Heterogeneous Gain Forecasting Using Historic Asset Information*

Nicolas Sippl-Swezey
Oberlin College Economics Honors Seminar 2010-2011

Abstract

Using historic return inputs in a stylized computational financial market, this paper explores how participant outcomes are affected by the degree to which their asset allocation behavior responds to new market information. Findings support the efficient market hypothesis in that no alternate trading rule shows consistent improved outcomes relative to a full market exposure buy-and-hold strategy over the given time period. The only exception occurs briefly at the bottom of the 2008 financial crisis. Market participants that drastically alter market exposure in response to volatile returns, however, do outperform those who alter their exposure less drastically. Furthermore, the trading rules used here appear to offer a tradeoff between risk, return volatility and wealth development that is in-part at odds with efficient market hypothesis.

* I’m very grateful for the support of my readers in the Oberlin Economics Department and would like to thank Ellis Tallman, Tobias Pfutze, Edward McKelvey and Alberto Ortiz for their critical contributions to my work. I would like to thank Richard Salter and Alexander Conway in the Oberlin Computer Science Department for their continued support in expanding and developing the NOVA modeling platform throughout this project.
Table of Contents
1. Introduction

2. Exploring a New Model Platform NOVA

3. Model Description
   3.1 Sequencing
   3.2 Assets
   3.3 Preferences
   3.4 Constant Variance Forecasts
   3.5 Expected Return Forecasts
   3.6 The Gain Parameter
   3.7 Initial Conditions

4. Results

5. Discussion
1. Introduction

Market participants are capable of determining market structure and market outcomes. In the case of financial markets, simulated financial market models have shown that participants’ perceptions of risk are important in generating artificial price, dividend and return series that replicate long range samples of US financial data (LeBaron, 2010). Furthermore, artificial markets consisting of participants that place large amounts of weight on recent past information, using forecasting rules applied in this paper, exhibit greater market volatility than those that spread weight across broader time series samples (LeBaron, 2010).

In this financial model, individuals allocate wealth between a risky asset with a stochastic return, and a safe asset with a known return. The risky asset’s return, price and dividend are inputted exogenously and follow historic S&P 500 monthly data between 1980 and 2010. Using a simple learning model based on adaptive expectations, market participants forecast the return of the risky asset, and use that forecast to determine a portfolio allocation for each time period. A unique gain parameter, which serves as a forecasting time horizon, is used to determine expected return. This leads to heterogeneous wealth allocation decisions, portfolio return outcomes and wealth histories over time.

Gain parameters are the integral component to participants forecasting models. A gain parameter determines the weight with which recent asset history affects agents forecasted expected returns, in turn affecting portfolio allocations. This might be interpreted as the backwards-looking time horizon that agents use in their future forecasting. In this paper, the risky and riskless asset return series are the information to which participants respond. A high-gain agent might be said to interpret returns as they are in the short run, while a low-gain agent draws its forecast based on longer run returns history. Another interpretation could be that high-gain agents are quicker to forget the past, something that low-gain agents might never forget. Low gain parameters correspond to an allocation that responds to new information, but is slow to change in both upward and downward
directions.

Embedded in gain learning are questions regarding the stationary or non-stationary nature of returns. A gain parameter could be described as the time window in which a market participant believes economic conditions to be unique. Where one might believe that conditions determining returns are consistent only over a five-year period, another might forecast solely on history of a one-year period, and another over a history of 30 years. Agents using an asset history window that best fits the underlying nature of returns are expected to have the most successful allocation strategy.

Gain parameters are also a known as a form of filter rule, or mechanical trading rule (Fama 1966), where asset information serves as an input into a system of functions which leads to a change in trading behavior. The original intent of filter rules was to increase expected future returns over buy-and-hold strategies. Filter rules were first studied at length in the mid 60’s as part of the development and support of random-walk theory for securities prices. The random-walk hypothesis, though simultaneously observed by many economists, was made famous by Eugene Fama’s 1965 “The Behavior of Stock Market Prices.” By 1965, the independence of price movements—arguably the integral assumption of the random walk hypothesis—had been upheld using standard serial correlation tests in Cootner 1962, Fama 1965, and Kendall 1953, where the coefficients of serial correlation in successive daily, weekly and monthly securities price series were extremely close to zero. Today’s prices, therefore, could not be used to any consistent effect to predict tomorrow’s.

The argument was further extended to test whether filtering mechanisms designed to uncover more complex correlations in market information could be used to gain returns-improving advantages in securities market trading. Though serial correlation was non-existent in price series, more complex trends in asset prices were suspected to exist. The counter hypothesis, consistent with the random-walk hypothesis, was that using historical price information inputs to make future returns-improving investing decisions would be, at most, marginally more successful than a “buy-and-hold” strategy.
This was the argument in Fama 1966, which tested the effectiveness of a filter rule somewhat more complex than the one used in this paper, proposed by Alexander 1961 and Alexander 1964. Initial findings by Alexander demonstrated real gains and improvements in returns. Fama, however, rigorously refuted the effectiveness of the Alexander rule finding that after correcting for the structural and institutional realities of securities trading including commissions, realistic purchase prices and details regarding the real-life structure of financial transaction, only in certain circumstances did the filter rule perform better than a buy-and-hold strategy, and even then only marginally so.²

Though this paper does not present the findings of an agent-based computation model, the model used here borrows much of its structure from the stylized agent-based financial market in LeBaron. Much in the same way as Fama scrutinized the filter rules in Alexander’s work, this paper explores the behavior of one filter rule that in part contributes to the findings of the LeBaron model.

This scrutiny is valuable in a few different ways. If in response to historic US financial data, the filter rule does not behave in a way that is congruent with random-walk theory, then one could conclude that the LeBaron results may not be based on an agent framework the conforms well with reality. Conversely, results that confirm random-walk theory and efficient market hypothesis using historic data, will strengthen the support for those findings. The LeBaron agent-based market does a good job of replicating the historic price and dividend features of the S&P 500 with the exception of what appears to be a divergence that emerges between 1980 and 2010. In the same way that Fama 1966 was able to use the Alexander filter to explore important divergences from random-walk theory, the findings of this paper seek to discover similarly minute divergences from what would be expected in a random-walk market. The cause of this divergence, however, is unclear and mostly speculative.

² Interestingly, Fama 1966 suggests that the added search and clearinghouse costs of operating a potentially returns-improve filter rule reduces the filter rule to return to parity with buy-and-hold. In a sense, brokers make the profit that could be detected by filter rules. This reaffirms the ability of market structure and participants to shape market conditions. This also suggests that moments of structural change in financial markets that reduce search and commission costs are potentially opportunities for return-improving filter rules as well as increased brokerage revenue for rendering services necessary to improve returns.
Above all, however, the motivation of this paper is a curiosity in the random-walk hypothesis, the efficient market hypothesis and volatility feedbacks in both real world assets as well as those modeled in agent-based economics.

Section two of this paper outlines a secondary objective in this work regarding a novel dynamic modeling platform. Section three describes the elements of the model in detail and section four explores the how different agents wealth changes in response to market dynamics. Section five discusses results, potential explanations, new questions that arise from the data, and concludes with future directions for the model.

2. Exploring a New Model Platform: NOVA

This general financial modeling framework has persisted for more than a decade. The economics of this market originate with the Santa Fe Institute’s artificial stock market. (Palmer et al. 1994, LeBaron 2001, LeBaron, 2002, Ehrentreich 2003, Ehrentreich 2009) It continues to be developed, critiqued and improved. (LeBaron, personal communication, November 11, 2010)

The lasting tractability of this model contributes to a secondary objective of this paper. The modeling software used here is a pre-beta dynamic hierarchical modeling platform in development at Oberlin College called NOVA. (Starfield & Salter, 2010) Attempting to apply NOVA to questions of economic relevance is intended to both discover potential for improving the modeling platform as well as expand the fields to which NOVA may be applied.

Much of the professional work in agent based computational modeling is performed using the numerical computing program Matlab. While both powerful and flexible enough for professional research, Matlab does not easily accommodate agent based modeling. (LeBaron, personal communication, November 11, 2010)

Other platforms for agent based economic modeling include Stella and Netlogo. Stella is limited both in its power and flexibility, and struggles to model agent based systems. Netlogo is capable of multi-agent modeling, but is similarly constraints in model complexity. Both are used
academically to produce tractable, though simplified, financial models. This project is the first application of NOVA to agent based computational modeling in the field of economics.

NOVA’s hierarchical structure accommodates agent-based modeling through its easy modularization of sub-models. An agent is essentially a reproduction of a general forecasting model responding to the same dataset of asset information. Each input into the general forecasting model can be changed, generating heterogeneity in the agent behavior. Therefore hypotheses regarding any component of agent forecasting can be explored. This flexibility allows a stable NOVA model to become a sort of economic laboratory/teaching tool, where agent and asset parameters can be adjusted and altered and results can be observed.

The complexity of NOVA modeling, however, is somewhat constrained. NOVA is limited in its ability to apply search algorithms to find values that solve dynamic mathematical systems. The most pertinent of these systems, which NOVA could not solve, is equilibrium pricing as determined by the aggregating demand of agents preferences. This prevents the model from exploring questions of equilibrium pricing, and agent behavior under equilibrium pricing conditions. The key issue is that participants expectations of the future affect current decisions, current decisions affect current prices and that current prices affect future expectations. The simultaneity of these features is the real difficulty. MATLAB offers the capacity to numerically solve period non-linear optimization and dynamic optimization problems subject to period constraints, something that NOVA in its current form cannot solve.

Instead of equilibrium pricing, the model is limited to partial equilibrium analysis of agents’ wealth, exogenous determinants of returns. In this way, I have found NOVA to be a modeling platform that integrates the dynamic systems functionality of Stella with agent-based capacity of Netlogo, with the complexity constraints of each. Regardless, as both Netlogo and Stella are both in final public form, and NOVA exists in a limited closed Beta, this work continues to support strong

---

3 There may exist a work-around solution for this platform constraint. For now, however, it will be left as a future challenge.
prospects for future development of NOVA and broader applicability to complex agent-based economic modeling.

3. Model description

This section describes the structure of the model being explored. The goal is to produce a partial equilibrium asset market and to observe how each participants' wealth develops over time given differing temporal forecasting rules.

3.1 Sequencing

The sequencing of events in an agent-based model is integral to understanding the conditions in which agent's make decisions, as well as the overall flow of information as the model progresses. Because this is a simplified model, is a relatively straightforward two-stage cycle:

First: Dividends, asset price, and asset returns are determined. Wealth increases or decreases according to allocations and returns. The model is initialized with a portfolio allocation for each participant.

Second: Agents use their forecast of expected future returns to determine an allocate of their portfolio to the risky asset for the next period, and the next time period begins.

3.2 Assets

The market consists of only two assets: one risky and the other safe. The risky asset pays a stochastic dividend, \( D_t \), and the risk-free asset pays a constant dividend, \( D_f \) at the rate of \( r_f \).

\( D_t \) is determined according to monthly real dividends from a historic dataset initially used in Shiller (1981), which has been maintained and update through 2011. The data inputted is the nominal S&P 500 monthly mean price and dividend, where the dividend is a linear interpolation from quarterly dividends. The return on the risk-free asset is drawn from the St.

\[^4\] See LeBaron (2010) for the model upon which this is based.
Louis FRED2 database and corresponds to what would be the monthly yield of a three-month treasury bill in that time period.

3.3 Preferences

Participants attempt to maximize next time period wealth using a simple power utility function. The portfolio problem corresponds to,

$$\max_{\alpha_{t,i}} \frac{E^i_{t} W_{t+1,i}^{1-\gamma}}{1-\gamma},$$

subject to

$$W_{t+1,i} = (1 + R^p_{t+1})(1 - \lambda)W_{t,i},$$

where

$$R^p_{t+1,i} = \alpha_{t,i}R_{t+1} + (1 - \alpha_{t,i})R_f,$$

where $$W_t$$ is the participants current wealth, $$\gamma$$ is the agent’s risk-aversion, and consumption, $$\lambda$$, is given as a constant fraction of wealth. The period return on the participant’s portfolio is given by $$R^p_{t+1}$$. The critical component here is $$\alpha_{t,i}$$, which represents agent i’s proportion of savings allocated to the risky asset. The solution to the above maximization problem follows Campbell and Vicera, 2002 and yields an optimal portfolio weight given by,

$$\alpha_{t,i} = \frac{E^i_t(r_{t+1}) - r_f + \frac{1}{2} \sigma_{t,i}^2}{\gamma \sigma_{t,i}^2}$$

where $$E^i_t(r_{t+1})$$ is the expected return forecast generated by the agent, $$\sigma_{t,i}^2$$ is agent i’s estimated conditional variance at time $$t$$ and. The return on the risk-free asset given by

$$r_f = \log(1+R_f).$$
Portfolio proportions \( \alpha_{t,i} \) are constrained to positive values \([0.00, 1.00]\), which omits leveraging and short sales from the market. Addressing bankruptcy and borrowing constraints would add complexity and implementation details to the model, and may be done later. Agent forecasting rules, however, often induce allocations well outside of these constraints.

The intertemporal budget constraint, which corresponds to the participants wealth, is given by,

\[
W_{t+1,i} = (1 + R^P_{t+1})(1 - \lambda)W_{t,i},
\]

where, once again \( R^P_{t+1} \) is next periods portfolio return, given in equation 3.

The current period budget constraint is then given by,

\[
P_tS_{t,i} + B_{t,i} = (1 - \lambda)((P_t + D_t)S_{t-1,i} + (1 + R_f)B_{t-1,i}),
\]

where \( P_t \) is the price of the risky asset, \( S_{t,i} \), referring to securities, is the agents holdings in the risky asset and \( B_{t,i} \), referring to bonds, is the agents holdings in the safe asset.

One assumption is made to justify this intertemporal model. The consumption wealth ratio is only fixed when intertemporal elasticity of substitution is one. This holds consumption constant regardless of agents' wealth.

3.4 Constant Variance Forecasts

Agents are given a constant value, \( \sigma^2_{t,i} \), for the variance of their returns forecast. This variable could be forecasted similar to expected returns, using the agents unique gain parameter. Holding this value constant and close to its true value, however, reduces the number of dynamic variables affecting agent allocation. This allows observations regarding agent behavior to be
mainly attributed to the gain parameter's effect on the return forecast. Relaxing this constraint would likely only amplify the volatility of portfolio allocations, but may also improve outcomes.

3.5 Expected Return Forecasts

Agents use a single simple forecasting rule to determine their expected return forecast, \( E_t(r_{t+1}) \). The forecasting rule is a form of adaptive expectations,

\[
\hat{r}_t = \hat{r}_{t-1} + g_j (r_{t-1} - \hat{r}_{t-1})
\]

(9)

where each period forecast, \( \hat{r}_t \), is updated based on the difference between last period returns, \( r_{t-1} \), and last periods forecast, \( \hat{r}_{t-1} \). The key parameter differentiating agents' forecasts, and the central source of agent heterogeneity, is the gain parameter \( g_j \).

3.6 The Gain Parameter

Gain levels, which are the critical source of heterogeneity in agent forecasts, are given by,

\[
g_j = 1 - 2^{-\frac{1}{m_h}}
\]

(10)

This form is taken directly from LeBaron to maintain consistency in agent structure. According to the LeBaron derivation, the discrete values of \( m_h \) correspond to the number of years over which agents view new information as significant to future outcomes. Discrete values,

\[
[0.1, 0.25, 0.5, 1, 2, 5, 10, 18, 25, 50]
\]

(11)

are used and intended to expand across a large enough range to provide sensitivity for agent performance. Though somewhat counterintuitive, high gain agents are captured in small \( m_h \).
values, and low gain agents are captured in larger values. This is best observed in the gain parameters for the corresponding $m_h$ values,

$$[0.99, 0.94, 0.75, 0.5, 0.29, 0.13, 0.07, 0.04, 0.03, 0.01] \text{, (12)}$$

where the lowest $m_h$ values correspond to the largest incorporation of last periods error into next periods forecast.

3.8 Initial Conditions

Table 1 shows initial values used in the model. The following values clarify the assumptions and inputs that will be used to generate the observed results. Each time period is to correspond to a month time. Agents update their forecasts at the beginning of each month.

Relative risk aversion $\gamma$ is set to 3.5, which suggest a moderate amount of risk aversion. The fraction of wealth consumed translates to a 1% annual rate of consumption which would be analogous to an annual flat rate brokerage fee. Consumption is held constant regardless of differences in the rate at which agents change the allocation of their portfolio. Agents are allocated starting wealth of 16,000 units.

4. Results

Verification of the Shiller data is the first step in describing results. Figure 1 shows a normal distribution of returns, characteristic of a random process. A Lilliefors test for normality also refutes evidence for normality. Furthermore, the distribution also shows the characteristic kurtosis, or fat-tailed nature of real asset returns. This would indicate that our input data is indeed congruent with random walk theory.

Results are separated into two runs. The first, shown in Figure 3 includes all gain
parameters from Table 1, and shows each parameter’s wealth development. This figure is generated using each time periods output of equation 9 from the model description. Two buy and hold strategies, one representing a constant full allocation to the risky asset and another representing a 50-50 allocation, are included for comparison. Under these model conditions, it appears that all filter rules underperform the 100% buy-and-hold allocation, as does the 50-50 buy-and-hold allocation. It also appears that the highest gain agents with $m_h$ values less than 2 consistently outperform lower gain agents. These results both congruent with Fama 1966 and efficient market hypothesis. Furthermore the findings that high-gain agents outcomes are improved over low-gain agent outcomes supports the agent based equilibrium results of LeBaron 2010, where high gain agents held larger fractions of wealth relative to lower gain agents. It appears that using historic data, higher gain agents also hold larger wealth when compared to low gain agents.

Figure 4 provides a more detailed look differences in wealth development between high-gain, low-gain and benchmark buy-and-hold strategies. During the strong growth market of the dot-com boom, and during the run-up prior to the 2008 financial crisis, the full market allocation exhibits huge gains relative to the high-gain filter rule. Those gains, however, dissipate in recessions, suggesting that the filter rule somewhat accurately forecast declines in returns, and limits market exposure during those periods. This is such the case that for a brief period in the bottom of the 2008 financial crisis, the high gain filter rule has outperformed the buy-and-hold strategy.

Figure 5, which shows the portfolio returns given in equation 2, suggests that the high-gain agent completely avoids the worst month of the 2008 financial crisis. This figure also shows how the high-gain agent is both insulated from strong monthly portfolio returns as well as strong negative portfolio returns.
Figure 6 shows the portfolio allocations, or \( \alpha_{t,i} \) given in equation 4, for the \( mh = 1 \) filter and the \( mh = 50 \) filter. Here it’s possible to see exactly why the high-gain agent does not suffer the negative returns of the worst month of the 2008 financial crisis. In response to a trend of negative returns, by the worst month of the crisis, the high gain agent has fully exited the market.

Table 2 provides general summary data from the two selected filter rules and the buy-and-hold strategy. Supporting Figure 5 and 6, the high-gain filter produces a higher monthly mean portfolio return and compounded ROI than both the 50-50 B&H and the low gain filter. The 100% B&H provides the strongest return. Curiously, the high gain filter shows both a lower volatility in returns and a lower mean market exposure than the 50-50 B&H strategy. This is remarkable because the terminal wealth and annual compounded ROI for the high-gain filter is greater than that of the 50-50 B&H strategy. This, in part, conflicts both with the efficient market hypothesis and the capital asset pricing model, where return is reduced to a function of risk. It would appear that these results suggest the over the last 30 years, the S&P may not have delivered efficient returns relative to risk.

5. Discussion

The results of the exercise presented here support both the random walk hypothesis (RWH) as well as the efficient market hypothesis (EMH). First, RWH appears to be supported by preliminary results of a Lillifor’s test for normality that suggests there is insufficient evidence against a non-normal random distribution. Second, RWH and the EMH are supported by the fact that a simple filter rule does not consistently improve returns over a buy-and-hold strategy.

It is unclear exactly why or how the high-gain filter rule produces a lower mean market portfolio allocation, and a slightly lower return volatility than the 50-50 B&H. Without a rolling sample return, or a more statistically rigorous analysis using a more expansive data set, it’s
impossible to make any definite conclusions. These findings do suggest a future investigation of
risk-return dynamics in a high gain adaptive expectation filter.

With regards to LeBaron 2010 agent-based financial model, which inspired this project,
LeBaron finds that agents utilizing the high-gain form of this filter rule control a large fraction of
wealth relative to low-gain agents. This observation is supported by the findings of this model.
Simultaneously, in a dynamic equilibrium, high-gain agents are also key figures in perpetrating
volatility. The high-gain filter’s exit prior to the worst month of the 2008 crisis, suggests that in
equilibrium, a high-gain response to new information filter could have contributed to more
extreme declines of the 2008 crisis. This is mostly speculative and may be an area for future
examination.

Finally, the findings of this model suggest that its current form is at least somewhat
tractable, and the assumptions made may adhere somewhat to real conditions. Furthermore, these
results support potential pedagogical value of this model. Though NOVA provides considerable
constraints in economic research on agent-based dynamic equilibrium, this model may be
resilient enough to serve as a sort of EMH testing laboratory, where students might be able to test
different forecasting rules and market participant assumptions, to see if they can beat the market.
Works Cited


LeBaron, B. "Building the Santa Fe artificial stock market." Physica A (Citeseer), 2002.


Table 1. Model parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>One month.</td>
</tr>
<tr>
<td>$r_f$</td>
<td>0.00</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3.5</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0007</td>
</tr>
<tr>
<td>$\sigma^2_{t,i}$</td>
<td>0.01</td>
</tr>
<tr>
<td>$I$</td>
<td>16000</td>
</tr>
<tr>
<td>$J$</td>
<td>11</td>
</tr>
<tr>
<td>$g_j$</td>
<td>[0.1, 0.25, 0.5, 1, 2, 5, 10, 18, 25, 50]</td>
</tr>
<tr>
<td>$[g_L, g_H]$</td>
<td>[1, 50]</td>
</tr>
<tr>
<td>$[\alpha_L, \alpha_H]$</td>
<td>[0.00, 1.00]</td>
</tr>
</tbody>
</table>

Figure 1. The distribution of S&P 500 returns using the Shiller database. The distribution is roughly normal, with the characteristic kurtosis, or “fat-tails”, US stock return distributions. A preliminary Lilliefors test statistic of 0.0971 further supports that there is no evidence against normality.
Table 2. The mean monthly portfolio proportion is larger for the high gain agent than the low gain. Both, however, are less than the 50-50 B&H. Furthermore, the variance of both filter rules returns are less than the 50-50 B&H.

<table>
<thead>
<tr>
<th></th>
<th>High Gain Filter</th>
<th>Low Gain Filter</th>
<th>50-50 B&amp;H</th>
<th>100% B&amp;H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Monthly Portfolio Return</td>
<td>0.633%</td>
<td>0.497%</td>
<td>0.617%</td>
<td>0.763%</td>
</tr>
<tr>
<td>Portfolio Return</td>
<td></td>
<td></td>
<td>[0.06, -0.1]</td>
<td>[0.07, -0.08]</td>
</tr>
<tr>
<td>[Max, Min]</td>
<td>[0.07, -0.08]</td>
<td>[0.04, -0.04]</td>
<td>[0.06, -0.1]</td>
<td>[0.07, -0.08]</td>
</tr>
<tr>
<td>StdDev PortReturn</td>
<td>0.014</td>
<td>0.010</td>
<td>0.018</td>
<td>0.037</td>
</tr>
<tr>
<td>Mean Portfolio Proportion</td>
<td>31.96%</td>
<td>24.94%</td>
<td>50%</td>
<td>100%</td>
</tr>
<tr>
<td>StdDev PortProportion</td>
<td>0.295</td>
<td>0.054</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Portfolio Allocation</td>
<td></td>
<td></td>
<td>[100%, 100%]</td>
<td>[100%, 0.00%]</td>
</tr>
<tr>
<td>[Max, Min]</td>
<td>[100%, 0.00%]</td>
<td>[50%, 50%]</td>
<td>[100%, 100%]</td>
<td>[39%, 0.00%]</td>
</tr>
<tr>
<td>End Wealth</td>
<td>59209.77</td>
<td>41186.71</td>
<td>48934.51</td>
<td>67310.79</td>
</tr>
<tr>
<td>Compounded Annual ROI</td>
<td>4.21%</td>
<td>3.10%</td>
<td>3.67%</td>
<td>4.74%</td>
</tr>
</tbody>
</table>
Figure 2 Shows price, dividend, monthly mean S&P500 returns, and monthly returns on a 3-month treasury bill.
Figure 3. Wealth development, as described in equation (2) using $g_j$ gain values, is compared here against two buy-and-hold strategies. The two buy-and-hold strategies included are a 100% risky asset allocation and a 50-50 risky-riskless asset allocation. With the exception of the 2008 financial crisis, the period wealth of all filter rules is less than the period wealth of the full risky asset allocation. This conforms to the efficient market hypothesis in that no trading rule dominates a buy-and-hold strategy consistently over time. Furthermore, though difficult to observe in this graph, the optimum gain parameter in terms of wealth accumulation over this period is between 0.25 and 1.00.
Figure 4. HGain corresponds to a \( mh = 1 \) filter, and LGain corresponds to an \( mh = 50 \) filter. Here it is easier to observe the similar performance between the high-gain filter and the buy-and-hold strategy, again up until the end of the dotcom boom around \( t = 250 \). Subsequently, wealth increases tend to be similar during periods of economic stability, followed by the losses of the late 2000s crisis near \( t = 335 \).
Figure 5. The portfolio return series determined in equation (3) is shown below for \( mh = 1 \) and B&H 100%. The 100% risky asset buy-and-hold strategy exhibits stronger positive and negative portfolio returns in comparison to the high gain filter rule. This is expected, as the portfolio returns for B&H 100% are the same as those for the S&P 500. One important and very strong divergence is that the filter rule manages to avoid the dramatic losses of the 2008 financial crises, which corresponds to \( t = 345 \). This coincides with the brief moment in Figures 3 and 4, where the \( mh = 1 \) filter rule wealth is greater than the full exposure B&H.
Figure 6. Here $\alpha_{t,i}$, given in equation 4, is shown for the high-gain agent and low gain agent. The second axis is return series to which the filter rules respond. At $t = 345$, the high gain agent responds to a series of negative returns that precede the month of -20% returns, by which time the high gain agent has fully exited from the market.