number decreases rapidly to a value of about 0.8 to 0.9 in the slug at a distance of 20 cm from the slug front. This also corresponds to the end of the mixing zone in this case. The Froude Number then begins to rise slowly in the slug body and tends to a value close to unity near the tail of the slug. The total length of the slug at which the Froude Number tends to unity is about 60 to 80 cm.
Figure 6.2a: Variation of Froude Numbers along length of slug
\( V_{sl} = 0.2 \text{ m/s}, \ V_{sg} = 1.07 \text{ m/s} \)

Figure 7.3b: Variation of Froude No. along length of slug
\( V_{sl} = 0.3 \text{ m/s}, \ V_{sg} = 1.07 \text{ m/s} \)
Figure 7.3c: Variation of Froude Nos. along length of slug

\[ V_{sl} = 0.4 \text{ m/s}, \quad V_{sg} = 1.43 \text{ m/s} \]
Figure 7.3b shows the Froude Number variation in the slug for a superficial liquid velocity of 0.3 m/s, and a superficial gas velocity of 1.07 m/s. The Film Froude Number in this case is also about 3.8. In this case again, it is seen that the Froude Number drops to a value between 0.8 and 0.9 at 20 cm into the slug. In this case, however, there is an oscillation of the Froude Number near the end of the mixing zone which is approximately 30 cm into the slug. This is due to a release of pulses of gas bubbles in the mixing zone as described in Figure 4.6b, and Figure 5.8d. The pulse of bubbles results in increased local void fraction, which gives an increase in the average velocity of the liquid there. In the slug body, these effects are dissipated and the Froude Number rises again to a value close to unity near the tail of the slug. This is at a distance of 40-50 cm into the slug.

Figure 7.3c shows the Froude Number variation in the slug for a superficial liquid velocity of 0.4 m/s, and a superficial gas velocity of 1.43 m/s. The Film Froude Number in this case is also about 4.6. In this case, the increase in the Froude Number value is rapid. As is seen the Froude Number drops rapidly below unity to a value of around 0.7. At this Froude Number, there are high levels of turbulence within the mixing zone. The Froude Number can therefore be expected to oscillate rapidly in this zone. At a distance of 20 cm into the slug, the Froude Number drops to 0.7. From this point, the Froude Number increases rapidly and reaches a value near unity at the tail of the slug. This occurs at approximately 55 cm.

Figure 7.4 shows the variation of the Froude Number from the film to the slug and beyond the tail of the slug for slug velocities of 1.27 m/s, 1.37 m/s, and 1.83 m/s. It is seen that the Froude Number in the film ahead of the slug in each case is of the order of 4 to 5. This decreases rapidly to a value of about 0.7-0.9 within the slug. The Froude Number then
Figure 7.4: Froude Number variations at different distances into the slug Water-CO₂ slug system
gradually increases within the body of the slug and reaches a value close to 1 at the tail of the slug. Beyond the slug tail, it is seen that the Froude Number begins to rise rapidly again. For slug velocities of 1.27 m/s and 1.37 m/s, the Froude Number in the tail rises to 2.5, while for a slug velocity of 1.83 m/s, the Froude Number beyond the tail rises back to about 4.5.

Figures 7.3 and 7.4 prove the existence of a transition in the flow characteristics based on Froude Number. It is seen that when viewed in a coordinate system moving with the front of the slug, the Froude Number in the film is greater than unity. It drops to below unity in the slug and rises again to above unity beyond the slug tail. There is a transition from supercritical flow in the film, to subcritical flow in the slug, and a transition back to supercritical flow at the end of the slug. This point of transition marks the end of the slug.

7.4 Mixing Length of Slug

Figures 7.5 shows a plot of mixing length in the slug as a function of Film Froude Number. From Equation (6.20), the values of the regression coefficients are found to be $m=0.13$, and $c = -0.31$. This gives the length of the mixing zone in metres.

The equation implies that below a Film Froude Number of 2.5, the mixing length will be zero. This is approximately true, since in reality a Froude Number of at least 2-2.5 is required for slugs to exist. It is seen that at a Film Froude Number of about 4.0, the mixing length is about 2 times the pipe diameter. However, at about a Froude Number of 8, the mixing length becomes 10 times the pipe diameter.

It is expected that as the Froude Number increases the mixing length will increase. This is predicted by Equation (6.20) and Figure 7.5. As the Reynolds Number increase for the
Figure 7.5: Variation of mean mixing length with Film Froude Number
Water-carbon dioxide system

\[ l_m = 0.13F_r - 0.31 \]
liquid, it is expected that the mixing length will decrease, owing to lesser turbulence effects. An increase in the Eotvos Number, which includes the effect of surface tension of the liquid, can also be expected to increase the length of the mixing zone.

7.5 Average Liquid Holdup Results

Figures 5.8a through 5.8e showed the variation of the average liquid holdup in the slug. From the definitions of the Reynolds Number, Film Froude Number and Eotvos Number given in sections (6.2) and (6.6), and the knowledge of the mixing length at any given slug velocity the values of $\zeta$ and $\tau$ in Equations (5.8), (5.9) and (5.10) may be estimated.

\[
\tau = \frac{L_m}{4} \\
\zeta = \frac{Fr}{\sqrt{Re/Eo}}
\]  (7.1)

Table 7.1 lists the comparison between the model estimations of $\zeta$ and $\tau$ with the mean values obtained by the nonlinear regression in Figures 5.8a through 5.8e.

Table 7.1: Comparison of model estimations of $\zeta$ and $\tau$ with mean nonlinear regression coefficients in Figures 5.8a-5.8e

<table>
<thead>
<tr>
<th>Slug Velocity m/s</th>
<th>Mean $\zeta$</th>
<th>Model $\zeta$</th>
<th>Mean $\tau$</th>
<th>Model $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.27</td>
<td>1.16</td>
<td>.75</td>
<td>6</td>
<td>7.1</td>
</tr>
<tr>
<td>1.37</td>
<td>.8</td>
<td>.93</td>
<td>6</td>
<td>5.7</td>
</tr>
<tr>
<td>1.63</td>
<td>.74</td>
<td>.71</td>
<td>5</td>
<td>4.34</td>
</tr>
<tr>
<td>1.73</td>
<td>1.3</td>
<td>1.1</td>
<td>9.6</td>
<td>5.4</td>
</tr>
<tr>
<td>1.83</td>
<td>.76</td>
<td>.87</td>
<td>6</td>
<td>6.5</td>
</tr>
</tbody>
</table>
It is seen that there is good agreement between the experimental values and the theoretical model estimations. Further experiments are needed with other fluids before definite conclusions can be drawn. However, it is felt that there is a reasonable trend here, and that the liquid holdup variations in slug flow can be described using this concept.

7.6 Total Slug Length

Figure 7.6a and 7.6b shows the variation of total slug length for water-carbon dioxide and ARCOPAC90™-carbon dioxide slug systems. The model agrees well with the experimental data for the mean slug length in the case of water-carbon dioxide slug systems. The mean slug length varies from about 0.7 m at a slug velocity of 1.3 m/s, to 1.3 m at a slug velocity of 3.2 m/s. This translates from about 10 pipe diameters to about 17 pipe diameters. It should be noted that there is a distribution of the slug length of approximately two standard deviations from the mean, and the model predictions are well within the range of this distribution.

Figure 7.6b shows the results for slug lengths for ARCOPAC90™-carbon dioxide slug system. The mean experimental slug length varies from about 0.6 m at a slug velocity of 1.6 m/s to about 1.2 m at a slug velocity of 6 m/s. The model agrees well with the mean slug length results up to a slug velocity of 3 m/s. Beyond that velocity, it begins to over predict the slug length. Again it is to be noted that there is a distribution of lengths around the mean slug length and the model predictions are within this range.

Figure 7.7 shows the model predictions for the data of Jepson and Taylor (1988) and Crowley et.al. (1988). The lengths vary from about 3 pipe diameters to 17 pipe diameters. It is seen that there is good agreement between the model and the experimental data.
Figure 7.6a: Variation of mean slug length with slug velocity
Water-CO\textsubscript{2} slug system

Figure 7.6b: Variation of mean slug length with slug velocity
ARCOPAC90\textsuperscript{TM}-CO\textsubscript{2} slug system
Figure 7.7: Model predictions for slug lengths in large diameter pipes
CHAPTER 8
CONCLUSIONS

Based on the results from the experimental measurements, visual observations, and mathematical models, the following conclusions are made.

Slug flow characteristics have been studied and modelled for two-phase gas-liquid systems. The effect of liquid properties were investigated by the use of two different liquids, water and ARCOPAC90™.

Experimental conditions maintained in this study were in the slug flow regime. The liquid superficial velocity ranged from 0.2 m/s to 0.7 m/s for water, and 0.15 m/s to 0.88 m/s for ARCOPAC90™. The superficial gas velocity ranged from 1 m/s to 5 m/s. Slug flow was observed for this entire range of velocities in a 7.6 cm I.D. pipe.

A novel flow visualization system has been developed to study the detailed characteristics of slug flow. The system involves the observation of the flow from two different distances using two different cameras. The images from the two cameras are then combined into a single image, which is digitized and analysed to obtain the necessary data.

Digital image processing algorithms have been developed involving edge-tracking routines. The coordinates of bubbles and voids within the slug were obtained using these routines. These coordinate data were then used to obtain data on slug translational velocity,
liquid film heights, local velocity of liquid within the slug and in the tail, and local void fraction distribution within the slug.

In the range of velocities studied for the water-carbon dioxide system, the ratio of the slug translational velocity to the slug velocity was found to decrease from 1.75 at a slug velocity of 1.2 m/s to 1.2 above a slug velocity of 2 m/s and up to 3.2 m/s. The higher ratio at the lower slug velocity was explained by using a drift velocity between gas and liquid. The drift velocity decreases to zero above a slug velocity of 2 m/s. The results agree with those of several other workers, e.g. (Crowley et.al. (1988), Kouba (1986)).

The data for ARCOPAC90™-carbon dioxide slugs fell within the same range as that for water. The range of slug velocities for this system were between 1.6 m/s and 5.88 m/s. From the data it was concluded that there was no significant effect of liquid properties for these two systems within the range of velocities studied.

From a plot of $v_l$ vs $v_p$, a model for the drift velocity was produced and this was found to be a function of the slug velocity.

In this study, the nondimensional liquid film height varied between 0.32 and 0.38 for water-carbon dioxide slug system, and between 0.27 and 0.36 for ARCOPAC90™-carbon dioxide slug system. Large amplitude roll waves on the liquid film between slugs were responsible for these film height variations in both systems.

The average nondimensional liquid film heights in all cases were between 0.3 and 0.4. This is in agreement with the results of Kouba (1986) and Jepson and Taylor (1988).
Velocity profiles in slugs at velocities ranging from 1.27 m/s to 1.83 m/s indicate that near the front of the mixing zone, there is a high degree of turbulence, resulting in the destruction of the hydrodynamic boundary layer and the development of a flat velocity profile. At the end of the mixing zone, the boundary layer has fully redeveloped and a one-seventh power law model provides a good estimate of the shape of the velocity profile in the slug. At the tail of the slug the effect of the gas pocket behind the slug begins to distort the boundary layer once more.

Different void structures in slugs were noted and these were divided into two types. In the mixing zone and near the top of the pipe were large voids that were distributed throughout the depth of the pipe. Beyond the mixing zone, discrete gas bubbles are formed and move towards the top of the pipe. The predominant form of voids in this region of the slug is spherical bubbles. Based on this classification, the void profile across the slug was computed. The data obtained using this technique was used to compute the overall void fraction and these agree well with the empirical model proposed by Gregory et.al. (1978).

The void fraction distribution across an entire section of the pipe in the slug body was seen to have a steady profile for all cases. The void fraction up to the center of the pipe was less than ten to fifteen percent. It then increased rapidly to values between fifty to ninety percent near the top of the pipe. A regression analysis was done with over two hundred images, and a single equation was proposed to describe the void fraction variation in the slug body.
The average liquid holdup in the slug was seen to vary substantially near the slug front from about seventy percent to thirty percent. This was due to the release of pulses of bubbles in the mixing zone. At the end of the mixing zone, the holdup increased monotonically to a steady value of about eighty to ninety percent.

The holdup variation in slugs was modelled as the response of a second order viscoelastic system to a step input. A nonlinear regression fit was done to obtain values of the process parameters, $\tau$ and $\zeta$ that characterise the system. A model was developed to predict the value of $\zeta$ using ratios of the Froude Number, Reynolds Number and Eotvos Number. $\tau$ was modelled as being one-fourth of the mixing length. The model incorporates the effects of liquid density, viscosity, and surface tension on the liquid holdup. Model results show good agreement with experimental data.

The mixing length was defined in this study as the first point where the void profile became steady in the slug. It was seen from video images that this also corresponded to the end of the frothy, turbulent part of the slug. The mixing length was then estimated as a linear function of the Film Froude Number ahead of the slug. It varied from two to ten pipe diameters. It was found that the model given by Dukler and Hubbard (1975) generally underpredicted the length of the mixing zone.

There was a wide variation of slug lengths for both water-carbon dioxide slugs and ARCOPAC90$^{TM}$-carbon dioxide slugs. The slug lengths were distributed within ±2 standard
The model gives a good prediction of the mean slug length for the data in this study. It also is able to predict the data of Jepson and Taylor (1988), and Crowley et.al. (1988).
Bibliography


Appendix A

Calibration of Liquid and Gas Flow Rates

Figures A.1a and A.1b show the calibration graphs for the orifice plate with water and ARCOPAC90\textsuperscript{TM}, respectively.
Figure A.1b: Calibration for ARCOPAC® flow through orifice plate
Flow Rate Corrections for Gas Flow Meter

Flowmeters are normally calibrated for use with air. The units are SCFM of air at 68°F (20°C) and 1 atm pressure. For compressible fluids, such as gases, the actual flow rate will differ from the gauge reading if the flow meter is at a different temperature or pressure. Further, if the gas flowing through the meter is different from air, then a correction for gas density is also required. The final calibration is given by:

\[
F = G \sqrt[{}]{\frac{P_1 T_0 d_o}{P_0 T_1 d_1}} \tag{A.1}
\]

where,

- \(F\) = actual flow rate (SCFM)
- \(G\) = flow meter reading (SCFM)
- \(T_0\) = absolute temperature of air for which gauge is calibrated
- \(T_1\) = absolute temperature of gas flowing through the meter
- \(P_0\) = absolute pressure of air for which gauge is calibrated
- \(P_1\) = absolute pressure of gas flowing through the meter
- \(d_o\) = density (at standard conditions) for air = 1.2928 g/l
- \(d_1\) = density (at standard conditions) for the gas flowing through meter

Using Equation (A.1), the flow rate of gas through the pipeline may be computed. Then, dividing the volumetric flow rate by the cross sectional area of the pipe, the superficial gas velocity may be obtained. This is shown in Figure A.2.
Figure A.2: Variation of superficial gas velocity with inlet pressure
Appendix B

Image Processing Programs

Film.c

#include <math.h>
#include <gl/gl.h>
#include <gl/device.h>
#include <stdio.h>
#include <gl/image.h>
#include "rct.h"

#define X 0
#define Y 1
#define Z 2
#define XYZ 3
#define XY 2
#define R 0
#define G 1
#define B 2
#define RGB 3

short redvec[RGB] = {255,0, 0};
short bluevec[RGB] = {0, 0, 255};
short greenvec[RGB] = {0, 255, 0};
short blackvec[RGB] = {0, 0, 0};

unsigned long *imgbuf;

float xmax, ymax, conv;
long xorig, yorig, xsize, ysize;
int i, datapoint, lowestpix, highestpix;
FILE *fp,*fp1;
char filename[20];
char datafile[20];
long vertex[4][XY];
long vert1[2],vert2[2];

main(argc,argv)
int argc;
char **argv;
{  
  register IMAGE *image;  
  short val;  
  int wx, wy, preforg, number, nn;  
  register int i;  
  extern long *longimagedata();  

  fp1 = fopen("image.dat", "r");  
  fscanf(fp1, "%f %d %d", &conv, &lowestpix, &highestpix);  
  fclose(fp1);  

  sprintf(datafile, "film.dat");  
  fp = fopen(datafile, "w");  

  if((image = fopen("f.rgb", "r")) == NULL) {  
    fprintf(stderr, "rpaste: can't open input file\n");  
    exit(1);  
  }  

  /* calculate the window size */  
  sizeofimage("f.rgb", &xsize, &ysize);  

  /* allocate the memory for the pixel data to be then fed to lrectwrite */  
  imgbuf = (unsigned long *) malloc(xsize * ysize * sizeof(long));  
  imgbuf = (unsigned long *) longimagedata("f.rgb");  

  /* open the window */  
  prefsize(xsize - 1, ysize - 1 + 20);  
  winopen(argv[0]);  
  wintitle("f.rgb");  

  /* set the display mode for the image */  
  RGBmode();  
  gconfig();  

  drawit();  
  drawtextline(xsize/2, ysize/2);  
  sleep(5);  

  scrcod();  

}
drawit()
{
    cpack(0x00808080);
    clear();
    reshapeviewport();
    Irectwrite(O, 16, xsize-1, ysize-1, imgbuf);
}
drawtextline(xpos, ypos)
long xpos, ypos;
{
    char str[256];

    viewport(0, xsize-1, 0, 16);
    grey(0.8);
    clear();
    sprintf(str, "xpos: %04d ypos: %04d", xpos, ypos);
    grey(0.0);
    cmov2i(IOO, 6);
    char str(str);
}
sccodt()
{
    short val, mval[2], lastval[2];
    float filmht;
    long org[2], size[2];
    Device dev, mdev[2], mdev1[2];
    Boolean run;
    int leftmouse_down = 0;
    lastval[X] = -1;

    getorigin(&org[X], &org[Y]);
    getsize(&size[X], &size[Y]);

    mdev[X] = MOUSEX;
    mdev[Y] = MOUSEY;
    getdev(2, mdev, lastval); /* initialize lastval[] */
    lastval[X] = org[X];
    lastval[Y] = org[Y];

    qdevice(LEFTMOUSE);
qdevice(ESCKEY);
qdevice(MOUSEX);
qdevice(MOUSEY);
qdevice(SPACEKEY);
qdevice(PADENTER);

while(1) {
    switch (dev = qread(&val)) {
    case LEFTMOUSE:
        getdev(2, mdev, mval);
        mval[X] = mval[X] - org[X];
        mval[Y] = mval[Y] - org[Y];
        vert1[X] = mval[X];
        vert1[Y] = mval[Y];
        mark(vert1[X], vert1[Y]);
        drawtextline(vert1[X], vert1[Y]);
        qread(&val);
        break;
    case SPACEKEY:
        filmht = (float)(vert1[Y]-lowestpix)/conv;
        printf("%f\n", filmht);
        fprintf(fp, "%f\n", filmht);
        qread(&val);
        break;
    case PADENTER:
        lowestpix = vert1[Y];
        qread(&val);
        break;
    case ESCKEY:
        gexit();
        return;
        break;
    }
}

mark(xpos, ypos)
long xpos, ypos;
{
    viewport(xpos-2, xpos+2, ypos-2, ypos+2);

c3s(bluevec); clear();
}
Filmfrd.c

#include <math.h>
#include <gl/gl.h>
#include <gl/device.h>
#include <gl/image.h>
#include <stdio.h>
#include <string.h>

float conv,sum=0,vt,filmht,vsl,vsg,vm,filmfr,vlf,mixl;
float els,elf,cls,al,a,hl,ratio,r=0.0381,T,heq,hj;

int MAX, number, x[10], y[10];

main(argc,argv)
int argc;
char **argv;
{

long x1,y1,n,i;
FILE *fp,*fp1,*fp2,*fp3,*fp4;
char string[81];

fp3 = fopen("image2.dat","r");
 fscanf(fp3,"%f",&conv);
fclose(fp3);

fp3 = fopen("param.dat","r");
 fscanf(fp3,"%d %f%f",&MAX,&vsl,&vsg);
fclose(fp3);

fp3 = fopen("film.dat","r");
 fscanf(fp3,"%f",&filmht);
fclose(fp3);

hl=filmht/100.;

fp = fopen("vtp.dat","r");
fp2 = fopen("filmfrd.dat","w");

n=0;
while (((fgets(string,81,fp)) != NULL)
{
sscanf(string, "%d %d", &xl, &yl);
x[n] = xl;
y[n] = yl;
n++;
}
fclose(fp);
number=n-1;

for (i=0; i<number; i++)
{
    sum += ((float)(labs(x[i+1] - x[i])))*60.0/conv;
}

vt=sum/((float)(number-1))/100.0;

vm=vsl+vsg;
printf("%f \n", vm);

els=1./(1+(powf((vm/8.66), 1.39)));
cls=els/(49./60. - (1-els)*5./6.);

ratio=(hl/r -1.);
al=r*r*(3.1416-facos(ratio)+ratio*sqrt(1.-ratio*ratio));
T=2*r*sqrt(1.-ratio*ratio);
a=3.1416*r*r;
heq=al/T;
elf = al/a;

vlf = vt -(vt-vm)*els/elf;

filmfr = (vt - vlf)/sqrt(9.8*heq);

hj=heq*0.5*(sqrt(1.+8.*filmfr*filmfr) -1.);

fprintf(fp2, "%f %f %f %f %f %f %f \n", vt, els, cls, heq, elf, vlf, filmfr, hj);
printf("%f %f %f %f %f %f %f \n", vt, els, cls, heq, elf, vlf, filmfr, hj);
```c
#include <math.h>
#include <gl/gl.h>
#include <gl/device.h>
#include <gl/image.h>
#include <stdio.h>
#include <string.h>

long region, reg;
long ybound[10];
float conv;
float dia = 7.62;
float crossarea;
float g = 9.8;
float els, elf, cls, heq, vt, filmfr, vlf;
float vsl;
float vsg;
float vsl;
int MAX, totali;

main(argc, argv)
int argc;
char **argv;
{
    long x1, y1, i, n, n, getout, volume, x[3][10], y[3][10], j, ii, i = 0, ij;
    char string[81], string1[81], filename[20], datafile[20], datafile2[20];
    char head1[10], head2[10], head3[10], head4[1], datafile3[20];
    float vfr[10], vfg[10], vofr[10], voidr, v[50], hod[50], vl[10], hd[10];
    float hi, ho, h, hib, hob, hb, ptho, pthi, fhi, fho, dist, temp1, temp2, temp3;
    float alo, al, delta, alib, delt, integral, vavg, vt, vm, fr1, fr2;
    float hid[10], velsum[10];
    long reg, voidvol[10], regvol[10], ylow[10];
    long min1, min2;
    long areavol[10], vvvol[10], lowestpix, highestpix, interv;

    if (argc < 2) {
        fprintf(stderr, "usage: %s frame number \n", argv[0]);
        exit(1);
    }

    fp = fopen("filmfrd.dat", "r");
```
fscanf(fp,"%f%f%f%f%f%f%n",&vt,&els,&cls,&heq,&elf,&vlf,&filmfr);
close(fp);

fp = fopen("image.dat","r");
fscanf(fp,"%f%d %d",&conv,&lowestpix,&highestpix);
fclose(fp);

interv = (highestpix-lowestpix)/10;

for (n=0;n<10;n++)
{
    ybound[n]=lowestpix+(n+1)*interv;
    hd[n] = (float)(ybound[n]-lowestpix)/conv/dia;
    if (hd[n] > 1.0) {hd[n]=1.0;}
    ylow[n]=lowestpix+n*interv;
    hid[n] = (float)(ylow[n]-lowestpix)/conv/dia;
}

fp4=fopen("param.dat","r");
fscanf(fp4,"%d %f%f",&MAX,&vsl,&vsg);
fclose(fp4);

nn = atoi(argv[1]);

sprintf(datafile,"%sd%s","t",nn,".frac");
sprintf(datafile2,"%sd%s","v",nn,".dat");
sprintf(datafile3,"froude.dat");

fp1 = fopen(datafile,"r");
fp2 = fopen(datafile2,"r");
fp3 = fopen(datafile3,"a");

fscanf(fp2,"%f",&dist);

reg=0;
while ((fgets(string,81,fp2)) != NULL)
{
    sscanf(string,"%f%f",&hod[reg],&v[reg]);
    v[reg]=v[reg]*100.0;
    reg++;
}
fclose(fp2);
reg=reg-1;
for (min1=0;min1<reg;min1++)
{
    for (min2=min1+1;min2<=reg;min2++)
    {
        if (hod[min1] > hod[min2])
        {
            temp1 = hod[min1];
            temp2 = v[min1];
            hod[min1]=hod[min2];
            v[min1]=v[min2];
            hod[min2]=temp1;
            v[min2]=temp2;
        }
    }
}

for (i=0;i<10;i++)
{
    fscan(fp1,"%d %d %d %f",&region,&voidvol[i],
        &regvol[i],&vfr[i]);
}

for (i=0;i<10;i++)
{
    vl[i] = 0;
    totali=0;
    velsum[i] = 0;

    for (j=0;j<=reg;j++)
    {
        if ( (hod[j] >= hid[i]) && (hod[j] < hd[i]) )
        {
            velsum[i]+=v[j];
            totali++;
        }
    }

    vl[i] = velsum[i]/totali;
}

for (i=0;i<10;i++)
{
    if (vl[i] > 0)
```c
{ 
    j=i;  
    for (jj=ii;jj<jjj++)  
    {  
        vvol[i]+=voidvol[jj];  
        areavol[i]+=regvol[jj];  
    }  
    ii=i+1;  
    vofr[i]=(float)vvol[i]/(float)areavol[i];  
    if (vofr[i] > 1.0) {vofr[i]=1.0;};  
}

while (((fgets(string,81,fp1)) != NULL)  
{  
    if ( (string[0] == ' ') & (string[1] == 'A') )  
    {  
        sscanf(string,"%s %s %s %s %s",  
            head1,head2,head3,head4,&voidr);  
    }  
}
fclose(fp1);  

hi = 0.0;  

fhi = 0.0;  

for (i=0;i<10;i++)  
{  
    if (vl[i] > 0)  
    {  
        ho=hd[i]*dia;  
        ptho = (2.*hd[i] - 1.0);  
        hib = hi/dia;  
        pthi = (2.*hib - 1.0);  
        alob = 0.25*(3.1416-facos(ptho)+ptho*sqrt(1.0-ptho*ptho));  
        alib = 0.25*(3.1416-facos(pthi)+pthi*sqrt(1.0-pthi*pthi));  
        alo = alob*dia*dia;  
        ali = alib*dia*dia;  
        deltar = (alo - ali);  
        h = ho - hi;  
        fho = vl[i]*(1.0-vofr[i]);  
        integral+=0.5*deltar*(fho + fhi);  
        hi = ho;  
    }  
}  
```
fhi = fho;
}
}
crossarea = 3.1416/4.0*dia*dia;
vavg = integral/crossarea/(1.0-voidr);
vavg = vavg/100.0;
vm = vsl + vsg;
dia = dia/100.0;
fr1 = (vt - vavg)/sqrt(g*dia*(1.0-voidr));
fr2 = (vt - vavg)/sqrt(g*dia);

printf("%f %f %f %f %f \n",dist,vt,vavg,voidr,fr1,fr2);
fprintf(fp3,"%f%f%f%f%f%f\n",dist,vt,vavg,voidr,fr1,fr2);

Frjump.c

#include <math.h>
#include <gl/gl.h>
#include <gl/device.h>
#include <gl/image.h>
#include <stdio.h>
#include <string.h>

#define X 0
#define Y 1
#define Z 2
#define XYZ 3

main(argc,argv)
int argc;
char **argv;
{
    FILE *fp,*fp1;
    char filename[20],datafile[20],string[81];
    float dist,vt,vavg,voidr,fr1,fr2,frj1,frj2,vj,al,filmht,ht,hj;
float els, cls, heq, elf, vlf, filmfr, frj3;
float d = 0.076;

al = 3.1416 * d * d / 4.;

fp = fopen("filmfri.dat", "r");
fscanf(fp, "%f %f %f %f %f %f %f %f\n", &vt, &els, &cls,
    &heq, &elf, &vlf, &filmfr, &hj);
fclose(fp);

hf = heq;

sprintf(datafile, "froude.dat");
sprintf(filename, "frjump.dat");

fp1 = fopen(datafile, "r");
fp = fopen(filename, "w");

while (!fgets(string, 81, fp1) != NULL)
{
    sscanf(string, "%f %f %f %f %f %f %f %f\n", &dist, &vt, &vavg, &voidr,
        &fr1, &fr2);

    vj = vavg * al / hj / d;

    frj1 = (vt - vj) / sqrt(9.8 * hj);
    frj2 = (vt - vavg) / sqrt(9.8 * hj);
    frj3 = (vt - vj) / sqrt(9.8 * d);

    fprintf(fp, "%f %f %f %f %f %f %f %f\n", dist, vj, voidr, fr2, frj1, frj2, frj3);
    printf("%f %f %f %f %f %f %f\n", dist, vj, voidr, fr2, frj1, frj2, frj3);
}
}
#include <math.h>
#include <gll/gl.h>
#include <gl/device.h>
#include <stdio.h>
#include <gll/image.h>
#include <string.h>

/* These are definitions to help in defining coordinates later */

#define X 0
#define Y 1
#define Z 2
#define XYZ 3
#define XY 2
#define R 0
#define G 1
#define B 2
#define RGB 3

int i, xo, yo, wx, wy;
long znear, zfar, xsize, ysize, xorig, yorig;
long dev;
short val, radius, x, y, z, delx, dely;
float xmax, ymax, conv;
char buf[100];

short vert1[XY], vert2[XY];
short vert[XY];

short redvec[RGB] = {255, 0, 0};
short bluevec[RGB] = {0, 0, 255};
short greenvec[RGB] = {0, 255, 0};
short blackvec[RGB] = {0, 0, 0};

unsigned long *imgbuf;

main(argc, argv)
int argc;
char **argv;
{
    long mval[2], lastval[2];
    long size[2], org[2], lowestpix, highestpix, interv;
    FILE *fp, *fp1;

    fp1=fopen("image.dat", "r");
    fscanf(fp1, "%f %d %d", &conv, &lowestpix, &highestpix);
    fclose(fp1);

    interv=(highestpix-lowestpix)/10;

    if ((fp = fopen("calib.dat", "r")) != NULL)
    {
        fscanf(fp, "%d %d", &lastval[X], &mval[X]);
    }
    else
    {
        printf("File : calib.dat containing values for mesh vertices \n");
        printf("does not exist \n");
        exit(0);
    }

    prefsize(640,480);
    winopen("Mesh");

    getsize(&size[X], &size[Y]);
    getorigin(&org[X], &org[Y]);

    xo = org[X];
    yo = org[Y];
    wx = xo + size[X]-1;
    wy = yo + size[Y]-1;

    RGBmode();
    gconfig();

    cpack(0x00000000);
    clear();

    linewidth(3);
    c3s(redvec);

    printf("%d %d \n", lastval[X], mval[X]);
vert1[X] = lastval[X];
vert1[Y] = lowestpix;
vert2[X] = vert1[X];
vert2[Y] = highestpix;
bgnline();
v2s(vert1);
v2s(vert2);
endline();
vert1[X] = mval[X];
vert1[Y] = lowestpix;
vert2[X] = mval[X];
vert2[Y] = highestpix;
bgnline();
v2s(vert1);
v2s(vert2);
endline();
for (i=lowestpix;i<=highestpix;i+=interv)
{
    bgnline();
    vert[X] = lastval[X];
    vert[Y] = i;
    v2s(vert);
    vert[X] = mval[X];
    v2s(vert);
    endline();
}
sprintf(buf, "/usr/sbin/scrsave %s %d %d %d %d",
"mesh.rgb", xo, wx, yo, wy);
system (buf);
}
Path.c

#include <math.h>
#include <gl/gl.h>
#include <gl/device.h>
#include <stdio.h>
#include <gl/image.h>
#include "rct.h"

#define X 0
#define Y 1
#define Z 2
#define XYZ 3
#define XY 2
#define R 0
#define G 1
#define B 2
#define RGB 3

short redvec[RGB] = {255, 0, 0};
short bluevec[RGB] = {0, 0, 255};
short greenvec[RGB] = {0, 255, 0};
short blackvec[RGB] = {0, 0, 0};

unsigned long *imgbuf;

float xmax, ymax;
long xorig, yorig, xsize, ysize;
int i, datapoint;
FILE *fp, *fp1;
char filename[20];
char datafile[20];
long vertex[4][XY];
long vert1[2], vert2[2];

main(argc, argv)
int argc;
char **argv;
{
    register IMAGE *image;
    short val;
int wx, wy, preforg, number, n, nn;
register int i;
extern long *longimagedata();

if( argc<2 ) {
    fprintf(stderr,"usage: scope inimage
    exit(1);
}

n = atoi(argv[1]);
nn = number + 3;

sprintf(datafile,"p%d.dat",n);
if( (fp=fopen(datafile,"r") != NULL )
    printf("This frame has a datafile already ! \n");
    exit(1);)
fp = fopen(datafile,"w");
fp1=fopen("bottom.dat","w");

sprintf(filename,"%s%d%s","t",D,.rgb");

if( (image=iopen(filename,"r") == NULL ) {
    fprintf(stderr,"rpaste: can't open input file %s\n",filename);
    exit(1);
}

/* calculate the window size */
sizeofimage(filename, &xsize, &ysize);

/* allocate the memory for the pixel data to be then fed to lrectwrite */
imbuf= (unsigned long *) malloc(xsize*ysize*sizeof(long));
imbuf= (unsigned long *) longimagedata(filename);

/* open the window */
prefsize(xsize-1,ysize-1+16);
winopen(argv[0]);
wintitle(filename);

/* set the display mode for the image */
RGBmode();
gconfig();
drawit();
drawtextline(xsize/2,ysize/2);
sleep(5);

scrcod(datafile);
}

drawit()
{
cpack(0x00808080);  
clear();  
reshapeviewport();
Irectwrite(0,16,xsize-1,ysize-1,imgbuf);
}

drawtextline(xpos,ypos)
long xpos, ypos;
{
    char str[256];

    viewport(0,xsize-1,0,16);
    grey(0.8);
    clear();
    sprintf(str,"xpos: %04d ypos: %04d",xpos,ypos);
    grey(0.0);
    cmov2i(100,4);
    charstr(str);
}

scrcod()
{
    short val, mval[2], lastval[2];
    long org[2], size[2];
    Device dev, mdev[2], mdev1[2];
    Boolean run;
    int leftmouse_down = 0;
    lastval[X] = -1;

    getorigin(&org[X], &org[Y]);
    getsize(&size[X], &size[Y]);

    mdev[X] = MOUSEX;
mdev[Y] = MOUSEY;
getdev(2, mdev, lastval); /* initialize lastval[] */
lastval[X] = org[X];
lastval[Y] = org[Y];

qdevice(LEFTMOUSE);
qdevice(ESCKEY);
qdevice(MOUSEX);
qdevice(MOUSEY);
qdevice(MIDDLEMOUSE);
qdevice(SPACEKEY);
qdevice(PADENTER);

while(1) {
switch (dev = qread(&val)) {
    case LEFTMOUSE:
        getdev(2, mdev, mval);
        mval[X] = mval[X] - org[X];
        mval[Y] = mval[Y] - org[Y];
        vert1[X] = mval[X];
        vert1[Y] = mval[Y];
        mark(vert1[X], vert1[Y]);
        drawtextline(vert1[X], vert1[Y]);
        qread(&val);
        break;
    case SPACEKEY:
        printf("%d %d \n", vert1[X], vert1[Y]);
        qread(&val);
        qread(&val);
        break;
    case MIDDLEMOUSE:
        getdev(2, mdev, mval);
        mval[X] = mval[X] - org[X];
        mval[Y] = mval[Y] - org[Y];
        vert1[X] = mval[X];
        vert1[Y] = mval[Y];
        printf("%d %d \n", vert1[X], vert1[Y]);
        fprintf(fp, "%d %d \n", vert1[X], vert1[Y]);
        qread(&val);
        qread(&val);
        break;
    case PADENTER:
        fprintf(fp1, "%d\n", vert1[Y]);
qread(&val);
bredit;
case ESCKEY:
exit(0);
}
}
}

mark(xpos,ypos)

long xpos, ypos;
{
viewport(xpos-2,xpos+2,ypos-2,ypos+2);

c3s(bluevec);
clear();
}

Vel.c

#include <math.h>
#include <gl/gl.h>
#include <gl/device.h>
#include <gl/image.h>
#include <stdio.h>
#include <string.h>

long region;

long ybound[10];

float conv;
float dia = 7.62;
float crossarea;
float g = 9.8;

float vsl;
float vsg;
int MAX, number,m;
main(argc, argv)
int argc;
char **argv;
{

long x1, y1, i, nn, n, getout, volume, x[4][50], y[4][50], jj, ii, ij;
char string[81], string1[81], filename[20], datafile[20], datafile2[20];
char head1[10], head2[10], head3[10], head4[1], datafile3[20];
float vfr[50], vfg[50], vofr[50], voidr, v[50], temp1, temp2, sumvel[50];
float hi, ho, h, hib, hob[50], hb, ptho, phi, fho, dist;
float alo, ali, delar, alob, alib, deltar, integral, vavg, vt, vm, fr1, fr2;
long reg, voidvol[10], regvol[10];
long areavol[10], vvol[10], lowestpix, min1, min2;

if (argc < 2) {
    fprintf(stderr, "usage: filename root 
")
    exit(1);
}

nn = atoi(argv[1]);

fp4 = fopen("param.dat", "r");
fscanf(fp4, "%d %f %f", &MAX, &vsl, &vsg);
fclose(fp4);

fp3 = fopen("image.dat", "r");
fscanf(fp3, "%f", &conv);
fclose(fp3);

fp3 = fopen("bottom.dat", "r");
fscanf(fp3, "%d", &lowestpix);
fclose(fp3);

if (lowestpix == 0)
{
    fprintf(stderr,"File: bottom.dat is empty
");
    exit(1);
}

dist = (vsl+vsg)*100.0/60.0*(float)nn;

printf("How many frames? 
");
scanf("%d",&m);

sprintf(filename,"%s%d%s","p",nn,".dat");
sprintf(datafile2,"%s%d%s","v",nn,".dat");

fp = fopen(filename,"r");
fp2 = fopen(datafile2,"w");

n=0;
while ((fgets(string,81,fp)) != NULL)
{
    sscanf(string,"%d %d",&xl,&yl);
    x[0][n] = xl;
    y[0][n] = yl;
    n++;
}
fclose(fp);
number=n-1;
i=1;
for (n=nn+1,n<=nn+m-1,n++)
{
    sprintf(filename,"%s%d%s","p",n,".dat");
    printf("%s\n",filename);
    fp = fopen(filename,"r");

    j=0;
    while ((fgets(string,81,fp)) != NULL)
    {
        sscanf(string,"%d %d",&xl,&yl);
        x[i][j] = x1;
        y[i][j] = y1;
        j++;
    }
}
fclose(fp);
i++;
}

for (i=0;i<=number;i++)
{
for (n=0; n<m-1; n++)
{
    sumvel[i] += (float)(x[n][i] - x[n+1][i])*60.0/conv;
}

    v[i] = sumvel[i]/(float)(m-1);

for (i=0; i<=number; i++)
{
    ho = (float)(y[0][i] - lowestpix)/conv;
    hob[i] = ho/dia;
    if (hob[i] >= 1.0) hob[i] = 1.0;
}

for (min1=0; min1<number; min1++)
{
    for (min2=min1+1; min2<=number; min2++)
    {
        if (hob[min1] > hob[min2])
        {
            temp1 = hob[min1];
            temp2 = v[min1];
            hob[min1] = hob[min2];
            v[min1] = v[min2];
            hob[min2] = temp1;
            v[min2] = temp2;
        }
    }
}

fprintf(fp2, "%f\n", dist);
for (i=0; i<=number; i++)
{
    fprintf(fp2, "%f %f\n", hob[i], v[i]/100.0);
}

}
#include <math.h>
#include <gl/gl.h>
#include <gl/device.h>
#include <gl/image.h>
#include <stdio.h>
#include <string.h>

long region;

long ybound[10];

float conv;
float dia = 7.62;
float crossarea;
float g = 9.8;

float vsl;
float vsg;
int MAX, number, m;

main(argc,argv)
int argc;
char **argv;
{

long x1,y1,i,nn,n,getout,volume,x[4][50],y[4][50],jj,ii,ij;
char string[81], string 1 [81],filename[20],datafile[20],datafile2[20];
char head1[10],head2[10],head3[10],head4[1],datafile3[20];
float vfr[50],vfg[50],vofr[50],voidr,v[50],temp1,temp2,sumvel[50];
float hi,ho,h,hib,hob[50],hb,ptho,pthi,phi,pho,dist;
float alo,ali,delar,a1ob,alib,deltar,integral,vavg,vt,vm,fr1,fr2;
long reg, voidvol[10], regvol[10];
long area vol[10], vvol[10], lowestpix, min1, min2;
FILE *fp,*fp1,*fp2,*fp3,*fp4;

if( argc<2 ) {
    fprintf(stderr,"usage: filename root \n");
    exit(1);
}

nn = atoi(argv[1]);
fp4=fopen("param.dat","r");
fscanf(fp4,"%d%f%f",&MAX,&vsl,&vsg);
fclose(fp4);

fp3 = fopen("image.dat","r");
fscanf(fp3,"%f",&conv);
fclose(fp3);

fp3 = fopen("bottom.dat","r");
fscanf(fp3,"%d",&lowestpix);
fclose(fp3);

if (lowestpix == 0)
{
    fprintf(stderr,"File: bottom.dat is empty\n");
    exit(1);
}

dist = (vsl+vsg)*100.0/60.0*(float)nn;

printf("How many frames?\n");
scanf("%d",&m);

sprintf(filename,"%s%d%s","p",nn,".dat");
sprintf(datafile2,"%s%d%s","vy",nn,".dat");

fp = fopen(filename,"r");
fp2 = fopen(datafile2,"w");

n=0;
while ((fgets(string,81,fp)) != NULL)
{
    sscanf(string,"%d %d",&xl,&yl);
    x[0][n] = xl;
    y[0][n] = yl;
    n++;
}
fclose(fp);
fclose(fp2);
number=n-1;

i=1;
for (n=nn+1;n<=nn+m-1;n++)
```
{
    sprintf(filename, "p%s%d%s", "p", n, ".dat");
    printf("
II
    filename);
    fp = fopen(filename, "r");

    j = 0;
    while (!fgets(string, 81, fp)) != NULL)
    {
        sscanf(string, "%d %d", &xl, &yl);
        x[i][j] = xl;
        y[i][j] = yl;
        j++;
    }

    fclose(fp);
    i++;
}

for (i = 0; i <= number; i++)
{
    for (n = 0; n < m - 1; n++)
    {
        sumvel[i] += (float)(y[n + 1][i] - y[n][i]) * 60.0 / conv;
    }
    v[i] = (sumvel[i]) / (float)(m - 1);
}

for (i = 0; i <= number; i++)
{
    ho = (float)(y[0][i] - lowestpix) / conv;
    hob[i] = ho / dia;
    if (hob[i] >= 1.0) {hob[i] = 1.0;}
}

for (min1 = 0; min1 < number; min1++)
{
    for (min2 = min1 + 1; min2 <= number; min2++)
    {
        if (hob[min1] > hob[min2])
        {
            temp1 = hob[min1];
```
temp2 = v[min1];
hob[min1] = hob[min2];
v[min1] = v[min2];
hob[min2] = temp1;
v[min2] = temp2;

fprintf(fp2, "%f\n", dist);
for (i=0; i<=number; i++)
{
    fprintf(fp2, "%f %f\n", hob[i], v[i]/100.0);
}

Voidn.c

#include <math.h>
#include <gl/gl.h>
#include <gl/device.h>
#include <stdio.h>
#include <string.h>
#include <gl/image.h>

#define X 0
#define Y 1
#define Z 2
#define XYZ 3
#define XY 2
#define R 0
#define G 1
#define B 2
#define RGB 3

short redvec[RGB] = {255, 0, 0};
short bluevec[RGB] = {0, 0, 255};
short greenvec[RGB] = {0, 255, 0};
short blackvec[RGB] = {0, 0, 0};

unsigned long *imgbuf;

float xmax, ymax;
long xorig, yorig, xsize, ysize;
int i, datapoint;
FILE *fp;
char filename[20];
char datafile[20];
long vertex[4][XY];
long vert1[2], vert2[2];

main(argc,argv)
int argc;
char **argv;
{
    register IMAGE *image;
    short val;
    int wx, wy, preforg;
    register int i;
    extern long *longimagedata();

    if( argc<2 ) {
        fprintf(stderr,"usage: scope inimage\n");
        exit(1);
    }

    preforg=1; /* Sets the origin of the image to 350,350 */
    wx=550;
    wy=250;

    sprintf(filename,"%s%s",argv[1],".rgb");
    sprintf(datafile,"%s%s",argv[1],".dat");

    if( (fp=fopen(datafile,"r")) != NULL )
    {
        printf("This frame has a datafile already ! \n");
        exit(1);
    }

    if( (image=iopen(filename,"r")) == NULL ) {
fprintf(stderr,"rpaste: can't open input file %s\n",filename);
exit(1);
}

/* calculate the window size */
sizeofimage(filename, &xsize, &ysize);

/* allocate the memory for the pixel data to be then fed to lrectwrite */
imbuf = (unsigned long *) malloc(xsize*ysize*sizeof(long));
imbuf = (unsigned long *) longimagedata(filename);

/* open the window */
if(preforg) {
    prefposition(wx,wx+xsize-1,wy,wy+ysize-1);
    prefsize(xsize-1,ysize-1);
    winopen(argv[0]);
    wintitle(filename);
} else {
    prefsize(xsize-1,ysize-1);
    winopen(argv[0]);
    wintitle(filename);
}

/* set the display mode for the image */
RGBmode();
gconfig();

drawit(); /* All this is part of the ipastelrw program */
sleep(5);

csrcod(datafile); /* My addition starts here */
}

drawit()
{
    cpack(0x00808080);
    clear();
    reshapeviewport();
    lrectwrite(0,0,xsize-1,ysize-1,imgbuf);
}

csrcod(filenn)
char filen[20];
{
    short val, mval[2], lastval[2], num;
    long org[2], size[2];
    Device dev, mdev[2], mdev1[2];
    Boolean run;
    int leftmouse_down = 0;
    lastval[X] = -1;

    getorigin(&org[X], &org[Y]);
    getsize(&size[X], &size[Y]);

    mdev[X] = MOUSEX;
    mdev[Y] = MOUSEY;
    getdev(2, mdev, lastval); /* initialize lastval[] */
    lastval[X] = org[X];
    lastval[Y] = org[Y];

    qdevice(LEFTMOUSE);
    qdevice(ESCKEY);
    qdevice(MOUSEX);
    qdevice(MOUSEY);
    qdevice(MIDDLEMOUSE);
    qdevice(SPACEKEY);
    qdevice(PADENTER);
    qdevice(AKEY);
    qdevice(BKEY);

    c3s(greenvec);    /* prepare to draw green lines */

    fp = fopen(filen,"w");

    while(1) {
        switch (dev = qread(&val)) {
            case LEFTMOUSE:
                leftmouse_down = val;
                break;
            case MOUSEX:
                mval[X] = val - org[X];
                break;
            case MOUSEY:
                mval[Y] = val - org[Y];
                if (leftmouse_down) {
                    break;
                } else {
                    leftmouse_down = lastval;
                }
        }
    }
}
bgnline();
v2s(lastval);
v2s(mval);
endline();
num++;
if (num > 3)
{ fprintf(fp,"%d %d \n",lastval[X],lastval[Y]);
 printf("%d %d \n",lastval[X],lastval[Y]);
 num=0; }
 lastval[X] = mval[X];
 lastval[Y] = mval[Y];
 break;
case SPACEKEY:
 printf("%s \n","more..");
 fprintf(fp,"%s \n","more..");
 qread(&val);
 break;
case PADENTER:
 fprintf(fp," \n");
 printf(" \n");
 qread(&val);
 break;
case AKEY:
 fprintf(fp," a \n");
 printf(" a \n");
 qread(&val);
 qread(&val);
 break;
case BKEY:
 fprintf(fp," b \n");
 printf(" b \n");
 qread(&val);
 qread(&val);
 break;
case ESCKEY:
 exit(0); 
} }
#include <math.h>
#include <gl/gl.h>
#include <gl/device.h>
#include <gl/image.h>
#include <stdio.h>
#include <string.h>

long region,r,interv;
long ybound[10],ylow[10],highestpix,lowestpix;
float mchord[10],lowt,hight;

main(argc,argv)
    int argc;
    char **argv;
{
    long x1,y1,x2,y2,i,area,n,getout,volume;
    char string[81],string1[81],filename[20],datafile[20];
    float ratio1,ratioh,conv,r1;
    FILE *fp,*fp1;

    if( argc<2 ) {
        fprintf(stderr,"usage: t filenumber \n");
        exit(1);
    }

    fp = fopen("image.dat", "r");
    fscanf(fp,"%f %d %d",&conv,&lowestpix,&highestpix);
    fclose(fp);

    interv = (highestpix-lowestpix)/10;
    r = (highestpix-lowestpix)/2;

    for (n=0;n<10;n++)
    {
        ybound[n]=lowestpix+(n+1)*interv;
        ylow[n]=lowestpix+n*interv;
        ratio1=((float)(ylow[n]-lowestpix)/(float)r-1.);
ratioh=\((\text{float})\text{ybound}[n]-\text{lowestpix})/\text{(float} r-1.); 
lowt=2.*r*sqrt(1.-\text{ratio}1*\text{ratio}1); 
hight=2.*r*sqrt(1.-\text{ratio}h*\text{ratio}h); 
mchord[n]=(\text{hight}+\text{lowt})/2.; 
\}

\text{sprintf(filename, "\%s\%s", argv[1], ".dat");} 
\text{sprintf(datafile, "\%s\%s", argv[1], ".area");} 
fp = fopen(filename, "r"); 
fp1 = fopen(datafile, "w"); 
i=1; 
region=0; 
while ((fgets(string,81,fp)) ! = NULL) 
{ 
if (string[0] != ' ') && (string[0] != 'a') && (string[0] != 'b') 
{ 
if (i<2) 
{ 
\text{sscanf(string, "\%d \%d", &x1, &y1);} 
i++; 
getout=0; 
for (n=0; n<10; n++) 
{ 
if (getout==0) 
{ 
if (y1<ybound[n]) 
{ 
getout=1; 
region=n+1; 
} 
} 
} 
} 
else 
{ 
\text{sscanf(string, "\%d \%d", &x2, &y2);} 
area+=areacalc(x1,y1,x2,y2); 
x1=x2; 
y1=y2; 
} 
}
else if (string[0] == 'a')
{
    area = labs(area);
    r1 = fsqrt((float)area/3.1416);
    volume = 4./3.*3.1416*r1*r1*r1;
    fprintf(fp1,"%d %d r# = %d \n",area,volume,region);
    i=1;
    area=0;
}
else if (string[0] == 'b')
{
    area = labs(area);
    volume = area*mchord[region-1];
    fprintf(fp1,"%d %d r# = %d \n",area,volume,region);
    i=1;
    area=0;
}
else
{
    fprintf(fp1," \n");
    i=1;
    area=0;
}
}

areacalc(xk,yk,x,y)
long xk,yk,x,y;
{
    long area;
    area=(xk*y - x*yk)/2;
    return(area);
}
#include <math.h>
#include <gl/gl.h>
#include <gl/device.h>
#include <gl/image.h>
#include <stdio.h>

long regionvolume[10], y[10], sectionarea[10];

main(argc, argv)
  int argc;
  char **argv;
  {
    long area, i, region, volume, totalvolume[10];
    long wholevolume, totalvoidvolume;
    float voidfrac[10], temp1, temp2, avgvoidfraction, temp3;
    char string[81], filename[20], datafile[20], str[3];
    FILE *fp, *fp1, *fp2;

    if (argc < 2) {
      fprintf(stderr, "usage: filename root \n");
      exit(1);
    }

    sprintf(filename, "%s%s", argv[1], "area");
    sprintf(datafile, "%s%s", argv[1], "frac");

    fp = fopen(filename, "r");
    fp1 = fopen(datafile, "w");
    fp2 = fopen("sectvol.dat", "r");

    i = 0;
    while ((fgets(string, 81, fp2)) != NULL)
      {
        sscanf(string, "%d %d %d %d \n", &i, &y[i], &sectionarea[i],
               &regionvolume[i]);
        i++;
      }

    while ((fgets(string, 81, fp)) != NULL)
      {
       // Code continues here
      }
if (string[0] != ' ')
{
    sscanf(string, "%d %d %s %d", &area, &volume, str, &region);
    fprintf(fp2, "%d %d %s %d\n", area, volume, str, region);
    totalvolume[region-1] += volume;
}
else
{
    temp1 = (float)totalvolume[region-1];
    temp2 = (float)regionvolume[region-1];
    voidfrac[region-1] = temp1/temp2;

    if (voidfrac[region-1] > 1.0)
    { voidfrac[region-1] = 1.0; }
}

    temp1 = (float)totalvolume[region-1];
    temp2 = (float)regionvolume[region-1];
    voidfrac[region-1] = temp1/temp2;

    if (voidfrac[region-1] > 1.0)
    { voidfrac[region-1] = 1.0; }

totalvoidvolume = 0;
wholevolume = 0;
for (i=0; i<10; i++)
{
    fprintf(fp1, "%d %d %d %f\n", i+1, totalvolume[i], regionvolume[i], voidfrac[i]);
    totalvoidvolume += totalvolume[i];
    wholevolume += regionvolume[i];
}

avgvoidfraction = ((float)totalvoidvolume)/((float)wholevolume);

if (avgvoidfraction > 1.0)
{
    avgvoidfraction = 1.0;
}

fprintf(fp1, "\n\nAverage Void Fraction = %f\n", avgvoidfraction);
printf("Average Void Fraction = %f\n", avgvoidfraction);
#include <math.h>
#include <gl/gl.h>
#include <gl/device.h>
#include <gl/image.h>
#include <stdio.h>
#include <string.h>

float conv, sum=0, vt;

int MAX, number, x[10], y[10];

main(argc, argv)
int argc;
char **argv;
{

long x1, y1, n, i;
char string[81];

fp3 = fopen("image2.dat", "r");
 fscanf(fp3, "%f", &conv);
fclose(fp3);

fp = fopen("vtp.dat", "r");
fp2 = fopen("vt.dat", "w");

n=0;
while ((fgets(string, 81, fp)) != NULL)
{
    sscanf(string, "%d %d", &x1, &y1);
    printf("%d %d \n", x1, y1);
    x[n] = x1;
    y[n] = y1;
    n++;
}
fclose(fp);
number=n-1;

for (i=0; i<number; i++)
{  
    sum += ((float)(labs(x[i+1] - x[i])))*60.0/conv;
}  
  
vt=sum/((float)(number-1));  
  
fpprintf(fp2,"%f\n",vt/100.0);  
printf("%f\n",vt/100.0);  
}

Vtp.c

#include <stdio.h>
#include <gl/gl.h>
#include <gl/device.h>
#include <gl/image.h>
#include <math.h>
#include "rct.h"

#define X 0
#define Y 1
#define Z 2
#define XYZ 3
#define XY 2
#define RGB 3

short bluevec[RGB] = {0, 0, 255};

unsigned long *imgbuf[10];
int num, nn, n, total;
char file[10][20], datafile[20];

float xmax, ymax;
long xorig, yorig, xsize, ysize;
int i, datapoint;
FILE *fp;
long vertex[4][XY];
long vert1[2], vert2[2];
main(argc, argv)
int argc;
char **argv;
{

    register IMAGE *image[10];
    short val;
    int wx, wy, preforg;
    register int i;
    extern long *longimagedata();

    if ( argc<2 ) {
        fprintf(stderr, "usage: %s inimage number \n", argv[0]);
        exit(1);
    }

    nn=atoi(argv[1]);

    printf("Enter no. of frames to paste, maximum 10 \n");
    scanf("%d", &total);

    sprintf(datafile,"vtp.dat");
    fp = fopen(datafile, "w");

    n=0;
    for (num=nn; num<nn+total; num++)
    {
        sprintf(file[num], "%s%d%S", "t", num, "rgb");

        if( image[n]=iopen(file[num], "rt") == NULL ) {
            fprintf(stderr, "rpaste: can't open input file %s\n", file[num]);
            exit(1);
        }

        /* calculate the window size */
        sizeofimage(file[num], &xsize, &ysize);

        /* allocate the memory for the pixel data for lrectwrite */
        imgbuf[n] = (unsigned long *) malloc(xsize*ysize*sizeof(long));
        imgbuf[n] = (unsigned long *) longimagedata(file[num]);

        /* open the window */
    }
prefsize(xsize-1,ysize-1+20);
winopen(argv[0]);
wintitle(file[num]);

/* set the display mode for the image */
RGBmode();
gconfig();
drawit(imgbuf[n]);
drawtextline(xsize/2,ysize/2);

scrcod(datafile);
n++;
}
}

drawit(img)
unsigned long *img;
{
cpack(0x00808080);
clear();
reshapeviewport();
lrectwrite(0,0,xsize-1,ysize-1,img);
}

drawtextline(xpos,ypos)
long xpos, ypos;
{
char str[256];

viewport(0,xsize-1,2,16);
grey(0.8);
clear();
sprintf(str,"xpos: %04d ypos: %04d",xpos,ypos);
grey(0.0);
cmov2i(100,4);
charstr(str);
}

scrcod()
{ short val, mval[2], lastval[2];
long org[2], size[2];
Device dev, mdev[2], mdevl[2];
Boolean run;
int leftmouse_down = 0;
lastval[X] = -1;

getorigin(&org[X], &org[Y]);
getsize(&size[X], &size[Y]);

mdev[X] = MOUSEX;
mdev[Y] = MOUSEY;
getdev(2, mdev, lastval);   /* initialize lastval[] */
lastval[X] = org[X];
lastval[Y] = org[Y];

qdevice(LEFTMOUSE);
qdevice(ESCKEY);
qdevice(MOUSEX);
qdevice(MOUSEY);
qdevice(SPACEKEY);
qdevice(PADENTER);

while(1) {
switch (dev = qread(&val)) {
case LEFTMOUSE:
    getdev(2, mdev, mval);
    mval[X] = mval[X] - org[X];
    mval[Y] = mval[Y] - org[Y];
    vert1[X] = mval[X];
    vert1[Y] = mval[Y];
    mark(vert1[X], vert1[Y]);
    drawtextline(vert1[X], vert1[Y]);
    qread(&val);
    break;

case SPACEKEY:
    printf("%d %d \n", vert1[X], vert1[Y]);
    fprintf(fp, "%d %d \n", vert1[X], vert1[Y]);
    qread(&val);
    break;

case PADENTER:
    printf(" \n");
}
fprintf(fp, "\n");
qread(&val);
break;
case ESCKEY:
gexit();
return;
break;
}
}
}

mark(xpos,ypos)
long xpos, ypos;
{
viewport(xpos-2,xpos+2,ypos-2,ypos+2);

c3s(bluevec);
  clear();
}

Vyavg.c

#include <math.h>
#include <gVgl.h>
#include <gVdevice.h>
#include <gVimage.h>
#include <stdio.h>
#include <string.h>

main(argc,argv)
int argc;
char **argv;
{

char string[81],datafile2[20],datafile3[20];
float v[50],hod[50],velsum;
float temp1,temp2,temp3,vyavg,dist;
long min1,min2,reg,nn;
FILE *fp2,*fp3;
if ( argc<2 ) {
    fprintf(stderr,"usage: %s frame number \n",argv[0]);
    exit(1);
}

nn = atoi(argv[1]);

sprintf(datafile2,"%s%d%s","vy",nn, ".dat");
sprintf(datafile3,"vyavg.dat");

fp2 = fopen(datafile2,"r");
fp3 = fopen(datafile3,"a");

fscanf(fp2,"%f", &dist);

reg=0;
while ((fgets(string,81,fp2)) != NULL) {
    sscanf(string,"%f %f", &hod[reg],&v[reg]);
    reg++;
}
fclose(fp2);
reg=reg-1;

velsum=0;
for (min1=0;min1<reg;min1++) {
    velsum+=v[min1];
}

vyavg=velsum/(float)reg;

printf("%f %f\n",dist,vyavg);
fprintf(fp3,"%f %f\n",dist,vyavg);
}
This program is intended to be a non-linear regression routine. The program is a minimization (optimization) of the least squares error computed as $y - \text{nonlinearmodel}$. The nonlinear optimization technique used in this program is the regula falsi.

* Given a nonlinear function, the program outputs the parameter
* The program is a one-dimensional routine.
* The program has been written to handle 100 datapoints.

```
c This is the main program in this case

      common xdata(100), ydata(100), number
      common /blk/holdup,tau
      character*10 filename
      character*10 datafile

      open(50,file='input.dat',status='unknown')
      read(50,*)vholdup
      read(50,*)filename
      read(50,*)datafile
      read(50,*)tauini,taud,step
      read(50,*)zinitia1
      read(50,*)mflag
      read(50,*)vt,vsl,vsg

      holdup=1. - vholdup
      call data(filename)

      2 do 5 i=1,number
        ydata(i)=1.0-ydata(i)
      5 continue

      if (mflag .eq. 1) then
        scale=vt/(vsl+vsg)
        do 6 i=1,number
          xdata(i)=xdata(i)*scale
        6 continue
      endif

      open(60,file=datafile,status='unknown')
```
do 10 tau=tauinit-taud,tauinit+taud,step
tflag=0

call regression(zinitial,zmin,tflag)

if (tflag .eq. 0) then
  sumsq=sumsqefl zmin)
  write(6,*)'Minimum error=',sumsq
  write(6,*)
  write( 60,4 )tau,zinitial,zmin,sumsq
  format(1x,f5.2,2x,f5.2,f8.4,2x,f10.6)
endif

10 continue
  stop
end

*****************************************************************

subroutine data(filename)
  common xdata(100), ydata(100), num
  character*10 filename

c Open the datafile and read the data
c num is the number of datapoints

c write(6,*)
c write(6,*)'Enter the datafile to be opened'
c write( 6, *)'The name should be within single quotes'
c read(5,*)filename

  open(90,file=filename,status='old')

  num=0
  3 read(90,1,end=2)xx,yy
  1 format(2(f9.6))
      num=num+1
      xdata(num)=xx
      ydata(num)=yy
      goto 3

2   return
end

*****************************************************************

c This is the subprogram cmodel that computes the value of
c the second order process model for zeta less than 1.0

c The function requires as input the following parameters:
c a : The value of the step height
c zeta : The value of zeta
c tau : The value of the time constant
c x : The independent variable ( can be t )

c This model assumes that a and tau have been declared
in a common block called blk in the main program.

********************************************************************************
function cmodel(zeta,t)
common /blk/a,tau

expterm = exp(-zeta*t/tau)

zsqrt = sqrt(1. - zeta**2)
lagterm = atan(zsqrt/zeta)

sinterm = sin(zsqrt*t/tau + lagterm)

cmodel = a*(1. - 1/zsqrt * expterm * sinterm)

return
end

********************************************************************************

The function to be optimized is the least square error
x is the parameter, e.g. zeta, or tau, or m etc.

function f(x)
common xdata(100), ydata(100), num

sum=0.0
do 10 i=1,num
    sum=sum+((ydata(i)-cmodel(x,xdata(i)))**2)
10 continue

f=sum

return
end

********************************************************************************

The derivative of the function
function fderiv(x)

deltax=0.001

fderiv = (f(x+deltax) - f(x))/deltax

return
end

*******************************************************************
c This is the program that executes the nonlinear regression

subroutine regression(xinitial,xmin,tflag)

common xdata(100), ydata(100), number

delta=0.1

1 call bracket(delta,xinitial,xprev,xleft,xright,iflag)

if (iflag .eq. 0) then
  if ((xright - xprev) .ge. 10) then
    delta=delta/2.0
    xinitial=xprev
    goto 1
  elseif (xright .eq. 0.0) then
    write(6,*)'Error .. zeta value exceeding unity'
    tflag=1
    goto 5
  elseif ((fderiv(xprev)*fderiv(xright)) .gt. 0.0) then
    write(6,*)'Error .. Function blew up'
    tflag=1
    goto 5
  else
    write(6,*)'Function successfully bracketed'
    endif
else
  write(6,*)'Error .. Function minimum at initial guess'
  tflag=1
  goto 5
endif

write(6,3) xprev,xleft,xright
3 format(1x,3(f10.3))
if (iflag .eq. 0) then
  call secant(xprev,xright,xmin,kflag)
if (kflag .eq. 1) then
  tflag=1
endif
endif
5 return
end

*******************************************************************************

subroutine bracket(delta,xinitial,xprev,xleft,xright,iflag)

c Bracketing the function

  x=xinitial
  func=f(xinitial)
  iflag=0

  i=1
  x=x+delta
  func1=f(x)

  if (func1 .ge. func) then
    iflag=1
    goto 100
  else
    xleft=x
    xprev=xinitial
  endif

1  i=i+1
  x=x+delta
  func2=f(x)
  if (func2 .le. func1) then
    xprev=xleft
    xleft=x
    func=func1
    func1=func2
    goto 1
  elseif (func2 .gt. func1) then
    xright=x
  endif
subroutine secant(xl,x2,xmin,kflag)

tol = 1.0e-4
k=0
kflag=0

fd1=fderiv(xl)
fd2=fderiv(x2)
write(6,4) fd1,fd2
4 format(1x,'Initial derivatives',/,1x,2(e10.3))

xl=xl
xr=x2
fdr=fderiv(xr)
fdl=fderiv(xl)

10 k=k+1
slope = (fdr - fdl)/(xr-xl)
x = xr - fdr/slope
fdx=fderiv(x)

if ( abs(fdx) .le. tol) then
   write(6,*)'Function minimum found at',x
   xmin=x
else
   if( (fdr*fdx) .lt. 0.0) then
      xl=x
      fdl=fdx
   else
      xr=x
      fdr=fdx
   endif
   if (k .eq. 5000) then
      write(6,*)'Error .. No convergence'
      write(6,*)'x=',x,'fdx=',fdx
      kflag=1
      goto 50
   endif
goto 10
endif

50 return
end
**Nomenclature**

**English Letters**

A  
cross sectional area of pipe

A  
amplitude of step input for second order system, Equation (5.7)

$A_f$  
area of front face of void structure, defined in Equation (4.6)

$A_s$  
area of a section of mesh, defined in Equation (4.4)

a  
empirical coefficient in Equation (5.5)

a  
area in the stratified zone, used with subscripts

$a_L$  
local liquid area, used in integral in Equation (6.16)

$\alpha$  
area fraction, used with subscripts

b  
empirical coefficient in Equation (5.5)

C  
constant used in Equations (2.1) and (5.1)

$C_D$  
drag coefficient

$C_G$  
constant used in Equation (2.8) to calculate friction factor

$C_L$  
constant used in Equations (2.7) and (6.23) to calculate friction factor

$c_v$  
used denote conversion from pixels to cm, Equation (4.1)

$c_1$  
used with subscript to denote constant used in Equation (5.3)

D  
pipe diameter

$D_L$  
hydraulic diameter for liquid, defined in Equation (2.8)

$D_G$  
hydraulic diameter for gas, defined in Equation (2.8)

$d_b$  
bubble diameter, used in Equation (6.25)

$E_o$  
Eotvos Number, defined in Equation (6.19)
$F_D$  drag force

$Fr$  Film Froude Number

$f_G$  gas phase friction factor

$f_i$  interfacial friction factor

$f_L$  liquid phase friction factor

$G_1$  dimensionless group defined in Equation (6.26)

$g$  acceleration due to gravity

$h$  height

$h_{EF}$  effective height of the liquid film, defined in Equation (6.3)

$h_L$  equivalent head of pressure, defined in Equation (4.8)

$h_F$  equivalent head of pressure in film, Equation (6.11)

$h_j$  equivalent head of pressure in slug

$h_{LF}$  height of liquid film

$h$  dimensionless height, $h / D$

$k_1$  constant used to determine $f_i$ over $f_G$, Equation (5.4)

$L$  length

$l$  length

$P$  pressure

$p$  pressure

$p'_o$  pressure defined in Equation (6.9)

$q$  volumetric flow rate

$Re$  Reynolds Number
$S_l$ width of gas-liquid interface, Equation (2.4)

$S_L$ perimeter of liquid contact with wall

$S_G$ perimeter of gas contact with wall

$T$ chord length at height, $h$, from the bottom

$t$ time

$u$ velocity in a coordinate system moving with the slug front

$v$ velocity in a stationary coordinate system, used with subscripts

$v(h)$ local velocity at height $h$ from the bottom, used with subscripts

$x$ axial coordinate

$Y$ denotes response of a system, Equation (5.7)

$y$ transverse coordinates

$y$ distance from the bottom of the pipe

**Greek Letters**

$\alpha$ void fraction

$\rho$ density

$\mu$ viscosity

$\zeta$ damping coefficient characterising second order system, Equation (5.6)

$\zeta'$ depth over which $p'$ acts

$\delta$ thickness of boundary layer

$\tau$ time constant characterising second order system, Equation (5.6)
$\sigma$ surface tension

**Subscripts**

b denoted the pipe bottom

b bubble

c cursor position in image analysis

d drift

EF effective film

f film

G gas

GF gas over liquid film

GS gas in the slug

i interface

$i$ index of number of points processed in image analysis

j hydraulic jump (slug)

L liquid

LF liquid film

LS liquid in the slug

o center line

0, 1 points in the liquid film and inside the slug

s slug
Superscripts

\(m\) exponent in the calculation of the liquid friction factor, Equations (2.7) and (6.23)

\(n\) exponent in the calculation of the gas friction factor, Equation (2.7)
ABSTRACT

GOPAL, MADAN Ph.D. August, 1994
Chemical Engineering

Visualization and Mathematical Modelling of Horizontal Multiphase Slug Flow

Director of Dissertation: W. Paul Jepson

This study involves an experimental and theoretical investigation of slug flow in two phase gas-liquid mixtures. Water and an oil of viscosity 15 cP were used for the liquid phase and carbon dioxide was used for the gas phase. Flow in a 75 mm I.D., 10 m long acrylic pipeline system is studied. The techniques utilize digital image analysis and computational fluid dynamics. Flow is recorded on video by S-VHS cameras, using an audio-visual mixer. The image is then digitized frame-by-frame and analyzed on a SGI™ workstation. Detailed slug characteristics including, liquid film heights, slug translational velocity, mixing length, slug length, axial and radial void distribution, and instantaneous velocity profiles are obtained.

It is seen that slug characteristics are strongly influenced by the Film Froude Number ahead of the slug. The gas is released into the slug body in the form of pulses whose frequency increases with Froude number. The hydrodynamic boundary layer is destroyed at the slug front but begins to redevelop in the mixing zone. It becomes fully developed at the end of the mixing zone and the one-seventh power law shape is applicable. The length of the mixing zone is directly proportional to the Film Froude Number. At the end of the mixing zone, the gas tends to move towards the top of the pipe and the void fraction distribution tends towards a steady profile.

The slug translational velocity is described using a drift velocity model. A Froude Number for the slug is defined, utilizing the equivalent pressure head. The variation of the Froude