This thesis entitled
ADAPTIVE DATA RATE MULTICARRIER DIRECT SEQUENCE SPREAD SPECTRUM IN RAYLEIGH FADING CHANNEL

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We propose adaptive data rate multicarrier direct-sequence spread spectrum (MC-DS-SS) systems for transmission over a frequency-nonselective Rayleigh fading channel. We investigate two different novel transformations to be used at the transmitter that switch between conventional Serial-to-Parallel (S:P) and split transformations based on the knowledge of channel state information, a pre-selected threshold and a majority-vote decision. The selection of threshold is crucial and it directly affects the performance of our adaptive systems. We provide results to show that our adaptive system is flexible as it allows the user-selection of operating points. Our results also show that our adaptive transformations are advantageous than S:P in terms of bit error ratio performance and advantageous than split in terms of data rate.
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# TABLE OF CONTENTS

Abstract ............................................................................................................................................. 3

Acknowledgements .......................................................................................................................... 4

Table of Contents ............................................................................................................................ 5

List of Tables ..................................................................................................................................... 7

List of Figures ................................................................................................................................. 8

CHAPTER 1 ....................................................................................................................................... 11

INTRODUCTION ............................................................................................................................... 11

  1.1 Digital Communications ........................................................................................................... 11
  1.2 Spread spectrum Communications ....................................................................................... 13
  1.3 Multicarrier DS-CDMA ........................................................................................................... 14
  1.4 Fading Channels ..................................................................................................................... 16
  1.5 Thesis Scope ........................................................................................................................... 18

CHAPTER 2 ....................................................................................................................................... 20

SYSTEM MODEL ............................................................................................................................... 20

  2.1 Literature Review .................................................................................................................... 20
  2.2 Transmitter .............................................................................................................................. 23
  2.3 Channel Model ....................................................................................................................... 27
  2.4 Receiver ................................................................................................................................... 28
      2.4.1 Receiver for S:P .............................................................................................................. 28
      2.4.2 Receiver for split ........................................................................................................... 29

CHAPTER 3 ....................................................................................................................................... 31

AN ADAPTIVE MC-DS-SS SYSTEM ............................................................................................... 31

  3.1 Adaptive Transformation ......................................................................................................... 31
  3.2 Bit Error Rate Computations ................................................................................................. 32
List of Tables

Table 2.1. Relationships between various quantities in different transformations ..........25

Table 4.1. Different transformations for the hybrid case; $a_T$ is the threshold .................60
List of Figures

Figure 1.1. Block diagram of a typical digital communication system ................................12

Figure 1.2. Power spectra of MC-DS-CDMA and MT-DS-CDMA.................................16

Figure 2.1. Power Spectrum of MC-DS-SS......................................................................23

Figure 2.2. MC-DS-SS transmitter ...................................................................................24

Figure 2.3. A frequency non-selective fading channel model ........................................28

Figure 2.4. Block diagram of the receiver for S:P transformation....................................29

Figure 2.5. Block diagram of the receiver for Split transformation.................................30

Figure 3.1. Order Statistics of channel fading vectors for a $M=3$ subcarrier system......38

Figure 3.2. Example Order statistics of an $M =3$ subcarrier system at two different time instants .......................................................................................................................39

Figure 3.3. Schematic depiction of MATLAB simulation model.........................................44

Figure 3.4. Performance of systems using S:P and split transformations with $M=3$ subcarriers and processing gain $N=10$ .........................................................................................................................47

Figure 3.5. Performance of an adaptive system as compared to the performances of S:P and split transformations with $M=3$ subcarriers, 2/3 majority-vote strategy and processing gain $N=10$ .........................................................................................................................49

Figure 3.6. Performance of an adaptive system $M=3$ subcarriers, 2/3 majority-vote strategy and processing gain $N=10$ for different thresholds .................................................................50

Figure 3.7. Data rate gain vs. threshold for a system using adaptive transformation with $M=3$ subcarriers, 2/3 majority-vote strategy and processing gain $N=10$ .........................51

Figure 3.8. Performance of an adaptive system with $M=4$ subcarriers, 2/4 majority-vote strategy and processing gain $N=10$ for different thresholds .........................................................52
Figure 3.9. Performance Comparison of an adaptive system with $M=4$ subcarriers for 2/4 and 3/4 majority-vote strategies with processing gain $N=10$ and for the same threshold of -3 dB .................................................................53

Figure 3.10. Performance of an adaptive system with $M=5$ subcarriers, 3/5 majority-vote strategy and processing gain $N=10$ for different thresholds ..............................................54

Figure 3.11. Performance Comparison of an adaptive system with $M=5$ subcarriers for 3/5 and 4/5 majority-vote strategies with processing gain $N=10$ and for the same threshold of -3 dB .............................................................................................................55

Figure 3.12. Performance of an adaptive system with $M=3$ subcarriers, 2/3 majority-vote strategy, using block-wise adaptation and with a processing gain $N=10$ for different thresholds ........................................................................................................................................56

Figure 3.13. Performance of an adaptive transformation of $M=3$ subcarriers with noisy channel estimates using processing gain of $N = 10$ and threshold of -3 dB .................57

Figure 4.1. Performance of a hybrid transformation with $M=3$ subcarriers and processing gain $N=10$, compared with the performances of S:P and split transformations..............70

Figure 4.2. Performance of a hybrid system with $M=3$ subcarriers, and processing gain $N=10$ for different thresholds .................................................................................................................................71

Figure 4.3. Comparing the performances of M-state and 2-state systems with $M=3$ subcarriers, and processing gain $N=10$ for a threshold of -3 dB ...............................................................72

Figure 4.4. Comparing the transmit power levels of hybrid and 2-state adaptive systems for 20 symbol durations with $M=3$ subcarriers, and processing gain $N=10$, using the same threshold of -3 dB ......................................................................................................................74
Figure 4.5. Comparing the average transmit power levels of $M$-state and 2-state adaptive systems with $M=3$ subcarriers, and processing gain $N=10$, for various thresholds...........75
Chapter 1

Introduction

In this chapter, we review basic concepts of digital communications systems that were made use of in this research. We present a brief overview of the spread spectrum varieties of digital communication systems, followed by a discussion of novel multi-carrier direct sequence spread spectrum systems used in this research. Then, we examine different mathematical models of fading channels before concluding the chapter with an outline of thesis.

1.1 Digital Communications

Digital communications is a branch of communications that utilizes discontinuous signals, i.e., signals which appear in discrete steps, for example 0 and 1 for binary. It is different from analog communications, which uses continuous waveforms for transmitting data. The advantages of digital communications techniques include the following: greater data processing options and flexibilities; robustness to transmission impairments such as noise; and the ability to use error-detection and error-correction codes which further improve the performance. The disadvantages are that it requires somewhat complex equipment, may in some instances have limited transmission speed, and for some cases may require more bandwidth.

The principal feature of a digital communication system (DCS) is that during a finite interval of time, the transmitter sends a waveform from a finite set of possible waveforms, in contrast to an analog communication system, which sends a waveform
from an infinite variety of waveform shapes with theoretically infinite resolution [13]. In a DCS, the objective at the receiver is not to reproduce a transmitted waveform with precision; instead, the objective is to determine from a noise-perturbed signal which waveform from the finite set of waveforms was sent by the transmitter. A block diagram of DCS is shown in Figure 1.1 [7].

![Block diagram of a typical digital communication system](image)

**Figure 1.1.** Block diagram of a typical digital communication system

The upper row in the Figure 1.1 depicts the various signal transformations from the information source to the transmitter output. The lower row of blocks denotes the signal transformations from the receiver input to the information sink. It can be observed that the processes undergone from the receiver input to information sink are basically opposite to the transformations undergone by the signal from the source to the transmitter output. These transformations are required to transmit the source information across a
communications channel. We provide more discussion on communication channels in Section 1.4. A detailed description and functionality of each block in Figure 1.1 can be found in [7].

1.2 Spread Spectrum Communications

Over the last 50 years, a class of digital modulation techniques called "Spread Spectrum" (SS) has been developed. The SS investigation was motivated primarily by the desire to achieve highly secure digital communication. These techniques are called spread spectrum because the transmission bandwidth employed is much greater than the minimum bandwidth required to transmit the information. Spreading is accomplished by means of a spreading signal, often called a code signal or pseudo-random (PR) or pseudo-noise (PN) signal. One important parameter of a SS system is the processing gain, defined as the ratio of transmission bandwidth to information bandwidth. The advantages of SS systems include the following: suppression of interference such as that which comes from multipath propagation; resistance to jamming; reduction of energy density; and use in multiple access techniques such as code-division multiple access (CDMA).

A SS system can primarily be implemented in one of two varieties: direct sequence (DS) and frequency hopping (FH). In a DS-SS system, the information sequence is multiplied by a high-speed PN signal for spreading. The PN signal is independent of the data and from a spectral perspective, it has noise-like properties. At the receiver, a replica of the PN signal from the transmitter is used for de-spreading and subsequent data recovery. The wideband signal required for a SS system is generated in
a different manner in a FH system. The FH system takes the data signal and modulates it with a carrier signal whose center frequency hops from frequency to frequency over a wide band. The specific order in which frequencies are occupied is a function of a code signal, and the rate of hopping from one frequency to another is a function of the information rate. In this work, we focus on DS-SS communication systems.

1.3 Multicarrier DS-CDMA

One multicarrier modulation scheme, called *orthogonal frequency-division multiplexing* (OFDM), has recently drawn a lot of attention in the field of radio communications [2]. This is mainly because of the need to transmit at high data rates in a mobile environment, which makes for a “hostile” radio channel. This technique distributes the incoming bit stream onto many orthogonal sinusoidal subcarriers so that the symbol durations on each subcarrier are much longer in time than the input data symbols, which reduces the possibility of *intersymbol interference* (ISI). OFDM can be very efficient in spectrum usage and can perform well in a frequency selective channel.

In 1993, different types of innovative multiple access schemes based on a combination of code division and OFDM techniques were proposed [2]; prominent among them are multicarrier DS-CDMA (MC-DS-CDMA) and multitone DS-CDMA (MT-DS-CDMA). In MC-DS-CDMA, the transmitter spreads the parallel data streams using a given spreading code in the time domain so that the resulting spectrum of each subcarrier can satisfy the orthogonality condition with the minimum frequency separation. In MT-DS-CDMA, spreading is done in time domain so that the spectrum of each subcarrier prior to spreading operation can satisfy the orthogonality condition with
the minimum frequency separation. So in MT-DS, the resulting spectrum of each subcarrier may not satisfy the orthogonality condition; this depends upon the spreading codes used on each of the subcarriers.

The fundamental difference in these two schemes is the amount of frequency overlap between subcarriers. The subcarriers in MC-DS-CDMA have little or no overlap, and are orthogonal over a chip duration, whereas the subcarriers in MT-DS-CDMA have much overlap, with orthogonality over a symbol duration. In this work, we investigate only the MC-DS-CDMA variety. These systems have the advantages of yielding frequency diversity, suppressing narrowband interference, being robust to multipath fading, and require lower chip rates than that of a single carrier system [1]. The MT-DS-SS systems can potentially accommodate a large number of users because of large processing gains. Typically, the bandwidth of each subcarrier is constant in both types of systems. Figure 1.2 shows the power spectra of MC and MT-DS-CDMA schemes.
1.4 Fading Channels

The physical medium between the transmitter and receiver, known as the channel, plays an important role in analyzing and designing a communication system. This is more so in wireless communications, as the channel is usually time varying in nature due to the motion between transmitter and receiver, which results in changes of propagation paths. In this section, we present an overview of fading channel manifestations and describe the channel model we are using in this research.

Mobile communications can be characterized by two types of fading effects: large-scale fading and small-scale fading. Large-scale fading represents the average signal power attenuation or path loss due to motion over large areas\[13\]. The statistics of large-scale fading provide a way of computing an estimate of path loss as a function of
distance. This is often described in terms of a mean-path loss and a log-normally distributed variation about the mean. Small-scale fading refers to the dramatic changes in signal amplitude and phase that can be experienced as a result of small changes in the spatial positioning between a receiver and transmitter. It manifests itself in two mechanisms: time spreading of the signal and time-variant behavior of the channel. Small-scale fading is often modeled as Rayleigh fading if there are multiple reflective paths that are large in number and of approximately equal amplitude in the absence of a line-of-sight signal component. The envelope of such a received signal is statistically described by a Rayleigh probability density function (PDF). In addition, when there is a dominant nonfading signal component present, such as a line-of-sight component, the small-scale fading envelope is described by a Rician PDF, and hence the fading is Rician fading.

Considering the signal time-spreading mechanism in small-scale fading, fading can be characterized as frequency-selective or frequency-nonselective, the latter of which is also termed flat fading. A frequency-flat degradation occurs when all of the signals’ spectral components will be affected by the channel in a similar manner, i.e., when the coherence bandwidth of the channel is greater than the signal bandwidth. Frequency selective distortion occurs whenever a signal’s spectral components are not all equally affected by the channel.

In terms of the time-variation mechanism of the channel, the small-scale fading can be either fast or slow. Fast fading describes a condition where the time duration in which the channel behaves in a correlated manner is short compared with the time
duration of a symbol. Fading is considered slow if the time duration in which the channel behaves in a correlated manner is long compared with the time duration of a symbol.

In this thesis, we consider a Rayleigh fading channel that is frequency nonselective and slow from the perspective of any single subcarrier, in an MC-DS system. We further assume that the fading on each subcarrier is independent and memoryless, meaning that the fading at one instant does not depend on the fading earlier.

1.5 Thesis Scope

In [1], the performance of a variety of MC-DS-CDMA systems was evaluated. Since fading channels can introduce an irreducible bit error rate, diversity techniques are used to improve performance. In frequency diversity, the same information bearing signal is transmitted on a number of subcarriers, separated by at least the coherence bandwidth of the channel. This diversity technique achieves very good performance but at the expense of data rates.

The main objective of the thesis is to design an adaptive MC-DS-SS system that facilitates user-selectable operating points, where an operating point is defined as a pair of bit error rate and throughput. We devise an adaptive transformation to switch between conventional serial-to-parallel (S:P) and split transformations. These transformations, about which more will be discussed in subsequent chapters, essentially describe how the data symbols are distributed across the multiple subcarriers. By keeping the symbol and chip rates constant irrespective of the transformation, we attain higher data rates when the system is in S:P mode and better performance while in split mode, due to the frequency diversity. The switching between modes is based on information about the channel, a
pre-chosen threshold, and a majority-vote decision. We consider the cases when the fading is instantaneous (or on a symbol basis) and the more practical scenario of block-wise fading, when the fading is approximately constant over a block of symbols.

In the second part of the thesis, we design an $M$-state adaptive or hybrid transformation, where $M$ is the number of subcarriers. This is different from the earlier adaptive transformations, which can have only two states: either S:P or split across all $M$ subcarriers. By incorporating hybrid states in these $M$-state adaptive systems, wherein we split across only some of the subcarriers while using S:P across the remaining subcarriers, still higher data rates can be obtained at comparable bit error rate performance. As in the 2-state adaptive systems, the switching between transformations is based on the channel state information and a pre-chosen threshold.

The outline of the thesis is as follows: in chapter 2, we present an overview of the related past research followed by a description of a system model. In chapter 3, we analyze the proposed adaptive system and present the results of analysis and simulation. Chapter 4 presents the analysis of the $M$-state adaptive or hybrid systems along with the results of analysis and simulation. In chapter 5, we summarize our work, and provide conclusions drawn from this work. We conclude by describing some suggestions for future work.
Chapter 2

System Model

In this chapter, we review the past research in this field followed by a discussion of the system models used in our research. The models of the transmitter, channel, and receiver are presented and the signals are mathematically described.

2.1 Literature Review

This research is primarily based on a paper by Kondo and Milstein [1], in which the performance of a variety of multicarrier DS CDMA systems in fading environments is discussed. The authors analyze the performance over a dispersive channel that is frequency non-selective over each subcarrier, when all the subcarriers have same data and same spreading code. Since same data is being transmitted on all subcarriers, maximal ratio combining is used to obtain frequency diversity. One of the key advantages of using multicarrier schemes is the frequency diversity attainable on frequency-selective fading channels. In [3], the author proposes a multitone DS CDMA systems and its performance is evaluated in Rician fading channel. Perfect power control and Gaussian multiuser interference statistics are assumed in this paper.

In one of the earlier papers in this field, Hara and Prasad discuss both the structure and performance of the multitone (MT) & multicarrier (MC) DS schemes [2]. This paper considers terrestrial cellular applications. In [2], the authors use simulation results to compare performance among several schemes on a chip-synchronous dispersive channel.
Receiver structures and algorithms are also discussed in [3], with its virtue being primarily illustration of the similarities of the various DS-CDMA approaches.

In [4], the authors assess the performance of generalized MC-CDMA systems over multipath Nakagami-m fading channels. It is proposed and proved that there is an optimum subcarrier spacing that results in a minimum bit error rate MC-DS-CDMA system. This spacing optimized system outperforms conventional MC and MT DS-CDMA schemes. When AWGN channels are considered, the orthogonal MC DS-CDMA scheme represents the spacing optimized system.

In [5] and [8], authors investigate the performance of MT and MC DS-SS signaling in the presence of narrowband interference and partial band pulse jamming/interference, respectively. The authors consider the tradeoff between the number of subcarriers and the per-subcarrier processing gain, for a fixed data rate and fixed bandwidth. In these references, coherent detection with MPSK modulation is used. It is found that in the presence of multiple tone jamming, the MT DS-SS system is most robust when compared to conventional single carrier and MC schemes. Reference [6] discusses the performance of MC and MT DS-SS systems in the presence of imperfect phase synchronization.

Different diversity combining techniques for Rayleigh fading channels are examined in [14]. A generalized selection combining (SC) technique is proposed and its performance is compared with that of maximal ratio combining (MRC) for coherent reception and equal gain combining (EGC) for non-coherent reception. In $m^{th}$ order selection combining, the $m$ signals with the largest amplitudes are combined, in contrast
to the selection combining case, wherein only the largest signal is selected. In this thesis, we use MRC as it gives the best performance with coherent detection at the receiver.

We have looked at various ways from past research work to obtain higher data rates, as this is one main objective of this thesis. Adaptive systems were under consideration as early as 1968, as is evident from [17]. In that paper, design of binary signals for transmission through a Rayleigh fading medium was considered. The transmitter uses the channel state information that is fed back to modify the transmission in such a manner that the probability of error is minimized. In [16], another variety of adaptive system for a Rayleigh fading channel is proposed, in which the adaptation is by varying the duration of the pulse that is being transmitted in response to signal strength variations in the channel.

More recently, for high-speed data transmission over fading channels, systems were proposed that adapt the signal constellation size. In [11], a variable-rate and variable-power MQAM modulation scheme is proposed. The spectral efficiency of the proposed scheme is compared with the capacity of fading channels with channel side information, derived in [12].

Based on the review of past research, we are convinced that adapting the transformation in order to achieve higher data rates and/or lower BER is a novel approach. We analyzed these proposed adaptive systems and results are presented in subsequent chapters.
2.2 Transmitter

In this MC-DS-SS system, the data streams on each subcarrier can be spread by using the same spreading code or a different spreading code on each subcarrier. Since the subcarriers are chosen to be orthogonal by virtue of frequency separation, the use of the same or different spreading codes will not make any significant difference when channel Doppler spreading is minimal, though we use different spreading codes in this research. In the frequency domain, the individual adjacent subcarriers are separated by the chip rate $R_c$. The bandwidth of each subcarrier signal after spreading is designed to be approximately equal to the coherence bandwidth of the channel. Fig 2.1 shows the frequency domain representation of the subcarrier spectra. This system model is similar to the one used in [1].

![Figure 2.1. Power Spectrum of MC-DS-SS](image)

The transmitter structure for an arbitrary number of subcarriers $M$ is shown in Figure 2.2. The transmitter uses a transformation to distribute the incoming serial data onto different subcarriers. The data on each subcarrier is then multiplied by a spreading code before being upconverted to a carrier frequency determined by the multicarrier scheme. The individual subcarriers’ signals are summed and transmitted by an antenna.
In this research, the basic transformations we use at the transmitter are Serial-to-Parallel (S:P) conversion, and Split or Replication. The S:P transformation converts the serial data into $M$ parallel streams, each of lower rate than the input signal, while the split transformation replicates each bit onto all the $M$ subcarriers. So, the S:P transformation increases the duration of the symbol on each subcarrier relative to the input data symbol duration, whereas the split transformation maintains the same rate, but reduces the symbol energy on each subcarrier. Table 1 shows the relationships between various quantities of the input and output sequences of the transformation block, for the two transformations. In the table, $R$, $E$, and $T$ are the rate, energy, and duration of the symbols, respectively, and the subscripts $b$ and $s$ stand for before and after the

**Figure 2.2.** MC-DS-SS transmitter
transformation, respectively. So, $R_s$ and $E_s$ correspond to the rate and energy per symbol on one subcarrier.

**Table 2.1.** Relationships between various quantities in different transformations

<table>
<thead>
<tr>
<th>Transformation</th>
<th>$R_b$</th>
<th>$E_s$</th>
<th>$T_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serial-to-Parallel</td>
<td>$MR_s$</td>
<td>$E_b$</td>
<td>$T/M$</td>
</tr>
<tr>
<td>Split</td>
<td>$R_s$</td>
<td>$E_s/M$</td>
<td>$T_s$</td>
</tr>
</tbody>
</table>

We note here that these relationships are developed primarily for binary data coming into the transmitter, though generalizations to $M$-ary modulated data are straightforward. The split transformation reduces the energy of each symbol by a factor $M$, for a system consisting of $M$ subcarriers.

In this research, we enforce a constant symbol rate condition on all the subcarriers. This is necessary to achieve higher data rates, equal to $MR_s$ when the S:P transformation is in use, compared to the date rate of $R_s$ obtainable using the split transformation. Another advantage of having the same symbol rate is that it simplifies the hardware implementation of the system. This condition requires the transmitter to have a data source capable of sending data at any required rate. This could be achieved for example with a large data buffer preceding the transmitter input. Finally, we also assume equal chip rates for the DS spreading waveforms on each subcarrier, which implies equal bandwidths that are also constant during transmission. The constant bandwidths of all subcarriers ensure that there is no overlap between the signals of different subcarriers in slowly fading channels.
We use different long codes with the same chip rate for all subcarriers. Spreading codes are called “long” if the period of the code is greater than the symbol period and “short” if the period of the code is equal to the symbol period. The short codes have the advantage of simplicity in hardware while the long codes have higher flexibility and greater security, and in some cases can support a larger number of users than can be accommodated with short codes.

Let the incoming data waveform \( d(t) \) be mathematically defined as

\[
d(t) = \sqrt{\frac{2E_b}{T_b}} \sum_{n} d_n(t) P_T(t - nT) \tag{2.1}
\]

where \( E_b \) and \( T_b \) are energy and duration of a bit, respectively. The variable \( d_n \) in the set \( \{ \pm 1 \} \) is the equiprobable binary data, and \( P_T \) is a rectangular pulse equal to unity from \( t - nT = 0 \) to \( t - nT = T \), and zero elsewhere. The data waveform on the \( i^{th} \) subcarrier after the transformation is denoted \( d_i(t) \); for the S:P transformation \( d_i(t) \) is of the same form as (2.1), with a different pulse duration and every \( M^{th} \) bit is used; for the split transformation, \( d_i(t) \) is again of the same form as (2.1), but with an amplitude scale factor of \( 1/M \) and the same pulse duration. The waveform \( d_i(t) \) is multiplied by a spreading code \( c_i(t) \) and a sinusoidal carrier signal of frequency \( f_{ci} = f_c + f_i \), where \( f_c \) is a common carrier frequency, and \( f_i \) is the frequency offset of the \( i^{th} \) subcarrier from \( f_c \); the sinusoids are used to upconvert the spread data signals. In multicarrier DS-SS, adjacent subcarriers are separated in frequency by the chip rate, so \( f_i = iR_c \), where \( R_c \) is the chip rate. We let \( N \) be the processing gain of the system so that \( R_c = R_s N \).

The transmitted signal on \( i^{th} \) subcarrier can then be expressed as
\[ s_i(t) = \sqrt{\frac{2E_{si}}{T_{si}}} \left\{ \sum_n d_i \left( \left\lfloor n/N \right\rfloor c_i(t) \right) \right\} \cos(2\pi(f_c + f_i)t) \]  \hspace{1cm} (2.2)

where \( c_i(t) \), the spreading code, is given by \( \Sigma c_n p_T(t-nT_c) \), with \( c_n \) in the set \( \{\pm 1\} \) and \( p_T(t-nT_c) \) is the rectangular pulse shape, equal to unity from \( t=nT_c \) to \( t=T_c+nT_c \). Also \( E_{si} \), \( T_{si} \) in the above equation denote the energy and duration of a symbol after the transformation on \( i^{th} \) subcarrier, respectively and \( \lfloor x \rfloor \) denotes the integer part of \( x \). The final transmitted signal is simply the summation of the signals of the individual subcarriers.

### 2.3 Channel Model

The channel impairments present in the system are additive white Gaussian noise (AWGN) and Rayleigh fading, as shown in Figure 2.3. It is assumed that the fading processes on different subcarriers are mutually statistically independent and memoryless [1]. Our channel model allows us to consider each subcarrier’s signal independently at the receiver. Further, initially we considered the case of slow and flat Rayleigh fading, which can be modeled as a multiplicative process. With this, the received signal on the \( i^{th} \) subcarrier can be expressed as

\[ r_i(t) = s_i(t)\alpha_i(t)e^{-j\phi_i(t)} + z_i(t) \]  \hspace{1cm} (2.3)

where \( \alpha_i(t) \), \( \phi_i(t) \) are the amplitude and phase distortions of the channel, and \( z_i(t) \) is the AWGN with zero mean and a variance of \( N_0/2 \) W/Hz. The actual noise variance depends on the receiver noise figure. Because of the orthogonality of the subcarriers, the noise
processes in various channels are statistically independent. The channel in consideration for any individual subcarrier is hence a non-dispersive channel.

![Figure 2.3](image)

**Figure 2.3.** A frequency non-selective fading channel model

### 2.4 Receiver

#### 2.4.1 Receiver for S:P

The receiver structure for a system using the S:P transformation with $M$ subcarriers is shown in Figure 2.4. The signal $r_i(t)$ is assumed to have already passed through an antenna and any RF dividing networks, plus any wideband noise limiting filters, plus the low noise amplifier (LNA). In the receiver, we employ coherent detection with perfect carrier and code synchronization. In this research, we focus our attention on a single user case. Thus, the optimum receiver consists of a bank of correlators, each correlator corresponding to one subcarrier. The received signal on $i^{th}$ subcarrier is multiplied by the corresponding spreading code $c_i(t)$ and sinusoidal carrier frequency $\cos(2\pi f_{c,i}t)$. The resulting signal is then integrated over a symbol duration $T_s$. The integration operation removes any double or high-frequency terms resulting from the
multiplication by the sinusoidal carrier signals, even though in practice, an additional lowpass filter after the downconversion would typically be used. The decision circuits sample the output of the correlators at symbol times to make hard bit decisions. Finally, a back transformation, such as a multiplexer for example, is used to convert the parallel data to serial data.

![Block diagram of the receiver for S:P transformation](image)

Figure 2.4. Block diagram of the receiver for S:P transformation

### 2.4.2. Receiver for Split

The receiver structure for a system that uses the split transformation is shown in Figure 2.5. It is similar to the receiver structure for the S:P transformation, except for a maximal ratio combiner (MRC) that is used to combine the decision statistics of the
individual subcarriers [1], [7], [14]. This receiver also employs coherent detection with perfect carrier and code synchronization. Integration is performed over symbol duration on the received signal after despaying and downconversion processes. Since the same data is transmitted on all subcarriers, we obtain frequency diversity by using a MRC [7]. Before combining, a MRC scales the outputs of the correlators at the symbol times by the corresponding complex-valued conjugate channel gains $\alpha_i(t)e^{j\phi_i(t)}$, where $i$ represents the index of the subcarrier [7]. In this coherent detection, phase is actually taken into account by the sinusoidal multiplication in down conversion process, but in practice, a part of this is done at baseband also. This scaling of the decision variables weights each subcarrier signal by a factor that is proportional to the signal strength. Hard bit decisions are then made on the output of the MRC. It is assumed that the estimates of the Channel State Information (CSI), i.e., estimates of $\{\alpha_i\}$ and $\{\phi_i\}$, used for MRC, contain no noise.

Figure 2.5. Block diagram of the receiver for Split transformation
Chapter 3

An Adaptive MC-DS-SS System

In this chapter, we propose an adaptive MC-DS-SS system, and its BER performance is analyzed. We corroborate the analysis with simulations. Adaptation is via the transformation that is used to distribute symbols across the $M$ subcarriers, and so is distinct from other schemes that adapt other transceiver parameters, e.g., signal alphabet size [11], [12].

3.1 Adaptive Transformation

As discussed in chapter 2, the basic transformations we use in this research are S:P conversion of the data and splitting each symbol across all the subcarriers. In chapter 4, we propose a “hybrid” transformation, wherein, some of the subcarriers may have S:P data while the remaining are in split mode. By keeping the symbol rate $R_s$ and chip rate $R_c$ constant on each subcarrier, we obtain a data rate of $MR_s$ and a performance equivalent to that on a flat Rayleigh fading channel [7] when S:P is used. When split is used, we obtain $M^{th}$ order frequency diversity and a data rate of $R_s$. Hence, the S:P has the highest data rate and split has the best BER performance. By using an adaptive transformation, we obtain higher data rates at comparatively low bit error rates, and can select the system operating points, where the operating point is defined as the $(BER, R_b)$ pair.

For the purpose of switching between transformations, it is assumed that the transmitter has complete and accurate knowledge of channel state information (CSI). We
assume that there is no delay in transferring the CSI to the transmitter. We use a majority-vote strategy and CSI, along with a pre-selected threshold, to switch between the transformations. In general, in an $M$ subcarrier system, if $M/2 \leq k \leq M$ channels are fading below a pre-chosen threshold, we use the split transformation, and S:P otherwise. Theoretically, $k$ can take any value from 1 to $M$, but since we are using a majority-vote decision, the lower limit on $k$ is $M/2$.

Initially, the adaptation in the transformation is instantaneous, i.e., for each bit. We have also investigated cases when the adaptation is for a block of bits, with similar results. In block-wise adaptation, the length of the block is the duration over which the channel fading is approximately constant and is more practical than the instantaneous case.

3.2 Bit Error Rate Computations

3.2.1 S:P Transformation

To evaluate the performance of the binary communication system described in the previous chapter, we evaluate the decision variables, and from these, determine the probability of error [13]. Since the channel is frequency-nonselective, the transmitted signal undergoes multiplicative distortion [7]. Furthermore, the condition that the channel fades slowly implies that the multiplicative process may be regarded as a constant during at least one symbol duration--for most applications it can be assumed constant for many (e.g., hundreds of) symbols. We first explore the case of
“instantaneous,” or per-symbol adaptation, as a best-case situation, and then by analogy, write the expressions for the block-wise case.

The received signal in one signaling interval is

\[ r(t) = ae^{-j\phi} s(t) + z(t) \quad 0 \leq t \leq T_s \] (3.1)

where \( a, \phi \) are the amplitude and phase distortions of the channel, respectively, and \( z(t) \) is the AWGN with zero mean and a variance of \( N_0/2 \) W/Hz.

The receiver structure for S:P transformation is shown in Figure 2.4. We demodulate the received signal for \( i^{th} \) subcarrier by multiplying with corresponding sinusoidal carrier frequency \( \cos(2\pi f_c t) \) and the spreading code \( c_i(t) \). The input signal to the correlator of the \( i^{th} \) subcarrier is

\[ r(t) \cos(2\pi(f_c + f_i)t) \]

\[ = \sum_{i=1}^{M} \left[ \sqrt{\frac{2E_{si}}{T_s}} \left\{ \sum_{n} d_i \left( \left\lfloor \frac{n}{N} \right\rfloor \right) c_i(t) \cos(2\pi(f_c + f_i)t) \right\} \right] \alpha_i(t) + z_i(t) \cos(2\pi(f_c + f_i)t) \] (3.2)

where \( c_i(t) \), the spreading code, is \( \sum_{n} c_n p_f(t-nT_c) \), where \( c_n \in \{ \pm 1 \} \) and \( p_f(t-nT_c) \) is the rectangular pulse shape, equal to unity from \( t=nT_c \) to \( t=T_c+nT_c \).

We make note of the fact here that we can consider the fading on each channel separately by virtue of our channel model, which states that each subcarrier undergoes independent fading. Even though the input contains the signals of all \( M \) subcarriers, with perfect frequency and phase synchronization, only the signal of interest, i.e., the \( i^{th} \) subcarrier signal, can get through the downconverter. After integrating the product of received signal and composite code signal over a symbol time, the decision statistic on the \( i^{th} \) channel is
\[ U_i = \sqrt{E_u} d_i \alpha_i + z_i. \] (3.3)

First, an expression for the probability of bit error for a time-invariant channel, i.e., for constant \( \alpha_i \), will be developed. If \( d_i(\lfloor n/N \rfloor) = 1 \) was transmitted, \( E(U_i) = \sqrt{E_{s,i}} \alpha_i \), where \( E(.) \) is the expectation operator. Similarly, when \( d_i(\lfloor n/N \rfloor) = -1 \) was transmitted, \( E(U_i) = -\sqrt{E_{s,i}} \alpha_i \).

Using the binary minimum error probability detector formulation developed in [13], the probability of bit error can be written as

\[ P_2(\gamma_b) = Q(\sqrt{2\gamma_b}) \] (3.4)

where \( \gamma_b = \alpha_i^2 E_s / N_0 \) and \( Q(.) \), the complementary error function, defined as

\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt, x \geq 0, \]

gives the area under the tail of the Gaussian probability density function (PDF).

In order to obtain an expression for the bit error ratio (BER) on a Rayleigh fading channel, \( P_2(\gamma_b) \) must be averaged over the PDF of \( \gamma_b \). When \( \alpha_i \) is Rayleigh distributed, \( \alpha_i^2 \) has a chi-square probability distribution with two degrees of freedom [19]. Consequently, \( \gamma_b \) is also chi-square distributed with two degrees of freedom, and its PDF is given by [7]

\[ p(\gamma_b) = \frac{1}{\gamma_b^2} e^{-\gamma_b/\gamma_b}, \gamma_b \geq 0 \] (3.5)

where \( \gamma_b^2 \) is the average signal-to-noise ratio, defined as \( \gamma_b^2 = E(\alpha_i^2) \frac{E_{bi}}{N_o} \).

The expression for BER is then obtained by averaging \( P_2(\gamma_b) \) over \( p(\gamma_b) \)
\[ P_{b,s} = \int_0^\infty P_s(\gamma_b) \gamma_b d\gamma_b \] (3.6)

Strictly speaking, the lower limit of integration in (3.6) for channels that are fading above the threshold is \( \gamma_T \) and for the channels that are fading below the threshold, it can be taken as 0, where \( \gamma_T = a_T^2 E_{si} / N_0 \) is the threshold SNR, with \( a_T \) being the threshold. But, for simplicity, we approximated the lower limit to be 0 for all the channels in our analysis. Substituting equations (3.4) and (3.5) and carrying out the integration, we obtain a closed form solution [7]

\[ P_{b,s} = \frac{1}{2} \left( 1 - \sqrt{\frac{\gamma_b}{1 + \gamma_b}} \right) \] (3.7)

### 3.2.2 Split Transformation

In the split case, in order to achieve frequency diversity, the receiver uses a maximal ratio combiner (MRC) to combine the \( M \) decision variables. In MRC, each matched filter output is multiplied by the corresponding complex-valued channel gain. This scaling of the decision variables weighs the signal by a factor that is proportional to the signal strength. It is assumed that the estimates of CSI used for MRC contain no noise, i.e., they are perfect. The output of the maximal ratio combiner, expressed as a single decision variable, is [7]

\[ U = 2E_{si} \sum_{i=1}^{M} \alpha_i^2 + \sqrt{E_{si}} \sum_{i=1}^{M} \alpha_i z_i(t) \] (3.8)

where \( M \) is the number of diversity channels.
As in the case of S:P, the probability of bit error conditioned on a fixed set of attenuation factors \( \{ \alpha_i \} \) is first obtained, and is then averaged over the PDF of the set \( \{ \alpha_i \} \).

For a fixed set of \( \{ \alpha_i \} \), the decision variable \( U \) is Gaussian with mean
\[
E(U) = 2E_{si} \sum_{i=1}^{M} \alpha_i^2
\]
and variance \( \sigma_U^2 = 2E_{si} N_0 \sum_{i=1}^{M} \alpha_i^2 \). The probability of bit error for these values of mean and variance is
\[
P_b(\gamma_b) = Q(\sqrt{2\gamma_b})
\]
where \( \gamma_b = \frac{E_{si}}{N_0} \sum_{i=1}^{M} \alpha_i^2 \) is the SNR per bit in this case.

The PDF of \( \gamma_b \) is determined using the characteristic functions of the individual squared Rayleigh random variables. For any single channel, the instantaneous SNR, given by \( \gamma_i = \frac{E_{si}}{N_0} \alpha_i^2 \), has a chi-square PDF and its characteristic function can be expressed as [7]
\[
\psi_{\gamma_i}(j\omega) = \frac{1}{1 - j\omega \overline{\gamma}_c}
\]
where \( \overline{\gamma}_c \), the average SNR per channel, assumed to be identical for all channels, is
\[
\overline{\gamma}_c = \frac{E_{si}}{N_0} E(\alpha_i^2).
\]

Since the individual random variables are independent, the characteristic function of the summation is the product of the individual characteristic functions [7], [19].
\[
\psi_{\gamma_s}(j\omega) = \frac{1}{(1 - j\omega \overline{\gamma}_c)^M}
\]
This is the characteristic function of a chi-square distributed random variable with $2M$ degrees of freedom. The PDF of $\gamma_b$, which is the inverse Fourier transform of the characteristic function in (3.11), is given by

$$p(\gamma_b) = \frac{1}{(M-1)!\gamma_c^M} \gamma_b^{M-1} e^{-\gamma_b/\gamma_c}$$  \hspace{1cm} (3.12)$$

While averaging $P_2(\gamma_b)$ over the fading channel statistics, the upper limit of the integration should be $M\gamma_T$ when $M$ channels are fading below the threshold. There is no closed form expression for a finite upper limit of the integration and hence it is approximated as $\infty$ and the bit error rate in the split case is obtained as [7]

$$P_{b,\text{Split}} = \left[\frac{1}{2} (1-\mu)\right]^{M-1} \sum_{i=0}^{M-1} \left(\frac{M-1+i}{i}\right) \left[\frac{1}{2} (1+\mu)\right]^i$$  \hspace{1cm} (3.13)$$

where, $\mu = \frac{\gamma_c}{1+\gamma_c}$, and $\binom{N}{k}$ is the combinatorial coefficient $N!/[k!(N-k)!]$. 

### 3.2.3 Adaptive Transformation

A “majority-vote” strategy is used to switch between transformations, wherein the S:P scheme is employed if the majority of channel amplitudes are above a pre-selected threshold and the split scheme is used otherwise. The selection of threshold is critical, and the performance of the system depends on this single parameter.

We employ order statistics to facilitate the analysis of the adaptive transformation. The order statistics of the random variables $X_i, i \in \{1, 2, \ldots, n\}$, are $n$ random variables $Y_k$ defined as follows: for a specific outcome $\zeta$, the random variables $X_i$ take the values $X_i(\zeta)$. 

37
Ordering these numbers, we obtain the sequence \( X_{r1}(\zeta) \leq \ldots \leq X_{rk}(\zeta) \leq \ldots \leq X_{rn}(\zeta) \). If we define the random variable \( Y_k \) such that

\[
Y_1(\zeta) = X_{r1}(\zeta) \leq \ldots \leq Y_k(\zeta) = X_{rk}(\zeta) \leq \ldots \leq Y_n(\zeta) = X_{rn}(\zeta)
\]

then, the density function \( f_k(y) \) of the \( k^{th} \) statistic \( Y_k \) is given by [19]

\[
f_k(y) = \frac{n!}{(k-1)!(n-k)!} F_{X}^{k-1}(y)(1-F_{X}(y))^{n-k} f_{X}(y)
\]

(3.14)

where \( F_{X}(x) \) is the distribution of the i.i.d. random variables \( X_i \), and \( f_{X}(x) \) is their density. For the Rayleigh random variables, the distribution function \( F_{X}(x) = 1 - \exp[-x^2/(2b^2)] \) and density function \( f_{X}(x) = x \exp[-x/(2b^2)]/b^2 \), where \( 2b^2 \) is the mean-square value of \( X \).

Figure 3.1 shows the PDFs of all the three order statistics along with the Rayleigh density function, in an \( M=3 \) subcarrier system. In the figure, order statistic one corresponds to the minimum order statistic and order statistic three corresponds to the maximum order statistic. It follows our intuition that the minimum order statistic is the leftmost plot and maximum is the rightmost plot.
Figure 3.1. Order Statistics of channel fading vectors for a $M=3$ subcarrier system

At any instant of time $t$, if $[a_{1t}, a_{2t}, \ldots, a_{Mt}]$ are the channel amplitudes for the $M$ subcarriers, let $[y_{1t}, y_{2t}, \ldots, y_{Mt}]$ be the ordered channel amplitudes such that $y_{1t} < y_{2t} < \ldots < y_{Mt}$. In Figure 3.2, we plotted example order statistics of a $M=3$ subcarrier system on the axis of real numbers at two different time instants $t_1$ and $t_2$. This figure illustrates how different the channel gains will be at different time instants. The minimum order statistics at times $t_1$ and $t_2$ are $a_2$ and $a_3$ respectively, while the maximum order statistics at these instants are $a_3$ and $a_2$. 
In this $M$ channel case and for a $k$-out-of-$M$ majority-vote switching strategy, we have calculated the probability that any $k$ channels will fade below the threshold, or, equivalently, the probability that the split transformation will be employed, as

\[
P_{\text{Split}} = P(y_{kt} < a_T) = 1 - \sum_{i=1}^{k} \frac{M!}{(k-i)!(M-k+i)!} (1 - e^{-\frac{a_T^2}{2b^2}})^{k-i} e^{-\frac{a_T^2}{2b^2}(M-k+i)}}
\]  

(3.15)

where $a_T$ is the threshold.

Then, the probability of bit error for the adaptive transformation can easily be formulated as

\[
P_b = (1 - P_{\text{Split}})P_{b,S:P} + P_{\text{Split}}P_{b,\text{Split}}
\]  

(3.16)

since the probability of being in the S:P transformation is $P_{S,P} = 1 - P_{\text{Split}}$, in this two-transformation case.

The same analytical results hold for systems using block-wise adaptation as the correlators in the receiver still perform integration over a symbol time. The only difference in the analysis of such systems is that all the symbols in a block undergo the
same fading. In these systems, the length of the block is the duration over which the channel fading is approximately constant.

If $R_s$ is the symbol rate on all subcarriers, the effective data rate of a S:P transformation is $MR$, and that of a split system is $R_s$. So, the gain in data rate over that of a pure split system, defined as the ratio of data rates in adaptive and split transformations can mathematically be given as

$$T = (1 - P_{\text{Split}})M + P_{\text{Split}}$$  \hspace{1cm} (3.17)

where $T$ stands for throughput. For a constant probability of split transformation, it is clear from the equation (3.17) that the throughput increases linearly as a function of number of subcarriers.

In the next two sections, we describe the method used to simulate the proposed system and present the simulation results.

### 3.3 Simulation Description

A MC-DS-SS simulation model was developed for this research. This model was developed primarily to study the MC-DS-SS and evaluate its performance for different transformations used at the transmitter. This simulation also serves as a platform for future study of system characteristics, and as an aid in the design of practical applications. One of the methods used for the performance evaluation of digital systems is estimation of bit error probability. We employ the Monte Carlo method for this estimation. The Monte Carlo method is a numerical method for statistical simulation which utilizes sequences of random numbers to perform the simulation. The simulation
computes an estimate for the bit error probability. The simulation model was developed in MATLAB®.

Throughout this research, we have used BPSK as our modulation scheme because of its simplicity in implementation. The processing gain and number of samples per chip are 10 and $2M$, respectively, where $M$ is the number of subcarriers. The simulations are for $M = 3$ subcarriers unless stated otherwise.

The following is a list of user-selectable parameters for the program

- $N$: The processing gain (number of chips per symbol)
- $M$: Number of subcarriers
- $N_s$: Number of samples per chip
- $E_b/N_0$ range
- Same or different spreading codes

Figure 3.3 shows a schematic description of the simulation. The above parameters are not shown in the figure for clarity.

### 3.3.1 Transmitter Description

The random binary data is generated using the `rand` function in MATLAB. The `rand` function generates random numbers chosen from a uniform distribution on the interval (0,1) and since we are using antipodal signaling, the random numbers between 0 and 0.5 are assigned a value of -1 and others, a value of 1. The data is distributed onto all the subcarriers depending on the transformation at that instant (either S:P or split). The use of a “hybrid” transformation, which can have three different states, is discussed in
chapter 4. The spreading codes of length $NN_b$ are generated in the same way as the data is generated, when $N_b$ is the number of transmitted bits. We have used long spreading codes in this simulation on all the subcarriers. Spreading codes are called “long” if the period of the code is greater than the symbol period and “short” if the period of the code is equal to the symbol period. In other words, if we have the same spreading code on each symbol in a sequence, it is called a short code, and if we have a different code on each symbol then it is called a long code.

\[ T \rightarrow \text{CSI} \]

Oversample By $N$

\[ T^{-1} \]

Combine

\[ P_b \text{ Estimate} \]

\[ \text{Hard Bit Decisions} \]

\[ \text{Maximal Ratio Combiner} \]

\[ \text{Check for T} \]

\[ \text{Accumulators} \]

\[ \text{CSI} \]

\[ \text{Split into M subcarriers} \]

\[ \text{Receiver despreading and sinusoid generators} \]

\[ \text{Transmitter} \]

\[ \text{Rayleigh Fading Generator} \]

\[ \text{Sum over Subcarriers} \]

\[ \text{Sinusoid Generators} \]

\[ \text{Spreading code Generators} \]

\[ \text{Random Binary Data Source} \]

\[ \text{Channel} \]

\[ \text{Compare} \]

\[ \text{Receiver} \]

\[ \text{Figure 3.3. Schematic depiction of MATLAB simulation model} \]
For up-converting the data on each subcarrier, sinusoidal carrier signals are generated. In this MC-DS-SS system, the frequency separation between subcarriers is $R_c$, the chip rate. The sinusoidal signal multiplied by the spreading code is called the composite code signal. This composite code is then multiplied by the data bits for each subcarrier and is then transmitted.

### 3.3.2 Channel Description

The impairments present in the channel are AWGN and Rayleigh fading. The fading in consideration is slow and frequency flat. The fading is slow because it is modeled as constant over at least a symbol time. The AWGN is generated by using the `randn` function in the MATLAB. The function `randn` generates random numbers chosen from a normal (Gaussian) distribution with mean zero and variance one. By keeping energy per bit $E_b$ constant and changing the variance of AWGN, desired values of $E_b/N_0$ are obtained.

The Rayleigh fading is generated by using the fact that the square root of the summation of two squared Gaussian random variables is a Rayleigh random variable, i.e., if $X$ and $Y$ are two Gaussian random variables, then $z = \sqrt{X^2 + Y^2}$, is a Rayleigh random variable, and it is characterized by the single parameter $E(z^2)$, the mean square value of $z$ where $E(.)$ is the expectation operator. Without losing generality, we have used a mean-square value of 1 for the Rayleigh distribution in the simulation. The unit mean-square value of Rayleigh fading keeps the average $E_b/N_0$ ratio constant for a constant value of
transmitted bit energy. The fading vectors generated for different subcarriers are memoryless and independent.

In the system model shown in Figure 2.2, we summed up the subcarriers’ signals before transmission. But, in the simulation, we treat each subcarrier signal separately, as if it were transmitted on a different channel. Our assumptions of independent fading on all the channels, and the perfect carrier and symbol synchronism at the receiver allow us to do this. The assumption of perfect carrier synchronization also eliminates the need to consider the phase distortion of the Rayleigh fading.

### 3.3.3 Receiver Description

In this single-user case, the conventional receiver structure of a bank of correlators is used. The receiver also uses a comparator for making hard bit decisions. We have assumed symbol and carrier synchronism in the receiver, as mentioned in the analysis. We also assumed that there is no multipath propagation and the transmitted signal arrives at the receiver via only a single path.

The signal that enters the receiver is despread and downconverted by the composite code signal. The signal is then integrated over the symbol time, and hard bit decisions are made on the decision variables in the S:P case; in the split case, the decision variables on different subcarriers are combined using a maximal ratio combiner (MRC) and bit decisions are made on the output of the MRC. The bit error rate calculations are performed by comparing these received bit estimates with transmitted bits. This process is different for the two transformations. In the S:P transformation case, the bits of each
transmitted subcarrier are compared with the corresponding received subcarrier bit estimates, and a bit error is recorded, if the transmitted and received bits do not match. The aggregate bit error rate of the system in this case is the average of the subcarrier bit error rates. When split transformation is used, the subcarrier signals are combined using a MRC, and then compared to the transmitted bit stream to calculate the bit error rate.

The code for this simulation was written in MATLAB® and the programs are attached in Appendix A.

### 3.4 Simulation Results

In this section, we present the results of the simulation and discuss how closely they match the analytical results. The communication system described in chapter 2 is simulated using the simulation method described in the previous section. The analytical expressions obtained for the bit error rate are also corroborated with the simulation results. The simulation results for a “hybrid” system, which can have three different states in a $M=3$ subcarrier system, are presented in chapter 4.

First, we have simulated systems that employ only one of the transformations: either the serial-to-parallel or the split. Figure 3.4 shows analytical and simulation results for an $M=3$ subcarrier system with a processing gain of $N=10$. It is clear from the figure that there is an excellent agreement between analytical and simulated results. The split systems achieve 3rd order frequency diversity with an average received SNR of $E_b/(3N_0)$. If the same symbol rate is maintained on the subcarriers in both the systems, serial-to-parallel will have a data rate of $3R_s$ as compared to $R_s$ in the split case. The basic S:P and split systems impose bounds on the performance of our adaptive system.
The advantage of the adaptive transformation is evident in Figure 3.5, in which the performance of an adaptive system with $M=3$ subcarriers, for a 2/3 majority-vote strategy and a threshold of $a_T=-3\,\text{dB}$ is plotted. The analytical and simulated performances of the adaptive system are in close agreement. This system has a performance advantage of 3 dB over a S:P system and a data rate gain of 2.31 over the split system. If instead we interpret in terms of losses suffered, for the same threshold of -3 dB, the adaptive system has a 4 dB loss in performance when compared to the performance of the pure split transformation at a bit error rate of 0.01, and data rate loss...
of 0.23 with respect to the pure S:P system, where the loss in data rate is defined as the
ratio of the difference between the data rates of the adaptive and S:P transformations to
the data rate of S:P. The gains in the data rate and performance obtained by adaptive
transformation are substantial, but most significantly make the system operating point
(BER, R_b) user selectable.

The adaptive system has two transmitted power levels, one corresponding to each
transformation. If E_s is the energy per symbol of incoming serial data, the sum of all
subcarrier energies transmitted during one symbol duration in split mode is E_s and that in
S:P mode is ME_s. So, the transmitted power is not constant in the systems using the
adaptive transformation.
Figure 3.5. Performance of an adaptive system as compared to the performances of S:P and split transformations with $M=3$ subcarriers, 2/3 majority-vote strategy and processing gain $N=10$

Figure 3.6 shows the performance of the above adaptive system for various thresholds. The numbers in the figure indicate the gain in data rate over the pure split case. The higher the threshold, the closer is the system performance to full $M^{th}$-order diversity performance, at the expense of lower data rates. Similarly, the lower the threshold, the higher the gain in data rate at higher bit error rates. So, by varying a single parameter threshold, the system user has the choice of selecting an operating point according to requirements.
In Figure 3.6, we have plotted gains in data rates for different values of threshold. It is evident from the figure that as the value of threshold increases, the data rate gains decrease, because the split transformation is used a larger fraction of the time. For adaptive systems with very low thresholds, we employ S:P for most of the time, and hence the data rate gain is $M$ (here, 3) and as the threshold increases, the data rate gains approach a value of 1, equivalent to the data rate obtainable with split transformation (the lowest throughput).
Figure 3.7. Data rate gain vs. threshold for a system using adaptive transformation with $M=3$ subcarriers, 2/3 majority-vote strategy and processing gain $N=10$

We have also simulated adaptive systems having a larger number of subcarriers. The results obtained are similar to those obtained using $M=3$ subcarriers. In Figure 3.8, the results obtained for an adaptive system using $M=4$ subcarriers and a 2-out-of-4 majority-vote strategy are plotted. The numbers in the figure indicate the gain in data rate of this adaptive system over the pure split case.
Figure 3.8. Performance of an adaptive system with \( M=4 \) subcarriers, 2/4 majority-vote strategy and processing gain \( N=10 \) for different thresholds.

In Figure 3.9, we compare the performance of the adaptive system with \( M=4 \) subcarriers for 2-out-of-4 and 3-out-of-4 majority-vote decisions. Since the threshold is same for both the cases, any three channels simultaneously fading below threshold is less probable than any two channels fading below the threshold. So, the probability of split transformation in 3-out-of-4 strategy is less than that in 2-out-of-4 strategy. Hence, the 2-out-of-4 strategy fares better than the 3-out-of-4, while the latter has throughput advantage.
Figure 3.9. Performance Comparison of an adaptive system with $M=4$ subcarriers for 2/4 and 3/4 majority-vote strategies with processing gain $N=10$ and for the same threshold of -3 dB

The results for an adaptive system with $M=5$ subcarriers and a 3-out-of-5 majority-vote strategy are plotted in Figure 3.10. As in the previous figures, the numbers in the plot indicate the gain in data rate of this adaptive system over pure split case. The results follow the similar trend as in the $M=3$ and $M=4$ cases. We note here that as $M$ increases, higher order frequency diversity is obtainable with an average per subcarrier energy of $E_b/MN_0$, yielding a “larger region” between S:P and split.
Figure 3.10. Performance of an adaptive system with $M=5$ subcarriers, 3/5 majority-vote strategy and processing gain $N=10$ for different thresholds.

The performance of an adaptive system with $M=5$ subcarriers for two different majority-vote schemes is plotted in Figure 3.11. The results follow the similar trend as in the $M=4$ subcarrier case, with the 3-out-of-5 strategy performing better than 4-out-of-5 strategy. Again, the latter has an advantage in terms of throughput.
Figure 3.11. Performance Comparison of an adaptive system with $M=5$ subcarriers for 3/5 and 4/5 majority-vote strategies with processing gain $N=10$ and for the same threshold of -3 dB

In the simulation results thus far presented, the adaptation in the transformation at the transmitter is instantaneous, i.e., on a bit-by-bit basis. A more practical scenario of block-wise adaptation has also been simulated, with similar results. In such systems, the length of the block is equivalent to the duration over which the channel fading is approximately constant. The advantages of block-wise adaptation are that it reduces the rate at which we switch transformations and it reduces the rate at which the channel state information has to be fed back to the transmitter. Figure 3.12 shows the performance of
an $M=3$ subcarrier system adapting on a block-by-block basis. Similar results are obtained for systems with higher number of subcarriers using block-wise adaptation.

![Graph](image)

**Figure 3.12.** Performance of an adaptive system with $M=3$ subcarriers, 2/3 majority-vote strategy, using block-wise adaptation and with a processing gain $N=10$ for different thresholds

The knowledge of CSI is used for switching between the transformations. All the results presented so far assumed that the transmitter and receiver have perfect knowledge of CSI, which is not very realistic. A more practical situation, wherein noisy channel estimates are used at both the transmitter and receiver, is simulated and the results are presented in Figure 3.13. The adaptive system simulated in Figure 3.13 is an $M=3$
subcarrier system at a threshold of -3 dB. The noise variance is formulated as being proportional to 1/SNR, and B is the proportionality constant. The performance is plotted for three different values of B and as expected, the performance worsens as the noise (or B) increases.

![Graph showing performance of an adaptive transformation of M=3 subcarriers with noisy channel estimates using processing gain of N = 10 and threshold of -3 dB](image)

**Figure 3.13.** Performance of an adaptive transformation of M=3 subcarriers with noisy channel estimates using processing gain of N = 10 and threshold of -3 dB
Chapter 4

*M*-State Adaptive MC-DS-SS Systems

In this chapter, we propose another variety of adaptive transformation, called *M*-state adaptive, or simply, hybrid systems, and discuss their performance and advantages over the 2-state adaptive system discussed in chapter 3. The proposed system is analyzed and the results of analysis and simulation are also presented in this chapter.

4.1 Hybrid Transformation

In the adaptive systems discussed in chapter 3, we have only two different states in the transformation: either the serial-to-parallel conversion of the incoming serial data or replicating each bit onto all subcarriers. So, at any instant of time, all the subcarriers either get a different bit or the same bit, depending on the number of channels fading below the pre-selected threshold. This arrangement does not make the best use of channel conditions as the channels with gains greater than threshold are also going to get a replicated bit, if the majority of channels are below the threshold. This led us to design another novel transformation scheme at the transmitter.

In this hybrid scheme, only some of the subcarriers can transmit the same bit while the remaining subcarriers can transmit different bits. In other words, at any instant, only some of the subcarriers can employ the split transformation while the remaining are using a S:P conversion. This hybrid scheme also employs S:P or split across all the subcarriers if all the channels’ gains are greater or less than the threshold, respectively. A hybrid system of *M* subcarriers can have a total of *M* different states, namely, S:P across
all $M$ subcarriers, and $M-1$ states starting from splitting over all $M$, to splitting over 2 subcarriers. The subcarriers to be used for splitting are the ones whose channel gains are less than the threshold; the subcarriers with gains greater than the threshold employ the S:P transformation. As a special case, if there is only one channel that is fading below the threshold, we would still split across the two channels with lowest gains, thereby “sacrificing” the performance of one of the channels with greater gain. As with the 2-state adaptive systems in chapter 3, we assume perfect channel state information at the transmitter and no delay is accounted for in feeding the channel information back to the transmitter. We also enforce the constant symbol and chip rates condition on all subcarriers for the hybrid systems.

We consider an $M=3$ subcarrier hybrid system and explain how we switch between the transformations to illustrate the concept clearly. This hybrid system has three different states, namely, splitting over all three channels, splitting over two channels with a different data symbol on third subcarrier, and using serial-to-parallel conversion across all subcarriers. We employ S:P or split across all the three subcarriers if all the channels’ gains are greater or less than the threshold, respectively. If two channels are fading below the threshold and the gain of the third channel is above the threshold, we send the same bit on the two channels that are fading below the threshold and a different second bit on the third subcarrier. The advantage of a hybrid system is that it increases the throughput when compared to the 2-state adaptive system discussed in chapter 3, wherein we would send the same bit on all the three channels, if two or more channels are below the threshold. In terms of performance, this novel system achieves a second order frequency diversity during the hybrid state and third order diversity during the split state,
whereas the adaptive system in chapter 3 can only obtain third order frequency diversity. Therefore, the hybrid system performs better than a 2-state adaptive under similar conditions. Table 4.1 illustrates how we vary the transformation for an $M=3$ subcarrier case, depending on the channel conditions.

The system models for hybrid systems are similar to the ones described for 2-state adaptive systems in chapter 2, and are essentially generalizations of the 2-state systems. We employ maximal ratio combining in the receiver when the split transformation is used to obtain frequency diversity.

### Table 4.1. Different transformations for the hybrid case; $a_T$ is the threshold.

<table>
<thead>
<tr>
<th>Channel 1</th>
<th>Channel 2</th>
<th>Channel 3</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&gt; a_T$</td>
<td>$&gt; a_T$</td>
<td>$&gt; a_T$</td>
<td>S:P on all 3 channels</td>
</tr>
<tr>
<td>$&gt; a_T$</td>
<td>$&gt; a_T$</td>
<td>$&lt; a_T$</td>
<td>Channel 1 – S:P Channels 2,3 – Split</td>
</tr>
<tr>
<td>$&gt; a_T$</td>
<td>$&lt; a_T$</td>
<td>$&gt; a_T$</td>
<td>Channel 1 – S:P Channels 2,3 – Split</td>
</tr>
<tr>
<td>$&gt; a_T$</td>
<td>$&lt; a_T$</td>
<td>$&lt; a_T$</td>
<td>Channel 1 – S:P Channels 2,3 – Split</td>
</tr>
<tr>
<td>$&lt; a_T$</td>
<td>$&gt; a_T$</td>
<td>$&gt; a_T$</td>
<td>Channels 3 – S:P Channels 1,2 – Split</td>
</tr>
<tr>
<td>$&lt; a_T$</td>
<td>$&gt; a_T$</td>
<td>$&lt; a_T$</td>
<td>Channel 2 – S:P Channels 1,3 – Split</td>
</tr>
<tr>
<td>$&lt; a_T$</td>
<td>$&lt; a_T$</td>
<td>$&gt; a_T$</td>
<td>Channel 3 – S:P Channels 1,2 – Split</td>
</tr>
<tr>
<td>$&lt; a_T$</td>
<td>$&lt; a_T$</td>
<td>$&lt; a_T$</td>
<td>Split on all 3 channels</td>
</tr>
</tbody>
</table>
4.2 State Probabilities

As in chapter 3, we use order statistics to analyze the hybrid transformation. First, we find the probability that we employ each transformation, and then the bit error rate expressions when the system is in those states are derived. We follow the same approach as used in chapter 3 to find the expressions for bit error ratio.

As mentioned earlier, in this hybrid variety of adaptive systems, we employ serial-to-parallel only when all the channels’ gains are greater than the threshold. At any instant of time \( t \), if \([a_{1t} \ a_{2t} \ \ldots \ a_{Mt}]\) are the channel amplitudes for the \( M \) subcarriers, let \([y_{1t} \ y_{2t} \ \ldots \ y_{Mt}]\) be the ordered channel amplitudes such that \( y_{1t} < y_{2t} < \ldots < y_{Mt} \). In terms of order statistics, the probability of being in the S:P state in the \( M \)-state adaptive system is the probability that the minimum order statistic \( y_1 \) is greater than the threshold. This probability is mathematically expressed as

\[
P_{S:P} = 1 - p(y_1 < a_T)
\]  
(4.1)

where \( p(y_k < a_T) \), for any \( k \) is given by (3.15) and \( a_T \) is the threshold.

We employ split across all the \( M \) subcarriers only when all the \( M \) channels are fading below the threshold. So, the probability of employing split transformation is the same as the probability that maximum order statistic \( y_M \) is less than the threshold, and is given by

\[
P_{\text{split}} = p(y_M < a_T)
\]  
(4.2)

In general, for the hybrid states of an \( M \)-state adaptive transformation, the probability that we split across \( k \)-out-of-\( M \) subcarriers while using S:P on the remaining
subcarriers is equal to the probability that exactly \( k \) channels will fade below threshold.

Using order statistics, the probability is calculated as

\[
P_{\text{hybrid},k} = p(y_k < a_r) - p(y_{k+1} < a_r)
\]  

(4.3)

As mentioned, we split across two subcarriers when either one or two subcarriers are fading below the threshold. Hence this is a special case and its probability is given by

\[
P_{\text{hybrid},2} = p(y_1 < a_r) - p(y_2 < a_r)
\]  

(4.4)

In the next section, we derive the bit error rate expressions using the state probabilities derived in this section.

**4.3 Bit Error Rate Computations**

We follow the same approach used in chapter 3 for computing the bit error ratio expressions. First, we derive the probability of bit error for a time-invariant channel using decision statistics and then, this expression is averaged over the probability density function (PDF) of the received SNR in the appropriate ranges to obtain the BER expression for our time varying channel.

**4.3.1 S:P State**

The analysis up to the point where we obtain the BER expression for a constant gain channel is similar to the analysis done in chapter 3, and hence is not repeated here. From (3.4), the probability of bit error for a constant gain channel is

\[
P_b(\gamma_b) = Q\left(\sqrt{2\gamma_b}\right)
\]  

(4.5)
where $\gamma_b = \alpha^2 E_s / N_0$ is the instantaneous received SNR and $Q(\cdot)$ is the complementary error function, defined in the previous chapter.

The BER expression in the S:P state is obtained by averaging the above expression for a constant gain channel over the PDF of $\gamma_b$. From chapter 3, the PDF of $\gamma_b$ is given by

$$p(\gamma_b) = \frac{1}{\gamma_b} e^{-\gamma_b / \bar{\gamma}_b} \gamma_b \geq 0$$

(4.6)

In order to make sure that the averaging process is over the appropriate regions of PDF, the channel conditions under which we employ the S:P transformation have to be taken into account for averaging. Since we employ S:P only when all the channels’ gains are greater than the threshold, we have to average the expression (4.5) only when the received SNR $\gamma_b$ is greater than the threshold SNR $\gamma_T$, defined as $\gamma_T = a_T^2 E_s / N_0$, with $a_T$ the threshold. The BER expression on a single subcarrier can now be written as

$$P_{b,S:P} = \int_{\gamma_T}^{\infty} P_2(\gamma_b) p(\gamma_b) d\gamma_b$$

(4.7)

By substituting (4.5) and (4.6) and carrying out integration, we obtain a closed form solution given by

$$P_{b,S:P} = Q(\sqrt{2 \gamma_T}) e^{\gamma_T / \bar{\gamma}_b} - \bar{\gamma}_b Q \left( \sqrt{\frac{2 \gamma_T}{\bar{\gamma}_b} \left( \frac{\bar{\gamma}_b}{\gamma_b} + 1 \right)} \right)$$

(4.8)

where $\bar{\gamma}_b$ is the average signal-to-noise ratio, defined as $\bar{\gamma}_b = E(\alpha^2) E_s / N_0$, identical for all subcarriers.
The above expression gives the BER of each subcarrier. The expression for the aggregate BER for the S:P state is the average of the above expressions for all $M$ subcarriers, which results in the same expression again.

### 4.3.2 Split State

As with the S:P state, the analysis is similar to the point where we obtain the BER expression for a fixed set of channel gains. The probability of bit error expression in (3.9) for fixed channel gains is reproduced here

$$P_z(\gamma_b) = Q(\sqrt{2\gamma_b})$$

(4.9)

where $\gamma_b = \frac{E_{ul}}{N_0} \sum_{i=1}^{M} \alpha_i^2$ is the SNR per received bit in this case.

As computed in chapter 3, the PDF of $\gamma_b$ is a chi-square density with $2M$ degrees of freedom, given by

$$p(\gamma_b) = \frac{1}{(M-1)!\gamma_c} \gamma_b^{M-1} e^{-\gamma_b/\gamma_c}$$

(4.10)

We employ split transformation across all $M$ subcarriers based on the condition that all the channels are fading below the threshold. So, the averaging of (4.9) to obtain the BER expression for time varying channels must consider this condition. Since each channel can have a maximum gain equal to the pre-chosen threshold, the sum of gains can not exceed $M_{tr}$, for a system using $M$ subcarriers. In terms of SNRs, the upper limit of the combined SNR is $M\gamma_T$. Accordingly, the bit error rate expression can be written as

$$P_{b,\text{split}} = \int_{0}^{M\gamma_T} P_z(\gamma_b) p(\gamma_b) d\gamma_b$$

(4.11)
A closed form solution for the above integral has not been found. However, for reasonable values of threshold, the upper limit of the above integration $M\gamma_T$ can be approximated as infinity, in which case, the solution to the above integral is the BER expression with $M^{th}$ order frequency diversity given by

$$P_{b,\text{split}} = \left[ \frac{1}{2} (1 - \mu) \right]^M \sum_{i=0}^{M} \binom{M}{i} \left( \frac{1}{2} (1 + \mu) \right)^i$$

(4.12)

where, $\mu = \gamma_b / (1 + \gamma_b)$ with $\gamma_b = \frac{E_{bi}}{N_0} E(\alpha_i^2) = \frac{E_b}{MN_0}$ is the average SNR per subcarrier, same for all subcarriers, and $\binom{N}{k}$ is the combinatorial coefficient $N!/[k!(N-k)!]$. The average SNR is scaled by a factor $M$, because of the splitting operation at the transmitter.

Based on the results of simulations, we note here that the performance loss incurred by approximating the upper limit of the above integral is negligible.

### 4.3.3 Hybrid State(s)

Initially, we derive the BER expressions for a system consisting of $M=3$ subcarriers because of its simplicity. The three subcarrier system can have only one hybrid state that uses a single data bit or split on the two channels with lowest gains with a different second bit or S:P over the third channel. Hence, we must analyze the simultaneous use of the S:P and split transformations to obtain the BER of the hybrid state.

First we calculate the probability of bit error of the split bit we sent on the two channels with lowest gains. As in the case of the pure split transformation, the two split
symbols are combined using a maximal ratio combiner (MRC) at the receiver. The receiver decision statistic is of the same form as (3.8), but with summing over only two channels. For a fixed set of \( \{ \alpha_i \} \), the probability of bit error \( P_2(\gamma_b) \) is the same as (3.8) and is reproduced here

\[
P_2(\gamma_b) = Q(\sqrt{2\gamma_b})
\] (4.13)

where \( \gamma_b = \frac{E_{si}}{N_0} \sum_{i=1}^{2} \alpha_i^2 \), the SNR per bit, is the summation of the SNRs of the two channels used for splitting. As in earlier cases, the expression (4.13) must be averaged over the PDF of \( \gamma_b \) to obtain the BER expression for the fading channel.

In this \( M = 3 \) channel case, if \([a_1t, a_2t, a_3t] \) are the channel amplitudes at any instant of time \( t \), and if we let \([y_{1t}, y_{2t}, y_{3t}] \) be the ordered channel amplitudes such that \( y_{1t} < y_{2t} < y_{3t} \), then the two channels that will be used for split are \( y_1 \) and \( y_2 \). Using order statistics and transformations of random variables, the densities of the random variables \( \gamma_1 = \frac{E_{si}}{N_0} y_1^2 \) and \( \gamma_2 = \frac{E_{si}}{N_0} y_2^2 \) are calculated as [18], [19]

\[
p(\gamma_1) = \frac{3}{\gamma_b} e^{-\frac{\gamma_1}{\gamma_b}}
\] (4.14)

\[
p(\gamma_2) = 6 \frac{e^{-\frac{\gamma_2}{\gamma_b}}}{\gamma_b} \left( 1 - e^{-\frac{\gamma_2}{\gamma_b}} \right)
\] (4.15)

where \( \gamma_b = \frac{E_{si}}{N_0} E(\alpha_i^2) \) is the average received SNR per subcarrier.

We are interested in finding the PDF of sum of \( \gamma_1 \) and \( \gamma_2 \). As these ordered random variables are not independent, we first find the joint PDF as [18]
The PDF of the summation $\gamma_b = \gamma_1 + \gamma_2$ is calculated using the joint PDF and is found to be

$$p(\gamma_b) = \frac{6}{\gamma_b^2} e^{-\frac{\gamma}{\gamma_b}} \left(1 - e^{-\frac{\gamma}{\gamma_b}}\right) \quad (4.17)$$

The limits of integration have to be defined carefully for the averaging process to obtain the BER expression. As mentioned earlier, we employ this hybrid state when either one or two channels are fading below the threshold. Hence $\gamma_1$, the minimum SNR of all the channels, will be less than the threshold SNR $\gamma_T$ for both the cases. But the SNR of the channel corresponding to second order statistic, i.e., $\gamma_2$ is less than the threshold SNR when two channels are fading below the threshold and is greater than the threshold SNR when there is only one channel that is fading below the threshold. So, the upper limit of the summation $\gamma_b$ is not a constant for these two cases. For simplicity, the upper limit is approximated as $2\gamma_T$ for both the cases in this work.

The equation for probability of bit error can now be formulated as

$$P_{b,H,\text{split}} = \int_{0}^{2\gamma_T} P_2(\gamma_b) p(\gamma_b) d\gamma_b \quad (4.18)$$

By substituting (4.13) and (4.17) and integrating, we obtain a closed form solution as

$$P_{b,H,\text{split}} = \frac{3}{2} + \sum_{i=1}^{2} (-1)^i 3 \left[ \frac{2}{i} Q\left(\sqrt{4\gamma_T}\right) e^{-\frac{2\gamma_T}{\gamma_b}} + \frac{2}{i} \sqrt{\frac{\gamma_b}{i+\gamma_b}} \text{erf}\left(\sqrt{\frac{2\gamma_T}{\gamma_b+i}}\right) \right] \quad (4.19)$$
where \( \text{erf}(.) \) is the error function, defined as \( \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \).

Next, we find the probability of error of the serial-to-parallel converted symbol transmitted on the channel with maximum gain. This channel corresponds to the maximum order statistic \( y_3 \). The BER expression for a time-invariant channel is derived first and is the same as (4.5) with \( \gamma_b = y_3^2 E_s / N_0 \). The BER expression is then integrated over the PDF of \( \gamma_b \). Using order statistics and a transformation of random variables, the PDF of \( \gamma_b \) is calculated as [18],[19]

\[
p(\gamma_b) = \frac{3}{\gamma_b} e^{-\frac{\gamma_b}{2\gamma_b}} \left( 1 + e^{-\frac{2\gamma_b}{2\gamma_b}} - 2e^{-\frac{2\gamma_b}{2\gamma_b}} \right)
\]

(4.20)

Since we employ this hybrid transformation only when one or two channels are fading below the threshold, the gain of the third channel is always greater than the threshold. So, the lower limit of integration is equal to threshold SNR. The probability of bit error is calculated as

\[
P_{b,H:S,P} = \int_{\gamma_T}^{\infty} P_2(\gamma_b) p(\gamma_b) d\gamma_b
\]

\[
= \sum_{i=1}^{3} (-1)^i \binom{3}{i} \left[ \frac{\gamma_b}{i + \gamma_b} Q \left( \sqrt{\frac{2\gamma_b}{\gamma_b} \left( \frac{\gamma_T}{\gamma_b} \right) + i} \right) - Q \left( \sqrt{2\gamma_T} e^{-\frac{2\gamma_b}{2\gamma_b}} \right) \right]
\]

(4.21)

The total probability of bit error when the system is in hybrid state is the average of the bit error rates of the S:P and split symbols.

\[
P_{b,\text{Hybrid}} = \frac{1}{2} \left( P_{b,H:\text{split}} + P_{b,H:S,P} \right)
\]

(4.22)
We have attempted to derive the BER expressions for a general hybrid state, wherein we employ split across any \( k \) subcarriers while using S:P conversion on the remaining \( M-k \) subcarriers. As the ordered random variables we are using are not independent, closed form expressions for the PDFs and probabilities of bit error are not obtainable. Though we have not completed this investigation, numerical integration techniques might be applicable.

We can now calculate the aggregate probability of bit error of a hybrid system with \( M=3 \) subcarriers using the state probabilities and the BERs corresponding to these states as

\[
    P_b = P_{S:P} P_{b,S:P} + P_{\text{split}} P_{b,\text{split}} + P_{\text{Hybrid,2}} P_{b,\text{Hybrid}} \tag{4.23}
\]

These results were derived for instantaneous switching between transformations, though the same results will hold for block-wise adaptation, as in 2-state adaptive systems.

### 4.4 Simulation Results

A model similar to the one described in chapter 3 is developed in MATLAB for simulating a hybrid system. We employ the same Monte Carlo method for evaluating the bit error ratios. The simulations are for an \( M=3 \) subcarrier system, unless otherwise stated. The source code is attached in Appendix A.

Figure 4.1 shows the performance of a hybrid system and the performance of pure S:P and pure split transformations. A threshold of -3 dB and a processing gain of 10 were used for this simulation. The advantage of hybrid transformation over conventional
transformations is evident from this figure. The hybrid system has a performance advantage of approximately 4 dB over the pure S:P performance. It outperforms the S:P system because it achieves third order diversity while in split state, and second order diversity while in hybrid state. The hybrid system also has a data rate gain of split 2.16 over a pure split system, where the data rate gain is defined as the ratio of throughput of hybrid system to that of a split system. The abovementioned gains in performance and data rate are substantial. From the figure, it can be seen that the analytical and simulated results are in close agreement.

![Graph showing performance comparison](image)

**Figure 4.1.** Performance of a hybrid transformation with $M=3$ subcarriers and processing gain $N=10$, compared with the performances of S:P and split transformations
Figure 4.2 shows the performance of the same hybrid system for three different thresholds. The numbers in the figure indicate the gain in data rate over the pure split case. It can be noticed from the figure that the higher the threshold, the closer is the system performance to full $M^{th}$-order diversity performance, at the expense of lower data rates. Similarly, the lower the threshold, the higher the gain in data rate at higher bit error rates. So, by varying a single threshold parameter, the system user has the choice of selecting an operating point according to requirements.

Figure 4.2. Performance of a hybrid system with $M=3$ subcarriers, and processing gain $N=10$ for different thresholds.
In Figure 4.3, the performances of a 2-state and 3-state hybrid system for the same threshold are plotted along with the performance of pure S:P and split transformations. Clearly, the hybrid system performs better than a 2-state adaptive transformation, though the latter has an advantage in terms of data rate. The hybrid system has performance advantage of approximately 1.5 dB over 2-state adaptive and the data rate gains of hybrid and 2-state are 2.16 and 2.31, respectively. For these comparable gains in data rate, hybrid system is more advantageous than a system using 2-state adaptive transformation.

![Figure 4.3](image_url)

**Figure 4.3.** Comparing the performances of M-state and 2-state systems with $M=3$ subcarriers, and processing gain $N=10$ for a threshold of -3 dB.
A hybrid system of $M=3$ subcarriers can have three different transmit power levels, one corresponding to each state, as shown in Figure 4.4. The transmit power levels of a 2-state adaptive system, which can have only two different power levels, are also plotted in the same figure. The power levels plotted were obtained with the same threshold of -3 dB for both the systems. For the hybrid system, if $E_s$ is the energy of each symbol entering the transformation, then the sum of all subcarrier energies during one symbol duration in S.P state is $3E_s$, in split state is $E_s$ and in hybrid state is $2E_s$. For better visualization, the plot shows the power levels for a brief period of 20 symbol durations.
Figure 4.4. Comparing the transmit power levels of hybrid and 2-state adaptive systems for 20 symbol durations with $M=3$ subcarriers, and processing gain $N=10$, using the same threshold of -3 dB.

Figure 4.5 clearly illustrates the power advantage of the hybrid systems. From the figure, it can be seen that the hybrid 3-state system requires less transmitter power than a 2-state system at lower thresholds, corresponding to left part of the plot. It is found that the hybrid 3-state transformation performs better than a 2-state system in terms of BER at these lower thresholds, for comparable gains in data rate. At higher thresholds, a hybrid transformation requires more power than a 2-state adaptive system. The performance in this case is comparable with higher gains in data rate for the hybrid. Since the energy per
symbol of the transmitted bit sequence is constant regardless of the transformation used, the advantage in data rates is directly translated into a power advantage. The operating points \( (P_b, R_b) \) of both the systems at arbitrary thresholds of -16 dB, -8 dB and 2 dB are indicated in the figure to highlight the selectability feature of our adaptive systems. These points were at an arbitrary received SNR value of 5 dB. The numbers on top correspond to 2-state adaptive system.

**Figure 4.5.** Comparing the average transmit power levels of \( M \)-state and 2-state adaptive systems with \( M=3 \) subcarriers, and processing gain \( N=10 \), for various thresholds
Chapter 5

Summary, Conclusions and Future Work

In this chapter, we summarize the work done during the course of this thesis before presenting conclusions drawn from this research. We conclude the chapter by listing some suggestions for future work.

5.1 Summary

In the first part of the thesis, we proposed a 2-state adaptive transformation to be used in the transmitter of a MC-DS-SS system to enable the user to achieve selectable operating points, where the operating point is defined as the Bit Error Ratio (BER), throughput pair. The proposed adaptive systems also aim to achieve higher data rates at relatively lower BERs than conventional systems. As the demand for high data rates is ever growing in cellular communications, our work may be of use toward this goal. Initially, we studied the conventional S:P and split transformations and their advantages. By adapting the transformation i.e., switching between transformations based on certain channel conditions, we tried to obtain the advantages of both the conventional transformations. The switching between transformations is based on channel state information (CSI), a pre-chosen threshold and a majority-vote switching strategy.

We analyzed the adaptive systems using order statistics. The expressions for BER were derived using the probabilities calculated by using order statistics. Initially, the adaptation at the transmitter is instantaneous or on a per-symbol basis, but we also
studied the performance of a system that adapts in a block-wise sense, where the length of the block is the duration over which the channel gain is approximately constant.

In the next part of the thesis, we proposed another novel adaptive transformation, called $M$-state adaptive or “hybrid” transformation. The objective of this system is to achieve still higher data rates than the 2-state case, at comparable performance, while still allowing the user to choose the operating point. The 2-state adaptive systems employ either S:P or split across all the subcarriers, at any particular instant. In contrast to this, the $M$-state adaptive systems can employ S:P across only some of the subcarriers, while the remaining subcarriers are used for split. Thus, the $M$-state hybrid system can attain any diversity order from 1 to $M$ during some part (bit or block) of transmission, whereas the 2-state system can attain only diversity order 1 or diversity order $M$ at any given part of transmission.

We developed simulation models for both 2-state and $M$-state adaptive systems in MATLAB to calculate the BERs for a range of SNR values. Both instantaneous and block-wise cases were simulated. The simulations are primarily for simple $M=3$ subcarrier systems, though we have also simulated 2-state adaptive systems of $M=4$ and $M=5$ subcarriers, with similar results. We found that there is an excellent agreement between analytical and simulation results.

### 5.2 Conclusions

The primary finding of our work is that both the 2-state adaptive and $M$-state adaptive or hybrid transformations perform better than the S:P in terms of BER, and
better than the split in terms of throughput over a fading channel, as expected. For example, in a 3 subcarrier and 2/3 majority-vote strategy, for a threshold of -3 dB, it is shown that the 2-state adaptive transformation has approximately a 3 dB performance advantage over S:P and a gain in data rate of 2.33 over split, where the gain is defined as the ratio of throughput obtained by using the 2-state adaptive transformation to that obtained using only split. For the above conditions, the hybrid system has performance advantage and data rate gains of 4 dB and 2.16, respectively. These gains in throughput and performance are significant and hence our system is useful when high data rates, and flexibility, are desired. The results are similar when the fading is block-wise and for a larger number of subcarriers.

In both the proposed systems, by varying a single threshold parameter, the user has the choice of selecting the operating point \((\text{BER}, R_b)\), according to the specific needs of system. The data rate gains vary from \(M\) to 1, corresponding to pure S:P and pure split, respectively. In terms of BER performance, the user can obtain any performance between the performance of pure S:P and pure split. So, the selection of threshold is critical in our adaptive systems and it directly translates into a performance/throughput point.

Between the two proposed adaptive systems, for equal gains in data rate, the hybrid system has lower bit error rate than a 2-state adaptive system. Similarly, for identical performance, the hybrid system has higher gains in data rate than the 2-state adaptive system. It was also found that the hybrid transformation is advantageous in terms of the amount of transmitted power required. At lower thresholds, even with lower average transmitter power, a hybrid transformation performs better than a 2-state adaptive
system in terms of BER, for comparable gains in data rate. At higher thresholds, a hybrid transformation requires more power than a 2-state system. The performance in this case is comparable, with higher gains in data rate for hybrid.

During the course of this thesis, we realized that our adaptive transformations work with any digital communications system employing multicarrier modulation and are not necessarily restricted to multicarrier direct-sequence spread spectrum systems (MC-DS-SS). Hence, our results are directly applicable to the non-spread spectrum case.

5.3 Suggestions for Future Work

In this work, we assumed that the fading on each channel is independent. In practical situations, however, complete statistical independence of the fading on different channels can rarely be achieved due to the nature of the propagation environment. So, one of the suggestions for future work is to use more practical correlated fading on subcarriers. Mathematically, the fading can be correlated in both the time and frequency domains.

For the $M$-state adaptive transformation, the hybrid state bit error rate expressions were derived for an $M=3$ subcarrier system. Another area of future work is to determine the probability density function of the received SNR and bit error rate expressions for any hybrid state that uses split across any $k$ channels with S:P over the remaining subcarriers. Thus, the derivation of performance expressions for larger values of $M$ could be done.

For switching between transformations, we used perfect estimates of channel state information (CSI) and assumed no delay was incurred in transmitting the CSI to the
transmitter and receiver. Another area of future research is to use noisy channel estimates at the transmitter and receiver and investigate the performance of adaptive systems. We have simulated an adaptive system with noisy channel estimates but the performance of such a system was not analyzed by BER calculations. Yet another suggestion is to incorporate feedback delay in CSI at transceivers.

All DS-SS systems have the inherent advantages of suppressing intentional jamming and interference. Since the focus of this work is on MC-DS-SS systems, it would be interesting to investigate the performance of these adaptive systems in the presence of jamming and interference. Both narrowband and partial band jamming are of interest.

We employed coherent detection with perfect carrier synchronization. So, one area of future research is to assess the performance of adaptive systems in the presence of phase noise. Another topic of future research would be to use different fading channel statistics, for example the generalized Nakagami-m fading channels.
References


Appendix: Matlab Source codes

%********************************************************************
%This program simulates a 2-state adaptive MC-DS-SS system
%********************************************************************

clear all;
tic;

M = 3;  % Set the number of subcarriers
Eb_No=[0:1:16];  % Set range of Eb/No
LE=length(Eb_No);

chip_samp = 2*M;  % Set the number of samples to be transmitted per chip

Ne=100;       % Number of errors to count
EPB=10.^(Eb_No/10);
offset = 0;

%Calculate the analytical probability of bit error of S:P transformation
for i=1:1:LE
    pea(i)=(1-sqrt(EPB(i)/(1+EPB(i))))/2;
end

%Calculate the analytical probability of bit error of split transformation
eb_sc = EPB./M; % SNR per subcarrier
mu = sqrt(eb_sc./(1 + eb_sc));
for i = 1:1:LE
    pb_div(i) = ((1 - mu(i)).^3) * (1/8 + (3/16 * (1 + mu(i))) + (3/ 16 * (1 + mu(i)).^2));
end

P = 10; % Set the processing gain

E_desired = 2*chip_samp;  % desired energy of mod and demod signals
E_total = M*E_desired; % total energy of transmitted signal

k = 1; % A Counter

%Adaptive transformation
threshold = [-3];  % Choose a threshold
len = length(threshold);
dc = 1;  % Set another two counters
datac = 1;
chip_Eb_No = Eb_No-10*log10(E_desired*P);
No= 1./(10.^((chip_Eb_No)/10));  % Noise density for different values of Eb_No

%***** Probability calculation from ORDER STATISTICS *****
j = 2;  % jth order statistic
syms x;
% Energy of the rayleigh fading vector is normalized to 1.
dens_fn = 2*x*exp(-x^2);  % Density function of Rayleigh random variable with a mean-
% square value of 1
dist_fn = 1-exp(-x^2);  % Distribution function of Rayleigh random variable with a
% mean-square value of 1
% Calculate the PDF of jth order statistic
fjos = (factorial(M)/(factorial(j-1)*factorial(M-j)))*dist_fn^(j-1)*(1-dist_fn)^(M-j)*dens_fn;

% Initialize vectors for Pbs, data rates and powers
pes=zeros(len,LE);
apes=zeros(1,LE);
rbg = [];
power_ada = [];

for count = 1:1:len  % Calculate the Pbs for given threshold(s)
    thresh = threshold(count)  % Current threshold
temp_power = [];

    for i = 1:LE
        current_Eb_No = i-1

        prepl(count,i) = double(int(fjos,0,sqrt(10^(thresh/10))));  % Probability that the
transformation will be split
        pb_comb(count,i) = (1 - prepl(count,i))*pea(i) + prepl(count,i) * pb_div(i);  % Calculating the combined probability of error

        Nb(i) = ceil(Ne./pb_comb(count,i));  % Number of data bits to be transmitted

        if Nb(i) > 50000
            % Upper limit on the number of bits that can be transmitted per
            % subcarrier is 50000. This choice is arbitrary
        end
    end
end
\[ \text{Nb}(i) = 50000; \]
\[ \text{end} \]

\[ \text{Nb} \]

% Generate the fading vectors for all channels
for \( j = 1:M \)
\[ [\text{mag}(j,:) \; \text{pha}(j,:)] = \text{rayleigh1}(\text{Nb}(i),0); \]
\[ \text{end} \]
\[ \text{clear pha}; % Delete pha from memory \]

\[ \text{data} = 2*\text{BIN01}(M*\text{Nb}(i),0.5) - 1; % Generate antipodal data \]
\[ \text{dc} = 1; \]
\[ \text{datac} = 1; \]

% Use split if \( j/M \) are fading below the threshold, use S:P otherwise
for \( i1 = 1:\text{Nb}(i) \)
\[ \text{if } ((10*\log10((\text{mag}(1,i1)^2) < \text{thresh}) \; \&\; (10*\log10((\text{mag}(2,i1)^2) < \text{thresh})) \; \| \; ((10*\log10((\text{mag}(2,i1)^2) < \text{thresh}) \; \&\; (10*\log10((\text{mag}(3,i1)^2) < \text{thresh})) \; \| \; ((10*\log10((\text{mag}(3,i1)^2) < \text{thresh}) \; \&\; (10*\log10((\text{mag}(1,i1)^2) < \text{thresh}))) \]
\[ \text{The transformation is SPLIT} \]
\[ \text{for } i2 = 1:M \]
\[ \text{d}(i2,\text{dc}) = (1/\sqrt{M})*\text{data}(:,:,\text{datac}); % Scale the energy in split} \]
\[ \text{end} \]
\[ \text{dc} = \text{dc} + 1; \]
\[ \text{datac} = \text{datac} + 1; \]
\[ \text{else} \]
\[ \text{The transformation is S:P} \]
\[ k = 0; \]
\[ \text{for } i4 = 1:M \]
\[ \text{d}(i4,\text{dc}) = \text{data}(:,:,\text{datac} + k); \]
\[ k = k + 1; \]
\[ \text{end} \]
\[ \text{dc} = \text{dc} + 1; \]
\[ \text{datac} = \text{datac} + M; \]
\[ \text{end} \]
\[ \text{end} \]

\[ \text{datac} = \text{datac}-1; % Total number of DIFFERENT bits transmitted \]
\[ \text{dc} = \text{dc} - 1; % Number of bits transmitted per subcarrier \]

\[ \text{data} = \text{data}(1:\text{datac}); \]
\[ \text{thrput}(i) = \text{datac}; \]
\[ \text{rbgain}(\text{count},i) = \text{thrput}(i)/\text{Nb}(i); % Calculate the gain in data rate over split} \]
% Calculate the amount of transmitted power
tx_power = zeros(1,Nb(i));
for i3 = 1:M
    tx_power = tx_power + d(i3,:).^2;
end
temp_power = [temp_power mean(tx_power)];

% Transmit data on each subcarrier
for ch=1:M
    long_code= rand(1,P*Nb(i)); % Generate Spreading code
    long_code(find(long_code>=0.5)) = 1; % convert random vector to antipodal elements
    long_code(find(long_code<0.5)) = -1;
    coderep = OverN(long_code,chip_samp); % Oversample the spreading code
dosp=OverN(d(ch,:),P*chip_samp); % Oversample data
    clear long_code; % delete long_code from memory

    sinmod(ch,:) = sqrt(2)*cos(2*pi*(ch/chip_samp)*(0:1:(chip_samp-1))); % Generate sinusoid vector for modulation
    E = sum(sinmod(ch,:).^2); % energy of the signal
    sinmod(ch,:) = sqrt(E/E_desired)*sinmod(ch,:); % scale so E=E_desired
    sinmodulation(ch,:) = reshape(sinmodulation(ch,:),1,Nb(i)*P*chip_samp); % Reshape sinemodulation vector equal to length of chips vector
    clear sinmod; % delete variables from memory
clear sinmodrep;
clear sinmodulate;

txc(ch,:) = dosp.*coderep; % Spreading
    clear dosp;

txc(ch,:) = txc(ch,:).*sinmodulation(ch,:); % Upconversion
    clear sinmodulation;

% Rayleigh Fading
rlf=OverN(mag(ch,:),P*chip_samp);
txc(ch,:) = txc(ch,:).*rlf; % Rayleigh fading causes multiplicative distortion
    clear rlf;

% AWGN
b = rbgain(count,i)/M;
    noise = sqrt(No(i)*.5)*randn(1,Nb(i)*P*chip_samp);
txc(ch,:) = txc(ch,:) + noise; % add AWGN
clear noise;

% Offset modulation to be used at the receiver
offsetsin(ch,:) = sqrt(2)*cos(2*pi*(ch/chip_samp)*(1-offset)*(0:1:(chip_samp-1)));
% Generate sinusoid vector for modulation
E = sum(offsetsin(ch,:).^2); % energy of the signal
offsetsin(ch,:) = sqrt(E_desired/E)*offsetsin(ch,:); % scale so E=E_desired
offsetrep = offsetsin(ch,:);
offsetmodulate = offsetrep*ones(1,Nb(i)*P);
offsetmodulation(ch,:) = reshape(offsetmodulate,1,Nb(i)*P*chip_samp);
clear offsetsin; % delete offsetsin from memory
clear offsetrep; % delete offsetrep from memory
despread = txc(ch,:).*coderep.*offsetmodulation(ch,:); % Multiplication of received bits by reshaped spreading code
clear txc;
dreshape = reshape(despread,P*chip_samp,Nb(i)); % Reshape received bits for detection
clear despread;
dintegrating(ch,:) = sum(dreshape,1); % Integration (accumulation) over symbol
clear offsetmodulation;
clear dreshape;
detection(ch,:) = mag(ch,:).*dintegrating(ch,:); % Use Maximal Ratio Combining
out(ch,:) = detection;
clear detection;
test = d(ch,:) - out(ch,:);
error1 = sum(abs(test))/2; % Calculating the probability of error on each subcarrier
pes(ch,i) = error1/Nb(i);
end

% Arrange the received bits - Use back transformations
rx = zeros(1,datac);
dc = 1;
datac = 1;
for i1 = 1:Nb(i)
if \(((10\times\log_{10}(\text{mag}(1,i1)^2) < \text{thresh}) \&\& (10\times\log_{10}(\text{mag}(2,i1)^2) < \text{thresh})) \\|\n\((10\times\log_{10}(\text{mag}(2,i1)^2) < \text{thresh}) \&\& (10\times\log_{10}(\text{mag}(3,i1)^2) < \text{thresh})) \\|\n\((10\times\log_{10}(\text{mag}(3,i1)^2) < \text{thresh}) \&\& (10\times\log_{10}(\text{mag}(1,i1)^2) < \text{thresh}))\)
%The transformation is SPLIT
for i2 = 1:M
    \(\text{rx(datac)} = \text{rx(datac)} + \text{detection}(i2,dc)\);
end
\(\text{dc} = \text{dc} + 1\);
\(\text{datac} = \text{datac} + 1\);
else
%The transformation is S:P
\(k = 0\);
for i4 = 1:M
    \(\text{rx(datac + k)} = \text{detection}(i4,dc)\);
    \(k = k + 1\);
end
\(\text{dc} = \text{dc} + 1\);
\(\text{datac} = \text{datac} + M\);
end
end

\text{out} = []; \\
\text{rx(find(rx>=0))}=1; \% Make hard bit decision \\
\text{rx(find(rx<0))}=-1;
\text{atest} = \text{data} - \text{rx};
\text{aerror1} = \text{sum(abs(atest))/2} \% Calculating the aggregate probability of error
\text{apes(count,i)} = \text{aerror1/datac};

%Delete Variables from memory
\text{clear noise};
\text{clear data};
\text{clear d};
\text{clear long_code};
\text{clear coderep};
\text{clear txc};
\text{clear tx};
\text{clear rx};
\text{clear dreshape};
\text{clear dintegrating};
\text{clear detection};
\text{clear offsetmodulation};
\text{clear despread};
\text{clear test};
clear error1;
clear adetection;
clear atest;
clear aerror;
clear mag;
clear agg_detection;
end

end

clear Nb

end

figure
semilogy(Eb_No, pea, Eb_No, pb_div, 'md-', Eb_No, pb_comb, 'r*-', Eb_No, apes, 'g>-')
legend('Analytical - Rayleigh Channel', 'Pb -- Diversity', 'Combined Analytical Pb', 'Simulated Pb - Threshold -3 dB');
xlabel('Eb/No(dB)')
ylabel('Probability of Error')
title('Probability of bit error on a Rayleigh fading channel for Adaptive Transformation');
grid

Totaltime = toc/60; % Stop timer.......Time in minutes
sprintf('Total time = %f Minutes', Totaltime)

%*****************************************************************
% Function BIN01.m generates a random binary vector x, with elements in set {0,1}
% Probability of a 1 is an input parameter p0, and length of x is N.
% Syntax y=BIN01(N,p0), where 0<= p0 <=1
%*****************************************************************

function xb = BIN01(N,p0)
xb=(rand(1,N) < p0);
% Function OverN oversamples input vector x by N
% thus, for example, if N=3,
% x=[x(1) x(2) ... x(M)] becomes y=[x(1) x(1) x(1) x(2) x(2) x(2) ... x(M) x(M) x(M)]
% not yet generalized for matrices

function y=OverN(x,N)
Lx=length(x);
y=zeros(1,N*Lx);  % initialize oversampled vector y
for kk=1:Lx  % loop to create y
    for jj=1:N
        y((kk-1)*N+jj)=x(kk);
    end
end

function [r, phi] = rayleigh1(Ns,u)
% [r, phi] = rayleigh1(Ns,u)
%
% A Rayleigh fading simulator based on a "conventional lowpass"
% response
% for the power spectrum (using a 2 pole elliptic filter), a reasonable
% approximation to the "Clarke" spectrum, which assumes isotropic
% scattering
% about the mobile, which has an isotropic antenna. This model more
% accurate
% in suburban/rural areas.
% A FILTER MUST BE SELECTED TO SET THE RELATIVE DOPPLER
% FREQUENCY
%
% INPUTS:
% Ns = # samples of the Rayleigh fading process to produce
% u = parameter to select filter; 0:no filtering, 1:fDT=0.01, 2:fDT=0.1
%
% OUTPUTS:
% r  = row vector containing Ns samples of the Rayleigh fading
% process
% phi= row vector containing Ns samples of the phase
j=sqrt(-1);

% SELECT ONE OF THE FILTERS BELOW
if u == 0; % no filter, for memoryless fading
    aa=[1]; bb=[1];
end
if u == 1 % Filter #1, fdT=0.01 (or, fd/fs=0.01)
    aa=[1 -1.9346 0.9376]; bb=[3.772 -4.7 3.772]*1e-3;
end
if u == 2 % Filter #2, fdT=0.1 (or, fd/fs=0.1)
    aa=[1 -1.295 0.5296]; bb=[5.765 0.1064 5.765]*1e-2;
end

% generate the Gaussian i.i.d. r.v.'s, w/100 "extra" samples for % initialization
Iin = randn(1,Ns+100);
Qin = randn(1,Ns+100);

% pass the input components through the filters
Iout = filter(bb,aa,Iin);
Iout = Iout(101:Ns+100);
Qout = filter(bb,aa,Qin);
Qout = Qout(101:Ns+100);

phi = angle(Iout + j*Qout); % Compute phase vector
r = sqrt(Iout.^2 + Qout.^2); % Compute Rayleigh magnitude vector

% normalize for E[r^2]=1
rms = sqrt( mean( r.*r ) );
r = r/rms;

% This program simulates an HYBRID system of M=3 subcarriers
clear all;
tic;

M = 3;  % Number of subcarriers
Eb_No=[0:1:16];  % Set range of Eb/No
LE=length(Eb_No);

chip_samp = 2*M;  % Number of samples per chip

Ne = 100;  % Number of errors to count
EPB=10.^(Eb_No/10);
offset = 0;  % Offset at the receiver

threshold = [2];  % Choose a threshold
len = length(threshold);
threshold_numeric = sqrt(10.^(threshold/10));

% Calculate the Analytical Probability of bit error for the S:P state in Hybrid transformation
pb_sp_ana = zeros(len,length(EPB));
for count = 1:len
    for i = 1:length(EPB)
        pb_sp_ana(count,i) = 0.5 * erfc(sqrt(EPB(i)*threshold_numeric(count)^2)) - 0.5 * sqrt(EPB(i)/(EPB(i)+1)) * erfc(sqrt(threshold_numeric(count)^2*(EPB(i)+1)));
    end
end

% Probability of bit error for S:P transformation
for i=1:1:LE
    temp(i)=(1-sqrt(EPB(i)/(1+EPB(i))))/2;
end

% Probability for split (3rd order diversity)
eb_sc3 = EPB./M;  % Energy per subcarrier
mu3 = sqrt(eb_sc3./(1 + eb_sc3));
for i = 1:1:LE
    pb_div3(i) = ((1 - mu3(i)).^3) * (1/8 + (3/16 * (1 + mu3(i)))) + (3/16 * (1 + mu3(i)).^2);
end

% Calculate the Pb of the HYBRID state in hybrid transformation
% Calculate the Pb of the S:P symbol when the system is in hybrid state first
pb_sp_h_hyb = zeros(len,length(EPB));
for count = 1:1:len
    for i = 1:length(EPB)
        for k = 1:M
            pb_sp_h_hyb(count,i) = pb_sp_h_hyb(count,i) + (-1)^k * factorial(M)/factorial(k)*factorial(M-k)*(0.5 * sqrt(EPB(i)/(EPB(i)+k)) * erfc(sqrt(threshold_numeric(count)^2*(EPB(i)+k))) - 0.5 * erfc(sqrt(EPB(i)*threshold_numeric(count)^2)) * exp(-k*threshold_numeric(count)^2));
        end
    end
end

%Pb of the Replicated symbols when the system is in hybrid state

pb_repl_h_hyb = zeros(len,length(EPB));
for count = 1:1:len
    for i = 1:length(EPB)
        temp1(i) = 0.5 + 0.5 * erfc(sqrt(2*EPB(i)*threshold_numeric(count)^2)) * (exp(-4*threshold_numeric(count)^2)-2*exp(-2*threshold_numeric(count)^2));
        temp2(i) = sqrt(EPB(i)/(EPB(i)+1)) * erf(sqrt(2*threshold_numeric(count)^2*(EPB(i)+1)));
        temp3(i) = 0.5 * sqrt(EPB(i)/(EPB(i)+2)) * erf(sqrt(2*threshold_numeric(count)^2*(EPB(i)+2)));
        pb_repl_h_hyb(count,i) = 3 * (temp1(i) - temp2(i) + temp3(i));
    end
end

%Take the average of the above two BERs to obtain the aggregate BER of %hybrid state
pb_hybrid_ana = (pb_sp_h_hyb + pb_repl_h_hyb)/2;

P = 10; % Processing gain

E_desired = 2*chip_samp; % desired energy of mod and demod signals
E_total = M*E_desired; % total energy of transmitted signal

chip_Eb_No = Eb_No-10*log10(E_desired*P);
No= 1./(10.^(chip_Eb_No/10)); % Noise density for different values of Eb_No

%Initialize vectors for storing Pb and power values
pb_sim = zeros(1,LE);
power_hyb = [];

for count = 1:1:len % Calculate the Pbs for given threshold(s)
    thresh = threshold(count) % Current threshold
% Analytical State Probabilities for hybrid transformation
thr = sqrt(10^(thresh/10));
lambda = exp(-thr^2);

j = 3; % jth order statistic
syms x;
%Energy of the rayleigh fading vector is normalized to 1.
dens_fn = 2*x*exp(-x^2); % Density function of Rayleigh random variable with a mean-square value of 1
dist_fn = 1-exp(-x^2); % Distribution function of Rayleigh random variable with a mean-square value of 1
% Calculate the PDF of jth order statistic
fjos3 = (factorial(M)/(factorial(j-1)*factorial(M-j))) * dist_fn^(j-1) * (1-dist_fn)^(M-j) * dens_fn;

j = 1; % jth order statistic
fjos1 = (factorial(M)/(factorial(j-1)*factorial(M-j))) * dist_fn^(j-1) * (1-dist_fn)^(M-j) * dens_fn;

p_repl_ana(count) = double(int(fjos3,0,thr)); % Probability that split will be employed (all three are below the threshold)
p_sp_ana(count) = 1 - double(int(fjos1,0,thr)); % Probability that S:P will be employed (all three are above threshold)
p_hybrid_ana(count) = 1 - (p_repl_ana(count) + p_sp_ana(count)); % Use hybrid otherwise

for i = 1:LE
    current_Eb_No = i-1
    % Combined analytical Pb
    pb_comb_ana(count,i) = p_repl_ana(count)*pb_div3(i) + p_sp_ana(count) * pb_sp_ana(count,i) + p_hybrid_ana(count) * pb_hybrid_ana(count,i);

    % Upper limit on the number of bits that can be transmitted per subcarrier is 50000. This choice is arbitrary
    Nb(i) = ceil(Ne/pb_comb_ana(count,i)); % Number of bits to transmit if (Nb(i) > 50000)
    Nb(i) = 50000;
end

% Generate Fading vectors of the 3 channels
for j = 1:M
    [mag(j,:) pha(j,:)] = rayleigh1(Nb(i),0);
end
clear pha; % Delete pha from memory
data = 2*BIN01(M*Nb(i),0.5) - 1; % Generate antipodal data
dc = 1; % Counter for number of bits on a subcarrier
datac = 1; % Counter for total throughput (number of different bits)

for j = 1:M
    mag_db(j,:) = 10*log10(mag(j,:).^2); % Convert magnitudes from absolute values to dBs
end

% Set counters for different states of hybrid transformations
hyb_cnt1 = 0;
hyb_cnt2 = 0;
sp_cnt = 0;
repl_cnt = 0;

% Initialize vectors for storing the transmitted bits in one particular state
hyb2_tx = [];
hyb1Tx = [];
hyb_tx = [];
sp_tx = [];
repl_tx = [];
sp_index_tx = [];

% Devise the transformation using the fading vectors, threshold and majority-vote strategy
for i1 = 1:Nb(i)
    if (mag_db(1,i1) > thresh)
        if (mag_db(2,i1) > thresh)
            if (mag_db(3,i1) > thresh)
                % The transformation is S:P
                T = 'S:P';
                k = 0;
                for i4 = 1:M
                    d(i4,dc) = data(datac + k);
                    sp_tx = [sp_tx data(datac + k)];
                    k = k + 1;
                end
                sp_index_tx = [sp_index_tx i1];
                dc = dc + 1;
                datac = datac + M;
                sp_cnt = sp_cnt + 1;
            else
                end
            else
                end
        else
            end
    else
        end
end
% The transformation is hybrid
T = '2 Greater - 1 - S:P and 2,3 - Repl';
d(1,dc) = data(datac);
d(2,dc) = (1/sqrt(M-1)) * data(datac + 1); % Scale the energy accordingly
d(3,dc) = d(2,dc);
% hyb2_tx = [hyb2_tx data(datac) data(datac+1)];
hyb_tx = [hyb_tx data(datac) data(datac+1)]; % Store the transmitted bits
in a vector

dc = dc + 1; % Increment the counters
datac = datac + (M-1);
hyb_cnt2 = hyb_cnt2 + 1;
end
else
if (mag_db(3,i1) > thresh)
% The transformation is Hybrid
T = '2 Greater - 1 - S:P and 2,3 - Repl';
d(1,dc) = data(datac);
d(2,dc) = (1/sqrt(M-1)) * data(datac + 1);
d(3,dc) = d(2,dc);
% hyb2_tx = [hyb2_tx data(datac) data(datac+1)];
hyb_tx = [hyb_tx data(datac) data(datac+1)];
dc = dc + 1;
datac = datac + (M-1);
hyb_cnt2 = hyb_cnt2 + 1;
else
% The transformation is Hybrid
T = '1 Greater - 1 - S:P and 2,3 - Repl';
d(1,dc) = data(datac);
d(2,dc) = (1/sqrt(M-1)) * data(datac + 1);
d(3,dc) = d(2,dc);
% hyb1_tx = [hyb1_tx data(datac) data(datac+1)];
hyb_tx = [hyb_tx data(datac) data(datac+1)];
dc = dc + 1;
datac = datac + (M-1);
hyb_cnt1 = hyb_cnt1 + 1;
end
end
else
if (mag_db(2,i1) > thresh)
if (mag_db(3,i1) > thresh)
% The transformation is Hybrid
T = '2 Greater - 1,2 - Repl and 3 - S:P';
d(1,dc) = (1/sqrt(M-1)) * data(datac);
d(2,dc) = d(1,dc);
d(3,dc) = data(datac + 1);
%hyb2_tx = [hyb2_tx data(datac) data(datac+1)];
hyb_tx = [hyb_tx data(datac) data(datac+1)];
dc = dc + 1;
datac = datac + (M-1);
hyb_cnt2 = hyb_cnt2 + 1;
else
%The transformation is Hybrid
T = '1 Greater - 1,3 - Repl and 2 - S:P';
d(1,dc) = (1/sqrt(M-1)) * data(datac);
%d(1,dc) = data(datac);
d(3,dc) = d(1,dc);
d(2,dc) = data(datac + 1);
%hyb1_tx = [hyb1_tx data(datac) data(datac+1)];
hyb_tx = [hyb_tx data(datac) data(datac+1)];
dc = dc + 1;
datac = datac + (M-1);
hyb_cnt1 = hyb_cnt1 + 1;
end
else
if (mag_db(3,i1) > thresh)
%The transformation is Hybrid
T = '1 Greater - 1,2 - Repl and 3 - S:P';
d(1,dc) = (1/sqrt(M-1)) * data(datac);
%d(1,dc) = data(datac);
d(2,dc) = d(1,dc);
d(3,dc) = data(datac + 1);
%hyb1_tx = [hyb1_tx data(datac) data(datac+1)];
hyb_tx = [hyb_tx data(datac) data(datac+1)];
dc = dc + 1;
datac = datac + (M-1);
hyb_cnt1 = hyb_cnt1 + 1;
else
%The transformation is Split
T = 'split';
for i2 = 1:M
   d(i2,dc) = (1/sqrt(M))*data(datac);
   %d(i2,dc) = data(datac);
end
repl_tx = [repl_tx data(datac)];
dc = dc + 1;
datac = datac + 1;
repl_cnt = repl_cnt + 1;
end
end
end
end
dc = dc - 1;
datac = datac - 1;
data = data(1:datac); % The portion of data that is transmitted

% State Probabilities from simulation
p_hybrid2_sim(count,i) = hyb_cnt2/Nb(i);
p_hybrid1_sim(count,i) = hyb_cnt1/Nb(i);
p_repl_sim(count,i) = repl_cnt / Nb(i);
p_sp_sim(count,i) = sp_cnt / Nb(i);
p_hybrid_sim(count,i) = p_hybrid2_sim(count,i) + p_hybrid1_sim(count,i);

% Calculate the amount of transmitted power
tx_power = zeros(1,Nb(i));
for i3 = 1:M
    tx_power = tx_power + d(i3,:).^2;
end
rbgain_hyb(count,i) = datac/Nb(i); % Calculate the data rate gain

% Transmit data on each subcarrier
for ch=1:M
    long_code = rand(1,P*Nb(i)); % Generate Spreading code
    long_code(find(long_code >= 0.5)) = 1; % convert random vector to antipodal elements
    long_code(find(long_code < 0.5)) = -1;
coderep = OverN(long_code,chip_samp); % Oversample the spreading code
dosp = OverN(d(ch,:),P*chip_samp); % Oversample the data
clear long_code;
sinmod(ch,:)=sqrt(2)*cos(2*pi*(ch/chip_samp)*(0:1:(chip_samp-1)))); % Generate sinusoid vector for modulation
E = sum(sinmod(ch,:).^2); % energy of the signal
sinmod(ch,:) = sqrt(E_desired/E)*sinmod(ch,:); % scale so E=E_desired
sinmodulate = sinmod(ch,:)*ones(1,Nb(i)*P); % Reshape sinemodulation vector equal to length of chips vector
sinmodulation(ch,:)=reshape(sinmodulate,1,Nb(i)*P*chip_samp);

txc(ch,:)=dosp.*coderep; % Spreading
txc(ch,:)=txc(ch,:).*sinmodulation(ch,:); % Upconversion
clear sinmod; % delete variables from memory
clear sinmodrep;
clear sinmodulate;
clear dosp;
clear sinmodulation;

% Rayleigh Fading is constant over at least a symbol duration
rlf = OverN(mag(ch,:),P*chip_samp); % Fading is modelled as a multiplicative process
txc(ch,:) = txc(ch,:).*rlf;
clear rlf;

%AWGN
noise = sqrt(No(i)*.5)*randn(1,Nb(i)*P*chip_samp); % Generate noise vector
txc(ch,:)=txc(ch,:)+noise; %Add AWGN to the transmitted vector
clear noise;

% Offset modulation to be used at the receiver
offsetsin(ch,:)=sqrt(2)*cos(2*pi*(ch/chip_samp)*(1-offset)*(0:1:(chip_samp-1)));
% Generate sinusoid vector for demodulation
E = sum(offsetsin(ch,:).^2); %energy of the signal
offsetsin(ch,:) = sqrt(E_desired/E)*offsetsin(ch,:); % scale so E=E_desired
offsetmodulate = offsetsin(ch,:)*ones(1,Nb(i)*P); % Reshape the offsetmodulation vector equal to length of chips vector
offsetmodulation(ch,:) = reshape(offsetmodulate,1,Nb(i)*P*chip_samp);

despread = txc(ch,:).*coderep.*offsetmodulation(ch,:); % Despread and downconvert the received signal
dreshape = reshape(despread,P*chip_samp,Nb(i)); % Reshape received bits for detection
dintegrating(ch,:) = sum(dreshape,1); % Integration (accumulation) over symbol
detection(ch,:) = mag(ch,:).*dintegrating(ch,:); % Maximal Ratio Combining

clear offsetsin;
clear offsetmodulation;
clear offsetmodulate;
clear dreshape;
clear despread; end

%Initialize vectors for storing received bits in each state and on
%a whole
rx = zeros(1,datac);
dc = 1;
datac = 1;
hyb2_rx = [];
hyb1_rx = [];
hyb_rx = [];
sp_rx = [];
repl_rx = [];
sp_index_rx = [];
sp_index_rx = [];

% Use an inverse transformation to get an estimate of the transmitted data
for i1 = 1:Nb(i)
    if (mag_db(1,i1) > thresh)
        if (mag_db(2,i1) > thresh)
            if (mag_db(3,i1) > thresh)
                % The transformation is S:P
                T = 'S:P';
                k = 0;
                for i4 = 1:M
                    rx(datac + k) = detection(i4,dc);
                    sp_rx = [sp_rx detection(i4,dc)];
                    k = k + 1;
                end
                sp_index_rx = [sp_index_rx i1];
                dc = dc + 1;
datac = datac + M;
            else
                % The transformation is Hybrid
                T = '2 Greater - 1 - S:P and 2,3 - Repl';
                rx(datac) = detection(1,dc);
                % Combine the two split symbols
                rx(datac+1) = rx(datac+1) + detection(2,dc) + detection(3,dc);
                % hyb2_rx = [hyb2_rx rx(datac) rx(datac+1)];
                hyb_rx = [hyb_rx rx(datac) rx(datac+1)];
                dc = dc + 1;
datac = datac + (M-1);
            end
        else
            if (mag_db(3,i1) > thresh)
                % The transformation is Hybrid
                T = '2 Greater - 1 - S:P and 2,3 - Repl';
                rx(datac) = detection(1,dc);
                % Combine the two split symbols
                rx(datac+1) = rx(datac+1) + detection(2,dc) + detection(3,dc);
                hyb_rx = [hyb_rx rx(datac) rx(datac+1)];
                dc = dc + 1;
datac = datac + (M-1);
            end
        end
    else
        % The transformation is Hybrid
        T = '2 Greater - 1 - S:P and 2,3 - Repl';
        rx(datac) = detection(1,dc);
        % Combine the two split symbols
        rx(datac+1) = rx(datac+1) + detection(2,dc) + detection(3,dc);
        hyb_rx = [hyb_rx rx(datac) rx(datac+1)];
dc = dc + 1;
datac = datac + (M-1);
else
%The transformation is Hybrid
T = '1 Greater - 1 - S:P and 2,3 - Repl';
rx(datac) = detection(1,dc);
rx(datac+1) = rx(datac+1) + detection(2,dc) + detection(3,dc);
%hyb1_rx = [hyb1_rx rx(datac) rx(datac+1)];
hyb_rx = [hyb_rx rx(datac) rx(datac+1)];
dc = dc + 1;
datac = datac + (M-1);
end
end
else
if (mag_db(2,i1) > thresh)
if (mag_db(3,i1) > thresh)
%The transformation is Hybrid
T = '2 Greater - 1,2 - Repl and 3 - S:P';
rx(datac) = rx(datac) + detection(1,dc) + detection(2,dc);
rx(datac + 1) = detection(3,dc);
%hyb2_rx = [hyb2_rx rx(datac) rx(datac+1)];
hyb_rx = [hyb_rx rx(datac) rx(datac+1)];
dc = dc + 1;
datac = datac + (M-1);
else
%The transformation is Hybrid
T = '1 Greater - 1,3 - Repl and 2 - S:P';
rx(datac) = rx(datac) + detection(1,dc) + detection(3,dc);
rx(datac + 1) = detection(2,dc);
%hyb1_rx = [hyb1_rx rx(datac) rx(datac+1)];
hyb_rx = [hyb_rx rx(datac) rx(datac+1)];
dc = dc + 1;
datac = datac + (M-1);
end
else
if (mag_db(3,i1) > thresh)
%The transformation is Hybrid
T = '1 Greater - 1,2 - Repl and 3 - S:P';
rx(datac) = rx(datac) + detection(1,dc) + detection(2,dc);
rx(datac + 1) = detection(3,dc);
%hyb1_rx = [hyb1_rx rx(datac) rx(datac+1)];
hyb_rx = [hyb_rx rx(datac) rx(datac+1)];
dc = dc + 1;
datac = datac + (M-1);
else
%The transformation is Hybrid
T = '1 Greater - 1,3 - Repl and 2 - S:P';
rx(datac) = rx(datac) + detection(1,dc) + detection(3,dc);
rx(datac + 1) = detection(2,dc);
%hyb1_rx = [hyb1_rx rx(datac) rx(datac+1)];
hyb_rx = [hyb_rx rx(datac) rx(datac+1)];
dc = dc + 1;
datac = datac + (M-1);
else

% The transformation is split
T = 'Replication';
% combine all the three symbols
for i2 = 1:M
    rx(datac) = rx(datac) + detection(i2,dc);
end
repl_rx = [repl_rx rx(datac)];
dc = dc + 1;
datac = datac + 1;
end
end
end

rx(find(rx>=0))=1; % Make hard bit decisions
rx(find(rx<0))=-1;

% Calculating the probability of errors

% Aggregate BER
atexit=data - rx;
aerror1=sum(abs(atexit))/2
pb_sim(count,i)=aerror1/datac;

hyb_rx(find(hyb_rx>=0))=1; % Make hard bit decisions
hyb_rx(find(hyb_rx<0))=-1;

sp_rx(find(sp_rx>=0))=1; % Make hard bit decisions
sp_rx(find(sp_rx<0))=-1;

repl_rx(find(repl_rx>=0))=1; % Make hard bit decisions
repl_rx(find(repl_rx<0))=-1;

% BER of hybrid state only
hyb_err(i) = sum(abs(hyb_tx - hyb_rx))/2;
ber_hyb(count,i) = hyb_err(i)/(2*(hyb_cnt1 + hyb_cnt2));

% BER of S:P state only
sp_err(i) = sum(abs(sp_tx - sp_rx))/2;
ber_sp(count,i) = sp_err(i)/(3*sp_cnt);

% BER of split state only
repl_err(i) = sum(abs(repl_tx - repl_rx))/2;
ber_repl(count,i) = repl_err(i)/repl_cnt;
%Delete variables from memory
clear data;
clear d;
clear coderep;
clear txc;
clear rx;
clear dintegrating;
clear detection;
clear test;
clear error1;
clear adetection;
clear atest;
clear aeror;
clear mag_db;
clear mag;
clear hyb2_tx;
clear hyb2_rx;
clear hyb1_tx;
clear hyb1_rx;
clear sp_tx;
clear sp_rx;
clear repl_tx;
clear repl_rx;
end
end

%plotting the calculated BERs

for count = 1:1:len
    figure
    semilogy(Eb_No, temp, Eb_No, pb_div3, 'md-',Eb_No,pb_comb_ana(count,:),r*-'
    ,Eb_No,pb_sim(count,:),g>-')
    legend('Analytical - Rayleigh Channel','Pb -- Diversity - 3rd order','Pb -- Diversity -
    2nd order','Combined Analytical Pb','Simulated Pb');
    xlabel('Eb/No(dB)');
    ylabel('Probability of Error');
    s = sprintf('Pb on a Rayleigh channel using Hybrid transformation; threshold = %2.1f
    dB',threshold(count));
    title(s);
    grid
end

Totaltime=toc/60; % Stop timer......Time in minutes
sprintf('Total time = %f Minutes',Totaltime)