ANALYSIS OF AIR-TO-AIR ROTARY ENERGY WHEELS

A dissertation presented to

the faculty of

the Russ College of Engineering and Technology

of Ohio University

In partial fulfillment

of the requirements for the degree

Doctor of Philosophy

Abdulmajeed S. Al-Ghamdi

June 2006
This dissertation entitled

ANALYSIS OF AIR-TO-AIR ROTARY ENERGY WHEELS

by

ABDULMAJEED S. AL-GHAMDI

has been approved for

the Department of Mechanical Engineering

and the Russ College of Engineering and Technology by

Khairul Alam

Moss Professor of Mechanical Engineering

Dennis Irwin

Dean, Russ College of Engineering and Technology
ABSTRACT

Al-GHAMDI, ABDULMAJEED S., Ph.D., June 2006. Integrated Engineering

ANALYSIS OF AIR-TO-AIR ROTARY ENERGY WHEELS (269 pp.)

Director of Dissertation: Khairul Alam

Integration of the energy recovery ventilator (ERV) with a traditional HVAC system has gained recent attention because of the potential for energy savings. The energy recovery ventilator selected for this research employs a rotary air-to-air energy wheel with a porous matrix as the heat and moisture transfer medium. The wheel is symmetrical and balanced, operating with counter-flow pattern. The primary goal of this research is to develop a mathematical and numerical model to predict the effectiveness of this energy wheel.

Three models were developed for the energy recovery ventilator (ERV) containing a porous exchange matrix: a sensible model, a condensation model, and an enthalpy model. The sensible model describes the heat transfer in a rotary wheel with a non-desiccant porous matrix. Two energy conservation equations were used to model this ERV. The condensation model describes the heat and mass transfer due to condensation and evaporation in a rotary wheel designed with non-desiccant porous matrix. The enthalpy model describes the heat and mass transfer in a rotary wheel using a desiccant porous media matrix in which adsorption and desorption can take place. Both the condensation and enthalpy models can be mathematically represented by two energy conservation equations with four basic thermodynamic relationships, and two mass conservation equations.
A non-dimensional representation for each model was derived and solved using the finite difference method with the integral-based method formulation scheme. The study examined the effect of wheel design parameters such as: rotational speed, number of transfer units, heat capacity, and porosity of the matrix. Operating parameters such as volume flow rate, inlet air temperatures and humidity ratios were studied, and the numerical results are compared with experimental results. Experimental data of effectiveness obtained from a commercial unit are found to be consistent with the numerical results.

Approved:

Khairul Alam

Moss Professor of Mechanical Engineering
ACKNOWLEDGMENTS

First of all, I would like to thank almighty God for helping and guiding me during my life and through my study and research. Then, I would like to express my sincere gratitude and appreciation to my adviser Professor M. K. Alam. Thank you for providing ideas, guidance, assistance, and editing throughout this research. I especially thank him for his thoughtful insights not only in scientific research but also in life issues.

I would like to extend my sincere appreciation to my Ph.D. committee members, Professor H. Pasic, Professor D. Gulino, Professor L. Herman, and Professor N. Pavel, for supporting and encouraging me to finish this work successfully.

Special thanks go to my wife, Fattmah, for all her patience, understanding, support, and love. I also wish to thank my family and friends for their support and encouragement.

Finally, I would like to acknowledge the help of Stirling Technology, Inc. for letting me use their experimental facility to conduct the experimental works required for my research. Thanks also go to their mechanical engineer, Jason Morosko.
TABLE OF CONTENTS

ABSTRACT ....................................................................................................................iv

ACKNOWLEDGMENTS ..................................................................................................vi

LIST OF TABLES .........................................................................................................xi

LIST OF FIGURES .......................................................................................................xiii

NOMENCLATURE .......................................................................................................xix

1. INTRODUCTION ...................................................................................................1
  1.1 Background ...........................................................................................................1
  1.2 Rotary Air-to-Air Energy Wheels .........................................................................3
    1.2.1 Principles of Operations ..............................................................................3
    1.2.2 Classification ..............................................................................................5
    1.2.3 Typical Wheel ..............................................................................................6
    1.2.4 Application ..................................................................................................7
  1.3 Review of Analysis of ERVs ..............................................................................11
    1.3.1 Non-Desiccant Wheels ..............................................................................11
      1.3.1.1 Sensible Wheels ...................................................................................12
      1.3.1.2 Condensation Wheels .........................................................................15
    1.3.2 Desiccant Wheels .......................................................................................16
      1.3.2.1 Sorption Wheels ..................................................................................17
      1.3.2.2 Enthalpy Wheels ..................................................................................18
  1.4 Research Objectives ..........................................................................................23
  1.5 Scope of Research ..............................................................................................24
  1.6 Overview of Thesis ............................................................................................26

2. THEORETICAL BACKGROUND ........................................................................27
  2.1 Energy Wheel Models .......................................................................................27
    2.1.1 Sensible Wheel: Willmott’s Model ..............................................................28
    2.1.2 Desiccant Wheel: Maclaine-Cross’s Model ...............................................37
    2.1.3 Enthalpy Wheel: Simonsons’s Model .........................................................41
    2.1.4 Discussion of Models .................................................................................45
  2.2 Effectiveness Correlations ................................................................................46
    2.2.1 The ε-NTU Method ....................................................................................46
  2.3 Performance Criteria .........................................................................................50
2.4 Desiccants Overview.................................................................53
2.4.1 Desiccant Isotherms..............................................................53
2.4.2 Desiccant Materials...............................................................54
2.4.3 Desiccant General Sorption Curve..........................................56
2.4.4 Desiccant Isotherm Model.......................................................57
2.5 Thermodynamics Relationships...............................................62
2.5.1 Properties of Moist Air.........................................................62
2.5.2 Properties of Matrix..............................................................66

3. MODELING OF ROTARY ENERGY WHEELS..............................68

3.1 Model Description.................................................................68
3.1.1 Coordinates System and Nomenclature..................................69
3.1.2 Surface Geometrical Properties..........................................71
3.1.3 Dimensionless Parameters..................................................73
3.2 Assumptions............................................................................75
3.3 Heat Transfer Model (Sensible Wheel Model).............................77
3.3.1 Governing Equations............................................................77
3.3.2 Dimensionless Representation..............................................81
3.3.3 Effectiveness Correlation for Limiting Cases...........................86
3.3.4 Finite Difference Equations..................................................92
3.4 Heat and Mass Transfer Model (Condensation Model).................102
3.4.1 Governing Equations............................................................102
3.4.2 Modeling of Condensation and Evaporation...........................116
3.4.3 Dimensionless Representation..............................................118
3.4.4 Finite Difference Equations..................................................124
3.4.5 Control Criteria for Condensation Model Computer Program.....128
3.5 Heat and Mass Transfer Model (Enthalpy Model).........................129
3.5.1 Governing Equations............................................................129
3.5.2 Dimensionless Representation..............................................134
3.5.3 Finite Difference Equations..................................................136
3.5.4 Modeling Mass Transfer in the Enthalpy Wheel.......................140
3.6 Computer Codes.................................................................142
3.6.1 Structure of Computer Programs.........................................142
3.6.2 Convergence and Stability..................................................145

4. MODELING RESULTS.............................................................147

4.1 Energy Wheel Characteristics and Boundary Conditions...............148
4.1.1 Energy Wheel.................................................................148
4.1.2 Boundary Conditions..........................................................152
4.2 Sensible Wheel Model............................................................153
4.2.1 Temperatures Profiles.................................................................154
4.2.2 Effect of Volume Flow Rate $\dot{Q}$ on the Wheel Effectiveness........163
4.2.3 Effect of Rotational Speed $\Omega$ on the Wheel Effectiveness..........165
4.2.4 Influence of $NTU$ on the Effectiveness of Wheel.........................167
4.2.5 Influence of Porosity $\phi$ on the Effectiveness of Wheel.................169
4.2.6 Summary..................................................................................173

4.3 Condensation Wheel Model ..........................................................174
4.3.1 Heating Mode Where the Condensation Might Take Place............174
4.3.2 Comparing the Sensible Effectiveness with Sensible Model Effectiveness.................................................................177
4.3.3 Effect of the Rotational Speed $\Omega$ on the Effectiveness..............179
4.3.4 Influence of the Inlet Conditions on the Wheel Performance.........182
4.3.5 Summary..................................................................................184

4.4 Enthalpy Wheel Model....................................................................185
4.4.1 Enthalpy Wheel Behavior..........................................................185
4.4.2 Effect of the Volume Flow Rate $\dot{Q}$ on the Effectiveness............192
4.4.3 Effect of the Rotational Speed $\Omega$ on the Effectiveness..............195
4.4.4 Effect of the Porosity $\phi$ on the Effectiveness of Wheel..............197
4.4.5 Effect of the Number of Transfer Unit $NTU$ and Lewis Number $Le$ on the effectiveness..........................................................200
4.4.6 Effect of the Isotherm Shape on the Effectiveness.......................204
4.4.7 Effect of the Supply Humidity on the Effectiveness.....................207
4.4.8 Summary..................................................................................208

5. EXPERIMENTAL DATA AND COMPARISON WITH THE ENTHALPY MODEL........................................................................................................211

5.1 Description of the Experimental Facility........................................212
5.1.1 Experimental Setup......................................................................212
5.1.2 RecoupAerator............................................................................214
5.1.3 Air Flow Measurement...............................................................216
5.1.4 Temperature and Humidity Measurement.....................................217
5.1.5 Pressure Measurement...............................................................217
5.1.6 Data Acquisition and Dasylab.....................................................217
5.2 Energy Wheel Under Investigation (RecoupAerator)........................219
5.3 Operation Conditions.................................................................220
5.4 Experimental Results and Comparison with the Enthalpy Model......222
5.4.1 Winter Tests................................................................................222
5.4.2 Summer Tests.............................................................................225
5.5 Summary......................................................................................228
LIST OF TABLES

Table 2.1 Range of parameters used in the Simonson’s effectiveness correlations….44
Table 2.2 Typical values of $H^*$ [93].………………………………………………52
Table 2.3 Values of the constants used in finding $P_{wv,s}$ according to ASHARE [24].………………………………………………………………63
Table 2.4 Value of the constants used in finding $T_{dew}$ according to ASHRAE [24]…64
Table 2.5 The specific enthalpies approximate input data……………………………65
Table 3.1 Effect of the number of spatial and time steps on the wheel effectiveness ………………………………………………………………………146
Table 4.1 Properties of material used as matrix core of the energy wheel..........149
Table 4.2 Fundamental air-to-air rotary energy wheel parameters [106]………..150
Table 4.2 Fundamental air-to-air rotary energy wheel parameters [106] (continued).151
Table 4.3 Wheel parameters and matrix properties used in numerical analysis……152
Table 4.4 Boundary conditions used in numerical analysis…………………….153
Table 4.5 Boundary conditions and wheel properties used in simulation………..153
Table 4.6 Simulation results and comparison with $\varepsilon – NTU$ method……….164
Table 4.7 Resulting sensible effectiveness from both sensible and condensation models………………………………………………………………177
Table 4.8 Wheel effectiveness validation for heating mode operation……………..194
Table 4.9 Wheel effectiveness validation for cooling mode operation……………..194
Table 4.10 Comparison of the wheel effectiveness for different R values for fixed $\sigma_{\text{max}} = 0.2$ (cooling mode)………………………………………204
Table 4.11 Comparison of the wheel effectiveness for different R values for fixed $\sigma_{\text{max}} = 0.2$ (heating mode)………………………………………206
| Table 5.1 | Measurement stations for the experimental work | 212 |
| Table 5.2 | Matrix properties and energy wheel parameters | 220 |
| Table 5.3 | Inputs data used by CAN/CSA-C439-00 | 221 |
| Table 5.4 | Experimental and numerical results for polyester during heating mode (Stirling Technology, Inc. experimental setup) | 223 |
| Table 5.5 | Comparison between experimental and numerical results for polyester during heating mode (Bodycote Materials Testing, Inc.) | 224 |
| Table 5.6 | Comparison between experimental and numerical results for polyester during cooling mode (Stirling Technology, Inc. experimental setup) | 226 |
| Table 5.7 | Comparison between experimental and numerical results for polyester during cooling mode (Bodycote Materials Testing, Inc.) | 227 |
LIST OF FIGURES

Figure 1.1  Schematic diagram of rotary energy wheel………………………………………..4
Figure 1.2  Space conditioning system with energy recovery unit [31]…………………8
Figure 1.2a  Using a sensible wheel (SW) in air cooling mode (summer) [31]………9
Figure 1.2a  Using an enthalpy wheel (EW) in air cooling mode (summer) [31]……9
Figure 1.3  Schematic chart of the ventilation cycle desiccant air conditioning system…………………………………………………………………………………11
Figure 2.1  Classification of energy recovery wheels based on mathematical modeling………………………………………………………………………………28
Figure 2.2  Schematic representation of rotary regenerator…………………………29
Figure 2.3  Finite difference mesh for the numerical solution for the regenerators [90]………………………………………………………………………………31
Figure 2.4  Coordinate diagram for the counter-flow rotary regenerator………………32
Figure 2.5  A schematic of the finite-difference element (Maclaine–cross [42])………41
Figure 2.6  A schematic of a typical air-to-air energy wheel layout for the four measuring stations……………………………………………………………………50
Figure 2.7  The Brunauer classification of gas physisorption isotherm [94]…………..53
Figure 2.8  General sorption isotherm shapes…………………………………………….57
Figure 2.9  Comparison of silica gel curve fitting isotherm shape with linear adsorption isotherm type…………………………………………………………60
Figure 2.10 Type 1 isotherm: adsorption isotherm shape for a molecular sieve……….61
Figure 2.11 Type III isotherm: adsorption isotherm shape…………………………61
Figure 3.1  Nomenclature and coordinate system for the energy wheel………………70
Figure 3.2  The energy balance of the unit volume element of the energy wheel……77
Figure 3.3  Boundary conditions for the flow streams in counter-flow regime..........85
Figure 3.4  Boundary conditions for the matrix in counter-flow regime..............86
Figure 3.5  Effect of $C_r^+$ on the rotary wheel effectiveness.........................90
Figure 3.6  Effect of low $C_r^+$ on the effectiveness of rotary sensible wheel........92
Figure 3.7  Sketch of grid representation of the numerical scheme for supply side (sweep in x-direction).................................................................94
Figure 3.8  Sketch of grid representation of the numerical scheme for exhaust side (sweep in x-direction).................................................................97
Figure 3.9  Finite difference grids for matrix boundary conditions at the entrance and exit.................................................................101
Figure 3.10  Heat and mass transfer control volume........................................102
Figure 3.11  Boundary conditions for the airflow in counter-flow energy wheel....122
Figure 3.12  Boundary conditions for the matrix in counter-flow energy wheel.....123
Figure 3.13  Sketch of grid representation of the numerical scheme for (a)- supply side and (b)- exhaust side.................................................................125
Figure 3.14  A flow diagram showing how control is transferred to the main parts of the condensation model computer program.................................129
Figure 3.15  Numerical switches between condensation and enthalpy models formulation.........................................................................................141
Figure 3.16  Flowchart outlining air-to-air rotary energy wheel effectiveness estimation program.................................................................144
Figure 4.1  Dimensionless temperature distributions for the air during hot and cold periods (summer run). The set of data lines shows progression to the steady state from initial point.................................................................155
Figure 4.2  Dimensionless temperature distributions for the matrix during hot and cold periods (summer run). The set of data lines shows progression to the steady state from initial point.................................................................155
Figure 4.3  Dimensionless temperature distributions for the air at the face of the wheel for the hot and cold periods (summer run).................................156
Figure 4.4  Dimensionless temperature distributions for the matrix at the face of the wheel for the hot and cold periods (summer run).................................157
Figure 4.5  Sensible effectiveness versus wheel cycle for (summer run)...........158
Figure 4.6  Dimensionless temperature distributions for the air during hot and cold periods (winter run). The set of data lines shows progression to the steady state from initial point.................................................................159
Figure 4.7  Dimensionless temperature distributions for the matrix during hot and cold periods (winter run). The set of data lines shows progression to the steady state from initial point.................................................................159
Figure 4.8  Dimensionless temperature distributions for the air at the face of the wheel for the hot and cold periods (winter run).................................160
Figure 4.9  Dimensionless temperature distributions for the matrix at the face of the wheel for the hot and cold periods (winter run).........................160
Figure 4.10 Sensible effectiveness versus cycle of the wheel (winter run).......161
Figure 4.11 Average supply and exhaust outlet conditions for heating and cooling modes (sensible wheel).................................................................162
Figure 4.12 Comparison of numerical and $\varepsilon - NTU$ method sensible effectiveness for cooling mode (summer).................................................................164
Figure 4.13 Variation of sensible effectiveness with high rotational speed......166
Figure 4.14 Variation of sensible effectiveness with low rotational speed......166
Figure 4.15 Variation of sensible effectiveness with rotational speed for two different NTU values.................................................................167
Figure 4.16 Variation of the sensible effectiveness with NTU....................168
Figure 4.17 Variation of the sensible effectiveness with NTU for different $Fo_m$ ....169
Figure 4.18 Variation of sensible effectiveness with porosity......................171
Figure 4.19 Variation of sensible effectiveness with porosity at different volume flow rates……………………………………………………………………171

Figure 4.20 Variation of sensible effectiveness with volume flow for different porosity ϕ…………………………………………………………172

Figure 4.21 Dimensionless temperature distribution of cold and hot gas as a function of wheel dimensionless length and period………………………175

Figure 4.22 Dimensionless temperature distribution of cold and hot matrix as a function of wheel dimensionless length and period……………………176

Figure 4.23 Dimensionless humidity ratio distribution of cold and hot gas as a function of wheel dimensionless length and period………………………176

Figure 4.24 Wheel effectiveness (sensible, latent, and total) as a function of wheel cycles…………………………………………………………….177

Figure 4.25 Comparison of sensible wheel effectiveness of both sensible and condensation models as a function of wheel cycles…………………..178

Figure 4.26 Average supply and exhaust outlet conditions for heating mode for (condensation wheel)……………………………………………………179

Figure 4.27 Comparison of wheel effectiveness of both sensible and condensation models as a function of wheel cycles for Ω = 20 rpm………………..180

Figure 4.28 Comparison of wheel effectiveness of both sensible and condensation models as a function of wheel cycles for Ω = 30 rpm……………….181

Figure 4.29 Comparison of wheel effectiveness of both sensible and condensation models as a function of wheel cycles for Ω = 40 rpm…………….…181

Figure 4.30 Performance variations under various rotational speeds with volume flow rate ̇Q = 250[CMH] and porosity φ = 90% …………………..182

Figure 4.31 Sensible and latent effectiveness of the condensation wheel for different inlet conditions……………………………………………………….183

Figure 4.32 Dimensionless temperature distribution of cold and hot gas as a function of wheel dimensionless length and period………………………186

Figure 4.33 Dimensionless temperature distribution of cold and hot matrix as a function of wheel dimensionless length and period………………………187
Figure 4.34  Dimensionless humidity ratio distribution of cold and hot gas as a function of wheel dimensionless length and period…………………...187

Figure 4.35  Wheel effectiveness (sensible, latent, and total) as a function of wheel cycles……………………………………………………………………...188

Figure 4.36  Dimensionless temperature distribution of cold and hot gas as a function of wheel dimensionless length and period……………………………...188

Figure 4.37  Dimensionless temperature distribution of cold and hot matrix as a function of wheel dimensionless length and period……………………………...189

Figure 4.38  Dimensionless humidity ratio distribution of cold and hot gas as a function of wheel dimensionless length and period……………………………...189

Figure 4.39  Wheel effectiveness (sensible, latent, and total) as a function of wheel cycles……………………………………………………………………...190

Figure 4.40  Enthalpy model outlets for both heating and cooling modes……...191

Figure 4.41  Wheel effectiveness versus volume flow rate for heating mode……...193

Figure 4.42  Wheel effectiveness versus volume flow rate for cooling mode……...193

Figure 4.43  Wheel effectiveness variations under various rotational speeds with $\varphi = 93\%$ for heating mode……………………………………………………195

Figure 4.44  Wheel effectiveness variations under various rotational speeds with $\varphi = 93\%$ for cooling mode……………………………………………………196

Figure 4.45  Effect of the wheel porosity on the wheel effectiveness for heating mode…………………………………………………………………………197

Figure 4.46  Effect of specific area on the wheel effectiveness for heating mode……...198

Figure 4.47  Effect of the wheel porosity on the wheel effectiveness for cooling mode…………………………………………………………………………199

Figure 4.48  Effect of specific area on the wheel effectiveness for cooling mode……...200

Figure 4.49  Effect of $NTU_{HT}$ on the wheel effectiveness for cooling mode……...201

Figure 4.50  Effect of $Le$ on the wheel effectiveness for cooling mode……...202
Figure 4.51 Effect of $NTU_{HT}$ on the wheel effectiveness for heating mode..........203
Figure 4.52 Effect of $Le$ on the wheel effectiveness for heating mode..................203
Figure 4.53 Effect of $\sigma_{\text{max}}$ on the wheel effectiveness for cooling mode with $R=1$..205
Figure 4.54 Effect of $\sigma_{\text{max}}$ on the wheel effectiveness for heating mode with $R=1$..206
Figure 4.55 Effect of $\phi_{r,i}$ on the wheel effectiveness for cooling mode.................207
Figure 4.56 Effect of $\phi_{r,i}$ on the wheel effectiveness for heating mode.................208
Figure 5.1 Picture of the test facility (Source: Stirling Technology, Inc.).................213
Figure 5.2 Instruments used to measure the flow rate, temperature, and humidity (Source: Stirling Technology, Inc.).................................................................214
Figure 5.3 Rotary energy wheel with matrix pies (Stirling Technology, Inc.)......215
Figure 5.4 RecoupAerator with air-to-air rotary energy exchanger (Source: Stirling Technology, Inc.).................................................................215
Figure 5.5 Experimental results shown by Dasylab (the results are updated every minute) at Stirling Technology, Inc. test facility.................................219
Figure 5.6 Supply and exhaust inlet conditions for both summer and winter operation (CAN/CSA-C439-00 standard).................................................................222
Figure 5.7 Comparison between experimental and numerical results for polyester during heating mode (Stirling Technology, Inc. experimental setup)......224
Figure 5.8 Comparison between experimental and numerical results for polyester during heating mode (Bodycote Materials Testing, Inc.).........................225
Figure 5.9 Comparison between experimental and numerical results for polyester during cooling mode (Stirling Technology, Inc. experimental setup)......227
Figure 5.10 Comparison between experimental and numerical results for polyester during cooling mode (Bodycote Materials Testing, Inc.).........................228
NOMENCLATURE

ACRONYMS

ASHRAE  American Society of Heating, Refrigerating and Air Conditioning Engineers, Inc.
CW     Condensation Wheel
DCS    Desiccant Cooling System
ERV    Energy Recovery Ventilators
EW     Enthalpy Wheel
HVAC   Heating, Ventilating and Air Conditioning
HRV    Heat Recovery Ventilator
IAQ    Indoor Air Quality
RH     Relative Humidity
STI    Stirling Technology Inc.
SW     Sensible Wheel

ENGLISH SYMBOLS

\[ A_w = \frac{\alpha}{100} \left( \frac{\pi D_w^2}{4} \right) \]
\[ A_g = \varphi A_w \]
\[ A_m = (1 - \varphi) A_w \]
\[ A_s = \frac{(1 - \varphi) \pi D_w^2 L_w}{2d_f} \]
\[ A_v = \frac{A_s}{V_{tot}} \]
\[ C_{p_g} \] specific heat of the gas \([J/kg^\circ K]\)
\[ C_{p_m} \] specific heat of the matrix \([J/kg^\circ K]\)
\[ C_{p_{wv}} \] specific heat of water vapor \([J/kg^\circ K]\)
\( C_{p_{wl}} \) specific heat of water liquid [\( J/kg\,°K \)]
\( C_g \) gas capacity rates [\( W/s \)], \( C_g = (\dot{m}C_p)_g \)
\( C_g^+ \) ratio of supply to exhaust gas capacity rate
\( C_r \) capacity-rate of the rotary (matrix) [\( W/s \)], \( C_r = (MCP)_m\Omega \)
\( C_r^+ \) heat capacity-rate ratio of the rotary wheel, \( C_r^+ = \frac{C_r}{C_g} \)
\( D_w \) diameter of the wheel [\( m \)]
\( d_f \) diameter of the fiber [\( m \)]
\( d_h \) hydraulic diameter [\( m \)]
\( F_{o_m} \) the Fourier number of matrix, \( F_{o_m} = \frac{\alpha_m\tau}{L_w^2} \)
\( h_{ad} \) differential heat of adsorption [\( KJ/Kg \)]
\( h_{vap} \) specific heat of vaporization [\( KJ/Kg \)]
\( h_{HT} \) convective heat transfer coefficient [\( W/m^2.°K \)], \( h_{HT} = \frac{NuK_g}{d_h} \)
\( h_{MT} \) mass transfer coefficient, [\( kg/s.m \)], \( h_{MT} = \frac{h_{HT}}{LeCp_g} \)
\( h_{fg} \) heat of vaporization [\( J/kg \)]
\( i \) index of stepping along x-direction
\( j \) index of stepping along y-direction
\( k \) integer
\( k_g \) thermal conductivity of the gas [\( W/m.°K \)]
\( k_m \) thermal conductivity of the matrix [\( W/m.°K \)]
\( L \) length of regenerator [\( m \)]
\( L_w \) length of the wheel [\( m \)]
\( Le \) lewis number, \( Le = \frac{NTU_{HT}}{NTU_{MT}} \)
M \text{ total mass of energy wheel matrix [kg]}

\dot{m}_g \text{ mass flow rate of gas [Kg,\,s]}

\dot{m}' \text{ rate of phase change per unit length [kg/s,m]}

NTU_{HT} \text{ number of the heat transfer unit, } NTU_{HT} = \frac{h_{HT}A_s}{\dot{m}_g C_p_g}

NTU_{MT} \text{ number of the mass transfer unit, } NTU_{MT} = \frac{h_{MT}A_s}{\dot{m}_g}

Nu \text{ Nusselt number, } Nu = \frac{h_{HT}d_h}{K_g}

n \text{ integer}

NTU_0 \text{ modified number of transfer units (for balanced wheel), } NTU_0 = \frac{NTU}{2}

P \text{ length of period and it is the sum of the hot-gas flow period } P_h, \text{ and cold-gas flow period } P_c

P_{atm} \text{ atmospheric pressure [Pa]}

P_{wv} \text{ partial pressure of water vapor in the air [KPa]}

P_{s-wv} \text{ partial pressure of water vapor at saturation [KPa]}

Pr \text{ Prandtl number, } Pr = \frac{\mu_g C_p_g}{K_g}

\dot{Q} \text{ volume flow rate [m$^3$/s]}

R \text{ gas constant for water [KJ/Kg$^\circ$K]}

R \text{ separation factor that defines the isotherm shape}

Re \text{ Reynolds number, } Re = \frac{\rho_g U D_f d_f}{\mu_g (1 - \varphi)}

T \text{ bulk temperature [K or \circ C]}

T_{g} \text{ temperature of the gas (air-stream) [\circ K]}

T_{g}^{+} \text{ non-dimensional temperature of gas, } T_{g}^{+} = \frac{T_g - T_{g,s}}{T_{g,s} - T_{g,c}}
\( T_m \) temperature of matrix \([ ^{\circ}K ]\)

\( T_m^+ \) non-dimensional temperature of matrix, \( T_m^+ = \frac{T_m - T_{g_{-s,i}}}{T_{g_{-s,i}} - T_{g_{-e,i}}} \)

\( t \) time \([ s ]\)

\( t^+ = \frac{t}{\tau} \) non-dimensional time

\( U_g \) superficial mean gas velocity \([ m/s ]\), \( U_g = \frac{\dot{Q}}{(\alpha/100)A_g} = \frac{U_D}{\varphi} \)

\( U_D \) Darcian velocity \([ m/s ]\), \( U_D = \frac{\dot{Q}}{(\alpha/100)A_w} \)

\( u \) moisture content of matrix \([ kg_{wv}/kg_{dm} ]\), \( \sigma_m = \frac{m_i}{m_{dm}} \)

\( V_{tot} \) total volume of half split wheel \([ m^3 ]\)

\( V_m \) solid volume of wheel (matrix) \([ m^3 ]\)

\( V_g \) fluid volume (air stream) \([ m^3 ]\)

\( x \) axial coordinate \([ m ]\)

\( x^+ = \frac{x}{L_w} \) non-dimensional length

\( y \) distance from regenerator entrance \([ m ]\)

\( Z \) temperature \( T_g \), humidity ratio \( \omega_g \), or enthalpy \( h_g \)

**GREEK SYMBOLS**

\( \alpha \) wheel split \( \alpha/100 \), (balance wheel: 50/50)

\( \alpha_g \) thermal diffusivity of the gas \([ m^2/s ]\), \( \alpha_g = \frac{K_g}{\rho_g C_{p_g}} \)

\( \alpha_m \) thermal diffusivity of the matrix \([ m^2/s ]\), \( \alpha_m = \frac{K_m}{\rho_m C_{p_m}} \)

\( \Delta \) difference
\( \Delta H_i \) latent energy difference between supply and exhaust inlet conditions 
\[ [kJ/kg_{da}] \]

\( \Delta H_s \) sensible energy difference between supply and exhaust inlet conditions 
\[ [kJ/kg_{da}] \]

\( \Delta T_g \) temperature difference between supply and exhaust inlet conditions \([\degree K]\)

\( \Delta \omega_g \) humidity ratio difference between supply and exhaust inlet conditions 
\[ [kg_{wv}/kg_{da}] \]

\( \omega_g \) humidity ratio of moist air (water content in dry air) \([Kg_{wv}/Kg_{da}]\),
\[ \omega_g = \frac{m_{wv}}{m_{da}} \]

\( \omega_s \) humidity ratio in equilibrium with matrix surface (saturated),
\[ [Kg_{wv}/Kg_{da}] \]

\( \sigma_m \) moisture content of matrix \([kg_{wv}/kg_{dm}]\),
\[ \sigma_m = \frac{m_i}{m_{dm}} \]

\( \sigma_{max} \) loading of desiccant at 100% relative humidity, \([kg_{wv}/kg_{dm}]\)

\( \rho_g \) density of the gas \([kg/m^3]\)

\( \rho_{da} \) density of the gas \([kg/m^3]\)

\( \rho_m \) density of matrix \([kg/m^3]\)

\( \rho_{wv} \) density of the water vapor in the air \([kg/m^3]\)

\( \rho_{v,m} \) water vapor density on the surface of the matrix \([kg/m^3]\)

\( \mu_g \) dynamic viscosity of the gas \([kg/m.s]\)

\( \phi \) relative humidity

\( \varepsilon_s \) sensible energy wheel effectiveness

\( \varepsilon_l \) latent energy wheel effectiveness

\( \varepsilon_{tot} \) total energy wheel effectiveness
\( \eta \)   dimensionless time, \( \eta = \frac{h_{HT} A_t t}{(mC_p)_m} \)

\( \eta_{\text{reg}} \)   thermal ratio (regenerator effectiveness)

\( \xi \)   dimensionless length, \( \xi = \frac{h_{HT} A_{x,y}}{(mC_p)_g L} \)

\( \phi \)   porosity of the matrix

\( \Gamma \)   dimensionless period, \( \Gamma = \frac{u_g \tau}{L_w} \)

\( \tau \)   length of one complete cycle of the wheel [s]

\( \Omega \)   rotational speed [rpm]

\( \Lambda \)   reduced length, \( \Lambda = \frac{h_{HT} A_s}{(mC_p)_g} \)

\( \Pi \)   reduced period, \( \Pi = \frac{h_{HT} A_s P}{(mC_p)_m} \)

**SUBSCRIPTS**

- \( a \)   air
- \( c \)   cold flow stream
- \( d \)   desiccant
- \( da \)   dry air
- \( e \)   exhaust flow to the energy wheel
- \( g \)   gas flow
- \( h \)   hot flow stream
- \( i \)   inside flow to the wheel or at a point “i” in x-space direction
- \( j \)   at point j in y-space direction of the regenerator
- \( l \)   water liquid
- \( m \)   matrix surface
- \( \text{min} \)   minimum value
- \( \text{max} \)   maximum value
outside flow from the wheel
supply flow to the energy wheel
saturation value
rotor matrix
regenerator
water vapor
water liquid

**SUPERSCRIPrTS**

cold flow stream
hot flow stream
the current time step
saturated
water vapor
non-dimensional quantity
non-dimensional quantity
average value
CHAPTER ONE
INTRODUCTION

1.1 Background

Indoor air quality (IAQ) which describes the nature of air inside the building is an important factor for occupants who spend 85-90% of their time indoors [1]. The American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) defines the indoor air quality as "an acceptable comfort level to 80% of the people exposed to it" [1]. Maintaining acceptable IAQ can be achieved by bringing fresh outside air into a building by either opening windows or using mechanical ventilation. For energy saving purposes, modern buildings are tightly built to insulate them from the outside environment. Therefore, mechanical ventilation is the appropriate way to bring fresh outdoor air into the sealed house and exhausts the contaminated indoor air while preserving energy [2]. Adequate ventilation rates are required to provide acceptable Indoor Air Quality (IAQ) [3].

In 1950, mechanical ventilation in buildings became common. However, mechanical ventilation dates backed to the early 1900s [4]. In 1973, the American Society of Heating, Refrigeration and Air Conditioning Engineers (ASHRAE) approved the first version of Standard 62, which was titled Standard 62-73, "Standard for Natural and Mechanical Ventilation" [5]. This Standard required a "minimum of 10 cfm per person of outside air, but recommended outside air ventilation rates in the 15 to 25 cfm
per person range" [5]. In 1981, ASHRAE Standard 62-73 was revised as Standard 62-81 and renamed "Ventilation for Acceptable Indoor Air Quality" [6]. ASHRAE Standard 62-1989 specified "minimum ventilation rates and indoor air quality that will be acceptable to human occupants and is intended to minimize the potential for adverse health effects" [7]. Since then, the Standard 62-1989 was under continuous maintenance, which allowed ASHRAE to update the standard on an ongoing basis through the addition of addenda and new sections [7,8]. A number of additional addenda are under development to convert the Standard 62 into a "code-intended language" [9].

Controlling humidity inside the house is an important factor measuring indoor environmental quality. Microbial growth and building material tear are the results of too humid air (above 60%), while human discomfort from dry skin to respiratory irritation results from too dry indoor air (below 30%) [10]. For these reasons, ASHRAE has developed a thermal comfort Standard 55-1992 [11], which identifies temperature and humidity conditions (30-60%) that will satisfy 80% or more of building occupants. Green [12] has conducted research in schools located in cold dry regions to examine the effect of humidity increase on student health. Results indicate that increasing the relative humidity from 20% RH to 40% RH can reduce absenteeism and upper respiratory inflections by 50%.

Importance of relative humidity as indicated in [13] can be observed from its effect on: thermal comfort [14], the perception of IAQ [15], occupant health [12,16], and energy consumption [17,18]. In normal weather conditions, well-designed heating, ventilation, and air-conditioning (HVAC) systems can save energy and provide an
acceptable indoor climate [19]. In hot-humid and cold-near-saturation weather, integrating heat/energy recovery devices to air-conditioning design is becoming a popular choice to meet the market requirements: control of humidity, energy, and IAQ [1,20].

In the 2002 ASHRAE equipment handbook [21], the most popular six types of air-to-air heat exchange devices are presented. Some are sensible heat exchangers and the rest are total energy exchangers (sensible and latent). The rotary air-to-air energy wheel is one of these types which can be designed as sensible or total. Using an energy wheel (sensible and latent) leads to increased comfort while reducing HVAC system capital and operating costs. In terms of energy recovery, the total heat recovery device is more superior to the sensible heat recovery device (three times more) [21]. From the typical effectiveness and pressure drop data for different recovery devices, it is found that the energy wheel has the highest effectiveness and least pressure drop at any face velocity, making it the most appropriate choice for energy recovery in comfort ventilation [21].

1.2 Rotary Air-to-Air Energy Wheels

1.2.1 Principles of Operations

Energy recovery technologies known as heat recovery ventilators (HRVs) or energy recovery ventilators (ERVs) employ a rotary air-to-air energy wheel as a main part of its components. The performances of the recovery ventilators are strongly dependent on the types and configurations of energy wheel. Exchange media of the energy wheel can be designed to recover sensible heat or total energy (sensible and latent). In typical applications, ventilators can be positioned in a duct system or as stand-
alone to supply the house with fresh outdoor air while exhausting indoor air out. Using energy recovery ventilators in HVAC applications can benefit the system by reducing energy consumption, improving indoor air quality (IAQ), and downsizing of HVAC systems [22].

A rotary air-to-air energy wheel is a rotating cylindrical wheel with frontal area designed with a small diameter flow passages for air to pass through from one side to another [22]. In a typical installation, the rotating wheel is positioned in a duct system such that it is divided into two equal sections. As the wheel rotates at constant rotational speed $\Omega$, half of the frontal area takes the fresh outdoor air and supplies it to the indoor while indoor air is drawn through the other half in a counter flow pattern, as shown in Figure 1.1 [23].

![Schematic diagram of rotary energy wheel](image)

**Figure 1.1** Schematic diagram of rotary energy wheel
In the rotary energy wheel, heat and moisture exchanges between supply and exhaust air stream are accomplished in an indirect manner. As the heat transfer media rotates between the supply and exhaust air stream the medium picks up the heat from the supply hot air, stores it and releases it to the cold exhaust air during the second half of wheel rotation [24]. Moisture transfer can be done in the same manner; the transfer medium captures the moisture from the high humid air stream during the first half of the wheel rotation and releases it to the low humid air stream during the second half. The wheel transfer medium captures the moisture from the high humid air stream by condensation when medium temperature is lower than the dew point temperature of the air stream or by adsorption when the medium is desiccant (material that can absorb moisture). During the second half of the wheel rotation, the medium releases the moisture to the low humid air stream through the evaporation in the case of condensation or by desorption process in the other case [24].

1.2.2 Classification

Three types of heat and mass rotary energy wheels are used in air conditioning system [25]. They are namely the sensible heat wheel, the dehumidifier wheel, and the enthalpy (sensible and latent heat) wheel. For the same inlet air temperatures and humidities, each wheel operates with the same principles however different outlet temperatures and humidities are expected due to the capability of the wheel to transfer heat and moisture. The performance characteristics of the wheel are largely determined by the physical properties of the porous matrix. Sensible heat wheels require a non-
sorbing matrix with large thermal capacity since only heating and cooling of air streams is desired. Maximum moisture transfer is important in dehumidifier wheels, so they utilize a sorbent matrix with large moisture capacity but small thermal capacity. Enthalpy wheels employ a sorbent matrix with large thermal and moisture capacity, since both heat and mass transfer are desired.

Sensible heat wheels are used for heat recovery from exhaust air in air conditioning, but total heat wheels give great total heat recovery in hot humid and cold areas [26]. Dehumidifier wheels are used when low humidities are required in air conditioning [26].

1.2.3 Typical Wheel

Jeong [27] reviewed data from several energy wheel manufactures catalogs and concluded that there are common features of enthalpy wheels in the market. The majority of commercial wheels are made up of a honeycomb type aluminum matrix coated by silica gel or 3–4 Å molecular sieves [27]. Tiny sinusoidal flutes of the matrix are designed by thin corrugated aluminum foil. Desiccants mass in matrix take about 20% of total enthalpy wheel mass. Also, enthalpy wheel matrix can be designed using non-metallic materials, such as paper, plastic, synthetic fiber or glass [27].

Moreover, configuration of the most available commercial wheel on the market is of balanced flow type, where the wheel is divided into half equal sections and the flow is ducted into indoor and outdoor equally. The speed of the wheel ranges from 5 rpm to 40 rpm.
Recently, there is a new trend in designing energy wheels. The exchange media (matrix) is designed using highly porous material supported by a plastic case. This style of design is expected to show better performance than the tubular one.

1.2.4 Application

For a typical U.S home, 56% of energy use is due to heating and cooling applications [19]. Traditional vapor compression air-conditioning and dehumidification systems were first introduced to the market for heating and cooling buildings. However, they are considered to be inefficient in a thermodynamic sense, which led to the introduction of a wide range of new technologies in the air-conditioning market [28].

Integrating the energy recovery exchangers into the space conditioning system improves the performance of the system and reduces the energy consumption. Rotary types are the most commonly used in the space conditioning systems. Specifically, they are the rotary sensible and enthalpy wheels. However, the rotary enthalpy wheel is superior to the sensible one due to its potential to transfer both heat and moisture.

Enthalpy wheels with desiccant coated matrix are characterized by transferring water in the adsorbed phase rather than condensed phase. This allows the enthalpy wheel to run at lower outdoor temperatures in the winter without ice blocking the matrix channels [29,30]. In the sensible wheel, ice blocking of matrix and running water are possible in such weather [29,30].

To discuss the basic performance differences between the sensible heat exchanger and the enthalpy exchanger, a typical application of these two exchangers in a space-
conditioning system is depicted in Figure 1.2 [31]. In summer, state-1 represents hot outdoor air which will be cooled by colder exhaust air state-5. In winter, state-1 represents cold outdoor air which will be preheated by warmer exhaust air state-5.

To demonstrate the performance of both sensible and enthalpy wheels in the air-conditioning application, the summer (cooling mode) season scenario will be used. In the air-conditioning application, plotting the inlets and outlets of such system on a psychrometric chart is a convenient way to investigate its performance. In the case of sensible wheel (Figure 1.2a), the humidity ratios of the two air streams remain constant throughout the process since only heat transfer is possible. In the case of enthalpy wheel (Figure 1.2b), the humidity ratios change due to the combined heat and mass transfer process. Because of this, the outlet states are on a straight line between the two inlet states.
Figure 1.2a Using a sensible wheel (SW) in the air cooling mode (summer) [31]

Figure 1.2b Using an enthalpy wheel (EW) in the air cooling mode (summer) [31]
One can see from Figures 1.2a and 1.2b, the amount of energy that can be recovered by the enthalpy exchanger ($\Delta H_{EW}$) is considerably greater than the amount of energy recovered by a sensible heat exchanger ($\Delta H_{SW}$) [31].

Moreover, Rengarajan et al. [32] and Shirley and Regenarajan [33] calculated the energy saving and operational benefits of energy wheels for large and small office buildings in Florida. In hot and humid climates, an enthalpy wheel can transfer 2 to 3 times as much energy as a sensible heat wheel of similar size. They found that the energy required to dehumidify outdoor air can be several times greater than the sensible energy required to cool the air.

Applications of energy wheels in the desiccant cooling cycles have received great attention recently. Variety of desiccant cooling cycles have been proposed and investigated extensively by Jurinak [34]. Figures 1.3 shows the schematic chart of the ventilation solid desiccant air cooling cycle with two regenerative components [34]. The rotary sensible heat wheel consists of a non-hygroscopic matrix whereas the rotary enthalpy contains a solid desiccant material (hygroscopic).

In the ventilation cycle [34], the desiccant dehumidifier wheel takes the fresh outside air (state 1) and humidifies it (state 2). During the sorption process (water vapor adsorbed by the desiccant material), the temperature of the air also rises due to the heat of sorption release. Sensible heat wheel is used to cool the air stream at (state 2) without changing its humidity ratio to (state 3) using evaporatively cooled exhaust air (state 6). The desired level of the temperature and humidity of supply air stream is not accomplished so the supply air stream is evaporatively cooled (state 4) and passed to the
space to be conditioned (state 5). The sensible heat wheel picks up the heat from hot air stream (state 2) and releases it to the exhaust cold air stream (state 7). If the temperature at (state 7) is not enough to regenerate the matrix of the dehumidifier wheel, additional heat source is needed to reach the required regeneration temperature (state 8). Finally, the desiccant matrix is regenerated (moisture removed) by the hot air stream (state 9).

Figure 1.3 Schematic chart of the ventilation cycle desiccant air conditioning system

1.3 Review of Analysis of ERVs

1.3.1 Non-Desiccant Wheels

A rotary air-to-air sensible wheel with non-desiccant matrix usually transfers only sensible heat from the hot to the cold air stream through the transfer media (matrix). On a psychrometric chart, the resulting outlet states are both on horizontal lines with inlet
states which represent the corresponding inlet humidity ratios. Therefore, the sensible wheel is used to heat or cool the air stream without changing its humidity ratio.

1.3.1.1 Sensible Wheels

Early on the development of modeling sensible heat regenerators, lack of a closed-form analytical solution to the deriving differential equations limited the advancements in the industry of regenerators. The available solutions were either an analytical solution to a simplified problem or graphical solutions using complicated graphical techniques. Therefore, wide investigation of design and operation parameters on the performance of regenerators was complicated. A comprehensive description of the periodic flow regenerator was first introduced by Coppage and London [35].

With advancements on the computer technology, Lamberston [36], Bahnke and Howard [37], and Mondt [38] developed more practical and complete solutions of regenerators by employing a finite difference method. They considered two coupled cross-flow heat exchangers with unmixed fluids. They used a calculation procedure in which a matrix temperature distribution was assumed on the left edge of matrix, and with this initial estimate of all element temperatures, the Gauss-Seidel point-by-point marching procedure was applied. This method has been a typical standard numerical calculation procedure for the rotary periodic-flow heat exchanger.

Lambertson [36] developed a numerical model for an ideal regenerator (no fluid carryover and no axial conduction in the matrix). He used a finite difference method to find out the temperature distributions in the matrix and air stream. The exchanger effectiveness was evaluated over a certain range of dimensionless parameters. Bahnke
and Howard [37] developed a numerical finite difference method of calculating the effectiveness of the rotary sensible heat exchanger accounting for the effect of longitudinal heat conduction in the direction of the fluid flow. The method considers the metal stream (rotary motion of the matrix) in cross-flow with each of the gas streams as two separate but dependent heat exchangers. Bahnke and Howard’s work was an extension of Lambertson's work. Mondt [38] studied the effect of axial conduction on the temperature distributions in the matrix and fluid to facilitate a thermal stress analysis.

Harper [39] developed a theory for the rotary heat exchanger for gas turbine application. Mainly, his work focused on predictions and counts for a carryover and pressure leakage. An analysis and experimental investigation of seal leakage were presented. Harper’s work was an excellent contribution to the subject of seal leakage in rotary regenerators.

London et al. [40] studied the transient response of the outlet fluid temperatures of a counter-flow regenerator of both types: direct-transfer and periodic-flow type. Step input change of one of the inlet fluid temperatures was used to investigate the transient response of the outlet temperatures. Design tables and charts (neglecting interstitial fluid heat capacity) have been given by Kays and London [41] to determine effectiveness. Maclaine-Cross [42] developed a finite difference program to calculate the temperature efficiency including the interstitial fluid heat capacity effect. He found that the effect of interstitial fluid capacity on regenerator performance is almost the same as the effect of an increase in matrix specific heat together with an increase in heat transfer coefficient.
Shah [43] and Kays and London [41] summarized the basic thermal design theory of the rotary regenerator which includes the effects of rotation, longitudinal and transverse conduction, pressure leakage, and carryover. Detailed step-by-step solution procedures were outlined for rating (performance) and sizing (design) of counter-flow rotary regenerator. The \( \varepsilon - NTU \) method of analysis first developed by Coppage and London [35] was used in the thermal design procedures of counter-flow regenerator. These summaries contain many design charts and design equations that allow a designer to predict the performance of sensible rotary heat exchangers under various operating conditions. Shah [44] has combined these charts and equations into a structured design methodology for sensible rotary heat exchangers.

Willmott [45-49] published a series of papers in which he developed an open method (numerical method) for solving the differential equations of the regenerative heat exchanger. Based on his method, several studies of the performance of the regenerator under different conditions were conducted. Iliffe [50] also developed a method which gave identical solutions.

Willmott [46,47] developed three-dimensional equations for regenerator taking the account of the thermal conductivity in the direction perpendicular to the direction of the gas flow. The effect of the simplifying assumption that converts the problem to two dimensions was examined.

More recently, Druma and Alam [51] developed a finite difference method for finding the effectiveness of the rotary counter-flow recovery ventilator using porous media matrix. The method was based on using an upwind integral-based method. The
results were compared with $\varepsilon - NTU_0$ method and experimental data. The numerical method shows a very close result to the $\varepsilon - NTU_0$, and is comparable to the experimental data. They showed that the porous matrix produces very high effectiveness due to high surface area. Buyukalaca and Yilmaz [52] developed numerical solutions of the governing differential equations of the regenerator for various values of the number of transfer units and heat capacity-rate of the matrix.

1.3.1.2 Condensation Wheels

A few heat and mass transfer models of the condensation wheels exists in the literature. A search of the literature available to the author yielded references [54-58] which explain the phenomena of condensation in the rotary sensible heat wheel.

Maclaine-Cross [42] first discussed the problem of condensate forming inside a sensible heat regenerator. As a continuation of Maclaine-Cross’s work, Van Leersum [53] investigated the performance of a sensible wheel in which condensation occurs.

Van Leersum [54] compared the performance of a numerical model with experimental results and obtained good agreement within the experimental limitations. He also pointed out that there was a lack of a fully non-dimensional model, which prevents the drawing of general conclusions of the model without performing a large number of experimental works.

Van leeersum and Banks [55] modeled the processes which occur when air blows through a non-sorbing matrix with conditions such that water condenses on the matrix
using a theoretical model in which air, matrix, and condensed water are in thermodynamic equilibrium at any location.

Holmberg [56] described numerical experiments with a model of heat and mass transfer in rotary regenerators having non-hygroscopic matrix material. Holmberg’s model permits both condensate and ice to form on the matrix. Also, it assumes the matrix to have a finite thermal resistance in the fluid flow direction. Comparing Holmberg’s model with the model used by Van leersum [54] shows that it is less restrictive.

In the operation of the sensible wheel, excess water and freezing on the matrix should be avoided. This is because excess water could result in leakage and undesirable mold growth while freezing could lead to poor performance by blocking the matrix [57].

1.3.2 Desiccant Wheels

Air dehumidification and enthalpy recovery can be accomplished by using desiccant wheel [58]. In the air dehumidification, supply air is dehumidified as it flows through sportive matrix (dry and hot). The wheel rotates continuously between the supply air and exhaust air stream (regenerative air stream). In the enthalpy recovery, the wheel operates differently in summer and winter due to different operation conditions. In the summer, excess heat and moisture would be transferred to the exhaust to cool and dehumidify the supply air. In winter, heat and humidity would be recovered from the exhaust air stream [58].

As a result of the above discussion, the wheels are classified into two different types based on its operation: (a) the sorption wheels (desiccant rotary dehumidifiers) in
which the emphasis is on air drying: this requires the use of external heat source to
regenerate the desiccant: and (b) the enthalpy wheels focus on the energy exchange
(sensible and latent energy) between the air streams and use no external source for
regeneration of the desiccant.

1.3.2.1 Sorption Wheels

There have been many works in modeling heat and moisture transfer in desiccant
wheels. Van Den Bulck et al. [59,60] introduced a wave analysis to establish a one-
dimensional transient model of the rotary heat and mass exchanger with infinite transfer
coefficients and \( \varepsilon - NTU \) method to analyze the same model with finite transfer
coefficients. Maclaine-Cross and Banks [61] analyzed a desiccant dehumidifier wheel
using the analogy between the heat and mass transfer process. Using the analogy method,
the heat transfer solution is used to predict mass transfer. Later, Maclaine-Cross [42]
developed the MOSHMX program using a finite-difference method to analyze the heat
and mass transfer in a desiccant wheel.

Jurinak and Mitchell [62] developed a mathematical and numerical mode of
dehumidifier wheel. Their model is one-dimensional and transient for heat and moisture
transfer between desiccant and air streams. Matrix sorption properties effect on the wheel
performance was studied in great detail.

Zheng and Worek [63] developed a new implicit finite-difference method for
simulating the combined heat and mass transfer process in a desiccant dehumidifier.
Zheng et al. [64,65] optimized the performance of a desiccant wheel by controlling the rotational speed.

Tauscher et al. [66] performed several experiments to investigate different materials that might be used in a rotary exchanger application. Investigation of this material with different flow channel geometries was the aim of his work. His numerical model, which describes the process of heat and mass transfer in different channel configurations, was developed to make a comparison with experimental results.

San and Hsiau [67] developed a mathematical model to investigate heat and mass transfer in a regenerator. Their focus was to examine the effects of heat conduction and mass diffusion on regenerator effectiveness. In their model, the storage terms are neglected. They developed a full-implicit finite difference method to solve the model’s equations. They concluded that zero resistance leads to poor performance.

1.3.2.2 Enthalpy Wheels

Several models have been developed for sorption wheels as discussed before. Most of the previous studies on the desiccant wheels have not considered the conditions relevant to the enthalpy wheels. The focus was instead on the analysis of the various solid desiccant cycles using the sensible heat wheels as well as dehumidifier wheels.

The first attempt to develop a numerical model for the enthalpy wheel was introduced by Holmberg [68]. He numerically solved the coupled governing equations for the enthalpy wheel using the finite difference method. In his model, the storage terms of energy and moisture in the air within the wheel were neglected due to its small value in
the enthalpy recovery application. Later Holmberg [69] experimentally investigated the performance of hygroscopic and non-hygroscopic rotary wheel. He discussed the prediction of condensation and frosting limits in rotary wheels for heat recovery in buildings and the frosting-defrosting process. His study establishes guidelines for prediction of condensation and frosting in wheels.

Klein et al. [30] developed a computationally simple model of a solid desiccant air-to-air enthalpy wheel. They established the minimum rotation speed under which maximum enthalpy exchange can be accomplished. Further, the equilibrium exchanger model they used was the basis for development of enthalpy wheel effectiveness. They found that the maximum possible effectiveness for a counter-flow direct type heat exchanger for both the temperature and humidity can be expressed by a simple correlation based on the appropriate number of transfer units. However, their model is not applicable at the reduced speeds necessary to prevent condensation or freezing of water in the exhaust stream under conditions experienced during winter operation.

Stiech et al. [70] developed a computationally simple model for determining the effectiveness of the enthalpy wheel as a function of the air inlet and matrix rotation speed. Their results were based on the desiccant model of Maclaine-Cross [42]. In the range of enthalpy wheel operation, their numerical results agree with the manufacture’s performance data. Banks [71,72] pointed out that the appropriate methods for modeling a desiccant dehumidifier and the enthalpy wheel may not be the same because the operating conditions are significantly different, even though the fundamental physics and surface chemistry of the desiccant drying and energy exchange are similar.
Simonson [73,74] developed a numerical model to investigate the importance of these assumptions for a specific application and validated his model using the experimental work performed by Ciepliski [75]. He concluded that all effectiveness decreases as the wheel speed decreases. Also he showed that axial conduction through the matrix is important for the type of energy wheel (aluminum desiccant coated matrix) that he investigated. Neglecting the axial conduction can lead to a 6% error in predicting the sensible effectiveness. In modeling the sensible and dehumidifier wheels, the transient storage of energy and moisture in the air stream is often neglected compared to the storage in the matrix. He concluded that neglecting the transient storage in the air can lead to the errors in the predicted effectiveness of 2.4% in hot-humid test conditions.

Recently, Simonson and Besant [76-83] published a series of papers in modeling the heat and mass transfer in a rotary energy wheel with tubular flow path. Simonson and Besant [76] developed a numerical model for a coupled heat and moisture in the rotary energy wheel. Their model is one dimensional, transient, and is formulated using the finite volume method with an implicit time discretization. The accuracy of the model has been confirmed using experimental data [75]. The agreement between the measured and simulated effectiveness (sensible, latent, and total energy) was close and within the experimental uncertainty.

Simonson and Besant [77] extended their model which models the adsorption and desorption processes to include condensation and frosting. They used two different numerical algorithms: one for the sorption process and one for saturation conditions. The
numerical switch between the two codes was based on the relative humidity and the rate of phase change.

Simonson and Besant [78] also investigated the sensitivity of condensation and frosting to the wheel speed and desiccant types using the above model [77]. They presented the performance of the energy wheel during both sorption process and saturation conditions. They selected two types of wheels for their study. Namely, they are the wheel with matrix coated with a molecular sieve (Type I sorption isotherm) and the wheel with matrix coated with silica gel desiccant (linear sorption isotherm type). Simulation results show that the desiccant with a linear sorption curve is favorable for the energy recovery because it has better performance characteristics. Moreover, less amounts of condensation/frosting can be formed for extreme operation conditions.

Simonson et al. [79,80] presented an experimental procedure and measurements for an energy wheel operating over a wide range of operating conditions. They concluded that the numerical model and experimental effectiveness values agree within the experimental and numerical uncertainties and the trends in the experimental data are the same as the trends in the simulated results. In later papers [81,82], they derived new dimensionless parameters and effectiveness correlations for air-to-air energy wheels that transfer both sensible heat and water vapor.

Simonson et al. [83,84] studied the potential of two standard methodologies (wheel speed and bypass) to control the heat and moisture transfer rate of the energy wheel.
Later, simple models for silica gel desiccant wheel were proposed by Beccalic et al. [85]. Their models strongly depend on a wide range of performance data which was gathered from manufacturers. Developed correlations were derived by interpolating these data. The models are designed to predict outlet temperature and absolute humidity for different kinds of rotors desiccant.

On the other hand, Jae-Weon et al. [86] proposed a linear regression equation of the sensible and latent effectiveness for the normal operating speed as a function of six wheel variables. These parameters are the supply temperature and relative humidity, the exhaust temperature and relative humidity, face velocity, and the air flow ratio. Their correlation, were derived for the common enthalpy wheel in the market. It was found that the face velocity and air flow ratio have very high contribution to both sensible and latent effectiveness [86]. The supply and exhaust conditions give relatively small contributions to the sensible effectiveness, but they show higher contributions to the latent effectiveness.

Freund et al. [87,88] developed a simple and generalized method to predict the enthalpy wheel effectiveness. The method is based on the $\varepsilon – NTU$ method which is designed for the standard counter flow heat exchanger. To mimic the counter flow heat exchanger, they assumed that the enthalpy wheels are operated at a sufficiently high rotational speed. The method is developed based on two reference data (sensible and latent effectiveness for two different volume flow rates). Knowing this effectiveness along with $\varepsilon – NTU$, the effectiveness of enthalpy wheel can be estimated.
1.4 Research Objectives

The central goal of this research is to develop an understanding of the heat and mass transfer processes which occur in a porous matrix rotary energy wheel. A review of the literature reveals that considerable information is available regarding energy wheels with tubes or channels. However, there is little information about modeling the condensation and evaporation processes in porous matrix energy wheels. The energy wheel selected for this research is a rotary wheel with a polymer fiber mat as a heat and moisture exchange medium operating in counter-flow arrangement.

The objectives of this research are to:

1. Develop a numerical model to predict the performance of a sensible heat wheel without condensation (sensible model).
2. Develop a numerical model to predict the performance of a sensible heat wheel with condensation (condensation model).
3. Develop a numerical model to predict the performance of a desiccant wheel for enthalpy recovery application (enthalpy model).
4. Compare the models with the existing effectiveness correlations and with new experimental data.
1.5 Scope of Research

In this study, geometrical properties, coordinates, and dimensionless parameters for the air-to-air rotary energy wheel under investigation will be first established to have a basis for the models that will be developed in this research (heat transfer model, condensation model, and enthalpy model). A mathematical model for a porous media based energy wheel will be developed based on the physical principles of the operation of the energy wheels. The model is one-dimensional and transient with space \((x)\) and time \((t)\) as the independent variables. The thermodynamic relationships will be discussed for each model since the models can have different thermodynamic relationships. Enthalpy and temperature formulation will be developed for each model. A complete dimensionless representation for each model will be accomplished. The initial and boundary conditions will be defined for the supply and exhaust side. In this phase of the research we will establish the governing equations of the models.

In the second step, we choose the numerical method and computer codes structures. The governing equations of the energy wheel will be solved numerically by the finite difference method. Because the equations are strongly coupled, iterations are necessary to get converged values for each time steps. The integral-based method [89] for solving the advection-diffusion equation was used because it has been shown to be effective and stable. The integral based method addresses both stability and inaccuracy issues while providing a reduction in the total computational time. The numerical formulation of the wheel was implemented in the Matlab code. The codes are designed to simulate the wheel from an initial condition until the cyclic equilibrium is reached.
The heat transfer numerical model will be used to examine the effectiveness of
the wheel under summer and winter conditions. Effects of the rotational speed \( \Omega \), the
number of transfer unit \( NTU_{HT} \), the heat capacity-ratio \( C_r^+ \), porosity \( \phi \), volume flow
rate \( \dot{Q} \), and the Fourier number \( Fo_m \) will be studied and compared with theoretical
results. In addition, an effectiveness correlation will be presented. The temperature
profiles of the wheel will also be investigated to study the behavior of the wheel.

Modeling the condensation and evaporation will also be performed in this study.
Theoretical and numerical models of condensation will be implemented in the Matlab
code. The analysis used to investigate the performance of the sensible model was used as
a starting point for the condensation model.

As the final modeling study, a numerical code will be developed for the enthalpy
wheel model. The sorption isotherm for the matrix material will be modeled. Results will
be compared with experimental and theoretical results. New experimental data was
obtained from Stirling Technology, Inc. (STI) (Athens, OH) for comparison with the
enthalpy model. Tests were done using the ‘RecopAreator’, which is manufactured by
STI. It is an energy recovery ventilator with an energy wheel using a novel polymer
matrix. The available experimental results (using different volume flow rates and
boundary conditions) for this unit were used to compare with the predicted results from
the enthalpy model.
1.6 Overview of the Thesis

This thesis contains five extra chapters. Chapter 2 provides a quick review of theoretical models. Chapter 3 is devoted to the mathematical models, including governing equations, dimensionless representation, numerical modeling, and computer codes. Chapter 4 presents the theoretical analysis and validation of the models using the effectiveness correlations. Chapter 5 discusses the experimental setup, results, and comparison with the enthalpy model. Finally, Chapter 6 provides conclusions and suggestions for future research.
CHAPTER TWO
THEORETICAL BACKGROUND

This chapter presents the theory of the rotary energy wheels. First, the model equations, the solution techniques, effectiveness correlations, and the performance criteria are presented. The second part of this chapter describes the desiccant materials along with the theory of sorption isotherm of desiccants. Finally, thermodynamic relationships for moist air and the wheel matrix are presented.

2.1 Energy Wheel Models

The rotary energy wheels can be described by either heat transfer model or heat and mass transfer models according to the process under investigation. To be precise, four possible models of the energy wheel can be considered, as shown in Figure 2.1. The heat transfer model can only be used to model the energy wheel with non-desiccant matrix while the heat and mass transfer model is suitable for desiccant wheels as well as non-desiccant wheels since both wheels can experience both mechanisms of heat and mass transfer.

In the model of a sensible heat wheel, the dew point temperature of the inlet air with the higher temperature is less than the inlet temperature of the other air flow. Therefore, only heat transfer takes place in the sensible heat wheel. The heat transfer mechanism in the sensible wheel can be described with two energy equations one for the air stream and one for the matrix.
In the models of heat and mass transfer, the source of mass transfer is due to the condensation/evaporation or adsorption/desorption process. The adsorption/desorption process can occur only in the desiccant matrix; however the condensation/evaporation can take place in any kind of matrix. Heat and mass transfer in an energy wheel (desiccant or non-desiccant) can be described with the same set of differential equations, but with different thermodynamic relationships since the desiccant material is more involved in the modeling due to its sorption characteristics [57]. The theory of isotherm is used to model the matrix behavior.

Figure 2.1 Classification of energy recovery wheels based on mathematical modeling

2.1.1 Sensible wheel: Willmott’s Model

Before discussing the methods of solution, the descriptive differential equations will first be introduced. Willmott [45] described the heat transfer between the fluids and the solid in the rotary regenerator (Figure 2.2) with the following simplified form of the partial differential equations.

\[
\frac{\partial T_g}{\partial y} = -\frac{h_{HT} A_s}{\dot{m}_g C_p g L} \left( T_m - T_g \right)
\]  

(2.1)
where

\[
\frac{\partial T_m}{\partial t} = \frac{h_{HT} A_s}{(mCp)_m} (T_g - T_m)
\]

(2.2)

- \( A_s \) the regenerator heating surface area \([m^2]\)
- \( C_{p_g}, C_{p_m} \) the specific heat of gas and matrix respectively \([J/kg.K]\)
- \( h_{HT} \) convective heat transfer coefficient \([W/m^2.K]\)
- \( m_g \) mass flow rate of the gas \([kg/s]\)
- \( m_m \) mass of the heat storing matrix \([Kg]\)
- \( L \) length of regenerator \([m]\)
- \( T_g, T_m \) the gas and matrix temperatures \([^\circ K]\)
- \( y \) distance from regenerator entrance \([m]\)
- \( t \) time \([s]\)

**Figure 2.2** Schematic representation of rotary regenerator

Willmott [45] stated that if the time and distance variables are non-dimensionised onto \([0,1]\) scales Equations (2.1 and 2.2) can be written as
\[
\frac{\partial T_g}{\partial y^*} = \Lambda (T_m - T_g) \tag{2.3}
\]
\[
\frac{\partial T_m}{\partial t^*} = \Pi (T_g - T_m) \tag{2.4}
\]

where
\[
\Lambda = \frac{h_{HT} A_s}{(mCp)_g} \text{ Reduced length} \quad \Pi = \frac{h_{HT} A_s P}{(mCp)_m} \text{ Reduced period}
\]

\(P\) length of period and it is the sum of the hot-gas flow period \(P_h\), and cold-gas flow period \(P_c\).

\(t^* = \frac{t}{P}\) dimensionless time \quad \(y^* = \frac{y}{L}\) dimensionless length

Then, Willmott [46] introduced the following dimensionless parameters by assuming constant thermo-physical properties of his model:

\[
\xi = \frac{h_{HT} A_s y}{(mCp)_g L} \quad \eta = \frac{h_{HT} A_s t}{(mCp)_m}
\]

which leads to

\[
\frac{\partial T_g}{\partial \xi} = T_m - T_g \tag{2.5}
\]
\[
\frac{\partial T_m}{\partial \eta} = T_g - T_m \tag{2.6}
\]

The dimensionless \(\xi\) and \(\eta\) variables have limiting values at \(y = L\) and \(t = P\), which lead to the reduced length and period, respectively.

Willmott [45,46] discretized the differential equations of the two-dimensional model by finite difference method. Willmott integrated these equations using the trapezoidal rule. The matrix and gas temperatures are evaluated at mesh points \((j,n)\) at a distance \(j\Delta \xi\) from the gas entrance and at a time \(n\Delta \eta\) from the start of the period of
operation under consideration. The $\Delta \xi$ is the distance step length and $\Delta \eta$ the time-step length. The finite difference mesh representation of Willmott’s method is illustrated in Figure 2.3.

![Finite difference mesh for the numerical solution for the regenerators](image)

Figure 2.3  Finite difference mesh for the numerical solution for the regenerators [90]

It is necessary to define the coordinate system of the regenerator in terms of dimensionless variables along with reduced length and period before describing the numerical method. Figure 2.4 shows the coordinate system of the regenerator in counterflow arrangement.
Equations (2.5) and (2.6) should be applied successively to the hot and then the cold period for each wheel cycle along with the reversal and boundary conditions.

Boundary conditions are given by

\[ T_g^h (0, \eta^h) = T_{g\_in}^h \] (2.7)

\[ T_g^c (0, \eta^c) = T_{g\_in}^c \] (2.8)

At the end of the hot period (start of cooler gas flow):

\[ T_m^h (\xi^h, \Pi^h) = T_m^c (\Lambda^h - \xi^h, 0) \] (2.9)

Figure 2.4 Coordinate diagram for the counter-flow rotary regenerator
and at the end of the cold period (start of hotter gas flow):

$$T_m^c(\xi^c, \Pi^c) = T_m^h(\Lambda^c - \xi^c, 0)$$  \hspace{1cm} (2.10)

Finite difference method starts by employing the trapezoidal rule to the gas phase equation which results into:

$$T_{g,j+1}^n = T_{g,j}^n + \frac{\Delta \xi}{2} \left\{ \frac{\partial T_g^n}{\partial \xi} \bigg|_{j+1} + \frac{\partial T_g}{\partial \xi} \bigg|_j \right\}$$  \hspace{1cm} (2.7)

using

$$\frac{\partial T_g^n}{\partial \xi} \bigg|_j = T_{m,j}^n - T_{g,j}^n$$  \hspace{1cm} (2.8)

$$A_1 = \frac{2 - \Delta \xi}{2 + \Delta \xi}$$  \hspace{1cm} (2.9)

$$A_2 = \frac{\Delta \xi}{2 + \Delta \xi}$$  \hspace{1cm} (2.10)

Equation (2.7) becomes

$$T_{g,j+1}^n = A_1 T_{g,j}^n + A_2 (T_{m,j+1}^n + T_{m,j}^n), \text{ for } \begin{cases} 1 \leq j \leq J - 1 \\ 0 \leq n \leq N \end{cases}$$  \hspace{1cm} (2.11)

Employing the trapezoidal rule to the matrix results into:

$$T_{m,j}^{n+1} = T_{m,j}^n + \frac{\Delta \eta}{2} \left\{ \frac{\partial T_m^{n+1}}{\partial \eta} \bigg|_j + \frac{\partial T_m}{\partial \eta} \bigg|_j \right\}$$  \hspace{1cm} (2.12)

using

$$\frac{\partial T_m^n}{\partial \eta} \bigg|_j = T_{g,j}^n - T_{m,j}^n$$  \hspace{1cm} (2.13)
\[ B_1 = \frac{2 - \Delta \eta}{2 + \Delta \eta} \] (2.14)

\[ B_2 = \frac{\Delta \eta}{2 + \Delta \eta} \] (2.15)

Equation (2.12) becomes

\[ T_{m_j}^{n+1} = B_1 T_{m_j}^n + B_2 \left( T_{g_j}^{n+1} + T_{g_j}^n \right), \text{ for } \left\{ \begin{array}{l} 0 \leq j \leq J \\ 0 \leq n \leq N - 1 \end{array} \right\} \] (2.16)

Now equation (2.16) can be used to find the matrix temperatures at \( j = 0 \) for time period \( n = 0, 1, 2, \cdots, N \). The dimensionless duration of the period under consideration (hot or cold) is defined by \( N \Delta \eta = \Pi \). In the case for \( j > 0 \), equation (2.16) cannot be used directly since it involves prior knowledge of \( T_{g_j}^{n+1} \) which is not known at this stage.

To overcome this, equation (2.11) can be written at \((j, n+1)\) as follows

\[ T_{g_j}^{n+1} = A_1 T_{g_j}^{n+1} + A_2 \left( T_{m_j}^{n+1} + T_{m_{j-1}}^n \right), \text{ for } \left\{ \begin{array}{l} 1 \leq j \leq J \\ 0 \leq n \leq N - 1 \end{array} \right\} \] (2.17)

Now, equation (2.17) can be used to replace \( T_{g_j}^{n+1} \) in equation (2.16) to get

\[ T_{m_j}^{n+1} = K_1 T_{m_j}^n + K_2 T_{g_j}^n + K_3 T_{m_{j-1}}^{n+1} + K_4 T_{g_{j-1}}^{n+1}, \text{ for } \left\{ \begin{array}{l} 1 \leq j \leq J \\ 0 \leq n \leq N - 1 \end{array} \right\} \] (2.18)

where

\[ K_1 = \frac{B_1}{1 - A_2 B_2} \] (2.19)

\[ K_2 = \frac{B_2}{1 - A_2 B_2} \] (2.20)
To start the first cycle process, the initial conditions are established:

- Constant inlet gas temperature (hot or cold) for all time intervals
  \[ T_{g_0}^n = T_{g_{-in}} \quad \forall \, n, \]

- The initial solid temperature (matrix) distribution is defined at the beginning of the first cycle usually by
  \[ T_{m_0}^h = \frac{T_{g_{-in}}^h + T_{g_{-in}}^c}{2}, \quad \text{for} \quad 0 \leq j \leq J \]

  and then by the counter-flow boundary conditions at the beginning of successive periods:
  \[ T_{m_j}^h = T_{m_{j-j}}^h \quad (2.23) \]
  \[ T_{m_j}^c = T_{m_{j-j}}^c \quad (2.24) \]

  Using the known initial matrix temperatures \( T_{m_j}^0 \mid j = 0,1,2,\cdots,J, \) (where \( J\Delta \xi = \Lambda, \) the dimensionless length of the regenerator for the period under consideration) together with the inlet gas temperature \( T_{g_0}^n = T_{g_{-in}} \), the fluid temperatures along the length of the regenerator are calculated using equation (2.11):

  \[ T_{g_{j+1}}^0 = A_1 T_{g_j}^0 + A_2 (T_{m_{j+1}}^0 + T_{m_j}^0), \quad \text{for} \quad j = 0,1,2,\cdots,J-1. \quad (2.25) \]

  The value of gas and matrix temperature at \( (n = 0) \) are known. The procedure now is to evaluate \( T_{m_j}^1 \) for \( j = 1,2,\cdots,J \) and \( T_{g_j}^1 \) for \( j = 1,2,\cdots,J \).
Equation (2.18) is used to calculate $T_{m1}^1$, that is

$$T_{m1}^1 = K_1 T_{m1}^0 + K_2 T_{g1}^0 + K_3 T_{m0}^1 + K_4 T_{g0}$$

(2.26)

The value of $T_{m0}^1$ in equation (2.26) is evaluated using equation (2.16)

$$T_{m0}^1 = B_1 T_{m0}^0 + B_2 (T_{g0}^0 + T_{g0}^0) = B_1 T_{m0}^0 + 2B_2 T_{g\_in}$$

(2.28)

Now it is possible to get $T_{g1}^1$ using equation (2.17)

$$T_{g1}^1 = A_1 T_{g0}^1 + A_2 (T_{m1}^1 + T_{m0}^1)$$

(2.29)

Using the same calculation procedures above, the temperatures of matrix and gas along the length of regenerator can be computed easily. More precisely, the values of $T_m$ and $T_g$ at any given point in the space-time mesh, namely $(j,n)$, $(j,n+1)$ and $(j+1,n)$ are known. Knowing that, the value of $T_m$ at $(j+1,n+1)$ is obtained explicitly using equation (2.18). Then for the same calculation level $(j+1,n+1)$, the value of $T_g$ is calculated using equation (2.17).

The above algorithm is used over the hot period $N^h \Delta \eta = \Pi^h$, that is for $n = 0,1,2,\cdots, N^h$. At the end of the period the values of gas and matrix temperatures are stored to be used after the flow reverses its direction. Then, for the cold period the boundary condition is applied and the integration proceeds to the end of the cold period.

The cyclic equilibrium of the regenerator is reached after a sufficient number of cycles. At the cyclic equilibrium, the sequential temperature variation at all points down the length of the regenerator becomes independent of the initial temperature distribution.
The thermal efficiency $\eta_{reg}$ for both periods is computed once equilibrium has been achieved.

$$\eta^h_{\text{reg}} = \frac{T^h_{\text{g,in}} - T^h_{\text{g, out}}}{T^h_{\text{g, in}} - T^c_{\text{g, in}}}, \text{ for the heating period and}$$

$$\eta^c_{\text{reg}} = \frac{T^c_{\text{g, out}} - T^c_{\text{g, in}}}{T^h_{\text{g, in}} - T^c_{\text{g, in}}}, \text{ for the cooling period.}$$  \hspace{1cm} (2.30)\hspace{1cm} (2.31)

Willmott [46] computed a thermal ratio at the end of each cycle using the time-mean exit gas temperatures $T^h_{\text{g, out}}$ and $T^c_{\text{g, out}}$. For the cooling period, the ratio is defined by

$$\Phi(n) = \frac{T((\Pi^c)_{\text{g, out, final}} - T^c_{\text{g, in}})}{T^h_{\text{g, in}} - T^c_{\text{g, in}}},$$

where $T((\Pi^c)_{\text{g, out, final}}$ is the final exit gas temperature at the end of the cooling period of the $n_{\text{cycle}}$ cycle. The $(n_{\text{cycle}} + 1)^{th}$ cycle is regarded as the equilibrium cycle, when

$$|\Phi(n_{\text{cycle}}) - \Phi(n_{\text{cycle}} - 1)|$$

is less than a pre-specified small number. At the equilibrium cycle, the time mean exit gas temperatures are computed and then the values of $\eta^h_{\text{reg}}$ and $\eta^c_{\text{reg}}$ calculated.

**2.1.2 Desiccant Wheel: Maclaine-Cross’s Model**

Based on the Maclaine-Cross’s model assumptions [42], the partial differential equations for describing the heat and mass transfer in the desiccant dehumidifier can be written as:
for conservation of energy in the matrix

$$\dot{m}_g L \frac{\partial h_g}{\partial x} + M_m \frac{\partial H_m}{\partial t} = 0$$

(2.33)

where

- $h_g$ enthalpy of gas (moist air) [J/Kg]
- $H_m$ enthalpy of matrix [J/Kg]
- $\dot{m}_g$ mass flow rate of gas [Kg/s]
- $M_m$ mass of desiccant matrix [Kg]
- $L$ passage length in fluid flow direction [m]
- $x$ distance from matrix inlet in fluid flow direction [m]
- $t$ time from beginning of period [s]

for conservation of mass in the matrix,

$$\dot{m}_g L \frac{\partial w_g}{\partial x} + M_m \frac{\partial W_m}{\partial t} = 0$$

(2.34)

where

- $w_g$ moisture content or humidity ratio of gas [kg/kg_d]
- $W_m$ moisture content of matrix per mass active desiccant [kg/kg_d, d]

for conservation of sensible heat in the gas,

$$\dot{m}_g L \frac{\partial T_g}{\partial x} = \frac{h_{HT} A_s}{Cp_g} (T_m - T_g)$$

(2.35)

where

- $A_s$ surface area associated with $h_{HT}$ and $h_{MT}$ [m^2]
- $Cp_g$ specific heat of gas [J/Kg.°K]
- $h_{HT}$ heat transfer coefficient [W/m^2.°K]
- $T_g, T_m$ temperature of gas and matrix, respectively [°K]

for conservation of moisture in the gas,
\[ m_g L \frac{\partial w_g}{\partial x} = h_{MT} A_s (w_m - w_g) \]  

(2.36)

where

- \( h_{MT} \): water vapor transfer coefficient of moist air stream \([ \text{kg}_{da}/\text{m}^2\cdot\text{s} \])
- \( w_g \): moisture content or humidity ratio of gas \([ \text{kg}/\text{kg}_{da} \])
- \( w_m \): moisture content of fluid in equilibrium with matrix \([ \text{kg}/\text{kg}_{da} \])

With the help of the non-linear equilibrium relations of matrix enthalpy and water content, the four linear partial differential equations defined above were solved. The first relation is the enthalpy of matrix \( H_m \), which is defined as a function of two independent variables; namely, the matrix temperature and moisture content of fluid in equilibrium with matrix.

\[ H_m = H(T_m, w_m) \]  

(2.37)

Second, moisture content of desiccant matrix \( W_m \) is defined as a function of matrix temperature and moisture content of fluid in equilibrium with matrix.

\[ W_m = W(T_m, w_m) \]  

(2.38)

Equations (2.33) - (2.36) may be transformed to a more convenient form for the finite difference solution using [43]:

\[ \xi = \frac{h_{MT} A_s x}{m_g L} \text{ as the dimensionless distance} \]  

(2.39)

\[ \eta = \frac{h_{MT} A_s t}{M_m} \text{ as the dimensionless time} \]  

(2.40)

and writing the equilibrium relations in differential form,
\[
\left( \frac{\partial h_g}{\partial T_g} \right)_{w_g} + \left( \frac{\partial h_g}{\partial T_g} \right)_{T_g} + \left( \frac{\partial H_m}{\partial T_m} \right)_{w_m} + \left( \frac{\partial H_m}{\partial T_m} \right)_{T_m} + \left( \frac{\partial \tilde{H}}{\partial \tilde{w}_m} \right)_{\tilde{T}_m} = 0 \quad (2.41)
\]

\[
\frac{\partial \tilde{w}_m}{\partial \tilde{\xi}} + \left( \frac{\partial \tilde{w}_m}{\partial \tilde{\xi}} \right)_{w_m} + \left( \frac{\partial \tilde{w}_m}{\partial \tilde{\xi}} \right)_{T_m} + \left( \frac{\partial \tilde{w}_m}{\partial \tilde{\xi}} \right)_{T_m} = 0 \quad (2.42)
\]

\[
\frac{\partial \tilde{w}_m}{\partial \tilde{\xi}} = \frac{\partial \tilde{w}_m}{\partial \tilde{\xi}} = \text{Le}(T_m - T_g) \quad (2.43)
\]

where the effective Lewis number is defined as

\[
\text{Le} = \frac{h_{HT}}{C_p \tilde{h}_{MT}} \quad (2.44)
\]

Finally, the conservation of moisture in the gas can be written as

\[
\frac{\partial w_m}{\partial \xi} = w_m - w_g \quad (2.45)
\]

To get the finite difference equations, Maclaine–Cross [42] defined the position of the dependent variables as in (Figure 2.5) and used the Taylor’s series expansion about the center of the grid which yields a second order accurate approximation of the outlet’s state.

Maclaine-Cross [42] characterized the regenerator as a rectangular grid of small cross-flow heat exchangers. He considered the matrix rotation as one of the flows and the others the flow stream. The finite difference equations are an approximate expression for the outlet states of these small exchangers. The coordinate system of the wheel and the matrix dimensionless length and the time period used by Maclaine-Cross are similar to that used by Willmott.
As a conclusion of Maclaine–Cross’s work [42], MOSHMX program was developed for desiccant dehumidified wheel. Axial heat conduction, air moisture content, and phase change energy distribution were neglected in Maclaine-cross’s model. Since the focus here is not in the method of the solution but in the equations, the method description will be omitted.

2.1.3 Enthalpy Wheel: Simonson’s Model

Finite volume method (FVM) was used by Simonson and Besant [74,76] to develop a numerical model for enthalpy wheels. They considered axial conduction, moisture storage in the air and diffusion of phase-change energy between desiccant and air processing. The governing equations for coupled heat and mass transfer were derived in a way that energy and mass balance equations for the air and matrix can be presented for one flute. The governing equations can be written as [74]:

Figure 2.5  A schematic of the finite-difference element (Maclaine–Cross [42])
Energy equation of the air:

\[ \rho_g \, C_{p_g} \, A_g \, \frac{\partial T_g}{\partial t} + U_g \, \rho_g \, C_{p_g} \, A_g \, \frac{\partial T_g}{\partial x} - m' h_{fg} \eta + h_{HT} \, \frac{A_x}{L} (T_g - T_m) = 0 \]  

(2.46)

Energy equation for the matrix:

\[ \rho_m \, C_{p_m} \, A_m \, \frac{\partial T_m}{\partial t} - m' h_{fg} (1 - \eta) - m' C_{p_v} (T_g - T_m) - h_{HT} \, \frac{A_x}{L} (T_g - T_m) = \frac{\partial}{\partial x} \left( k_m \, A_m \, \frac{\partial T_m}{\partial x} \right) \]  

(2.47)

Conservation of mass for water vapor:

\[ A_g \, \frac{\partial \rho_v}{\partial t} + \frac{\partial}{\partial x} \left( \rho_v U_g \, A_g \right) + m' = 0 \]  

(2.48)

Conservation of mass for dry air:

\[ \frac{\partial \rho_d}{\partial t} + \frac{\partial}{\partial x} \left( \rho_d U_g \right) = 0 \]  

(2.49)

Conservation of mass for matrix:

\[ m' = \rho_{d,\text{dry}} A_d \, \frac{\partial u}{\partial t} \]  

(2.50)

The fraction of phase change energy that convected directly to the air is given by \( \eta \). This has been shown [74] to be equal to

\[ \eta = \frac{k_g / \sqrt{\alpha_g}}{k_g / \sqrt{\alpha_g} + k_m / \sqrt{\alpha_m}} \]  

(2.51)

The value \( \eta \) is expected to range between \([0,0.1]\), which means each enthalpy wheel will be characterized by its \( \eta \) value. To complete the formulation of the enthalpy model, additional thermodynamics relationships of moist air and matrix are required.
The main contribution of Simonson and Besant’s work was developing an effectiveness correlation for balanced enthalpy wheel by correlating their numerical model simulation data [81,82]. This model gives good results for various desiccant materials.

\[
\varepsilon_s = \frac{NTU_0}{1 + NTU_0} \left( 1 - \frac{1}{7.5C_{r_0}^*} \right) \frac{H^*}{(C^*)^{0.33}} \left[ (0.26) \frac{C_{r_0}^*}{Wm^2.Crm^*} \right]^{0.28} + \frac{0.31\eta}{NTU_0^{0.68}} + \frac{210}{7.2(Cr_0^*)^{0.53}} + \frac{5.2}{NTU_0^{2.9}} \right]
\]

(2.52)

\[
\varepsilon_t = \frac{NTU_0}{1 + NTU_0} \left( 1 - \frac{1}{0.54(C_{rmt,0}^*)^{0.86}} \right) \left( 1 - \frac{1}{NTU_0^{0.51}(C_{rmt,0}^*)^{0.54}.H^*} \right)
\]

(2.53)

\[
\varepsilon_t = \frac{\varepsilon_s + \varepsilon_t H^*}{1 + H^*}
\]

(2.54)

where

the overall number of transfer units \( NTU_0 \) is defined by

\[
NTU_0 = \frac{1}{(\dot{m}Cp)_{\min}} \left[ \frac{1}{(h_{HT} A_s)_{\text{sup,pf}} + (h_{HT} A_s)_{\text{exhaust}}} \right]^{-1}
\]

(2.55)

the overall matrix heat capacity ratio \( C_{r_0}^* \) is defined by

\[
C_{r_0}^* = \frac{(MCp)\Omega}{(\dot{m}Cp)_{\min}}
\]

(2.56)

the overall matrix moisture capacity ratio \( Crm_{r_0}^* \) is defined by

\[
Crm_{r_0}^* = \frac{M_{d,\text{dry}} \Omega}{(\dot{m})_{\min}}
\]

(2.57)

the storage of moisture in the desiccant is defined by
\[ Cr_{m,0}^{*} = \left( Crm_0^* \right)^{0.58} Wm^{0.33} \left( \frac{\partial u}{\partial \phi} \right)_{\phi_{ave}}^{0.2} \left( Cr_0^* \right)^{1.13} \left( \frac{1482}{T_{ave}} \right)^{0.5} \left( \frac{47.9}{\left( \phi_{ave} \right)^{0.5}} \right)^{4.66} \] (2.58)

The operating condition factor \( H^* \) which represents the ratio of latent to sensible enthalpy differences between the inlets of the energy wheel is defined by

\[ H^* = \frac{\Delta H_l}{\Delta H_s} = \left( \frac{\Delta H_s}{\Delta H_l} \right)^{-1} - 1 = 2500 \frac{\Delta W}{\Delta T} \] (2.59)

Table 2.1 summarizes the range of parameters used in the Simonson’s effectiveness correlations.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction of the phase change energy that is delivered directly to the air: ( \eta )</td>
<td>( 0 \leq \eta \leq 0.1 )</td>
</tr>
<tr>
<td>Operating condition factor that represents the ratio of latent to sensible enthalpy differences between the inlets of the energy wheel: ( H^* )</td>
<td>( -6 \leq H^* \leq 6 )</td>
</tr>
<tr>
<td>Overall number of transfer units: ( NTU_0 )</td>
<td>( 2 \leq NTU_0 \leq 10 )</td>
</tr>
<tr>
<td>Overall matrix heat (or moisture) capacity ratio: ( Cr_0^* )</td>
<td>( 3 \leq Cr_0^* \leq 10 )</td>
</tr>
<tr>
<td>Ratio of overall matrix heat capacity ratio to overall matrix moisture capacity ratio: ( Cr_0^<em>/Crm_0^</em> )</td>
<td>( 1 \leq \frac{Cr_0^<em>}{Crm_0^</em>} \leq 5 )</td>
</tr>
<tr>
<td>Maximum moisture capacity of desiccant: ( Wm )</td>
<td>( 0.1 \leq Wm \leq 0.5 ), and ( C = 1 )</td>
</tr>
<tr>
<td>Shape of the sorption curve: ( C )</td>
<td>( C^* = 1 )</td>
</tr>
<tr>
<td>Ratio of the minimum to maximum heat capacity rate of the air streams: ( C^* )</td>
<td>( C^* = 1 )</td>
</tr>
</tbody>
</table>
2.1.4 Discussion of Models

After careful discussion of the models presented above, one can conclude several issues about modeling concepts. Willmott’s model provides a well defined numerical method. It is easy to comprehend and implement in models described in this research. The numerical method and the structure of the code were modified to be adopted in this current research. Willmott’s model describes the phenomena of heat transfer process in the wheel; however, it will be extended to cover both heat and mass transfer.

Maclaine-Cross’ model was aimed at the application of the desiccant wheel dehumidifier, and as a conclusion of his work, MOSHMX program was developed. Discussion of model equations and thermodynamics relations was of great help in developing the models equations in this research. Maclaine-Coress’ model neglects the axial conduction, air moisture content, and phase change energy distribution in the desiccant wheel.

Finally, Simonson’s model was developed for heat and mass transfer in enthalpy wheels using finite volume method. It covers most of the issues in the modeling of the energy wheel, so it can be considered an appropriate model for enthalpy wheel application. Phase energy diffusion between the air and the matrix is still under investigation. He introduced the constant $\eta$ which describes the friction of phase energy convected directly to air. However, most researchers [29,53,63,70] assumed that the phase changes take place in the matrix, which will also be assumed in this study. Equations in the current research follow, to a degree, Simonson’s model. Effectiveness
correlation was the main focus of Simonson’s work and it will be used as a validation tool in this research.

All models discussed above were designed using different configurations of flow channel coated with desiccant as exchange media. However, the wheel under investigation here consists of a novel polymer porous media through which the heat and mass transfer take place.

2.2 Effectiveness Correlations

In the operation of regenerator (energy wheel), the regular periodic flow conditions are assumed to be achieved for design purposes. The solution to the governing differential equations is commonly presented in term of the regenerator effectiveness as a function of the dimensionless parameters. The two most common forms are the \((\varepsilon - NTU_o)\) method (generally used for rotary regenerators) and the reduced length reduced period \((\Lambda - \Pi)\) method (generally used for fixed matrix regenerators) [91].

2.2.1 The \(\varepsilon\)-NTU Method

This method is primarily used for designing and analyzing of rotary regenerators and was first developed by Coppage and London [35]. The dimensionless groups in this method are formulated in such a way that to be parallel to the recuperators when the influence of additional groups is negligible [91]. These dimensionless groups were derived by first making the governing differential equations in dimensionless form for both the hot and the cold period of regenerator.
The effectiveness is expressed as a function of four non-dimensional parameters given by

\[ \varepsilon = f(NTU_h, NTU_c, C_{r,h}^*, C_{r,c}^*) \]  

(2.60)

where

\[
NTU_h = \left( \frac{hA}{C} \right)_h \quad NTU_c = \left( \frac{hA}{C} \right)_c \\
C_{r,h}^* = \left( \frac{C_r}{C} \right)_h \quad C_{r,c}^* = \left( \frac{C_r}{C} \right)_c
\]  

(2.61)

In the above equation (2.16), \( h \) is the heat transfer coefficient, \( A \) is the surface area, \( C = \dot{m}C_p \) is the heat capacity of the gas, and \( C_r = M_rC_p \Omega \) is the matrix heat capacity rate. Since these independent dimensionless groups defined above do not parallel those of a recuperator (a direct transfer type exchanger), Shah [43,44] has defined a related set, as follows:

The modified number of transfer units \( NTU_0 \) is defined as

\[
NTU_0 = \frac{1}{C_{\text{min}}} \left[ \frac{1}{1/(hA)_h + 1/(hA)_c} \right]
\]  

(2.61)

The heat capacity ratio \( C^* \) is simply the ratio of the smaller to the larger heat capacity rate of the gas streams so that \( C^* \) should be less than or equal to 1.

\[
C^* = \frac{C_{\text{min}}}{C_{\text{max}}}
\]  

(2.62)

The \( C_{r}^* \) is the matrix heat capacity rate \( C_r \) normalized with respect to \( C_{\text{min}} \)

\[
C_{r}^* = \frac{C_r}{C_{\text{min}}}
\]  

(2.63)
The \((hA)^*\) is the ratio of convective conductance on the \(C_{\text{min}}\) side to that on the \(C_{\text{max}}\) side

\[
(hA)^* = \frac{(hA) \text{ on the } C_{\text{min}} \text{ side}}{(hA) \text{ on the } C_{\text{max}} \text{ side}}
\]

(2.64)

Now, it is right to express the effectiveness as

\[
\varepsilon = f\{NTU_0, C^*, C_{r^*}, (hA)^*\}
\]

(2.65)

The behavior of rotary regenerator and counter-flow direct-type exchanger becomes identical for the case \(C_{r^*} \to \infty\) and its effectiveness is given by

\[
\varepsilon_{cf} = \begin{cases} 
1 - \exp(-NTU_0(1-C^*)) & \text{for } C^* < 1 \\
1 - C^* \exp(-NTU_0(1-C^*)) & \text{for } C^* = 1 
\end{cases}
\]

(2.66)

Lamberston [36] developed the first empirically correlated effectiveness of the rotary regenerator and then later modified by Kays and London [41]:

\[
\varepsilon = \varepsilon_{cf} \left[ 1 - \frac{1}{9(C^*)^{1.93}} \right]
\]

(2.67)

For the case of \(C^* < 1\), Razelos [92] proposed a simple method to calculate the regenerator effectiveness \(\varepsilon\) of unsymmetric regenerator \(((hA)^* \neq 1)\). For the specific values of \(NTU_0\), \(C^*\) and \(C_{r^*}\), the equivalent balanced regenerator \((C^* = 1)\) values can be calculated as follows:

\[
NTU_{0,m} = \frac{2NTU_0 C^*}{1 + C^*}
\]

(2.68)

\[
C_{r,m} = \frac{2C_{r^*} C^*}{1 + C^*}
\]

(2.69)
With these values of $NTU_{0\_m}$ and $C_{r\_m}^*$, one can obtain the value $\varepsilon_r$ from the following approximate equation:

$$
\varepsilon_r = \frac{NTU_{0\_m}}{1 + NTU_{0\_m}} \left[ 1 - \frac{1}{9(C_{r\_m}^*)^{1.93}} \right] \tag{2.70}
$$

Finally, the actual value the $\varepsilon$ can be calculated from

$$
\varepsilon = \frac{1 - \exp(\varepsilon_r(C^* - 1)/2C^*(1 - \varepsilon_r))}{1 - C^\ast \exp(\varepsilon_r(C^* - 1)/2C^*(1 - \varepsilon_r))} \tag{2.71}
$$

To include the influence of longitudinal heat conduction, Bahnk and Howard [37] defined the longitudinal conduction parameter $\lambda$ as

$$
\lambda = \frac{K_w A_{k,t}}{LC_{\text{min}}} \tag{2.72}
$$

where $K_w$ is the thermal conductivity of the matrix wall, and $A_{k,t}$ is the total solid area available for longitudinal conduction.

Using the conduction parameter, the conduction effect is defined as

$$
\frac{\Delta \varepsilon}{\varepsilon} = \frac{\varepsilon_{\lambda=0} - \varepsilon_{\lambda}}{\varepsilon_{\lambda=0}} \tag{2.73}
$$

For the case $C^\ast = 1$, the effectiveness can be accurately expressed by [37]

$$
\varepsilon = C_{\lambda}\varepsilon_{\lambda=0} \tag{2.74}
$$

The axial conduction correction factor $C_{\lambda}$ is given by the following equation where $\varepsilon_{\lambda=0}$ is given by equation (2.67)

$$
C_{\lambda} = \frac{1 + NTU_0}{NTU_0} \left[ 1 - \frac{1}{1 + NTU_0(1 + \lambda \psi)/(1 + \lambda NTU_0)} \right] \tag{2.75}
$$
in which

$$\psi = \left( \frac{\lambda NTU_0}{1 + \lambda NTU_0} \right)^{0.5} \tanh \left[ \frac{NTU_0}{\sqrt[0.5]{\lambda NTU_0/(1 + \lambda NTU_0)}} \right]$$

$$\approx \left( \frac{\lambda NTU_0}{1 + \lambda NTU_0} \right)^{0.5} \text{ for } NTU_0 \geq 3$$ (2.76)

### 2.3 Performance Criteria

The performance of the energy wheel is characterized by several parameters, but the most common one is the effectiveness. The effectiveness of air-to-air energy wheels is commonly measured in terms of temperature $T_g$, humidity ratio $\omega_g$, and enthalpy $h_g$.

The energy wheel that operates in counter-flow arrangement (Figure 2.6) is the most commonly used in the market. In the Figure 2.6, four measuring stations are used to define the effectiveness of the energy wheels.

![Air-to-air rotary energy wheel diagram](image-url)
ASHRAE [24] defines effectiveness as

\[
\varepsilon_{\text{wheel}} = \frac{\text{Actual transfer (of moisture or energy)}}{\text{Maximum possible transfer between airstreams}} \tag{2.78}
\]

\[
\varepsilon_{\text{wheel}} = \frac{\dot{m}_{\text{supply}} \left( \overline{Z}_{s,e} - \overline{Z}_{s,i} \right)}{\min(\dot{m}_{\text{supply}}, \dot{m}_{\text{exhaust}}) \left( \overline{Z}_{e,i} - \overline{Z}_{s,i} \right)} = \frac{\dot{m}_{\text{exhaust}} \left( \overline{Z}_{e,i} - \overline{Z}_{e,o} \right)}{\min(\dot{m}_{\text{supply}}, \dot{m}_{\text{exhaust}}) \left( \overline{Z}_{e,i} - \overline{Z}_{s,o} \right)} \tag{2.79}
\]

where

- \(\varepsilon_{\text{wheel, s}}\) sensible effectiveness, \(\varepsilon_{\text{wheel, l}}\) latent effectiveness, or \(\varepsilon_{\text{wheel, t}}\) total effectiveness
- \(Z\) temperature \(T_g\), humidity ratio \(\omega_g\), or enthalpy \(h_g\)
- \(\dot{m}_{\text{supply}}\) supply air mass flow
- \(\dot{m}_{\text{exhaust}}\) exhaust air mass flow
- \(Z_{i,j}\) \(i = s, e\) (s) supply air, (e) exhaust air and \(j = i, o\) (i) supply and exhaust side in-flow, (o) supply and exhaust side out-flow
- \(\overline{Z}_{i,j}\) the time averaged value

\[
\overline{Z}_{s,o} = \frac{1}{\tau_1} \int_{0}^{\tau_1} Z_{s,o} \, dt \quad \text{and} \quad \overline{Z}_{e,o} = \frac{1}{\tau_2} \int_{0}^{\tau_2} Z_{e,o} \, dt \tag{2.80}
\]

The total effectiveness is somewhat between the sensible and latent effectiveness. The specific enthalpy of air is calculated by

\[
h_g = C_p \, T_g + \omega_g \left( h_{\text{vap}}^0 + C_p \, T_g \right) \tag{2.81}
\]

The third term in the equation (2.102) can be neglected, since it has only less than 3% effect [93]. Therefore, the enthalpy effectiveness is

\[
\varepsilon_{\text{tot}} = \frac{\left( C_p \, T_{g,s,i} + h_{\text{vap}}^0 \omega_{g,s,i} \right) - \left( C_p \, T_{g,s,o} + h_{\text{vap}}^0 \omega_{g,s,o} \right)}{\left( C_p \, T_{g,s,i} + h_{\text{vap}}^0 \omega_{g,s,i} \right) - \left( C_p \, T_{g,e,i} + h_{\text{vap}}^0 \omega_{g,e,i} \right)} \tag{2.82}
\]
\[
\varepsilon_{\text{tot}} = \frac{h_{\text{vap}}^0}{C_p d a} \left( T_{g_{-s,i}} - T_{g_{-s,o}} \right) + \frac{h_{\text{vap}}^0}{C_p d a} \left( \omega_{g_{-s,i}} - \omega_{g_{-s,o}} \right) \\
(2.83)
\]

Further simplification leads to

\[
\varepsilon_{\text{tot}} = \frac{\varepsilon_S + H^* \varepsilon_L}{1 + H^*} \\
(2.84)
\]

where

\[
H^* = \frac{h_{\text{vap}}^0}{C_p d a} \left( \frac{T_{g_{-s,i}}}{T_{g_{-s,i}} - T_{g_{-e,i}}} \right) \approx \frac{h_{\text{vap}}^0}{C_p d a} \frac{\Delta \omega_g}{\Delta T_g} \approx 2501 \frac{\Delta \omega_g}{\Delta T_g} \\
(2.85)
\]

The \( H^* \) is a ratio of latent to sensible energy differences between the inlets of two air stream flowing through the rotary energy wheel. Possible values of \( \varepsilon \) are presented in Table 2.2 as a function of \( H^* \). As you can see, the total effectiveness is not a simple algebraic average of \( \varepsilon_S \) and \( \varepsilon_L \).

**Table 2.2  Typical values of \( H^* \) [93]**

<table>
<thead>
<tr>
<th>( H^* )</th>
<th>( \varepsilon_{\text{tot}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>HVAC application: -6 to +6</td>
<td>0.0 to 1</td>
</tr>
<tr>
<td>( H^* = 1 )</td>
<td>( \varepsilon_{\text{tot}} = \frac{\varepsilon_S + \varepsilon_L}{2} )</td>
</tr>
<tr>
<td>( H^* \rightarrow \infty )</td>
<td>( \varepsilon_{\text{tot}} \rightarrow \varepsilon_L )</td>
</tr>
<tr>
<td>( H^* \rightarrow 0 )</td>
<td>( \varepsilon_{\text{tot}} \rightarrow \varepsilon_S )</td>
</tr>
<tr>
<td>( H^* \rightarrow -1 )</td>
<td>( \varepsilon_{\text{tot}} \rightarrow \pm \infty )</td>
</tr>
</tbody>
</table>
2.4 Desiccants Overview

2.4.1 Desiccant Isotherms

Desiccant materials in the air-conditioning applications are defined by the capability of the material to adsorb and desorb water at a given temperature. They are characterized mainly by their adsorption isotherm shape. At a given temperature, the relationship between the amount adsorbed by unit mass matrix and the equilibrium pressure is defined as adsorption isotherm. The adsorption isotherm is typically presented in graphic format in the case of experimental studies. Burnnauer et al. [94] gather the experimental work of various isotherms of gas-solid systems into five categories, as shown in Figure 2.7.

![Figure 2.7 The Brunauer classification of gas physisorption isotherm [94]](image-url)
Research performed by Jurinak [62] indicates that Type III, IV, and V are uncommon, while Type I and II are common. In comparing the five isotherm types shown in (Figure 2.7), the Type I isotherm yields the most favorable characteristic (maximum water uptake at lower relative pressure range) [62]. Detailed discussions of isotherm shapes are presented in section 2.5.

2.4.2 Desiccant Materials

The choice of desiccant is one of the key elements in the energy wheel technology. Silica gel and molecular sieves are the desiccants currently being most widely used for the enthalpy wheels. However, research conducted by Staton [95] shows that the polymer can be a good candidate material in the application of the enthalpy wheels. Indeed, the wheel under investigation in this research is the polymeric wheel type. Therefore, a discussion on the polymer is included next.

Polymers are a giant organic macromolecules created by linking a large number of small molecules, or monomers, in a chain (chain-like molecules) [96]. Long molecules can be formed by chemically reacting of small monomers repeating units. They can be formed as linear, branched, or interconnected to form three-dimensional networks [96]. Simple commercial methods, such as fiber spinning, drawing and annealing can be used to process the polymers [96].

Gzanderna [97] investigated some polymers that might be used in the desiccant cooling system and concluded that they meet the same general criteria as any other
desiccant material. They show a favorable isotherm and performance for use in Desiccant Cooling System (DCS).

Czanderan et al. [98] noted that the following attribution of the available polymeric desiccants:

- Polymers have the potential to be modified so that both the desired isotherm shape and heat of adsorption (2508 KJ/Kg) can be obtained.
- Polymers have a capability of adsorbing water from 5 to 80% of their own weight compared to silica-gels’ 5% to 40% adsorbing capability.
- Polymers fabricated into shape required for DCS is possible.
- Polymers structures can be synthesized to provide high diffusivities of water vapor through the material.
- Polymers have the capability being regenerated at temperature below 80°C for thermal desorbing water.
- Long-stability and low-cost are major characteristic of polymers.

One of the polymer candidates, namely polyester, is a synthetic polymer with long molecules. In the polyester matrix, the space between the packed strings can hold condensed water, which gives the polymer a greater sorbing capacity than that of many other solid desiccants.
2.4.3 Desiccant General Sorption Curve

The general sorption curve presented in equation (2.86) is now widely used to model the sorption characteristics of the most available desiccants in the market such as silica gel and molecular sieves.

The desiccant isotherm shape describes how a desiccant material adsorbs moisture at different levels of relative humidity ($\phi$). The isotherm equation used here [62] is

$$\frac{\sigma_m^+}{\sigma_{max}} = \frac{\phi}{R + (1 - R)\phi}$$  \hspace{1cm} (2.86)

where

$\sigma_m$ \hspace{1cm} moisture content of matrix
$\sigma_{max}$ \hspace{1cm} loading of desiccant at 100% relative humidity
$\sigma_m^+$ \hspace{1cm} ratio of actual matrix water content to maximum matrix water content
$R$ \hspace{1cm} separation factor that defines the isotherm shape
$\phi$ \hspace{1cm} relative humidity

For different values of $\sigma_{max}$ and $R$, different types of desiccant isotherm shapes can be produced. For example, the silica gel’s isotherm shape can be described by $\sigma_{max}$ and $R$ of value 0.4 and 1.0, respectively. In the case of molecular sieves, $\sigma_{max}$ and $R$ take value of 0.2 and 0.01, respectively.

The most common isotherms shapes are a Type I extreme isotherm (Type IE with R=0.01), a Type I moderate isotherm (Type IM with R=0.1), and a linear isotherm (Linear with R=1) [64]. Figure 2.8 shows five different types of sorption isotherm shapes based on this general equation.
2.4.4 Desiccant Isotherm Model

In practice, the Jurinak’s adsorption isotherm model is widely used to model the desiccant matrix. The model was developed based on the Clausius-Clapeyron equation [34,62]. The concept of the model rises from the differences between the thermodynamic characteristics of adsorbed water vapor by the desiccant matrix and the condensed water vapor on the surface of the matrix. Therefore, the heat of sorption, heat released when the air changes from a gaseous phase to an adsorbed phase, is different than the heat of
vaporization, heat resealed when air changes from gaseous a phase to a condensed layer of bulk water phase.

The heat of sorption and the heat of vaporization can be defined, using the Clausius-Clapeyron equation [34], as

\begin{align}
    h_{ad} &= RT^2 \frac{d \ln P_{wv}}{dT} \\
    h_{vap} &= RT^2 \frac{d \ln P_{s-wv}}{dT}
\end{align}

(2.87)

(2.88)

where

- \( h_{ad} \) differential heat of adsorption [KJ/Kg]
- \( h_{vap} \) specific heat of vaporization [KJ/Kg]
- \( R \) gas constant for water [KJ/Kg K]
- \( T \) thermodynamic temperature [K]
- \( P_{wv} \) partial pressure of water vapor in the air [KPa]
- \( P_{s-wv} \) partial pressure of water vapor at saturation [KPa]

The change from water vapor to adsorbed phase can occur in unsaturated or saturated air, while the vaporization/condensation occurs only if the air is saturated as can be seen from equations (2.87) and (2.88) respectively.

The energy effects of the two phenomena can be related as:

\[
\left( \frac{\partial \ln P_{wv}}{\partial \ln P_{wv-s}} \right)_{\sigma_s} = \frac{h_{ad}}{h_{vap}} = h^*
\]

(2.89)

Jurinak [34,62] assumed that \( h^* \) is independent of temperature and can be only a function of matrix water content. Based on this, he derived the following empirical correlation which can be used to reproduce all isotherm types:
\[
\frac{h_{ad}}{h_{vap}} = h^* (\sigma_m) = 1 + \Delta h^* \frac{e^{K \sigma_m^*}}{1 - e^K} \tag{2.90}
\]

where \(\sigma_m\) is the actual matrix water content, and \(\sigma^*\) is \(\sigma_m\) normalized by the maximum water content \(\sigma_{\text{max}}\) of the matrix. The constants \(\Delta h^*\) and \(K\) are used to determine the isotherm shapes.

To find relative humidity \(\phi\), equation (2.90) can be integrated to show that if the adsorption isotherm is a function \(G(\sigma_m)\) at temperature \(T_0\), then at any temperature \(T_m\) the relative humidity of moist air in equilibrium with the desiccant is

\[
\phi = G(\sigma_m) \left[ \frac{P_{wv-s}(T_m)}{P_{wv-s}(T_0)} \right]^{h^*-1} \tag{2.91}
\]

To find the function \(G(\sigma_m)\) the general isotherm relation is used [62]

\[
\sigma_m^* = \frac{\sigma_m}{\sigma_{\text{max}}} = \frac{\phi}{R + (1 - R)\phi} \tag{2.92}
\]

Finally, the relative humidity in equilibrium with a desiccant surface can be expressed as

\[
\phi = \frac{R \sigma_m^* \left[ \frac{P_{wv-s}(T_m)}{P_{wv-s}(T_0)} \right]^{h^*-1}}{1 + (R - 1)\sigma_m^*} \tag{2.93}
\]

The local equilibrium (isotherm) relation is an empirical relation, different for each desiccant. Brillhar [99] performed an experimental work in silica gel and found the equilibrium isotherm by linear fitting of uptake data, as indicated in Figure 2.9. The curve-fitting data is compared with isotherm shape produced by using equations (2.92) and (2.93). As on can see, there is a good agreement between the two methods.
As an example, the adsorption isotherm of a molecular sieve (Type I isotherm) is shown in Figure 2.10 [57,100]. Normally, the differential heat of adsorption is higher than the heat of vaporization. For some adsorbents the case is opposite which leads to negative scaling parameter $\Delta h^*$ [57,100]. Type III isotherm which is shown in Figure 2.11 is an example of negative scaling parameter [57]. The isotherm shapes used in these simulations were produced by using the parameters presented in [57,101].
Figure 2.10  Type I isotherm: adsorption isotherm shape for a molecular sieve

Type I isotherm
\[ \sigma_{\text{max}} = 0.23 \]
\[ \Delta h^* = 0.3 \]
\[ R = 0.05 \]
\[ K = -5.0 \]
\[ T_0 = 60^\circ C \]

Figure 2.11  Type III isotherm: adsorption isotherm shape

Type III isotherm
\[ \sigma_{\text{max}} = 0.3 \]
\[ \Delta h^* = -0.3 \]
\[ R = 10 \]
\[ K = -5.0 \]
\[ T_0 = 60^\circ C \]

\[ T_m = 20^\circ C \]
2.5 Thermodynamics Relationships

2.5.1 Properties of Moist Air

Moist air is a mixture that consists of dry air and water vapor. Conditions of moist air can be described by the quantity of water vapor that exists in it. Water vapor in the moist air can be varied from zero (dry-air) to maximum (saturated-air). This section was heavily drawn from ASHRAE [24].

Moist air properties, such as saturation humidity ratio and relative humidity, are required to describe the air stream states. These properties can be found using the water vapor saturation pressure $P_{wv,s}$. The values of $P_{wv,s}$ can be calculated using the polynomial expression presented in ASHRAE [24]. The saturation pressure over ice for the temperature range of [-100 to 0°C] and over liquid water for the temperature range of [0 to 200°C] is given by

$$\ln(P_{wv,s}) = \begin{cases} 
\frac{C_0}{T} + C_1 + C_2T + C_3T^2 + C_4T^3 + C_5T^4 + C_6 \ln(T) & ;173^0 < T < 273^0 K \\
\frac{C_9}{T} + C_{10} + C_{11}T + C_{12}T^2 + C_{13}T^3 + C_{14} \ln(T) & ;273^0 < T < 473^0 K 
\end{cases} \tag{2.94}$$

The constants in equation (2.94) are shown in Table 2.3.

In the air-conditioning and ventilation applications, humid air can be treated as an ideal gas mixture of dry air and water vapor as indicated by ASHRAE [24]. To specify the amount of water vapor in the air, the humidity ratio is used in this study. Humidity ratio is defined as:
\[ \omega = \frac{\text{mass of the water vapor in the air}}{\text{mass of the dry air}} = \frac{m_{\text{wv}}}{m_{\text{da}}} = 0.62198 \frac{p_{\text{wv}}}{p - p_{\text{wv}}} \]  

(2.95)

**Table 2.3** Values of the constants used in finding \( P_{\text{wv},s} \) according to ASHARE [24]

<table>
<thead>
<tr>
<th>Condition</th>
<th>Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 173^0 &lt; T &lt; 273^0 K )</td>
<td>( C_1 = -5674.5359, C_2 = 6.3925247 )</td>
</tr>
<tr>
<td></td>
<td>( C_3 = -9.677843 \times 10^{-3}, C_4 = 6.22115701 \times 10^{-7} )</td>
</tr>
<tr>
<td></td>
<td>( C_5 = 2.0747825 \times 10^{-9}, C_6 = -9.484024 \times 10^{-13} )</td>
</tr>
<tr>
<td></td>
<td>( C_7 = 4.1635019 )</td>
</tr>
<tr>
<td>( 273^0 &lt; T &lt; 473^0 K )</td>
<td>( C_8 = -5800.2206, C_9 = 1.3914993 )</td>
</tr>
<tr>
<td></td>
<td>( C_{11} = 4.1764768 \times 10^{-5}, )</td>
</tr>
<tr>
<td></td>
<td>( C_{12} = -1.4452093 \times 10^{-8}, )</td>
</tr>
<tr>
<td></td>
<td>( C_{13} = 6.5459673 )</td>
</tr>
</tbody>
</table>

In both expression \( P_{\text{wv},s} \) in Pa, and the absolute temperature \( T \) in Kelvin

Other important parameters are the humidity parameters involving saturation.

They are degree of saturation \( \mu \), relative humidity \( \phi \), and saturation humidity ratio \( \omega_s \).

They are defined as:

\[ \mu = \frac{\omega}{\omega_s} = \frac{\phi}{1 + (1 - \phi)\omega_s / 0.62198}, \text{ where} \]

(2.96)

\[ \phi = \frac{P_{\text{wv}}}{P_{\text{wv},s}}, \quad \omega_s = 0.62198 \frac{P_{\text{wv},s}}{P - P_{\text{wv},s}} \]

(2.97)

The values of \( \phi \) and \( \mu \) are zero for the dry air and one for saturated moist air.
The dew-point temperature $T_{dew}$ of moist air with humidity ratio $\omega$ and $P$ is defined for the perfect gases as

$$P_{wv-z}(T_{dew}) = P_w = \frac{P \omega}{0.612198 + \omega} \quad (2.98)$$

where $P_w$ in [KPa] is the water vapor partial pressure for the moist air sample and $P_{wv-z}(T_{dew})$ is the saturation vapor pressure at temperature $T_{dew}$. The saturation vapor pressure is derived from equation (2.94). Alternatively, the dew-point temperature can be calculated directly from the following equation [24]:

$$T_{dew} = \begin{cases} C_{14} + C_{15} \ln P_w + C_{16} (\ln P_w)^2 + C_{17} (\ln P_w)^3 + C_{18} (P_w)^{0.1984} & 0 < T_{dew} < 93^\circ C \\ C_{19} + C_{20} \ln P_w + C_{21} (\ln P_w)^2 & T_{dew} < 0^\circ C \end{cases} \quad (2.99)$$

The value of constants used in equation (2.99) is shown in Table 2.4.

**Table 2.4 Value of the constants used in finding $T_{dew}$ according to ASHRAE [24]**

<table>
<thead>
<tr>
<th>$T_{dew}$ = $F(P_{wv})$ - polynomial with constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; $T_{dew}$ &lt; 93°C</td>
</tr>
<tr>
<td>$C_{14} = 6.45, C_{15} = 14.526$</td>
</tr>
<tr>
<td>$C_{16} = 0.7389, C_{17} = 0.09486$</td>
</tr>
</tbody>
</table>
| $C_{18} = 0.4569$ | }
The enthalpy of a mixture of the perfect gas can be written as follows [24,102]

\[ h_g = h_{da} + \omega_g h_{wv} \]  \hspace{1cm} (2.100)

where \( h_{da} \) is the specific enthalpy for the dry air in \([KJ/Kg(dry air)]\), and \( h_{wv} \) is the specific enthalpy for saturated water vapor in \( KJ/Kg \) (water) at the temperature of the mixture. These enthalpies are given by the following expressions [102]:

The specific enthalpy of dry air is calculated as:

\[ h_{da}(T_g) = h_{da}^0 + C_{pa}(T_g - T_g^0) \]  \hspace{1cm} (2.101)

The specific enthalpy of water vapor is calculated using:

\[ h_{wv}(T_g) = h_{wv}^0 + C_{pwv}(T_g - T_g^0) \]  \hspace{1cm} (2.102)

Moreover, the specific enthalpy of pure water at saturation is calculated from:

\[ h_{wl}(T_g) = h_{wl}^0 + C_{pw}(T_g - T_g^0) \]  \hspace{1cm} (2.103)

Finally, the heat of vaporization is calculated using:

\[ h_{vap} = h_{wv} - h_{wl} = h_{vap}^0 + (C_{pwv} - C_{pw})T_g \]  \hspace{1cm} (2.104)

The approximate values of these enthalpies are shown in Table 2.5.

<table>
<thead>
<tr>
<th>Table 2.5</th>
<th>The specific enthalpies approximate input data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enthalpy of dry air</td>
<td>( h^0 [J/kg] )</td>
</tr>
<tr>
<td>Enthalpy of water vapor</td>
<td>2502500</td>
</tr>
<tr>
<td>Enthalpy of water liquid</td>
<td>100</td>
</tr>
<tr>
<td>Heat of vaporization</td>
<td>2502400</td>
</tr>
</tbody>
</table>
2.5.2 Properties of Matrix

In the modeling of energy wheel, the matrix can be designed by utilizing non-desiccant or desiccant materials. The dry non-desiccant matrix can be completely described with the specific heat capacity of the matrix. However, in the case of the wet matrix the liquid water condensed on the matrix surface should be included to describe the matrix.

For the sensible model, the enthalpy of a dry matrix is defined as:

\[ h_{dn} = C_p\ dm\ T_m \quad [KJ/Kg_{dm}] \quad (2.105) \]

For the condensation model, the enthalpy of the matrix is defined as the sum of the enthalpy of the dry matrix and the enthalpy of liquid water condensed on the surface of the matrix:

\[ h_{dm} = C_p\ dm\ T_m + C_p\ w/l\ T_m\ \omega_m \quad [KJ/Kg_{dm}] \quad (2.106) \]

For a desiccant matrix, the equilibrium relations for the water content and the enthalpy of the matrix are used. The enthalpy of the matrix \( h_m \) can be represented by a function of temperature of matrix \( T_m \) and the water content of the matrix \( \omega_m \) as follows:

\[ h_m = h(T_m, \omega_m) \quad (2.107) \]

The water content of the matrix \( \omega_m \) is related to the humidity ratio of the air in the equilibrium with matrix surface \( \omega_s \) through adsorption isotherm correlation, which can be expressed as [34].

\[ \omega_m = \sigma(T_m, \omega_s) \quad (2.108) \]
As mentioned above, Jurinak’s isothermal model is used in order to describe the equilibrium relations of desiccant materials [34].

Therefore, for the enthalpy model the enthalpy of the matrix is defined as the sum of the enthalpy of the dry matrix and the enthalpy of adsorbed water:

\[ h_m = h_{dm} + h_{ad} \]  \hspace{1cm} (2.109)

The difference between the enthalpy of water liquid and adsorbed liquid is denoted by the integral heat of wetting. The integral heat of wetting [57,62] can be written as:

\[ \Delta h_{wet}(\sigma_m, T_m) = \int_0^{\sigma_m} (h_{vap}(T_m) - h_{ad}(\sigma_m, T_m)) d\sigma = \int_0^{\sigma_m} h_{vap}(T_m) \left( 1 - \frac{h_{ad}(\sigma, T_m)}{h_{vap}(T_m)} \right) d\sigma \]  \hspace{1cm} (2.110)

Combining the above definition of integral heat of wetting and the definition of water vapor at matrix temperature, the enthalpy of matrix [62] can be written as:

\[ h_m = (C_p_m + C_{p,wl}\sigma_m)T_m + \Delta h_{wet} \]  \hspace{1cm} (2.111)

Moreover, for Jurinak’s model the ratio \( \frac{h_{ad}}{h_{vap}} \) is assumed to be a function of the matrix water content only and the heat of vaporization is dependent on matrix temperature, so the enthalpy of the matrix can be computed as:

\[ h_m = C_p_m T_m + C_{p,wl} T_m \sigma_m + h_{vap}(T_m) \int_0^{\sigma_m} (1 - h^*) d\sigma \]  \hspace{1cm} (2.112)
CHAPTER THREE
MODELING OF ROTARY ENERGY WHEELS

In this chapter, mathematical and numerical models for an air-to-air rotary energy wheel are presented. This chapter focuses on three models of the rotary wheel: sensible wheel mode, condensation model, and enthalpy model.

The configuration of the rotary energy wheel, coordinates, operation parameters, and assumptions are first introduced. Next, the governing equations, dimensionless representation, and numerical formulations of each model are discussed in separate sections. There are common issues for all three models; however, a specific consideration for modeling condensation/evaporation and adsorption/desorption is discussed. After completing the modeling of each wheel, the computer codes structure and programming issues are presented in section 3.6.

3.1 Model Description

The air-to-air rotary energy wheel under investigation is a counter-flow balanced and symmetric wheel. It is a rotating cylindrical wheel of length $L_w$ and diameter $d_w$ and it is divided into two equal sections: supply and exhaust section. While the wheel is rotating the equal supply and exhaust air-streams in equal quantities are supplied to the wheel in a counter-flow arrangement. The wheel available in the market consists of a matrix of tubular flow channels. Rectangular, triangular, or sinusoidal shapes of matrices
are commonly used. However, the unit under investigation consists of a porous matrix which is made of polyester filaments. The filaments comprising the polyester core are typically of the order of 50 microns in diameter. The core is highly porous, with air passages occupying most of the volume.

The differential equations in one cycle of energy wheel operation can be developed by considerations of the exchange of heat and moisture between the gas flowing through the matrix and the matrix itself.

3.1.1 Coordinates System and Nomenclature

For the formulation of the model, the nomenclature and coordinate system for the rotary energy wheel with diameter $d_w$ and matrix thickness $L_w$ are defined and illustrated in Figure 3.1. The supply side flow is denoted by subscript $s$ and the exhaust side flow by $e$. The in-flow and out-flow for both sides are distinguished by subscripts $i$ and $o$, respectively. The additional subscripts $m$ and $g$ can be used to differentiate between matrix and air flow stream. As an example, $T_{g,s,i}$ is the in-flow gas temperature of the supply side.

The axial coordinate, $x$, is defined as positive in the direction of flow, so that in a counter-flow system the axial coordinate reverses direction at the beginning of each side. The entry point of the supply or exhaust flow to the energy wheel is considered as $x = 0$. In the counter-flow situation, when the supply period finishes and exhaust period starts, the starting point $x$ changes. The point $x = L_w$ in the supply period becomes $x = 0$ in the
exhaust period. Thus, this enables us to use the same differential equations for both hot and cold periods.

Figure 3.1  Nomenclature and coordinate system for the energy wheel
The circumferential or azimuthal coordinate, $\theta$, originates at the beginning of the first air flow supply to the wheel and increases in the direction of the wheel rotation. There is a direct relationship between the angle, $\theta$, and time, $t$, through $t = \theta/\Omega$; therefore, either variable can be used to indicate rotational position. The wheel used in this research is of balanced flow type ($\Phi_1 = \Phi_2 = 180^\circ$). Therefore, if the time required for one complete cycle is $\tau$, then the time required for each period is equal ($\tau_1 = \tau_2 = \tau/2$).

### 3.1.2 Surface Geometrical Properties

The porous heat transfer media being studied is in the shape of a wheel that rotates between two counter-flow air streams. The wheel contains fiber mats, which serve as the heat and mass exchanger core. Since the wheel is symmetric and balanced, the wheel frontal area, free flow area, and heat transfer area on each side are the same. Therefore, they can be derived as follows:

- **Frontal cross-sectional area of the wheel (for each flow):**
  \[
  A_w = \left(\frac{\alpha}{100}\right)\left(\frac{\pi d_w^2}{4}\right) = \frac{\pi d_w^2}{8} \tag{3.1}
  \]

- **Cross-section area of the air stream:**
  \[
  A_g = \left(\frac{\alpha}{100}\right)\left(\phi\left(\frac{\pi d_w^2}{4}\right)\right) = \frac{\phi \pi d_w^2}{8} \tag{3.2}
  \]

- **Cross-section area of matrix:**
\[
A_m = \left( \frac{\alpha}{100} \right) (1 - \varphi) \left( \frac{\pi d_w^2}{4} \right) = \frac{(1 - \varphi) \pi d_w^2}{8}
\]

(3.3)

where:

- \( d_w \) diameter of the wheel [m]
- \( \varphi \) porosity
- \( \alpha \) wheel split 50%

Definition of porosity:

Porosity (\( \varphi \)) of the matrix which is the ratio of the air stream volume \( V_g \) to the total (gas \( V_g \) + matrix (solid) \( V_m \)) volume \( V_{tot} \), can be written as [102]

\[
\varphi = \frac{V_g}{V_{tot}} = \frac{V_g}{V_m + V_g} \quad \text{or} \quad \frac{V_m}{V_m + V_g} = 1 - \varphi
\]

(3.4)

Convective heat and mass transfer surface area:

The porous matrix is used in order to increase the available surface area \( A_s \) for heat and mass transfer. The density area or specific surface area \( \frac{A_s}{V_{tot}} \) is used to describe the compactness of the wheel [103]. The matrix has the polyester filaments with an average diameter \( d_f \) and porosity \( \varphi \). We can write \( A_s \) in terms of \( \varphi \) and \( d_f \) as

\[
\frac{A_s}{V_m} = \frac{\pi d_f L_w}{\left( \frac{\pi d_f^2}{4} \right) L_w} = \frac{4}{d_f}
\]

(3.5)

In the above equation (3.5), \( \frac{A_s}{V_m} \) is the volumetric or specific area based on the matrix volume (the solid volume faction is \( 1 - \varphi \)).

Using the definition of \( V_m \) and \( V_{tot} \), one can get
Hydraulic diameter

The hydraulic diameter is defined as [103]

\[ d_h = \frac{4(\text{void volume})}{\text{surface area}} = \frac{4V_g}{A_s} \]  

Equation (3.7) can be written in following form

\[ d_h = \frac{4(V_g/V_{tot})}{(A_s/V_{tot})} \]  

Substituting equation (3.4) and (3.5) into equation (3.8), yields

\[ d_h = \frac{\varphi d_f}{(1 - \varphi)} \]  

3.1.3 Dimensionless Parameters

A dimensional surface heat transfer rate can be obtained by dividing the convective heat flux expressed, \( h_{HT} \Delta T \) by the conduction heat flux, \( K_g (\Delta T/d_h) \), where \( d_h \) is characteristic width of the channel [103]. This yields the Nusselt number, \( Nu \), namely

\[ Nu = \frac{h_{HT} d_h}{K_g} \]  

The Reynolds number is defined by relation [103]:

\[ Re = \frac{\rho_g u d_h}{\mu_g} = \frac{\rho_g (u_g/\varphi)}{\mu_g} \frac{\varphi d_f}{(1 - \varphi)} = \frac{\rho_g u_g}{\mu_g} \frac{d_f}{(1 - \varphi)} \]  

where
\[ \rho_g \] density of the gas \([\text{Kg m}^{-3}]\) \\
\[ u_g \] superficial mean gas velocity \([\text{ms}^{-1}]\) \\
\[ d_f \] diameter of fiber \([\text{m}]\) \\
\[ \mu_g \] dynamic viscosity of the air \([\text{Kg m}^{-1}\text{s}^{-1}]\) \\
\[ \phi \] porosity

The thermal effects within the gas arise as a consequence of there being a temperature profile within the gas across the channel width. This is addressed by Prandtl number

\[
\text{Pr} = \frac{V_g}{\alpha_g} = \frac{\mu_g C_p_g}{K_g}
\]  

The surface heat transfer coefficient is computed, frequently using a correlation typically of the form \([103]\)

\[
Nu = \text{Nu}(\text{Re, Pr, flow regime, etc.}) = c \text{ Re}^n \text{ Pr}^m
\]  

where \(c, n\) and \(m\) are determined by the relevant experimental work. One of the correlations for the Nusselt number, \(Nu\), for porous media is defined based on the curve fit to the experimental results \([103]\). It is given by

\[
Nu = 2 + \left(0.4 \text{Re}^{1/2} + 0.2 \text{Re}^{2/3}\right)\text{Pr}^{0.4}
\]  

Therefore,

\[
h_{HT} = \frac{NuK_g (1 - \phi)}{\phi d_f}
\]  

For similar geometry and boundary conditions, convective mass transfer \(h_{MT}\) is analogous to convective heat transfer \(h_{HT}\). The design parameters of the energy wheel that govern the heat and mass transfer mechanisms are the number of transfer units for the heat transfer between one air stream and the matrix, defined as \([62]\)
and the number of transfer units for mass transfer:

\[ NTU_{MT} = \frac{h_{MT} A_s}{m_g C_p g} \]  \hspace{1cm} (3.17)

The simultaneous heat and mass transfer problems are linked using an overall Lewis number which is defined as the ratio of the heat and mass transfer numbers:

\[ Le = \frac{NTU_{HT}}{NTU_{MT}} = \frac{h_{HT}}{h_{MT} C_p g} \]  \hspace{1cm} (3.18)

### 3.2 Assumptions

The mathematical model is based on the idealization of a physical air-to-air rotary energy wheel system.

In particular, the model assumes that:

1. The matrix is assumed to be a porous homogenous solid with constant characteristic and porosity though which an air-water vapor mixture flows with constant velocity.
2. A mixing of the flow between the supply and exhaust section does not take place.
3. The pressure drop along the axial flow length of the wheel is small compared to the total pressure.
4. The effect of a small pressure drop on the changes of thermodynamic properties of the air and the matrix is negligible.
5. The carry-over of air due to the wheel switching from one period to the other is neglected because the mass of this air is small compared to the mass of the matrix.

6. The radial gradients of temperature and moisture are neglected for both the matrix and the gas phase.

7. The axial heat conduction and axial mass diffusion in the air are negligible.

8. The axial diffusion of moisture within the matrix is negligible.

9. The heat and mass transfer coefficients for both periods are considered to be identical and do not vary spatially or with time.

The differential equations describing the operation of the models stated above will be derived with the above assumptions. Two sets of differential equations can be used to describe the three previously mentioned models: one for the heat transfer only and the other for the heat and mass transfer. The heat transfer or heat and mass transfer balance can be performed on an elemental volume of the energy wheel. The elemental volume will consist of both matrix and flow stream, with air occupying a fraction $\phi$ of the volume. The porosity $\phi$ will be considered constant throughout the matrix.

An energy balance on this segment can be performed in one of two ways: with either the control volume fixed in space as the air and matrix move through the volume or the control volume fixed on the matrix and rotated with the matrix. In the first case, the dependent variables are the axial coordinate, the angle of rotation, and the time. For the second case, the dependent variables are only the axial coordinate and the time. The analyses of this thesis will employ equations derived from an elemental volume which rotates with the matrix.
3.3 Heat Transfer Model (Sensible Wheel Model)

3.3.1 Governing Equations

The mathematical formulation of the heat transfer for the energy wheel is based on the energy balance for an elemental volume of matrix and flow stream. The governing equations are derived for the control volume of Figure 3.2.

![Figure 3.2: The energy balance of the unit volume element of the energy wheel](image)

**In the flow region:**

If the first law of thermodynamics is applied to the unit volume element of the wheel in Figure 3.2, then the energy balance of the flow will be

\[
q_{\text{stored_flow}} = q_{\text{convective}}(x) - q_{\text{convective}}(x + dx) - dq_{\text{conv}}
\]

(3.19)

Here, \(q_{\text{stored_flow}}\) represents the rate of heat capacity storage of the flow in the unit volume, and \(q_{\text{conv}}\) represents the heat transfer rate from the flow to the matrix or from matrix to the flow by convection, while the \(q_{\text{convective}}\) represents the heat transfer rate by
convective through the unit volume. If the necessary changes are applied in equation (3.19) above then,

\[ q_{\text{stored flow}} = q_{\text{convective}}(x) - \left[ q_{\text{convective}}(x) + \frac{\partial q_{\text{convective}}}{\partial x} \right] dx - dq_{\text{conv}} \]  

(3.20)

can be obtained. In this equation \( q_{\text{stored flow}} \), \( dq_{\text{conv}} \) and \( q_{\text{convective}} \) can be expressed as follows:

\[ q_{\text{stored flow}} = \rho_x A_x dx C_{p_x} \frac{\partial T_g}{\partial t} \]  

(3.21)

\[ dq_{\text{conv}} = h_{HT} A_s \frac{dx}{L} (T_g - T_m) \]  

(3.22)

\[ q_{\text{convective}} = m C_{p_g} T_g = U_g A_g \rho_g C_{p_g} T_g \]  

(3.23)

Thus, if equations (3.21), (3.22) and (3.24) are substituted in equation (3.20), then

\[ \rho_g A_g dx C_{p_g} \frac{\partial T_g}{\partial t} = U_g A_g \rho_g C_{p_g} T_g - U_g A_g \rho_g C_{p_g} \left( T_g + \frac{\partial T_g}{\partial x} \right) dx \]

\[ - h_{HT} A_s \left( \frac{dx}{L} \right) (T_g - T_m) \]  

(3.24)

can be obtained. Further simplification yields

\[ \rho_g A_g dx C_{p_g} \frac{\partial T_g}{\partial t} = -U_g A_g \rho_g C_{p_g} \left( \frac{\partial T_g}{\partial x} \right) dx - h_{HT} A_s \left( \frac{dx}{L} \right) (T_g - T_m) \]  

(3.25)

\[ \rho_g A_g C_{p_g} \frac{\partial T_g}{\partial t} = -U A_g \rho_g C_{p_g} \frac{\partial T_g}{\partial x} - h_{HT} \frac{A_s}{L} (T_g - T_m) \]  

(3.26)

\[ \phi \rho_g C_{p_g} \frac{\partial T_g}{\partial t} = -\phi U_g \rho_g C_{p_g} \frac{\partial T_g}{\partial x} - h_{HT} A_v (T_g - T_m) \]  

(3.27)
Equation (3.27) is used to calculate the temperature distribution of the flow inside the wheel.

**In the matrix:**

Similarly, if the first law of thermodynamics is applied to the matrix element of the wheel, then the energy balance

\[ q_{\text{stored}_\text{matrix}} = q_{\text{cond}}(x) - q_{\text{cond}}(x + dx) + q_{\text{conv}} \quad (3.28) \]

can be obtained. If some changes are made in the above equation, then

\[ q_{\text{stored}_\text{matrix}} = q_{\text{cond}}(x) - \left( q_{\text{cond}}(x) + \frac{\partial q_{\text{cond}}}{\partial x} dx \right) + q_{\text{conv}} \quad (3.29) \]

is obtained. In equation (3.29), \( q_{\text{stored}_\text{matrix}} \) represents the heating rate of matrix unit volume of the wheel, \( q_{\text{cond}}(x) \) represents the heat transfer rate in the direction of \( x \) in the matrix unit volume. \( q_{\text{stored}_\text{matrix}} \) and \( q_{\text{cond}}(x) \) can be expressed as follows:

\[ q_{\text{stored}_\text{matrix}} = \rho_m A_m dx Cp_m \frac{\partial T_m}{\partial t} \quad (3.30) \]

\[ q_{\text{cond}}(x) = -K_m A_m \frac{\partial T_m}{\partial x} \quad (3.31) \]

So, if equations (3.22), (3.30) and (3.31) are substituted in equation (3.29) and some necessary changes are made in this equation, one can get the differential equation used for calculating the temperature distribution of the matrix of the regenerator:

\[ \rho_m A_m dx Cp_m \frac{\partial T_m}{\partial t} = -K_m A_m \frac{\partial T_m}{\partial x} - K_m A_m \left( -\frac{\partial T_m}{\partial x} - \frac{\partial^2 T_m}{\partial x^2} dx \right) + h_{HT} A_e \left( \frac{dx}{L} \right) \left( T_g - T_m \right) \quad (3.32) \]
To complete the formulation of the problem, the boundary and the initial conditions are the following:

**Supply side (period 1):** \((k-1)(\tau) \leq t \leq (k-1)(\tau) + \tau/2, \ k = 1,2,3,\ldots,N_{\text{cycle}}\)

\[
T_g(x=0,t) = T_{g-s,i} \tag{3.36}
\]

Reversal condition at the start of supply side

\[
T_{m-s}(x,(k-1)\tau) = T_{m-e}(x,k\tau) \tag{3.37}
\]

\[
T_{g-s}(x,(k-1)\tau) = T_{g-e}(x,k\tau) \tag{3.38}
\]

**Exhaust side (period 2):** \((k-1)(\tau) + \tau/2 \leq t \leq (k)(\tau), \ k = 1,2,3,\ldots,N_{\text{cycle}}\)

\[
T_g(x=0,t) = T_{g-e,i} \tag{3.39}
\]

Reversal condition at the start of exhaust side:

\[
T_{m-e}(x,(k-1)\tau + \tau/2) = T_{m-s}(x,(k-1)\tau + \tau/2) \tag{3.40}
\]

\[
T_{g-e}(x,(k-1)\tau + \tau/2) = T_{g-s}(x,(k-1)\tau + \tau/2) \tag{3.41}
\]

The boundary conditions for the matrix are chosen to be adiabatic. Mathematically this is

\[
\frac{\partial T_m(x=0,t)}{\partial x} = \frac{\partial T_m(x=L_w,t)}{\partial x} = 0 \tag{3.42}
\]
The common initial conditions for starting the simulation are

\[
T_g(x,0) = T_{g,0}, \quad \text{and} \quad T_m(x,0) = \frac{T_{g,s,t} + T_{g,c,t}}{2}
\]  

(3.43)

After solving the differential equations that were expressed in equation (3.27) and equation (3.35) together with the boundary and initial conditions, the flow and the matrix temperatures can be found and, hence, the wheel effectiveness. Those differential equations are valid for both cold and hot regions. For this, the coordinates of the starting point \( x \), which represents the coordinate of the direction of the flow, should be chosen so that the starting point \( x \) should change while shifting from one period to another in a counter flow. Thus, this eliminates the need for different differential equations for hot and cold periods.

### 3.3.2 Dimensionless Representation

Applying the first law of thermodynamics to the unit element of the matrix and the flow stream, the differential equations for the flow stream and the matrix temperature were obtained as discussed in the above section. For the sake of completeness, these equations can be express as follows:

\[
\varphi \rho_g C_p \frac{\partial T_g}{\partial t} + \varphi u_g \rho_g C_p \frac{\partial T_g}{\partial x} + h_{HT} A_v (T_g - T_m) = 0
\]  

(3.44)

\[
(1 - \varphi) \rho_m C_p \frac{\partial T_m}{\partial t} - (1 - \varphi)K_m \frac{\partial^2 T_m}{\partial x^2} - h_{HT} A_v (T_g - T_m) = 0
\]  

(3.45)

They are linear coupled partial differential equations. By solving equations (3.44) and (3.45), the temperature of the flow stream and the matrix in the wheel can be
calculated. It is possible to use dimensionless variables and parameters in order to make the differential equations dimensionless. This will enable us to use less parameters in these differential equations. The solution and analysis of the dimensionless equations become easy and understandable. For this reason, in order to make equations (3.44) and (3.45) dimensionless, the following dimensionless variables and parameters are defined.

\[ t^+ = \frac{t}{\tau} \quad \text{non-dimensional time} \]  

\[ x^+ = \frac{x}{L_w} \quad \text{non-dimensional length} \]  

\[ T^+_g = \frac{T_g - T_{g,e,i}}{T_{g,s,i} - T_{g,e,i}} \quad \text{non-dimensional temperature of gas} \]  

\[ T^+_m = \frac{T_m - T_{g,e,i}}{T_{g,s,i} - T_{g,e,i}} \quad \text{non-dimensional temperature of matrix} \]  

\[ Fa = \frac{\alpha_m \tau}{L_w^2} \quad \text{the Fourier number, where } \alpha_m = \frac{K_m}{\rho_m C_p m} \]  

\[ \Gamma = \frac{u \tau}{L_w} \quad \text{dimensionless period} \]  

The gas capacity rates are defined by

\[ C_{g,s} = (\dot{m} C_p)_{g,s} = (\dot{\rho} A u C_p)_{g,s} \]  

\[ C_{g,e} = (\dot{m} C_p)_{g,e} = (\dot{\rho} A u C_p)_{g,e} \]  

where \( \dot{m}_g \) is the mass flow rate and \( C_p \) is the specific heat capacity of the gas. \( C_{g_{\text{min}}} \) is defined as the smaller of the supply \( (C_{g,s}) \) and exhaust \( (C_{g,e}) \) gas capacity rates. The relation \( C_{g,s} = C_{g,e} \) is valid since the wheel under investigation is balanced flow wheel.
The number of the heat transfer unit is defined as:

\[ NTU_{HT} = \frac{h_{HT} A_s}{(\rho Au C_p)_g} = \frac{h_{HT} A_s}{C_{g_{min}} A_s} = \frac{h_{HT} A_s}{C_g} \]  

(3.54)

where \( h_{HT} \) is overall heat transfer coefficient and \( A_s \) is the heat transfer surface area of the rotary energy wheel.

Heat capacity-rate ratio of the rotary regenerator \( C_r^+ \) is defined as

\[ C_r^+ = \frac{(\rho ALC_p)_m}{(\rho Au C_p)_g} \Omega = \frac{(MCp)_m}{(mCp)_g} \Omega = \frac{C_r}{C_g} \]  

(3.55)

where \( C_r \) is the heat capacity-rate of the rotary wheel matrix:

\[ C_r = (\rho ALC_p)_m \Omega = (MCp)_m \Omega \]  

(3.56)

where \( \Omega \) is the rotational speed of the wheel. Here \( M_m \) and \( C_{p_m} \) are mass and specific heat capacity of the matrix respectively.

Using these dimensionless parameters, the dimensionless form of the coupled partial differential equations (3.44) and (3.45) are:

\[ \frac{1}{\Gamma} \frac{\partial T^+_g}{\partial t^+} + \frac{\partial T^+_g}{\partial x^+} + NTU_{HT} (T^+_g - T^+_m) = 0 \]  

(3.57)

\[ \frac{\partial T^+_m}{\partial t^+} - F_{om} \frac{\partial^2 T^+_m}{\partial x^+^2} \frac{NTU_{HT}}{C^+_r} (T^+_g - T^+_m) = 0 \]  

(3.58)

These equations are valid for wheels that have parallel or counter flow and for cold and hot periods at the same time. The above equations (3.57) and (3.58) can be expressed in different formats, in which the first side of the equation depends on the temperature of the flow and the other side depends on the matrix temperature. By solving
these equations together, the temperature of the flow in the wheel and the temperature of
the matrix can be obtained in a dimensionless form. There exists four dimensionless
parameters in the wheel differential equations (3.57) and (3.58) given with dimensionless
temperatures and space-time variables. There is an apparent advantage of exploring the
effects on the solutions of those parameters over the solutions according to each and
every dimensional parameters of the model. Dimensionless model can produce a very
broad and general conclusion of the rotary wheel under investigation. To solve the
dimensionless differential equations the initial and boundary conditions should also be
expressed in the dimensionless form.

**Boundary and periodic equilibrium conditions:**

**For the gas flow part (Figure 3.3):**

During the supply period (heating period):

\[
x^+ = 0
\]

\[
(k - 1) < t^+ < (k - 1) + \frac{1}{2} , k = 1, 2, ..., N_{cycle}
\]

\[
\Rightarrow T_g^+(x^+ = 0, t^+) = 1
\]

(3.59)

During the exhaust period (cooling period):

\[
x^+ = 0
\]

\[
(k - 1) + \frac{1}{2} < t^+ < k , k = 1, 2, ..., N_{cycle}
\]

\[
\Rightarrow T_g^+(x^+ = 0, t^+) = 0
\]

(3.60)

During reversal time:

The gas temperature at the beginning of the heating period is equal to the gas
temperature at the end of the cooling period:

\[
T_{g_{x^-}}^+(x^+, k - 1) = T_{g_{x^-}}^+(x^+, k) , \text{ for } k = 1, 2, ..., N_{cycle}
\]

(3.61)
Figure 3.3  Boundary conditions for the flow streams in counter-flow regime

For the matrix part (Figure 3.4):

Entrance of the matrix: \( x^+ = 0 \) \( \Rightarrow \frac{\partial T^+_m(x^+ = 0, t^+)}{\partial x^+} = 0 \)  \( (3.62) \)

Exit of the matrix: \( x^+ = 1 \) \( \Rightarrow \frac{\partial T^+_m(x^+ = 1, t^+)}{\partial x^+} = 0 \)  \( (3.63) \)

During reversal time:

The matrix temperature at the beginning of the heating period is equal to the matrix temperature at the end of the cooling period:

\[ T^+_{m,e}(x^+, k - 1) = T^+_{m,s}(x^+, k), \text{ for } k = 1, 2, \ldots, N_{\text{cycle}} \]  \( (3.64) \)
Initial conditions:

\[
T^+_g(x^+,0) = T^+_g(0), \quad \text{and } T^+_m(x^+,0) = \frac{T^+_{g,s,0} + T^+_{g,0}}{2}
\]  

(3.65)

3.3.3 Effectiveness Correlation for Limiting Cases

The outlet temperatures of the wheel are conveniently expressed by the wheel effectiveness

\[
\varepsilon_{\text{wheel}} = \frac{q}{q_{\text{max}}}
\]

(3.66)

the actual heat transfer rate in the wheel is
\[ q = C_{g-s} \left( T_{g-s,i} - \overline{T}_{g-s,o} \right) = C_{g-e} \left( \overline{T}_{g-e,o} - T_{g-e,i} \right) \]  

(3.67)

where

\[ \overline{T}_{g-s,o} = \frac{1}{\tau_1} \int_0^1 T_{g-s,o}(x = L_w, t) dt, \quad \overline{T}_{g-e,o} = \frac{1}{\tau_2} \int_0^1 T_{g-e,o}(x = L_w, t) dt \]  

(3.68)

the thermodynamically limited maximum transfer rate is defined as

\[ q_{\text{max}} = C_{g-min} \left( T_{g-s,i} - T_{g-e,i} \right) \]  

(3.69)

where

\[ C_{g-min} = \min(C_{g-s}, C_{g-e}) \]  

(3.70)

the wheel effectiveness is thus

\[ \varepsilon_{\text{wheel}} = \frac{C_{g-s} \left( T_{g-s,i} - \overline{T}_{g-s,o} \right)}{C_{g-min} \left( T_{g-s,i} - T_{g-e,i} \right)} = \frac{C_{g-e} \left( \overline{T}_{g-e,o} - T_{g-e,i} \right)}{C_{g-min} \left( T_{g-s,i} - T_{g-e,i} \right)} \]  

(3.71)

then for the case \( C_{g-min} = C_{g-e} \)

\[ \varepsilon_{\text{wheel}} = \frac{\overline{T}_{g-e,o} - T_{g-e,i}}{T_{g-s,i} - T_{g-e,i}} = \overline{T}_{g-e,o}^{+} \]  

(3.72)

and for the case \( C_{g-min} = C_{g-s} \)

\[ \varepsilon_{\text{wheel}} = \frac{T_{g-s,i} - \overline{T}_{g-s,o}}{T_{g-s,i} - T_{g-e,i}} = 1 - \overline{T}_{g-s,o}^{+} \]  

(3.73)

Coppage and London [35] developed \( \varepsilon - NTU_o \) for design and analysis of the regenerator (rotary sensible heat wheel). They showed that the effectiveness of the wheel is a function of four non-dimensional parameters as follows

\[ \varepsilon_{\text{wheel}} = f \left( NTU_o, C_g^+, C_r^+, (h_{HT} A_s)^+ \right) \]  

(3.74)

the modified number of transfer units \( NTU_o \) is defined as
the heat capacity ratio \( C^+_g \) is simply the ratio of the smaller to the larger heat capacity rate of the gas streams so that \( C^+_g \) should be less than or equal to 1.

\[
C^+_g = \frac{C_{g_{\min}}}{C_{g_{\max}}} \tag{3.76}
\]

the capacity rate \( C^+_r \) is the matrix heat capacity rate \( C_r \) normalized with respect to the minimum heat capacity rate of the flow \( C_{g_{\min}} \)

\[
C^+_r = \frac{C_r}{C_{g_{\min}}} \tag{3.77}
\]

the ratio of convective conductance \((h_{HT\ A_s})^+\) is defined by the \((h_{HT\ A_s})\) on the \(C_{g_{\min}}\) side to that on the \(C_{g_{\max}}\) side

\[
(h_{HT\ A_s})^+ = \frac{(h_{HT\ A_s}) \text{ on the } C_{g_{\min}} \text{ side}}{(h_{HT\ A_s}) \text{ on the } C_{g_{\max}} \text{ side}} \tag{3.78}
\]

For a special case of \( C^+_r \to \infty \), the effectiveness of the wheel approaches that of the counter-flow direct-type exchanger (recuperator). The influence of the \( C^+_r \) on the \( \varepsilon_{\text{wheel}} \) can be presented by an empirical correlation as suggested by Lamberston [36] and later modified by Kays and London [41]:

\[
\varepsilon_{\text{wheel}} = \varepsilon_{\text{cf-recuperator}} \left[1 - \frac{1}{9(C^+_r)^{1.93}}\right] \tag{3.79}
\]

where \( \varepsilon_{\text{cf-recuperator}} \) is the counter-flow recuperator effectiveness as follows:
For a balanced air-to-air rotary sensible heat wheel, the convective heat transfer coefficient and convective heat transfer area on the cold and hot side are assumed equal. Therefore, \( C_g^+ = 1 \) and \( (h_{HT} A_e)^+ = 1 \). So, the equation becomes

\[
\chi_{cf\rightarrow recuperator} = \begin{cases} 
\frac{1 - e^{-NTU_0 (1-C_r^+)}}{1 - C_g^+ e^{-NTU_0 (1-C_r^+)}} & \text{for } C_g^+ < 1 \\
\frac{NTU_0}{1 + NTU_0} & \text{for } C_g^+ = 1
\end{cases} 
\] (3.80)

Figure 3.5 shows the effect of the \( C_r^+ \) on the sensible effectiveness of a rotary sensible wheel. One can note that the \( \chi_{cf\rightarrow recuperator} \) is approached with increasing values of \( C_r^+ \). Equation 3.79 cannot be used for very low rotation speeds, which is equivalent to low values of \( C_r^+ \). A close look at equation (3.79) shows that the equation does not produce zero for \( C_r^+ = 0 \); the equation is zero when \( C_r^+ = 0.32 \). A new equation has been derived by [52], which accounts for the influence of low rotational speed on the wheel effectiveness. Discussion of derivation of this equation and a new equation for large rotational speed will be presented next.

If the rotational speed of the rotary wheel approaches zero (\( \Omega \rightarrow 0 \Rightarrow C_r^+ \rightarrow 0 \)), it is reasonable to assume that the matrix of the wheel is heated to the maximum temperature of the hot air flow (supply flow) immediately and cooled to the cold air stream (exhaust flow) temperature instantly. The amount of the heat transferred in a half cycle can be found by:
\[ q = \frac{C_m}{2} \left( T_{g_{s,i}} - T_{g_{e,i}} \right) \]  

where

\[ C_m = m_mC_p \] is the thermal capacitance of the matrix \([J/K]\)

Figure 3.5  Effect of \(C_r^+\) on the rotary wheel effectiveness

The maximum heat that can be transferred in a half cycle is given by:

\[ q_{\text{max}} = C_{g_{s,s}} \left( T_{g_{s,i}} - T_{g_{e,i}} \right)P_1 \]  

(3.83)

where

\[ C_{g_{s,s}} = (mC_p)_{g_{s,s}} \] is the heat capacity rate of hot gas \([J/s.k]\)
\[ P_1 = \frac{\tau}{2} \]  

is the time for the hot period for balanced wheel \([s]\)

Substituting equations (3.82) and (3.83) into equation (3.66) yields

\[
\varepsilon_{wheel} = \frac{C_m}{C_{g-s}} \frac{1}{\tau} = \frac{C_m}{C_{g-s}} \frac{\Omega}{r} = \frac{C_r}{C_{g-s}} = C_r^+ \quad (3.84)
\]

Therefore, the effectiveness of the wheel as the rotational speed approaching zero \((\Omega \to 0, \text{and therefore } C_r^+ \to 0)\) can be expressed as:

\[
\varepsilon_{wheel} = C_r^+ \quad \text{for} \quad C_r^+ \to 0 \quad (3.85)
\]

A more accurate expression for the effectiveness has been derived by [52] from the differential equations for the wheel for the case \(\Omega \to 0\), assuming constant temperatures for hot and cold air streams flowing within the wheel. The justification of this assumption is that the effectiveness of the wheel approaches zero \((\varepsilon_{wheel} \to 0 \text{ as } \Omega \to 0)\).

\[
\varepsilon_{wheel} = C_r^+ \frac{\alpha}{\alpha + 1} \left[1 - \frac{2C_r^+}{NTU_{HT}} \left(1 - \frac{1}{\alpha}\right)\right] \quad (3.86)
\]

where

\[
\alpha = e^{-\frac{NTU_{HT}}{2C_r^+}} \quad (3.87)
\]

For the case of \(C_r^+ \to 0\), \(\alpha\) approaches infinity \((\alpha \to \infty)\) and equation (3.86) simplifies to

\[
\varepsilon_{wheel} = C_r^+ \quad (3.89)
\]

Maclaine-Cross [42] showed that for \(C_r^+ \leq 0.4\) the above effectiveness correlation holds. Figure 3.6 illustrates the effect of small \(C_r^+\) on the effectiveness of the rotary
sensible wheel using equations (3.86) and (3.89). There is a good agreement between the two equations for $C_r^+ \leq 0.4$; however, for a larger value of $C_r^+$ the trend of effectiveness values is diverged and is evident that equation (3.89) cannot be used for large $C_r^+$ [52].

![Figure 3.6](image)

**Figure 3.6** Effect of low $C_r^+$ on the effectiveness of rotary sensible wheel

### 3.3.4 Finite Difference Equations

For the purpose of numerical computations, differential equations of the sensible wheel can be written more concisely as in the following form:

$$
\frac{1}{\Gamma} \frac{\partial T_g^+}{\partial t^+} + \frac{\partial T_g^+}{\partial x^+} + NTU_{HT} (T_g^+ - T_m^+) = 0
$$

(3.90)
The mathematical model of heat transfer in sensible heat wheel are coupled partial differential equations with the given boundary and periodic equilibrium conditions. The available analytical solutions are restricted to simple model equations. Therefore, the numerical solutions of the mathematical model are required to have an accurate characteristic of wheel performance.

This section describes the method used to perform the numerical calculations by finite difference techniques. The method is based on that used by Willmott [45] for general scheme and the one used by Tsai et al. [89] for solving the advection-diffusion equations.

An integral-based method developed by Tsai et al. [89] was applied with some modifications to both gas and matrix phase. A computational element consisting of two consecutive nodes \((x_i, x_{i+1})\) is used to integrate the differential equations for supply side (hot), exhaust side (cold) period, and matrix. Using the integral-based method to solve the governing differential equations offers two key advantages: stability and a reduction in the total computational time [51]. The sketch of grid representation of the numerical scheme for supply side direction is shown in Figure 3.7.

Integrating equation (3.90) over the computational element \((x_i, x_{i+1})\) in the direction \(\Rightarrow\) of the supply flow yields:

\[
\int_{x_i}^{x_{i+1}} \frac{1}{\Gamma} \frac{\partial T_g^+}{\partial t^+} dx^+ + \int_{x_i}^{x_{i+1}} \frac{\partial T_m^+}{\partial x^+} dx^+ + \int_{x_i}^{x_{i+1}} NTU_{HT} (T_g^+ - T_m^+) dx^+ = 0
\]

(3.92)
Rearranging equation (3.92) yields:

$$\frac{1}{\Gamma} \int_{x_i}^{x_i + \Delta x^+} \frac{\partial T_g^+}{\partial t^+} dx^+ + \int_{x_i}^{x_i + \Delta x^+} \frac{\partial T_g^+}{\partial x^+} dx^+ + NTU_{HT} \int_{x_i}^{x_i + \Delta x^+} (T_g^+ - T_m^+) dx^+ = 0$$  \hspace{1cm} (3.93)$$

Interchanging the integration with partial derivative for the first term and solving the second and the last integral term yields:

$$\frac{1}{\Gamma} \frac{\partial}{\partial t^+} \left[ \int_{x_i}^{x_i + \Delta x^+} T_g^+ dx^+ \right] + T_g^+ \bigg|_{x_i}^{x_i + \Delta x^+} + NTU_{HT} \int_{x_i}^{x_i + \Delta x^+} (T_g^+ - T_m^+) dx^+ = 0$$  \hspace{1cm} (3.94)$$
\[
\frac{1}{\Gamma} \frac{\partial}{\partial t^+} \left( \int_{x_i^+}^{x_{i+1}^+} T_{g}^+ \, dx^+ \right) + \left[ T_{g, i+1}^+ - T_{g, i}^+ \right] + NTU_{HT} \int_{x_i^+}^{x_{i+1}^+} (T_{g, i}^+ - T_{m}^+) \, dx^+ = 0 \quad (3.95)
\]

Now it is possible to define \( \bar{T}_{g, i+\frac{1}{2}}^+ \) as the average temperature over the computational element \((x_i, x_{i+1})\):

\[
\frac{1}{\Delta x^+} \int_{x_i^+}^{x_{i+1}^+} T_{g}^+ \, dx^+ = \bar{T}_{g, i+\frac{1}{2}}^+ \quad (3.96)
\]

Substituting equation (3.96) into equation (3.95) produces:

\[
\frac{1}{\Gamma} \frac{\partial \bar{T}_{g, i+\frac{1}{2}}^+}{\partial t^+} \Delta x^+ + \left[ T_{g, i+1}^+ - T_{g, i}^+ \right] + NTU_{HT} (\bar{T}_{g, i+\frac{1}{2}}^+ - \bar{T}_{m, i+\frac{1}{2}}^+) \Delta x^+ = 0 \quad (3.97)
\]

Using the forward difference in time (FT) of gas temperature \((n \uparrow)\) yields:

\[
\frac{\partial \bar{T}_{g, i+\frac{1}{2}}^+}{\partial t^+} = \frac{\bar{T}_{g, i+1}^+ - \bar{T}_{g, i}^+}{\Delta t^+} \quad (3.98)
\]

Using fully-implicit discretization in time to equation (3.97), one obtains:

\[
\frac{1}{\Gamma} \frac{\bar{T}_{g, i+\frac{1}{2}}^{n+1} - \bar{T}_{g, i+\frac{1}{2}}^n}{\Delta t^+} \Delta x^+ + \left[ T_{g, i+1}^{n+1} - T_{g, i}^{n+1} \right] + NTU_{HT} (\bar{T}_{g, i+\frac{1}{2}}^{n+1} - \bar{T}_{m, i+\frac{1}{2}}^{n+1}) \Delta x^+ = 0 \quad (3.99)
\]

To find the average temperature over the computational element \([x_i^+, x_{i+1}^+]\), the trapezoidal rule is used.

\[
\int_{x_i^+}^{x_{i+1}^+} T_{g}^+ \, dx^+ = \frac{\Delta x^+}{2} \left( T_{g, i+1}^+ + T_{g, i}^+ \right) \quad (3.100)
\]

The averaged temperature over the interval \([x_i^+, x_{i+1}^+]\), is found by replacing equation (3.100) into equation (3.96):
The supply gas temperature can be found by replacing the average temperature in equation (3.99) by equation (3.101)

\[
\frac{T_{g_{i+\frac{1}{2}}}^+}{2} = \frac{T_{g_{i+1}}^+ + T_{g_{i}}^+}{2} \quad (3.101)
\]

Rearranging equation (3.103) yields the final form of the finite difference equation for the supply gas temperature:

\[
\frac{1}{\Gamma \Delta t^+} \left[ \left( \frac{T_{g_{i+1}}^+ + T_{g_{i}}^+}{2} \right)^{n+1} - \left( \frac{T_{g_{i+1}}^+ + T_{g_{i}}^+}{2} \right)^n \right] + \frac{1}{\Delta x^+} [T_{g_{i+1}}^+ - T_{g_i}^+] \\
+ NTU_{HT} \left[ \left( \frac{T_{m_{i+1}}^+ + T_{m_i}^+}{2} \right)^{n+1} - \left( \frac{T_{m_{i+1}}^+ + T_{m_i}^+}{2} \right)^n \right] \Delta x^+ = 0 \quad (3.102)
\]

\[
\frac{1}{\Gamma \Delta t^+} \left[ \left( \frac{T_{g_{i+1}}^+ + T_{g_{i}}^+}{2} \right)^{n+1} - \left( \frac{T_{g_{i+1}}^+ + T_{g_{i}}^+}{2} \right)^n \right] + \frac{1}{\Delta x^+} [T_{g_{i+1}}^+ - T_{g_i}^+] \\
+ NTU_{HT} \left[ \left( \frac{T_{m_{i+1}}^+ + T_{m_i}^+}{2} \right)^{n+1} - \left( \frac{T_{m_{i+1}}^+ + T_{m_i}^+}{2} \right)^n \right] = 0 \quad (3.103)
\]

Rearranging equation (3.103) yields the final form of the finite difference equation for the supply gas temperature:

\[
\left[ A_g + B_g + C_g \right] T_{g_{i}}^{n+1} = -\left[ A_g - B_g + C_g \right] T_{g_{i-1}}^{n+1} + \left[ A_g \right] T_{g_{i+1}}^{n+1} + T_{g_{i}}^{n+1} + \left[ C_g \right] T_{m_{i}}^{n+1} + T_{m_{i}}^{n+1} \quad (3.104)
\]

where

\[
A_g = \frac{1}{2\Delta t^+ \Gamma}, B_g = \frac{1}{\Delta x^+}, \text{ and } C_g = \frac{NTU_{HT}}{2}
\]

In the case of exhaust air-stream side, the discretized energy equation of the air stream on the direction of exhaust side is the same as the equation on the supply side since the direction of the air-flow goes with positive x-axis for exhaust side. The integration is performed on the computational element \([x_{i+1} \leftarrow x_i]\). The sketch of grid
representation of the numerical scheme for exhaust side direction is shown in Figure 3.8. Therefore, equation (3.90) is integrated over \([x_{i+1} \leftarrow x_i]\) using integral-based method as shown below \((\iff)\):

\[
\begin{align*}
\int_{x_i}^{x_{i+1}} \frac{1}{\Gamma} \frac{\partial T_g^+}{\partial t^+} \, dx^+ + \int_{x_i}^{x_{i+1}} \frac{\partial T_g^+}{\partial x^+} \, dx^+ + \int_{x_i}^{x_{i+1}} NTU_{HR} (T_g^+ - T_m^+ ) \, dx^+ = 0
\end{align*}
\] (3.105)

Figure 3.8 Sketch of grid representation of the numerical scheme for exhaust side (sweep in x-direction)
Applying the same approach as for solving the equation for the temperature of the gas on the supply side, the temperature of the gas on the exhaust side becomes:

\[
\begin{align*}
[A_g + B_g + C_g]T_{g,i}^{n+1} &= -[A_g - B_g + C_g]T_{g,i-1}^n + [A_g]T_{g,i}^n + T_{g,i}^n
+ [C_g]T_{m,i}^n + T_{m,i}^n\quad(3.106)
\end{align*}
\]

In the case of matrix, applying the integral-based method finite difference method on equation (3.91), the temperature of the matrix can be expressed as:

\[
\int_{x_i}^{x_{i+1}} \frac{\partial T_m^+}{\partial t^+} dx^+ - \int_{x_i}^{x_{i+1}} \frac{\partial^2 T_m^+}{\partial x^+\partial t^+} dx^+ - \int_{x_i}^{x_{i+1}} \frac{NTU_{HR}}{C_r} (T_g^+ - T_m^+) dx^+ = 0 \quad(3.107)
\]

\[
\frac{\partial}{\partial t^+} \left( \int_{x_i}^{x_{i+1}} T_m^+ dx^+ \right) - \int_{x_i}^{x_{i+1}} \frac{\partial}{\partial x^+} \left( \frac{\partial T_m^+}{\partial x^+} \right) dx^+ - \int_{x_i}^{x_{i+1}} \frac{NTU_{HR}}{C_r} (T_g^+ - T_m^+) dx^+ = 0 \quad(3.108)
\]

Defining \(\bar{T}_{m,i+1/2}^+\) as the average temperature over the computational element \([x_i, x_{i+1}]\) as in flow stream equations produces

\[
\frac{\partial}{\partial t^+} \left( \bar{T}_{m,i+1/2}^+ \right) \Delta x^+ - \int_{x_i}^{x_{i+1}} \frac{\partial T_m^+}{\partial x^+} \left|_{x_i}^{x_{i+1}} \right. dx^+ - \frac{NTU_{HR}}{C_r} \left[ \bar{T}_{g,i+1/2}^+ - \bar{T}_{m,i+1/2}^+ \right] \Delta x^+ = 0 \quad(3.109)
\]

The approximations of the first derivatives at points \(x_i^+\) and \(x_{i+1}^+\) using the backward difference can be expressed as:

\[
\left. \frac{\partial T_m^+}{\partial x^+} \right|_{x_i}^{x_{i+1}} = \frac{T_{m,i+1}^+ - T_{m,i}^+}{\Delta x^+} \quad(3.110)
\]

\[
\left. \frac{\partial T_m^+}{\partial x^+} \right|_{x_i}^{x_{i+1}} = \frac{T_{m,i}^+ - T_{m,i-1}^+}{\Delta x^+} \quad(3.111)
\]
Substituting equations (3.110) and (3.111), and discretizing the temperature of the matrix in time using forward finite difference and applying fully-implicit discretization in time to equation (3.109), one obtains:

\[
\left[ \frac{T_{m, i+\frac{1}{2}}^{n+1} - T_{m, i+\frac{1}{2}}^n}{\Delta t^+} \right] \Delta x^+ - F_{Om} \left[ \frac{T_{m, i+1}^+ - 2T_{m, i}^+ + T_{m, i-1}^+}{\Delta x^+} \right]^{n+1} \\
- \frac{NTU_{HT}}{C_r^+} \left[ \frac{T_{g, i+\frac{1}{2}}^+ - T_{m, i+\frac{1}{2}}^+}{\Delta t^+} \right]^{n+1} \Delta x^+ = 0 \quad (3.112)
\]

Now, applying the trapezoidal rule in calculating the average temperature over the computational element \([x_i^+, x_{i+1}^+]\) produces:

\[
\left[ \frac{T_{m, i+1}^+ + T_{m, i}^+}{2} \right]^{n+1} - \left[ \frac{T_{m, i+1}^+ + T_{m, i}^+}{2} \right]^n \Delta x^+ - F_{Om} \left[ \frac{T_{m, i+1}^+ - 2T_{m, i}^+ + T_{m, i-1}^+}{\Delta x^+} \right]^{n+1} \\
- \frac{NTU_{HT}}{C_r^+} \left[ \frac{T_{g, i+1}^+ + T_{g, i}^+}{2} - \frac{T_{m, i+1}^+ + T_{m, i}^+}{2} \right]^{n+1} \Delta x^+ = 0 \quad (3.113)
\]

\[
\frac{1}{\Delta t^+} \left[ \frac{T_{m, i+1}^+ + T_{m, i}^+}{2} \right]^{n+1} - \left[ \frac{T_{m, i+1}^+ + T_{m, i}^+}{2} \right]^n \Delta x^+ - F_{Om} \left[ \frac{T_{m, i+1}^+ - 2T_{m, i}^+ + T_{m, i-1}^+}{\Delta x^+} \right]^{n+1} \\
- \frac{NTU_{HT}}{C_r^+} \left[ \frac{T_{g, i+1}^+ + T_{g, i}^+}{2} - \frac{T_{m, i+1}^+ + T_{m, i}^+}{2} \right]^{n+1} = 0 \quad (3.114)
\]

Rearranging equation (3.114) yields

\[
\left[ - B_m \right]^{n+1} T_{m, i-1}^+ + \left[ A_m + 2B_m + C_m \right] T_{m, i}^+ + \left[ A_m - B_m + C_m \right] T_{m, i+1}^+ = \\
\left[ A_m \right] \left[ T_{m, i}^+ + T_{m, i+1}^+ \right]^n + \left[ C_m \right] \left[ T_{g, i}^+ + T_{g, i+1}^+ \right]^{n+1} \quad (3.115)
\]

where
\[ A_m = \frac{1}{2\Delta t^+}, \quad B_m = \frac{F_{om}}{\Delta x^+}, \quad C_m = \frac{(NTU_{HT}/2)}{C_r^+} \]

The resulting tri-diagonal system of matrix equations is due to using the integral-based scheme with full-implicit time discretization. This system can be solved efficiently by using the Thomas algorithm [89].

Finite difference equation (3.115) of the matrix was used for both supply side (hot) and exhaust side (cold) flow. The time index \( n \) and spatial index \( i \) changes when the flow reverses its direction.

**Numerical representation of the boundary conditions:**

During the supply side (heating period): \( n \uparrow i \Rightarrow \)

\[
\begin{align*}
\quad i &= 1, \\
1 \leq n \leq N &\Rightarrow T_{1,1}^+ = 1
\end{align*}
\]  
(3.116)

During the exhaust side (cooling period): \( \Leftarrow i \uparrow n \)

\[
\begin{align*}
\quad i &= 1, \\
N \leq n \leq 2N &\Rightarrow T_{1,1}^- = 0
\end{align*}
\]  
(3.117)

During reversal time:

At the start of supply side:

\[
T_{g\_e, i}^+ = T_{g\_e, i+1\_e}^N, \quad T_{m\_e, i}^+ = T_{m\_e, i+1\_e}^N, \quad i = 1, 2, 3, \ldots, I
\]  
(3.118)

At the start of exhaust side:

\[
T_{g\_e, i}^+ = T_{g\_e, i+1\_e}^N, \quad T_{m\_e, i}^+ = T_{m\_e, i+1\_e}^N, \quad i = 1, 2, 3, \ldots, I
\]  
(3.119)
Boundary conditions at entrance and exit of matrix (adiabatic conditions):

The derivative boundary conditions will be implemented in the FDE using the application of the interior point FDE [104] at the boundary point, as shown in Figure 3.9.

At \( i = 1 \Rightarrow T_{m0}^+ = T_{m2}^+ \), matrix equation becomes

\[
\begin{bmatrix}
A_m + 2B_m + C_m T_{m1}^{n+1} + [A_m - 2B_m + C_m T_{m2}^{n+1} = [A_m] [T_{m1}^+ + T_{m2}^+]^n \\
+ [C_m] [T_{g1}^+ + T_{g2}^+]^{n+1}
\end{bmatrix}
\tag{3.120}
\]

Figure 3.9  Finite difference grids for matrix boundary conditions at the entrance and exit

At \( i = I \Rightarrow T_{mI+1}^+ = T_{mI-1}^+ \), matrix equation becomes

\[
\begin{bmatrix}
A_m - 2B_m + C_m T_{mI-1}^{n+1} + [A_m + 2B_m + C_m T_{mI}^{n+1} = [A_m] [T_{mI}^+ + T_{mI-1}^+]^n \\
+ [C_m] [T_{gI}^+ + T_{gI-1}^+]^{n+1}
\end{bmatrix}
\tag{3.121}
\]

Solving equations (3.104), (3.106) and (3.115) together with boundary conditions and initial conditions determine the effectiveness of the sensible energy wheel. These finite difference equations were implemented in Matlab code. Several test cases were
studied to gain insight into sensible wheel model’s behavior. Results of these test cases are presented in Chapter 4.

### 3.4 Heat and Mass Transfer Model (Condensation Model)

#### 3.4.1 Governing Equations

The equations of a heat and mass transfer of the energy wheel are derived in the same manner as those for the heat transfer model. The mathematical formulation of the combined heat and mass transfer for the energy wheel is based on the energy and mass balance for an elemental volume of matrix and flow stream. The governing equations are derived for the control volume of Figure 3.10.

![Figure 3.10 Heat and mass transfer control volume](image)

The governing equations for simultaneous heat and moisture transfer in the flow and matrix region are shown next.
In the flow region:

The conservation of the mass in the air results in two continuity equations one for dry air and one for water vapor. These equations include storage, convection, and phase change.

Mass balance of the gas phase

The mass balance equation for the dry air is defined by

\[
\frac{\partial m_{da}}{\partial t} = \dot{m}_{da} - \left[ m_{da} + \left( \frac{\partial m_{da}}{\partial x} \right) dx \right] \quad (3.122)
\]

\[
\frac{\partial (\rho_{da} A_g dx)}{\partial t} = -\left( \frac{\partial (\rho_{da} A_g U_g)}{\partial x} \right) dx \quad (3.123)
\]

\[
\frac{\partial (\rho_{da})}{\partial t} = -\left( \frac{\partial (\rho_{da} U_g)}{\partial x} \right) \quad (3.124)
\]

\[
\frac{\partial \rho_{da}}{\partial t} + \frac{\partial (\rho_{da} U_g)}{\partial x} = 0 \quad (3.125)
\]

Mass balance of water vapor in the gas phase

The rate change of water vapor mass \( m_{wv} \) with respect to time changes due to the convective of the water vapor in the gas phase \( \dot{m}_{wv} \), and water vapor transport between the phases \( d\dot{m}_{wv} \). This is expressed by

\[
\frac{\partial m_{wv}}{\partial t} = \dot{m}_{wv} - \left[ m_{wv} + \left( \frac{\partial \dot{m}_{wv}}{\partial x} \right) dx \right] - d\dot{m}_{wv} \quad (3.126)
\]

using the \( \omega_g \) as a mass transfer potential, as follows:

\[
d\dot{m}_{wv} = h_{MT} \left( A_s \frac{dx}{L} \right) (\omega_g - \omega_i) \quad (3.127)
\]
Equation (3.127) represents the mass transfer rate of the water vapor due to its diffusion from bulk air to the condensing surface. Inserting equation (3.127) into equation (3.126) and using \( m_{sv} \) and \( \dot{m}_{sv} \) yields:

\[
\frac{\partial (\omega_g \rho_{da} A_g \frac{dx}{dt})}{\partial t} = - \left( \frac{\partial (\omega_g \rho_{da} A_g U_g \omega_g)}{\partial x} \right) dx - h_{MT} \left( \frac{A_s}{L} \right) (\omega_g - \omega_s) \tag{3.128}
\]

\[
\rho_{da} A_g \frac{\partial (\omega_g)}{\partial t} + \rho_{da} U_g \frac{\partial (\omega_g)}{\partial x} = - \rho_{da} A_g U_g \frac{\partial (\omega_g)}{\partial x} - \rho_{da} A_g U_g \frac{\partial (\rho_{da} A_g U_g \omega_g)}{\partial x} \tag{3.129}
\]

\[
\rho_{da} A_g \frac{\partial (\omega_g)}{\partial t} + \rho_{da} U_g \frac{\partial (\omega_g)}{\partial x} = - \rho_{da} A_g U_g \frac{\partial (\omega_g)}{\partial x} - h_{MT} \left( \frac{A_s}{L} \right) (\omega_g - \omega_s) \tag{3.130}
\]

Finally, the mass balance of water vapor in the gas phase yields to

\[
\frac{\partial \omega_g}{\partial t} + \frac{\partial (\omega_g \rho_{da} A_g)}{\partial x} + h_{MT} A_s \left( \omega_g - \omega_s \right) = 0 \tag{3.134}
\]

where
\( A_s \) cross-sectional area of the air stream \([\text{m}^2]\)

\( A_w \) frontal cross-sectional area of the wheel \([\text{m}^2]\)

\( A_v = \frac{A_s}{V} \) geometrical surface to volume ratio \([\text{m}^2/\text{m}^3]\)

\( A_s \) convective heat and mass transfer surface area \([\text{m}^2]\)

\( h_{MT} \) mass transfer coefficient of water vapor based on the humidity ratio potential difference \([\text{Kg/s m}^2]\)

\( L \) length of the rotary energy wheel \([\text{m}]\)

\( \varphi \) matrix porosity [-]

\( \rho_{da} \) density of the air \([\text{Kg/m}^3]\)

\( \omega_g = \frac{m_{wv}}{m_{da}} \) humidity ratio of moist air (water content in dry air) \([\text{Kg}_{wv}/\text{Kg}_{da}]\)

\( \omega_s \) humidity ratio in equilibrium with matrix surface \([\text{Kg}_{wv}/\text{Kg}_{da}]\)

**Energy balance of the gas phase**

Application of the first law of thermodynamics on the control volume in Figure 3.10 leads to the following equation:

\[
\frac{\partial H}{\partial t} = \dot{H}(x) - \dot{H}(x + dx) - h_{wv} \rho_{wv} \, dm_{wv} - d\dot{Q}
\]

\[
(3.135)
\]

\[
\frac{\partial H}{\partial t} = -\frac{\partial \dot{H}}{\partial x} \, dx - h_{wv} \rho_{wv} \, dm_{wv} - d\dot{Q}
\]

\[
(3.136)
\]

where

\( \dot{H} \) convection enthalpy in gas phase
enthalpy transport with water vapor between the air and matrix

heat flux exchange between air stream and matrix

The enthalpy of air stream is defined by

\[ H = m_{da} h_{da} + m_{wv} h_{wv} = m_{da} (h_{da} + \omega \rho\ h_{wv}) = m_{da} h_g \]  

(3.137)

In the air stream enthalpy is transferred through convection of the water vapor:

\[ \dot{H} = \dot{m}_{da} h_g \]  

(3.138)

The specific enthalpy \( h_g = h_{da} + \omega \rho\ h_{wv} \) is based on the mass of dry air. With these correlations equation (3.167) yields

\[
\frac{\partial}{\partial t}(m_{da} h_g) = -\frac{\partial}{\partial x} (\dot{m}_{da} h_g) dx - h_{wv} \dot{m}_{wv} - \dot{Q}
\]  

(3.139)

Defining the energy associated with the convective transfer of heat by the temperature difference between the air stream and the matrix by following equation

\[
d\dot{Q} = h_{HT} dA_x (T_g - T_m) = h_{HT} \left( A_x \frac{dx}{L} \right) (T_g - T_m)
\]  

(3.140)

One can get

\[
\frac{\partial}{\partial t} (\rho_{da} A_g dx h_g) = -\frac{\partial}{\partial x} (\rho_{da} A_g U_g h_g) dx - h_{HT} \left( A_x \frac{dx}{L} \right) (T_g - T_m) - h_{wv} \dot{m}_{wv}
\]  

(3.141)

The only term left to define is the last one, which represents the energy exchange associated with convective mass transfer. Enthalpy associated with the transfer of water vapor is keeping up by mass transfer coefficient. Therefore, the driving force for the flux of water vapor from moist air to the wetted matrix using the \( \omega \) as a mass transfer potential is defined as follows
\[
\begin{aligned}
\frac{\partial}{\partial t} (\rho_{ad} A_s dx h_g) &= - \frac{\partial}{\partial x} \left( \rho_{ad} U A_s h_g \right) dx - h_{wv} \frac{\partial}{\partial x} \left( \frac{A_s}{L} \right) \left( \omega_g - \omega_s \right) - h_{HT} \left( \frac{A_s}{L} \right) \left( T_g - T_m \right) \\
A_g &\frac{\partial}{\partial t} (h_g) + A_g h_g \frac{\partial}{\partial t} (\rho_{da}) = - A_g \rho_{da} U \frac{\partial}{\partial x} (h_g) - A_g h_g \frac{\partial}{\partial x} (\rho_{ad} U) \\
&\quad - h_{wv} \frac{\partial}{\partial x} \left( \frac{A_s}{L} \right) \left( \omega_g - \omega_s \right) - h_{HT} \left( \frac{A_s}{L} \right) \left( T_g - T_m \right) \\
A_g \rho_{da} &\frac{\partial}{\partial t} (h_g) = - A_g \rho_{da} U \frac{\partial}{\partial x} (h_g) - A_g h_g \left\{ \frac{\partial}{\partial t} (\rho_{da}) + \frac{\partial}{\partial x} (\rho_{da} U) \right\} \\
&\quad - h_{wv} \frac{\partial}{\partial x} \left( \frac{A_s}{L} \right) \left( \omega_g - \omega_s \right) - h_{HT} \left( \frac{A_s}{L} \right) \left( T_g - T_m \right) \\
\varphi &\rho_{da} \frac{\partial h_g}{\partial t} = - \varphi \rho_{da} U \frac{\partial h_g}{\partial x} - h_{MT} \left( \frac{A_s}{A_s L} \right) \left( \omega_g - \omega_s \right) h_{wv} - h_{HT} \left( \frac{A_s}{A_s L} \right) \left( T_g - T_m \right) \\
\varphi &\rho_{da} \frac{\partial h_g}{\partial t} = - \varphi \rho_{da} U \frac{\partial h_g}{\partial x} - h_{MT} A_s \left( \omega_g - \omega_s \right) h_{wv} - h_{HT} A_s \left( T_g - T_m \right)
\end{aligned}
\]

(3.142)Inserting the above correlation into equation (3.141) and using the mass balance of the gas phase, one can obtain:

\[
\begin{aligned}
dim_{wv} &= h_{MT} dA \left( \omega_g - \omega_{g,m} \right) = h_{MT} \left( A_s \frac{dx}{L} \right) \left( \omega_g - \omega_s \right) \\
&\quad - h_{HT} \left( A_s \frac{dx}{L} \right) \left( T_g - T_m \right) \\
&\quad - h_{wv} \frac{\partial}{\partial x} \left( \frac{A_s}{L} \right) \left( \omega_g - \omega_s \right) - h_{HT} \left( \frac{A_s}{L} \right) \left( T_g - T_m \right)
\end{aligned}
\]

(3.143)

\[
\begin{aligned}
A_g &\frac{\partial}{\partial t} (\rho_{da} h_g) = - A_g \frac{\partial}{\partial x} (\rho_{da} U h_g) - h_{wv} h_{MT} \left( \frac{A_s}{L} \right) \left( \omega_g - \omega_s \right) - h_{HT} \left( \frac{A_s}{L} \right) \left( T_g - T_m \right) \\
&\quad - h_{wv} h_{MT} \left( \frac{A_s}{L} \right) \left( \omega_g - \omega_s \right) - h_{HT} \left( \frac{A_s}{L} \right) \left( T_g - T_m \right)
\end{aligned}
\]

(3.144)

\[
\begin{aligned}
A_g &\frac{\partial}{\partial t} \left( \frac{h_g}{\rho_{da}} \right) + A_g h_g \frac{\partial}{\partial t} (\rho_{da}) = - A_g \rho_{da} U \frac{\partial}{\partial x} (h_g) - A_g h_g \frac{\partial}{\partial x} (\rho_{da} U) \\
&\quad - h_{wv} h_{MT} \left( \frac{A_s}{L} \right) \left( \omega_g - \omega_s \right) - h_{HT} \left( \frac{A_s}{L} \right) \left( T_g - T_m \right)
\end{aligned}
\]

(3.145)

\[
\begin{aligned}
A_g \rho_{da} &\frac{\partial}{\partial t} (h_g) = - A_g \rho_{da} U \frac{\partial}{\partial x} (h_g) - A_g h_g \left\{ \frac{\partial}{\partial t} (\rho_{da}) + \frac{\partial}{\partial x} (\rho_{da} U) \right\} \\
&\quad - h_{wv} h_{MT} \left( \frac{A_s}{L} \right) \left( \omega_g - \omega_s \right) - h_{HT} \left( \frac{A_s}{L} \right) \left( T_g - T_m \right)
\end{aligned}
\]

(3.146)

\[
\begin{aligned}
\varphi &\rho_{da} \frac{\partial h_g}{\partial t} = - \varphi \rho_{da} U \frac{\partial h_g}{\partial x} - h_{MT} \left( \frac{A_s}{A_s L} \right) \left( \omega_g - \omega_s \right) h_{wv} - h_{HT} \left( \frac{A_s}{A_s L} \right) \left( T_g - T_m \right) \\
&\quad - \varphi \rho_{da} U \frac{\partial h_g}{\partial x} - h_{MT} A_s \left( \omega_g - \omega_s \right) h_{wv} - h_{HT} A_s \left( T_g - T_m \right)
\end{aligned}
\]

(3.147)

\[
\begin{aligned}
\varphi &\rho_{da} \frac{\partial h_g}{\partial t} = - \varphi \rho_{da} U \frac{\partial h_g}{\partial x} - h_{MT} A_s \left( \omega_g - \omega_s \right) h_{wv} - h_{HT} A_s \left( T_g - T_m \right)
\end{aligned}
\]

(3.148)

In the matrix region:

Mass balance of liquid water on the matrix

The conservation of the mass of the matrix relates the manner in which the matrix accumulates the condensed water vapor and releases it during evaporation. It represents
the change in moisture content with respect to time, while the mass of the matrix material remains constant in time. The conservation of the mass of the water liquid $m_i$ in the matrix reduces to

$$\frac{\partial}{\partial t}(m_i) = dm_{wv}$$ \hspace{1cm} (3.149)

differential which is controlled by the convective mass transfer coefficient

$$dm_{wv} = h_{MT} \left( A_s \frac{dx}{L} \right) (\omega_g - \omega_s)$$ \hspace{1cm} (3.150)

The amount of water vapor adsorbed by the matrix can be related to the mass of the matrix material using the mass fraction of the water matrix $\sigma_m$.

$$\sigma_m = \frac{m_i}{m_{dm}}$$ \hspace{1cm} (3.151)

Resolving the mass of the water adsorbed by the matrix from equation (3.151) yields:

$$m_i = \sigma_m m_m = \sigma_m \rho_{dm} A_m dx$$ \hspace{1cm} (3.152)

Inserting equations (3.151) and (3.152) into equation (3.149) the following equation can be obtained:

$$\frac{\partial}{\partial t} \left( \sigma_m \rho_{dm} A_m dx \right) = h_{MT} \left( A_s \frac{dx}{L} \right) (\omega_g - \omega_s)$$ \hspace{1cm} (3.153)

$$(1 - \varphi) A_w \rho_{dm} \frac{\partial \sigma}{\partial t} = h_{MT} \left( \frac{A_s}{L} \right) (\omega_g - \omega_s)$$ \hspace{1cm} (3.154)

$$(1 - \varphi) \rho_{dm} \frac{\partial \sigma}{\partial t} = h_{MT} A_s (\omega_g - \omega_s)$$ \hspace{1cm} (3.155)

**Energy balance of the matrix**

Energy conservation in the matrix yields
\[
\frac{\partial H_m}{\partial t} = q_m - \left[ q_m + \frac{\partial q_m}{\partial x} \right] + h_{wv} \dot{m}_{wv} + d\dot{Q}
\] (3.156)

The energy of the wet matrix can be calculated as

\[
H_m = m_{dm} h_{dm} + m_{w} h_{w} = m_{dm} \left( h_{dm} + \rho_m h_{w} \right) = m_{dm} (h_m)
\] (3.157)

Defining the axial heat conduction by \( q_m = -A_m K_m \frac{\partial T_m}{\partial x} \), leads

\[
\frac{\partial}{\partial t} (m_{dm} h_m) = -\frac{\partial}{\partial x} \left( -A_m K_m \frac{\partial T_m}{\partial x} \right) dx + h_{wv} \dot{m}_{wv} + d\dot{Q}
\] (3.158)

Using

\[
m_{dm} = \rho_{dm} A_m dx
\] (3.159)

\[
d\dot{Q} = h_{HT} dA_s (T_g - T_m) = h_{HT} \left( A_s \frac{dx}{L} \right) (T_g - T_m)
\] (3.160)

\[
\dot{m}_{wv} = h_{MT} dA_s \left( \omega_g - \omega_s \right) = h_{MT} \left( A_s \frac{dx}{L} \right) \left( \omega_g - \omega_s \right)
\] (3.161)

Substituting equations (3.159), (3.160) and (3.161) into equation (3.158) leads to

\[
\frac{\partial}{\partial t} (\rho_{dm} A_m dx h_m) = \left( A_m K_m \frac{\partial^2 T_m}{\partial x^2} \right) dx + h_{wv} h_{MT} \left( A_s \frac{dx}{L} \right) \left( \omega_g - \omega_s \right)
\]

\[
+ h_{HT} \left( A_s \frac{dx}{L} \right) (T_g - T_m)
\] (3.162)

\[
(1 - \varphi) \rho_{dm} \frac{\partial}{\partial t} \left( h_m \right) = (1 - \varphi) K_m \frac{\partial^2 T_m}{\partial x^2} + h_{MT} \left( \frac{A_s}{A_w} L \right) (\omega_g - \omega_s) h_{wv}
\]

\[
+ h_{HT} \left( \frac{A_s}{A_w} L \right) (T_g - T_m)
\] (3.163)

\[
(1 - \varphi) \rho_{dm} \frac{\partial}{\partial t} \left( h_m \right) = (1 - \varphi) K_m \frac{\partial^2 T_m}{\partial x^2} + h_{MT} A_s (\omega_g - \omega_s) h_{wv} + h_{HT} A_s (T_g - T_m)
\] (3.164)
The energy conservation equations are similar to that for the rotary sensible wheel. However, there is an extra term added to the equation which indicates that energy is transferred in two different ways. As in the sensible case, there is an energy transfer due to the temperature gradient between the air stream and the matrix. Energy is also transferred with the water vapor. The energy stream is called latent energy and is due to the enthalpy $h_{wv}$ of the water vapor.

In summary, the governing equations are:

**In the flow region:**

\[(1 - \varphi) \rho_{dm} \frac{\partial}{\partial t} (h_m) = (1 - \varphi) K_m \frac{\partial^2 T_m}{\partial x^2} + (1 - \varphi) \rho_{dm} \frac{\partial \sigma_m}{\partial t} h_{wv} + h_{HT A_v} (T_g - T_m) \quad (3.165)\]

The energy conservation equations are similar to that for the rotary sensible wheel. However, there is an extra term added to the equation which indicates that energy is transferred in two different ways. As in the sensible case, there is an energy transfer due to the temperature gradient between the air stream and the matrix. Energy is also transferred with the water vapor. The energy stream is called latent energy and is due to the enthalpy $h_{wv}$ of the water vapor.

In summary, the governing equations are:

**In the flow region:**

\[
\frac{\partial \rho_{da}}{\partial t} + \frac{\partial (\rho_{da} U_g)}{\partial x} = 0 \quad (3.166)
\]

\[
\varphi \rho_{da} \frac{\partial \omega^g_{h_g}}{\partial t} + \varphi \rho_{da} U_g \frac{\partial \omega^g_{h_g}}{\partial x} + h_{MT A_v} (\omega^g - \omega^x) = 0 \quad (3.167)
\]

\[
\varphi \rho_{da} \frac{\partial h^g_{h_g}}{\partial t} = -\varphi \rho_{da} U_g \frac{\partial h^g_{h_g}}{\partial x} - h_{MT A_v} (\omega^g - \omega^x) h_{wv} - h_{HT A_v} (T_g - T_m) \quad (3.168)
\]

**In the matrix region:**

\[
(1 - \varphi) \rho_{dm} \frac{\partial \sigma_m}{\partial t} = h_{MT A_v} (\omega^g - \omega^x) \quad (3.169)
\]

\[
(1 - \varphi) \rho_{dm} \frac{\partial h_m}{\partial t} = (1 - \varphi) K_m \frac{\partial^2 T_m}{\partial x^2} + (1 - \varphi) \rho_{dm} \frac{\partial \sigma_m}{\partial t} h_{wv} + h_{HT A_v} (T_g - T_m) \quad (3.170)
\]

Equations (3.166) - (3.170) form a set of transient one-dimensional coupled hyperbolic differential equations. The coupling is a result of two different effects. First, as described above, there is both a sensible and a latent term in the energy equations.
Furthermore, the equations are coupled through the following thermodynamic relationships:

\[ h_g = h(T_g, \omega_g) \]  
\[ \omega_m = \omega(T_m, \omega_s) \]  
\[ h_m = h(T_m, \sigma_m) \]  
\[ h_{wv} = h(T_g) \]

Up to this point, the set of differential equations (3.166) - (3.170) are the same for heat and mass transfer models. The difference between the performance of a desiccant and a wet non-desiccant is the different thermodynamic behavior of the matrix surface.

**Energy equations in temperature formulation**

Before we derive the energy equations, the thermodynamic relationships for the condensation model should be defined as follows:

\[ h_g = C_p \delta T_g + \omega_g h_{wv} \]  
\[ h_m = C_p \delta T_m + \sigma_m C_p \delta T_m \]  
\[ h_{wv}(T_g) = h_{wv}^0 + C_p \omega_{wv} T_g \]  
\[ \omega_x = \omega_g^* (T_m) \]

Using the above relations, the energy equations can be written in terms of the temperatures and humidity ratio as the independent variables.

**Energy balance of gas phase:**

Starting from the following equation:
The specific enthalpy $h_g$ depends on the temperature $T_g$ and the water content $\omega_g$ of the air, whereas the specific enthalpy $h_{wv}$ of the water vapor depends only on the temperature. The differential forms of the enthalpy are obtained after applying the product rule:

$$\frac{\partial h_g}{\partial t} = \left(\frac{\partial h_g}{\partial T_g}\right)_{\omega_g} \frac{\partial T_g}{\partial t} + \left(\frac{\partial h_g}{\partial \omega_g}\right)_{T_g} \frac{\partial \omega_g}{\partial t}$$

$$= \left(C_{p_{da}} + \omega_g C_{p_{wv}}\right)\frac{T_g}{\partial t} + \left(h_{wv}^0 + C_{p_{wv}}T_g\right)\frac{\partial \omega_g}{\partial t} \tag{3.180}$$

$$\frac{\partial h_g}{\partial x} = \left(\frac{\partial h_g}{\partial T_g}\right)_{\omega_g} \frac{\partial T_g}{\partial x} + \left(\frac{\partial h_g}{\partial \omega_g}\right)_{T_g} \frac{\partial \omega_g}{\partial x}$$

$$= \left(C_{p_{da}} + \omega_g C_{p_{wv}}\right)\frac{T_g}{\partial x} + \left(h_{wv}^0 + C_{p_{wv}}T_g\right)\frac{\partial \omega_g}{\partial x} \tag{3.181}$$

The enthalpy changes in equation (3.179) can be replaced by these correlations such that the following energy balance for the gas phase can be obtained:

$$\varphi \rho_{da} \left[ \left(C_{p_{da}} + \omega_g C_{p_{wv}}\right)\frac{\partial T_g}{\partial t} + \left(h_{wv}^0 + C_{p_{wv}}T_g\right)\frac{\partial \omega_g}{\partial t} \right] =$$

$$-\varphi \rho_{da} U_g \left[ \left(C_{p_{da}} + \omega_g C_{p_{wv}}\right)\frac{\partial T_g}{\partial x} + \left(h_{wv}^0 + C_{p_{wv}}T_g\right)\frac{\partial \omega_g}{\partial x} \right]$$

$$- h_{MT} A_r (\omega_g - \omega_s) \left[h_{wv}^0 + C_{p_{wv}}T_g\right] - h_{HT} A_r (T_g - T_m) \tag{3.182}$$
\[
\varphi \rho_{da} \left[ (C_{p_{da}} + \omega g C_{p_{wv}}) \frac{\partial T_g}{\partial t} \right] = - \varphi \rho_{da} U_g \left[ (C_{p_{da}} + \omega g C_{p_{wv}}) \frac{\partial T_g}{\partial x} \right] - h_{HT} A_v \left( T_g - T_m \right) \\
+ \left[ h_{wv}^0 + C_{p_{wv}} T_g \right] \left\{ \varphi \rho_{da} \frac{\partial \omega g}{\partial t} + \varphi \rho_{da} U_g \frac{\partial \omega g}{\partial x} + h_{HT} A_v \left( \omega_g - \omega \right) \right\}
\]

(3.183)

Since the last term is zero in equation (3.183), one can get

\[
\varphi \rho_{da} \left[ (C_{p_{da}} + \omega g C_{p_{wv}}) \frac{\partial T_g}{\partial t} \right] = - \varphi \rho_{da} U_g \left[ (C_{p_{da}} + \omega g C_{p_{wv}}) \frac{\partial T_g}{\partial x} \right] - h_{HT} A_v \left( T_g - T_m \right) \tag{3.184}
\]

\[
\varphi \rho_g C_{p_g} \frac{\partial T_g}{\partial t} = - \varphi \rho_g C_{p_g} U_g \frac{\partial T_g}{\partial x} - h_{HT} A_v \left( T_g - T_m \right) \tag{3.185}
\]

where

\[
\rho_g C_{p_g} = \rho_{da} C_{p_{da}} + \rho_{wv} C_{p_{wv}}, \quad C_{p_g} = \frac{(C_{p_{da}} + \omega g C_{p_{wv}}) \rho_{da}}{\rho_g}, \quad \text{and} \quad \omega_g = \frac{\rho_{wv}}{\rho_{da}}
\]

The phase changes like evaporation or condensation take place on the matrix surface, therefore no affect to temperature \( T_g \) of the air was observed.

**Energy balance of the matrix**

Starting from the following equation

\[
(1 - \varphi) \rho_{dm} \frac{\partial}{\partial t} \left( h_m \right) = (1 - \varphi) K_m \frac{\partial^2 T_m}{\partial x^2} + (1 - \varphi) \rho_{dm} \frac{\partial \sigma_m}{\partial t} h_{wv} + h_{HT} A_v \left( T_g - T_m \right) \tag{3.186}
\]

The specific energy \( h_m = h(T_m, \sigma_m) \) depends on the temperature \( T_m \) and the water content \( \sigma_m \).

\[
\ h_m = C_{p_{dm}} T_m + C_{p_{wv}} T_m \sigma_m \tag{3.187}
\]

The change of the internal energy can be expressed as
\[
\frac{\partial h_m}{\partial t} = \left( \frac{\partial h_m}{\partial T_m} \right)_m \frac{\partial T_m}{\partial t} + \left( \frac{\partial h_m}{\partial \sigma_m} \right)_m \frac{\partial \sigma_m}{\partial t} \\
= (C_p_{dn} + \sigma_m C_p_{wl}) \frac{\partial T_m}{\partial t} + (C_p_{wl} T_m) \frac{\partial \sigma_m}{\partial t}
\]
(3.188)

Using the above equations (3.186) and (3.187), equation (2.186) can be written as

\[
(1 - \varphi) \rho_{dn} \left[ (C_p_{dn} + \sigma_m C_p_{wl}) \frac{\partial T_m}{\partial t} + (C_p_{wl} T_m) \frac{\partial \sigma_m}{\partial t} \right] = (1 - \varphi) K_m \frac{\partial^2 T_m}{\partial x^2} \\
+ (1 - \varphi) \rho_{dn} \frac{\partial \sigma_m}{\partial t} h_{vw} + h_{HT} A_v (T_g - T_m)
\]
(3.189)

\[
(1 - \varphi) \rho_{dn} \left[ (C_p_{dn} + \sigma_m C_p_{wl}) \frac{\partial T_m}{\partial t} \right] = (1 - \varphi) K_m \frac{\partial^2 T_m}{\partial x^2} \\
+ (1 - \varphi) \rho_{dn} \frac{\partial \sigma_m}{\partial t} [h_{vw} - C_p_{wl} T_m] + h_{HT} A_v (T_g - T_m)
\]
(3.190)

Equation (3.221) can be arranged to be:

\[
(1 - \varphi) \rho_m C_p_m \frac{\partial T_m}{\partial t} = (1 - \varphi) K_m \frac{\partial^2 T_m}{\partial x^2} \\
+ (1 - \varphi) \rho_{dn} \frac{\partial \sigma_m}{\partial t} \left[ h_{vw} + C_p_{wl} T_g - C_p_{wl} T_m \right] + h_{HT} A_v (T_g - T_m)
\]
(3.191)

\[
(1 - \varphi) \rho_m C_p_m \frac{\partial T_m}{\partial t} = (1 - \varphi) K_m \frac{\partial^2 T_m}{\partial x^2} \\
+ (1 - \varphi) \rho_{dn} \frac{\partial \sigma_m}{\partial t} h_{vw} + (1 - \varphi) \rho_{dn} \frac{\partial \sigma_m}{\partial t} C_p_{wl} (T_g - T_m) + h_{HT} A_v (T_g - T_m)
\]
(3.192)

where

\[
C_p_m = \frac{(C_p_{dn} + \sigma_m C_p_{wl}) \rho_{dn}}{\rho_m}, \text{ where } \sigma_m = \frac{\rho_{wl}}{\rho_{dn}}
\]
(3.193)
The last two terms in equation (3.192) describe the temperature change of the matrix due to condensation and evaporation. It contains the total difference between the enthalpy of water vapor and enthalpy of liquid water on the matrix.

**Boundary and initial conditions:**

The above system of equations is subjected to the following boundary and initial conditions.

**Initial conditions:**

\[
T_g(x,0) = T_{g,0} \quad (3.194)
\]

\[
T_m(x,0) = T_{m,0} \quad (3.195)
\]

\[
\omega_g(x,0) = \omega_{g,0} \quad (3.196)
\]

\[
\omega_m(x,0) = \omega_{m,0} \quad (3.197)
\]

**Boundary conditions:**

**Supply side (period 1):** \((k-1)(\tau) \leq t \leq (k-1)(\tau) + \tau / 2, \quad k = 1,2,3,\ldots,N_{cycle}\)

\[
T_g(x = 0,t) = T_{g,x,i} \quad (3.198)
\]

\[
\omega_g(x = 0,t) = \omega_{g,x,i} \quad (3.199)
\]

Reversal condition at the start of supply side

\[
T_{m,x}(x,(k-1)\tau) = T_{m,e}(x,k\tau) \quad (3.200)
\]

\[
\sigma_{m,x}(x,(k-1)\tau) = \sigma_{m,e}(x,k\tau) \quad (3.201)
\]

**Exhaust side (period 2):** \((k-1)(\tau) + \tau / 2 \leq t \leq (k)(\tau), \quad k = 1,2,3,\ldots,N_{cycle}\)

\[
T_g(x = 0,t) = T_{g,e,i} \quad (3.202)
\]
\[ \omega_g (x = 0, t) = \omega_{g_{-c,i}} \quad (3.203) \]

Reversal condition at the start of exhaust side

\[ T_{m_{-e}} (x, (k-1) \tau + \tau/2) = T_{m_{-s}} (x, (k-1) \tau + \tau/2) \quad (3.204) \]

\[ \omega_{m_{-e}} (x, (k-1) \tau + \tau/2) = \omega_{m_{-s}} (x, (k-1) \tau + \tau/2) \quad (3.205) \]

Matrix for both sides (adiabatic):

\[ \frac{\partial T_m (x = 0, t)}{\partial x} = \frac{\partial T_m (x = L_w, t)}{\partial x} = 0 \quad (3.206) \]

### 3.4.2 Modeling of Condensation and Evaporation

Under usual operating condition, warm moist air enters the energy wheel during the supply/exhaust part of the cycle; water vapor will condense if the temperature of the matrix is below the dew point temperature of the air. In this period, the matrix is wetted with a thin film of liquid water. During the supply/exhaust period, the water may evaporate into the dry air stream. On the matrix, dry and wet parts may exist next to each other. Depending in whether the matrix is wet, water can evaporate into air. Therefore, the dry and wet parts of the matrix must be described by different sets of model equations.

Condensation/evaporation over the matrix surface depends on the value of actual vapor pressure of water in the air stream \( P_{w} \) and saturation vapor pressure \( P_{w_{-s}} \) over the liquid water film. For specific point \( x \), condensation occurs when this condition meets \( P_w (t, x) > P_{w_{-s}} \) \[54,105\]. Evaporation of liquid water occurs when matrix surface is wet and \( P_w (t, x) < P_{w_{-s}} \) \[54,105\].
Water content in the air \( \omega_g \) is a more convenient way to describe the condensation and evaporation in wheel matrix. \( \omega_g \) is defined as [24]:

\[
\omega_g = \frac{\text{mass of the water vapor in the air}}{\text{mass of the dry air}} = \frac{m_{\text{wv}}}{m_{\text{da}}} = 0.622 \frac{P_{\text{wv}}}{p - P_{\text{wv}}} 
\] (3.207)

The energy wheel matrix surface is a porous media which has the potential to accumulate all the condensed water vapor. Hence, no liquid movement will be considered in the \( x \) direction. The water content \( \sigma_m \) of the matrix is defined by [54]:

\[
\sigma_m = \frac{\text{mass of the liquid water}}{\text{mass of the dry solid matrix}} = \frac{m_l}{m_{dm}} 
\] (3.208)

The difference between the water content \( \omega_g \) of the air stream and the water content \( \omega_s(T_m) \) on wetted matrix represents the driving force of water vapor from the moist air to the matrix surface [54]. This linear driving force is defined by

\[
dim_{\text{wv}} = h_{MT} dA_s (\omega_g - \omega_s) = h_{MT} \left( A_s \frac{dx}{L} \right) (\omega_g - \omega_s) 
\] (3.209)

When a moisture film is present on matrix, the air in equilibrium with matrix is necessarily saturated, therefore [56]

\[
\omega_s = \omega_g^s(T_m) 
\] (3.210)

In this case, \( \omega_s \) is defined as

\[
\omega_s = \omega_g^s(T_m) = 0.622 \frac{P_{\text{wv}}(T_m)}{P_{\text{atm}} - P_{\text{wv}}(T_m)} 
\] (3.211)

The saturation vapor pressure can be calculated using [24]

\[
P_{\text{wv}} = e^{F(T_m)} 
\] (3.212)
where the function \( F(T_m) \) is given in Table 2.5.

Using the above equations with a set of the system equations that describe the heat and mass transfer of the condensation model, one can describe the process of heat and mass transfer in the sensible wheel when condensation occurs. The distinction between whether the sensible wheel operates as a heat transfer wheel or a heat and mass transfer wheel (condensation or evaporation) should be made. This can be described in the system of the equations using the sign function \( \text{sign}(\omega_g - \omega_s) \) [54,105]. The \text{sign} function is a nonlinear and can be expressed as

\[
\text{sign}(\omega_g - \omega_s) = \begin{cases} 
\pm 1 & \text{if } \sigma_m \geq 0, \text{ and } \omega_g \neq \omega_s \text{ (condensation/evaporation)} \\
0 & \text{if } \sigma_m = 0, \text{ and } \omega_g \leq \omega_s \text{ (heat transfer only)}
\end{cases} 
\] (3.213)

In case of the solution, this source of nonlinearity of the system of equations will be eliminated by deciding which sign should be used for the entire spatial domain. The implementation of the sign function will be shown next.

### 3.4.3 Dimensionless Representation

The descriptive differential equations of the condensation model can be expressed as follows:

**In the flow region:**

\[
\begin{aligned}
\frac{\partial \rho_{du}}{\partial t} + \frac{\partial (\rho_{du} U_g)}{\partial x} &= 0 \\
\varphi \rho_{du} \frac{\partial \omega_g}{\partial t} + \varphi \rho_{du} U_g \frac{\partial \omega_g}{\partial x} + h_{MT} A_s \text{sign}(\omega_g - \omega_s) \left| \omega_g - \omega_s \right| &= 0 \\
\varphi \rho_g Cp_g \frac{\partial T_g}{\partial t} &= - \varphi \rho_g Cp_g U_g \frac{\partial T_g}{\partial x} - h_{HT} A_s \left( T_g - T_m \right) 
\end{aligned}
\] (3.214-3.216)
In the matrix region:

\[ (1 - \phi) \rho_{dm} \frac{\partial \sigma_m}{\partial t} = h_{MT} A_i \text{sign}(\omega_g - \omega_i) \left| \omega_g - \omega_i \right| \]

(3.217)

\[ (1 - \phi) \rho_m C_p_m \frac{\partial T_m}{\partial t} = (1 - \phi) K_m \frac{\partial^2 T_m}{\partial x^2} + h_{MT} A_i \left(T_g - T_m\right) \]

\[ + h_{MT} A_i \text{sign}(\omega_g - \omega_i) \left| \omega_g - \omega_i \right| h_{\text{vap}} \]

\[ + h_{MT} A_i \text{sign}(\omega_g - \omega_i) \left| \omega_g - \omega_i \right| C_p_{\text{w}} \left(T_g - T_m\right) \]

(3.218)

The system of equations above is nonlinear coupled partial differential equations describing the heat and mass transfer in a rotary heat and mass wheel (condensation model). The set of governing equations can be simplified by introducing its dimensionless form. First, the dimensionless parameters can be defined as follows:

\[ t^+ = \frac{t}{\tau} \quad \text{non-dimensional time} \]

(3.219)

\[ x^+ = \frac{x}{L} \quad \text{non-dimensional length} \]

(3.220)

\[ T_g^+ = \frac{T_g - T_{g,e,i}}{T_{g,x,i} - T_{g,e,i}} \quad \text{non-dimensional temperature of gas} \]

(3.221)

\[ T_m^+ = \frac{T_m - T_{g,e,i}}{T_{g,x,i} - T_{g,e,i}} \quad \text{non-dimensional temperature of matrix} \]

(3.222)

\[ \rho_{da}^+ = \frac{\rho_{da} - \rho_{da,e,i}}{\rho_{da,x,i} - \rho_{da,e,i}} \quad \text{non-dimensional density of air} \]

(3.223)

\[ \omega_g^+ = \frac{\omega_g - \omega_{g,e,i}}{\omega_{g,x,i} - \omega_{g,e,i}} \quad \text{non-dimensional humidity ratio of air} \]

(3.224)

\[ \sigma_m^+ = \frac{\sigma_m}{\sigma_{\text{max}}} = \omega_s \left(T_{g,x,i}\right) \quad \text{non-dimensional matrix water content} \]

(3.225)
\[ \omega_i^* = \frac{\omega_i - \omega_{g,e,i}}{\omega_{g,s,i} - \omega_{g,e,i}} \] non-dimensional humidity ratio in equilibrium with matrix surface (3.226)

\[ NTU_{HT} = \frac{h_{MT} A_s}{m_g} \] number of mass transfer unit (3.227)

\[ F_{0_m} = \frac{\alpha_m T}{L} \] the Fourier number, where \( \alpha_m = \frac{K_m}{\rho_m C_p m} \) (3.228)

\[ \Gamma = \frac{U_g T}{L} \] dimensionless period (3.229)

\[ NTU_{HT} = \frac{h_{HT} A_s}{(m C_p)_g} = \frac{h_{HT} A_s}{C_p g} \] number of the heat transfer unit (3.230)

Heat capacity-rate ratio of the rotary regenerator \( C_r^+ \) is defined as:

\[ C_r^+ = \frac{(\rho A L C_p)_m \Omega}{(\rho A U C_p)_g} = \frac{(M C_p)_m \Omega}{(m C_p)_g} = \frac{C_r}{C_g} \] (3.231)

An overall Lewis number is defined as the ratio of the heat and mass transfer \( NTU' \)'s

\[ Le = \frac{NTU_{HT}}{NTU_{MT}} = \frac{h_{HT}}{h_{MT} C_p g} \] (3.232)

Using these dimensionless parameters, the dimensionless representations of the governing equations of the condensation model are shown next:

**In the flow region:**

\[ \frac{1}{\Gamma} \frac{\partial \rho_0^+}{\partial t^*} + \frac{\partial \rho_{da}^+}{\partial x^*} = 0 \] (3.233)

\[ \frac{1}{\Gamma} \frac{\partial \omega_{g}^+}{\partial t^*} + \frac{\partial \omega_{g}^+}{\partial x^*} + NTU_{MT} \text{sign}(\omega_g^+ - \omega^+ s) \left| \omega_g^+ - \omega_s^+ \right| = 0 \] (3.234)
\[
\frac{1}{\Gamma} \frac{\partial T_g^+}{\partial t^+} + \frac{\partial T_g^+}{\partial x^+} + NTU_{HT} (T_g^+ - T_m^+) = 0
\]  
(3.235)

In the matrix region:

\[
\frac{\partial \sigma_m^+}{\partial t^+} = \frac{NTU_{MT}}{C_r^+} \frac{C_p_m}{C_p_g} \frac{\Delta \omega_g}{\omega_{max}} \text{sign}(\omega_g^+ - \omega_s^+) \left| \omega_g^+ - \omega_s^+ \right|
\]
(3.236)

\[
\frac{\partial T_m^+}{\partial t^+} - F_{O_m} \frac{\partial^2 T_m^+}{\partial x^2} - \frac{NTU_{MT}}{C_r^+} \left( T_g^+ - T_m^+ \right) - \frac{NTU_{MT}}{C_r^+} \frac{h_{\text{vap}}}{\Delta T_g^+} \frac{\Delta \omega_g}{\omega_{max}} \text{sign}(\omega_g^+ - \omega_s^+) \left| \omega_g^+ - \omega_s^+ \right|
\]

\[
- \frac{NTU_{MT}}{C_r^+} \frac{C_{p_m}}{C_{p_g}} \Delta \omega_m \text{sign}(\omega_m^+ - \omega_s^+) \left| \omega_m^+ - \omega_s^+ \right| (T_g^+ - T_m^+) = 0
\]
(3.237)

The above system of equations is subjected to the following initial conditions:

\[
T_g^+ (x^+, t^+ = 0) = T_{g_{-0}}
\]
(3.238)

\[
T_m^+ (x^+, t^+ = 0) = T_{m_{-0}}
\]
(3.239)

\[
\omega_g^+ (x^+, t^+ = 0) = \omega_{g_{-0}}
\]
(3.240)

\[
\sigma_m^+ (x^+, t^+ = 0) = \sigma_{m_{-0}}
\]
(3.241)

The required boundary conditions for the wheel are supply (heating period) and exhaust (cooling period) air stream inlet conditions. Boundary conditions are listed below:

**For the gas flow part (Figure 3.11):**

During the heating period

\[
\begin{align*}
    x^+ & = 0  \\
    (k-1) < t^+ < (k-1) + \frac{1}{2}, k = 1, 2, \ldots, N_{\text{cycle}} \quad & \Rightarrow \quad T_g^+ (x^+ = 0, t^+) = 1  \\
    & \quad \omega_g^+ (x^+ = 0, t^+) = 1
\end{align*}
\]
(3.241)

During the cooling period...
\[ x^+ = 0 \]
\[
(k - 1) + \frac{1}{2} < t^+ < k \quad , k = 1, 2, \ldots, N_{\text{cycle}} \]

\[
T^+_g(x^+, 0, t^+) = 0 \quad , \quad \omega^+_g(x^+, 0, t^+) = 0 \quad \text{(3.242)}
\]

During reversal time:

The gas temperature and humidity ratio at the beginning of the heating period is equal to the gas temperature and humidity ratio at the end of the cooling period:

\[
T^+_g(x^+, k - 1) = T^+_g(x^+, k) \quad \text{for} \quad k = 1, 2, \ldots, N_{\text{cycle}} \quad \text{(3.243)}
\]

\[
\omega^+_g(x^+, k - 1) = \omega^+_g(x^+, k) \quad \text{for} \quad k = 1, 2, \ldots, N_{\text{cycle}} \quad \text{(3.244)}
\]

Figure 3.11 Boundary conditions for the airflow in counter-flow energy wheel
For the matrix part (Figure 3.12):

Entrance of the matrix: $x^+ = 0 \Rightarrow \frac{\partial T_m^+(x^+ = 0, t^+)}{\partial x^+} = 0$ (3.245)

Exit of the matrix: $x^+ = 1 \Rightarrow \frac{\partial T_m^+(x^+ = 1, t^+)}{\partial x^+} = 0$ (3.246)

During reversal time:

The matrix temperature and water content at the beginning of the heating period is equal to the matrix temperature and water content at the end of the cooling period:

$$T_{m-e}^+(x^+, k-1) = T_{m-s}^+(x^+, k), \text{ for } k = 1, 2, \ldots, N_{\text{cycle}}$$

$$\sigma_{m-e}^+(x^+, k-1) = \sigma_{m-s}^+(x^+, k), \text{ for } k = 1, 2, \ldots, N_{\text{cycle}}$$

Figure 3.12 Boundary conditions for the matrix in counter-flow energy wheel
3.4.4 Finite Difference Equations

To solve the system of differential equations of condensation wheel, a mixed numerical scheme is used. The governing equations of the air stream are solved by using an integral–based scheme with implicit discretization in time. The matrix equations are approximated by using a full-implicit finite-difference scheme. The full-implicit finite-difference scheme is verified to be effective for solving the equation with a diffusion term and no convection. Numerical discretization of the governing equations is similar to that of the sensible wheel model. Because of the way that the coordinates of wheel are defined, the same equations can be used for both supply side and exhaust side period. The sketch of grid representation of the numerical scheme is shown in Figure 3.13.
The final results of the finite difference representation of the condensation model equations are shown next.

**Finite difference equation for gas temperature:** \( T_{g, i}^{n+1} \) : 

\[
\begin{align*}
[A_g + B_g + C_g] T_{g, i}^{n+1} &= -[A_g - B_g + C_g] T_{g, i-1}^{n+1} + [A_g] T_{g, i-1}^{n+1} + T_{g, i}^{n+1} \\
&+ [C_g] T_{m, i-1}^{n+1} + T_{m, i}^{n+1}
\end{align*}
\]  \( (3.249) \)

where
\[ A_g = \frac{1}{2\Delta t^+ \Gamma}, \quad B_g = \frac{1}{\Delta x^+}, \quad \text{and} \quad C_g = \frac{NTU_{HT}}{2} \]

**Finite difference equation for humidity ratio:** \( \omega_{g,i}^{n+1} \implies \)

\[
\begin{bmatrix} A_{w_g} & B_{w_g} & C_{w_g} \end{bmatrix} \omega_{g,i}^{n+1} = \left[ A_{w_g} - B_{w_g} + C_{w_g} \right] \omega_{g,i}^{n+1} + \left[ A_{w_g} \right] \left( \omega_{g,i-1}^{n+1} + \omega_{g,i}^{n+1} \right) \\
+ \left[ C_{w_g} \right] \left( \omega_{s,i-1}^{n+1} + \omega_{s,i}^{n+1} \right)
\]

(3.250)

where

\[ A_{w_g} = \frac{1}{2\Delta t^+ \Gamma}, \quad B_{w_g} = \frac{1}{\Delta x^+}, \quad C_{w_g} = \text{sign}(\omega_{g,i}^{n+1} - \omega_{s,i}^{n+1}) \frac{NTU_{MT}}{2} \]

**Finite difference equation for humidity ratio in equilibrium with matrix:** \( \omega_{s,i}^{n+1} \implies \)

\[
\omega_{s,i}^{n+1} = \frac{0.622 \frac{P_{wv\cdot s\cdot s}}{P_{wv\cdot s\cdot s}} T_{m,i}^{n+1} - \omega_{g_{-e,i}}^{n+1}}{\omega_{g_{-s,i}}^{n+1} - \omega_{g_{-e,i}}^{n+1}}
\]

(3.251)

**Finite difference equation for matrix water content:** \( \sigma_{m,i}^{n+1} \implies \)

\[
\sigma_{m,i}^{n+1} = \sigma_{m,i}^{n} + \left[ A \sigma_{m} \right] \left( \omega_{g_{-i}}^{n} - \omega_{s_{-i}}^{n} \right) + \left( \omega_{g_{-i}}^{n+1} - \omega_{s_{-i}}^{n+1} \right)
\]

(3.252)

where

\[ A \sigma_{m} = \text{sign}(\omega_{g_{-i}}^{n+1} - \omega_{s_{-i}}^{n+1}) \frac{NTU_{MT}}{C_{r}^{+}} \frac{\Delta C_{p_m}}{C_{p_g} \sigma_{\max}^{+}} \frac{\Delta t^{+}}{2} \]

**Finite difference equation for matrix temperature:** \( T_{m,i}^{n+1} \implies \)

\[
\begin{bmatrix} - B_m \end{bmatrix} T_{m,i}^{n+1} + \left[ A_m + 2B_m + C_m \right] T_{m,i}^{n+1} + \left[ A_m - B_m + C_m \right] T_{m,i+1}^{n+1} = \\
\left[ A_m \right] \left( T_{m,i}^{n+1} + T_{m,i+1}^{n+1} \right) + \left[ C_m \right] \left( T_{g_{-i}}^{n+1} + T_{g_{-i}}^{n+1} \right) + [D_m]
\]

(3.253)

where

\[ A_m = \frac{1}{2\Delta t}, \quad B_m = \frac{Fo_m}{\Delta x^2} \]
\[ C_m = \frac{NTU_{HT}}{2C_r^+} \text{sign}(\omega^*_{g+1} - \omega^*_{s+1}) \frac{NTU_{MT}}{2C_r^+} \frac{C_{p_g}}{C_p} \Delta T_g \left( \omega^*_{g+1} - \omega^*_{s+1} \right) + \left( \omega^*_{g} - \omega^*_{s} \right) \]

\[ D_m = \text{sign}(\omega^*_{g+1} - \omega^*_{s+1}) \frac{NTU_{MT}}{2C_r^+} \frac{h_{sup}}{\Delta T_g C_p} \left( \omega^*_{g+1} - \omega^*_{s+1} \right) + \left( \omega^*_{g} - \omega^*_{s} \right) \]

The resulting tri-diagonal system of matrix equations is due to using the integral-based scheme with full-implicit time discretization. This system can be solved efficiently by using the Thomas algorithm [89].

**Numerical representation of the boundary conditions:**

During the supply side (heating period): \( n \uparrow i \Rightarrow \)

\[
i = 1 \quad \Rightarrow \quad T_{g 1}^{* n} = 1 \quad \omega_{g 1}^{* n} = 1 \quad (3.254)
\]

During the exhaust side (cooling period): \( \Leftarrow i \uparrow n \)

\[
i = 1 \quad \Rightarrow \quad T_{g 1}^{* n} = 0 \quad \omega_{g 1}^{* n} = 0 \quad (3.255)
\]

During reversal time:

At the start of supply side:

\[
T_{m-s_1}^{+ 1} = T_{m-s_{1-1}}^{+ N}, \quad \omega_{m-s_1}^{+ 1} = \omega_{m-s_{1+1}}^{+ N}, \quad i = 1,2,3,\ldots,I \quad (3.256)
\]

\[
T_{g-s_1}^{+ 1} = T_{g-s_{1-1}}^{+ N}, \quad \omega_{g-s_1}^{+ 1} = \omega_{g-s_{1+1}}^{+ N}, \quad i = 1,2,3,\ldots,I \quad (3.257)
\]

At the start of exhaust side:

\[
T_{m-e_1}^{+ 1} = T_{m-e_{1-1}}^{+ N}, \quad \omega_{m-e_1}^{+ 1} = \omega_{m-e_{1+1}}^{+ N}, \quad i = 1,2,3,\ldots,I \quad (3.258)
\]

\[
T_{g-e_1}^{+ 1} = T_{g-e_{1-1}}^{+ N}, \quad \omega_{g-e_1}^{+ 1} = \omega_{g-e_{1+1}}^{+ N}, \quad i = 1,2,3,\ldots,I \quad (3.259)
\]
Boundary conditions at the entrance and exit of matrix (adiabatic conditions) are derived based on the method used in sensible model.

At $i = 1 \Rightarrow T_{m,0}^+ = T_{m,2}^+$, matrix equation becomes

$$
[A_m + 2B_m + C_m]T_{m,i}^{n+1} + [A_m - 2B_m + C_m]T_{m,i+1}^{n+1} = 
[A_m](T_{m,1}^+ + T_{m,2}^+) + [C_m](T_{g,1}^+ + T_{g,2}^+) + [D_m] \tag{3.260}
$$

At $i = I \Rightarrow T_{m,I+1}^+ = T_{m,I-1}^+$, matrix equation becomes

$$
[A_m - 2B_m + C_m]T_{m,N_{m,I}}^{n+1} + [A_m + 2B_m + C_m]T_{m,I}^{n+1} = 
[A_m](T_{m,1}^+ + T_{m,I-1}^+) + [C_m](T_{g,1}^+ + T_{g,I-1}^+) + [D_m] \tag{3.261}
$$

Solving equations (3.249)-(3.261) determines the effectiveness of the condensation wheel. As in the study of the sensible wheel model, the finite difference equations were implemented in Matlab code. Several test cases were studied to gain insight into the condensation wheel model’s behavior. The results of these test cases are presented in Chapter 4.

### 3.4.5 Control Criteria for Condensation Model Computer Program

As discussed earlier, at a specific coordinate $x$, the model’s equation should be defined. Matrix surface condition (dry or wet) should be checked at each grid point. Water content is a control method, if the surface is wet $\varpi_m > 0$ or dry $\varpi_m = 0$. In the case of dry matrix, the direction or sign of the driving gradient for the water vapor
transfer \( \text{sign}(\omega_g - \omega_s) \) should be controlled [105]. The flow chart of the particular part of the main program is shown in Figure 3.14.

**Figure 3.14** A flow diagram showing how control is transferred to the main parts of the condensation model computer program

### 3.5 Heat and Mass Transfer Model (Enthalpy Model)

#### 3.5.1 Governing Equations

Combining the concept of sensible energy wheels with desiccant dehumidifier wheels in enthalpy wheels has been used widely for air enthalpy recovery. Enthalpy
wheels are used when both the heat and moisture transfer are in the same degree-of-importance.

In application, the major difference between the desiccant dehumidifier wheels and the enthalpy wheel is that the degree of importance of heat and mass transfer. The heat and moisture are parallel in importance in the enthalpy wheels, while heat transfer is typically less important in desiccant dehumidifier wheels.

Typically, the matrix in the desiccant dehumidifier wheels does not reach saturation conditions because it is designed to adsorb the water in one period (process period) and give it up to the hot flow stream in the second period (regenerative period). Therefore, there is a low possibility of condensation to occur in the desiccant dehumidifier wheels, while there is some chance of the condensation to take place in the enthalpy wheel for some extreme operation conditions.

For a non-desiccant matrix with the possibility of condensation, two sets of partial differential equations are necessary to describe the sensible wheels because the matrix can be dry or wet. However, if a desiccant matrix is exposed to humid air, the maximum matrix water content is always greater than zero. Therefore, a set of heat and mass conservation equations is required to describe a desiccant system, no matter if condensation occurs or not. The thermodynamic relationships for a desiccant matrix wheel are different, which results in different equations. The equations for the process flow are similar; however, the equations for the matrix are different due to the use of the isotherm relation. These equations are discussed next.
Energy balance of the gas phase

The enthalpy of humid air is:

\[ h_g = C_p \, d_T \, T_g + \omega_g \left( h_{wv}^0 + C_p \, w_T \right) \]  
(3.262)

Since the air inlet states are usually below saturation [105]. Therefore, the energy equation of the gas phase will be similar to that of condensation model.

Energy balance of the matrix

The specific energy \( h_m = h(T_m, \sigma_m) \) depends on the temperature \( T_m \) and the water content \( \sigma_m \). The differential form of the matrix enthalpy can be written in terms matrix water content \( \sigma_m \) and matrix temperature \( T_m \) as:

\[ dh_m = \left( \frac{\partial h_m}{\partial T_m} \right)_{\sigma_m} dT_m + \left( \frac{\partial h_m}{\partial \sigma_m} \right)_{T_m} d\sigma_m \]  
(3.263)

the enthalpy of matrix is given by

\[ h_m = C_p \, d_m \, T_m + C_p \, w_m \, \sigma_m + h_{vap}(T_m) \int_0^{\sigma_m} (1 - h^*) d\sigma \]  
(3.264)

the next analysis is performed to find the derivatives \( \left( \frac{\partial h_m}{\partial T_m} \right)_{\sigma_m} \) and \( \left( \frac{\partial h_m}{\partial \sigma_m} \right)_{T_m} \)

\[ \left( \frac{\partial h_m}{\partial T_m} \right)_{\sigma_m} = C_p \, d_m + C_p \, w_k \sigma_m + \frac{dh_{vap}(T_m)}{dT_m} \int_0^{\sigma_m} (1 - h^*) d\sigma \]  
(3.265)

\[ h_{vap} = h_{vap}^0 - C_p \, vap \, T_m \]  
(3.266)

\[ \frac{dh_{vap}}{dT_m} = -C_p \, vap \]  
(3.267)

using the definition of \( h^* \) from Chapter 2

\[ \frac{h_{ad}}{h_{vap}} = h^*(\sigma_m) = 1 + \Delta h^* \frac{e^{\kappa \sigma_m} - e^\kappa}{1 - e^\kappa} \]  
(3.268)
The integration in the equation (3.265) can be found as follows:

\[
\int_0^{\sigma_m} (1-h^*) d\sigma = \int_0^{\sigma_m} (-\Delta h^* \frac{e^{K\sigma_m^*}}{1-e^K} - e^K) d\sigma = -\Delta h^* \frac{\sigma_{max} e^{K\sigma_m^*} - \sigma_m^* e^K}{1-e^K}
\]  

(3.269)

The final form of \(\frac{\partial h_m}{\partial T_m}\) and \(\frac{\partial h_m}{\partial \sigma_m}\) is

\[
\left(\frac{\partial h_m}{\partial T_m}\right)_{\sigma_m} = C_p_{dm} + C_p_{wl} \sigma_m + C_p_{vap} \Delta h^* \frac{\sigma_{max} e^{K\sigma_m^*} - \sigma_m^* e^K}{1-e^K}
\]

(3.270)

\[
\left(\frac{\partial h_m}{\partial \sigma_m}\right)_{T_m} = C_p_{wl} T_m + h_{vap} (1-h^*) = C_p_{wl} T_m + (h_{vap}^0 - C_p_{vap} T_m) (-\Delta h^* \frac{e^{K\sigma_m^*} - e^K}{1-e^K})
\]

(3.271)

Starting from the energy balance of the matrix

\[
(1-\varphi) \rho_{dm} \frac{\partial}{\partial t} (h_m) = (1-\varphi) K_m \frac{\partial^2 T_m}{\partial x^2} + (1-\varphi) \rho_{dm} \frac{\partial \sigma_m}{\partial t} h_{wv} + h_{HT} A_v (T_g - T_m)
\]

(3.272)

and using the above analysis, one can arrive to the final form of the energy equation

\[
(1-\varphi) \rho_{dm} \left[ \left(\frac{\partial h_m}{\partial T_m}\right)_{\sigma_m} \frac{\partial T_m}{\partial t} + \left(\frac{\partial h_m}{\partial \sigma_m}\right)_{T_m} \frac{\partial \sigma_m}{\partial t} \right] = (1-\varphi) K_m \frac{\partial^2 T_m}{\partial x^2} + (1-\varphi) \rho_{dm} \frac{\partial \sigma_m}{\partial t} h_{wv} + h_{HT} A_v (T_g - T_m)
\]

(3.273)

\[
(1-\varphi) \rho_{dm} \left[ \left(\frac{\partial h_m}{\partial T_m}\right)_{\sigma_m} \frac{\partial T_m}{\partial t} \right] = (1-\varphi) K_m \frac{\partial^2 T_m}{\partial x^2} + (1-\varphi) \rho_{dm} \left[ h_{wv} - \left(\frac{\partial h_m}{\partial \sigma_m}\right)_{T_m} \right] + h_{HT} A_v (T_g - T_m)
\]

(3.274)
Mass balance of liquid water on the matrix

The conservation of the mass of the matrix relates to the manner in which the matrix surface adsorbs or desorbs water during the sorption process. It represents the change in moisture content with respect to time, while the mass of the matrix material remain constant in time. The equation of the mass conservation of water liquid of the condensation model is used to describe the mass conservation of liquid water in the enthalpy model but with different representation of the humidity ratio in the equilibrium with matrix surface.

The sorption isotherm of the matrix plays an important role in the water vapor exchangers between the matrix and air stream. Based on the isotherm shape and maximum water content of matrix, the capability of matrix to adsorb or to desorb water is
determined. The equilibrium humidity ratio $\omega_s$ with a matrix using the isotherm shape relationship can found by the following equations:

$$\omega_s = \frac{0.622 \phi_s}{P_{atm}/P_{s_{wv}} - \phi_s} \tag{3.277}$$

where

$$\phi_s = \frac{R \sigma_m}{\sigma_{max} + (R-1)\sigma_m} \left( \frac{P_{s_{wv}(T_m)}}{P_{s_{wv}(T_0)}} \right)^{(R-1)} \tag{3.278}$$

where $\sigma_{max}$ is the maximum water content $[Kg.Kg^{-1}]$, $R$ is a constant that determines the isotherm shape, $\omega_s$ and $\phi_s$ are the humidity ratio and relative humidity in equilibrium with matrix surface.

As in section 3.4.1, the governing equations discussed above were subjected to boundary and initial conditions during the solution process. Since the solution method is the same for both models, the same boundary and initial conditions are implemented in the enthalpy model.

### 3.5.2 Dimensionless Representation

The dimensionless representation of the enthalpy model was accomplished using the following dimensionless parameters which are analogous to that introduced in the condensation model

$$t^* = \frac{t}{\tau} \quad \text{non-dimensional time} \tag{3.279}$$

$$x^* = \frac{x}{L} \quad \text{non-dimensional length} \tag{3.280}$$
\[ T^+_g = \frac{T_g - T_{g,e,i}}{T_{g,s,i} - T_{g,e,i}} \] non-dimensional temperature of gas (3.281)

\[ T^+_m = \frac{T_m - T_{g,e,i}}{T_{g,s,i} - T_{g,e,i}} \] non-dimensional temperature of matrix (3.282)

\[ \rho^+ = \frac{\rho_{da} - \rho_{da,e,i}}{\rho_{da,s,i} - \rho_{da,e,i}} \] non-dimensional density of air (3.283)

\[ \omega^+ = \frac{\omega_g - \omega_{g,e,i}}{\omega_{g,s,i} - \omega_{g,e,i}} \] non-dimensional humidity ratio of air (3.284)

\[ \sigma^+ = \frac{\sigma_m}{\sigma_{\text{max}}} \] non-dimensional matrix water content (3.285)

\[ \omega^+ = \frac{\omega_s - \omega_{g,e,i}}{\omega_{g,s,i} - \omega_{g,e,i}} \] non-dimensional humidity ratio in equilibrium with matrix surface (3.286)

\[ NTU_{HT} = \frac{h_{HT} A_s}{\dot{m}_g} \] number of mass transfer unit (3.287)

\[ Fo_m = \frac{\sigma_m T}{L^2} \] the Fourier number (3.289)

\[ \Gamma = \frac{U_g T}{L} \] dimensionless period (3.290)

\[ NTU_{HT} = \frac{h_{HT} A_s}{(mCp)_g} = \frac{h_{HT} A_s}{C_g} \] number of the heat transfer unit (3.291)

\[ C_r^+ = \frac{(MCp)_m \Omega}{(mCp)_g} = \frac{C_r}{C_g} \] heat capacity-rate ratio of the wheel (3.292)

\[ Le = \frac{NTU_{HT}}{NTU_{HT}} = \frac{h_{HT}}{h_{MT} C_p g} \] an overall Lewis number (3.293)
The dimensionless representation of the enthalpy wheel model can be presented as follows

**In the flow region:**

\[
\frac{1}{\Gamma} \frac{\partial \rho^+_{da}}{\partial t^+} + \frac{\partial \rho^+_{ds}}{\partial x^+} = 0
\]  

(3.294)

\[
\frac{1}{\Gamma} \frac{\partial \omega^+_g}{\partial t^+} + \frac{\partial \omega^+_s}{\partial x^+} + NTU_{MT} (\omega^+_g - \omega^+_s) = 0
\]  

(3.295)

\[
\frac{1}{\Gamma} \frac{\partial T^+_g}{\partial t^+} + \frac{\partial T^+_s}{\partial x^+} + NTU_{HT} (T^+_g - T^+_m) = 0
\]  

(3.296)

**In the matrix region:**

\[
\frac{\partial \sigma^+}{\partial t^+} = \frac{NTU_{MT}}{C^+_r} \frac{C_p^+_m}{C_p^+_g} \frac{\Delta \omega^+_g}{\sigma^+_{\text{max}}} (\omega^+_g - \omega^+_s)
\]  

(3.297)

\[
\frac{\partial T^+_m}{\partial t^+} - \omega^+_m \frac{\partial^2 T^+_m}{\partial x^+} - \frac{NTU_{MT}}{C^+_r} \frac{C_p^+_m}{C_p^+_g} \left( T^+_g - T^+_m \right) - \frac{NTU_{MT}}{C^+_r} \frac{h_{ad} \Delta \omega^+_g}{\Delta T^+_g} \left( \omega^+_g - \omega^+_s \right)
\]

\[
- \frac{NTU_{MT}}{C^+_r} \frac{C_p^+_m}{C_p^+_g} \frac{\Delta \omega^+_g}{\sigma^+_{\text{max}}} \left( \omega^+_g - \omega^+_s \right) T^+_g = 0
\]  

(3.298)

**3.5.3 Finite Difference Equations**

Numerical discretization of the governing equations is similar to that of the condensation wheel model. To solve the system of differential equations of enthalpy wheel, a mixed numerical scheme is used. The governing equations of the air stream with energy equation of matrix are solved by using an integral–based scheme with implicit discretization in time. The matrix water content equation and humidity ratio in equilibrium with matrix are approximated by using an implicit finite-difference scheme.
Because of the way that the coordinates of wheel are defined the same equations can be used for both supply side and exhaust side period. The sketch of grid representation of the numerical scheme is similar to that of condensation model (see Figure 3.13).

**Finite difference equation for gas temperature:** \( T_{g,i}^{n+1} \rightleftharpoons \\

\[ \left[ A_g + B_g + C_g \right] T_{g,i}^{n+1} = -\left[ A_g - B_g + C_g \right] T_{g,i-1}^{n+1} + \left[ A_g \right] \left[ T_{g,i}^+ + T_{g,i}^- \right]^{n+1} + \left[ C_g \right] \left[ T_{m,i}^+ + T_{m,i}^- \right]^{n+1} \]

\( (3.299) \)

where

\[ A_g = \frac{1}{2\Delta t^+ \Gamma}, B_g = \frac{1}{\Delta x^+}, \text{ and } C_g = \frac{NTU_{HT}}{2} \]

**Finite difference equation for humidity ratio:** \( \phi_{g,i}^{n+1} \rightleftharpoons \\

\[ \left[ A_w + B_w + C_w \right] \phi_{g,i}^{n+1} = -\left[ A_w - B_w + C_w \right] \phi_{g,i-1}^{n+1} + \left[ A_w \right] \left[ \phi_{g,i}^+ + \phi_{g,i}^- \right]^{n+1} + \left[ C_w \right] \left[ \phi_{s,i}^{n+1} + \phi_{s,i}^{n+1} \right] \]

\( (3.300) \)

where

\[ A_w = \frac{1}{2\Delta t^+ \Gamma}, B_w = \frac{1}{\Delta x^+}, C_w = \delta_{MT} \frac{NTU_{MT}}{2} \]

**Finite difference equation for humidity ratio in equilibrium with matrix:** \( \phi_{s,i}^{n+1} \rightleftharpoons \\

\[ \phi_{s,i}^{n+1} = \frac{0.622}{P_{wv,x} \left( T_{m,i}^{n+1} \phi_{s,i}^{n+1} \right)} \phi_{s,i}^{n+1} - \phi_{s,i}^{n+1} \]

\( (3.301) \)

\[ \phi_{s,i}^{n+1} = \frac{R \phi_{s,i}^{n+1}}{1 + (R-1) \phi_{s,i}^{n+1}} \left( \frac{P_{wv,x} \left( T_{m,i}^{n+1} \right)}{P_{wv,x} \left( T_0^{n} \right)} \right)^{n+1} \]

\( (3.302) \)

**Finite difference equation for matrix water content:** \( \phi_{m,i}^{n+1} \rightleftharpoons \\

\[ \phi_{m,i}^{n+1} = \frac{R \phi_{m,i}^{n+1}}{1 + (R-1) \phi_{m,i}^{n+1}} \left( \frac{P_{wv,x} \left( T_{m,i}^{n+1} \right)}{P_{wv,x} \left( T_0^{n} \right)} \right)^{n+1} \]

\( (3.303) \)
\[
\omega_{m_i}^{n+1} = \omega_{m_i}^n + \left[ A \sigma_m \right] \left( \omega_{g_i}^{n+1} - \omega_{s_i}^n \right) + \left( \omega_{g_i}^{n+1} - \omega_{s_i}^{n+1} \right)
\]

\[(3.303)\]

where

\[
A \sigma_m = \delta_{MT} \frac{NTU_{MT}}{C_r^+} \frac{C_p_m \Delta \omega_g}{C_p_g \sigma_{\text{max}}} \frac{\Delta t^+}{2}
\]

Finite difference equation for matrix temperature: \( T_{m_i}^{n+1} \) :

\[
\begin{align*}
- B_m [T_{m_{i-1}}^{n+1} + [A_m + 2B_m + C_m]T_{m_i}^{n+1} + [A_m - B_m + C_m]T_{m_{i+1}}^{n+1} = \\
[A_m]T_{m_i}^{n+1} \left[ T_{m_i}^+ + T_{m_{i+1}}^+ \right] + [C_m]T_{g_i}^{n+1} + T_{g_{i+1}}^{n+1} + [D_m]
\end{align*}
\]

\[(3.304)\]

where

\[
A_m = \frac{1}{2\Delta t}, \quad B_m = \frac{F_o}{\Delta x^2}
\]

\[
C_m = \frac{NTU_{HT}}{2C_r^+} - \delta_{MT} \frac{NTU_{MT}}{2C_r^+} \frac{C_p_{\text{avg}}}{C_p_g} \Delta \omega_g \left( \omega_{g_{i+1}}^{n+1} + \omega_{g_i}^{n+1} \right) - \omega_{s_i}^{n+1}
\]

\[
D_m = \delta_{MT} \frac{NTU_{MT}}{2C_r^+} \frac{h_{ad}}{\Delta T_g} \frac{C_p_{\text{avg}}}{C_p_g} \left( \omega_{g_{i+1}}^{n+1} + \omega_{g_i}^{n+1} \right) - \omega_{s_i}^{n+1}
\]

The resulting tri-diagonal system of matrix equations is due to using the integral-based scheme with full-implicit time discretization. This system can be solved efficiently by using the Thomas algorithm [89].

**Numerical representation of the boundary conditions:**

During the supply side (heating period): \( n \uparrow i \Rightarrow \)

\[
\begin{align*}
i = 1 & \Rightarrow T_{g_{1}}^{n+1} = 1 \\
1 \leq n \leq N & \Rightarrow \omega_{g_{1}}^{n+1} = 1
\end{align*}
\]

\[(3.305)\]

During the exhaust side (cooling period): \( \Leftarrow i \uparrow n \)
\begin{align*}
i = 1 & \quad N \leq n \leq 2N \quad \Rightarrow \quad T_{s1}^{n} = 0 \quad \omega_{s1}^{n} = 0 \tag{3.306}
\end{align*}

During reversal time:

At the start of supply side:

\begin{align*}
T_{m_s}^{i+1} &= T_{m_s}^{i} \quad N, \quad \omega_{m_s}^{i+1} = \omega_{m_s}^{i} \quad i = 1, 2, 3, \ldots, I \tag{3.307}
\end{align*}

\begin{align*}
T_{g_s}^{i+1} &= T_{g_s}^{i} \quad N, \quad \omega_{g_s}^{i+1} = \omega_{g_s}^{i} \quad i = 1, 2, 3, \ldots, I \tag{3.308}
\end{align*}

At the start of exhaust side:

\begin{align*}
T_{m_e}^{i+1} &= T_{m_e}^{i} \quad N, \quad \omega_{m_e}^{i+1} = \omega_{m_e}^{i} \quad i = 1, 2, 3, \ldots, I \tag{3.309}
\end{align*}

\begin{align*}
T_{g_e}^{i+1} &= T_{g_e}^{i} \quad N, \quad \omega_{g_e}^{i+1} = \omega_{g_e}^{i} \quad i = 1, 2, 3, \ldots, I \tag{3.310}
\end{align*}

Boundary conditions at the entrance and exit of matrix (adiabatic conditions) are derived based on the method used in sensible model.

At \(i = 1 \Rightarrow T_{m_0}^{+} = T_{m_2}^{+}\), matrix equation becomes

\begin{align*}
\left[ A_m + 2B_m + C_m \right] T_{m1}^{n+1} + \left[ A_m - 2B_m + C_m \right] T_{m3}^{n+1} &= \\
\left[ A_m \right] \left( T_{m1}^{n} + T_{m2}^{n} \right) + \left[ C_m \right] \left( T_{m1}^{n} + T_{m2}^{n} \right) + \left[ D_m \right] \tag{3.311}
\end{align*}

At \(i = I \Rightarrow T_{m_{I+1}}^{+} = T_{m_{I-1}}^{+}\), matrix equation becomes

\begin{align*}
\left[ A_m - 2B_m + C_m \right] T_{m_{N+1}}^{n+1} + \left[ A_m + 2B_m + C_m \right] T_{m_{I+1}}^{n+1} &= \\
\left[ A_m \right] \left( T_{m1}^{n} + T_{m2}^{n} \right) + \left[ C_m \right] \left( T_{m1}^{n} + T_{m2}^{n} \right) + \left[ D_m \right] \tag{3.312}
\end{align*}

Solving equations (3.299)-(3.312) determines the effectiveness of the enthalpy wheel. As in the study of the condensation wheel model, the finite difference equations
were implemented in Matlab code. Several test cases were studied to gain insight into enthalpy wheel model’s behavior. Results of these test cases are presented in Chapter 4.

### 3.5.4 Modeling Mass Transfer in the Enthalpy Wheel

If the matrix under some operation conditions adsorbs water up to its maximum water content \( \sigma_{\text{max}} \), then the matrix becomes saturated. The saturated matrix cannot adsorb water anymore, so further water uptake \( \sigma_m \geq \sigma_{\text{max}} \) results in a film of liquid water on the matrix surface (condensation model) [54]. Whether the matrix water content is below or above the saturated matrix state, determines the thermodynamic behavior of the matrix.

To describe the enthalpy model in such cases, two numerical models should be combined. One describes the enthalpy wheel in sorption conditions (matrix below saturation) while another one describes the enthalpy wheel in saturation conditions (matrix above saturation). The algorithm should be changed explicitly, as indicated in Figure 3.15.

The following sets of the thermodynamic equilibrium relationships describe the behavior of each model [30]:

**Enthalpy model for the case of matrix below saturation (\( \sigma_m \leq \sigma_{\text{max}} \))**

\[
\sigma_m = \sigma_m(T_m, \omega_s) \tag{3.313}
\]

\[
\omega_s = 0.622 \frac{\phi_s}{P_{\text{atm}}/P_{\text{wv,s}} - \phi_s}, \text{ where} \tag{3.314}
\]
\[
\phi_s = \frac{R \sigma_m}{\sigma_{\text{max}}} + (R - 1) \sigma_m \left[ \frac{P_{\text{wv,s}}(T_m)}{P_{\text{wv,s}}(T_0)} \right]^{\frac{h^* - 1}{h^*}}
\]  
(3.315)

\[
h_m = C_p_{\text{dm}} T_m + C_p_{\text{w}} T_m \sigma_m + h_{\text{vap}}(T_m) \int_0^{\sigma_m} (1 - h^*) d \sigma
\]  
(3.316)

Enthalpy model for the case of matrix above saturation (\( \sigma_m > \sigma_{\text{max}} \))

\[
\omega_s = \omega_{g-s}(T_m)
\]  
(3.317)

\[
\omega_s = \omega_{g-s}(T_m) = 0.622 \frac{P_{\text{wv,s}}(T_m)}{P_{\text{atm}} - P_{\text{wv,s}}(T_m)}
\]  
(3.318)

\[
h_m = C_p_{\text{dm}} T_m + C_p_{\text{w}} T_m \sigma_m + h_{\text{vap}}(T_m) \int_0^{\sigma_{\text{max}}} (1 - h^*) d \sigma
\]  
(3.319)

![Diagram](image)

**Figure 3.15** Numerical switches between condensation and enthalpy models formulation
3.6 Computer Codes

3.6.1 Structure of Computer Programs

Even though the wheel models are different in modeling heat or heat and mass transfer, the air-to-air rotary wheels codes (sensible model code, condensation model code, and enthalpy model code) implemented the same logic in their solutions methods. The codes are arranged in modular form based in the operation of the wheel. The structure of the codes benefits from Willmott’s [45] arrangement with some modification to be applicable to the models presented in this research. Average outlet conditions of the supply and exhaust period was the fundamental notation that drove each rotary energy wheel code [95]. Knowing the average outlets conditions, the effectiveness of the energy wheel can be calculated.

The flowchart in Figure 3.16 illustrates the logic applicable to the codes, and general description of the code sequence is presented next:

1. Starting with supply air side: temperature and moisture content of both the air and the matrix are computed along the length of the wheel (x direction) at each time-step. This process is repeated until the end of the supply period. To speed up the algorithm, convergence should be checked at each time-step.

2. At the end of supply period (half of the wheel cycle): the air and matrix conditions are recorded and stored for effectiveness and next period calculations. At this time, flow is reversed and the wheel starts its exhaust period. The stored conditions are used to start the exhaust process.
3. Next, the exhaust period: same analysis as outlined in step 1. The temperature and moisture content are calculated for both air and matrix during the exhaust rotation.

4. After another 180° rotation (exhaust period analysis completed), air and matrix conditions are recorded and stored. At this point, the wheel has completed the first cycle. Now effectiveness of the first cycle can be found and convergence criteria can be applied to either continue the process or stop the code and accept the resulting effectiveness. The stored conditions are used as starting conditions for the wheel when the supply process is begun.

5. If the convergence criterion is not satisfied, the flow reverses and preparation for the next cycle is made. The matrix and air stream conditions are both updated and the code continues to the cycle. At this time, effectiveness are stored for each cycle.

6. Steps 1-5 are repeated for each cycle until cyclic equilibriums is reached.

7. Outputs of the code are stored and plotted as needed. Temperature and humidity ratio variation with wheel length and effectiveness against cycle are commonly needed to analyze the wheel performance.
Figure 3.16  Flowchart outlining air-to-air rotary energy wheel effectiveness estimation program
The code has been designed in modular format based on the wheel operation. The wheel cycle function is the core of the code. It contains four functions: wheel supply period, reverse, wheel exhaust period, and reverse. This form of construction of the cycle function enables any discussion and modification for different models. Each function can be easily understood and modified by user as the need might arise.

3.6.2 Convergence and Stability

The energy wheel will reach the steady-state condition after specific cycles of operation. The cycle’s number is a function of wheel types and operation conditions. There are two conditions to be satisfied to have the wheel operate in the steady state conditions. First, the matrix and the air states must not vary in time. Second, the reversal conditions in model’s equations must be satisfied. These two conditions are implemented in the finite difference scheme to describe steady-state solution of the wheel. Of course, this will be checked for every two complete consecutive numerical cycles. Since the solution of the wheel is commonly presented in the effectiveness, the convergence criterion is based on the value of effectiveness at each cycle step. It is defined as

\[
\| (\varepsilon)^k - (\varepsilon)^{k-1} \| \leq \delta
\]

(3.320)

When the convergence criteria above are met, the \( (k)^{th} \) cycle is regarded as the equilibrium cycle. The time mean exit air stream properties are then computed and the value of the effectiveness for both wheel periods is calculated.
To get more rapid convergence of the solution, the sequence of the solution plays an important role. Simonson [74] performed trial-and-error analysis to choose the appropriate order of solution and found that $T_g$ should be solved last.

The convergence criteria within each period of the wheel’s cycle are based on the change of the dependent variables from iteration to iteration as indicated by Simonson [74]. Therefore, before the code advances in time, the following condition should be satisfied [74]:

$$\frac{\sum_{i=1}^{I}|X_i^n - X_i^{n-1}|}{I(X_{max} - X_{min})} \leq \delta$$

(3.321)

The stability of any numerical schemes is one of the main properties that must first be investigated before those can be positively considered for application. The program has been validated by changing the step sizes in both space and time, as shown in Table 3.1. Table 3.1 shows that the increasing the number of spatial and time steps above 100 has little effect on the predicted sensible effectiveness of the wheel.

<table>
<thead>
<tr>
<th>Table 3.1</th>
<th>Effect of the number of spatial and time steps on the wheel effectiveness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet conditions</td>
<td>Supply side</td>
</tr>
<tr>
<td>$T = 0 \ ^\circ C$</td>
<td>$\phi = 75%$</td>
</tr>
<tr>
<td>Fixed time step=100</td>
<td>Fixed spatial step=100</td>
</tr>
<tr>
<td>Variable spatial steps</td>
<td>$\Delta \varepsilon_s$</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>150</td>
<td>1.2%</td>
</tr>
<tr>
<td>200</td>
<td>0.9%</td>
</tr>
<tr>
<td>300</td>
<td>0.8%</td>
</tr>
</tbody>
</table>
CHAPTER FOUR

MODELING RESULTS

The purpose of this chapter is to analyze the three numerical models presented in Chapter 3. The air-to-air rotary energy wheel under investigation is of the balanced and symmetric type. A brief description of the wheel under investigation along with operation conditions will be presented first. The solution of the air-to-air rotary energy wheel model equations is often presented in terms of the wheel effectiveness (sensible effectiveness $\varepsilon_s$, latent effectiveness $\varepsilon_L$, and total effectiveness $\varepsilon_{TOT}$). Therefore, the analysis focused on the influence of wheel design and operating parameters on the effectiveness. In addition, the variation of dependent variables along the wheel length and at the wheel outlet face was also investigated. Inlet conditions for both heating and cooling modes were used in this analysis. Theoretical validation was performed using $\varepsilon - NTU$ method and Simonson’s model effectiveness correlation.

In this chapter, the modeling results are divided into four sections. Section 4.1 will describe the wheel parameters, matrix properties, and operation conditions used in this study. Section 4.2 deals with the results and discussion of the sensible heat wheel. Section 4.3 describes the results and discussion of the condensation model. Finally, the results and discussion of the enthalpy model are presented in sections 4.4.
4.1 Energy Wheel Characteristics and Boundary Conditions

4.1.1 Energy Wheel

The air-to-air rotary energy wheel under investigation is a counter-flow balanced and symmetric wheel. It is a rotating cylindrical wheel of length $L_w$ and diameter $d_w$ and it is divided into two equal sections: supply and exhaust section. While the wheel is rotating the equal supply and exhaust air-streams in equal quantities are supplied to the wheel in a counter-flow arrangement. From the definition, the wheel is assumed to be balanced and symmetric if the heat capacity rate ratio

$$C_{g_{\text{supply}}}^+ = \frac{C_{g_{\text{supply}}}}{C_{g_{\text{exhaust}}}} = \frac{(mCp)_{g_{\text{supply}}}}{(mCp)_{g_{\text{exhaust}}}} = 1$$  \hspace{1cm} (4.1)$$

and the ratio of the convective conductance

$$\left(\frac{h_{HT}A_s}{h_{HT}A_s}\right)_{\text{on the } C_{g_{\text{supply}}}} = 1$$  \hspace{1cm} (4.2)$$

The energy wheel under investigation consists of a novel porous matrix. It is made of polyester filaments. The filaments comprising the polyester core are typically of the order of a micron in diameter. Since polyester can absorb small amounts of the moisture, isotherm shape and maximum water uptake of polyester should be defined. This energy wheel is the main part of the energy recovery ventilator (ERV) manufactured by Stirling Technology, Inc. (Athens, OH). The properties of the polyester matrix material are given in Table 4.1.
The basic physical parameters and relations describing the rotary energy wheel are summarized in Table 4.2. The relations and parameters in Table 4.2 are derived for the wheel under investigation in this current research.

<table>
<thead>
<tr>
<th>Table 4.1 Properties of material used as matrix core of the energy wheel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix material</td>
</tr>
<tr>
<td>Porosity (%)</td>
</tr>
<tr>
<td>Density (Kg/m³)</td>
</tr>
<tr>
<td>Specific heat (J/Kg.K)</td>
</tr>
<tr>
<td>Thermal conductivity (W/m².K)</td>
</tr>
<tr>
<td>Isotherm shape</td>
</tr>
<tr>
<td>Maximum water content (Kg ev/Kg dm)</td>
</tr>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Wheel depth</td>
</tr>
<tr>
<td>Wheel diameter</td>
</tr>
<tr>
<td>Wheel split</td>
</tr>
<tr>
<td>Wheel face area</td>
</tr>
<tr>
<td>Porosity</td>
</tr>
<tr>
<td>Flow area</td>
</tr>
<tr>
<td>Matrix area</td>
</tr>
<tr>
<td>Surface area</td>
</tr>
<tr>
<td>Specific surface area</td>
</tr>
<tr>
<td>Hydraulic diameter</td>
</tr>
<tr>
<td>Superficial mean gas velocity</td>
</tr>
<tr>
<td>Interfacial mean gas velocity</td>
</tr>
<tr>
<td>Reynolds number</td>
</tr>
<tr>
<td>Prandtl number</td>
</tr>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>----------------------------</td>
</tr>
<tr>
<td>Nusselt number</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Convective transfer</td>
</tr>
<tr>
<td>coefficients</td>
</tr>
<tr>
<td>Number of heat</td>
</tr>
<tr>
<td>transfer units</td>
</tr>
<tr>
<td>Lewis number</td>
</tr>
<tr>
<td>Wheel speed</td>
</tr>
<tr>
<td>Volume flow rate</td>
</tr>
<tr>
<td>Air properties</td>
</tr>
<tr>
<td>Matrix properties</td>
</tr>
<tr>
<td>Effectiveness</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
For the base line case calculation, the volume flow rate $\dot{Q}$ of 300 [CMH] and the wheel rotational speed $\Omega$ of 30 rpm are used. The RecoupAerator, which is the trademark of the energy recovery ventilator (ERV) manufactured by Stirling Technology Inc., operates at constant rotational speed (30 rpm) with volume flow rates ranging from 120 to 300 CMH. Using the formula in Table 4.2, the matrix properties and the wheel parameters used in the numerical analysis are given in Table 4.3.

Table 4.3  Wheel parameters and matrix properties used in numerical analysis

<table>
<thead>
<tr>
<th></th>
<th>Air :volume flow rate: $\dot{Q}<em>{sup} = \dot{Q}</em>{exch} =$300 [CMH]</th>
<th>Rotational speed of wheel $\Omega = 30$ [RPM]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_w$ = 0.463 m</td>
<td>$L_w$ = 0.0395 m</td>
<td>$\varphi = 92.5%$</td>
</tr>
<tr>
<td>$d_h = 900 \mu m$</td>
<td>$Re = 63$</td>
<td>$A_v = 4026.5 m^{-1}$</td>
</tr>
<tr>
<td>$NTU_0 = 14.5$</td>
<td>$h_{HT} = 200 W/(m^2.K)$</td>
<td>$Nu = 7$</td>
</tr>
<tr>
<td></td>
<td>$Le = 1$</td>
<td>$C_r^* = 4.6$</td>
</tr>
<tr>
<td></td>
<td>$C_{p_m} = 1340 J/Kg.K$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\rho_m = 1380 Kg/m^3$</td>
<td>$K_m = 0.16 w/m.k$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{max} = 0.2$</td>
<td></td>
</tr>
</tbody>
</table>

| Matrix properties: Linear Isotherm |                     |

4.1.2 Boundary Conditions

Boundary conditions were chosen for the numerical analysis to represent a typical operating condition for the (ERV) in the winter (heating mode) and summer (cooling mode) seasons. The boundary conditions are given in Table 4.4.

All data in Tables 4.2-4.4 will be used in the numerical analysis for all three models unless otherwise indicated.
Table 4.4  Boundary conditions used in numerical analysis

<table>
<thead>
<tr>
<th>Test Mode</th>
<th>Supply air conditions</th>
<th>Exhaust air conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_{air} (^{0}C)$</td>
<td>$\phi_{air} (%)$</td>
</tr>
<tr>
<td>Heating Mode</td>
<td>0</td>
<td>75</td>
</tr>
<tr>
<td>Cooling Mode</td>
<td>35</td>
<td>50</td>
</tr>
</tbody>
</table>

4.2 Sensible Wheel Model

The solution of the sensible wheel model equations is often presented in terms of a sensible heat transfer effectiveness, $\varepsilon_s$, which is defined as the ratio of the actual heat transfer rate to the thermodynamically limited maximum possible heat transfer rate [21]. First, the analysis of the wheel performance under summer and winter operations will be investigated. Then, the influence of the wheel parameters $\dot{Q}$, $NTU_{HT}$, $Cr^+$, and $\phi$ on the wheel effectiveness will also be presented. To verify the validity of the numerical model, the sensible wheel effectiveness values predicted by the numerical model are compared with $\varepsilon−NTU$ method. The simulation was performed on a rotary sensible wheel having the characteristics presented in Table 4.5.

Table 4.5  Boundary conditions and wheel properties used in simulation

<table>
<thead>
<tr>
<th>Air</th>
<th>Summer: $T^+_{g,s,i} = 1$</th>
<th>Winter: $T^+_{g,e,i} = 0$</th>
<th>$\dot{Q}<em>{\text{supply}} = \dot{Q}</em>{\text{exhaust}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T^+_{g,e,i} = 0$</td>
<td>$T^+_{g,s,i} = 1$</td>
<td>Range: 150-300 [CMH]</td>
</tr>
<tr>
<td>Wheel</td>
<td>$D_w = 0.463$ m</td>
<td>$L_w = 0.0395$ m</td>
<td>$\varphi = 92.5%$</td>
</tr>
<tr>
<td></td>
<td>$Cp_m = 1340$ J/Kg.K</td>
<td>$\rho_m = 1380$ Kg/m$^3$</td>
<td>$K_m = 0.16$ w/m.k</td>
</tr>
<tr>
<td></td>
<td>$\Omega = 30$ rpm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.2.1 Temperatures Profiles

Air stream temperatures and matrix temperatures variation along the axial direction and the wheel exit face are important to determine the effectiveness of the wheel. The results of the mathematical model runs for the rotary sensible wheel are shown in Figures (4.1 - 4.11) for both summer and winter operations. The dimensionless representation of the wheel will be used to demonstrate the results for both runs. The dimensionless temperatures for air and matrix are defined as

\[
T_g^+ = \frac{T_g - T_{g\_e,i}}{T_{g\_s,i} - T_{g\_e,i}} \quad \text{and} \quad T_m^+ = \frac{T_m - T_{g\_e,i}}{T_{g\_s,i} - T_{g\_e,i}}
\]  

(4.3)

Figures 4.1 and 4.2 show the dimensionless temperature distribution for the air stream and the matrix for both hot and cold periods until steady-state condition is achieved. These multiple curves in Figures 4.1 and 4.2 for both hot and cold periods are describing the temperature behavior from initial conditions until it reaches the steady state. It can be seen that the air and matrix profiles have a transient that lasts for about 30-40 rotations. The steady state condition is reached when the difference in the effectiveness of each cycle is within the specification. Also, it can be noted that the temperature is linear at the steady state condition for most of the matrix and air; there is a change in the slope at the two ends which is due to the insulated boundary conditions.

According to Figures 4.1 and 4.2, the temperature distribution in the hot gas flow period for the matrix are lower than the air stream which means the heat transfer is from the hot air to the matrix surface. In the case of the cold period, the scenario is opposite which leads the heat transfer to flow from the matrix surface to the cold air flow.
Figure 4.1  Dimensionless temperature distributions for the air during hot and cold periods (summer run). The set of data lines shows progression to the steady state from initial point.

Figure 4.2  Dimensionless temperature distributions for the matrix during hot and cold periods (summer run). The set of data lines shows progression to the steady state from initial point.
Figures 4.3 and 4.4 show the variation of the air and matrix temperature at the outlet of the wheel for both the hot and cold periods. Results are recorded when the wheel reaches the steady-state. Figure 4.3 shows that the variation of the gas temperatures is almost linear for both periods with the rotation angle. As the wheel rotates from the cold to hot period, the temperature increases from the lowest cold point to a certain point before it switches to the hot period. The same behavior for the exit temperature for the cold period was observed. The temperature starts form highest point and varies linearly with rotating angle until it is switched to the hot period.

Figure 4.3 Dimensionless temperature distributions for the air at the face of the wheel for the hot and cold periods (summer run)
Figure 4.4 shows the variation of the matrix temperature. It starts from the lowest temperature and is heated up to a point where the wheel is switched to the cold period and then it cools down again. During the steady state operation, the matrix state at the end of the hot period must be equal to the matrix state at the beginning of the cold period.

Figure 4.4  Dimensionless temperature distributions for the matrix at the face of the wheel for the hot and cold periods (summer run)

Figure 4.5 illustrates the variation of the sensible effectiveness with the number of cycles of the wheel. It is clear from the Figure 4.5 that the effectiveness value changes in exponential fashion until it reaches the cyclic equilibrium. The wheel reaches the steady state after about 30 cycles. A fast rotating sensible wheel requires more rotations before reaching the steady-state operation because the time for each revolution is small.
As in the sensible wheel summer operation, the same analysis was performed for the winter run. Figures 4.6 - 4.10 illustrate the performance of the wheel for the winter operation. These figures are similar to Figures 4.1 - 4.4 for the summer run. Figures 4.6 and 4.7 illustrate the dimensionless temperature distribution for the air stream and the matrix at the end of hot and cold periods until steady state condition is reached.

Figures 4.8 and 4.9 show the variation of the air and matrix temperature at the outlet of the wheel for both the cold and hot periods. Figure 4.8 shows that the variation of the gas temperatures is almost linear for both periods with the rotation angle. Figure 4.9 shows the cyclic matrix temperature fluctuations.
Figure 4.6  Dimensionless temperature distributions for the air during hot and cold periods (winter run). The set of data lines shows progression to the steady state from initial point.

Figure 4.7  Dimensionless temperature distributions for the matrix during hot and cold periods (winter run). The set of data lines shows progression to the steady state from initial point.
Figure 4.8  Dimensionless temperature distributions for the air at the face of the wheel for the hot and cold periods (winter run)

Figure 4.9  Dimensionless temperature distributions for the matrix at the face of the wheel for the hot and cold periods (winter run)
Figure 4.10 illustrates the variation of the sensible effectiveness with the cycle of the wheel. It is clear from the Figure 4.10 that the effectiveness value changes in an exponential fashion until it reaches the cyclic equilibrium. The wheel reaches the steady-state after about 30 cycles.

![Figure 4.10 Sensible effectiveness versus cycle of the wheel for (winter run)](image)

At the conclusion of the summer and winter operations, the sensible wheel model shows that the wheel can achieve the same effectiveness for both the summer and winter seasons. The non-dimensional model of the sensible wheel allows us to draw very broad and general conclusions.

Since the sensible wheel transfers only sensible heat from the hot to cold stream, the resulting outlet states are both on horizontal lines that represent the corresponding
inlet humidity ratios. Using the actual wheel parameters presented in Tables 4.3 and 4.5, the average simulated outlet conditions for the heating and cooling modes using a balanced volume flow rate of 250 \( CMH \) are shown in Figure 4.11.

![Figure 4.11 Average supply and exhaust outlet conditions for heating and cooling modes (sensible wheel)](image)

It is clear that if we want to keep the exhaust air outlet of heating mode from condensing, the exhaust air temperature cannot fall below the dew point (100 % R.H.) line. If this happens, which is the case when supply air temperature is smaller than the
dew point temperature of exhaust air, sensible effectiveness using the sensible model in such operation conditions is not correct because the condensation is not modeled. The need for developing the condensation model then becomes critical to precisely estimate the wheel effectiveness and account for the moisture transfer. In such operation conditions, to avoid condensation in the sensible wheel, the effectiveness is controlled by reducing the rotational speed such that exhaust outlet is no colder the dew point.

4.2.2 Effect of Volume Flow Rate $\dot{Q}$ on the Wheel Effectiveness

Response of the energy wheel to the change of the volume flow rate is a popular way to determine the performance of the energy wheel in the HVAC industry. By increasing and decreasing the volume flow rate of both supply and exhaust flow; i.e. the velocity of the air passed though the wheel matrix, calculations were carried out to determine the effectiveness of the wheel. The residence time of the air within the wheel is determined by the volume flow rate and how fast the wheel is rotating.

To study the effect of the volume flow rate on the effectiveness, the results of the model runs are shown in Table 4.6 and plotted in Figure 4.12. As one can see, there is reasonable agreement between the simulated and $\varepsilon - NTU$ method results for all volume flow rates. The difference is probably due to the approximation in equation (3.79). Also, Figure 4.1 indicates that as the residence time of the air inside the matrix increased, more heat transfer takes place. It was observed that as the volume flow rate (face velocity) increased, the sensible effectiveness goes down as expected.
Table 4.6  Simulation results and comparison with $\varepsilon - NTU$ method

<table>
<thead>
<tr>
<th>Volume flow rate [CMH]</th>
<th>Boundary conditions [$^\circ$C]</th>
<th>$\varepsilon_s$ (%)</th>
<th>$\varepsilon_s$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_{g_{-i,j}}$</td>
<td>$T_{g_{-e,j}}$</td>
<td>Simulation</td>
</tr>
<tr>
<td>150</td>
<td>35</td>
<td>24</td>
<td>93.10</td>
</tr>
<tr>
<td>250</td>
<td>35</td>
<td>24</td>
<td>90.24</td>
</tr>
<tr>
<td>300</td>
<td>35</td>
<td>24</td>
<td>89.45</td>
</tr>
</tbody>
</table>

Figure 4.12  Comparison of numerical and $\varepsilon - NTU$ method sensible effectiveness for cooling mode (summer)
4.2.3 Effect of Rotational Speed $\Omega$ on the Wheel Effectiveness

To use the rotational speed of the wheel as a control method, extensive study the influence of $\Omega$ on wheel performance should be performed. Numerical investigation of the effect of the rotational speed of the wheel effectiveness is investigated next. An effectiveness correlation using limiting cases ($\Omega \rightarrow 0, \infty$), and $\varepsilon - NTU$ method are used for comparison.

Figure 4.13 shows the effect of the rotational speed $\Omega$ on the effectiveness with specified value of $NTU_{HT}$. It is concluded from the numerical results that the effectiveness increases with increasing $C_r^+$ up to a certain limit and then no effect after that limit can be detected, which agrees with the theory.

To validate the numerical model, comparison with limiting cases is presented by plotting the effectiveness with low and high $C_r^+$ in Figures 4.13 and 4.14. The limiting cases are given by equations (3.79) and (3.86). Other parameters are kept at the values indicated in table 4.5 which resemble the actual values for the wheel under investigation for normal operation.

Figure 4.15 shows the variation of the sensible effectiveness with rotational speed for a fixed heat transfer number. The same trend of the effectiveness for each NTU is observed but with higher effectiveness for the large NTU.
Figure 4.13  Variation of sensible effectiveness with high rotational speed

Figure 4.14  Variation of sensible effectiveness at low rotational speed
4.2.4 Influence of $NTU$ on the Effectiveness of Wheel

Number of transfer units ($NTU$) is defined as the ratio of convective heat transfer at a given matrix-to-air temperature potential to the thermal capacity of the air [106]:

$$NTU_{HT} = \frac{h_{HT} A_s \Delta T_g}{m_g C_{p,g} \Delta T_g} = \frac{h_{HT} A_s}{(mCp)_g}$$ (4.3)

The energy recovery ventilator under this investigation is characterized by a large $NTU$ number due to its high heat transfer coefficient. The simulation was carried out for the range of $NTU$ and the results for the sensible effectiveness are plotted in Figure 4.16.
Comparison of the numerical results with the \( \varepsilon - NTU \) method shows that the numerical model can produce a good estimate for the sensible wheel effectiveness. It can be seen that the effectiveness increases as the number of transfer units increases.

![Variation of the sensible effectiveness with NTU](image)

**Figure 4.16** Variation of the sensible effectiveness with NTU

Numerical investigation of the effect of matrix conduction in the sensible wheel model was also carried out. The Fourier number is a measure of the effect of the heat conduction in the numerical model. Figure 4.17 investigates the change of the effectiveness with the NTU for two different values of \( Fo_m \). For a low value of NTU, the
effect of heat conduction is less pronounced. As the value of $F_{o_m}$ increases, the effectiveness decreases. This is due to the effect of temperature becoming more uniform due to heat conduction.

![Figure 4.17 Variation of the sensible effectiveness with NTU for different $F_{o_m}$](image)

4.2.5 Influence of Porosity $\varphi$ on the Effectiveness of Wheel

There are several wheels in the market equipped with different types of the matrix to enhance the heat recovery. Parallel tubular channels are the most widely used in the market and are available in various configurations. The wheel under investigation
contains a fibrous media as a heat exchanger core. Fibrous porous media can be designed to be compact in volume with large surface area (heat exchange area). A matrix in this shape can produce a reasonable pressure drop and a very effective heat transfer mechanism. Since the flow path is through a porous matrix, the available surface area for heat transfer is higher than that of tubular one. Due to this, the results in this type of wheel are expected to be different from the rotary sensible wheel designed with a tubular matrix.

The numerical model was used to determine the influence of the porosity on the sensible effectiveness. As one can observe from Figure 4.18, there is a significant decrease in the effectiveness as the porosity of the wheel increases. This result is predictable since the increase in porosity causes significant reduction of the heat transfer area. However, for the porosity below 90% there is little influence of the porosity on the effectiveness value. The optimal value of the porosity is expected to be around the value of 90%. This conclusion is reached based on the numerical and experimental pressure drop studies.

Figure 4.19 illustrates the effect of the porosity on the wheel effectiveness at different volume flow rate. It can be observed that at a value of porosity $\varphi = 90\%$, the wheel effectiveness decreases from 0.94 to 0.90 with an increasing volume flow rate from 150 to 300 [CMH], respectively. However, at a higher value of porosity $\varphi = 93\%$, the effectiveness decreases from 0.91 to 0.86 with increasing volume flow rate from 150 to 300 [CMH]. The sensitivity of the wheel effectiveness to the volume flow rate increases as the porosity of matrix increases.
Figure 4.18 Variation of sensible effectiveness with porosity

\[ \dot{Q} = 300 \text{ [CMH]} \]
\[ \Omega = 30 \text{ [rpm]} \]
\[ C_r = 4.6 \]

Figure 4.19 Effect of the matrix porosity on the sensible effectiveness at different volume flow rates

\[ \dot{Q} = 150 \text{ [CMH]} \]
\[ \dot{Q} = 250 \text{ [CMH]} \]
\[ \dot{Q} = 300 \text{ [CMH]} \]

\[ \Omega = 30 \text{ [rpm]} \]
\[ C_r = 4.6 \]
\[ NTU = 14.5 \]
Figure 4.20 illustrates the effect of the volume flow rate on the wheel effectiveness at different matrix porosity. It can be observed that at a low value of volume flow rate 150 [CMH], the wheel effectiveness decreases from 0.94 to 0.90 with increasing matrix porosity from $\varphi = 90\%$ to 93\%, respectively. However, at a high value of volume flow rate 300 [CMH], the effectiveness decreases from 0.91 to 0.86 with increasing matrix porosity from $\varphi = 90\%$ to 93\%, respectively. The sensitivity of the wheel effectiveness to the matrix porosity increases as the volume flow rate increases.
4.2.6 Summary

It is concluded from this theoretical and numerical development of the sensible wheel model that:

- During summer and winter operations, the sensible wheel can achieve the same effectiveness.
- In startup mode, the effectiveness values change in an exponential fashion until it reaches the cyclic equilibrium after approximately 30 cycles.
- There is a good agreement between the numerical and $\varepsilon - NTU_o$ method.
- As expected, the sensible effectiveness goes down as the volume flow rate $\dot{Q}$ increases.
- The model can represent the sensible wheel for both low and high rotational speed. Increasing the rotational speed results in increasing of the sensible effectiveness.
- The sensible wheel model is characterized with a large NTU value number due to its high heat transfer area, and it shows a good agreement with the previously established theory. As we increase $NTU_{HT}$, the resulting effectiveness increases.
- Due to low thermal conductivity of the matrix, the effect of heat conduction is less pronounced. As the value of $Fo_m$ increases, the effectiveness decreases.
- The effectiveness decreases as the porosity increases. This is expected since the heat transfer area is reduced as the porosity is increased. However, the influence of the porosity on the effectiveness is not noticeable when the porosity decreases below 90%.
• The sensitivity of sensible wheel effectiveness to the matrix porosity increases as the volume flow rate increases.

• Based on numerical investigation of the sensible model, it is thought that the optimal parameters for achieving high effectiveness can be forced in this range:

\[ \dot{Q} \approx 150 - 250, \quad NTU_{HT} \approx 25 - 100, \quad C_{r}^{+} \approx 2.5 - 5, \quad \Omega \approx 20 - 30, \quad \varphi \approx 89 - 92 \]

4.3 Condensation Wheel Model

The condensation model was developed to precisely estimate the sensible wheel effectiveness when the condensation starts to form in the wheel matrix. Condensation can occur in the sensible wheel when the dew point temperature of the hot and humid air stream (exhaust air) is higher than the dry bulb temperature of the cold and dry stream (supply air). Because of this condition, water vapor may condense on the matrix surface during one period and evaporate into the other air stream during the next period.

In the case of condensation, two sets of partial differential equations (3.233)-(3.237) are used to describe the condensation model: energy and moisture conservation equations. In most cases, condensation occurs during the winter operation, where the wheel cools down the warm and humid indoor air. Therefore, sensible wheel performance will be examined based on these operation conditions.

4.3.1 Heating Mode Where the Condensation Might Take Place

Using the same operation conditions (heating mode) and parameters of Figure 4.11, the new figures are regenerated and presented in the Figures 4.21 - 4.4.24 using the
condensation model. The hot and cold gas dimensionless temperatures are shown in Figure 4.21 as a function of the flow dimensionless length and the rotational period as parameters. Figure 4.22 shows the hot and cold matrix dimensionless temperatures as a function of matrix dimensionless length and wheel period (end of hot and cold period). The dimensionless humidity ratio variation of cold and hot gas as a function of matrix dimensionless length at the end of the supply and exhaust period are shown in Figure 4.23. After 50 cycles, the wheel effectiveness ($\varepsilon_s$, $\varepsilon_t$, and $\varepsilon_{tot}$) is shown in Figure 4.24.

![Figure 4.21](image.png)  
**Figure 4.21** Dimensionless temperature distribution of cold and hot gas as a function of wheel dimensionless length and period
Figure 4.22  Dimensionless temperature distribution of cold and hot matrix as a function of wheel dimensionless length and period

Figure 4.23  Dimensionless humidity ratio distribution of cold and hot gas as a function of wheel dimensionless length and period
4.3.2 Comparing the Sensible Effectiveness with Sensible Model Effectiveness

Since the sensible model is designed to transfer heat transfer only, it is logical to investigate the effect of the moisture transfer on the sensible effectiveness. This can be done by running both the sensible and condensation models using the same wheel parameters and then to compare the sensible effectiveness of both models. The resulting sensible effectiveness is shown in Table 4.7 and Figure 2.25.

<table>
<thead>
<tr>
<th>$C_r$</th>
<th>$e_s^{Wheel_SM}$</th>
<th>$e_s^{Wheel_CM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{Q} = 250$ [CMH] $\varphi = .90$ $\Omega = 30$ [rpm]</td>
<td>0.92</td>
<td>0.89</td>
</tr>
</tbody>
</table>
Results from a computer code show that the resulting sensible effectiveness from the condensation model is lower than the value of sensible model. This means that the condensation/evaporation (moisture transfer) processes consume some of the thermal energy which is not available for sensible transport.

The average simulated outlet conditions for the heating mode using a balanced volume flow rate of 250 [CMH] with wheel parameters of the wheel that was used to generate Figure 4.11 are shown in Figure 4.26. The leaving supply air conditions at the exit of the unit are calculated by

\[ T_{s,o} = T_{s,i} - \varepsilon_s (T_{s,i} - T_{e,i}) \]  \hspace{1cm} (4.4)

\[ \bar{\omega}_{s,o} = \omega_{s,i} - \varepsilon_l (\omega_{s,i} - \omega_{e,i}) \]  \hspace{1cm} (4.5)

Similarly, exhaust air conditions leaving the unit can be calculated in the same way.
It is clear that the line connecting the inlet conditions with outlet conditions is not horizontal due to the moisture transfer in the wheel matrix. It can be seen that the entering exhaust air is first cooled at constant humidity until it almost reaches dew point, and then the actual condensation/evaporation process starts from this point on.

Figure 4.26 Average supply and exhaust outlet conditions for heating mode (winter) for (condensation wheel)

4.3.3 Effect of the Rotational Speed $\Omega$ on the Effectiveness

In the sensible wheel, higher sensible effectiveness can be achieved by increasing either or both wheel speed and the number of transfer units. In the case of the condensation model, the same scenario is true for both sensible and latent effectiveness.
However, to obtain the same sensible effectiveness as in the sensible model, the values of the rotational speed and the number of transfer unit will be higher than the one in the heat transfer case.

Figures 4.27-4.29 show the variation of the wheel effectiveness as the wheel rotates from period to period for the specific wheel rotational speed for both sensible wheel model and condensation model. Figure 4.27 shows the variation of wheel effectiveness as a function of the wheel cycle for rotational speed $\Omega = 20$ rpm. The sensible model produces the 91% of effectiveness while the condensation model gives around 86% sensible effectiveness. High latent and total effectiveness can be achieved by the condensation model due to the matrix high surface area.

![Figure 4.27 Comparison of wheel effectiveness of both sensible and condensation models as a function of wheel cycles for $\Omega = 20$ rpm](image)

Figure 4.28 shows the variation of wheel effectiveness as a function of the wheel cycle for rotational speed $\Omega = 30$ rpm. The same conclusion can be observed from
Figure 4.28 with higher sensible and latent effectiveness. Figure 4.29 shows the variation of wheel effectiveness as a function of the wheel cycle for rotational speed $\Omega = 40$ rpm.

Figure 4.28  Comparison of wheel effectiveness of both sensible and condensation models as a function of wheel cycles for $\Omega = 30$ rpm

Figure 4.29  Comparison of wheel effectiveness of both sensible and condensation models as a function of wheel cycles for $\Omega = 40$ rpm
Figure 4.30 shows the effect of the rotational speed $\Omega$ on the effectiveness with a specified value of volume flow rate and porosity. It is concluded from the numerical results that the effectiveness increases with increasing of $\Omega$ (i.e., $C_r^*$) up to a certain limit ($\approx 50$ rpm). After that limit, the effectiveness changes are small.

4.3.4 Influence of the Inlet Conditions on the Wheel Performance

For a given wheel, with fixed parameters operating in heating mode with desired indoor conditions, the amount of moisture transfer depends on the wheel inlet state’s
proximity to the saturation line in a psychrometric chart. An investigation of this dependence was carried out and results are given below.

Figure 4.31 shows the variation of sensible and latent effectiveness for the typical supply air state values in the air conditioning case. It is noted that the latent effectiveness increases rapidly by decreasing the supply air temperature. The moisture effectiveness \( \varepsilon_L \) reaches an upper limit (which is lower than \( \varepsilon_S \)). The sensible effectiveness \( \varepsilon_S \) is nearly constant for large \( \Delta T \) (\( \Delta T = T_{g,s,i} - T_{g,e,i} \)), and increases when \( \Delta T \) decreases.

![Figure 4.31](image.jpg)

Figure 4.31  Sensible and latent effectiveness of the condensation wheel for different inlet conditions.
The simulation also showed that if one of the wheel inlet states is sufficiently wet, the wheel will operate in an unstable manner. We believe that the reason is that more moisture is being deposited in one period than is evaporated in the other. Experimental investigation should be done to confirm this situation.

4.3.5 Summary

It is concluded from this theoretical and numerical development of the condensation wheel model that:

- The resulting sensible effectiveness from the condensation model is lower than the value of sensible model.
- The effectiveness depends on the extent of condensation
- The effectiveness increases with increasing of $\Omega$ (i.e., $C^*_r$) up to a certain limit.
- To obtain the same sensible effectiveness as in sensible model, the values of rotational speed and the number of transfer units for condensation wheel model need to be higher.
- High latent and total effectiveness can be achieved by the condensation model due to high surface area of the matrix.
- It is noted that the latent effectiveness increases rapidly with decreasing the supply air temperature.
- Simulation shows that if one of the wheel inlet states is sufficiently wet, the wheel will operate in an unstable manner.
4.4 Enthalpy Wheel Model

A rotary enthalpy wheel can be manufactured by integrating the characteristics of both the rotary sensible wheel and the desiccant dehumidifier wheel in one wheel. An enthalpy wheel employs a sorbent matrix with large thermal and moisture capacity, since both heat and mass transfer are desired. Integrating the enthalpy wheel in air-conditioning system leads to: high effectiveness, low material cost, and the down-sizing of the air conditioning system. During winter operation, the energy wheel warms and humidifies the incoming outdoor air. During summer operation, the energy wheel cools and dehumidifies incoming outdoor air.

Using the numerical model presented in Chapter 3, the sensible, latent, and total effectiveness of the enthalpy wheel can be determined. As mentioned before, the wheel under investigation is balanced and symmetric, which reduces the number of model parameters. Test cases were run to demonstrate the wheel’s performance with respect to the most influential wheel parameters volume flow rate $\dot{Q}$, rotational speed $\Omega$ (i.e. heat capacity-ratio $C^+_r$), number of mass transfer $NTU_{MT}$, porosity $\varphi$, and matrix isotherm correlation. In this study, as one parameter was varied, the other parameters were held constant. Default values for each parameter are given in Table 4.2. The results of the model run will be presented in the following sections.

4.4.1 Enthalpy Wheel Behavior

An enthalpy wheel model was used to simulate the enthalpy wheel behavior under the heating and cooling modes. Steady-state operation is reached as soon as the wheel
effectiveness for the last two cycles is within a specified limit determined by the numerical code. Figures 4.32 - 4.35 show the enthalpy wheel behavior under heating mode. In the case of cooling mode, the results are shown in Figures 4.36 - 3.39. The profiles in the figures show the dimensionless gas and matrix states at the end of the supply and exhaust period as a function of the dimensionless length of the enthalpy wheel. The effectiveness is plotted as a function of the wheel cycles. The enthalpy wheel operates at the steady-state after about 50 rotations.

Figure 4.32 Dimensionless temperature distribution of cold and hot gas as a function of wheel dimensionless length and period
Figure 4.33  Dimensionless temperature distribution of cold and hot matrix as a function of wheel dimensionless length and period

Figure 4.34  Dimensionless humidity ratio distribution of cold and hot gas as a function of wheel dimensionless length and period
Figure 4.35  Wheel effectiveness (sensible, latent, and total) as a function of wheel cycles

Figure 4.36  Dimensionless temperature distribution of cold and hot gas as a function of wheel dimensionless length and period
Figure 4.37  Dimensionless temperature distribution of cold and hot matrix as a function of wheel dimensionless length and period

Figure 4.38  Dimensionless humidity ratio distribution of cold and hot gas as a function of wheel dimensionless length and period
Figure 4.39  Wheel effectiveness (sensible, latent, and total) as a function of wheel cycles

In the enthalpy model, the average simulated outlet conditions (temperature and humidity ratio) for the heating and cooling modes using a balanced volume flow rate with actual wheel parameters (Tables 4.3 and 4.5) are shown in Figure 4.40. As can be seen, including the latent component in the enthalpy model gives it better performance. The outlets of the enthalpy wheel become close to the indoor conditions. For normal heating and cooling modes, the condensation is unlikely to happen due to the ability of the matrix of the enthalpy wheel to adsorb the moisture of high-humid air stream during the first period and release it to low-humid air stream of the second period.

In the case of heating mode, comparing the behavior of condensation model (sensible wheel with condensation) and the enthalpy model leads to the importance of enthalpy wheel in the air-conditioning application. It is easy to notice that more energy can be recovered by the enthalpy wheel due to its potential to recover latent heat.
Moreover, there is less possibility of condensation to form in the wheel’s matrix. However, a sensible wheel operating in such condition will reach the saturated moisture condition easily which implies that the condensation will form in the wheel. The condense moisture might damage the matrix material and may run out in the system to cause mold growth.

Figure 4.40 Enthalpy model outlets for both heating and cooling modes
In conclusion, using the enthalpy wheel is more realistic when the heat and moisture recovery are required. Recently, most of stand-alone ERVs are designed with the enthalpy wheel. In some cases, the sensible wheel is needed to just cool or heat the incoming air-stream. For desiccant cooling cycle units, the sensible wheel is used for heating or cooling air stream as a part of the cycle process.

4.4.2 Effect of the Volume Flow Rate $\dot{Q}$ on the Effectiveness

Response of the energy wheel to the change of the volume flow rate is a popular way to determine the performance of the energy wheel in the HVAC industry. Figure 4.41 shows the effect of volume flow rate on the sensible, latent, and total effectiveness of the enthalpy wheel for the heating mode of operation. The decreasing effectiveness with an increasing volume flow rate is expected. Differences in the effectiveness are higher at large volume flow rate.

Figure 4.42 shows the effect of volume flow rate on the sensible, latent, and total effectiveness of the enthalpy wheel for the cooling mode of operation. The decreasing effectiveness with increasing volume flow rate is expected as in heating mode. One can see the effectiveness is more sensitive to volume flow rate changes for the heating mode than for the cooling mode.
Figure 4.41  Wheel effectiveness versus volume flow rate for heating mode

Figure 4.42  Wheel effectiveness versus volume flow rate for cooling mode
To verify the validity of the enthalpy wheel numerical model, the effectiveness values predicted by the numerical model are compared with Simonson’s enthalpy wheel mode. Simonson’s model effectiveness correlations are given in equations (2.53) - (2.55). Validation was performed for both heating and cooling modes. Using the same wheel parameters in Table 4.3, validation of the enthalpy wheel with Simonson correlation for both heating and cooling mode is presented in Tables 4.7 and 4.8 respectively. Results in Tables 4.8 and 4.9 show that the numerical enthalpy wheel mode produces reasonably close effectiveness values to Simonson’s correlation results.

Table 4.8  Wheel effectiveness validation for heating mode operation

<table>
<thead>
<tr>
<th>Volume flow rate [CMH]</th>
<th>$\varepsilon_s$ (%)</th>
<th>$\varepsilon_i$ (%)</th>
<th>$\varepsilon_t$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Numerical model</td>
<td>Simonson correlation</td>
<td>Numerical model</td>
</tr>
<tr>
<td>150</td>
<td>91.58</td>
<td>90.32</td>
<td>87.88</td>
</tr>
<tr>
<td>200</td>
<td>89.95</td>
<td>88.45</td>
<td>85.98</td>
</tr>
<tr>
<td>250</td>
<td>88.32</td>
<td>87.12</td>
<td>83.87</td>
</tr>
</tbody>
</table>

Table 4.9  Wheel effectiveness validation for cooling mode operation

<table>
<thead>
<tr>
<th>Volume flow rate [CMH]</th>
<th>$\varepsilon_s$ (%)</th>
<th>$\varepsilon_i$ (%)</th>
<th>$\varepsilon_t$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Numerical model</td>
<td>Simonson correlation</td>
<td>Numerical model</td>
</tr>
<tr>
<td>150</td>
<td>90.79</td>
<td>89.55</td>
<td>86.28</td>
</tr>
<tr>
<td>200</td>
<td>86.80</td>
<td>86.05</td>
<td>82.49</td>
</tr>
<tr>
<td>250</td>
<td>83.18</td>
<td>82.75</td>
<td>79.00</td>
</tr>
</tbody>
</table>
4.4.3 Effect of the Rotational Speed $\Omega$ on the Effectiveness

By increasing and decreasing the rotational speed $\Omega$ of the wheel, i.e. matrix capacity rate ratio $C_r^\ast$, changes were made to the effectiveness of the wheel. This is shown in Figure 4.43 where the effectiveness is plotted as a function of the rotational wheel speed for heating mode and in Figure 4.44 for the cooling mode.

These figures indicate that the wheel effectiveness increases as the rotational speed increases up to a certain limit and then has little effect on wheel performance. A minimum rotational speed of $\approx 40$ [rpm] is needed for the wheel to operate at optimal performance for heating mode.

![Graph showing wheel effectiveness variations under various rotational speeds with $\varphi = 93\%$ for heating mode](image)
Figure 4.44 shows the effects of the wheel rotational speed on the effectiveness. The sensible effectiveness over 90% can be obtained at around a rotational speed $\Omega$ of 40 [rpm]. As can be seen, the sensible, latent, and total effectiveness for both heating and cooling modes are equally sensitive to the changes in wheel rotational speed. The large changes in the effectiveness are for the speeds in the range of 10 to 40 rpm.

Figure 4.44 Wheel effectiveness variations under various rotational speeds with $\varphi = 93\%$ for cooling mode
4.4.4 Effect of the Porosity $\phi$ on the Effectiveness of Wheel

The wheel under investigation contains a fibrous media as the heat exchanger core. Fibrous porous media provides a large heat exchange area with reasonable pressure drop for effective heat and mass transfer in a compact volume. The effect of the porosity and the specific area of the wheel on the wheel performance are shown in Figures 4.45 - 4.46 for heating mode and in Figures 4.47 - 4.48 for cooling mode.

Figure 4.45 shows the effect of the porosity on the effectiveness for heating mode. As can be seen, the effectiveness is almost constant for low porosity and then decreases for large porosity.

Figure 4.45  Effect of the wheel porosity on the wheel effectiveness for heating mode
Figure 4.46 shows the effect of the specific area on the wheel effectiveness for heating mode. As can be seen, the effectiveness rises almost linearly with increasing specific area and then becomes constant in the values of $A_v = 7000 \ [m^2/m^3]$. Enthalpy wheel with large $A_v$ can be more compact, which implies both large surface area and heat and mass transfer coefficients. All these factors together can improve the wheel performance significantly.

Figure 4.46  Effect of specific area on the wheel effectiveness for heating mode
Figure 4.47 shows the effects of the porosity on the effectiveness for cooling mode. As can be seen, the effectiveness is almost constant for low porosity and then decreases gradually as steep as in the heating mode for large porosity.

![Figure 4.47 Effect of the wheel porosity on the wheel effectiveness for cooling mode](image)

Figure 4.48 shows the effects of the specific area on the wheel effectiveness for cooling mode. As can be seen, the effectiveness rises almost linearly with increasing specific area and then becomes constant in the values of $A_e = 7000 \ [m^2/m^3]$. It is noted that the latent effectiveness is lower than the sensible effectiveness for all cases.
4.4.5 Effect of the Number of Transfer Unit $NTU$ and Lewis Number $Le$ on the Effectiveness

The design parameters of the energy wheel that govern the heat and mass transfer mechanisms are the number of transfer units for the heat and mass transfer between one air stream and the matrix. They are related through the Lewis number. An overall Lewis number is defined as the ratio of the heat and mass transfer unit’s numbers:

$$Le = \frac{NTU_{HT}}{NTU_{MT}} = \frac{h_{HT}}{h_{MT}C_p g}$$  \hspace{1cm} (4.4)
The simulation was carried out for the range of \( NTU \) and the results for the wheel effectiveness are plotted in Figure 4.49 for the cooling mode. It can be seen that the effectiveness increases as the number of transfer units is increased.

![Figure 4.49 Effect of \( NTU_{HT} \) on the wheel effectiveness for cooling mode](image)

Effect of the Lewis number on the wheel effectiveness is shown in the Figure 4.50 for the cooling mode. As can be seen, the wheel effectiveness decreases as the \( Le \) number increases for both latent and total effectiveness; however, for sensible effectiveness, small changes can be noticed. For a large \( Le \) number, the wheel can operate as a sensible
The enthalpy wheel can produce equal effectiveness (sensible, latent, and total) for a Le number of approximately 1.6.

The effect of the NTU and Le numbers on the effectiveness for the heating mode is shown in Figures 4.51-52. The same conclusion of cooling mode simulations can be reached for the heating mode. However, higher effectiveness can be obtained. As one can observe, the pattern of the profile of the effectiveness changes at a Le number of around 1.6. For a low Le number, a higher latent effectiveness can be obtained.
Figure 4.51  Effect of $NTU_{HT}$ on the wheel effectiveness for heating mode

Figure 4.52  Effect of $Le$ on the wheel effectiveness for heating mode.
4.4.6 Effect of the Isotherm Shape on the Effectiveness

The effects of the isotherm correlation on the wheel effectiveness can be examined by studying the influences of the moisture content of the matrix and the separation factor that defined the isotherm shape on the values of the effectiveness.

Table 4.10 and Figure 4.53 show the effects of the sorption curve constant $R$ and maximum water content of matrix $\sigma_{\text{max}}$ on the wheel effectiveness, respectively, for the cooling mode (summer run). Results in Table 4.10 show that the model under investigation can achieve good performance with $R$ values between 1 and 10. Also, the results in Table 4.10 show that a linear sorption curve ($R = 1$) has the highest effectiveness. Many hygroscopic materials, such as silica have a linear sorption isotherm. There is a significant difference in the effectiveness values for a small value of $R$. In the case of a low value of $R$, the model produces a higher value of sensible effectiveness and low latent effectiveness, which may indicate that the wheel cannot be modeled with a low value of $R$.

<table>
<thead>
<tr>
<th>$R$</th>
<th>$\varepsilon_s$</th>
<th>$\varepsilon_l$</th>
<th>$\varepsilon_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.643</td>
<td>0.573</td>
<td>0.456</td>
</tr>
<tr>
<td>0.1</td>
<td>0.839</td>
<td>0.792</td>
<td>0.754</td>
</tr>
<tr>
<td>1.0</td>
<td>0.878</td>
<td>0.857</td>
<td>0.806</td>
</tr>
<tr>
<td>10</td>
<td>0.844</td>
<td>0.802</td>
<td>0.786</td>
</tr>
<tr>
<td>100</td>
<td>0.705</td>
<td>0.690</td>
<td>0.674</td>
</tr>
</tbody>
</table>
Figure 4.53 shows the effect of maximum water content on the wheel effectiveness for the linear type. As can be seen, the effectiveness can be improved with a higher value of maximum moisture content of the matrix. Increasing moisture content of the matrix can prevent condensation from forming on the wheel matrix surface.

Table 4.11 and 4.54 show the effects of the sorption curve constant $R$ and the maximum water content of matrix $\sigma_{max}$ on the wheel effectiveness, respectively, for the heating mode (winter run). Table 4.11 shows that the separation factor $R$ becomes more significant in a cold region operation than a hot one due to the difference in moisture content between the two inlets of the wheel.
Table 4.11  Comparison of the wheel effectiveness for different R values for fixed $\sigma_{\text{max}} = 0.2$ (heating mode)

<table>
<thead>
<tr>
<th>$R$</th>
<th>$\varepsilon_x$</th>
<th>$\varepsilon_i$</th>
<th>$\varepsilon_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.768</td>
<td>0.694</td>
<td>0.537</td>
</tr>
<tr>
<td>0.1</td>
<td>0.871</td>
<td>0.855</td>
<td>0.822</td>
</tr>
<tr>
<td>1.0</td>
<td>0.869</td>
<td>0.850</td>
<td>0.829</td>
</tr>
<tr>
<td>10</td>
<td>0.859</td>
<td>0.842</td>
<td>0.806</td>
</tr>
<tr>
<td>100</td>
<td>0.839</td>
<td>0.805</td>
<td>0.745</td>
</tr>
</tbody>
</table>

Figure 4.54 shows the effects of the maximum moisture content of the matrix on the effectiveness for the linear type. As the maximum water content of the matrix increases, notable increases in the latent effectiveness can be obtained, while sensible effectiveness shows smaller changes.

Figure 4.54  Effect of $\sigma_{\text{max}}$ on the wheel effectiveness for heating mode with $R = 1$
4.4.7 Effect of the Supply Humidity on the Effectiveness

Effects of the supply humidity on the effectiveness enthalpy wheel for both cooling and heating mode operations are investigated by setting the indoor condition to a desired one recommended by ASHRAE ($T_{i,e} = 24^\circ C$ and $\phi_{i,e} = 50\%$) and varying the supply outdoor humidity $\phi_{s,i}$.

Figure 4.55 shows the influence of the supply humidity $\phi_{s,i}$ on the wheel effectiveness for cooling mode (summer). As can be seen, the effectiveness decreases as the supply humidity increases. Moreover, sensible effectiveness is strongly influenced by $\phi_{s,i}$.

![Figure 4.55](image-url)
Figure 4.56 shows the influence of the supply humidity $\phi_{s,i}$ on the wheel effectiveness for heating mode (winter). As can be seen, the effectiveness decreases as the supply humidity increases. Also, effectiveness has equal sensitivity over the given range of supply humidity.

![Effect of $\phi_{s,i}$ on the wheel effectiveness for heating mode](image)

**4.4.8 Summary**

It is concluded from this theoretical and numerical development of the enthalpy wheel model that:

- A computer code can perfectly represent the enthalpy wheel for typical operation conditions.
- Using the enthalpy wheel instead of the sensible wheel in application of an ERV is more realistic when the heat and moisture recovery are required.
• From analysis of the outlets of the wheel, the enthalpy wheel provides more energy recovery and it is less likely to produce condensation while the sensible wheel will approach the condensation conditions easily.

• A decrease in effectiveness with increasing volume flow rate is expected. Also, effectiveness is more sensitive to a large volume flow rate.

• Results show that the numerical enthalpy wheel mode produces reasonably close effectiveness values to Simonson’s correlation results.

• Wheel effectiveness increases as the rotational speed increases up to a certain limit and then a slight effect after that limit can be detected. The sensible, latent, and total effectiveness for both heating and cooling modes are equally sensitive to the changes in wheel rotational speed.

• Results indicate that the effectiveness is nearly constant for low porosity and decreases for high porosity. The effectiveness gradually decreases with high porosity for cooling mode while the decrease in heating mode is much steeper. Moreover, effectiveness rises almost linearly with increasing specific area and then becomes constant at the value of about \( A_s = 7000 \, [m^2/m^3] \).

• Results indicate that the effectiveness increases as the number of transfer units increases for both heating and cooling mode. In the case of the effect of the Le number on the effectiveness results indicate that effectiveness decreases as the Le number increases. For a large Le number, the wheel can operate as a sensible one. For a Le number of approximately 1.6, equal effectiveness can be obtained
• Results show that the model under investigation can achieve good performance with $R$ values between 1 and 10. Also, as the maximum moisture content of matrix increases, notable increases on the latent effectiveness can be obtained while sensible effectiveness shows smaller changes.

• Simulations show that the effectiveness decreases as the supply humidity increases. Moreover, sensible effectiveness is strongly influenced by $\phi_{s,j}$. For heating mode, effectiveness has equal sensitivity over the given range of supply humidity.
CHAPTER FIVE
EXPERIMENTAL DATA AND COMPARISON WITH THE ENTHALPY MODEL

As part of this research, the author had the opportunity to conduct experimental studies on the energy recovery ventilator (ERV) manufactured by Stirling Technology, Inc. (Athens, OH). Also, the author was given access to the experimental data on the ERV on the basis of tests conducted by the Canadian test facility, BodyCote Materials Testing, Inc., Mississauga, Canada. The energy recovery ventilator with the trademark name RecoupAerator is designed to exhaust room air while maintaining indoor moisture levels and supplying the inside space with conditioned and filtered air. The ERV accomplishes this through a patented rotary random matrix polymer. The energy wheel in the ERV can be best described by the enthalpy model considered in Chapters 3 and 4.

The purpose of this chapter is to compare the numerical results generated by the enthalpy model with experimental results. A comparison is performed for both heating and cooling modes. Effectiveness as a function of volume flow rate $\dot{Q}$ and inlet conditions is a common way of representing the performance of commercial wheels.

A brief description of the test facility and the experimental results provided by Stirling Technology, Inc. are presented in section 5.1. Section 5.2 describes the experimental results and gives a comparison with the enthalpy wheel model.
5.1 Description of the Experimental Facility

5.1.1 Experimental Setup

The stand-alone energy recovery ventilators (ERVs) equipped with the air-to-air rotary heat/energy wheel are commercially available in different sizes and shapes. Recently, they have gained more attention due to an increasing demand for ventilation along with saving energy. Therefore, testing performance of ERV’s becomes essential to get the right performance of the unit. The two supply and two exhaust inlet and outlet flow stations must be instrumented to obtain accurate airflow properties [107]. The experimental apparatus has four measurement stations. The measurement stations consist of three different measurements: volume flow rates, temperature, and humidity.

<table>
<thead>
<tr>
<th>Station (1)</th>
<th>Station (2)</th>
<th>Station (3)</th>
<th>Station (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply inlet</td>
<td>Supply outlet</td>
<td>Exhaust inlet</td>
<td>Exhaust outlet</td>
</tr>
<tr>
<td>Volume flow rate</td>
<td>Volume flow rate</td>
<td>Volume flow rate</td>
<td>Volume flow rate</td>
</tr>
<tr>
<td>Temperature</td>
<td>Temperature</td>
<td>Temperature</td>
<td>Temperature</td>
</tr>
<tr>
<td>Humidity</td>
<td>Humidity</td>
<td>Humidity</td>
<td>Humidity</td>
</tr>
</tbody>
</table>

Units: Volume flow rate [CMH], Temperature °C, Humidity Kg_{wv}/Kg_{da}

The experimental test facility shown in Figure 5.1 was designed by Stirling Technology, Inc. [108] to investigate the performance of the energy recovery ventilator with the trademark “RecoupAreator”. The unit is positioned between the supply and exhaust flow ducts with a diameter of 6” . The supply and exhaust ducts contain all the
equipment and controls necessary to supply two air streams (of a given volume flow rate, temperature, and humidity ratio) to the test unit.

Figure 5.1 Picture of the test facility (Source: Stirling Technology, Inc.)

Figure 5.2 shows the instruments used to measure the flow rate, temperature, and humidity for station-1. All four stations are connected to the computer and the computation is performed using Dasylab software. Both winter and summer tests are carried out.
Figure 5.2 Instruments used to measure the flow rate, temperature, and humidity (Source: Stirling Technology, Inc.)

5.1.2 RecoupAerator

RecoupAerator [107] is a trademark name of the energy recovery ventilator (ERV) manufactured by Stirling Technology, Inc. (Athens, OH). The energy recovery ventilator is designed to supply well sealed places with fresh air while maintaining a desired level of temperature and humidity [107]. The ERV draws fresh air from the outdoors through a patented rotary random matrix polymer [107].

The main parts of the RecoupAerator are the patented energy transfer wheel, two GE blower motors to control the air flow, and one motor to drive the energy wheel. Six removable pies can be installed in the wheel casing to construct the heat and mass transfer media [107, 108]. They are shown in Figure 5.3.

Airflow is controlled by two blower motors: one for supplying air flow to the building (blower 1) and the other one for exhaust the air flow out of the building (Blower 2) [107, 108]. The unit has the capability of different airflows for the exhaust and inlets,
but the testing is performed for the balanced airflow, which means equal airflow supply and exit the building [107,108]. Constant rotational speed ($\Omega = 30 \text{ rpm}$) is used by the energy recovery wheel drive motor [107,108]. Figure 5.4 shows the layout of the RecoupAerator.

![Energy wheel with matrix pies](Stirling Technology, Inc.)

**Figure 5.3** Energy wheel with matrix pies (Stirling Technology, Inc.)

![RecoupAerator with air-to-air rotary energy exchanger](Source: Stirling Technology, Inc.)

**Figure 5.4** RecoupAerator with air-to-air rotary energy exchanger (Source: Stirling Technology, Inc.)
5.1.3 Air Flow Measurement

Air flow rates are obtained by two methods. The first, uses the Model 36 FMS flow measuring station provided by Nailor Industries, Inc. [109] which is a multi-point averaging airflow sensor. In their catalog, they have described the model, “as it can provide accurate sensing by sampling air velocities in four quadrants of a round duct”. Moreover “the differential pressure flow sensor provides an averaged reading at an amplification of approximately 2.5 times the velocity pressure”. To read the differential pressure, a manometer provided by (Dwyer Instruments, Inc.) is used [110]. Pressure is measured in inches of water column and converted to the volume flow rate in CFM using the flow chart provided by the supplier.

A second, more precise way of measuring the volume flow rate is using an air velocity transducer. TSI air velocity transducer [111] is used as a calibrator. In their catalog, they have described the model as “a precision instrument designed to measure air velocity in fixed installations or test applications”. First, the desired location of the transducer is determined in the four stations of the duct systems, and then a small hole in each station is drilled to place the probe. The probe is centered in the middle of the duct to get the right signal. The transducer is interfaced with data acquisition board and the volume flow rates are calculated using the Dasylab. Volume flow rate for the four stations is displayed on a computer screen during the entire test.
5.1.4 Temperature and Humidity Measurement

Temperature and humidity measurements are made by using the duct-mounted humidity and temperature transmitters HMD60/70 manufactured by Vaisala [112]. When accurate and stable measurements of humidity and temperature is required, HMD60/70 model is the one to use [112]. The transmitters incorporate the Vaisala HUMICAP sensor as mentioned in their catalog[112], which is “selected for its accuracy and long-term stability and resistance to dust”. Four such instruments are used: two to monitor the bulk inlet conditions and two for the outlet conditions. The four instruments are interfaced to the data acquisition board which contains a card for receiving and transferring the data signals to the PC. The data analysis is performed by Dasylab [113].

5.1.5 Pressure Measurement

Pressure measurements are required to study pressure drop characteristics of the wheel matrix materials. Different matrices with different porosity were investigated by plotting the pressure against the volume flow rate. Four dampers for restriction and controlling the flow rate for all ducts are used to balance the static pressure.

5.1.6 Data Acquisition and Dasylab

Dasylab is “a data acquisition, process control, and analysis system” [113]. The experimental setup including all the modules and data channels can be displayed graphically using the Dasylab software [113]. This enables us to visualize the experimental setup and to correct the error easily.
Data acquisition is coordinated through a Omega data acquisition unit called the DAQ [114]. The DAQ contains specific cards for receiving and sending control signals [114]. DasyLab and the data acquisition unit are connected through the computer parallel port [114].

After an experiment has begun, the transducers sense the temperature, humidity, and airflow speed. The four transducers are interfaced to the data acquisition board, which contains a card for receiving the data signals and transferring them to the PC. Then, DasyLab uses the DAQ board “to read an analog input signal and generates analog as well as digital output signals”. Once the necessary data feeds into the computer, the Dasylab analysis modules make it easy to process and manipulate the data.

Two worksheets were designed one for the winter test and the other for summer test. Twelve data readings for the four stations with sampling rates of 60 seconds are the inputs for each worksheet. Results are shown on the screen of the computer (Figure 5.5) in the graphical and numerical format. In Figure 5.5, there are twelve indicators for measuring the temperature and humidity ratio for all the four measuring stations. Also, four reading of the volume flow rates are shown and finally three indicators for the ERV effectiveness are presented on the left side of the screen.

Also, the results are stored on the computer to be ready for analysis. The measurements were performed in accordance with the HVI standard, but with slight modifications according to the test facility capability.
5.2 Energy Wheel Under Investigation (RecoupAerator)

The air-to-air rotary energy wheel under investigation is a counter-flow balanced and symmetric wheel. It is a rotating cylindrical wheel of length \( L_w = 0.0395 \) [m] and diameter \( d_w = 0.463 \) [m] and it is divided into equal supply and exhaust sections. While the wheel is rotating, the supply and exhaust air-streams are supplied equally to the wheel in a counter-flow arrangement.

Currently, there are two types of matrix materials being used in the RecoupAerator as heat and mass exchange media. It was decided that the RecoupAerator with polymer matrix material would be the focus for this experimental work. The reason of choosing the polyester matrix, is because of the availability of the experimental data from a third party (BodyCote).
Matrix properties and wheel parameters are given in Table 5.2. The material properties in Table 5.2 were provided by the material manufacturer (TEX TECH industries), and the fiber diameter \( d_f \) and the porosity \( \phi \) were calculated by the author. The polyester, believed to have a linear isotherm shape with maximum water uptake at a relative humidity of, 100% is about 20% of the total matrix weight [108].

<table>
<thead>
<tr>
<th>Table 5.2</th>
<th>Matrix properties and energy wheel parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air : Volume flow rate : ( \dot{Q}<em>{up} = \dot{Q}</em>{exh} = 120-300 ) [CMH]</td>
<td>Wheel rotational speed ( \Omega = 30 ) [RPM]</td>
</tr>
<tr>
<td>( D_w = 0.463 ) m</td>
<td>( L_w = 0.0395 ) m</td>
</tr>
<tr>
<td>( d_h = 900 ) ( \mu )m</td>
<td>( C_{p_m} = 1340 ) J/Kg.K</td>
</tr>
<tr>
<td>Linear Isotherm</td>
<td>( \sigma_{\text{max}} = 0.2 )</td>
</tr>
</tbody>
</table>

5.3 Operation Conditions

The cold and hot tests were done with operation conditions that can be attained in the test facility at Stirling Technology, Inc., and BodyCote Materials Test Facility, Inc. In the Stirling Technology test facility, there was no control of the operation conditions. Therefore, as one can notice the operation conditions are not fixed for the different volume flow rates as in the Bodycote tests facility. The supply and exhaust air conditions are averaged for the three different volume flow rates.

To get a more accurate data for comparison, the experimental works conducted at BodyCote Materials Testing, Inc. was used. Testing was carried out in general accordance
with CAN/CSA-C439-00 “Standard Laboratory Methods of Test For Rating the Performance of Heat/Recovery Ventilators” [115]. The ERV was tested for both heating and cooling modes.

For the heating mode, supply air temperature (station 1) were maintained at a nominal average temperature of $T_{s,j} = 0^\circ C$, while exhaust air conditions (station 3) were fixed at $T_{e,j} = 22^\circ C$, with $\phi_{e,j} = 40\%$.

For the cooling mode, tests were carried out with supply air conditions (station 1) at nominally $T_{s,j} = 35^\circ C$ and $\phi_{s,j} = 50\%$. Exhaust air conditions (station 3) were kept at nominally $T_{e,j} = 24^\circ C$, and $\phi_{e,j} = 50\%$.

Table 5.3 summarizes the inputs data used by standard CAN/CSA-C439-00 to test the ERV for both heating and cooling modes. Also, the inputs data are shown on a psychrometric chart (Figure 5.6) to be able to compare with other input data.

<table>
<thead>
<tr>
<th>Test Mode</th>
<th>Supply air conditions</th>
<th>Exhaust air conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_{air} (^\circ C)$</td>
<td>$\phi_{air} (%)$</td>
</tr>
<tr>
<td>Heating Mode</td>
<td>0</td>
<td>75</td>
</tr>
<tr>
<td>Cooling Mode</td>
<td>35</td>
<td>50</td>
</tr>
</tbody>
</table>
Figure 5.6 Supply and exhaust inlet conditions for both summer and winter operation (CAN/CSA-C439-00 standard)

5.4 Experimental Results and Comparison with the Enthalpy Model

5.4.1 Winter Tests

During winter operation, the energy wheel provides the air-conditioning space with warm and humid air-flow. The enthalpy wheel transfers not only sensible heat but also moisture between the supply and exhaust air-streams. Table 5.4 and Figure 5.7 illustrate the results for heating mode for the enthalpy model. Results are shown for three different volume flow rates with different boundary conditions because it was difficult to
obtain the same boundary conditions are based on outdoor conditions. Also, Table 5.5 and Figure 5.8 present the results for fixed boundary conditions with three different volume flow rates using the experimental work conducted at Bodycote. The numerical results were generated using the enthalpy wheel model with the same inputs (material, volume flow rate, and boundary conditions) of the experimental work.

It can be seen that the sensible effectiveness is in good agreement with the experimental data. The results in Tables 5.4 and 5.5 and Figures 5.7 and 5.8 show that $\varepsilon_s > \varepsilon_{TOT} > \varepsilon_L$ occurs for all volume flow rates. The general trend is a decrease in the effectiveness as the flow rate increases. However, the theoretical value for the latent and total effectiveness slightly overestimates the actual values obtained in the experiments.

Table 5.4  
Experimental and numerical results for polyester during heating mode (Stirling Technology, Inc. experimental setup)

<table>
<thead>
<tr>
<th>Supply side $T (^\circ C)$</th>
<th>$\phi$ (%)</th>
<th>Exhaust side $T (^\circ C)$</th>
<th>$\phi$ (%)</th>
<th>$\varepsilon_s$ Exp.</th>
<th>$\varepsilon_s$ Num.</th>
<th>$\varepsilon_L$ Exp.</th>
<th>$\varepsilon_L$ Num.</th>
<th>$\varepsilon_T$ Exp.</th>
<th>$\varepsilon_T$ Num.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume flow rate $Q = 135$ [CMH]</td>
<td>Rotational speed $\Omega = 30$ [RPM]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.95</td>
<td>66</td>
<td>22.61</td>
<td>44</td>
<td>0.91</td>
<td>0.90</td>
<td>0.71</td>
<td>0.73</td>
<td>0.82</td>
<td>0.85</td>
</tr>
<tr>
<td>Volume flow rate $Q = 220$ [CMH]</td>
<td>Rotational speed $\Omega = 30$ [RPM]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.51</td>
<td>77</td>
<td>22.65</td>
<td>43</td>
<td>0.88</td>
<td>0.89</td>
<td>0.65</td>
<td>0.68</td>
<td>0.79</td>
<td>0.81</td>
</tr>
<tr>
<td>Volume flow rate $Q = 297$ [CMH]</td>
<td>Rotational speed $\Omega = 30$ [RPM]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.61</td>
<td>79</td>
<td>22.77</td>
<td>42</td>
<td>0.86</td>
<td>0.87</td>
<td>0.56</td>
<td>0.60</td>
<td>0.77</td>
<td>0.79</td>
</tr>
</tbody>
</table>
Figure 5.7  Comparison between experimental and numerical results for polyester during heating mode (Stirling Technology, Inc. experimental setup)

Table 5.5  Comparison between experimental and numerical results for polyester during heating mode (Bodycote Materials Testing, Inc.)

<table>
<thead>
<tr>
<th>Supply side $T$ (°C) $\phi$ (%)</th>
<th>Exhaust side $T$ (°C) $\phi$ (%)</th>
<th>$\varepsilon_s$</th>
<th>$\varepsilon_L$</th>
<th>$\varepsilon_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>72.1  22  38.8</td>
<td>0.93</td>
<td>0.92</td>
<td>0.74</td>
</tr>
<tr>
<td>Volume flow rate $Q=241$  [CMH]  Rotational speed $\Omega = 30$  [RPM]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>72.8  22.1  39.1</td>
<td>0.90</td>
<td>0.91</td>
<td>0.64</td>
</tr>
<tr>
<td>Volume flow rate $Q=350$  [CMH]  Rotational speed $\Omega = 30$  [RPM]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>75.1  22  39.2</td>
<td>0.85</td>
<td>0.88</td>
<td>0.55</td>
</tr>
</tbody>
</table>
5.4.2 Summer Tests

During summer operation, the energy wheel supplies the house with cool and dehumidified air-flow. In cooling applications, air temperatures and humidities are usually within ranges that assure no condensation in the warm supply air stream. The enthalpy wheel transfers both sensible and latent heat by sorption process, where most of the time no condensation takes place. Validation of the enthalpy model will be based on the experimental results conducted at Stirling Technology and Bodycote.
For the tests studied here, the normal supply and exhaust inlet conditions for a hot day were used in both experiments. Experimental results along with numerical results are presented in Tables 5.6 and 5.7 and shown in Figures 5.9 and 5.10. The agreement between the experimental and the numerical results is reasonably good, especially at lower volume flow rates.

<table>
<thead>
<tr>
<th>Supply side $T , (^\circ\text{C})$</th>
<th>Supply side $\phi$ (%)</th>
<th>Exhaust side $T , (^\circ\text{C})$</th>
<th>Exhaust side $\phi$ (%)</th>
<th>$\varepsilon_S$ Exp.</th>
<th>$\varepsilon_S$ Num.</th>
<th>$\varepsilon_L$ Exp.</th>
<th>$\varepsilon_L$ Num.</th>
<th>$\varepsilon_T$ Exp.</th>
<th>$\varepsilon_T$ Num.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume flow rate $Q = 135 , [\text{CMH}]$</td>
<td>Rotational speed $\Omega = 30 , [\text{RPM}]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35.23</td>
<td>52</td>
<td>24.52</td>
<td>56</td>
<td>0.92</td>
<td>0.91</td>
<td>0.63</td>
<td>0.62</td>
<td>0.71</td>
<td>0.73</td>
</tr>
<tr>
<td>Volume flow rate $Q = 220 , [\text{CMH}]$</td>
<td>Rotational speed $\Omega = 30 , [\text{RPM}]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31.11</td>
<td>48</td>
<td>25.60</td>
<td>57</td>
<td>0.84</td>
<td>0.86</td>
<td>0.51</td>
<td>0.55</td>
<td>0.68</td>
<td>0.69</td>
</tr>
<tr>
<td>Volume flow rate $Q = 297 , [\text{CMH}]$</td>
<td>Rotational speed $\Omega = 30 , [\text{RPM}]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.87</td>
<td>47</td>
<td>25.77</td>
<td>55</td>
<td>0.82</td>
<td>0.86</td>
<td>0.40</td>
<td>0.60</td>
<td>0.52</td>
<td>0.66</td>
</tr>
</tbody>
</table>
Figure 5.9  Comparison between experimental and numerical results for polyester during cooling mode (Stirling Technology, Inc. experimental setup)

Table 5.7  Comparison between experimental and numerical results for polyester during cooling mode (Bodycote Materials Testing, Inc.)

<table>
<thead>
<tr>
<th>Supply side $T (°C)$</th>
<th>$\phi (%)$</th>
<th></th>
<th>Exhaust side $T (°C)$</th>
<th>$\phi (%)$</th>
<th></th>
<th>$\varepsilon_S$</th>
<th></th>
<th>$\varepsilon_L$</th>
<th></th>
<th>$\varepsilon_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume flow rate $Q = 130$ [CMH]</td>
<td>Rotational speed $\Omega = 30$ [RPM]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34.5</td>
<td>50.7</td>
<td>24.4</td>
<td>47.8</td>
<td>0.88</td>
<td>0.89</td>
<td>0.50</td>
<td>0.53</td>
<td>0.62</td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>Volume flow rate $Q = 241$ [CMH]</td>
<td>Rotational speed $\Omega = 30$ [RPM]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35.1</td>
<td>49.9</td>
<td>23.9</td>
<td>49.9</td>
<td>0.86</td>
<td>0.87</td>
<td>0.45</td>
<td>0.48</td>
<td>0.58</td>
<td>0.60</td>
<td></td>
</tr>
</tbody>
</table>
5.5 Summary

The experimental work was conducted on the energy recovery ventilator (ERV) manufactured by Stirling Technology, Inc. (Athens, OH) to validate the enthalpy wheel which was developed to simulate the heat and mass transfer for the supply and exhaust processes. Validation was performed for both heating and cooling mode. Results show that there is a good agreement between the numerical model and experimental data. The results also show that the general trend is a decrease in the effectiveness as the flow rate increases and $\varepsilon_s > \varepsilon_{TOT} > \varepsilon_L$ happens for all volume flow rates.

Figure 5.10  Comparison between experimental and numerical results for polyester during cooling mode (Bodycote Materials Testing, Inc.)
CHAPTER SIX

RESEARCH SUMMARY, CONCLUSIONS, AND FUTURE WORK

6.1 Summary

For the cases of ventilation requirements and energy saving, integrating the energy (heat and moisture) recovery ventilator and heat recovery ventilator in HVAC applications has gained more attention recently. The ventilator selected for this research employs the rotary air-to-air energy wheel with a porous matrix as a heat or heat-and-moisture transfer medium. Such an energy wheel can operate with high effectiveness by using a low-cost porous matrix energy core. To determine the effectiveness of the energy wheel, mathematical and numerical models have been developed for a symmetric and balanced wheel operating in a counter-flow arrangement.

For the modeling of the air-to-air rotary wheel, three models were developed: the sensible model, the condensation model, and the enthalpy model. The models were based on the physical principles of the operation of the energy wheels and they were considered to be one-dimensional and transient with space \( x \) and time \( t \). One set of partial differential equations (energy equations of air and matrix) with boundary and initial condition was derived to describe the heat transfer in the energy wheel, while two sets of partial differential equations (energy equations and mass conservation equations) with boundary and initial conditions were derived to describe the heat and mass transfer.
A complete, dimensionless representation (equations and boundary conditions) for each model was derived and solved using the finite difference method with the integral-based method formulation scheme. The resulting finite difference equations for each model were implemented in computer code to investigate the performance of each wheel separately.

The sensible model describes the heat transfer in a rotary wheel designed with non-desiccant porous matrix. Two energy conservation equations were used to find the model’s effectiveness. The condensation model describes the heat and mass (condensation/evaporation) transfer in a rotary wheel designed with non-desiccant porous matrix. The enthalpy model describes the heat and mass (adsorption/desorption) transfer in a rotary wheel using a desiccant porous media matrix. In the enthalpy model, desiccant matrix in a saturated condition can be treated as non-desiccant, so the same formulation as the condensation model can be used. Both derived models (condensation and enthalpy models) can be mathematically represented by two energy conservation equations with four basic thermodynamic relationships and two mass conservation equations.

Effects of wheel design parameters such as: rotational speed $\Omega$, number of transfer unit $NTU_{HT}$, heat capacity-ratio $C'_r$, porosity $\varphi$ and the operating parameters such as volume flow rate $\dot{Q}$, supply inlet air temperature and humidity ratio $(T_{s,j}, \omega_{s,j})$ and exhaust inlet air temperature and humidity ratio $(T_{e,j}, \omega_{e,j})$ on the wheel effectiveness were studied and a comparison with experimental results and effectiveness correlation was presented. Also, the temperature and humidity profiles were investigated for summer and winter operating conditions.
Finally, experimental work was conducted on the energy recovery ventilator (ERV) manufactured by Stirling Technology, Inc. (Athens, OH). The purpose of the experimental work was to validate the enthalpy model. A comparison was performed for both heating and cooling modes. The numerical model compared reasonably well with the values obtained in experiments and demonstrated trends similar to experimental results. Effectivenesses was given as a function of volume flow rate $\dot{Q}$.

6.2 Conclusions

As indicated above, during the modeling of the air-to-air rotary wheel, three models were developed: the sensible model, the condensation mode, and the enthalpy model. The following lists include the general conclusions and the most specific conclusions for each model analysis completed through the process of this research:

**General conclusions:**

- Three theoretical and numerical models were developed to estimate the effectiveness of a symmetric-balanced air-to-air energy wheel with a novel porous matrix as the heat and mass exchange media.
- The models are based on the physical principles of the operation of the energy wheels and they are one-dimensional and transient with space ($x$) and time ($t$) as the independent variables.
- A completed dimensionless and finite difference representation for each model was accomplished and implemented in computer code.
- The effectiveness of the ERV decreases with increase in volume flow rate $\dot{Q}$. 
- Both heating and cooling modes are equally sensitive to the changes in wheel rotational speed
- Effectiveness is almost constant for low porosity and then decreases for high porosity.
- The optimal effectiveness of the wheel can be achieved using the porosity of about $\approx 90\text{-}92\%$.
- Effectiveness rises linearly with $A_v$ and becomes constant above around $A_v = 7000 \left[ \frac{m^2}{m^3} \right]$.

**Sensible model:**
- During summer and winter operations, the sensible wheel can achieve the same effectiveness.
- There is a good agreement between the numerical results and $\varepsilon - NTU$ method.

**Condensation model:**
- Resulting sensible effectiveness from the condensation model is lower than the value of sensible model.
- The effectiveness depends on the extent of the condensation.
- For fixed indoor conditions, the sensible effectiveness is nearly constant for lower supply temperatures and increases when supply temperature goes up.

**Enthalpy model:**
- The enthalpy model predictions are found to be consistent with experimental results and the results of Simonson.
• Using the enthalpy wheel instead of the sensible wheel in application of an ERV is more realistic when the heat and moisture recovery are required.

• The enthalpy wheel provides enhanced performance and closed outlets (temperatures and humidity ratios) to the indoor conditions.

• For the heating mode operation, enthalpy wheel provides more energy recovery and it is less likely to produce condensation while the sensible wheel will approach the condensation conditions easily.

**Experimental data:**

New experimental data was obtained from Stirling Technology, Inc. (STI) (Athens, OH) for comparison with the enthalpy model. The model’s predictions of the effectiveness are found to be consistent with experimental results. The general trend is a decrease in the effectiveness as the flow rate increases and $\varepsilon_s > \varepsilon_{TOT} > \varepsilon_L$ occurs for all volume flow rates.

### 6.3 Future Work

During this research, the focus was limited to estimate the effectiveness of a symmetric-balanced air-to air energy wheel with a novel porous matrix as heat and mass exchange media. Three theoretical and numerical models were developed to achieve these objectives. Many considerations were covered in this research; however, there are still many other issues that need to be investigated. Recommended future studies are as follows:
• Improve the model by allowing wheel split and volume flow rate to be variables.

• Experimental investigations of adsorption isotherm for the matrix material in this research are needed.

• Effectiveness and pressure drop are the most important performance factors of the energy wheel. Therefore, numerical and experimental investigations of pressure drop are needed.

• Numerical modeling and experimental investigation of crossover leakage are needed which might be important for some applications that need a high speed rotating wheel.

• Numerical and experimental investigations of frost growth within energy wheels are still needed. Frost control strategies can be also investigated.
REFERENCES


