Spread Spectrum Communication
Over a Fading Multipath HF Channel
Using Transform Domain Signal Processing
and a Transmitted Reference Signal

A Dissertation Presented to
The Faculty of the College of Engineering and Technology
Ohio University

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

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November, 1992
to Jack and Murt
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Acknowledgements

I would like to express my deepest thanks to Dr. Jeff Dill for his guidance, patience and friendship. There is no question in my mind that without Jeff, all this would not have happened. His blend of scientific knowledge and farmboy wisdom made me see thing in a different light and will forever be invaluable to me, especially his uncanny ability to put me in my place even when it comes to major appliances.

Thanks also go to the other members of my committee, especially Dr. John Brown, Dr. John Tague, Dr. Dennis Irwin for their time and advice which contributed to this dissertation.

I would also like to thank my family and friends for their love and support over the years. Special thanks go to my parents who started me along this wonderful journey.

And lastly I would like to thank my wife-to-be and best friend Dawn for her endless help, support, and love through this period in my life. She has shown me what truly matters.
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Chapter 1

INTRODUCTION

1.1. History of Transform Domain Signal Processing

With the introduction of digital computers into the mainstream in the 1960s, Digital Signal Processing (DSP) emerged as a new branch of signal processing. DSP's main advantage was the flexibility it provided though the use of digital computers to replace the analog signal processing of electronic and mechanical devices. However, the main limitation to more useful DSP algorithms (e.g., spectral analysis) was the inability to achieving real-time signal processing, due to the large processing requirements of DSP algorithms and the limited processing power of early microprocessors.

For the more useful DSP operations, it was easier to process or extract information from a signal in the frequency domain rather than in the time domain. These operations, when implemented digitally, were initially thought too intensive for real-time applications, since they required the evaluation of Discrete Fourier Transforms (DFTs). With the introduction of the Fast Fourier Transform (FFT) by Cooley and Tukey [1] in the mid-1960s, it now seemed possible to perform a wide range of useful DSP operations in real-time. However, it was not until the mid-1980s that advances in integrated circuit technology made it possible to develop real-time DSP implementations. More recently, with the development of more sophisticated algorithms using FFTs, as well as the...
development of other types of transform algorithms, the field of Transform Domain (TD) signal processing has emerged from DSP.

1.2. History of Rake Correlator

Classical digital communication over a High Frequency (HF) channel has been investigated since the 1930s [2]. The ability to communicate over the horizon using an HF channel made it very attractive. However, the problems of fading and multipath that occurs naturally on an HF channel make it difficult to communicate reliably. Not until 1958, with the development of the Rake correlator by Green and Price [2], did it become theoretically feasible to attempt highly reliable digital communication over a wideband HF channel. Since 1958, research in the development of the analog Rake correlator has continued to the present. However, these analog versions are limited physically both in their size and sensitivity.

1.3. Current Research in TD Signal Processing

Transform Domain (TD) signal processing for digital communication signals has recently been employed on existing communication waveforms (e.g., Binary Phase Shift Keying) to preprocess received signals before demodulation [3]-[7]. These techniques sample the received signal, transform it into the frequency domain using a DFT, perform the TD operation, and transform back into a time domain signal using an inverse DFT. After returning to the time domain, the received signal is then demodulated by conventional means.

Other TD signal processing techniques have been suggested for use in demodulation and matched filtered operations performed by the receiver [7]-[9]. In these papers, the communication waveform is primarily designed to facilitate the direct demodulation of the signal in the transform domain.
1.4. Contribution of this Research

The dissertation brings together the three concepts of TD signal processing, Rake correlation, and Cyclic Code Shift Keying (CCSK), to design a feasible real-time digital HF communication system. The real-time processing goal of this work is reached by processing the signal almost entirely in the transform domain. In addition, other TD signal processing operations are introduced that enhance the basic system performance while still achieving the goal of real-time implementation.

The combination of the CCSK waveform processing and the Rake correlation with the TD receiver described in this dissertation has a number of useful properties which make this general class of waveforms exceptionally robust and attractive for a wide variety of potential applications. In this dissertation, we will show the following:

1. The \( \text{Eb/N}_0 \) performance of TD/CCSK modulation is nearly identical to coherent M-ary orthogonal signalling, with large values of M easily achievable.

2. This performance can be maintained despite severe multipath or other channel distortion.

3. The processing gain of TD/CCSK modulation is given by \( M/\log_2(M) \), where \( M \) is the length in samples of the base CCSK symbol, and \( \log_2(M) \) represents the number of bits per M-ary symbol.\(^1\)

4. TD/CCSK demodulation has an advantage over classical Rake correlators in that its implementation is entirely digital, and thus can achieve a very large number of Rake taps with no increase in hardware complexity.

5. TD/CCSK is relatively immune to imperfect synchronization, and thus rough symbol synchronization is sufficient for near optimal performance, as opposed to the fine chip synchronization required in direct-sequence (DS) systems.

---

\(^1\) It is assumed here that each cyclic shift of the base vector represents a valid symbol. It is possible, and in some cases may be desirable, to only use some binary fraction (i.e. \( M/2, M/4, \) etc.) of the \( M \) possible symbols as the signal set.
1.5. Outline of this Dissertation

Chapter 2. In this chapter we examine the fundamental concepts of HF communication in general. The physical properties and modelling of the HF skywave channel are discussed. The basic principles of various frequency diversity techniques and the use of Rake correlators to exploit these techniques are examined.

Chapter 3. In this chapter we discuss the use of TD signal processing. The use of CCSK waveforms and Rake correlation in the TD are discussed. The basic discrete signals and channel model used in this dissertation are also presented. Finally, we introduce the use of a transmitted reference signal for estimating the channel impulse response.

Chapter 4. In this chapter we calculate the performance of the various communication systems described in chapter 3. These include traditional Rake systems for comparison with the TD systems. Also, the performance of the TD/CCSK Rake receiver under ideal conditions with and without reference signals is derived. The performance of the TD/CCSK Rake receiver using only the instantaneous reference signal to estimate the channel impulse response is also derived.

Chapter 5. In this chapter we introduce variations of the basic TD/CCSK Rake receiver. This includes the concepts of frequency domain interleaving, tail clipping, center clipping and time-averaging to improve the estimate of the channel impulse response. The description as well as the theoretical performance improvements achieved over the basic TD/CCSK Rake receiver using these algorithms are presented.

Chapter 6. Here we discuss the implementation factors of the TD/CCSK Rake receiver. The use of binary maximum length sequences, chirps, and filtered impulse trains as potential CCSK waveforms are examined. Also, the performance effects of both time and frequency synchronization errors are calculated and discussed.
Chapter 7  In this chapter the computer simulation model structure, operation and limitations are discussed. The use of the Block Oriented System Simulation (BOSS) application used to simulate the TD/CCSK system is described. The BOSS modules built to model the transmitter, channel, and receiver are described. Also, the input files, parameters, and the performance measurement data are discussed.

Chapter 8. This chapter presents the simulation results for the various simulation model described in chapter 7. We then summarize the theoretical and simulation results of this research and draws conclusions. Further research in a number of specific areas is also recommended.
Chapter 2

BACKGROUND

2.1. HF Communication

The High Frequency (HF) radio communication band is designated in the 3- to 30-MHz frequency band. A unique property of HF signals is that communication cannot only be achieved via ground wave channel but also via skywave channel. HF ground wave signals, which propagate along the earth surface, attenuate rapidly. This rapid attenuation is mainly due to rough terrain and/or the curvature of the earth. Typically, HF ground wave signals are useful only up to 50 miles. However, HF skywave signals, which propagate towards space and then refract back towards earth, make communication possible up to several thousand miles (i.e., over the horizon). Therefore, radio communication over great distances can be achieved via the HF skywave channel.

This refraction of HF signals is caused by ionized layers in the ionosphere. These ionospheric layers are formed from solar electromagnetic radiation over the ultra-violet and X-ray spectrum ionizing gases in the atmosphere. This process is countered by the collisional recombination of the free electrons and the positive charged ions. However, the low air pressure in the ionosphere hinders the recombination and allows for the charged particles to exist over short periods of time[10].
The ions form into layers over three region of the ionosphere which are designated D, E and F layers (see figure 2.1). The D layer does not refract HF signals well, and acts primarily as an absorber of HF signal energy. The E layer does refract HF signal with the exception of signals at the low end of the HF band. The F layer also refracts HF signals and is the primary mode of communication. During the daytime, the F layer actually separates into two layers, F1 and F2. During the nighttime, the F1 and F2 layers combine to form one layer.[10]

![Figure 2.1 Abstract Illustration of the Ionospheric Regions](image)

Recombination of these layers differs since they reside at different altitudes and therefore are at different air pressures. The ionization of each of these levels vary as a function of time of day, reaching a maximum at noon and dissipating at night. Only the F layer maintains sufficient ionization levels for HF communication at night. The layers
are also in constant motion rising in the day and falling at night. For example, the altitude of the F layer varies between 200 and 250 miles depending on the time of day [11]. The fluctuations of these layers makes the attenuation and phase of HF signals skywave channel a time variant process.

2.2. Fading and Frequency Diversity

The fluctuations of the ionospheric layers result in time variations in the signal strength, which is known as fading. Fading occurs primarily when two or more signals that are propagating over different paths (i.e., multipath) arrive with time variant phase changes and effectively cancel or reinforce each other. A typical fading rate for a narrowband HF channel is 10-15 fades per minute with an average fading depth of approximately 10 dB. This channel fading is a major limiting factor for digital communication over the HF band since typically error rates are highly sensitive to signal strength.

To combat fading, various diversity techniques (e.g. interleaving) can be employed. One commonly used method is called frequency diversity. In frequency diversity the signal is spread over a wide band such that the probability of different signal components fading simultaneously is reduced significantly. Frequency diversity is achieved by either sending the same information over different carriers (i.e., cascaded narrowband channels) simultaneously to form a wide band or by intentionally spreading the signal bandwidth using spread spectrum techniques. The CCSK modulation scheme, developed in this paper, uses spread spectrum as a frequency diversity technique.

To understand the effect of frequency diversity, we must go back to the channel model. The refraction of the signal by an ionospheric layer causes signals to spread over time. The duration of this multipath spread is defined to be $T_m$. Typical values for $T_m$ range from 1-5 milliseconds, but during disturbed condition, $T_m$ can be as high as 10
milliseconds[10][12]. The coherence bandwidth of the channel, $W_c$, is defined to be the minimum separation in frequency at which two signal components are affected independently by the channel. From [12], the coherence bandwidth can be approximated by

$$W_c = \frac{1}{T_m}$$  \hspace{1cm} (2.1)

If the transmitted signal bandwidth, $W_s$, is smaller in comparison to the coherence bandwidth, the channel is termed frequency-nonselective. If the opposite is true, then the channel is referred to as frequency-selective. In applying frequency diversity techniques, it is the intent to make the channel frequency-selective, and therefore we wish to make $W_s \gg W_c$.

For a frequency-selective channel, the channel can be modeled as a tapped delay line with tap spacing of $1/W_s$. Let $L$ be the number of taps in the channel model, then

$$L = T_m W_s$$  \hspace{1cm} (2.2)

In other words, the multipath spread can be divided into $L$ resolvable frequency components spaced at intervals of $1/W_s$. Figure 2.2 is a pictorial example of a frequency-selective channel model. Note that the tap weights $\{h(n,t)\}$ are functions of time and consist of both an attenuation and phase shift factor (e.g. complex-valued).
Since the layer characteristics are time variant, and since these variations are unpredictable, the multipath structure must be described as time variant random processes. A commonly used time variant random process version of the frequency-selective HF skywave channel model is the Rayleigh fading channel model [12]. In this model, the time variant channel tap weights \( h(n,t) \) are assumed to be slowly fading, zero mean, complex-valued Gaussian random processes. The envelope \(|h(n,t)|\) is Rayleigh-distributed, thus the name Rayleigh Fading channel. In various references, it is assumed that the scattering of the multipath is uncorrelated [12]-[14]. Therefore, \( \{h(n,t)\} \) are also considered mutually uncorrelated independent random processes.

### 2.3. Rake Correlator

After employing frequency diversity at the transmitter, the receiver must now coherently recombine the different signal components that were intentionally spread over the frequency-selective (i.e., multipath) channel. The concept of a Rake correlator was originally introduced in 1958 by R. Price and P. Green [2] as a means of achieving this
task. The Rake correlator is simply a tapped delay line that attempts to "rake in" the signal components that were intentionally spread over a signal bandwidth. If we assume the Rayleigh fading channel model, then the Rake correlator aligns the phase of each arriving multipath signal component and then delays each component such that they reinforce each other (i.e., coherently combines them) forming one strong signal.

2.3.1. Binary Signalling

One frequency diversity technique that has been widely used is Direct Sequence Spread Spectrum (DSSS) using Binary Phase Shift Keying (BPSK)[2][12]. A spread spectrum modulation scheme is defined to be a scheme where the signal bandwidth is much greater than the information bandwidth. Therefore, spread spectrum provides a means of intentionally spreading the signal bandwidth without changing the information bandwidth. If the signal bandwidth is spread such that $W_s \gg W_c$ then the frequency diversity required for the Rake correlator is achieved. Figure 2.3 illustrates the implementation of a Rake correlator. For DSSS/BSPK signalling, the signal $u_2(t) = -u_1(t)$ and $u_1(t)$ is the direct sequence used to spread the signal. It can also be shown that for the Rake correlator to achieve optimum performance, its tap weights must be the same as the tap weights of the Rayleigh fading channel model [12].
2.3.2. M-ary Orthogonal Signalling

Another common signalling scheme which can be employed for frequency diversity is Frequency Shift Keying (FSK). In FSK, M symbols are represented by M tones that are mutually orthogonal. For frequency diversity we expand the set to \( L \cdot M \) equally spaced tones having a minimum spacing equal to the coherence bandwidth of the channel [7][12]. Therefore, the signal set is mutually independent statistically as well as mutually orthogonal. If the FSK tones are subdivided into M subsets of L tones each, then an M-ary FSK signal set can be transmitted on appropriate sets of L tones. This will achieve both frequency diversity and orthogonality. If we use binary FSK signalling, then a Rake correlator similar to DSSS/BPSK can be used with the exception that \( u_1(t) \) and \( u_2(t) \) are...
now the sum of the two subsets of L tones. Therefore, we can use the Rake correlator structure in figure 2.3.

For conditions where the multipath phase components are rapidly changing, the FSK demodulation can use a noncoherent detection scheme by implementing a square-law detection algorithm. Figure 2.4 illustrates the implementation of noncoherent binary FSK Rake correlator using square-law detection. Note that this system does not require a channel estimator but depends only on the instantaneous signal attenuation. This allows for easier implementation (i.e., no channel estimator) but squares the amount of noise on the decision variables resulting in degraded performance.

Figure 2.4 Block Diagram of Binary Non-Coherent Rake Correlator [12].
The ideal bit error performances (i.e., perfect Rake tap weights) for binary BPSK, coherent binary FSK, and noncoherent binary FSK can be seen in figure 2.5. The set of curves for \( L=1 \) represents the upper performance bound, where the set of curves for \( L=1024 \) is approximately the lower performance bound. The derivation of these performance curves can be found in [12]. Note that with an increasing number of diversity channels, \( L \), the performance improves. However, the limiting factor to DSSS/BPSK and FSK systems is the high cost of implementing the Rake correlator for a large number of diversity channels (i.e., taps). Analog implementation of the tapped delay line and tap weight estimators is bulky and sensitive to temperature and tuning.

![Figure 2.5 Bit Error Rate Performance of Systems Using Classical Rake Correlation](image_url)
In the following chapter, we will describe a way of implementing a Rake correlator using Digital Signal Processing (DSP) which will eliminate many of the problems of the analog versions. However, the limiting factor now becomes the correlation computational load required for a large number of Rake taps and the speed of the radio’s microprocessor. It will be shown that this can be overcome for large L by performing the correlation in the transform domain.

2.4. Cyclic Code Shift Keying

CCSK is a form of spread spectrum in which the signal is intentionally spread in a manner that is similar to direct sequence (DS), but is an M-ary orthogonal or near-orthogonal signalling scheme. While CCSK\(^2\) by itself has been studied and used previously [12][15]-[17], demodulation of a CCSK waveform using TD signal processing has not been studied. CCSK is accomplished by first generating a base-vector of length M, denoted \(s_0\). An M-ary signal set, \(\{s_i\}\), is generated by taking cyclic shifts of \(s_0\), thus the name cyclic code shift keying. Data is modulated on the CCSK waveform by taking \(b=\log_2(M)\) source bits, interpreting these b bits as a binary number, m, and cyclically shifting \(s_0\) by m positions, such that the transmitted signal vector, \(s_m\) is given by

\[
s_m(n) = s_0(n - m)_M \quad |n, m = 0, 1, 2, \ldots, M-1\]

where \((\ )_M\) indicates a modulo M operation. For example, if the base-vector were an impulse, this modulation scheme would be equivalent to Pulse Position Modulation (PPM).

A block diagram representation of a general CCSK transmitter can be seen in figure 2.6. Assuming that a suitable \(s_0\) has been selected, \(s_m\) is generated by a cyclic shift

---

\(^2\)CCSK is a general term for a class of waveform which include maximum length codes, shift register sequences, pulse position modulation.
operation. Let $T_c$ be the time spacing between successive discrete time components of the transmitted CCSK vector. Borrowing a term from DS, $T_c$ is the "chip rate" of the signal [18]. The discrete time vector $s_m$ is passed through a D/A converter (if $s_m$ is has non-binary components) at a rate of $T_c$, filtered (in this example a zero order hold is used), and upconverted to the desired transmission band. The data is thus contained in the relative position, (i.e., the cyclic shift) of the base-vector.

![Block Diagram of CCSK Transmitter](image)

Figure 2.6 Block Diagram of CCSK Transmitter

At the receiver, the signal is downconverted and matched filtered at base band (see figure 2.7). The matched filter operation can be implemented by first performing integrate and dump operations at a rate of $T_c$, for a period of $MT_c$ seconds to create a discrete received vector of length $M$, denoted $r$. The remainder of the match filter operation is the dot product of $r$ with all possible symbols vectors $\{s_i\}$ yielding a set of decision variables $\{U_j\}$ respectively. Note that since the set $\{s_i\}$ is all possible circular shifts of $s_0$, then the output, $U_j$, an also be defined as the circular cross-correlation of $r$ and $s_0$. The symbol decision is then the maximum real value of these dot products (i.e., $\max(\Re[U_j])$) or the highest real-valued peak in the circular cross-correlation.
2.4.1. Properties of CCSK

To maximize the performance of the CCSK waveform, \( \{ s_i \} \) should be an orthogonal signal set. By definition, a discrete signal set is orthogonal if

\[
\sum_{n=0}^{M-1} s_i^*(n)s_j(n) = \begin{cases} E_s, & i = j \\ 0, & i \neq j \end{cases} \quad \forall i,j
\]  

(2.4)

where \( E_s \) is the energy per symbol and * denotes complex conjugation. Let \( R_{s_0}(\tau) \) be defined as the circular auto-correlation of \( s_0(n) \) where

\[
R_{s_0}(\tau) = \sum_{n=0}^{M-1} s_0^*(n)s_0(n+\tau)\mod{M}
\]  

(2.5)

Since the CCSK waveform set, \( \{ s_i \} \), are all circular shift of \( s_0 \), then the orthogonality condition is simply the circular auto-correlation of \( s_0 \), that is

\[
R_{s_0}(\tau) = \begin{cases} E_s, & \tau = 0 \\ 0, & \tau = 1, 2, \ldots, M-1 \end{cases}
\]  

(2.6)

In general, the base-vector space need not be limited to the binary field as in DS, and can be expanded to the complex number field. This higher degree of freedom (i.e., large vector space), allows us to achieve better auto-correlation properties of \( s_0 \). In fact, since \( s_0 \) can now take on complex values, one could use 90 degree phase rotations of the same \( s_0 \) and obtain a \((4M)\)-ary bi-orthogonal signal set. For clarity of exposition, we defer this
possibility for future research. The CCSK system that will be investigated here will have an M-ary signal base on a complex vector, \( s_0 \), of length \( M \). Some possible families of near-orthogonal base-vectors for \( s_0 \) are examined in chapter 6.

The spread-spectrum processing gain of this system is calculated by noting that for an M-ary signal set (i.e., an M-component discrete transform) the spread spectrum bandwidth is given by \( W_{ss} = 1/T_c \), where \( T_c \) is the "chip rate". The data rate of the system, \( R_b \), in bits per symbol duration is given by \( R_b = b/MT_c \). Thus, the spread spectrum processing gain, \( \varphi_s \), is found as

\[
\varphi_s = \frac{W_{ss}}{R_b} = \frac{MT_c}{bT_c} = \frac{M}{b}
\]

(2.7)

If all shifted positions of \( s_0 \) are used in the symbol set, then \( b = \log_2(M) \), and the processing gain is given by

\[
\varphi_s = \frac{M}{\log_2(M)}
\]

(2.8)
Chapter 3

TD/CCSK RAKE RECEIVER

3.1. Transform Domain Signal Processing

For CCSK Rake receivers, TD signal processing can reduce the complexity of the digital implementation of the receiver. The CCSK demodulation operation and the Rake correlation are each cross-correlation operations of a stored base-vector with the sampled received signal vector, all of length M. Processed digitally, these operations can be intensive for large M (i.e., a total of $2M^2$ complex multiplications). In the TD, these operations can be implemented by component-wise multiplication of one TD vector by the complex conjugate of the other TD vector. If the FFT and IFFT operations are employed to perform the DFT and the inverse DFT, the total demodulation now requires $2M(1+\log_2(M))$ complex multiplications. This is a significant reduction in the computational load of the demodulation operation making it possible to do real-time digital demodulation for large M-ary CCSK signals.

3.2. TD/CCSK Rake System Description

The TD/CCSK transmitter/receiver pair differs from conventional systems in that the CCSK demodulation is now implemented in the TD (see figure 3.2 and 3.4). Since the receiver processing requires a transformation into the TD, it is convenient to also
implement Rake correlation operations (to combine the multipath components) in the transform domain as well, thus economizing on the required DFT operation.

**Figure 3.1 Block Diagram of TD/CCSK Transmitter**

**Figure 3.2 Block Diagram of TD/CCSK Receiver**

**Figure 3.3 Block Diagram of TD/CCSK Transmitter with Transmitted Reference Signal**
For now, we assume a multipath channel. The receiver RF front end is the same as described in section 2.4 (see figure 2.6) and therefore the generation of the received baseband vector, \( \mathbf{r} \), is the same. If we assume a Rayleigh-fading channel with \( L \) diversity channels (i.e., multipath) as in [12], the received baseband vector, \( \mathbf{r} \), can be expressed as

\[
\mathbf{r}(n) = \sum_{l=0}^{L-1} s_m(l) h_o(n-l) + z(n), \quad n = 0, 1, \ldots, L+M-1
\]

(3.1)

where \( h_o(n) \) is the sampled channel impulse response and \( z(n) \) is a lowpass sampled Additive White Gaussian Noise (AWGN) sequence.

The vector \( \mathbf{r} \) is truncated to a length of \( M \), and transformed, yielding the TD received vector \( \mathbf{R} \) (see figure 3.2). If narrowband excision is required, it occurs at this point in the receiver operation. The TD/CCSK demodulation is implemented as a component-wise multiplication of \( \mathbf{R} \) by the complex conjugate of the TD base-vector \( \mathbf{s}_0 \). The TD/Rake correlator is also implemented as a component-wise multiplication of the output of the TD/CCSK demodulation by the complex conjugate of the TD channel impulse response estimate \( \hat{h}_o \). The output of the TD/Rake correlator, \( \mathbf{C} \), is simply

\[
\mathbf{C} = \mathbf{R} \mathbf{s}_0^* \hat{h}_o^* 
\]

(3.2)
For example, if the channel was a single time-invariant path with unity attenuation and the receiver had perfect timing synchronization, then $\hat{H}_0 = I$ (i.e., all ones) and therefore the Rake correlation is not required (i.e., $C = RSS^*_o$). The vector output $C$ is inverse transformed to the time domain, $c$. The vector $c$ is the received signal vector $r$ circularly correlated with the unshifted base-vector $s_0$ which is circularly correlated with the channel tap weight estimation $\hat{h}_0$, all of length $M$. Thus,

$$c(n) = \sum_{k=0}^{M-1} \left( \sum_{p=0}^{M-1} r(n+p+k)MS^*_o(p) \right)\hat{h}_0(k), \quad n=0,1, \ldots, M-1$$ (3.3)

For a noiseless channel this will produce a strong correlation peak in $c$ at position $m$ (i.e., the transmitted symbol). Each component of the output vector $c$ can be thought of as one of $M$ decision variables, $U_i$, $i=0, 1, \ldots, M-1$. The optimum receiver chooses the $U_i$ with the largest positive real part as the symbol decision, and therefore,

$$U_i = \text{Re}[c(n)\delta(n-i)]$$ (3.4)

There are several possible methods for determining the estimate of the channel impulse response, $\hat{H}_0$. If the channel is well known and time invariant, then $\hat{H}_0$ is fixed (i.e., $\hat{H}_0 = H_0$). If the channel is time varying, however, then $H_0$ must be estimated by the receiver. Two possible methods for estimating $H_0$ are shown in figures 3.2 and 3.4. The first method is to use decision feedback. From (3.1) and (3.3), the output of the TD/CCSK demodulation can shown to be the instantaneous $H_0$ circularly shifted by $m$ positions (i.e., the transmitted data symbol) plus a noise term. Assuming correct symbol decisions, this can be used to update the channel estimate. This is somewhat analogous to a classical DS binary Rake receiver [2] discussed in section 3.3, and works well if the channel is slowly time varying relative to the symbol rate, and the probability of symbol

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3 Under the stated assumptions of this paper, largest refers to the real part of $u(k)$.
4 A hybrid of the two is also possible.
error is small. For a channel which varies rapidly, however, such as disturbed HF channels, the transmitted reference signal described below may be preferable. A similar transmitted reference approach for conventional Rake receivers, called a parallel channel probe, can be found in [14].

In the transmitted reference approach, a reference base-vector, $s_r$, is added component-wise to the modulated CCSK base-vector, $s_m$, at the transmitter before the shift register (see figure 3.3.), and thus is transmitted simultaneously with the data signal. The vector $s_r$ is also a CCSK base-vector, but it is not modulated (i.e., shifted). Since this is known a priori to the receiver, no decision feedback is required to update the channel estimator. The base-vector of the reference signal, $s_r$, is chosen such that it has the following three properties:

1) The reference base-vector $s_r$ is a CCSK base-vector, and therefore

$$R_{s_r}(\tau) = \begin{cases} E_s, & \tau = 0 \\ 0, & \tau = 1, 2, ..., M-1 \end{cases}$$

(3.5)

2) The magnitude spectrum of $s_r$ is approximately equal to the magnitude spectrum of $s_0$, i.e.

$$|S_0(k)| \approx |S_r(k)|, \quad k = 0, 1, ..., M-1$$

(3.6)

3) The reference base-vector and the data base-vector are uncorrelated, and therefore

$$R_{s_0s_r}(\tau) \equiv E_s, \quad \tau = 0, 1, ..., M-1$$

(3.7)

where $R_{s_0s_r}(\tau)$ is defined as the circular cross-correlation of $s_0(n)$ and $s_r(n)$

$$R_{s_0s_r}(\tau) = \sum_{n=0}^{M-1} s_0^*(n)s_r(n+\tau)$$

(3.8)

Properties 1 and 2 ensure that the channel will have the same effect on $s_0$ and $s_r$, and thus that the receiver can use the received reference signal (after the reference CCSK demodulation) as an estimate of $H_0$. With an appropriate choice of $s_0$ and $s_r$ (see
appendix A), and some noise cancellation operations (see section 4.4), the transmitted reference signal produces a good estimate of the instantaneous transfer function of the channel, measured over the exact symbol transmission time and spectrum. Thus, the receiver can attain acceptable performance without tracking the past history of the channel, and operate efficiently even on a rapidly time-varying channel. Property 3 effectively states that the two base-vectors $s_0$ and $s_r$ are Code Division Multiple Access (CDMA) separated, and thus can be easily separated at the receiver.
Chapter 4

Theoretical Performance of TD/CCSK Rake Systems

4.1. Generalized Theoretical Performance

With the basic CCSK and TD/CCSK Rake systems defined, we now compare the theoretical performance of these systems with conventional Rake systems. The probability of bit error for conventional Rake systems using binary and M-ary orthogonal signalling over frequency selective slowly-fading channel has been thoroughly investigated in [12] and [19] (see section 2.3). However, the differences in the signalling and the receiver processing for the CCSK and TD/CCSK systems introduced in this research require a new derivation.

Typically, the theoretical performance of a communication system is measured in probability of bit error, $P_{BE}$, as a function of ratio of received bit energy, $E_b$, to the average noise energy, $N_0$. In this dissertation we will divide the theoretical derivation for $P_{BE}$ into the following three parts: first, the derivation of a consistent definition for $E_b/N_0$ from the received signal; second, the derivation of the decision variable (i.e., $\{U_i\}$) Probability Density Functions (PDFs) as a function of $E_b/N_0$; and third, the derivation of $P_{BE}$, as a function of the decision variable PDFs. The first and third derivation can be generalized for all the system variations. However, each system will
differ in the second derivation since this is dependent on the signal processing at the receiver. We will investigate the second derivation for each individual system later and for now address the first and third derivation.

Let $s_m(t)$ be the baseband transmitted CCSK signal, where $m$ indicates the $m$th symbol in the symbol set. The signal is transmitted over a frequency selective HF skywave channel. The mathematical model for a frequency selective HF channel has been discussed in an chapter 2. It is assumed that there are $L$ diversity channels, over which the signal is received, where $L$ is approximately the ratio of the signal bandwidth to the coherence bandwidth. We will assume that $M \gg L \gg 1$, which is true for the CCSK waveform we are investigating. Each diversity channel is assumed to be a slowly fading channel with Rayleigh-distributed envelope statistics. The fading processes of each channel are also assumed to be mutually independent [12],[13]. Also, the channel is corrupted by AWGN with zero mean. Note that for simplicity of this initial model, intersymbol interference is ignored. Therefore, we can use (4.1) to describe the sampled baseband received signal, $X$. Let $\gamma_s = E_s/N_0$, then by definition

$$\gamma_s = \frac{\text{Average Received Signal Power} \times \text{Symbol Duration}}{\text{Average Received Noise Power} / \text{Noise Bandwidth}}$$  \hspace{1cm} (4.1)

The average received signal and noise power can be derived from $X$. The symbol duration is $NT_c$ and the noise bandwidth is the reciprocal of the sample rate (i.e., $1/T_c$). Therefore, $\gamma_s$ becomes

$$\gamma_s = \frac{\left( \frac{1}{N} \mathbb{E} \left[ \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} s_m(l) h^*(n-l) \right]^2 \right) \times NT_c}{\left( \frac{1}{N} \mathbb{E} \left[ \sum_{n=0}^{N-1} |z(n)|^2 \right] / (1/T_c) \right)}$$  \hspace{1cm} (4.2)

where $\mathbb{E}[\ ]$ is defined to be the expected value. Cancelling terms and simplifying, we get
Let \( \sigma^2 \) and \( \sigma_h^2 \) be the underlying Gaussian variances for the additive channel noise and the Rayleigh fading channel attenuation. Since \( |h(n)| \) and \( |z(n)| \) are both Rayleigh-distributed, their squares are chi-square-distributed. It follows from [20] that

\[
\gamma_s = \frac{2L \sigma_h^2 E_s}{2 \sigma^2}
\]

The term \( N_0 \) is also called the power spectral density of the AWGN received noise random process. By definition [19],

\[
\sigma^2 = \frac{N_0}{2}
\]

Substituting equation (4.5) into equation (4.4) we get

\[
\gamma_s = \frac{2L \sigma_h^2 E_s}{N_0}
\]

In the later calculations, it will be easier to solve in terms of \( \gamma_s \). However, the conversion to the \( E_b/N_0 \) where \( E_b \) is the average received bit energy, for \( M \)-ary signal set, is simply

\[
\frac{E_b}{N_0} = \frac{M \gamma_s}{\log_2(M)}
\]

We thus have a means of computing \( E_b/N_0 \) from the discrete baseband received signal vector \( \mathbf{r} \).

Once \( \mathbf{r} \) is generated, it is demodulated into a set of decision variables from which a symbol decision is made. As stated above, each \( M \)-ary orthogonal system will differ in the actual derivation of these PDFs from the received SNR and the processing of the signal. However, some general assumptions can be made for all decision variables.
1) The decision variables are Gaussian-distributed random variables.

2) The decision variables are mutually independent.

3) The decision variable corresponding to the transmitted symbol have a nonzero mean.

4) All other decision variables have a mean of zero.

The first assumption relies on the Central Limit Theorem [20] and the fact that $L \gg 1$. The remaining assumptions are a consequence of the orthogonality of the signal set\(^5\). From these assumptions a general solution can be derived for all the systems as follows.

Let $\bar{X}$ and $\sigma_X^2$ be the mean and variance of the correct (i.e., $i=m$) symbol decision variable. Assume that the remaining decision variables have equal variance, $\sigma_Y^2$. For example, if $s_0(t)$ is the transmitted symbol, then the correct and incorrect symbol decision variable PDFs are

\[
f_X(U_0) = \frac{1}{\sqrt{2\pi} \sigma_X} e^{-\frac{(U_0-\bar{X})^2}{2\sigma_X^2}}
\]

\[
f_Y(U_y) = \frac{1}{\sqrt{2\pi} \sigma_Y} e^{-\frac{U_y^2}{2\sigma_Y^2}} \quad y = 1, 2, \ldots, M-1
\]

(4.8)

(4.9)

The probability that the receiver makes the correct decision, $P_C$, is the probability that $U_0$ exceeds all the other decision variables and can be expressed as

\[
P_C = \int_{-\infty}^{U_0} P(U_1<U_0, U_2<U_0, \ldots, U_{M-1}<U_0|U_0)P(U_0)dU_0
\]

(4.10)

where $P(U_1<U_0, U_2<U_0, \ldots, U_{M-1}<U_0|U_0)$ denotes the joint probability that $U_y$ for $y=1, 2, \ldots, M-1$ are all less than $U_0$, conditioned on $U_0$.

The probability that $U_0$ exceeds one of the other decision variables given $U_0$ can be expressed as

\[^5\text{It is assumed here that even with a near orthogonal signal set the channel noise will dominate the decision variable statistics over the bit error rates of interest.}\]
\[ P(U_y < U_0 | U_0) = \int_{-\infty}^{U_0} f_Y(U_y) \, dU_y \]
\[ = \int_{-\infty}^{U_0} \frac{1}{\sqrt{2\pi} \sigma_Y} e^{-\frac{U_y^2}{2\sigma_Y^2}} \, dU_y \]
\[ = 1 - \frac{1}{2} \text{erfc} \left( \frac{U_0}{\sqrt{2\pi} \sigma_Y} \right) \]  
(4.11)

Since the probabilities for \( U_y \) are independent, identically distributed, and there are \( M-1 \) of these decision variables, the conditional probability can be written as

\[ P(U_1 < U_0, U_2 < U_0, \ldots, U_{M-1} < U_0 | U_0) = \left[ 1 - \frac{1}{2} \text{erfc} \left( \frac{U_0}{\sqrt{2\pi} \sigma_Y} \right) \right]^{M-1} \]  
(4.12)

Solving for the joint probability, we get for \( M \) symbols

\[ P_c = \frac{1}{\sqrt{2\pi} \sigma_X} \int_{-\infty}^{\infty} \left[ 1 - \frac{1}{2} \text{erfc} \left( \frac{U_0}{\sqrt{2\sigma_Y}} \right) \right]^{M-1} e^{-(U_0 - \overline{X})^2/2\sigma_X^2} \, dU_0 \]
\[ = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \left[ 1 - \frac{1}{2} \text{erfc} \left( \frac{\sigma_X x - \overline{X}}{\sqrt{2\sigma_Y}} \right) \right]^{M-1} e^{-x^2} \, dx \]  
(4.13)

Let \( P_{SE} \) be the probability of symbol error, then \( P_{SE} = 1 - P_c \) and therefore

\[ P_{SE} = 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \left[ 1 - \frac{1}{2} \text{erfc} \left( \frac{\sigma_X x - \overline{X}}{\sqrt{2\sigma_Y}} \right) \right]^{M-1} e^{-x^2} \, dx \]  
(4.14)

The conversion of symbol error to bit error performance is done by computing the average number of bit errors per symbol error. Since the symbols are orthogonal, all symbol errors occur with equal probability. The average number of bit errors per symbol, \( \overline{B} \), is derived from the number of bit errors in an individual symbol error weighted by the probability of that symbol. Since all symbols have equal probability, the average number of bit errors per symbol becomes
where $b$ is the number of bits per M-ary symbol (i.e., $b = \log_2(M)$). Dividing the average number of bit errors per symbol by the number of bits per symbol gives the average bit error probability, and expressed as

$$P_{BE} = \frac{MP_{SE}}{2(M-1)}$$ (4.16)

Substituting (4.14) into (4.16), we finally get

$$P_{BE} = \frac{M}{2(M-1)} \left[ 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \left[ 1 - \frac{1}{2} \text{erfc} \left( \frac{\sigma_X}{\sigma_Y} - \frac{\bar{X}}{\sqrt{2} \sigma_Y} \right) \right]^{M-1} e^{-x^2} dx \right]$$ (4.17)

It can be seen that $P_{BE}$ is a function of the two ratios, $\sigma_X/\sigma_Y$ and $\bar{X}/\Sigma \sigma_Y$. In this dissertation, we will use these ratios as our figures of merit to compare the different signalling schemes. In turn, these figures of merit are functions of $\gamma_s$, and will differ for each system. Let $f_1(\gamma_s) = \sigma_X/\sigma_Y$ and $f_2(\gamma_s) = \bar{X}/\Sigma \sigma_Y$. Then for the TD/CCSK systems, we can generalize the $P_{BE}$ as a function of $\gamma_s$ where

$$P_{BE}(f_1(\gamma_s), f_2(\gamma_s)) = \frac{M}{2(M-1)} \left[ 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \left[ 1 - \frac{1}{2} \text{erfc} \left( f_1(\gamma_s) x - f_2(\gamma_s) \right) \right]^{M-1} e^{-x^2} dx \right]$$ (4.18)

### 4.2. PPM and Coherent FSK

The performance of Pulse Position Modulation (PPM) over a frequency-selective, slowly fading channel is calculated as a baseline for the CCSK scheme. Though M-ary Coherent FSK is a more practical and more widely used orthogonal signaling scheme (see section 3.4), PPM is examined since it is in fact one possible implementation of CCSK (though it is impractical for obvious reasons). However, the performance calculation for
Coherent FSK and PPM do not differ since both are perfect M-ary orthogonal signalling schemes. This derivation will facilitate comparison of different variations of CCSK with each other, as well as with more conventional modulation schemes.

Having generated \( \mathbf{r} \) at the receiver, the signal is demodulated to obtain the decision variables. The demodulation consists of the signal CCSK demodulation followed by the Rake correlation. For PPM the CCSK signal set \( \{ \mathbf{s}_i \} \) is defined as \( s_i(n) = \sqrt{E_s} \delta(n-i), \) where \( \delta(n) \) is a unit impulse function and \( i = 0, 1, 2, \ldots, M-1 \). Therefore, the CCSK demodulation operation for the \( i \) th symbol decision variable is a shift of the received signal vector \( \mathbf{r} \) (see figure 2.7). Assuming we have a perfect estimation of the channel impulse response, and again letting \( s_m(n) \) be the transmitted symbol, then from (4.1), (4.3) and (4.4) the PPM decision variables become

\[
U_i = \text{Re} \left[ \sum_{k=0}^{L-1} \sqrt{E_s} r(n+i) h^*(k) \right]
\]

The output of the signal matched filter can be truncated to the maximum number of taps required for the Rake correlator (i.e., \( L \)). Substituting and solving for the decision variables, we get

\[
U_i = \text{Re} \left[ \sqrt{E_s} \sum_{k=0}^{L-1} \sum_{l=0}^{L-1} s_m(l) h(k+i-l) + z(k+i) h^*(k) \right]
\]

\[
= \text{Re} \left[ \sqrt{E_s} \sum_{k=0}^{L-1} \sum_{l=0}^{L-1} \sqrt{E_s} \delta(l-m) h(k+i-l) + z(k+i) h^*(k) \right]
\]

\[
= \text{Re} \left[ E_s \sum_{k=0}^{L-1} h(k+i-m) h^*(k) + \sqrt{E_s} \sum_{k=0}^{L-1} z(k+i) h^*(k) \right]
\]

Since we assume that \( M \gg L \), then \( h(n) \) and the truncated vector \( z(k+i) \) can be zero padded to a length of \( M \) without a loss of generality. Therefore, substituting equations (2.3) and (3.8) into equation (4.20), we get
Then from Appendix A, the statistics for the circular auto-correlation sequence can be computed as

$$\Re [R_{I}(\tau)] = \begin{cases} G[2L\sigma_h^2, 4L\sigma_h^4] & \tau=m \\ G[0, 2(L-m-t)\sigma_h^4] & m-L<\tau<L+m \\ 0 & \text{else} \end{cases}$$

and for the circular cross-correlation,

$$\Re [R_{h}(\tau)] = G[0, 2L\sigma_h^2\sigma_x^2]$$

where $G[\overline{X}, \sigma_X^2]$ is the Gaussian probability density function defined as

$$G[\overline{X}, \sigma_X^2] = f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_X} e^{-\frac{(x-\overline{X})^2}{2\sigma_X^2}}$$

Simplifying the mean and variances by assuming worst case in equation (4.22) and substituting equations (4.5), (4.22) and (4.23) into (4.21), the statistics for the decision variables now become

$$U_i = \begin{cases} G[2E_s\sigma_h^2, 4E_s^2L\sigma_h^4 + E_sN_0L\sigma_h^2] & i=m \\ G[0, 2E_s^2L\sigma_h^4 + E_sN_0L\sigma_h^2] & \text{otherwise} \end{cases}$$

Substituting the decision variable theoretical statistics into the figures of merit defined in section 4.1 (i.e., $f_1$ and $f_2$), we have

$$f_1(\gamma_s) = \frac{\overline{X}}{\sqrt{2\sigma_Y}} = \sqrt{\frac{2E_s\sigma_h^2}{\sqrt{2\sigma_Y^2}2E_s^2L\sigma_h^4 + E_sN_0L\sigma_h^2}} = \sqrt{\frac{\gamma_s}{L+1}}$$

and
Equations (4.26) and (4.27) represent the lower bound of performance for the general M-ary orthogonal signalling with a Rake receiver for large L. This performance can be achieved in practice if the channel is time-invariant, and the channel transfer function is known. In addition, since PPM is a form of perfect CCSK, this lower performance bound also applies for all CCSK systems.

4.3. TD/CCSK with Decision Feedback

Next we examine the general TD/CCSK system with decision feedback as illustrated in figure 3.1 and 3.2. The difference in ideal theoretical performance between the CCSK and TD/CCSK receivers is due to truncation of the received multipath signal. In basic CCSK, the signal is truncated after the discrete signal matched filter operation, and thus all the signal energy resides in the remaining samples. However for TD/CCSK, L is truncated before the CCSK demodulation. The truncation of the received signal energy will reduce the signal energy and distort the data signal at the output of the demodulation operation. However, if M ≫ L, then this distortion is negligible (i.e., most of the signal energy arrives within the received symbol period), and therefore we need only consider the reduction of the signal energy. If we assume a trapezoidal envelope (see figure 4.1) for the received data signal, then the truncation removes the triangular tail of the trapezoid. Therefore, if we approximate that the truncation processing gain, φt, as a function of the receiver signal energy reduction, then

$$\phi_t = 1 - \frac{L}{2M}$$

(4.28)
The curve in figure 4.2 shows the truncation processing gain as a function of \( L/M \). The received signal vector, \( \mathbf{r} \), is transformed into the frequency domain, \( \mathbf{R} \). The TD/CCSK demodulation operation is a component-wise multiplication of \( \mathbf{R} \) by the complex conjugate of the TD data base-vectors, \( \mathbf{S}_0 \). Using equation (4.1) and expanding the output of the TD/CCSK demodulation we get

\[
R(k)S_0^*(k) = \phi_1 S_m(k)H(k)S_0^*(k) + Z(k)S_0^*(k)
\]  
(4.29)

Note that the truncating processing gain is assessed on the multipath received symbol (i.e., \( \mathbf{S}_m \mathbf{H}_o \)). Also note that all vector multiplications are component-wise multiplications. From equation (2.4), it can be shown that

\[
S_m(k)S_0^*(k) = E_s e^{j\pi km}
\]  
(4.30)

Substituting (4.30) into (4.29), we get

\[
R(k)S_0^*(k) = \sqrt{\phi_1} E_s e^{j\pi km}H(k) + Z(k)S_0^*(k)
\]  
(4.31)
We again assume that the receiver has perfect knowledge of the channel impulse response. Assigning \( \hat{H}_o = H_o \), the transform domain output of the TD/Rake correlator, \( \mathbf{C} \), becomes

\[
C(k) = R(k)S_o^*(k)(\hat{H}(k))^* = \left(\sqrt{\varphi_i} E_s e^{j\pi kmH(k)} + Z(k)S^*_o(k)\right)H^*(k)
\]

\[= \sqrt{\varphi_i} E_s e^{j\pi kmH(k)H^*(k)} + Z(k)S^*_o(k)H^*(k) \tag{4.32}
\]

It can be seen that the first term is actually a delayed (i.e., circularly shifted) circular auto-correlation, \( R_h(\tau-m) \), of the channel impulse response. This term is delayed such that the circular auto-correlation peak in the time domain (i.e., \( R_h(\tau)|\tau=0 \)) is in the transmitted data impulse position (i.e., \( m \)). The second term in the time domain is the cross-correlation of the impulse response of the channel, \( H_o \), and the received noise sequence, \( Z \).
correlated with the data base-vector, $s_0$. Since $s_0$ has an approximately constant complex magnitude, $\sqrt{E_s}$, and since the components of $Z$ are independent random variables with uniform phase, then $Z s_0^*$ generates another scaled transform domain AWGN sequence $\sqrt{E_s}\tilde{Z}$, whose transform, $\tilde{Z}$, has a variance such that $\sigma_\tilde{Z}^2 = \sigma^2$. From (4.32), recalculating for the circular complex correlation $C(k)$ as follows

$$C(k) = \sqrt{\phi_t} E_s e^{jkmH(k)}H^*(k) + \sqrt{E_s}\tilde{Z}(k)H^*(k)$$

(4.33)

and taking the DFT yields

$$c(n) = \sqrt{\phi_t} E_s R_h(n-m) + \sqrt{E_s}R_{hz}(n)$$

(4.34)

Substituting equation (3.4) into equation (4.34), we get for the decision variables:

$$U_1 = \sqrt{\phi_t} E_s \text{Re}[R_h(i-m)] + \sqrt{E_s} \text{Re}[R_{hz}(i)]$$

(4.35)

But this result is the same as PPM with the exception of $\tilde{Z}$ and the processing gain, $\phi_t$. Since $\tilde{Z}$ and $z$ have equal variance, then statistically $R_{hz}(\tau)$ and $R_{hz}(\tau)$ are equivalent. Comparing (4.21) to (4.35), it follows from (4.25) that the figures of merit for the ideal TD/CCSK with decision feedback are

$$f_1(\gamma_s) = \sqrt{\frac{\phi_t \gamma_s}{\phi_t \gamma_s + 1}}$$

(4.36)

and

$$f_2(\gamma_s) = \sqrt{1 + \frac{f_1(\gamma_s)^2}{L}}$$

(4.37)

Thus, the difference in ideal performance between CCSK (e.g. PPM) and TD/CCSK with decision feedback is the truncation processing gain (i.e., $\phi_t$) from equation (4.26) and (4.36) respectively.
4.4. **TD/CCSK with Transmitted Reference**

4.4.1. **Ideal Case**

The TD/CCSK signal with transmitted reference can be viewed as being comprised of data and reference vectors, $s_m$ and $s_r$, transmitted simultaneously over $L$ diversity channels. Equations (3.4), (4.5), and (4.7) describe the properties of both vectors. Assuming that data and reference signal are transmitted at equal strength, then the transmitted energy of each is halved. Rederiving the baseband sampled received signal, $r$, from (4.1), we get for the CCSK signal with transmitted reference

$$r(n)=\sum_{l=0}^{L-1} \left(\frac{s_m(l) + s_r(l)}{\sqrt{2}}\right) h_0^*(n-l) + z(n) \mid n = 0, 1, 2, ..., L+M-1$$

(4.38)

As in the decision feedback case, the received signal is truncated at the receiver after 1 symbol period (i.e., $M$ points), and therefore the received signal $r(n)$ ranges only for $n=0, 1, 2, ..., M-1$. The truncation of the received signal energy distorts both the data and reference signals. Therefore, the truncation processing loss $\varphi_t$, as in section 3.3, applies for the transmitted reference case as well.

As illustrated in figure 2.4, the received signal vector, $r$, is transformed into $R$. The data and reference TD/CCSK demodulation operations are a component-wise multiplication of $R$ by the complex conjugate of the data and reference base-vectors, $S_0$ and $S_r$, respectively. After the reference TD/CCSK demodulation, the output, $RS_r^*$, is normally used to update the TD channel estimate $\hat{H}_0$. For the ideal case, we assume that the channel impulse response, $H_0$, is known by the receiver and therefore $\hat{H}_0 = H_0$.

Expanding the output of the data TD/CCSK demodulation, we get

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5In general, equal energy reference and data signals are not required. The optimum energy fraction to be used in the reference signal depends in a complex fashion on the expected stationarity of the channel and the structure of the channel estimator. This is addressed in [15] for conventional BPSK Rake implementations.
Since $S_o$ and $S_r$ are also known, the receiver can remove the second term in the summation of (21). Therefore (4.39), in this case, becomes

$$R(k)S_o^*(k) = \frac{\Phi_t}{2} (S_m(k)S_o^*(k) + S_r(k)S_o^*(k))H(k) + Z(k)S_o^*(k)$$  \hspace{1cm} (4.39)$$

Note that this is similar to the results for TD/CCSK with decision feedback (4.34) except for a constant. If we follow the same logic to derive the decision variable PDFs, we can conclude from (4.40) that for the ideal transmitted reference case,

$$U_i = \frac{\Phi_t E_s}{2} \text{Re}[R_h(i-m)] + \sqrt{\frac{\Phi_t E_s}{2}} \text{Re}[R_{hi}(i)]$$  \hspace{1cm} (4.41)$$

Substituting and scaling equation (4.22) and (4.23) into equation (4.41), we get for the decision variables

$$U_i = \begin{cases} G \left[ \frac{\phi_1 E_s L \sigma_h^2}{2} + \frac{\phi_1 E_s N_o L \sigma_h^2}{2} \right] & \text{i=m} \\ G \left[ 0, \frac{\phi_1 E_s L \sigma_h^2}{2} + \frac{\phi_1 E_s N_o L \sigma_h^2}{2} \right] & \text{else} \end{cases}$$  \hspace{1cm} (4.42)$$

As is section 4.2, we calculate the figures of merit $\sigma_x/\sigma_y$ & $\bar{X}/\sqrt{2}\sigma_y$ for this scheme. From the above calculations, it follows that

$$f_1(\gamma_s) = \frac{\bar{X}}{\sqrt{2}\sigma_y} = \frac{\phi_1 E_s L \sigma_h^2}{\sqrt{\phi_1^2 E_s^2 L \sigma_h^4 + \phi_1 E_s N_o L \sigma_h^2}} = \sqrt{\frac{\phi_1 \gamma_s}{L} + 2}$$  \hspace{1cm} (4.43)$$

and

$$f_2(\gamma_s) = \frac{\sigma_x}{\sigma_y} = \sqrt{\frac{2\phi_1^2 E_s^2 L \sigma_h^4 + \phi_1 E_s N_o L \sigma_h^2}{\phi_1 E_s^2 L \sigma_h^4 + E_s N_o L \sigma_h^2}} = \sqrt{1 + \frac{f_1(\gamma_s)^2}{L}}$$  \hspace{1cm} (4.44)$$
Let $\varphi_r$ be the processing loss due to sending a transmitted reference. From equation (4.18) it can be seen that

$$\varphi_r = \frac{1}{2}$$

(4.45)

Substituting this into (4.43), $f_1(\gamma_s)$ becomes

$$f_1(\gamma_s) = \sqrt{\frac{\varphi_1 \varphi_r \gamma_s}{\varphi_1 \varphi_r \gamma_s / L + 1}}$$

(4.46)

In comparing the ideal theoretical performance for decision feedback with this result, the difference performance is $\varphi_r$ (i.e., a process loss of 3 dB).

4.4.2. Instantaneous Case

The instantaneous system differs from the ideal system in that the channel impulse response estimate is no longer assumed to be perfect, but is rather the instantaneous received channel estimate (i.e., the output of the reference TD/CCSK demodulation). This means that there is no time averaging or filtering of the channel impulse response estimation. This can be seen as the guaranteed worst case performance under any fading condition, since no Rake type channel estimates are performed.

The calculation for the performance on the instantaneous TD/CCSK for transmitted reference requires the calculation of the received signal cross-correlation variance and the channel impulse response estimation. For the instantaneous case, CCSK demodulated reference signal is used as the channel estimate (i.e., $\hat{H}_0 = \text{RS}_r$). From section 4.1, the TD Rake correlation output vector, $\mathbf{C}$, in the instantaneous case, becomes the component-wise vector multiplication of the outputs of the data and reference TD/CCSK demodulation (i.e., $\mathbf{C} = \text{RS}_d \times (\text{RS}_r^*)^\dagger$). Note that these operations are commutative since they all involve only component-wise multiplication. Rearranging these operations, the demodulation can be rewritten as
\[ C(k) = R(k)S^*_k(k) \left[ R(k)S^*_k(k) \right]^* \]
\[ = |R(k)|^2 S^*_k(k)S_k(k) \]
\[ (4.47) \]

which now takes the form of an auto-correlation of the received signal followed by a double CCSK demodulation to produce the data impulse.

With the assumption that \( L \) is large and the channel consists of \( L \) Rayleigh-fading diversity channels, and applying the Central Limit Theorem, we can approximate \( \xi \) by an independent Gaussian sequence. The variance of the received signal is then the sum of the variances associated with the average signal and noise power, given by

\[ \sigma^2 = \phi_0 E_s^* \sigma_n^2 + \sigma_z^2 \]
\[ (4.48) \]

From Appendix A it can be shown that the probability density function for the received signal circular auto-correlation is

\[ \text{Re}[R_f(\tau)] = \begin{cases} 
G[2M\sigma^2, 4M\sigma^2] & \tau = 0 \\
G[0, 2M\sigma^2] & \text{else} 
\end{cases} \]
\[ (4.49) \]

The noise energy contained in \( R_f(0) \) is relatively large compared to the noise energy in the \( R_f(\tau) : \tau \neq 0 \) components. Since before the TD/CCSK demodulation operations only a small fraction of the data signal energy is contained in \( R_f(0) \), then zeroing this component will reduce the potential noise variance on the decision variables considerably, with negligible effect on the output signal energy. This noise reduction operation can easily be implemented in the frequency domain between the TD received signal auto-correlation operation and the TD/CCSK demodulation by applying the following formula. Let \( \mathbf{V} \) be the vector output of the noise reduction operation, then

\[ V(k) = |R(k)|^2 - \frac{1}{M} \sum_{k=0}^{M-1} |R(k)|^2 \]
\[ (4.50) \]

From (4.49) and (4.50),
After this noise reduction operation, the TD/CCSK demodulation operations are applied, and (4.47) becomes

\[ C(k) = V(k)S_v^*(k)S_r(k) \quad (4.52) \]

Since only the delayed auto-correlation spike, \( R_h(\tau-m) \), \( \tau = m \), associated with the data impulse will coherently combine, the statistical change to the output sequence resulting from this operation must be considered. Conceptually, if the energy contained in \( R_h(\tau-m) \), is separated from \( R_v(\tau) \) and then added back after the CCSK demodulation operations, then the correlation output yields

\[ U_i = \text{Re}[c(i)] = \begin{cases} G[\phi_iE_s \sigma_h^2, 2M\sigma_r^4] & i=m \\ G[0, 2M\sigma_r^4 - \frac{\phi_i^2E_s^2\sigma_h^4}{2}] & \text{else} \end{cases} \quad (4.53) \]

Substituting (4.48) into (4.53) we get

\[ U_i = \begin{cases} G[\phi_iE_s \sigma_h^2, 2M(\phi_iE_s \sigma_h^2/M + \sigma_z^2)] & i=m \\ G[0, 2M(\phi_iE_s \sigma_h^2/M + \sigma_z^2)^2 - \frac{\phi_i^2E_s^2\sigma_h^4}{2}] & \text{else} \end{cases} \quad (4.54) \]

Calculating the figures of merit, \( f_1 \) and \( f_2 \) from (4.54), for the instantaneous TD/CCSK scheme, it can be shown that

\[ f_1(\gamma_s) = \sqrt{\frac{(\phi_i\phi_r\gamma_s)^2}{(4M - \frac{1}{L})(\phi_i\phi_r\gamma_s)^2 + 4\phi_i\phi_r\gamma_s + M}} \quad (4.55) \]

---

6This energy is assumed to be evenly spread across all M points before the matched filtering.
Note that $f_1$ is now in terms of $\gamma_i$ rather than $\sqrt{\gamma_i}$ as in the ideal case, and therefore a significant performance degradation is noted. This is mainly due to the noisy estimation of the channel IR which is dominated by the auto-correlation of the received noise sequence. This system however, represents a guaranteed worst case bound on the performance of TD/CCSK with transmitted reference, even on a rapidly varying channel. In practice, even a relatively simple reference estimator can substantially improve on this performance bound, provided that the time variation of the channel is slow relative to the symbol duration.

4.5. Comparison of Systems

Figure 4.3 shows bit error performance of selected TD/CCSK systems presented in this dissertation, as well as some ideal FSK and BPSK Rake systems [12] over a frequency-selective, slowly fading channel. For all curves in this figure, the following values are used: $L=1024$, $M=8192$, and $E_b/N_0=\gamma_s/\log_2(M)$. The same general behavior will hold for other values provided that $M \gg L$. From figure 4.3, we can summarize the results of this section as follows:

1) Ideal CCSK modulation performance is equivalent to ideal coherent FSK (or any other coherent, M-ary orthogonal signalling scheme).

2) The processing loss for implementing ideal TD/CCSK with decision feedback is approximately equal to the percentage of the signal truncated (i.e., $\phi_t$). This is roughly on the order of a fraction of a dB for $M \gg L$.

3) The processing loss for implementing ideal TD/CCSK with equal power transmitted reference is an additional 3 dB (i.e., $\phi_r$).

4) The processing loss for implementing TD/CCSK with transmitted reference and using the instantaneous reference as a channel estimate is approximately an additional 9 dB.
Figure 4.3 Bit Error Performance of Basic TD/CCSK Rake Systems

In a comparison of the TD/CCSK system to ideal FSK systems, the processing loss due to truncating the received signal (necessary for TD processing) is the main cost. However, TD processing makes it feasible to do real-time processing of the signal. In addition, the truncation processing loss is minimal when the signal length is much greater than the multipath duration (i.e., $M \gg L$). From an implementation standpoint, the construction of an analog FSK (or any other $M$-ary orthogonal signal scheme) Rake receiver, with a large number of taps is unfeasible. On the other hand, it is feasible to construct a BPSK Rake receiver with large number of taps. However, from figure 4.3 it can be seen that the ideal TD/CCSK Rake receiver will outperform an ideal BPSK Rake
receiver. Since, it is feasible to construct a real-time TD/CCSK Rake receiver, and with the DSP benefits of flexibility, size, and reliability, it appears that the TD/CCSK Rake receiver is preferable.

The processing loss of 3 dB in the implementation of a transmitted reference is a significant performance difference in the TD/CCSK system. However, the robustness it provides is of importance on a highly disturbed HF skywave channel. Without reference symbols, the receiver must use data symbols to update the channel impulse response estimate. Correct symbol decision feedback is thus required to update the channel estimate properly. In the case of a highly disturbed channel, a system using decision feedback can become unstable if a burst of incorrect symbol decision occurs. With reference symbols, the receiver is no longer dependent on correct symbol decisions. In addition, if the channel is so highly disturbed that past symbols cannot be used in the estimation of the channel impulse response, communication with the TD/CCSK system with transmitted reference is still possible. This is because the data and reference are sent simultaneously over the same channel providing an instantaneous estimate of the channel impulse response. However, the system is required to operate 9 dB over the ideal performance. In the following chapter, we will introduce several signal processing improvements to the TD/CCSK system with transmitted reference that can improve this upper bound in significantly, while still maintaining the robustness achieved with the transmitted reference signal.
Chapter 5

IMPROVEMENTS TO THE TD/CCSK RAKE SYSTEM

5.1. Frequency Domain Interleaving

As stated in previous sections, it is desirable (if not essential) to have the vector length of the symbol, M, be much greater than the number of diversity channels, L. In theory, this is possible by simply making M as large as is practically feasible. However, note that the number of bits per symbol in the basic CCSK signal is a logarithmic function of M (i.e., \( b = \log_2(M) \)). If the length of the CCSK symbol is increased such that the \( M \gg L \), then the data rate will be reduced substantially. For example, if the symbol length is doubled from 1024 to 2048, then the number of bits per symbol only increases by one (i.e., 10 to 11). In this example, the data rate is reduced by a factor of 11/20 (i.e., the number of bits sent by one 2048-length CCSK symbol compared to the number of bits sent by two 1024-length CCSK symbols). This inflexibility in the basic TD/CCSK modulation scheme could cause problems in meeting communication system requirements.

Frequency Domain Interleaving (FDI) is a method of eliminating the dependency between data rate and CCSK symbol length. This should not be confused with time domain interleaving which is used primarily in communication system to reduce the effects of burst errors. As indicated by its name, in FDI the symbol samples are interleaved in the
frequency domain rather than in the time domain. This could be thought of as tone interleaving since each point of a frequency domain vector represents a magnitude and phase of a specific complex tone. Let $Q$ be the number of interleaved symbols. Then figure 5.1 illustrates an example of interleaving in which 4 TD/CCSK symbols (i.e., $Q = 4$) of length $M$ are interleaved in the frequency domain. The interleaved symbols form a "super"-symbol of length $QM$. The position of the interleaving and deinterleaving operation in the transmitter and receiver can be seen in figure 5.2 and 5.3, respectively.

![Figure 5.1 Example of Frequency Domain Interleaving.](image)

![Figure 5.2 Block Diagram of TD/CCSK Transmitter with Frequency Domain Interleaving](image)
It should be noted that the interleaving process occurs after the generation of Q CCSK symbols. At first glance, the transmitter must interleave these Q symbols in the frequency domain. Therefore, the transmitter would be required to compute an M-point FFT for each of the Q symbols, interleave, and then compute a QM-point IFFT to get back to the time domain with the super-symbol. However, it is computationally more efficient to perform FDI directly in the time domain.

Let \( \mathbf{s}_m q \) be the \( q \) th CCSK symbol of length M to be interleaved into the super-symbol and \( \mathbf{s}_m \) be the discrete Fourier pair of \( \mathbf{s}_m q \), where by definition

\[
\mathbf{s}_m q(k) = \sum_{n=0}^{M-1} s_{mq}(n) e^{j2\pi kn}
\]

(5.1)

Now, let \( \mathbf{s}'_m q \) be the \( q \) th CCSK symbol of length M after the interleaving process (see figure 5.1). In other words \( \mathbf{s}'_m q \) is \( \mathbf{s}_m q \) spaced every \( Q \) th point in the super-symbol starting at the \( q \) (i.e., \( q, q+Q, q+2Q, \ldots, q+(M-1)Q \), then

\[
S'_{mq}(k) = \begin{cases} 
\frac{\mathbf{s}_m q(k-q)/Q}{Q} & k= q, q+Q, q+2Q, \ldots, q-(M-1)Q \\
0 & \text{else} 
\end{cases}
\]

(5.2)

where \( \mathbf{s}_m q \) has a length of QM. Let \( \mathbf{s}'_m q \) be the Fourier pair of \( \mathbf{s}'_m q \) then by definition

\[
\mathbf{s}'_{mq}(n) = \sum_{k=0}^{Q M-1} S'_{mq}(k) e^{j\frac{2\pi}{QM} kn}
\]

(5.3)

Figure 5.3 Block Diagram of TD/CCSK Rake Receiver with Frequency Domain Interleaving.
Substituting equation (5.2) into (5.3) and letting $p=(k-q)/Q$, then

$$s'_{mq}(n) = \sum_{p=0}^{M-1} S_{mq}(k) e^{i \frac{2\pi p}{Q} q}$$

(5.4)

Substituting equation (5.1) into (5.4), we get

$$s'_{mq}(n) = s_{mq}(n) e^{i \frac{2\pi q}{Q} n}$$

(5.5)

From equation (5.5), it can be seen that an individual interleaved symbol can be generated in the time domain by cascading it $Q$ times and then modulating the result with a complex tone. Note that the complex tone is different for each interleaved symbol since the $q$th interleaved symbol in the frequency domain is shifted by $q$ points. Therefore, the transmitter can generate the super-symbol by cascading each symbol $Q$ times, modulating them by the proper complex tones, and performing a component-wise vector addition of the $Q$ symbols. Let $s'$ be the transmitted super-symbol such that

$$s'(n) = \sum_{q=0}^{Q-1} s_{mq}(n) e^{i \frac{2\pi q}{Q} n}$$

(5.6)

In the case when a transmitted reference is used, the reference signal can be added before the interleave process. The TD/CCKS demodulations, TD/Rake correlation and deinterleaving operations are all component-wise operations (i.e., linear operations) and are therefore interchangeable. This leads to another advantage of FDI. If we deinterleave last (see figure 5.3), then the channel estimation has $QM$ components (i.e., taps) rather than $M$ components. If the sampling rate, $T_c$, remains constant regardless of interleaving, then FDI results in improved frequency resolution of the channel. Specifically, we achieve $Q$ times the frequency resolution of the channel estimation using FDI. The improved channel estimate will be important for a noise reduction technique called Tail Clipping (TC) which
is discussed later in this chapter. For the systems analyzed in this paper, the deinterleave operation will be the last of the TD operations.

As in other interleaving schemes, the process of interleaving and deinterleaving should in theory be transparent (i.e., no performance loss) at the symbol level. This of course is with the exception of a delay resulting from storing $Q$ symbols before interleaving. However, the use of different length FFT and IFFTs has an effect on the AWGN of the channel. This effect can be seen by examining how the FDI deinterleaving process distorts the received time domain super-symbol, $r(n)$. This deinterleaving effect, as will be shown, is equivalent to aliasing.

Let $Q$ be the number of interleaved symbols, and let $M$ be the length of the symbols to be interleaved, then $r(n)$ has length $QM$. By definition the DFT of $r(n)$ is,

$$ R(k) = \sum_{n=0}^{QM-1} r(n) e^{-j\frac{2\pi}{QM}kn} \quad (5.7) $$

Let $R_q(k)$ be the $q$th deinterleaved received symbol of length $M$, which is derived by decimating $R(k)$ such that

$$ R_q(k) = R(kQ + q) \quad (5.8) $$

Next we compute the deinterleaved symbol in the time domain where

$$ r_q(n) = \frac{1}{M} \sum_{k=0}^{M-1} R_q(k) e^{j\frac{2\pi}{M}kn} \quad (5.9) $$

substituting equation (5.7) and (5.8) into (5.9) we get

$$ r_q(n) = \frac{1}{M} \sum_{k=0}^{M-1} \sum_{p=0}^{QM-1} r(p) e^{-j\frac{2\pi}{QM}(kQ+q)p} e^{j\frac{2\pi}{M}kn} \quad (5.10) $$

Reversing the summations and rearranging equation (5.10), we get
Solving the inner summation, we get

\[
 r_q(n) = \sum_{p=0}^{QM-1} \left[ \sum_{k=0}^{M-1} \sum_{p=0} \frac{2\pi}{QM} (p - n)Q_k \right] r(p) e^{-j2\pi \cdot qp}
\]

\[(5.11)\]

But \(\delta(n-p)M = 1\) only when \(n-p = 0, M, 2M, (Q-1)M\), therefore equation (5.12) becomes

\[
 r_q(n) = \sum_{p=0}^{QM-1} \left\{ \sum_{k=0}^{Q-1} r(kM + n) e^{-j\frac{2\pi}{QM} q(kM + n)} \right\}
\]

\[(5.12)\]

Thus, to generate the deinterleaved \(q\)th received symbol we modulate the received signal with the proper complex tone and then sum every \(Q\)th point. Note that for \(q=0\), equation (5.13) becomes

\[
 r_q(n) = \sum_{k=0}^{Q-1} r(kM + n)
\]

\[(5.13)\]

\[(5.14)\]

From equation (5.14), it is evident that FDI deinterleaving causes aliasing in the time domain. This time domain aliasing also applies to the received AWGN. Therefore, the variance of the noise (i.e., the average noise power) increases by a factor equal to the interleave ratio, \(Q\). This increase in the symbol decision-variable noise will cause a degradation in bit error performance on individual interleaved symbols.

However, to be consistent, we must also consider only the average received signal power for an individual interleaved symbol. Since there are \(Q\) interleaved symbols, it is seen that the average received signal power for an individual interleaved symbol (see equation 5.5) is \(1/Q\)th of the total average received signal power for the super-symbol. An alternative view is that each individual interleaved symbol is repeated \(Q\) times, as stated earlier in this section. Therefore, the average signal power must be reduced by a factor of
Q. Thus the increase in average noise power (i.e., FDI processing loss) is offset by the decrease in average signal power (i.e., FDI processing gain) when computing $E_b/N_0$. Therefore, there is no performance loss in using FDI.

With FDI, we can now increase the symbol length to achieve the $QM \gg L$ requirement without effecting the data rate. It would seem that we could increase the symbol length without bound since there is theoretically no processing loss. However, there are tradeoffs. First, a larger QM results in a larger FFT at the receiver, which will increase the computational load per symbol. Second, with a larger FFT we will increase either the bandwidth or frequency resolution of the transmitted signal. If we fix the sample or chip rate, $T_c$, then FDI results in finer frequency resolution. This could make the system more sensitive to frequency errors (e.g., Doppler Shift and Spread) (see section 7.3). Finally, large super-symbols increase the memory requirements of the system implementation, and also increase system delay. Therefore, there are a number of limiting factors to implementing FDI that must be considered in the overall system design.

5.2. **Channel Estimation Noise Reduction**

In the cases examined thus far, we have either assumed a perfect noiseless estimate of the channel (best case), or the noisy instantaneous channel measurement (worst case). Note that the lower bound (best case) is only achievable if the channel impulse response is known. However, on an HF skywave channel, the impulse response is a time-variant random process. Therefore, we cannot realistically achieve this bound. This does not mean that we have to operate at the upper bound (worst case), however. We have not yet taken advantage of various properties of the channel to reduce the noise on the channel estimate, $\hat{h}$. The more noise reduction we can achieve, the more closely we will approach the lower performance bound.
Figures 5.4a through 5.4d present an abstract example of the noise reduction operations presented in this dissertation. In these figures the black area represents the noiseless signals and the gray area represents these signals with Additive White Gaussian Noise (AWGN). As stated in section 4.3, we assume a trapezoidal envelope for the received signal, $r$, as seen in figure 6.4a. The data and reference symbols are demodulated separately from the received signal. As stated in section 4.4, the CCSK demodulated reference symbol is used as a the instantaneous channel impulse response estimate, $\hat{h}_o$. Figure 5.4b illustrates $\hat{h}_o$ as the instantaneous channel impulse response $h_o$ buried in AWGN. In the instantaneous TD/CCSK Rake system, the channel estimate in the frequency domain, $\hat{H}_o$, is the CCSK demodulated reference symbol. In the ideal TD/CCSK Rake system, the channel estimate is perfect (i.e., $H_o$).

The purpose of the noise reduction techniques is to improve $\hat{H}_o$ in the instantaneous case by processing the demodulated reference symbol before it is used in the Rake operation. The three channel estimate noise reduction techniques described in this dissertation are tail clipping, center clipping and time averaging. Figure 5.5 illustrates these operations.

Tail clipping is a blind noise reduction technique that zeros out the majority of channel taps known to contain no signal energy (i.e., $n > L$). In figure 5.4c, the tail clipping operation zeros out 75 percent of the taps. Center clipping is a finer noise reduction technique that examines the remain taps (after tail clipping) that could contain signal energy and zeros out those determined to contain no signal energy. In this case, a simple threshold is used as seen in figure 5.4d[21]. Time averaging uses past channel estimates to average out the noise assuming that the channel is at least somewhat stationary. Thus time averaging effectively lowers the noise floor as seen in figure 5.4e. These noise reduction techniques have been suggested before for use in Rake correlators [14] [10]. However, for
the TD/CCSK Rake receiver, these operations are implemented in the frequency domain, thus requiring further analysis.

Figure 5.4a Abstract Example of Received CCSK Signal in Noise

Figure 5.4b Abstract Example of Channel Estimation Before Noise Reduction

Figure 5.4c Abstract Example of Channel Estimation After Tail Clipping

Figure 5.4d Abstract Example of Channel Estimation After Center Clipping
5.2.1. General Theoretical Performance for Estimation Noise Reduction

Let $\sigma_{zc}^2$ be the variance of the AWGN on the instantaneous channel estimate $\hat{h}_0$ and let $\chi$ be the effective noise reduction on the estimate, such that the noise variance after the noise reduction equals $\chi \sigma_{zc}^2$. The parameter $\chi$ ranges from 1 (i.e., no reduction or instantaneous case) to 0 (i.e., ideal case).

The Estimation Noise Reduction BER performance can be derived from the analysis of the ideal and instantaneous TD/CCSK modulation schemes. Let $U_i$ be the decision variable PDFs for the instantaneous reference case described by (4.54). Let $A_i$ be the decision variable PDFs for the ideal transmitted reference case described by (4.42). The difference between the two cases is the channel and self noise that is added to the channel estimate in the instantaneous case. If we let $U_i$ be separated by superposition into an ideal transmitted
reference case and an additive channel and self noise sequence, then \( U_i = A_i + B_i \). From equation (4.42)

\[
A_i = \begin{cases} 
G \left[ \phi_i E_s L \sigma_h^2, \phi_i^2 E_s^2 L \sigma_h^4 + \frac{\phi_i E_s N_0 L \sigma_h^2}{2} \right] & i=m \\
0, \phi_i^2 E_s^2 L \sigma_h^4 + \frac{\phi_i E_s N_0 L \sigma_h^2}{2} & \text{else} 
\end{cases}
\]

(5.15)

\( B_i \) contains the decision variable noise induced by the cross-correlation between the sum of the channel and self noise on the demodulated reference signal and the data signal. Using superposition, this is simply the difference between (4.54) and (4.42), and therefore for the instantaneous case without noise reduction

\[
B_i = G \left[ 0, \left( \frac{2M - L}{L} \right) \phi_i^2 E_s^2 L^2 \sigma_h^4 + \frac{3\phi_i E_s L \sigma_h^2 N_0}{2} + \frac{MN_0^2}{2} \right]
\]

(5.16)

From our assumption, the noise on the channel estimation is reduced by a factor of \( \chi \). Therefore, the noise reduction cross-correlation vector, \( B'_i \) becomes

\[
B'_i = G \left[ 0, \chi \left( \frac{2M - L}{L} \right) \phi_i^2 E_s^2 L^2 \sigma_h^4 + \frac{3\phi_i E_s L \sigma_h^2 N_0}{2} + \frac{MN_0^2}{2} \right]
\]

(5.17)

The new noise reduction decision variable, \( U'_i \), can be expressed as \( U'_i = A_i + B'_i \), and thus from (5.15) and (5.17)

\[
U'_i = \begin{cases} 
G \left[ E_s L \sigma_h^2, \left( \frac{2M - \chi L}{L} \right) \phi_i^2 E_s^2 L^2 \sigma_h^4 + \frac{(1+3\chi)\phi_i E_s L \sigma_h^2 N_0}{2} + \frac{2\chi MN_0^2}{2} \right] & i=m \\
0, \left( \frac{2M - 2\chi L}{2L} \right) \phi_i^2 E_s^2 L^2 \sigma_h^4 + \frac{(1+3\chi)\phi_i E_s L \sigma_h^2 N_0}{2} + \frac{2\chi MN_0^2}{2} & \text{else} 
\end{cases}
\]

(5.18)

Solving for the figures of merit, similar to instantaneous TD/CCSK with transmitted reference,

\[
\rho_1(\gamma_s) = \sqrt{\frac{(\phi \phi_r \gamma_s)^2}{\left( \frac{4\chi - 2\chi - L}{L} \right)(\phi \phi_r \gamma_s)^2 + (1+3\chi)\phi \phi_r \gamma_s + \chi M}}
\]

(5.19)
If there is no noise reduction (i.e., \( \chi = 1 \)), the above result will be the same as the instantaneous TD/CCSK scheme result. Also, if we let \( \chi \) approach zero, then the above result will reduce to the ideal TD/CCSK results. Next, we derive \( \chi \) for clipping and time averaging. This result holds for any noise reduction scheme, and depends only on the factor \( \chi \). The dramatic improvement which can be achieved by noise reduction techniques is shown in figure 5.6 for various values of \( \chi \).

\[
f_2(\gamma) = \sqrt{1 + \frac{f_1(\gamma)^2}{L}}
\]

Figure 5.6 Bit Error Performance for TD/CCSK Rake System with Channel Estimate Noise Reduction
5.2.2. **Tail Clipping**

With the implementation of FDI, we can assume that the CCSK vector length is much greater than the number of multipath channels, (i.e., $QM > L$). Therefore, after the TD/CCSK demodulation of the data and reference signals, a majority of the time domain vector components contain only noise. Tail clipping reduces the noise on the channel estimation by zeroing out the last $M-L$ noisy time domain components that are known to contain no signal energy (thus the name tail clipping).

For now, let us assume that we have some form of coarse synchronization and let $\nu$ be the percentage of remaining taps after tail clipping. Under perfect synchronization, $\nu$ should equal $L/M$. However, to allow for some synchronization errors as well as some uncertainty in the channel, we will let $\nu$ range from $L/M$ (i.e., perfect tail clipping) to 1 (i.e., no tail clipping). If $\nu$ is preselected for some worst case scenario such that none of the signal energy is clipped, then tail clipping does not require any real-time feedback to be implemented.

For now, we will assume that the tail clipping operation is used on the instantaneous CCSK with transmitted reference scheme (i.e., no time averaging). If we assume that we are using FDI, then as discussed in the last section, the deinterleaving process causes time domain aliasing. Tail clipping can be thought of as an anti-aliasing filter before the deinterleaving process. From our assumption on selecting $\nu$, the zeroed time domain taps contain no signal energy. Therefore, the signal is unaffected by the tail clipping (see figure 5.4c). However, the channel IR estimation noise sequence, which had an original length of $M$, is shortened to a length of $\nu M$. This shortening of the noise vector and the use of FDI will effectively reduce the noise on the channel estimation by a factor of $\nu$. Therefore, the tail clipped noise reduction, $\chi_{TC}$, is given by

$$\chi_{TC} = \nu$$

(5.21)
From equations (5.19), (5.20), and (5.21) then the BER performance curves for tail clipping can be calculated.

In concept, the tail clipping operation is a time domain operation. Unfortunately, the implementation of the tail clipping operation would occur between two frequency domain operations (i.e., the TD/CCSK demodulation and TD/Rake correlation operations). At first glance, implementation of the tail clipping operations could be computationally intensive, since it would require an additional IFFT and FFT to the overall demodulation operations. For large M-ary signals, these additional transforms may prove unfeasible in a real-time system. It is thus desirable to find an efficient frequency domain implementation of tail clipping.

One such approach is to use filter operations in the opposite (i.e., frequency) domain. Since time and frequency domain operations have symmetry (e.g., a windowing function in one domain is a convolution in the other), the tail clipping operation can be visualized as filtering in the time domain from the frequency domain. In other words, by using classical IIR or FIR filter design techniques, but filtering the frequency domain vector, we can get the desired time domain bandpass response.

One important property required for this filtering approach is that no phase distortion occur in the passband (i.e., the portion of the time domain vector not tail clipped). This is required by the Rake correlator in order for the multipath signal components to coherently combine. There are simple methods of achieving this with FIR filters as found in [22], and some slightly more sophisticated methods for IIR filters (e.g., Bessel IIR filters).

The best approach and tradeoffs to the filter design and implementation are highly dependent on the channel model and beyond the scope of this paper. For this research, we
will implement a simple rectangular window FIR filter of order $\Omega$ to achieve a sinc-like\textsuperscript{8} shaped tail clipping passband. To remove the phase distortion, the filter output sequence is reversed and filtered again with the same filter. As shown in figure 5.7, the tail clipping operation using rectangular window FIR filters is performed by the following equation:

\[
Y(k) = \frac{1}{\Omega} \sum_{i=0}^{\Omega-1} X(k - i)_{M}
\]

(5.22)

\[
Z(k) = \frac{1}{\Omega} \sum_{i=0}^{\Omega-1} Y(k+1)_{M}
\]

(5.23)

\textbf{Figure 5.7 Block Diagram of FIR filter TD/Tail Clipping Operation}

\textbf{Figure 5.8 Window Function of Rectangular FIR Filter TD/Tail Clipping}

\textsuperscript{8}Sinc-like meaning it has the form $\sin(x)/\sin(Ax)$ rather than $\sin(x)/x$
This results is a Sinc^2-like shaped tail clipped passband with no phase distortion (see figure 5.8) where

\[ \hat{h}_e(n) = w(n) \times (n) \]  

(5.24)

and where \( w \) is the filter shape in the time domain given by

\[ w(n) = \frac{1}{Q} \left| \frac{\sin\left(\frac{\pi}{Q} \Omega \frac{n}{2}\right)}{\Omega \sin\left(\frac{\pi}{Q} \frac{n}{2}\right)} \right|^2 \]  

(5.25)

This tail clipping operation is also computationally efficient since it requires no complex multiplications\(^9\). Its speed could be improved even further by implementing it in a recursive "pipeline" manner. For example,

\[ Y(0) = \frac{1}{Q} \sum_{l=0}^{\Omega-1} X(-1)_M \]  

\[ Y(k) = Y(k-1) + \frac{1}{Q} (X(k)_M - X(k-J+1)_M) \quad k=1,2,...,M-1 \]  

(5.26)

One modification required for this algorithm to achieve its maximum efficiency is that the assumed energy in the channel IR estimation be centered around \( n=0 \). This is because this filter is a low-pass filter and therefore the passband is centered at the origin. This can easily be accomplished by circularly shifted backward \( \Omega/2 \) (i.e., half the width of the main lobe) the received vector, \( \mathbf{x} \), immediately before transforming the vector to the frequency domain (i.e., before the FFT) (see figure 5.9). This will have no effect on the demodulated signal since both the data and reference symbols are shifted by the same amount.

\(^9\) Note that the \( 1/\Omega \) term is added only to normalize the equation for analysis purposes and is not actually required for implementation.
An example of before and after the rectangular FIR filter tail clipping operation is illustrated in figure 5.10. Note that the signals in figure 5.10 represents the channel estimate in the time domain for illustrative purposes only, and in actuality the receiver is processing these signals in the frequency domain. In this example there are two channel modes which have been centered around the origin by the vector rotate operation. In this example the components of these modes all have a magnitude of one. Using the tail clipping operation with $\Omega=8$ (see figure 5.8), the majority of the noncontributing taps are zeroed out. However, there is some magnitude distortion on the actual channel impulse response since the rectangular FIR filter operation is not perfect. In order to calculate the effective noise reduction of rectangular FIR filter tail clipping, we must account for both the noise reduction and signal distortion.
The effective noise reduction of rectangular FIR filtering tail clipping, $\chi_R$, is the ratio of the channel estimation SNR before tail clipping over SNR after tail clipping, and therefore

$$\chi_R = \frac{S_i/N_i}{S_{ic}/N_{ic}}$$  \hspace{1cm} (5.27)

The main lobe of the tail clipping window is of width $2/\Omega$. In order to keep the all active taps in the main lobe, $\Omega$ should be no greater than $2QM/L$. Therefore, the selection of $\Omega$ should be such that $1 < \Omega < 2QM/L$. The first step is to compute the amount of noise reduction of the tail clipping. Let $\sigma_e^2$ be the variance of the AWGN on the channel IR estimation. Then the average noise power on the estimate $N_i$ is
Computing the average noise power on the estimate after tail clipping, $E_{z(n)}$, we get

$$ N_i = \frac{1}{Q^M} \sum_{n=0}^{Q^M-1} E \left[ |z(n)|^2 \right] = \frac{1}{Q^M} \sum_{n=0}^{Q^M-1} 2\sigma_{z(n)}^2 $$

(5.28)

\[ N_{tc} = \frac{1}{Q^M} \sum_{n=0}^{Q^M-1} E \left[ |w(n)|^2 \right] \]
\[ = \frac{1}{Q^M} \sum_{n=0}^{Q^M-1} |w(n)|^2 E \left[ |z(n)|^2 \right] \]
\[ = \frac{N_i}{Q^M} \sum_{n=0}^{Q^M-1} |w(n)|^2 \]  

(5.29)

By Parseval’s Theorem,

$$ \sum_{n=0}^{Q^M-1} |w(n)|^2 = \frac{1}{Q^M} \sum_{k=0}^{Q^M-1} |W(k)|^2 $$

(5.30)

where $W(k)$ is the discrete Fourier transform pair of $w(n)$ and $W(k)$ is described by

$$ W(k) = \begin{cases} \frac{1}{\Omega^2} & k, \lfloor k/\Omega \rfloor \leq k \leq \Omega \\ 0 & \text{else} \end{cases} $$

(5.31)

which is a triangle centered at the origin. Substituting equation (5.31) into equation (5.29) and simplifying we get

$$ \frac{N_{tc}}{N_i} = \frac{1}{Q^M} \sum_{k=0}^{M-1} |W(k)|^2 = \frac{1}{Q^M \Omega^4} \sum_{k=-\Omega}^{\Omega} k^2 $$

(5.32)

The finite summation in equation (5.32) can be bounded by a parabolic curve. Therefore, we can get a closed form solution for the bound on the noise gain of
To compute processing loss due to the distortion of the channel IR by tail clipping, we must first derive the signal power of the estimation, which from chapter 5, was

\[ S_i = \frac{1}{QM} \sum_{n=-L/2}^{L/2} 2\sigma_h^2 E_{se} \]

where \( E_{se} \) is the symbol energy in the estimation. Next, derive the signal power of the estimation after the distortion from tail clipped estimation which can be found by,

\[ S_{tc} = S_i \left( \sum_{n=-L/2}^{L/2} w(n) \right)^2 \]  

(5.35)

The main lobe of the tail clip window, \( w(n) \), can be approximated by an equilateral triangle of width \( 2/\Omega \) and height of \( 1/QM \) and \( L < 2QM/\Omega \), and therefore,

\[ S_{tc} = \left( \frac{1}{QM} \sum_{n=-L/2}^{L/2} 1 - \frac{\Omega}{QM} n \right)^2 \]

(5.36)

Substituting equation (5.33) and (5.39) into (5.27), we get

\[ \chi_R > \frac{2(\Omega+0.5)^3}{3\Omega^4 \left( 1 - \frac{L\Omega}{4QM} \right)^2} \]  

(5.37)

Figure 5.11 illustrates the performance of rectangular filtering for the case of \( \Omega = 8 \) and 32. The performance bound of perfect tail clipping (i.e., \( \chi = L/QM \)) is also added for
reference. It can be seen that rectangular filter TD/tail chippers, though not perfect, achieve most of the performance of a perfect tail clipper. Near perfect tail clipping can be achieved at higher values of $\Omega$ (i.e., $\Omega=32$), but only over a short range of $L/QM$. The higher the value for $\Omega$, the more sensitive the tail clipper becomes to the number of active channel taps. This is evident by the difference in slope of $\chi$ for $\Omega$ equal to 8 and 32.

![Figure 5.11 Noise Reduction Performance of Rectangular FIR Filter Tail Clipping](image)

5.2.3. Center Clipping

If sufficient processing power is available, the TD/tail chipping can be replaced by a perfect clipping process referred to as center clipping. In this process, a QM-point IFFT is performed on the TD Channel Estimate. All non-contributing taps can then be zeroed out (i.e., center chipped). This is then followed by a QM-point FFT to return to the frequency
domain. The problem here is that the transforms alone would essentially double the required processing at the receiver. However, if we TD/tail chip first, we have effectively lowpass filtered the frequency domain vector. This vector can then be thought of as being oversampled. Therefore we can decimate before transforming back to the time domain and interpolate after returning to the frequency domain. Figure 5.12 illustrates the block diagram for this entire clipping operation using the decimation/interpolation technique. Figure 5.13 through 5.15 shows an example channel estimate being processed by the clipping operation.

If the rectangular filter described in section 5.2.1 is used with $\Omega = 16$ for this example, then the time domain tail clipping passband has a width $2QM/\Omega$ (see figure 5.13). Therefore, we can decimate by a factor of $\Omega/2$ in the frequency domain and thus only require a $2QM/\Omega$-point (i.e., 1024-point) IFFT to get to the time domain (see figure 5.14 signal:a). The decimation in the frequency domain causes aliasing in the time domain of the tail clipped channel estimate. However, since the tail clipping significantly reduces the stopband components, only a small amount of aliasing occurs.

Once in the time domain, we have a $2QM/\Omega$ vector of the passband to center chip. Note the passband still has the sinc$^2$-like envelope from the tail clipping. Since the tail clipping was a component-wise vector multiply (i.e., a window operation) in the time domain, we can remove this distortion by multiplying by the inverse of the sinc$^2$-like envelope (see figure 5.14 signal:b). If we still assume that the L multipaths are centered around the origin in contiguous taps, then the tail clipped channel estimate should only have these taps remaining (i.e., not zeroed out). Let $L_2$ be the next power of 2 greater than L. Then after the center clipping operation (see figure 5.14 signal:c), the clipped vector is returned to the frequency domain with a $L_2$-point FFT (in this example $L_2 = 1024$). The QM point channel estimate can be reconstructed by a simple interpolation algorithm with a $QM/L_2$ sampling.
rate expander and lowpass filter [22]. We can also pre-compensate for any distortion of the interpolation lowpass filter (see figure 5.14 signal:d) before returning to the frequency domain using the same method as in the tail clipper compensation (i.e., the inverse of the interpolator filter window in the pass-band). If an FIR filter, with an order several times the sampling expanded rate, is used (in this example a 31st-order Chebyshev FIR filter), then the processing loss of the interpolator will be negligible (see figure 5.15).

Figure 5.12 Block Diagram of Tail/Center Clipping Operation
Figure 5.13 Example of Channel Estimate Before and After Tail Clipping Operation ($\Omega=16$)
Figure 5.14 Example of Channel Estimation Processing During Center Clipping Operation.

Figure 5.15 Example of Channel Estimate After Center Clipping Operation.
Let $\chi_{CC}$ be the effective noise reduction for the combined operation of tail and center clipping. Then the aliasing from the decimation and imperfect tail chipping is the only remaining major factor influencing the effective noise reduction $\chi_{CC}$. The aliasing adds to the level of AWGN in the passband components which is then scaled up after the tail clipping window compensation operation. Let $w_c$ be the tail clipping compensation vector, which is the first and last $QM/\Omega$ subvectors of $w$ concatenated. We can express $w_c$ as

$$w_c(n) = \begin{cases} w(n) & 0 < n < \frac{QM}{\Omega} - 1 \\ w\left(\frac{n-QM(\Omega-2)}{\Omega}\right) & \frac{QM}{\Omega} < n < \frac{2QM}{\Omega} - 1 \end{cases}$$  \hspace{1cm} (5.38)$$

Let $w_a$ be the aliasing caused by the decimation. The vector $w_a$ can be derived from calculating and IFFT on $W$ decimated by a factor of $\Omega/2$, which gives us

$$w_a(n) = \left(\frac{\sin(2\pi n)}{2\sin(\pi n)}\right)^2$$  \hspace{1cm} (5.39)$$

To compute the center clipping effective noise reduction, $\chi_{CC}$, we must take the sum of the ratio $w_a/w_c$ for the $L$ points center at the origin, then

$$\chi_{CC} = \frac{1}{L/2} \sum_{n=-L/2}^{L/2} \frac{w_a(n)}{w_c(n)}$$  \hspace{1cm} (5.40)$$

Figure 5.16 shows the effective noise reduction of the tail clipping only (i.e., TC only) and the tail and center clipped (i.e, TC & CC) noise reduction for $\Omega = 16$.

Note that the noise aliasing increases in the taps as we move away from the origin. If we select a smaller value for $\Omega$ such that most of the active signal taps are near the center of the tail clipped region (i.e., $\Omega \ll QM/L$), then the aliasing noise becomes negligible since it would be clipped. For example, if we had selected a value of 8 instead of 16 for $\Omega$ then the tail clipping window's main lobe would double in width and thus there would be less active signal taps near the edges (compare figure 5.10 and 5.13). However for $\Omega=8$, we
can only decimate by a factor of 4 (instead of 8 for $\Omega=16$). This in turn makes the center clipping operation more computationally intensive since a longer IFFT is required. A tradeoff analysis between the amount of tail clip and the processing load of the center clipping must be performed at a system level.

![Graph](image-url)

**Figure 5.16 Noise Increase on Channel Estimation for Tail/Center Clipping**

### 5.2.4. Time Averaging

With a slowly fading channel, past data symbols and reference signals can be used to reduce the noise on the decision variables by time averaging the channel IR estimation, $\hat{H}$. This can be performed by either low-pass filtering the tap weights or by using some parameter estimation techniques (e.g., Kalman filtering). By filtering some of the noise on the channel estimate, we should be able to achieve BER performance which is bounded by
the ideal and instantaneous performance curves. The actual BER performance will depend somewhat on the average number of symbol errors (if decision feedback is used) and on whether a reference signal is used. However, the BER performance will depend significantly on the fading rate of the channel.

The fading rate will determine how to weight past channel estimates. The greater the fading rate, the less weight or information a past channel estimate should contribute toward the current channel impulse response estimate. Even if an optimum solution can be found for a specific fading rate, the fading rate can vary since the HF channel conditions are nonstationary in general, and change over periods of minutes, hours and days. The analysis required for varying fading rates and the optimum Time Averaging (TA) operation is beyond the scope of this paper. However, we will obtain preliminary first order results from simulation of the system. For this we will use a simple recursive time averaging algorithm.

A pictorial example of the estimator for one vector component can be seen in figure 5.17. Depending on the fading rate, the weighting constant, $\alpha$, which ranges from 0 to 1, can theoretically be set for the optimum solution. For example, the slower the fading rate, the lower $\alpha$ is set. For a fixed fading rate, this approach is equivalent to a steady-state vector Kalman filter. For this research, we will assume $\alpha$ is selected high enough that the channel fading does not effect the channel estimation. In other words, by the time a past channel measurement is out of date (i.e., contains no information), its contribution to the channel estimate is insignificant. Thus, we can bound the problem.
The TA noise reduction on the estimate will have the same theoretical effect as tail clipping. Therefore we will assume that the TA noise reduction on the estimation will increase by a factor of $\chi_{TA}$. If we assume for now that there is no fading, we can calculate the $\chi_{TA}$ from $\alpha$. Let $\mathbf{H}_o$ and $\mathbf{G}$ be the current time averaged channel estimation and instantaneous channel estimate respectively. Therefore

$$\mathbf{H}_o = (1- \alpha)\mathbf{H}_{-1} + \alpha \mathbf{G}_o$$  \hspace{1cm} (5.41)

rewriting this equation in summation form

$$\mathbf{H}_o = \sum_{i=-\infty}^{0} \alpha(1- \alpha)^{i} \mathbf{G}_i$$  \hspace{1cm} (5.42)

Let the current received channel estimate, $\mathbf{G}$ be comprised of a constant process (i.e., the tap weights), $\mathbf{S}$ and white random process (i.e., the noise), $\mathbf{N}$, such that $\mathbf{G} = \mathbf{S} + \mathbf{N}$, then at steady-state

$$\mathbf{S}_{ta} = \sum_{i=-\infty}^{0} \alpha(1- \alpha)^{i} \mathbf{S}$$

$$= \mathbf{S}$$  \hspace{1cm} (5.43)
Therefore, the time averaging will not affect the constant process. Let the $\sigma_N^2$ be the variance of the current time average $N_i$ and $\sigma_{N_{ta}}^2$ be the variance of the noise after time averaging, $N_{tc}$. We will assume that the noise of each time sample is independent and of the same variance. Since each noise time sample is a random variable, multiplying by a scalar changes the variance by the square of the scalar. Then summing the samples, we get

$$\sigma_{N_{ta}}^2 = \sigma_N^2 \sum_{i=-\infty}^{0} (\alpha(1-\alpha)^i)^2$$

$$= \sigma_N^2 \frac{\alpha}{2-\alpha}$$

Therefore for the noise reduction is given by

$$\frac{N_{ta}}{N_i} = \frac{\alpha}{2-\alpha}$$

(5.45)

To determine the amount of noise reduction we get from time averaging we use

$$\chi_{TA} = \frac{S_i/N_i}{S_{ta}/N_{ta}}$$

(5.46)

Substituting equation (5.43) and (5.45) into (5.46) we get

$$\chi_{TA} = \frac{\alpha}{2-\alpha}$$

(5.47)

Figure 5.18 illustrates the effective noise reduction of tail clipping over a range of $\alpha$. From this curve, it is seen that choosing small values of $\alpha$ can significantly reduce the noise. However, extreme caution must be used in the system design, since the channel is nonstationary. A small value of $\alpha$ in combination with a rapidly varying channel could result in unstable behavior, since the estimator could not keep up with channel variations.
5.2.5 Combination of Noise Reduction Techniques

Individually, the clipping and time averaging noise reduction techniques all can be effective in improving the channel estimate. Applying a combination of these techniques, however, can significantly improve system performance, since they reduce the estimation noise in different ways. If we combine rectangular FIR filter tail chipping and time averaging of the channel estimation, we get an overall effective noise reduction of

\[ \chi = \chi_r \chi_{\text{TA}} \]  

(5.48)

For example, if we use tail clipping with \( \Omega = 8 \) and assume the channel estimate model in figure 5.10, then figure 5.19 illustrates the additional performance gain time averaging provides to tail clipping for different values of \( \alpha \). In this example, integrating over a few symbols (i.e., \( \alpha = 0.5 \)) provides a significant improvement over just tail clipping alone.
(i.e., $\alpha = 1$). At $\alpha$ equal to 0.25, we approximately double the processing gain of tail clipping alone.

If we now include the center clipping operation and again assume the channel model in figure 5.10, then we can center clip such that 500 out of 8192 taps remain (i.e., not zero out). From figure 5.16 this results in a $\chi_{CC}$ of 0.061. Figure 5.20 shows the performance expected for different values of $\alpha$ and where

$$ \chi = \chi_{CC} \chi_{TA} $$ (5.49)

In this case, when center clipping is used in conjunction with tail clipping, time averaging does not provide a significant of a performance improvement. This is due primarily to the fact that center clipping removes virtually all the noise from the channel estimate. As a practical matter, however, the time averaging operation would be implement prior to the center clipping operation. The noise on the channel estimate achieved by time averaging will cause the HF channel modes to "pop-out" of the noise and thus greatly enhance the accuracy of the center clipping algorithm. Thus, the three algorithms in conjunction provide an effective balance of near-optimum noise reduction, robustness, and efficient processing.

It should as be noted that when center clipping is used the computation load can be reduce by taking advantage of the decimation operation. First, the tail clipping operation need only generate the decimated vector points (i.e., $Z(k\Omega/2)$ in equation (5.23)) instead calculating the entire tail clipped vector in the frequency domain and then decimating. Second, if time averaging is performed before center clipping, it too can operate at the decimated vector length rather then the vector length of QM. These enhancements will not effect the performance, but will significantly reduce the amount of calculation required to perform these operations.
Figure 5.19 Bit Error Performance of Tail Clipping and Time Averaging Channel Estimate Noise Reduction for Various Values of Alpha
Figure 5.20 Bit Error Performance of Tail Clipping, Center Clipping and Time Averaging Channel Estimate Noise Reduction for Various Values of Alpha
Chapter 6

TD/CCSK IMPLEMENTATION FACTORS

6.1. CCSK Waveforms

In this section we examine several different possible CCSK waveforms and their properties. These include the Binary Maximum Length Sequences, Chirps, Filtered Impulse Trains, as well as a combination of these waveforms.

6.1.1. Binary Maximum Length Sequences

Binary Maximum Length Sequences (BMLS) are a class of cyclic code used in Direct Sequence Spread Spectrum (DSSS) systems [12] [17] because of their good circular autocorrelation properties. These sequences are easily generated using a shift register with feedback loops corresponding to the generating polynomial (see figure 6.1). In this example it can be seen from the table that (except for the first sequence) all are circular shifts of each other.
Let the binary base vector, $h_0$, be a BMLS used for the CCSK modulation, and the reference vector, $h_r$, be a BMLS generated in the same manner, but from a different generating polynomial (i.e., base sequence). For example in this section $h_0$ was constructed with a generating polynomial of $g(x) = x^{10} + x^3$, and $h_r$ was constructed with a generating polynomial of $g(x) = x^{10} + x^5 + x^3 + 1$, and therefore both sequences are of length $2^{10} - 1$ (i.e., 1023). The actual transmitted signal is the CCSK modulated data vector, $h_m$, on the in-phase component and $h_r$ on the quadrature component of the signal. Thus the
transmitted signal characteristics are similar to Quadrature Phase Shift Keying (QPSK). The normalized circular auto-correlation of $h_o$ can be seen in figure 6.2, and is typical for a BMLS. In this example, the circular cross-correlation properties between $h_o$ and $h_r$ are fairly good. (see figure 6.4).

BMLS are well suited for DSSS system and have been used in binary Rake systems, but there are some implementation problems in using BMLs in a CCSK system. One problem is that BMLs are of length $2^{b-1}$ where $b$ is the number of stages in the shift register. This means that BMLs need to be augmented by one component to make the sequence length a power of 2 for the FFT operation at the receiver. This destroys the perfect circular auto-correlation property of the BMLs (see figure 6.3). Another potential problem is that small time synchronization errors can degrade performance and thus synchronization is a major consideration in DSSS systems [12]. This is primarily due to the fact that the transmitted baseband signals are sequences of rectangular pulses and therefore not strictly bandlimited. Timing feedback could be used to correct this problem. However the spacing of the multipaths is not guaranteed to be at chip intervals, $T_c$, and therefore, performance loss will occur if the multipath structure is not perfectly aligned.

\[10\] In the next section, we will discuss strictly bandlimited signal properties dealing with fine-time synchronization errors.
Figure 6.2 Circular Auto-Correlation of a BMLS of Length 1023.

Figure 6.3 Circular Auto-correlation of an Augmented BMLS of Length 1024.

Figure 6.4 Circular Cross-Correlation of Two BMLS of Length 1024.
6.1.2. Chirps

Linear Frequency Modulated (FM) signals, also called chirps, are another type of waveform used for spread spectrum systems, mainly in radar applications [15][23]. Chirps also have excellent circular auto-correlation properties. At baseband, a single-rate chirp sweeps linearly from lowest to highest frequency exactly over one symbol period (see figure 6.5). A triple-rate chirp sweeps through the signal band three times in a symbol period (see figure 6.6). Since these chirp signals are a form of frequency modulation, they are well suited for use with non-linear amplifiers (i.e., Class C amplifiers) which are efficient from a power consumption standpoint. This makes them attractive for use in radio communication systems.

Let \( \{c_a\} \) be the set of sampled linear odd-rate chirps of length \( M \). The vectors can be described by the equation

\[
\begin{align*}
    c_a(n) &= e^{-j\pi a n^2 / M} \\
    \text{where } a &= 1, 3, 5, \ldots, M-1
\end{align*}
\]

(6.1)

The circular auto-correlation for any vector in this set is perfect (see figure 6.7).

\[
R_{c_0}(\tau) = \begin{cases} 
1, & \tau = 0 \\
0, & \tau = 1, 2, \ldots, M-1
\end{cases}
\]

(6.2)

Let the reference vector be chosen as a different vector from the same set. For best results an adjacent odd-rate chirp from the set is selected. For example, \( c_0 = c_1 \), \( c_r = c_3 \) (see figure 6.8). This results in a near perfect cross-correlation of

\[
|R_{c_0 c_3}(\tau)| = \begin{cases} 
\frac{1}{\sqrt{2M}}, & \text{odd } \tau \\
0, & \text{even } \tau
\end{cases}
\]

(6.3)

The chirp provides the optimum solution for the circular auto- and cross-correlation properties of the CCSK waveform. In addition, chirps also are strictly bandlimited signals which will be shown in the next section to be important for time synchronization.
error correction. However, they are a limited set of vectors, which could cause potential problems for large multiuser broadcast networks, in which many different orthogonal CCSK codes are required.

Figure 6.5 Single Rate Sampled Chirps

Figure 6.6 Triple Rate Sampled Chirp
6.1.3. Filtered Impulse Trains

A more general waveform for use in the TD/CCSK system is a Filtered Impulse Train (FIT) which is based on the Nyquist sampling theorem \[22\]. If a discrete CCSK vector is considered to be a strictly bandlimited signal sampled at the Nyquist sampling rate, then the original continuous time bandlimited signal can be constructed from the samples and sampling rate. This process is referred to by most texts as the reconstruction algorithm in which the sampled signal is modulated on an impulse train at the sampling rate, \(T_s\), and filtered by a lowpass filter with a bandwidth of \(2\pi/T_s\). For perfect
reconstruction, the lowpass filter is a rectangular window in the frequency domain which corresponds to a Sinc shaped filter impulse response. Let \( s_m(t) \) be the impulse train signal of \( s_m(n) \) such that

\[
s_m(t) = \sum_{n=0}^{N-1} s_m(n) \delta(t-nT_s)
\]  

(6.4)

Let \( u_m(t) \) be a FIT-CCSK symbol generated by strictly bandlimiting \( s_m(t) \) to the interval of \((-\pi/T_s, \pi/T_s)\) with an ideal filter. Then \( u_m(t) \in \beta(-\pi/T_s, \pi/T_s) \), where \( \beta(-\pi/T_s, \pi/T_s) \) is the set of all signals bandlimited to the interval of \((-\pi/T_s, \pi/T_s)\). In the time domain

\[
u_m(t) = \sum_{n=0}^{N-1} s_m(n) \text{Sinc}\left(\frac{\pi}{T_s}(t-nT_s)\right)
\]  

(6.5)

This process is similar to the interpolation process discussed in chapter 5.

The chirp waveforms can be constructed in this manner since they are strictly bandlimited signals \( \text{c}_b(t) \in \beta(-\pi/T_s, \pi/T_s) \) and are therefore an optimum subset of the FIT waveforms. However, the FIT class of waveforms is a much broader class, and their performance (i.e., auto-correlation properties) is dependent solely on the selection of the discrete CCSK waveform. The only problem foreseen with using FIT waveforms is that in most instances a linear amplifier is required. However, there is no processing loss from fine time synchronization, as with the BMLS waveform.

As an example, a combination of the BMLS and chirp waveforms could be used as suggested in [24]. In this case, we can modulate the BMLS sequence onto a chirp which will give us good spectral containment required for FIT[24]. This will also provide us with a large base of CCSK waveforms required for large multiuser broadcast networks, and will still maintain good auto and cross-correlation properties. Figure 6.9 illustrates the auto-correlation of a BMLS sequence overlaid on a single rate chirp. Figure 6.10 illustrates the same signal cross correlated with a different BMLS sequence overlaid on a
single rate chirp. These figure demonstrate the good CCSK properties of these example waveforms.

6.2. Time Synchronization Error

As stated earlier, fine timing synchronization errors in a Rake system cannot always be compensated for in a multipath environment. This is due to the fact that multipath signals do not necessarily have delays on intervals equal to the chip or sample rate. In the
following analysis, we will examine the effects of fine timing synchronization errors in a Rake system which uses a FIT-CCSK waveform.

For simplicity, we will only examine a single path with complex-valued attenuation of $A$ and a timing offset $t_0$. This can be easily be expanded to include a multipath channel by including a summation of paths. For now, however we are interested primarily in the effect of time synchronization error on any particular path. We are also interested in the self noise induced on the matched filter outputs by fine-time synchronization error, and will initially assume that the channel is also noiseless. At the receiver, the received signal is matched filtered with all possible symbols. let $r(n)$ be the output to the $n$th matched filter. Therefore

$$r(n) = A \int_{-\infty}^{\infty} u_m(t-t_0)u_n^*(t) \, dt$$

(6.6)

Expanding equation (6.6) we get

$$r(n) = A \sum_{p=0}^{M-1} \sum_{q=0}^{M-1} s_m(p)s_n^*(q) \int_{-\infty}^{\infty} \text{Sinc} \left( \frac{\pi}{T_s} (t-pT_s-t_0) \right) \text{Sinc} \left( \frac{\pi}{T_s} (t-qT_s) \right) \, dt$$

(6.7)

Nyquist sampling theorem [22], states that if $f(t), g(t) \in \beta(-\pi/T_s,\pi/T_s)$, then

$$\int_{-\infty}^{\infty} f(t) g^*(t) \, dt = \sum_{l=-\infty}^{\infty} f(lT_s) \, g^*(lT_s)$$

(6.8)

Since $u_m(t), u_n(t) \in \beta(-\pi/T_s,\pi/T_s)$, then by applying Nyquist Sampling Theorems equation (6.7) becomes

$$r(n) = A \sum_{p=0}^{M-1} \sum_{q=0}^{M-1} s_m(p)s_n^*(q) \sum_{l=-\infty}^{\infty} \text{Sinc} \left( \frac{\pi}{T_s} ((l-p)T_s-t_0) \right) \text{Sinc} \left( \frac{\pi}{T_s} (l-q)T_s \right)$$

(6.9)

and rearranging and reducing equation (6.9), we get
\[ r(n) = A \sum_{p=0}^{M-1} \sum_{l=-\infty}^{\infty} s_m(p) \text{Sinc}\left(\frac{\pi}{T_s} ((l-p)T_s-t_0)\right) \sum_{q=0}^{M-1} s^*_n(q) \text{Sinc}\left(\frac{\pi}{T_s} (l-q)T_s\right) \]  
\hspace{1cm} (6.10)

But, it can be shown that

\[ \text{Sinc}\left(\frac{\pi}{T_s} (n)T_s\right) = \begin{cases} 1 & n=0 \\ 0 & n\neq 0 \end{cases} \]  
\hspace{1cm} (6.11)

Substituting equation (6.11) into (6.10), we get

\[ r(n) = A \sum_{p=0}^{M-1} \sum_{l=-\infty}^{\infty} s_m(p)s^*_n(l) \text{Sinc}\left(\frac{\pi}{T_s} ((l-p)T_s-t_0)\right) \]  
\hspace{1cm} (6.12)

Now, expanding \( s_m(p) \) and \( s_n(l) \) further

\[ r(n) = A \sum_{l=-\infty}^{\infty} \sum_{p=0}^{M-1} s(p+m)M^* s^*(l+n)M \text{Sinc}\left(\frac{\pi}{T_s} ((l-p)T_s-t_0)\right) \]  
\hspace{1cm} (6.13)

Letting \( q = l-p \) and substituting into equation (6.13), then

\[ r(n) = A \sum_{q=-\infty}^{\infty} \sum_{p=0}^{M-1} s(p+m)M^* s^*(p+q+n)M \text{Sinc}\left(\frac{\pi}{T_s} (qT_s-t_0)\right) \]  
\hspace{1cm} (6.14)

Let \( \phi_s(n) \) be the circular auto-correlation of \( s_0(n) \) (i.e., the CCSK data base vector).

\[ r(n) = A \sum_{q=-\infty}^{\infty} \phi_s(q+n-m) \text{Sinc}\left(\frac{\pi}{T_s} (qT_s-t_0)\right) \]  
\hspace{1cm} (6.15)

From chapter 3, if \( s_0(n) \) is an ideal CCSK waveform, then \( \phi_s(n) \) is a Dirac impulse (i.e., \( \phi_s(n) = \delta(n) \)). From equation (6.15) if \( s_0(n) \) is an ideal CCSK waveform, then the received despread signal in this case is a sampled Sinc function. Further, it can be seen that for \( t_0 = 0 \) (i.e., no timing error) that

\[ r(n) = A\phi_s(n-m) \bigg|_{t_0 = 0} \]  
\hspace{1cm} (6.16)
The Rake correlation operation for the CCSK waveform as described in chapter 3 is a circular cross-correlation between the \( r(n) \) and channel reference signal. Let \( h_e(n) \) be the noiseless channel estimate but with the same timing error, since we will assume that the channel estimate is generated from decision feedback on past symbols or from a known transmitted reference signal. In either case, \( r(n) \) is circularly correlated with reference vector \( h_e(n) \). Therefore, \( h_e(n) \) has the same channel attenuation and fine-time synchronization error as \( r(n) \), and can be expressed as

\[
h_e(n) = A \sum_{q=-\infty}^{\infty} \phi_d(q+n) \text{Sinc}\left(\frac{\pi}{T_s} (qT_s-t_0)\right)
\]

(6.17)

Note that, as would be expected, the channel estimate is identical to the zeroth CCSK symbol demodulated (i.e., equation (6.15) with \( m=0 \)).

Let \( c(n) \) be the output decision variable vector from the Rake correlator, where

\[
c(n) = \sum_{l=0}^{M-1} r(l)_M h_e^*(n+l)_M
\]

(6.18)

It will be easier mathematically to solve this equation by taking discrete Fourier transform, \( R(k) \) and \( H_e(k) \), computing \( C(k) \) and taking an inverse DFT to reduce \( c(n) \).

First we compute \( R(k) \), which is by definition

\[
R(k) = \sum_{n=0}^{M-1} r(n) e^{-j\frac{2\pi}{M} kn}, \quad \frac{M}{2} < k \leq \frac{M}{2}
\]

(6.19)

Substituting equation (6.15) into (6.19), we get

\[
R(k) = \sum_{n=0}^{M-1} A \sum_{q=-\infty}^{\infty} \phi_s(q+m-n) \text{Sinc}\left(\frac{\pi}{T_s} (qT_s-t_0)\right) e^{-j\frac{2\pi}{M} kn}
\]

(6.20)

Next we rearrange the summations and collect terms for \( n \).
\[
R(k) = A \sum_{q=-\infty}^{\infty} \text{Sinc} \left( \frac{\pi}{T_s} (qT_s - t_0) \right) \sum_{n=0}^{M-1} \phi_s(q+n-m) e^{-j\frac{2\pi}{M}kn}
\]  

(6.21)

Let \( \Phi_s(k) \) be the discrete Fourier pair of \( \phi_s(n) \) and from the time shifting property equation (6.21) becomes

\[
R(k) = A \Phi_s(k) e^{j\frac{2\pi}{M}km} \sum_{q=-\infty}^{\infty} \text{Sinc} \left( \frac{\pi}{T_s} (qT_s - t_0) \right) e^{-j\frac{2\pi}{M}kq}
\]  

(6.22)

The function, \( \text{Sinc} \left( \frac{\pi}{T_s} (qT_s - t_0) \right) \) ∈ \( \beta(-\pi/T_s, \pi/T_s) \) and since \(-M/2 < k < M/2\) then \( e^{-j\frac{2\pi}{M}kq} \) ∈ \( \beta(-\pi/T_s, \pi/T_s) \).

By applying Nyquist sampling theorem to equation (6.22), we get

\[
R(k) = A \Phi_s(k) e^{j\frac{2\pi}{M}km} \int_{-\infty}^{\infty} \text{Sinc} \left( \frac{\pi}{T_s} (t-t_0/T_s) \right) e^{-j\frac{2\pi}{M}kt} \, dt
\]  

(6.23)

Let \( F(\omega) \) be the Fourier transform of a Sinc function such that,

\[
F(\omega) = \int_{-\infty}^{\infty} \text{Sinc} \left( \frac{\pi}{T_s} (t-t_0/T_s) \right) e^{-j\omega t} \, dt
\]  

(6.24)

Then using equation (6.24) we can rewrite equation (6.23) as

\[
R(k) = A \Phi_s(k) e^{j\frac{2\pi}{M}km} F \left[ \frac{2\pi k}{M T_s} \right]
\]  

(6.25)

Calculating the Fourier transform of equation (6.24), we get

\[
F(\omega) = \begin{cases} e^{j\omega t_0} & -\frac{\pi}{T_s} < \omega < \frac{\pi}{T_s} \\ 0 & \text{else} \end{cases}
\]  

(6.26)

Now, letting \( \omega = 2\pi k/T_s \), equation (6.25) becomes
\[
F\left(\frac{2\pi k}{MT_s}\right) = \begin{cases} 
\frac{e^{j2\pi k t_o}}{MT_s} & \frac{M}{2} < k < \frac{M}{2} \\
0 & \text{else} 
\end{cases}
\]  

But \(-M/2 < k < M/2\), and therefore substituting equation (6.27) into equation (6.25)

\[
R(k) = A\Phi_s(k)e^{j\frac{2\pi k t_o}{MT_s}} \left(1 - \frac{k}{M}\right) 
\]  

Similarly, it can be shown for \(H_e(k)\),

\[
H_e(k) = A\Phi_d(k)e^{j\frac{2\pi k t_o}{MT_s}} 
\]  

We can now compute \(C(k)\) for \(R(k)\) and \(H_e(k)\) by the expression

\[
C(k) = R(k)H_e^*(k) 
\]  

Substituting equation (6.28) and (6.29) into equation (6.30) and solving, we get

\[
C(k) = |A|^2\Phi_s(k)\Phi_d^*(k)e^{j\frac{2\pi km}{MT_s}} 
\]  

Let \(s_o(n)\) be selected for optimum performance, such that, \(\phi_s(n) = E_s\delta(n)\). Substituting this into equation (6.31), we get

\[
c(n) = |AE_s|^2\delta(n-m) 
\]  

From equation (6.32), it can be seen that output of the Rake operation is no longer a function of \(t_o\). Therefore, FIT-CCSK Rake receivers are not effected by fine-time synchronization errors. This solution can be easily be expanded to the multipath case, by representing the Rake operation as a combining or summation of \(L\) paths where each multipath is matched (in time) with the channel impulse response estimation.

In the above derivation, it is assumed that an ideal lowpass filter was used to generate the FIT-CCSK waveform. This of course, is unrealizable. However, if the ideal filter is approximated by a good high-order lowpass IIR or FIR filter such that \(u_m(t)\) is very close to being an element of \(\mathbb{B} (-\pi/Ts, \pi/Ts)\), then the effect should be negligible and the results should hold.
As an example, let the lowpass filter be a 7-th order elliptical filter. Let \( s_0(t) \) be single rate chirp and \( M=128 \). For this example, we select the worst case fine time synchronization error which is when \( t_0 = T_c/2 \). The data symbol, \( m \), in this example is equal to 10 and the received symbol energy is equal to 1. Figure 6.11 illustrates the noiseless CCSK demodulated received symbol (i.e., \( r(n) \)). In this figure, it can be seen that without any compensation for time synchronization that the signal energy is spread over several decision variables. Figure 6.12 illustrates the noiseless channel impulse response estimate (i.e., \( h(n) \)). Note that \( h(n) \) has the same basic structure as \( r(n) \) but has a majority of the signal energy at the origin. In figure 6.13, the output of the TD/Rake operation (i.e., \( c(n) \)) is illustrated. Note the near perfect compensation for the time synchronization error as most of the symbol energy is located in the correct symbol decision variable (i.e., \( c(10) \equiv 1 \)). If an ideal reconstruction filter were used, then the result would have all the signal energy in the correct symbol decision variable with no fringe.
Figure 6.11 TD/CCSK Data Signal with Time Synchronization Error

Figure 6.12 TD/CCSK Reference Signal with Time Synchronization Error

Figure 6.13 TD/CCSK Rake Output Signal with Time Synchronization Error
6.3. Frequency Synchronization Errors

Typically in a communication system, frequency synchronization errors arise when the carrier frequencies or sampling rates are misaligned between the transmitter and receiver. Usually, this form of error is corrected by some form of feedback loop at the receiver. However, the HF skywave channel can also introduce Doppler shift and Doppler spread. As discussed in chapter 2, these phenomena result from the fluctuation of the ionospheric layers over time. These forms of frequency synchronization errors are more difficult, if not impossible to correct at the receiver since they are non-stationary random processes.

Since the receiver does most signal processing in the frequency domain, then there is a potential that the TD/CCSK system is sensitive to these forms of frequency synchronization errors. Reviewing the basic TD/CCSK demodulation operation, assume that a discrete CCSK base vector, $\mathbf{s}_0$, is selected such that it has good circular auto-correlation properties. The base vector is TD/CCSK to modulated, $\mathbf{s}_m$, FIT modulated and upconverted. The symbol is received through the channel, downconverted and sampled. Let $\mathbf{r}$ be the received sampled signal vector to be demodulated. On a noiseless channel with perfect synchronization, $\mathbf{r} = \mathbf{s}_m$. Let $\mathbf{R}$ and $\mathbf{S}_0$ be the FFT pairs of $\mathbf{r}$ and $\mathbf{s}_0$, respectively. As stated in chapter 3, the TD/CCSK demodulation is component-wise multiplication of $\mathbf{R}$ and the complex conjugate of $\mathbf{S}_0$. Taking this vector product back into the time domain, we could get an impulse function in the $m$th position.

The critical operation of the TD/CCSK demodulation is the component-wise multiplication of $\mathbf{R}$ and the complex conjugate of $\mathbf{S}_0$. The frequency domain vectors $\mathbf{R}$ and $\mathbf{S}_0$ are representations of $\mathbf{r}$ and $\mathbf{s}_0$ as a sum of harmonically related complex tones with frequencies that are integer multiples of the fundamental frequency $(2\pi/QMT_3)[22]$. Let $\omega_s$ be the fundamental frequency of the system. These frequency domain vectors are
the sequence of complex weights of this set of complex tones that, when summed, reconstruct the original time domain vectors.

If we were to modulate \( \mathbf{x} \) by a complex tone, then the harmonics of \( \mathbf{x} \) would be modulated up resulting in a circular shift of the vector \( \mathbf{R} \). For example, if we modulated the signal by 1/4 of the fundamental frequency (i.e., \( \omega_s/4 \)) then figure 6.14b illustrates the shift of the tones in the frequency domain. The effect of frequency error on the TD/CCSK demodulation process is the misalignment of the components of \( \mathbf{R} \) to the components of \( \mathbf{s}_0 \). If this erroneous modulation were equal to the \( \omega_s \), then the result would be a perfect circular vector rotation of \( \mathbf{R} \).

Assuming, for example, that there is a carrier offset of \( \omega_s \) (which results in a circular vector rotate of \( \mathbf{R} \)), then if there is no correlation between adjacent components of \( \mathbf{R} \) the TD/CCSK demodulation will not produce a strong impulse. On the other hand, if the adjacent components are correlated, then the shifted vector should not differ much from the unshifted vector, and therefore will still produce an impulse of equal or lesser strength. Therefore, the correlation between adjacent components of \( \mathbf{R} \), determines the sensitivity of the waveform to frequency synchronization errors.

The correlation property of the frequency domain vector is not only dependent on the base vector \( \mathbf{s}_0 \) but also the data symbol, \( m \), that is transmitted. For example if \( \mathbf{s}_0 \) frequency components are highly correlated and \( m=0 \), then frequency synchronization errors will have little effect. However, as \( m \) is increased, and \( \mathbf{s}_0 \) is circularly shifted by \( m \) positions, a linear phase shift in the frequency domain occurs. The frequency components of some data symbol (i.e., different values for \( m \)) could be less correlated than that of other data symbols even if they were generated by the same base vector. To determine the extent of the correlation properties among different data symbols and base-vectors is non-trivial and beyond the scope of this paper. It is possible, however, to bound
this problem by assuming that in the worst case the adjacent frequency components are uncorrelated.

As stated earlier, a frequency domain vector is the sequence of complex weights of harmonically related complex tones with frequencies that are integer multiplies of \( \omega_s \).
The effects of spectral shifting of these components in the frequency domain have been thoroughly investigated [11]. This shift in frequency will cause a leakage of the signal power of the tone across all the vector components. As explained in [25] this is due to the fact that the DFT operates on a truncated sequence.

The degree of leakage in this case can be represented by a sampled Sinc function in the frequency domain (see figure 6.15). If there is no frequency error then signal power is contain in the original frequency bin as indicated by the '*' in figure 6.15. However, as the frequency error is increased the power loss increases. For example, in figure 6.15 for a frequency offset of $\omega_s/4$, only 0.9 of the original signal energy remains in the original frequency bin while the rest of the energy is lost due to spectral leakage. At a frequency error equal to $\omega_s$, it can be shown that all the signal power resides in the adjacent component. For this analysis of frequency error, we will initially concentrate on the power loss of the original complex tone, and will ignore the effects of spectral leakage.

Figure 6.15 Leakage of TD/CCSK Signal Frequency Component Due to Frequency Synchronization Error
If $\omega_k$ represents the frequency error of the $k$th frequency component, then the signal loss, $\xi_k$, (i.e., the remaining original complex tone energy in the original frequency bin) for that component will be,

$$\xi_k = \frac{\sin(\omega_k \pi)}{\sin(\omega_k \pi / M)}$$  \hspace{1cm} (6.32)

For the example in figure 6.15, $\omega_k$ equals $\omega_s / 4$ and $\xi_k = 0.9$. Now using the various frequency synchronization error models presented in figure 6.14, we can calculate a first order approximation of their effects on the TD/CCSK system performance. The total signal loss is equal to the sum of the average component signal loss or

$$\xi = \frac{1}{M} \sum_{k=0}^{M-1} \xi_k$$  \hspace{1cm} (6.33)

6.3.1. Carrier Frequency Error

As stated previously, carrier frequency error results in the circular shift of each of the frequency components of $R$ be the same amount. Let $\omega_{cf}$ be the carrier frequency error in unit of $\omega_s$. Then for carrier frequency error

$$\omega_k = \omega_{cf} \forall k$$  \hspace{1cm} (6.34)

Using equation (6.33) and (6.34), we get

$$\xi = \frac{\sin(\omega_{cf} \pi)}{\sin(\omega_{cf} \pi / M)}$$  \hspace{1cm} (6.35)

6.3.2. Doppler Shift

Doppler shift can be modelled as the compression or expansion of the set of harmonic complex tones of $R$. In this dissertation, we will assume that the Doppler shift causes
compression of group the harmonic complex tones, and that the first complex tone is aligned properly. Figure 6.14c illustrates the model used in this dissertation for the doppler shift. From this model it can be seen that $\omega_k$ increase linearly as $k$ increases. Let $\omega_{sh}$ be the overall compression of the signal bandwidth, then

$$\omega_k = k\omega_{sh} \ \forall k \tag{6.36}$$

Using equation (6.33) and (6.36), we get

$$\xi = \frac{1}{M} \sum_{M_k=0}^{M-1} \frac{\sin(k\omega_{sh}\pi)}{\sin(k\omega_{sh}\pi/M)} \tag{6.37}$$

6.3.3. Doppler Spread

Doppler spread broadens the frequency components and can be modelled as random modulation of the harmonic complex tones (see figure 3.14d). For this dissertation, we will model the doppler spread as a rectangular dispersion of the tones. Let $\omega_{sp}$ be the width of doppler spread of an individual tone. Then

$$\xi_k = \int_{-\omega_{sp}/2}^{\omega_{sp}/2} \frac{\sin(\omega\pi)}{\sin(\omega\pi/M)} d\omega \tag{6.38}$$

Using equation (6.33) and (6.38), we get

$$\xi = \int_{-\omega_{sp}/2}^{\omega_{sp}/2} \frac{\sin(\omega\pi)}{\sin(\omega\pi/M)} d\omega \tag{6.39}$$

Figure 6.16 shows the worst case signal loss of the frequency synchronization errors as the models in this section. Doppler shift and spread do not appear to be a significant problem. Carrier offset could potentially be a problem; however, there are commonly used feedback methods which can be employed at the receiver to correct this problem. In should again be noted that the independent axis is based on $\omega_s$ where $\omega_s = 2\pi/QMT_s$. 
Thus, if we fix the sampling rate $T_s$, an increase the super-symbol length, $Q_M$, the $\omega_s$ is reduced. This will in effect make the system more sensitive to frequency synchronization errors. Therefore, a system level tradeoff is required to balance frequency synchronization errors with the improvements resulting from choosing a large $Q$ or $M$.

Figure 6.16 Worst Case Signal Loss Due To Frequency Synchronization Errors.
Chapter 7

TD/CCSK SYSTEM COMPUTER SIMULATION MODEL

7.1. TD/CCSK System Simulation Overall Description

To verify the performance in chapter 4 and 5, a detailed computer simulation was developed to precisely model the TD/CCSK and FIT modulation schemes, TD/RAKE operation, frequency domain interleaving operations, and channel estimation algorithms developed in this dissertation. This TD/CCSK system simulation model was developed using the Block Oriented System Simulation (BOSS) as part of this research, and its operation, use and limitations are described in this chapter.

The BOSS software package is a product of Comdisco, Inc, and is a subset of Comdisco Signal Processing Workbench (SPW). The BOSS software package used for this dissertation operated on a Sun Sparc2 Workstation. BOSS has an user interface that takes full advantage of mouse-driven protocols and the graphical capacity of the Unix operating system. BOSS provides a complete interactive environment for simulation based analysis and design of systems which can be represented in block diagram form where each block performs a signal processing operation. The simulation program, which is generated by BOSS, is written in FORTRAN based on the block diagrams and their connections specified in the user interface.
Whereas BOSS can perform a time-domain (waveform-level) simulation of any system, the current model library contains functional blocks most suitable for communication system simulation and needs of this dissertation. The BOSS model library is also easily expandable to handle the specialized functions required for the TD/CCSK system simulation. These specialized operations are written as FORTRAN subroutines and attached to the new BOSS modules using the BOSS primitive generation application. All BOSS documentation of the block diagrams and primitive module code generated for this dissertation is listed in appendix C.

The TD/CCSK system model can simulate a transmitter/receiver pair over a fading HF skywave channels at the physical layer. The simulation executes at baseband to reduce the required sampling rate. It executes at twice the system sampling clock rate, which can be specified at runtime. This enables the simulation to accurately model the effects of the TD/CCSK systems down to the waveform level. Executing at twice the system sampling clock rate, the model requires 45 seconds of CPU time to simulate one symbol period. Typically, the simulation will require 1000 symbols to compute a BER data point on the performance curve, and thus require 12.5 hours to perform a single system analysis (most of this computation occurs in the multipath HF channel model). Therefore, the various simulation cases must be carefully selected to minimize the amount of simulation time required.

7.2. Bit Error Rate (BER) Estimation

Two techniques have been employed to directly and indirectly estimate the Bit Error Rate (BER) of the TD/CCSK systems. The Monte Carlo technique calculates the BER directly from symbol errors count made after the receiver symbol decision operations. To extrapolate to lower BERs, we shall use a semi-analytical technique which is discussed in appendix B. This technique computes the BER indirectly by computing the decision
variables statistics defined in chapter 4 (i.e., $\bar{X}, \sigma_x, \sigma_y$) before the symbol decision operation. The BER is then calculated offline using equation (4.18) to extrapolate the BER from these statistics. With this technique, it is possible to estimate BER performance at much lower error rates than the Monte Carlo techniques, without increasing the computational load.

7.3. Structure of the Model

To improve the execution time, the model is broken down into two executable programs (i.e., the transmitter-channel and the receiver simulation) as illustrated in figure 8.1. The transmitter-channel program simulates the transmitter, the HF skywave multipath channel, and the receiver frontend,\textsuperscript{11} and records the received noiseless symbols in a unformatted disk file. In addition, this simulation records the average received signal power for the entire run. Since the simulation operates at baseband, the receiver front end will consist only of the FIT demodulator. This transmitter-channel program execution is the majority of the CPU time required for a particular scenario since it includes the computationally intensive modelling of the transmitter, channel and receiver at the waveform level. The receiver program reads the recorded noiseless received symbol files, adds the channel noise, and performs the TD/CCSK demodulation, symbol decision, and symbol error data recording. The receiver programs executes much more quickly than the transmitter-channel program since it is operating at the symbol rate and consists mainly of vector operations. With the receiver program and the recorded data, numerous simulation can be run at different SNRs to generate entire BER performance curves without duplicating the computationally intensive modelling at the waveform level. The receiver

\textsuperscript{11}The receiver frontend is included only to reduce the amount of data stored from this program, since the data output of the receiver frontend is at the sampling rate rather than the simulation rate (i.e., waveform level).
program uses both the Monte Carlo and semi-analytical techniques to concurrently generate BER performance data.

![Block Diagram for the FIT-TD/CCSK System Simulation](image)

**Figure 7.1:** Block Diagram for the FIT-TD/CCSK System Simulation.

### 7.3.1. TD/CCSK Transmitter Module

The TD/CCSK Transmitter Module is consistent with transmitter configuration described in chapter 5 and illustrated in figure 5.2. In addition, the TD/CCSK Transmitter Module will also contain a pseudo-random symbol generation module. The generation of these symbols requires a random number seed making the symbol sequence repeatable. The base vectors for the data and reference symbol are generated offline and read in at the beginning of the simulation. The frequency domain interleaving operation is performed in the transform domain to take advantage of existing BOSS modules, though this operation is more efficiently executed in the time domain. However, the time domain version would require writing of a specialized primitive module, which was avoided whenever possible.
Since the simulation operates at baseband the transmitter frontend will consist only of the FIT modulator as described in chapter 5.

7.3.2. TD/CCSK Receiver Modules
The TD/CCSK Receiver Module is consistent with the receiver configuration described in chapter 5 and illustrated in figure 5.3. As stated above, the receiver model is divided into two modules to achieve data compression. The first section consists of the RF frontend and the shift register, and is used in the transmitter-channel simulation before recording the symbol data. The second section consists of the remaining operations that are unique to the TD/CCSK receiver. This module includes the FFT, TD/CCSK data and reference demodulation, TD/CCSK RAKE operation, frequency domain deinterleaving, coherent noise canceling operation, IFFT, and symbol decision operation. The coherent noise canceling operation, which is not shown in figure 5.3, is described by equation (4.50). Again the base vector for the CCSK data and reference symbols are generated offline and read in at the beginning of the simulation. The channel estimation module include tail clipping and time averaging operation, with the option of deactivating either operation at runtime.

7.3.3. HF Channel Module
The HF channel module is consistent with channel model described in chapter 2 and illustrated in figure 2.2. An HF channel mode is modeled as a tapped delay line, where the taps are independent complex (i.e., 2-degrees of freedom) Gaussian random variables. The top-level multi-modal HF channel is simulated by using three modes each with a separate group delay. Fading is simulated by modulating the three modes by different complex tones, thus rotating the phase of each mode independently. In this manner phase fading is introduced while the channel attenuation (i.e., magnitude of the taps) remain constant. This
allows us to introduce some degree of fading while still maintaining a constant received signal power. It was necessary to maintain a constant received signal power level to get an accurate BER performance, since the simulation time was significantly shorter than the fading rate. The mode length, group delays, and fading rate are selected at runtime.

7.4. BOSS Simulation Files and Parameters

7.4.1. BOSS Initialization Files

The TD/CCSK simulation requires the offline generation of the data and reference base vectors (i.e., $S_0$ and $S_r$). These files contain the time domain base vectors for the super-symbol. For the data vector, the base vector is equal to the modulated vector when all symbols in the super-symbol first symbol (i.e., $m=0$ for all symbols). These files are read automatically by BOSS modules at initialization, and are in standard BOSS format.

7.4.2. BOSS Simulation Parameters

In setting up a BOSS simulation the user is queried for simulation parameters. Figures 7.2 and 7.3 are example simulation parameter lists for the transmitter-channel and receiver simulation models. For the transmitter-channel simulation this list includes such fundamental parameters as simulation rate (i.e., DT), clock rate, symbol rate, symbol length, interleave ratio, channel mode length, channel mode delays and fading rates, random symbol generation seed, data and reference base vector files, and the signal recording file. For the receiver simulation this list includes such fundamental parameters as number of symbols in simulation, symbol length, interleave ratio, random symbol generation seed (for symbol error counting), data and reference base vector files, and the signal recorded file and average signal power and the Signal to Noise Ratio (SNR).
7.4.3. BOSS Output files

BOSS has a built-in capacity to probe and record signals at any point in the top-level simulation. The BOSS Post-processor allows the user to generate both time and spectrum plots of the recorded signal upon completion of a simulation. In addition, a standard BOSS text file can be generated with each simulation. For the TD/CCSK simulation programs, the text file will be used to record the average signal power (i.e., for the transmitter-channel simulation) and the BER performance statistics as a function of SNR (i.e., receiver simulation). The BER performance statistics will be processed using Matlab software package to generate the BER curves. An example of these files can be seen in figure 7.4.
STOP-TIME = 1.00
DT = 5.0E-7
CLK RATE (HZ) = 1000000.0
SYM RATE (HZ) = 60
SYMBOL LENGTH = 16384
INTERLEAVE RATIO = 16
SRC SEED = 3030311
FREQ. SHIFT (HZ) 1 = 0.43
TAP WEIGHT FILE 1 = /home/jsmal/data/tapfile105.dat
GROUP DELAY (SEC.) 1 = 0
PATH GAIN 1 = (1, 0.0)
FREQ. SHIFT (HZ) 2 = -3.1
TAP WEIGHT FILE 2 = /home/jsmal/data/tapfile90.dat
GROUP DELAY (SEC.) 2 = 3.0E-4
PATH GAIN 2 = (0.7, 0.0)
FREQ. SHIFT (HZ) 3 = 1.7
TAP WEIGHT FILE 3 = /home/jsmal/data/tapfile100.dat
GROUP DELAY (SEC.) 3 = 6.0E-4
PATH GAIN 3 = (0.6, 0.0)
DBV FILE = /home/jsmal/chirp16.3x
RBV FILE = /home/jsmal/chirp16.1x
SIGNAL FILE = /home/jsmal/boss/data/tdccsk1.nf

Figure 7.2 Example of Parameter Inputs for TD/CCSK Transmitter/Channel BOSS Simulation.

STOP-TIME = 900
DT = 1
SNR (DB) = ITERATION: from -8, by 2, for 3 steps
OMEGA = 8
RX VECTOR ROTATE = 50
ALPHA = (0.5, 0.0)
REAL NOISE SEED = 123231777
IMAG NOISE SEED = 213331111
SYMBOL LENGTH = 16384
INTERLEAVE RATIO = 16
SRC SEED = 3030311
AVE. SIGNAL PWR = 1.679E-5
SIGNAL FILE = /home/jsmal/boss/data/tdccsk1.nf
RBV FILE = /home/jsmal/chirp16.1x
DBV FILE = /home/jsmal/chirp16.3x

Figure 7.3 Example of Parameter Inputs for TD/CCSK Receiver BOSS Simulation.
ITERATION STEP = 1 SNR (DB) = -14.0000
-14.00000 2.22723
-14.00000 0.51609
ITERATION STEP = 2 SNR (DB) = -13.0000
-13.00000 2.69097
-13.00000 0.30522
ITERATION STEP = 3 SNR (DB) = -12.0000
-12.00000 3.22976
-12.00000 0.10766
ITERATION STEP = 4 SNR (DB) = -11.0000
-11.00000 3.84827
-11.00000 0.02553
ITERATION STEP = 5 SNR (DB) = -10.00000
-10.00000 4.54944
-10.00000 0.00333
ITERATION STEP = 6 SNR (DB) = -9.00000
-9.00000 5.33401
-9.00000 0.00000
ITERATION STEP = 7 SNR (DB) = -8.00000
-8.00000 6.20035
-8.00000 0.00000

Figure 7.4 Example of TD/CCSK BOSS Simulation Text Output File.
Chapter 8

RESULTS AND CONCLUSIONS

8.1. Simulation Results

Several simulations of the TD/CCSK Rake system were performed to verify the theoretical performance predicted in this dissertation. In these simulations, sampled single and triple rate chirps for the data and reference CCSK base symbols were used respectively. A symbol length of 1024, frequency domain interleave rate of 16, and a signal bandwidth of 1 MHz were also used. In the transmitter-channel simulation, a three mode channel was used, where each mode has a duration of 0.1 milliseconds (i.e., 100 taps). The modes were staggered for a total delay (i.e., from first to last path) of 0.7 milliseconds. Also, the channel had a phase fading rate of approximately 3 Hz. The simulation duration was 1 second of real-time. For the receiver simulation, three cases were simulated with different values of $\Omega$ and $\alpha$ (see table 8.1).

<table>
<thead>
<tr>
<th>Case</th>
<th>$\Omega$</th>
<th>$\alpha$</th>
<th>$\chi^*$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.00</td>
<td>1.00</td>
<td>No Tail Clipping, No Time Averaging</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>1.00</td>
<td>0.12</td>
<td>Tail Clipping, No Time Averaging</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>0.50</td>
<td>0.04</td>
<td>Tail Clipping, Time Averaging</td>
</tr>
</tbody>
</table>

* - Predicted

Table 8.1 TD/CCSK System Simulation Cases
For the generation of a Monte Carlo data point above and below $P_{be} = 10^{-3}$, 900 and 90,000 symbols were simulated respectively. For the 90,000 symbol simulation, the 900 symbol disk file from the transmitter-channel simulation was read 100 times by the receiver simulation, but a different AWGN sequence was generated for each run. For all semi-analytical results, 900 symbols were used to generate a data point with the receiver simulation. The results of the simulation can be seen in figures 8.1 through 8.3.

Figure 8.1 Simulation Results for Case 1 (No Tail Clipping and Time Averaging).
Figure 8.2 Simulation Results for Case 2 (Tail Clipping and No Time Averaging)

Figure 8.3 Simulation Results for Case 3 (Tail Clipping and Time Averaging)
In all three cases, the simulation results of the instantaneous TD/CCSK system with transmitted reference verified the predicted performance though both Monte Carlo and semi-analytical techniques. The slight discrepancy for case 3 was probably due to the simplified fading of the channel and/or the use of the same CCSK base vectors throughout the simulation. Both conditions could cause the noise on the channel estimate to be artificially correlated from symbol to symbol thus reducing the effectiveness of the time-averaging operation. However, the discrepancy is minimal since it is on the order of 0.1 dB.

Again, this is just a first order approximation of the predicted performance. For a more rigorous verification of the theoretical predictions, a much longer simulation is required to get more reliable Monte Carlo results for low bit error rates. However, this is too computationally intensive for the scope of this dissertation. Though the semi-analytical approach used in this dissertation was not as rigorous a verification of the system performance, it did provide a first order approximation for the low bit error rates. The results were similar for the Monte Carlo and semi-analytical techniques in the case when both could be used. Thus, this reinforces the validity of both techniques.

8.2. Conclusions

8.2.1. Summary of Results

In this dissertation, we have proposed a digital implementation of CCSK Rake system using transform domain signal processing. We have shown that the theoretical performance of the basic TD/CCSK Rake system for any channel with a known, time invariant impulse response achieves a bit error performance equivalent to M-ary orthogonal signalling. For a time varying channel (e.g., wideband HF channel), we have shown the
addition of a reference signal may be beneficial. We have also determined an upper performance bound for using the instantaneous reference signal as a channel estimate.

We have presented the transform domain signal processing techniques of frequency domain interleaving, tail clipping, center clipping and time averaging which improve the channel estimation of the basic system using the instantaneous reference signal. The individual and combined theoretical improvements achieved by these techniques were derived. It was shown that these noise reduction techniques complement each other, and yield a significant improvement over the basic system. They are also implementable in a computationally efficient manner.

BMLS, chirps, and FIT waveforms were proposed as possible CCSK waveforms. A FIT waveform based on a combination of BMLs and chirps was suggested as the best candidate for implementation. This combination of waveforms provides for a large class of CCSK waveforms with excellent properties. The theoretical system's sensitivity to time and frequency synchronization was also investigated. It was shown that performance of any FIT waveform is virtually unaffected by fine time synchronization errors. A first order estimate of the effects of doppler shift and doppler spread of less than fundamental frequency of the system was shown to be negligible. Carrier offset of the same magnitude was shown to be a potential problem, this type of frequency synchronization error can easily be corrected with a feedback loop at the receiver.

A detailed computer simulation of the system with a HF skywave channel model was developed to get a first order approximation on the performance of several variations of the TD/CCSK Rake System with transmitted reference signal. In each case, the simulation verified the theoretical performance of the basic system, the basic system with tail clipping and the basic system with tail clipping and time averaging.
8.2.2. Strengths and Weakness

The strength of the TD/CCSK Rake system is in its digital implementation and the use of transform domain signal processing techniques. The transform domain signal processing allows for the real-time implementation of the system. This allowed us to achieve a very large number of Rake taps with no increase in hardware. The addition of the reference signal gives the system the robustness to operate under severe multipath and channel distortion, as well as time-varying channel conditions, and also makes the system relatively immune to imperfect time synchronization.

The digital implementation is also a weakness of the system in that the computation load is a limiting factor. Thus, the information bandwidth is limited by the processing power at the receiver. Also, the system is highly sensitive to frequency synchronization errors with the implementation of a transform domain signal processing operations.

8.2.3. Future Research

Coding Scheme - A M-ary coding scheme could be implemented with the symbol decision operation to correct symbol errors. In addition, soft decisions could be based on the relative magnitude of the decision variable selected by the symbol decision stage and the other decision variables. An erasure scheme could also be introduced based on this soft decision.

Reference Signal and Decision Feedback Hybrid - The reference signalling and decision feedback could be combined to improve the channel estimate. The output of the decision feedback could also be weighted based on the soft decision made by the symbol decision operation as suggested above.
Variable Power Reference Signal - Under good channel conditions, the use of a half of the transmitted signal energy on a reference signal is suboptimal. A variable power reference could be employed to optimize the signal resources. However, the less power used in the reference signal the less robust the system would become. A tradeoff study would be required to analyze the performance benefits and stability issues.

Adaptive Time Averaging and Tail Clipping - As presented in this dissertation, both the tail clipping and time averaging parameters (i.e., $\alpha$ and $\Omega$ respectively) are fixed value for the worst case channel. If the current characteristics of the channel (e.g., the duration of the multipath or the amount of fading) could be estimated then these parameters could be adaptively change to optimize the performance.

Center Clipping - The actual implementation of a center clipping operation and its simulated performance was ignored in this dissertation though the theoretical performance improvement by center clipping make it extremely attractive.

Acquisition and Initial Synchronization - A weak point in this dissertation is the assumption of the system having achieved acquisition and some form of course time and frequency synchronization. These assumptions need to be address both in theory and in simulation.

Other Environments - The robustness of the TD/CCSK Rake system under severe multipath and channel distortion make it an attractive approach in other environments. For example, wireless office Local Area Networks (LANs) and urban mobile communication environments also have severe multipath and channel distortion problem that make it
difficult to use classical digital communication schemes, but could be feasible with TD/CCSK Rake systems.

**BCH Codes** - The Bose, Chaudhuri, and Hocquenghem (BCH) codes are a large class of codes developed primary for error correction. The algebraic properties that make them well suited for error correcting codes also make them excellent potential candidates for use as CCSK base vectors. In addition, nonbinary BCH codes, such as Reed-Solomon Codes, have error correcting algorithms that are quite similar to the DFT operation. Potentially, these codes could be directly connected to the transform domain signal processing.
References


Appendix

A. Theorems

**Theorem 1:** Given two independent complex Gaussian random variables \( X : \mathcal{N}(0, \sigma_X^2) \) and \( Y : \mathcal{N}(0, \sigma_Y^2) \), their product, \( Z = XY \), is a complex random variable with a Laplacian probability density function

\[
f_Z(z) = \frac{1}{2\sigma_X\sigma_Y} e^{-|z|^2/(2\sigma_X^2\sigma_Y^2)}
\]  

(A.1)

**Proof:** Let \( X \) and \( Y \) be two independent complex Gaussian random variables where \( X = X_r + jX_i \) and \( Y = Y_r + jY_i \). Let \( Z \) be a new random variable that is the complex product of \( X \) and \( Y \). Therefore

\[
Z_r = \text{Re}(Z) = X_rY_r - X_iY_i \\
Z_i = \text{Im}(Z) = X_rY_i + X_iY_r
\]  

(A.2)  

(A.3)

Let \( Z_1 = X_rY_r \), \( Z_2 = X_iY_i \), \( Z_3 = X_rY_i \) and \( Z_4 = X_iY_r \), and where \( Z_1 \) and \( Z_2 \) are mutually independent, as are \( Z_3 \) and \( Z_4 \). Since the probability density functions for \( X_i \) and \( Y_i \) are even, then the probability density function for \( Z_2 \) is even. Therefore,

\[
Z_r = Z_1 + Z_2 \\
Z_i = Z_3 + Z_4
\]  

(A.4)  

(A.5)

The joint probability density function for \( X_r \) and \( Y_r \) can be expressed as

\[
f_{X_r,Y_r}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right)}
\]  

(A.6)

Performing a transformation of variables where the new variables are \( z=xy \) and \( y=y \) yields
\[ f_{Z_1 Y_1}(z, y) = \frac{1}{|y|} f_{X_1 Y_1}(z/y, y) \]

Now solving for \( f_{Z_1}(z) \) by integrating over \( y \) we get

\[ f_{Z_1}(z) = \frac{1}{2\pi \sigma_1 \sigma_y} \int_{-\infty}^{\infty} \frac{1}{|y|} e^{-\frac{z^2}{2\sigma_1^2} - \frac{y^2}{2\sigma_y^2}} dy = \frac{1}{2\pi \sigma_1 \sigma_y} \int_{-\infty}^{\infty} \frac{1}{w} e^{-\frac{z^2}{2w\sigma_1^2} - \frac{w}{2\sigma_y^2}} dw \]

Let \( \phi_{Z_1}(u) \) be the characteristic function of \( Z_1 \) then

\[ \phi_{Z_1}(u) = \int_{-\infty}^{\infty} f_{Z_1}(z) e^{izu} dz = \frac{1}{2\pi \sigma_1 \sigma_y} \int_{-\infty}^{\infty} \frac{1}{w} e^{-\frac{w}{2w\sigma_1^2}} e^{iuz} e^{-\frac{w}{2\sigma_y^2}} dw \]

\[ = \left( \sigma_1^2 \sigma_y^2 u^2 + 1 \right)^{-1/2} \]

By the same method,

\[ \phi_{Z_2}(u) = \phi_{Z_3}(u) = \phi_{Z_4}(u) = \left( \sigma_2^2 \sigma_y^2 u^2 + 1 \right)^{-1/2} \]

Since \( Z_T \) and \( Z_I \) are the sums of two independent random variables, then the characteristic functions for \( Z_T \) and \( Z_I \) are the product of the characteristic functions of the two random variables, and therefore

\[ \phi_{Z_T}(u) = \phi_{Z_I}(u) \phi_{Z_T}(u) = \left( \sigma_2^2 \sigma_y^2 u^2 + 1 \right)^{-1} \]

\[ \phi_{Z_I}(u) = \phi_{Z_T}(u) \phi_{Z_I}(u) = \left( \sigma_2^2 \sigma_y^2 u^2 + 1 \right)^{-1} \]

By definition the probability density functions for these two characteristic functions are

\[ f_{Z_T}(z) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{Z_T}(u) e^{izu} du \]

\[ f_{Z_I}(z) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi_{Z_I}(u) e^{izu} du \]

and solving these equations yields
Therefore $Z$ is a complex random variable with a Laplacian probability density function for the real and imaginary parts.

**Theorem 2:** Given two independent complex Gaussian random vectors, $X(n) : \mathcal{G}[0, \sigma_X^2]$ and $Y(n) : \mathcal{G}[0, \sigma_Y^2]$, of length $N$ (where $N$ is large) in which all vector components are independent, the dot product $Z = X(n) \cdot Y(n)$ is a complex random variable with a Gaussian probability density function where $Z \sim \mathcal{G}[0, 2N\sigma_X^2\sigma_Y^2]$.

**Proof:** Let $X(n)$ and $Y(n)$ be two independent complex Gaussian random variable vectors on length $N$, were all components are mutually independent and have a mean of zero and a variance of $\sigma_X^2$ and $\sigma_Y^2$ respectively. Let $Z$ be the dot product of $X(n)$ and $Y(n)$, and therefore

$$Z = \sum_{n=1}^{N} X(n) \cdot Y(n) = \sum_{n=1}^{N} Z_n = \sum_{n=1}^{N} Z_{nr} + jZ_{ni}$$

(A.14)

By Theorem 1,

$$f_{Z_n}(z) = f_{Z_m}(z) = \frac{1}{2\sigma_X\sigma_Y} e^{-\frac{|z|}{\sigma_X\sigma_Y}}$$

$$\phi_{Z_n}(u) = \phi_{Z_m}(u) = \left(\sigma_X^2\sigma_Y^2u^2 + 1\right)^{-1}$$

(A.15)

Since $\{Z_{nr}\}$ and $\{Z_{ni}\}$ are independent for all $n$ then

$$\text{Re}\{\phi_Z(u)\} = \prod_{n=1}^{N} \phi_{Z_n}(u)$$

$$\text{Im}\{\phi_Z(u)\} = \prod_{n=1}^{N} \phi_{Z_n}(u)$$

$$\text{Re}\{\phi_Z(u)\} = \text{Im}\{\phi_Z(u)\} = \left(\sigma_X^2\sigma_Y^2u^2 + 1\right)^{-N}$$

(A.16)
The mean of \( Z_r \) and \( Z_i \), \( m_Z \), can be calculated from characteristic function by

\[
m_Z = \frac{1}{j} \frac{d}{du} \phi_Z(u) \bigg|_{u=0} = \frac{1}{j} \frac{d}{du} \left( \sigma_x^2 \sigma_y^2 u^2 + 1 \right)^N \bigg|_{u=0} = -\frac{1}{j} 2u\sigma_x^2\sigma_y^2 N \left( \sigma_x^2 \sigma_y^2 u^2 + 1 \right)^{N-1} \bigg|_{u=0} = 0
\]

(A.17)

Since the mean is zero the variance of \( Z_r \) and \( Z_i \), \( \sigma_Z^2 \), is the second moment, and is calculated from the characteristic function by

\[
\sigma_Z^2 = \frac{1}{j^2} \frac{d^2}{du^2} \phi_Z(u) \bigg|_{u=0} = -\frac{d^2}{du^2} \left( \sigma_x^2 \sigma_y^2 u^2 + 1 \right)^N \bigg|_{u=0} = 2\sigma_x^2\sigma_y^2 N \left( \sigma_x^2 \sigma_y^2 u^2 + 1 \right)^{N-1} + u (N+1) \left( \sigma_x^2 \sigma_y^2 u^2 + 1 \right)^{N-2} \bigg|_{u=0} = 2\sigma_x^2\sigma_y^2 N
\]

(A.18)

Since \( N \) is large, the Central Limit Theorem can be applied. Therefore \( Z_r \) and \( Z_i \) are Gaussian random variables with mean, \( m_Z \) and variance \( \sigma_Z \). Thus \( Z \) is a complex Gaussian random variable such that \( Z : G[0, 2N\sigma_x^2\sigma_y^2] \).

**Theorem 3:** Given two independent complex Gaussian random vectors, \( X(n) : G[0, \sigma_x^2] \) and \( Y(n) : G[0, \sigma_y^2] \) of length \( N \), their circular cross-correlation is a complex Gaussian random vector such that

\[
\text{Re} [R_{XY}(\tau)] = \text{Im} [R_{XY}(\tau)] = G[0, 2N\sigma_x^2\sigma_y^2] \forall \tau
\]

(A.19)

**Proof:** Since the two vector are independent, then any circular shift of one vector to another is independent. Therefore, by theorem 2,

\[
\text{Re} [R_{XY}(\tau)] = \text{Im} [R_{XY}(\tau)] = N(0, 2N\sigma_x^2\sigma_y^2) \forall \tau
\]

(A.20)
Theorem 4: Given an independent complex Gaussian random vector, \( X(n) \): \( G[0, \sigma_X^2] \), the auto-correlation is a complex Gaussian random vector such that

\[
\begin{align*}
\text{Re} \{ R_X(\tau) \} &= \begin{cases} 
G[2N\sigma_X^2, 4N\sigma_X^2] & \tau = 0 \\
G[0, 2N\sigma_X^2] & \text{else}
\end{cases} \\
\text{Im} \{ R_X(\tau) \} &= \begin{cases} 
0 & \tau = 0 \\
G[0, 2N\sigma_X^2] & \text{else}
\end{cases}
\end{align*}
\] (A.21)

Proof: Let \( \tau = 0 \). Then from the definition of the auto-correlation function,

\[
R_X(0) = \sum_{n=1}^{N} |x(n)|^2
\] (A.22)

Therefore, \( \text{Re} \{ R_X(0) \} \), is a random variable with a chi-squared probability density function with \( 2N \) degrees of freedom, with a mean of \( 2N\sigma_X^2 \) and a variance of \( 4N\sigma_X^4 \). \( R_X(0) \) is also a real number. Therefore \( \text{Im} \{ R_X(0) \} = 0 \). Since \( N \) is large, the Central Limit Theorem can be applied. Thus

\[
\begin{align*}
\text{Re} \{ R_X(0) \} &= G[2N\sigma_X^2, 4N\sigma_X^2] \\
\text{Im} \{ R_X(0) \} &= 0
\end{align*}
\] (A.23)

Let \( \tau \neq 0 \), then

\[
R_X(\tau) = \sum_{n=1}^{N} x^*(n)x(n+\tau)
\] (A.24)

Since, the components of \( X(n) \) are independent, then \( R_X(\tau) |_{\tau \neq 0} \) can be considered a dot product of two independent random vectors. Therefore, by applying Theorem 2,

\[
\begin{align*}
\text{Re} \{ R_X(\tau) \} &= \text{Im} \{ R_X(\tau) \} = G[0, 2N\sigma_X^4] |_{\tau \neq 0}
\end{align*}
\] (A.25)
B. Semi-Analytical Performance Measurement

The Semi-Analytical (SA) performance measurement is a method of estimating the Bit Error Rate (BER) of a system directly from the decision variable statistics (i.e., the means and variances of the correct and incorrect symbol decision variables) rather than from actual symbol decisions. The basic SA approach used in this dissertation is to compute the probability of bit error, $P_{BE}$, directly from the decision variable statistics using equation (4.17). With this approach it is possible to extrapolate the bit error rate to values much lower than the reciprocal number of simulated bits.

As in chapter 4, we will let $X$ and $\sigma_X^2$ be the mean and variance of the correct (i.e. $i=m$) symbol decision variable. We will assume that all the remaining incorrect decision variables have zero mean and a variance of $\sigma_Y^2$. We will also assume that the all the decision variables are independent Gaussian random variables. These assumptions are reasonable since the TD/CCSK Rake System is an M-ary orthogonal signalling system.

These variables can be directly calculated from simulation data recorded after the final IFFT operation of the receiver. If we run the simulation with no noise, then $X$ is a constant and can be calculated directly. Note that $\sigma_X^2$ should be slightly greater than $\sigma_Y^2$ only because of the additional variance from magnitude fading of the channel taps. Since the fading channel model in this dissertation is only phase fading, the we can assume that $\sigma_X^2$ and $\sigma_Y^2$ are the same, thus we need only compute one variance, $\sigma^2$ (i.e, $\sigma^2 = \sigma_X^2 = \sigma_Y^2$). Though this is only a first order approximation of the performance, we will be able to achieve a tighter confidence interval. With $X$, $\sigma^2$, and the number of simulated symbols, we will be able to calculate a BER confidence interval for a given confidence level. Since $X$ is a constant, the confidence interval will only depend on the statistical variations of $\sigma^2$.

\footnote{For a given M-ary CCSK symbol, there are M-1 samples of $\sigma_Y^2$ and only 1 sample of $\sigma_X^2$.}
The determination of a confidence interval for the variance \( \sigma^2 \) of a Gaussian distribution, whose mean need not be known, is derived [26], and the algorithm used to determine the confidence interval as given by [26] is listed in table (B.1).

1st Step: Choose a confidence level \( \lambda \).
2nd Step: Determine solutions \( c_1 \) and \( c_2 \) for \( n > 100 \) using formulas
\[
\begin{align*}
c_1 &= \frac{1}{2} \left( \sqrt{2n - 1} - A \right)^2 \\
c_2 &= \frac{1}{2} \left( \sqrt{2n - 1} + A \right)^2
\end{align*}
\]
where \( n \) is the number of samples and \( A \) is determined as follows:
\[
\begin{array}{c|c}
\lambda & A \\
0.99 & 2.58 \\
0.98 & 2.33 \\
0.95 & 1.96 \\
0.90 & 1.64
\end{array}
\]
3rd Step: Compute \( k_1 \) and \( k_2 \) from
\[
\begin{align*}
k_1 &= \frac{(n - 1) \tilde{\sigma}^2}{c_1} \\
k_2 &= \frac{(n - 1) \tilde{\sigma}^2}{c_2}
\end{align*}
\]
where \( \tilde{\sigma}^2 \) is the variance of the samples.

Confidence Interval \( \{k_2 \leq \sigma^2 \leq k_1\} \)

Table B.1 Algorithm for Determining Confidence Interval of the Variance for a Gaussian Distribution.[26]

Let \( k_1 \) and \( k_2 \) be the confidence limits for \( \sigma^2 \). Then, the confidence interval for \( \bar{X}/\sigma_Y \) is

\[
\text{Confidence Interval } \left\{ \frac{\bar{X}}{\sqrt{k_1}} \leq \frac{\bar{X}}{\sigma_Y} \leq \frac{\bar{X}}{\sqrt{k_2}} \right\}
\]

For a given simulation let \( m \) be the number of symbols simulated. Under the assumptions above, there are \( M \) samples of \( \sigma^2 \) per symbol (i.e., one for each symbol decision variable in a symbol). Thus, for a given simulation, the number of samples of \( \sigma^2 \), which we call \( n \), is given by \( n = m M \) samples. For example, if we simulate 1,000 symbols and
M=1024, then $n = 1,024,000$. Let $\hat{\sigma}^2$ be the estimate for $\sigma^2$ from a given simulation. Figure B.1 shows the confidence intervals for confidence levels of 0.99 as a function of $\bar{X}/\hat{\sigma}$ (by application of equation (4.17) and (B.6)). From figure B.1, it can be seen that over the BER range of interest (i.e., $10^{-6} \leq P_{BE} \leq 10^{-3}$) a tight confidence interval can be achieved using the semi-analytical approach with the simulation of 1,000 1024-ary symbols.

![Figure B.1 TD/CCSK Bit Error Performance with 99% Confidence Interval.](image-url)
C. BOSS Simulation Block Diagrams and Primitive Code

This appendix contains the BOSS documentation set for the simulations developed in this dissertation. To minimize the length of this dissertation, only the key portions of the documentation set are printed here. This includes the entire documentation of the top level systems, the block diagrams for all lower level modules, and the primitive code of specialized algorithms related to this dissertation.
C.1. BOSS Systems
Lower Limit: 3.0E-39
Upper Limit: 1.7E38

Time between discrete simulation signal samples (in seconds). DT must be small enough to satisfy the Nyquist Sampling Theorem for all signals at all points in the simulation.

NOTE:
If the specified period or rate of a periodic function results in a period that is not a multiple of DT, then BOSS will round the period to the nearest value that IS a multiple of DT. For example, if DT=0.125(sec) and a rate was specified as rate=1.4(hz) which corresponds to a period of T=0.714(sec) which is 5.7*DT, then BOSS will round this period to be 6.0*DT and thus the effective rate will be 1.333(hz).

Because of this, the choice for DT can affect the periods of the periodic signals in the simulation.

CLK RATE (HZ) Type: REAL
Lower Limit: 2.999998E-39
Upper Limit: 1.7E38
Determines the rate of the output signal. The output signal will be TRUE every (1/RATE) seconds.

SYN RATE (HZ) Type: REAL
Lower Limit: 3.0E-39
Upper Limit: 1.7E38
overall symbol rate

SYMBOL LENGTH Type: INTEGER
Lower Limit: 1
Upper Limit: 2147483647
The length of the vectors to be added. The output vector will be the same length.

INTERLEAVE RATIO Type: INTEGER
Lower Limit: 1
Upper Limit: 2147483647
number of symbols to be interleaved

SRC SEED Type: INTEGER
Lower Limit: 1

Initial seed for the uniform random number generator. Should be a large odd integer.

FREQ. SHIFT (HZ) 1 Type: REAL
Lower Limit: -1.7E38
Upper Limit: 1.7E38
Frequency of complex tone (Hertz). This frequency can be negative.

TAP WEIGHT FILE 1 Type: OLD-FILE
this is the name of a standard boss tabular data file containing the desired time domain input xfer function.

GROUP DELAY (SEC.) 1 Type: REAL
Lower Limit: -1.7E38
Upper Limit: 1.7E38
delay of mode

PATH GAIN 1 Type: COMPLEX
Lower Limit: (-1.7E38 -1.7E38)
Upper Limit: (1.7E38 1.7E38)
output = gain constant * input

FREQ. SHIFT (HZ) 2 Type: REAL
Lower Limit: -1.7E38
Upper Limit: 1.7E38
Frequency of complex tone (Hertz). This frequency can be negative.

TAP WEIGHT FILE 2 Type: OLD-FILE
this is the name of a standard boss tabular data file containing the desired time domain input xfer function.
GROUP DELAY (SEC.) 2
Lower Limit: -1.7E38
Upper Limit: 1.7E38
Type: REAL
delay of mode

PATH GAIN 2
Lower Limit: (-1.7E38 -1.7E38)
Upper Limit: (1.7E38 1.7E38)
Type: COMPLEX
output = gain constant * input

FREQ. SHIFT (HZ) 3
Lower Limit: -1.7E38
Upper Limit: 1.7E38
Type: REAL
Frequency of complex tone (Hertz). This frequency can be negative.

TAP WEIGHT FILE 3
Lower Limit: NIL
Upper Limit: NIL
Type: OLD-FILE
this is the name of a standard boss tabular data file containing the desired time domain input xfer function.

GROUP DELAY (SEC.) 3
Lower Limit: -1.7E38
Upper Limit: 1.7E38
Type: REAL
delay of mode

PATH GAIN 3
Lower Limit: (-1.7E38 -1.7E38)
Upper Limit: (1.7E38 1.7E38)
Type: COMPLEX
output = gain constant * input

DBV FILE
Lower Limit: NIL
Upper Limit: NIL
Type: OLD-FILE

Old data file name which contains the vector values.

RBV FILE
Lower Limit: NIL
Upper Limit: NIL
Type: OLD-FILE

Old data file name which contains the vector values.

SIGNAL FILE
Lower Limit: NIL
Upper Limit: NIL
Type: NEW-FILE
this is the name of the unformatted output file name

MODULES USED IN BLOCK DIAGRAM:
SINK
CIR. BUFFER W/LATCH
WRITE UNFORMAT VECTOR
AND
IMPULSE TRAIN
AVG CMPLX POWER
PRINT SIGNAL
STOP TIME TRUE
FIT_CCSK TXER
3_MODE HF_SKYWAVE CHANNEL

PARAMETER VALUES FOR INSTANCES IN BLOCK DIAGRAM:
3_MODE HF_SKYWAVE CHANNEL (key 1)
PATH GAIN 3 == $PATH GAIN 3
GROUP DELAY (SEC.) 3 == $GROUP DELAY (SEC.) 3
TAP WEIGHT FILE 3 == $TAP WEIGHT FILE 3
FREQ. SHIFT (HZ) 3 == $FREQ. SHIFT (HZ) 3
PATH GAIN 2 == $PATH GAIN 2
GROUP DELAY (SEC.) 2 == $GROUP DELAY (SEC.) 2
TAP WEIGHT FILE 2 == $TAP WEIGHT FILE 2
FREQ. SHIFT (HZ) 2 == $FREQ. SHIFT (HZ) 2
PATH GAIN 1 == $PATH GAIN 1
GROUP DELAY (SEC.) 1 == $GROUP DELAY (SEC.) 1
TAP WEIGHT FILE 1 == $TAP WEIGHT FILE 1
FREQ. SHIFT (HZ) 1 == $FREQ. SHIFT (HZ) 1

FIT_CCSK TXER (key 2)
INTERLEAVE RATIO == $INTERLEAVE RATIO
RBV FILE == $RBV FILE
DBV FILE == $DBV FILE
SYM RATE (HZ) == $SYM RATE (HZ)
STOPBAND EDGE (HZ) == 'CLK RATE (HZ)' * 0.6
PASSBAND EDGE (HZ) == 'CLK RATE (HZ)' * 0.5
PASSBAND RIPPLE (DB) == 1
FILTER ORDER == 7
CLK RATE (HZ) == $CLK RATE (HZ)
SRC SEED == $SRC SEED
SYMBOL LENGTH == $SYMBOL LENGTH

PRINT SIGNAL (key 3)
LEADING STRING == ave signal power =
TRAILING STRING == )

AVG CMPLX POWER (key 4)
ON TIME == 0

IMPULSE TRAIN (key 5)
RATE (Hz) == $CLK RATE (HZ)

WRITE UNFORMAT VECTOR (key 6)
VECTOR LENGTH == $SYMBOL LENGTH
OUTPUT FILE NAME == $SIGNAL FILE

CIR. BUFFER W/LATCH (key 7)
VECTOR LENGTH == $SYMBOL LENGTH

INITIALIZATION CODE:
(none)
FIT_CCSK 1.0.0R

MODULE NAME: FIT_CCSK 1.0.0R
GROUP: SYSTEM
DATABASE: /home/jsmal/boss/dissys/
AUTHOR: jsml
CREATION DATE: 31-Jul-1992 18:07:11

DESCRIPTION:

This module is pair to TD/CCSK 1.0.0t.
This system extrapolates the BER of a TD/CCSK system with ref. signal and tail clipping time ave.

REVISIONS:

Author : jsml
Date : 31-Jul-1992 18:07:11
Description:
31-Jul-1992 18:07:13
Module CREATION,

INPUT SIGNALS:
(none)

OUTPUT SIGNALS:
(none)

PARAMETERS:

STOP-TIME
Type: REAL
Lower Limit: 3.0E-39
Upper Limit: 1.7E38

Specifies the maximum value of time for the simulation (in seconds).
The simulation clock runs from time=0.0 to time=STOP-TIME in steps of DT, all in seconds.

DT
Type: REAL
Lower Limit: 3.0E-39
Upper Limit: 1.7E38

Time between discrete simulation signal samples (in seconds).
DT must be small enough to satisfy the Nyquist Sampling Theorem for all signals at all points in the simulation.

NOTE:
If the specified period or rate of a periodic function results
in a period that is not a multiple of DT, then BOSS will round the period to the nearest value that IS a multiple of DT.

For example, if DT=0.125(sec) and a rate was specified as rate=1.4(hz) which corresponds to a period of T=0.714(sec) which is 5.7*DT, then BOSS will round this period to be 6.0*DT and thus the effective rate will be 1.33(hz).

Because of this, the choice for DT can affect the periods of the periodic signals in the simulation.

-------

SNR (DB) Type: REAL
Lower Limit: -1.7E38
Upper Limit: 1.7E38
The desired signal to noise power ratio in decibels.

-------

OEGA Type: INTEGER
Lower Limit: 1
Upper Limit: 2147483647
The number of points in the foward and rev. filters.

-------

RX VECTOR ROTATE Type: INTEGER
Lower Limit: -2147483648
Upper Limit: 2147483647
amount vector need to rotated fro tail clipping. should probably be equal to half the channel IR if synced on first path.

-------

ALPHA Type: COMPLEX
Lower Limit: (-1.7E38 -1.7E38)
Upper Limit: (1.7E38 1.7E38)
weight of incoming estimate

-------

REAL NOISE SEED Type: INTEGER
Lower Limit: 1
Upper Limit: 2147483647
Initial seed for the random number generator. This seed should be a large odd integer.

-------

IMAG NOISE SEED Type: INTEGER
Lower Limit: 1
Old data file name which contains the time domain ccssk base vector values.

-------

DBV FILE Type: OLD-FILE
VECTOR ADDER (key 9)

VECTOR LENGTH == SYMBOL LENGTH

OLD DATA FILE NAME WHICH CONTAINS THE TIME DOMAIN ccsk BASE VECTOR VALUES.

MODULES USED IN BLOCK DIAGRAM:
RANDOM M-ARY SYMBOL GENERATOR
VECTOR ADDER
BURST IMPULSE TRAIN
DIRAC DELTA
READ UNFORMAT VECTOR
CMPLX WHITE NOISE VECTOR
MATLAB ERR_CNT
CCSK DECISION STAGE
MATLAB SENT_ANLY BER_EST
TD/CCSK DEMOD

PARAMETER VALUES FOR INSTANCES IN BLOCK DIAGRAM:

TD/CCSK DEMOD (key 1)
ALPHA == $ALPHA
RX VECTOR ROTATE == $RX VECTOR ROTATE
OMEGA == $OMEGA
SYMBOL LENGTH == $SYMBOL LENGTH
SUB-SYMBOL LENGTH == SYMBOL LENGTH / 'INTERLEAVE RATIO'
RBV FILE == $RBV FILE
DBV FILE == $DBV FILE

MATLAB SENT_ANLY BER_EST (key 2)
SNR (DB) == $SNR (DB)
VECTOR LENGTH == SYMBOL LENGTH / 'INTERLEAVE RATIO'

CCSK DECISION STAGE (key 3)
VECTOR LENGTH == SYMBOL LENGTH / 'INTERLEAVE RATIO'

MATLAB ERR_CNT (key 4)
SNR == $SNR (DB)

CMPLX WHITE NOISE VECTOR (key 5)
IMAG NOISE SEED == $IMAG NOISE SEED
REAL NOISE SEED == $REAL NOISE SEED
VECTOR LENGTH == SYMBOL LENGTH
AVE. SIGNAL PWR == $AVE. SIGNAL PWR
SNR (DB) == $SNR (DB)

READ UNFORMAT VECTOR (key 6)
VECTOR LENGTH == SYMBOL LENGTH
INPUT FILENAME == $INPUT FILENAME

DIRAC DELTA (key 7)
TIME OF IMPULSE == 0

BURST IMPULSE TRAIN (key 8)
C.2. BOSS Modules

CMPLX WHITE NOISE VECTOR

MATLAB ERR CNT

BURST IMPULSE TRAIN
C.3. Primitive Code

SUBROUTINE INTERLEAVER (SUPERSYMBOL_LENGTH, & SYMBOL_LENGTH, INPUT_VECTOR, OUTPUT_VECTOR)

! interleaver - takes concatenated symbols and interleaves them to form a super symbol

INTEGER SYMBOL_LENGTH, SUPERSYMBOL_LENGTH, ISYMRATE
INTEGER I, J
COMPLEX INPUT_VECTOR(SUPERSYMBOL_LENGTH)
COMPLEX OUTPUT_VECTOR(SUPERSYMBOL_LENGTH)

ISYMRATE = SUPERSYMBOL_LENGTH/SYMBOL_LENGTH
DO I = 1, ISYMRATE
  DO J = 1, SYMBOL_LENGTH
    OUTPUT_VECTOR((I-1)*SYMBOL_LENGTH + J) = & INPUT_VECTOR(I + (J-1)*ISYMRATE )
  END DO
END DO
RETURN
END
SUBROUTINE DEINTERLEAVER(SUPERSYMBOL_LENGTH,
& SYMBOL_LENGTH,INPUT_VECTOR, OUTPUT_VECTOR)

c deinterleaver - takes interleaved vector and returns the dinterleaved
symbol concatenated form

INTEGER SYMBOL_LENGTH, SUPERSYMBOL_LENGTH, ISYMRATE
INTEGER I,J
COMPLEX INPUT_VECTOR(SUPERSYMBOL_LENGTH)
COMPLEX OUTPUT_VECTOR(SUPERSYMBOL_LENGTH)

ISYMRATE = SUPERSYMBOL_LENGTH/SYMBOL_LENGTH
DO I = 1, ISYMRATE
  DO J = 1, SYMBOL_LENGTH
    OUTPUT_VECTOR(I + (J-1)*ISYMRATE ) =
& INPUT_VECTOR((I-1)*SYMBOL_LENGTH + J)
END DO
END DO
RETURN
END
subroutine tailclip(vin,vlen,omega,vout)
c
transform domain tail clipper - filters input vector with omega-order
rectangular FIR filter in forward and reverse directions
c
integer vlen,omega
complex vin(vlen)
complex vout(vlen)
complex vtemp(32768)
c
integer n,l,ix1,ix2
c
do n=1,vlen
  vtemp(n)=0.0
  do l=1,omega
    ix1 = MOD(n-l+vlen,vlen)+1
    vtemp(n) = vtemp(n) + vin(ix1)
  enddo
enddo

do n=1,vlen
  vout(n)=0.0
  do l=1,omega
    ix2 = MOD(l+n+vlen-2,vlen)+1
    vout(n)=vout(n) + vtemp(ix2)
  enddo
enddo
return
end
subroutine ccsk_dec(vector_in,vector_length,index)

c ccsk decision stage - selects the large positive real value
C of input vector and return index

c

ing integer vector_length
integer index
complex vector_in(vector_length)
real element

c
integer i

c
index = 0
element=vector_in(1)
do i=2,vector_length
if (real(vector_in(i)) .gt. element) then
  element = vector_in(i)
  index = i-1
endif
enddo
return
end