ANALYZING WAREHOUSE – RETAILER INTERACTION USING A MODIFIED ECONOMIC ORDER QUANTITY (EOQ) MODEL

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This thesis entitled

ANALYZING WAREHOUSE – RETAILER INTERACTION USING A MODIFIED
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This thesis analyzes the interaction between the warehouse and the retailer to find the characteristics that affect the time between orders and the order quantities of the retailer. The classic EOQ model was altered to explicitly include the operating costs at the warehouse and the transportation costs and this, accounts for multiple SKUs and multiple unit loads. This modified EOQ model is referred to as the Economic Order Frequency (EOF) model.

Using the design of a typical warehouse, equations for detailed calculation of the individual unit load costs at the warehouse were developed. These equations were then incorporated into the total cost equation of the EOF model. This model was tested by changing values of time between orders using Excel and VBA and the change in optimal order frequencies and minimum total costs were analyzed. ANOVA was used to analyze the effect that different costs had on the order frequency and the minimum total cost.

The holding cost per item was found to have the most significant effect on the order frequency and the minimum total cost. Therefore, if the holding cost at the retailer was too high, he/she would order more frequently causing more work at the warehouse but preventing the retailer from incurring high inventory costs.
The transportation cost also has a significant effect on the order frequency and the total cost. This indicates that the distance of the retailer from the warehouse is an important factor to be considered while determining orders. The quantity ordered also has an effect on the transportation cost, since a little more than the truck capacity will require an additional truck that will increase the minimum total cost. Hence a balance between the transportation costs and the order quantity has to be considered as well.

The labor rate has been found to have no significant effect on the order frequency but has an effect on the minimum total cost when considered with other interactions thus indicating that for a given order frequency as the labor cost increases, the total cost increases (i.e. the retailer will pay more for the same quantity as the warehouse costs increase)

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1 INTRODUCTION

A supply chain comprises the different entities that contribute to transforming a product from raw material to a finished product in the hands of the end customer. An example of a simple supply chain for a single product is shown in Figure 1-1.

![Figure 1-1: The Supply Chain for a Single Product (adapted from [1])]({{image}})

Supply Chain Management (SCM) is planning, organizing and controlling the flow of the product from the suppliers to the customer. A more formal definition can be obtained from the Council of Logistics Management [2], as "the systemic, strategic coordination of the traditional business functions and tactics across these business functions within a particular company and across businesses within the supply chain for the purposes of improving the long-term performance of the individual companies and the supply chain as a whole." Since coordination across businesses and interaction between them is a key aspect of SCM, this research focuses on this aspect of SCM for two entities of the chain: warehouses and retailers.
Traditionally all the operations in a company such as purchasing, manufacturing, distribution and marketing were independent of one another. Each department had its own agenda and objectives and very often these were in conflict with the other departments’ objectives. Because of this there were as many policies for the company as there were sections/departments. Therefore, a methodology was needed to integrate all these functions so as to improve the company performance as a whole. Supply chain management can be used for such integration. Supply chain management has four main decision areas: location, production, inventory and transportation [37]. This research delves into the inventory decision area of supply chain management.

For any manufacturing company, inventory in any form typically translates to cost. This includes the cost of physically storing the products, costs associated with product damage due to long term storage and the loss incurred by the company when the value of the product reduces due to a change in the market trend (i.e. the cost of obsolescence) [11]. Therefore, reducing inventory along the supply chain is a good way to reduce cost.

However, inventories are needed to act as buffers for the variation in demand or the variation in transit time from the supplier or the warehouse/distribution center. Also, inventories exist in all stages of the supply chain: as raw material, as work waiting to be processed and as finished products [30]. Most companies view these inventories as a necessary evil and prefer to have enough inventories to account for variation. To
minimize cost, efforts are being made to arrive at a reasonable trade off between inventory to be held and cost incurred by the company.

1.1 Background

The ideal supply chain would be one which worked entirely on the basis of Just-In-Time (JIT). The suppliers would supply the raw materials just in time for the manufacturer and they in turn would have the products ready to ship to the customer completely eliminating the need for a warehouse anywhere along the supply chain. Unfortunately, this is not possible because of inadequate information lead-time, variation in the production of the products and uncertainty in the demand. Therefore the existing supply chain has to be made as efficient as possible by optimizing the interaction between the entities: improving communication between the suppliers, manufacturers and customers and ensuring that the entities interact efficiently.

This research deals with the latter part of the supply chain: warehouses and retailers. Warehouses or distribution centers are vital to the smooth working of the supply chain. Warehouses are used for storage, either by retailers who store products that they buy and sell or by manufacturers to store raw materials and supplies. Warehouses can also be used by a manufacturer or retailer to expand their business beyond their local markets, by placing the warehouses in different locations. This study deals with distribution warehouses where finished goods are stored in large quantities and
organized and separated into smaller quantities as per the orders received from the customers.

After the warehouse receives orders from different customers, the items in the order have to be picked either manually or with the help of automated pickers. Typically, each picker is assigned a set of storage locations and the orders are distributed among the pickers according to the locations they work in. Depending on the number of items of each SKU to be picked, the items could be retrieved on pallets, in cartons, or as individual items. The items are then sorted into the different orders either manually or using an automated sorting system. The sorted pallets, cartons or items are then packed as per the requirements for each type of item. These packed items are then loaded into vehicles (typically trucks) to be transported to the customer.

1.2 Motivation

A multi-echelon supply chain is one in which the product passes through more than one facility before reaching the customer. Since there are a number of facilities involved, coordination and scheduling will always be a problem in such a supply chain. This research is limited to studying the interaction between a warehouse and the retailers it supplies with respect to the ordering policies involved and the order fulfillment costs for the retailers.
1.2.1 Problem Description

In the situation being studied, there is a single warehouse, which supplies products to retailers as shown in Figure 1-2. The warehouse receives orders from the retailers based on the demand that retailers expect. The warehouse then picks the items, sorts them and packs the orders. These orders are then loaded into the trucks taking into consideration the capacity of the trucks. The costs involved in this scenario are:

- Order handling costs at the warehouse
- Fixed costs for the transportation (e.g., the cost of renting a vehicle)
- Variable costs (e.g., the transportation depending on mileage).
- Holding costs for the retailers

![Diagram](image-url)

Figure 1-2: Depicting the Warehouse-Retailer relationship
This thesis does not include the inventory cost at the warehouse because this is a cost that is incurred either by the warehouse or the retailers that store their products at the warehouse. The retailers do not have to contend with this cost and thus this cost is not included in their ordering cost. This thesis also does not include the vehicle routing between retailers. Each retailer’s order is treated separately and hence delivered to that retailer independent of the other retailers. Therefore this scenario can be treated as one with multiple single warehouse-retailer transactions as shown in Figure 1-2.

1.2.2 Importance to the Warehouse

The warehouse receives the orders and they are sent to the storage areas for fulfillment. Based on the quantities ordered the workers retrieve pallets, cartons, or individual items. Efficient picking schedules have to be followed to optimize the usage of employees and also to keep the order handling cost in the warehouse to a minimum. Therefore it would be advantageous for the warehouse if they receive large orders from the retailers. Large orders would enable the picking of pallets rather than cartons or individual items which would be an advantage for the warehouse because of the efficiency of pallet picking when compared to that of cartons and items.

Though pallets have the highest cost of picking per unit load (since they are the largest unit loads available), they have the lowest cost of picking per item because handling each pallet amounts to handling a large number of individual items in one move thus making the handling of that many individual items efficient and cost effective.
Picking individual items involves the additional task of opening pallets and cartons and thus the cost incurred for this operation is high, which in turn contributes to a higher order processing cost within the warehouse. In addition to this, handling pallets during packing, loading and unloading is also more efficient.

1.2.3 Importance to the Retailer

Retailers like to store as little inventory as possible. This is aimed at lowering their inventory holding costs, which is taken as a multiple of the number of units held in inventory. Thus, they prefer to store only the amount that is needed to satisfy demand for a particular period of time and then reorder when the need arises. Many retailers have begun ordering partial pallets or partial cases in order to satisfy a short time demand at their premises but this has in turn increased the workload at the warehouse because the warehouse has to handle smaller unit loads more frequently.

Thus a trade off between the order handling costs at the warehouse and the inventory costs will have to be considered for optimal order quantities for the entire system.

1.3 Objective

The objective of this thesis is to minimize the total cost involved in storing inventory at the store and fulfilling the orders from a retailer in a One Warehouse–One Retailer
system, determine optimal order frequency and total cost and study the effect of various costs on the order frequency and the total cost for the retailer when the ordering cost is split over different unit loads and includes multiple SKUs.

The model that has been developed in this research, called the Economic Order Frequency (EOF) is based on the Economic Order Quantity (EOQ) model used to determine optimal order sizes. The classic EOQ model considers the demand, inventory holding cost and the order fulfillment cost to minimize the total cost for a single SKU. In the model developed in this research, the simple ordering cost variable is comprised of the order handling cost at the warehouse and the transportation cost from the warehouse to the retailers.

The classic EOQ model also assumes that the order fulfillment cost is directly proportional to the quantity ordered. In the EOF model, however, this direct proportionality will not hold good because the order is composed of different unit loads and thus the order cost per unit is determined by the unit load ordered.
2 LITERATURE REVIEW

There has been a great deal of work done in the areas of logistics, warehousing, and supply chain management. This chapter summarizes the work done relevant to this research, categorizing the papers under different sections. The first section reviews literature about the change in the operating procedure of the manufacturing industry that led to the beginning of supply chain management. The next section reviews the practices and strategies used in warehousing and distribution. The final section discusses different types of warehouse-retailer problems and applications developed for these problems and their results.

2.1 Supply Chain Management

Supply Chain Management (SCM) evolved soon after lean manufacturing and Just-In-Time (JIT) techniques were implemented in the 1970s [1]. This was after the manufacturers realized the impact carrying excess inventory and WIP had on the quality of products and lead-time. Excess inventory along the manufacturing line leads to congestion and consequently affects the quality of the products. Once the quality is affected, the rework rate increases and hence lead-time increases. Carrying smaller inventories required fostering a better relationship with the suppliers so that the manufacturers could expect a better response time from the suppliers. This led to development of supplier partnerships. The manufacturers also realized that close relationships with the customers helped the manufacture of products that conformed to customer’s needs and helped the manufacturers decide on their next product line based
on what the customer wanted. Thus customer partnerships were promoted. These new dimensions in the manufacturing chain led to SCM.

SCM thus originated in the late 1980s and the early 1990s. Since then researchers have studied this management concept extensively. The literature present in the field ranges from the different definitions coined to explain and categorize SCM, to the different principles and algorithms needed to apply it to the manufacturing and distribution industries.

Harland [2] states that SCM is the technique of managing business practices and relationships within and outside the organization including all the suppliers and the customers. Scott and Westbrook [3] define SCM as material management for the products until they reach the end of their life in the supply chain (i.e. until they reach the customer). The definition of New and Payne [4] emphasizes the importance of the transportation and logistics function of SCM.

Tan [6] emphasizes that SCM literature spans different aspects of manufacturing, but as they developed their summary, they detected two distinct perspectives that were more prominent than the others: the purchasing and supply perspective and transportation and logistics perspective. The purchasing and supply perspective refers to integration and standardization of the suppliers for a manufacturing company to make the purchasing function more effective [7]. The transportation and logistics perspective
refers to the area of integration of the transportation providers with the manufacturing company to make their transportation and distribution function more effective [8].

Fredendall and Hill [9] present a comprehensive view of the supply chain and the reasons for it being a focus of research for the last ten years. They also discuss reasons for the change in the operating policy of the manufacturing environment. Lead-time and customer satisfaction gained importance as the traditional policy of “make as much as possible to fulfill any amount of demand to gain profit” took a back seat.

The book also describes different aspects of the supply chain such as management basics, performance measures, purchasing and distribution. Fredendall and Hill [9] explain the logistics cost analysis as shown in Equation 2-1 in detail:

\[
C_l = C_t + C_w + C_o + C_{LQ} + C_i
\]  

(2-1)

where,

\(C_l\) = Total Logistics Cost

\(C_t\) = Transportation Cost

\(C_w\) = Warehousing Cost

\(C_o\) = Order Processing Cost

\(C_{LQ}\) = Lot Quantity Cost

\(C_i\) = Inventory Carrying Cost
The authors point out that most analysts treat the order processing costs as constant and only take into consideration the inventory holding costs and the transportation costs in the process of minimizing the total cost. However, this assumption does not lead to optimal solutions.

Each part of the logistics cost has to be considered during the optimization process and the order processing costs (per unit) depend on the size of the order being filled. This thesis uses this principle in developing an optimization methodology.

2.2 Warehousing and Distribution

Frazelle [10] emphasizes the warehousing and distribution function in his work. Though efforts are being made to minimize the presence of warehouses and eliminate them wherever possible, as long as variations are present during production, warehouses will be needed as buffers to fulfill demand. Because of the pressures of managing the supply chain the perspective of a warehouse has changed a lot since its inception. From being merely a storage place for excess inventory, a warehouse now plays a pivotal role in the efficient distribution of products to the customers. Frazelle gives a detailed explanation about the storage policies, warehousing operations, strategies used to optimize these activities and popular practices followed by warehouses.

Since warehouses and the associated holding costs cannot be completely avoided, companies strive to ensure that they operate efficiently and fulfill customer orders on
time and accurately. Warehousing is an expensive operation to perform taking up about 2% to 5% of the sales of a corporation. Increasing the effectiveness of the warehouse planning and operations, improving the design of the warehouse to increase operating efficiency and enhancing the internal and external logistics functions are some of the issues that are often researched in this area.

Berry [11] and Bassan, et al. [12], both analyze the layout of a conventional warehouse and suggest mathematical models which minimize the material handling distance, handling time, utilization of space or operating costs and results in the optimal dimensions for the warehouse. Rosenblatt and Roll [13] also present a model for the optimal size and layout of a conventional warehouse, but this model concentrates on the storage capacity of the warehouse for the objective function of the model. Gray, et al. [14] present a hierarchical model that minimizes the setup and operating costs of an order consolidation warehouse. These papers all result in an optimal layout for a warehouse with each paper studying and evaluating a different aspect of the warehouse.

Bozer and White [15], [16], study the picking and retrieval operations within the warehouse and present models for the performance of specific storage and AS/RS systems to come up with expressions for optimal configurations and time. Spee [17] analyzes the carousel system in a warehouse and presents a mathematical model, which maximizes throughput.
2.2.1 Order Fulfillment

Order fulfillment is the term given to the processing of orders within the warehouse. The warehouse receives the order, reviews it and assigns employees for the picking, sorting and packing of the orders. Once the orders have been packed, they are prepared for shipping and then shipped to the respective customer. Each of these operations consumes resources in the warehouse: time, money and manpower. Therefore, these operations have to be optimized for the warehouse to perform efficiently and effectively. The research done in the field of order fulfillment deals with improving and optimizing the various operations that occur within the warehouse.

Daniels, et al. [18] present a model for order picking in a warehouse. Order picking is a sequence of operations to be performed by the employees to obtain the items contained in the aisles according the order given to them. Order picking can be modeled as a traveling salesman problem (TSP) because the TSP finds the shortest route to visit all the assigned locations. They present a model to minimize the cost of retrieving items from their locations when similar SKUs are stored in different locations. The constraints considered are the locations of the SKU, the quantity of each SKU that has to be retrieved and the total stock of each item available at the warehouse. The authors also discuss several heuristic approaches to the same problem so as to present different alternatives to solving this problem in areas where a feasible assignment is sufficient and low cost approaches are preferred.
Ratliff and Rosenthal [19] present a model for order picking in a warehouse in minimum time. The authors consider a rectangular warehouse, where the aisles are parallel to the length of the warehouse and crossover is only allowed at the ends of the aisles. The TSP is used to solve the problem. The authors detail all the possible types of minimum length routes that can be formed with this aisle configuration and also present a detailed graphical representation of each. A loading dock can be present along any aisle without requiring any change in the model. A dock or items present along the crossover can also be accommodated without any major alterations. The model can be implemented with crossovers within the aisles, but this would greatly increase the time taken to solve the problem. The authors feel that it would not be practical to use this technique for more than 2 or 3 crossovers within the aisle.

Petersen [20], [21], discusses the problem of routing pickers in a warehouse. He considers the storage policies and the different aisle configurations and compares the performance of different heuristics on the basis of the time to complete a pick list. The time to complete a pick list consists of the travel time to the location, the time to identify each storage location and SKU, the time to retrieve each unit load, the time to cross check the pick list and the time to place the unit loads on the cart.

The impact of the travel speed, the picking rate and the shape of the warehouse and the pick-up/drop-off point on route performance were also studied. The results of the different heuristics used were compared to the result obtained by using the model developed by Ratliff and Rosenthal [19] and it was found that the travel speed and
picking rate had a significant effect on the performance of the route. Among the picking situations considered, the composite strategy (i.e. combination of the best features of the return and traversal strategies which minimizes the travel distance between the two farthest picks in 2 adjacent aisles) is found to work the best.

As noted by Fredendall et al. [9], order processing costs are generally taken to be a constant per item, when considering the ordering costs for retailers. But as this section suggests, the operations that occur in a warehouse form a significant part of the ordering process of a retailer. It is also evident that the amount of work involved during order fulfillment is heavily dependent on the quantity ordered by the retailers and the pallets, cartons or items required to fulfill the order. Thus the costs incurred during order fulfillment will be constant only if all the orders the warehouse received were identical, which is an unlikely situation in the real world. These costs cannot be assumed to be constant and it is imperative to include these costs in the ordering cost assigned to a retailer.

2.2.2 Inventory Management

Inventory management deals with decisions regarding supply levels: the correct amount of material and the correct time to reorder. There are many reasons for a company to hold excess inventory; variation in demand and production; poor quality; and unreliable suppliers and shippers. However, there are also good reasons to cut down the amount held in inventory: carrying cost, storage space and material handling. Thus a trade off has to be considered between the two situations.
Manufacturers are moving towards lean manufacturing and JIT, so companies are decreasing the amount of inventory being held. Retail stores are also applying the philosophy of JIT to reduce inventories and in turn reduce the associated costs at the store. But determining the exact amount that is needed to cover contingencies changes based on the situation faced by the company. There is one model that is commonly used to determine optimal order size. This is the Economic Order Quantity (EOQ) model. Another model that is also used is one that includes purchasing cost per unit in the total cost equation and aims at offering quantity discounts to customers that order large quantities. This is called the EOQ with Quantity Discounts.

2.2.2.1 Classic EOQ

The Order Quantity in the EOQ model is determined by minimizing the total annual cost incurred by the company by virtue of its ordering cost and carrying cost. The expression for total annual cost is shown in Equation 2-2.

\[
TC = \frac{Q}{2} h + \frac{D}{Q} s
\]  

(2-2)

where,

\( TC \) = Total annual cost

\( Q \) = Order quantity

\( D \) = Annual demand

\( s \) = Ordering cost
This model is based on the assumption that there is a single item, with deterministic demand and lead-time, no shortages, and inventory is replenished in batches rather than continuously over a period of time.

The first component of equation 2-2 represents the inventory management costs and the second component represents the ordering costs. Differentiating with respect to order quantity, the expression for EOQ is obtained as indicated by Equation 2-3.

\[ Q^* = \sqrt{\frac{2DS}{h}} \]  

(2-3)

where,

\[ Q^* = \text{Economic Order Quantity} \]

The literature in the area of inventory management includes different types of inventory models dealing with different real-world constraints. Many of these models are variations of the basic EOQ model where the alterations include the conditions that are encountered in the situation being studied. Despite these new conditions these models still try to determine the optimal order quantity, which is one area where the model developed in this research differs from other models for the EOQ.
Liberatore [22] discusses an EOQ model, with a few alterations to the assumptions on the basis of which the traditional EOQ model has been developed. Typically, demand always follows a pattern that can be traced by a probability distribution for analysis. The basic EOQ model, however, assumes that this demand is deterministic to simplify the calculations involved.

The traditional EOQ model also assumes that if the inventory is zero when the order is received then that particular order is lost. This is not the scenario in real life as orders may be backordered and fulfilled when the inventory is available. Therefore, Liberatore considers a more realistic situation for his model and develops an equation for the order size based on stochastic lead-times and backlogged demand. The traditional equations of inventory theory with deterministic lead-times and no backlogging are special cases of this model.

Kim and Benjaafar [23] analyze the suitability of using the Order Quantity-Reorder point (Q, R) model where Q is the order quantity and R is the reorder point, for different situations in production/inventory systems. The authors present a Production/Inventory (Q, R) model that includes the production lead times and the order replenishment lead times explicitly with the inventory costs. Comparisons between this model and the traditional (Q, R) model show that the optimal order quantity and reorder point are different for each of the models. This indicates that the average inventory and backorders would also be different and in turn, the estimated costs would also be different.
Thus the value of lead-time used in the models would make a substantial difference in the costs. If the lead-times were fixed then the costs in both the models would be the same. But in an actual manufacturing environment, the lead-times are rarely constant and therefore the traditional model could severely overestimate or underestimate the order quantity and the reorder point. The authors also portray the impact of setup times on the order quantity and they show that the system stability depends on the order sizes. The authors conclude by presenting the extensions that can be done to make this research more generic.

2.2.2.2 EOQ with Quantity Discounts

Quantity discounts are price reductions that are offered to the retailer when he/she places an order that is above a specific threshold level. It is an incentive to the retailer to buy larger quantities. When quantity discounts are offered the retailer is forced to consider the possible benefit of ordering larger number of items with a lower price per item over the increase in the inventory costs that would be incurred by him/her [23]. The quantity discount model can be written as shown in Equation 2-1. The total cost curve with quantity discounts is shown in Figure 2-1.

\[
TC = \left( \frac{Q}{2} \right) \cdot HC + \left( \frac{D}{Q} \right) \cdot S + P \cdot D
\]  

(2-4)

where

\[ P = \text{unit price} \]
Including the purchasing cost in the total cost equation does not change the EOQ point but changes the total cost for the retailer since the unit costs for certain ranges are different. There are two cases of this model:

- **Carrying costs are constant:** When carrying costs are constant, the EOQ remains the same for all the curves.
- **Carrying costs are a percentage of the purchasing cost:** When the carrying costs are a percentage of purchasing cost per unit, the EOQ starting with the lowest price range is found. If this EOQ is feasible (i.e. falls in the correct quantity-cost range),
range), it is the EOQ for that model. If the EOQ found is infeasible, then the EOQ for the other prices are calculated starting from the next highest one. This procedure is continued until a feasible solution is reached.

There is a large body of research that has dealt with quantity discounts in the case of single supplier-single buyer situations and single supplier-multiple buyer situations. Benton and Park [24] have compiled a paper that reviews the literature in determining lot sizes using the principle of quantity discounts. Benton and Park categorize the literature based on whether the quantity discounts are all-units or incremental and also categorize from buyer’s or the seller’s perspective. This section of the literature focuses on some of the research that has been done regarding the quantity discount model and modifications of the EOQ model in this regard.

Khouja [25] proposes an algorithm that determines the EOQ with a demand that has been adjusted to consider the effects of the increased demand in the previous period due to discounted costs. This paper considers the situation where suppliers that have excess inventory sell these by the end of the period at discounted costs. Taking advantage of this situation, when products can be stored for more than a single period, buyers buy larger quantities at discounted prices so that this would decrease their costs for the next period. If the supplier does not consider the effect of such large order quantities, the classic EOQ will be suboptimal. This paper thus suggests a technique that would help suppliers calculate the true order quantity and true profit.
Guder, et al. [26] present a heuristic that determines order quantities for multiple items when incremental quantity discounts and a single resource constraint are given. The results obtained by this heuristic are compared with the results obtained by a combinatorial algorithm, which considers all price levels for all items, used to find the optimal solution for small problems. This combinatorial algorithm assumes that the reorder times for each item are independent. However when the number of items is large and there are many price breaks, this algorithm cannot solve the problem to optimality. This is when the heuristic comes into play. This heuristic uses the Lagrangian relaxation technique. The heuristic works well when compared to the optimal algorithm for small problems and hence can be used to solve large problems to optimality.

Ghandforoush and Loo [27] present a non-linear procurement model which considers quantity discounts in order to reduce the total procurement cost. This model was developed for a multinational oil company and compared with the technique currently used by the company. The authors use the non-linear programming technique for this model. The model considers all combinations of shipments to all the customers in its cost minimization function. The constraints include those of supplier capacity, customer demand, price to volume relationship and order requirement. This model was found to be flexible and can adapt to changes in the objective and can consider multiple objectives as well.

Dada and Srikanth [29] study quantity discounts from a seller’s point of view. The authors characterize the range of order quantities and prices that would lower costs for
both the buyer and the seller. Pricing policies that help with balancing the savings for both the buyer and the seller are developed according to these characteristics.

This principle of offering quantity discounts is similar to the principle discussed in this research but the benefit of ordering large quantities is implicitly included in the model as opposed to explicitly considering the purchasing cost per unit and providing discounted rates to buyers when they order large quantities. The discount is obtained by the retailer when large quantities are ordered that larger unit loads are used.

2.3 Warehouse-Retailer Interaction

The most common warehouse-retailer problem is the One Warehouse/\textit{N}-Retailer problem. Therefore it is not surprising that most of the research focuses on this problem. Since the model developed in this thesis can be extended to include many retailers by either considering the orders sequentially, or by incorporating the vehicle routing solution in it, this section of the literature review focuses on different aspects of this issue such as inventory, routing and costs. Different models are available which consider each of these aspects as decision variables.

2.3.1 Inventory Analysis

Inventory analysis within the One-Warehouse/\textit{N}-Retailer problem involves evaluating the policies used to maintain a certain quantity of inventory within the warehouse and at the premises of the retailers. Since these inventories translate to cost, the warehouse and
the retailers should store optimal quantities. The literature available presents different models that minimize the costs incurred due to holding and replenishing inventory.

Ganeshan [30] considers a scenario of multiple suppliers, a single warehouse and multiple retailers. He analyzes this system with respect to the inventory held at the retailers, the demand process at the warehouse and the inventory held at the warehouse. He then combines all these analyses and depicts a synthesized model that minimizes the total annual cost in this system. This annual cost is comprised of the ordering cost, the holding costs and the transportation costs. This model is subject to the constraints regarding the expected units short at the warehouse and retailers, the order quantity at the warehouse and the retailers, and the reorder points at the warehouse and the retailers. This model is applicable to determine the safety stock levels for the warehouse and the retailers and is flexible to include changes with respect to number of suppliers, lead-times or modes of transportation. But this model is limited to two echelons and assumes identical suppliers and identical retailers and a fixed order processing cost at the warehouse.

Ahire and Schmidt [31] analyze a One-Warehouse/N-Retailer system for a single, perishable product. This model was essentially developed to study the working of an electric utility for its transmission and distribution parts. Each of the retailers faces independent and stationary demand and they follow the \((Q, r)\) policy for inventory replenishment where \(r\) represents the reorder point. The warehouse has a review interval period \(T\) where they review the retailers who have placed the orders and then
decide which retailers’ order is fulfilled. Each retailer is given a specific reorder period interval and the warehouse ships the products to that retailer only in that period (even if the warehouse has the inventory needed to replenish the retailer).

The warehouse considers a lead-time composed of two stochastic components in addition to the customary fixed time consisting of the transit time from the warehouse to the retailers. One of these components is the delay that occurs if the warehouse is short of stock and the other is the time for which the particular retailers’ order is reviewed ($T$). This model is found to predict the operational measures of the system when the retailer fill rates are high but is found to deteriorate when these fill rates were low.

2.3.2 Vehicle Routing

The Vehicle Routing Problem (VRP) deals with transportation of products from a warehouse(s) to retailers with a fleet of vehicles with limited payload capacity. The objective of this problem could be different based on the situation: cost minimization and time minimization being the typical goals. Vehicle routing is one of the main aspects of Distribution Management. It is a major factor in determining total cost because out of the annual logistics cost, it is said that transportation costs contribute to higher than 50% of the cost [32].

Vehicle routing plays a very important role in determining transportation costs. With an efficient vehicle routing algorithm, a company can obtain the best possible route in terms
of the distance traveled by each of the vehicles and thus reduce the total cost incurred during distribution. An inefficient algorithm will do just the opposite. To date there have been very few algorithms that can solve the VRP to optimality. Small-scale problems are generally solved with ease, but solutions to real world problems are generally found to be lacking.

Kim and Kim [33] consider a multi-period vehicle-scheduling problem where a single depot supplies multiple retailers with a single type of product. The objective of the paper is to minimize costs with respect to transportation of products from the depot to the retailers and inventory held by the retailers. The vehicles are identical with identical capacities. The paper suggests a two-phase heuristic solve this problem. The first phase considers each retailer as independent and alternate delivery schedules are generated for each retailer. It is assumed that replenishments occur only when the inventory is zero.

In the first phase, the model considers a quantity-based transportation cost, a distance-based transportation cost and the cost incurred in holding inventory. In the second phase, these alternate schedules are studied and the best schedule for each retailer is selected. The model used for this phase is a generalized assignment problem (GAP), which when solved results in the best possible delivery schedules for each retailer with minimum cost incurred.
Clarke and Wright [35] propose an iterative method to route a number of vehicles from a single warehouse to multiple retailers. The objective is to minimize the total distance traveled by the trucks and assign all the loads to the trucks, given the shortest distances between any two points in the problem. The technique used in this paper is similar to that used by Dantzig and Ramser [34] except that the method developed by Clarke and Wright is for a specific application. This algorithm can be run on digital computers and finds an optimal or a near optimal solution of assigning customers and trucks to routes. But the authors suggest that once the final allocation is obtained, it is advisable to solve the traveling salesman problem for each truck allocation so that an optimal solution can be found for the route.

Fisher and Jaikumar [41] develop a generalized heuristic to generate the route that each vehicle has to take while simultaneously minimizing the delivery cost. A fleet of vehicles supplies retailers with products from a single warehouse. The authors first formulate a mixed integer-programming problem, which consists of constraints involving a GAP. The objective function minimizes the cost of the traveling salesman problem tours that each of the vehicles makes to service the retailers in their tour. Once this assignment has been made then the authors apply a traveling salesman algorithm to generate the sequence in which the vehicles deliver to the retailers.

Fisher and Jaikumar propose a method to select seed customers from all the customers present and generate tours with them. These seed customers are selected either by an automatic rule or an experienced scheduler in the company. Then the remaining
customers are taken and introduced in these tours and their optimality is tested. In a similar, iterative manner the rest of the tours are developed. Once all these tours have been generated, the authors apply the traveling salesman optimizing algorithm to obtain the final sequence. This heuristic is found to give better solutions than solutions given by Clarke and Wright [35].

Campos and Mota [40] deal with the capacitated VRP. This problem is a combination of an assignment problem and a traveling salesman problem. The authors present two heuristic procedures to solve this problem. One of the procedures starts from scratch while the second procedure uses information from the first method. The first heuristic generates an initial feasible solution, which is inspired by the heuristic proposed by Fisher and Jaikumar [41]. The initial customers are selected based on their demand and distance from the warehouse.

The greater the product of the demand and the distance from the warehouse, the greater is the importance of the retailer. Thus the last retailers to be routed are the ones close to the warehouse and having a small demand. The second heuristic does not begin from scratch but uses the initial solution obtained by the algorithm proposed by Clarke and Wright [35]. Both solutions are improved iteratively using the Tabu Search technique. The procedures were found to give nearly optimal results in some cases and feasible but sub-optimal solutions in other cases.
2.3.3 Logistics Costs

Logistics is the “procurement, maintenance, distribution and planning of material and personnel” [9]. The costs involved in these operations are subject to extensive research. The literature that is present in this area deals with different situations that are encountered in the physical distribution of products from the warehouse(s) to the retailer(s) and models that have been developed to minimize these costs.

Forsberg [42] studies a two-level inventory system in his paper, to determine the exact holding and shortage costs incurred. The situation studied is a One-Warehouse/\textit{N}-Retailer system. Transportation times are taken to be constant and considered part of the lead-time. The retailers face independent Poisson demand. The unsatisfied orders are considered backordered and Forsberg states the shortage costs as linear functions of time until the shipment is delivered. Using these assumptions, Forsberg develops exact cost evaluations for the different policies and these costs are expressed as a weighted mean of those costs that are obtained from one on one ordering policies.

Vishwanathan and Mathur [43] integrate the inventory and routing decisions and develop a heuristic that minimizes the total cost of transportation and inventory while simultaneously generating efficient delivery schedules for the retailers. The authors consider a One-Warehouse/\textit{N}-Retailer system in which the warehouse only reorganizes the shipments it receives into orders for individual retailers. The warehouse does not stock any inventory. The uncapacitated vehicle scenario is studied first and a heuristic is developed for it, which is then extended to include the capacitated vehicle scenario. This
heuristic generates nested order replenishment schedules and is found to perform efficiently for single product problems.

Swenseth and Godfrey [32] discuss the importance of the inclusion of accurate transportation costs in decisions regarding inventory replenishment without compromising the complexity of the model. The authors suggest different alterations to the basic EOQ model discussed in Section 2.2.2, based on different freight rates that are available in the industry. The paper presents the various models that the authors put together and then compares all of them to reach a decision regarding the appropriate model. The first model, the EOQ model incorporates freight rate per pound for a given shipping weight over a given route and the weight per unit of the shipment.

\[
L = \frac{Q}{2} C_h + \frac{R}{Q} C_o + F_y R w
\]  

(2-4)

where,

\( L \) = Total cost

\( Q \) = Order quantity

\( C_h \) = Inventory holding cost for one unit for a year (obtained by taking the product of the unit cost and the holding cost rate)

\( R \) = Annual requirements (units)

\( C_o \) = Cost to place one order

\( F_y \) = Freight rate per pound for weight \( y \) over a given route
When this expression is differentiated with respect to \( Q \), the same EOQ expression as the basic EOQ model (Section 2.2.2) is obtained.

\[
Q^* = \frac{2RC_o}{C_h} \sqrt{\frac{h}{w}} 
\]  

(2-5)

This is the same result as not incorporating the freight rates in the total cost expression.

The second model, the Inverse Model, considers a ratio between the freight rate per pound at truckload (TL) shipments and the actual shipping weight.

\[
F_y = \frac{F_xW_x}{W_y} \]  

(2-6)

where,

\( F_x = \) True truckload freight rate per pound (at the full shipping weight)

\( W_x = \) Full truckload shipping weight

\( W_y = \) Actual shipping weight

Substituting this equation in the expression for total cost:

\[
L = \frac{Q}{2}C_h + \frac{R}{Q}C_o + \left[ \frac{F_xW_x}{Qw} \right]Rw 
\]  

(2-7)

Differentiating this equation with respect to \( Q \), the next expression for \( Q \) is obtained.
The last model, the Adjusted Inverse Model, takes into consideration the less than truckload (LTL) shipments.

\[ Q = \sqrt{\frac{2R(C_o + F_x W_y)}{C_h}} \]  \hspace{1cm} (2-8)

\[ F_y = F_x + \alpha F_x \left[ \frac{(W_x - W_y)}{W_y} \right] \]  \hspace{1cm} (2-9)

where,

\[ \alpha = \text{Constant between 0 and 1. This parameter } \alpha \text{ is used to determine the actual freight rate that is to be charged when the carrier orders LTL shipments based on the rate for TL shipments and a discount that is applied based the amount by which the actual weight is less than a TL shipment.} \]

This expression was substituted in the equation for total cost.

\[ L = \frac{Q}{2} C_h + \frac{R}{Q} C_o + \left[ F_x + \alpha F_x \left[ \frac{(W_x - Qw)}{Qw} \right] \right] R w \]  \hspace{1cm} (2-10)

Since \( W_y \) is a function of \( Q_w \). The above equation, when differentiated with respect to \( Q \), generates the following expression for optimal order quantity.

\[ Q^* = \sqrt{\frac{2R(C_o + \alpha F_x W_x)}{C_h}} \]  \hspace{1cm} (2-11)
These three models were tested using a wide range of experimental inventory replenishment situations. The inverse model was found to work well for TL shipments and the adjusted inverse model was found to work well for LTL shipments. Because of this scenario, the authors proposed a heuristic, which combined the Inverse and the Adjusted Inverse model. This thesis uses the EOQ formula in a similar fashion as done by Swenseth and Godfrey [32]. The ordering cost is defined explicitly as being comprised of the order fulfillment cost within the warehouse and the cost of transporting goods to the retailers.
3 METHODOLOGY

The objective of this research is to develop an optimization model (based on the classic EOQ Model) to minimize the sum of two costs: the ordering cost and the inventory carrying cost, in a One-Warehouse/One-Retailer system. The model will then be used to study the impact of several the variables on the cost and the order frequency. The ordering costs incurred by the retailers include the order processing costs within the warehouse and the cost of delivering the products from the warehouse to the retailers. In order to accurately determine the costs needed for each of these components it is essential to determine the time taken for each of the corresponding operations. Hence this makes the model more complicated than the classic EOQ model.

The order processing cost within the warehouse is also called order fulfillment cost. This is the cost incurred by the warehouse to process an order received from the retailer. The order fulfillment costs considered in this model are further broken down to explicitly define the picking costs, packing costs and loading costs. Each of these costs is determined by the costs required to handle the appropriate unit load (a pallet, a carton or an item). The transportation costs are the costs incurred by the warehouse or by a logistics provider to deliver products from the warehouse to the retailer.

The classic EOQ model considers the ordering and holding cost for a single product. The model developed in this research considers the all the SKUs going to a particular retailer in the warehouse, to calculate the order quantities.
3.1 System Description

The scenario being studied in the research is a two-echelon distribution system comprised of a single warehouse that fulfills the demand for retailers who receive the products. The warehouse holds multiple SKUs and it is assumed that these SKUs are similar in shape and size. This assumption simplifies the estimation of the time taken to pack and load the SKUs because the time taken to pack and load each unit load of these SKUs will be the same.

This assumption is reasonable for a small to mid-sized warehouse. For example, a warehouse storing spare parts for electronic goods. A number of spare parts for electrical and electronic devices are approximately the same size. The SKUs are stored in three different locations that comprise a designated area for pallet picking, carton picking and item picking. A schematic drawing of the warehouse is shown in Figure 3-1.

When orders arrive from the retailers, they are separated according to the retailer. Then the combination of pallets, cartons and items for the different SKUs that make up an order is determined. Assuming the order quantity for one of the retailers is 578 items of a particular SKU. Then, the largest unit load i.e. a pallet (assumed capacity = 100 items) is chosen. The number of pallets that have to be shipped to this retailer is 5. Of the remaining 78 items, 7 cartons (assumed capacity = 10 items) can be shipped. The remaining 8 items are shipped as they are. The same procedure is followed until the entire order is broken down into the number of unit loads required for each SKU and pickers are
given information about the SKUs and the unit loads that have to be retrieved for each of them.

It is assumed that each order is handled separately and each retailer’s orders are shipped separately. This assumption is why only a single retailer is considered in this analysis, the above statement is redundant but has to be mentioned to explain other aspects of this research. This assumption simplifies the analysis of the picking operation and the transportation since routing between the retailers will not have to be considered. It also
eliminates the need for a sorting operation and the associated costs. This assumption is reasonable for warehouses which handle large orders containing several SKUs from customers and would therefore like to reduce the work undertaken by the warehouse workers by picking the orders separately and eliminating the sorting of large orders later.

The warehouse operates vehicles for the transportation of the products to the retailers. These vehicles are assumed to have identical capacity to simplify the calculation of the transportation costs. This is a factor that is common to many warehouses and hence is a reasonable assumption to consider.

### 3.2 Derivation of the Modified EOQ Model

As explained in Section 2.2.2, the EOQ model is obtained by minimizing the total cost involved in placing an order. The total cost expression used for the classic EOQ model is shown in Equation 3-1.

\[
TC = \frac{Q}{2} h + \frac{D}{Q} s 
\]  

(3-1)

This equation can be generalized as shown in Equation 3-2.

\[
TC = IC + OC 
\]

(3-2)

where,
IC = Inventory costs per week incurred by the retailers in storing products at their facilities.

OC = Ordering cost per week incurred by the retailers when their request for products is satisfied.

TC = Total cost per week

The Ordering Cost shown in Equation 3-2 comprises two components as shown in Equation 3-3.

\[ OC = WHC + TRC \]  

(3-3)

where,

\( WHC \) = Order fulfillment cost incurred at the warehouse when the orders from the retailers are being processed.

\( TRC \) = Transportation cost incurred when the orders are being transported to the retailers.

3.2.1 Order Fulfillment Cost Analysis/Warehouse Cost

The Order Fulfillment Cost shown in Equation 3-3 refers to the costs incurred by the warehouse while processing the orders obtained from the retailers. The different costs that make up this cost are shown in Equation 3-4.

\[ WHC = R + Pa + L \]  

(3-4)
where,

\[ R = \text{Retrieval cost to retrieve the pallets, cartons or items needed to fulfill the order.} \]

\[ Pa = \text{Packing cost to pack and prepare the orders for transportation.} \]

\[ L = \text{Loading cost to place the packed orders in the vehicles prior to transportation.} \]

The retrieval, packing and the loading costs specified in Equation 3-4 comprise different components to account for different unit load costs. These components are shown in Equations 3-5, 3-6 and 3-7.

\[ R = R_P + R_C + R_{IT} \]  \hspace{1cm} (3-5)

\[ Pa = Pa_P + Pa_C + Pa_{IT} \]  \hspace{1cm} (3-6)

\[ L = L_P + L_C + L_{IT} \]  \hspace{1cm} (3-7)

where,

\[ R_i = \text{Picking cost per unit load} \]

\[ Pa_i = \text{Packing cost per unit load} \]

\[ L_i = \text{Loading cost per unit load} \]

\[ i = P,C,IT \]

\[ P = \text{Pallets} \]

\[ C = \text{Cartons} \]

\[ IT = \text{Items} \]
3.2.2 Transportation Cost Analysis

The *Transportation Cost* shown in Equation 3-3 refers to the cost incurred to transport the products from the warehouse to the retailers. This cost is comprised of different components as shown in Equation 3-8.

\[
TC = (\text{Dist} \cdot c + \text{RentCost}) \cdot Tr
\]  

(3-8)

where,

\(\text{Dist}\) = The distance of the retailer from the warehouse in miles

\(c\) = Cost of fuel per mile

\(\text{Rent Cost}\) = Cost of renting one vehicle for transportation

\(Tr\) = Number of vehicles needed to transport the order of the retailer

The number of vehicles needed for transporting the products from the warehouse to the retailer is obtained using the order quantity that determined by the modified EOQ model. This order quantity is the cumulative order quantity formed by the individual order quantities of all the SKUs in the order. This is shown in Equation 3-9.

\[
Q = \sum Q_j
\]  

(3-9)

where,

\(Q\) = Cumulative order quantity (for all SKUs)

\(Q_j\) = Order quantity per SKU
Once the cumulative order quantities for the retailer are known, the number of trucks can be calculated. This is shown in Equation 3-10.

\[
Tr = \left( \frac{Q}{Vcap} \right)^+ 
\]

(3-10)

where,

\( Tr \) = The number of vehicles needed to transport the products of an order.

\( Q \) = Order Quantity for a retailer (converted to an equivalent number of pallets)

\( Vcap \) = The capacity of the vehicle (# pallets)

3.2.3 Determining Order Frequency/Time between Orders

This modified EOQ model deals with multiple SKUs instead of a single one. Therefore, the time between orders has to be the same for all the SKUs. The reason this cannot be accomplished with a traditional EOQ with the demand being replaced by a cumulative demand for each SKU is that this would result in a cumulative order quantity which would not result in order quantities for each SKU. For example if an order resulted in 10 cartons, it would show as a pallet irrespective of whether these cartons belonged to the same SKU or not and this would not be correct because if these belonged to different SKUs then the retrieval time and cost for these 10 cartons is different from the retrieval time and cost for 1 pallet.
The time between orders for an EOQ model is given by Equation 3-11.

\[ T = \frac{Q}{D} \]  

(3-11)

where,

\[ T \] = Time between orders

\[ Q \] = Order Quantity

\[ D \] = Demand

Since order quantities are different for each SKU, it is apparent that each SKU would have a different \( T \) value. This would not allow the warehouse to fulfill the demand for all the SKUs for each of the retailers simultaneously. Thus the model developed in this research optimizes the \( T \) value for each retailer. This optimal or economic \( T \) value is then used to calculate the EOQ for each of the SKUs.

Once this is done, the model determines the appropriate number of pallets, cartons and items that are to be picked in order to minimize the cost incurred by the warehouse in processing the order. Correspondingly the number of orders for a retailer can be calculated using the reciprocal of Equation 3-11, as shown in Equation 3-12.

\[ NO = \frac{1}{T} \]  

(3-12)

where,
\[ NO = \text{Number of orders} \]

\[ T' = \text{Optimal Time between orders} \]

### 3.3 Deriving Expressions for Costs

As shown in Section 3.2, the ordering costs for the retailer are comprised of different components such as the warehouse costs or order fulfillment costs and the transportation costs. The order fulfillment costs are in turn comprised of different elements: picking costs, packing costs and the loading costs. As is evident from Equations 3-5, 3-6 and 3-7 each of these costs can be split into different parts depending on the number of pallets, number of cartons and the number of items involved in each of these operations.

Since the number of pallets, cartons or items that are picked, packed and loaded are dependent on the demand of a particular retailer and because the work involved in picking/packing/loading each of these unit loads is different, it is essential that separate models for the unit loads be developed. These models involve the layout of the warehouse and techniques used to perform the picking, packing and loading operations. In order to calculate the costs incurred during the above-mentioned operations, it is necessary that time taken to perform each of these operations is estimated and time in combination with the cost of labor at the warehouse will result in the necessary costs.
3.3.1 Picking Costs

Picking costs refer to the cost of retrieving a pallet, carton or item from its storage location and transporting it to the staging area to be packed and prepared for transportation. During the picking operation the workers receive information regarding the SKUs and the number of pallets, cartons or items of each SKU that they have to pick. The workers then proceed to the respective storage locations and locate the SKUs that have to be picked.

3.3.1.1 Pallet Picking

Retrieval of pallets typically occurs using a vehicle owing to the size of the pallet. It is assumed in this study that a forklift is used to retrieve pallets one at a time in the warehouse. In large warehouses, a combination of vehicles maybe used to retrieve pallets and therefore different speeds for each of these types of vehicles have to be estimated. Therefore, the assumption that only forklifts are used to retrieve pallets is reasonable for most warehouses and also reduces the number of speeds that have to be estimated to calculate the time taken to retrieve pallets.

In order to calculate the time taken to pick a pallet, it is necessary to know the average distance traveled by the worker to pick a pallet. The path used to retrieve a pallet is shown in Figure 3-2. The figure is that of a typical rectangular warehouse.
The average distance traveled by a picker to retrieve a pallet from its location in the warehouse can be determined by considering the path traveled by the picker in terms of different elements: average distance to the aisle, average distance within the aisle and a constant distance ($e_p$) as shown in Figure 3-2. This distance can also be referred to as the expected distance and is shown in Equation 3-13.

$$E[D_p] = 2 \left[ d_p + \frac{A_p - 1}{2} X_p + \frac{L_p}{2} + \frac{W_p}{2} + e_p \right]$$

(3-13)

where,
$E[D_p] = \text{Expected distance traveled to pick a pallet}$

$A_p = \text{Number of aisles storing pallets}$

$L_p = \text{Length of each aisle storing pallets}$

$d_p = \text{Distance from the entrance of the storage area to the beginning of the first aisle}$

$e_p = \text{Distance from the end of each aisle to the line joining the entrance of the docks and the center of the last aisle}$

$X_p = \text{Center to Center distance between aisles}$

$W_p = \text{Width of an aisle}$

Each of the components mentioned before can be seen in Equation 3-13:

$E \text{ [to aisle]} = \left[ d_p + \frac{A_p - 1}{2} X_p \right]$ \hspace{1cm} (3-13-a)

$E \text{ [in aisle]} = \left[ \frac{L_p}{2} + \frac{W_p}{2} \right]$ \hspace{1cm} (3-13-b)

The rest of the elements are the constants considered to ensure that the distance calculated from Equation 3-12 is practically accurate.
3.3.1.2 Carton Picking

Cartons are retrieved in many different ways depending on the size of the warehouse, and the number of cartons that are stored. Often, a conveyor aids the workers in the retrieval. The worker begins the retrieval by locating a SKU in the order, retrieves it from the storage places and places it on the conveyor. The worker then moves along the aisle looking for the next SKU, retrieves it and places it on the conveyor. This continues until all the cartons have been retrieved. The conveyor aids the retrieval process by transferring the cartons to the staging area, where the cartons are prepared for transportation. Figure 3-3 is the schematic representation of the carton picking area.

Figure 3-3: Schematic of the carton picking area
Since the orders from different retailers are being processed separately, an expression to calculate the time taken to pick the cartons in an order is needed, which includes the time traveled by the worker to reach the SKU storage location, retrieve the carton and then place it on the conveyor. The details of the carton picking operation are shown in Figure 3-4. The conveyor then transfers the cartons from the pick area to the staging area where they are prepared for transportation.

![Figure 3-4: Path of a worker during carton picking](image-url)
In order to calculate this time it is necessary that the average distance traveled by the picker to pick cartons be estimated. This average or expected distance is calculated by considering the number of aisles storing pallets used for carton picking and the length and width of each of these aisles. The expression used to calculate this expected distance is shown in Equation 3-14.

\[ E[D_c] = 2d_c + A_c[2L_c + 4e_c + X_c] \]  

(3-14)

where,

- \( E[D_c] \) = Expected distance traveled by the picker to pick a carton
- \( L_c \) = Length of the aisle used for storing pallets for carton picking
- \( A_c \) = Number of aisles used for carton picking
- \( d_c \) = Distance from the entrance of the storage area to the beginning of the first aisle used for carton picking.
- \( e_c \) = Distance from the end of each aisle to the line joining the entrance of the docks and the center of the last aisle
- \( X_c \) = Center-to-center distance between the aisles

### 3.3.1.3 Item Picking

Workers generally perform retrieval of items manually with the aid of a cart. The worker locates the SKU in the retailers' order and the required number of items is retrieved from their storage location and loaded into the cart. The worker then locates the next SKU
along the aisle and the necessary number of items of that SKU is placed in the cart. This process continues till all the items are picked. One of the paths that could be taken by the worker during the picking operation is shown in Figure 3.5.

In this case as well the time taken to pick items is to be estimated. In order to estimate this time it is essential to know the average distance traveled by the picker to pick items. This distance is calculated by considering the path traveled by the picker as composed of different elements: average distance to the storage racks, average distance traveled within the aisle and required constants. The distance traveled by the picker back and forth across the aisle is assumed to be insignificant when compared to the rest of the
segments, since the aisles are closer than in the case of the previous two storage systems. This distance is calculated by the expression shown in Equation 3-14.

\[ E[D_{IT}] = 2d_{IT} + A[L_{IT} + 2e_{IT} + X_{IT}] \]  

(3-15)

where,

\[ E[D_{IT}] = \text{Expected traveled by the picker to pick an item} \]
\[ L_{IT} = \text{Length of the aisles used to store items} \]
\[ A_{IT} = \text{Number of aisle used to store items} \]
\[ d_{IT} = \text{Distance from the entrance of the docks to the edge of the first aisle} \]
\[ e_{IT} = \text{Distance from the end of each aisle to the line joining the entrance of the docks and the center of the last aisle} \]
\[ X_{IT} = \text{Center to Center distance between the aisles} \]

3.3.1.4 Calculating Picking Cost

The expected distances as shown in Equations 3-12, 3-13 and 3-14 are then combined with the time to physically retrieve a pallet/carton/item from its storage spot and place it on the forklift/conveyor/cart, the number of pallets/cartons/items in an order; and the labor cost per hour to arrive at the cost incurred by the warehouse to retrieve the pallets, cartons or items in an order. These expressions are shown in Equations 3-16, 3-17 and 3-18.
\[ RC_P = \left( \frac{E[D_P]}{V_{FL}} + T_{RP} \right) \cdot N_P \cdot C_H \] (3-16)

\[ RC_C = \left( \frac{E[D_C]}{V_{WC}} + T_{RC} \cdot N_C \right) \cdot C_H \] (3-17)

\[ RC_{IT} = \left( \frac{E[D_{IT}]}{V_{WT}} + T_{RIT} \cdot N_{IT} \right) \cdot C_H \] (3-18)

where,

- \( RC_P \) = Retrieval cost per order for pallets
- \( RC_C \) = Retrieval cost per order for cartons
- \( RC_{IT} \) = Retrieval cost per order for items
- \( V_P \) = Velocity of the forklift
- \( V_C \) = Velocity of worker while picking cartons
- \( V_{IT} \) = Velocity of worker while picking items
- \( C_H \) = Hourly Rate for the warehouse employee
- \( T_{RP} \) = Retrieve Time for Pallets
- \( T_{RC} \) = Retrieve Time for Cartons
- \( T_{RIT} \) = Retrieve Time for Items
- \( N_P \) = The number of pallets required for the order
- \( N_C \) = The number of cartons required for the order
\[ N_{IT} = \text{The number of items required for the order} \]

The equations discussed above are however subtly different from each other. With respect to pallet retrieval, only a single pallet can be retrieved for every trip of the forklift. Hence, the retrieval cost for pallets is a multiple of the number of pallets retrieved for the order. However, when cartons and items are retrieved, the pickers move through the storage racks and pick all the cartons or items as they pass by the SKU located in the racks. Therefore, a single trip through the storage racks results in the picking of cartons and items for the entire order. Hence, the retrieval cost for cartons and items is a linear function of the number of cartons and items in the order and not a direct multiple of them as in the case of pallets.

3.3.2 Packing Costs

During packing the workers prepare the cartons and items for transportation. The pallets are generally stored in a form that is easy for them to be transported; the cartons are placed on a pallet and they are stretch wrapped in place before they are placed in their storage location. Therefore if a pallet is picked for an order the work involved in its packing is negligible. Cartons that are picked are placed on a pallet and then stretch wrapped into place. Items are placed in a carton and then to make the transporting easier these cartons are also stretch wrapped into place on a pallet.
Typically most of the packing is done in the form of pallets to enable easy loading and unloading of orders. It is assumed that the time taken to pack the carton or the item of any SKU is the same as all the SKUs are of the same shape and size. Thus the expressions for the packing cost per order for each type of unit load are shown in Equations 3-19, 3-20 and 3-21.

\[ PaC_p = 0 \]  \hfill (3-19)

\[ PaC_c = \left( \frac{N_c + \left( \frac{N_{IT}}{n_{IC}} \right)^+}{n_{cp}} \right) T_{WRAP} + T_{CP} \left( N_c + \left( \frac{N_{IT}}{n_{IC}} \right)^+ \right) C_H \]  \hfill (3-20)

\[ PaC_{IT} = \left( \frac{N_{IT}}{n_{IC}} \right)^+ T_{WRC} + T_{ITC} N_{IT} C_H \]  \hfill (3-21)

where,

\[ PaC_p = \text{Packing cost per order for pallets} \]

\[ PaC_c = \text{Packing cost per order for cartons} \]

\[ PaC_{IT} = \text{Packing cost per order for items} \]

\[ T_{CP} = \text{Time taken to place a carton on a pallet} \]

\[ T_{ITC} = \text{Time taken to place an item in a carton} \]

\[ T_{WRAP} = \text{Time taken to wrap a pallet} \]

\[ T_{WRC} = \text{Time taken to seal a carton} \]
\[ n_{cp} = \text{Number of cartons in one pallet} \]
\[ n_{ic} = \text{Number of items in one carton} \]

Equation 3-19 is valid in this thesis because pallets are stored already packed and are transported as they are. The cartons and the items on the other hand as shown in equations 3-20 and 3-21 result in packing costs because these unit loads are transported as pallet loads and thus have to be placed on pallets and packed.

3.3.3 Loading Costs

The loading operation involves placing the packed orders into the vehicles transporting them. Typically the loading is performed using vehicles or conveyors, in general the loading equipment used depends on the unit loads that are being loaded. Since the loading operation is a straightforward operation and different warehouses use different types of equipment the expressions used to determine the time taken to load a truck is taken as a variable and not as a function of the distance traveled and the speed.

Section 3.3.2 details how the cartons and the items in the orders are packed to form mixed pallets for ease in transportation. Hence the loading operation involves only pallets. Consequently, the time taken to load a carton and an item are functions of the time taken to load a whole pallet. It is assumed that the pallets are loaded one at a time. The distribution industry typically loads the pallets one at a time or two at a time.
Therefore, this assumption is a valid one. If the forklift is capable of transporting 2 pallets at a time, then the number of pallet trips per order will just have to be reduced by half in the model and corresponding time and cost can be calculated. The expression used to calculate loading costs per order is shown in Equation 3-22. Therefore Equation 3-7 is simplified to Equation 3-22 because the orders are loaded as pallets.

\[
\text{Loading Cost} = LC_p = N_p \left( N_C + \left( \frac{N_{IT}}{n_{lc}} \right)^+ \right) \frac{LT_p C_H}{n_{cp}}
\]  

(3-22)

where,

\[ LC_p = \text{Loading cost per order for pallets} \]

\[ LT_p = \text{Loading time for a pallet in hours} \]

There are no loading costs associated with cartons and items because all the cartons and items in the individual orders are packed in the form of pallets and hence only pallets are loaded on to the vehicles and transported.

3.3.4 Cost per Unit Load

The cost equations for each of the warehouse operations discussed in the previous sections can now be summarized in order to obtain the total cost per unit load in the warehouse.
Equations 3-23, 3-24 and 3-25 are composed of all the cost equations shown previously from equation 3-15 to 3-21.

\[
C_p = \left( \frac{E[D_p]}{V} + T_{RP} \right) N_p C_H
\]

(3-23)

\[
C_c = \left( \frac{E[D_c]}{V_{WC}} + T_{RC} N_C + \left( N_c + \left( \frac{N_{IT}}{n_{itc}} \right)^+ \right) T_{WRAP} \right) C_H
\]

(3-24)

\[
T_{CP} \left( N_c + \left( \frac{N_{IT}}{n_{itc}} \right)^+ \right) C_H
\]

\[
C_{IT} = \left( \frac{E[D_{IT}]}{V_{WIT}} + T_{RIT} \right) N_{IT} + \left( N_{IT} \left( \frac{N_{IT}}{n_{itc}} \right)^+ \right) T_{WRAP} + T_{CIT} N_{IT} \right) C_H
\]

(3-25)

\[
\text{Loading Cost} = \left( N_p + \left( N_c + \left( \frac{N_{IT}}{n_{itc}} \right)^+ \right) \right) L_{T_p}
\]

(3-26)

The loading cost is considered per order but is not combined with the total cost of a pallet, because though the loading is done completely in the form of pallets, these pallets are comprised of pallets that have been picked from the pallet storage area and pallets that have been formed by combining the cartons and the items that have been picked for the order. Therefore it is a cost for order, not exclusively for the pallets.
This cost is directly considered a part of the total warehouse cost for that order and consequently for the corresponding retailer.

### 3.4 Optimization Model

The model in this research analyzes the relationship between the warehouse and the retailer with the aid of the EOQ model as stated in the earlier sections. The optimization for this modified EOQ model was done performed using the equations explained in Sections 3.2 and 3.3.

#### 3.4.1 The Objective Function

The objective function of the model minimizes the total cost incurred by the retailer during the process of ordering and receiving the products from the warehouse. This objective function is shown in Equation 3-27.

\[
TC = IC + OC
\]  
(3-27)

where,

\[TC = \text{Total Cost for the retailer}\]

\[IC = \text{Inventory cost for each unit per week}\]

\[OC = \text{Order cost}\]
The inventory carrying costs and the ordering costs are calculated while considering the multiple SKUs that the retailer orders as shown in Equations 3-28 and 3-29.

\[
IC = \left( \frac{\sum_{j=1}^{n} Q_j}{2} \right) \cdot HC
\]  

(3-28)

and

\[
OC = \left( \frac{\sum_{j=1}^{n} D_j}{\sum_{j=1}^{n} Q_j} \right) \cdot (WHC + TRC)
\]  

(3-29)

where,

- \( Q_j \) = Order Quantity for SKU \( j \)
- \( D_j \) = Demand/period for SKU \( j \)
- \( j \) = Index to indicate the SKU

The Order Cost for the retailer can also be written as a function of the time between orders and the Order Handling Cost. This is shown in Equation 3-30.

\[
OC = \frac{1}{T} \cdot (WHC + TRC)
\]  

(3-30)

where,

- \( T \) = Time between orders for a given retailer
Since the model in this research is dealing with multiple SKUs, the order frequency was considered a decision variable instead of the order quantity. If the order quantity were to be considered, then it would be for each SKU separately. This would then result in a different order frequency for each SKU and the warehouse would have to deliver different SKUs to the retailer at different times. This kind of order fulfillment would only be useful if there is demand variation in the SKUs to such an extent that some SKUs would have to be delivered more frequently than the other SKUs.

Hence, it is reasonable to assume that the order frequencies for each of the SKUs is the same and thus use the Time between orders ($T^*$) as the decision variable and calculate the Order Quantity ($Q_j^*$) from this value. The model is therefore optimized with respect to the time between orders ($T^*$). The OQ for each SKU for a retailer is then determined from the time between orders that is determined by the optimization model as shown in Equation 3-31.

$$Q_j^* = D_j T^*$$

(3-31)

where,

$Q_j^*$ = Economic Order Quantity for SKU $j$

$D_j$ = Demand for SKU $j$

$T^*$ = Time between orders (Economic) for a retailer
The order handling cost is the cost incurred to pick, pack and load the orders on to the vehicles and ship them to the retailers. Therefore this cost, shown in equation 3-30, is composed of the order fulfillment cost incurred at the warehouse and the transportation cost incurred in delivering the products to the retailer. The order fulfillment costs are obtained by combining Equations 3-23, 3-24 and 3-25 as shown in Equation 3-32.

\[ WHC = C_p + C_C + C_{IT} \]  
(3-32)

Substituting the values for the Cost/order of the unit loads from equations 3-23, 3-24 and 3.25 in equation 3-32, Equation 3-33 is obtained.

\[ WHC_i = \left( \frac{E[D_p]}{V} + T_{RP} \right) \cdot N_{jp} + \left( N_{js} + \left( \frac{N_{js}}{n_{ITC}} \right)^+ \right) \cdot T_{IT} + \left( \frac{N_{jC} + \left( \frac{N_{jC}}{n_{ITC}} \right)^+}{n_{CP}} \right) \cdot T_{CP} \cdot C_H + \left( \frac{E[D_C]}{V_{WC}} + T_{RC} \cdot N_{jc} + \left( \frac{N_{jC} + \left( \frac{N_{jC}}{n_{ITC}} \right)^+}{n_{CP}} \right) \cdot T_{WRAP} + T_{CP} \cdot \left( \frac{N_{jC} + \left( \frac{N_{jC}}{n_{ITC}} \right)^+}{n_{ITC}} \right) \cdot C_H \]  
(3-33)

The transportation cost is calculated as per Equation 3-8 in Section 3.2.2. Each of the terms in Equation 3-31, are calculated as per the equations detailed in Section 3.3.
3.4.2 Constraints

Since the model used in this research is an EOF model, the constraints of the model remain the same as those used for the classic EOQ model. All the values in the model should result in positive numbers hence; the non-negativity constraints are enforced as shown in Equations 3-34, 3-35, 3-36, 3-37 and 3-38.

\[ T^* \geq 0 \quad (3-34) \]
\[ Q_j \geq 0 \quad (3-35) \]
\[ N_{jp} \geq 0 \quad (3-36) \]
\[ N_{jc} \geq 0 \quad (3-37) \]
\[ N_{jt} \geq 0 \quad (3-38) \]

\[ \forall j = 1 \text{ to } n \]

where,

\[ T_i \] = Time between orders for the retailer

\[ Q_j \] = Order Quantity for SKU \( j \)

\[ N_{jp} \] = Number of Pallets of SKU \( j \)

\[ N_{jc} \] = Number of Cartons of SKU \( j \)

\[ N_{jt} \] = Number of Items of SKU \( j \)
Since the order quantity in the model is represented as a combination of unit loads, i.e. pallets, cartons and items, the variables representing each of these quantities also are restricted to integer as shown in Equation 3-39, 3-40 and 3-41.

\[ N_{jp} = \text{Integer} \quad (3-39) \]

\[ N_{jc} = \text{Integer} \quad (3-40) \]

\[ N_{jt} = \text{Integer} \quad (3-41) \]

\[ \forall j = 1 \text{ to } n \]

Determining the number of pallets, cartons and items needed to fulfill an order is based on the number of items held on a pallet and the number of items held in a carton. Once these quantities are determined it is also essential to ensure that the number of items on pallets, the number of items in cartons and the individual items are equal to the order quantity. These two constraints can be combined into a single expression and form one of the constraints of this model as shown in Equation 3-42.

\[ N_{jp} n_{cp} + N_{jc} n_{tc} + N_{jt} = Q_j \quad (3-42) \]

where,

\[ n_{cp} = \text{Number of Items on a Pallet} \]

\[ n_{tc} = \text{Number of Items in a Carton} \]
3.4.3 **Execution of the Model**

The model described above is a non-linear integer-programming model. The Solver in Excel was used to run this model initially. The model was executed with different values of holding costs used to study the behavior of the model in different conditions.

Since the model being analyzed dealt with order quantities of different SKUs as well as different unit loads, the costs involved in this model changed according to the number of unit loads in the order. For a specific holding cost and constant demand, as the time between orders increased, the order quantities increased. As is typical of an inventory model, this caused an increase in the inventory costs incurred and a decrease in the ordering costs because of the decrease in the number of trips to and from the warehouse to the retailers. The total cost, which is a sum of both the above-mentioned costs, decreased gradually until it reached a minimum and then increased.

The Excel Solver failed to provide a reliable answer to the optimization and hence a manual optimization technique was adopted for execution of this model. The model was then run for different values of holding costs and demands. The decision variable $T$ was varied in steps of 0.1 time periods from 0.1 to 10 and then in steps of 0.01 time periods from 0.01 to 5.5, to study the behavior in a detailed manner.

Results were thus obtained for all the cases mentioned above. The number of iterations required for the analysis was too tedious to be done manually. Therefore, Excel Macros
were used to automate the analysis process. The Macros were coded as generically as possible so as to be able to test different scenarios and therefore study the situation under different conditions.

3.5 Comparison of the EOQ Model and the EOF Model

It would have been ideal if the model developed in this research could be compared with the classic EOQ model to test its accuracy in obtaining the order frequencies and correspondingly the order quantities. However, it was very difficult to determine the ideal cost per order to be used for the model.

In typical manufacturing situations, the cost per order is assumed to be a variable percentage of the purchasing cost of the items. Since there were different SKUs here, a different percentage would have to be used for each of them. This need not be absolutely necessary but would have given a good estimate of the cost per order in a real-world situation. Once the cost per order was obtained, this would be used in a cumulative classic EOQ model with a specified holding cost to find the optimal cumulative order quantity and correspondingly the order frequency.

This order frequency would then have to be substituted into the modified EOQ model developed in this research with the holding cost used to obtain the individual order quantities and the unit loads for each of the order quantities. The cost per order would then be determined from this.
This was tested using an assumed purchase cost for each of the SKUs. When this procedure was used, the cost per order obtained from the classic EOQ proved too high to be compared to the one obtained by the modified EOQ. Thus, obtaining the correct purchase cost and the right percentage that would provide the correct numbers for the testing proved very hard.

Hence comparing this modified EOQ model with the classic EOQ proved to be too complicated to be included in this research.
4 RESULTS

4.1 Warehouse Model used for Testing

To evaluate the model developed in this research it was tested with a typical scenario. Therefore a rectangular warehouse was considered as shown in Figure 3-1. The warehouse has designated areas for pallet, carton and item storage and handles 100 SKUs. The dimensions used for the analysis were selected from typically used warehouse parameters. These dimensions were used to draw the warehouse, shown Figure 4-1, in FactoryCAD.

These dimensions were then used to calculate the other quantities needed for the analysis. Single deep pallet racks were used to store both pallets and cartons. The items were also stored in single deep pallet racks that were of a smaller size to accommodate cartons instead of pallets.
4.1.1 Expected Distance Calculation for Pallet Picking

Pallet picking is performed with the aid of forklifts. It is assumed that each forklift can carry only one pallet at a time. When a worker is given the pick list to use in the pallet picking area, he drives the forklift to the appropriate location, retrieves the pallet and then takes it back to the staging area. Therefore, every pallet will need a trip from the forklift dock to the pallet location and then to the staging area.
The pallet storage area in this warehouse has single deep pallet racks arranged in 5 aisles, each aisle having 5 high racks on either sides. Therefore, the entire pallet-pick area stores 5 pallets of each SKU (stacked 5 high).

Figure 4-2 shows the dimensions of this pallet storage area followed by the calculation of the expected pallet pick distance. Table 4-1 contains all the values used in the pallet area.
\[ E[D_p] = 2 \left[ d_p + \frac{A_p - 1}{2} X_p + \frac{L_p}{2} + \frac{W_p}{2} + e_p \right] \] (from Equation 3-13)

Table 4-1: Parameters in the Pallet Storage Area

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Aisles:</td>
<td>( A_p )</td>
<td>5</td>
</tr>
<tr>
<td>Length of each Aisle:</td>
<td>( L_p )</td>
<td>50 ft</td>
</tr>
<tr>
<td>Offset from the Aisle to path:</td>
<td>( e_p )</td>
<td>5.5 ft</td>
</tr>
<tr>
<td>Distance from the entrance to the edge of the first aisle:</td>
<td>( d_p )</td>
<td>10 ft</td>
</tr>
<tr>
<td>Center to center distance:</td>
<td>( X_p )</td>
<td>18 ft</td>
</tr>
<tr>
<td>Width of the aisle:</td>
<td>( W_p )</td>
<td>10 ft</td>
</tr>
</tbody>
</table>

Using the numbers in Table 4-1,

\[ E[D_p] = 2 \left[ 10 + \frac{5 - 1}{2} \times 18 + \frac{50}{2} + \frac{10}{2} + 5.5 \right] = 163 \text{ feet} \]

4.1.2 Expected Distance Calculation for Carton Picking

Carton picking is done by workers who locate the carton required in the pick list, retrieve the quantity needed and then place it on the conveyor. This conveyor carries the cartons to the staging area. Since each order is assumed to be done separately, the worker assigned to a particular pick list will travel from aisle to aisle locating and placing the cartons on the conveyor, until all the cartons in an order, are picked. Once the workers are done they will walk back to the staging area as indicated by Figure 4-3.
The carton storage area assumed in this testing also consists of single-deep pallet rack arranged in 5 aisles, each aisle having 5 pick faces on the floor on either side. Thus the carton storage area has one pallet of each SKU and the pickers pick the required number of cartons from the pallet. Once a pallet on the floor has been completely picked, a reserve pallet (which is stacked above the floor pallets) is lowered and the pickers pick from the new pallet. Figure 4-3 shows the dimensions of this carton storage area followed by the calculation of the expected carton pick distance for an order. The distances are in Table 4-2.
\[ E[D_C] = 2d_c + A_c[2L_c + 4e_c + X_c] \] (from Equation 3-14).

**Table 4-2: Parameters in the Carton Storage Area**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Variable</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Aisles:</td>
<td>( A_c )</td>
<td>5</td>
</tr>
<tr>
<td>Length of each Aisle:</td>
<td>( L_c )</td>
<td>25 ft</td>
</tr>
<tr>
<td>Offset from the Aisle to path:</td>
<td>( e_c )</td>
<td>2.5 ft</td>
</tr>
<tr>
<td>Distance from the entrance to the edge of the first aisle:</td>
<td>( d_c )</td>
<td>10 ft</td>
</tr>
<tr>
<td>Center to center distance:</td>
<td>( X_c )</td>
<td>18 ft</td>
</tr>
</tbody>
</table>

Using the distances from Table 4-2,

\[ E[D_C] = 2(10) + 5[2(25) + 4(2.5) + 18] = 410 \text{ feet} \]

### 4.1.3 Expected Distance Calculation for Item Picking

Items are picked with the aid of a worker and a cart. The worker locates the SKUs that are needed and then retrieves the correct quantity of items off the racks and then puts them in the cart. The item storage area consists of single deep racks arranged in 5 aisles similar to the other two storage areas. Since the aisle in the item storage area is smaller than the aisle in the carton picking area and there is no conveyor the worker can pick items from both locations on both sides of the aisle. This reduced the travel required since the worker needs to walk only once through the aisle to get all the items in the aisle.
The traversal method of picking cannot be used to pick cartons in the layout that is shown in Figure 4-3. As in the case of picking cartons, the worker goes from aisle to aisle retrieving the items needed in an order and then puts them in a cart. Once the items in the order are picked, the worker takes them to the staging area, where they may be packed for transportation.

Once the cartons in the racks have been emptied, they are replenished from the reserve area, typically during the third shift. Figure 4-4 shows the dimensions of this item storage area followed by the calculation of the expected item pick distance for an order. The distances are shown in Table 4-3.
Table 4-3: Parameters in the Item Storage Area

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Variable</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Aisles:</td>
<td>$A_{IT}$</td>
<td>5</td>
</tr>
<tr>
<td>Length of each Aisle:</td>
<td>$L_{IT}$</td>
<td>12.9 ft</td>
</tr>
<tr>
<td>Offset from the Aisle to path:</td>
<td>$e_{IT}$</td>
<td>1.5 ft</td>
</tr>
<tr>
<td>Distance from the entrance to the edge of the first aisle:</td>
<td>$d_{IT}$</td>
<td>10 ft</td>
</tr>
<tr>
<td>Center to center distance:</td>
<td>$X_{IT}$</td>
<td>8 ft</td>
</tr>
</tbody>
</table>

\[ E[D_{IT}] = 2d_{IT} + A_{IT} [L_{IT} + 4e_{IT} + X_{IT}] \] (from Equation 3-15)

Using the numbers from the above table,

\[ E[D_{IT}] = 2(10) + 5[12.9 + 4(1.5) + 8] = 154.58 \text{ feet} \]

Note; the distances that have been calculated thus far are only expected distances and not the actual distances, which will be determined by the location of SKUs in an order. These distances are only used to provide an estimate of how much the pickers have to travel.

Once all the pallets, cartons and items of an order are collected in the staging area, the cartons and items are accommodated on pallets and the retailer’s order is packed, loaded on the corresponding vehicle(s) and then transported. Apart from the retrieval costs depicted in the tables shown above, a number of other parameters had to be either
estimated or determined based on typical warehouse and retailer operations. These values have been split into multiple sections to explain them in a piecewise manner.

4.1.4 Retailer Parameters

The number of SKUs ordered by the retailer has been limited to a small value of 10 SKUs, to simplify the analysis of the problem. The demand for each of the SKUs ordered was assumed to be random and normally distributed with expected values as shown in Table 4-4. Standard deviation used was 5%.

<table>
<thead>
<tr>
<th>SKU</th>
<th>Demand per week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500 items</td>
</tr>
<tr>
<td>2</td>
<td>650 items</td>
</tr>
<tr>
<td>3</td>
<td>375 items</td>
</tr>
<tr>
<td>4</td>
<td>450 items</td>
</tr>
<tr>
<td>5</td>
<td>400 items</td>
</tr>
<tr>
<td>6</td>
<td>580 items</td>
</tr>
<tr>
<td>7</td>
<td>750 items</td>
</tr>
<tr>
<td>8</td>
<td>300 items</td>
</tr>
<tr>
<td>9</td>
<td>600 items</td>
</tr>
<tr>
<td>10</td>
<td>700 items</td>
</tr>
</tbody>
</table>

The retailer was taken to be at a distance of 200 miles from the warehouse. This research discusses the interaction of the warehouse with only a single retailer and therefore the cost of transportation cannot be decreased, as would be the case with a multiple retailer scenario where vehicle routing would also be considered. In that case the transportation
cost would not necessarily be in direct relation with the distance of the retailer from the warehouse. But this model can be extended to include the vehicle routing algorithm and the transportation cost could change depending on that. In general, trips to a retailer who is closer would always be less expensive than one who is further away.

4.1.5 Warehouse Parameters

Typical values were used for warehouse parameters as shown in Table 4-5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle Capacity</td>
<td>$V_{\text{CAP}}$</td>
<td>40 pallet loads</td>
</tr>
<tr>
<td>Number of Cartons/pallet</td>
<td>$n_{\text{cp}}$</td>
<td>10</td>
</tr>
<tr>
<td>Number of Items/carton</td>
<td>$n_{\text{itc}}$</td>
<td>10</td>
</tr>
<tr>
<td>Hourly wage</td>
<td>$C_{\text{H}}$</td>
<td>$10/hr$</td>
</tr>
<tr>
<td>Retrieve time (pallets)</td>
<td>$T_{\text{RP}}$</td>
<td>20 sec</td>
</tr>
<tr>
<td>Retrieve time (cartons)</td>
<td>$T_{\text{RC}}$</td>
<td>8 sec</td>
</tr>
<tr>
<td>Retrieve time (items)</td>
<td>$T_{\text{RIT}}$</td>
<td>6 sec</td>
</tr>
<tr>
<td>Picking speed for a pallet (forklift speed)</td>
<td>$V$</td>
<td>4 mph</td>
</tr>
<tr>
<td>Picking speed for a carton (worker speed)</td>
<td>$V_{\text{WC}}$</td>
<td>1.705 mph</td>
</tr>
<tr>
<td>Picking speed for an item (worker speed)</td>
<td>$V_{\text{WIT}}$</td>
<td>1.36 mph</td>
</tr>
<tr>
<td>Time to place cartons on a pallet (labor time)</td>
<td>$T_{\text{CP}}$</td>
<td>15 sec</td>
</tr>
<tr>
<td>Time to place an item in a carton (labor time)</td>
<td>$T_{\text{ITC}}$</td>
<td>4 sec</td>
</tr>
<tr>
<td>Time to wrap a pallet (labor time)</td>
<td>$T_{\text{WRAP}}$</td>
<td>60 sec</td>
</tr>
<tr>
<td>Time to wrap a carton (labor time)</td>
<td>$T_{\text{WRC}}$</td>
<td>5 sec</td>
</tr>
<tr>
<td>Time to load a pallet onto a truck (labor time)</td>
<td>$T_{\text{LOAD}}$</td>
<td>3 min</td>
</tr>
</tbody>
</table>

In order to calculate the packing and the loading costs for the orders, equations 3-19 to 3-22 were used. The values shown in Table 4-5 were substituted and the corresponding costs were calculated. These calculated costs are shown below.
Packing costs = \( PaC_P + PaC_C + PaC_{IT} \)

Substituting the values from Table 4-5 in equations 3-19 to 3-21,

\[
\text{Packing costs} = \left( \frac{N_{\text{Cartons}}}{10} + \frac{N_{\text{items}}}{10} \right)^+ \left( \frac{60}{3600} \right) + \left( \frac{15}{3600} \right) \left( \frac{N_{\text{Cartons}}}{10} + \frac{N_{\text{items}}}{10} \right)^+ \left( \$10 \right) + \left( \frac{N_{\text{items}}}{10} \right)^+ \left( \frac{5}{3600} \right) + \left( \frac{4}{3600} \right) \cdot N_{\text{items}} \left( \$10 \right)
\]

Loading Costs = \# Pallets. \( LT_P \cdot C_H \)

Substituting the values from Table 4-4 in equation 3-22,

\[
\text{Loading Costs} = \left( \frac{N_{\text{Pallets}}}{10} + \frac{N_{\text{items}}}{10} \right)^+ \left( \frac{3}{60} \right) \left( \$10 \right)
\]

Therefore, the warehouse costs can be calculated by adding all these costs.

Transportation costs are calculated based on the distance of the retailer from the warehouse and the number of vehicles needed to fulfill the order for that particular retailer. Equation 3-8 is used to compute the transportation costs for the retailers.
4.1.6 Transportation Parameters

Some typical transportation values are shown in Table 4-6. These values were calculated by utilizing a company website [44], which allows the user to calculate the approximate transportation cost from one city to another depending on distance and load to be transported. This company provides freight, trucking and shipping services throughout USA.

Table 4-6: Transportation values used for testing

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent Cost per vehicle</td>
<td>$C_V$</td>
<td>$195</td>
</tr>
<tr>
<td>Mileage</td>
<td>$C_m$</td>
<td>$1.53 per mile</td>
</tr>
</tbody>
</table>

4.2 Testing with Excel Solver and Manual Optimization

As stated in Section 3.4.3, this model is a non-linear model. Excel Solver was initially used to test the model with the values given in Section 4.1. Unfortunately this optimization technique was not able to solve the model satisfactorily. The demand was kept constant while the model was optimized for values of time between orders ($T$), but this resulted in different minimum values for each run.

The reason for this was that the optimization of the model was resulting in a local optimum because of the change in unit loads and the change in the number of trucks used for transportation. This local optimum changed each time the optimization was performed and thus Solver was considered inappropriate as a solution technique for this
model. A manual optimization was then considered and implemented as stated in Section 3.4.3.

4.2.1 Optimal Order Frequency

As an initial test of the manual optimization a single holding cost was considered to study the behavior of the entire EOQ curve with the modifications described in Chapter 3. This single holding cost of $0.75/item/week was chosen because it is a reasonable estimate for a weekly holding cost per item. For this cost, the ordering costs and the inventory costs were calculated and graphed against time between orders and order quantity. These graphs are shown in Figure 4-5 and Figure 4-6.

![Figure 4-5: Time between orders against Cost when HC = $0.75/item/week.](image-url)
The graphs in both the figures are the same, which shows the appropriateness of choosing the time period, $T$ as the decision variable. (Traditional EOQ on the other hand uses order quantity $Q$ as the decision variable). Once again the steps in the curve depict the quantities where the trucks were increased. The optimal time period $T$ and corresponding cumulative $Q$ are shown in Table 4-7.
Table 4-7: Optimal order sizes for HC = $0.75/item/week

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>5246 items/week (cumulative)</td>
</tr>
<tr>
<td>HC</td>
<td>$0.75</td>
</tr>
<tr>
<td>$T^*$</td>
<td>0.49 weeks</td>
</tr>
<tr>
<td>Minimum Cost</td>
<td>$2,029.23</td>
</tr>
<tr>
<td>$Q^<em>$ (from $T^</em>$)</td>
<td>2571 Items/week (cumulative)</td>
</tr>
</tbody>
</table>

As Figures 4-5 and 4-6 show, the holding cost has a great impact on the total cost incurred by the retailers. When the holding cost is increased, the total cost also increases. This change in the holding cost also increases the rate at which the total cost increases which can be seen from the curve.

4.2.2 Changes in Total Cost with respect to Change in Unit Loads

In order to see the change in the total cost at the unit loads changed it was necessary to scrutinize the total cost curve as shown in Figure 4-7. The clear trend in the change in the total cost can be seen in Table 4-8. This is only a small section of the total cost which has been used to depict the change in the total cost. Such trends occur throughout the graph.
Figure 4-7: Decrease in total cost (as unit loads change) when HC = 0.75/item/week

Table 4-8: Change in total cost with change in unit load

<table>
<thead>
<tr>
<th>T (weeks)</th>
<th>OQ (#items)</th>
<th>TC</th>
<th>#Pallets</th>
<th>#Cartons</th>
<th>#Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.11</td>
<td>577</td>
<td>$4,862</td>
<td>0</td>
<td>52</td>
<td>57</td>
</tr>
<tr>
<td>0.12</td>
<td>630</td>
<td>$4,499</td>
<td>0</td>
<td>57</td>
<td>60</td>
</tr>
<tr>
<td>0.13</td>
<td>683</td>
<td>$4,193</td>
<td>0</td>
<td>64</td>
<td>43</td>
</tr>
<tr>
<td>0.14</td>
<td>734</td>
<td>$3,931</td>
<td>1</td>
<td>59</td>
<td>44</td>
</tr>
<tr>
<td>0.15</td>
<td>788</td>
<td>$3,708</td>
<td>2</td>
<td>54</td>
<td>48</td>
</tr>
<tr>
<td>0.16</td>
<td>840</td>
<td>$3,517</td>
<td>2</td>
<td>60</td>
<td>40</td>
</tr>
</tbody>
</table>

TC = Total Cost
OQ = Cumulative Order Quantity/week
D = Cumulative Demand/week
T = Time period
HC = Holding Cost/week
It is apparent from Table 4-8 that as the order quantity increases, the unit load changes from items to cartons and then to pallets. When this happens the cost reduces because picking a pallet would involve picking 100 items in one trip rather than traveling to the items storage area and picking 100 individual items (of the same SKU). Of course, this is only possible if the order is large enough for the retailer to need a whole pallet of one SKU. Similarly, it is less expensive to pick a carton because this is would involve picking 10 items simultaneously.

The difference in retrieving and preparing pallets when compared to cartons or items can be seen very clearly from the example shown below. For a given time period, \( T \) (in this case, \( T = 0.12 \) weeks), the difference in warehouse costs (i.e. retrieval + packing + loading costs) when the workers have to pick 99 items (as 9 cartons and 9 items) and when the workers have to pick 100 items (in 1 pallet) is shown in Table 4-9.

<table>
<thead>
<tr>
<th>Time period (T)</th>
<th># items in order</th>
<th>Pallets</th>
<th>Cartons</th>
<th>Items</th>
<th>Warehouse Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12 weeks</td>
<td>99</td>
<td>0</td>
<td>9</td>
<td>9</td>
<td>$2.25</td>
</tr>
<tr>
<td>0.12 weeks</td>
<td>100</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$0.63</td>
</tr>
</tbody>
</table>

Thus, Table 4-9 shows clearly that picking pallets are beneficial to the retailer in terms of cost and beneficial to the warehouse in terms of work effort. But one aspect that has to be considered is that the advantage of picking pallets and/or cartons can be utilized
when the order for the retailer involves large quantities of the same SKU. When the number of SKUs increases, picking pallets requires multiple trips to different SKU locations while picking cartons of different SKUs can be performed in the same trip. Therefore, this steep difference in warehouse costs will reduce as the SKUs in the order increase and there maybe some situations where picking cartons may be preferable to picking pallets in order to reduce the extra travel.

One aspect that has to be considered is that the retailers will order quantities that are greater than what they require if the cost of holding the products at the store is less expensive when compared to the cost of ordering the products from the warehouse. However when the holding cost is greater than what it costs for the retailer to order from the warehouse, the retailer will order as frequently as necessary. Thus the trade off between the holding cost and the order cost is what drives the order frequency and correspondingly the order quantity.

4.3 Testing the EOF Model for Different values of Holding Cost and Time Period ($T$)

To test other aspects of the model, the time period, $T$ was varied in steps of 0.1 weeks from $T = 0$ to 10 weeks, and then in steps of 0.01 weeks from $T = 0$ to 3 weeks for different changing values of holding cost since this was the main parameter that would determine the reorder time and quantity. Detailed optimization was then performed by varying time period $T$ from 0.01 week to 5.5 weeks in steps of 0.01 week.
The total cost curves for both ranges of $T$ are shown in Figure 4-8 and Figure 4-9, respectively. Table 4-10 and Table 4-11 that follow the above-mentioned figures explicitly tabulate the optimal time periods ($T^*$) with the corresponding cumulative order quantities, weekly demand and the total cost for each of these time periods. The order quantity per SKU is also shown.

Figure 4-8: Total Cost when $T$ is in steps of 0.1 week and HC is varied
Table 4-10: Least Cost and the corresponding Order Quantities and Time period

<table>
<thead>
<tr>
<th>HC</th>
<th>$0.01</th>
<th>$0.05</th>
<th>$0.10</th>
<th>$0.50</th>
<th>$1</th>
<th>$2.5</th>
<th>$5</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min Cost</td>
<td>$744</td>
<td>$847</td>
<td>$939</td>
<td>$1,663</td>
<td>$2,348</td>
<td>$3,686</td>
<td>$5,186</td>
<td>-</td>
</tr>
<tr>
<td>$T^* (wk)</td>
<td>1.5</td>
<td>0.7</td>
<td>0.7</td>
<td>0.6</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>OQ</td>
<td>7869</td>
<td>3672</td>
<td>3672</td>
<td>3148</td>
<td>2098</td>
<td>1574</td>
<td>1049</td>
<td>-</td>
</tr>
<tr>
<td>SKU1</td>
<td>768</td>
<td>358</td>
<td>358</td>
<td>307</td>
<td>205</td>
<td>154</td>
<td>102</td>
<td>512</td>
</tr>
<tr>
<td>SKU2</td>
<td>936</td>
<td>437</td>
<td>437</td>
<td>374</td>
<td>250</td>
<td>187</td>
<td>125</td>
<td>624</td>
</tr>
<tr>
<td>SKU3</td>
<td>585</td>
<td>273</td>
<td>273</td>
<td>234</td>
<td>156</td>
<td>117</td>
<td>78</td>
<td>390</td>
</tr>
<tr>
<td>SKU4</td>
<td>666</td>
<td>311</td>
<td>311</td>
<td>266</td>
<td>178</td>
<td>133</td>
<td>89</td>
<td>444</td>
</tr>
<tr>
<td>SKU5</td>
<td>620</td>
<td>289</td>
<td>289</td>
<td>248</td>
<td>165</td>
<td>124</td>
<td>83</td>
<td>413</td>
</tr>
<tr>
<td>SKU6</td>
<td>804</td>
<td>375</td>
<td>375</td>
<td>322</td>
<td>214</td>
<td>161</td>
<td>107</td>
<td>536</td>
</tr>
<tr>
<td>SKU7</td>
<td>1107</td>
<td>517</td>
<td>517</td>
<td>443</td>
<td>295</td>
<td>221</td>
<td>148</td>
<td>738</td>
</tr>
<tr>
<td>SKU8</td>
<td>474</td>
<td>221</td>
<td>221</td>
<td>190</td>
<td>126</td>
<td>95</td>
<td>63</td>
<td>316</td>
</tr>
<tr>
<td>SKU9</td>
<td>858</td>
<td>400</td>
<td>400</td>
<td>343</td>
<td>229</td>
<td>172</td>
<td>114</td>
<td>572</td>
</tr>
<tr>
<td>SKU10</td>
<td>1052</td>
<td>491</td>
<td>491</td>
<td>421</td>
<td>280</td>
<td>210</td>
<td>140</td>
<td>701</td>
</tr>
</tbody>
</table>

Figure 4-9: Total Cost when T is in steps of 0.01 week and HC is varied

Holding Cost
/item/week

- $0.01
- $0.05
- $0.1
- $0.5
- $1
- $2.5
- $5
Table 4-11: Least Cost and the corresponding Order Quantities and Time period

<table>
<thead>
<tr>
<th>HC</th>
<th>$0.01</th>
<th>$0.05</th>
<th>$0.1</th>
<th>$0.5</th>
<th>$1</th>
<th>$2.5</th>
<th>$5</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min Cost</td>
<td>719</td>
<td>798</td>
<td>898</td>
<td>1,663</td>
<td>2,337</td>
<td>3,676</td>
<td>5,186</td>
<td>-</td>
</tr>
<tr>
<td>T* (wk)</td>
<td>0.76</td>
<td>0.76</td>
<td>0.76</td>
<td>0.61</td>
<td>0.43</td>
<td>0.28</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>OQ</td>
<td>3986</td>
<td>3986</td>
<td>3986</td>
<td>3201</td>
<td>2256</td>
<td>1470</td>
<td>1049</td>
<td>-</td>
</tr>
<tr>
<td>SKU1</td>
<td>389</td>
<td>389</td>
<td>389</td>
<td>312</td>
<td>220</td>
<td>143</td>
<td>102</td>
<td>512</td>
</tr>
<tr>
<td>SKU2</td>
<td>474</td>
<td>474</td>
<td>474</td>
<td>381</td>
<td>268</td>
<td>175</td>
<td>125</td>
<td>624</td>
</tr>
<tr>
<td>SKU3</td>
<td>296</td>
<td>296</td>
<td>296</td>
<td>238</td>
<td>168</td>
<td>110</td>
<td>78</td>
<td>390</td>
</tr>
<tr>
<td>SKU4</td>
<td>337</td>
<td>337</td>
<td>337</td>
<td>271</td>
<td>191</td>
<td>124</td>
<td>89</td>
<td>444</td>
</tr>
<tr>
<td>SKU5</td>
<td>314</td>
<td>314</td>
<td>314</td>
<td>252</td>
<td>178</td>
<td>116</td>
<td>83</td>
<td>413</td>
</tr>
<tr>
<td>SKU6</td>
<td>407</td>
<td>407</td>
<td>407</td>
<td>327</td>
<td>231</td>
<td>150</td>
<td>107</td>
<td>536</td>
</tr>
<tr>
<td>SKU7</td>
<td>561</td>
<td>561</td>
<td>561</td>
<td>450</td>
<td>317</td>
<td>207</td>
<td>148</td>
<td>738</td>
</tr>
<tr>
<td>SKU8</td>
<td>240</td>
<td>240</td>
<td>240</td>
<td>193</td>
<td>136</td>
<td>89</td>
<td>63</td>
<td>316</td>
</tr>
<tr>
<td>SKU9</td>
<td>435</td>
<td>435</td>
<td>435</td>
<td>349</td>
<td>246</td>
<td>160</td>
<td>114</td>
<td>572</td>
</tr>
<tr>
<td>SKU10</td>
<td>533</td>
<td>533</td>
<td>533</td>
<td>428</td>
<td>301</td>
<td>196</td>
<td>140</td>
<td>701</td>
</tr>
</tbody>
</table>

The steps seen in Figure 4-9 indicate the increase in the number of trucks needed to transport the order. Since the transportation cost is a multiple of the number of trucks needed, there is a sudden increase in the cost when the order quantity requires more trucks for transportation. When the $T^*$ values in Table 4-10 and Table 4-11 are observed, there seems to be a large difference in $T^*$ when $T$ is in steps of 0.1 weeks and 0.01 weeks, while this difference does not occur for the other values of $T^*$. This occurs because, the minimum when $T$ is in steps of 0.1 weeks occurs at 1.5 weeks and the minimum cost at that point is $744.

If the pattern for the next values of $T^*$ were to be taken as a standard, then the minimum should occur at $T = 0.7$ weeks. The minimum cost at this point is $774. When $T$ is
studied in steps 0.01 weeks, $T^*$ occurs at 0.76 weeks where the minimum cost is $719. If the minimum cost at $T = 1.52$ weeks is observed, it is $736. Therefore the costs at these values are all close and what seems like a discrepancy is actually consistency with some variation.

Figure 4-10 shows the change in the time between orders as the holding cost increases. As is expected the time between orders decreases as the high holding cost makes it expensive for the retailer to hold products. Thus the retailer begins to order products more often because it is less expensive to order. It can also be seen that as the time between orders decreases and tends to zero, the rate at which the total cost decreases and slowly levels out, asymptotically tending to infinity, indicating that in the hypothetical situation where the holding cost was zero, the retailer would order and store an infinite amount of products.
The sudden increase in the cost when the number of trucks required for transporting the order quantity is also seen in Figure 4-10. In keeping with the conclusion that can be drawn from the slowly decreasing and leveling out pattern of the time between orders, it can be seen that when the holding cost is very low such as $0.01 and $0.05 shown in Figure 4-10, the order quantity is higher than the weekly demand. But, despite the fact the order quantity is high enough to warrant two trucks which basically doubles the transportation cost and increases the ordering cost, the total cost is actually less than the order handling cost because of the $D/Q$ ratio that is used in calculating the total cost. This also demonstrates the fact that if the holding costs at the retailer are very low relative to the ordering cost, the retailer would order a large amount initially, and thus
reduce the order frequency to a minimum value. The Excel sheet and the macros used to run the model to obtain the results shown in this Section and in Section 4.2 are included in Appendix A and Appendix B.

4.4 Effect of the Different Cost Components on the Total Cost

The holding cost at the retailer has a large impact on the total cost and hence the order quantity for the retailer. Since the order handling cost in this modified EOQ model is comprised of the warehouse cost and the transportation cost, these would also have an effect on the total cost. In order to study the extent of the effects of these costs, fractional factorial design was considered.

Five factors were chosen that were expected to have a prominent effect on the total cost and the values of these factors was tested at predetermined high and low values and the effects of different combinations of these factors on total cost were studied. The factors chosen were holding cost (HC), labor rate (LR), mileage (M), vehicle rental cost (RC) and distance of the retailer from the warehouse (D). The high and low values used for these factors are shown in Table 4-12.
Table 4-12: Notation and High, Low values of factors used for factorial design

<table>
<thead>
<tr>
<th>Factors</th>
<th>Variable</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holding Cost</td>
<td>HC</td>
<td>$0.05/item</td>
<td>$5/item</td>
</tr>
<tr>
<td>Labor Rate</td>
<td>LR</td>
<td>$4/hr</td>
<td>$75/hr</td>
</tr>
<tr>
<td>Mileage</td>
<td>M</td>
<td>$0.6/mile</td>
<td>$4/mile</td>
</tr>
<tr>
<td>Rental Cost</td>
<td>RC</td>
<td>$40</td>
<td>$400</td>
</tr>
<tr>
<td>Distance of the Retailer from the warehouse</td>
<td>D</td>
<td>15 miles</td>
<td>250 miles</td>
</tr>
</tbody>
</table>

The reason to perform a fractional factorial instead of a full factorial was not to perform too many trials. Since there were 5 factors, when a full factorial was performed, there would have been 32 treatment combinations to be tested and each of these needed to be calculated for each value of $T$. Typically, when there are so many factors the higher order interactions tend not to have an effect on the dependent variable and hence it is practical to not include them in this analysis.

Therefore a fractional factorial of $2^{5-1}$ was performed. This required 16 treatments and five replications were carried out. The demand was taken to be normally distributed around random means as shown in Table 4-4. The demands used for the runs were randomly generated. The outputs analyzed were the optimal time between orders ($T^*$) and the minimum total cost for each of the combinations, using MINITAB 14. The reason for studying the effects of the factors on both the output variables (i.e. time and cost) was to ensure that conclusions drawn about the effect that each of these factors has on the model is complete.
It is quite possible that some of the control factors—either alone or in combination with one or more of the other factors—may have an effect on one or both output variables. If such a situation occurs, and if the variable chosen is one that seems not to be affected by the factors, then the conclusions drawn from this result would not be very reliable because they do not look at the effect on the other output variable, which could lead to a totally different conclusion.

However, since both variables (i.e. time and cost) are chosen in this research, if a control factor (alone or in combination with other factors) does not cause a change in the output variables then it is possible to conclude that this factor or combination of factors does not have a significant effect on both the output variables. The input data and the results obtained are shown in detail in Appendix C.

4.4.1 Analysis Results - ANOVA

The ANOVA performed in MINITAB studied the effects of the main factors and the two-way interactions between the factors. As explained in the previous section, the higher order interactions tend not to have an effect and hence were not included. The result of this analysis is shown in the form of an ANOVA in Table 4-13 and Table 4-14.
Table 4-13: ANOVA for Optimal Time Period (T*)

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<tr>
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Table 4-14: ANOVA for Minimum Total Cost

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</table>

Table 4-13 and Table 4-14 show the ANOVA for only the main factors and it is apparent from the p-values of both these tables that all factors except the warehouse cost are statistically significant and affect both T* and minimum total cost when only the effects of the main factors are considered. When the main factors are studied with the two-way interactions, the ANOVA that is obtained is shown in Table 4-14 and Table 4-15.
Table 4-15: ANOVA for single factor and two-way interactions for Optimal Time Period 

\((T^*)\)

<table>
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<tr>
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Table 4-16: ANOVA for single factor and two-way interactions for Minimum Total Cost

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</table>

As can be seen from the p-values in Table 4-15 certain factors and their interactions show a statistically significant effect while others do not. When the p-values in Table 4-16 are studied, the reason why both output variables have to be studied can be seen. When the effect of the factors and their interactions on cost is considered, it can be seen that all the factors and their two-way interactions have a statistically significant effect. Thus varying any of these factors would cause a change in the minimum cost of the model and
therefore these have to be considered when decisions regarding time between orders
and consequently order quantity are made.

4.4.2 Analysis Results – Main Effects and Interaction Plots

The effect of the main factors and the two-way interactions can be seen graphically
using the main effects plots and the interaction plots. The main effects plots are shown
in Figure 4-11 and Figure 4-12.

Figure 4-11: Main Effects Plot for the Factors for Time Period (T)
The graphs displayed in Figure 4-11 and Figure 4-12, corroborate what Table 4-13 and Table 4-14 show. The warehouse cost does not have a significant effect on both the time period (T) and minimum total cost as can be seen by the line which is almost parallel to the X-axis. The other factors have lines with high slopes indicating a significant change in the output variable when their value is changed from a predetermined low to high value.

The interaction graphs shown in figure 4-13 and figure 4-14 depict those relationships that significantly affect optimal time period $T^*$ and minimum total cost.
(a) (b) (c) (d)
Figure 4-13 (a), (b), (c), (d), (e), (f), (g): Interaction plots for the factors for Optimal Time Period ($T^*$)
Figure 4-14: (a), (b), (c), (d), (e), (f), (g), (h), (i), (j): Interaction plots for the factors for Minimum Total Cost

As is apparent from Figure 4-13 and Figure 4-14, all these interactions are significant in affecting the time period and minimum total cost and hence should be considered during ordering decisions.
5 CONCLUSIONS

This research was aimed at analyzing the interaction between a retailer and a distribution center to find the aspects that affect the order frequencies and order quantities of the retailer. The classic inventory model was modified to explicitly include warehousing and transportation cost in the equation instead of a simple ordering cost. The warehouse cost was further split into its individual constituents of retrieval cost, packing cost, and loading cost. This was aimed at studying the effect of these individual costs on the ordering costs and ultimately the total cost of the order considering different unit loads and multiple SKUs.

5.1 Summary of Results

The relationship between the retailer and the warehouse was studied in terms of the ordering frequency between the two. The classic EOQ model was changed to explicitly to specify the warehouse cost and the transportation cost that make up the ordering cost in the EOQ model. The costs in this modified EOQ model were then defined by considering operational aspects of the warehouse and the transportation of the products from the warehouse to the retailer.

The applicability of the EOQ model for a multi-item system was also studied. The classic EOQ is generally used to determine the order quantity for a single item. This would then determine the order frequency or the time between orders for that item. When there is more than one item or SKU as in the case of a warehouse, an order from a retailer will
typically entail orders for multiple different SKUs in different quantities. Thus if the traditional EOQ model was to be applied separately for each of these SKUs, this would result in a different order frequency or time between orders for each SKU. This would not be practical for a warehouse, which has many SKUs to deliver and each would have to be delivered at a different time. Thus the time between the orders was chosen as the decision variable. Thus the time between orders was the same for all the SKUs and for the optimal time between orders \( T^* \), the economic order quantity for each SKU \( Q_j^* \) was equal to the product of the demand and the optimal time between orders \( T^* \).

5.1.1 Optimization Method

The optimization performed in this research considers the time between orders as the decision variable. The Excel Solver was used as a possible optimization tool but unfortunately it did not find an optimal solution to this model because it was getting stuck in a local optimum and could not find a global optimum. To overcome this difficulty, Excel Macros were used to test a range of values.

A macro was written to change the value of the time between orders to find the total cost at each time. A search for the minimum total cost in this range of values resulted in the optimal order frequency or time between orders. This macro is shown in Appendix B and can be used in the future for such similar applications when optimization tools do not produce satisfactory results. The macro uses values from specified cells in the
accompanying Excel sheet to calculate the order quantity and all the associated costs for a specific time period.

5.1.2 Effect of Costs

From the data that was collected and analyzed, the cost of picking items as pallets and cartons is less than picking individual items for one SKU. This would however change as the number of SKUs increased. Thus it would be beneficial to the warehouse to pick and deliver pallets rather than cartons or items. This aspect is also profitable to the retailer because he/she will be able to obtain a larger quantity of products at a lower price than if they had to pay for each on a per item basis. But this depends on the holding cost at the retailers and the transportation cost. If the holding cost is too high, the retailer would prefer to order more frequently rather than storing large quantities and incurring inventory costs, causing more work at the warehouse. Thus a trade off is necessary between these two costs so that a balance can be obtained that is beneficial to both parties.

The relationship that exists between the warehouse costs and the transportation costs is also important to the ordering decision. This model does not consider less than truckloads and therefore irrespective of whether the retailer uses 10% of the truck capacity or 90% of the truck capacity, he/she will have to pay for the entire truck.
Therefore an order quantity that is a little above a truck’s capacity will include an additional truck, which in turn would increase the transportation costs (as a multiple of the number of trucks) and consequently increase the ordering costs and the corresponding minimum total costs as can be seen in Chapter 4. Hence a balance has to be identified between the warehouse costs and the transportation costs as well to ensure that orders are delivered to the retailer at an optimal rate.

From the analysis done using the fractional factorial experiment it is clear that holding cost has the largest influence on the time between the orders and the cost. As the holding cost increased the order frequencies increased, which clearly indicated that the retailer would rather order as frequently rather than storing any item at the store. The main effects plots shown in Section 4-4 corroborate this. But the distance of the retailer from the warehouse and the transportation costs influence the time and the cost in a significant but decreasing order and hence have to be carefully considered during the ordering process.

The warehouse cost has been seen to have no significant effect on the time between orders. This is because the other costs are much higher than the warehouse costs and hence drive the time period ($T^*$) and consequently the minimum total cost. But the warehouse cost has an effect on the cost for the optimal order quantity, which indicates that for the same quantity of items, the retailers have to pay more for a given time period as the labor costs increase.
The fractional factorial experiment also shows the effect of the other factors chosen such as mileage, rental cost of the vehicle and distance of the retailer from the warehouse. Since the model was tested with some extreme real world values, the behavior of the model under other similar conditions can be estimated easily and thus the effect of these factors on total cost can be studied.

5.2 Future work

The EOF model developed in this research can be modified to incorporate different warehousing and distribution situations in the industry. Some different aspects of the warehouse-retailer interaction that could be included are given below.

Instead of using the same time between orders for all the SKUs, it could be possible to use a different time between orders for each of the SKUs, i.e. have some SKUs delivered once a week, some twice a week and some once in two weeks, and study the behavior of the model and the changes to minimum total costs under these conditions.

It is possible to extend this to a one warehouse-\(N\) retailer problem by considering other retailers and their costs and then the total cost would be the sum of all the total costs of the individual retailers. The behavior of the retailers would be identical to what is shown in this research because the orders are considered independently and so it would scale down to multiple one warehouse – one retailer transactions.
The vehicle routing algorithm could be included in this model to recalculate the transportation costs in this algorithm which would make this a one warehouse – n retailer problem.

The less than truckload (LTL) policy could also be incorporated in the model to test the applicability and the behavior of the model in such a scenario.
6 BIBLIOGRAPHY


[23] Kim, Joon-Seok and Benjaafar, Saifallah, “Is the (Q, R) Model appropriate for Production-Inventory Systems?”, *IE Graduate Program*, University of Minnesota.


7 APPENDIX A

Model as used in Excel

| Number of Retailers: | 1 |
| Number of SKUs: | 10 |
| Distance: | Retailers 1 Warehouse 200 |
| Vcap: | 40 |
| Holding_Cost: | 0.001 |
| Data: |
| Number of Items/Pallet: | 100 |
| Number of Items/Carton: | 10 |
| Number of Cartons/Pallet: | 10 |
| Hourly Wage: | 10 |
| Retrieve Time (Pallet): | 0.00556 |
| Retrieve Time (Carton): | 0.002222 |
| Retrieve Time (Item): | 0.001667 |
| Picking Speed (Pallet): | 4 |
| Picking Speed (Carton): | 1.704545 |
| Picking Speed (Item): | 1.136364 |
| Demand: |
| SKUs | Retailers 1 |
| 1 | 512 |
| 2 | 624 |
| 3 | 390 |
| 4 | 444 |
| 5 | 413 |
| 6 | 536 |
| 7 | 738 |
| 8 | 316 |
| 9 | 572 |
| 10 | 701 |
| Data for Transportation: |
| Mileage: | 1.53 |
| Rent Cost for each vehicle: | 195 |
Expected Distance (Pallet): 0.03087121
Expected Distance (Carton): 0.07765152
Expected Distance (Item): 0.02927715
Retrieval Cost/order (Pallet): 19.9167045
Retrieval Cost/order (Carton): 1.52222222
Retrieval Cost/order (Item): 1.04097222
Packing Cost/order (Pallet): 0
Packing Cost/order (Carton): 3.07416667
Packing Cost/order (Item): 0.5875
Loading Cost/order: 77.635

Pallet Pick Area

# of Aisles: 5
Length of each Aisle: 50
Offset from the Aisle to path: 5.5
distance from the entrance to the edge of the first aisle: 10
Center to center distance: 18
Width of the aisle: 10
Carton Pick Area

# of Aisles: 5
Length of each Aisle: 25
Offset from the Aisle to path: 2.5
Distance from the entrance to the edge of the first aisle: 10
Center to center distance: 18

Item Pick Area

# of Aisles: 5
Length of each Aisle: 12.91667
Offset from the Aisle to path: 3
Distance from the entrance to the edge of the first aisle: 10
Center to center distance: 8

Data for Packing:

Time to wrap a pallet: 0.016667
Time to place cartons on a pallet: 0.004167
Time to place an item in a carton: 0.001111
Time to wrap a carton: 0.001389

Data for Loading:

Time to load a pallet (hrs): 0.05
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| WHC:    | 103.77657 |
| TF:     | 3.88175   |
| TR:     | 4         |
| TRC:    | 2004      |
| OHC:    | 2107.7766 |
| TC:     | 719.82899 |
| Shipments: | 0.3378274 |
| Time Period between Orders: | 2.9600 |
8 Appendix B

Macros used for testing EOQ curves and effect of Change in Holding cost.

Sub HoldingCostsChange ()

' HoldingCostsChange Macro

'Macro recorded 9/7/2003 by meghanap

'To make the model run faster, 5 cells in a row were calculated and the values were transposed and pasted in column form for further analysis in Excel

'value for Unit Holding Cost - this is where different holding cost values were inserted
Worksheets("Model").Cells(21, 3).Value = "0.1"

'time between orders

For i = 1 To 60

'Value for the time between orders
Worksheets("Model").Cells(77, 20).Value = t

'Values of total cost were taken from the Model in the Excel sheet and pasted in the required sheet
Worksheets("Model").Activate
Range(Cells(33, 20), Cells(33, 24)).Select
Selection.Copy
Worksheets("T0.01").Activate
Cells(2 + p, 15).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks :=False, Transpose:=True

'Values of order quantity were taken from the Model in the Excel sheet
and pasted in the required sheet
Worksheets("Model").Activate
Range(Cells(7, 19), Cells(7, 23)).Select
Selection.Copy
Worksheets("OQ").Activate
Cells(3 + p, 8).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks:=False, Transpose:=True

t = t + 0.05
p = p + 5

Next i

End Sub
9 APPENDIX C

Macro used to develop data for MINITAB Analysis and the data used for MINITAB Analysis

Sub MINITABTrials()

' MINITABTrials Macro
' Macro recorded 7/23/2004 by meghanap

For i = 3 To 82
p = 0
t = 0.01

For j = 1 To 550
'activate worksheet that has the model
Worksheets("ModelFD").Activate

'assign the T value to the required cell
Worksheets("ModelFD").Cells(77, 20).Value = t

'value for Unit Holding Cost
Worksheets("ModelFD").Cells(21,3).Value =
Worksheets("FDRunsMINITABDesign").Cells(2, i).Value

'Factor A - Holding cost
Worksheets("ModelFD").Cells(39, 4).Value =
Worksheets("FDRunsMINITABDesign").Cells(3, i).Value
'Factor B - Warehouse cost

Worksheets("ModelFD").Cells(56, 4).Value =
Worksheets("FDRunsMINITABDesign").Cells(4, i).Value
'Factor C - Mileage

Worksheets("ModelFD").Cells(58, 4).Value =
Worksheets("FDRunsMINITABDesign").Cells(5, i).Value
'Factor D - Rental cost

Worksheets("ModelFD").Cells(11, 3).Value =
Worksheets("FDRunsMINITABDesign").Cells(6, i).Value
'Factor E - Distance of the retailer

'Demand randomization
Worksheets("ModelFD").Activate
Range("I15").Select
ActiveCell.FormulaR1C1 = "=round(norminv(rand(),500,0.05*500),0)"

Range("I16").Select
ActiveCell.FormulaR1C1 = "=round(norminv(rand(),650,0.05*650),0)"
Range("I17").Select
ActiveCell.FormulaR1C1 = "=round(norninv(rand(),375,0.05*375),0)"

Range("I18").Select
ActiveCell.FormulaR1C1 = "=round(norninv(rand(),450,0.05*450),0)"

Range("I19").Select
ActiveCell.FormulaR1C1 = "=round(norninv(rand(),400,0.05*400),0)"

Range("I20").Select
ActiveCell.FormulaR1C1 = "=round(norninv(rand(),580,0.05*580),0)"

Range("I21").Select
ActiveCell.FormulaR1C1 = "=round(norninv(rand(),750,0.05*750),0)"

Range("I22").Select
ActiveCell.FormulaR1C1 = "=round(norninv(rand(),300,0.05*300),0)"

Range("I23").Select
ActiveCell.FormulaR1C1 = "=round(norninv(rand(),600,0.05*600),0)"

Range("I24").Select
ActiveCell.FormulaR1C1 = "=round(norninv(rand(),700,0.05*700),0)"
'copy the total cost value from the model sheet
Worksheets("ModelFD").Activate
Range("T33").Select
Selection.Copy

'paste it into the minitab runs sheet
Worksheets("FDRunsMINITABDesign").Activate
Cells(7 + p, i).Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks:=False, Transpose:=False

'increment t and p

    p = p + 1
    t = t + 0.01

Next
Next i
End Sub