The Effects of Cognitively Guided Instruction and Cognitively Based Assessment on Pre-Service Teachers' Learning, Instruction, and Dispositions

A project completed in partial fulfillment of the requirements for the Honors Program

by

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Honors Thesis
Abstract

The mathematics education research community is focused on changing the preparation of mathematics teachers. The adoption of the Common Core State Standards for Mathematics (Mathematics Standards, 2010) and associated Mathematical Practices in many states requires a different type of preparation for teachers who will be held accountable for helping K-12 students meet the learning demands of the curriculum. This research particularly focuses on the preparation of pre-service mathematics teachers to teach geometry-related standards.

The study examines the phenomenon of teacher candidates learning to teach geometry to middle school students. The study focuses on an undergraduate pre-service teacher methods course that emphasizes reformed teaching and learning. The researcher examines how the methods used in the course impact the pre-service teachers as learners and teachers.

This study employs a grounded theory approach to data collection and data analysis. Data sources include pre-service teachers’ written case study assignments, surveys of pre-service teachers’ opinions and dispositions related to teaching mathematics, and pre- and post-assessments of pre-service teachers’ geometry content knowledge.
Research Background

Over the past 20 years, there has been an urgent call for a reform movement in mathematics education in the United States (Battista, 1994; Carpenter & Fennema, 1996; Clements & Sarama, 2011; Franke & Kazemi 2001, Sarama & Clements 2009). Traditional mathematics instruction had long been treated as a rigid set of rules and regulations, and as various isolated skill sets which students must master through memorization and rote. This way of mathematics teaching and learning suggests that education, or producing understanding, had long been confused with training, or producing some desired performance. The National Council of Teachers of Mathematics (NCTM) states that students who learn with a focus on conceptual understanding rather than a focus on developing skill are more able to adapt their knowledge and apply it to new kinds of problems (NCTM, 2016). When students are “taught” mathematics by memorizing formulas and procedures and having to follow, to the letter, teachers’ instructions of which skills must be used in which situations, the students do not actually learn how to do mathematics on their own. This builds a mathematical disposition in that math consists of an assigned set of rules and one correct procedure which must be communicated by a teacher and then reproduced by students. This damaging mathematical disposition results in a view that school mathematics learning is not easily applicable or related to non-school mathematical situations and that mathematics itself is not a tool for making sense of the world.

Students approach and attempt to solve a problem posed inside of school differently than they approach and solve that same problem when posed outside of school (Battista, 1994). A 1985 seminal study titled “Mathematics in the Streets and in Schools” found that computational strategies used in the everyday lives of young people in Brazil when acting as street vendors in
marketplaces were far superior to their strategies on similar context-free, school-type problems (Carraher, 1985). Since the publication of this study, the term “street mathematics” has become commonly used in the area of mathematics education to refer to the way many people are able to perform calculations in a real-life context with no problem, but are unable to perform calculations on similar context-free problems. This phenomenon has since been heavily researched, and implications for schools have been investigated. Further studies widely agree that mathematics instruction in schools must employ real-life contexts and learning with meaningful understanding rather than rote and memorization (Battista, 1994; Carpenter & Fennema, 1996; Franke & Kazemi, 2001).

Learning with understanding is learning that is personally meaningful to students. Rather than knowledge of isolated facts and skill sets which cannot be extended or justified by the student, knowledge gained from learning with understanding has four distinct characteristics (Carpenter et al., 2015). First, knowledge is interconnected. Skills are not all isolated from one another, but form a rich network of connections, which makes knowledge easier to recall and apply to new situations. Second, knowledge is generative, or able to be extended and applied to new types of problems (Carpenter et al., 2015; Franke & Kazemi, 2001). Students who learn procedures only by rote and do not truly grasp concepts have little flexibility when determining when a procedure is appropriate and when another strategy may be more efficient. For example, when calculating the difference of 300 and 298, a student who has learned subtraction with regrouping by rote will use this tedious algorithm strategy rather than observing that the strategy of “counting up” may be used to quickly determine that the difference must be 2. Third, students who learn with understanding are able to describe, explain, and justify their reasoning. They know why a certain procedure works, not just that it works (Carpenter et al., 2015). Finally,
students who learn with understanding see themselves as mathematical thinkers with the ability to make sense of mathematics by their own actions. These students see themselves as capable of doing mathematics and therefore do not seek verification for every idea they have from a source of mathematical authority such as a teacher or a textbook—verification comes from convincing themselves and each other that their ideas are valid (Carpenter et al., 2015; Franke & Kazemi, 2001).

In order to reap the benefits of learning with understanding, mathematics instruction in the classroom must shift from traditional, teacher-centered to reform, student-centered. Battista (2001) states “[a]ll current major scientific theories describing mathematics learning agree that mathematical ideas must be personally constructed by students as they intentionally try to make sense of situations.” In order to learn with true understanding, students must construct meaning for themselves. Teachers must learn to support creative thinking in mathematics and let children come up with their own strategies for solving different types of problems (Sarama & Clements, 2009). The role of the teacher must no longer be “conductor,” but “guide”—to help, not force, students along to more sophisticated levels of reasoning. For example, rather than just moving a whole class from section 2.1 to 2.2 in a textbook as the next day’s lesson, whether or not every student in the class truly understands the ideas in lesson 2.1, a teacher must meet students where they are in their understanding of a topic.

Despite the now decades old call for schools to shift to conceptual-based, student-centered classrooms with a focus on learning with understanding, there is still an overwhelming amount of traditional instruction in schools today. There have been programs and responses to the call, however, and one response to the call for this type of reform and the push for learning with understanding came through the Common Core State Standards for Mathematics
(CCSSM). The goal of the CCSSM is to “build understanding and to help the student realize that conceptual learning is a way of thinking and a skill” [emphasis added] (Dreambox Learning, 2016). In fact, beyond particular mathematical content standards, the CCSSM outlays particular mathematical practices like “applying mathematical ways of thinking to real-world issues and challenges,” as key to mathematics teaching and learning (Common Core State Standards Initiative, 2016). These types of practices encourage thought beyond the traditional “teacher-assigned, student-regurgitated” model of classroom practice. Teachers are charged to emphasize learning itself as a skill and students must be able to use their mathematical knowledge to choose methods of problem solving appropriately.

It is difficult for seasoned and master teachers alike to move towards reforming their teaching practices: reform methods were not used in the mathematics classes when they themselves were students, and were perhaps only discussed, and surely not modeled, in recent professional development programs. Even if covered generally in pre-service teacher programs, specific classroom methods that can be used to make sense of students’ individual cognition, individualize each student’s instruction, and meet the high demands of the CCSSM-aligned curricula were absent. College math professors still widely use traditional methods in their courses. Though the Common Core has begun to try to shift the focus for all teachers to reform, it was not implemented in time for current college students to experience it themselves in high school or elementary school. As a result, pre-service teachers must now be prepared to teach using reform methods as soon as they enter the field, despite having very little to no experience with those methods themselves. Those looking to move mathematics teaching forward, however, have been researching the best methods for teachers to use to help students construct meaning for themselves.
Researchers widely agree that in order to foster meaningful learning in students, instruction must be highly individualized, and most importantly, teachers must have a strong basis of student cognition, or the way students think (Battista 1994, 2001, 2011, 2012; Carpenter et al. 2015; Carpenter & Fennema 1996; Clements & Sarama 2011; Franke & Kazemi 2001). Further, “[e]arly childhood teachers’ knowledge of young children’s mathematical development is related to their students’ achievement,” which emphasizes the importance of teachers’ knowledge of students’ cognitive functions (Clements & Sarama, 2011). Research on student cognition in the field of mathematics education has yielded various frameworks for teaching mathematics based on the evolution of students’ reasoning. For example, Cognitively Guided Instruction (CGI) and Cognitively Based Assessment (CBA) are used primarily in elementary education and secondary education, respectively, in order to individualize teaching and learning.

CGI is not a curriculum or a template for a classroom, and it does not include instructions for how to implement instruction—it does provide a framework of the progression of student knowledge on a topic, which teachers should bear heavily in mind when implementing their own teaching styles and practices (Carpenter et al., 2015). This framework, or learning progression, is defined by the National Research Council as a description of “the successively more sophisticated ways of thinking about a topic” (NRC, 2007). Battista defines a learning progression as a detailed description of general student cognition and ways of reasoning about a particular topic (Battista 2011). This description is broken down into “cognitive plateaus” which students progress through to get from the everyday knowledge of a topic they bring to school to mastery of the topic. For example, Battista’s (2012) CBA Learning Progression for Geometric Shape (Appendix E) describes students’ entry knowledge as visual/holistic reasoning, and
gradually builds and culminates at formal deductive proof. Teachers who have a solid understanding of the ways children think about mathematics “are well equipped to teach for understanding,” because understanding a progression for a particular topic—for example, measurement or geometric shapes—provides teachers with some of the knowledge they need about the ways students make sense of the topic (Carpenter et al., 2015). Teachers should then be able to locate a particular student on the progression and individualize instruction to move that student along to the next level on the progression.

CBA is “a cognition-based, needs-sensitive framework to support teaching that enables all students to understand, make personal sense of, and become proficient with mathematics” (Battista, 2012). There are three critical components of CBA: (1) clear, research-based descriptions of the way students develop meaning for key ideas in school mathematics; (2) assessments which determine the ways each student is reasoning about those key ideas; and (3) the types of instructional tasks that will help students at each level of reasoning about the ideas (Battista, 2012). These three things—descriptions of student thought processes, assessments, and suggested instructional tasks—all work together as a powerful tool for teachers to use for individualized, high-quality instruction which results in learning with understanding.

One area for which the CBA framework is particularly needed is middle school geometry. International studies show that there is a weakness in students’ geometry achievements, and the Trends in International Mathematics and Science Study (TIMSS) has shown that international performance on geometry is low (Clements & Sarama, 2011). Pre-service teachers especially are not adequately prepared to teach geometry, as many are not able to reach the level of recognizing and characterizing shapes by their properties themselves, and one study found that almost two-thirds of prospective teachers could not define a square.
correctly (Clements & Sarama, 2011). These findings indicate an unmistakable call for professional development in the area of geometry (Clements & Sarama, 2011).

Pre-service teachers, in addition to geometry content knowledge and professional development, need to have knowledge of the reform movement in mathematics. They must learn how to individualize instruction, teach for understanding, emphasize the importance of connecting new knowledge to existing knowledge, and allow students to construct meaning for themselves while acting as a guide. Pre-service mathematics teachers today have not experienced this reformed classroom in their own high school or elementary school careers, and therefore must be taught how to create a reformed math classroom for themselves. Giving pre-service math teachers exposure to the CGI and CBA frameworks and teaching them how to make sense of learning progressions and use them effectively is crucial at the university level.

**Study Rationale and Research Questions**

Preparing pre-service teachers to meet both the changing demands of mathematics teaching and to incorporate best practices found in research is recognized in the mathematics community as both vitally important to create needed change in mathematics teaching and especially difficult. Given the needs outlined above to prepare pre-service teachers to effectively teach geometry, individualize instruction, uncover student cognition, and teach for understanding, this study generally examines pre-service teacher learning in an undergraduate pre-service teacher methods course that emphasizes reformed teaching and learning. The specific research questions considered are:

1. How does studying CGI in general and CBA for geometry in particular impact pre-service teachers’ geometry content knowledge and pedagogical dispositions?
2. In what ways are pre-service teachers able or unable to effectively use the CBA framework to accurately identify student levels of reasoning?

3. In what ways are pre-service teachers able or unable to effectively use the CBA framework to increase student levels of reasoning?

Methodology

Data Collection

An outline of the study methodology can be found in Appendix A. The course examined in the study is a mathematics methods course for undergraduate secondary (grades 7-12) pre-service teachers at a Catholic liberal arts university in the Midwest. The course took place in the 15-week Fall 2016 semester, and the study was conducted after the course had concluded. In the outline of study methodology (Appendix A), the pre-service teachers’ course requirements as pertaining to the study can be found in Column C, Rows 1 and 2. At the beginning of the semester, the five pre-service teachers enrolled in the mathematics methods course (including the researcher) took a written geometry content knowledge assessment (see cell C1; Appendix B) and a multiple choice and open-response mathematical disposition survey (see cell C1; Appendix C) about pedagogical dispositions towards mathematics teaching in general, the subject area of geometry specifically, and the approach of individualizing instruction. Pre-service teachers in the course then learned about Cognitively Guided Instruction (CGI) in general and geometry-specific Cognitively Based Assessment (CBA) (see cell B2). Next, each of the five pre-service teachers completed a case study of three students in his or her field placement (see cell C2; Case Study Assignment Rubric and Description, Appendix D). The case study consisted of the following:
Pre-service teachers first administered the geometry content knowledge assessment (the same assessment the pre-service teachers themselves took at the beginning of the semester) to each of the three students they had chosen to study. Pre-service teachers administered the assessment to each of their three students according to the CBA framework, meaning that in addition to written responses, pre-service teachers were to ask the student probing questions while he or she was answering the questions in order to gain more insight into his or her levels of reasoning. The pre-service teachers then assigned levels from Battista’s (2012) Learning Progression for Geometric Shapes (Appendix E) to each student’s thinking on each of the nine problems on the assessment, based on the students’ written and verbal responses. Bearing these levels of student cognition in mind, the pre-service teachers then set goals for each of their students with regards to the learning progression (for example, to move a student from using informal and insufficient formal language to describe rectangles to using complete and formal language to describe rectangles) and constructed three individualized lessons with which the pre-service teachers aspired to meet those goals. The pre-service teachers then taught the three lessons to each of their students. After the lessons were completed, the pre-service teachers administered the geometry content knowledge assessment once more as a post-assessment of student geometry content knowledge. Pre-service teachers then leveled each student’s thinking on each problem on the post-assessment. Finally, pre-service teachers completed a written summary of each student’s pre-assessment, lessons, post-assessment, and learning gains and reported all findings, with all of their students’ work de-identified (a complete rubric with requirements for the written case study assignment can be found in Appendix D).

After the case study assignment was finished and submitted, pre-service teachers took the geometry content knowledge assessment as a post-assessment of geometry content knowledge,
and the mathematical disposition survey as a follow-up on pre-service teachers’ pedagogical dispositions at the beginning of the course. It is important to note that pre-service teachers did not take the geometry content knowledge assessment in the same manner in which they administered it to their students. Pre-service teachers took the assessment (both pre- and post-) only in written form—the course instructor did not administer the assessment individually to each pre-service teacher or ask pre-service teachers any probing questions while they took the assessment.

The study therefore consists of qualitative analysis of the following three data sources:

1. **Geometry Content Knowledge Assessments** (administered to pre-service teachers before and after conducting the case study)

2. **Mathematical Disposition Surveys** (administered to pre-service teachers before and after conducting the case study)

3. **Case Study Assignments** (including each K-12 student’s pre- and post-Geometry Content Knowledge Assessments)

It is important to emphasize the researcher’s role as a student in the course; therefore, her own personal geometry content knowledge assessments, mathematical disposition surveys, and case study assignment were not included in data analysis. It is also important to note that course grades were submitted by the course instructor prior to the researcher receiving any data for analysis. Therefore, the researcher’s work *in no way* affected any pre-service teachers’ grades in the course.

**Description of Data**
The geometry content knowledge assessment taken by both pre-service teachers and their K-12 students can be found in Appendix B. There is a notable difference in the format of the data collected by the assessment from pre-service teachers and from their K-12 students. Pre-service teachers took both their pre-assessment and their post-assessment only in written form, fully understanding that the only way for them to show their knowledge on the assessment was by their written work. Students who took the pre- and post-assessment as a part of the pre-service teachers’ case studies were encouraged to write down their thinking, but also had the opportunity to explain and elaborate on their thinking verbally while taking the assessment. Therefore, the only source of data to consider pre-service teachers’ geometry content knowledge is their written pre- and post-assessments, whereas data on K-12 students’ geometry content knowledge comes from both their written assessments included in the case study assignments, and pre-service teachers’ descriptions of students’ verbal responses and/or gestures that accompanied their written answers.

The mathematical disposition survey taken by pre-service teachers can be found in Appendix C. The survey consisted of both multiple choice questions with fixed answers and open-ended questions mostly to offer opportunity for pre-service teachers to explain their answer selections to the multiple choice questions.

The rubric for the case study assignment can be found in Appendix D.

**Analysis of Data**

Because the study employed qualitative research methods, the research findings are admittedly subjective and not generalizable to any population wider than the individual cases studied. However, the use of a grounded theory approach to study this phenomenon of learning
how to teach geometry is important to better understand how pre-service teachers make sense of their learning and apply the new mathematics pedagogy. The study is not a case study; rather, it is a qualitative approach to analyzing set, written data sources. The researcher was a participant in the learning settings, so the analysis is situated within the personal framework of a participant who was also learning and applying the methodology. The position of a participant researcher has both strengths and weaknesses (see Kawulich 2005); however, the researcher believes this role allows for a more informed understanding of the complex process of learning. To analyze the data, the researcher used a grounded theory approach (Trochim, 2006). The researcher considered each of the three research questions individually before reading through the data sources, and then utilized Glaser and Strauss’s method of memoing. Memoing involves making simple notations that progress to extensive, focused observations over the course of reading and re-reading the data sources to look for patterns and outliers (Trochim, 2006).

Findings

Mathematical Disposition Surveys

Overall, the mathematical disposition surveys showed a favorable shift in pre-service teachers’ dispositions towards geometry in particular (see Appendix F for a summary of survey responses to multiple choice questions). The first part of the survey focused on pre-service teachers’ personal opinions about geometry. There seemed to be a small shift in pre-service teachers’ personal opinions about geometry from pre- to post-. For example, the number of pre-service teachers who felt most prepared to teach geometry among other content areas increased from 1 to 2, and the number who felt least prepared to teach geometry decreased from 1 to 0. Three pre-service teachers indicated on the pre-survey that the content area they
believe students struggle with most is geometry, and only one indicated the same on the post-
survey after conducting the case study.

The second part of the survey focused on pre-service teachers’ opinions about teaching
gometry, and pre-service teachers seem to have a more favorable outlook on teaching geometry
from pre- to post-. For example, on the pre-survey, only one pre-service teacher indicated that
“For geometry in particular, you: Really like it. Can’t wait to teach it” while the other three
chose “other,” and specified that they struggle with geometry, are “okay” with it, or need work
refreshing their own knowledge of the content. On the same question on the post- survey, three
pre-service teachers chose “Really like it. Can’t wait to teach it” while only one indicated that
they “Struggle with it, but will teach it if necessary.” Also on the pre-survey, when asked, “How
prepared do you feel to teach geometry?” one pre-service teacher chose “Not at all prepared” and
three chose “Somewhat prepared”. On the same question on the post-survey, no pre-service
teachers chose “Not at all prepared,” only one chose “Somewhat prepared,” and three chose
“Fairly well prepared.” On the post-survey, all four pre-service teachers explained their answers
by saying that they are in need of more review with the content and more experience teaching
gometry in a classroom setting. Not one of the four pre-service teachers had any field
experience work in geometry classrooms prior to their work on the case study assignment.

This indication that pre-service teachers need more experience with geometry content
became even more prevalent in the third part of the survey. On the pre-survey, all pre-service
teachers explained that they had very little or no experience teaching geometry. On the post-
survey, all four indicated that they had some experience teaching geometry (from conducting the
case study). In the various questions about how prepared pre-service teachers feel to
differentiate to meet the needs of all students, to teach students with special needs, to teach ELL
students, and to teach students with exceptionalities, pre-service teachers often explained on the
post-survey that they need more experience in the respective areas to be able to feel prepared. It
is clear from analyzing the pre- and post-surveys that pre-service teachers recognize their own
need for more experience—with geometry content, methods of teaching geometry, and with
dynamic geometry software—to feel comfortable with and enjoy teaching geometry effectively.

**Pre-Service Teacher Geometry Content Knowledge Assessments**

Pre-service teachers’ pre- and post- geometry content knowledge assessments were
leveled by question by the researcher. A summary of pre-service teachers’ levels of reasoning
from pre- to post- is as follows:

<table>
<thead>
<tr>
<th>Question</th>
<th>Pre-Service Teacher Pre-Assessment Levels</th>
<th>Pre-Service Teacher Post-Assessment Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PST A</td>
<td>PST B</td>
</tr>
<tr>
<td>1</td>
<td>2.2/2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>2</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>3</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>4</td>
<td>2.3</td>
<td>2.3</td>
</tr>
<tr>
<td>5</td>
<td>3.3</td>
<td>2.3</td>
</tr>
<tr>
<td>6</td>
<td>Inc.*</td>
<td>3.3</td>
</tr>
<tr>
<td>7</td>
<td>Inc.</td>
<td>2.3</td>
</tr>
<tr>
<td>8</td>
<td>3.2</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>3.4</td>
<td>n/a</td>
</tr>
</tbody>
</table>

*Pre-Service Teacher C did not submit a pre-assessment.

**The letters “Inc.” indicate that the written response was incomplete, and there was not
sufficient evidence to accurately level the PST’s reasoning.

The highlighted boxes indicate a change in reasoning level from pre- to post-assessment.
**Pre-Service Teacher A**

Pre-Service Teacher (PST) A showed some growth in his precision of language and articulation of reasoning from pre- to post-, particularly on questions 1, 6, and 7.

For question 1, when describing how all the shapes made by Drawing Machine 1 (above) are alike, PST A shifted from language like “all the shapes have a similar angle” to more strong claims, such as “They are all right angles that have exactly a measurement of 90°.” This shift in language use demonstrates an increase in precision, and thus brings him up to a 2.3 on the post-assessment.
6. The measurements for a shape are given below.

For each statement about this shape, circle T if the statement is True, or F if the statement is False. If you can't tell if the statement is true or false, circle Can't tell.
For each statement, describe how you would convince someone that your answer is correct.

a. The shape is a square.  
   T  F  Can't tell

b. The shape is a rectangle.  
   T  F  Can't tell

c. The shape is a parallelogram.  
   T  F  Can't tell

d. The shape is a rhombus.  
   T  F  Can't tell

7. The measurements for a shape are given below.

For each statement about this shape, circle T if the statement is True, or F if the statement is False. If you can't tell if the statement is true or false, circle Can't tell.
For each statement, describe how you would convince someone that your answer is correct.

a. The shape is a square.  
   T  F  Can't tell

b. The shape is a rectangle.  
   T  F  Can't tell

c. The shape is a parallelogram.  
   T  F  Can't tell
For questions 6 and 7 on his pre-assessment (above), PST A only circled “T” or “F” for each statement, but did not explain his reasoning at all. Though his responses were all correct with only one exception, without knowledge of PST A’s reasoning, it is not possible to level his thinking accurately. On his post-assessment, however, PST A indicated that the shape in question 6 is not a square for a reasons like “not all sides are equal,” that the shape is a rectangle because it is a “closed shape with four 90° angles and 2 sets of opposite sides that are equal and parallel.” On question 7, PST A used even more formal, precise language about angles to claim that the shape is a parallelogram (e. g. “opposite interior angles are equal”). These responses merit a level 3.3—PST A used logical inference about formal properties to decide whether or not a shape meets the criteria to be a square, rectangle, parallelogram, or rhombus.

The most notable finding in PST A’s content knowledge assessments is from his work on problem 8, below:

PST A, on his pre-assessment, answered “false,” accompanied with a drawing which he believed to be a counter example. The shape looked similar to the following two shapes, with one pair of opposite sides equal and only one right angle:

Semantically, in mathematics, “opposite sides” indicates all sets of opposite sides (two pairs of opposite sides in quadrilateral shapes, for example). PST A’s interpretation of “opposite sides”
was “one pair of opposite sides.” The answer he provided merits a level 3.2, due to his thinking about shape construction. On his post-assessment, PST A acknowledged what he believed to be ambiguity in the wording in question 8. PST A indicated that the statement could be either true or false, depending on the interpretation of the words “opposite sides.” He further explained that in the case of both pairs of opposite sides equal, the statement would be true, and in the case of one set of opposite sides equal, the answer would be false. PST A’s work on problem 8 on the post-assessment also includes several partial drawings of shape constructions. His response to problem 8 on the post-assessment, while still leveled at 3.2, indicates that his thinking has deepened. He recognizes that the phrase “opposite sides” may have two different meanings, and therefore not only considers one case, but both—and is able to answer both cases based on shape construction. PST A’s work on problem 8 shows a deepening of thinking about language, specifically.

**Pre-Service Teacher B**

The most significant change for PST B from pre- to post- was his increased use of formal mathematical language and terms. For example, on the pre-assessment, PST B described the following shapes with words like “four sides” and “all the sides are equal”:
In his post-assessment, PST B described the same shapes with words like “four congruent sides” and “they are all rhombuses.” This shift in language use is a shift to more formal and precise language and perhaps even a shift in conceptualization and knowledge.

PST B became not only more formal and precise in his language use, but more thorough. For example, on his pre-assessment, PST B decided whether shapes were rectangles based on two sets of criteria: four sides, four right angles. On his post-assessment, PST B decided whether a shape was a rectangle or not based on three criteria: opposite sides congruent, opposite sides parallel, and four right angles. This definition, while not a minimal definition of a rectangle, is more precise, and is based on a larger and more specific set of criteria than his original definition.

PST B’s work on problem 8 highlights a particular limitation of this study. In his pre-assessment, PST B justified his answer to problem 8 with a proof.

If a quadrilateral has opposite sides equal and at least one right angle, then the quadrilateral is a rectangle.

Circle One: True False Prove your answer or tell why your answer is correct.
PST B claimed that the statement is true, and gave a proof using triangle congruency, based on his construction of a figure with two sets of opposite sides equal and one right angle. This response is leveled at 4: PST B gave a formal proof of the statement. On his post-assessment, however, to back up his claim that the statement is true, PST B simply constructed a shape with two pairs of opposite sides equal and four right angles, similar to this:

![Shape](image)

Because there was no accompanying proof or explanation, this response must be leveled at a 3.2, because of the use of shape construction to reason out a claim. Had the researcher been able to ask probing questions of PST B while he answered question 8, more light may have been shed on PST B’s reasoning.

**Pre-Service Teacher C**

Pre-Service Teacher C did not submit a pre-assessment, so it was not possible to discover growth in his responses. However, on his post-assessment, PST C used formal language often and employed hierarchical reasoning and formal proof to back up his claims. PST C used language such as “vertical symmetry” and “opposite sides parallel,” and explained that the following shape in question 6 is a parallelogram because it is a rectangle, and therefore meets the criteria to be a parallelogram:
He also used formal proof to show that “If a quadrilateral has opposite sides equal and at least one right angle, then the quadrilateral is a rectangle” (problem 8) and to show that all squares are rectangles (problem 9).

**Pre-Service Teacher D**

Pre-Service Teacher D also showed an increased use of formal and precise language from pre- to post-. When describing the rules for triangles in problem 4b, below;

\[
b. \text{Describe exactly how you decide if a shape is a triangle or not.}
\]

PST D used language on the pre-assessment like “three lines and three angles.” The informal word “lines” gave way to the word “sides” in the post-assessment.

PST D’s biggest jump in reasoning came in problem 8, below.

On the pre-assessment, PST D incorrectly believed that the statement was false, for reasons such as “the other two sides don’t have to be equal, and if the other three angles aren’t right angles then the statement isn’t true.” Similar to PST A, PST D believed “opposite sides equal” referred to “one pair of opposite sides equal.” However, on the post-assessment, PST D used shape
construction on problem 8 to show that the statement must be true. He mapped out the steps one must take to construct a quadrilateral with opposite sides equal and one right angle, and used this as justification that the quadrilateral must have four right angles and is therefore a rectangle. This shift in PST D’s understanding of the phrase “opposite sides equal” shows a shift from level 2.3 reasoning to level 3.2 reasoning.

Case Studies

Each of the four pre-service teachers submitted a case study. Notable findings from each case study are as follows:

Pre-Service Teacher A

PST A’s lessons which he created and administered to students were centered on three areas: formal language use, properties of shapes, and hierarchy of shapes. PST A was able to demonstrate that he moved his students forward in each of these three areas. His students went from using language such as “square-ish shapes” and “four lines” to more formal and precise language such as “all sides are equal” and “right angles.” After learning about properties of shapes in more depth in their lessons, PST A’s students seemed to look very closely at the particular properties a group of shapes have and not as much about what a group of shapes had in common. An example of this is one of PST A’s students’ answer to question 2, below. We will call her Abigail.
In her pre-assessment, Abigail claimed that the rule for the Drawing Machine was that all of the shapes have four equal sides. She indicated that shape H cannot be made by the Drawing
Machine because it does not meet that criteria, and she circled shapes J and L in part (c). On her post-assessment, Abigail’s rule in part a consisted of four different parts. She claimed that all the shapes had four sides, four angles, were all quadrilaterals and all had two sets of parallel lines. Abigail based her answers to parts (b) and (c) on this set of rules, claiming that shape H cannot be made by the Drawing Machine because it does not have two sets of parallel sides, and this time claiming that all four shapes in part c (I, J, K, and L) can be made by the Drawing Machine. The criteria she used in the post-assessment, while valid, is missing the one more unifying characteristic of all seven shapes which she recognized in her pre-assessment. After learning about specific properties of shapes in the lessons after she took the post-assessment, Abigail was more focused on attending to all the properties of shapes she could think of, rather than looking closely for a rule that could narrow down the characteristics of the group of shapes even further.

PST A also placed a significant emphasis on hierarchies of shapes in his lessons. He had his students think deeply about the properties each shape has, specifically squares and rectangles, and facilitated discussions about which shapes could be made by each other and which could not (for example, a rectangle maker can make any square, but a square maker cannot make every rectangle). The impact these discussions had on PST A’s students can be seen in his students’ answers to question 9, below.

9. Tell whether the statement in the box is true or false.
   Circle your answer.  True  False
   All squares are rectangles.
   What would you say to convince other students that your answer is correct?
On their pre-assessments, all three of PST A’s students incorrectly answered “False.” They did not have an understanding of the formal properties that make a square a rectangle, but a rectangle not always a square. However, on their post-assessments, two of his three students answered “True,” and were able to explain why all squares must be rectangles, by using properties. These two students moved up in the learning progression—one of them from a level 2.1 to a level 2.3, and one from a level 2.2 to a level 2.3.

A notable finding in PST A’s case study is his emphasis on whether his students’ answers were correct or incorrect. In his summary tables of each of his students’ levels of thinking on each question from pre- to post-, PST A included a column not just for the level of reasoning each student attained on each question, but whether or not their answer was correct. This focus on correct/incorrect answers did not hinder PST A’s ability to think deeply and accurately about his students’ reasoning, but is not something typically seen in Cognitively Based Assessment, given its emphasis on student cognition rather than accuracy. This is a significant finding which merits further investigation.

**Pre-Service Teacher B**

After leveling all three of his students’ thinking on the pre-assessment, PST B chose to place his focus on the properties of and hierarchy between squares, rectangles, rhombuses, and parallelograms. The most notable finding in PST B’s case study is his realization of the way his students were thinking about those four shapes. PST B understood after conducting the case study that his students did not think parallelograms and rhombuses could have right angles, because they were accustomed to seeing only specific types of traditional parallelograms and rhombuses (those that are not also rectangles or squares). He also explained that he believes his
students were not able to articulate all the properties of a rectangle (opposite sides equal and four right angles) because rectangles are commonly seen and recognized by students, and therefore students are not often challenged to think deeply about them or their properties.

PST B’s work with his students on the properties of these shapes and the hierarchical relationship between them allowed his students to move toward an understanding of the properties of the four quadrilaterals and how they are interrelated. On question 9 (below), each of PST B’s students moved up in their reasoning at least two sublevels, and were all able to interrelate the properties of squares and rectangles to explain why all squares are rectangles.

![Image of question 9](image)

**Pre-Service Teacher C**

The most notable finding in PST C’s case study work was his ability to think critically not only about his students’ responses to the questions and their levels of reasoning, but about the questions on the assessment themselves and the ways in which they elicited student thinking. This is specifically illustrated in PST C’s descriptions of his students’ responses to question 2, below.
PST B analyzed his students’ responses to this question deeply. None of his students, on the pre-assessment or the post-assessment, described the “rule” for part a as “all four sides equal” or “all
rhombuses” or something similar. Instead, his students’ “rules” consisted of properties like “opposite sides equal,” “opposite sides parallel,” and “opposite angles equal.” These properties are all valid, but not conventional in formal mathematics because they are incomplete. PST B explains that he believes shape H in part (b) (a kite) was an interesting choice, because it does not guide students to see the one unifying property (equal sides) that shapes A-G have. He claimed that his students’ rules could have been more specific and included the property of four sides equal, but the kite in part (b) did not challenge any of his students’ rules in part (a). If the shape in part (b) that cannot be made by the Drawing Machine had looked like either of the two shapes below, his students’ “rules” of “opposite sides equal,” “opposite sides parallel,” and “opposite angles equal” would have been challenged.

If the Drawing Machine could not make either of the two shapes above, the rule for the Drawing Machine would have to be something more specific.

**Pre-Service Teacher D**

PST D’s case study showed an emphasis in two major areas: (1) his expectation that his students use an exhaustive list of properties of shapes to back up their claims, and (2) his use of “correct or incorrect” to categorize student responses, similar to PST A. The first area of expecting students to name every property of a shape to explain their reasoning is illustrated by PST D’s explanation of one of her student’s reasoning on problem 6a below. We will call this student Matthew.
PST D explains that Matthew answered “False” because all the lines are not the same length. PST D indicates that Matthew is correct, but forgets to mention angle measures in his answer, and his thinking merits a level 2.1. Matthew’s reasoning in this sub-question involves the use of a formal property, side length, to determine whether a shape is a square. The researcher levels this type of reasoning at a level 2.2. Though Matthew’s explanation, while informal, was sufficient to explain why the shape is not a square, PST D made mention of the fact that Matthew did not use angle measurements in his reasoning. This pattern was common to PST D’s explanations for why he leveled student reasoning the way he did—he made frequent mention of properties of shapes that his students did not use in their responses on the pre- and post-assessment, even when students’ responses were sufficient to answer the question. Mentioning these details in his students’ pre-assessments is valid because it was not clear to PST D that his students (e.g. Matthew) had complete knowledge of shapes or were giving the sufficient, minimal ideas intentionally. With more questioning during the pre-assessment, PST D may have clarified this gap in his knowledge of his students’ cognition.
However, on PST D’s students’ post assessments, PST D appears to have asked more probing questions about these extraneous properties. In Matthew’s response to question 6a on the post-assessment, PST D explains that Matthew answered “False” and gave an explanation very similar to his explanation on the pre-assessment—not all sides are the same length. PST D goes on to explain that he asks Matthew about the angles, to which Matthew responds that the angles are all 90° and do not make a difference in this problem. PST D indicates that Matthew’s reasoning is correct and merits a level 2.3. It appears that the acknowledgement of angle measures on problem 6a caused Matthew’s reasoning to go from a level 2.1 to a level 2.3 in PST D’s eyes. Had PST D asked Matthew about the angles on the pre-assessment, perhaps he would have leveled Matthew’s thinking on this question of the pre-assessment at a level 2.3 as well. Though the acknowledgement of angles in problem 6c was not necessary to answer the prompt and therefore PST D leveled Matthew’s thinking on the pre-assessment inaccurately, it appears that PST D was able to extract more information about his students’ knowledge about the properties of shapes on the post-assessment than on the pre-assessment.

As seen in the examples of Matthew’s responses to question 6a on the pre-assessment and the post-assessment, indications of the whether the answers were correct or not were present in the majority of PST D’s descriptions of his students’ responses on the assessment. Similar to PST A, correctness seemed to be a critical factor in determining levels of student reasoning for PST D, though it did not appear to hinder PST D’s ability to think critically about his students’ cognition.

Limitations
The present study is a qualitative study conducted with a small number of participants, and therefore the findings are not generalizable. The researcher was unable to corroborate the findings and analysis with the participants in the study, and therefore was inferring pre-service teachers’ learning, instructional practices, and dispositions from only written data sources. Leveling of pre-service teacher geometry content knowledge assessments was done only from written data sources, and it was therefore not possible to use cognitively based assessment methods such as questioning or gestures in order to determine pre-service teachers’ levels of reasoning. It may be useful in future studies to have participants (pre-service teachers) analyze their own data, reflect on their own learning gains, and explain what they believe their work shows about their learning.

Conclusions and Implications

It is clear from the survey responses that the pre-service teachers’ pedagogical dispositions were altered from the pre-survey to the post-survey. After conducting the case study, the pre-service teachers knew that they needed more experience with geometry—the content, methods of teaching it, and specific technologies to aid in instruction. The case study assignment allowed the pre-service teachers to gain some first experiences with teaching geometry and thinking about student cognition, and as a result, pre-service teachers felt more prepared to teach geometry. At the same time, they grew more aware of how much more they needed to learn to be able to more effectively teach geometry.

From the pre-service teachers’ own geometry content knowledge assessments, it is clear that pre-service teachers’ geometry content knowledge levels also changed from pre- to post-. All pre-service teachers showed an increased knowledge of content-specific language and its
particulars in some way from their pre-assessments to their post-assessments. Further research should be conducted to determine the effects of pre-service teachers’ increasingly formal language use on student learning.

In addition to pre-service teachers’ geometry content knowledge assessments, their written case study assignments also suggest that the pre-service teachers were able to think critically about not only geometry-specific language as it relates to geometry content, but also as it relates to geometry-specific questions being asked of students. Pre-service teachers showed deep thinking about the phrasing of questions on the geometry content knowledge assessment by their discussions of the ways in which the phrasing of questions may affect student responses. Further research should focus on pre-service teachers’ abilities to critically evaluate geometry-specific questions posed to geometry students in Cognitively Based Assessments.

The pre-service teachers identified their students’ levels of reasoning with nearly completely consistent accuracy. As reported by pre-service teachers and confirmed by the researcher, student geometry content knowledge did increase as a result of pre-service teacher instruction from the case study. Though pre-service teachers were able to think about student cognition critically and deeply, the labels of “correct” and “incorrect” appeared in pre-service teachers’ explanations of student reasoning frequently. While not in itself problematic, this mentality of “correct/incorrect” without any deeper thinking about student cognition is not desirable for cognition-based frameworks. In Cognitively Based Assessment, the focus must be on student reasoning and its complexities in order to move the student forward, rather than simply identifying whether or not student reasoning is “correct” or “incorrect”. The present study is unable to determine the effects of these labels on pre-service teachers’ ability to level student reasoning or increase student knowledge. Further research specifically aimed at
understanding the terminology of “correct/incorrect” may give insight into how teachers use the CBA appropriately.

There are many implications of the results of this study for pre-service teacher preparation especially. There is debate among researchers that pre-service teachers are unable to handle such mature thinking about student cognition as the case study assignment required. However, though not generalizable to any wider population, the results of the study suggest that it was effective to teach cognition-based frameworks and learning progressions to pre-service teachers because it positively affected their teaching effectiveness and increased their own geometry content knowledge. Teaching learning progressions to the pre-service teachers and having them work to understand student cognition in ways the case study assignment required seemed to also positively impact the pre-service teachers’ dispositions towards geometry and especially towards teaching geometry.

Suggestions for Future Research

For future studies, larger sample sizes ought to be used to study the effects of CGI and CBA on pre-service teachers’ learning, instruction, and dispositions in a wider manner. In a more widespread study, it would also be possible to look quantitatively at the significance of learning gains made by both pre-service teachers and their students as a result of assignments meant to teach pre-service teachers to effectively use learning progressions. Research ought also to be done on the methods of teaching learning progressions and cognition-based frameworks to pre-service teachers in order to determine which methods are most effective.

Reflection
Learning to teach is a complex task which requires ongoing reflection and exploration. As a participant in the study as well as a researcher, I have increased my own geometry content knowledge, dispositions, and knowledge of geometry-specific instructional strategies by conducting regular reflection on the data I worked with. The cognition-based frameworks for teaching geometry are complex and difficult to grasp, but finding that the pre-service teachers studied (myself included) were able to understand and use them effectively to further student learning is promising for the future of the reform movement in mathematics education. Learning that pre-service teachers’ attitudes towards teaching geometry and feelings of preparedness to teach geometry changed positively after conducting the case study assignment was also encouraging. Studying cognition-based frameworks through many layers of data has equipped me with the tools to be able to maintain an awareness of students’ varying levels of reasoning as well as the strategies to use to move students’ levels of reasoning forward.
References


Retrieved February 17, 2016, from http://web.b.ebscohost.com/ehost/pdfviewer/pdfviewer?vid=28&sid=b4128281-1f44-4e52-a45f-10ceb3a93b3b@sessionmgr110&hid=125


Appendix A

Outline of Study Methodology

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<tr>
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<th>A</th>
<th>B</th>
<th>C</th>
</tr>
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<td></td>
<td>Researcher</td>
<td>Course Instructor</td>
<td>Pre-Service Teachers (PSTs)</td>
</tr>
<tr>
<td>1</td>
<td>Administer Geometry Content Knowledge Assessment</td>
<td>Complete Geometry Content Knowledge Assessment</td>
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<td></td>
<td>and Mathematical Disposition Survey at beginning</td>
<td>and Mathematical Disposition Survey at beginning</td>
<td></td>
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<td></td>
<td>of course (Pre) and at end of course (Post)</td>
<td>of course (Pre) and at end of course (Post)</td>
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<tr>
<td>2</td>
<td>Provide course instruction</td>
<td>Conduct Case Study Assignment</td>
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<tr>
<td>3</td>
<td>Grade Case Study Assignment, submit final grades</td>
<td>Receive final grades for the course</td>
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<td></td>
<td>for the course</td>
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<tr>
<td>4</td>
<td>Receive IRB approval to begin Research Project</td>
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<tr>
<td>5</td>
<td>Obtain participant (PST) and Course Instructor</td>
<td>Give consent to participate in the study</td>
<td>Give consent to participate in the study</td>
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<td>Consent</td>
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<tr>
<td>6</td>
<td>Collect de-identified data sources from Course</td>
<td>De-identify all data sources and share with</td>
<td></td>
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<tr>
<td></td>
<td>Instructor</td>
<td>researcher</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Analyze all data collected from data sources</td>
<td></td>
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</tbody>
</table>
Appendix B

Geometry Content Knowledge Assessment

1. Drawing Machine 1 made these shapes. All shapes that can be made by Drawing Machine 1 are alike in some way. They follow a rule.

   A
   B
   C

   D
   E

   a. Describe how the shapes made by Drawing Machine 1 are alike. Tell what the rule is for this drawing machine.

   b. Drawing Machine 1 cannot make this shape. Why not?

   F

   c. Circle the shapes that Drawing Machine 1 can make. For each shape, tell why Drawing Machine 1 can or cannot make it.

   G
   H
   I
   J
2. Drawing Machine 2 made these shapes. All shapes that can be made by Drawing Machine 2 are alike in some way. They follow a rule.

   A   B

   C   D   E

   F   G

a. Describe how the shapes made by Drawing Machine 2 are alike. Tell what the rule is for this drawing machine.

b. Drawing Machine 2 cannot make this shape. Why not?

   H

c. Circle the shapes that Drawing Machine 2 can make. For each shape, tell why Drawing Machine 2 can or cannot make it.

   I   J   K   L
3. Drawing Machine 3 made these shapes. All shapes that can be made by Drawing Machine 3 are alike in some way. They follow a rule.

![Diagram of shapes A to I]

a. Describe how the shapes made by Drawing Machine 3 are alike. Tell what the rule is for this drawing machine.

b. Drawing Machine 3 cannot make these shapes. Why not?

![Diagram of shapes J, K, L]

c. Circle the shapes that Drawing Machine 3 can make. For each shape, tell why Drawing Machine 3 can or cannot make it.

![Diagram of shapes M, N, O, P]
4. a. Circle each triangle.

b. Describe exactly how you decide if a shape is a triangle or not.

c. Is Shape d a triangle? Explain why.

d. Describe everything you know about triangles.
5. a. Circle each rectangle.

b. Describe exactly how you decide if a shape is a rectangle or not.

c. Is Shape c a rectangle? Explain why.

d. Describe everything you know about rectangles.
6. The measurements for a shape are given below.

For each statement about this shape, circle T if the statement is True, or F if the statement is false. If you can't tell if the statement is true or false, circle Can't tell.

For each statement, describe how you would convince someone that your answer is correct.

a. The shape is a square.

b. The shape is a rectangle.

c. The shape is a parallelogram.

d. The shape is a rhombus.
7. The measurements for a shape are given below.

![Shape Diagram]

For each statement about this shape, circle T if the statement is True, or F if the statement is False. If you can’t tell if the statement is true or false, circle Can’t tell.

For each statement, describe how you would convince someone that your answer is correct.

a. The shape is a square.  
   T  F  Can’t tell

b. The shape is a rectangle.  
   T  F  Can’t tell

c. The shape is a parallelogram.  
   T  F  Can’t tell

8. A quadrilateral is a closed shape with four straight sides. Examples of quadrilaterals are squares, rectangles, rhombuses, parallelograms, kites, and trapezoids.

Circle True if the statement is True, or False if the statement is False. Describe how you would prove or show that your answer is correct.

If a quadrilateral has opposite sides equal and at least one right angle, then the quadrilateral is a rectangle.

Circle One: True  False  Prove your answer or tell why your answer is correct.
9. Tell whether the statement in the box is true or false.

Circle your answer.  True  False

All squares are rectangles.

What would you say to convince other students that your answer is correct?
Appendix C

Mathematical Disposition Survey

Part 1:

1. What content area/course in secondary mathematics is your personal favorite?
   - Algebra 1
   - Geometry
   - Algebra 2
   - Trigonometry
   - Pre-Calculus
   - Calculus
   - Statistics
   - Other (please specify)

2. Explain your choice.

3. What content area/course in secondary mathematics is your least favorite?
   - Algebra 1
   - Geometry
   - Algebra 2
   - Trigonometry
   - Pre-Calculus
   - Calculus
   - Statistics
   - Other (please specify)

4. Explain your choice.

5. What content area/course(s) in secondary mathematics do you feel most prepared to teach?
   - Algebra 1
   - Geometry
THE EFFECTS OF COGNITIVELY GUIDED INSTRUCTION

☐ Algebra 2
☐ Trigonometry
☐ Pre-Calculus
☐ Calculus
☐ Statistics
Other (please specify)

*6. Explain your choice.

*7. What content area/course(s) in secondary mathematics do you feel least prepared to teach?
☐ Algebra 1
☐ Geometry
☐ Algebra 2
☐ Trigonometry
☐ Pre-Calculus
☐ Calculus
☐ Statistics
Other (please specify)

*8. Explain your choice.

*9. What content area/course(s) in secondary mathematics do you think students struggle with the most?
☐ Algebra 1
☐ Geometry
☐ Algebra 2
☐ Trigonometry
☐ Pre-Calculus
☐ Calculus
☐ Statistics
Part 2:

1. For geometry in particular, you:
   - [ ] Really like it. Can’t wait to teach it.
   - [ ] Really like it. Don’t want to teach it.
   - [ ] Struggle with it, but will teach it if necessary.
   - [ ] Struggle with it, but will avoid teaching it.
   - [ ] Dislike it, but it makes sense and is easy.
   - [ ] Dislike it. It’s confusing and difficult to understand.
   - [ ] Other (please specify)

2. Explain your choice.

3. Geometry is ______ pertinent to the real work than other areas of mathematics.
   - [ ] Less
   - [ ] The same
   - [ ] More
   - [ ] Other (please specify)

4. Explain your choice.

5. How prepared do you feel to teach geometry?
   - [ ] Not at all prepared.
   - [ ] Somewhat prepared.
   - [ ] Fairly well prepared.
   - [ ] Completely prepared.
   - [ ] Other (please specify)
THE EFFECTS OF COGNITIVELY GUIDED INSTRUCTION

6. Explain your choice.

7. What do you know about US students' performance in geometry specifically as compared to other areas of mathematics?

8. What do you know about Learning Progressions in mathematics in general?

9. What do you know about Learning Progression in geometry specifically?

10. What experiences do you have with dynamic geometry software?

Part 3:

1. What experiences do you have with teaching geometry?

2. What are your experiences as a student in geometry? (talk about middle school/junior high, high school, and college) What did you like? What didn't you like? Was there anything unique about the pedagogy?

3. How prepared do you feel to differentiate your instruction to meet the needs of ALL students?
   - Not at all prepared.
   - Somewhat prepared.
   - Fairly well prepared.
   - Completely prepared.
   - Other (please specify)

4. Explain your choice.

5. How prepared do you feel to teach students with special needs (e.g., IEP)?
   - Not at all prepared.
   - Somewhat prepared.
   - Fairly well prepared.
   - Completely prepared.
   - Other (please specify)
6. Explain your choice.

7. How prepared do you feel to teach ELL students?
   - Not at all prepared.
   - Somewhat prepared.
   - Fairly well prepared.
   - Completely prepared.
   - Other (please specify)

8. Explain your choice.

9. How prepared do you feel to teach students with exceptionalities (e.g., gifted)?
   - Not at all prepared.
   - Somewhat prepared.
   - Fairly well prepared.
   - Completely prepared.
   - Other (please specify)

10. Explain your choice.
Appendix D

Case Study Assignment Rubric and Description

<table>
<thead>
<tr>
<th>CATEGORY</th>
<th>Exceeds Expectation</th>
<th>Meets Expectation</th>
<th>Below Expectation</th>
<th>Unacceptable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length and APA</td>
<td>More than 10 pages. Follows APA formatting. All appropriate materials are cited and in appendices.</td>
<td>10 pages &amp; follows APA formatting. All appropriate materials are cited and in appendices.</td>
<td>Below 5 and 9 pages OR loosely followed APA format.</td>
<td>Below 5 pages OR did not follow APA format.</td>
</tr>
<tr>
<td>Student Background</td>
<td>Includes personal interests, abilities, age, grades, mathematical disposition.</td>
<td>Does not include one or two of the &quot;meets expectation&quot; category items.</td>
<td>Does not include 3 or more of the &quot;meets expectation&quot; category items.</td>
<td></td>
</tr>
<tr>
<td>Relationship to Geometry Learning</td>
<td>Thoroughly explained where the student should be in the learning progression relative to CCSSM and curriculum in the school district.</td>
<td>Curiously explained where the student should be in the learning progression relative to CCSSM and curriculum in the school district.</td>
<td>Did not completely explain where each student should be in the learning progression relative to CCSSM and curriculum in the school district.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Includes pre-assessment as appendix.</td>
<td>Correctly identifies student reasoning from pre-assessment data AND justifies with examples from student work and transcription.</td>
<td>Does not include every pre-assessment or incomplete pre-assessments.</td>
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<td>-------------------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------</td>
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<tr>
<td>Pre-Assessment</td>
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<td></td>
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</tr>
<tr>
<td>Student Reasoning on Pre-Assessment</td>
<td>Correctly identifies student reasoning from pre-assessment data but lacks convincing proof from examples of student work and transcriptions.</td>
<td>Incorrectly identifies student reasoning or does not use LP to identify reasoning.</td>
<td></td>
<td></td>
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<tr>
<td>Lesson Plans</td>
<td>Fully completes the equivalent of 3 ODU formatted lesson plans, include as appendix, and explains lesson plan goals appropriate to case study.</td>
<td>Fully completes the equivalent of 3 ODU formatted lesson plans, include as appendix, but lacks explanations of lesson plan goals appropriate to case study.</td>
<td>Incomplete lesson plan(s).</td>
<td></td>
</tr>
<tr>
<td>Student Interaction with lessons</td>
<td>Describes student learning from lesson activities, shows evidence (quotes, student work, etc) of teaching and learning.</td>
<td>Describes student learning from lesson activities but lacks evidence (quotes, student work, etc) of teaching and learning.</td>
<td>Does not describe student learning.</td>
<td></td>
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<tr>
<td>Post-Assessment</td>
<td>Includes post-assessment as appendix.</td>
<td></td>
<td>Does not include postassessment or incomplete post-assessments.</td>
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</table>
Field Experience Case Study

**Case Study:** "a process or record of research in which detailed consideration is given to the development of a particular person, group, or situation over a period of time." For your Case Study project for this class, you will work with at least one student, identifying their level of reasoning for a particular mathematical content strand, plan and teach them a few lessons, and then re-evaluate their reasoning to see if your teaching was effective. You will utilize Battista’s (2007, 2012) Learning Progression for Shape.

**Process:**

1. You, with input from your mentor teacher, will select at least three students to work with. If at all possible, 1 student should have an IEP in mathematics and 1 student should be classified as an ELI. Otherwise, the students do not have to have any particular abilities, but I suggest selecting students who will be easy to talk to and who are willing to communicate. Get permission from the student to work with them one-on-one, and then contact the parent for permission. You may not need permission from the parent to work...
with the student, but you for sure need permission to audio/video record the student. Gain written permission. You will not share the videos of the students with anyone other than your classmates and instructor.

2. After you have selected your students, you need to work with your CT to determine how and when you'll conduct pre and post interviews and implement lessons. It would be awesome to align the lessons to the whole class, but you may work with just your student, students as a small group, etc.

3. Use Battista’s pre-assessment. Give the pre-assessment to each student one-on-one. Audio and video record this assessment.

4. Evaluate your pre-assessment data to “level” each student’s reasoning. Create a learning goal and design a few lessons to give to the student that will move from the current level to a more sophisticated level.

5. Use Battista’s post-assessment. Give the post-assessment to each student one-on-one. Audio and video record this assessment. Some of the items from your pre-assessment should be on the post (those will directly show lots of growth). Some new items that prove your student is at a more sophisticated level of reasoning should also be included.

6. Collect all work the students complete for the pre-assessment, lessons, and post-assessment.

7. Prepare your written Case Study and your presentation. See components of each below.

**Products:**

**Written**

- Minimum 10-page paper (but most are significantly longer)
- Discuss background of student (personal, interests, abilities, age, grade, mathematical dispositions)
- Explain where the student should be in their knowledge of geometry relevant to the learning progression
- Include pre-assessment (as Appendix)
- Discuss student reasoning on pre-assessment, include LP level and justification with specific quotes, figures of student work
- Explain lesson plan goals, number of lesson plans, when and how they were taught, etc., include all Lesson Plans as appendices
- Talk about student interaction with lessons and content, what changes occurred? What did you observe? What did the student struggle with? What new things did you add to your lessons that you hadn’t planned for? What was effective? Ineffective?
- Include post-assessment (as Appendix)
- Discuss student reasoning on post-assessment, include LP level and justification with specific quotes, figures of student work
- Final discussion should include overall discussion of student growth and plans for future learning. What should instruction focus on now for continued growth? What did the student learn? What did you learn?

- Consider each of your students. What similarities and differences did you observe? What can you attribute those to if anything?
- Follow APA guidelines, include citations and sources, incorporate research on student learning, CGI, CBA, and LPs
## Appendix E

Battista’s (2012) Learning Progression for Geometric Shapes

<table>
<thead>
<tr>
<th>Level</th>
<th>Sublevel</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td><strong>Student identifies shapes as visual wholes.</strong></td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>Student incorrectly identifies shapes as visual wholes.</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>Student correctly identifies shapes as visual wholes.</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td><strong>Student describes parts and properties of shapes.</strong></td>
</tr>
<tr>
<td></td>
<td>2.1</td>
<td>Student informally describes parts and properties of shapes.</td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td>Student uses informal and insufficient formal descriptions of shapes.</td>
</tr>
<tr>
<td></td>
<td>2.3</td>
<td>Student formally describes parts and properties of shapes.</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td><strong>Student interrelates properties and categories of shapes.</strong></td>
</tr>
<tr>
<td></td>
<td>3.1</td>
<td>Student uses empirical evidence to interrelate properties and categories of shapes.</td>
</tr>
<tr>
<td></td>
<td>3.2</td>
<td>Student analyzes shape construction to interrelate properties and categories of shapes.</td>
</tr>
<tr>
<td></td>
<td>3.3</td>
<td>Student uses logical inference to relate properties and understand minimal definitions.</td>
</tr>
<tr>
<td></td>
<td>3.4</td>
<td>Student understands and adopts hierarchical classifications of shape classes.</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td><strong>Student understands and creates formal deductive proofs.</strong></td>
</tr>
</tbody>
</table>
Appendix F
Partial Summary of Survey Results

<table>
<thead>
<tr>
<th>Pre-Survey Results</th>
<th>Post-Survey Results</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PART 1</strong></td>
<td><strong>PART 1</strong></td>
</tr>
<tr>
<td>What content area/course in secondary mathematics is your personal favorite?</td>
<td>What content area/course in secondary mathematics is your personal favorite?</td>
</tr>
<tr>
<td>Algebra 1</td>
<td>1</td>
</tr>
<tr>
<td>Geometry</td>
<td>0</td>
</tr>
<tr>
<td>Algebra 2</td>
<td>1</td>
</tr>
<tr>
<td>Trigonometry</td>
<td>0</td>
</tr>
<tr>
<td>Pre-Calculus</td>
<td>0</td>
</tr>
<tr>
<td>Calculus</td>
<td>2</td>
</tr>
<tr>
<td>Statistics</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>What content area/course in secondary mathematics is your least favorite?</td>
<td>What content area/course in secondary mathematics is your least favorite?</td>
</tr>
<tr>
<td>Algebra 1</td>
<td>0</td>
</tr>
<tr>
<td>Geometry</td>
<td>1</td>
</tr>
<tr>
<td>Algebra 2</td>
<td>0</td>
</tr>
<tr>
<td>Trigonometry</td>
<td>0</td>
</tr>
<tr>
<td>Pre-Calculus</td>
<td>1</td>
</tr>
<tr>
<td>Calculus</td>
<td>0</td>
</tr>
<tr>
<td>Statistics</td>
<td>2</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>What content area/course(s) in secondary mathematics do you feel most prepared to teach?</td>
<td>What content area/course(s) in secondary mathematics do you feel most prepared to teach?</td>
</tr>
<tr>
<td>Algebra 1</td>
<td>3</td>
</tr>
<tr>
<td>Geometry</td>
<td>1</td>
</tr>
<tr>
<td>Algebra 2</td>
<td>3</td>
</tr>
<tr>
<td>Trigonometry</td>
<td>0</td>
</tr>
<tr>
<td>Pre-Calculus</td>
<td>1</td>
</tr>
<tr>
<td>Calculus</td>
<td>0</td>
</tr>
<tr>
<td>----------</td>
<td>---</td>
</tr>
<tr>
<td>Statistics</td>
<td>0</td>
</tr>
</tbody>
</table>

### What content area/course(s) in secondary mathematics do you feel least prepared to teach?

<table>
<thead>
<tr>
<th>Algebra 1</th>
<th>0</th>
<th>Algebra 1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td>1</td>
<td>Geometry</td>
<td>0</td>
</tr>
<tr>
<td>Algebra 2</td>
<td>0</td>
<td>Algebra 2</td>
<td>0</td>
</tr>
<tr>
<td>Trigonometry</td>
<td>0</td>
<td>Trigonometry</td>
<td>0</td>
</tr>
<tr>
<td>Pre-Calculus</td>
<td>1</td>
<td>Pre-Calculus</td>
<td>1</td>
</tr>
<tr>
<td>Calculus</td>
<td>1</td>
<td>Calculus</td>
<td>1</td>
</tr>
<tr>
<td>Statistics</td>
<td>4</td>
<td>Statistics</td>
<td>4</td>
</tr>
</tbody>
</table>

### What content area/course(s) in secondary mathematics do you think students struggle with the most?

<table>
<thead>
<tr>
<th>Algebra 1</th>
<th>1</th>
<th>Algebra 1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td>3</td>
<td>Geometry</td>
<td>1</td>
</tr>
<tr>
<td>Algebra 2</td>
<td>1</td>
<td>Algebra 2</td>
<td>0</td>
</tr>
<tr>
<td>Trigonometry</td>
<td>0</td>
<td>Trigonometry</td>
<td>1</td>
</tr>
<tr>
<td>Pre-Calculus</td>
<td>1</td>
<td>Pre-Calculus</td>
<td>1</td>
</tr>
<tr>
<td>Calculus</td>
<td>2</td>
<td>Calculus</td>
<td>3</td>
</tr>
<tr>
<td>Statistics</td>
<td>0</td>
<td>Statistics</td>
<td>3</td>
</tr>
</tbody>
</table>

### PART 2

**For geometry in particular, you:**

<table>
<thead>
<tr>
<th>Really like it. Can't wait to teach it.</th>
<th>1</th>
<th>Really like it. Can't wait to teach it.</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Really like it. Don't want to teach it.</td>
<td>0</td>
<td>Really like it. Don't want to teach it.</td>
<td>0</td>
</tr>
<tr>
<td>Struggle with it, but will teach it if necessary.</td>
<td>0</td>
<td>Struggle with it, but will teach it if necessary.</td>
<td>1</td>
</tr>
<tr>
<td>Struggle with it, but will avoid teaching it.</td>
<td>0</td>
<td>Struggle with it, but will avoid teaching it.</td>
<td>0</td>
</tr>
<tr>
<td>Dislike it, but it makes sense and is easy.</td>
<td>0</td>
<td>Dislike it, but it makes sense and is easy.</td>
<td>0</td>
</tr>
<tr>
<td>Dislike it. It's confusing and difficult to understand.</td>
<td>0</td>
<td>Dislike it. It's confusing and difficult to understand.</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>3</td>
<td>Other</td>
<td>1</td>
</tr>
<tr>
<td>Geometry is ______ pertinent to the real work than other areas of mathematics.</td>
<td>Geometry is ______ pertinent to the real work than other areas of mathematics.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less</td>
<td>0</td>
<td>Less</td>
<td>0</td>
</tr>
<tr>
<td>The same</td>
<td>4</td>
<td>The same</td>
<td>3</td>
</tr>
<tr>
<td>More</td>
<td>0</td>
<td>More</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
<td>Other</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>How prepared do you feel to teach geometry?</th>
<th>How prepared do you feel to teach geometry?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not at all prepared.</td>
<td>1</td>
</tr>
<tr>
<td>Somewhat prepared.</td>
<td>3</td>
</tr>
<tr>
<td>Fairly well prepared.</td>
<td>0</td>
</tr>
<tr>
<td>Completely prepared.</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PART 3</th>
<th>PART 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>How prepared do you feel to differentiate your instruction to meet the needs of ALL students?</td>
<td>How prepared do you feel to differentiate your instruction to meet the needs of ALL students?</td>
</tr>
<tr>
<td>Not at all prepared.</td>
<td>1</td>
</tr>
<tr>
<td>Somewhat prepared.</td>
<td>2</td>
</tr>
<tr>
<td>Fairly well prepared.</td>
<td>1</td>
</tr>
<tr>
<td>Completely prepared.</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>How prepared do you feel to teach students with special needs (e.g., IEP)?</th>
<th>How prepared do you feel to teach students with special needs (e.g., IEP)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not at all prepared.</td>
<td>2</td>
</tr>
<tr>
<td>Somewhat prepared.</td>
<td>2</td>
</tr>
<tr>
<td>Fairly well prepared.</td>
<td>0</td>
</tr>
<tr>
<td>Completely prepared.</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>How prepared do you feel to teach ELL students?</th>
<th>How prepared do you feel to teach ELL students?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not at all prepared.</td>
<td>1</td>
</tr>
<tr>
<td>How prepared do you feel to teach students with exceptionalities (e.g., gifted)?</td>
<td>How prepared do you feel to teach students with exceptionalities (e.g., gifted)?</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Not at all prepared.</td>
<td>Not at all prepared.</td>
</tr>
<tr>
<td>Somewhat prepared.</td>
<td>Somewhat prepared.</td>
</tr>
<tr>
<td>Fairly well prepared.</td>
<td>Fairly well prepared.</td>
</tr>
<tr>
<td>Completely prepared.</td>
<td>Completely prepared.</td>
</tr>
<tr>
<td>Other</td>
<td>Other</td>
</tr>
</tbody>
</table>

| Somewhat prepared. | 3 | Somewhat prepared. | 3 |
| Fairly well prepared. | 0 | Fairly well prepared. | 0 |
| Completely prepared. | 0 | Completely prepared. | 0 |
| Other | 0 | Other | 0 |