FADING MULTIPATH BIAS ERRORS IN GLOBAL POSITIONING SYSTEM
RECEIVER TRACKING LOOPS

A Thesis Presented to
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by
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<thead>
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<th>SYMBOL/ACRONYM</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>SPS</td>
<td>Standard Positioning Service</td>
</tr>
<tr>
<td>GPS III</td>
<td>Long-term modernization effort for GPS</td>
</tr>
<tr>
<td>DGPS</td>
<td>Differential GPS</td>
</tr>
<tr>
<td>SA</td>
<td>Selective Availability</td>
</tr>
<tr>
<td>PRN</td>
<td>Pseudo-Random Noise (used for ranging codes)</td>
</tr>
<tr>
<td>DSSS</td>
<td>Direct-Sequence Spread-Spectrum</td>
</tr>
<tr>
<td>IF</td>
<td>Intermediate Frequency</td>
</tr>
<tr>
<td>DLL</td>
<td>Delay-Lock Loop</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase-Lock Loop</td>
</tr>
<tr>
<td>$\varphi_e$</td>
<td>carrier phase tracking error</td>
</tr>
<tr>
<td>$t$</td>
<td>time vector</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular carrier frequency</td>
</tr>
<tr>
<td>$T_{\text{dump}}$</td>
<td>accumulate and dump interval of correlators</td>
</tr>
<tr>
<td>$C(t)$</td>
<td>PRN spreading code in general model</td>
</tr>
<tr>
<td>$C'_c(t)$</td>
<td>demodulated spreading code in coherent DLL</td>
</tr>
<tr>
<td>$C'_i(t), C'_q(t)$</td>
<td>in-phase and quadrephase demodulated spreading code in noncoherent DLL</td>
</tr>
<tr>
<td>$\tau$</td>
<td>lag vector in correlation function (code offset)</td>
</tr>
<tr>
<td>$I_E, I_L, I_P$</td>
<td>early, late, and prompt in-phase correlation components</td>
</tr>
<tr>
<td>$Q_E, Q_L, Q_P$</td>
<td>early, late, and prompt quadrephase correlation components</td>
</tr>
<tr>
<td>CDLL</td>
<td>Coherent Delay-Lock Loop</td>
</tr>
<tr>
<td>NCDLL</td>
<td>NonCoherent Delay-Lock Loop</td>
</tr>
</tbody>
</table>
\( R_i, R_Q \) in-phase and quadrature correlation functions
\( D, D_C, D_{NC} \) discriminator function/output for general model, CDLL, and NCDLL
\( f_{\text{rec}} \) frequency at which a signal is received by a user
\( f_{\text{nom}} \) unperturbed transmission frequency
\( V_{\text{LOS}} \) rate of change in signal path length
\( c \) speed of light in a vacuum
\( f_{\text{DOP}} \) Doppler shift
\( h \) height above ground
\( \alpha \) satellite elevation angle
\( \Delta p \) path length difference between multipath and direct signal
\( f_m \) reception frequency of multipath signal
\( f_d \) reception frequency of direct signal
\( p_l \) line-of-sight path length
\( p_m \) reflected signal path length
\( f_{\text{diff}} \) relative multipath Doppler shift
\( \text{CW} \) Continuous Wave
\( \text{RF} \) Radio Frequency
\( \text{CDMA} \) Code-Division Multiple Access
\( \text{C/A code} \) Coarse/Acquisition code
\( d \) GPS receiver correlator chip spacing
\( a_i, \tau_i, \theta_i \) relative amplitude, delay, and phase of the \( i \)th received signal
\( \phi \) phase of locally generated IF carrier
\( \text{VCO} \) Voltage Controlled Oscillator
\( \text{SMR} \) Signal-to-Multipath Ratio
\( \lambda \) chip length of PRN code (293 meters for C/A code)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADR</td>
<td>Accumulated Delta Range (carrier phase measurement)</td>
</tr>
<tr>
<td>PR</td>
<td>PseudoRange</td>
</tr>
<tr>
<td>R_{true}</td>
<td>geometric range from user to satellite</td>
</tr>
<tr>
<td>T_{rec}, T_{sv}</td>
<td>receiver and satellite clock errors</td>
</tr>
<tr>
<td>N_{code}, N_{car}</td>
<td>noise on code and carrier measurements</td>
</tr>
<tr>
<td>MP_{code}, MP_{car}</td>
<td>multipath error on code and carrier measurements</td>
</tr>
<tr>
<td>SNR</td>
<td>signal-to-noise ratio</td>
</tr>
<tr>
<td>L1</td>
<td>primary GPS link frequency (1575.42 MHz +/- 12 MHz)</td>
</tr>
<tr>
<td>\phi_m</td>
<td>multipath relative phase in phasor model</td>
</tr>
<tr>
<td>\phi_c</td>
<td>composite signal relative phase in phasor model</td>
</tr>
</tbody>
</table>
1. **INTRODUCTION**

In the new era of a truly dual-use Global Positioning System (GPS), the users of the standard positioning service (SPS) recently have been granted greater access and capabilities that will drive the development of yet broader and higher-accuracy applications of the system. An expansion of protected bandwidth, a performance increase stemming from the removal of Selective Availability, and the promise of additional signals to aid in higher-precision ranging, better standalone performance, and added resistance to interference guarantee that GPS has a future in a myriad of markets. Earmarked funds from the civilian sector and the United States Department of Defense have insured that development of all system segments will continue and long-term plans for the third generation of the system (GPS III) are in place to provide service through the third decade of this century [1].

Of the numerous error sources that plague the core operation of GPS, none has received so much recent attention as multipath interference. Generally speaking, multipath is the phenomenon of a signal propagating through more than one spatial channel between a transmitter and a receiver. The effect is seen in any real wireless transmission scheme but, because of different priorities in communications and radio-navigation systems, the perceived severity of system degradation caused by multipath varies. The focus of this thesis is the effect of multipath on navigation systems, specifically the user segment of GPS. In previous years, multipath typically was a concern only for high-accuracy differential GPS (DGPS) users because the low-frequency errors from Selective
Availability (SA) tended to dominate the error budget of the standalone system user. The error budget for the standard service is given in Table 1.1 [2] for standalone use prior to SA being shut off.

Table 1.1: Error budget for standalone GPS.

<table>
<thead>
<tr>
<th>Error Source</th>
<th>GPS 1-sigma error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selective Availability</td>
<td>32.3</td>
</tr>
<tr>
<td>Ionospheric Delay</td>
<td>5</td>
</tr>
<tr>
<td>Tropospheric Delay</td>
<td>1.5</td>
</tr>
<tr>
<td>Receiver Noise</td>
<td>1.5</td>
</tr>
<tr>
<td>Multipath</td>
<td>2.5</td>
</tr>
<tr>
<td>Other (Clock effects, orbital errors, etc.)</td>
<td>5.4</td>
</tr>
<tr>
<td>Total</td>
<td>33.3</td>
</tr>
</tbody>
</table>

These errors assume a standard correlator spacing in the receiver. For differential use, Selective Availability and atmospheric effects would ideally be reduced to zero for short baselines and thermal noise can be suppressed to below the decimeter level with carrier-aiding and smoothing techniques in modern receivers. Now, with the removal of SA from the GPS signal, the accuracy potential of even the standalone user has dramatically increased and composite errors of such users consist mainly of receiver thermal noise, atmospheric effects, and multipath. The anticipated addition of a third GPS ranging signal for civilian use will allow less-sophisticated receivers (i.e. not having to employ codeless techniques or P-code tracking on L2) to remove the largest component of atmospheric with a high degree of accuracy using knowledge of the dispersive nature of the ionospheric plasma [3]. These delays can then be removed
from measured pseudoranges to negate the error source. Thus, noise and multipath will represent the largest error sources for virtually all GPS users. Further, carrier aiding techniques and higher chipping rates in the new signal structure will allow further reduction in the effect of thermal noise on position estimation, leaving multipath as the single largest barrier to guaranteed sub-meter positioning accuracy under nominal system operating criteria. The severity of multipath errors is dependent on several factors. Of most obvious concern are the defining parameters of the multipath signal itself, which will be examined thoroughly in Chapter 3. Briefly, these are the power and delay of the multipath relative to the direct line-of-sight signal. Additionally, while phase rate is closely tied to relative delay, a conceptual benefit is gained in this thesis by treating it as a separate parameter. Multipath interferes with the direct signal and causes the tracking loops to synchronize incorrectly to the pseudo-random noise (PRN) ranging code by introducing added apparent signal delay. Additionally, platform-dependent factors such as antenna gain pattern and receiver architecture influence the effect that multipath will have on pseudorange measurements. This document examines a specific facet of the latter.

Many of the current models and all published validation efforts regarding multipath error are concerned with bounding multipath errors by defining the envelope of the worst-case performance based on characteristics of the incident multipath. There are three apparent methods used to obtain these error envelopes. The first is to formulate a mathematical model of core operations in a GPS receiver and to find the collection of multipath parameters that return the maximum PRN code-phase estimation errors in
that model. The second approach is to simulate the operation of a GPS receiver in software and to vary a defining parameter of the multipath in such a way that allows the GPS receiver tracking loops to settle on a steady-state error before the parameter is advanced to the next value. The error is then recorded as a function of the variable parameter and tabulated or plotted so that conclusions about the effects of multipath can be made. The third method involves using actual GPS signals in a test environment where a multipath reflection can be injected, monitored, and controlled. In this case, data collection is similar to that in the simulation. While these techniques are very effective in determining error envelopes, they inherently assume a static or very slowly changing multipath relative phase and consequently allow all of the multipath power to get through the tracking loops. In reality, a GPS user generally is in motion and, as a result, the path between satellite and user is subject to dynamics that differ from those of the satellite-reflector-user path. From a classical communications perspective, it can be seen that different dynamics between the two signals allow some of the multipath power to be electronically blocked from entering various parts of the receiver. This thesis will examine GPS range measurement performance in situations where the relative phase of a multipath signal is more rapidly changing. Many of the preliminary results discussed further in this document were reported by the author and M. Braasch to the Institute of Navigation and are available in the relevant conference proceedings [4, 5]. For this thesis, the initial task is to study specific attributes of the GPS receiver tracking loops that will be modeled and evaluated.
2. TRACKING LOOP ARCHITECTURES

In spread-spectrum communications, a staple component in the receiver is the feedback-tracking loop. The covert nature and limited power of spread-spectrum schemes make synchronization very important. Reliance on nominal signal properties can cause poor quality or even total failure in the channel because of deviations due to clock errors and transmitter/receiver motion. In a direct-sequence spread-spectrum (DSSS) receiver, it is the task of the feedback structure to replicate the carrier waveform and to synchronize the local and incident spreading codes so that the baseband signal can be used to form a decision about the modulated data. In communications, this is the entire point of the system and extensive effort is placed on increasing the data rate and reducing the bit-error probability to the best possible performance. For radio-navigation systems, the data is important, but not the top priority. Cutting edge speed of data through the channel is not required but in a trade-off, precise synchronization becomes crucial. Assuming that a DSSS signal at its final intermediate frequency (IF) is used as the input to the loop of Figure 2.1, two further stages of down-conversion are required.
Figure 2.1: Simplified tracking procedure for spread-spectrum signal.

For this model, and all to follow, it is assumed that the receiver is maintaining frequency lock on the incoming carrier. In the diagram, a more precise estimate of the IF carrier signal, made by a separate phase-lock loop (PLL), is multiplied with the input in an effort to strip the carrier from the received signal completely. A delay-lock loop (DLL) then aligns the locally generated replica of the spreading code with the demodulated signal and multiplies the two. The outputs are used for data symbol decision and for feedback to the DLL. This thesis will concentrate on the dependence of certain multipath errors on the type of DLL built into the receiver architecture. A more detailed representation of the conceptually simpler type of receiver channel is given in Figure 2.2. Here, the carrier estimate is simply a sinusoid that is in error by some phase $\varphi_c$, and the DLL uses the down-converted signal component, which results from multiplication with this local waveform, to perform code synchronization.
For this architecture, consider the noise-free, data-free DSSS input signal:

\[ S(t) = C(t) \sin(\omega t) \]  \hspace{1cm} (2.1)

where \( S(t) \) is the input to the loop, \( C(t) \) is the spreading code after data has been stripped off, and \( \omega \) is the angular frequency of the modulated carrier. Since the correlation process will filter out high-frequency terms, only the signal components around the lower code frequency need be considered. The demodulated code, \( C'(t) \), for a carrier phase error of \( \varphi_e \) then becomes:

\[ C'_e(t) = \text{LOWPASS}[C(t) \sin(\omega t) \sin(\omega t + \varphi_e)] \]

\[ = \text{LOWPASS}[C(t)(\frac{1}{2})\{\cos(-\varphi_e) - \cos(2\omega t + \varphi_e)\}] \]

\[ = \frac{1}{2} C(t) \cos(\varphi_e) \]  \hspace{1cm} (2.2)

In the general architecture of Figure 2.1, this output is then multiplied by the locally generated spreading code and accumulated over a certain period of time before it is
sampled for feedback and decision. The maximum output magnitude of this process occurs when the code alignment is perfect and tapers off to nearly zero for synchronization errors greater than 1 PRN chip. For a local code offset, \( \tau \), and no phase tracking error, the ideal normalized output may be expressed as:

\[
R(\tau) = \frac{T_{\text{dump}}}{T_{\text{dump}}} \int_0^{T_{\text{dump}}} C(t)C(t + \tau)dt
\]  

(2.3)

where \( T_{\text{dump}} \) is the time of the accumulation. Mathematically, this is the autocorrelation function of the spreading code and additional information on this procedure and its properties may be found in [2, 6, 7]. Referring again to the architecture of Figure 2.2, if the phase error, \( \phi_e \), is introduced, the resulting correlation function, \( R_C \), is then

\[
R_C(\tau, \phi_e) = R(\tau) \cos(\phi_e).
\]  

(2.4)

The last term in the equation implies that the correlation output is strongest when the phase error in the carrier estimate is low and degrades with larger values of \( \phi_e \). The DLL feedback process is designed ideally to keep this output at its maximum value, i.e. so that the code offset, \( \tau \), is zero. The actual mechanism used to accomplish this is depicted in the second stage of Figure 2.2. Advanced and delayed versions of the local PRN code are generated with reference to the actual estimated code phase and are correlated with the demodulated signal. These signal paths are appropriately termed early and late correlators. Since the autocorrelation of the code is an even function of the relative offset, \( \tau \), the synchronization with the error-free signal is accurate when the outputs of the early and late correlator legs are equal. The difference in early and late correlation values forms a discriminator signal that can be driven to zero in the
feedback loop to align the local code with the incident waveform. The discriminator, \( D \), for the loop in Figure 2.2 is:

\[
D = I_E - I_L
\]  

(2.5)
prior to filtration, where \( I_E \) denotes the in-phase correlation energy from the early channel and \( I_L \) is in-phase energy from the late channel. Additionally, the discriminator may be multiplied by the sign of a prompt correlation channel output in order to remove dependence on the BPSK-modulated navigation data. This difference is filtered to eliminate high-frequency noise and used as the driving signal to a voltage/numerically controlled oscillator to adjust the code generation rate until lock is achieved. In a practical GPS receiver, the synchronized (prompt) code is tapped off and used in the carrier PLL to strip the modulation from the signal. Often, information from carrier tracking is used to aid the code loop since carrier noise is about two orders of magnitude less than code noise. For simplicity, these features have been omitted in Figure 2.2 and related diagrams, but will be considered later in this thesis. Since good reception in this architecture requires that the demodulating sinusoid be generated in phase-lock with the IF input, this loop is appropriately referred to as a coherent DLL (CDLL).

For added tracking robustness, most spread-spectrum receivers use not only in-phase signal energy, but also that which is in phase quadrature. Figure 2.3 shows how this is done by using a phase shifted version of the local carrier IF estimate so that two orthogonal signals are used to demodulate the input and ensure that all of the energy in the phase plane can be captured and used for tracking. This also ensures that the
relative sign of the navigation data bit, D(t), on the signal will not corrupt signal tracking.

**Figure 2.3: Unaided code loop for non-coherent DLL channel.**

The demodulated signal for this loop consists of two components. The first, C'I(t), is the in-phase portion and is identical to C'(t). The signal on the second leg, C'Q(t), is the quadrature portion and is given as follows:

\[
C'Q(t) = \text{LOWPASS}[C(t)\sin(\omega t)\cos(\omega t + \varphi_e)]
\]

\[
= \text{LOWPASS}[C(t)(\frac{1}{2})\{\sin(-\varphi_e) + \sin(2\omega t + \varphi_e)\}]
\]

\[
= -\frac{1}{2} C(t) \sin(\varphi_e)
\]

(2.6)

The correlation process may be applied to each of these I and Q signals and the respective results versus code offset and carrier phase error are as follow:

\[
R_1(\tau, \varphi_e) = R(\tau)\cos(\varphi_e)
\]

(2.7)
To relieve the effect of the phase tracking error, the correlator outputs may be squared and added so that the composite correlation function reduces to:

\[ R_{Q}(\tau, \varphi_{e}) = -R(\tau) \sin(\varphi_{e}) \]  \hspace{1cm} (2.8)

The peak of this new function is at \( \tau = 0 \) since the maximum of \( R(\tau) \) is at \( \tau = 0 \) and so the code tracking philosophy of adjusting the local code for maximum correlator output remains the same. Also, this function is still even and so a procedure analogous to the discriminator operation in the coherent loop can be used. The signal given by

\[ D = I_{E}^{2} + Q_{E}^{2} - I_{L}^{2} - Q_{L}^{2} \]  \hspace{1cm} (2.10)

can be driven to zero to synchronize the local PRN code where these components are the outputs of the four main legs in Figure 2.3. Since the carrier phase error, \( \varphi_{e} \), is no longer of concern, it can be allowed to drift over time. In other words, phase lock is not required and signal tracking can proceed under the less strict condition of frequency lock. Since phase lock is not required in this architecture, it is termed a non-coherent DLL (NCDLL) and is a more robust and often-used tracking method in spread-spectrum systems.

In both cases, the DLL uses advanced and delayed versions of the local replica code to maintain synchronization with the signal and extract navigation data. The specific operation and consequences of this process for different types of DLLs will be discussed further in Chapter four, as understanding this feedback process is essential for appreciation of the main points of this thesis.
In practice, the code loop discriminator may be constructed in a number of ways so long as the result provides some unambiguous indication of the error in the phase of the local PRN code generation. The discriminator for the coherent loop is rather straightforward in that the simple early-late difference is used to drive the code generation oscillator. However, one useful modification of this is to normalize the output by correlation power in the prompt channel and use

\[ D = \frac{I_E - I_L}{I_P} \]  

(2.11)
in closing the loop. This modification makes the discriminator more stable under input power fluctuations and removes the navigation data polarity from the signal. For the non-coherent loop, different forms of the discriminator function are apparent since the receiver designer may wish to use the squared correlation components as they come from the correlation legs, or convert back to amplitude to keep the entire process as linear as possible. Other forms use correlation energy from the prompt channel to form products that are similar to the squared values in equation 2.10. Some possible non-coherent DLL discriminators and their mathematical definitions are shown in Table 2.1 [2].
Table 2.1: Examples of NCDLL discriminator functions

<table>
<thead>
<tr>
<th>Type (name of discriminator)</th>
<th>Mathematical Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dot Product Power</td>
<td>((I_E - I_L)I_P + (Q_E - Q_L)Q_P)</td>
</tr>
<tr>
<td>Early - Late Power (used as default)</td>
<td>(I_E^2 + Q_E^2 - I_L^2 - Q_L^2)</td>
</tr>
<tr>
<td>Early - Late Envelope</td>
<td>(\sqrt{I_E^2 + Q_E^2} - \sqrt{I_L^2 + Q_L^2})</td>
</tr>
<tr>
<td>Normalized Early - Late Power</td>
<td>(\frac{I_E^2 + Q_E^2 - I_L^2 - Q_L^2}{I_E^2 + Q_E^2 + I_L^2 + Q_L^2})</td>
</tr>
</tbody>
</table>

It should be noted that normalization can be used for any of the discriminator types and a large number of combinations are possible. For the sake of consistency in comparison, this thesis will use the normalized early minus late discriminator when discussing a CDLL and the normalized dot product discriminator when discussing a NCDLL.

Further discussion of the operation of each of these loop types will continue in Chapter four. It is first necessary, however, to explore the relationship between the operating environment of GPS and characteristics of multipath interference so that a study of their effect on receiver performance can commence.
3. **Fading Multipath**

3.1 *Doppler-Shifted Multipath*

The elements that make up the Global Positioning System are constantly in motion. Even for a user who is stationary with respect to the earth or some other reference point, the perturbed Keplerian mechanics of the satellite constellation make it necessary for the reception equipment to update constantly the calculated location of transmitters in order to establish its own position in space. For a user undergoing general motion, such as an automobile, a seagoing vessel, or an aircraft, the relative dynamics between receiver and satellite become more complex. Consider the case in which a GPS range measurement is made to a particular satellite by a receiver on the earth, as depicted in Figure 3.1.

![Figure 3.1: Line-of-sight GPS signal.](image)
If all error sources and clock offsets are assumed to be zero, the pseudorange measured by the receiver will be equal to the true geometric range between the user and the satellite vehicle. As either the user or the satellite moves in space, the range between the two changes. Although all satellites currently broadcast on the same frequency, the relative velocity between transmitter and receiver directly translates into a perceived frequency shift in the signal received by the user. This is identical to the phenomenon causing red shift in distant stars and is consequently termed a Doppler shift on the GPS signal. Since this effect is explored and used extensively in optics [8], radar [9], and acoustics [10], an equation involving the frequency shift will be presented here without proof. When the first derivative of the line-of-sight range is $v_{\text{LOS}}$, the expression relating the nominal transmitted frequency to the received frequency is:

$$f_{\text{rec}} = f_{\text{nom}} (1 - v_{\text{LOS}}/c)$$  \hspace{1cm} (3.1)

where $f_{\text{rec}}$ is the frequency of the signal at the receiver, $f_{\text{nom}}$ is the transmitted frequency from the satellite, and $c$ is the speed of light through the medium between the two, usually assumed to be a vacuum. This means that the frequency change, or the Doppler shift, is

$$f_{\text{Dop}} = f_{\text{nom}} (-v_{\text{LOS}}/c).$$  \hspace{1cm} (3.2)

Now, consider the case in which a reflection from the earth is being picked up by the user equipment in addition to the direct signal from the satellite. The purely illustrative geometry of Figure 3.2 allows one to visualize what would happen if the user adopted a positive velocity with respect to the earth.
Figure 3.2: Line-of-sight signal and a ground reflection with moving user.

Referring to equation 3.1, the direct line-of-sight velocity in this case is negative and so this signal acquires a positive Doppler shift prior to reception. The reflected signal, however, has a growing path length and so $v_{\text{los}}$ in this case is positive and thus resulting in a negative Doppler shift. For this case, in which the satellite is directly overhead and the reflection is from the ground, the difference in the frequencies of the two received signals is double the Doppler shift caused on the direct signal by platform motion. In general, the frequency difference is dependent on geometry of the user velocity vector, the satellite velocity vector, and the relative position of the reflecting body. In all cases, the frequency difference can be calculated by comparing the dynamics in the direct and reflected signal paths.
Knowing that the general case of multipath includes a Doppler shift from the frequency of the direct signal, the next step is to examine the magnitudes of the frequency differences. This will allow a thorough investigation of the nature of multipath-induced errors, a more enlightened path toward reducing their effect, and an understanding of the circumstances under which certain candidate mitigation techniques can be employed. First consider the case just shown where the airborne GPS receiver is picking up a ground reflection multipath. Since the satellite transmitter is a great distance away from the user and reflection point on the earth, the parallel-ray approximation shown in Figure 3.3 is valid. Here, $h$ is the aircraft height above ground and $\alpha$ is the elevation angle of the satellite for both the receiver and reflection point.

Figure 3.3: Geometric analysis for frequency-shifted multipath with ground reflection.
Under the assumption of plane wave incidence, the following relationship is a commonly accepted result:

\[ \Delta p = 2h \sin \alpha \]  

(3.3)

where \( \Delta p \) is the path difference for the multipath signal. Over a short period of time, \( \alpha \) is nearly constant and the time-derivative of the path length difference is

\[ \frac{d[\Delta p]}{dt} = (d[h]/dt)\sin \alpha \]  

(3.4)

Since the Doppler shifts on each signal are linearly proportional to the relative velocities along the signal paths from equation 3.2, the following logic holds:

\[ f_{\text{diff}} = f_m - f_d = f_{\text{nom}} \left( \frac{v_d - v_m}{c} \right) = f_{\text{nom}} \frac{d[p_l - p_m]}{c} = \frac{d[\Delta p]}{dt} \frac{f_{\text{nom}}}{c} \]  

(3.5)

In this expression, \( f_{\text{diff}} \) is the frequency difference, \( f_m \) is the Doppler shift of the multipath, \( f_d \) is the Doppler shift of the direct signal, \( v_d \) is the relative closing velocity of the direct signal path, \( v_m \) is the relative closing velocity of the reflected signal path, \( p_l \) is the line-of-sight path length, and \( p_m \) is the reflected path length. To summarize this result and show the magnitudes of the relative multipath Doppler shifts, Figure 3.4 presents the frequency difference between direct and multipath signals versus satellite elevation angle for a descending aircraft experiencing a ground bounce multipath. The vertical velocity of the reception antenna is used as a parameter.
Figure 3.4: Relative multipath Doppler shift vs. satellite elevation angle, descent speeds of 5, 10, and 20 feet per second.

These descent rates are common for civil and commercial aviation. It is apparent from the figure that multipath relative Doppler shifts on the order of tens of Hertz will not be uncommon for a user tracking high-elevation satellites. Lower elevation satellites and more conservative vertical speeds yield frequency differences of several Hertz.

For multipath coming from sources other than the ground, the elevation dependence may vary. Consider, for example, a vertical reflecting surface such as a building. Figure 3.5 depicts an aircraft traveling directly away from such an object and tracking a satellite with a certain elevation, $\alpha$. 
Figure 3.5: Geometric analysis for frequency-shifted multipath with a vertical reflector.

While the two-dimensional situation in the Figure 3.5 is not likely to occur on the scale of the drawing, it serves to illustrate conceptually the types of multipath signals likely to be received when a user is operating near large natural or man-made obstructions. In this situation, the greatest shared component between the velocity vector of the taxiing aircraft and the satellite line-of-sight vector occurs for low-elevation satellites and thus, the Doppler shift should be highest in these cases. Figure 3.6 verifies this notion. Here, the relative multipath Doppler is plotted against satellite elevation angle for some common speeds of aircraft final approach and landing using the geometry of Figure 3.5.
In reality, of course, these values are worst-case and actual observations will depend on the azimuth of the satellite and the geometric relationship among velocity vectors of all bodies involved. Still, the relative Doppler shifts for low-elevation satellites, which are the most likely to be corrupted by multipath, can reach several hundred Hertz.

3.2 Signal Fading

Now that the existence of a frequency disparity between direct and reflected signals from GPS satellites has been established, it is possible to begin a study of the phenomenon of multipath fading. While the ultimate goal of this document is to explore the contributions of the two receiver architectures defined in the previous
chapter, the best understanding of signal fading will come from a high-level analysis of continuous wave (CW) and DSSS interference properties. From basic signal theory, it is known that the superposition of CW signals at two frequencies results in a modulation of one-half the sum-frequency sinusoid by one-half the difference-frequency sinusoid or, mathematically,

$$\cos(at) + \cos(bt) = 2 \cos\left[\frac{1}{2}(a + b)t\right] \cos\left[\frac{1}{2}(a - b)t\right]$$  \hspace{1cm} (3.6)$$

For two sinusoids that are close in frequency, such as the 1-Hz ($x_1(t)$) and 1.1-Hz ($x_2(t)$) signals in Figure 3.7a and 3.7b, the difference frequency will be small and the resultant wave has a modulation frequency of 0.05-Hz as shown in Figure 3.7c ($x_3(t)$).

Figure 3.7: Continuous-wave (CW) fading phenomenon from superposition of similar sinusoids.
Notice that the envelope of the modulation is periodic at 0.1 Hz, the difference frequency. This is due to the cancellation of phases from the two component frequencies at the 180-degree point of the half-difference frequency sinusoid. If an average power measurement is taken over an interval that is much smaller than the beat interval (10 seconds in this case), there is an apparent fading of signal power that occurs at the difference frequency.

Examination of signal fading in a DSSS scheme is very similar, but with the addition of an extra step of modulation. If the assumption is made that a receiver has already obtained frequency-lock on a signal and synchronized the local copy of the spreading code, the ideal autocorrelation function that is used for code tracking is given by equation (2.3) and shown graphically in Figure 3.8. Although this simplified form of the function assumes an ideal spreading code, infinite bandwidth, and perfect signal lock, it is still useful for illustrative purposes.

![Autocorrelation Main Lobe](image)

**Figure 3.8: Ideal autocorrelation main lobe for DSSS code tracking.**

The correlators responsible for forming the outputs described by this autocorrelation function count on similar components in the locally generated and incoming signals.
For a superposition of a similar signal with a slightly different carrier frequency, the phase of the new carrier rotates with respect to the phase of the original carrier at the difference frequency. In regard to the tracking mechanism, this implies that there would often be partial or negative correlation between this new signal and a local copy that is perfectly locked on the original signal. With respect to the local code, the correlation function of the second signal would oscillate between the triangular peak of Figure 3.8 and an inverted version of the same. Since the correlation process is additive, the frequency-shifted second signal would cause the net correlation function to oscillate as shown in Figure 3.9.

![Figure 3.9: Net correlation function oscillating at carrier difference frequency.](image.png)

It is apparent that, just as in the CW case, the addition of a second signal of equal power causes an oscillation between a double-amplitude phase and a null phase of the received signal.

To examine the specific case of Doppler-shifted multipath on DSSS systems, it is necessary to augment this model to include two general attributes of multipath signals. First, since the path length of the reflected signal is longer than that of the direct by
definition, the multipath will always arrive later than the line-of-sight signal. This causes a path-dependent shift in the correlation function component from the multipath reflection. Secondly, since a reflecting surface is likely, very likely in fact, to be non-infinite and have a certain non-zero resistivity, the amplitudes of most specular multipath signals upon reception are significantly less than the respective direct signal amplitudes. Incorporating these two features leads to the superposition of the two correlation lobes shown in Figure 3.10a.

Figure 3.10: Ideal traces of a) correlation contributions from direct and multipath signal components, and b) phase boundaries of composite correlation lobe.
In this figure, the original triangular function is generated by correlation of the direct GPS signal with the phase-locked local waveform. The smaller, shifted version of the autocorrelation function and its inverted counterpart represent the boundaries of the correlation between the attenuated and delayed multipath signal and the local waveform. It is this part of the function that contains time dependent terms and causes the composite correlation function to oscillate within the bounds shown in Figure 3.10b.

The severity of the variation in the correlation function will depend on relative strength and delay of the multipath, but the effect is constantly present for any real CDMA transmission channel. The frequency of the oscillation in this case is determined by the Doppler shift in the single multipath signal. In general, a number of multipath reflections may be received by the user antenna and require a means of characterization. The fading bandwidth of the signal is used to describe the spread of fading frequencies in the same way that a radio frequency (RF) signal may be assigned a bandwidth. It implies that a specific portion of all fading multipath power may be bound to within certain frequency limits. To allow for more useful interpretation of results and greater clarity, further analysis in this thesis will focus on the case of one specular, dominant multipath reflection so that the term fading frequency will suffice in a description of the signal.

3.3 Doppler Crossover

In addition to the common notion of multipath, which is based on a signal propagating through more than one spatial path between a transmitter and receiver, a second effect
specific to code-division multiple-access (CDMA) communication systems produces results that are very similar to multipath in effect. To study this, it is necessary to abandon the previous notion of an autocorrelation function based on perfect spreading codes. In most real systems, the autocorrelation function does not have values of identically zero for all code offsets greater than one PRN chip. For the ten-bit Gold codes used for ranging in GPS, these unwanted correlation levels are guaranteed to be at least 21 dB below the peak value of the function, regardless of relative Doppler. Nevertheless, their impact is not negligible. Further, since there are currently 28 GPS satellite transmitters in service, the cross-correlation among different pairs of ranging codes is a concern. At times, this gap can also close to as little as 21 dB for equal signal strengths. Figure 3.11 depicts typical autocorrelation and cross-correlation functions for GPS coarse/acquisition (C/A) codes using an analysis of PRN 1 and PRN 2. To make an equivalent comparison, the peak value of the autocorrelation has been truncated in Figure 3.11a, but would have a value of 1023 in this representation. It is clear that the sidelobes of the autocorrelation function (Figure 3.11a) and the cross-correlation values (Figure 3.11b) have approximately the same statistical properties, with the maximum of each function 24 dB below the peak autocorrelation energy for these two codes.
Figure 3.11: a) Autocorrelation function of PRN 1. b) Cross-correlation between PRN 1 and PRN 2.

Under nominal operating conditions, this is sufficient isolation to ensure that minimal error results from distortion of the correlation function. However, for a user that is experiencing substantial signal blockage or power loss on a satellite measurement due to a low-elevation satellite or antenna nulls, the cross-correlation energy from other PRN codes can quickly grow in relative strength. It has been demonstrated in [2] that the combination of atmospheric effects, receiver antenna gain, and power profiles of GPS signals in space can cause a worst-case attenuation of about 16 dB in a tracked satellite relative to the nominal power level. This means that, at times, the isolation between PRN codes can close to as little as 5 dB. Given that the multipath-induced distortion of Figure 3.10 is caused by a signal that is 3 dB down from the direct
strength, it can be surmised that a -5-dB interference of a similar nature can cause substantial errors in the shape of the correlation function.

In order to understand why this type of interference will cause fading in the same way as some multipath, one needs to consider the nature of the system geometry. Relative dynamics among the user and satellite vehicles allow true Doppler shifts (neglecting oscillator errors) of around +/- 5 kHz, depending on the specific user platform [6]. As a result, the Doppler frequency experienced by a receiver tracking a satellite changes over time and depends on satellite elevation and azimuth angles, and on its own motion along the line-of-sight path. For a satellite being tracked with some Doppler shift, say +3 kHz for argument, the stronger signal from a second satellite may have a Doppler shift that is near, or even crossing, the 3 kHz mark. This event is termed a Doppler crossover for obvious reasons. When the frequencies of the wanted and unwanted satellites are sufficiently close that the first stage of filtration in the receiver cannot discriminate between them based on Doppler shift alone, the cross-correlation energy from the interfering satellite becomes an issue. At this point the tracking loop may experience fading correlation power just as in the case of the Doppler-shifted multipath. To illustrate the frequency of occurrence of Doppler crossover, Figures 3.12 and 3.13 present satellite Doppler frequency profiles for visible satellites when two or more signals are within 50 Hertz Doppler shift. The frequencies of interfering satellites are plotted over a twenty-four hour period for vehicles below approximately 35° elevation (Figure 3.12) and above 35° elevation (Figure 3.13) [11] in the mid-latitudes.
Figure 3.12: Doppler crossovers of low-elevation satellites. Shading used to denote individual Doppler traces of interfering satellites. (Source: de Bruijn, et al [11])

Figure 3.13: Doppler crossovers of high-elevation satellites. Shading used to denote individual Doppler traces of interfering satellites. (Source: de Bruijn, et al [11])
These plots were presented by the original authors in color to depict Doppler traces of satellites as their frequencies near each other. The purpose will be served here, however, if one simply realizes that each group of lines represents two or more transmitting vehicles with similar Doppler shifts, i.e. within 50 Hertz. This gives the reader an appreciation for the number of opportunities for multipath-like effects to occur in a typical day even in a perfect free-space operating environment.

Thus far, GPS receiver platform dynamics have been shown to cause a relative Doppler shift in multipath and multipath-like signals due to certain CDMA signal properties, which in turn leads to a time variation of the form of the correlation function. It is the time-varying nature of this correlation function which forms the basis for the theoretical tracking error models presented in the remainder of this thesis. The information discussed in chapters two and three will now be combined to develop the existing theory of the effect of fading multipath on GPS code-tracking performance.
The second chapter discussed how two general loop types can be used in a GPS receiver's code-tracking channel to estimate PRN code phase. The goal of tracking is to maintain synchronization between the incoming GPS transmission and a locally generated replica of that signal in order to extract timing information. The transmission time and reception time estimates are used in conjunction with deterministic models of constellation dynamics and Earth rotation to calculate a pseudorange to a particular satellite. A collection of pseudorange measurements is then used to solve for the position of the phase center of the reception antenna. This process will be discussed no further, as this thesis claims only to examine GPS pseudorange errors caused by multipath in the user equipment, which can be equated to the error in PRN code-phase estimation. This code-phase error is conceptually represented by an offset from zero-lag in the peak of the correlation function from Figure 3.8. In other words, for an extra delay introduced into a pseudorange measurement, the error can be visualized as the entire triangular correlation function of Figure 3.8 moving to the right along the axes. The shift of the estimated peak location maps directly into a range error by multiplying the shift in chips by the spatial chip length. Since the correlation function itself is not used for tracking, but rather the discriminators defined in Equation 2.5 and Equation 2.10, the code-phase estimation error is more accurately discussed as the offset in the zero-crossing of these discriminator functions. Ideally, these should occur at the zero-lag point since early and late correlators will be of equal magnitude here and the difference between these forms the discriminator output.
4.1 Mathematical Analysis

This document now turns to a study of the current body of knowledge regarding pseudorange errors caused by fading multipath. The dominant presence in such published research for the past decade has been R. D. J. van Nee from the Delft University of Technology, The Netherlands. Dr. van Nee’s work has focused on formulating mathematical descriptions of discriminator functions for the general cases of coherent and non-coherent delay lock loops [12, 13, 14]. General assumptions are then made about the ability of the code-tracking loop in the GPS receiver to follow the resultant discriminator zero crossing. Based on this notion, fading frequencies are grouped into two categories.

The first, slow-fading multipath, is that which causes time-varying range errors due to the relative Doppler shift being smaller than the bandwidth of the tracking loop. The other proposed region, fast-fading multipath, occurs when the loop filter effectively removes variation in the multipath-induced error through an averaging process and causes a pseudorange error bias to emerge. Equivalently, the multipath relative Doppler shift is much larger than the code tracking loop bandwidth in this case. Theoretical results have been backed up by field data for the more observable case of slower fading multipath. This section deals mainly with reviewing the available theoretical material on the subject to the extent that it relates to the bench test results for the case of fast-fading multipath presented later in this thesis. A third, but less important, case of multipath fading is also defined. Very-fast-fading multipath is that which has a relative Doppler shift larger than the accumulate and dump rate of the
correlators, between 50 and 1000 Hz for classical receiver design. Multipath of this type is averaged out before it ever enters further processing and so it is not considered a problem.

In order to adhere to the chronology behind the performed research, the performance of a receiver employing a NCDLL with an early-late power discriminator is examined first. This is intended to serve as the token example of a non-coherent receiver for analysis purposes even though many various forms are seen in operation. Adjusting the \( \tau \) variable in Equation 2.9 by \( d/2 \) where \( d \) is the chip spacing between early and late correlators, the in-phase (I) and quadraphase (Q) components for the error-free reception case become:

\[
I^2_E(\tau) + Q^2_E(\tau) = R_{NC}(\tau + d/2) = R^2(\tau + d/2)
\]

(4.1)

\[
I^2_L(\tau) + Q^2_L(\tau) = R_{NC}(\tau - d/2) = R^2(\tau - d/2)
\]

(4.2)

making the NCDLL discriminator function:

\[
D_{NC}(\tau) = I^2_E(\tau) + Q^2_E(\tau) - I^2_L(\tau) - Q^2_L(\tau) = R^2(\tau + d/2) - R^2(\tau - d/2)
\]

(4.3)

from application of Equation 2.10. Using the triangular approximation for the autocorrelation function, \( R(\tau) \), it can be shown (Appendix A) that the ideal discriminator function is linear on the interval \(-d/2 < \tau < d/2\). This simplifies the determination of feedback loop gains in the DLL. The full non-zero region of the ideal discriminator function, or S-curve as van Nee terms it for obvious reasons, is shown in Figure 4.1 for cases of 1-chip and 0.1-chip early-late correlator spacings under unperturbed conditions in the noncoherent loop.
Figure 4.1: Nominal NCDLL discriminator functions for correlator spacings of a) 1 chip, b) 0.1 chips.

Under error-free conditions, the receiver will settle on the code estimate that produces minimal output from the discriminator in the linear region. Graphically, this can be interpreted as operation on the down-going zero crossing of the discriminator function.

The task of defining the CDLL is much simpler. Since the discriminator is of the form shown in Equation 2.5, the relationship between the autocorrelation function and the code offset and phase error from Equation 2.4 can be used to show:

\[
I_E = R_C(\tau + d/2, \varphi_e) = R(\tau + d/2)\cos(\varphi_e) \quad (4.4)
\]

\[
I_L = R_C(\tau - d/2, \varphi_e) = R(\tau - d/2)\cos(\varphi_e) \quad (4.5)
\]

\[
D_C = I_E - I_L = \cos(\varphi_e)[R(\tau + d/2) - R(\tau - d/2)] \quad (4.6)
\]

If one sees the simple subtraction of two straight line segments here, it is quite obvious that the region of the function where \(-d/2 < \tau < d/2\) is linear for this case as well. If
there are no errors in the system, the phase term becomes unity and the CDLL discriminator function, $D_c$, takes the form shown in Figure 4.2 for 1-chip and 0.1-chip correlator spacings.

![Nominal CDLL discriminator functions for correlator spacings of a) 1 chip, b) 0.1 chips.](image)

Figure 4.2: Nominal CDLL discriminator functions for correlator spacings of a) 1 chip, b) 0.1 chips.

Because of the significant performance improvement seen in GPS receivers employing tighter correlator spacings, and because this is the nature of the receiver to be tested, only the 0.1-chip discriminator will be considered for the remainder of this thesis.

The operation of these loop types in the presence of one or more multipath signals is now considered. Assuming the presence of $M$ distinct multipath reflections, Van Nee [13] derives the resultant discriminator curves as:

$$D_{NC}(\tau) = \int_{t-T_0}^{t} \left| \sum_{i=0}^{M} a_i R(\tau - \tau_i + \frac{d}{2})e^{j\theta} - \sum_{i=0}^{M} a_i R(\tau - \tau_i - \frac{d}{2})e^{j\theta} \right|^2 dt \quad (4.7)$$
where \( a_i, \tau_i, \) and \( \theta_i \) are the relative amplitude, delay, and phase of the \( i^{th} \) signal. The \( i = 0 \) case corresponds to the direct signal and so the delay and phase are zero and the amplitude is unity for \( i = 0 \). The time variable, \( t \), is present only to show that these curves may have a time-varying characteristic, depending on the relationship between the dynamics of the input signals and the tracking loop filtration properties. \( T_{LF} \) is the filter time constant determined by bandwidths of the loop filter, the voltage controlled oscillator (VCO) and any other components introducing lag in the feedback loop. In the coherent discriminator, \( \phi \) is the reference phase used for demodulation.

When the dynamic characteristics of the multipath signals cause fading that is slow in comparison to the loop bandwidth, a resultant code phase can be tracked. This type of interference is termed slow-fading multipath and leads to time variations in the pseudorange measurements since the integration in Equations 4.7 and 4.8 does little to suppress changes in the discriminator zero crossing from multipath distortion. Since the focus here is on modern carrier-aided receivers with very narrow code loop bandwidths, a great deal of multipath, especially that in GPS receivers on moving platforms, can be classified as another type: fast-fading multipath. Heavy relative Doppler shifts on multipath signals prevent the DLL from tracking the exact discriminator zero crossing. As a result, the time-averaged composite must be examined. Referring again to Equation 4.7, an assumption that all multipath relative
phase rates are well outside of the DLL bandwidth results in the fast-fading NCDLL discriminator function:

\[ D_{\text{NC-FF}}(\tau) = \sum_{i=0}^{M} [a_i R(\tau - \tau_i + \frac{d}{2})]^2 - [a_i R(\tau - \tau_i - \frac{d}{2})]^2 \]  

(4.9)

which is simply the sum of the error-free discriminator function and M discriminator functions stemming from tracking M multipath signals. Note that all time and relative phase dependence has been removed. The additional functions behave as if the multipath is perfectly tracked by some separate loop. Since multipath takes a longer path to the receiver by definition, the additional discriminator zero crossings inherently come later than that from the line-of-sight signal. The summation of these will then always have a zero crossing that is further along the lag (\(\tau\)) axis than the truth signal. The NCDLL receiver will thus settle on a constant range bias that is necessarily positive. Van Nee continues on to show these expected biases for the NCDLL under various signal-to-multipath ratios (SMRs) for the 0.1 chip-spacing receiver. These traces are presented in Figure 4.3. It should be noted that these biases are not equivalent to the mean range errors observed under slow-fading multipath interference. These mean errors exist for both coherent and non-coherent loops under such conditions and are separately calculated by Van Nee in [12].
The predictions of pseudorange bias shown here have a few very important characteristics. First, under a strong multipath that is only 3dB below the strength of the direct signal, the worst case error is about 0.024 code chips, or 7 meters for the C/A code. Secondly, the occurrence of the peak at short delays is a direct consequence of using the 0.1 chip separation in the DLL. Additionally, for the $i^{\text{th}}$ multipath signal to have a noticeable effect in this model, it must satisfy Equation 4.10.

$$\tau_i < \lambda(1 + d/2)$$

(4.10)

where $\lambda$ is the chip-length of an ideal spreading code. Contributions from multipath with a relative delay outside of this interval only result from imperfect sidelobe properties in the autocorrelation function of the PRN code.
On examination of the coherent loop under fast-fading multipath conditions, one will notice that a significant difference exists from the non-coherent case. When the phase dynamics get very large, the cosine term in Equation 4.8 contains several periods over one integration interval, resulting in a zero average for all signals with those high phase dynamics. The time-averaged approximation for Equation 4.8 is shown here in Equation 4.10 for fast fading multipath in the CDLL.

\[
\cos(\phi_0)\left[R(\tau + \frac{d}{2}) - R(\tau - \frac{d}{2})\right]
\]  

(4.11)

One will notice that this is nearly identical to the ideal coherent loop discriminator function of Equation 4.6, with the exception that the phase tracking error has been replaced by the reference phase, \(\phi_0\). This implies that, under fast-fading multipath conditions, the CDLL performs error-free. It is this potential for multipath rejection that motivates all of the research discussed in this document.

4.2 Simulation

To verify Van Nee's results, a series of software simulations was constructed in Matlab™. Although full simulations of GPS receiver channels are available, it eases computational requirements and offers more insight if one begins by examining only the channel components of interest. Since the main focus of the theory is on the code DLL, the software will be built to model tracking loops starting at the outputs of the baseband correlators in the receiver. A lookup table of correlation values versus current tracking error may be built and used by the feedback structure in the simulation program to determine contributions from both direct and multipath signals to the
composite discriminator output. Since the correlation operation is additive, the effect of the time-varying multipath may be added in on a point-by-point basis through the program run time. A filter may then be used at the output to compensate for lags and gains from both the loop filter in the real receiver, and for those introduced by the VCO. In other words, a single filter can be used to set the bandwidth and feedback characteristics of the entire tracking loop. Figure 4.4 compares the DLL structure in a real GPS receiver with the equivalent structure used for this simulation.

Figure 4.4: a) Actual GPS DLL structure. b) Simulated DLL structure.

For the simulated DLL, a designated tracking point is created relative to the error-free pseudorange and its deviation from the ideal discriminator zero crossing at $\tau = 0$ is
recorded as the pseudorange error. A time-domain error along the τ axis equates to a
time-of-arrival estimation error, which causes a range error in the real GPS receiver.
The determination of loop type lies in the link between the correlator output table and
the discriminator block. The noncoherent loop uses in-phase and quadrature
components while the coherent loop only uses in-phase components. For these
simulations, an initial tracking point is chosen at the truth reference and the system is
allowed to converge to its steady-state operation before measurements are taken. The
filter is chosen very simply here as a first-order Butterworth filter with cutoff frequency
equal to the code tracking loop bandwidth of the GPS receiver to be tested, about 0.03
Hz. A sample estimation path for the NCDLL is shown in Figure 4.5 to demonstrate
multipath error for a single delay value under two fast-fading conditions. The fading
frequencies here are 0.2 Hz and 5 Hz. It will be seen later that having a distinctly fast-
fading multipath is necessary for isolating the bias errors in a real-world GPS receiver.

Figure 4.5: Tracking error vs. time in simulation for NCDLL with SMR = 3 dB,
delay = 0.1 chips. Fading frequencies = a) 0.2 Hz, and b) 5 Hz.
Clearly, variation in the error on the slower fade (Figure 4.5a) is greater than for the faster fade (Figure 4.5b). Therefore, a more definite estimate of the steady-state tracking bias can be made for higher fading frequencies. As a nominal test case, the 5-Hz fade is chosen for use in the initial simulations. To allow convenient comparison with both Van Nee’s models and with test results to be discussed later, analyses are made of NCDLL performance with SMR values of 3, 4, 6, and 10 dB. For noncoherent operation, the normalized dot product discriminator is employed. The steady-state pseudorange biases versus multipath delay for these power levels are shown in Figure 4.6.

![Figure 4.6: NCDLL tracking biases vs. multipath delay for varying multipath power levels.](image-url)
Converting van Nee's traces (Figure 4.3) to the range domain, it is seen that the 3, 6, and 10 (interpolated) dB SMR cases show errors of 7.0 meters, 3.7 meters, and 1.5 meters for worst case delay, respectively. Simulation results seem to give slightly lower predictions, which may be attributed to initial transients not fully dying out before bias estimates are made. Still, the agreement is sufficient to conclude that the NCDLL simulation and the mathematical model are predicting the same behavior. A quick look at the performance of the same loop under a 10 Hz fading frequency in Figure 4.7 will show that the invariance with respect to frequency is yielded by both methods as well.

![Figure 4.7: NCDLL tracking error for 10 Hz fading frequency, SMR = 6 dB.](image)

This trace is virtually identical to the 6 dB / 5 Hz case since the two test frequencies are well into the fast-fading region. Agreement also exists in the prediction of a nearly
linear taper from the maximum error at short delay to zero error at delays greater than one code chip. The last piece of the theory to be verified is the perfect attenuation of fast-fading multipath in the CDLL. Under the same conditions as in preceding trials, the simulation is run using the coherent early-minus-late discriminator. A sample result is shown in Figure 4.8.

![Figure 4.8: CDLL bias for SMR = 3 dB, fade = 5 Hz.](image)

Here, even with a very strong multipath, the steady-state bias is well under 1 centimeter. For any real GPS code phase applications, this error is effectively zero. This is the case for the CDLL regardless of SMR or fading frequency with the exception that the multipath must be weaker than the direct signal so that the loops do not settle on an incorrect solution.
The outcome of these high-level simulations seems to verify Van Nee’s theoretical model to within reasonable tolerances and the interested reader can find code listings for the software simulations in Appendix B. Both methods, however, implicitly make the assumption that the locally generated carrier can be approximated by a pure sinusoid. In other words, that the $\phi_b$ factor from Equation 4.10 remains constant over the integration interval. It will be shown in later chapters that the slight variations in relative phase of the estimated carrier signal with respect to that of the true direct signal lead to large differences between the current model and actual performance data.
5. **Bench Test**

Thus far, the analysis of GPS receiver tracking performance under fading multipath interference has been tied to mathematical approximations and high-level software simulation. For any model or system to prove its mettle, it should be tested in the field. An optimal field test of the effect of fast-fading multipath on a receiver tracking loop would require a location with one very strong reflecting surface and a predictable dynamic geometry between satellite, reflector, and user. Since multipath parameters are so dependent on specific environmental factors such as time of day, obstacle placement, obstacle constitution and antenna gain pattern, a true field test may not give the required insight into the particular range errors sought in this work. A slight compromise must be made to ensure observable and useful results. A real-time commercial GPS receiver can be tested under controlled conditions using high-fidelity GPS signal emulators for precise control over the incident signal. This chapter will discuss the specifics of the laboratory bench setup, the GPS receiver to be tested, and the results obtained from data collection and processing. First, an investigation of measurement products is made to illustrate how pseudorange biases can be extracted from the data.

5.1 *Isolation of Pseudorange Bias*

Detection of systematic biases in any process can be a difficult task. A true bias may be virtually indistinguishable from certain very low frequency error sources if constraints are placed on observation time. The task of finding biases in the pseudorange
measurements of GPS becomes even more complicated due to the other errors inherent in determining a valid truth reference point to which biases can be gauged. Additionally, for a system like GPS with additional known biases such as atmospheric effects and orbital errors, the need for isolation of the bias error from fading multipath alone renders any simple averaging technique inadequate.

Since the theory discussed in the previous chapter predicts errors in a single pseudorange measurement, a validation effort would be served by exploring ways in which these errors can be calculated from data without the need for a full position solution. This would thus exclude errors from any signals save the one of interest. It has been presented in [15] that a code-minus-carrier range solution can be used to cancel many errors present on a single pseudorange measurement since pseudorange and accumulated Doppler range (ADR, sometimes presented as accumulated delta range, integrated delta range, etc.) share several common errors. Equation 5.1 describes a model of the GPS pseudorange under the assumption that SA and atmospheric effects are not present in the signal. It will be seen in the next section that the bench testing process employed for this research will legitimize this assumption.

\[
PR_{\text{meas}} = R_{\text{true}} + cT_{\text{rec}} + cT_{\text{sv}} + N_{\text{code}} + MP_{\text{code}}
\]  

(5.1)

In this equation, the net measurement, \( PR_{\text{meas}} \), consists of the true geometric range between user and satellite, \( R_{\text{true}} \), clock errors in the receiver and satellite, \( T_{\text{rec}} \) and \( T_{\text{sv}} \), which are multiplied by the speed of propagation, \( c \), a code noise term, \( N_{\text{code}} \), and a pseudorange error from multipath, \( MP_{\text{code}} \). Note that satellite orbital errors may be
justifiably included in the $T_{sv}$ term for all purposes in this thesis. In a similar manner, the accumulated Doppler range may be broken down as in Equation 5.2.

$$\text{ADR}_{\text{meas}} = R_{\text{true}} - \Delta + cT_{rec} + cT_{sv} + N_{\text{car}} + MP_{\text{car}}$$ \hspace{1cm} (5.2)

In this case, the measurement is composed of the same true range and clock terms. Since the ADR is based on an ambiguous carrier phase measurement, the integer-valued $\Delta$-term appears to denote that the measurement is only a fraction of a carrier wavelength. Additionally, the ADR has its own noise and multipath, $N_{\text{car}}$ and $MP_{\text{car}}$. The code-carrier difference can then be calculated as shown in Equation 5.3.

$$\text{PR}_{\text{meas}} - \text{ADR}_{\text{meas}} = \Delta + N_{\text{code}} - N_{\text{car}} + MP_{\text{code}} - MP_{\text{car}}$$ \hspace{1cm} (5.3)

Once common terms are removed, the difference consists of the phase ambiguity, code and carrier noise, and code and carrier multipath. A general analysis in [16] shows that the maximum carrier multipath for a single dominant reflection is 4.8 centimeters. The short period and narrow-band character of the carrier signal constrain the phase-tracking noise to the centimeter level as well, and often better. Since the theoretical results obtained in Chapter four suggest that pseudorange multipath errors of several meters may be present, very little harm is done by ignoring carrier error components. Additionally, modern carrier aiding and smoothing techniques can significantly reduce pseudorange noise by allowing code-tracking loops to be tightened and code noise to be estimated and compensated. The data presented in the next chapter will verify that this noise component is indeed small in comparison with the observed biases. The remaining elements in the code-carrier difference are thus the integer phase ambiguity and the error from pseudorange multipath. The final part of this section will discuss
how the ambiguity term, which may seem hopelessly mixed in with the pseudorange multipath bias, can be removed to leave the multipath bias as the sole residual in the differencing scheme.

The method most widely used to separate constant variables in a system is to take additional measurements until the number of unknowns is met, or exceeded, by the number of system equations. In the situation at hand, not only would it be undesirable to introduce further measurements for the reason that new error sources would be created, but at first glance, it would also be ineffective since doing so would only bring in more unknowns. A more reasonable approach to separate the constants, and thus the one chosen here, is to change the test space in such a way that one of the unknown variables is no longer a constant. At that point, the problem becomes the relatively simple task of separating a constant and a time-varying term. The argument was cited in chapter four that multipath signals of delay greater than $\lambda(1 + d/2)$ code chips have little effect on tracking performance. This notion is further explored by both theory and experiment for general spread-spectrum systems in [14, 16, 17, 18, and 19]. It is also reasonable that a multipath signal with no relative path delay will not cause errors in tracking since the code timing is identical to that of the direct signal. At worst, a multipath signal with no delay could cause a reduction in signal-to-noise ratio (SNR) because of the time variation it imposes on the correlation outputs when a relative carrier frequency difference is imposed.
Using these two features of multipath in spread-spectrum, the predicted dependence of the pseudorange bias on multipath delay can be exploited to separate the bias term from the carrier phase ambiguity. During the test, multipath delay in the simulated signal is varied from zero out to some value greater than $\lambda(1 + d/2)$ chips. If the receiver can maintain lock on the signal through the entire process of data collection, the endpoints of the data set should contain errors that are small in comparison with the sought bias magnitudes. While only one true error-free section of the data must be known in order to separate the delay-dependent bias from the phase ambiguity, having both endpoints of the data set bound to small error levels allows detection of any unanticipated drifts in the code-carrier solution.

5.2 Generating the Signal

The ultimate goal of this research is to show how fast-fading multipath will affect steady-state tracking in an off-the-shelf GPS receiver. This makes the accurate synthesis of the input signal absolutely crucial since any loss of fidelity in the simulation of the multipath will automatically degrade confidence in the test results. There are two readily apparent ways in which to emulate a spatial signal reflection in a laboratory. Each has its benefits and limitations. The highest-fidelity method, from an analytical standpoint, is to generate one signal as if coming from a satellite transmitter and then split it in two in the transmission channel. One of these legs is attenuated by a certain amount and then delayed by the desired time before being added back in to the original signal. With well-designed circuitry this technique guarantees that the multipath signal will be an exact copy of the simulated line-of-sight signal. However,
construction of attenuation and delay filter circuitry that allows smooth continuous control would be a complex task in itself. Any mismatched impedances in the additional components could cause standing waves and reflections that appear to the receiver as additional multipath signals, which, in fact, they are. This would defeat the intended purpose of the bench test: to analyze the effect of a single, specular multipath reflection. Also, discrete jumps in the multipath parameters from the use of digitally-controlled components could cause a loss of lock on the signal, thus violating the assumption that the carrier phase ambiguity, \( \Delta \), remains constant over the entire test. A less complex method of producing the composite signal is depicted in Figure 5.1.

\[\text{Figure 5.1: Bench set-up block diagram depicts two GPS signal emulator cards being driven by a common oscillator.}\]

In this diagram, one can see that a single ten-MegaHertz rubidium oscillator is used to drive all code and carrier timing on two separate GPS signal emulators. The devices chosen are Global Simulation Systems’ (GSS) STR2775 L1, C/A code peripheral component interconnect (PCI) mounted cards. The hard limit block is required to
generate the clocking waveform needed for code synthesis from the sinusoidal output of
the oscillator. The carrier input uses the oscillator output directly. Each card has its
own controlling software that allows precise command of the signal generation within
certain parameters. For this research, one of these emulators will be used to output the
same transmission for every test scenario. The second card will be used to simulate the
reflected signal and will thus be used as the variable. The user control features will
allow the simulation of necessary multipath parameters such as delay, strength, and
fading frequency as functions of time. These two signals are combined and used as the
radio-frequency input to the receiver under evaluation. The common clock used
between the two emulators ensures that no unintentional relative drift occurs between
the two signals and that any differences will be due to deliberate intervention. One
additional provision is that the cards have a one pulse-per-second link generated by one
device and locked onto by the other. This allows for simultaneous triggering. The
slight delay in the multipath card trigger caused by propagation speed is not substantial
in the scope of this test. If such a delay were much larger, it would be observable when
the code minus carrier product did not return close to its initial value for very long
multipath delays.

In order to generate a multipath signal with the desired properties, it is first necessary to
point out three options available in the laboratory signal emulator cards. First, the
signal power level is variable. This allows the SMR to be determined and set prior to
running the test. The variable signal is attenuated by the desired amount and
maintained constant through a run of the test. Second, profiles of signal dynamics may
be written and used to effect a change in the imaginary transmitter and receiver line-of-sight velocity. Finally, a feature is in place that allows a change in the carrier frequency while code rate is kept constant. The intent of such an offset in the modulated carrier is simulation of code-carrier divergence from signal propagation through dispersive media. The true effects of this divergence as they would be observed in field operation are discussed in detail in [15], but the purpose here is simply to gain independent control over relative multipath carrier frequency.

If the GPS RF signals are initially being generated synchronously, then imposing a certain constant velocity on the multipath signal leads to a slightly accelerated code rate and mimics the effect of a changing relative delay. This is done slowly until around a microsecond of lag is introduced to the lower-power signal. This is equivalent to approximately 310 meters, or 1.05 C/A code chips, which satisfies the requirement of Equation 4.10 for a receiver using NovAtel Inc.'s Narrow Correlator™ technology.

Under the conditions imposed by this effect alone, a scenario in which the gamut of multipath delay is swept in two hours would only experience a fading frequency of 0.2 Hz on L1 (see Equation 5.4). Such a fading frequency would likely still cause substantial variations in the pseudorange measurements since it may not be sufficiently outside of the code tracking loop bandwidth, around 0.03 Hz (see Figure 4.5). Such fluctuations could be on the order of the residual noise and could mask the bias that this test aims to analyze.
While this may indeed fall within the realm of a fast-fading multipath signals for many modern receivers, it is one goal of this thesis to examine the dependence of tracking performance on fading frequency without changing the fundamental signal generation or data collection processes. Limiting attention to signals that are unquestionably fading outside of the loop bandwidth ensures that an equal comparison of pseudorange biases can be accomplished. For this reason, the signal generation scheme takes advantage of the third of the features listed previously. The introduction of a large relative Doppler shift on the multipath is accomplished by taking advantage of a carrier frequency offset option in the control software. While this was originally included to reproduce the effects of frequency dispersion in the ionosphere, it may be used here to simulate a fast-fading signal without sweeping through multipath delays too rapidly or changing the sweep rate from test to test. Since the input for the carrier frequency offset must be in terms of velocity, the following guideline is used to select the proper fading frequency:

\[
\nu_{\text{CAR}} = \frac{f}{5}
\]

(5.5)

where \(\nu_{\text{CAR}}\) is the velocity used in the controlling software in meters-per-second and \(f\) is the desired fading frequency in Hz. This equation is a sufficient approximation since one spatial meter contains roughly five \(\text{L}_1\) wavelengths.

In order for this test method to be valid, one must initially assume that the signal outputs from the cards are nominally identical so that the multipath signal is, in fact, a
copy of the direct signal. If this is too gross an assumption, divergences in the data should act as flags. The added complexity due to SA and atmospheric effects mentioned earlier are turned off in the generator cards so that they need not be considered in processing the collected data.

5.3 GPS Receiver

The receiver used in this investigation is chosen to be a NovAtel Beeline Propak GPSCard™. The motivation behind this choice is NovAtel’s corporate focus on receiver research and development. Close ties with the company’s engineers resulted in a willingness to alter a portion of a stock Beeline receiver, which would allow separate channels to use disparate algorithms for signal tracking. Specifically, this nominally NCDLL-based receiver was modified to make a single channel operate with a coherent tracking loop. Flexibility in the receiver design allowed the entire change to be made by uploading new software to the receiver’s on-board processing. The benefits of this method are obvious. Clearly, a difference between the two tracking channels of the same physical GPS receiver that is defined solely by a software change will eliminate all dependence that the test results may have on the hardware of a particular model. Differences in signal processing methods or discrete component values that vary from manufacturer to manufacturer, model to model, or even unit to unit, will be avoided. Not only can it be guaranteed that the channel paths will be electrically similar, but also that the two tracking legs under analysis will simultaneously obtain their input signal from a common receiver front end. Thus, the mathematical form of the code discriminator is isolated as the lone variable for which differences in bench results will
be seen. A block diagram depicting the general signal path in the receiver is shown in Figure 5.2.

![Block diagram of modified Beeline receiver signal path](image)

**Figure 5.2: High-level signal path of modified Beeline (RF through Navigation Processing)**

Here, one notices that only a single RF path is used from the input through the analog-to-digital conversion (ADC). At this point, the path splits off so that numerous channels process the incoming data, which has been converted to a digital intermediate frequency. In this test, only two channels are allowed to track, and only on a single satellite PRN. One of these is the noncoherent channel, employing the normalized dot product discriminator in the DLL, and the other is the coherent channel, which uses the normalized early minus late discriminator. Both channels incorporate the same method for carrier tracking. This type of comparison is far preferable to finding one receiver each from companies that employ non-coherent and coherent code-tracking loops.
Data collection from this receiver takes advantage of the RGEA data output format defined in the user manual for the NovAtel Beeline receiver [20]. The key parameters and measurements contained in this data set are: receiver lock time in seconds, PRN code number, pseudorange in meters, and integrated carrier phase in L1 cycles for each channel tracked. A sorting routine was written in Matlab™ to convert these receiver outputs into a normalized code-carrier difference versus multipath delay for each loop type. The normalization in this case consists of the removal of the large constant component of the difference product, which should contain only the carrier cycle ambiguity. This allows straightforward comparison of the remaining transient component, dominated by the effect of multipath on the pseudorange measurement.

One more point that needs to be made prior to showing results from these laboratory tests is that no study is made of the dependence of these multipath errors on the chosen PRN code. Since the mathematical model and the software simulation both assume perfect autocorrelation properties, it is assumed that a validation effort using any of the 32 designated Gold codes for measurement will be equally effective. PRN 1 is arbitrarily chosen and used for all measurements in this research.

5.4 Bench Test Results

The first point of this analysis is to verify the existence of pseudorange biases in a NCDLL receiver and compare their magnitudes versus multipath parameters to those predicted by the equations and simulations from chapter four. Since both models
predict that the tracking bias is frequency independent, a nominal case of 5-Hz fading frequency is arbitrarily chosen. The initial test injects a relatively strong multipath that is only 4 dB below the power of the direct signal. Taking the code-carrier difference and subtracting off the carrier cycle ambiguity surmised from the endpoints of the data set, the plot of Figure 5.3 emerges.

Figure 5.3: Code-carrier product for NCDLL channel with SMR = 4dB, fade = 5Hz.

The abscissa in the figure is initially recorded as the time tag on the recorded data but, because of the linearly drifting code rate on the multipath signal, it has been mapped here as the relative delay of the multipath reflection for comparison with the simulation outputs. For the reasons outlined in section 5.1, this curve should contain only the multipath-induced pseudorange bias and minor contributions from carrier effects. Clearly, the effect of receiver thermal noise on the pseudorange measurements is small
in comparison with the bias component as well. One might expect that the presented model would naturally be correct in predicting that weaker multipath signals would cause less severe range biases. Figure 5.4 and Figure 5.5 verify this for the noncoherent loop by using SMR levels of 6 dB and 10 dB, respectively.

![Figure 5.4: Code-carrier product for NCDLL with SMR = 6 dB, fade = 5 Hz.](image)
Indeed, the pseudorange bias magnitude is reduced with higher SMR, but all three tests at this frequency give results that are only around 70% as severe as the model predicts. An explanation for this difference is reserved for Chapter 6. A study of the error performance versus fading frequency while holding a constant SMR of 6 dB gives the code carrier plots of Figure 5.6 for a two-Hertz fade and Figure 5.7 for a ten-Hertz fade.
Figure 5.6: Code-carrier product for NCDLL with SMR = 6 dB, fade = 2 Hz.

Figure 5.7: Code-carrier product for NCDLL with SMR = 6 dB, fade = 10 Hz.

Upon comparison of Figures 5.6 and 5.7 with Figure 5.4, one will see that the range biases certainly depend on the fading frequency of the signal. While a numerical
discussion will be reserved for the next section, it suffices to say that the NCDLL performance exhibits two main differences from the current model. First, the severity of biases is less than predicted in all cases and second, there is an unanticipated reduction in errors at higher fading frequencies.

Focus now turns to results obtained from the coherent receiver channel, which operated simultaneously with the noncoherent channel from which all presented data through this point was taken. These results will be given in the same order as the NCDLL for the sake of easy comparison. First, the code-carrier product for the CDLL with the strong multipath and a 5-Hz fading frequency is shown in Figure 5.8.

![Figure 5.8: Code-carrier product for CDLL with SMR = 4 dB, fade = 5 Hz.](image)

The most crucial feature to be seen here is that the coherent loop performance very closely resembles that of the noncoherent loop. This coherent loop tracking bias is a
violation of the previous model and consequently is in disagreement with simulation results. An explanation for the CDLL performance is one of the goals of this thesis. Figures 5.9 and 5.10 show the pseudorange errors for the weaker multipath cases and also appear to follow the corresponding NCDLL traces.

Figure 5.9: Code-carrier product for CDLL with SMR = 6 dB, fade = 5 Hz.
Figure 5.10: Code-carrier product for CDLL with SMR = 10 dB, fade = 5 Hz.

As in the strong multipath case, there is close agreement between the pseudorange errors from each type of receiver architecture. The frequency dependence, too, carries over to the coherent receiver, as shown in Figures 5.11 and 5.12 for medium-strength multipath fading frequencies of two and ten Hertz, respectively.
One will notice the limited span of the testing space in terms of fading frequency and multipath power. This is due to the necessity of maintaining phase lock on the signal.
through the entire test in order to avoid discontinuities in the carrier cycle ambiguity. Multipath signals of power higher than 4 dB below the direct tended to cause temporary loss of lock and tainted the code-minus-carrier processing results. Biases from multipath weaker than −10 dB tended to be lost in the residual noise and carrier multipath of the signal. Multipath reflections of reasonable strength with higher fading frequencies stressed the carrier tracking loop and so they too caused cycle slips and would not yield clean data. The test space is still adequate for comparison with model predictions and simulation results. Clearly, in the collected data, trends are similar for both receiver types with one important difference. Although it is slight in the available testing range, a definite relative reduction in bias is seen in the CDLL at higher fading frequencies. This performance advantage and other attributes of the data presented here will now be discussed.

5.5 Data Analysis

The first result to be observed in testing is that the biases seen in the noncoherent receiver channel are not as severe as the model would predict. For the strongest multipath in the real data, with an SMR of 4dB, the Matlab™ simulations predict a maximum pseudorange bias of around 5.5 meters. Figure 5.2 shows the real bias to be more on the order of 4 meters for the 5-Hz fading frequency case. This is still a very significant error in terms of high-precision GPS applications. While the errors do scale down proportionately with the model predictions for weaker multipath, the reduction in bias magnitude seen at higher fading frequencies is not anticipated. Both loop types exhibit lower error from multipath of higher spectral content but the coherent channel
actually improves more than the noncoherent channel. For the low frequency test, the coherent loop gave a maximum bias error of 2.4 meters compared to 2.5 meters in the noncoherent loop. This represents a difference of only 4%, hardly the drastic improvement initially sought with the CDLL. At 5 Hz fading frequency, this difference rose slightly to 4.2%, but at 10 Hz, with CDLL error at 1.4 meters and NCDLL error at 1.8 meters, the relative bias disparity was 23%. A distinct advantage seems to emerge in the coherent loop only at higher fading frequencies. A study of the fundamental mechanism driving the observed performance in the CDLL is the target of Chapter 6.
6. **New Model**

Traditionally, carrier phase measurements in GPS have been treated as an independent source of information about range rate. Making this assumption allows the use of certain intelligent filters in determining a higher-accuracy navigation solution from code and carrier measurements. It was decided that, in formulating the existing model for fast-fading multipath in a DLL, the code-phase measurement could be treated as something that is effectively immune to carrier-phase effects. This is reflected in the decision to assume a constant sinusoidal output from the carrier PLL in both the mathematical model and in simulation. One must realize that multipath does not only occur on the signal modulation, but also on the carrier itself. Distortion of the estimated carrier phase is inevitable and even though it tends to be very small and zero-mean, the effects of this distortion are most certainly not negligible nor are they unbiased. The results of including carrier phase effects in this model were presented in [21] and will be discussed in detail in this chapter. Consider the phasor representation of two additive signals shown in Figure 6.1.

![Figure 6.1: Phasor representation of multipath distortion in PLL tracking.](image)
Here, the vector $D$ represents the carrier of the direct line-of-sight signal. The multipath signal, $M$, has a time-varying relative phase denoted by $\phi_m$ and the geometric sum is the composite vector $C$. The difference between the phase angle of the composite vector and the truth (direct signal) is $\phi_c$. This composite phase angle is given by a geometric analysis of Figure 6.1 as:

$$\phi_c = \tan^{-1}\left(\frac{\sin \phi_m}{d/u + \cos \phi_m}\right)$$

where $d/u$ is the post-correlation signal-to-multipath amplitude ratio. It is approximately equal to the ratio of the incident signal strengths for short delay multipath since the correlation process does not attenuate such signals. To illustrate how the carrier PLL reacts under multipath interference, a first order loop is simulated in software with a bandwidth of 20 Hz, the cutoff frequency of a typical GPS carrier tracking loop. A relatively strong multipath signal (-3dB) is injected and the tracking performance is recorded. Figure 6.2 and 6.3 show the PLL estimated phase of the waveform and the phase obtained from the vector composite of Equation 6.1 for a low frequency signal (Figure 6.2) and for one on the order of the loop bandwidth (Figure 6.3).
Figure 6.2: Phase estimates under 5-Hz fading multipath in a) a first-order PLL, and b) the phasor representation.

Figure 6.3: Phase estimate under 20-Hz fading multipath in a) a first-order PLL, and b) the phasor representation.
These figures accomplish two tasks. First, they validate the accuracy of the phase plane representation of carrier multipath phase distortion by showing that a PLL capable of tracking the composite signal dynamics will estimate the signal phase predicted by that model. Second, they confirm the intuitive conclusion that more heavily-Doppler-shifted multipath causes less error in the estimate of the true signal phase. While the composite signal is tracked nearly perfectly for the 5-Hz case, the estimated variation in the PLL is only 70% of the true vector composite at the cutoff frequency. This is consistent with the expected performance of a filter of this type. If the frequency were raised yet higher, the phase variation would approach zero, approximating a pure sinusoid. Such the assumption initially made in the DLL models from Chapter 4. This is clearly not the case, however, for fading frequencies below or near the carrier tracking loop bandwidth. The coherent DLL uses correlation components that are "in-phase" with the reference signal from the carrier PLL. Any time-varying rotation of the reference phase about the phase plane implies the incorporation of components that were previously used only in the noncoherent DLL.

It is therefore necessary to incorporate this new phase variation knowledge into the CDLL simulation to bring about a higher-fidelity model of tracking performance. The modification incorporates an inverse-tangent discriminator PLL that uses prompt correlation components from the lookup table into the simulation along with a filter that is intended to contain parameters from the carrier loop filter and oscillator. The new architecture can be shown as a modification to Figure 4.4b, which is depicted in Figure
6.4. The addition of a simulated carrier loop ensures that the I and Q components are referenced to the phase of the composite signal, to the extent that it can be tracked by the PLL.

![Simulated DLL architecture incorporating carrier multipath effects.](image)

**Figure 6.4: Simulated DLL architecture incorporating carrier multipath effects.**

It is now possible to gain a conceptual understanding of why the CDLL performs relatively better at higher frequencies. As the fading frequency approaches the bandwidth of the carrier loop, the demodulation signal becomes closer and closer to a constant sinusoid. This means that the coherent loop performs more like the previous model at higher frequencies. That model predicted an unbiased pseudorange measurement under the condition that the estimated carrier phase does not change at all. It is this ideal that is approximated by heavily Doppler shifted multipath. In the absence of other effects, the CDLL bias would go to zero and the NCDLL bias would
remain unchanged as the fading frequency rose arbitrarily high. The following simulation results verify this notion. First examined is the base case of a strong –3dB multipath signal with a five-Hertz fade. Figure 6.5 reveals that the higher-fidelity simulation yields substantial bias errors for a CDLL under these conditions.

Figure 6.5: CDLL bias errors in revised simulation for SMR = 3 dB, fade = 5 Hz.

Further, to illustrate the power and frequency dependence, Figure 6.6 shows bias traces for a multipath with SMR = 6 dB and fading frequencies of 2, 5, and 10 Hz.
Since this model does indeed provide an explanation for real observations, it makes it necessary to parse a third region in the existing slow/fast fading dichotomy. A composite signal that is much greater in fading frequency than the code tracking loop bandwidth will cause the receiver channel to settle on a constant bias rather than following a time-varying resultant code phase estimate. The channel may still, however, be tracking the composite carrier since the corresponding loop bandwidth is often much larger. This means that the time-varying phase reference will effectively enter the discriminator and cause the CDLL bias seen in Figures 6.5 and 6.6. This new region of fading frequencies will be termed medium-fading multipath. A truly fast-
fading multipath, then, that will yield the results predicted by the original model, must be well beyond the carrier tracking loop bandwidth in its relative Doppler shift.

It can also be verified that the reference phase in the new model has no effect on the NCDLL bias. Figure 6.7, which plots the results of the new simulation with a 3-dB SMR and a 5-Hz fade, is identical to Figure 4.6, which runs the same multipath parameters through the original program.

![Figure 6.7: NCDLL bias in new model for SMR = 3 dB, fade = 5 Hz.](image)

A quick visual comparison shows that no obvious change is seen in the NCDLL. This is reasonable since its intended purpose is to capture all energy in the phase plane regardless of the reference.
The intent of this chapter was to show that carrier multipath has a dramatic effect in pseudorange bias determination for the coherent DLL. The previous model had provisions for determining error induced by carrier multipath, but only that on the carrier-phase measurement (ADR). This result shows that the effects of code and carrier multipath are not so easily decoupled when evaluating channel biases. As loop integration in more modern GPS receivers becomes tighter, the effect may become even more drastic. The pseudorange distortion discussed here exists independently of any modern carrier aiding or smoothing techniques, but still depends intimately on the ability of the carrier tracking loop to follow the phase of the true multipath-free GPS signal.

The second result obtained in bench testing is that both the CDLL and NCDLL improve in their pseudorange measurement accuracy with rising multipath relative Doppler shift. From an accuracy standpoint, this observation is perhaps more substantial than the relative benefit of the CDLL, although the justification for it does not necessarily violate the predictions made by the standing model. The reason lies in a specific processing technique used by the test receiver that is not accounted for in simulation. It has recently been revealed to the author by the receiver developers that the NovAtel Beeline actually feeds back the code loop at a much slower rate than the few-hundred Hertz assumed in a traditional model. Summed discriminator outputs are only fed back to the code generator five times per second while the carrier oscillator is updated at 100 Hz. Although the exact technique used to get around the natural 50-Hz limit imposed by data bit transitions is not known, it is suspected that some combination of
normalization and/or navigation data prediction is used. Regardless of the process, this 5-Hz code loop feedback rate makes multipath signals with substantial Doppler shifts average out before getting to the code generator and distorting the code phase estimate. In effect, the multipath errors do not complete the full cycle around the tracking loop. This is essentially the same reason that the previous model allowed the exclusion of multipath reflections with Doppler shifts greater than a few hundred Hertz. In this case, the accumulation rate caused an averaging before the multipath even entered the tracking loop. This effect is thought to account for the general reduction in multipath bias errors at higher fading frequencies observed in bench tests for the noncoherent loop as well as the coherent loop. The 70% difference between the model and observed data for all SMRs at a 5-Hertz fading frequency is just an indication of the magnitude of the transfer characteristic in this filtering scheme.

Combined, these two effects serve to explain nearly all observed multipath errors in the NovAtel Beeline GPS receiver. Frequency dependencies and the non-zero pseudorange bias of the coherent DLL are both products of the revised loop model that are not predicted by either initial approach. For completeness, final simulation results incorporating carrier multipath effects and the specific timing structure from the Beeline receiver are compared with the bench observations for a -6-dB multipath in Figures 6.8 – 6.11. The results from the base case of a 5-Hz fading frequency are shown in Figure 6.8 and Figure 6.9 for the noncoherent and coherent receiver simulations, respectively. Here, one can surmise which traces represent real data and
which are simulated since the bench data naturally carry noise and other residual effects such as carrier multipath.

Figure 6.8: Comparison of final simulation data and bench data for NCDLL receiver, SMR = 6dB, fade = 5Hz.
Figure 6.9: Comparison of final simulation data and bench data for CDLL receiver, SMR = 6dB, fade = 5Hz.

The nearly-identical behavior of the two receiver types is reflected by the model with the addition of this last piece of information. At higher frequencies, the additional consideration in the new model gives results shown in Figure 6.10 and Figure 6.11.
Figure 6.10: Comparison of final simulation data and bench data for NCDLL receiver, SMR = 6dB, fade = 10Hz.
Figure 6.11: Comparison of final simulation data and bench data for CDLL receiver, SMR = 6dB, fade = 10Hz.

Here, the simulation results adequately predict the error bias for short-delay multipath, but the drop in bias at longer delays seen in the bench data remains unexplained. It is thought that perhaps higher-order tracking loops in the test receiver lead to effects not yielded by the first-order loops used in simulation. Still, adequate agreement between simulated data and bench test results has been achieved in the final model.
7. CONCLUSIONS

Every GPS user will experience some level of multipath interference due simply to the fact that the operating environment is cluttered with potential reflecting surfaces. Because the entire system is inherently dynamic, all multipath signals generally have some Doppler shift that causes their frequencies at reception to differ from those of the direct line-of-sight signal. For a user in motion with respect to the surface of Earth, the rapidly changing geometry among the receiver, the satellites, and any reflectors typically causes relative multipath Doppler shifts to be large with respect to the code tracking loop bandwidth of a GPS receiver. For this reason, pseudorange errors from multipath become invariant in time since the receiver code DLL cannot follow the rapidly varying discriminator output. Additionally, interference from GPS satellite transmissions other than the desired one can cause multipath-like effects due to imperfect correlation properties of the GPS Gold codes. Satellites with similar Doppler shifts tend to distort the output of the baseband correlation process in the same way as multiple reflections from the same satellite. The result of each of these effects is a constant bias in the range measurement and so the two can be treated equivalently as multipath.

The model that has been used to predict this bias for the past decade was presented in [12] and calculates performance for receivers employing coherent and noncoherent delay-lock loops in their architecture. This research examined a specific set of predictions from that theory and intended provide real data to validate the mathematical
results. The model shows that the nonlinear operations used in a noncoherent loop lead to non-zero bias errors in the range measurement that can approach ten meters for very strong multipath reflections. Certain symmetries in the coherent loop, however, were predicted to allow an unbiased estimate of the PRN code phase, and thus, a bias of zero in the range measurement. Simulations were constructed in the Matlab™ programming language that approximated the operation of the DLL in a real GPS receiver while making the same assumptions that were used for the existing mathematical model. Their results showed nearly perfect agreement with the theory. NCDLL range biases from 7 meters for −3dB multipath down to 1.2 meters for a −10 dB multipath were observed with no dependence on relative multipath Doppler shift except for the requirement that it be much greater than the bandwidth of the code tracking loop. Biases for the modeled CDLL were well below the centimeter level; effectively zero for all practical current uses of pseudorange in GPS.

A laboratory-based performance test of a NovAtel Beeline Propak GPSCard GPS receiver showed reasonable agreement with the standing theory for the noncoherent loop bias. Although error magnitudes were overestimated by around 30%, the existence and characteristic form of such biases were verified. Such was not the case for the test of a coherent DLL-based receiver. Contrary to the unbiased pseudorange measurement that was predicted by the initial model and software simulation, the CDLL performed only slightly better than the NCDLL under identical conditions. A curiosity arose when the data was examined and it was discovered that the coherent receiver channel gained a relative advantage to the noncoherent channel as the fading
frequency of the multipath signal became greater. Also, it was observed that both loops produced less-severe bias at higher fading frequencies.

A more thorough picture of the CDLL was constructed to explain the most significant result of the unpredicted bias in the coherent channel of the hardware receiver. In this case, characteristics of the carrier tracking loop were incorporated into the previous simulation to account for the phase distorting effects of carrier multipath. The result of this modification was an ability of the simulation to mimic the actual observations from bench tests of the coherent receiver. The significant range bias from the CDLL, as well as its relative reduction under increasing multipath fading frequency, came out as results of the new model.

The essence of the coherent receiver is that it uses only components of the incoming signal that are in-phase with a reference signal in the receiver hardware. This reference is determined by the carrier PLL, as its output is used to translate the final GPS IF signal down to baseband for processing. Carrier phase multipath distorts the phase of the PLL output so that it is not a pure sinusoid, but rather a phase-modulated waveform. This time-varying signal causes the definition of “in-phase” signal components to differ from that used in the initial model. The difference leads to the bias in the coherent loop. Under the condition that a multipath fading frequency grows large even in comparison with the carrier PLL bandwidth, the composite phase is no longer tracked and the output of the carrier loop becomes closer to the true, sinusoidal reference signal modeled in the preliminary simulations. Since the NCDLL takes advantage of both in-
phase and quadrature signal components, its operation is not affected by this phase variation and it only requires adequate frequency-lock for good performance.

While the standing theory in the field assumes that multipath interference may be grouped into two general categories of fast-fading and slow fading, tests show that it is necessary to separate these two regions with a third partition. Medium-fading multipath is that which has a relative Doppler shift much larger than the code loop bandwidth, but can still be tracked by the carrier loop. It is this type of multipath that dominated the points inside the available test space used to evaluate the commercial GPS receiver. Under this medium-fading multipath, the CDLL-receiver offers little advantage over the NCDLL. Also, the greatly-reduced code feedback rate of the test receiver tended to mitigate all multipath of higher spectral content regardless of the architecture used.

While the revision to the current model for fading-multipath error analysis offers great insight into the actual performance of real receiver hardware, further tests on more specific receiver architecture elements may yield even more clues as to how multipath bias errors could be further reduced in receiver processing. It is suggested that, for this model to be more useful, manufacturers be consulted as to their exact implementation practices and that they be encouraged to work with full system designers toward an end product. Techniques such as external aiding and navigation data bit estimation may allow filter parameters to be tightened to the point that virtually all multipath is in the very-fast-fading region. Judging by results observed in the test space, this could allow
complete elimination of multipath bias errors even for the NCDLL receiver. Emerging
time-frequency techniques could also be used to process data in blocks of varying size
to bring about the same result. An expansion of the test space could be effected with
higher-resolution GPS signal emulators and more sensitive receiver hardware, thus
allowing further examination of multipath parameter dependence and determination of
the adequacy of this new model.
REFERENCES


APPENDIX A: LINEARITY OF NCDLL DISCRIMINATOR FUNCTION

Consider the discriminator function given by Equation 4.3 and repeated here:

\[ D_{NC}(\tau) = I_{e}^2(\tau) + Q_{e}^2(\tau) - I_{l}^2(\tau) - Q_{l}^2(\tau) = R^2(\tau + d/2) - R^2(\tau - d/2) \]  (A.1)

In order to model the region of the discriminator function inside of the correlator spacing, it is useful to first present expressions for the leading and lagging side of the squared correlation function, \( R(\tau) \), shown in Figure A.1.

![Figure A.1: Squared autocorrelation function](image)

The leading, or early, edge can be expressed as \((\tau + 1)^2\) and the lagging, or late, edge can be expressed as \((\tau - 1)^2\). On the interval of interest, where \(|\tau| < d/2\), the components in the discriminator function are the lagging edge of the early correlation function, \((\tau - 1 + d/2)^2\), and the leading edge of the late correlation function, \((\tau + 1 - d/2)^2\). The difference of these two components forms the early-minus-late power discriminator.
function for the noncoherent loop along the same set of code offsets (lags).

Substituting these components into Equation A.1 and expanding gives:

\[ D_{NC} = R^2 (\tau + d/2) - R^2 (\tau - d/2) = (\tau - 1 + d)^2 - (\tau + 1 - d)^2 \]

\[ = \tau^2 - \tau + d\tau/2 - \tau + 1 - d/2 + d\tau/2 - d/2 + d^2/4 \]

\[ - (\tau^2 + \tau - d\tau/2 + \tau + 1 - d/2 - d\tau/2 - d/2 + d^2/4) \]

\[ = -4(1 - d/2)\tau \quad \text{(A.2)} \]

Thus, for the ideal early-minus-late power discriminator, the output along the interval \{-d/2, d/2\} is always a linear function of the code offset, \(\tau\).
APPENDIX B: MATLAB™ SIMULATION SOURCE CODE

% PROGRAM:     BEELINE_model.env
% PROGRAMMER:  JOSEPH M. KELLY
% work was partially funded by FAA/NASA JUP and Stocker Family Foundation
% continuing work funded under FAA 33-08-8780
% PURPOSE:    MODELS CODE-PHASE MULTIPATH IN A NOVATEL BEELINE RECEIVER FOR
%             NONCOHERENT AND COHERENT OPERATION
% ERROR BOUNDS AND BIASES ARE PRESENTED
% uses information about 5 Hz code feedback and 100 Hz carrier feedback.
% clear
%chip_sam = 300000; %samples per chip determines sim resolution, 300K yields ~1mm
trii = zeros(1,chip_sam); %build lookup table of correlator outputs
trii(2*chip_sam:3*chip_sam) = [1:chip_sam+1]/(1:1);
trii(3*chip_sam+1:4*chip_sam) = (chip_sam:-1:1)/(chip_sam+1);
SSBW = 8e6; %single-sided FE bandwidth (~8.5e6 for beeline)
half_rate = chip_sam*1023*1000/2; %half the effective sampling rate
[b,a] = butter(1,SSBW/half_rate); %filtration of correlation peak is equivalent to filtering input signal
tri = filtfilt(b,a,tri); %index of direct correlation peak
% ***
% ff = 5; %fading frequency (Hz)
% smr = 3; %dB signal-to-multipath ratio
% ***
% f_code = 5; %Hz
dump rate of code accumulators (make sure f_acc is a factor of f_loop)
f_acc = 100; %Hz
dump rate of PLL accumulators
code_BW = .03; %Hz
car_BW = 25;

a2wo_code = 1.414*code_BW/0.53; %filter coefficient notation used in Kaplan GPS text
wo2_code = (code_BW/0.53)^2; %2nd order code loop
a2wo_car = 1.414*car_BW/0.53; %filter coefficient notation used in Kaplan GPS text
wo2_car = (car_BW/0.53)^2; %2nd order car loop
d_corr = round(0.1*chip_sam/2); %correlator spacing offset (1/2 of chip spacing)
msr = sqrt(1/(10^g(smrr/10))); %amplitude ratio of multipath strength
del_chip = [0:0.05:1.2]; %multipath delay in samples
del_sam = del_chip*chip_sam; %allow time for code loop filter to settle down and multiple fading periods
tmax = max([300, 3/f + 280]); %smallest time step in simulation (car feedback rate)
tlen = length(t);
um_dump = f_acc/f_code; %number of discriminator calculations per dump
slen = floor(tlen*(f_code/f_acc)); %number of correlation dumps in simulation

% *** CYCLE THROUGH DELAY VALUES ***
for l = 1:length(del_sam),
loc_phase = 0; %phase of locally generated carrier (reference phase)
track = truth;
bar1 = waitbar(0,'tracking... ');
err_met = zeros(1,num_dump);

% *** SLOW LOOP (CODE FEEDBACK) ***
for k = 1:slen,
edumsl_acum = 0; %dump accumulated discriminator contents
edumsl_acumq = 0;
prompt_accum = 0;
prompt_accumq = 0;
%
% *** FAST LOOP (CARRIER FEEDBACK) ***
for i = (k-1)*num_dump+1:(k*num_dump).
  delay = del_sam(i);
  le_dir = tri(track - d_corr)*cos(-loc_phase);
  ll_dir = tri(track + d_corr)*cos(-loc_phase);
  lp_dir = tri(track)*cos(-loc_phase);
  Qe_dir = tri(track - d_corr)*sin(-loc_phase);
  Ql_dir = tri(track + d_corr)*sin(-loc_phase);
  Qp_dir = tri(track)*sin(-loc_phase);
  % sub-accumulation to account for mp phase change in fast fading
  t_mp = t(i) + [-0.009: 0.001: 0];
  mp_ph = mod(2*pi*ff*t_mp,2*pi);
  le_mp = sum(tri(track - d_corr - delay)*cos(-loc_phase-mp_ph)*msr)/10;
  ll_mp = sum(tri(track + d_corr - delay)*cos(-loc_phase-mp_ph)*msr)/10;
  lp_mp = sum(tri(track - delay)*cos(-loc_phase-mp_ph)*msr)/10;
  Qe_mp = sum(tri(track - d_corr - delay)*sin(-loc_phase-mp_ph)*msr)/10;
  Ql_mp = sum(tri(track + d_corr - delay)*sin(-loc_phase-mp_ph)*msr)/10;
  Qp_mp = sum(tri(track - delay)*sin(-loc_phase-mp_ph)*msr)/10;
  %
  le = le_dir + le_mp; % early, late, and prompt terms for I and Q components
  ll = ll_dir + ll_mp;
  lp = lp_dir + lp_mp;
  Qe = Qe_dir + Qe_mp;
  Ql = Ql_dir + Ql_mp;
  Qp = Qp_dir + Qp_mp;
  %
  np = atan2(Qp lp);
  [hold1_car,car_loop_out] = second_ord_loop(np,hold1_car,wo2_car,a2wo_car,1/lf_acc);
  loc_phase = loc_phase + (car_loop_out/lf_acc);
  %
  eminusl_accum = eminusl_accum + (le - ll)/num_dump; % normalize to 1 accumulation period
  eminusl_accumq = eminusl_accumq + (Qe - Ql)/num_dump;
  prompt_accum = prompt_accum + (Qp)/num_dump;
  prompt_accumq = prompt_accumq + Qp/num_dump;
%
end
%
% *** SELECT DISCRIMINATOR TYPE, NONCOHERENT/COHERENT, NORMALIZED/UNNORMALIZED ***
%disc_out = (eminusl_accum)/prompt_accum + eminusl_accumq*prompt_accumq/typ = 'NCDLL';
%disc_out = (eminusl_accum)*prompt_accum + ...
%eminusl_accumq*prompt_accumq/(prompt_accum^2+prompt_accumq^2)/typ = 'NCDLLn';
%disc_out = (eminusl_accum)/(prompt_accum)/typ = 'CDLLn';
%
% *** UPDATE PSEUDORANGE BASED ON DISC OUTPUT ***
if ((strcmp(typ,'NCDLL'))|(strcmp(typ,'NCDLLn'))),
  outpt = disc_out/(4*(1-0.112))*chip_sam;
else
  outpt = disc_out*(1-0.112)*chip_sam;
end

[hold1_code,code_innov] = second_ord_loop(outpt,hold1_code,wo2_code,a2wo_code,1/lf_code);
% adjustment = round(code_innov/lf_code); % number of samples to shift tracking point
%
track = track - adjustment;
% final integrator in code loop
%
err_chip = (track-truth)/chip_sam;
err_met(k) = err_chip*293;
waitbar(k/slen);
%
end
%
close(bar1);
%  *** RECORD ERROR STATISTICS ***

err_mean(I) = mean(err_met(280*f_code:end));
err_dev(I) = std(err_met(280*f_code:end));
err_max(I) = max(err_met(280*f_code:end));
err_min(I) = min(err_met(280*f_code:end));

fprintf(’completed %d of %d
’, I, length(del_sam))

end

%  *** FIGURES/RESULTS ***

figure(1)
plot(del-chip,err-mean)
title([typ,’ Error Performance: smr = ‘,num2str(smr),’ dB, fade = ‘,num2str(ff),’ Hz’])
xlabel(’delay (chips)’), ylabel(’bias (m)’)
orient landscape
grid
zoom on

figure(2)
plot(del-chip,err-dev)
title([typ,’ Error Performance: smr = ‘,num2str(smr),’ dB, fade = ‘,num2str(ff),’ Hz’])
xlabel(’delay (chips)’), ylabel(’s. deviation (m)’)
orient landscape
grid

figure(3)
plot(del-chip,err_max,del-chip,err_min)
title([typ,’ Error Bounds: smr = ‘,num2str(smr),’ dB, fade = ‘,num2str(ff),’ Hz’])
xlabel(’delay (chips)’), ylabel(’error bounds (m)’)
zoom on

clear tri
clear outpt