ABSTRACT

ON THE NATURE OF RADIAL DISPERSION PROFILES FOR DWARF SPHEROIDAL GALAXIES IN THE LOCAL GROUP ACCORDING TO MOND

by Matthew James Walentosky

Modified Newtonian Dynamics (MOND) has long been the only proposed plausible alternative to dark matter to explain mass discrepancies observed in a wide array of systems. In this thesis, the ability of MOND to accurately predict the radial velocity dispersion profile of Milky Way and Andromeda dwarf spheroidal galaxies (dSphs) is investigated. Using a Hermite Individual Timestep Scheme based algorithm, the motion of 10,000 stars is tracked in a Hernquist potential with a MONDian correction applied to the acceleration of each star. The observable properties are compared to present day observations and used to generate radial velocity dispersion profiles for 5 Milky Way dSphs which can be compared to present day observations to demonstrate the accuracy of the procedure. Our calculations are in good agreement with observations. In addition, predictions are made for the radial velocity dispersion profiles for 17 Andromeda dSphs. The results of this study will show agreement between simulated, observed and calculated bulk dispersion and radial dispersion profiles. These predictions will serve as a test on the validity of MOND to either be verified or discredited by future observations.
This thesis titled

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by

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And III and And VI dSph radial dispersion profiles

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1 Introduction

Section 1.1 of this thesis will introduce the origin of the central discussion, the missing matter problem. It will address the discrepancies between the observed rotation curves for galaxies and the Newtonian predicted rotation curves. Section 1.2 will introduce the theory of Modified Newtonian Dynamics (MOND) and its predictions. Sections 1.3 and 1.4 will discuss dwarf spheroidal galaxies and their observable properties which can be tested using MOND. Finally, Section 1.5 will clearly state the problem being presented in this thesis and will outline how this problem is answered throughout the duration of this document.

1.1 Missing Matter Problem

At the forefront of scientific inquiry in physics and astronomy today is the missing matter problem. The discrepancy between Newtonian predicted dynamical masses in galaxies and actual observations dates back to astronomer Fritz Zwicky’s observations of the Coma cluster of galaxies [22] in the early 1930s. Zwicky, an astronomer at Caltech, noted that the outlying galaxies in the Coma cluster were moving much faster than the mass calculated for the visible galaxies seemed to indicate. Based on Zwicky’s observations, these clusters should have flown apart long ago because the visible stars in the galaxies could not account for all the gravity necessary to keep the cluster together. Zwicky’s calculations suggested that as much as 90% of the cluster’s total mass was dark matter or “missing matter.”

This mystery remained unsolved and relatively untouched for several decades until Vera Rubin first presented observational evidence that most stars in the outer part of a spiral galaxy orbit at the same speed [18]. An example of such observations is seen in Figure 1. Newtonian physics predicts that stars on the edge of a spiral galaxy should have velocities that decrease inversely proportional to the square root of their distance from the center of the galaxy.
It was concluded by astronomers at the time that there must be some missing mass, which drove the stars at further distances to move at the same velocities. This missing mass has come to be known as dark matter and remains the accepted theory in science to explain these observations. However, no direct observational evidence for dark matter has ever been observed. To quote Sanders, 2002 review article [19]

“The appearance of discrepancies between the Newtonian dynamical mass and the directly observable mass in large astronomical systems has two possible explanations: either these systems contain large quantities of unseen matter, or gravity (or the response of particles to gravity) on these scales is not described by Newtonian theory.”

While the theory of dark matter remains the consensus explanation for the rotation curve observation discrepancy, one viable alternative theory does exist. This theory, Modified Newtonian Dynamics, will be outlined in the proceeding section.

1.2 Proposal of MOND

In 1983 Israeli physicist Mordehai Milgrom proposed his theory of Modified Newtonian Dynamics (MOND), which is founded on the principle that the discrepancy between the Newtonian predicted falloff for rotational velocity at the edge of a spiral galaxy is not due to dark matter, but instead is a result of a subtle correction to
Newtonian mechanics at extremely small accelerations, on the order of $10^{-10}$ m/s$^2$ [16]. This acceleration is referred to as $a_o$ [19]. Viewed as a modification of inertia, MOND suggests that Newton’s Second Law is:

$$\vec{F} = m\vec{a} \mu \left( \frac{a}{a_o} \right).$$  \hspace{1cm} (2)

In this formula $\vec{a}$ is the acceleration and $\mu(x)$ is a matching function which obeys the following properties:

$$\lim_{x \to \infty} \mu(x) = 1,$$

$$\lim_{x \to 0} \mu(x) = x. \hspace{1cm} (3)$$

![Acceleration vs. Acceleration](image)

Figure 2: Plotted above is an acceleration vs. acceleration plot of MOND acceleration vs. Newtonian acceleration. The blue line is MOND acceleration via Equation (5) and the green line is the Newtonian acceleration. Notice how the MOND acceleration diverges at extremely low accelerations.

Figure 2 shows how MOND acceleration differs from purely Newtonian acceleration at $10^{-10}$ m/s$^2$ using the specific matching function described in Section 2.4. However, when MOND is viewed as a modification of inertia the principle of conservation of momentum is not obeyed [8]. MOND as a modification of gravity implies that gravitational acceleration is defined as:

$$\vec{g}_n = \vec{g} \mu \left( \frac{g}{a_o} \right).$$  \hspace{1cm} (4)
where \( \vec{g}_n \) is the Newtonian acceleration and \( \vec{g} \) is the actual acceleration. In either the modification of gravity or inertia, it can be shown that the gravitational force when \( g \ll a_o \) is described as:

\[
g = \sqrt{g_n a_o}.
\]  

When the gravitational acceleration \( g \) is set equal to \( v^2/r \) as in the case of centripetal acceleration (for say an object moving in a circular motion around mass \( M \)), it can be shown that:

\[
g_n = \frac{GM}{r^2}.
\]

and

\[
g = \frac{v^2}{r},
\]

\[
= \sqrt{g_n a_o}.
\]

From this, it can be seen that MOND requires a constant velocity for stars rotating around the edge of a galaxy:

\[
v = (GMa_o)^{1/4}.
\]  

This is the deep MOND rotation speed for a spiral galaxy [19]. The proportion between \( M \) and \( v^4 \) is the well known as the Tully Fisher relationship and is absolute in MOND. This \( M \propto v^4 \) relationship predicts an asymptotically flat rotation curve for stellar velocities as a function of distance from the center [4].

1.3 Dwarf Spheroidal Galaxies as a Testing Ground

Dwarf Spheroidal galaxies (dSph) provides an excellent testing ground for both dark matter and MOND. These systems contain such a low baryonic mass (\( \sim 10^5 - 10^7M_\odot \)) that they should not be gravitationally bound according to Newtonian physics. As a comparison, the Milky Way Galaxy has an estimated mass of \( 10^{12}M_\odot \). Dwarf spheroidal galaxy systems are thought to contain a heavy presence of dark matter, or if dark matter doesn’t exist they are in the deep MOND regime.

Both the Milky Way and Andromeda dSph satellites are very low in surface density, pressure supported stellar systems, that are viable tests for MOND predictions. Observations of the classical Milky Way dSphs from Walker et. al., (2007) and Walker et. al., (2009) provide measurements of velocity dispersion as a function of radius. In addition to these Milky Way galaxies, observations from McGaugh and Wolf, (2010),

From McGaugh and Milgrom, (2013a) an analytical estimate for the MOND line of sight velocity dispersion where $g < < a_o$ is provided below.

$$MGa_o = \frac{9}{4} \langle v^2 \rangle^2.$$  \hspace{1cm} (7)

where $\langle v^2 \rangle$ is the 3D mass weighted mean-square velocity, and $M$ is the total mass of the system.

From Equation (7) $\langle v^2 \rangle^{1/2}$ can be calculated to find $\sigma$, assuming the dSphs are isotropic. By representing $\langle v^2 \rangle$ as $3\sigma^2$, we can write the line of sight dispersion predicted by MOND as:

$$\sigma_{iso} \approx \left( \frac{4}{81} MGa_o \right)^{1/4}.$$  \hspace{1cm} (8)

Where the subscript $iso$ means no external forces are acting on the galaxy [14].

1.4 Observable Properties

Dwarf spheroidal galaxies have multiple observable properties of particular interest to this study. The first is the observed line of sight bulk dispersion of a galaxy, the dispersion is defined as:

$$\sigma = \sqrt{\langle v_x^2 \rangle - \langle v_x \rangle^2}.$$ 

Where $v_x$ is the velocity in the $x$ direction for all the stars in the galaxy, assuming $x$ is in the direction of Earth.
Figure 3: Position of stars in Leo I dSph at 1500 Myr. The color bar represents the velocity of the stars in the $x$ direction, $v_x$ (km/s).

Figure 3 is provided to clearly display and define the cartesian coordinate system referenced in this work. The second observable property is the radial dispersion profile. This measures the radial dispersion of a galaxy as a function of distance from the center. In addition astronomers are able to observe the luminosity and half light radius, both of which are critical to the results presented in the preceding sections.

1.5 Statement of Problem

This thesis will address the effectiveness of MOND to predict both the bulk line of sight velocity dispersion and dispersion profile for dwarf spheroidal galaxies. Five Milky Way and seventeen Andromeda dSph galaxies will be simulated to match present day (stable state) conditions using MOND theory. First, the galaxies will be simulated at a variety of mass to light ratios based on luminosities from McGaugh and Milgrom, (2013a), McGaugh and Milgrom, (2013b), and Walker et. al., (2009). Then the stable state bulk dispersion will be calculated for each galaxy. Using the simulation whose mass to light ratio most closely matches the observed bulk dispersion, a radial velocity dispersion profile will be generated in a manner which replicates current
observational techniques. These radial dispersion profiles will serve as predictions for the Andromeda dSph galaxies until observed radial dispersion profiles are able to be observed, and will serve as a test to the validity of MOND as an alternative to dark matter. Chapter 2 will discuss the methodology behind the galaxy simulation code, Chapter 3 will discuss the techniques used to analyze the data produced by the galaxy simulation code, Chapter 4 will present the results for the line of sight bulk dispersion and bulk dispersion profile for the Milky Way and Andromeda dSph galaxies, the Milky Way radial dispersion profiles will be compared to observations to demonstrate the accuracy of our technique, finally these results will be discussed in Chapter 5.
2 Computational Model

Section 2.1 of this thesis will explain the Hernquist density profile and how it is used in this galaxy simulation. Section 2.2 will introduce and reference the Hermite Individual Timestep Scheme, or HITS Integrator. Section 2.3 will discuss how the acceleration and its time derivative, the jerk, is calculated for a Hernquist density profile. Section 2.4 will discuss the constant density core used in the galaxy simulation and why it is necessary. Section 2.5 will discuss the MOND correction for the acceleration and jerk. Section 2.6 will provide examples of orbits with MOND in a Hernquist profile. Finally, Section 2.7 will discuss the initial conditions used for the 17 Andromeda dSph galaxies and 5 Milky Way dSph galaxies being simulated.

2.1 Hernquist Density Profile

The density profiles used in these simulations are based on the Hernquist Model [9], [3]. The density function for a spherical galaxy of total mass $M$ is described by the following formula:

$$\rho_{\text{Hern}}(r) = \frac{M}{2\pi} \frac{a_s}{r(r+a_s)^3}.$$  \hspace{1cm} (9)

Where $r$ is the distance from the center and $a_s$ is the scale length. The interior mass can be calculated as a function of $r$.

$$M_{\text{Hern}}(r) = 4\pi \int_0^r r^2 \rho_{\text{Hern}}(r)dr,$$ \hspace{1cm} (10)

$$M_{\text{Hern}}(r) = M \frac{r^2}{(r+a_s)^2}.$$ \hspace{1cm} (11)

For the Hernquist model the relationship between $r_{1/2}$, the half mass radius and $a_s$, the scale length is:

$$r_{1/2} = (1+\sqrt{2})a_s.$$ \hspace{1cm} (12)

2.2 Hermite Individual Timestep Scheme

The algorithm for calculating the evolution of stellar positions is based on the Hermite Individual Timestep Scheme [12, 1, 2, 6]. The procedure by which the HITS Integrator works is to calculate the Newtonian acceleration and jerk for each body is outlined below. The procedure used in this code is not a pure N-body in that the calculation for the acceleration of each star is not dependent on the accompanying stars in the galaxy, but instead is solely dependent on the interaction between the star with a baryonic mass cloud, which approximates the interaction with a large number of stars. This is necessary to provide an appropriate mass for
a dwarf spheroidal galaxy ($\sim 10^7 M_\odot$).

### 2.3 Integration Technique

The acceleration of a point mass interacting with the baryonic mass cloud, and the interior mass $M_{\text{int}}$ in a Hernquist potential are defined:

\begin{equation}
\vec{a} = -\frac{GM_{\text{int}} \vec{r}}{r^3},
\end{equation}

\begin{equation}
M_{\text{int}} = M \frac{r^2}{(r + a_s)^2}.
\end{equation}

The derivative of the acceleration, commonly known as the jerk is generally defined as:

\begin{equation}
\dot{\vec{a}} = \frac{d\vec{a}}{dt},
\end{equation}

\begin{equation}
= -G \left( M_{\text{int}} \frac{d}{dt} \left( \frac{\vec{r}}{r^3} \right) + \frac{\vec{r}}{r^3} \frac{d}{dt} (M_{\text{int}}) \right).
\end{equation}

The first term in the above expression gives the standard jerk for a point mass from the HITS Integrator can be described as:

\begin{equation}
- \frac{GM_{\text{int}}}{r^3} \frac{d}{dt} \left( \frac{\vec{r}}{r^3} \right) = - \frac{GM_{\text{int}}}{r^3} \left[ \frac{\vec{v}}{r^2} - \frac{3\vec{r}}{r^3} (\vec{r} \cdot \vec{v}) \right].
\end{equation}

This is the standard equation for the jerk of point masses [12, 1, 2]. The second term arises due to the interaction with a distributed mass and is described as:

\begin{equation}
- \frac{GM_{\text{int}}}{r^3} \frac{d}{dt} \left( \frac{\vec{r}}{r^3} \right) = - \frac{GM_{\text{int}}}{r^3} \frac{d}{dt} \left[ \frac{r^2}{(r + a_s)^2} \right].
\end{equation}
The derivative in Expression (17) can be further simplified by the following steps:

\[
\frac{d}{dt} \left[ \frac{r^2}{(r+a_s)^2} \right] = \frac{1}{(r+a_s)^2} \frac{dr}{dt} r^2 + r^2 \frac{d}{dt} \frac{1}{(r+a_s)^2},
\]

\[
= \frac{1}{(r+a_s)^2} \frac{d}{dt} (\vec{r} \cdot \vec{r}) + r^2 \frac{d}{dt} \frac{1}{(r+a_s)^2},
\]

\[
= \frac{1}{(r+a_s)^2} \left[ \vec{r} \cdot \frac{d\vec{r}}{dt} + \frac{d\vec{r}}{dt} \cdot \vec{r} \right] + r^2 \left[ -2(r+a_s)^{-3} \frac{dr}{dt} \right],
\]

\[
= \frac{2\vec{r} \cdot \vec{v}}{(r+a_s)^2} - \frac{2r^2}{(r+a_s)^3} \frac{d}{dt} (\vec{r} \cdot \vec{r})^{1/2},
\]

\[
= \frac{2\vec{r} \cdot \vec{v}}{(r+a_s)^2} - \frac{2r^2}{(r+a_s)^3} \frac{(\vec{r} \cdot \vec{r})^{-1/2}}{2} \left( \vec{r} \cdot \frac{d\vec{r}}{dt} + \frac{d\vec{r}}{dt} \cdot \vec{r} \right),
\]

\[
= \frac{2(\vec{r} \cdot \vec{v})}{(r+a_s)^2} - \frac{2r^2}{2(r+a_s)^3} \frac{1}{r} (2\vec{r} \cdot \vec{v}),
\]

\[
= \left[ \frac{2}{(r+a_s)^2} - \frac{2r}{(r+a_s)^3} \right] (\vec{r} \cdot \vec{v}).
\]

This leaves the second term in Equation (16) as:

\[-GM \frac{\vec{r}}{r^3} \left[ \frac{2}{(r+a_s)^2} - \frac{2r}{(r+a_s)^3} \right] (\vec{r} \cdot \vec{v}).\]

The entire expression for \( \ddot{a} \) in a Hernquist potential may now be rewritten as:

\[
\ddot{a} = -\frac{GM_{\text{int}} r^2}{(r+a_s)^2} \left[ \frac{\vec{v}}{r^3} - \frac{3\vec{r}}{r^5} (\vec{r} \cdot \vec{v}) \right] - \frac{2GM_{\text{int}} r}{r^3(r+a_s)^2} \left[ 1 - \frac{r}{r+a_s} \right] (\vec{r} \cdot \vec{v}),
\]

\[
= -\frac{GM_{\text{int}} r^2}{(r+a_s)^2} \left[ \frac{\vec{v}}{r^3} - \frac{3\vec{r}}{r^5} (\vec{r} \cdot \vec{v}) \right] - \frac{2GM_{\text{int}} r}{r^3(r+a_s)^2} \left[ 1 - \frac{1}{r+a_s} \right] (\vec{r} \cdot \vec{v}),
\]

\[
= -\frac{GM_{\text{int}}}{r^3} \left[ \frac{\vec{v}}{r^3} - \frac{3\vec{r}}{r^5} (\vec{r} \cdot \vec{v}) \right] - \frac{2GM_{\text{int}} r}{r^3(r+a_s)^2} \left[ 1 - \frac{r}{r+a_s} \right] (\vec{r} \cdot \vec{v}).
\]
2.4 Constant Density Core

An unavoidable consequence of the Hernquist profile is that it gives a non-zero acceleration for an object at \( r = 0 \). To correct for this, a constant density core is placed at the center of the galaxy. In this core, the interior mass is defined as:

\[
M_{\text{int}}(r) = \frac{Mr^3}{R^3}
\]

Where \( R \) is the core radius, which is defined as a fraction of the scale length, \( a_s \), for these simulations \( R = \alpha a_s \) with \( \alpha = 0.1 \). The acceleration, \( \ddot{a} \), and the jerk, \( \dot{\ddot{a}} \), are still defined by Equations (13) and (15). The variation of mass with respect to time can be re-written and simplified:

\[
\frac{dM_{\text{init}}}{dt} = \frac{M}{R^3} \frac{d}{dt} (r^3),
\]

\[
= \frac{M}{R^3} 3 r^2 \frac{dr}{dt},
\]

\[
= \frac{3M}{R^3} r (\vec{r} \cdot \vec{v}).
\]

This leads to a jerk for the case of a constant density core as:

\[
\dot{\ddot{a}} = -GM_{\text{int}} \left[ \frac{\vec{v}}{r^3} - \frac{3 \vec{r}}{r^5} (\vec{r} \cdot \vec{f}) \right] + \frac{\vec{r}}{r^3} \frac{3M}{R^3} r (\vec{r} \cdot \vec{v}),
\]

\[
= -GM_{\text{int}} \frac{\vec{v}}{r^3} - \frac{GM_{\text{int}} 3 \vec{r}}{r^5} (\vec{r} \cdot \vec{v}) + \frac{\vec{r}}{r^2} \frac{3M}{R^3} \left( \frac{r^3}{r^3} \right),
\]

\[
= -GM_{\text{int}} \frac{3 \vec{r}}{r^5} (\vec{r} \cdot \vec{v}),
\]

\[
= -GM_{\text{int}} \frac{\vec{v}}{r^3}.
\]

2.5 MOND Correction to the Acceleration and Jerk

A chosen matching function which will meet the limits outlined in Equation (3) used to simulate MOND is:

\[
\mu(x) = \frac{x}{1 + x}.
\]
MOND as a modification of gravity is defined as:

\[ \vec{g} \mu \left( \frac{\vec{g}}{a_o} \right) = \vec{g}_n. \]  

(19)

where \( \vec{g} \) is the actual acceleration, \( \vec{g}_n \) is the Newtonian acceleration, and \( a_o \) is a MOND parameter of \( 1 \times 10^{-10} \) m/s\(^2\). We may now rewrite Equation (19) in the following format:

\[ \vec{a}_n = \vec{a} \mu \left( \frac{\vec{a}}{a_o} \right). \]  

(20)

From this formula, an expression for the MONDian acceleration and jerk is derivable by the following steps:

First, the vector magnitude of both sides of Equation (20) is taken

\[ |\vec{a}_n| = |\vec{a}| \mu \left( \frac{|\vec{a}|}{a_o} \right). \]

This is the equivalent of stating that:

\[ |\vec{a}_n| = \frac{|\vec{a}| |\vec{a}|}{a_o \left(1 + \frac{|\vec{a}|}{a_o}\right)} . \]

Through the proceeding algebraic steps, the MOND acceleration \( \vec{a} \) can be calculated:

\[ \frac{|\vec{a}|^2}{a_o} = |\vec{a}_n| + \frac{|\vec{a}_n| |\vec{a}_n|}{a_o} , \]

\[ = a_o |\vec{a}_n| + a_o |\vec{a}_n| . \]

This formula can be solved quadratically as such:

\[ |\vec{a}| = \frac{|\vec{a}|}{2} \pm \frac{1}{2} \sqrt{|\vec{a}|^2 + 4 |a_o| |\vec{a}_n|} . \]

By taking the positive root, we are left with the final expression for the MONDian acceleration:

\[ |\vec{a}| = \frac{|\vec{a}_n|}{2} \left[1 + \sqrt{1 + 4 \frac{a_o}{|\vec{a}_n|}} \right] . \]
The time derivative of $\ddot{a}$ can be taken to give the MONDian jerk:

$$\ddot{a} = \frac{1}{2} \ddot{a}_n \left( 1 + \sqrt{1 + 4 \frac{a_o}{a_n}} \right) + \ddot{a}_n \frac{d}{dt} \left( 1 + 4 \frac{a_o}{a_n} \right)^{1/2},$$

$$= \frac{1}{2} \ddot{a}_n \left( 1 + \sqrt{1 + 4 \frac{a_o}{a_n}} \right) + \frac{\ddot{a}_n}{2} \left( 1 + 4 \frac{a_o}{a_n} \right)^{-1/2} 4 \frac{d}{dt} \left( \frac{1}{a_n} \right),$$

$$= \frac{1}{2} \ddot{a}_n \left( 1 + \sqrt{1 + 4 \frac{a_o}{a_n}} \right) + 2a_n \left( 1 + 4 \frac{a_o}{a_n} \right)^{-1/2} a_o \frac{d}{dt} \left( \frac{1}{a_n} \right).$$

(21)

To further elaborate:

$$\frac{d}{dt} \left( \frac{1}{a_n} \right) = \frac{d}{dt} (\ddot{a}_n \cdot \ddot{a}_n)^{-1/2},$$

$$= - \frac{1}{2} (\ddot{a}_n \cdot \ddot{a}_n)^{-3/2} (\ddot{a}_n \cdot \dddot{a}_n + \dddot{a}_n \cdot \ddot{a}_n),$$

$$= - \frac{2 \dddot{a}_n}{2a_n^3},$$

$$= \dddot{a}_n \cdot \ddot{a}_n \frac{1}{a_n^3}.$$

At long last we are left with the following expression for $\ddot{a}$, the MONDian correction for the jerk:

$$\ddot{a} = \frac{1}{2} \ddot{a}_n \left( 1 + \sqrt{1 + 4 \frac{a_o}{a_n}} \right) - 2 \frac{d}{dt} \left( 1 + 4 \frac{a_o}{a_n} \right)^{1/2} \frac{d}{dt} \left( \frac{1}{a_n} \right).$$

2.6 Orbits in a Hernquist Profile With MOND

The orbits for two randomly selected stars at various distances are plotted below in Figures 4, 5, and 6. These orbits are plotted in the $y,z$ plane to show how they would appear from Earth. It can be seen that a star in a MONDian orbit will have a larger acceleration and will complete more orbits than the same star with a
Newtonian acceleration over the same period of time.

Figure 4: Orbits of two randomly selected stars with initial positions between 200 and 400 pc. The MOND orbits are shown in the left figures and the Newtonian orbits in the right figures. The initial position is represented by a blue marker.
Figure 5: Orbits of two randomly selected stars with initial positions between 400 and 600 pc. The MOND orbits are shown in the left figures and the Newtonian orbits in the right figures. The initial position is represented by a blue marker.
Figure 6: Orbits of two randomly selected stars with initial positions between 600 and 800 pc. The MOND orbits are shown in the left figures and the Newtonian orbits in the right figures. The initial position is represented by a blue marker.
2.7 Initial Positioning of Stars

The initial positioning of the stellar bodies is based on the Hernquist Density and Mass Profiles described in Equations (9) and (11) respectively. A radial density probability function is defined by the following formula:

\[ P(r) = 4\pi r^2 \rho_{\text{Hern}}dr , \]

\[ = 2M \frac{a_s r}{(r + a_s)^3} dr . \]  \hspace{1cm} (22)

This function divided by a given solar mass will give the number of stars per width \( dr \). Using Equation (22), a trial \( r \) value, \( r_{\text{trial}} \), is calculated such that:

\[ r_{\text{trial}} = (r_{\text{max}} - r_{\text{min}})(c) + r_{\text{min}} \]

Where \( r_{\text{min}} = 0 \text{ pc} \), \( r_{\text{max}} = 10,000 \text{ pc} \), and \( c \) is a random number between 0 and 1 from a uniform distribution. Next, a second random number is compared to \( P(r_{\text{trial}}) \). If \( P(r_{\text{trial}}) \) is larger than \( P(r_{\text{trial}}) \), then the \( r \) value for the star is set equal to \( r_{\text{trial}} \).

![Histogram of Initial Radial Positions](image)

Figure 7: Histogram of the initial positions of 10,000 stars in Leo I dSph.

Each star is also given a random initial \( \theta \) between 0 and \( 2\pi \) and \( \phi \) between 0 and \( \pi \). From these initial
\((r, \theta, \phi)\) coordinates the initial \((x, y, z)\) coordinates are calculated. All simulations were run with 10,000 stars for 2500 Myr, the initial velocities \((v_x, v_y, v_z)\) for each star is randomly selected between -15 and 15 km/s.

2.8 Initial Conditions

The observational data and analytical predictions used in these simulations were primarily taken from Walker et. al. (2009), McConnachie et. al. (2012), McGaugh and Milgrom, (2013a), and McGaugh and Milgrom (2013b). The properties for the Milky Way dSph galaxies and the Andromeda dSph galaxies are presented in Tables 1 and 2 respectively.

<table>
<thead>
<tr>
<th>Dwarf</th>
<th>(r_{MW}) (kpc) (^b)</th>
<th>(r_{1/2}) (pc) (^c)</th>
<th>(L_V (10^7 L_\odot)) (^d)</th>
<th>(\sigma_{obs}) (km/s) (^e)</th>
<th>(\sigma_{iso}) (km/s) (^f)</th>
<th>Ref. (^g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carina</td>
<td>841</td>
<td>137 ± 22</td>
<td>2.4</td>
<td>6.6 ± 1.2</td>
<td>4.4 (^{+0.5}_{-0.7})</td>
<td>1.2</td>
</tr>
<tr>
<td>Fornax</td>
<td>772</td>
<td>339 ± 36</td>
<td>140</td>
<td>11.7 ± 0.9</td>
<td>12.2 (^{+2.3}_{-2.0})</td>
<td>1.2</td>
</tr>
<tr>
<td>Leo I</td>
<td>922</td>
<td>133 ± 15</td>
<td>34</td>
<td>9.2 ± 1.4</td>
<td>8.6 (^{+1.6}_{-1.4})</td>
<td>1.2</td>
</tr>
<tr>
<td>Leo II</td>
<td>901</td>
<td>123 ± 27</td>
<td>5.9</td>
<td>6.6 ± 0.7</td>
<td>5.5 (^{+1.1}_{-0.9})</td>
<td>1.2</td>
</tr>
<tr>
<td>Sculptor</td>
<td>765</td>
<td>94 ± 26</td>
<td>14</td>
<td>9.2 ± 1.1</td>
<td>6.8 (^{+1.3}_{-1.1})</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Table 1: The known observational properties of the 5 Milky Way dSph galaxies are presented above. References: (1) [21], (2) [13]

Column a: dSph name
Column b: Distance to Milky Way Galaxy
Column c: 3D Half light radius
Column d: Luminosity
Column e: Observed bulk dispersions
Column f: MOND predicted bulk dispersions structured in the format \(M/L = 2^{+2}_{-1}\) based on Equation (8)
Column g: References
Table 2: The known observational properties of the 17 Andromeda dSph galaxies are presented above. **References:** (1) [20], (2) [11], (3) [10], (4) [17], (5) [13], (6) [7], (7) [20], (8) [5], (9) [14], (10) [15].

<table>
<thead>
<tr>
<th>Dwarf</th>
<th>$r_{M31}$ (kpc)</th>
<th>$r_{1/2}$ (pc)</th>
<th>$L_V$ ($10^3 L_\odot$)</th>
<th>$\sigma_{obs,1}$ (km/s)</th>
<th>$\sigma_{obs,2}$ (km/s)</th>
<th>$\sigma_{iso}$ (km/s)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>And I</td>
<td>58.4$^{+2.4}_{-3.4}$</td>
<td>832</td>
<td>45</td>
<td>10.2$^{+1.9}_{-1.0}$</td>
<td>10.6$^{+1.1}_{-1.0}$</td>
<td>9.2$^{+1.5}_{-1.4}$</td>
<td>1.2</td>
</tr>
<tr>
<td>And II</td>
<td>184</td>
<td>1660</td>
<td>93</td>
<td>7.3$^{+0.8}_{-0.7}$</td>
<td>10.0$^{+1.7}_{-1.1}$</td>
<td>11.0$^{+2.1}_{-1.1}$</td>
<td>2.3</td>
</tr>
<tr>
<td>And III</td>
<td>75.2$^{+3.5}_{-3.4}$</td>
<td>525</td>
<td>10</td>
<td>9.3$^{+1.4}_{-1.1}$</td>
<td>4.7$^{+1.8}_{-1.1}$</td>
<td>6.3$^{+1.2}_{-1.1}$</td>
<td>1.2</td>
</tr>
<tr>
<td>And VI</td>
<td>269</td>
<td>440</td>
<td>34</td>
<td>9.4$^{+2.4}_{-2.4}$</td>
<td>...</td>
<td>8.6$^{+2.1}_{-2.1}$</td>
<td>1.4</td>
</tr>
<tr>
<td>And VII</td>
<td>218.3$^{+3.5}_{-3.4}$</td>
<td>977</td>
<td>178</td>
<td>13.0$^{+1.0}_{-1.1}$</td>
<td>9.7$^{+1.6}_{-1.1}$</td>
<td>13.0$^{+2.4}_{-2.5}$</td>
<td>1.2</td>
</tr>
<tr>
<td>And IX</td>
<td>40.5$^{+3.9}_{-3.9}$</td>
<td>552</td>
<td>1.5</td>
<td>10.9$^{+2.0}_{-2.0}$</td>
<td>4.5$^{+3.4}_{-3.4}$</td>
<td>3.9$^{+0.6}_{-0.6}$</td>
<td>1.5</td>
</tr>
<tr>
<td>And X</td>
<td>109.4$^{+7.9}_{-9.0}$</td>
<td>309</td>
<td>0.76</td>
<td>6.4$^{+1.4}_{-1.3}$</td>
<td>3.9$^{+1.2}_{-1.1}$</td>
<td>3.0$^{+0.5}_{-0.5}$</td>
<td>1.2</td>
</tr>
<tr>
<td>And XIV</td>
<td>162.3$^{+1.3}_{-1.6}$</td>
<td>537</td>
<td>2.1</td>
<td>5.3$^{+1.0}_{-1.0}$</td>
<td>5.4$^{+1.3}_{-1.3}$</td>
<td>4.3$^{+0.7}_{-0.7}$</td>
<td>1.2</td>
</tr>
<tr>
<td>And XV</td>
<td>93.6$^{+7.4}_{-7.4}$</td>
<td>355</td>
<td>7.1</td>
<td>4.0$^{+1.4}_{-1.4}$</td>
<td>...</td>
<td>5.8$^{+1.1}_{-0.9}$</td>
<td>1.9</td>
</tr>
<tr>
<td>And XVI</td>
<td>279.4$^{+3.6}_{-5.5}$</td>
<td>178</td>
<td>4.1</td>
<td>3.8$^{+2.9}_{-2.9}$</td>
<td>...</td>
<td>5.0$^{+1.0}_{-1.0}$</td>
<td>1.9</td>
</tr>
<tr>
<td>And XIX</td>
<td>189</td>
<td>2244</td>
<td>4.1</td>
<td>4.7$^{+1.6}_{-1.4}$</td>
<td>...</td>
<td>5.0$^{+1.0}_{-1.0}$</td>
<td>5.6</td>
</tr>
<tr>
<td>And XXI</td>
<td>149.2$^{+5.7}_{-3.9}$</td>
<td>1023</td>
<td>4.6</td>
<td>7.2$^{+5.5}_{-5.5}$</td>
<td>4.5$^{+1.2}_{-1.0}$</td>
<td>5.2$^{+1.0}_{-1.0}$</td>
<td>1.9</td>
</tr>
<tr>
<td>And XXII</td>
<td>220.6$^{+2.4}_{-2.4}$</td>
<td>340</td>
<td>0.35</td>
<td>3.5$^{+4.2}_{-2.5}$</td>
<td>2.8$^{+1.9}_{-1.4}$</td>
<td>2.7$^{+0.5}_{-0.5}$</td>
<td>1.8</td>
</tr>
<tr>
<td>And XXIII</td>
<td>127</td>
<td>1372</td>
<td>10</td>
<td>7.1$^{+1.0}_{-1.0}$</td>
<td>...</td>
<td>6.4$^{+1.2}_{-1.0}$</td>
<td>5.6</td>
</tr>
<tr>
<td>And XXV</td>
<td>88</td>
<td>945</td>
<td>6.5</td>
<td>3.0$^{+1.2}_{-1.2}$</td>
<td>...</td>
<td>5.7$^{+1.1}_{-1.0}$</td>
<td>5.6</td>
</tr>
<tr>
<td>And XXVIII</td>
<td>368</td>
<td>284</td>
<td>2.1</td>
<td>6.6$^{+2.1}_{-2.1}$</td>
<td>4.9$^{+1.6}_{-1.6}$</td>
<td>4.3$^{+0.8}_{-0.7}$</td>
<td>4.5</td>
</tr>
<tr>
<td>And XXIX</td>
<td>188</td>
<td>481</td>
<td>1.8</td>
<td>...</td>
<td>5.7$^{+1.2}_{-1.2}$</td>
<td>4.1$^{+0.8}_{-0.7}$</td>
<td>5.7</td>
</tr>
</tbody>
</table>

The initial half mass radii, used in Equation (12), are taken from Column C in Tables 1 and 2. At each timestep, $r_{1/2}$ is re-calculated based on the current position of all of the stars and used to adjust $a_t$. All MOND models reach a stable state by at least 500 million years. The stable state is defined as when $\frac{ddt}{dt} \approx 0$, initially $\sigma$ changes with respect to time because stars with a velocity above the escape velocity threshold are still being calculated in the bulk dispersion. Over time, these stars leave the system and are no longer in the dispersion calculation. Table 2 includes 17 of the 27 galaxies discussed in McGaugh and Milgrom, 2013a, and McGaugh and Milgrom, 2013b. 10 of the 27 galaxies are excluded for various reasons discussed below:

**And V**

And V is at a distance of 110 kpc from M31, this galaxy may be experiencing tidal effects. There is a low sample size and inconsistency between observations [17].

**And XI**

And XI has a low luminosity ($4.9 \times 10^4 L_\odot$) and is experiencing tidal stripping [11].

**And XII**
And XII has a low sample size (Collins, 2011) and low luminosity ($3.1 \times 10^4 \, L_\odot$). This galaxy is more metal rich then expected and is experiencing tidal stripping [11].

**And XIII**

And XIII has a low sample size [7] and low luminosity ($4.1 \times 10^4 \, L_\odot$), and is experiencing tidal stripping [11].

**And XVII**

And XVII’s observed dispersion ($2.9^{+2.2}_{-1.9}$) is based on only four stars [13, 7, 20].

**And XVIII**

And XVIII’s observed dispersion ($\geq 2.7$) is based on only four stars (McConnachie, 2012, Collins, 2013, and Tollerud, 2012). There remains large photometric uncertainty in this galaxy as documented in McGaugh and Milgrom, 2013b.

**And XX**

And XX’s observed dispersion of ($7.1^{+3.9}_{-2.5}$) is based on only four stars [11].

**And XXIV**

And XXIV’s observed dispersion is based on three probable member stars, and may be influenced by the Milky Way [7].

**And XXVI**

And XXVI has a low luminosity ($5.9 \times 10^4 \, L_\odot$), only six potential member stars, and likely contaminated by the Milky Way [7].

**And XXVII**

And XXVII likely contains tidal disruptions and may no longer be gravitationally bound [7].
3 Analysis of Data

Section 3.1 will explain how the line of sight bulk dispersion is calculated for a galaxy. Section 3.2 will explain why each galaxy is simulated at a variety of different mass to light ratios and how the results of this are used to calculate the bulk dispersion. Section 3.3 will explain how a radial dispersion profile is generated. Finally, Section 3.4 will explain how radial dispersion profiles at each timestep are time averaged to produce the simulated radial dispersion profile.

3.1 Calculating Bulk Dispersion

The bulk dispersion is defined as the radial (line of sight) dispersion for the entire galaxy. The “galaxy” is defined as all stars within a 2500 pc radius. At every timestep from 0 to 2500 Myr the bulk dispersion is calculated. The average bulk dispersion during the stable state (defined between 500 and 2500 Myr) is the average of the bulk dispersion at every timestep. The simulated bulk dispersions ($\sigma_{\text{sim}}$) for each galaxy are presented in Chapter 4.

3.2 Simulating at a Variety of $M/L$

For all of the dSph’s that we consider here, we know the luminosity, but not the mass. Thus, for each galaxy a simulation is run at a variety of mass to light ratios based on the luminosity of the galaxy. The simulated bulk dispersion is then compared to the observed bulk dispersion for the galaxy.

Figure 8: The bulk dispersion is calculated at a variety of $M/L$ ratios over time. As can be seen $M/L = 2$ is the ratio which most closely matches the observed bulk dispersion. Note: this calculation is only considered at time steps between 500 and 2500 Myr.

By finding the simulated bulk dispersion which most closely matches the observed bulk dispersion we are able to find a mass to light ratio which will produce the most accurate results. Figure 8 shows an example for
Leo I dSph galaxy, notice that $M/L = 2.0$ most closely matches the observed dispersion.

3.3 Generating Radial Dispersion Profile

To generate radial dispersion profiles, the observations are divided into circular annuli of radius $r$ as in the method of subdividing seen in Walker, 2009, here $r$ is:

\[ r = \sqrt{y^2 + z^2}. \]

in a cartesian coordinate system. The bin sizes are then set so that each bin contains the same number of stars. An illustration of the binning technique is provided in Figure 9. This generates the dispersion as a function of radius at a given time.

![Figure 9: Example of binning technique used for Leo I dSph galaxy at $M/L = 2$ at 2500 Myr. This technique is used at each timestep.](image)

3.4 Time Averaging of Data

The radial dispersion plots are time averaged by taking the output of each radial dispersion profile during every timestep between 500 and 2500 Myr and plotting them at their defined distances. These dispersions are then
“re-binned” by taking all of the bins and dividing them into a final set of time averaged bins equal to the square root of the total number of radial dispersion bins, where the time averaged bin sizes are scaled so that each time averaged bin has the same number of initial radial bins.

Figure 10: The top figure shows a plot of the radial dispersion profile at every timestep between 500 and 2500 Myr. The bottom figure shows how this data is “re-binned” into equal population sizes. The blue lines represent the inner and outer limits for each spatial bin.
After the techniques described above and illustrated in Figure 10 are carried out. The final result is a time averaged and spatially averaged radial dispersion profile. An example for the Leo I galaxy is presented below using both Newtonian and MOND physics. The Newtonian profile is added to show that even though a $M/L$ ratio which matches the observed line of sight bulk dispersion can be calculated, the radial dispersion profiles, for the Newtonian and MONDian cases are quite different.

![Simulated line of sight radial dispersion profile for Leo I dSph.](image)
4 Results

Section 4.1 will provide the results for the simulated Milky Way dSph galaxy radial dispersion profiles, these results are compared to observations from Walker et al. (2009). Section 4.2 provide predicted radial dispersion profiles for 17 Andromeda dSph galaxies. Relevant statistics are provided in the tables within each of these sections.

4.1 Milky Way dSph Galaxies

<table>
<thead>
<tr>
<th>dSph</th>
<th>Stellar Bin Population</th>
<th>σ Bin Population</th>
<th>c^d</th>
<th>r_{MW} (pc)^f</th>
<th>Refs^f</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carina</td>
<td>98.7 ±0.46</td>
<td>198</td>
<td>0.33</td>
<td>831</td>
<td>1</td>
</tr>
<tr>
<td>Fornax</td>
<td>98.0 ±0.07</td>
<td>197</td>
<td>0.30</td>
<td>772</td>
<td>1</td>
</tr>
<tr>
<td>Leo I</td>
<td>99.0 ±0.18</td>
<td>199</td>
<td>0.21</td>
<td>922</td>
<td>1</td>
</tr>
<tr>
<td>Leo II</td>
<td>99.0 ±0.18</td>
<td>198</td>
<td>0.13</td>
<td>901</td>
<td>1</td>
</tr>
<tr>
<td>Sculptor</td>
<td>99.0 ±0.00</td>
<td>199</td>
<td>0.32</td>
<td>765</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: The statistical properties for the simulated radial dispersion profile are shown above.

References: (1). Walker et. al., 2009

Column a: dSph name
Column b: Mean number of stars per bin, see Figure Section 3.3 for an illustration of this technique. Column c: Number of combined bins during time averaging, see Section 3.4 for an illustration of this process.
Column d: Ellipticity of dSph galaxy
Column e: Distance to center of Milky Way
Column f: References

The accuracy of the simulated line of sight bulk dispersion can be compared to both the analytically calculated bulk dispersion based on Equation (8), and the observed line of sight bulk dispersion [21]. Table 4 Columns d and e show extremely accurate relative error between the analytically predicted and observed bulk dispersion and the MOND simulated bulk dispersion for the Fornax, Leo I, and Leo II dSphs (<15%), and less accurate results for the Carina dSph. Here, relative error is defined as:

$$\text{Relative Error} = \frac{\sigma_s - \sigma_{approx}}{\sigma_s}$$

Here, $\sigma_s$, is either the analytically calculated bulk dispersion ($\sigma_{iso}$) or the observed bulk dispersion ($\sigma_{obs}$) and $\sigma_{approx}$ is simulated bulk dispersion ($\sigma_{sim}$). All simulated $M/L$ ratios are to within a limit of 0.5, it is assumed however, that a smaller limit would produce a more accurate simulated bulk dispersion. An upper limit for the simulated $M/L$ ratio is established for the Carina dSph galaxy, because the relative error between the simulated bulk dispersion and the observed bulk dispersion is > 10%.
<table>
<thead>
<tr>
<th>dSph</th>
<th>Simulated M/L</th>
<th>( \sigma_{\text{sim}} ) (km/s)</th>
<th>( \sigma_{\text{Mc}} ) (km/s)</th>
<th>Relative Error, 1 (%)</th>
<th>Relative Error, 2 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carina</td>
<td>4.0</td>
<td>6.6</td>
<td>5.2</td>
<td>24.5</td>
<td>26.9</td>
</tr>
<tr>
<td>Fornax</td>
<td>1.5</td>
<td>11.7</td>
<td>11.3</td>
<td>4.9</td>
<td>3.5</td>
</tr>
<tr>
<td>Leo I</td>
<td>2.0</td>
<td>9.2</td>
<td>8.6</td>
<td>2.1</td>
<td>6.9</td>
</tr>
<tr>
<td>Leo II</td>
<td>3.5</td>
<td>6.6</td>
<td>6.3</td>
<td>1.5</td>
<td>4.8</td>
</tr>
<tr>
<td>Sculptor</td>
<td>4.0</td>
<td>9.2</td>
<td>8.1</td>
<td>0.0</td>
<td>13.6</td>
</tr>
</tbody>
</table>

Table 4: The simulated observable properties for the 5 Milky Way dSph galaxies are shown above.

- **Column a**: dSph name
- **Column b**: Simulated \( M / L \)
- **Column c**: Simulated Bulk Dispersion
- **Column d**: Analytical Bulk Dispersion from Equation (8)
- **Column e**: Relative Error between Column c and Column b in Table 3
- **Column f**: Relative Error between Column c and Column d, where Column b is the approximate value

Figure 12: Milky Way Dwarf Galaxy Positions in Sky. The positions are given by Right Ascension and Declination.

Figure 12 shows the positions of the Milky Way dSph galaxies in the night sky relative to Earth. The results for the Milky Way dSph galaxy line of sight dispersion profile predictions using MOND are presented below. These results are compared to observations from Walker et al. (2009). The radial dispersion profile for Carina in Figure 13 shows consistent agreement with the observations. The simulated radial dispersion profile is essentially flat, but is well within the range of error for the observed radial dispersion profile. Also in Figure 13, the simulated Fornax dSph radial dispersion profile replicated the observed trend, but is consistently higher than the observations, particularly from 1200+ pc. One possible explanation for this is that the Fornax has a relatively high ellipticity, and our simulation explicitly assumes spherical symmetry.
In the case of the Leo I and Leo II dSphs, which have the lowest ellipticities of any of the simulated Milky Way dSphs, the observed radial dispersion profile and the simulated radial dispersion profile are in Figure 14. These show the best agreement with the observed profile. It can be noticed that the Leo I dSph simulation shows a gradual increase then decrease of the radial dispersion in the inner portion of the galaxy, similar to that observed in the Fornax dSph simulated dispersion. This trend is also observed in the Sculptor dSph radial dispersion profile, but not in the results from the Leo II dSph. There is not enough spatial resolution in the observations to reveal this feature.

![Figure 13: Comparison of the bulk dispersion values for the Milky Way dSph galaxies. The red open circle represents $\sigma_{\text{obs}}$ from [21], the black open diamond represents $\sigma_{\text{sim}}$, and the magenta star represents $\sigma_{\text{iso}}$.](image)

Figure 13 shows a comparison of the calculated dispersion from Equation (8) matches the observed, and simulated bulk dispersion values for the Milky Way dSph galaxies. In particular, the simulated dispersion, ($\sigma_{\text{sim}}$), calculated dispersion, ($\sigma_{\text{iso}}$), and observed dispersion, ($\sigma_{\text{obs}}$), are shown to be nearly identical in the case of Leo II. In the case of the Carina dSph galaxy the calculated dispersion and observed dispersion are nearly identical, while the simulated dispersion is near the upper limit of uncertainty. All five dSph galaxies show that the simulated dispersion, and calculated dispersion are within the observed error. The accuracy of the simulated
dispersion relative to the calculated and observed dispersion should serve as a testament to the accuracy of our method. Shown below are the simulated radial dispersion profiles for each of the 5 Milky Way dSph galaxies in comparison to the observed radial dispersion profiles from Walker et. al., (2009).

Figure 14: Carina and Fornax dSph radial dispersion profiles
Figure 15: Leo I and Leo II dSph radial dispersion profiles
Galaxy Name: Sculptor
M/L: 4.0
Simulated $\sigma$ (km/s): 9.2 km/s
Observed $\sigma$ (km/s): 9.2 km/s
Mean Stars per Bin: 99.0 +/- 0.00
$\sigma$ Bin Count: 199

Figure 16: Sculptor dSph radial dispersion profile
4.2 Andromeda dSph Galaxies

The stellar bin and $\sigma$ bin populations from Figures 19 through 27 are summarized in the tables below. The stellar bin population ranges between 80.6 and 91.9, which implies that the number of stars in these simulations whose positions stayed within the 2500 pc radius (i.e. they did not escape the galaxy) ranges between 6,496 and 8,445, or 64.96% to 84.45% of the initial 10,000 stars. The $\sigma$ bin populations for each galaxy, which varies depending on the stellar bin populations, varies between 179 and 198.

![Figure 17: Andromeda Dwarf Galaxy Positions in Sky. The positions are given by Right Ascension and Declination](image)

Figure 17 shows the positions of the Andromeda dSph galaxies in the night sky relative to Earth. The results for the Andromeda dSph galaxy line of sight dispersion profile predictions using MOND are presented in Figure 17 through 25. In the And I, II, VI, and VII galaxies an initial increase, then slight decrease in the radial dispersion for the inner-most part of the galaxy is observed. The And III, IX, XIV, XV, XVI, XIX, XXIII, XXV, XXVIII, and XXIX galaxies show an essentially flat radial dispersion profile. The And X, and XXII galaxies show a consistent decrease in the radial dispersion over distance from the center of the galaxy.
Table 5: The simulated observable properties for the 17 Andromeda dSph galaxies are shown above.  
Column a,d: Name of dSph Galaxy  
Column b,d: Mean number of stars per bin, see Figure Section 3.3 for an illustration of this technique.  
Column c,e: Number of combined bins during time averaging, see Section 3.4 for an illustration of this process.

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<th>dSph</th>
<th>Best Fit M/L</th>
<th>$\sigma_{ave}$ (km/s)</th>
<th>$\sigma_{ave}$ (km/s)</th>
<th>Relative Error, 1 (%)</th>
<th>$\sigma_{ave}$ (km/s)</th>
<th>Relative Error, 2 (%)</th>
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Table 6: The simulated observable properties for the 17 Andromeda dSph galaxies are shown above.  
Column a: dSph name  
Column b: Mass to light ratio that produced a bulk dispersion most closely aligned with $\sigma_{ave}$  
Column c: MOND simulated stable state bulk dispersion  
Column d: Calculated bulk dispersion from Formula (8) based on $M/L$ ratio from column a  
Column e: Relative error between column error c column d  
Column f: Average dispersion of $\sigma_{obs,1}$ and $\sigma_{obs,2}$ from Table 2  
Column g: Relative error between column c and column f

Table 6 displays the accuracy of the simulated bulk dispersions. By comparing the average of $\sigma_{obs,1}$ and $\sigma_{obs,2}$, $\sigma_{ave}$ to the analytically predicted $M/L$ from Table 2, the relative error of our simulated bulk dispersions show excellent agreement with McGaugh’s calculated dispersion. The And I, II, III, VI, VII, IX, X, XV, XVI, XIX, XXII, XXIII, XXVIII, and XXIX galaxies all show incredibly accurate agreement between the simulated radial dispersion and McGaugh’s analytically predicted radial dispersion, with only the And XXVIII dSph galaxy showing an extremely high relative error of 42.6%. In the case of And IX, X, XIV, XV, XVI, XXV, XXVIII, and XXIX, where the relative error between $\sigma_{sim}$ and $\sigma_{ave}$ is $> 10%$ an upper and lower limit is
established for the M/L ratio and is attributed to inconsistencies in the quality of observations.

Figure 18: Comparison of the bulk dispersion values for the Andromeda dSph galaxies. The red open circle represents $\sigma_{obs,1}$ and its error, the blue open circle represents $\sigma_{obs,2}$ from [14] and [15] and its error, the yellow star represents $\sigma_{ave}$, the average of these observations for dSphs with multiple observations, the black diamond represents $\sigma_{sim}$, the simulated dispersion, and the magenta star represents $\sigma_{iso}$, the dispersion calculated from Equation (8).

Figure 18 shows a comparison of the calculated, observed, and simulated bulk dispersion values for the Andromeda dSph galaxies. The observations show similar dispersion calculations from three entirely different methods. These results imply that the analytical calculations of McGaugh and Milgrom, (2013a) and McGaugh and Milgrom, (2013b), the observed bulk dispersion from McGaugh and Milgrom (2013a) and McGaugh and Milgrom, (2013b) match the simulated dispersions in this report. Three bulk dispersions from entirely different methods are shown to be in excellent agreement in this report. dSph galaxies such as And I, VI, XIV, XVI, XIX, and XXIII show incredibly accurate agreement between the simulated, observed and calculated dispersion. dSph galaxies such as And III, and XXV do not show good agreement between the observed, calculated and simulated dispersions. In the case of And III, this is likely attributed to inconsistencies between the two observations. And XXI shows such wildly varying observations with relatively large error that make it difficult...
to predict a simulated dispersion. Below, the radial dispersion profile predictions for the Andromeda dSph galaxies are shown.

![Radial Dispersion Profiles](image)

**Figure 19:** And I and And II dSph radial dispersion profiles
Figure 20: And III and And VI dSph radial dispersion profiles

Galaxy Name: And III
M/L: 3.0
Simulated \( \sigma \) (km/s): 7.0 km/s
Mean Observed \( \sigma \) (km/s): 7.0 km/s
Mean Stars per Bin: 97.0 +/- 0.00
Mean \( \sigma \) Bin Count: 196

Galaxy Name: And VI
M/L: 2.5
Simulated \( \sigma \) (km/s): 9.4 km/s
Mean Observed \( \sigma \) (km/s): 9.4 km/s
Mean Stars per Bin: 97.9 +/- 0.34
Mean \( \sigma \) Bin Count: 197
Galaxy Name: And VII
M/L: 1.0
Simulated $\sigma$ (km/s): 11.3 km/s
Mean Observed $\sigma$ (km/s): 11.3 km/s
Mean Stars per Bin: 95.0 $\pm$ 0.16
$\sigma$ Bin Count: 195

Galaxy Name: And IX
M/L: 4.0
Simulated $\sigma$ (km/s): 4.6 km/s
Mean Observed $\sigma$ (km/s): 7.7 km/s
Mean Stars per Bin: 94.8 $\pm$ 0.42
$\sigma$ Bin Count: 194

Figure 21: And VII and And IX dSph radial dispersion profiles
Figure 22: And X and And XIV dSph radial dispersion profiles
Figure 23: And XV and And XVI dSph radial dispersion profiles
Figure 24: And XIX and And XXI dSph radial dispersion profiles
Figure 25: And XXII and XXIII dSph radial dispersion profiles
Galaxy Name: And XXV
M/L: 3.5
Simulated $\sigma$ (km/s): 7.2 km/s
Mean Observed $\sigma$ (km/s): 7.1 km/s
Mean Stars per Bin: 91.3 $\pm$ 0.67
$\sigma$ Bin Count: 191

Galaxy Name: And XXVIII
M/L: 3.5
Simulated $\sigma$ (km/s): 5.0 km/s
Mean Observed $\sigma$ (km/s): 5.8 km/s
Mean Stars per Bin: 98.0 $\pm$ 0.00
$\sigma$ Bin Count: 197

Figure 26: And XXV and And XXVIII dSph radial dispersion profiles
Figure 27: And XXIX dSph radial dispersion profile.
5 Conclusions

Section 5.1 will discuss where MOND was successful in replicating observations or making predictions. Section 5.2 will emphasize the importance of the quality of observations to the simulated radial dispersion profile.

5.1 Success of MOND

In the case of the Milky Way dSph galaxies the MOND simulated radial dispersions show excellent to moderate agreement for each galaxy’s observation within the error bars. It should be noted that galaxies with a lower ellipticity (e) show better agreement than galaxies with a higher ellipticity. Our model can in no way model the ellipticity and so should be treated as an approximation. For instance Leo I and Leo II have an ellipticity of 0.21 and 0.13 respectively, these galaxies show a better fit to observational data then Carina and Fornax which have an ellipticity of 0.33, and 0.30 respectively. The differences between the predictions from the spherically symmetric dSph galaxy simulation and the actual observations becomes more noticeable at the edge of the galaxy. In the case of the Andromeda dSph galaxies we produce simulated predictions for the radial dispersion profiles for 17 different Andromeda dSph galaxies. These predicted radial dispersions will serve as a test for the validity of MOND and be left to future observations to either show MOND is a viable predictor for the radial dispersion profiles of And dSph galaxies, or if the opposite is shown to be true could expose a critical miscalculation in MOND theory.

5.2 Importance of Accuracy of Observations

The accuracy of the radial dispersion profile prediction is dependent on the accuracy of the observed bulk dispersion. As observations continue to become more accurate, the method demonstrated here will be able to produce more accurate mass to light ratios, which in turn will produce more accurate radial dispersion profile predictions.
References


