ABSTRACT

THE EFFECTS OF METACOGNITIVE STRATEGIES ON MATH PROBLEM SOLVING ABILITY IN GIFTED SECOND GRADE STUDENTS

by Caroline Elizabeth Houston

It is critical that schools continue to advance the skills of high ability students through tasks that require complex thinking processes and work that goes beyond computational mathematics. The study examined the effects of metacognitive strategy instruction on the problem-solving mathematics achievement and mathematical agency of 2nd grade mathematically gifted students. Results showed that metacognitive strategies can be utilized with gifted students to extend their ability to problem-solve. Metacognitive strategy instruction also had a positive effect on the mathematical agency of the students. The mathematical agency rubric which was utilized examined the opportunity to express and contribute mathematical ideas, whether ideas are built upon, and if group members recognize and support the contributor of the idea.
THE EFFECTS OF METACOGNITIVE STRATEGIES ON MATH PROBLEM SOLVING ABILITY IN GIFTED SECOND GRADE STUDENTS

A Thesis

Submitted to the

Faculty of Miami University

in partial fulfillment of

the requirements for the degree of

Education Specialist

by

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Oxford, Ohio

2017

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This Thesis titled

THE EFFECTS OF METACOGNITIVE STRATEGIES ON MATH PROBLEM
SOLVING ABILITY IN GIFTED SECOND GRADE STUDENTS

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has been approved for publication by

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Introduction

According to the National Association for Gifted Children (n.d.), six to ten percent of the total student population is classified as academically gifted and talented. While high ability students benefit from more individualized educational plans, in most schools these students are taught in the general education classroom by educators who have not been formally trained to educate this population of students. Furthermore, the 2012-2013 State of the States in Gifted Education survey indicated that only three states mandate training in gifted education for general education teachers. Additionally, eight of the fifty states estimated that of all the general education teachers in the state, five percent or less received some sort of professional development related to gifted education (National Association for Gifted Children, n.d.).

America is currently producing students that are behind same age peers in other countries (OECD, 2014). Research has also shown that the United States is not detecting and retaining the best students in the areas of science, technology, engineering, or mathematics for careers associated with these fields (Trinter, Moon, & Brighton, 2015). Gifted students are capable of performing at high levels and may not be appropriately challenged in the general education setting (Colangelo, Assouline, & Gross, 2004). Tomlinson (1997) addresses ways to teach a gifted student well but there are also many instructional methods that are inappropriate for instruction. This study focuses on the use of metacognitive strategies instruction to improve mathematics problem solving in a group of high-ability mathematics students.

Literature Review

I’Anson (2014) states that our schools are often unable to support the educational needs of gifted students. These students may lack support due to the amount of resources that are available to school districts. In order to efficiently serve the school population, the Triage Model of Education is used. The model is similar to the medical triage model where the treatment, or in this case focus, in schools is based on the severity of the condition of the student. For example, students who have cognitive disabilities, physical disabilities, or are performing below grade level will be served before gifted learners. With such a model, the needs of gifted students may not be adequately met (I’Anson, 2014). In order to contribute to gifted student research, prior research involving effective and appropriate instructional strategies, metacognitive strategy instruction, and problem solving was explored.

Proper Instruction of High Ability Students

According to Trinter, Moon, and Brighton (2015) a “curriculum designed with high expectations for students is critical in fostering mathematical promise and…many schools do not provide this experience” (p. 26). Tomlinson (1997) suggests the proper instruction for gifted learners created through good curriculum and instruction, pacing based on the individual, difficult content, and an understanding of the risk that gifted learners will experience. The risk can be further described by looking at the history of the academic background of high ability students. Typically, these students receive high grades with ease and succeed without failures faced by average-ability peers. When gifted students are presented with challenging tasks, they may face failure, which is a risk to high-ability students (Tomlinson, 1997). Wilkins, Wilkins, and Oliver (2006) stated that proper challenges for high ability students consist of work that stretches beyond simple computations and require complex cognitive processes possibly involving novel mathematics, problem solving, or real-world problems. Furthermore, despite their challenging nature, novel but appropriate level tasks can help develop the cognitive abilities...
of students (Zhang, Xin, and Si, 2013). Lastly, Diezmann and Watters (2001) showed that utilizing collaboration as an instructional method can be beneficial to high ability students. When students were faced with tasks that were considered appropriately challenging, collaboration was effective at producing “cognitive, metacognitive, and affective benefits for students (p. 25).”

Inappropriate instruction focuses on the same content completed at a faster pace; on content that has already been mastered, removing the student from peers or teacher instruction; or requires students to function as a tutor for an extensive amount of time. Also, instructing high ability students to complete various tasks or “classroom chores” in order to fill time while waiting for others to finish is also an inappropriate instruction which is also a poor use of educational time (Tomlinson, 1997). Sometimes students may be asked just to sit quietly until others finish the necessary work (Wilkins, Wilkins, & Oliver, 2006).

High-ability students cannot be expected to function fully by themselves or to direct their own learning without guidance. Based on the wide range of ability levels involved in a classroom, it is difficult for a teacher to provide appropriate instruction to all ability-levels at once (Wilkins, Wilkins, & Oliver, 2006). Additionally, high-stakes testing limits the amount of differentiated experiences for students. The decontextualized learning environment provides limited experiences for students to be actively engaged in solving mathematical problems, making it difficult for teachers to identify student potential (Trinter, Moon, & Brighton, 2015).

Mathematics Agency & the Common Core State Standards (CCSS)

The CCSS were first introduced in 2009 in an effort to better prepare students for careers, college, and life beyond school. The mathematics standards were developed to improve achievement in mathematics. Their development emphasized a shift in mathematics instruction towards abstract reasoning and deep conceptual understanding (Common Core State Standards Initiative, 2010; Key shifts in mathematics). Not only must students be able to reason, but also they must be able to make justifications and be able to review the reasoning process of other students (Common Core State Standards Initiative, 2010; Standards for mathematical practice). Essentially, the CCSS emphasize the importance of students as active learners and problem solvers. In addition to learning procedures and developing fluency, students also need to negotiate meaning and understanding.

The development of CCSS and the changes that they demand directly connects to mathematics agency. Agency can be described as “the capacity and willingness to engage mathematically” (Schoenfeld & Floden, 2014, (document suite), p. 2). Agency can be built through experiences that require students to “conjecture, explain, make mathematical arguments, and build on one another’s ideas” (Schoenfeld & Floden, 2014, (document suite), p. 2). Ultimately, the CCSS promotes mathematics agency and recommends teachers develop rich tasks to allow similar experiences to build mathematics achievement and powerful classrooms.

Metacognitive Strategies & Mathematics Problem Solving

Metacognition was first described by Flavell (1976) as “one’s knowledge concerning one’s own cognitive processes and products or anything related to them” (p. 232). Since this time, many studies have examined the effects of metacognitive strategies on mathematical ability of various populations of students. Desoete, Roevers, and Buysse (2001) stated metacognition is important for students faced with challenging mathematics. Metacognition is helpful through being able to essentially not strain the child when faced with a difficult task (as cited in Carr, Alexander, & Folds-Bennett, 1994; Carr & Jessup, 1995). Ozsoy and Ataman (2009) added
Metacognition instruction benefits students by allowing them to learn an effective problem solving process and to have the ability to use this process under a wide variety of conditions. This type of instruction allows students to reach “high-level cognitive process” (p. 70. as cited in Victor, 2004).

Teaching metacognitive strategies has also had a positive effect on student mathematics problem solving ability (Montague, 1992). Problem solving research has indicated that it is not enough to just to know what to do when problem solving in the area of mathematics, it is also necessary to also know when to apply strategies to the tasks at hand (Ozsoy & Ataman, 2009; as cited in McLoughlin & Hollingworth, 2001). The metacognitive strategies that will be used in the current study focus on self-instruction, self-question, and self-monitor, which is supported by Montague (1992). Effective problem solvers are known to use these strategies in order to plan, execute, and regulate these strategies and their performance (Montague, 1992).

Rationale and Purpose

With an increase in the number of identified gifted and talented students and often a lack of appropriate challenges in the general education setting, it is critical that schools help those students to access tasks that require complex thinking processes and work that goes beyond just computational mathematics. According to Ozsoy & Ataman (2009), metacognition training has been shown to be successful in positively affecting problem solving ability (as cited in Lucangeli, Galderisi, and Cornoldi, 1995). More research is warranted on the effects of using metacognitive strategies with high-ability students since this research is generally conducted using older students and/or students with math related disabilities and struggling learners. Also, inconsistent results have been found with younger students (Desoete, Roeyers, & Buysse, 2001).

The purpose of the study was to examine the effect of metacognitive strategy instruction on the mathematics problem solving achievement of mathematically gifted second grade students. Secondly, the study examined the effects of metacognitive strategy instruction on the mathematical agency of gifted second grade students.

Research questions.

What effects does metacognitive strategy instruction have on the problem solving achievement of gifted second grade students? What effect does the metacognitive strategy instruction have on their mathematical agency?

Research Methodology

Participants

High-ability students were identified through a partnership with the researcher and teachers in a suburban elementary school. Subjects were three 2nd grade students from one class at the school. Two students were female, and one was male. These students were selected during the spring semester of the previous year by their teacher through their performance on the STAR assessment. The students that were selected earned scores in the 94th percentile and above in the area of mathematics.

Protection of Human Subjects

When studying human subjects, precautions were taken to minimize risks and maximize benefits for participants. The researchers obtained IRB approval to conduct the study.
study, there were minimal risks likely to affect participants. Participants were required to take a pre-test, post-test, and participate in biweekly group lessons which focused on big ideas from the Common Core State Standards (CCSS). There was a risk that students would have difficulty responding to open ended questions or a challenging time with the pre and post-tests. These difficult tasks could have caused some frustration for the students. Also, when working in a group, students had the option to agree to the thoughts of one student but not the other and this may have caused participants to become further frustrated or hesitant to interact again in the future during this instruction. Lastly, in order to code for the Teaching for Robust Understanding of Mathematics, students were audiotaped during the three baseline lessons and on the last day of the intervention. Audiotaped lessons could have made the students anxious about what they may say or do while being videotaped.

In order to address these concerns, background information about the study and what it entails was sent home to parents who had the option to consent or forbid the student from participating in the study. Throughout the eight-week study period, parents and children were allowed to withdraw from the study at any time without any consequences. Also, the researcher focused on maintaining a cooperative group experience during this study in order to minimize distress during the lessons.

**Measurements**

The *KeyMath-3 Diagnostic Assessment* (KeyMath-3 DA) is a measure designed to help identify the strengths and weaknesses in the area of mathematics and to also guide instruction. The recommended age ranges for this assessment are 4 to 21 years old. The diagnostic assessment was normed using 4,000 people with an age span from 4 years, 5 months, to 21 years, 11 months. The assessment is designed in parallel forms (Form A and Form B), meaning that students can be tested and then assessed again after a period of at least six weeks. However, due to the lack of availability of the Form B assessment, Form A was utilized for the pre and post intervention assessments. Although Form A was used for both pre and post measures, it was unlikely the participants recalled information presented from the pre to post assessment given the time between measures. For this study, students were administered one subtest from the Basic Concepts area and one subtest from the Applications area (Connolly, 2007) pre and post intervention.

From the Basic Concepts area, the student took the Numeration subtest which measured student capabilities to demonstrate basic math concepts and abilities to understand whole and rational numbers. According to Connolly (2007) this subtest focuses on place value and percentages, which “serves as a foundation for math concepts involving estimation and computation, measurement, data interpretation, and problem solving” (p. 6). For example, for one problem a student could be shown a set of objects and asked how many more objects are needed to reach a specific number of objects. This subtest must be completed prior to testing subjects in other mathematical areas because this subtest determines the starting item on other subtests.

Once the Numeration subtest was completed, students were tested in the area of Applications with the Applied Problem Solving subtest. Applied Problem Solving measures the ability to “interpret problems set in a context and to apply computational skills and conceptual knowledge to produce a solution” (Connolly, 2007, p. 8). For example, some questions from this subtest require students to interpret pictures, understand and create patterns, and identify ways objects are similar.
The reliability indexes for this measure indicate it is reliable based on the various indexes provided in Connolly (2007). The split-half reliabilities for Grade 2 of the Numeration subtest are .82 for Form A. The Applied Problem Solving subtest had a split-half reliability in Grade 2 of .84 for Form A. Also, in Grades Pre-Kindergarten to Grade 5, Test-Retest Reliability Coefficients were measured and a correlation of .85 was found for the Numeration subtest and a correlation of .84 was found for the Applied Problem Solving Subtest.

Connolly (2007) also provided validity indexes for this assessment. A correlation of .67 was found between the Numeration subtest and the Applied Problem Solving subtest for Pre-Kindergarten through Grade 2. The KeyMath3 Diagnostic Assessment was also compared to several other similar measures. Overall, the KeyMath3 demonstrated consistent validity among measures compared. Please see Connolly (2007) for additional details. Several specific populations were examined with measuring the validity of the index. When giftedness was examined, the individuals performed about one standard deviation above the mean. This will be important to be aware of when examining the scores of the 2nd grade students.

The Teaching for Robust Understanding of Mathematics (TRU Math) is part of the Mathematics Assessment Project (MAP) at the University of California, Berkeley. TRU Math was created to help assess and evaluate learners and their environment with a focus on five dimensions: 1) The Mathematics, 2) Cognitive Demand, 3) Access to Mathematical Content, 4) Agency, Authority, and Identity, and 5) Uses of Assessment (Schoenfeld & Floden, 2014, (document suite)). However, the dimension that the current study focused on is Agency, Authority, and Identity. Please see the TRU Math Rubric under Figure 1, which was retrieved from the Mathematics Assessment Project website (Schoenfeld & Floden, 2014, (document suite)).

The dimension aims to measure the opportunity to express and contribute mathematical ideas, whether ideas are built upon, and if group members recognize and support the contributor of the idea. The small group was scored from 1 being the least powerful or ideal to 3 being the most beneficial for a productive classroom. The term agency for the dimension can be further described as the perception the student has about whether they can or cannot complete mathematics problems and the confidence he or she has with the solutions formed. Mathematical authority refers to the feeling that the student has about specific parts of mathematics. A student may feel knowledgeable about certain areas about mathematics and peers may view her as adept. However, the opposite may be true. A student may feel as if he or she are lacking knowledge in this area and peers may be aware of this deficit as well. The third segment of the dimension is identity. A mathematics identity is established through prior experiences with the subject. Identity may encompass thoughts that are counterproductive to learning mathematics or positive beliefs about the subject that will aid students in their thought process in mathematics (Schoenfeld & Floden, 2014, (dimensions)).

**Metacognitive Intervention Procedure**

Questions from Marian Small’s “Good Questions” book (2012) designed for grades pre-kindergarten through 8th grade were used within the intervention. The book separates instructional problems into three different grade levels: pre-kindergarten through 2nd grade, 3rd grade through 5th grade, and 6th through 8th grade. In order to design lessons that focus on using open questions that align with the CCSS (2010) and fundamental ideas as outlined by the National Council of Teachers of Mathematics (NCTM), the parallel task content for Pre-K through Grade 2 and 3rd through 5th grade of Small (2012) was used.
Operations and Algebra areas were focused on for the intervention phase of the project. Please see Table 1 for example problems. Nineteen lessons were created from the questions focused on in this text.

The metacognitive strategies intervention that was used for the study was acquired from interventioncentral.org and is known as the ‘Say-Ask-Check’ Metacognitive Prompts tied to Word-Problem Cognitive Strategy (Montague, 1992). The intervention consisted of cognitive strategies and metacognitive prompts. There are seven cognitive strategy steps: (1) Read the problem, (2) paraphrase the problem, (3) ‘draw’ the problem, (4) create a plan to solve the problem, (5) predict/estimate the answer, (6) compute the answer, and (7) check the answer. For each cognitive step, there are three metacognitive prompt targets, which are the (1) self-instruction target ‘Say,’ the (2) self-question target ‘Ask,’ and the (3) self-monitor target ‘Check’ (Montague, 1992). During the current study, students were encouraged to self-question, but also say, ask, and check as a group. The researcher utilized this intervention process in order to complete the questions identified from Small (2012) for each lesson completed during the intervention period.

**Procedure**

First, the researcher obtained IRB approval from Miami University and the Talawanda School District through an amendment to an IRB that was previously approved for the 2015-2016 school year. Next, the three identified students were sent home with an information sheet about the project and consent forms for participating in the study. These forms included the possible benefits and risks that are involved in the study. The researcher provided her contact information in order to answer additional questions. Once permission forms were received, the pre-assessments commenced.

Before baseline, the three students were assessed individually using the Numeration and Applied Problem subtests from the KeyMath3 Form A. After the assessments were finished, the researcher conducted three lessons on separate days during a one-week period. The three lessons were audiotaped and scored by the researcher and her advisor for the Agency, Authority, and Identity section of the small group work TRU Math rubric. After the baseline week was completed, the intervention phase of the study commenced.

The researcher met with students biweekly for an eight-week period for the intervention phase of the study. The intervention sessions focused on explicitly teaching and utilizing the Say-Ask-Check metacognitive strategies to solve open ended math questions requiring high levels of critical thinking skills. First, the researcher modeled how to solve a specific problem using the metacognitive strategy. Then the students interacted with each other and through guided practice, used the strategy to solve a problem similar to the one that was modeled. Students took turns modeling with researcher support the necessary metacognitive steps for each cognitive strategy step. After the first intervention day, turn-taking instruction was phased out. On the second intervention day, students completed two problems, taking turns using the cognitive and metacognitive steps and one problem on the third intervention day. During these sessions, students completed 3-6 problems using these strategies. In the last week of the intervention phase, the last session was audiotaped and then student agency was scored using the same categories of the TRU Math Rubric that were used during baseline.

Once the intervention phase was completed, the three students were assessed individually using Form A of the KeyMath3 Diagnostic Assessment. The Numeration and Applied Problem
Solving subtests were utilized for post intervention. The researcher completed these assessments after the intervention phase during the most convenient times for the teachers of the students.

Data Analysis

The primary purpose of the study was to see whether metacognitive strategies used in tutoring sessions for a half an hour twice a week affected student math problem solving performance on the KeyMath3 Diagnostic Assessment. The data collected was analyzed by using SPSS to compare problem solving pre and posttest scores. Due to the limited number of participants involved in this study, inferential statistics will not be utilized to compare the Form A Pre-Intervention Performance to the Form A Post-Intervention Performance. Descriptive statistics will be utilized to analyze the data collected from the KeyMath3 assessments.

The secondary purpose of this study was to examine the impact of cognitive strategy instruction on the overall agency of students using the TRU Math data rubric as a measure. In order to analyze the TRU Math data, scores on the dimension utilized were compared. A 3-minute interval recording technique was utilized to observe student mathematical understanding based on the TRU rubric. The highest numerical rating possible was given based on the discussions during the interval. The researcher and her advisor scored the audiotaped lessons, and then interobserver agreement was assessed. The scores obtained were averaged for the pre-intervention lessons and the lesson at the end of the intervention period. A percentage of times student shows agency and authority based on the audio recordings using the TRU rubric were computed for the group for each audiotaped lesson.

In order to better understand the mathematical agency of participants, the audiotapes were reviewed for examples of agency at various stages of the study. Using the coding from the rubric, researchers identified samples of thinking at the 1st, 2nd, and 3rd levels of agency. Examining the qualitative samples of agency will help teachers to more effectively identify similar stages of development among their students to determine their levels of agency.

Results

The first research question aimed to examine the effects metacognitive strategy instruction would have on the problem solving achievement of gifted second grade students. The second research question asked what effect the metacognitive strategy instruction would have on the students’ mathematical agency.

KeyMath3 - DA

The participants were given a pre and post test utilizing Form A of the KeyMath3 assessment. The Numeration and Applied Problem Solving subtests were given to each student during the pre and post assessment. Due to the small number of participants, an inferential statistical analysis was not completed. Table 8 provides the scaled scores on each of the subtests for the pre and post assessments. For the purpose of this study, only the problem solving subtest was examined. An Applied Problem Solving mean of 13.7 (scaled score) was exhibited by the participants on the pre KeyMath3 assessment. After the metacognitive strategy instruction intervention, the students exhibited a mean of 15.7 (scaled score) on the post KeyMath3 assessment, showing growth from the pre-intervention assessment. From the pre-intervention to the post-intervention assessments, students gained an average of 2 points in scaled scores, with one student gaining 1 point, one student gaining 2 points, and one student gaining 3 points.
Mathematical Agency

One purpose of the study was to examine what effect metacognitive strategy instruction had on the students’ mathematical agency. In order to examine this, the Agency, Authority, and Identity dimension rubric of the TRU Math Rubric from the Mathematics Assessment Project was utilized. The researcher and advisor rated each three-minute interval from the audio recordings collected during the three baseline lessons and the last lesson. The highest rating exhibited by the students during each interval was recorded. At the end of a recording, if the last interval did not last a complete three minutes, it was still included and rated. To score the intervals, the researcher and advisor scored the recordings individually and reconvened to discuss inter-rater reliability. An inter-rater reliability of 1.00 was established. Due to the age of the participants, the Agency, Authority, and Identity rubric was updated to reflect the verbal ability of younger participants. With the updated rubric, for a score of 1, “The teacher initiates conversations. Students’ speech turns are short and constrained by what the teacher says or does. Students give answers with no justification, or was prompted by the teacher.” For a score of 2, “Students have a chance to explain some of their thinking, but “the student proposes, the teacher disposes”: in class discussions, student ideas are not explored or built upon. The response is short— one sentence that is not built upon by other students.” For a score of 3, Students explain their ideas and reasoning. The teacher may ascribe ownership for students’ ideas in exposition, AND/OR students respond to and build on each other’s ideas, reasoning, or negotiate meaning. Please see Table 2 for the updated rubric descriptions.

Table 3 reports the ratings for all the intervals. During baseline, participants were audiotaped during the lessons on three separate days. On the first day of baseline, students scored six ratings of 1, four ratings of 2, and zero ratings of 3. On the second day of baseline, students scored eight ratings of 1, five ratings of 2, and zero ratings of 3. On the final day of baseline, students scored nine ratings of 1, four ratings of 2, and zero ratings of 3. However, during the last day of intervention, students were audiotaped again, and scored two ratings of 1, two ratings of 2, and eight ratings of 3. Please see Table 4 for the frequency of ratings for each audiotaped lesson. From baseline to the last day of intervention, the frequency of scores of 1 and 2 decreased while the frequency of scores of 3 increased. Also, when scores were averaged for the baseline lessons and the last audiotaped intervention lesson, the average score indicates an increase in agency. The following displays the average score during each lesson audiotaped: day one of baseline (1.40), day two of baseline (1.38), day three of baseline (1.31), and last day of intervention (2.50). The average ratings reported from the last day of intervention, was over one rating higher than any average rating reported during the baseline period. Please see Table 5 for the average ratings during each audiotaped lesson.

Table 7 displays transcripts for examples of each level of agency. It can be seen that the ratio of researcher to student talk changes among the score levels. The following were the ratios of researcher statements to student statements by score: Score of 1 (14:17), Score of 2 (13:13), and Score of 3 (10:46). There was more researcher guidance during scores of 1 and 2 but for scores of 3 there was less research guidance and more discussion among the three participants.

Discussion

Overview

Overall, the results indicate that metacognitive strategy instruction had a positive effect on the mathematics problem solving achievement of mathematically gifted second grade
students. Also, the metacognitive strategy instruction had a positive effect on the mathematical agency of the students.

It is important to consider that certain research questions and ideas cannot be evaluated through quantitative research. According to Trainor and Graue (2014), qualitative methods are beneficial when the ideas and thoughts of stakeholders related to the research questions are pivotal to the understanding and practice of the research area. The current study may be an example of such an area. In school systems, there are numerous individuals that play a role in the academic growth of students, and in this case, gifted students. Without considering these stakeholders, it would be difficult to understand if intervention protocols or groups would be feasible in the school setting with the resources that are available. If stakeholders do not consider interventions to be reasonable, the interventions may be less likely to continue over time (Leko, 2014; as cited in Greenwood & Abbott, 2001). Thus, it is important to consider how research questions or ideas cannot always be answered with quantitative research and it important to consider the growth of students in a variety of ways. The students in this case example had the opportunity to explain their thinking and develop their problem solving skills. The following section will aim at summarizing and interpreting the results of the study. Limitations, implications, and suggestions for future research will also be evaluated.

KeyMath3 - DA

The first purpose of the study was to examine the effect of metacognitive strategy instruction on the mathematics problem solving achievement of mathematically gifted second grade students. The KeyMath3 - DA measured the problem solving ability of gifted students pre intervention and post intervention with Form A. Students were given as much time as they needed to complete the items on the diagnostic assessment. The students were also given a pencil and paper to help them work out items at their discretion. Results indicated an increase in scaled scores from the pre-assessment (13.7) to the post assessment (15.7). Students increased an average scaled score of two points. This change in scaled scores suggests a growth in problem solving based on an increase of correct problems post intervention. However, the students displayed a slightly varied increase of scores. Students displayed the following increases: one student gained 1 point, another student gained 2 points, and the last student gained 3 points. It is important to note that student 1 who gained one scaled score was less actively involved with group discussion throughout the audiotaped lessons and during the other intervention lessons. The student completed a similar amount of problems during the intervention period compared to the other students in the group, but infrequently shared ideas without being prompted and critiqued the thinking of others. This suggests that in order to increase mathematical problem solving to a greater extent, it may be beneficial to be a more active learner when it comes to participation, sharing ideas, and critiquing the ideas of others.

Mathematical Agency

The other purpose of the present study was to examine the effects of metacognitive strategy instruction on the mathematical agency of gifted second grade students. The researcher and advisor rated each three minute interval from the audio recordings collected during the three baseline lessons and the last lesson. Results showed a decrease of 1 and 2 ratings and an increase of 3 ratings. When ratings were averaged for each lesson audiotaped, the averages indicated an increase in agency when comparing the baseline lessons and the last lesson. There was also changes in the teacher to student talk ratio. During the last lesson there was not only less
researcher guidance, but there was more student discussion and an increase in agency. Table 7 shows an example of each score possible on the TRU Math rubric. The examples provided in this table show that within the discussion that was rated a 1 on the rubric was much more teacher directed. In comparison, the discussion that was rated a 3 on the rubric primarily consisted of student conversation about the problem. During this conversation, students monitored the thinking of their peers in the group and worked together to solve the problem. Based on the Common Core State Standards Initiative, students must be able to reason, make justifications and review the reasoning process of other students (Common Core State Standards Initiative, 2010; Standards for mathematical practice). Results of the present student suggest that metacognitive strategy instruction may be beneficial to creating active learners and problem solvers. Students in the present study took a more active role in discussion by the end of the intervention period and were able to negotiate meaning with peers in the small group setting.

Limitations of the Study

One limitation of the study was the size of the group involved in the case study research. Although, this group provided benefits for in-depth study of mathematics problem solving and metacognitive strategies, it was not possible to make generalizations to a larger population. However, it was possible to formulate theories or additional hypothesis from the data that will be collected. Another limitation of the study was that due to the grouping of the high-ability students, it is possible that the grouping and tutoring sessions of these students affected the changes in the scores on the KeyMath3 Diagnostic Assessment.

The last limitation identified for the study was that students were assessed using Form A of the KeyMath3 assessment for the pre-intervention and post-intervention measures. It is possible that the participants recalled some of the questions from the pre-intervention assessment, when they were assessed after the intervention phase. Despite this, during assessment the assessor gives the student limited feedback whether she or he gets an item correct. Thus, although students may have recalled the questions, they do not know the correct answer due to the assessor only providing praise for attention and working hard on each item, not for a correct answer. Also, due to the eight week period between the pre and post assessments, students may not recall the information presented.

Despite the limitations of the current study, there are also several strengths. The small size of the participants involved in the study allows the researcher to study gifted students and their problem solving in depth. This may not be possible with a large group of participants. The researcher was able to observe through both observational and formal assessment and dissect how the use of cognitive strategy interventions may support the development of agency. While many gifted students perform above average in the area of mathematics on standardized assessments, this tells us little about the ability for gifted learners to negotiate meaning and communicate their thinking effectively. Utilizing mathematical practices as outlined in the CCSSM (2010) are considered critical standards for all learners, and are necessary to develop advanced mathematicians.

Implications

The results of the current study suggest several implications. First, it is important to consider previous studies that examined the effectiveness of metacognitive strategy instruction. Based on the literature that was examined prior to the present study, the majority of the studies examined the effectiveness with different populations; the participants were primarily older
students and students with learning disabilities. The present study suggests that metacognitive strategy instruction can be beneficial for gifted students to increase their problem solving ability and mathematical agency.

While it is typical of students who are considered gifted to score highly on math achievement measures it is also critical that we examine ways to increase their ability to verbalize their thinking. Training teachers to find ways to encourage verbalizations and student responses in math is needed, as well as, identifying ways to measures this type of student learning. This study suggests that there could be a correlation between increased agency and increased math achievement. Future research in this area is warranted.

Lastly, the average weekly or yearly growth was not available for the KeyMath3 - DA or the agency rubric. However, based on the descriptive and qualitative data that was examined, the students made positive growth during the eight-week intervention. Based on their growth in such a small intervention period, it can be concluded that a longer intervention period or a more intensive intervention when it comes to duration or frequency, may be beneficial for gifted students.

**Future Research**

Based on the current study, several things can be concluded. The study was able to produce positive effects on the mathematics problem solving ability of second grade gifted students. While metacognitive strategies have been helpful in increasing skills in students, the majority of studies have focused on increasing the abilities of older students or students who have skill deficits. The current study was also successful in generating positive effects on the mathematical agency of the young students.

Additional research is warranted based on the results from the current study. First, this study highlighted one metacognitive intervention that produced positive results for three mathematically gifted students. Additional research needs to be completed to examine the effects of teaching metacognitive strategies on other mathematically gifted students at the primary level due to the small number of participants in the present study. If positive effects are produced in future studies, there will be more evidence in the positive effects of using metacognitive strategies as an intervention to increase the problem solving ability of young mathematically gifted students.

Secondly, it may be beneficial to utilize the KeyMath3 form B for the post assessment instead of the KeyMath3 form A. By using a different form for the post assessment, researchers can eliminate the possibility of the students recalling the questions from the pre assessment. Additionally, it may be beneficial to track the problem solving ability with a different measure throughout the intervention period in order to examine the progress of students during the intervention. In a school system that utilizes response to intervention, it is a beneficial practice to examine the effectiveness of the intervention during the intervention and not just when the intervention has concluded. An example of measure that could be utilized for this purpose is a curriculum based measure in the area of math application or problem solving that is appropriate for the grade-level of the students.

Lastly, in the future it would be beneficial for research groups to include not only gifted students but average ability students and students with deficits in the area of math problem solving. By doing this, and examining the effects of teaching metacognitive strategies, researchers can eliminate one of the limitations of the present study. The grouping of the high-
ability students may have produced the positive effects in agency and problem solving rather than the teaching of metacognitive strategies.
References


Figure 1
TRU Math Rubric: Small Group Work

Table 1
Good Questions (Small, 2012) Example Problems

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>____ X ____ = 36. How many pairs of numbers can you use to fill in the blanks? What are they?</td>
</tr>
<tr>
<td>2.</td>
<td>Ethan has 30 stickers and Rebecca has 12. Ethan gives Rebecca 3 stickers at a time. Will Rebecca and Ethan ever have the same number of stickers? How many stickers would that be?</td>
</tr>
<tr>
<td>3.</td>
<td>You know that 20 + 25 = ___. What other equations have to be true if this one is?</td>
</tr>
<tr>
<td>4.</td>
<td>A pattern begins at 4. Choose an amount to keep adding to get future terms. Tell something you notice about your pattern and why you think what you observed occurs.</td>
</tr>
<tr>
<td>5.</td>
<td>Create a word problem that can be solved by subtracting two large numbers.</td>
</tr>
<tr>
<td>6.</td>
<td>Model two fractions with the same denominator. Tell which is greater and why.</td>
</tr>
</tbody>
</table>

Table 2
Updated Agency Rubric

| Agency, Authority, Identity Rubric |
|---|---|
| Score | Description |
The teacher initiates conversations. Students’ speech turns are short and constrained by what the teacher says or does. Students give answers with no justification, or was prompted by the teacher.

Students have a chance to explain some of their thinking, but “the student proposes, the teacher disposes”: in class discussions, student ideas are not explored or built upon. The response is short--- one sentence that is not built upon by other students.

Students explain their ideas and reasoning. The teacher may ascribe ownership for students’ ideas in exposition, AND/OR students respond to and build on each other’s ideas, reasoning, or negotiate meaning.

Table 3
*Mathematical Agency Ratings*

<table>
<thead>
<tr>
<th>Interval</th>
<th>Baseline 1</th>
<th>Baseline 2</th>
<th>Baseline 3</th>
<th>Post Intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 3 m</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3 - 6 m</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>6 - 9 m</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>9 - 12 m</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
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<tr>
<td>12 - 15 m</td>
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</tr>
<tr>
<td>15 - 18 m</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>18 - 21 m</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>21 - 24 m</td>
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</tr>
<tr>
<td>24 - 27 m</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>27 - 30 m</td>
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<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>30 - 33 m</td>
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<td>33 - 36 m</td>
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<tr>
<td>36 - 39 m</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4
*Frequency of Ratings During Each Lesson Audiotaped*

<table>
<thead>
<tr>
<th>Score</th>
<th>Baseline 1</th>
<th>Baseline 2</th>
<th>Baseline 3</th>
<th>Post Intervention</th>
</tr>
</thead>
</table>
Table 5
Average Ratings During Each Lesson Audiotaped

<table>
<thead>
<tr>
<th></th>
<th>Baseline 1</th>
<th>Baseline 2</th>
<th>Baseline 3</th>
<th>Post Intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Rating</td>
<td>1.40</td>
<td>1.38</td>
<td>1.31</td>
<td>2.50</td>
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</table>

Table 6
Percentages of Ratings During Baseline and Post-Intervention

<table>
<thead>
<tr>
<th></th>
<th>1s Percent</th>
<th>2s Percent</th>
<th>3s Percent</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0.6</td>
<td>0.4</td>
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<td></td>
<td>0.62</td>
<td>0.38</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.69</td>
<td>0.31</td>
<td>0.67</td>
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<tr>
<td></td>
<td>0.17</td>
<td>0.17</td>
<td></td>
</tr>
</tbody>
</table>

Table 7
Agency Scoring Examples with Questions and Dialogue

<table>
<thead>
<tr>
<th>AGENCY SCORE: 1</th>
<th>AGENCY SCORE: 2</th>
<th>AGENCY SCORE: 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline 1 (Minutes 9-12)</td>
<td>Baseline 3 (Minutes 12-15)</td>
<td>Post Intervention (Minutes 3-6)</td>
</tr>
<tr>
<td>Question: The sum of the digits of a 3-digit number is 12. How many base 10 blocks could you use to model that number?</td>
<td>Question: Choose two three-digit numbers to add for a total close to 400. How can you use base ten blocks to help you add?</td>
<td>Question: You multiply two numbers and the result is 54. What numbers might you have multiplied?</td>
</tr>
<tr>
<td>Student 2: One ten stick, two ones! Researcher: Okay, so let’s focus on the first part of the question. The sum of the digits of a 3-digit number is 12. What does that mean? Student 2: I don’t get that. Researcher: Yeah, Student 3? Student 3: It’s like… I don’t know. Researcher: You don’t know? Does anyone have an idea for</td>
<td>Student 3: I thought of what the answer could have been. Over here I kind of did one that… Researcher: Okay, so kind of like trial and error? Student 3: Yeah, and then and then I put an X through it and then did this one. Researcher: Okay, that makes sense. I think that’s good! How about you, Student 2? So how did you select the two numbers to make that total and what was</td>
<td>Student 2 &amp; T: 36, Student 3: 40 Student 2: then 44! Student 3: It’s 54! Student 2: Oh! Oh… Student 3: 54, 58 Student 2: You’re going past our number! Student 3: Oh it’s 40, it’s 48. Student 2 &amp; 3: 52, 50--No Student 2: No you can’t do it by 4’s Student 3: No you can’t do it</td>
</tr>
</tbody>
</table>
the first part?
All students: No…
Researcher: I’m going to show you an example and then you can go from there
Student 3: 3 digits…Oh wait I think I know what it means. Does it…It’s like something that equals 3?
Student 3: A three digit… Oh!
Researcher: So we’re adding these three digits
Student 2: But do they need to be the same number?
Researcher: No
Student 3: They can.
Student 2: 5
Researcher: So 5 for this place?
Student 1: Ohhhh
Researcher: I’ll show you my example. How about that? For my example, I chose 1, 9, and 2 because 1 plus 9 is 10 and 10 plus 2 is 12. Does that make sense?
Student 2: 192! Oh oh I know. Oh. 5 plus 5 plus 2.
Student 1: Awh man. Student 2 took my answer.
Student 3: I have one!
Researcher: Alright You can write your answers down for the first part of it. Three numbers that add up to 12 so that’ll be your 3-digit number.
Student 3: 5 plus three plus 4
Researcher: Okay. That’ll work. For the second part of the question, how many base ten blocks could you use to model that number?
Student 3: Uhm…Model…
Researcher: Do you know what base ten blocks are?
Student 3: Yes I know what base ten blocks are. You could use a tens stick and two ones?
Student 2: My two numbers were 200 and 215 and I got 415.
Researcher: Awesome! And how did you select those two numbers?
Student 2: Well I know that 200 plus 200 is exactly 400. So if I just take 15 more it would be close enough.
Researcher: Okay, okay. So it’s close enough. And how did you model the total you were trying to get close to? I think you did a good job doing that as well. You didn’t have to do that. You didn’t have to do that but I’m just wondering how did you do that.
Student 2: Well it said base ten blocks.
Researcher: Okay. Yeah. That’s good. And how did you know your number was close to 400? Like what does that mean?
Student 2: It’s only 15 more. Researcher: It’s only 15 off so that kind of made you think it was close enough?
Student 2: Yeah. I don’t know why.
Researcher: Alright, Student 1. So how did you select your two numbers to make that total and what was your total?
Student 1: My total was 410.
Researcher: Good. I think that’s close.
Student 1: And I said 200 plus 210 equals 410. And I used 4 hundreds blocks, and then I ten to get the number 410.
Researcher: Good! I think you did a great job modeling. So guys, I just want to discuss as a whole so what do you think would be too big of a number for close to 400? Yes, Student 3?
Student 3: 4, well 500.
Researcher: 500? Why do you that way
Student 2: Can you do it by 4’s?
Student 3: No
Researcher: Maybe draw it out? Would that help? Drawing out the problem?
Student 2: No…Okay so 4, 8, 12, 16, 20, 24.
Student 2 & 3: 28, 32, 36, 40, 44, 48, 52
Student 2: No. Can’t do it by 4’s
Researcher: Good job!
Student 2: Let’s try by 3’s
Researcher: I like the way you’re working together
Student 2: 3’s. So 3, 6----
Student 3: Yes! You can because if 4’s just
Student 2: Don’t work
Student 3: Yeah it does. 3, 6, 9,
Student 2: 4’s don’t work but 3’s do
Student 3: 12, 15, 17, no 18, 21, 24, 27, 30
Researcher: What are you thinking. Student 1?
Student 3: 33, 36
Researcher: I like what you have so far. Keep on working.
Student 3: 39, 41, 44! 15. 15 times 3
Researcher: But that’s 44.
Student 3: Oh. 44
Student 2 & 3: 44, 47, No.
Student 2: 3’s can’t
Student 3: because 3 more would be…
Student 2: 5’s! No 5’s can’t work
Student 3: 5. Yes it--- No.
Student 1: It doesn’t
Student 3: 2’s worked, 1 works, (laughs) of course 1 works,
Researcher: I have two more that will work.
Student 1: Huh?
Student 2: Let me see.
Researcher: No. Keep on working. You guys are close!
Student 1: Awh.
Researcher: A tens stick and two ones? But remember you added these three numbers together. Your number is 534.
Student 3: Oh. Uhm.
Researcher: So I’ll let you think about that for a second.

Researcher: Say 500? Seems like a big number.
Student 3: Well cause it’s not close at all to 400.
Researcher: Okay but I’m talking close to 400. So 500 would be too far?
Student 3: Oh. Yeah, too far.
Researcher: I think that’s a pretty good answer. How about you, Student 2? You had something to say?
Student 2: Uhm.
Researcher: How big would be too big for close to 400?

Student 3: 12’s?
Student 1: 12’s?
Researcher: No. I’ll tell you that so you don’t have to count up that high.
Student 2: 12’s. 24,
Student 3: Yeah. We’re getting close.
Student 2: 24
Researcher: You tried 4’s. 4’s didn’t work---
Student 2: No! I’m trying to do 12’s, 12,
Student 3: That would take us to 60...some.
Student 2: 24
Student 3: That would take us to 68 or 69
Student 2: I’m trying to think!

Table 8
*KeyMath3 Pre-test Form A and Post-test Form A Scaled Scores by Participant*

<table>
<thead>
<tr>
<th>Student</th>
<th>Pre Numeration Scaled Score</th>
<th>Post Numeration Scaled Score</th>
<th>Pre Applied Problem Solving Scaled Score</th>
<th>Post Applied Problem Solving Scaled Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>16</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>14</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>16</td>
</tr>
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</table>