THE IMPACT OF STANDARDS-BASED CURRICULA ON UNDERREPRESENTED GROUPS

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by

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ABSTRACT

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The standards-based reform has changed the way mathematics is taught in the United States. The emerging research shows that the reform movement has brought about greater student achievement in mathematics. This paper looks at specific examples of student performance as a result of using reform-based curricula while concentrating on a group that has largely been ignored in mathematics—students from a lower socioeconomic background. There is evidence that suggests that the reforms are helping these students achieve in mathematics, but others have found that not all aspects of the reform are beneficial to them. The paper then emphasizes the important role that teachers play in making mathematics accessible to all students, no matter their background.
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Introduction

In 1954 *Brown vs. Board of Education* found that the accepted educational practice of providing “separate but equal” opportunities in education was in and of itself unconstitutional. “Separate” educational opportunities cannot result in equal ones for students. Thus schools were reconfigured to provide the same opportunities for all students. Unfortunately, while it changed the way schools were structured, it did not actually change the opportunities students got in their classrooms. Many students were not given a fair chance to learn important material, including mathematics. The recent reforms in mathematics have tried to right this wrong.

Like other content areas, mathematics is deeply entrenched in the standards movement. With the standards-based reform movement came the emphasis on the fact that “mathematics can and must be learned by all students” (NCTM, 2000, p. 13). This important statement challenges the myth that mathematical ability is inherited so only certain people are capable of learning mathematics. This means that no matter what a child’s gender, race, or economic background, he or she has the ability to become mathematically competent and should, therefore, be given opportunities to learn important mathematics.

While there has been a large amount of research done on the role of gender in mathematics, minority students and students from a low socioeconomic status (SES) have been largely ignored until recently. While these two groups identify different segments of the population, many of the societal factors that affect one also have an impact on the
other. “One hardly needs to provide evidence of the correlations between race and SES or race and opportunity to learn” (Schoenfeld, 2002, p. 15). This is not to say that all minorities have a low SES or that all people with a low SES are minorities. That is definitely not the case. That assumption does a great injustice to many people in society.

But since race and SES often result from similar environmental situations, I have chosen to focus primarily on SES. Race is discussed separately when the research clearly delineates between the two.

This paper outlines the standards movement and what it entails. Examples of student achievement are discussed with a focus on minorities and low-SES students. This provides a framework for an analysis of ways in which some students are disenfranchised by the reform mathematics. Finally, the teacher’s role is examined to show what he or she can do to help all students learn mathematics while using a reform curriculum.

**How the standards-based reform came about**

From the 1940s-1980s, mathematics instruction changed very little. In this traditional mathematics curriculum, the students learn facts in isolation and practice these new facts by completing a worksheet where the students go through the same steps over and over (Brahier, 2000). Then in 1983, the National Commission on Excellence in Education published *A Nation at Risk*, which found that students were falling further behind students from other countries in their basic mathematical competencies. Perhaps the traditional curriculum was not working. “If a large proportion of K-12 students had been successful in the traditional curriculum, the impetus for change might have been
muted. But that was not the case. Large numbers of students failed or left mathematics…” (Schoenfeld, 2002, p.14).

In 1989 the National Council of Teachers of Mathematics published the *Curriculum and Evaluation Standards for School Mathematics*, which laid out the path NCTM thought mathematics education should take in this country. This path, called the Standards-based\(^1\) reform, is the current movement in mathematics education in which students focus on learning mathematical concepts in contextual situations and apply processes like how to communicate and reason mathematically. Proponents of this movement are trying to replace the traditional curriculum. NCTM was the first national organization in history to publish a list of topics that states could model their curriculum around (Brahier, 2000). There were originally thirteen standards for students in grades 5-8 and fourteen standards for 9-12 students that were divided into two areas—content and process. The content standards illustrate the subject matter all students should learn while the process standards describe how students should acquire this subject matter (NCTM, 1989). Over the next decade, NCTM published two more standards documents: the *Professional Standards for School Mathematics* in 1991 that instructed how to teach mathematics and the *Assessment Standards for School Mathematics* in 1995 that told how mathematical learning could be assessed. In 2000, the *Principles and Standards for School Mathematics* was published. This document was a compilation of the previous three and provided research for its recommendations. The standards were edited so there

\(^1\) A standard is a benchmark that a school district, state, or country can use to see if their curriculum meets a list of recommendations or items that are considered important for every child to learn.
are now five content and five process standards as seen in table 1. These standards should be learned by all students pre-school through twelfth grade.

<table>
<thead>
<tr>
<th>Content Standards</th>
<th>Process Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and Operations</td>
<td>Problem Solving</td>
</tr>
<tr>
<td>Algebra</td>
<td>Reasoning and Proof</td>
</tr>
<tr>
<td>Geometry</td>
<td>Communication</td>
</tr>
<tr>
<td>Measurement</td>
<td>Connection</td>
</tr>
<tr>
<td>Data Analysis and Probability</td>
<td>Representation</td>
</tr>
</tbody>
</table>

Table 1. Content and Process Standards

NCTM realized our society was now entering an information age that is driven by technology. In this information age, there are four goals NCTM feels that the traditional curricula did not address: the need for a mathematically literate workforce, lifelong learning for citizens, opportunities for all, and an informed electorate (Brahier, 2000). Citizens now need to be able to solve problems, think on their feet, adapt to new situations, and reason logically both in and out of the workplace.

NCTM believed that students needed to be able to do more than go through the motions of solving exercises like adding fractions on a worksheet to accomplish the above four goals. A change from just being able to do isolated facts was needed. Joseph Martinez, an education professor at the University of New Mexico stated, “Standards for both curriculum and assessment call for students to go beyond what and how to why, how come, and what if. In other words, it is not enough to present a ‘mathematically correct’ answer; students must also understand the processes, meanings, and implications...
involved” (Martinez & Martinez, 1998, p. 747). Students should gain a conceptual understanding of mathematical topics with the ability to explain why something is so and how it can be generalized to other ideas. For example, students have been taught to add fractions by “getting a common denominator” with many not ever understanding why a common denominator was needed. Under the standards-based reform, students should know why a common denominator is needed and be able to explain it to others. This can be done by changing how mathematics is taught in the classroom.

The reform is based on the idea of constructivism which states that students build knowledge based on their prior knowledge, even in complex areas like mathematics (Silver, Smith & Nelson, 1995). Instead of the teacher standing at the chalkboard lecturing the entire time, the students engage in carefully selected activities that allow them to explore and make conjectures about mathematical topics based on their prior knowledge and what they discover in the explorations. “Compared to mathematics instruction commonly observed in American classrooms today, standards-based curriculum programs place less emphasis on memorization…Teachers are encouraged to spend less time on formal lecture and demonstration” (Riordan & Noyce, 2001, p. 369). The students often work in small groups where they collaborate with each other in order to solve problems. The students are then asked to explain how and why they solved a problem a certain way. Many times this involves writing or using different representations like charts or diagrams. The problems the students work on are based on real-life situations and are often more complex than problems students solve in traditional
classrooms. The basic skills that students learn in traditional classrooms are embedded in the problems the students work on.

**New Curricula**

After the publication of *Curriculum and Evaluation Standards for School Mathematics* in 1989, textbooks needed to be written that could be used in the reform-based mathematics classrooms. The National Science Foundation (NSF) funded selected teams of authors to write curricula that were aligned with the NCTM standards. This was the first time that curricula, supported by research based on an understanding of mathematical thinking, teaching, and learning, became available (Schoenfeld, 2002). Thirteen projects were funded to develop these materials for students in all grades as shown in table 2 (COMPASS, 2003).
<table>
<thead>
<tr>
<th>Everyday Mathematics</th>
<th>Connected Mathematics Project</th>
<th>Contemporary Mathematics in Context: A Unified Approach (CPMP) (Core-Plus)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Trailblazers: A Mathematical Journey Using Science and Language Arts</td>
<td>Mathematics in Context</td>
<td>Interactive Mathematics Program (IMP)</td>
</tr>
<tr>
<td>Investigations in Number, Data, and Space</td>
<td>MathScape</td>
<td><strong>MATH Connections</strong>: A Secondary Mathematics Core Curriculum</td>
</tr>
<tr>
<td><strong>MATH Thematics</strong></td>
<td></td>
<td>Integrated Mathematics: A Modeling Approach Using Technology (SIMMS)</td>
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<tr>
<td>Middle School Mathematics through Applications Project (MMAP)</td>
<td></td>
<td>Mathematics: Modeling Our World (ARISE)</td>
</tr>
</tbody>
</table>

**Table 2. NSF Curricula**

The textbooks are written so that students encounter topics from the content standards each year. “The topical areas of algebra, geometry, discrete mathematics, and so forth, are merely content organizers that were never intended to build walls between those areas of study. Virtually any algebra problem can be represented geometrically, and geometry problems can be represented by algebraic expressions or equations” (Brahier, 2000, p. 73-74). This allows students to see connections between the content areas just like they exist in the real world.

An examination of examples of the new curricula shows how these sample lessons differ from traditional textbooks. (See appendix for a sample Core-Plus lesson on exponential functions) A typical traditional text shows the procedure of how to solve the problem, gives several examples, and then has the student solve around twenty of
these same problems. The NSF-funded textbooks are written using a different approach. While each textbook has a slightly different format, they all use problems to introduce mathematical concepts to students. The students are guided through a series of problems that help them discover or understand a mathematical concept. These problems are designed to be real-world problems so the students learn mathematics in context (Huntley, Rasmussen, Villarubi, Sangtong & Fey, 2000).

**Characteristics that need to be in place**

In order for a curriculum to be successful, there needs to be support for it. In the late 1990s, a study was done with one hundred middle-school mathematics teachers and administrators in Missouri. They learned about the *Standards* documents, and then these teachers implemented a curriculum aligned with the standards. At the end of the third year, the teachers reported what they encountered with using these curricula. Results showed that in order for the reform curricula to be successful, a number of things need to be in place. These include support from the administration and the parents and professional development so that teachers can learn how to use the curriculum. (Bay, Reys & Reys, 1999). Attending workshops and training sessions have been found to be the best ways to achieve the latter (Schoen, Cebulla, Finn & Fi, 2003).

Specific examples of student achievement will be discussed later, but in a study done in Pittsburgh, the students who were enrolled in mathematics classes that strongly implemented the reform curriculum far outperformed the students enrolled in classes where the reform was weakly implemented (Schoenfeld, 2002). Strong implementation classrooms were ones in which the teachers were familiar with activities and instruction
methods that are promoted by reform-based curriculum while weak implementation classrooms barely used the curriculum or did not use the curriculum at all. Therefore, students performed better when their teachers implemented the curriculum as it was designed. However, proper implementation of the curriculum is more likely to occur if the teachers receive training on it. Reform often does not work because the teachers never receive training. No matter how good a curriculum is, it can still fail if a teacher does not implement it properly.

**Importance of Mathematical Literacy**

With society changing to a more technological age, it is important that all people become competent in mathematics. Unfortunately, the prevailing attitude in this country is that it is okay to be bad at mathematics. “It is acceptable in our society to be mathematically inept. Although hardly anyone will admit being unable to read and write, Americans often matter of factly comment on their limited mathematical skills” (Ladson-Billings, 1997, p. 699). In 1989 the publication of another important document showed why the reform movement was very important. The National Research Council published a book on its findings of the state of mathematics in the United States. It stated:

More than any other subject, mathematics filters students out of programs leading to scientific and professional careers. From high school through graduate school, the half-life of students in the mathematics pipeline is about one year; on average, we lose half the students from mathematics each year, although various requirements hold some students in class temporarily for an extra term or a year. Mathematics is the worst curricular villain in driving students to failure in school. When mathematics acts as a filter, it not only filters students out of careers, but frequently out of school itself (NRC, 1989, p. 7).
It is imperative that all students receive a quality mathematical education that challenges them to think, solve problems, and reason. Technology and mathematics often go hand-in-hand. “Mathematics has long been recognized as a critical filter. Course work in mathematics has traditionally been a gateway to technological literacy and higher education” (Schoenfeld, 2002, p. 13). About forty percent of students fail calculus in universities throughout this country (Moses & Cobb, 2001). Since calculus is often a pre-requisite for many scientific and technological jobs, students are often forced to change their major.

Further, the Department of Labor reports that around seventy percent of all jobs require a person to be technologically literate. By 2010 all jobs will require a person to be proficient in technology and use technical skills on the job (Moses & Cobb, 2001). The people who do not possess these skills will often wind up in the lowest paying jobs. With a disproportionate number of minorities and students of low SES in lower-level or no mathematics classes at all, this will “force” many into low-level jobs (Schoenfeld, 2002). This is why some find it so important for all students to get a first-rate mathematical education that they refer to mathematics education as a “civil rights issue” (Moses & Cobb, 2001). Robert Moses argues, “Children who are not quantitatively literate may be doomed to second-class economic status in our increasingly technological society” (Schoenfeld, 2002, p. 13).

Even if students do not choose a mathematical or technological career, mathematical thinking is needed more and more to function in everyday life. Decisions that occur in a person’s personal life, at the workplace, or in society ask for more
quantitatively sophisticated reasoning (Schoenfeld, 2002). “Citizenship now requires not only literacy in reading and writing but literacy in mathematics and science” (Moses & Cobb, 2001, p. 12). In today’s society, nobody can afford to be left behind in the study of mathematics.

**Achievement Using Reform-Based Curricula**

The results of the standards-based curricula being used around the country have been promising. With the emerging data, students’ performances on different assessments have shown that the new curricula are helping students achieve in the study of mathematics. In general, Schoenfeld summarized three main results he found while studying the data that has just been released on standards-based reform:

1. On tests of basic skills, there are no significant performance differences between students who learn from traditional or reform curricula.
2. On tests of conceptual understanding and problem solving, students who learn from reform curricula consistently outperform students who learn from traditional curricula by a wide margin.
3. There is some encouraging evidence that reform curricula can narrow the performance gap between Whites and underrepresented minorities (Schoenfeld, 2002, p. 16).

The first result demonstrates that the students still learn the basic skills while using a reform-based curricula. Many opponents of the standards-based reform thought that the students would be learning “fuzzy math” and accused NCTM of “dumbing down to promote classroom equality” (Martinez & Martinez, 1998, p. 746). They feared students would no longer learn the basic skills that have traditionally been taught in mathematics classrooms. As was mentioned earlier, the basic skills typically taught in mathematics classes are embedded in the problems so students still learn and practice these skills. The second finding is very important as this was one of the main goals of the standards-based
reform. It shows that students are gaining a deep understanding of mathematical concepts instead of just a surface-level knowledge of ideas which often occurred in many traditional classrooms. The final result gives hope that many students who have traditionally been ignored in mathematics classrooms can achieve under the new curriculum. NCTM made it very clear that mathematics is for all students, and all students need to be taught important mathematics. If given a chance, all students will be able to succeed.

In the following section, a summary of various studies will take place. Most of these studies will be referenced later in different sections. A quick examination of results from different states will be touched on before exploring two examples in more detail.

In Illinois, students in a fifth-grade class using the Everyday Mathematics curriculum outperformed students in traditional classes on twenty-four of twenty-five mental computation questions (Riordan & Noyce, 2001). This study shows that students using a reform-based curriculum still learn the basic computations as well, if not better, than students in traditional classes. Another study done in Illinois involved students that used the Math Trailblazers curriculum. Test results of third-grade students in eight Chicago schools were examined to see the effect of using a reform curriculum. After only two years of using the curriculum, the students performed as well or better than they had using a traditional curriculum (Carter et al., 2003).

Similar achievements were found with students using a middle-school curriculum. In five Minneapolis schools that are fully implementing Connected Mathematics, most of the eighth-grade students outscored other eighth graders at schools that did not use a
reform-based curricula by a large margin on the State Basic Standards Tests (Riordan & Noyce, 2001). Similar results were found with students in Missouri who were field-testing the MATH Thematics curriculum. They performed as well as students using traditional curricula on standardized tests that emphasized traditional content. They performed better than the traditional group on problem-solving abilities (Billstein & Williamson, 2003). The Michigan Government News from April 2001 reported how eighth graders in the state of Michigan performed on the Third International Mathematics and Science Study-Repeat (TIMSS-R). The TIMSS was given in 1995 and was used to compare the performance and curricula of different countries around the world. The TIMSS-R was given as a follow-up to that study. Michigan has mathematics standards that are aligned with NCTM’s Standards and advocate the use of reform curricula at all levels. Eighth graders in Michigan performed among the best in mathematics compared to the other states that participated in the TIMSS-R. Twenty-one schools from the state of Michigan that were included in the study had curricula that aligned with the state’s standards. These schools scored even better than the rest (Schoenfeld, 2002).

Like the students in the middle grades, high school students also show an improvement in mathematical achievement when they use a reform curriculum. In 1997, a study of six high schools that piloted the Core-Plus curriculum was done to see how well students had mastered algebra. These students were compared with peers in traditional algebra classes. In open-ended problems and applied algebra problems, the students that used Core-Plus outperformed the students from the traditional group (Huntley et al., 2000). In this study, the students did not do as well on the computation
section as their traditional counterparts. However, it should be noted that these students were involved in piloting the program and changes have been made to remedy this (Huntley et al., 2000). Another study using the Core-Plus curriculum involved around 1400 students from around the country that were using this curriculum for the first time. In September and again in May, the students took the Ability to Do Quantitative Thinking section of the Iowa Test of Educational Development (ITED). This section tests the students’ mathematical reasoning, conceptual understanding, and problem solving (Schoen et al., 2003). Student percentile\(^2\) scores were then compared with other students’ scores around the country who had taken the ITED at the same time. The percentiles of students using Core-Plus went from 52% to 57% in one year (Schoen et al., 2003). While these scores are not optimal, it does show a fairly good improvement over just one year.

In 1999 a study was conducted in Massachusetts comparing schools using reform curricula as their primary curricula with schools that used traditional curricula in elementary and middle school mathematics classrooms. The reform schools used Everyday Mathematics and Connected Mathematics. The way the schools implemented the reform curricula was not examined as the researchers wanted to see the results that occurred in ordinary conditions (Riordan & Noyce, 2001). The researchers examined how the students did on the Massachusetts Comprehensive Assessment System (MCAS), which contains multiple-choice, short answer, and open-response items. This test is constructed around the state standards that were developed based on NCTM’s Standards (Riordan & Noyce, 2001). The groups were divided by how long the schools had used

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\(^2\) A student percentile is the percentage of students from the national norm group who scored below the
the reform curricula. The “early implementers” refers to the schools that had used a reform curricula for four or more years while the “later implementers” are schools that had only used the curricula for two or three years. The results from the 1999 MCAS showed that the reform groups outscored the traditional groups. When broken down into achievement quartiles based on prior performance on state achievement tests, students in all quartiles that used the reform curricula did better than the students in the quartiles who were taught using traditional curricula. In addition, the longer a school used a reform curriculum, the better the students did compared to the traditional groups (Riordan & Noyce, 2001).

It could be argued that the MCAS was aligned with the Standards so students using reform curricula should do better on tests like these. However, by examining how students have historically performed on national and international tests, it has been shown that American students are lacking when it comes to mathematical skills, especially problem solving. Results on the 1995 TIMSS showed the United State’s curriculum is less rigorous than other countries. In 1992 the National Assessment of Educational Progress (NAEP), which is often referred to as the nation’s report card, found that students in all levels of school lack mathematical understanding (Brahier, 2000). The Massachusetts study helps show that reform curricula may be a solution to helping all students obtain problem-solving skills and a conceptual understanding of mathematical topics. In regards to the study done in Massachusetts, the MCAS had only been used since 1998. Before that, the Massachusetts Educational Assessment Program student’s score.
(MEAP) had been used to test students. This test was not aligned with any standards. When comparing the performance of reform students who used *Everyday Mathematics* in 1996 to the traditional group, it was discovered that the students using the reform curricula were already showing greater gains on this test (Riordan & Noyce, 2001). As with many studies, this research shows that no matter which state tests were examined, the reform groups performed better than the traditional groups.

The final example deals with the schools of Pittsburgh, Pennsylvania. The Pittsburgh schools educate around 40,000 students in ninety-seven public schools (Schoenfeld, 2002). Pittsburgh has implemented standards-based curricula since the mid-1990s and has used both reform (New Standards Reference Examinations) and traditional (Iowa Test of Basic Skills) assessments to test the students (Schoenfeld, 2002).

The results of students in fourth grade will be addressed in detail. In the following discussion of results, students in the 1996 and 1997 cohorts used traditional curricula while all students in the 1998-2000 cohorts used reform curricula. In 1997 approximately ten percent of students using traditional curricula met or surpassed the standards for concepts or problem solving on the New Standards Reference Examinations as can be seen in figure 1. In 2000 around twenty-five percent of students met or exceeded these standards (Schoenfeld, 2002). While educators would like the numbers to be higher, it is still important to focus on the fact that many more students met or exceeded the standards in concepts or problem solving, two areas in which American students have typically performed poorly.
In addition, the use of reform curricula in Pittsburgh has shown that it helps students at all levels, not just the top students. Data indicate that fewer students scored well below the standard in 1998-2000 compared to 1996 and 1997 (Schoenfeld, 2002). Like the previous examples, the Pittsburgh example shows how standards-based curricula have helped student improve in mathematics. These studies have shown an increase in performance on conceptual understanding, problem solving, and skills.

Some of the best evidence of the impact of standards-based curricula comes from comments students have made. In the beginning, students may favor the old, traditional style because they are more familiar with it. It takes time for the students to adjust to a new curriculum, but many find they prefer the new one after they became accustomed to it. For example, a male student who had originally complained about the new style of teaching commented on the traditional, substitute teacher they had one day. “It was
terrible. We had this teacher who acted like he knew all the answers and we just had to find them” (Boaler, 2002, p. 253). Students from a class in Michigan commented on the new mathematics they were learning and how it was beneficial. One girl remarked that in her old mathematics classes she would forget what she learned over the summer, but she remembers the concepts she learned using the reform curriculum longer. “

[Connected Mathematics] sticks with you—it just stays with you” (Lubienski, 2000, p. 466). Another girl said, “[In Connected Mathematics] I’ve learned new stuff and I’m able to think more…stuff that will help in real life in the future…I can probably figure out a problem now, without having someone just tell me the rule-like if we’re doing integers again, I could probably figure out a rule” (Lubienski, 2000, p. 466).

**What about minority students and students from a low (SES)?**

After seeing how the standards-based reform has helped many different students, one can ask whether it is helping ALL students! In the past, many students, like minorities and low-SES students, have not often had the same opportunities in mathematics classes. While there is a reasonable amount of research showing how the reform has helped minorities and students from a low SES, there is also research saying that it might not help them as much as originally thought. Both sides will be discussed as well as issues teachers need to be aware of in order for the reform curricula to work for ALL students.

In the past, a large number of students that dropped out of school were minorities. According to findings by the National Research Council in 1989, the dropout rate for African Americans and Hispanics was higher than other groups and low expectations for
these students and limited opportunities for them were two reasons cited (NRC, 1989). Unfortunately, these two reasons that caused many students to drop out in the past are still pervasive in many schools in the United States today. African American students that are also from a low SES are more likely to be in low-ability mathematics classes (Ladson-Billings, 1997). Others have found that, in general, students from poor communities are disproportionately represented in the low-level or remedial track (Silver et al., 1995). The cycle is set so “working-class kids” are trained for “working-class jobs” (Willis, 1977, p. 1). Therefore, a large number in the lowest track are minorities or low SES students. “A large number of studies have pointed to the lack of access and equity that students of color experience in the classroom…. Sorting, grouping or tracking students into lower levels where they receive minimal or no instruction from the teacher…is an example of how such inequity is structured” (Ladson-Billings, 1995, p. 130).

Unfortunately, the students are not given much opportunity in these lower-level mathematics classes. They usually receive less instruction from the teacher compared to their classmates in the upper tracks. The instruction the students do receive focuses on low-level information and skills with the students completing basic computations over and over (Silver et al., 1995).

With such low expectations and limited opportunities, it should not come as a surprise that these students do not perform well on mathematics exams. The TIMSS data show that SES correlates with performance. It has already been discussed how SES also
correlates with race. Unfortunately, most of the poorest areas around the country contain a large minority population.

When it comes time for enrolling in college mathematics classes, a large percentage of minorities have to take a remedial mathematics class for no credit before they are able to take the mathematics courses that count for credit (Moses & Cobb, 2001). For example, at the University of Kentucky at Louisville, the head of the academic advising center for minority students said that nearly ninety percent of entering minority students had to enroll in a remedial algebra class because they did not possess the skills to enroll in a college credit mathematics class straight out of high school (Moses & Cobb, 2001). Looking at recent NAEP scores by seventeen-year-olds, the age when students are about ready to graduate, shows that the Louisville example is not uncommon. According to the NAEP, more than two thirds of Caucasians scored at benchmark levels, levels showing the students knew mathematics at an appropriate level. However, only about forty percent of Latinos and less than one third of African Americans met these benchmark levels (Schoenfeld, 2002).

Taking the remedial class does not mean the students do not have the ability to learn mathematics. Many were never given the chance to learn important mathematics as they went through school. Research has shown that achievement levels are about the same for all racial groups in kindergarten, but the gap widens as students continue their schooling (Walker & McCoy, 1997). Looking at scores on the NAEP for 9-, 13-, and 17-year-olds also shows that the gap in performance between Caucasians and minorities continues to grow as students progress through school (Schoenfeld, 2002).
Standards-based mathematics hopes to reverse this trend and give all students the opportunity to learn mathematics at a high level. The inequities that minorities and low SES students have seen in the past would no longer be present in the reform. The NCTM has urged schools to establish a core curriculum, a three year sequence of classes at the high school level that have common objectives for all students (Brahier, 2000). This would mean that all students learn important mathematical topics no matter what their future plans. This does not mean higher-achieving students will be left behind with reform mathematics either. NCTM suggests that these students study many concepts at a greater depth, but a core curriculum helps ensure all students have the opportunity to study different areas of mathematics and not just arithmetic (Brahier, 2000). Having a core curriculum means eliminating or modifying the tracking system that many schools have so that all students will be studying the same or similar topics. Since the lower tracks are often disproportionately represented with minorities and low SES students, this will allow these students a chance to study the important mathematics. With this, students would receive the same instruction.

As previously noted, the standards-based mathematics calls for students to solve problems within a contextual situation. This can be helpful for students as learning becomes meaningful for people when it has a connection to their lives (Ladson-Billings, 1995). This may especially be helpful for low SES students as they have a more contextual orientation to life than middle-class families (Lubienski, 2000). Therefore, learning mathematics situated in real-life contexts should be beneficial for these students.
Positive Impact on Minorities and Low SES Students

Earlier, examples of how standards-based mathematics affected all students were discussed. Now, a deeper look at these and other examples will be examined to see how they positively impact minorities and students from a low SES. Examples of minorities and low SES will be discussed separately unless the example was drawn from a population where they did not distinguish between the two groups.

Positive Impact on Minorities

The first example to be looked at is the Pittsburgh School District. Of Pittsburgh’s 40,000 students, fifty-six percent are African American (Schoenfeld, 2002). As mentioned earlier, the researchers divided schools into two categories—strong and weak implementers. A sample of strong and weak implementation schools were matched based on socioeconomic and test scores on the Iowa Test of Basic Skills the year before the reform was implemented to compare how students performed (Schoenfeld, 2002). Looking again at the fourth-grade students, the minorities in strong implementing schools far outperformed the minorities in the weak implementing schools as figure 2 shows.
As the graph shows, a gap still exists between Caucasians and African Africans on problem solving and concepts, but the results are promising. At weak implementing schools about four percent of African Americans performed at or above the appropriate standard level on the problem solving and concepts sections of the test. At the strong implementing schools, thirty percent of African Americans met the standards on problem solving while almost forty percent met it on the concepts section of the test (Schoenfeld, 2002). While there is still much room for improvement, that is a tremendous difference in performance between strong and weak implementing schools. Note that this comparison was done in 1998 and the scores continued to rise in 1999 and 2000. In fact, at the weak implementing schools, the ratio of Caucasian to African-American students who performed at the standard level was greater than four to one while at strong implementing schools, the ratio dropped to around three to two (Schoenfeld, 2002).
Another remarkable finding is that at strong implementing schools, the performance on the skills part of the New Standards Mathematics exam by Caucasian and African-American students was very similar (Schoenfeld, 2002).

Returning to the Massachusetts example, recall that all schools that used a reform curriculum as their primary curriculum were compared to schools with similar characteristics that did not use a reform curriculum. The reform groups were then divided into two groups: the early implementers who had used the reform curricula for four or more years and the later implementers that had only used the curricula for two or three years. The early implementers of *Everyday Mathematics* and later implementers of *Everyday Mathematics* and *Connected Mathematics* were broken down into three racial groups for comparison: Asian Americans, African Americans, and Hispanics. (No comparison was made for later implementers of *Connected Mathematics* as there were not enough students to make a comparison.) In general, students who used a reform curricula outperformed their counterparts who used the traditional curricula on the Massachusetts Comprehensive Assessment System (MCAS) except for one group, later implementers of *Everyday Mathematics*. However, the difference was not significant (Riordan & Noyce, 2001). The positive results of the reform groups show that the standards-based mathematics is helping minorities achieve better results in mathematics compared to the traditional curricula.
Positive Impact on Students from a Low SES

The Massachusetts study also compared students in the early and later implementing groups by who received free and reduced-priced lunch\(^3\). While the results between the later implementing group of *Everyday Mathematics* and the comparison group was negligible, the other two groups showed positive results. The early implementing *Everyday Mathematics* group and the later implementing *Connected Mathematics* students outscored the comparison groups by about six points (Riordan & Noyce, 2001). The differences between the above two groups and their comparison groups was greater than the difference between the students who paid full price for lunch and their comparison groups (Riordan & Noyce, 2001). Again, this shows that the reform procedures are helping students of a low SES more than the traditional curricula.

Recall that on the TIMSS-R, eighth graders in Michigan scored among the best in mathematics compared to the other states that participated. Twenty-one schools that participated had curricula that aligned with the *Standards*, and these schools scored even better than the rest. A large number of these standards-based schools were low-SES school districts (Schoenfeld, 2002).

A study done in Chicago in the early 1990s compared students using *Everyday Mathematics* to a suburban county using a traditional curriculum by examining the state scores on the Illinois statewide standardized test. This test consisted of sixty multiple choice questions and is not aligned with the standards (Riordan & Noyce, 2001). Schools that had the largest number of disadvantaged students scored higher than the comparison
group and the state average (Riordan & Noyce, 2001). No matter if the test is aligned with the standards like the Massachusetts study or not, the students using reform materials score higher than their peers who do not.

The next example is not directly a result of the NCTM Standards, but it is a project based on the same ideas used in the Standards documents and combines minorities and students from a low SES. Initiated in 1989, the Quantitative Understanding: Amplifying Student Achievement and Reasoning Project (QUASAR) is a program specifically for middle schools in economically disadvantaged neighborhoods that helps develop mathematical programs that focus on thinking, reasoning, and problem solving (Silver et al., 1995; Lane & Silver, 1999). This program concentrates on the development of quality mathematics instruction and the implementation of high quality mathematics programs in areas that typically do not have such programs. There are six schools involved the program that are located in California, Georgia, Massachusetts, Oregon, Pennsylvania, and Wisconsin. Two of the schools serve predominantly African-American students, two serve Hispanic or Latino students, and the final two are of mixed racial backgrounds. Since this program was developed for poorer communities, the majority of students came from families that have a yearly income of $20,000 or less, live in low-income housing projects, and qualify for free and reduced-priced lunch (Silver et al., 1995).

As has been discussed, it is the students who come from this type of background who often get a second-rate mathematics education. Therefore, the teachers underwent

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3 Free and reduced-priced lunch is one characteristic looked at when determining a student’s SES.
training on the program and how to implement it in their classroom. The curriculum was
designed to focus on problem solving and use a variety of assessments so the students
could demonstrate what they learned in multiple ways. Communication between teacher
and students, as well as between the students, was important along with the mathematics
classroom becoming a community that allowed students to share their thinking and
reasoning in a safe environment. The curriculum also included topics like statistics and
probability for all students (Silver et al., 1995). These are all ideas that NCTM believes
are important in all mathematics classrooms.

The following is an example of a lesson from a QUASAR classroom that used
ideas the NCTM reform advocates—communication and discussion among students to
learn mathematical ideas. The students are learning how to go between different metric
units. On this day, the students are trying to convert 17.5 cm\(^2\) to mm\(^2\).

As students worked with partners, Ms. Healy conversed with various student pairs,
asking them to explain and justify their work. After a few minutes, Ms. Healy asked a student,
Larry, to explain his solution. Larry indicated that he obtained the answer of 170 by multiplying
17 by 10, because for “every one centimeter there is 10 millimeters.” Six students indicated that
they also had obtained the answer 170, but most students did not get the same answer. Ms. Healy
encouraged students to ask Larry questions about his reasoning and solution process…. Another
student thought that multiplication was the correct idea, but that the factor should be 100 rather
than 10, since each centimeter square was 10 mm x 10 mm. This student argues that the answer
should be 1,750 mm\(^2\). …another student, Natalie, returned to an observation made previously by
another student and asserted that each square centimeter was actually 100 square millimeters, after
which Ms. Healy asked her to explain her reasoning. At the overhead projector, Natalie illustrated
her assertion with a 10 x 10 square, which she asked to classmates to think of as one square
centimeter ‘blown up’ to make it clear that it was also a 10 mm x 10 mm square. Natalie argued
that since each centimeter square was equivalent to 100 mm squares, 17.5 cm\(^2\) was equivalent to
1,750 mm\(^2\)…. However, there was still some disagreement in the class. Ms. Healy indicated that
“it’s okay to think differently,” and allowed even further discussion (Silver et al., 1995, p. 37-38).

Ms. Healy helped the students develop a conceptual understanding of the topic—linear
versus area measure. She did not stand at the front of the classroom and lecture, but
helped guide the students in a rich discussion of the topic.
Looking at the performance of eighth graders in one QUASAR school also showed promising results. Comparing the performance gains of African-American students to Caucasian students showed that the gains were similar for the students who used the reform curriculum. For students who did not have access to this curriculum, the performance gains were higher for the Caucasian students (Lane & Silver, 1999). Again, this helps show that when students are taught using ideas similar to NCTM, communication, problem solving, etc., students are capable of learning mathematics at a higher level—even students who have traditionally been left behind.

While the following example did not occur in the United States, it is one study that had been carried out for a number of years and produced some interesting results in regards to SES. The curriculum used in the reform school is very similar to the ideas of NCTM. A three-year study done in England compared two secondary schools that were located in low-income areas. One school, Amber Hill, used ability grouping and a traditional approach that emphasized procedures and skills while the other school, Phoenix Park, used an open-ended approach in their mathematics classes while having mixed ability classes. The curriculum at Phoenix Park was designed by teachers both in and outside the school. The outside teachers were from the Association of Teachers of Mathematics which is equivalent to NCTM of the United States (Boaler, 2002). Before attending these two secondary schools, the students had attended traditional schools with ability grouping for the Phoenix Park students and mixed ability grouping for the Amber Hill students. There were about three hundred students at these schools, and they were matched by social class, race, and gender for this study (Boaler, 2002).
When the students started school at Phoenix Park, class disparities already were noticeable in their mathematics achievement levels. At age thirteen, the correlation between mathematics achievement and social class was 0.43 (Boaler, 2002). However, when the students left this school at age sixteen, the correlation was much lower. After controlling for initial attainment, the correlation was only 0.15 (Boaler, 2002). This is a significant drop in only three years. Unfortunately, the Amber Hill students had a different experience. The initial correlation between mathematics achievement and class was only 0.19 (Boaler, 2002). Using the traditional curricula, the correlation between achievement and social class strengthened (became worse). After three years at Amber Hill, the correlation was 0.30 after controlling for initial attainment (Boaler, 2002).

At the beginning of the study, there was not a significant difference in the students’ mathematics achievement at the two schools. At the end of the three years, the students at Phoenix Park far outscored their counterparts on many different assessments, including the national examination given to English students (Boaler, 2002). The students at this reform school also scored above the national average, even with the school being located in one of the poorest parts of England. In addition to attaining higher levels of mathematics achievement at Phoenix Park, there was more of an equal attainment between different social classes. When the researcher compared students’ achievement at age thirteen and then sixteen at both schools, there was another interesting result. Eighty percent of students at Amber Hill who were achieving above their projected potential were members of the middle class where 80% of underachievers
at this same school were lower class. At Phoenix Park, overachievers and underachievers were found equally among the middle class and lower class (Boaler, 2002).

This study shows that reform curricula can help lower SES students achieve better results in mathematics. The students at Phoenix Park outperformed their peers at a traditional school even though their initial achievement had been comparable. The performance levels between the middle- and lower-class students also were similar after three years of using a reform curricula.

**Does the Reform Really Help ALL Students?**

There have also been studies done showing that standards-based reform may not help ALL students. There are aspects of it that may hinder some students. An examination of the problems and the students’ approaches to them show that just implementing the reform curricula will not solve all the problems that exist in mathematics education.

**Low SES Students**

Sarah Theule Lubienski, one of the leading researchers of the effects of standards-based curricula on low SES students, has found problems with the reform. She worked with a class of thirty seventh-graders in a medium-sized city in the Midwest. The school was socioeconomically diverse, but its 500 students were mainly Caucasian. This school was a pilot site for *Connected Mathematics* (CMP), and the students in the class had used this curriculum the year before (Lubienski, 2000). In order to find out the SES of the students in the class, she sent home surveys to the parents asking about their occupations, incomes, number of books and computers in their home, and which newspapers they read.
regularly (Lubienski, 2000). From the data collected, she was able to classify eighteen of the thirty students into two groups—lower and higher SES. The lower SES students contained mainly working class with a few lower-class students while the higher SES group consisted of middle-class students with a few upper-middle class. Each group contained nine students (Lubienski, 2000). Throughout the year she collected surveys and interviewed the students about their experience with CMP. She found that more of the higher SES students preferred CMP, and that six of the nine lower SES said they were better at mathematics using the traditional lecture approach (Lubienski, 2000).

Since she interviewed the students, she was able to find out why many of them felt this way. Eight of the nine lower SES said the problems were too confusing or hard. One student said, “I don’t like this math book because it doesn’t explain EXACTLY!” (Lubienski, 2000, p. 463). These lower-SES students felt that the curriculum lacked specific directions. They had difficulty in deciding what the problems were asking them to do partly because of the vocabulary and sentence structure. While the higher SES complained at the beginning of the year when it was “cool” to do so, their complaints had to do with specific problems. Unlike the lower SES students, the higher SES did not complain about having no idea how to solve the problems. Several of the higher SES students said CMP was easier than traditional curricula, but none of the lower SES students made that comment even though there were high achievers among the lower
SES group (Lubienski, 2000). The problem is not with the CMP curricula being a second-rate curriculum⁴.

By examining the types of questions asked and the students’ responses to them, she found that the lower SES students often got preoccupied with the context of the problem. While it was thought the standards-based reform would help the students from a low SES because they have more of a contextual orientation, this may not be the case. The contextualized problems may do more harm than good. While working with the students, Lubienski (2000) coded the students’ responses for contextualized language, talking about objects, places, etc. that the problems mentioned. She found that the lower SES group used this language more than the higher SES. This shows that the students often get caught up in what each problem is saying without always seeing the mathematical ideas embedded. Barry Cooper and Máiréad Dunne (1998) found that working class children use their everyday knowledge to solve problems, but this knowledge is often used inappropriately. For example, when a problem asked the students to find how much a soft drink and popcorn cost at a certain movie theater, the students made the mistake of using the current cost of a soft drink instead of solving for the cost of it. Lubienski (2000) also found that the lower SES students often focused on each problem without recognizing the mathematical ideas that connected different problems. In general, the lower SES students would often focus their attention on the

⁴ In 1999 the U.S. Department of Education created a panel to review mathematics curricula. Connected Mathematics was given an exemplary rating, the highest rating possible. In addition, it was the only middle school curriculum to earn this rating.
real-world constraints of the problem missing the mathematical ideas whereas the higher SES students could separate the mathematical ideas from the contexts.

Lubienski is not the only one to have found that the contextual problems may hinder some students and not just students of a low SES. Deborah Ball found that real-world problems and contexts can present difficulties because not all students have the same access to things in the real world (Ball, 1995). Students can interpret problems differently, or incorrectly, because of their experience with what the problem is discussing. In addition, the context of a problem may discuss ideas that are not interesting to a group of students. If this happens frequently, the students become disinterested (Ball, 1995).

The following is an example that occurred in one of the QUASAR schools and illustrates what Lubienski and Ball have found. While the problem and answer may appear straightforward, that was not the case when this problem was presented in the classroom.

Yvonne is trying to decide whether she should buy a weekly bus pass. On Monday, Wednesday, and Friday, she rides the bus to and from work. On Tuesday and Thursday, she rides the bus to work, but gets a ride home from her friends. Should Yvonne buy a weekly bus pass? Explain your answer.

<table>
<thead>
<tr>
<th>Busy Bus Company Fares</th>
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<tbody>
<tr>
<td>One Way</td>
</tr>
<tr>
<td>Weekly Pass</td>
</tr>
</tbody>
</table>

...many students indicated that Yvonne should purchase the weekly pass rather than paying the daily fare, which the teachers believed to be the more economical choice. Curious about this unexpected answer to what the teachers believed to be a rather straightforward question - a multistep arithmetic story problem involving multiplication of whole numbers - they decided to discuss the problem in class and ask students to explain their thinking. The ensuing discussion with students provided an interesting illustration of their application of out-of-school knowledge and problem-solving strategies to a mathematics problem. Many students argued that purchasing the weekly pass was a much better decision because the pass could allow many members of a family to use it (e.g., after work and in the evenings), and it could also be used by a family member on weekends. Students’ reasoning about this problem - situated in the context of urban
living and the cost-effective use of public transportation - demonstrated to the teachers that there
was more than one “correct” answer to this problem (Silver et al., 1995, p. 41).

The students’ real-life experiences caused them to give the “wrong” mathematical
answer, but their answer is justified in the real world. When working with real-world
problems, it is important to have the students explain their answers. As the above
situation shows, “what might appear as irrational reasoning might be rational in lower-
class culture” (Lubienski, 1997, p. 57).

Another influence that has an impact on students’ approach to problem solving
and mathematics in general is how the students are raised. Differences in parenting styles
have been found with regard to different SES levels and different ethnicities. While these
may not exist for all families, it is characteristic of many of them. While growing up,
middle-class students discuss ideas with their parents and learn to interpret their parents’
statements while lower-class students are often told what to do and how to do it.
Communication is much more direct for low-SES students (Boaler, 2002). This carries
over into the classroom. Under traditional instruction, the teacher is the authority. He or
she stands in front of the classroom and tells the class how to solve the exercises. This is
not the case with the reform curricula. Discussion and thinking through ideas are key
aspects used to solve problems. A higher SES student in the CMP pilot class said the
problems “are a lot easier [for me because] I guess our family’s just—we are word-
problem kind of people” (Lubienski, 2000, p. 463).

In the seventh-grade classroom piloting the CMP materials, the lower SES
students said they liked the teacher to have a more direct role. Only the lower SES
students said they preferred it when the teacher would just tell them the rules in the
mathematics classroom (Lubienski, 2000). With all of her work with SES at the CMP pilot school, Lubienski concluded:

> Overall, more of the higher SES students than the lower SES students seemed to possess orientation skills that allowed them to actively interpret the open problems, believe their interpretations were sensible, and follow their instincts in finding a solution. The lower SES students seemed more concerned with having clear direction that enabled them to complete their work and were less apt to creatively venture toward a solution (Lubienski, 2000, p.465).

Lubienski also observed Ms. Stilwell, a veteran teacher in Minneapolis who had been using an integrated, problem-solving curriculum quite successfully in her classroom. The school where Ms. Stilwell taught had a student population where the majority of students were African American, Hispanic, and Liberian or Hmong immigrants, and 74% of the students qualified for free and reduced-priced lunch (Lubienski & Stilwell, 2001). Ms. Stilwell also found that the students like a teacher who will take charge and direct the class as opposed to the discussion-oriented approach the reform calls for (Lubienski & Stilwell, 2001).

**A Call for Examination**

With the above discussion of results—both positive and negative—that have emerged from the reform movement in mathematics, there are areas of concern that need to be addressed. Some are results of the standards movement while others are inequities that may be remedied by the movement.

An examination of the data from the 1990, 1996, and 2000 National Assessment of Educational Progress (NAEP) provides evidence of how students in fourth, eighth, and twelfth grade have performed in mathematics across the country. There are two NAEP assessments that are used to test students. One is the Long-Term Trend NAEP, which
was created in 1973 and has stayed the same over the years. This can help people draw conclusions of groups like race-related achievement over time to see the differences in achievement. The other test, the NAEP, is constructed around national trends that are occurring in the country. Since 1990, the framework for the assessment has been NCTM’s *Standards* (Dossey, 2000).

Although both minorities and students of a low SES have historically been in lower-level mathematics classes and have not performed well in the subject, disparities even exist between these two groups of students. Looking at the 2000 NAEP scores, broken down by race (Caucasians, Hispanics, and African Americans) and SES, an alarming fact can be observed at the eighth and twelfth grades. Looking at table 3, the Caucasian students who were eligible for free and reduced-priced lunch and Hispanics and African-American students who are not eligible showed that these eligible students scored significantly higher than the other two non-eligible groups (Lubienski, 2002).
Another issue that the reform could and should address is the performance gap between different groups. Schoenfeld noted that there is evidence to show the reform could narrow the gap between Caucasians and underrepresented minorities, which is a very promising result. Unfortunately, the majority of schools across the country have not yet adopted a reform curriculum. According to NAEP data, mathematics achievement has increased between 1990 and 2000, but the gap has not closed between different racial and SES groups, and in some instances, appears to be growing (NCES, 2004).

### Table 3. Mean Achievement by Race and Lunch Eligibility, 2000.

<table>
<thead>
<tr>
<th></th>
<th>Fourth Grade</th>
<th>Eighth Grade</th>
<th>Twelfth Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eligible for</td>
<td>Not eligible</td>
<td>Eligible for</td>
</tr>
<tr>
<td>White</td>
<td>221</td>
<td>238</td>
<td>270</td>
</tr>
<tr>
<td>Hispanic</td>
<td>205</td>
<td>221</td>
<td>246</td>
</tr>
<tr>
<td>Black</td>
<td>200</td>
<td>219</td>
<td>242</td>
</tr>
<tr>
<td>White-Hispanic Gap</td>
<td>16</td>
<td>17</td>
<td>24</td>
</tr>
<tr>
<td>White-Black Gap</td>
<td>21</td>
<td>19</td>
<td>28</td>
</tr>
<tr>
<td>Eligible White/non-eligible Hispanic Gap</td>
<td>0</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Eligible White/Non-eligible Black Gap</td>
<td>2</td>
<td>15</td>
<td>19</td>
</tr>
</tbody>
</table>
The biggest difference in performance is seen when cross-grade comparisons are made. The NAEP-scale is designed so that these cross-grade comparisons can be made (Lubienski, 2002). As figure 3 shows, the performance of Caucasian fourth graders is closer to Hispanic and African-American eighth graders than to their fourth-grade peers. Their achievement has gotten closer to the eighth grade Hispanic and African Americans from 1990 to 2000. In 1990, eighth-grade Caucasian students scored similarly to twelfth-grade African Americans. By 2000, the eighth graders scored eight points higher than African Americans in the twelfth grade (Braswell et al., 2001). “These data do imply that, on average, black students are leaving high school with less mathematical knowledge than white eighth graders possess, at least as measured by the NAEP assessment (Lubienski, 2002, p. 15).

![Mathematics Achievement by Race on NAEP, 1990-2000.](image)

Lubienski (2000) and her work with SES in the CMP pilot classroom also pointed out that while reform-oriented teaching could improve all students’ mathematics
performance, it could also increase the gap between lower and higher SES students. Others have found that the reform curricula can help narrow the gap (Schoenfeld, 2000). One difference could be in how the curriculum is presented and how the teacher runs his or her classroom. More studies will have to be conducted before a definitive answer can be given in how the reform will impact the achievement gap.

**It Must be a Reform for ALL**

As noted earlier in different studies, some minority and low-SES students had trouble with problem solving. They had difficulty with problems that were based on real-world situations. They often got caught up in the context of the question and missed the mathematics. This may have had something to do with how they are raised as white, middle-class students learn to interpret what their parents say while many minority and low-SES parents transmit rules and facts directly. That is why some people say the reform is in line with white-middle class students (Boaler, 2002). When using the reform curricula, care must be taken when working with contextual questions. Assuming that all students will interpret the questions the same or have the same experiences will hinder them and their learning.

**Implementation of Reform-based Mathematics**

One main reason why the curricula may not help students or help all students equitably is because of how the curricula is implemented. It has been shown that partial implementation of standards-based curricula does not help students as much as full implementation. Two earlier examples, Minneapolis and Pittsburgh, showed that the students in the partial implementing schools scored only a little better or at the same level
as schools not using reform curricula on different assessments. They also did not perform as well as the full implementation schools (Riordan & Noyce, 2001; Schoenfeld, 2002). Remember that also held true for minorities in the Pittsburgh example. From 1992 to 1995, a study was conducted in the state of Illinois in which a close relationship between student achievement and the level of implementation of reform mathematics was found. The schools with the highest implementation had increases in achievement. The same could not be said for the non-reform schools. The schools determined to have the highest level of reform scored above the state mean by almost one standard deviation (Martinez & Martinez, 1998). Another reason is improper use of the curriculum. Teachers are not using problem solving as a context to learn mathematical ideas but just as an application of concepts which is how it has traditionally been used in the past (Sztajn, 2003). In order for the standards-based mathematics to work, the curriculum has to be fully implemented using the framework of NCTM’s Standards.

When Lubienski worked with Ms. Stilwell, a teacher who used a problem-centered curriculum in her class of minority and low-SES students, Lubienski noticed how much structure was still in place in the classroom (Lubienski & Stilwell, 2001). The students still had unit study guides in which they filled in blanks in response to their teacher’s prompts. The teacher also worked at the overhead showing her students what she wanted them to do. The reform calls for more of an open structure where the teacher serves as a guide and not just a person standing at the front of the room telling students how to do something. In response to a more structured classroom, Ms. Stilwell said:

“I sometimes add more structure so that students can focus on the meaning of what they are doing instead of being distracted by the extraneous details…I used to stay closer to the teacher’s guide,
but I found it didn’t give enough structure to students who were struggling with their behavior and with the mathematics” (Lubienski & Stilwell, 2001, p. 8-9).

Lubienski (2000) faced similar challenges while working with low-SES students in a pilot *Connected Mathematics* classroom. The students wanted more structure and a more authoritative teacher even though the reform does not advocate that. Lubienski did not give in to the students’ requests for more structure and direction because she thought the open-ended approach would help the students in the end (Lubienski & Stilwell, 2001). By working with open-ended questions, Lubienski felt the students would be able to develop critical thinking skills. “I continue to be haunted by the knowledge that higher-SES students are more likely to be taught critical thinking skills, whereas lower-SES students are more likely to be taught obedience to rules their teachers give them” (Lubienski & Stilwell, 2001, p. 11).

**The Teacher’s Role in Reform**

While examining some potential problems of the reform, someone that could be a large part of the solution has often been ignored—the teacher. The teacher has a *significant role* in what occurs in the classroom. This includes whether the standards-based reform and its different curricula will be successful in helping all students succeed. Jo Boaler (2002) found that equality was achieved at the low-SES England school using an open curriculum because of the teaching and learning practices used. Many have reduced “the complexity of teaching and learning to a question of curriculum, leaving the teaching of the curriculum relatively unexamined” (Boaler, 2002, p. 240). Therefore, a discussion of teachers and what they can do is warranted.
The attitudes and beliefs that teachers hold heavily influence their teaching practices. Unfortunately, not all teachers hold the same or similar expectations for all their students. For example, a teacher who worked with both low and high-SES students said:

When I taught at a school where children were poorer, [the kids] needed to become literate and numerate….their low socioeconomic background went hand in hand with their skills. So I had to work on all these things with those low kids. With wealthier students it’s different because they have that background before school (Sztajn, 2003, p. 67).

In the England example, the school that used a curriculum that used ideas in line with NCTM showed more equitable attainment for the low-SES students. The school that still used a traditional approach continued to perpetuate the achievement levels and gap between the higher- and lower-SES students. The teachers at this school said they had to teach using a structured, procedural approach because the students were from disadvantaged backgrounds so they would not be able to handle the open-ended work (Boaler, 2002). However, the open-ended school showed this was not the case.

No reform, no matter how promising, will be able to help all students unless people in charge of schools and classrooms change their beliefs and thinking (Khisty, 1997). In order for all students to succeed, teachers like the ones above need to change their views on believing that only a limited number of students can perform well in a mathematics classroom. If a teacher thinks certain students cannot do well in mathematics because of their race or SES status, they consciously or subconsciously limit the type of mathematics presented to the students. By interpreting teachers’ words and actions, students can tell when teachers believe in them and their abilities. Walker and McCoy (1997) note that mathematics teachers can have a large impact on African-
American students’ self-images. The same argument can be made for all students. Teachers need to realize how big of an impact they have on their students’ lives.

The profession of teaching is complex (Lortie, 1975). Teachers have to have an understanding of their subject area, how students learn, pedagogical strategies, as well as how to manage twenty or more different personalities. Woven into all these facets of teaching needs to be the understanding of different groups of students. This should not be done to develop negative attitudes or stereotypes of these groups but to get an idea of how to educate these students in the best way possible. For example, Lubienski found that students of a low SES often have trouble with contextualized questions. Instead of concluding that these students will never be able to solve these problems, teachers can come up with ways to help these students in order to make the classroom more equitable (Boaler, 2002). By learning about different groups of students, teachers can take a proactive approach in their classroom to make it equitable for all students. This can help teachers deal with potential problems or confusions they may have in the long run. For instance, some students gave what teachers thought was an incorrect answer to the bus question posed earlier. If teachers did not understand the experiences of the students, they might believe the students did not or would not be able to understand the mathematics. That is why it is so important to learn how low SES students’ cultures have an impact on their beliefs and approaches to mathematics (Lubienski, 1997). Different groups of students learn differently based on their background and life experiences. Khisty (1997) points out that teachers need to have an understanding of different strategies that lead to effective learning by Latino students. Whether it be Latino or any
other group of students, it is fundamental that teachers take the time to learn about them. One reason the low-SES students at Phoenix Park in England succeeded was because of their teachers. The mathematics teachers at this school believed that the open-ended approach to learning mathematics was valuable to all students. Therefore, it was their job to make sure that it was equally accessible to the students (Boaler, 2002).

Having this type of attitude will go a long way in helping students succeed, but knowing effective teaching strategies will help it go even further. The following strategies are ones that teachers of minorities and low-SES students have used and have found to be effective. They are all centered around four main ideas: 1) treating the students as competent mathematics students, 2) teaching students how to use the new curriculum, 3) introducing activities and problems through discussion, and 4) discussing the context of the problem with the students.

**Competent Mathematics Students**

Ladson-Billings (1997) has done work with African-American students, and she has found that teachers who treat their students as competent learners are likely to demonstrate competence in their mathematics. The focus of the classroom should be instructional and not on keeping students occupied by working on “busy” work. This does not mean the teacher stands at the front of the room and lectures the whole time. It means that the students should be focusing on important mathematics and learning new concepts instead of working on worksheets or thirty identical problems.
Teach Students to use the Curriculum

Working with minority and low-SES students for many years in Minneapolis, Ms. Stilwell found how important it was for mathematics teachers to collaborate with each other from year to year. She said it often took her students a couple of years to really develop the problem-solving and discussion skills called for by NCTM (Lubienski & Stilwell, 2001). By collaborating with the others, teachers can make sure the students are given support that is consistent to what they received in years past in order to really develop their problem-solving and discussion skills. If the students are unable to learn everything in one year, this allows them to continue to develop the necessary skills with a teacher who knows their past development.

Introduce Activities and Problems Through Discussion

At the school in Phoenix Park, the teachers made sure they introduced the activities through discussion, taught all students how to explain and justify, and they made the real-world contexts accessible (Boaler, 2002). The standards-based movement is a departure from the traditional teaching style that has been around for years. The students cannot be expected to make the switch on their own. It will take a while for them to get used to the new way of learning mathematics. Teachers who used a reform curriculum for three years found that it took their students from several months to an entire year to adjust to the new curriculum (Bay et al., 1999). Introducing the activities through a discussion will help the students understand what they are supposed to do and clear up any questions or misconceptions that exist. Second, explaining and justifying their answers and thought processes will be new to students. These two ideas were not
emphasized with the traditional curricula so the students will have to be taught how to do this. It cannot be assumed that students know how to do this because many may not have any experience with having to justify their thoughts or actions.

Discuss the context of the Problems

Finally, the teachers at Phoenix Park realized the importance of addressing the real-world contexts that occurred with the problems. They did not assume the students were familiar with what the problem was discussing. Since many low-SES and minority students had trouble separating the context from the mathematics in other studies, this can help clear up problems the students may have.

QUASAR Example

Different teachers at QUASAR schools also used different strategies, many of which can be used to help address the open nature and contextualized questions that may trouble students. One teacher at a QUASAR school also introduced the different activities with a class discussion. She would have the students read the problems out loud and then discuss the context of the problems as well as any vocabulary the students did not know. Then she would have the students talk about what they thought the problems were asking. While they worked in groups, the teacher walked around the room to make sure the students did understand what they were supposed to do (Boaler, 2002). To deal with real world contexts, various QUASAR teachers discussed the different contexts presented in the problems with their students (Boaler, 2002). This way, the teacher and students can determine how each is interpreting the problems. “One of the problems presented by real-world contexts is that they often require familiarity
with the situation that is described, but such familiarity cannot always be assumed” (Boaler, 2002, p. 251). That is why it is a good idea to discuss the different contexts presented in problems. It will help ensure that all students understand what is being addressed in the problems and help level the playing field.

**Conclusion**

For the first time, curricula based on research in how mathematics is learned and taught became available. “Therein lies the key. There is now a solid curricular base to work” (Schoenfeld, 2002, p. 20). The curricula currently have some flaws, but these can be corrected. For example, some of the “real-world” problems discuss situations that are not familiar to all students. These questions found in the different curricula need to be reworded so that students will not get caught up in the context and miss the mathematics.

Numerous studies have shown that the standards-based reform has helped students make greater gains in mathematics. Looking at achievement results, the students’ problem-solving abilities have improved and in most cases, their performance on skill-based questions has not declined. That means the students are performing just as well on material that was emphasized under the traditional method while becoming more proficient in an area where U.S. students typically perform poorly—problem-solving. Therefore, there is no need to replace the standards-based reform with another movement. Minor adjustments just need to be made so that ALL students can become mathematically proficient.

Studies have shown that the reform has helped minorities and low-SES students, but there are areas of the reform that seem to hinder these students. In general, they have
a harder time with “real-world problems” compared to some of their classmates. However, getting rid of these questions and stressing skill-based questions that are not set in any context will not help alleviate the problem. After all, that is what the traditional method emphasized and this did not help minorities or low-SES students.

Ultimately, the success or failure of the reform is going to depend on the teacher. That is why teachers need proper training on how to use a reform-based curriculum. In addition, there needs to be follow-up with the teachers to make sure they are implementing what they learned in their training (Loucks-Horsley, Hewson, Love & Stiles, 1998).

The teachers also need to be aware of the students in their classrooms. All students bring a different set of experiences with them into the classroom. In order to help minorities and low-SES students with the new curriculum, teachers should implement the strategies discussed earlier. They should not deviate from how the reform emphasizes the material be taught, but the teachers must ensure that all students understand the questions. Traditionally, teachers have come to the conclusion that teaching makes them autonomous beings in the classroom (Lortie, 1975). They are able to make their own decisions when it comes to ideas like how the material is taught. Therefore, the teaching model is separate from the curriculum. Teachers must understand that the authors of the reform curricula have worked hard to dispel this idea. With the reform, the teaching model is connected to the curricula. Teachers have to follow this model in order for the curricula to be successful.
References


In 1989, the oil tanker Exxon Valdez ran aground in waters near the Kenai peninsula of Alaska. Over 10 million gallons of oil spread on the waters and shoreline of the area, endangering wildlife. That oil spill was eventually cleaned up—some of the oil evaporated, some was picked up by specially equipped boats, and some sank to the ocean floor as sludge.

For scientists planning environmental cleanups, it is important to be able to predict the pattern of dispersion in such contaminating spills. Think about the following experiment that simulates pollution of a lake or river by some poison and the cleanup.

- Mix 20 black checkers (the pollution) with 80 red checkers (the clean water).
- On the first “day” after the spill, remove 20 checkers from the mixture (without looking at the colors) and replace them with 20 red checkers (clean water). Count the number of black checkers remaining. Then shake the new mixture. This simulates a river draining off some of the polluted water and a spring or rain adding clean water to a lake.
- On the second “day” after the spill, remove 20 checkers from the new mixture (without looking at the colors) and replace them with 20 red checkers (more clean water). Count the number of black checkers remaining. Then stir the new mixture.
- Repeat the remove/replace/mix process for several more “days.”
Think About This Situation

The graphs below show two possible outcomes of the pollution and cleanup simulation.

The pollution cleanup experiment gives data in a pattern that occurs in many familiar and important problem situations. That pattern is called **exponential decay**.

**INVESTIGATION 1: More Bounce to the Ounce**

Most popular American sports involve balls of some sort. In playing with those balls, one of the most important factors is the bounciness or **elasticity** of the ball. For example, if a new golf ball is dropped onto a hard surface, it should rebound to about $\frac{2}{3}$ of its drop height.

Suppose a new golf ball drops downward from a height of 27 feet onto a paved parking lot and keeps bouncing up and down, again and again.
1. Make a table and plot of the data showing expected heights of the first ten bounces.

<table>
<thead>
<tr>
<th>Bounce Number</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rebound Height</td>
<td>27</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. How does the rebound height change from one bounce to the next? How is that pattern shown by the shape of the data plot?

b. What equation relating \( \text{NOW} \) and \( \text{NEXT} \) shows how to calculate the rebound height for any bounce from the height of the preceding bounce?

c. Write an equation beginning \( y = \ldots \) to model the rebound height after any number of bounces.

d. How will the data table, plot, and equations for calculating rebound height change if the ball drops first from only 15 feet?

As is the case with all mathematical models, data from actual tests of golf-ball bouncing will not match exactly the predictions from equations of ideal bounces. You can simulate the kind of quality control testing that factories do by running some experiments in your classroom.

2. Get a golf ball and a tape measure or meter stick for your group. Decide on a method for measuring the height of successive rebounds after the ball is dropped from a height of at least 8 feet. Collect data on the rebound height for successive bounces of the ball.

a. Compare the pattern of your data to that of the model that predicts rebounds which are \( \frac{2}{3} \) of the drop height. Would a rebound height factor other than \( \frac{2}{3} \) give a better model? Explain your reasoning.

b. Write an equation using \( \text{NOW} \) and \( \text{NEXT} \) that relates the rebound height of any bounce of your tested ball to the height of the preceding bounce.

c. Write an equation beginning \( y = \ldots \) to predict the rebound height after any number of bounces.

3. Repeat the experiment of Activity 2 with some other ball such as a tennis ball or a basketball.

a. Study the data to find a reasonable estimate of the rebound height factor for your ball.

b. Write an equation using \( \text{NOW} \) and \( \text{NEXT} \) and an equation beginning \( y = \ldots \) that model the rebound height of your ball on successive bounces.
Checkpoint

Different groups might have used different balls and dropped the balls from
different initial heights. However, the patterns of \((\text{bounce number}, \text{rebound height})\) data should have some similar features.

\textbf{a} Look back at the data from your two experiments.

\begin{itemize}
  \item How do the rebound heights change from one bounce to the next in each case?
  \item How is the pattern of change in rebound height shown by the shape of the data plots in each case?
\end{itemize}

\textbf{b} List the equations relating \textit{NOW} and \textit{NEXT} and the rules \((y = \ldots)\) you found for predicting the rebound heights of each ball on successive bounces.

\begin{itemize}
  \item What do the equations relating \textit{NOW} and \textit{NEXT} bounce heights have in common in each case? How, if at all, are those equations different and what might be causing the differences?
  \item What do the rules beginning \textit{“y = ...”} have in common in each case? How, if at all, are those equations different and what might be causing the differences?
\end{itemize}

\textbf{c} What do the tables, graphs, and equations in these examples have in common with those of the exponential growth examples in the beginning of this unit? How, if at all, are they different?

\textit{Be prepared to share and compare your data, models, and ideas with the rest of the class.}

On Your Own

When dropped onto a hard surface, a brand new softball should rebound to about \(\frac{2}{3}\) the height from which it is dropped. If a foul-tip drops straight down onto concrete after achieving a height of 25 feet, what pattern of rebound heights can be expected?

\textbf{a} Make a table and plot of predicted rebound data for 5 bounces.

\textbf{b} What equation relating \textit{NOW} and \textit{NEXT} and what rule \((y = \ldots)\) giving height after any bounce match the pattern of rebound heights?
c. Here are some data from bounce tests of a softball dropped from a height of 10 feet.

<table>
<thead>
<tr>
<th>Bounce Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rebound Height</td>
<td>3.8</td>
<td>1.5</td>
<td>0.6</td>
<td>0.2</td>
<td>0.05</td>
</tr>
</tbody>
</table>

- What do these data tell you about the quality of the tested softball?
- What bounce heights would you expect from this ball if it were dropped from 20 feet instead of 10 feet?

d. What equation would model rebound height of an ideal softball if the drop were from 20 feet?