GLOBAL POSITIONING SYSTEM INTERFERENCE AND SATELLITE
ANOMALOUS EVENT MONITOR

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This dissertation entitled
GLOBAL POSITIONING SYSTEM INTERFERENCE AND SATELLITE
ANOMALOUS EVENT MONITOR

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Global Positioning System satellite Signal Quality Monitoring (SQM) is required to ensure the integrity of the received signal for aviation safety-critical systems. Failure mitigation is not addressed since failure detection ensures system integrity. The GPS Anomalous Event Monitor (GAEM) is introduced, consisting of a GPS receiver serving as an anomaly sensor, and the Software Defined Radio, allowing for a thorough analysis of signal malfunction modes through advanced signal processing techniques. Algorithms to monitor the GPS signal by the anomaly sensor are developed and in case of possible signal inconsistencies the signal is analyzed by the Software Defined Radio.

For the purpose of quality monitoring it is essential to understand the impact of the radio frequency front-end on the received signal, and implicitly onto the signal parameter estimation process; otherwise a signal inconsistency may be flagged which is induced by the monitoring system. Thus, radio frequency front-end induced errors are examined and the statistics for signal parameter estimators are derived.

As the statistics of an anomalous signal are unknown, a non-parametric, non-homoscedastic (uncommon variance of sample space) statistical test is developed. Berry-Esseen bounds are introduced to quantify convergence and to establish confidence levels. The algorithm is applied to the detection of signal anomalies, with emphasis on interference detection.
The algorithms to detect GPS signal anomalies are verified with experimental data. The performance of the interference detection algorithms is demonstrated through data collection in a shielded measurement chamber. Actual GPS signals in combination with interference sources such as narrowband, wideband and pulsed interference were broadcast in the chamber. Subsequently, case studies from continuous GPS monitoring are included and observed anomalies are discussed. The performance demonstration of the GPS anomalous event monitor is concluded with a field experiment to investigate the effects of aircraft overflights on GPS signal distortions.

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<td>ADC</td>
<td>Analog-to-Digital Converter</td>
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<tr>
<td>ADR</td>
<td>Accumulated Doppler Range</td>
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<td>AGC</td>
<td>Automatic Gain Control</td>
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<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
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<tr>
<td>C/A</td>
<td>Coarse Acquisition</td>
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<tr>
<td>CAT</td>
<td>Category</td>
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<tr>
<td>cdf</td>
<td>Cumulative Distribution Function</td>
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<td>CLT</td>
<td>Central Limit Theorem</td>
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<tr>
<td>CNR</td>
<td>Carrier-to-Noise Ratio</td>
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<td>DD</td>
<td>Double Difference</td>
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<td>DFT</td>
<td>Discrete Fourier Transform</td>
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<td>DGPS</td>
<td>Differential GPS</td>
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<td>DoD</td>
<td>Department of Defense</td>
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<td>DQM</td>
<td>Data Quality Monitoring</td>
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<td>DSP</td>
<td>Digital Signal Processor</td>
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<td>DT</td>
<td>Detection Threshold</td>
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<td>FAA</td>
<td>Federal Aviation Administration</td>
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<td>FCC</td>
<td>Federal Communications Commission</td>
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<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>FM</td>
<td>Frequency Modulation</td>
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<tr>
<td>FPGA</td>
<td>Field Programmable Gate Array</td>
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<tr>
<td>GAEM</td>
<td>GPS Anomalous Event Monitor</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<tr>
<td>$H_0$</td>
<td>Null Hypothesis</td>
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<tr>
<td>$H_A$</td>
<td>Alternative Hypothesis</td>
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<tr>
<td>HZA</td>
<td>High Zenith Antenna</td>
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<tr>
<td>IF</td>
<td>Intermediate Frequency</td>
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<tr>
<td>IID</td>
<td>Independent Identically Distributed</td>
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<tr>
<td>Abbreviation</td>
<td>Full Form</td>
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<tr>
<td>ILS</td>
<td>Instrument Landing System</td>
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<td>IMLA</td>
<td>Integrated Multipath-Limiting Antenna</td>
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<td>INS</td>
<td>Inertial Navigation System</td>
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<tr>
<td>IOC</td>
<td>Initial Operational Capability</td>
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<tr>
<td>JNR</td>
<td>Jamming to noise ratio</td>
<td></td>
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<tr>
<td>L1</td>
<td>Link 1</td>
<td></td>
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<tr>
<td>L2</td>
<td>Link 2</td>
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<tr>
<td>LAAS</td>
<td>Local Area Augmentation System</td>
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<td>LGF</td>
<td>LAAS Ground Facility</td>
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<td>LNA</td>
<td>Low Noise Amplifier</td>
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<td>LNAV</td>
<td>Lateral Navigation</td>
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<tr>
<td>LO</td>
<td>Local Oscillator</td>
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<tr>
<td>LOS</td>
<td>Line of sight</td>
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<tr>
<td>MA</td>
<td>Moving Average</td>
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<tr>
<td>MASPS</td>
<td>Minimum Aviation System Performance Standards</td>
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<tr>
<td>Mcps</td>
<td>Mega chips per second</td>
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<td>MCS</td>
<td>Master Control Station</td>
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<tr>
<td>MDB</td>
<td>Minimum Detectable Bias</td>
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<tr>
<td>MI</td>
<td>Misleading Information</td>
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<tr>
<td>MLA</td>
<td>Multipath-Limiting Antenna</td>
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<tr>
<td>MLS</td>
<td>Microwave Landing System</td>
<td></td>
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<tr>
<td>MOPS</td>
<td>Minimum Operational Performance Standards</td>
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<tr>
<td>MQM</td>
<td>Measurement Quality Monitoring</td>
<td></td>
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<tr>
<td>Msps</td>
<td>Mega samples per second</td>
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<tr>
<td>MVUE</td>
<td>Minimum Variance Unbiased Estimator</td>
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<tr>
<td>NBP</td>
<td>Narrow Band Power</td>
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<tr>
<td>P Code</td>
<td>Precise Code</td>
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<tr>
<td>pdf</td>
<td>Probability Density Function</td>
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<tr>
<td>P_{FA}</td>
<td>Probability of False Alarm</td>
<td></td>
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<tr>
<td>P_{MD}</td>
<td>Probability of Missed Detection</td>
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<tr>
<td>PPS</td>
<td>Precise Positioning Service</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>PR</td>
<td>Pseudorange</td>
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<td>PRN</td>
<td>Pseudo Random Noise</td>
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<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
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<td>QM</td>
<td>Quality Monitoring</td>
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<tr>
<td>RF</td>
<td>Radio Frequency</td>
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<tr>
<td>RFI</td>
<td>Radio Frequency Interference</td>
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<tr>
<td>RV</td>
<td>Random Variable</td>
<td></td>
</tr>
<tr>
<td>SA</td>
<td>Selective Availability</td>
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<tr>
<td>SDR</td>
<td>Software Defined Radio</td>
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<td>SDRFE</td>
<td>Software Defined Radio Front End</td>
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<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
<td></td>
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<tr>
<td>SPS</td>
<td>Standard Position Service</td>
<td></td>
</tr>
<tr>
<td>SQM</td>
<td>Signal Quality Monitoring</td>
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<tr>
<td>SV</td>
<td>Space Vehicle</td>
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</tr>
<tr>
<td>TiD</td>
<td>Time Difference</td>
<td></td>
</tr>
<tr>
<td>TOA</td>
<td>Time of Arrival</td>
<td></td>
</tr>
<tr>
<td>TOT</td>
<td>Time of Transmission</td>
<td></td>
</tr>
<tr>
<td>TTA</td>
<td>Time to alert</td>
<td></td>
</tr>
<tr>
<td>VDB</td>
<td>VHF Data Broadcast</td>
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<tr>
<td>VHF</td>
<td>Very High Frequency</td>
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<tr>
<td>VNAV</td>
<td>Vertical Navigation</td>
<td></td>
</tr>
<tr>
<td>WAAS</td>
<td>Wide Area Augmentation System</td>
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<tr>
<td>WBP</td>
<td>Wide Band Power</td>
<td></td>
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<tr>
<td>WJHTC</td>
<td>William J. Hughes Technical Center</td>
<td></td>
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<tr>
<td>WSS</td>
<td>Wide Sense Stationary</td>
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1 INTRODUCTION

The Global Positioning System (GPS) achieved Initial Operational Capability (IOC) on December 8, 1993, signifying that it is no longer a developmental system [1]. A 100 m horizontal position accuracy (95%) and worldwide availability for Standard Positioning Service (SPS) users was declared. Since then, GPS has gained enormous attention in the civil user community and a growing number of civil applications rely on GPS. On June 2, 1994 the termination of the Microwave Landing System (MLS) was announced in the United States for Category II and III landings [1] and research efforts were focused on satellite-based landing systems instead. Standalone GPS does not satisfy the stringent aircraft approach and landing requirements. To improve the performance of standalone GPS, measurement data from multiple reference receivers installed at an airport, are used to calculate corrections and determine the integrity of the satellite signals. This monitoring and correction technique is the basis for the Local Area Augmentation System (LAAS). Since a GPS-based navigation system relies on a Radio Frequency (RF) link, signal quality monitoring is performed to detect any signal distortion, which may lead to the loss of integrity. Therefore, the GPS satellite signals are continuously monitored with great scrutiny for high performance applications. Multiple approaches to this task are found in the literature. The approach introduced in this document is based the Software Defined Radio (SDR), which captures a digitized image of the received RF signal and implements all processing in software. This allows for application of advanced signal processing algorithms. In the context of quality monitoring, the SDR implementation is regarded as superior over conventional sequential receiver architectures as a signal anomaly may induce a loss-of-lock condition in the tracking loops of a conventional receiver, thus not allowing a further examination of the signal anomaly. No such loss of observability occurs for the SDR, as all information in the signal is stored as raw samples.

This document is structured as follows. Chapter 2 provides a brief introduction of the GPS and the LAAS. It is not meant to be comprehensive; it only outlines the important aspects of GPS and LAAS in the context of signal quality monitoring. Chapter 2 also discusses
previous research efforts in the field of the SDR and Signal Quality Monitoring (SQM). Chapter 3 introduces the GPS Anomalous Event Monitor (GAEM), which is a SDR-based implementation intended to perform the task of monitoring the received satellite signal. Since the GPS signal is distorted by noise, any quality monitoring requires a statistical estimation process to identify potential signal distortions. Chapter 4 discusses the effects of the SDR front-end induced errors that affect signal estimation. Chapter 5 introduces a large sample T-test to perform a statistical decision process at a given level of confidence. Since the statistical properties of the signal under investigation are unknown, an \textit{a priori} Minimum Detectable Bias cannot be defined. However, an \textit{a posteriori} Minimum Detectable Bias is introduced in order to perform a post detection validation. As interference is a major threat to a GPS-based system, Chapter 6 is dedicated to pre-correlation and post-correlation interference detection algorithms. Chapter 7 concludes the document with experimental results. The interference detection algorithms are validated in an experimental setup in a shielded measurement chamber. Further, data from continuous operation of the GAEM are presented and a case study from aircraft overflight-induced GPS anomalies are discussed.
2 BACKGROUND

2.1 GLOBAL POSITIONING SYSTEM

The GPS offers four-dimensional navigation by providing position, velocity and timing information worldwide. Initially designed for military use, GPS also made its way into a variety of civil applications and specifically into aeronautical use. Two kinds of services are offered, the SPS for civil use and the Precise Positioning Service (PPS) for military use. The GPS is based on the Space Segment, Control Segment and User Segment. The Space Segment consists of a minimum of 24 space vehicles (SVs) in 6 nearly circular orbits with a radius of 26,560 km [2]. The SV constellation is designed to provide worldwide navigation service with at least 4 SVs. The satellites provide time-of-transmission (TOT) ranging information and orbital data. The Control Segment consists of worldwide distributed monitor stations and the Master Control Station (MCS) located in Colorado Springs, Colorado [2]. The Control Segment monitors the satellite orbits and through small initiated maneuvers maintains the orbits, maintains the satellite health and the GPS system time as well as each satellite’s individual time base, and predicts the future orbital data. The user segment consists of the civil and military user community using SPS or PPS enabled receiver equipment. U.S. Department of Defense (DoD) authorized users, which are in possession of the cryptographic key, achieve a guaranteed 22 m horizontal accuracy and a 27.7 m vertical accuracy 95% of the time [3]. Throughout the 1990s, the civil user community only received a degraded SPS signal through the introduction of intentional degradation, called Selective Availability (SA). With SA, SPS users achieved a 100 m horizontal accuracy and 156 m vertical accuracy (95% of the time) [2]. A Presidential order on May 2nd, 2000 deactivated SA. Without SA, an improved performance as good as 20 m horizontal and 30 m vertical (95% of the time) is achieved [2]. No specification of SPS without SA has been issued.

The GPS satellites transmit two signals in the L-Band, the Link 1 (L1) with a carrier frequency of 1575.42 MHz and the Link 2 (L2) with a carrier frequency of 1227.60 MHz. The L1 frequency broadcasts the navigation data and the SPS Pseudorandom Noise (PRN) code signals defined as the Coarse Acquisition (C/A) code in order to perform the ranging
measurement. Further, the precise (P) code for the PPS user is modulated onto the L1 carrier. The C/A code consists of 1023 chips and is repeated every 1 ms, yielding a chipping rate of 1.023 Mcps. The P code is a PRN sequence of approx. $10^{14}$ chips and the chipping rate is 10.23 Mcps. This leads to a much higher ranging measurement precision for authorized PPS users. Every satellite is identified through its unique PRN sequence. The navigation information consisting of the satellite health status, ephemeris and almanac data (satellite position and velocity) and satellite clock parameters are modulated as Binary Phase Shift Keying (BPSK) onto the L1 and L2 carrier. The P code is also modulated onto the L2 carrier. Through difference measurement between the L1 and L2 P codes, the ionosphere-induced delay is determined and a PPS user is able to accomplish a more accurate ranging measurement.

The user equipment measures the pseudorange to a satellite through the alignment of a receiver-generated PRN sequence with the received satellite signal, yielding both the time-of-transmission (TOT) and the time-of-arrival (TOA) measurements. The difference between TOA and TOT is corrected for the satellite clock offset and multiplied by the speed of light, and is called the pseudorange. The term pseudorange is introduced because of the unknown clock bias of the user equipment (receiver clock offset), which is contained in the pseudorange. Since the receiver clock offset is common to all SV measurements performed from the same user equipment, the receiver clock bias is resolved through an additional range measurement. Trilateration [4] yields the user position, since the satellite position at TOT and the ranging measurement to the user equipment are determined. At least four ranging measurements are needed to determine the three-dimensional user position and the clock bias of the receiver.

As PPS is not available to the civil community, the work presented in this document only addresses SPS-related research.
2.1.1 Wide Area Augmentation System and Local Area Augmentation System

Until recent years, civil aviation solely relied on on-board navigation systems and ground-based radio navigation systems such as the Instrument Landing System (ILS). Due to the high operation cost, and the limited distribution of those navigation systems throughout the world, a GPS-based navigation-aid for civil aviation is widely regarded as a promising candidate to replace current operational navigation and landing systems. It has the potential to improve economy and safety [2]. Thus, the Federal Aviation Administration (FAA) is investigating the transition to a GPS-based navigation system for all phases of flight. Standalone GPS provides some level of protection against system anomalies and failures by relying on satellite self-checks and monitoring by the DoD MCS. However, for aviation use, standalone GPS as described in the previous Section does not satisfy the stringent navigation requirements [5], [6]. Thus, in order to meet the requirements for all phases of flight, the LAAS and the Wide Area Augmentation System (WAAS) are researched. Both systems are differential GPS (DGPS) architectures and enhance standalone GPS through the removal of spatial and temporal correlated ranging errors [3]. The WAAS is based on geostationary satellites, transmitting ranging corrections and integrity information for the GPS satellites. The airborne subsystem applies the received information from the WAAS satellite in order to satisfy the accuracy, integrity, availability and continuity requirements of en route and terminal phases of flight, non-precision approaches and LNAV/VNAV (Lateral Navigation/Vertical Navigation) precision approaches [2]. The LAAS is a DGPS precision approach and landing system that is based on a carrier-smoothed code architecture. The LAAS Ground Facility (LGF) estimates the error components of each satellite range measurement based on the surveyed location of the reference antennas. This error estimate, which is referred to as the differential correction, precision path point data, and integrity parameters are sent via the Very High Frequency (VHF) Data Broadcast (VDB) to the airborne subsystem. The airborne subsystem calculates a differentially-corrected position by combining the airborne pseudorange measurements with the received differential corrections from the ground subsystem. It has been demonstrated that such a system has the potential to provide the required accuracy, continuity, availability and integrity for a CAT III landing system [7], [8], [9].
2.1.1.1 Ohio University LGF Prototype

For development and test purposes, Ohio University has implemented a prototype LGF, which employs four Integrated Multipath-Limiting Antennas (IMLA) [10], each consisting of a Multipath-Limiting Antenna (MLA) and a High Zenith Antenna (HZA). The MLA consists of a 14-element dipole array, which features an omni-directional radiation pattern in the azimuth and a directional pattern in elevation, with maximum gain between 5° and 35° measured upward from the horizon. The HZA also exhibits a nearly omni-directional pattern in azimuth and a directional pattern in elevation, with maximum gain between 35° and 90°. The dipole array antenna has a sharp roll-off radiation characteristic in elevation from +5° to -5° to limit multipath signals reflecting off the surface of the earth. To enable detection of antenna/receiver pair malfunctions, four IMLAs, spaced by approximately 80 m, are situated at the prototype LGF. The four dual-spigot antennas are connected to eight Novatel OEM-4 GPS receivers. The Novatel receivers itself are connected to a CPU running the LAAS ground software, which performs several monitoring functions to ensure integrity and which calculates the pseudorange corrections to be broadcast over the VDB. The monitoring functions are categorized into five sub functions. Signal Quality Monitoring (SQM) is implemented to detect satellite signal anomalies, which are not mitigated by differential corrections. Data Quality Monitoring (DQM) continuously verifies the transmitted clock and satellite data for every space vehicle tracked. Measurement Quality Monitoring (MQM) checks the pseudorange measurement and the carrier phase measurement to detect anomalous steps and impulses in the measured quantities. Multiple Reference Consistency Check (MRCC) verifies the differential corrections through the comparison of each LGF reference receiver correction. To monitor the error characteristics of the ground subsystem over time, the Sigma Monitor is implemented at the LGF.
2.2 LGF Monitoring Functions

Signal quality monitoring may be categorized into failure detection and failure mitigation. This document focuses on failure detection because it ensures system integrity. Due to the potentially disastrous consequences in case of loss of integrity for aeronautical use, the LGF is classified as a safety-critical system [11]. As with any safety-critical application, the LAAS must meet the strict integrity, continuity, availability and accuracy requirements as stated in the previous Section. The LGF must ensure that the transmitted data to the airborne user from both the GPS satellites and the LGF meet the stringent integrity requirement as specified in [11]. This is driven through the LAAS architecture, which assigns the responsibility of detecting a system failure mode to the LGF, since standalone GPS is not able to satisfy the requirements. The inability of standalone GPS to satisfy the requirement is illustrated through a case study. One of the primary tasks of the Operational Control Segment (OCS) of the GPS is to monitor the satellites as they fly overhead the Monitor Stations by measuring the pseudorange and carrier phase on the L1 and L2 downlink frequencies. These data are processed and the clock correction, almanac and ephemeris data are uplinked to each SV. In some circumstances the OCS can take up to 10 min to resolve integrity issues of an SV [12], which is significantly longer than the required time to alarm (TTA) of 3 s or better for the LAAS as specified in [11].

In order to satisfy the requirement of detecting any system failure mode that could affect the performance of the airborne user, the received GPS signals are continuously verified by the Quality Monitor (QM) implemented at the LGF. In case of detected Misleading Information (MI), the LGF must alert a user in the specified amount of time of 3 s. It is emphasized that the primary goal of the QM is only to detect a system failure mode in order to declare the system inoperable. This guarantees safe operation, but comes at the trade-off of losing continuity and/or availability.

In the literature, multiple approaches to the general problem of QM are published. Real time vs. non-real time implementations are found. This is due to the fact that a non-real time implementation is applicable, because the TTA as specified in [11] of 3 s allows for a
processing time. A non-real time implementation is of interest because it adds more flexibility to the processing methodology, e.g. non-causal algorithms are applicable.

The quality monitor as presented in [13] is aimed at a broad range of possible failure conditions. The real time SQM as presented in [13] applies to a receiver where all correlators are assigned to one channel. These measurements are analyzed by the SQM algorithms to determine if any signal deformation anomaly has altered the ideal triangular-shaped correlation function [14]. Signal deformation failure modes are also known as evil waveforms [15]. The real time SQM also estimates the incoming signal power and checks if it is within the specifications. Further, the code-minus-carrier divergence is analyzed [14]. The research presented in this document focuses on SQM and MQM.

The thermal noise floor of a receiver is at -110 dBm for a 2 MHz bandwidth. Because the received GPS signal is at a nominal power level of -130 dBm and thus implicitly 20 dB below the receiver noise floor, the task of detecting any anomaly in the GPS signal requires a statistical decision process to draw conclusions about the deterministic part of the signal. This requires that any signal parameters need to be associated with detection thresholds (DT), which is driven by $P_{FA}$, the probability of missed detection $P_{MD}$ and the Minimum Detectable Bias (MDB) as specified by the requirements in [11]. The derivation of any DT hence involves the knowledge of the underlying distribution functions of the signal parameters. However, by the nature of the problem statement, no a priori knowledge on the signal failure mode is available, e.g. the properties of a possible interference source may be a composite of a variety of scenarios. Any assumption regarding the distribution would reduce the integrity of QM. Therefore the task of QM requires the development of criteria which allow inferring on signal failure modes independent of the underlying statistical processes with a given level of confidence. It is important to recognize that the association of confidence bounds to the detection algorithms is a fundamental criterion for QM and presents the challenge of the research. The detection schemes cannot be too conservative in order not to unreasonably decrease the system’s continuity and/or availability by a high rate of false alerts, and vice-versa the safety of the system needs to be guaranteed through a low rate of missed detections.
The approach to the task of QM pursued by Ohio University is based on the SDR implementation [16], [17], [18] as introduced in the following Section. It captures a digitized version of the GPS RF signal and stores it in memory. In earlier days this approach was uninteresting because of the extraordinary computational burden that this approach requires. However as computer technology advances, especially Field Programmable Gate Array (FPGA) technology, the approach becomes more practical [19]. As opposed to any conventional real time implementation, the SDR stores its data for post-evaluation by block processing techniques. Inherent to this approach is the fact that the processed data are always going to be available with a deterministic amount of delay. However as [11] allows for a specified TTA, a delay of the processed solution is tolerable as long as the criteria of TTA can be met.

2.3 SOFTWARE DEFINED RADIO

Over the past ten years the SDR evolved rapidly and received enormous recognition in the telecommunication industry. The basic concept of a SDR is to migrate the communication tasks, which predominantly used to be implemented by dedicated hardware resources, into software modules and to run it on a generic hardware platform consisting of processors and Digital Signal Processors (DSP). The radio functions such as the generation of signals and the demodulation of received signals are implemented through software. This approach has enormous advantages over conventional receiver architectures, such as flexibility for adaptation to new standards, e.g. the same hardware platform for a Galileo (future European satellite navigation system) and GPS receiver could be employed. Importantly, a SDR allows the implementation of advanced signal processing algorithms to perform the modulation and demodulation of a radio signal, leading to better system performance. Further, the processing capability is not bound to the receiver architecture. Processing techniques such as adaptive filtering and different domain processing techniques can be implemented. In the scope of QM as presented in this document, the SDR enables a thorough analysis of possible signal failure modes. Conventional receiver architectures do not allow a direct signal failure evaluation, since it relies on the fact that a failure mode introduces an error on the demodulation mechanization such as the tracking loops, correlator, etc. Further, in case of a
severe failure mode, a conventional receiver often loses lock and does not allow any insight into the signal behavior following the loss-of-lock condition. A SDR, on the other hand, has data stored for the time periods before and after the disturbance, enabling a better characterization of the anomaly. The SDR has drawbacks such as higher requirements for memory and processing power. However, in the context of QM, where these requirements are not a primary factor, the SDR is superior to conventional receiver architectures.

As depicted in Figure 1, Ohio University’s SDR employs a downconvert-and-digitize front-end implementation [20].

![Figure 1, RF Front-End implementation](image)

The RF input signal received by the antenna passes pre-amplification and filtering stages. The modified signal is translated to an intermediate frequency (IF) of 21.27 MHz by frequency mixing. This requires a local oscillator (LO) at \( f_{LO} = 1554.15 \text{ MHz} \). The downconverted signal centered at IF is sampled at 5 Msps with an analog-to-digital converter (ADC) resolution of 12 bits. This digital band folding technique (band aliasing) [16] translates the GPS signal to a center frequency of 1.27 MHz, and it is stored in memory for processing. It is important to notice that the GPS signal is below the noise floor, therefore the amplification stages need to be adjusted in order to match the received signal level to the ADC input.

All further demodulation steps of the captured RF sequence are implemented in software.
The incoming RF signal is modeled as [3]:

\[ r(t) = s(t) + n(t) \]  \hspace{1cm} (1)

where \( n(t) \) is the received noise as examined in Section 4.1.1 and the signal \( s(t) \) for a particular satellite is of the form [3]:

\[ s(t) = A G(t) D(t) \sin(2\pi(f_0 + \Delta f_{SR}) t + \phi_0) \]  \hspace{1cm} (2)

Where \( A \) denotes the signal amplitude; \( G(t) \) the C/A code PRN sequence; \( D(t) \) the transmitted data; \( f_0 \) the L1 carrier frequency at 1575.42 MHz; \( \Delta f_{SR} \) the frequency offset due to Doppler shift, satellite frequency offset, local oscillator offset, and \( \phi_0 \) the phase uncertainty.

The signal acquisition is performed by using block processing techniques as opposed to conventional sequential processing and the method is depicted in Figure 2.

![Figure 2, Signal acquisition](image)

The block length for the data processing is 1 ms, which accommodates the full length of one Coarse-Acquisition (C/A) period, ignoring any Doppler shift on the code. The acquisition
of the code is achieved by use of the frequency domain circular convolution [21], which performs the code phase search in parallel, versus correlation, which is a computationally intensive serial search. The incoming RF signal is translated to final IF in two stages, i.e. by frequency mixing and subsequent band aliasing [16] (see also Figure 1).

Based on the SDR front-end configuration the sampled signal $r_{i,k}$ is represented as follows [17]:

$$r_{i,k} = AG_i D_i \sin\left(2\pi\left(f_{iw} + \Delta f\right)\frac{m_i + k}{f_s} + \phi_0\right) + n_{i,k}$$  \hspace{1cm} (3)

where $f_{iw}$ is the down-aliased signal at 1.27 MHz, $m_i$ the $(i-1)\text{th}$ ms data block and $k$ the $k\text{th}$ sample in the $i\text{th}$ ms data block.

The obtained IF signal is split into the in-phase and quadrature-phase baseband components by multiplication with the local carrier signal, which is adjusted for Doppler frequency shifts and local oscillator frequency offset. This yields for the in-phase $I_{i,k}$ channel by multiplication with the sine of the local carrier [17]:

$$I_{i,k} = \frac{1}{2} AG_i D_i \cos\left(2\pi\left(f_{iw} + \Delta f - f_i\right)\frac{m_i + k}{f_s} + \phi_0\right) + n_{i,k} \sin\left(2\pi f_i \frac{k}{f_s}\right)$$  \hspace{1cm} (4)

where $f_i$ denotes the local frequency, $\phi_0$ combines $\phi_0$ and the constant phase difference between the local carrier and the incoming signal. Further, the quadrature-phase $Q_{i,k}$ channel is equivalent to Equation (4) except that the received signal is multiplied by the cosine of the local carrier.

The code removal is performed by correlating the locally-generated replica code with the in-phase and quadrature-phase channels. Again only the in-phase channel is shown.
\[
I_i = \sum_{i=1}^{m} \left\{ \frac{1}{2} AG_i L_i D_i \cdot \cos \left( 2\pi (f_{w} + \Delta f - f_i) \frac{m_i + k}{f_s} + \phi_0 \right) + n_{i,k} L_i \sin \left( 2\pi f_i \frac{k}{f_s} \right) \right\}
\] (5)

Where \( L_i \) denotes the locally-generated replica code. Equation (5) is simplified by neglecting the sign of the data bit and assuming accurate local frequency \( f_i \):

\[
I_i = \frac{1}{2} A \cos(\phi_0) R(q) + \sum_{i=1}^{m} n_{i,k} L_i \sin \left( 2\pi f_i \frac{k}{f_s} \right)
\] (6)

where \( R(q) \) denotes the autocorrelation function of the C/A-code as a function of the time shift \( q \).

In the energy detector as shown in Figure 2, the frequency \( f_i \) closest to \( f_{w} + \Delta f \) will yield the maximum energy. Assuming that the SDR is stationary (no dynamics) and uses an accurate sampling clock, the frequency search can be performed within a range of \( f_{\text{max}} = 10 \text{ kHz} \). The chosen block length of \( T_B = 1 \text{ ms} \) provides a frequency resolution of 1 kHz. A search for the maximum power yields the code phase delay and the coarse carrier frequency at a resolution of 1 kHz.
The navigation data bit detection is accomplished through a hypothesis test procedure. As the acquisition provides the code phase delay, it inherently also defines the time instance of a possible navigation data bit transition. This information is used to define the two hypotheses:

\[ H_0: \text{No navigation data bit transition occurred} \]
\[ H_1: \text{A navigation data bit transition occurred.} \]

The correlation energy (refer to Equation (6)) for the in-phase and quadrature-phase is calculated over a C/A code period preceding and following the possible navigation data bit transition and the data bit wipe-off is performed under \( H_0 \) and \( H_1 \).

The test is formed as follows:

\[
I_{i-1,i}^2|_{H_0} + Q_{i-1,i}^2|_{H_0} < I_{i-1,i}^2|_{H_1} + Q_{i-1,i}^2|_{H_1} \tag{7}
\]

If the inequality in (7) yields the left side greater than the right side, no navigation data bit at the transition point occurs. If the right side is greater than the left side a navigation data bit transition is existent. Plotting the left and the right side of inequality (7) yields Figure 3.
As depicted in Figure 3, the navigation data bit transition is detected through the comparison of the acquisition energy under $H_0$ and $H_1$. Hypothesis isolation, i.e. energy separation between $H_0$ and $H_1$, on the order of a factor of $10^2$ is observed. Depending on the signal strength, the hypothesis isolation can be increased through a longer integration time.

The frequency resolution of 1 kHz obtained from the block length of 1 ms is refined through using the phase of the signal [16]. Finer frequency estimates can be obtained over a sequence of 1-ms blocks by calculating relative phase of each block using the baseband in-phase and quadrature-phase components and then taking successive differences.

The phase of the received signal is given by:

$$\phi_i = \tan\left(\frac{Q}{I}\right)$$

Thus the relative frequency offset from $f_{IF}$ is given by:

$$\Delta f = \frac{\phi_i - \phi_{i-1}}{2\pi \cdot T_s}$$

Using fractional cycle differences over 1-ms blocks, a frequency estimation noise of $225\sigma_\phi$ Hz ($1\sigma$) can be achieved, where $\sigma_\phi$ is the noise in the phase differences in radians. With a GPS signal of a power level of 45 dB-Hz, phase estimates in the Ohio University setup contain approximate 0.1 rad of phase variation, giving frequency noise of approximately 22 Hz. Averaging over additional ms data blocks can reduce this number substantially. When continuous, coherent phase measurements are made, the block processor is “locked” onto the incoming signal. If phase jumps approach the half-cycle mark between 1-ms blocks, the signal can no longer be tracked reliably and the situation is flagged by frequency noise nearing 500 Hz, or equivalently, 0.5 cycle per 0.001 s. While one expects to see these jumps for strong signals at 45 dB-Hz at multiples of 20 ms where navigation data bit transitions may occur, a higher rate than this indicates the presence of an anomaly in the collected RF
data. This metric will be used in the analysis of the field tests that follow. For the evaluation of weak signals, i.e. a CNR lower than 40 dB-Hz, the block size of 1 ms is not sufficient and needs to be increased through block addition [22]. This is the equivalent of increasing the integration time in a sequential receiver. It has been shown that signals down to 15 dB-Hz can be tracked [23] if the integration time is increased to 1 s. However, a larger block size comes at the trade-off of less accurate anomaly observability in time. Since the case studies presented in this document investigate sufficiently strong satellite signals, the use of 1-ms data blocks is adequate.
3 GPS ANOMALOUS EVENT MONITOR

For development and test purposes, Ohio University has developed a GAEM to enable detailed analyses of GPS anomalies. As discussed in Section 2.2 the GAEM has its foundation in the SDR, which captures a digitized version of the RF GPS L1 data continuously at 5 MspS and keeps the latest 10 s in memory. In comparison to “conventional” QM as discussed in [13], [14], [15] and introduced in Section 2.2, The GAEM allows for a thorough evaluation of failure modes by means of post-processing techniques (refer to Section 2.3).

The GAEM consists of the SDR (described in Section 2.3), an anomaly sensor and signal processing equipment. The real time part of the GAEM is formed by the anomaly sensor and the GPS data acquisition CPU. Figure 4 illustrates the sub-systems interconnected together.

Figure 4, Ohio University GPS Anomalous Event Monitor
The real time software module that automates the capture of signal anomalies is connected to a Novatel Beeline GPS receiver [24], which serves as a sensor for the GPS signal anomaly detection. At the time of the initiation of this research, computer technology did not allow a real time implementation of the SDR. Thus a regular GPS receiver is employed for the real time part of the system. However as computer technology advances, especially FPGA technology, the implementation of the anomaly sensor on the SDR has become feasible [19]. In case of a signal failure mode, the anomaly sensor sends a trigger command to the signal processing equipment for capturing the RF sequence surrounding the signal anomaly.

The RF front-end limits the received signal to a bandwidth of 2 MHz, which hosts the main lobe of the C/A code. This is essential to avoid aliasing of the image frequencies down into the baseband signal, since the RF sequence is sampled at 5 Msps. This configuration is based on the assumption that an anomaly in the GPS signal triggers one of the tracking flags in the Novatel Beeline GPS receiver or a general receiver failure mode as elaborated in Section 3.1. Tracking flags such as loss of code lock, loss of phase lock, automatic gain control (AGC) out of range, jamming signals and low CNR are monitored. Given that a failure mode occurs, an interrupt is initiated that triggers the software radio acquisition CPU with a 5-s delay to record the latest 10 s of RF data. The 5 s delay places the anomaly in the center of the 10-s interval. The captured sequence is processed and analyzed for possible failure modes. Signal acquisition is performed through the use of block processing techniques (refer to Section 2.3) as opposed to conventional sequential processing techniques. The block length chosen is 1 ms, which yields 1000 data points per second for performing the proposed anomaly detection as introduced in Section 2.3.

The following Section introduces the algorithms to perform the anomaly detection on the Novatel Beeline receiver to capture anomalous GPS events.
3.1 ANOMALY SENSOR

In order to accomplish the goal of flagging GPS anomalies, two basic approaches were selected:

- Monitor receiver built-in flags
- Monitor receiver measurements and evaluate for inconsistencies

The following Sections summarize the two methods, which are performed in real time in order to capture GPS signal anomalies.

3.1.1 Monitor Receiver Flags

A very basic method to verify the status of the received signal is accomplished through monitoring receiver tracking status and self test status flags. The flags as listed below of the tracking status message [24] of the Novatel Beeline receiver are continuously monitored under the condition of a locktime and minimum elevation threshold:

- Code lock flag
- Phase lock flag.

The following receiver self test status flags [24] are also monitored:

- AGC out of range
- Jamming signal.

It is important to recognize that these flags only provide basic monitoring capability, since the sensitivity of these flags is very limited (see also to Chapter 6).

3.1.2 Monitor Receiver Measurements

More sensitive GPS signal monitoring is accomplished through the implementation of monitoring functions of receiver measurements in order to transform certain anomalies into an observable state and to detect signal inconsistencies.
3.1.2.1 Channel Lost Event

The examination of continuous GPS data collected with the Novatel Beeline receiver revealed data discontinuities in the sense of a sudden loss of a receiver channel, which is the equivalent of losing code lock and phase lock at the same time. A GPS signal anomaly may initiate such a receiver failure mode and therefore needs to trigger the GAEM for further analysis. In order to minimize the rate of false alert, an interrupt to the data collection system is created under the condition of a minimum elevation threshold and a receiver lock time threshold. Since satellite signals may encounter blockage or multipath for low elevation satellites or simply just pass below the horizon, a channel lost event is only triggered for satellites above 10° elevation. To ensure that the observed tracking discontinuity is not only a signal acquisition fluctuation, a minimum satellite lock time of 500 s is specified. The Novatel Beeline receiver provides a 16 channel receiver card with a dual L1 antenna configuration. In Ohio University’s GAEM system, both RF inputs were connected to the same antenna. The antenna is mounted on the roof of Stocker Engineering Center to minimize signal blockage and signal multipath. In order to further increase the confidence of triggering a signal anomaly, the condition that both receivers need to encounter the same failure mode is set forth.

3.1.2.2 Phase Distortion Measurement

Due to the utilization of the Accumulated Doppler Range (ADR) measurement [3] in a variety of applications [2], [4], it is of interest to detect a failure mode of the carrier phase. A discontinuity in the carrier phase measurement, i.e. large phase distortion, is generally known as a cycle slip. Two independent mechanisms may induce a measurement cycle slip:

- Measurement inconsistency on the receiver side, such as loss of lock, clock error, etc.
- Satellite signal transmission inconsistency.

In the context of QM it is of interest to detect satellite signal transmission inconsistencies. An algorithm is required, which separates satellite errors from receiver-induced errors. The most common sources for an observed phase distortion in a GPS receiver are a receiver
clock inconsistency, low CNR and multipath. It is noted that a clock inconsistency translates into an ADR measurement inconsistency. Therefore it is essential to develop a clock independent cycle slip detection algorithm.

The ADR to satellite $j$ is written in the form

$$\text{ADR}_j(n) = \text{ADR}_j(n-1) + \Delta R_j(n) + \epsilon(n) + \psi_j(n)$$

(10)

where

- $\Delta R_j(n)$ is the Line Of Sight (LOS) range difference from the user to satellite $j$ from time $n-1$ to time $n$.
- $\epsilon(n)$ is the receiver clock induced ADR measurement error, which is satellite independent.
- $\psi_j(n)$ is the measurement error to satellite $j$, which is a composite of ionospheric delay, tropospheric delay, noise, multipath and satellite clock error.

In order to mitigate the clock error $\epsilon(n)$ for the cycle slip detection, the time difference of the ADR measurement (TiD) is introduced as defined in Equation (11) (see also [25]):

$$\text{TiD}_j = |\text{ADR}_j(n) - \text{ADR}_j(n-1)| - |\text{ADR}_j(n-1) - \text{ADR}_j(n-2)|$$

(11)

It is important to recognize that three successive measurements are required. Substituting Equation (10) into Equation (11) yields

$$\text{TiD}_j = \epsilon(n) + \Delta R_j(n) + \psi_j(n) - \epsilon(n-1) - \Delta R_j(n-1) - \psi_j(n-1).$$

(12)
Since the GAEM is stationary, the term $\Delta R_i(n) - \Delta R_i(n-1)$ is calculated from the ephemeris data and the satellite motion term can be subtracted out (neglecting differential ephemeris errors), which yields Equation (13).

$$TiD'_j = \varepsilon(n) + \psi_j(n) - \varepsilon(n-1) - \psi_j(n-1)$$  \hspace{1cm} (13)

Performing differencing between the TiD’ of satellite i and satellite j yields Equation (14).

$$TiD'_j - TiD'_i = \psi_j(n) - \psi_j(n-1) - \psi_i(n) + \psi_i(n-1)$$  \hspace{1cm} (14)

As indicated in Equation (14) the receiver clock-induced ADR measurement error term $\varepsilon(n) - \varepsilon(n-1)$ cancels out since it is common to all satellite measurements. The remaining term depicts a composite of the ADR measurement error to satellite i and j. Since the decorrelation from time $n-1$ to time $n$ is negligible, the remainder of the differences $\psi_j(n) - \psi_j(n-1)$ and $\psi_i(n) - \psi_i(n-1)$ is approximately bound by the ADR measurement noise. A current receiver such as a Novatel Beeline receiver provides an ADR measurement with a standard deviation on the order of 10 mm. In general, the measurement error is satellite elevation dependent; the higher the elevation the lower the measurement error. In order to keep the noise on $TiD'_j - TiD'_i$ as low as possible, it is desirable to pick the highest elevation satellite out of a constellation of K satellites as the reference satellite. Subsequently, K-1 differences of $TiD'_j - TiD'_i$ are formed.

The detection of a phase distortion is accomplished through the monitoring of the time-satellite difference $TiD'_j - TiD'_i$. If the value of a certain satellite is significantly greater than the expected standard deviation of the phase measurement, then a phase distortion in the ADR measurement is suspected.
To outline the performance of the TiD technique, a data set was collected on 02/24/04. Satellite 18 was the highest elevation satellite at 71 deg elevation and was selected as the reference satellite. The time differences for satellite 15 at 65 deg elevation and for satellite 5 at 12 deg elevation are evaluated as shown in Figure 5.

The standard deviation for TiD\(_{18}\)-TiD\(_{15}\) is 0.0122 cycles, which translates into 2.318 mm composite phase measurement error of SV 18 and SV 15. Due to the lower elevation of SV 5 the standard deviation for TiD\(_{18}\)-TiD\(_{5}\) is 0.1052 cycles (=19.88 mm), which is significantly higher. Further, as depicted in Figure 5, satellite 5 encounters a phase distortion at time 140 s, which was induced through a short loss of phase lock condition by the receiver, due to the low elevation of the satellite. The cycle slip is transformed observable through the time differencing technique, which illustrates the functionality of the algorithm. However, it also outlines the inherent limitation of the algorithm that the integrity in detecting a satellite induced phase distortion is directly dependent on the tracking reliability of the receiver. Receiver induced errors can not be completely decomposed from satellite transmission inconsistencies. Further it is noted, that in case of the reference satellite measurement being

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**Figure 5, TiD differences for satellites 15 and 5 vs. time with respect to satellite 18**
distorted through a phase inconsistency, all the K-1 time differences will indicate a phase
distortion.

3.1.2.3 Pseudorange Step Measurement

To ensure the consistency of the pseudorange measurement, pseudorange step detection is
implemented as researched in [26]. An impulse or a step error on the pseudorange may
directly translate into satellite ranging errors. Therefore it is of interest to flag such a failure
mode scenario. Further, as examined in [27], a dependency of pseudorange variance and
Radio Frequency Interference (RFI) exists. The relationship between pseudorange variance
and broadband RFI can be expressed as published in [27]:

$$\sigma_r^2 = \frac{d(cT_c)^2 BW_c}{2} \left[ 1 + \frac{2}{(2-d)(cT_c)^2 T_{square}} \right]$$

(15)

Where

- $d$ is the correlator spacing [chips]
- $c$ is the speed of light [m/s]
- $T_c$ is the C/A code chip width [s]
- $BW_c$ is the 1 sided tracking loop bandwidth [Hz]
- $T_{square}$ is the squaring loss from early-late power [4] detector [unitless]
- $C(k)$ is the carrier power of the $k$th satellite [dBm]
- $N_0$ is the noise power [dBm/Hz]
- $I_0$ is the interference power [dBm/Hz].

Equation (15) thus implies that the pseudorange step detection may also aid the RFI
detection as will be discussed in Chapter 6.
The detection is based on the Innovation Test, which is introduced in [14]. The innovation to satellite $j$ is defined in Equation (16):

$$ Innovation^i(k) = PR^i(k) - (PR^i(k-1) + \phi^i(k) - \phi^i(k-1)) $$

(16)

where

$PR^i(k)$ is the pseudorange measurement at time $k$ to satellite $j$

$PR^i(k-1)$ the smoothed pseudorange at time $k-1$ to satellite $j$ as defined in Equation (18)

$\phi^i(k)$ the ADR measurement at time $k$ to satellite $j$.

As indicated in [14], the derived innovation in Equation (16) is verified against the threshold as defined in Equation (17).

$$ Threshold^i(k) = 6.82\sigma_{theoretical}(k) = 6.82 \left(0.1 + 2.0e^{-3.0a^i(k)}\right) $$

(17)

Where $a^i(k)$ is the elevation angle of satellite $j$ at time $k$ and the constant 6.82 is determined from experimental results [14].

The expression for the smoothed pseudorange is specified in [11], [14]:

$$ PR^i_s(k) = \frac{1}{N_s} PR^i(k) + \frac{N_s - 1}{N_s} PR^i_{proj}(k) $$

(18)

Where the projected pseudorange is defined as the composite of the sum of the pseudorange at time $k-1$ and the change in the accumulated Doppler measurement from time $k-1$ to $k$ [11], [14]:

$$ PR^i_{proj}(k) = PR^i(k-1) + \phi^i(k) - \phi^i(k-1) $$

(19)
A data set was collected on October 10\(^{th}\), 2003 and the innovation as specified in Equation (16) and the threshold as specified in Equation (17) for satellite 11 are illustrated in Figure 6.

![Figure 6, PR step detection vs. time](image)

As depicted in Figure 6 the innovation does not cross the threshold and therefore no PR step is detected.

To gain further insight into the mechanization of the pseudorange detection scheme, the autocorrelation function for carrier-smoothed code is evaluated. The smoothed pseudorange as defined in Equation (18) \([11]\) is rewritten in the form

\[
PR_i'(k) = \frac{1}{N_s} PR_i'(k) + \frac{N_s - 1}{N_s} (PR_i'(k-1) + \phi'(k) - \phi'(k-1))
\]  (20)

Equation (20) is rewritten in the form of an Autoregressive (AR) Process \([28]\) as

\[
PR_i'(k) = \frac{N_s - 1}{N_s} PR_i'(k-1) + \frac{1}{N_s} \left[ PR_i'(k) + (N_s - 1)(\phi'(k) - \phi'(k-1)) \right]
\]  (21)
The autocovariance function \([28]\) is defined as:

\[
\gamma(h) = \text{COV}(PR_i(k), PR_i(k - h))
\]  

(22)

where \(h\) depicts the lag. Expanding Equation (22) yields:

\[
\gamma(h) = \text{COV}\left(\frac{N_e - 1}{N_e} PR_i(k - 1), PR_i(k - h)\right) + \text{COV}\left(\frac{1}{N_e} [PR_i(k) + (N_e - 1)(\phi(k) - \phi(k - 1))], PR_i(k - h)\right)
\]  

(23)

Simplifying under the assumption that \(h \neq 0\), yields

\[
\gamma(h) = \frac{N_e - 1}{N_e} \text{COV}(PR_i(k - 1), PR_i(k - h))
\]  

(24)

because it is given that \(PR(k) + (N_e - 1)(\phi(k) - \phi(k - 1))\) is uncorrelated with \(PR_i(k - h)\) for \(h \neq 0\) under the assumption that no dynamics are involved. The autocovariance is written in the form:

\[
\gamma(h) = \frac{N_e - 1}{N_e} \gamma(h - 1) = \left(\frac{N_e - 1}{N_e}\right)^{|h|} \gamma(0)
\]  

(25)

Also it is noted that

\[
\text{COV}\left(\frac{PR_i(k)}{N_e}, \frac{1}{N_e} [PR_i(k) + (N_e - 1)(\phi(k) - \phi(k - 1))]\right)
\]

\[
= \text{COV}\left(\frac{N_e - 1}{N_e} PR_i(k - 1) + \frac{1}{N_e} [PR_i(k) + (N_e - 1)(\phi(k) - \phi(k - 1))], \frac{1}{N_e} [PR_i(k) + (N_e - 1)(\phi(k) - \phi(k - 1))]\right)
\]

(26)

which is

\[
\text{VAR}\left(\frac{1}{N_e} [PR_i(k) + (N_e - 1)(\phi(k) - \phi(k - 1))]\right).
\]  

(27)
Further

\[ \gamma(0) = \text{COV}(PR'(k), PR'(k)) \] (28)

which is expanded into Equation (29)

\[ \gamma(0) = \frac{N_s - 1}{N_s} \text{COV}(PR'(k-1), PR'(k)) + \text{COV}\left( \frac{1}{N_s} \left[ PR'(k) + (N_s - 1)(\phi'(k) - \phi'(k-1)) \right], PR'(k) \right) \] (29)

Using the results from Equation (25) and Equation (28) yields

\[ \gamma(0) = \frac{\text{VAR}\left( \frac{1}{N_s} \left[ PR'(k) + (N_s - 1)(\phi'(k) - \phi'(k-1)) \right] \right)}{1 - \left( \frac{N_s - 1}{N_s} \right)^2} \] (30)

Therefore, the expression for the autocovariance function under the assumption that \( PR(k) + (N_s - 1)(\phi(k) - \phi(k-1)) \) is Wide Sense Stationary (WSS), i.e. no dynamics are involved in the system, yields Equation (31):

\[ \gamma(h) = \left( \frac{N_s - 1}{N_s} \right)^h \frac{\text{VAR}\left( \frac{1}{N_s} \left[ PR'(k) + (N_s - 1)(\phi'(k) - \phi'(k-1)) \right] \right)}{1 - \left( \frac{N_s - 1}{N_s} \right)^2} \] (31)

A current receiver under nominal operation provides an ADR measurement standard deviation on the order of 1 cm and a pseudorange measurement standard deviation on the order of 0.5 m. Both measurement variations increase as the elevation of a satellite decreases. Implementing the autocorrelation function, which is defined as \( \rho(h) = \gamma(h)/\gamma(0) \) and assuming a time constant \( N_s \) of 100 s yields Figure 7.
As indicated in Figure 7, the smoothed pseudorange shows significant correlation out to 200 s. Therefore, the innovation test as introduced in Equation (16) is based on the comparison of a highly correlated measurement in time $(PR'(k-1) + \phi'(k) - \phi'(k-1))$ with an almost independent measurement $PR'(k)$. 

$\rho(h)$ [unitless]
4 EFFECTS OF THE RADIO FREQUENCY FRONT END ON SIGNAL ESTIMATION

As described in Section 2.2, the task of QM requires a statistical decision process to identify potential failures of the deterministic part of the GPS signal. The focus of this Chapter is on the RF front-end induced errors that affect the signal parameter estimators used for QM.

The Software Defined Radio Front-End (SDRFE) amplifies the incoming signal, i.e. the composite of the GPS signal and the noise, by approximately 100 dB to achieve sufficient signal strength in order to sample the incoming signal by the ADC. The signal sampled after it is translated to an IF by frequency mixing. Current SDR implementations (refer to Section 2.3) are limited in the sampling frequency due to the computer speed, which implicitly also limits the bandwidth of the SDRFE.

The subsequent Sections introduce the statistical models of the SDRFE components. Subsequently, these models are applied to estimate the errors induced by the SDRFE. The Chapter concludes by applying the results to the GPS signal acquisition and quantifying the induced errors by a case study.

4.1 STATISTICAL EVALUATION OF SOFTWARE RADIO PROCESSING

The following Section presents the modeling of the noise. Subsequently, the effects of the hardware elements in the receiver front-end on the noise statistics are examined as the received signal (i.e. composite of noise and GPS signal) passes through amplification, bandpass filter and mixer stages.

4.1.1 Model of Incoming Noise

The received noise is modeled to have an Additive White Gaussian Noise (AWGN) density across the GPS band, i.e. the noise \( n(t) \) has a constant power spectral density (PSD) [2]. This model has its foundation in the central limit theorem (CLT), which states that the
summation of independent Random Variables (RV) converges to a standard normal
distribution [29]. The independent noise sources are thermal receiver noise, natural noise
received by the antenna, and man-made wideband noise. It is emphasized that this model is
not valid in case of signals distorted by multipath or in the presence of narrowband
interference.

The received AWGN sequence is of the form $N(0, \sigma^2)$. Since it describes a zero-mean
process, the noise variance and the received noise power are related as follows, noting that
the second moment $E[X^2(t)]$ expresses the signal power [30]:

$$\sigma^2 = E[X^2(t)] - E[X(t)]^2 = E[X^2(t)] = kTB$$

(32)

where

$k$ is Boltzmann’s constant of $1.38 \cdot 10^{-23} [J/K]$

$T$ is the equivalent noise temperature $[K]$

$B$ is bandwidth of the receiver bandpass filter $[Hz]$. 

As defined in Equation (32) the variance of the received noise is proportional to the receiver
bandwidth.

As the received sequence is modeled as AWGN, it is shown that the second moment is a
sufficient statistic [29]. Defining $Q_t = X_t^2$, where $X_t \sim N(0, \sigma^2)$ out of a sequence with $n$ samples, then

$$p_0(q) = \frac{1}{\sigma \sqrt{2\Gamma(\frac{1}{2})}} q^{\frac{\sigma^2}{2}} e^{-\frac{q}{2\sigma^2}}$$

(33)

where

$p_0(q)$ is the Probability Density Function (pdf) of a ChiSquare distribution

$\Gamma(\frac{1}{2})$ the gamma function [29].
The likelihood \( L \) of the samples is the conditional joint density function as indicated in Equation (34).

\[
L(q_1, q_2, ..., q_n | \sigma^2) = f(q_1, q_2, ..., q_n | \sigma^2)
\]  

(34)

Under the condition of independency as will be examined in Section 4.2, Equation (34) is rewritten:

\[
f(q_1 | \sigma^2) \cdot f(q_2 | \sigma^2) \cdot ... \cdot f(q_n | \sigma^2) = \frac{1}{\sigma \sqrt{2\pi}} q_i e^{-\frac{q_i^2}{2\sigma^2}} \cdot \frac{1}{\sigma \sqrt{2\pi}} q_2 e^{-\frac{q_2^2}{2\sigma^2}} \cdot ... \cdot \frac{1}{\sigma \sqrt{2\pi}} q_n e^{-\frac{q_n^2}{2\sigma^2}}
\]  

(35)

Rearranging Equation (35) yields

\[
L(q_1, q_2, ..., q_n | \sigma^2) = \left( \frac{1}{\sigma \sqrt{2\pi}} \right)^n e^{-\frac{1}{2\sigma^2} \sum q_i} \left( \frac{\sigma \sqrt{2\pi}}{\sigma \sqrt{2\pi}} \right)^n e^{\frac{1}{2\sigma^2} \sum q_i} = \left( \frac{1}{\sigma \sqrt{2\pi}} \right)^n e^{\frac{1}{2\sigma^2} \sum q_i}
\]  

(36)

Where \( \overline{q} \) denotes the second moment of \( X_i \), since per definition \( Q_i = X_i^2 \).

Using the factorization criterion [31] it is shown that \( \overline{q} \) is a sufficient estimator for the estimation of the energy \( \sigma^2 \), since \( \left( \frac{1}{\sigma \sqrt{2\pi}} \right)^n e^{\frac{1}{2\sigma^2} \sum q_i} \) is only a function of \( \overline{q} \) and \( \sigma^2 \), and \( \left( \frac{1}{\sigma \sqrt{2\pi}} \right)^n e^{\frac{1}{2\sigma^2} \sum q_i} \) is not a function of \( \sigma^2 \).

The next paragraph relates the noise model to the frequency mixer.
4.1.2 Frequency Mixing

The incoming GPS signal is modeled [3] as indicated in Equation (37)

\[ r(t) = s(t) + n(t) \]  

(37)

where \( s(t) \) depicts the GPS signal as defined in Equation (2) and \( n(t) \) the received noise as discussed in the previous paragraph, Section 4.1.1. Since the received GPS signal is at approx. -130 dBm, and in a 2 MHz bandwidth, the noise floor is at -110 dBm, the expression given in Equation (38) holds:

\[ S << N \]  

(38)

where \( S \) is the signal power and \( N \) the noise power.

Thus, it is justified to approximate \( r(t) \) to be a real, bandpass, zero mean, continuous WSS Gaussian process \( X(t) \) [28]. It can be represented by its complex envelope [32]

\[ X(t) = \text{Re}[r(t)e^{j\omega t}] \]  

(39)

The signal \( X(t) \) as specified in Equation (39) is fed into the mixer as shown in Figure 8.

![Figure 8, Frequency mixer](image-url)
This yields for the mixer output $Y(t)$

$$Y(t) = A_{to} \text{Re}\{r(t)e^{j\omega_1 t}\}\cos(\omega_2 t)$$  \hfill (40)

Using common trigonometric identities, the expression in Equation (40) is rearranged as:

$$Y(t) = \frac{A_{to}}{4} \left\{ r(t)e^{j\omega_1 t} + r^*(t)e^{-j\omega_1 t}\right\}\left\{ e^{j\omega_2 t} + e^{-j\omega_2 t}\right\}$$  \hfill (41)

Equation (41) is expanded into the form

$$Y(t) = \frac{A_{to}}{2} \left\{ r(t)e^{j(\omega_1 + \omega_2) t} + r^*(t)e^{-j(\omega_1 + \omega_2) t} + r(t)e^{j(\omega_1 - \omega_2) t} + r^*(t)e^{-j(\omega_1 - \omega_2) t}\right\}$$  \hfill (42)

which yields the expression as introduced in Equation (43)

$$Y(t) = \frac{A_{to}}{2} \text{Re}\{r(t)e^{j(\omega_1 + \omega_2) t}\} + \frac{A_{to}}{2} \text{Re}\{r(t)e^{j(\omega_1 - \omega_2) t}\}$$  \hfill (43)

The process $X(t)$ is translated to the frequencies $\omega_1 + \omega_2$ and $\omega_1 - \omega_2$ and scaled by $A_{to}/2$. Assuming an ideal mixer, the incoming random process is translated to a different frequency but no other effects are modifying the signal. Thus the mixing operation does not change the statistical properties of the signal.

Next the impact of the amplification stages on the noise properties is discussed.
4.1.3 Amplifier

The signal $r(t)$ is fed into the amplifier as shown in Figure 9.

$$r(t) \rightarrow v \rightarrow u(t)$$

\text{Figure 9, Amplifier}

If the amplifier is not driven into saturation, a linear amplification can be assumed. This defines the transfer function as introduced in Equation (44).

$$u(t) = v \cdot r(t) \quad (44)$$

By use of the method of transformations [29], the pdf of $u(t)$ is derived as:

$$f_u = f_r \left| \frac{dr}{du} \right| \text{ where } r = h^{-1}(u) \quad (45)$$

Assuming $\nu > 0$, the pdf is given by

$$f_u(u) = f_r \left( \frac{u}{\nu} \right) \cdot \frac{1}{\nu} \quad (46)$$

Thus as derived in Equation (46) the amplifier does not change the statistical properties of the received signal, it just rescales the received time series.

Finally, the effect of bandpass filtering on the incoming noise sequence is analyzed.
4.1.4 Bandpass Filtering

Based on the characterization of the received signal as AWGN as presented Section 4.1.1, the received signal is a WSS process.

When $X(t)$ is defined as a continuous time WSS random process, it is common to refer to the second moment of $X(t)$ as the average power [30].

$$E[X^2(t)] = R_X(0) = \int_{-\infty}^{\infty} S_X(f) df$$  \hspace{1cm} (47)

Where $S_X(f)$ is the PSD and the autocorrelation function $R_X(\tau)$ is defined as

$$R_X(\tau) = E[X(t+\tau)X(t)].$$  \hspace{1cm} (48)

The received AWGN sequence is fed into a bandpass filter, which modifies the PSD of the received noise as depicted in Figure 10. Again it is important to note that the received GPS signal is significantly below the noise floor as defined in Equation (38) and thus the PSD of the received noise, i.e. the composite of the GPS signal plus noise, is a constant over all frequencies.

![Figure 10, Bandlimited AWGN](image-url)
Based on Figure 10 the noise power is determined as

\[
\int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} N_u df + \int_{f_i}^{f_c} N_u df = kTB
\]  

(49)

where the bandwidth \( B \) of the receiver bandpass filter is defined as \( B = f_u - f_i \).

As AWGN passes through the bandpass filter, its properties are altered as shown in Figure 10. For characterization of the underlying time series process, the autocorrelation function is of interest as given in Equation (50).

\[
R(\tau) = \mathbb{E} \left[ X(f) X(f + \tau) \right] = \int_{-f_c}^{f_c} \int_{-f_i}^{f_i} e^{j2\pi f \tau} df
\]

(50)

Solving the integrals as introduced in Equation (50) yields

\[
R(\tau) = \frac{KT}{2\pi} \left( e^{-j2\pi f_c \tau} - e^{-j2\pi f_i \tau} \right) + \frac{KT}{2\pi} \left( e^{j2\pi f_c \tau} - e^{j2\pi f_i \tau} \right)
\]

(51)

Equation (51) is extended into the form:

\[
R(\tau) = \frac{KT}{2\pi} \left( e^{-j2\pi f_c \tau} - e^{-j2\pi f_i \tau} \right) e^{j2\pi f_c \tau} + \frac{KT}{2\pi} \left( e^{j2\pi f_c \tau} - e^{j2\pi f_i \tau} \right) e^{-j2\pi f_i \tau}
\]

(52)

Where \( f_c \) depicts the center frequency of the bandpass filter and is specified as \( f_c = \frac{f_u + f_i}{2} \).

Further defining \( \Delta f = \frac{f_u - f_i}{2} \) and performing the substitutions yields

\[
R(\tau) = \frac{KT}{2\pi} \left( e^{j2\pi f_c \tau} - e^{-j2\pi f_i \tau} \right) + \frac{KT}{2\pi} \left( e^{j2\pi f_i \tau} - e^{-j2\pi f_c \tau} \right)
\]

(53)
Expression (53) is rewritten and yields for the autocorrelation function of bandpass filtered AWGN:

\[ R(\tau) = \frac{kT}{\pi \tau} \sin(\pi B \tau) \cos(2\pi f_c \tau) = kTB \cdot \text{sinc}(\pi B \tau) \cos(2\pi f_c \tau) \]  

(54)

As seen in Equation (54) the autocorrelation consists of a sinc-function, which is driven by the filter bandwidth \( B \), and a cosine function, which is established by the center frequency of the filter.

Verifying the result in Equation (54) for \( \tau \rightarrow 0 \) yields:

\[ \lim_{\tau \rightarrow 0} R(\tau) = \lim_{\tau \rightarrow 0} \frac{kT \sin(2\pi f_c \tau) \cos(2\pi f_c \tau)}{\pi \tau} \]  

(55)

By using de l’Hôpital’s [33] rule:

\[ \lim_{\tau \rightarrow 0} R(\tau) = \lim_{\tau \rightarrow 0} kT \left\{ \frac{\sin(2\pi f_c \tau)(-1) \sin(2\pi f_c \tau) 2\pi f_c}{\pi} + \frac{\cos(2\pi f_c \tau) 2\pi f_c \cos(2\pi f_c \tau)}{\pi} \right\} \]  

(56)

yields

\[ \lim_{\tau \rightarrow 0} R(\tau) = kTB \]  

(57)

Which depicts the signal power as a function of the front end bandwidth and is in agreement with Equation (32).

The theoretical correlation function in Equation (54) is evaluated using the properties of Ohio University’s SDRFE. The incoming signal is down-converted to an IF of 21.27 MHz and filtered to a bandwidth of 2 MHz. The autocorrelation function \( \rho(\tau) \) of the bandpass-filtered AWGN is shown in Figure 11.
As depicted in Figure 11, the envelope of the autocorrelation function is shaped by the sinc-function and the inner oscillation is driven by the cosine term.

It is important to note that as the filter bandwidth $B$ is increased, the induced correlation is reduced. As the bandwidth goes to infinity, the autocorrelation converges to [30]:

$$\lim_{B \to \infty} R(\tau, B) = \frac{kT}{2} \delta(\tau)$$

(58)

Which is the autocorrelation function of an uncorrelated process. Also, the process remains a zero-mean sequence.

The bandpass filter introduces correlation as given by Equation (54), which may affect signal parameter estimation. As general characteristic behavior, it is observed that as the bandwidth
is increased, the serial correlation is reduced and conversely, as the bandwidth is reduced, the correlation is increased.

4.2 IMPACT OF SDRFE-INDUCED EFFECTS ON SIGNAL ESTIMATION

As the induced effects of the SDRFE elements have been derived, this Section applies the results to GPS signal acquisition. This is important in the context of QM, where the SDRFE-induced effects must be taken into account to derive accurate thresholds for anomaly detection.

In any signal acquisition it is of interest to separate the deterministic part, i.e. the signal, from the non-deterministic part, i.e. the noise. It is desired to obtain a highly accurate estimate of the deterministic part. As the received signal is fed through the RF chain of the SDRFE, the statistical properties are altered. The following Section will discuss the SDRFE-induced effects due to bandpass filtering. Note that mixing and amplification do not alter the statistical properties as derived in Sections 4.1.1 and 4.1.2. The discussion is divided into noise estimation and signal estimation paragraphs.

4.2.1 Noise only Estimation

After the signal passes through the SDRFE, it is digitized. The autocorrelation function given in Equation (54) is rewritten in the form:

$$R(q) = \frac{kTf_r}{\pi q} \sin \left( \frac{\pi B q}{q} \right) \cos \left( 2\pi f_s \frac{q}{q} \right)$$

(59)

where

- $f_s$ is the sampling frequency
- $q$ is the lag in samples
The energy of a signal is evaluated by the second moment. If the signal has $M$ samples and each sample has a distribution of the form $Z \sim N(0, kTB)$ based on the noise model of Section 4.1.1 as AWGN sequence, then the signal energy is estimated as:

$$E\left[\sum_{i=1}^{M} Z_i^2\right] = \sum_{i=1}^{M} E[Z_i^2] = M \cdot E[Z_i^2] = M \hat{\sigma}_Z^2$$

(60)

Thus, the energy estimator using the second moment is unbiased. Further, $\hat{\sigma}_Z^2$ has a ChiSquare distribution since $\sum_{i=1}^{M} Z_i^2 \sim \chi^2$ with $M - 1$ degrees of freedom if and only if $\{Z_i\}$ is independent. However, it is important to recognize that the bandpass filter of the SDRFE introduces serial correlation of the data as derived in Equation (54) and illustrated in Figure 11. Therefore, dependent on the correlation function, a different distribution function for $\hat{\sigma}_Z^2$ is obtained.

In general, the sample variance of the noise power estimator is evaluated as given by Equation (61).

$$\text{VAR}\left(\sum_{i=1}^{M} Z_i^2\right) = \sum_{i=1}^{M} \text{VAR}(Z_i^2) + 2 \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \text{COV}(Z_i^2, Z_j^2)$$

(61)

If the second term in Equation (61) approaches zero, the second moment approaches a ChiSquare distribution. However, due to the serial correlation of the bandpass filtered AWGN, the sample variance is also driven by a covariance addend, which instigates the change of the distribution function of $\hat{\sigma}_Z^2$. The impact of the covariance term is evaluated numerically and the results are introduced in the last Section of this Chapter.

In summary, the pdf of the noise after the filter is a function of the correlation structure of the process.
4.2.2 Signal plus Noise Estimation

Next, the analysis of the signal energy estimator is performed. The received signal is a composite of the GPS signal plus the channel noise as modeled in Equation (37).

For GPS, the signal $s(t)$ is of the form as specified in Equation (2).

$$s(t) = AG(t)D(t)\sin(2\pi(f_0 + \Delta f_{sv})t + \phi)$$  \hspace{1cm} (62)

The received composite of AWGN and the GPS signal is fed into the SDRFE and sampled at $f_s$. This yields the discrete representation for the received signal $r(kT_s)$ [18]:

$$r(kT_s) = \frac{A_m}{2} \{AG(kT_s)D(kT_s)\sin(2\pi f_{rf}kT_s + \phi) + n(kT_s)\}$$  \hspace{1cm} (63)

where

$k$ denotes the $k^{th}$ sample (no to be confused with Boltzmann’s constant). The double frequency term from the mixer is neglected, since it goes to zero after the correlation. Further, $f_0 + \Delta f_{sv} - f_{Lo}$ represents the translation to the IF which is substituted by $f_{rf}$. The received noise is given by $n(t)$.

The demodulation is performed by multiplication by the sine and cosine functions to extract the in-phase and quadrature-phase components. After the correlation, the I channel is given by

$$\sqrt{E_I} = \frac{A_m}{2} \sum_{i=1}^{M} \{AG_i(kT_s)D_i(kT_s)\frac{1}{2} \cos(2\pi(f_{rf} - f_i)kT_s + \phi) + n(kT_s)G_i(kT_s)\sin(2\pi f_{rf}kT_s)\}$$  \hspace{1cm} (64)

Where $\sqrt{E_I}$ denotes the square root of the correlation energy of the I channel, $f_i$ denotes the local frequency for generation of the replica code and carrier frequency, and $G_i$ the locally generated replica code.
Assuming a perfectly acquired carrier signal, i.e. \( f_{c} - f_{i} = 0 \), Equation (64) is simplified as follows:

\[
\sqrt{E_t} = \frac{A_{e}}{2} \left\{ AR(pT_{c}) \text{sign}(D(kT_{c})) \frac{1}{2} \cos(\phi) + \sum_{i=1}^{M} n(kT_{c}) G_{i}(kT_{c}) \sin(2\pi f_{c} kT_{c}) \right\}
\]  

(65)

Where \( p \) denotes the lead/lag in samples of the locally generated C/A replica code relative to the incoming code.

The expression for the Q channel after correlation is given by

\[
\sqrt{E_q} = \frac{A_{e}}{2} \sum_{i=1}^{M} \left\{ AG_{i}(kT_{c}) D(kT_{c}) \frac{1}{2} \sin(2\pi f_{c} kT_{c} + \phi) + n(kT_{c}) G_{i}(kT_{c}) \cos(2\pi f_{c} kT_{c}) \right\}
\]  

(66)

Where \( \sqrt{E_q} \) denotes the square root of the correlation energy of the Q channel.

Assuming a perfectly acquired carrier signal, i.e. \( f_{c} - f_{i} = 0 \) Equation (66) is simplified as follows:

\[
\sqrt{E_q} = \frac{A_{e}}{2} \left\{ AR(pT_{c}) \text{sign}(D(kT_{c})) \frac{1}{2} \sin(\phi) + \sum_{i=1}^{M} n(kT_{c}) G_{i}(kT_{c}) \cos(2\pi f_{c} kT_{c}) \right\}
\]  

(67)

To show the influence of the correlation induced by the bandpass filter, the expected value and the sample variance of the square-root energy detector defined in Equation (65) and (67) is derived. To obtain the variance of \( \sqrt{E_t} \) and \( \sqrt{E_q} \), the identity as derived in Equations (68) through (70) is used.
Define \( a \) as constant and \( X \) as a random variable with a given pdf. Then

\[
VAR[a + X] = E[(a + X)^2] - E[a + X]^2
\]  

(68)

This is simplified into

\[
VAR[a + X] = a^2 + 2aE[X] + E[X^2] - a^2 - 2aE[X] - E[X]^2
\]  

(69)

which reduces to

\[
VAR[a + X] = E[X^2] - E[X]^2 = VAR[X]
\]  

(70)

By applying the identity of Equation (70), the sample variance of \( \sqrt{E_i} \) (also refer to Equation (65)) is rewritten as

\[
VAR(\sqrt{E_i}) = VAR\left[\frac{A_i}{2} \sum_{k=1}^{M} n(kT_s) G_i(kT_s) \sin(2\pi f_s kT_s)\right]
\]  

(71)

Equation (71) is simplified into the form:

\[
VAR(\sqrt{E_i}) = \frac{A_i^2}{4} \left\{ \sum_{k=1}^{M} G_i^2(kT_s) \sin^2(2\pi f_s kT_s) Var(n(kT_s)) \right. \\
+ \sum_{k=1}^{M-1} \sum_{\ell=1}^{M} G_i(kT_s) G_j(\ell T_s) \sin(2\pi f_s kT_s) \sin(2\pi f_s \ell T_s) Cov(n(kT_s), n(\ell T_s)) \right\}
\]  

(72)

Where \( Cov(n(kT_s), n(\ell T_s)) \) is given by the autocorrelation function in Equation (16).
As the variance of the noise is known, Equation (72) is rewritten as

\[
\text{VAR}(E_i) = \frac{A_c^2}{4} \left\{ \sum_{i=1}^{m} \sin^2(2\pi f_i T_i) \sigma^2 + \sum_{k=1}^{M} \sum_{l=1}^{M} G(kT_i)G(lT_i) \sin(2\pi f_i T_i) \sin(2\pi f_l T_l) \frac{kTf_{\phi}}{\pi(l-k)} \cos(2\pi f_i \frac{l-k}{T_i}) \right\}
\]

(73)

As shown by Equation (73) the sample variance is driven by two summation terms, i.e. the ‘modulated’ sample variance of independent RV and the correlation terms, which are induced by the SDRFE bandpass filter. The sample variance of the quadrature-phase channel is given by:

\[
\text{VAR}(E_i^Q) = \frac{A_c^2}{4} \left\{ \sum_{i=1}^{m} \cos^2(2\pi f_i T_i) \sigma^2 + \sum_{k=1}^{M} \sum_{l=1}^{M} G(kT_i)G(lT_i) \cos(2\pi f_i T_i) \cos(2\pi f_l T_l) \frac{kTf_{\phi}}{\pi(l-k)} \sin(2\pi f_i \frac{l-k}{T_i}) \right\}
\]

(74)

Further, the derivation of the expected value of \( \sqrt{E_i} \) yields Equation (75)

\[
E[\sqrt{E_i}] = E \left[ \frac{A_c}{2} AR(pT, \text{sign}(D(kT_i))) \frac{1}{\pi} \cos(2\pi(f_o - f_i)kT_i + \phi) \right]
\]

(75)

And Equation (76) for the expected value of \( \sqrt{E_i^Q} \):

\[
E[\sqrt{E_i^Q}] = E \left[ \frac{A_c}{2} AR(pT, \text{sign}(D(kT_i))) \frac{1}{\pi} \sin(2\pi(f_o - f_i)kT_i + \phi) \right]
\]

(76)

which represents the signal amplitudes in the in-phase and quadrature-phase channels, respectively.

In conclusion, the bandpass-induced correlation does not affect the expected value of the signal power, however it is driving the sample variance, which is the equivalent behavior as observed in the noise estimation case.
The influence of the covariance terms in Equation (73) and (74) is numerically quantified in the following paragraph.

### 4.3 Simulation Results

To illustrate the significance of the results derived in Section 4.2, a simulation for the noise and signal estimation error was performed. This was implemented by evaluating the ratio of the covariance term to the variance term as a function of the filter bandwidth. This indicates the amount of the sample variance that is driven by the covariance introduced by the bandpass filter bandwidth.

For the signal estimation, the covariance to variance ratio of the I and Q channels is obtained from Equations (73) and (74).

\[
\alpha = \frac{\sum_{k=1}^{M-1} \sum_{i=1}^{M} G(kT_s)G(lT_s)\sin(2\pi f_jkT_s)\sin(2\pi f_jlT_s)\frac{kT_f}{\pi l-k} \sin(\pi B\frac{i-k}{f}) \cos(2\pi f_j\frac{i-k}{f})}{\sum_{i=1}^{M} \sin^2(2\pi f_jkT_s)\sigma^2} \tag{77}
\]

The computation of Equation (77), neglecting the varying Doppler shift and oscillator drift, yields Figure 12.
Figure 12 depicts an asymptotic shape of the covariance to variance ratio $\alpha$ as a function of the bandpass filter bandwidth. The larger the bandpass filter bandwidth, the less significant becomes the role of the filter-induced correlation onto signal estimation. Further, Figure 12 also illustrates filter bandwidth ranges, where the covariance terms are negative, i.e. the bandwidth induced correlation decreases the sample variance of the estimator. However it is important to recognize that the covariance term as introduced in Equation (77) is a function of the incoming signal frequency. As a satellite is passing overhead, the correlation function changes as a direct cause of the Doppler frequency shift. Thus, it is not possible to keep the covariance function stationary in one point. Therefore, the results illustrated in Figure 12 represent a specific realization of the process and the upper bound values describe the worst case values.
For the noise energy estimation, the covariance to variance ratio of the I and Q channels is obtained from Equation (61):

$$\beta = \frac{2 \sum_{i=1}^{M} \sum_{j=1}^{M} \text{COV}(Z_i^2, Z_j^2)}{\sum_{i=1}^{M} \text{VAR}(Z_i^2)} \quad (78)$$

Due to the fact that the covariance structure for the noise energy estimation is a function of a ChiSquare distribution as shown in Equation (61), no symbolic expression for the covariance is derived. Therefore, the covariance to variance ratio is implemented by filtering a random sequence with a Chebyshev filter and by subsequent evaluation of the sample variance at the input and output of the filter. Defining X as the input sequence and Y as the output sequence, then an equivalent of the covariance to variance ratio as introduced in Equation (78) is obtained as

$$\hat{\beta} = \frac{\text{VAR}(E[Y^2]) - \text{VAR}(E[X^2])}{\text{VAR}(E[X^2])} \quad (79)$$

The implementation of Equation (79) yields

![Figure 13, Covariance to variance ratio of noise vs. bandwidth](image)

As shown in Figure 13 the covariance to variance ratio $\beta$ is negative and linearly approaches zero as the bandwidth increases.
This indicates that the smaller the filter bandwidth, the smaller is the sample variance on the noise estimation. This is explained by the fact that the smaller the filter bandwidth, the less distortion energy is added into the system. Thus, to attain an optimal signal to noise ratio estimation, the bandpass filter bandwidth should be chosen in the range of the signal bandwidth, as expected. If it is too large, more distortion energy degrades the process.

4.4 Case Study

Finally, the obtained results are applied to the signal acquisition process. As elaborated in the introductory Section, fine-frequency estimates are obtained over a sequence of 1 ms data blocks by calculating the relative phase of each block by using the in-phase and quadrature-phase components and then taking the successive differences. With a strong GPS signal (CNR ~ 45 dB-Hz), phase estimates in the Ohio University setup contain approx. 0.1 rad of phase variation, giving frequency noise of approximately 22 Hz.

The obtained results of the signal covariance to variance ratio as defined in Equation (77) vs. the induced frequency noise due to serial correlation of the data are implemented. The simulation yields Figure 14.

![Figure 14](image)
As depicted in Figure 14 a covariance to variance ratio of 0.5, which is obtained by a filter bandwidth of 2 MHz, may introduce up to 10 Hz of additional frequency noise. This is explained by the fact that serial correlation of the data increases the sample variance on the in-phase and quadrature-phase energy estimators. Since the frequency estimates are obtained by calculating the relative phase of each block using the baseband in-phase and quadrature-phase components and then taking the successive differences, a higher sample variance on the energy estimators introduces a higher variance on the frequency estimates. Therefore decreasing the serial correlation by increasing the filter bandwidth, reduces the covariance to variance ratio (refer to Figure 12) and thus reduces the variance on the in-phase and quadrature-phase components leading to a lower sample variance on the frequency estimation. It is important to emphasize that the results shown above illustrate a worst case scenario. Referring to Figure 12 indicates that the covariance to variance ratio also may be negative, which yields a reduction of the sample variance of the energy estimation leading to decreased frequency noise. Further, a larger filter bandwidth increases the distortion energy in the system, which affects the signal to noise ratio estimation (refer to Figure 13). This indicates that the bandpass filter bandwidth is a trade-off between signal estimation errors and noise distortion energy.

4.5 IN SUMMARY

Chapter 4 discussed the effects of the software receiver front-end components on the signal statistics. It was shown that amplification and frequency mixing do not change the statistical properties of the received signal. However, bandpass filtering introduces serial correlation into the data that leads to estimation errors. A simulation was performed that demonstrates that a receiver bandwidth of 2 MHz can introduce additional frequency noise up to 10 Hz. It was shown that the bandpass filter bandwidth is a trade-off between signal estimation errors and noise distortion energy. As computer technology advances, wide-band SDRFE will be introduced and thus the bandpass filter induced correlation can be neglected.

Although the motivation of this Chapter resulted from the implementation of the SDR, the concepts introduced also apply to GPS receivers in general.
5 BIAS DETECTION AND ITS CONFIDENCE ASSESSMENT

As outlined in Section 2.2 the task of detecting an anomaly in the GPS signal requires a statistical decision process to estimate the deterministic part of the signal. The goal of QM is to establish statistical estimation procedures which are capable of revealing a signal anomaly at a given level of confidence. There is no a priori knowledge available on the characterization of a signal failure mode and, implicitly, on the statistics of a distorted signal. Any assumption would reduce the integrity of the detection procedure.

Any statistical decision on the existence of a signal failure mode inherently is subject to a probability of false alert $P_{FA}$ and a probability of missed detection $P_{MD}$. $P_{FA}$ describes the probability that a failure mode is detected, however in truth no failure exists. $P_{MD}$ describes the probability that a failure exists, but is not detected by the monitor. As illustrated in Figure 15, the decision on the existence of a system error is accomplished through the definition of a DT. The comparison of the parameter under investigation to the DT yields the decision mechanism.

![Diagram](Image)

**Figure 15, Minimum detectable bias**

The MDB describes the minimum bias that can be detected under the condition of specified $P_{FA}$ and $P_{MD}$. Important to recognize is that the MDB is directly related to the distribution functions of the anomaly-free and anomalous signal. However, since the statistical behavior
of the anomalous signal is unknown, there is no *a priori* information available of the anomaly to be investigated. This prevents the mathematical derivation of an *a priori* MDB.

The first Section of this Chapter outlines the problem statement. The following Section discusses the assumptions of the bias detection algorithm. The subsequent Section introduces a large sample T-test to perform confidence assessment of an anomaly. The last Section discusses application considerations and algorithm validation through simulations.

### 5.1 Problem Statement

As outlined above, the objective of statistics is to make an inference about unknown parameters based on information contained in sampled data. The received GPS signal as introduced in Equation (37) depicts a random process. In the context of QM, the statistical inference on signal parameters is used to test the received GPS signals for possible anomalies. For a possible failure mode, the question arises of what level of confidence is placed on the existence of a detected error. This task is accomplished by means of hypothesis test procedures.

This is further illustrated in Figure 16. A target parameter $Y$, which is chosen to be $Y=0$, is estimated over time. The process is represented in Equation (80):

\[ X_i = Y + \varepsilon_i \]  

(80)

A normally-distributed error term of the form $\varepsilon_i \sim N(0,2)$ is added to the process as shown in Equation (80). Further, a bias of 1 is induced onto the target parameter $Y$ at $T=500$. 
The primary QM task is to detect the induced bias at T=500 with a given level of confidence.

In the context of QM, the LGF specification [11] allows for a TTA of 3 s. Hence, the primary objective of an anomaly detection scheme is not tailored towards precise detection in time, but detection at a high level of confidence.

5.2 Assumptions

The proposed bias detection scheme is based on the assumptions outlined below:

- The samples are sufficiently independent (refer to Chapter 4).
- An anomaly will persist for an unknown time span and is not just a single sample time event.
- The distribution functions of the estimated signal parameters are unknown.
- The distributions between the assessment and evaluation window may not be identical, including the variances between the two population groups.
• An anomaly-free signal period is available to perform a statistical assessment of the process.
• A WSS time process, i.e. no dynamics are present in the region of investigation.

Numerous methods such as Analysis of Variance (ANOVA) [29], Mann-Whitney U Test [34], Kolmogorov-Smirnov Test [35], Wilcoxon Test [35], Kruskal-Wallis Test [34], etc. are published and well documented in literature. The statistical inference procedures are distinguished between parametric and non-parametric tests. Parametric tests such as ANOVA require a normally distributed population and common variance of the data sets, i.e. homoscedasticity. Non-parametric tests are considered distribution independent of the underlying population. However they do require other assumptions that must be met in order to achieve a reliable test statistic. The Mann-Whitney U Test, which denotes a special case of the Kruskal-Wallis Test, both require homoscedasticity and identical distributions among the groups. The Kolmogorov-Smirnov Test is non-parametric and does not require homoscedasticity at the cost of reduced statistical efficiency. However it requires identical distributions among the groups.

In the context of anomaly detection, it is important to recognize that a non-parametric test is required and further, the criteria of identical distributions and homoscedasticity are not necessarily satisfied. Any assumption would implicitly reduce the integrity of the algorithm and thus defeat the purpose of the implementation. Since the SDR provides a large amount of samples, a non-parametric large sample T-test is introduced, which is capable of providing an over bound error on the test statistic by the Berry-Esseen theorem [36]. Some of the tests as presented above might still perform acceptably well even under considerable heterogeneity of the underlying assumptions. However since the performance prediction highly relies on the properties of the anomaly, the large sample T-test is regarded to be superior.
5.3 Approach

The large sample T-test has its foundation on the advantage of the SDR in providing samples at a very high data rate. If in a given scenario the algorithm does not provide satisfying results, i.e. too short term anomalies, increasing the sampling rate leverages more power to the large sample T-test. The algorithm presented in this document incorporates an *assessment window* (see Figure 17) consisting of \( n \) samples, which allows for assessment of the statistical properties of the random process. In the time period of gathering the statistical properties of the process, it is assumed that no anomaly is present in the signal. The larger \( n \) is selected, the more accurate the assessment of the statistical properties of the process, i.e. the more accurate the calibration. The upper limit for \( n \) is determined by the time span in which the process can be considered to be WSS. In addition, the proposed method also incorporates an *evaluation window* of size \( k \) samples, which is shifted over the incoming data stream as shown in Figure 17.

![Assessment and evaluation window](image)

The estimated value of the evaluation window is tested on a possible inconsistency against the assessment window through a statistical hypothesis test introduced in Section 5.4. Since the distribution functions for the signal under investigation are unknown, the introduced statistical test makes use of the CLT. To address the overbound error on the CLT and
inherently the subsequent error on the test statistic, the Berry-Esseen bound [36] is discussed in Section 5.4.1, which allows for a quantification under which the test statistic is capable of operating reliably.

5.4 LARGE SAMPLE T-TEST

For recovering the signal parameter to be estimated, the expected mean value estimator as defined in Equation (81) is chosen.

\[ Y_n = \frac{1}{n} \sum_{i=1}^{n} X_i \to E[X] \]  

(81)

This is based on two criteria:

a) \( Y_n \) is a sufficient estimator [29].

b) As \( n \) is chosen large enough, \( Y_n \) converges to a normally-distributed estimator based on the CLT. The characterization of the approximation error and the speed of convergence in the CLT are further documented in Section 5.4.1.

Criterion (b) is important for the purpose of QM, since the distribution function of the signal under investigation is generally unknown. Therefore, the CLT translates the bias detection algorithm as presented in this document into a distribution-independent algorithm and, hence, generalizes the algorithm into a non-parametric test.

In summary it is important to recognize that the statistical inference is performed on \( Y_n \) as defined in Equation (81), since \( Y_n \) converges to a normal distribution. This is not to be confused with performing a statistical inference on \( X_{n+1} \) based on \( X_1 \ldots X_n \) because the distribution of the population \( X \) is unknown.

The following Section reviews the CLT and addresses the speed of convergence towards a normal distribution.
5.4.1 The Central Limit Theorem and the Berry-Esseen Bound

In the context of QM, it is important to associate confidence bounds on the detection of anomalies. However, since the distribution functions of the anomalies under investigation are unknown, self normalizing sums [36] are introduced, which translate the unknown distribution functions into a quantifiable distribution. The foundation of this transformation process is the CLT [36] as defined in the following paragraph.

Defining \( Y_n = \sum_{i=1}^{n} X_i \) be the sum of independent identically distributed (IID) [30] random variables, where the mean and variance of \( X_i \) is \( \mu \) and \( \sigma^2 \), respectively. Further, the distribution function \( F \) of \( X_i \) is unknown. Given that \( \sigma^2 < \infty \) then the CLT states [36]:

\[
\sqrt{n}(Y_n - \mu) \xrightarrow{L} N(0, \sigma^2)
\]  

(82)

While the normal distribution in the CLT is independent of the distribution \( F \) of the process \( X_i \), the required sample size \( n \) to adequately approximate a normal distribution and the speed of convergence are unknown.

This dilemma is resolved by the Berry-Esseen theorem [31]. It allows the determination of a bound for the expected error when approximating \( \phi(y) \) by \( H_n(y) \), where \( \phi(y) \) is the unit normal pdf and \( H_n(y) \) denotes the Cumulative Distribution Function (cdf) of \( \frac{(Y_n - E[Y_n])}{\sigma \sqrt{n}} \), and \( \Phi(y) \) is the unit normal cdf. If \( X_1, \ldots, X_n \) are IID with an unknown distribution \( F \), which has a finite third moment, then there exists a constant \( C \), which is independent of \( F \), such that the inequality as introduced in Equation (83) holds for any value of \( y \) [31]:

\[
|H_n(y) - \Phi(y)| = \left| \int_{-\infty}^{y} h_n(x) \, dx - \int_{-\infty}^{y} \phi(x) \, dx \right| \leq \frac{C \cdot \rho}{\sigma^3 \sqrt{n}}
\]  

(83)
Where $\rho$ is defined as the third moment $E[X_n - \mu]^3$. The important aspect of inequality (83) is not the explicit value of $C$, however the fact that $C$ is independent of $F$. The Berry-Esseen theorem asserts for the convergence the order $O\left(n^{-\frac{1}{2}}\right)$, where the order $a_n = o(b_n)$ is defined as $n \to \infty$ then $\frac{a_n}{b_n} \to 0$ [36]. The smallest value of $C$ for which equality (83) holds is unknown. Shiganov introduced the constant to be $C = 0.7655$ [37], which is generally considered to be the best result obtained thus far. It was shown by van Beek that equality (83) does not hold when $C < 0.4097$ [38]. Further refinement of $C$ is published in [39] through deriving a sample-size-dependent coefficient instead of targeting an absolute constant $C$.

To establish a better understanding of the Berry-Esseen theorem, Equation (83) is rewritten in the form

$$\left[ P \left( \frac{1}{\sigma \sqrt{n}} \sum_{i=1}^{n} X_i \leq y \right) - \Phi(y) \right] \leq \frac{C \cdot E[X_n - \mu]}{\sigma^3 \sqrt{n}}$$

(84)

The Berry-Esseen theorem requires the knowledge of $\mu$ and $\sigma^2$ of the process. In the context of QM the process properties are not available. Thus the substitutions $\mu$ by $\hat{\mu}$ and $\sigma^2$ by $\hat{\sigma}^2$ are introduced.

$$|H_n(y) - \Phi(y)| \leq \frac{C \cdot E[X_n - \hat{\mu}]}{\hat{\sigma}^3 \sqrt{n}}$$

(85)

Rewriting Equation (85) for $X_n - \hat{\mu} > 0$ yields

$$|H_n(y) - \Phi(y)| \leq \frac{C \cdot \left\{ E[X_n^2] - 3E[X_n]E[X_n^2] + 2E[X_n^4] \right\}}{\left\{ E[X_n^2] - E[X_n] \right\}^2 \sqrt{n}}$$

(86)
and accordingly, for $X_n - \hat{\mu} < 0$

$$|H_n(y) - \Phi(y)| \leq \frac{C \cdot \left\{ E[X_n^3] - 3E[X_n]E[X_n^2] + 2E[X_n]^3 \right\}}{\left\{ E[X_n^2] - E[X_n]^2 \right\}^{3/2} \sqrt{n}}$$  \hspace{1cm} (87)

Therefore the inequality as defined in Equations (86) and (87) represents an estimated Berry-Esseen bound on the CLT approximation. Since the Berry-Esseen bound is used to quantify the validity of the CLT approximation no confidence level on the estimated Berry-Esseen bound is introduced.

To illustrate the performance of the estimated Berry-Esseen theorem, a simulation of a ChiSquare process [30] is implemented with $\mu = 4$ and $\sigma = 4$ and the estimated Berry-Esseen bound $\frac{C \cdot E|X_n - \hat{\mu}|}{\sigma^3 \sqrt{n}}$ of Equation (85) is evaluated as indicated in Equations (86) and (87) versus the number of samples $n$.

\[ \frac{C \cdot E|X_n - \hat{\mu}|}{\sigma^3 \sqrt{n}} \]

\[ 10^0 \]
\[ 10^{-1} \]
\[ 10^{-2} \]
\[ 0 \quad 500 \quad 1000 \quad 1500 \quad 2000 \quad 2500 \quad 3000 \quad 3500 \quad 4000 \quad 4500 \quad 5000 \]

Number of samples

Figure 18, Estimated Berry-Esseen bound vs. samples $n$
As depicted in Figure 18, the estimated Berry-Esseen bound indicates the convergence of $H_n(y)$ towards $\Phi(y)$, or differently expressed the convergence of $Y_n$ (see Equation (81)) towards a normal distribution $\phi(y)$.

The Edgeworth correction [36], [31] introduced in Equation (88) provides the distribution of the correction term for $H_n(y)$ with respect to $\Phi(y)$ as a function of $y$. It only holds if $X_1,...,X_n$ are IID, $E|X_\mu - \bar{\mu}| < \infty$ and the distribution $F$ of $X_1,...,X_n$ is not a Lattice type distribution [36].

$$H_n(y) = \Phi(y) + \frac{E|X_\mu - \bar{\mu}|}{\sigma_3^{1/2}}(1 - y^2)\phi(y) + a\left(\frac{1}{\sqrt{n}}\right)$$

(88)

$\phi(y)$ is the pdf defined by $\phi(y) = \frac{d\Phi(y)}{dy}$. The term $\frac{E|X_\mu - \bar{\mu}|}{\sigma_3^{1/2}}(1 - y^2)\phi(y) + a\left(\frac{1}{\sqrt{n}}\right)$ expresses the correction term of $H_n(y)$ towards $\Phi(y)$ as a function of $y$.

Implementing Equation (88), assuming convergence of $\frac{1}{\sqrt{n}}$, yields Figure 19.
As depicted in Figure 19 the significant correction terms are located around the one-sigma region. The correction term approaches 0 as $y \to \infty$, i.e. the approximation error of $h_n(y)$ with respect to $\phi(y)$ converges to zero. This is an important aspect in the context of QM, since the tail probabilities are of special interest. Further as stated in [36], if the distribution under investigation is roughly symmetric, then the remainder $\rho(j_{(n)})$ in Equation (88) is of smaller order.

The limitations of the CLT need to be mentioned. The normal tendency of $Y_n$ asserted by the CLT requires some assumptions beyond IID [36]. If $X$ is Cauchy [36] distributed, lattice distributed or more extreme, a heavy tail distribution process, in which $X$ is more variable than a single observation, then the CLT will not converge as $n \to \infty$ [31]. Apparently, in these cases the application of the proposed detection scheme is not adequate. Further, the Berry-Esseen theorem and the Edgeworth correction only hold, if $E|X_1 - \mu|^4 < \infty$. 

Figure 19, Edgeworth probability density correction and unit normal density function
5.4.2 In Summary

The Berry-Esseen theorem and the Edgeworth correction yield the overbound distribution for \( y_n \). The Edgeworth correction describes the distribution of the approximation error of \( \Phi(y) \) by \( H_n(y) \). The Berry-Esseen theorem establishes the over bound value of \( |H_n(y) - \Phi(y)| \).

Further, the importance of distinguishing between statistical inferences on \( X_n \) with unknown distribution \( F \) versus statistical inference on \( Y_n \) is emphasized. Assuming a sample space of \( n \) samples and performing a statistical inference on \( X_n \) yields in general hypothetical results for a probability \( p(x) \ll \frac{1}{n} \), i.e. any assumptions of the tail probabilities delineates an extrapolation. Since the received GPS signal in the timeframe of investigation represents an IID process, the introduction of \( Y_n \) transforms the process into a normally distributed RV, where the convergence is described by the Berry-Esseen theorem and the approximation error by the Edgeworth correction.

5.5 Evaluation of the Variance and the Mean of \( Y_n \)

Section 5.4.1 introduces \( Y_n \) as foundation to perform statistical inference because of the convergence property of \( Y_n \) towards a normally distributed RV by the CLT. In order to establish test statistics to perform statistical inference, \( \mu \) and \( \sigma \) of \( Y_n \) need to be assessed.

Defining \( Y_n = \frac{1}{n} \sum_{i=1}^{n} X_i \) to be the sum of IID random variables, then

\[
\text{VAR}(Y_n) = \text{VAR}\left(\frac{1}{n} \sum_{i=1}^{n} X_i\right) = \frac{1}{n^2} \text{VAR}\left(\sum_{i=1}^{n} X_i\right) = \frac{1}{n} \sigma_n^2 + \frac{2}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \text{COV}(X_i, X_j)
\] (89)

As shown in Equation (89) the variance of \( Y_n \) depicts a composite of the variance of the process variance of \( X \) and a covariance term depending on the correlation structure of the process \( X \).
As shown in Chapter 4, as long as the front-end bandwidth is chosen sufficiently large
(bandwidth $\gg 2$ MHz), the correlation of the received AWGN sequence remains
insignificant and thus the term
$$\frac{2}{n^2} \sum_{i=1}^{n} \sum_{j=i}^{n} \text{COV}(X_i, X_j)$$
approaches zero.

Since $\sigma_x^2$ of the process $X$ is generally unknown, $\hat{\sigma}_x^2$ is determined by the unbiased estimator
defined in Equation (90) [29]:

$$\hat{\sigma}_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$$  \hspace{1cm} (90)

The sample variance of $\hat{\sigma}_x^2$ is then given by:

$$\text{VAR}[\hat{\sigma}_x^2] = \frac{1}{(n-1)^2} \text{VAR} \left[ \sum_{i=1}^{n} (X_i - \bar{X})^2 \right]$$  \hspace{1cm} (91)

To exemplify the characteristic of Equation (91), an independent, normally-distributed time
series is assumed, such that

$$(X_i - \bar{X}) \sim N(0, \sigma_x^2)$$  \hspace{1cm} (92)

From [40], it is given that:

$$\sum_{i=1}^{n} (X_i - \bar{X})^2 \sim \sigma_x^2 \chi^2(n-1)$$  \hspace{1cm} (93)

Equation (93) for the sample variance of $\hat{\sigma}_x^2$ is rewritten as Equation (94).

$$\text{VAR}[\hat{\sigma}_x^2] = \frac{2}{(n-1)} \sigma_x^4$$  \hspace{1cm} (94)
As shown in Equation (94), as $n \to \infty$, the sample variance approaches 0. Thus, as long as $n$ is chosen large enough, $\hat{\sigma}^2$ converges to the most efficient estimator for $\sigma^2$.

Further $\mu$ is approximated through the estimator $\bar{Y}_n$. Since

$$E[\bar{Y}_n] = E\left[\frac{1}{n} \sum_{i=1}^{n} X_i \right] = \frac{1}{n} \sum_{i=1}^{n} E[X_i] = \mu$$

(95)

and thus $\bar{Y}_n$ is an unbiased estimator for $\mu$.

In order to establish the test statistics of the assessment and evaluation window (see Figure 17), the variance of $\bar{Y}_n$ needs to be determined. As specified in Equation (55), under the assumption of negligible correlation and the substitution of $\sigma^2$ through $\hat{\sigma}^2$, the variance of $\bar{Y}_n$ is given as

$$VAR(\bar{Y}_n) = \frac{1}{n} \left( \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 \right)$$

(96)

The next Section addresses the derivation of the test statistics to perform the hypothesis test of the assessment window estimated mean value against the evaluation window estimated mean value.

### 5.6 Assessment Window Characterization

A statistical test consists of a pivotal quantity [29] and an associated rejection region. The pivotal quantity is a function of the sampled process.
The test statistics for the expected value of the random process $X$ with $n$ samples is formed as

$$\frac{\hat{\mu}_{x,\text{Asses}} - \mu_x}{\hat{\sigma}_{x,\text{Asses}}} \rightarrow T(n-1)$$  \hspace{1cm} (97)

where $\hat{\mu}_{x,\text{Asses}}$ is the estimated mean of the process $X$ for the assessment window, $\mu_x$ is the true mean of the population $X$ and $\hat{\sigma}_{x,\text{Asses}}$ is the sample standard deviation of $\hat{\mu}_{x,\text{Asses}}$ determined from $n$ samples and defined as (also refer to Equation (96))

$$\hat{\sigma}_{x,\text{Asses}} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2}$$  \hspace{1cm} (98)

Further, important to recognize is the fact that

$$\hat{\mu}_{x,\text{Asses}} \rightarrow N\left(\mu_{x,\text{Asses}}, \frac{1}{n} \sigma_x^2\right)$$  \hspace{1cm} (99)

as established in Section 5.4.1 by the CLT. The error component is over bound by the Berry-Esseen theorem as introduced in Section 5.4.1.

Based on Equation (97) the confidence interval for $\mu_{x,\text{Asses}}$ can be derived by employing the pivotal method [29].

$$P\left[\hat{\mu}_{x,\text{Asses}} - t_{\alpha/2} \hat{\sigma}_{x,\text{Asses}} \leq \mu_{x,\text{Asses}} \leq \hat{\mu}_{x,\text{Asses}} + t_{\alpha/2} \hat{\sigma}_{x,\text{Asses}}\right] = 1 - \alpha$$  \hspace{1cm} (100)

where $\alpha$ denotes the significance level [29] of the confidence interval, and $t_{\alpha/2}$ denotes the value for which the distribution function $\tau\left(t_{\alpha/2}\right)$ with $n-1$ degrees of freedom has a tail probability of $\alpha/2$. 
For a specified level of significance $\alpha$, an increase in the assessment window size $n$ leads to a narrower confidence interval as indicated in Equation (100).

### 5.7 Evaluation Window Characterization

The evaluation window is of size $k$. Performing the same analysis on the evaluation window as the analysis used for the assessment window yields for the test statistic:

\[
\frac{\hat{\mu}_{\text{Eval}} - \mu}{\hat{\sigma}_{s, \text{Eval}}} \sim T(n-1) \tag{101}
\]

Where the standard deviation of $\hat{\mu}_{s, \text{Eval}}$ is accordingly defined

\[
\hat{\sigma}_{s, \text{Eval}} = \sqrt{\frac{1}{k-1} \sum_{i=1}^{k} \left( X_i - \bar{X} \right)^2} \tag{102}
\]

The confidence interval of the evaluation window is given by:

\[
P\left( \hat{\mu}_{s, \text{Eval}} - t_{\frac{\alpha}{2}} \hat{\sigma}_{s, \text{Eval}} \leq \mu_{s, \text{Eval}} \leq \hat{\mu}_{s, \text{Eval}} + t_{\frac{\alpha}{2}} \hat{\sigma}_{s, \text{Eval}} \right) = 1 - \alpha \tag{103}
\]

### 5.8 Hypothesis Test

Based on the statistical inference of the assessment window and the evaluation window, a hypothesis test is introduced to verify if an inconsistency exists between the assessment window and the evaluation window. The null hypothesis $H_0$ is shown in Equation (104):

\[
H_0 : \hat{\mu}_{s, \text{Eval}} = \hat{\mu}_{s, \text{Assess}} \tag{104}
\]
The alternative hypothesis \( H_A \) is given by Equation (105)

\[
H_A: \mu_{\text{Eval}} \neq \mu_{\text{Asses}}
\]  

(105)

The pivotal quantity to perform the hypothesis test is introduced in Equation (106).

\[
\frac{\mu_{\text{Eval}} - \mu_{\text{Asses}}}{\sqrt{\sigma_{\text{Eval}}^2 + \sigma_{\text{Asses}}^2}} \sim T(k + n - 2)
\]  

(106)

A \( T \)-distribution with more than 30 degrees of freedom can be approximated by a normal distribution [29]. Therefore as long as the assessment window and evaluation window chosen are sufficiently large, Equation (106) is rewritten as (again over bound by Berry-Esseen):

\[
\frac{\mu_{\text{Eval}} - \mu_{\text{Asses}}}{\sqrt{\sigma_{\text{Eval}}^2 + \sigma_{\text{Asses}}^2}} \sim Z
\]  

(107)

Where \( Z \) is standard normally distributed.

### 5.9 Property of the Test Statistic

The test statistic as introduced in Equation (107) is characterized as a Moving Average process [41] of order \( k \) (abbreviated MA(\( k \)) process), where \( k \), in the context of the introduced algorithm, is the evaluation window length. An MA(\( k \)) process is of the form:

\[
X_i = \beta_0 Q_i + \beta_1 Q_{i-1} + \ldots + \beta_k Q_{i-k}
\]  

(108)

Where \( \{\beta_i\} \) are constants and \( \{Q_i\} \) the test statistic of sample \( i \). Given that the front-end bandwidth is chosen sufficiently large enough \( Q_i \) is sufficiently independent and assuming
for the evaluation window $\{\beta\} = \frac{1}{k}$, the autocorrelation function of the MA(k) process is given by [41]

$$\rho(q) = \begin{cases} 
1 & q = 0 \\
\frac{k-q+1}{q+1} & q = 1, \ldots, k \\
0 & q > k \\
\rho(-k) & q < 0
\end{cases}$$  \hspace{1cm} (109)

Therefore the test statistics as presented in Equation (107) is correlated as given by Equation (109).

5.10 Minimum Detectable Bias

The question is raised if the proposed technique allows for determination of the MDB. As illustrated in Figure 15, the MDB is a function of $P_{FA}$ and $P_{MID}$. Since no a priori knowledge of the distribution functions can be assumed, only an estimated a posteriori MDB can be evaluated; under the criterion that an anomaly has been detected beforehand to assess knowledge of the anomalous signal.

An a posteriori MDB estimation technique under $H_A$ is demonstrated in the following paragraph.

![Figure 20, Minimum Detectable Bias under $H_A$](image-url)
Figure 20 illustrates the evaluation window distribution function $f_{\mu_{\text{eval}}}$ and the assessment window distribution function $f_{\mu_{\text{ass}}}$ under the condition of $H_A$. As shown in Section 5.4.1, $f_{\mu_{\text{eval}}}$ converges to $N(\mu_{\text{eval}}, \sigma_{\text{eval}})$ and respectively $f_{\mu_{\text{ass}}}$ converges to $N(\mu_{\text{ass}}, \sigma_{\text{ass}})$. Assuming that $P_{FA}$ and $P_{MD}$ are given, then the DT is derived as:

$$\int_{-\infty}^{y} \hat{f}_{\mu_{\text{ass}}} (y) dy = 1 - P_{FA}$$  \hspace{1cm} (110)$$

Where $Y$ depicts the relative DT with respect to a translated normal distribution by $\hat{\mu}_{\text{ass}}$, i.e. $\hat{f}_{\mu_{\text{ass}}}$ is approximated through $N(0, \hat{\sigma}_{\text{ass}})$. Accordingly the offset $Q$ of the evaluation window distribution function under the condition of $H_A$ is evaluated:

$$\int_{Q}^{y} \hat{f}_{\mu_{\text{eval}}} (y) dy = 1 - P_{MD}$$  \hspace{1cm} (111)$$

Where $\hat{f}_{\mu_{\text{eval}}}$ is approximated through $N(0, \hat{\sigma}_{\text{eval}})$ as justified in Section 5.4.1.

Finally, the a posteriori MDB is formed as

$$\text{MDB}_{\text{aposteriori}} = Y - Q$$  \hspace{1cm} (112)$$

To gain further insight into the test statistics, the concept of p-values is introduced [29]. The p-values specify the probability $p$ that a certain test statistic is satisfied, under the assumption that the null hypothesis $H_0$ is fulfilled. Therefore, the smaller the p-values become, the more compelling the evidence that the null hypothesis $H_0$ is not fulfilled and that $H_0$ should be rejected. The higher the p-values become the more confidence is given that the null hypothesis $H_0$ should be accepted.
As discussed in Equations (110) through (112), the \textit{a posteriori} MDB is derived under H\textsubscript{A}. However, it is the inherent goal of the problem statement to decide on H\textsubscript{A} or H\textsubscript{0} by observing the p-value, under the assumption of H\textsubscript{0}. Therefore the p-value under which H\textsubscript{0} is rejected defines the \textit{a posteriori} DT, or differently expressed, the \textit{a posteriori} DT defines p-value\textsubscript{FA/MD} under which P\textsubscript{FA} and P\textsubscript{MD} are satisfied (113).

\[
p\text{-value}_{FA/MD} = \int_{y}^{\infty} f_{H_{0}}(y) dy \tag{113}
\]

Where

\[
f_{H_{0}} \sim N\left(\mu_{\text{Asses}} - \mu_{\text{Eval}}, \sqrt{\sigma_{\text{Asses}}^2 + \sigma_{\text{Eval}}^2}\right) = N\left(0, \sqrt{\sigma_{\text{Asses}}^2 + \sigma_{\text{Eval}}^2}\right) \tag{114}
\]

as justified in Section 5.4.1.

In summary, the data of the evaluation and assessment window are used to perform a statistical inference on the underlying distribution functions. This allows for calculation of the \textit{a posteriori} DT and subsequently the \textit{a posteriori} MDB. It is noted that those statistical quantities are only hypothetical quantities because they are derived under the assumption of H\textsubscript{A} and H\textsubscript{0}. However the importance of the results is given through the fact that it allows an \textit{a posteriori} performance evaluation of the algorithm.
5.11 Result Verification through Simulation

To verify the performance of the bias detection algorithm a case study is examined. A non-central ChiSquare process [40] is chosen, because it is a common process characteristic of an energy estimator [18].

The process is modeled of the form $X_i = Y_i^2$, where $Y_i \sim N(\mu_i, \sigma_i)$ and $\mu_i = 4$ and $\sigma_i = 2$. From sample 3000 to 4000, a bias is induced by setting $\mu_i = 4.5$ as illustrated in Figure 21. The mean of the process is given through $\mu_x = \sigma_i^2 + \mu_i^2$ [31].

As depicted in Figure 21 the implemented ChiSquare process is evidently non-normal. Further the induced bias is not obvious and the need for a detection scheme is apparent.
A simulation with an assessment window size of 2000 samples and an evaluation window size of 500 samples is performed.

Figure 22 shows the p-value of the test statistic vs. the location of the evaluation window (where the location is denoted by the first sample of the evaluation window). The induced bias is detected, indicated by the decrease of the p-value. Further p-valueFA/MD (established in Equation (113)) by specifying $P_{\text{FA}}=10^{-4}$ and $P_{\text{MD}}=10^{-7}$ is also plotted in Figure 22. The region where p-value is smaller than p-valueFA/MD, the specified missed detection and false alert rates are satisfied.

The evaluation of the \textit{a posteriori} MDB is implemented as established in Equation (110), (111), (112) and yields a value of approx. 4.4 under the criterion of $P_{\text{FA}}=10^{-7}$ and $P_{\text{MD}}=10^{-4}$. A bias smaller than 4.4 may still be observable in the test statistic but may initiate too many false alerts or missed detections.
5.11.1 Choosing the Evaluation and Assessment Window Size

It is a fundamental problem of the proposed algorithm to determine the size of the evaluation window and the assessment window. The \textit{a posteriori} MDB and \textit{a posteriori} $p_{FA/MD}$ provide an instrument to accomplish this goal by putting it into perspective with the Berry-Esseen bound as introduced in Section 5.4.1. The evaluation of the Berry-Esseen bound for different evaluation window sizes on the above process $X$ is introduced in Table 1. It is assumed that enough data for the assessment window are available.

<table>
<thead>
<tr>
<th>Evaluation Window Size [samples]</th>
<th>Berry-Esseen Bound [unitless]</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.14</td>
</tr>
<tr>
<td>500</td>
<td>0.07</td>
</tr>
<tr>
<td>1000</td>
<td>0.045</td>
</tr>
</tbody>
</table>

Clearly indicated in Table 1, the larger the evaluation window size the smaller the Berry-Esseen bound and hence more accurate the approximation of $H_*(y)$ with respect to $\Phi(y)$ (refer to Equation (84)). The test statistic introduced in Equation (107) can only deliver results as accurate as the over bound value provided by the Berry-Esseen theorem. Recognizing that the Berry-Esseen bound provides values on the order of $10^{-2}$ as presented in Table 1 for a 500-sample evaluation window yields that the test statistic as introduced in Equation (107) only delivers reliable values within this bound. This is further corroborated by the Edgeworth correction term (refer to Equation (88)) as $y \to \infty$ then $\left| H_*(y) - \Phi(y) \right| \to o(y^2)$. As shown in [36] the order immediately decreases depending on the population characteristic of the underlying process. The important aspect of the test statistic is shown in Figure 22; the simulated non-central ChiSquare process determines a p-value$_{FA/MD}$ on the order of $10^{-2}$ to satisfy $P_{FA}$ of $10^{-4}$ and $P_{MD}$ of $10^{-7}$. Therefore the test statistic does not have to operate on the same tail probabilities as $P_{MD}$ and $P_{FA}$. As long as the Berry-Esseen bound is on the same order as p-value$_{FA/MD}$, then enough estimation accuracy is given.
The determination of the assessment and evaluation window sizes is approached by defining a MDB, which needs to be satisfied by the anomaly detection requirement. This is based on the motivation that signal distortion only above a certain level will introduce a significant error component, which may lead to a system failure mode; e.g. a failure current in a 1 kA-power line of 1 mA will probably not introduce an anomalous system behavior. The evaluation of the Berry-Esseen bound yields the decision accuracy of the test statistics, where the p-value decision threshold of the test statistic needs to be equal or larger than the Berry-Esseen over bound error. Based on those assumptions the underlying \( P_{FA} \) and \( P_{MD} \) as a function of the simulated process \( X \) are estimated as described in Equations (110) and (111). Assuming the ChiSquare process as introduced in Section 5.11 and varying the MDB from 2 through 8 yields Table 2 for the estimated \( P_{FA} \) and \( P_{MD} \), where the estimated Berry-Esseen bound defines the p-value on which a reliable decision is made. It is noticed that this defines the detection threshold.

<table>
<thead>
<tr>
<th>Eval. Win. size</th>
<th>Ass. Win. size</th>
<th>Berry Esseen</th>
<th>( P_{FA} )</th>
<th>( P_{MD} )</th>
<th>( P_{FA} )</th>
<th>( P_{MD} )</th>
<th>( P_{FA} )</th>
<th>( P_{MD} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100</td>
<td>0.1394</td>
<td>0.0614</td>
<td>0.6220</td>
<td>0.0209</td>
<td>0.3361</td>
<td>0.0049</td>
<td>0.0042</td>
</tr>
<tr>
<td>500</td>
<td>500</td>
<td>0.0646</td>
<td>0.0658</td>
<td>0.2274</td>
<td>0.0182</td>
<td>0.0018</td>
<td>0.0044</td>
<td>4.6051e-16</td>
</tr>
<tr>
<td>2000</td>
<td>2000</td>
<td>0.0327</td>
<td>0.0750</td>
<td>0.0323</td>
<td>0.0156</td>
<td>2.3321e-8</td>
<td>0.0043</td>
<td>1.1208e-38</td>
</tr>
<tr>
<td>MDB = 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MDB = 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MDB = 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MDB = 8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As depicted in Table 2, the evaluation and assessment window is varied from 100 samples to 2000 samples, which defines the Berry-Esseen over bound error as indicated. Further, acknowledging that the Berry-Esseen bound defines the order of the p-value, which allows the decision on the test statistic, yields \( P_{FA} \) and \( P_{MD} \) as presented in Table 2. As evident, the smaller the evaluation window, the larger the Berry-Esseen bound. This initiates less decision accuracy for the test statistic (see Equation (107)) leading to a higher value of \( P_{FA} \) and \( P_{MD} \). Further recognizable, the Berry-Esseen bound is mainly driving \( P_{FA} \) where as \( P_{MD} \) is mainly influenced by the MDB. The general trend is apparent that the larger the evaluation and assessment windows chosen, the smaller \( P_{FA} \) and \( P_{MD} \) for a constant MDB. As introduced by
the Edgeworth correction term (see Equation (88)), the major error component in the convergence of \( H_y(y) \) towards \( \Phi(y) \) is located around the 1-sigma region. As elaborated in Section 5.4.1, as \( y \to \infty \) the Edgeworth correction converges towards \( o(\sqrt{\alpha}) \). If the process in focus is roughly symmetric, then the remainder \( o(\sqrt{\alpha}) \) immediately decreases to a smaller order [36].

### 5.11.2 The Detection Process as a Function of the Window Size

The previous paragraph addressed confidence levels associated with the size of the evaluation and assessment window. The detection of a failure mode is also associated with a TTA (refer to [11]). Simulations with an assessment window size of 2000 samples and an evaluation window size of 20 samples and 100 samples are performed to delineate the performance difference of the algorithm. Figure 23 shows the p-value calculation of the ChiSquare process as specified in Section 5.11.

![Figure 23, p-value for evaluation window size with 100 samples and 20 samples](image)

Figure 23, p-value for evaluation window size with 100 samples and 20 samples
As depicted in Figure 23, the induced bias is detected in both cases. It is noted that ‘evaluation window location’ as indicated in Figure 23 denotes the first sample of the window. Thus the time of detection is given by the evaluation window location plus the evaluation window length. Setting an arbitrary threshold of $p\text{-value}_{\text{FA/MD}}$ of $10^{-4}$ yields the indicated acceptance regions for $H_A$ and $H_0$. Comparing the $p$-values for the evaluation window size 20 and 100 samples confirms the results as discussed in Section 5.11.1. The smaller the evaluation window is chosen, the larger the rate of false alerts indicated in Figure 23 by $p$-values crossing the arbitrary threshold. The $p$-value estimation with an evaluation window size of 20 samples has much more false alerts than the $p$-value estimation with an evaluation window size of 100 samples.

Comparing the detection statistics illustrated in Figure 23 for different evaluation window sizes not only shows that the Berry-Esseen over bound is decreased the larger the window size is chosen, but also that the detection statistic becomes more observable. This is explained by rewriting Equation (107) and assuming a homoscedastic process:

$$z \sim \frac{\hat{\mu}_{\text{Eval}} - \hat{\mu}_{\text{Assess}}}{\sqrt{\hat{\sigma}_{\text{Eval}}^2 + \hat{\sigma}_{\text{Assess}}^2}}$$

where $\hat{\sigma}_x$ is the standard deviation of the underlying process, which is defined in Equation (90); $n$ the assessment window size and $k$ the evaluation window size. Hence, as indicated by Equation (115) the larger the evaluation window is chosen, i.e. the larger $k$, the more observable the test statistic becomes.

To illustrate the performance of the proposed bias detection scheme, Monte Carlo trials were implemented to gauge the bias detection sensitivity of the established algorithm. Again two cases were implemented, one for an evaluation window size of 10 samples and for an evaluation window size of 100 samples. The same non-central process as specified earlier was simulated with an induced bias at sample 3000. The decision threshold for accepting or rejecting $H_0$ was set at $p\text{-value}_{\text{FA/MD}} = 10^{-4}$. 

Figure 24 illustrates the estimated sample at which the induced bias is detected for an evaluation window size of 10 samples.

As recognizable in Figure 24, a significant number of the Monte Carlo trials detect the induced bias at the evaluation window location 3000. However the small evaluation window size of 10 samples initiates weak p-values leading to a late detection of the induced bias for certain Monte Carlo trials. With respect to the evaluation window size as indicated in Figure 24, a significant amount of missed detections at evaluation window location 3000 are observed. Hence the histogram shows an exponential shape. A closer look at Figure 24 also reveals detected biases before sample 3000, which are false alerts.
Figure 25 illustrates the estimated sample at which the induced bias is detected for an evaluation window size of 100 samples.

![Bias detection histogram for evaluation window size 100](image)

As depicted in Figure 25 the evaluation window size with 100 samples detects the induced bias at a much higher level of confidence, since the test statistic becomes more observable as indicated by Equation (115). With respect to the evaluation window size as indicated in Figure 25 there are almost no missed detections observed. Again, it is noted that ‘evaluation window location’ as indicated in Figure 25 denotes the first sample of the window. Thus the time of detection is given by the evaluation window location plus the evaluation window length. The bell shaped missed detection histogram is initiated through the variation in the bleeding effect into the evaluation window as it is sliding over the sequence. Almost no false detections were encountered. However the detection of the start of the induced bias is less precise due to the longer evaluation window size. This is mainly due to the fact that an anomaly starts bleeding into the evaluation window as it is sliding over the data stream. Therefore, the detection accuracy in time is directly related to the evaluation window size.
5.11.3 Performance on Non-Homoscedastic Population

To verify the performance of the test statistic on a non-homoscedastic time series, a process X1 with $\mu = 4$ and $\sigma = 2$ is generated. From sample 3000 to sample 4000, the standard deviation is inflated to a value of $\sigma = 5$. In a second generated process X2, from sample 3000 to sample 4000, not only the standard deviation is inflated to a value of $\sigma = 5$ but also the mean of the time series is altered to a value of $\mu = 5$. Performing the large sample T-test with an evaluation window size of 500 samples and an assessment window with 1000 samples yields Figure 26.

As depicted in Figure 26, the large sample T test performs satisfactory under the criterion of non-homoscedastic populations and the test statistic detects the inserted bias from sample 3000 to sample 4000 of the process X2. On the other hand, process X1 does not indicate any bias even though the variance is inflated. This verifies the performance of the test statistic on non-homoscedastic processes.
5.12 The Fine Print

For the application of the algorithm, it is important to recognize a few important considerations.

The size of the evaluation window is crucial to the performance of the bias detection scheme. A small evaluation window size, given that the p-values are sufficiently observable, establish a sharper detection in time. However, a smaller evaluation window size always initiates less observable p-values, which may lead to missed detection or late detection, or false detection. On the other hand a large evaluation window size provides more observable p-values leading to a higher detection confidence at the cost of a less precise detection of the start of the bias in time. Therefore it is acknowledged that the proposed algorithm is shaped by the trade-off, that a bias detection at a higher level of confidence comes at the cost of less precise detection of the start of the bias in time and vice versa, a lower level of confidence leads to a more precise detection of the start of the bias in time. For the purpose of QM, this trade-off is not a disadvantage as long as the TTA criterion as specified in [11] can be met. This is per definition fulfilled, as long as the evaluation window size is chosen smaller than TTA. Considering that the SDR provides a data stream at 5 Msps, the TTA criterion is satisfied by a generous margin through the proposed detection technique.

Furthermore, as the process under investigation needs to satisfy the criterion of being WSS, the size of the assessment and evaluation window need to be significantly smaller than the time period of dynamics in the system. Otherwise trend estimation is required to translate the process into a WSS process.

As outlined earlier in this document, a signal anomaly may not only lead to an estimation bias but also to a change in the distribution function of the estimator. Such an anomaly type can not be directly detected by the proposed algorithm. However it can be translated into a bias detection problem by establishing estimators, which e.g. measure the standard deviation of the estimated parameter.
5.13 In Summary

This Chapter discusses an algorithm to detect anomalies in the GPS signal and to address confidence levels on the detected failure mode. In the context of QM it is a primary requirement to establish confidence bounds on statistical inference. The detection scheme is based on the expected value estimator, which converges to a normally-distributed estimator due to the CLT. This makes the algorithm distribution-independent, i.e. non-parametric and hence pertinent to a wider area of applications. The over bound error of the test statistic is quantified by the Berry-Esseen theorem.
6 INTERFERENCE DETECTION

As opposed to a self contained navigation system such as an Inertial Navigation System (INS), a GPS-based navigation system relies on externally received information via an RF link. This poses vulnerability to RF interference (RFI) and may initiate failure modes ranging from degraded navigation accuracy to a complete signal loss-of-lock condition. In the literature, analyses of various types of interference are published, and a distinction is made between non-intentional and intentional RFI. Table 3 summarizes RFI and typical sources as listed in [42].

<table>
<thead>
<tr>
<th>Interference Type</th>
<th>Typical Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wideband-Gaussian</td>
<td>Intentional noise jammers</td>
</tr>
<tr>
<td>Wideband phase/frequency modulation</td>
<td>Television transmitter's harmonics</td>
</tr>
<tr>
<td>Wideband spread spectrum</td>
<td>Intentional spread spectrum jammers or nearfield of pseudolites</td>
</tr>
<tr>
<td>Wideband pulse</td>
<td>Radar transmitters</td>
</tr>
<tr>
<td>Narrowband phase/frequency modulation</td>
<td>AM stations transmitter's harmonics, or CB transmitter's harmonics</td>
</tr>
<tr>
<td>Narrowband swept continuous wave</td>
<td>Intentional CW jammers or FM stations transmitter's harmonics</td>
</tr>
<tr>
<td>Narrowband continuous wave</td>
<td>Intentional CW jammers or near-band unmodulated transmitter’s carriers</td>
</tr>
</tbody>
</table>

It is shown in [43] that the effects of RFI in conventional GPS receivers on code correlation and loop filtering are CNR reduction of all the signals below the tracking threshold. This causes the receiver to lose lock on the signal. Unintentional interference may arise from harmonics of mobile cellular, satellite, TV and radio [44].

RFI detection and mitigation techniques as proposed in [42], [43], [45], [46] such as front-end filtering techniques, tracking-loop techniques, etc., are well suited to a conventional receiver architecture. In the context of the QM the main task is to perform RFI detection at a high level of confidence. RFI mitigation is not of primary interest in QM. Further, the GAEM architecture used for QM allows additional freedom by removing the causality restriction imposed on conventional receivers.
The approach for interference detection is divided into two Sections, a pre-correlation and post-correlation interference detection. Pre-correlation interference detection focuses on detection techniques before the correlation stage within the receiver. This has the advantage that an interference source may be observed undistorted leading to more detection energy. E.g. a CW interference source which has a considerable frequency offset to the satellite carrier frequency may not have any impact on signal acquisition and tracking, and thus the interference source may not be observable in the post-correlation interference detection. However for QM, it is of general interest to detect any interference in the L-band, which may potentially lead to a degraded system performance. Conversely, post-correlation interference detection is aimed at detecting interference that is not observed in pre-correlation interference detection, and is targeted at monitoring signal parameters such as CNR, frequency noise, acquisition margin (being a measure of confidence in the proper estimate of code phase), etc.

As a preliminary examination, the following Section discusses discrete time and discrete amplitude effects introduced by the digitization process of the SDR. Following this discussion, pre-correlation and post-correlation interference detection are analyzed.

6.1 Discrete Spectrum

Evaluation of the PSD is a well established method for detecting RFI [42], [45]. At the receiver, the incoming signal is represented by the composite of the GPS signal and noise introduced in Equation (1). As established in Section 4.1.1, the GPS signal is approximately 20 dB weaker than the noise power (~2 MHz bandwidth), or mathematically expressed $E[s(t)^2] << E[n(t)^2]$. Thus, it is justified to approximate $s(t)$ to be a real, bandpass, zero mean, continuous WSS Gaussian process $\chi(t)$ [28].
The PSD of such a signal is defined in Equation (116) [30].

\[
S_x(f) = \mathcal{F}\{R_x(\tau)\}
\]  
(116)

Where \( S_x(f) \) is the PSD and \( R_x(\tau) \) the autocorrelation function of the process \( X \) defined in Equation (117).

\[
R_x(\tau) = X(t) \ast X(-t) = \mathcal{F}^{-1}\{\mathcal{F}\{X(t)\} \cdot \mathcal{F}\{X(t)\}^*\}
\]  
(117)

Therefore the PSD is obtained by

\[
S_x(f) = \mathcal{F}\{X(t)\} \cdot \mathcal{F}\{X(t)\}^*
\]  
(118)

The captured data sequence through the SDR represents a discrete-time and discrete-amplitude signal. The Discrete Fourier Transform (DFT) is applied to transform the signal into the frequency domain leading to a discrete-frequency representation. For the narrowband interference detection it is a necessity to examine the estimation errors due to the discrete frequency representation. Figure 27 provides an example.

![Figure 27, Power spectral density vs. frequency components](image-url)
As indicated in Figure 27, a CW interference source is assumed. As an example the CW interference is transmitted at a frequency of 2.5 kHz (between two frequency bins). A data block length of 1 ms is chosen yielding a DFT frequency resolution of 1 kHz. Since the interference source falls in between bins, an exact frequency domain representation is not possible. Hence, it is essential to study the mapping effect of the CW interference energy into the discrete spectrum for the latter derivation of the test statistics.

The total received signal is defined as a composite of the GPS signal $r(t)$ and CW interference $j(t)$ of the form

$$j(t) = A \cdot \cos((\omega_0 + \Delta\omega)t + \alpha)$$

(119)

Where $A$ is the amplitude of the CW interference, $\omega_0 = 2\pi \frac{f_0}{T_0}$, $T_0$ the data block length, $\alpha$ the phase angle and $\Delta\omega$ the frequency offset of the CW interference to $\omega_0$.

The spectrum is evaluated as presented in Equation (118).

$$S_x(j) = |\Im(j(t)) + \Im(r(t))|^2 + |\Re(j(t)) + \Re(r(t))|^2$$

(120)

Expanding Equation (120) yields for the PSD Equation (121).

$$S_x(j) = |\Im(j(t))|^2 + |\Re(r(t))|^2 + 2|\Re[\Im(j(t))]|\Re[\Im(r(t))] + |\Im[\Im(j(t))]|\Im[\Im(r(t))]|$$

(121)

As documented in Equation (121), the composite PSD of the signal and the interference source is a function of the signal spectrum and the interference source spectrum plus an interaction term of the signal spectrum and the interference spectrum.
The behavior of the discrete spectrum of the signal $r(t)$ is well documented in [2], [4]. Therefore the examination is focused on the Fourier transformation of the interference source $\mathcal{A}(j\omega)$. The Fourier coefficients are defined in [47]:

$$D_n = \frac{1}{T_0} \int g(t)e^{-jn\omega_0} dt \quad (122)$$

Substituting $g(t)$ with the CW interference source introduced in Equation (119) yields:

$$D_n = \frac{1}{T_0} \int A\cos((\omega_0 + \Delta\omega)t + \alpha)e^{-jn\omega_0} dt \quad (123)$$

Rewriting Equation (123) using Euler’s representation of cosine and sine, and solving the integral leads to Equation (124) for the discrete representation of the spectrum.

$$D_n = \frac{A}{2T_0} \left\{ \left[ \sin(\Delta\omega T_0 + \alpha) - \sin(\alpha) \right] (L + M) - j\left[ \cos(\Delta\omega T_0 + \alpha) - \cos(\alpha) \right] (L - M) \right\} \quad (124)$$

Where $L = \frac{1}{\omega_0 + \Delta\omega - n\omega_0}$ and $M = \frac{1}{\omega_0 + \Delta\omega + n\omega_0}$.

Implementing Equation (124) and solving for $|D_n|$ yields Figure 28. $T_0$ is chosen to be 1 ms resulting in a frequency resolution of 1 kHz.
Figure 28, Power Spectral Density vs. discrete and continuous representation

Figure 28 depicts in the y-axis the CW interference \( j(t) \) being shifted through the frequency range of \( f=1\text{-}4 \) kHz. The signal energy \( |D_n|^2 \) is distributed over the 1 kHz frequency bins (along x-axis) as indicated with the color pattern. When the frequency of the CW source coincides with a frequency bin of the DFT, all the energy is accumulated in the specified bin. As the CW interference falls between two frequency bins of the DFT, the energy is asymptotically distributed over the whole spectrum, where the main energy is absorbed in the two adjacent frequency bins. Further insight is provided by illustrating two specific cases as shown in Figure 29.
Figure 29 depicts the DFT spectrum for the case of CW interference at 3 kHz and 3.5 kHz. All energy of the CW interference at 3 kHz is represented in the according frequency bins at ±3 kHz. The CW interference at 3.5 kHz shows the energy being spread over the whole spectrum as an asymptotic function $\gamma'$. Further, 82% of the energy is absorbed by the two most adjacent frequency bins closest to the CW interference source. This leads to a maximum attenuation of 3.86 dB of the two most adjacent frequency bins as a result of the discrete spectrum representation. Increasing the block length $T_0$ leading to a higher frequency resolution decreases the discrete spectrum representation error.

### 6.2 Discrete Amplitude Through Quantization

The impact of the amplitude-discrete signal representation on the test statistics for the interference detection also needs to be examined.
The mean of a random process is defined in Equation (125) [30]:

$$\mu = \sum_j x_j p(x_j)$$  \hspace{1cm} (125)

The variance is defined as [30]:

$$\sigma^2 = \sum (x_j - \mu)^2 p(x_j) = \sum x_j^2 p(x_j) - \left( \sum x_j p(x_j) \right)^2$$  \hspace{1cm} (126)

Since the quantization process tiles the analog signal into discrete amplitude steps, the mean of the quantized signal for uniform quantization [47] is derived as established in Equation (127).

$$\mu_{\text{quantized}} = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}} k \int f_x(\nu) d\nu$$  \hspace{1cm} (127)

Where $f_x(\nu)$ depicts the pdf of the analog signal to be quantized and $N$ depicts the number of quantization levels, i.e. a 4-bit ADC yields $N = 2^4$.

The variance is derived accordingly as introduced in Equation (128).

$$\sigma^2_{\text{quantized}} = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}} \left\{ k^2 \int f_x(\nu) d\nu \right\} - \mu_{\text{quantized}}^2$$  \hspace{1cm} (128)

In context of the received GPS signal, the analog signal is characterized as AWGN sequence (refer to Section 4.1.1). Implementing Equation (127) and (128) for the pdf given in Equation (129),

$$f_x(\nu) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \left( \frac{\nu - \mu}{\sigma} \right)^2 \right\}$$  \hspace{1cm} (129)
results in a relative error $\frac{\mu_{\text{quantized}} - \mu}{\mu}$ on the mean on the order of $10^{-14}$, which is negligible. The evaluation of the relative error $\frac{\sigma^2_{\text{quantized}} - \sigma^2}{\sigma^2}$ on the variance reveals a significant quantization induced error as a function of the quantization levels as shown in Figure 30.

![Relative variance error](image)

**Figure 30, Relative error of standard deviation vs. quantization levels**

As depicted in Figure 30 a 2-bit ADC quantization introduces a relative error on the variance of approximately 50% assuming a Gaussian process and scaling the received signal to the ADC range as $\sigma = 0.1 \cdot N$. Shown in Figure 30, an 8-bit ADC introduces 0.01% quantization error. The Ohio University SDR employs a 12-bit ADC and thus the quantization induced error is negligible.
6.3 Pre-Correlation Interference Detection

Pre-correlation interference detection is performed in both the time and frequency domains. Time domain interference detection in current receiver architectures is accomplished by monitoring the AGC and monitoring the noise process in the tracking loops [43]. This allows for the detection of excessive signal power fluctuations. However, the methodology only works reliably for strong interference sources, i.e. at or above the same power level as the noise floor. Since the SDR allows capturing of a digitized version of the received RF signal, algorithms with much higher estimation sensitivity are possible and will be discussed in Section 6.4.

The pre-correlation interference detection is based on the proposed bias detection algorithm introduced in Chapter 5 and is applied on time domain estimators and frequency domain estimators. This approach assumes that at the startup time of the system no RFI is emitted in the L-band, in order to perform the assessment of the statistical signal properties. The algorithm therefore depicts a relative interference detection procedure, i.e. only signal fluctuations induced by the RFI are detectable. However, relative RFI detection is able to operate at much higher sensitivity levels. Observing the Novatel Beeline receiver used as anomaly sensor (refer to Section 3.1), which consists of a dual L1 antenna configuration, sometimes reveals a CNR estimation difference on the two RF inputs of 2 dB-Hz for a specific strong CNR satellite. Even though the two RF inputs are connected to the same antenna, a significant measurement inconsistency of the absolute signal measurements prevail. This outlines the limitations of an absolute interference detection scheme. Factors such as hardware inconsistencies, e.g. non-linearity of certain components, acquisition and tracking inconsistencies put an upper bound on the performance. Thus a relative interference detection scheme is regarded superior, because system driven inconsistencies can be mitigated. However it requires that at the startup of the system, absolute measurement quantities such as the CNR, PSD, frequency noise, etc. are monitored to judge if the statistical assessment is performed on undistorted data.
Time domain analysis is accomplished through detection of fluctuations in the distribution function of the received signal. Since the received signal is a zero mean process, a distribution function fluctuation is equivalent to a power fluctuation, and is aimed at the detection of wide band interference. Narrowband interference detection is predominantly carried out through the search of irregularities in the PSD due to the increased power density at the frequency component of the narrow band interference source. It is important to recognize that this method only provides reliable results under the condition that no AGC is employed and the amplification in the signal path remains constant.

6.3.1 Time Domain Energy Fluctuation Detection

In general, any interference source increases the received signal power. The objective of the second moment energy estimator as given in Equation (130)

$$\hat{U} = E[X^2] = \frac{1}{n} \sum_{i=1}^{n} X_i^2$$

is to reveal fluctuations of the received signal power. This is accomplished by means of the bias detection algorithm presented in Chapter 5. The RFI $j(t)$ is assumed to be introduced at time $T_{RFI}$. For $t < T_{RFI}$, the received signal is defined as $x(t) = r(t)$, where $r(t)$ denotes the received GPS signal defined in Equation (1) as $r(t) = s(t) + n(t)$. For $t \geq T_{RFI}$ the received signal is defined as $x(t) = r(t) + j(t)$. It is important to recognize that $\hat{U}$ converges to a normal distribution, and is over bound as described by the Berry-Esseen theorem (see Section 5.4.1).

Since the GPS signal is approximately 20 dB below the noise floor, for $t < T_{RFI}$ the received signal signifies a composite noise sequence of atmospheric and receiver noise therefore characterizing a WSS process (refer to Section 4.1.1).
The assessment window of the energy fluctuation detector is introduced as shown in Equation (131)

\[
\hat{U}_{\text{Assessment Window}} = \frac{1}{p} \sum_{i=0}^{p-1} X_{(n+i)T_s}
\]

Where \( p \) is the assessment window length in samples, \( T_s \) the sampling interval, \( X_{(n+i)T_s} \) is the received signal at time \((n+i)T_s\).

The evaluation window is defined accordingly in Equation (132).

\[
\hat{U}_{\text{Evaluation Window}} = \frac{1}{k} \sum_{i=0}^{k-1} X_{(n+i)T_s}
\]

Where \( k \) is the evaluation window length in samples.

The pivotal quantity [31] is established in Equation (133) and is based on the concept of self normalizing sums [48] introduced in Section 5.8. The Berry-Esseen over bound on the approximation error is elaborated in Section 5.4.1.

\[
Z \sim \frac{\hat{U}_{\text{Evaluation Window}} - \hat{U}_{\text{Assessment Window}}}{\sqrt{\hat{\sigma}^2_{\text{Assessment Window}} - \hat{\sigma}^2_{\text{Evaluation Window}}}}
\]

Where \( \hat{\sigma}^2_{\text{Assessment Window}} \) is the estimated variance of the assessment window and \( \hat{\sigma}^2_{\text{Evaluation Window}} \) the estimated variance of the evaluation window.

As specified in Section 5.5, \( \hat{\sigma}^2 \) for the variance of \( \hat{U}_{\text{Assessment Window}} \) and \( \hat{U}_{\text{Evaluation Window}} \) is approximated through Equation (90).

Since \( \hat{U}_{\text{Assessment Window}} \) and \( \hat{U}_{\text{Evaluation Window}} \) approximately represent non-central ChiSquare processes [31] the variance of \( \hat{\sigma}^2 \) approaches \( 2\sigma^4 \) [49].
However, inherent to the problem of interference detection, the underlying interference source may or may not be correlated in time. Depending on the statistical property of the interference source \( j(t) \), the process \( x(t) \) may be transformed into a correlated time series, which would introduce an estimation error on the bias detection algorithm described in Chapter 5. This is further examined. Defining the sequence \( X_n \) being a composite of the received signal \( R \) and a jamming signal \( J \), i.e. \( X_n = R_n + J_n \). Assuming that \( \text{COV}(J_n, J_m) \neq 0 \), i.e. the jamming signal \( J \) is not serially correlated, and \( \text{COV}(R_n, J_m) = 0 \), i.e. the jamming signal \( J \) is uncorrelated with the received signal \( R \), then Equation (89) is rewritten as introduced in Equation (134).

\[
\text{VAR}(Y_n) = \text{VAR}\left( \frac{1}{n} \sum_{i=1}^{n} X_i \right) = \frac{1}{n^2} \left\{ \text{VAR}\left( \sum_{i=1}^{n} R_i \right) + \text{VAR}\left( \sum_{i=1}^{n} J_i \right) \right\} 
\]

(134)

Since any received RF sequence characterizes a zero mean process, the variance represents the energy and Equation (134) is rewritten as

\[
\text{VAR}(Y_n) = \frac{1}{n^2} \left\{ E\left[ \sum_{i=1}^{n} R_i^2 \right] + E\left[ \sum_{i=1}^{n} J_i^2 \right] + 2 \sum_{i=1}^{n} \sum_{j \neq i} \text{COV}(J_i, J_j) \right\} 
\]

(135)

As is apparent in Equation (135), given that \( E[\sum R_i^2] \gg E[\sum J_i^2] \), the correlation of the interference will introduce a non-significant error on the variance estimation of \( Y_n \). If an interference source is emitting at a very strong signal power level, i.e. \( E[\sum R_i^2] \approx E[\sum J_i^2] \), then a significant estimation error on \( Y_n \) is induced. This characteristic does not introduce a constraint on the detection characteristic, since any RFI at a strong power level will distort the received signal so noticeably that no interference-sensitive algorithm for the detection is necessary.
6.3.2 Power Spectral Density Fluctuation Detection

To convert narrowband interference into a more observable domain [42], [45], the received signal is transformed into the frequency domain as established in Section 6.1. The interference detection is performed as illustrated in Figure 31.

![Figure 31, Time-frequency power spectral density](image)

As depicted in Figure 31 for every data block at time instant \( nT_0 \) the discrete PSD is evaluated as introduced in Section 6.1. The test-statistic is established over time for every frequency component of the PSD as indicated in Figure 31.

The pivotal quantity for each frequency component \( f_j \) is established in Equation (136). Again the Berry-Esseen over bound on the approximation error is established in Section 5.4.1.

\[
Z = \frac{\hat{E}_{\text{Evaluation Window}, f_j} - \hat{E}_{\text{Assessment Window}, f_j}}{\sqrt{\hat{\sigma}_{\text{Assessment Window}, f_j}^2 - \hat{\sigma}_{\text{Evaluation Window}, f_j}^2}}
\] (136)
Where

\[ \hat{E}_{\text{assessmentWindow}, f_j} = \frac{1}{p} \sum_{i=n}^{p+n} PSD_{T_p \cdot (n+p)}(f_j) \]

(137)

And

\[ \hat{E}_{\text{evaluationWindow}, f_j} = \frac{1}{k} \sum_{j=1}^{k} PSD_{T_e \cdot (m+1)}(f_j) \]

(138)

Where \( T_o \) is the data block length, \( n \) the first sample of the assessment window, \( p \) the length of the assessment window, \( f_j \) the frequency component to be investigated, \( m \) the first sample of the evaluation window, and \( k \) the length of the evaluation window. Since the Fourier Transformation is a linear transformation, the correlation properties remain unaffected.

### 6.3.3 Algorithm Validation

To verify the proposed pre-correlation time and frequency domain interference detection technique and the examined algorithm limitations, a simulated interference source is added onto the GPS data collected by the SDR as shown in Figure 32.

![Interference verification setup](image)

**Figure 32, Interference verification setup**
As illustrated in Figure 32 the simulated interference source is generated on the *processing CPU* and is fed through a digital bandpass filter. At $T_{\text{RFI}}$ the simulated RFI source is added onto the collected real GPS data. This permits accurate algorithm verification, since the interference power is scalable and it also allows the implementation of multiple interference sources such as narrowband CW, wideband Gaussian, pulsed interference, etc. Figure 33 illustrates the generation of the composite signal, i.e. the GPS signal and the induced RFI. $T_{\text{RFI}}$ is arbitrary chosen at 2000.

**Figure 33, GPS Signal, interference source and composite signal vs. time**

Since the received GPS signal is passing through the SDRFE with an inherent amplification of approx 100 dB the signal power at the ADC is unknown. To mitigate the uncertainty of SDRFE amplification, the post RF front-end Jamming-to-Noise ratio (JNR) is introduced to accurately relate the interference power to the received GPS signal power.
Where the jamming and noise power at the ADC are calculated through the second moment [30].

Further, the SDRFE bandwidth is measured to be 2.15 MHz as shown by the spectrum analyzer output in Figure 34.

Figure 34, Narrow band RF front-end bandwidth
Based on this bandwidth, the received noise power is evaluated as

\[ kTB = 1.38 \times 10^{-23} \cdot 293.15 \cdot 2.15 \times 10^6 = 8.7 \times 10^{-15} W = -110.6 dBm \]  (140)

where

- \( k \) is Boltzmann’s constant of \( 1.38 \times 10^{-23} \text{ J/K} \)
- \( T \) is the equivalent noise temperature of 293.15 K
- \( B \) is bandwidth of the receiver bandpass filter of 2.15 MHz

The result of Equation (140) and the JNR as defined in Equation (139) allows for the estimation of the equivalent jamming power at the antenna.

\[ RFIPower_{\text{antenna}} = -110.6 dBm + JNR \]  (141)

The estimated equivalent jamming power (Equation (141)) is an important measure to validate the proposed algorithm. The Minimum Aviation System Performance Standards (MASPS) for the LAAS [50] and the Minimum Operational Performance Standards (MOPS) for GPS Local Area Augmentation System Airborne Equipment [51] specify the maximum interference power levels that must be tolerated by the GPS receiver as a function of frequency and interference bandwidth. The interference mask for interference emitted at the L1 frequency is shown in Figure 35.
The requirements as introduced in Figure 35 serve as a benchmark for the algorithm verification.

### 6.3.3.1 Narrowband Interference

A simulation is performed with a data sequence of 1.2 sec length resulting in $6 \cdot 10^6$ data points leveraging the Berry-Esseen theorem. Narrowband CW interference is injected onto the GPS data at $T_{\text{rfi}}=600$ ms at a power level of -120.1 dBm. The noise floor of the received signal is at -110.6 dBm (refer to Equation (141)). The RFI mask requires the detection of interference greater than -120.5 dBm at the antenna port [51]. The evaluation window is chosen 200 ms and the assessment window 800 ms.
The spectrum of the sampled received signal without interference is shown in Figure 36.

In Figure 36, the measured SDRFE bandwidth as shown in Figure 34 is apparent in the digital spectrum. The variance on the PSD around the center frequency of 1.27 MHz is significant.

Injecting the CW interference at 1.27 MHz and evaluating the frequency domain RFI detector p-values (for p-value definition refer to Section 5.10) yields Figure 37.
As shown in Figure 37 the CW interference is detected at p-values smaller than $10^{-14}$ suggesting the rejection of $H_0: \hat{E}_{\text{EvaluationWindow}, f_i} = \hat{E}_{\text{AssessmentWindow}, f_i}$ and accepting the alternative hypothesis $H_1: \hat{E}_{\text{EvaluationWindow}, f_i} \neq \hat{E}_{\text{AssessmentWindow}, f_i}$. The interference source is detected with an approximate delay of 50 ms (p-value drops at 650 ms for interference starting at 600 ms). This is due to the evaluation window length of 200 ms.

Figure 38 illustrates the test statistics (refer to Equation (136)) along the 1.27 MHz frequency component.
As the evaluation window slides over the received data sequence, per definition of the problem statement, it traverses through a Non-WSS transition period, where the independence of $\mu$ with respect to $t$ is not satisfied, since $\mu_{\tau_{n1}} \neq \mu_{\tau_{n2}}$. Thus the criterion of IID is not met in the transition interval, and the results of the proposed technique are not reliable. However as soon as the evaluation window predominantly acquires the interference signal the process transforms WSS again as justified in Section 5.5. As the test statistic illustrated in Figure 38 represents a MA-process, it is correlated as introduced in Section 5.9.

In order to exemplify the characteristics of the mapping process of the CW interference into the discrete spectrum as examined in Section 6.1, a narrowband CW source with a power level equivalent to -120 dBm at the SDRFE is injected at 1.2705 MHz, i.e. between two discrete frequency bins. A block length of 1 ms for estimating the PSD is chosen, yielding a DFT frequency resolution of 1 kHz. The p-value estimation for the frequency RFI detector is shown in Figure 39.
Evident in Figure 39 with respect to Figure 37, the smearing effect discussed in Section 6.1 is nicely recognizable. The CW interference is still detected in the frequency region of 1.27 MHz at theoretical p-values lower than $10^{-14}$ supporting the alternative hypothesis

$$H_A: \hat{E}_{EvaluationWindow,f_i} \neq \hat{E}_{AssessmentWindow,f_i}^{*}$$

### 6.3.3.2 Wideband Interference

A simulation with Gaussian wideband interference of 2 MHz bandwidth is performed. The interference mask implies the need for detection of RFI at a power level greater than -107.5 dBm (for a 2 MHz bandwidth) [51]. The interference source is injected at $T_{RFI}=600$ ms at a power level equivalent to -107.2 dBm at the SDRFE. Figure 40 depicts the p-value estimation for the frequency domain RFI detector.
In Figure 40, the proposed algorithm detects the wideband interference at p-value levels smaller than $10^{-14}$. The interference is detected with a lag of approximately 30 ms.

As outlined in the above example, it is necessary to establish performance measures of the proposed technique as a function of window length, interference power, interference bandwidth etc., and the according detection performance in the time and frequency domains.

### 6.3.3.3 Performance Evaluation Narrowband Interference

To attain a comprehensive performance measure of the introduced RFI detection algorithm, a CW narrowband interference source at 1.27 MHz at different power levels is injected onto the GPS signal at $T_{\text{RFI}}=1.6$ s. The assessment window is chosen to be of length 1 s and the evaluation window of length corresponding to 400 ms. This implies a worst case TTA of 400 ms. The interference mask implies the need for the detection of signals above -120.5 dBm.
The implementation yields Figure 41 for the p-value evaluation of time domain RFI detection and Figure 42 for frequency domain RFI detection.

Figure 41 depicts the RFI power in the y-axis, the time in the x-axis and the color pattern indicates the p-value evaluation. The CW interference is detected down to a simulated power level equivalent to -120 dBm at the SDRFE in the time domain RFI detection algorithm.

The frequency domain RFI detection results are included in Figure 42.
Figure 42 shows the detection of the CW interference down to a power level equivalent to -137 dBm at the SDRFE. Important to recognize is that the frequency domain RFI detector provides more sensitivity for CW than the time domain RFI detector. The algorithm is able to detect CW interference at level greater than -120.5 dBm by a margin of 16.5 dB. Further it needs to be emphasized that the test statistics does not show any false alert, when using a p-value threshold of $10^{-4}$. To attain more interference sensitivity the evaluation window size may be increased at the cost of a less precise detection in time.

6.3.3.4 Performance Evaluation Wideband Interference

To attain a performance measure of the introduced algorithm, a 2 MHz wideband interference source centered around 1.27 MHz at different power levels was injected onto the GPS signal at $T_{RFI}=1.6\ s$. The assessment window is chosen to be of length 1 s and the evaluation window of length corresponding to 400 ms, yielding a maximum TTA of 400 ms. The interference mask implies the need for detection of interference sources with 2 MHz...
bandwidth above -107.5 dBm. The implementation yields Figure 43 for the time domain RFI detector and Figure 44 for the frequency domain RFI detector.

Figure 43, Time domain RFI detector p-values for wideband interference vs. RFI power

Figure 43 shows the RFI detection in the time domain down to a power level equivalent to -127 dBm at the SDRFE. With respect to the interference mask of -107.5 dBm an approximate margin of 20 dB is achieved.

Figure 44 depicts the p-value statistics of the PSD in the frequency domain.
Figure 44, Frequency domain RFI detector p-values for wideband interference vs. RFI power

Figure 44 shows that the wideband interference is not detected in the frequency domain reliably, only a few minor variations are recognizable. Therefore it is evident, that for wideband RFI detection, the time domain detector is superior.

### 6.4 Post-Correlation Detection

Post-correlation interference detection focuses on detection techniques after the receiver correlation stage. It is aimed at monitoring signal parameters, such as the CNR, to measure signal inconsistencies.

The following Section addresses CNR estimation techniques and the subsequent Section applies CNR monitoring for interference detection.
6.4.1 The Carrier-to-Noise Ratio

The CNR is defined as [3]:

\[
CNR = SNR \cdot B
\]  

(142)

Where SNR depicts the Signal-to-Noise Ratio and B the noise bandwidth.

In [43] it is examined that RFI introduces a decrease of the CNR. Therefore, as primary post-correlation interference detector, monitoring the CNR is introduced. Multiple algorithms for CNR estimation are found in the literature. One well known estimator is published in [3] and thoroughly analyzed in [52] and it has its foundation on estimating the ratio of narrow-band power (NBP) over wide-band power (WBP). However as shown in [3], [52] this estimator represents a biased estimator and requires an averaging time of approximately 1 s to achieve desirable results, i.e. to converge to a Minimum Variance Unbiased Estimator (MVUE). Due to the constraint on TTA as specified in [11] an estimator with satisfying short term performance (integration time smaller 1 s) is preferred.

Two concepts of short term estimators are introduced at the trade-off of an increased estimation variance in this document. It is mentioned that the integration time sets the lower threshold on the signal energy that can be acquired.

6.4.1.1 Signal and Noise Energy Estimation

The composite signal energy \( E \) of the in-phase and quadrature-phase of the acquired signal for the \( i^{th} \) ms-block is obtained as

\[
E_i = \sqrt{E_{i,i} + E_{Q,i}}
\]

(143)

Where the in-phase energy \( E_{i,i} \) is obtained as specified in Equation (65) and the quadrature-phase energy \( E_{Q,i} \) as specified in Equation (67).
Two different noise energy estimators are examined in [53], i.e. pre-correlation and post-correlation noise energy estimation as shown in Figure 45. It is noticed that both cases exhibit a post-correlation CNR estimation, since the signal energy can only be acquired in a post-correlation manner.

![Figure 45, Signal and noise energy estimation](image)

The pre-correlation noise energy estimator introduced in Figure 45 is accomplished through the second moment estimator as established in Equation (32). It is based on the enclosed derivation (Equation (33) through (36)) that the second moment is a sufficient estimator for evaluating the noise energy. The post-correlation noise estimation is implemented through performing the correlation of the received sequence with an unused PRN code (e.g. PRN 34), which per definition yields the noise energy. Subsequently, the CNR is evaluated as specified in Equation (142). For the post-correlation noise energy detector, the bandwidth is given by the integration bandwidth, and for the pre-correlation noise energy detector the bandwidth is determined by the front-end bandwidth. Evidently, as the post-correlation CNR estimator is independent of the hardware defined front-end bandwidth, the pre-correlation CNR estimator is directly dependent on the front-end bandwidth. Both, the pre-correlation and post-correlation noise energy estimators are associated to error sources. Since the signal energy estimation is inherently obtained after the correlation process, it is consistent to evaluate the noise energy through the suggested post-correlation technique. This keeps the error sources on the signal and noise energy estimators consistent. Error sources such as the cross-correlation energy, due to the non-perfect orthogonality of the Gold codes within the integration bandwidth may induce estimation errors on the post-
correlation noise estimator. As filters are not ideal, the pre-correlation noise energy estimator is degraded through the filter approximation.

6.4.2 Interference Detection through CNR Observation

The pre-correlation interference detection as discussed in Section 6.3 applies a power fluctuation detection technique, which represents a relative detection algorithm. On the other hand, monitoring the CNR (which is post-correlation detection) enhances an absolute RFI detection scheme, which does not require an undistorted signal sequence for the assessment of the statistical properties. However, the limitations of this technique need to be addressed. Generally, a spread spectrum system increases interference immunity with respect to other conventional RF communication systems. Thus post-correlation interference detection does not provide a sensitive RFI detector. Further, as the CNR estimation is directly dependent on the signal acquisition and tracking performance, post-correlation interference detection confidence is a conditional probability that is a function of acquisition and tracking probabilities of the receiver. This is demonstrated by observing tracking performance of the two Novatel Beeline receiver RF inputs connected to the same antenna, which sometimes revealed an inconsistent CNR measurement of up to 2 dB-Hz. It is further noted that multiple failure scenarios may map into the same failure condition, e.g. a sharp decrease of the CNR may be initiated through RFI, receiver tracking abnormalities or through SV power fluctuations.

In order to provide reliable results, post-correlation interference detection needs to address decomposition of receiver induced anomalies from signal induced anomalies and needs to operate at a high level of confidence as accomplished in [22] by the SDR.
7 EXPERIMENTAL RESULTS

The performance demonstration and validation of the introduced algorithms is split into three Sections:

- The interference detection validation is performed in an experiment in Ohio University’s shielded measurement chamber located in Morton Hall, in order not to disrupt GPS operation for user equipment within the vicinity of the experiment.
- Continuous GPS signal monitoring was performed over a seven-month period, from October 2003 through April 2004 at the Stocker Engineering building, Ohio University. A few case studies of captured signal failure modes are presented.
- In a field study the GAEM was employed to study the effects of aircraft over flights on GPS signal reception at the Ohio University Airport in Albany Ohio.

The results are presented in the following Sections.

7.1 INTERFERENCE EXPERIMENT IN SHIELDED CHAMBER

7.1.1 Experiment Setup

In order to attain a comprehensive performance assessment of the proposed algorithm an experiment in a shielded chamber was carried out as illustrated in Figure 46. The outside L-band signal was reradiated into the chamber, which guarantees a realistic scenario representation. An interference source was also transmitted. The SDR was connected to a receiving antenna, to collect the distorted signal and to perform the RFI detection verification.
Figure 46, Interference experiment in shielded measurement chamber

As shown in Figure 46, the RFI was generated by a function generator and transmitted into the shielded chamber in Morton Hall at Ohio University through a Sensor Systems passive antenna. Further, the GPS signal is received through a Novatel Pinwheel GPS-600 antenna located outside of Morton. Since the antenna could not be placed on the roof but only in front of the building, a limited constellation of satellites was received. In the context of the interference algorithm validation, this does not introduce a loss of generality. The received signal was amplified by a GPS networking LA40 amplifier with 40 dB gain and fed into a 100 ft cable leading into the shielded chamber. To acquire sufficient power for the reradiation of the GPS signal, a second GPS networking LA40 amplifier was inserted into the signal path. The DC power for the LA40 amplifiers and the GPS Pinwheel antenna was provided through a Bias-T DC power injector. In order to mitigate signal reflections through an unmatched antenna load, an isolator was inserted. The GPS signal was transmitted inside the chamber through a Sensor Systems passive patch antenna. A third Sensor Systems...
passive patch antenna received the composite signal, i.e. interference source and the reradiated GPS signal, and was connected to a spectrum analyzer and the GAEM. In order to reproduce an accurate measurement scenario, the interference source power was calibrated as discussed in the following Section.

7.1.1.1 Calibration of the Measurement Setup

For the validation of the pre-correlation interference detection algorithm and to reproduce a realistic scenario, the interference source power was calibrated to the reradiated measured noise floor in the shielded chamber. The Novatel GPS-600 Pinwheel antenna is equipped with a Low Noise Amplifier (LNA) with a Noise Figure $\leq 2.6$ dB and a gain of 26 dB [54]. Since the attenuation of the 100 ft cable (from outside into the shielded chamber) was much smaller than the gain of Pinwheel LNA and the first LA40 amplifier preceding the cable (see Figure 44), the overall Noise Figure was mainly determined by the Pinwheel LNA. Thus the reradiated GPS signal remains approximately 23 dB below the reradiated noise floor.

Measuring the reradiated noise floor with the spectrum analyzer and a resolution bandwidth of 100 kHz yielded -68 dBm. This implies that the reradiated noise floor was 56 dB above the noise floor of the chamber and approximately 20 dB above the noise floor of the spectrum analyzer. The received noise power by the SDRFE with respect to a 2 MHz bandwidth as measured in Figure 34 is evaluated as:

$$\text{NoisePower} = -68 \text{dBm} + 10 \log_{10}(\frac{2}{1}) = -55 \text{dBm}$$  \hspace{1cm} (144)

The attenuation of the RF link, i.e. transmitting antenna to receiving antenna, was measured to be 30 dB. Therefore, to generate an interference source at an equivalent power level as the reradiated noise floor, an interference source with -25 dBm power was generated.

Further, as theoretically elaborated in Section 6.2, the received signal needs to be scaled to match the quantization levels of the ADC, since the SDR is not equipped with an AGC. However, for the RFI detection as introduced in Section 6.3, the constant gain-chain is the
foundation for performing the pre-correlation interference detection. The employed 12-bit ADC yields a quantization range from -2048 to +2048 levels. Therefore the received signal through the Rx antenna (see Figure 46) was attenuated by 36 dB in order not to saturate the ADC. The evaluation of the histogram of the received signal yields Figure 47.

![Figure 47, pdf of quantized signal vs. ADC levels](image)

The received signal as shown in Figure 47 has a standard deviation of 502 quantization levels and nicely approximates a Gaussian distribution. The probability of saturating the ADC is approximately $10^{-5}$. Referring to the results obtained in Section 6.2, a quantization induced estimation error of less than $10^{-2}$ % is expected, which is negligible.

To estimate the bandpass filter induced correlation, the autocorrelation of the received data sequence is evaluated as shown in Figure 48. Further, to verify the results of Section 4.2, the overbound sinc-function of the autocorrelation function of Equation (54) is also plotted.
As illustrated in Figure 48, the measured autocorrelation function corroborates the theoretical results of bandpass induced correlation. However, the measured correlation is slightly less significant than indicated by the theoretical overbound autocorrelation. This is mainly due to the following aspects:

- As stated in Section 4.1.4, the theoretical results indicate an upper bound. Depending on the interaction of the cosine term of the theoretical autocorrelation function (refer to Equation (54)) with the sampling process, a smaller correlation may be observed.
- As the SDRFE filters are not ideal, out of band noise energy is also present in the signal.

For the data analysis following in the next Section, the data is sufficiently independent.

### 7.1.1.2 Interference Algorithm Validation

In order to validate the performance of the interference detection algorithm, different RFI sources, such as wideband, narrowband and pulsed interference were emitted into the chamber. The following Sections illustrate the experimental results and verify the RFI detection algorithm.
7.1.1.2.1 Narrowband Interference

To evaluate the RFI algorithm detection sensitivity to narrowband interference, the injected interference power was varied from -55 dBm down to -91 dBm (referenced to the receiving antenna). CW interference at the center frequency of the L1 GPS signal of 1575.42 MHz was generated. As calculated in Equation (144), the reradiated noise floor was at -55 dBm (referenced to the receiving antenna) at the receiving antenna. Since the GPS signal is 23 dB below the 2 MHz noise floor, the interference source power with respect to the GPS signal power varied from +23 dB to -13 dB. The interference mask [11] suggests the detection of narrowband interference above -120.5 dBm in association to a noise floor of -110 dBm, i.e. 10.5 dB below the noise floor. Therefore, the interference mask for the experiment calibration in the shielded chamber translates into -65.5 dBm at the reception antenna with respect to the reradiated noise floor.

To verify the algorithm, a collected data set with a CW interference source at a power level of -91 dBm was selected. This interference source is 25.5 dB below the interference mask; and implicitly 36 dB below the reradiated noise floor and 13 dB below the GPS signal.

At the time of the experiment, the outside antenna was receiving three strong satellites: SV9, SV18 and SV21. To verify the experimental setup, the estimation of the CNR and the downconverted carrier frequency for SV18 and SV21 are presented in Figure 49.
As depicted in Figure 49, the variance of the CNR is significant due to the use of 1-ms blocks for the CNR estimation. SV21 is at approximately 44 dB-Hz and SV18 at 42 dB-Hz. However, no tracking anomaly is evident in either the downsampling carrier frequency estimates or the CNR for SV18 and SV21.

The pre-correlation interference detection as described in Section 6.3 is applied by choosing an assessment window size of 1000 ms and an evaluation window size of 300 ms. The evaluation of the frequency domain RFI detector p-values as a function of time and frequency yields Figure 50.
Depicted in Figure 50, the RFI source is detected at the downsampled frequency of 1.27 MHz, which translates into 1575.42 MHz in the L-band spectrum. This is the value being dialed at the function generator and is thus in agreement with the experimental scenario. The interference was injected after approximately 5 s, which is corroborated in Figure 50. Calculating the a posteriori MDB for the frequency domain RFI detector (refer to Section 5.10) yields a MDB under the condition of $P_{F_A} = 10^{-7}$ and $P_{MD} = 10^{-4}$ of -37 dB/kHz. Therefore the detected interference source at -91 dBm as indicated in Figure 50 satisfies the required confidence levels as specified above. To illustrate the compliance of the confidence levels using an alternate method, a MDB of -91 dBm is specified. Considering the Berry-Esseen overbound error on the order of $10^{-3}$, which determines the DT (refer to Section 5.10), yields an a posteriori $P_{F_A} = 1.1 \cdot 10^{-10}$ and an a posteriori $P_{MD} = 6.6 \cdot 10^{-10}$. Therefore the RFI source can be identified with sufficient probability of false alert and missed detection. As corroborated in Table 2, it is important to recognize that under $H_A$, the smaller the p-value, the lower is $P_{MD}$.
To outline the time domain RFI detector performance on narrowband interference, the *a posteriori* MDB for the time domain RFI detector under the condition of $P_{fa} = 10^{-7}$ and $P_{as} = 10^{-4}$ is calculated. Based on the received process an *a posteriori* MDB of 3 dB, i.e. an RFI source at -58 dBm, is evaluated. Therefore the injected CW RFI is not visible in the time domain RFI detector as corroborated in Figure 51.

![Figure 51, Time domain RFI detector p-values for CW interference](image)

As shown in Figure 51, the p-values do not show a large variation. Based on the time domain Berry-Esseen overbound on the order of $10^{-4}$, which determines the p-value decision accuracy and implicitly the DT, an *a posteriori* $P_{fa} = 5.5 \cdot 10^{-15}$ and an *a posteriori* $P_{as} = 1.00$ is evaluated. This substantiates the observation that narrowband RFI is not detected in the time domain. However, it is important to recognize that the time domain RFI detector does not initiate false alerts.

Finally, to illustrate the impact of RFI on signal acquisition and tracking, a collected data set with a strong CW interference source at the same power level as the reradiated noise floor is evaluated (RFI power of -55 dBm). The normalized histogram of the received signal with and without RFI is presented in Figure 52.
As depicted in Figure 52 the RFI distorted signal (1:1 ratio of noise-to-RFI power) yields noticeable distortion of the histogram of the received signal. The standard deviation of the undistorted signal is 501 ADC levels and the distorted signal 685 ADC levels. The histogram of the RFI distorted signal nicely approximates the convolution of AWGN noise plus a sinusoidal CW RFI source [45]. To illustrate the impact on signal acquisition and tracking, the CNR and the carrier frequency estimate for SV 26 are shown in Figure 53.
As depicted in Figure 53, after 4.8 s the induced RFI source reduces the CNR and leads to an increased variance on the carrier frequency estimation. As was observed on the Novatel Beeline receiver serving as anomaly sensor, an RFI source at a 1:1 ratio of noise-to-RFI power led to multiple inconsistent failure modes such as code lock, phase lock and channel lost flags not allowing RFI detection at a high level of confidence. Further, it was observed that RFI sources at a lower power level are not detected by the Novatel Beeline receiver.

7.1.1.2.2 Frequency Modulated Interference

The algorithm performance with respect to wider band RFI is examined. A Frequency Modulated (FM) signal with a bandwidth of 0.8 MHz was transmitted (maximum modulation depth of the function generator). The same power calibration as described in paragraph 7.1.1.2.1 was maintained. The FM interference power was injected into the chamber at a power range of -55 dBm down to -88 dBm with respect to the receiving
antenna at the center frequency of the L1 GPS signal of 1575.42 MHz. The interference mask [11] suggests the detection of interference (with a bandwidth of 0.8 MHz) above -107.5 dBm, in association to a noise floor of -110 dBm, i.e. 2.5 dB above the noise floor. Therefore the interference mask for the experiment calibration in the shielded chamber translates into -52.5 dBm at the reception antenna with respect to the reradiated noise floor.

An FM interference source at a power level of -76 dBm was chosen. This interference source is 23.5 dB below the interference mask. An assessment window size of 1000 ms and an evaluation window size of 300 ms are chosen. The evaluation of the time domain p-value as a function of time yields Figure 54.

The interference source was injected after approximately 5 s, which is corroborated in the time domain p-value estimator as shown in Figure 54. The computation of the frequency domain RFI detector is illustrated in Figure 55.
Figure 55, Frequency domain RFI detector p-values for FM interference

Depicted in Figure 55, the frequency domain RFI detector does not reveal significant p-values. Therefore, FM RFI is more reliably detected in the time domain, which will be substantiated in the following paragraph.

Calculating the *a posteriori* MDB (refer to Section 5.10) for the time domain RFI detector under the condition of $p_{fa} = 10^{-7}$ and $p_{md} = 10^{-4}$ based on the received process yields a MDB of 7 dB. Thus, wideband RFI signals at a power level of -62 dBm (with respect to the experimental setup) are detectable under the confidence level as specified above. This leaves a margin to the interference mask of -9.5 dB. Therefore the results presented in Figure 54 for the time domain RFI detector do not meet the required false alert and missed detection rate.

Further, the frequency domain RFI detector performance is illustrated for a 2 MHz Gaussian RFI source. Specifying the MDB of -9.5 dB and considering the Berry-Esseen over
bound error on the order of $10^{-3}$ yields $P_{fa} = 7 \cdot 10^{-10}$ and $P_{fa} = 0.65$. Therefore as indicated, the time domain estimator is capable of revealing the RFI source at the required level of confidence, whereas the frequency domain RFI detector does not achieve a satisfactory missed detection rate.

To illustrate the performance of the frequency domain RFI detector for FM interference, a collected data set with an FM modulated wideband interference source at a power level of -64 dBm is chosen (11.5 dB below the required interference mask). It is mentioned that the *a posteriori* MDB discussed in the previous paragraph is in context of truly Gaussian interference and an FM modulated signal reveals a higher PSD at the edge of the interference bandwidth. This is due to the fact that the PSD of a FM modulated sinusoid takes the shape of the pdf of a sinusoid defined as [45], [55]

$$p(x) = \begin{cases} 
\frac{1}{\pi \sqrt{A^2 - x^2}} & |x| \leq A \\
0 & \text{otherwise}
\end{cases} \quad (145)$$

Where $A$ is the amplitude of the signal. Equation (145) is plotted in Figure 56.

![Figure 56, pdf of sinusoid](image)

As depicted in Figure 56, strong PSD are expected at the bandwidth edge of the FM modulated sinusoid.
An assessment window size of 1000 ms and an evaluation window size of 300 ms are chosen. The evaluation of the p-values as a function of time and frequency yields Figure 57.

As depicted in Figure 57, since the PSD at the bandwidth edge of the FM modulated interference source is strong, the frequency domain RFI detector reveals the induced RFI.

### 7.1.1.2.3 Intermittent CW Interference

To outline the detection sensitivity on intermittent CW pulsed interference, the interference power was injected in a power range of -55 dBm down to -79 dBm with respect to the receiving antenna at the center frequency of the L1 GPS signal of 1575.42 MHz. Again the reradiated noise floor was at -55 dBm. The interference mask [11] suggests the detection of narrowband interference above -120.5 dBm, in association to a noise floor of -110 dBm, i.e. 10.5 dB below the noise floor. Therefore the interference mask for the experiment
calibration in the shielded chamber translates into -65.5 dBm at the reception antenna with respect to the reradiated noise floor.

A collected data set with an intermittent CW interference source at a power level of -73 dBm is chosen, which leaves a 7.5 dB margin to the interference mask. An assessment window size of 1000 ms and an evaluation window size of 300 ms is chosen. The evaluation of the frequency domain RFI detector p-values as a function of time and frequency yields Figure 58.

![Figure 58, Frequency domain RFI detector p-values for intermittent CW interference](image)

As shown in Figure 58 the intermittent CW RFI is detected at the downconverted frequency of 1.27 MHz. The injected RFI is detected at approximately 5 s, which is in agreement with the experiment.
The time domain RFI detector provides the p-value as illustrated in Figure 59.

![Figure 59, Time domain RFI detector p-values for intermittent interference](image)

As depicted in Figure 58 and Figure 59, both the time domain and frequency domain p-value interference detectors detect the intermittent interference at high levels of confidence.

### 7.1.1.2.4 Variable Frequency Interference

As presented in the case studies of narrowband, wideband and pulsed interference, the pre-correlation interference detection provides an approximate margin of 20 dB to the interference mask. This is an asset for the evaluation of interference with variable frequency. An interference source at -73 dBm was swept through the spectrum as shown in Figure 60.
As depicted in Figure 60, the sweeping interference source is detected through the evaluation of the p-values.

### 7.1.1.2.5 RFI Summary

The experiment in the shielded chamber corroborates the RFI detection sensitivity as obtained in Section 6.3.3. It is shown that wideband interference is more reliably detected in the time domain, where as narrowband interference is clearly more visible in the frequency domain detector. The pre-correlation detection, which is based on a constant amplification chain, is capable of detecting interference at or above the interference mask [11] at the required missed detection and false alert rate. Because the pre-correlation interference detection does not rely on acquisition and tracking performance of the receiver, generally higher detection confidence is accomplished. It is important to recognize that post-
correlation interference detection confidence is a conditional probability that is a function of acquisition and tracking probabilities of the receiver.

### 7.2 Anomaly Statistics from Continuous Operation

To obtain a performance history of Ohio University’s GAEM as described in Chapter 3 and to gain an insight into the integrity of the GPS signal, the GAEM system, as shown in Figure 61, was continuously run in Stocker Engineering Center Laboratory Room 323.

The GAEM is monitoring the GPS signal, which is received through an L1/L2 right-hand circular polarized patch antenna on the roof of the Stocker Engineering. For transmission of the GPS signal through the cable to the laboratory, a JCA12-4189T amplifier is mounted at the antenna site. The whole system is running off a backup power supply, including the antenna power supply, to keep the GAEM system independent of possible power outages.
The system was online from October 2003 through April 2004. Table 4 summarizes the collected signal anomalies that were flagged by the system.

<table>
<thead>
<tr>
<th>Event Type</th>
<th>Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel Lost</td>
<td>15</td>
</tr>
<tr>
<td>Jamming Signal Present</td>
<td>10</td>
</tr>
<tr>
<td>Pseudorange step</td>
<td>8</td>
</tr>
<tr>
<td>AGC out of range</td>
<td>16</td>
</tr>
<tr>
<td>Cycle Slip</td>
<td>12</td>
</tr>
</tbody>
</table>

The receiver-indicated AGC out of range flag almost always was triggered along with the receiver-indicated jamming flag. The analysis of these events revealed no GPS signal anomaly and the indicated flags just triggered a false alarm. The analysis of the pseudorange step events and cycle slip events also discovered false alert events. Further examination indicated that the limiting factor and cause of those events is the insufficient stability of the Novatel Beeline clock, which indirectly, through induced measurement inconsistencies, triggers the monitoring algorithms. The algorithms as presented in Section 3.1 only yield satisfactory results if an oscillator with a short-term stability on the order of $10^{-9}$ to $10^{-10}$ is employed, e.g. an oven-controlled crystal oscillator as used by the SDR.

However, the channel lost events revealed successful results, which are presented in the following Sections. As described in Section 3.1.2.1 the channel lost events are triggered by a sudden loss-of-lock condition of a satellite, given that a satellite is at a sufficient elevation and satisfied a sufficient lock time.
The US Coast Guard performs monitoring of the GPS signals and publishes status information of the space vehicles [56]. Interestingly, some of the channel lost events were acknowledged by the US Coast Guard and some were not detected. Multiple reasons may provide an explanation:

- The collected event was only a local event, e.g. multipath.
- The collected event may have been a false alert due to a tracking anomaly of the Novatel Beeline receiver serving as anomaly sensor. Analysis of the software radio data will provide a resolution to this uncertainty.
- A missed detection event by the US Coast Guard.

The following Sections are dedicated to the analysis of these events.

7.2.1 Detected Satellite Outage by the Coast Guard and Ohio University’s GAEM

As published by the US Coast Guard, on December 12\textsuperscript{th}, 2003, satellite 26 encountered an unscheduled outage as declared [56]:

GPS OPERATIONAL ADVISORY  
SUBJ: GPS STATUS  12 DEC 2003

B. ADVISORIES:
NANU MSG DATE/TIME PRN TYPE SUMMARY (JDAY/ZULU TIME START – STOP)

| 2003112 | 120615Z DEC 2003 | 26 | UNUSUFN | 346/0415–/ |
| 2003113 | 120701Z DEC 2003 | 26 | UNUSABLE | 346/0415–346/0648 |

As indicted above, satellite 26 was declared unusable at 4:15 GMT. The observed outage was also detected by Ohio University’s GAEM and data surrounding this event was captured. The analysis of the CNR, carrier frequency estimates and the calculation of the p-values on the CNR yields Figure 62.
As observed in Figure 62, at t=1700 ms the CNR drops below the tracking threshold and the GPS signal is lost. This is corroborated through the carrier frequency estimates, which encounter loss of estimation accuracy, since the GPS signal of satellite 26 is vanished. The p-value estimation on the CNR evidently indicates the existence of an anomaly at convincing confidence levels.

### 7.2.2 Detected Satellite Outage by the GAEM but not by the Coast Guard

On November 21st, 2003, satellite 8 encountered a sudden loss of tracking event, however none of the Novatel-implemented error flags indicated a signal anomaly. The satellite was at 61 Deg elevation and 208 Deg azimuth. Further, no signal failure mode was observed by the US Coast Guard. The analysis of the CNR and the carrier frequency estimate is presented in Figure 63.
As depicted in Figure 63 the signal of satellite 8 walks through an attenuation process and the CNR is reduced below the tracking threshold, and accordingly, the carrier frequency estimate loses its estimation accuracy (i.e. the equivalent of loosing lock in a sequential receiver). Satellite 28 is also evaluated and included in Figure 63. As satellite 28 displays no signal inconsistency, failure-free operation of the RF front-end is proven. Multiple error sources may initiate the presented signal anomaly. Attributing the obtained data to possible RFI by applying the detection scheme as introduced in Chapter 6, yields Figure 64 for the frequency domain p-value estimation.
As indicated in Figure 64, the p-values in the range from 2000 ms through 4000 ms in the frequency band of 1 MHz through 1.8 MHz encounter a sudden decrease from $10^{-2}$ down to $10^{-8}$, suggesting the presence of RFI. It is important to note that the receiver internal jamming detection did not provide a jamming warning. However, the provoked signal distortion was severe enough to induce a loss of lock condition for satellite 8 on the anomaly sensor.

Further signal anomalies on satellite 7 and satellite 28 were discovered on the same day. SV 7 was at 54 deg elevation and 314 deg azimuth, and SV 7 was at 42 deg elevation 261 deg azimuth. Figure 65 illustrates the CNR for the observed distortion.
As depicted in Figure 65 the CNR encounters a strong decrease on SV 28 and SV 7. Based on experimental results, a tracking threshold of 38 dB-Hz is assumed. This leads to an expected loss of lock condition of approximately 100 ms for SV 28 and 1.8 s for SV 7. This is confirmed by the carrier frequency estimates for SV 28 and SV 7 as illustrated in Figure 66.
As indicated in Figure 66, SV 7 encounters a loss of lock condition of approximately 1 s and SV 28 of approximately 100 ms.

7.2.3 In Summary

A conventional receiver such as the Novatel Beeline receiver does not work satisfactory to provide enough integrity for anomaly detection and too many false alerts are initiated. One of the major limiting factors is the receiver oscillator short term stability. Any significant clock jitter can initiate a false alert in the anomaly detection algorithms. In order to mitigate those uncertainties, a receiver serving as anomaly sensor may be driven with an oven-controlled crystal oscillator. On the other hand, as the SDR real time implementation is advancing [19], the implemented algorithms may be migrated onto the SDR, which is currently driven by an oven-controlled oscillator with higher short term stability. These results also highlight the importance of the GAEM to correctly identify the source of a GPS
receiver tracking anomaly. Without the GAEM, many anomalies observed by the Novatel Beeline receiver could not have been explained.

7.3 The Effects of Aircraft Overflights on GPS Measurements

In a field experiment, the GAEM was employed to study aircraft overflight induced signal anomalies.

Observation of LGF prototypes at the William J. Hughes Technical Center (WJHTC) and Ohio University revealed the presence of unexplained short signal anomalies leading to receiver tracking errors and signal loss several times per day. These brief anomalies cause violations of the continuity requirements of the LGF. These requirements are defined in [11], Section 3.1.3.2, “The probability that the number of valid B-values is reduced below three (3) for any valid ranging source within the reception mask (Section 3.2.1.2.6.1) shall not exceed $2.3 \cdot 10^{-6}$ in any 15-second interval.” A closer observation of the anomalies by WJHTC personnel resulted in the observation that the anomalies may be correlated to aircraft overflights of the LGF GPS reception antennas.

A controlled experiment was designed to evaluate the correlation between a loss-of-lock condition at an LGF receiver and aircraft overflights of the LGF GPS reception antennas. Further, the observed anomalies are characterized to allow an inference on LAAS LGF continuity implications.
7.3.1 Overflight Data Collection Setup

7.3.1.1 Ground Based Data Collection Setup

The entire ground-based hardware data collection setup is depicted in Figure 67.

![Figure 67, Overflight data collection setup](image)

The Ohio University prototype LGF (refer to Section 2.1.1.1) is connected to three designated IMLAs at FLD1, FLD2 and FLD3 locations. The GAEM RF front-end is connected to the HZA antenna of FLD2 as is the Novatel Beeline receiver serving as the anomaly sensor. As described in Chapter 3, the Anomaly Sensor CPU monitors the Novatel Beeline receiver outputs and generates an interrupt in case of a detected failure mode. This interrupt is connected to the GAEM ADC and CPU, which stores the down-converted GPS RF signal.

7.3.1.2 Airborne Data Collection Setup

The true trajectory of the overflying aircraft is of interest in the investigation of the correlation between aircraft overflights and observed RF signal outages at the LGF. An Ashtech Z12 system is employed as the truth reference in the DC-3 research aircraft. Two receivers are used that both track the L1 and L2 GPS frequencies to resolve carrier phase ambiguities and to mitigate ionospheric refraction effects. One of the Z12 receivers, located
at a surveyed location, serves as the reference receiver. Post-processing of the ground and airborne data files yields a position solution better than 0.1 m root-mean-square (RMS).

### 7.3.2 Flight Test Experiment

A flight test was performed in order to examine the effect of an aircraft overflight on GPS signal reception at the LGF as shown in Figure 68, and to evaluate whether overflights are correlated to the reported anomalies.

Ohio University’s DC-3 was flown over the LGF antennas located at FLD1 and FLD2 as illustrated in Figure 69. This aircraft was selected since tracking anomalies had been observed during previous approach and landing flight tests. The antennas at FLD1 and FLD2 have a lateral offset of approx. 30 m with respect to the runway centerline. Therefore, the aircraft approach was offset by 30 m to the right of the runway centerline.
Ten overflights of heights between 100 ft and 500 ft with a 100-ft spacing were flown. The Ashtech Z12 truth reference system provided the height profile as presented in Figure 70.

While conducting the experiment, small General Aviation (GA) aircraft such as Cessna 172’s were also overflying the LGF. A few anomalies initiated by those airplanes were also captured by the GAEM. In contrast to the DC-3 overflights, these planes were approaching along the runway centerline.

Figure 69, Prototype LGF setup at KUNI

Figure 70, Flight profile
7.3.3 Data Analysis of Overflights

The data analysis was performed by first evaluating the effect of the observed anomalies on the LGF, followed by post-processing the captured RF sequences on the GAEM by utilizing block processing techniques.

7.3.3.1 Observed Effect at the LGF for Overflight at 100 ft

The first overflight at 100 ft over FLD1 and FLD2, as illustrated in Figure 69, is presented. The HZA antennas of FLD1 and FLD2 were tracking satellites SV6, SV10, SV17, SV18, SV23, SV24 and SV26.

Figure 71 shows outage lengths at each station for PRNs that experienced loss-of-tracking. The LGF Novatel 3951 receivers connected to the HZA antenna of FLD1 loses lock on satellites SV6, SV10, SV23 and SV26, and FLD2 loses lock on satellites SV6, SV10 and SV23. The minimum time to reacquire the signal is 2 s for SV26 and SV23 of FLD1 where in contrast, SV6 of FLD2 required 6 s for reacquisition.

![Figure 71, Observed LGF HZA antenna outages at FLD1 and FLD2](image-url)
It is important to notice that the HZA antenna of FLD1 loses 57% of its satellite constellation and the HZA antenna of FLD2 only loses 42%. Thus, the overflight at 100 ft shows a significant impact on the LGF. Without understanding the nature and the reason of the outage, the continuity requirement as specified in [11] is not satisfied.

To investigate the correlation between the flight path and the observed outages, an azimuth/elevation plot referenced to FLD2 is introduced. The satellite constellation is plotted as a function of azimuth and elevation. Also, the DC-3 flight trajectory is translated into the same format. To indicate the dimensions of the aircraft, the fuselage denoting the center line and the right and left wingtips are incorporated as shown in Figure 72.

![Figure 72, DC-3 overflight at 100 ft in elevation vs. azimuth with respect to FLD2](image)

The center point of the polar plot denotes the location of FLD2. Note that, as the aircraft approaches FLD2, its wingspan subtends a larger azimuth range at higher elevation angles. Since the aircraft does not pass directly over FLD2, the plot shows a small asymmetric characteristic. The interpretation of Figure 72 yields that the region between the left and right wingtip depicts the line-of-sight (LOS) coverage at FLD2, which is induced by the DC-
The region outside denotes the LOS range that is not intercepted by the aircraft.

As can be recognized in Figure 72, SV6, SV10, SV23 and SV26 fall between the cross-section of the right wing and left wing. SV18 is right on the border. Comparing those results with the observed outages at the LGF (see Figure 71) reveals a correspondence. It is interesting that the Novatel Receiver at FLD2 did not lose lock on SV26 (see Figure 71) even though the LOS was intercepted by the aircraft.

By application of post-processing techniques (refer to Section 2.3), the induced anomaly by the aircraft overflight at 100 ft of SV10 is further analyzed. As can be seen in Figure 73, the right wing followed by the right stabilizer intercepts the LOS. The circles in Figure 73 denote the 1-s measurements reported by the LGF receiver at FLD2.

![Figure 73, SV 10 interception in North [m] vs. East [m] local coordinates](image-url)
By analyzing the GAEM data, the impact on the acquisition margin due to LOS interception is examined.

The acquisition margin serves as a measure of confidence that the sought SV has been correctly identified. It is defined as the ratio of the highest correlation peak to the second highest correlation peak in the acquisition grid. The highest peak should identify the correlation peak of the acquired signal and the locally-generated C/A code [3]. All other peaks are caused by noise, cross-correlation, or C/A code sidelobes. Referring to Section 2.3, the acquisition margin is calculated as

\[ \text{Acquisition Margin} = \frac{\max_i \sqrt{I_i^2 + Q_i^2}}{\max_i \sqrt{I_i^2 + Q_i^2}} \]  
(146)

Where \( \max_i \) indicates the highest correlation peak and \( \max_i \) the second highest peak in the sequence. If the dominant component in Equation (146) is the signal term then the acquisition margin expresses a ratio larger than 1. An acquisition margin ratio approaching \( \approx 1 \) indicates a weak signal component and therefore shows less certainty to acquire the correct correlation peak.

For this experiment, the noise and the cross-correlation dominate the C/A code sidelobe levels, but neither one will cause consistent correlation peaks over time. A detailed look at the acquisition margin for SV10 with the 100-ft overflight yields Figure 74.
As can be observed from Figure 9, two regions of low signal energy are present. Overall, the event lasts approximately 300 ms. Considering the length of the DC-3 of 20 m and an approximate approach speed of 60 m/s yields a LOS interception of 333 ms. Therefore it is evident that the physical dimensions of the DC-3 are delineated in the observed outage.

The evaluation of the CNR is shown in Figure 75.
As can be observed from Figure 75, two regions of low signal energy are present, corroborating the results of the acquisition results as evaluated in Figure 74.

The p-value evaluation for the observed anomaly in the CNR estimation is illustrated in Figure 76. To attain a detection time precision in the ms-range, an evaluation window size of 10 ms is chosen.
As depicted in Figure 76, the p-value indicates an anomaly in the data starting at 4570 ms and suggests the rejection of $H_0$. It can be seen that the algorithm reveals the certainty of an existing anomaly in the received data starting at 4570 ms and ending at 4850 ms up to a theoretical confidence level of $\beta = 1 - 10^{-12}$. Setting the arbitrary decision threshold at $10^{-4}$ yields the acceptance regions for $H_0$ and $H_A$ as indicated in Figure 76.

The data are further investigated by performing the discrete tracking with 1 ms blocks for SV10 and extracting the frequency estimates as shown in Figure 77.
It is important to point out that the regions of high frequency disturbance indicate a loss-of-carrier-lock region. The estimates are obtained by inducing code-lock prior to the occurrence of the anomaly and subsequent frequency estimation.

\[ \Delta \Phi_i = \frac{\dot{\phi}_i - \dot{\phi}_{i-1}}{2\pi \cdot T_n} \]  \hspace{1cm} (147)

Where the estimate for \( \dot{\phi}_i \) is based on the code lock. Frequency estimates with high disturbances therefore indicate that no carrier was found in the received signal.

It was previously presented (Figure 73) that the right wing first intercepts the LOS to SV10 and subsequently, the right stabilizer. Given an approximate wing obstruction size of 5 m and an approach speed of 60 m/s, the LOS is obstructed for 83 ms. The wing and the stabilizer are separated by 8 m, i.e. the signal does not experience an obstruction for 133 ms. Thus the evaluation of the frequency estimates and the comparison of the loss-of-carrier-
lock region shows a high correlation to the physical dimensions of the DC-3. Therefore, it can be concluded that the LOS obstruction of the satellite signal by an aircraft initiates a loss of lock condition. The actual signal propagation mechanism is complicated as factors such as specular multipath, diffraction from edges and shielding by the airframe all contribute to the failure condition.

7.3.3.2 Observed Effect at the LGF for Overflight at 200ft

The effect on tracking outages due to an approach altitude of 200 ft is presented. The satellite constellation and the flight trajectory referenced to FLD2 are illustrated in Figure 78.

![Figure 78, DC-3 overflight at 200 ft in elevation vs. azimuth with respect to FLD2](image)

At the LGF receiver connected to FLD2, SV6 and SV10 experience a failure condition. Comparing these results to blockages indicated in Figure 78, shows that losses of SV6 and SV10 are likely caused by LOS obstruction from the aircraft. Compared to Figure 72, the higher approach altitude introduces a smaller LOS coverage, which is indicated by the shaded region.
The discrete tracking of SV6 yields the frequency estimates as illustrated in Figure 79.

![Doppler Frequency vs Time Graph](image)

**Figure 79, Doppler frequency estimates satellite 6 for overflight at 200 ft**

The same conditions prevail as presented in the previous case for an approach altitude of 100 ft. The right wing obstructs the LOS of SV6. The wing initiates a loss of carrier-lock condition for approximately 100 ms. After 150 ms, the stabilizer intercepts the LOS to SV6. The duration of the overall disturbance is approximately 300 ms. Again, at an approach speed of 60 m/s the physical dimensions of the DC-3 are delineated in the outage.

The evaluation of collected data surrounding an outage for an approach altitude of 500 ft led to the same inference that the duration of a loss-of-lock condition is mainly a function of aircraft speed and size and that the altitude is rather insignificant.
7.3.3.3 Observed Effect at the LGF for General Aviation (GA) Aircraft Overflight

Finally, while conducting the experiment, events due to GA overflights were also recorded. Figure 80 illustrates the captured event initiated by a Cessna 172.

![Doppler Frequency vs Time](image)

**Figure 80, Cessna overflight**

The duration of the observed carrier phase disturbance in the data is 180 ms. Comparing this to an approach speed of 70 knots (36 m/s), and considering the overall length of 8.28 m, yields a LOS blockage of 230 ms. Since this anomaly was only observed and recorded at the LGF, no data on the actual flight trajectory is available. However, the assumption that either the left or right wing followed by the stabilizer intercepts the LOS seems reasonable. This also explains the slightly shorter outage duration of 180 ms since the measure from the wing to the stabilizer is slightly shorter than the fuselage length (i.e. approx. 7 m).
7.3.3.4 Simulation of Operational Scenario

In order to predict the impact of overflight-induced outages on LGF performance, a simulation of nominal system operation was constructed. A full nominal satellite constellation was simulated over 24 h and different LGF offsets to the runway threshold were simulated. An aircraft with a 30 m wingspan (Boeing 737) was flown in a missed–approach pattern over the runway centerline. Further, a mask angle of 5° was applied at the ground station. The LOS blockage time periods were calculated over the length of the simulation and logged.

Figure 81 depicts the expected number of intercepted LOS occurrences per approach for a variable offset of the LGF perpendicular abeam to the runway centerline as a function of the approach altitude.

![Figure 81, Simulated average LOS blockage with variable offset](image-url)
As depicted in Figure 81, an increasing offset of the LGF to the runway centerline and an increasing approach altitude lowers the expected value to intercept a LOS.

According to [11], Section 3.1.3.2, “The probability that the number of valid B-values is reduced below three (3) for any valid ranging source within the reception mask (Section 3.2.1.2.6.1) shall not exceed $2.3 \cdot 10^{-6}$ in any 15-second interval.” Thus, a simulation was performed to estimate the scenario condition that more than 1 reception antennas can be affected by overflight-induced outages. Two reception antennas, separated by 80 m in distance, were placed on the ground and an aircraft was flown over at a simulated altitude. Rather than choosing a specific aircraft body size, the minimum size which may cause a LOS interception for both antennas simultaneously was evaluated.

As depicted in Figure 82, an aircraft flying at an altitude of 10,000 m and in a straight flight that is capable of causing a LOS interception for both antenna locations at the same time is within the range of 75.5 to 80 m, depending on the satellite elevation and azimuth. Considering a spacing of the reception antennas of 80 m, an Airbus...
A340-600, which is currently the longest commercial airplane with an overall length of 75.3 m, can still be tolerated and no multiple LOS blockage is initiated. This is based on the assumption that a receiver could immediately recover from an aircraft-induced outage. At a minimum approach speed of 80 m/s, not considering any head wind, the maximum outage duration is 0.94 s.

However, experimental data showed that a receiver needs 20 ms up to 0.5 s to recover from an overflight induced loss-of-lock condition. Therefore, the results obtained in Figure 82 are extended and the recovery time of the receiver is also incorporated. The receiver recovery time can simply be interpreted as an equivalent extension of the airframe measurement, and is determined as $\text{groundspeed} \times \text{receiver recovery time}$. This effect may lead to multiple simultaneous LGF receiver outages, even though the LOS interception does not occur at the same instant of time. The criteria determining the likelihood of such a failure condition are aircraft size, aircraft orientation, aircraft ground speed, receiver recovery time and antenna spacing.

Again considering the longest airplane with a fuselage length of 75.3 m, the equivalent increased airframe measurements are obtained as shown in Table 5.

<table>
<thead>
<tr>
<th>Aircraft speed [m/s]</th>
<th>Receiver recovery time [s]</th>
<th>Equivalent increased body measurement [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>250*</td>
<td>0.5</td>
<td>125</td>
</tr>
<tr>
<td>250</td>
<td>0.02</td>
<td>5</td>
</tr>
<tr>
<td>70**</td>
<td>0.5</td>
<td>30</td>
</tr>
<tr>
<td>70</td>
<td>0.02</td>
<td>1.4</td>
</tr>
</tbody>
</table>

*crusing speed A340
**landing speed A340

As depicted in Table 5 the worst case equivalent increased body measurement is 125 m for a receiver recovery time of 0.5 s. This would require a minimum antenna spacing of approximately 200 m to mitigate a multiple receiver loss-of-lock condition.
7.3.4 Summary

A strong correlation has been measured between aircraft overflights of LGF reference antennas and tracking anomalies that cause brief loss-of-lock conditions in GPS reference receivers. Further, the GAEM setup has allowed for thorough analyses of these signal anomalies. Strong correlation was found between the phase/frequency distortion signature and the surfaces of the overflying aircraft that intercept the LOS to a satellite vehicle. While the duration of the anomalies was found to be mostly dependent only on aircraft speed and size, altitude becomes a factor in determining the probability of these events occurring. Additionally, the line-of-sight breach criterion used greatly simplifies any propagation modeling that would be required to produce statistical estimates for use in continuity evaluations. Knowledge of the source of previously unexplained outages in LGF data may allow for bridging of aircraft overflight-related outages so that continuity requirements for LGF tracking can be met. To mitigate the possibility of having aircraft-induced outages at multiple antenna locations at the same time, it can be concluded that the antenna spacing should be sufficiently large relative to the largest aircraft dimension, assuming quick receiver recovery from an overflight-induced outage. For receiver recovery times of up to 20 ms, LGF antenna spacings of 80 m are sufficient to present the loss of more than one B-value at the time, while LGF antenna spacings of up to 200 m are required for receiver recovery times up to 0.5 s.
This document explores interference and satellite anomalous event monitoring of Global Positioning System signals. In order to enable detailed analyses of signal anomalies, the GPS anomalous event monitor was introduced, which has its foundation in the Software Defined Radio. Since the anomalous event detection requires a statistical decision process at a high level of confidence, error sources of the estimation process were investigated; in particular the effects of the software receiver RF front-end components. It was shown that amplification and frequency mixing do not change the statistical properties of the received signal. However, bandpass filtering introduces serial correlation into the data that may initiate estimation errors. It was illustrated that the bandpass filter bandwidth is a trade-off between signal estimation errors and noise distortion energy. Although the motivation for this analysis resulted from the implementation of the Software Defined Radio, the concepts introduced also apply to GPS receivers in general.

Since the received GPS signal is distorted by noise, any signal parameter determination translates into a statistical estimation process. In the context of signal monitoring, it is important to recognize that there is no *a priori* knowledge available on the characterization of a signal failure mode. Any assumption would reduce the integrity of the detection procedure. Thus, to accomplish the task of anomaly detection at a high level of confidence, a non-parametric, non-homoscedastic statistical test was developed. To establish confidence levels of the statistical test, the convergence was quantified by Berry-Esseen bounds. The statistical test is leveraged through the high sampling rate of the Software Defined Radio. The derived algorithm was verified through simulations.

The established technique was applied on the detection of GPS signal anomalies. Since Radio Frequency Interference is a major threat to GPS, the detection of Radio Frequency Interference was emphasized in this document. It is shown that pre-correlation interference detection is able to detect interference that is below the interference mask as specified in [11], under the condition of an initial undistorted signal period. The method of detecting
pre-correlation power variations proved advantageous over ‘absolute’ interference detection, not only since system induced inconsistencies of acquisition and tracking are mitigated, but also since no conditional probability of acquisition and tracking are influencing the overall false alert and missed detection rates.

Further, a several case studies of observed GPS anomalies from continuous operation over seven months were presented. Finally, in a field experiment the effects of aircraft overflights on GPS measurements were investigated. The GPS anomalous event monitor revealed strong correlation between aircraft overflights of LAAS Ground Facility reference antennas and tracking anomalies that cause brief loss-of-lock conditions in GPS reference receivers.

The following recommendations are provided for future work:

- As limited performance of the detection algorithms due to the limited anomaly sensor clock stability was discovered, better performance may be accomplished by an anomaly sensor driven with an oven-controlled crystal oscillator.
- At the initiation of the performed research, the implementation of the Software Defined Radio in real time was not feasible, due to the excessive computational burden required. Therefore, a regular receiver as anomaly sensor was employed. However as tremendous advances in the field of real time Software Defined Radio processing have been accomplished [19], the detection algorithms may be migrated onto the Software Defined Radio.
- Using a wideband SDRFE introduces less estimation errors and increases observability on signal anomaly analyses.
9 REFERENCES


[39] Chen, P., Asymptotic Refinement of the Berry-Esseen Constant, Department of Communications Engineering, National Chiao Tung University, Taiwan 30056, R.O.C.


