This thesis focuses on the study of size-scale structural effects on paper fracture toughness. Three analyses, including tensile strength ratio, consumed energy ratio and stress intensity factor ratio were used to explore the sensitivity of paper to the propagation of cracks. The results of the comparison were utilized to focus on stress ratios for the new modified LEFM approach presented in appendix A. The modified LEFM can be used as a simple method to explore fracture toughness relative to tensile strength. Both commercial papers and handsheets were tested and analyzed for their fracture behavior using double-notched tensile specimens. The combination of experimental testing and analysis suggest that paper structure plays an important role on relative fracture toughness. The results indicate that paper towel, the least homogenous material tested, had the lowest sensitivity to notches. The differences in relative fracture toughness are related to size-scale of structural features. Relative fracture toughness varied for handsheets made under various process conditions to yield different structures. It is concluded that material relative fracture toughness increases with material heterogeneity.
SIZE-SCALE STRUCTURAL EFFECTS ON THE FRACTURE TOUGHNESS OF PAPER

A Thesis
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Contents

1 INTRODUCTION .................................................................................................. 1
2. HYPOTHESIS ...................................................................................................... 9
3 MATERIALS ....................................................................................................... 10
4 METHODS .......................................................................................................... 17
5 RESULTS ............................................................................................................ 21
6. DISCUSSION ..................................................................................................... 36
7. CONCLUSIONS ................................................................................................. 49
8. REFERENCES .................................................................................................... 51
List of Tables

Table 1 Properties of Commercial Papers ................................................................. 10
Table 2 Properties of Handsheets .............................................................................. 12
Table 3 Mechanical Properties of Commercial Papers CD ............................... 14
Table 4 Mechanical Properties of Commercial Papers MD ............................... 14
Table 5 Mechanical Properties of Handsheets ......................................................... 15
Table 6 Slopes of Commercial Papers .................................................................... 42
Table 7 Pore Percentage of Commercial Papers ................................................... 46
Table 8 Effective Fracture Process Zone d and Structural Zone Length ds for
Commercial Papers .................................................................................................. 46
Table 9 Correlation Coefficients of Analyses for Commercial Papers .................... 47
List of Figures

Figure 1 Modes of crack surface displacement [4] ..................................................... 2
Figure 2 Crack Configurations [4] ............................................................................. 2
Figure 3 Micrographs of paper samples .................................................................... 11
Figure 4 Sample geometry for testing ...................................................................... 13
Figure 5 Stress vs. Strain Curve of Paper Towel CD Fracture Process ...................... 16
Figure 6 Stress vs. Strain Curve of Paper Towel CD Fracture Process ...................... 16
Figure 7 Sample geometry for testing ...................................................................... 18
Figure 8 Stress vs. Strain Curve of Paper Towel Fracture Process ......................... 21
Figure 9 Tensile Strength vs. Notch Size for Paper Towel ........................................ 22
Figure 10 Tensile Strength vs. Notch Size for Copy Paper ........................................ 22
Figure 11 Tensile Strength Ratio σ2 (CD) vs. Notch Size......................................... 23
Figure 12 Tensile Strength σ2 Ratio vs. Notch Size for News Print and Cellophane. 24
Figure 13 TEA Ratio vs. Notch Size for Commercial Paper .................................... 25
Figure 14 Energy Release Rate Ratio vs. Notch Size for Commercial Paper .......... 25
Figure 15 Critical Stress Intensity Factor Ratio vs. Notch Size for Commercial Paper .......................................................... 26
Figure 16 Tensile Strength vs. Notch Size for Paper Towel ...................................... 27
Figure 17 Tensile Strength vs. Notch Size for Paper Board .................................... 28
Figure 18 Tensile Strength vs. Notch Size for Copy Paper ...................................... 28
Figure 19 Critical Stress Intensity Factor Ratio vs. Notch Size ................................ 29
Figure 20 Critical Stress Intensity Factor Ratio vs. Notch Size ................................ 29
Figure 21 Critical Stress Intensity Factor Ratio vs. Notch Size ................................. 29
Figure 22 Tensile Strength vs. Notch Size for Hand Sheet ..................................... 30
Figure 23 Tensile Strength vs. Notch Size for Hand Sheet ..................................... 32
Figure 24 Critical Stress Intensity Factor Ratio vs. Notch Size ................................ 32
Figure 25 Tensile Strength vs. Notch Size for Hand Sheet ..................................... 33
Figure 26 Critical Stress Intensity Factor Ratio vs. Notch Size ................................ 34
Figure 27 Tensile Strength vs. Notch Size for Hand Sheet ..................................... 35
Figure 28 Critical Stress Intensity Factor Ratio vs. Notch Size ................................ 35
Figure 29 Tensile Strength and Stress Intensity Factor Ratio vs. Notch Size for Paper Towel .................................................................................................................. 36
Figure 30 Tensile Strength and Stress Intensity Factor Ratio vs. Notch Size for Paper Board .................................................................................................................. 37
Figure 31 Tensile Strength and Stress Intensity Factor Ratio vs. Notch Size for Copy Paper .................................................................................................................. 37
Figure 32 Tensile Strength and TEA Ratio vs. Notch Size for Paper Towel .......... 38
Figure 33 Tensile Strength and TEA Ratio vs. Notch Size for Paper Board .......... 38
Figure 34 Tensile Strength and TEA Ratio vs. Notch Size for Copy Paper .......... 39
Figure 35 Critical Stress Intensity Factor and Remote Load Ratio vs. Notch Size for Paper Towel .................................................................................................................. 39
Figure 36 Critical Stress Intensity Factor and Remote Load Ratio vs. Notch Size for
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1. INTRODUCTION

Tissue papers, including towels, are distinguished from other grades of paper by low grammage and high bulk. These products typically have open and inhomogeneous structure. Processes such as creping and novel drying technologies (Through Air Dryer (TAD), etc) are used to produce a complex structure [1]. Bonding between fibers is low. Paper towels are often two-ply and the grammage of each ply is between 16 to 24 g/m$^2$. These tissue papers are usually made from chemical pulp and recycled pulp [2]. The different size-scale levels of structure in these papers, will impact the resulting mechanical behavior.

The relatively special structure of tissue paper may magnify certain physical properties compared to more traditional papers. Preliminary results indicated that relative fracture toughness of tissue papers highlights the role of structure in developing mechanical properties. Fracture mechanics is widely applied to quantitatively predict the initiation and propagation of cracks in materials. In 1920, Griffith [3] studied fracture properties of brittle solid materials and introduced a theory for fracture. He opened the door to the use of fracture mechanics to study crack propagation in materials. Fracture toughness is used to quantify the resistance of a material to fracture, but can be evaluated in many ways. In general ductile materials have high fracture toughness and brittle materials have low fracture toughness. One way to express the fracture toughness is the stress intensity factor resulting from a stress analysis [4].

In fracture mechanics research, different configurations are usually employed in the experiment to study the fracture process of a material. There are three distinct modes for fracture as shown in Figure 1. This study focused on mode I. Three configurations shown in Figure 2 can be used to study the mode I fracture process. The left one has center crack, the middle one has a single notch and the right one has double edge notch.
There are two main approaches for fracture analysis. One uses an energy criterion and the other is a stress analysis to determine the distribution of stress [4]. Energy analysis focuses on the energy change of the sample during the process of initiation and propagation of a crack. The crack propagates only if the energy supplied to the sample is sufficient to overcome the fracture toughness of the material. The energy of fracture toughness includes the surface energy, plastic deformation energy, and other energy dissipation when the crack extends [1]. One example in the energy analysis is the method of energy release rate, which is usually expressed as $G$ [5]. It is defined as the rate of change in potential energy of material with a crack under the load and given as
\[ G = \frac{\pi \sigma^2 a}{E} \]  

(1)

Where \( E \) is Young’s modulus, \( \sigma \) is the remotely applied stress, and \( a \) is the half crack length.

When \( G \) reaches the critical energy release rate of material, the crack propagates. Therefore, the critical energy release rate is a measure of fracture toughness and is indicated as:

\[ G_c = \frac{\pi \sigma^2 a_c}{E} \]  

(2)

where \( a_c \) is the half crack length when propagation occurs.

In a fracture process, the energy release rate \( G \) is the driving force and \( G_c \) is the resistance to fracture. The term \( G_c \) is considered to be independent of size and geometry of sample and applicable for fracture with predominantly linear elastic responses.

In the stress analysis approach, the distribution of stresses is determined near the crack and stress intensity factor, \( K \), is determined. The larger the stress intensity factor is, the lower the load that will cause crack propagation. For an ideal elastic material the energy analysis and stress analysis can be related as \[5\].

\[ G = \frac{K^2}{E} \] .  

(3)

Therefore, the energy release rate can be determined from the stress intensity factor.

However, in other kinds of materials, such as elastic-plastic material and viscoelastic materials, \( G_c \) is usually not useful. So other approaches have been developed to study fracture mechanics of these materials. Cottrell and Wells \[6-7\] used the crack tip
opening displacement (CTOD) to study the fracture process. Rice \cite{8} introduced the method of $J$-integral, which is used to characterize nonlinear materials. The $J$-integral is defined as the value equal to the decrease in the energy when a crack extension occurs for a nonlinear material. It is noted the energy concept of $J$-integral is similar to Griffith’s approach. To simplify the experiment to find the $J$-integral, Liebowitz and Efits \cite{9} adopted a new parameter $G^*$ to approximate the $J$ integral and it is given as

$$G^* = \left[ 1 + \frac{2nk}{n+1} \left( \frac{P}{M} \right)^{n-1} \right] J_e,$$  \hspace{1cm} (4)

Where $P$ is the load, $M$ is the initial stiffness of the specimen with a crack, $n$ and $k$ can be derived from the Ramberg-Osgood model that fits the load-displacement curve \cite{1}. $J_e$ is the linear part of the $J$-integral. It is equal to the energy release rate of the linear part under the same configuration. It can also be obtained from the stress intensity factor, which was given in Equation (3). The term $J_c$ is the critical value of $J$-integral. When $J$ is equal to $J_c$, the crack propagates. So $J_c$ is used to characterize the fracture toughness of nonlinear materials.

Another approach of energy analysis is the essential work of fracture. Broberg \cite{10} introduced the concept of essential work of fracture, which is indicated as $W_e$. The work during the fracture process $W_f$, which is the area under the apparent stress-strain curve, can be divided into two parts. The first part is the essential work of fracture, which is used to promote the crack extension. The other part is the nonessential work of fracture, which is dissipated in the process of plasticity of material. It is indicated as $W_p$. The relation between $W_e$ and $W_p$ is obtained as

$$W_f = W_e + \beta W_p l,$$  \hspace{1cm} (5)

Where $\beta$ is used to consider the shape of plastic zone and $l$ is the ligament length.

Therefore, $W_e$ is obtained from the plot of fracture work versus crack length. $W_e$ is
typically used as a preliminary study of the energy absorption at the crack tip as the crack extends.

Another common method in energy analysis is tensile energy absorption (TEA) [11]. The absorbed energy is equal to the area under the load-displacement curve of fracture process.

Though energy analysis provides a simple and effective way to analyze fracture toughness, it does not offer details about the stresses at the crack tip of materials. Stress analysis approaches to fracture mechanics been developed. Airy introduced the concept of using a stress function when analyzing two-dimensional stress fields. The Airy’s stress function, \( \phi(x, y) \), satisfies equilibrium when defined as

\[
\sigma_x = \frac{\partial^2 \phi}{\partial y^2}; \quad \sigma_y = -\frac{\partial^2 \phi}{\partial x^2}; \quad \tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}
\]

(6)

For an isotropic linear-elastic material, \( \phi \) must satisfy the bi-harmonic equation

\[
\frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0
\]

(7)

for the strains to be compatible with a real displacement field.

\[
\phi = \text{Re} \bar{\phi}(z) + y \cdot \text{Im} \bar{\phi}(z);
\]

(8)

Westergaard adopted a complex function for the Airy’s stress function [12] where \( z = x + i \cdot y \); \( \bar{\phi}(z) \) and \( \bar{\phi}(z) \) are the first and second order integrals of function \( \phi \).

Using Equation (8) with Equation (6), the stresses can be described as

\[
\sigma_x = \text{Re} \phi(z) - y \cdot \text{Im} \phi'(z);
\]
\[
\sigma_y = \text{Re} \phi(z) + y \cdot \text{Im} \phi'(z);
\]
\[
\tau = -y \cdot \text{Re} \phi'(z).
\]

(9)

The above equations describe a general solution process for stresses in
two-dimensional plate. For a specific problem (a crack in the middle of plate, etc), the boundary condition must be established. A specific function $\phi$, which satisfies the boundary conditions must be found. Using this procedure the stresses in an infinite plate with the sharp crack tip can be expressed as

$$\sigma_{ij} = \frac{\sigma_x \sqrt{\pi a}}{2\pi r} f_{ij}(\theta)$$

(10)

Where $\sigma_{ij}$ is $ij$ stress component at the point of $(r, \theta)$ and $f_{ij}(\theta)$ defines the dependence on angle.

It is noted that at the crack tip there is a singularity since the stress tends to infinity at the crack tip. The intensity of the stress singularity is called the stress intensity factor, which is equal to $\sigma_x \sqrt{\pi a}$ in Equation (10) and indicated as $K_I$. It is usually used to study the change of stress at crack tip. The above equation is applied to crack with infinitely sharp tips. Creager and Paris [13] developed the expressions for the crack with finite tip radius as

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) - \frac{K_I}{\sqrt{2\pi r}} (\rho \theta) g_{ij}(\theta)$$

(11)

where $\rho$ is radius of the crack tip.

For the application of a stress analysis to a sample with a finite width the analysis is much harder. The stress components are affected by geometry of crack and the edges of the strip. For these cases, a modified intensity factor is given. The general expression of $K_I$ is given as

$$K_I = C \sigma \sqrt{\pi a}$$

(12)

where $C$ is geometry correction factor determined by the configuration.
Brown defined $C$ for a plate containing a crack in the middle [4] as

$$C = 1 + 0.256\left(\frac{a}{w}\right) - 1.152\left(\frac{a}{w}\right)^2 + 12.2\left(\frac{a}{w}\right)^3,$$

(accurate to 0.5% for $\frac{a}{w} \leq 0.35$)

and for the single edge notched plate as

$$C = 1.122 - 0.231\left(\frac{a}{w}\right) + 10.55\left(\frac{a}{w}\right)^2 - 21.71\left(\frac{a}{w}\right)^3 + 30.382\left(\frac{a}{w}\right)^4,$$

(accurate to 0.5% for $\frac{a}{w} \leq 0.6$)

Tada [14] introduced the geometry correction factor for double edge notched plate as

$$C = \frac{1.122 - 1.122\left(\frac{a}{w}\right) - 0.82\left(\frac{a}{w}\right)^2 + 3.768\left(\frac{a}{w}\right)^3 - 3.04\left(\frac{a}{w}\right)^4}{\sqrt{1 - \frac{2a}{w}}},$$

(accurate to 0.5% for any $\frac{a}{w}$)

The stress at the crack tip approaches infinity when the radius of the tip approaches zero. Some researchers [15-17] give assumptions to overcome this singularity in the analysis. Another way to eliminate the singularity is use of a numerical method such as finite-element analysis.

In the history of application of fracture mechanics in paper, the Elmendorf tear is widely used as one of strength indexes in paper industry. It is used as a reference parameter to predict the fracture failure of the paper and grade paper. However, it has been called to question frequently [18-19]. Therefore, though it is still used to monitor the quality of paper, it is not suitable for a detailed research of principle of fracture mechanics of paper. The second method adopted to study the fracture mechanics of paper is the in-plane tear developed by Van den Akker [20]. The concept of this method is to set specific test configuration to directly measure the crack growth resistance. However, Seth and Page [21] reported that the in-plane is not equal or
proportional to the stress intensity factor. This result limits its use in the application of
fracture mechanics of paper. The third method is the tensile energy absorption
describes above. Besides this way, energy release rate \( (G) \), \( J \)-integral \( (J) \) and essential
work of fracture \( (W_e) \) [22-33] are also used for the analysis of different kinds of paper.

Many kinds of paper have been studied for their mechanical properties. For example,
a summary of fracture toughness for some papers was drawn by Bither and
Waterhouse [34]. These authors studied the influence of refining and wet pressing on
tensile strength and fracture toughness [34]. Fracture toughness was found to be
related to tensile strength. Refining and wet pressing affects inter-fiber bond strength,
stress concentration levels. Higher beating degree and pressing pressure, which lead
to higher tensile strength of handsheets, increase samples’ sensitivity to single-edge
notches and decrease its fracture toughness.
2. HYPOTHESIS

Paper being inhomogeneous on various size-scale levels, will exhibit fracture behavior that varies with the geometry of cracks and the structure of paper. It is hypothesized that for paper towel products, the size-scale of inhomogeneity is sufficient to yield fracture toughness measures that are greater than that of an equivalent homogenous material. It is further hypothesized that as the degree of bonding increases, the size-scale of defects decreases, and the fracture toughness relative to the tensile strength decreases.
3. MATERIALS

For this study, commercial paper, a regenerated cellulose film, and handsheets were utilized. Handsheets were made from fully bleached kraft softwood fibers. More details are given below for these materials.

3.1 Commercial Paper

A range of commercial papers, including paper towel, paper board, copy paper and newsprint were used for the fracture toughness study. These papers had various size-scale levels of structure due to their different manufacturing processes. Their fracture toughness was studied and compared. The physical properties of commercial papers are shown in the Table 1 below.

<table>
<thead>
<tr>
<th></th>
<th>Grammage, g/m²</th>
<th>Caliper, mm</th>
<th>Density, Kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>copy paper</td>
<td>73 (0.6)</td>
<td>0.1 (0.001)</td>
<td>731</td>
</tr>
<tr>
<td>paper board</td>
<td>202 (2.7)</td>
<td>0.32 (0.009)</td>
<td>638</td>
</tr>
<tr>
<td>paper towel</td>
<td>22 (0.3)</td>
<td>0.13 (0.004)</td>
<td>167</td>
</tr>
<tr>
<td>newsprint</td>
<td>48 (0.4)</td>
<td>0.08 (0.003)</td>
<td>621</td>
</tr>
</tbody>
</table>

*Standard deviation given in parentheses

3.1.1 Assessment of Structure.
To help assess the homogenous level of structure in the sheet (pore space, percent surface fibers), quantities such as grammage and density were measured. Also, as part of this study, a direct observation of structure surface was implemented using SEM to microphotograph of the materials. Their SEM images are shown in Figure 3.

![Figure 3 Micrographs of paper samples](image)

From the SEM graphs, it appears that the paper towel had the least homogeneous level of structure among all types of paper, followed by paperboard, and then copy paper. Newsprint has the most homogenous level of structure.

### 3.2 Regenerated Cellophane

A plastic film will have only molecular-level structural features. A film was obtained and reported to be made from regenerated cellulose. It is used in this study to represent an extremely homogenous material for this study. Fracture toughness results for the films will be used to provide a comparison to the inhomogeneous paper samples.
3.3 Handsheets

Using a single source of fibers, softwood BKP, handsheets were produced for a full range of grammages from 25 to 100 g/m². The density of the sheet was used as one indicator for its structural homogeneity. In addition, handsheets with various beating degree were produced. Various beating degrees, which affect surface area and inter-fiber bonding, were expected to change the level of homogeneity in the structure. In addition, the level of pressing pressure was varied.

The handsheets were produced using a twelve-inch square Noble and Wood sheet former and cut to size of 9 inches long and 3 inches wide. Their physical properties are shown in the Table 2 and 3 below.

Table 2 Properties of Handsheets

<table>
<thead>
<tr>
<th>CSF</th>
<th>Grammage, g/m²</th>
<th>Pressing</th>
<th>Caliper, mm</th>
<th>Density, kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>465</td>
<td>49 (0.9)</td>
<td>low</td>
<td>0.07 (0.002)</td>
<td>712</td>
</tr>
<tr>
<td>160</td>
<td>49 (0.9)</td>
<td>low</td>
<td>0.07 (0.003)</td>
<td>728</td>
</tr>
<tr>
<td>705</td>
<td>52 (1.2)</td>
<td>low</td>
<td>0.09 (0.002)</td>
<td>588</td>
</tr>
<tr>
<td>705</td>
<td>95 (2.7)</td>
<td>low</td>
<td>0.15 (0.004)</td>
<td>634</td>
</tr>
<tr>
<td>705</td>
<td>24 (1.0)</td>
<td>low</td>
<td>0.05 (0.001)</td>
<td>473</td>
</tr>
<tr>
<td>705</td>
<td>50 (1.1)</td>
<td>none</td>
<td>0.09 (0.009)</td>
<td>558</td>
</tr>
</tbody>
</table>

*Standard deviation given in parentheses

3.4 Sample Preparation

Before mechanical testing, all paper samples were cut into strips with a size of 9 inches long and 3 inches wide. For DENT testing, two cracks, each of length $a$ were cut midpoint in the length direction from both sides. Crack length, $a$, was varied for different tests of each sample type. The sample geometry is shown in Figure 4. After
its nondestructive physical and mechanical properties were measured, the paper strip was used for a fracture test. At least five same samples were tested for each condition.

3.5 Stress-strain curves of materials

The stress-strain curves of materials in the tables are given in Figures 14, 15 and 16 of appendix A. In those figures, the stress is normalized with the elastic modulus so one can compare the relative shape of the curves. It is observed that paper towel has the highest stretch among the commercial papers. Paper in MD is more brittle than that in CD. Hand sheets with appropriate beating degree (CSF 465), grammage (50 g/m²) and pressing has the highest stretch. Mechanical properties of papers are list in Table 3, 4 and 5.
### Table 3 Mechanical Properties of Commercial Papers CD

<table>
<thead>
<tr>
<th></th>
<th>Tensile Index CD, m*N/g</th>
<th>Stretch CD, %</th>
<th>TEA CD, m*N</th>
</tr>
</thead>
<tbody>
<tr>
<td>copy paper</td>
<td>28.0</td>
<td>4.7</td>
<td>1.2</td>
</tr>
<tr>
<td>paper board</td>
<td>28.2</td>
<td>4.2</td>
<td>3.0</td>
</tr>
<tr>
<td>paper towel</td>
<td>12.5</td>
<td>16.1</td>
<td>0.3</td>
</tr>
<tr>
<td>newsprint</td>
<td>13.2</td>
<td>2.6</td>
<td>0.2</td>
</tr>
</tbody>
</table>

### Table 4 Mechanical Properties of Commercial Papers MD

<table>
<thead>
<tr>
<th></th>
<th>Tensile Index MD, m*N/g</th>
<th>Stretch MD, %</th>
<th>TEA MD, m*N</th>
</tr>
</thead>
<tbody>
<tr>
<td>copy paper</td>
<td>60.1</td>
<td>1.8</td>
<td>0.8</td>
</tr>
<tr>
<td>paper board</td>
<td>73.5</td>
<td>2.3</td>
<td>3.0</td>
</tr>
<tr>
<td>paper towel</td>
<td>16.2</td>
<td>17.2</td>
<td>0.6</td>
</tr>
<tr>
<td>newsprint</td>
<td>63.7</td>
<td>1.2</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Table 5 Mechanical Properties of Handsheets

<table>
<thead>
<tr>
<th>CSF</th>
<th>Grammage, g/m²</th>
<th>Pressing</th>
<th>Tensile Index, m*N/g</th>
<th>Stretch, %</th>
<th>TEA, m*N</th>
</tr>
</thead>
<tbody>
<tr>
<td>465</td>
<td>50</td>
<td>low</td>
<td>91.8</td>
<td>4.1</td>
<td>1.4</td>
</tr>
<tr>
<td>160</td>
<td>50</td>
<td>low</td>
<td>96.6</td>
<td>3.1</td>
<td>1.9</td>
</tr>
<tr>
<td>705</td>
<td>50</td>
<td>low</td>
<td>38.1</td>
<td>1.8</td>
<td>0.3</td>
</tr>
<tr>
<td>705</td>
<td>100</td>
<td>low</td>
<td>32.3</td>
<td>2.2</td>
<td>0.8</td>
</tr>
<tr>
<td>705</td>
<td>25</td>
<td>low</td>
<td>23.7</td>
<td>0.8</td>
<td>0.0</td>
</tr>
<tr>
<td>705</td>
<td>50</td>
<td>none</td>
<td>14.3</td>
<td>1.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Tensile energy absorption (TEA) and energy release rate Gc of paper towel in cross direction are shown as shaded area in Figures 5 and 6. TEA and Gc of other paper grades are given from Figure B1 to Figure B26 in Appendix B.
Figure 5 Stress vs. Strain Curve of Paper Towel CD Fracture Process

Figure 6 Stress vs. Strain Curve of Paper Towel CD Fracture Process
4. METHODS

4.1 Experimental Methods

4.1.1 Experimental Methods for Fracture Behavior

Experimental data for determining fracture toughness was obtained by performing double-notched tensile tests on an Instron 3344 universal testing machine. Sample sizes were chosen as 3 inches wide by a minimum of 9 inches gauge length. Notches were placed as a sharp cut in the paper at the middle of the gauge length to a pre-determined length. The notch was produced with a pair of precision scissors before mounting.

All tests were conducted at Tappi Standard Conditions (50% RH and 72 F). The test speed was 25.4 mm/min, while load and deformation were recorded from the start of the test until sample failure (99% decrease in maximum load).

The resulting data for the load-deformation curve was saved for each sample. By saving the entire load-deformation curve for each sample, re-analysis of data was possible to account for the development of methods during the study. From the collected data, quantities such as tensile strength ratio, tensile energy absorption and critical stress intensity factor were calculated.

4.2 Fracture Toughness Analysis Methods

4.2.1 Stress Analysis Approach

Paper tensile strength is the maximum stress that paper can withstand before failure. For the double notched test, tensile stresses along the cracked ligament are different from those far from the cracks. Here $\sigma_1$ is the average stress at failure away from the crack and $\sigma_2$ is the average stress in the ligament. The value of $\sigma_2$ was recalculated from $\sigma_1$ by multiplying the ratio of paper sample edge width to ligament width. Observed from tensile tests, most of paper samples broke at or close to the edge of ligament.
The stress, $\sigma_2$ was considered as one indication of fracture toughness. $\sigma_1$ and $\sigma_2$ are shown on Figure 7.

![Figure 7 Sample geometry for testing](image)

Although this method has limitations and scaling to different sample sizes would affect the measure, it is a direct method to explore the fracture toughness of paper. To compare relative fracture toughness of various materials, the notched tensile strength ratio is used in this study. Tensile strength ratio calculation is shown as Equation (16).

$$NTSR = \frac{TS_2/(1-2a/W)}{TS_1}$$

(16)

Where $TS_1$ is tensile strength of unnotched paper sample and $TS_2$ is tensile strength of notched paper sample. Calculated based on the ligament length (It is shown as ratio of $1-2a/W$ for notched sample), $TS_2/(1-2a/W)$ is equal to its $\sigma_2$. 
It should be noted that the same type of ratio is also used for other analysis methods like critical stress intensity factor and TEA.

In this study, another measure for evaluating fracture toughness was the critical stress intensity factor, Equation (12). It was extracted from tensile strength to represent fracture toughness of different materials and used to study the fracture stress near the crack tip. Calculation for $C$ that corresponds to the testing in this study is from Equation (15).

From fracture mechanics, it is known that stress intensity factor is used to amplify the applied stress. Critical stress intensity factor is often used to determine fracture toughness from tensile tests. Equation (12) indicates that for a linear elastic material, the stress approaching crack tip can be magnified to extremely high value. However, plastic deformation occurring near the crack tip reduces the magnification of the stress intensity during tensile test. Stress intensity factor is therefore approximated by Equation (12), which is adopted in this analysis for critical stress intensity factor. Mode I of loading is the only mode considered in this analysis. Equation (12) is further converted to Equation (18), from which it is understood that higher stress intensity factor could lead to longer critical crack length where the crack could propagate for a given load. Longer critical crack lengths obtained under prescribed load therefore could be considered as an indication for insensitivity of the material to crack.

$$a_c = \frac{K_c^2}{(C\sigma)^2\pi}$$  \hspace{2cm} (18)

Equation (12) can also be converted to Equation (19), from which it is interpreted to indicate that higher load is needed to reach the critical stress intensity factor required to propagate a defined length crack. This describes tensile test process during which external load keeps increasing on paper sample with pre-cut notches until it breaks. Equation (19) is shown below.
\[
\sigma_c = \frac{K_c^2}{C\sqrt{\pi a}}
\]  

(19)

To compare sensitivity of different materials to existing crack, critical stress intensity factor and certain load were adopted to calculate critical crack length based on Equation (18). Then critical stress intensity factor ratio, which is shown as Equation (20), will be used to show the sensitivity of different samples to crack.

\[
\text{Critical Stress Intensity Factor Ratio} = \frac{K_c}{K_{c(1mm)}}
\]

(20)

Where \( K_c \) is the critical stress intensity factor of notched paper sample and \( K_{c(1mm)} \) is the critical stress intensity factor of one millimeter notched paper.

4.2.2 Energy analysis approach

Stored and consumed energy are other measures that can be used to characterize fracture toughness of paper. Tensile energy absorption (TEA), which is the total energy consumed after paper completely breaks, is used as an indicator of paper fracture toughness. It is calculated from the stress vs. strain curve of the DENT and its value is equal to the shaded area shown as Figure 5.

Critical energy release rate \( G_c \), which is energy required for a crack to start propagating during the tensile test, is the other appropriate method to explore a samples’ fracture toughness \( G_c \). This quantity is difficult to measure because one needs to know how much of the energy is stored in the sample. The shaded area in Figure 6 gives the total work required but that included energy dissipated. In this study, the energy represented in Figure 6 was used as a measure of \( G_c \), knowing that it is an over-prediction. Therefore, both TEA and \( G_c \) are used as energy methods to explore paper samples’ fracture toughness in this study.
5. RESULTS

5.1 Tensile Strength Ratio Analysis

Commercial papers, a polymer film, and handsheets were pre-conditioned to 50% humidity and temperature of 22°C for fracture resistance testing. These papers can represent properties that one can expect from most paper grades in use. The physical properties of all papers are shown from Table 1 and Table 2. All papers’ stress-strain curves from testing are given in Appendix B and used for further analyses in this chapter.

Paper towel critical tensile strength, was determined as the maximum stress achieved during a notched tensile test as shown in Figure 9.

\[
\sigma_{\text{max}} = \begin{cases} 
\sigma_1 & \text{if } \sigma_1 > \sigma_2, \\
\sigma_2 & \text{if } \sigma_1 \leq \sigma_2.
\end{cases}
\]

Figure 8  Stress vs. Strain Curve of Paper Towel Fracture Process

\(\sigma_1 \) and \(\sigma_2\), which were defined in Figure 7 as average tensile stress in different locations of paper sample, are used for analysis. These strength values versus crack size are shown in Figure 9 and Figure 10 for paper towel and copy paper respectively. Figure 11 shows trends of critical tensile strength ratio versus notch size for all commercial papers used in this study. The Error bars are given in all the Figures this
thesis represents. (One standard deviation for five repetitions)

Figure 9 Tensile Strength vs. Notch Size for Paper Towel

Figure 10 Tensile Strength vs. Notch Size for Copy Paper
First it is noted that both paper towel and copy paper are sensitive to notches as shown by the decrease in strength with notch size in Figure 9 and Figure 10. Second, comparison of the results in Figure 11 reveals that paper towels are less sensitive than copy paper to small notches as noted by the relatively low change in ligament stress. Third, it is noted that tensile strength stops dropping for large notches. (Notch size is larger than 10 mm on both edges and sample size is 76.2 mm). Interestingly, both samples show less sensitivity to cracks for large notches.

To compare different papers with various scales of strength properties, the ratio of notched critical tensile strength to the unnotched tensile strength is calculated. Figure 11 shows trends of critical tensile strength ratio versus notch size for the cross direction of different commercial papers.

It is observed from Figure 11 that paper towel’s fracture resistance is the least sensitive to the increasing crack length, while newsprint has the biggest decreasing rate of critical tensile strength as crack length increases. Copy paper has the second biggest decreasing rate followed by paper board. These obvious differences support the hypothesis that paper structure could change relative fracture resistance. It is noticed that one major structural difference of these commercial paper is their heterogeneity from void space, which are shown in the Figure 3 above.
It is found that paper towel had the least homogeneous level of structure among all types of paper, followed by paperboard, and then copy paper. Newsprint has the most homogenous level of structure. Therefore, one preliminary result is drawn that the homogenous scale level of paper structure could increase sensitivity of paper critical fracture resistance as crack length increases.

Relative fracture toughness for cellophane, which has a small size scale level of structure, is compared with news print in Figure 12. Note the cellophane is much more sensitive to fracture than the papers.

As shown in Figure 12, fracture resistance of cellophane experiences a sharp decrease in strength even with a small notch size. This result further supports the conclusion that material with very homogenous structure would have low fracture resistance. Homogeneity in paper structure could affect paper fracture resistance negatively when cracks exist.

**5.2 Tensile Energy Absorption (TEA) and Energy Release Rate Analysis**

The same tensile load-deformation data used for strength analysis is adopted here for TEA and energy release rate analysis. TEAs and energy release rates are calculated as the shaded area in Figure 5 and Figure 6 for different papers. Then for each kind of
paper, TEA and energy release rate of DENT samples are divided by those of unnotched samples to get the ratio. Similar to strength analysis, this ratio can be used to compare papers’ sensitivity to cracks. The trends of TEA and energy release rate ratio versus notch size for different paper are shown on Figure 13 and Figure 14.

From Figure 13 and Figure 14, it is observed that energy consumed during the fracture of the paper towel changes little compared to the other papers.
However, there is no obvious difference for the behavior of all the other commercial papers. The cellophane did show more sensitivity. Because energy analysis did not differentiate between copy, newsprint, and paperboard, this approach was not further pursued. The calculation of critical stress intensity factor was derived from the energy balance during fracture process and its results and analysis is shown in the following section.

5.3 Stress Concentration Analysis

The critical stress intensity factor was derived from the tensile test results. Then the critical stress intensity factor of DENT samples are divided by that of samples with double notches of one millimeter size to get critical stress intensity factor ratio. Similar to the previous analysis, this critical stress intensity factor ratio can be used to reflect materials’ insensitivity to cracking. Figure 15 shows trends of critical stress intensity factor ratio versus notch size for different paper.

![Figure 15 Critical Stress Intensity Factor Ratio vs. Notch Size for Commercial Paper](image)

Figure 15 showed that under loads, paper towel has the lowest drop of critical stress intensity factor when making propagation of crack among all commercial papers. Paper’s stress intensity factor could reflect its fracture resistance to cracking and larger critical stress intensity factor ratio as crack increases could reflect more insensitivity of paper sample to existing crack. Therefore it is observed that paper
towel has the highest relative fracture resistance while news print has the lowest. The same result was obtained from tensile strength ratio analysis. This result further proves the conclusion that an inhomogeneous level of paper structure could result in high resistance to crack during fracture process.

5.4 Fracture Toughness in Machine Direction and Cross Direction

Another interesting point is the difference of relative fracture between machine direction and cross direction of the same paper. There is a difference of typical paper strength properties in the two directions. The process used to manufacture paper, results in preferentially more fibers aligned in MD than CD and those fibers are dried under tension. The number of other fibers crossing an MD fiber is lower than one in CD. These differences might affect the relative fracture toughness. One would expect CD to be relatively tougher than MD because the CD fibers that are better held in the sheet and can experience much more stretching before fracture.

Figure 16 provides the tensile strength ratio versus crack length for paper towel in MD and CD. As expected, it is observed that paper towel in cross direction has less sensitivity to cracks than that in machine direction. The similar trends of other commercial paper are shown in Figure 17 and 18.
The critical stress intensity factor analysis is also applied and critical crack length ratio is shown in Figure 19, 20, and 21. The results are the same with previous analysis.
Figure 19 Critical Stress Intensity Factor Ratio vs. Notch Size

Figure 20 Critical Stress Intensity Factor Ratio vs. Notch Size

Figure 21 Critical Stress Intensity Factor Ratio vs. Notch Size
Based on the results, it is therefore that structural differences in MD and CD lead to directional relative fracture toughness.

5.5 Insensitivity to Small Cracks for Fracture

Very small cracks appear to have no effect on the tensile strength of paper. The largest crack length where there is no significant change in tensile strength was investigated.

Figure 22 shows the trend of tensile strength ratio versus notch size for small cracks. It is shown in the figure that crack with length of 0.5mm have no effect on the strength of paper towel, paperboard and copy paper. In contrary, newsprint is sensitive to this small length crack and its fracture toughness decreases. Also, it was found that when crack size increases from 0.5 mm to 1 mm, fracture toughness of paper board and copy paper become sensitive to the crack. At the same time, the paper towel is still insensitive to the crack. This again suggests that the different homogenous levels of paper structure in different commercial papers influences fracture resistance. More heterogeneity in the structure results in less sensitivity to small cracks.

5.6 Handsheets Analysis

In previous section, the conclusion was drawn that the structure of commercial paper
affects relative fracture toughness. Commercial paper with less homogeneous levels of structure had higher relative fracture toughness. In this section, results of hand sheets made in the lab are used to further discuss how the structure of paper affects fracture resistance. It was expected that structure of a hand sheet would vary for different process parameters. Sheetmaking parameters, including paper grammage, beating degrees and pressing pressure were employed to give different structures in the handsheets. Grammage was divided by thickness to determine paper density, which was used as an indicator for homogenous level of paper structure. Therefore handsheets with denser fiber distribution are considered to have a more homogenous structure. Different parameters of grammage, beating degree and pressing pressure are expected to result in different fiber distribution and paper formation.

5.6.1 Grammage Analysis

Grammage is the measurement of mass per area. Three types of handsheets were made with different grammages. They are all unbeaten and unpressed. Their grammages and densities properties are shown in table 1, which show that the higher grammage of handsheet is leading to higher density. From the results in the table, it is expected that handsheet 25 should be the most affected by inhomogeneity while handsheet 100 should be the least affected. Both tensile strength ratio analysis and critical stress intensity factor are applied for further fracture toughness study.

Figure 23 shows the trend of tensile strength ratio versus notch length for these three types of handsheets, indicating that hand sheet 25 is the least sensitive while sheet 100 is the most sensitive to the cracks. This expected result further proves that paper structure affects relative fracture toughness.
A similar result is obtained from critical stress intensity factor analysis. The trend of critical stress intensity factor ratio is shown in Figure 24. It is found that hand sheet 25 requires the highest stress intensity factor, suggesting highest fracture toughness for the same length crack.

Both analyses give the conclusion that different grammage of paper could lead to different fracture resistance. Less grammage of paper could give higher fracture resistance to crack.
5.6.2 Beating Degree

Beating of pulp by refiner could give fiber more surface area and more relative bonded area. It is expected that with more beating, paper should have better bonding and therefore behave as if it is more homogenous. Three types of hand sheets were made with the same grammage and pressing pressure but different beating degrees. Their density properties are shown in table 2 and 3. From tables, it is found that paper with higher beating degree has higher density, which also means more homogenous level of structure. Therefore, it is expected that CSF 705 should have the highest fracture resistance to crack. Tensile strength ratio analysis and critical stress intensity factor are applied for further fracture toughness study.

Figure 25 shows the trend of tensile strength ratio versus notch size for these three hand sheets, and it shows that handsheet CSF 705 has the least decreasing rate while handsheet CSF 160 has the most decreasing rate. This is the expected result and it further proves that homogenous level of paper structure affects its fracture toughness to crack.

![Figure 25 Tensile Strength vs. Notch Size for Hand Sheet](image)

Result for critical stress intensity factor analysis is showed in Figure 26. It is found that hand sheet CSF 706 requires the highest stress intensity factor, which means highest fracture toughness for the same length crack.
It is concluded that beating degree can affect homogenous level of paper structure. Lower beating degree could lead to less homogeneous level of paper structure and higher fracture resistance.

### 5.6.3 Pressing Pressure

Pressing pressure is the often used parameter to make paper denser provide for more bonding and hence a stronger sheet. Therefore, it is expected that with higher pressing pressure, paper should have behave more like a homogenous material. Two types of handsheets are made with pressure and no pressure applied. Both of them have the same grammage and beating degree. It is expected that unpressed handsheet should have higher fracture resistance. Tensile strength ratio analysis and critical stress intensity factor are applied for further fracture toughness study.

Figure 27 is the trend of tensile strength ratio versus notch size for both two hand sheets, and it shows that hand sheet unpressed has lower decreasing rate while hand sheet pressed has higher decreasing rate. This is the expected result and it further proves that homogenous level of paper structure affects its fracture toughness.
Results for critical stress intensity factor analysis are shown in Figure 28. It is found that unpressed handsheets requires the highest stress intensity factor, which means higher fracture toughness for the same length crack.

It is concluded that pressing pressure can affect paper loose structure. Lower pressing pressure could give more loose structure, which lead to the higher fracture resistance to crack.
6. DISCUSSION

6.1 Evaluation of Analyses for Papers’ Relative sensitivity to Fracture

Relative sensitivity to fracture was investigated with three measures (1) tensile stress ratio, (2) consumed energy ratio and (3) stress intensity factor ratio. It is found from the results that tensile stress ratio and stress intensity factor ratio can effectively differentiate the sensitivity to crack between different paper grades while consumed energy analysis only separated the two extreme cases (paper towel, cellophane) from the other papers. A comparison of the tensile strength and stress intensity factor ratio for three commercial papers are given in Figure 29, 30 and 31.

![Graph showing Tensile Strength and Stress Intensity Factor Ratio vs. Notch Size for Paper Towel](image)

**Figure 29** Tensile Strength and Stress Intensity Factor Ratio vs. Notch Size for Paper Towel
It is observed that stress intensity factor analysis can differentiate MD and CD more effectively than tensile strength ratio analysis. To further compare the sensitivity of three methods, the TEA and tensile strength ratio curves for paper towel, paper board and copy paper are shown from Figure 32 to Figure 34.
Figure 32  Tensile Strength and TEA Ratio vs. Notch Size for Paper Towel

Figure 33  Tensile Strength and TEA Ratio vs. Notch Size for Paper Board
The tensile strength ratio analysis above is calculated based on the $\sigma_2$. The remote load ratio calculated based on the $\sigma_1$ is also introduced. The comparison of stress intensity factor ratio and remote load ratio for paper towel, paper board and copy paper are shown from Figure 35 to Figure 37.
It is observed that remote load ratio could not differentiate paper’s sensitivity to crack as stress intensity factor ratio analysis does even both of them are calculated from $\sigma_1$. Another comparison between tensile strength ratio analysis and remote load ratio analysis is shown from Figure 38 to Figure 40.
Figure 38 Tensile Strength and Remote Load Ratio vs. Notch Size for Paper Towel

Figure 39 Tensile Strength and Remote Load Ratio vs. Notch Size for Paper Board
The ratio curves are fitted to linear equation to get their slopes. Those slopes may represent the relative sensitivity. The slopes for three commercial papers in both MD and CD are shown in the Table 6.

### Table 6 Slopes of Commercial Papers

<table>
<thead>
<tr>
<th>Samples</th>
<th>Tensile Strength Ratio Slope</th>
<th>TEA Ratio Slope</th>
<th>Remote Load Ratio Slope</th>
<th>Kc Ratio Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper Towel CD</td>
<td>-0.0025</td>
<td>-0.0505</td>
<td>-0.0346</td>
<td>0.5178</td>
</tr>
<tr>
<td>Paper Towel MD</td>
<td>-0.0315</td>
<td>-0.0767</td>
<td>-0.0491</td>
<td>0.2932</td>
</tr>
<tr>
<td>Paper Board CD</td>
<td>-0.0099</td>
<td>-0.0848</td>
<td>-0.0335</td>
<td>0.4702</td>
</tr>
<tr>
<td>Paper Board MD</td>
<td>-0.0328</td>
<td>-0.0849</td>
<td>-0.0498</td>
<td>0.2776</td>
</tr>
<tr>
<td>Copy Paper CD</td>
<td>-0.0223</td>
<td>-0.0934</td>
<td>-0.0429</td>
<td>0.3972</td>
</tr>
<tr>
<td>Copy Paper MD</td>
<td>-0.0337</td>
<td>-0.092</td>
<td>-0.0527</td>
<td>0.2367</td>
</tr>
</tbody>
</table>

From Figures and Table 6, it is observed that stress intensity factor ratio is the most sensitive method to differentiate paper’s sensitivity to cracking while remote load...
ratio analysis is the least sensitive method. However, according to the Appendix A, the intensity factor is proportional to stress so the same information should be contained in both. The difference in sensitivity is because the intensity factor includes the correction factors, Equations (13) to (15). As noted in Appendix A, LEFM is not adequate to describe the fracture behavior of paper so the stress intensity factors calculated from LEFM are misleading. One must modify the LEFM theory to allow for a constant stress intensity factor as a function of crack length. Thus, it is concluded that even though the LEFM calculated stress intensity factor shows high sensitivity it cannot be applied to paper. This result indicates tensile strength ratio analysis and remote load ratio analysis as correct tools to explore paper’s sensitivity to crack and that papers are relatively tough materials when it comes to fracture propagation.

6.2 Comparison with modified LEFM method

However, stress intensity factor is calculated from far away remote load, $\sigma_1$, based on Equations (12) and (18). This calculation from traditional LEFM can not address the singularity at the crack tip, and it does not seem reasonable to have the stress intensity factor change with crack length. Thus, development of a modified LEFM model completed in the same research group (see appendix A) has been developed as Equation (21) below.

$$\sigma = \frac{K_i}{f(d/w)\sqrt{\pi d}}$$  \hspace{1cm} (21)

Assuming critical stress intensity factor is the constant for the material, the modified LEFM method introduced a length $d$, which represents a fracture process zone of the paper web. Length $d$ consists of both a structural length ($d_s$) and a material component ($d_m$). From Equation (12) and length $d$, the relationship between stress ratio (or remote load, $\sigma_1$), sample size and crack length has been developed as equation (22) and (23).
\[ F = \begin{cases} F_0 & \quad \forall a \leq d_s \\ \frac{1}{f\left(\frac{d}{\sqrt{\frac{a+d-d_s}{w}}}\right)} & \quad \forall d_s < a < w - 2d \end{cases} \]  

\[ \sigma_1 = TS \begin{cases} \frac{1}{f\left(\frac{d}{\sqrt{\frac{a+d-d_s}{w}}}\right)} & \quad \forall a \leq d_s \\ \frac{d}{\sqrt{\frac{a+d-d_s}{w}}} & \quad \forall d_s < a < w - 2d \end{cases} \]  

\[ F \] is the fracture load of notched sample and \( F_0 \) is that of unnotched sample. \( \sigma_1 \) is the far field stress and \( TS \) is the tensile strength. As shown in the Appendix A, this modified LEFM fit both published data and simulation results well. Shown as Figure (12) and (13) in the appendix A, the new method can predict behavior of paper better than the traditional LEFM. It was also able to differentiate various paper’s sensitivity to cracking. Therefore, Equation (22) and (23) can be considered as an adequate and proper method to determine the sensitivity of paper based on the stress analysis.

When compared to the values investigated in this thesis, the modified LEFM suggests that energy measures are not necessary and would be difficult to use in the characterization of the fracture behavior.

### 6.3 Load Concentration with Deep Notch

It is noted in this study and Appendix A that for some commercial papers and handsheets, \( \sigma_2 \) begins to increase again for larger cracks. Figure 23 shows this for the handsheets as well. This result might stem from a decrease in the ability of the deeply notched papers’ ability to concentrate load around crack tip. Thus, higher remote load is needed to make crack propagate on deeply notched paper resulting in higher calculated \( \sigma_2 \). It is also noted that for some deeply notched paper samples, \( \sigma_2 \) can be even higher than that of unnotched tensile strength. This suggests that the measured tensile strength of paper is in itself a result of a fracture process affected by the concentration stress somewhere along the edges of the sample.
6.4 Correlation of Papers’ Properties and Sensitivity to Cracking

In this thesis, physical properties of grammage and resulting sheet density of commercial papers were measured as physical or structural properties. To characterize the heterogeneity, the SEM images given Figure 3 were analyzes by thresholding with the result shown in Figure 41.

![Micrographs of paper samples](image)

The black pixels represent open spaces and white pixels represent the fibers in Figure 41. The pore percentages are thus calculated and shown in the Table 7.
Table 7 Pore Percentage of Commercial Papers

<table>
<thead>
<tr>
<th>Samples</th>
<th>Pore Percentage, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper Towel</td>
<td>60.5</td>
</tr>
<tr>
<td>Paper Board</td>
<td>20.1</td>
</tr>
<tr>
<td>Copy Paper</td>
<td>16.5</td>
</tr>
<tr>
<td>News Print</td>
<td>4.0</td>
</tr>
</tbody>
</table>

The effective fracture process zone $d$ and structural zone length $ds$ for commercial papers taken from Appendix A are given in Table 8.

Table 8 Effective Fracture Process Zone $d$ and Structural Zone Length $ds$ for Commercial Papers

<table>
<thead>
<tr>
<th></th>
<th>$d$, mm</th>
<th>$ds$, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper Towel CD</td>
<td>7</td>
<td>2.5</td>
</tr>
<tr>
<td>Paper Board CD</td>
<td>6.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Copy Paper CD</td>
<td>5</td>
<td>0.25</td>
</tr>
<tr>
<td>Paper Towel MD</td>
<td>4.2</td>
<td>1.7</td>
</tr>
<tr>
<td>Paper Board MD</td>
<td>2.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Copy Paper MD</td>
<td>2.3</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Correlation coefficients were determined between the papers’ sensitivity to cracking as measured by the ratio of sensitivity slopes (Table 6), fracture process zone and structural zone length in the Table 9 to pore percentage, grammage, and density. The correlation coefficients are shown in Table 9. One correlation curve between structural zone length CD and pore percentage is in Figure 42.
Table 9 Correlation Coefficients of Analyses for Commercial Papers

<table>
<thead>
<tr>
<th></th>
<th>Pore Percentage</th>
<th>Density</th>
<th>Grammage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fracture Process Zone in CD</td>
<td>0.745</td>
<td>0.797</td>
<td>0.003</td>
</tr>
<tr>
<td>Fracture Process Zone in MD</td>
<td><strong>0.997</strong></td>
<td><strong>0.9999</strong></td>
<td>0.609</td>
</tr>
<tr>
<td>Structure Zone Length in CD</td>
<td><strong>0.9999</strong></td>
<td>0.995</td>
<td>0.676</td>
</tr>
<tr>
<td>Structure Zone Length in MD</td>
<td><strong>0.9999</strong></td>
<td>0.998</td>
<td>0.650</td>
</tr>
<tr>
<td>Tensile Strength Ratio Slope in CD</td>
<td>0.828</td>
<td>0.871</td>
<td>0.133</td>
</tr>
<tr>
<td>Tensile Strength Ratio Slope in MD</td>
<td><strong>0.941</strong></td>
<td><strong>0.965</strong></td>
<td>0.374</td>
</tr>
<tr>
<td>TEA Ratio Slope in CD</td>
<td><strong>0.993</strong></td>
<td><strong>0.999</strong></td>
<td>0.574</td>
</tr>
<tr>
<td>TEA Ratio Slope in MD</td>
<td><strong>0.918</strong></td>
<td><strong>0.947</strong></td>
<td>0.315</td>
</tr>
<tr>
<td>Remote Load Ratio Slope in CD</td>
<td>0.471</td>
<td>0.541</td>
<td>0.345</td>
</tr>
<tr>
<td>Remote Load Ratio Slope in MD</td>
<td>0.705</td>
<td>0.760</td>
<td>0.061</td>
</tr>
</tbody>
</table>

Figure 42 Correlation of Structural Zone Length Versus Pore Percentage

It is found from the Table 9 that pore percentage and density of papers has excellent
correlation with several of the parameters while grammage has little correlation with any of the parameters. Structure zone length $d_s$ developed in the Appendix A has the best correlation with pore percentage. It has good correlation with density as well. Fracture process zone in MD has good correlation with pore percentage and density while fracture process zone in CD has relative worse correlation with pore percentage and density. This is probably because commercial papers would experience more plasticity in CD than in the MD as shown in Appendix A. Concerning the ratio analyses, tensile strength ratio in MD analysis and TEA ratio analysis in both directions has relative good correlation with pore percentage and density. Remote load ratio analysis does not show good correlation with pore percentage and density.

It appears that the fracture process zone developed in the Appendix A can be used as good method to explore the sensitivity of paper cracking and that the structural component, $d_s$, correlates well with density and the SEM pore percentage, which are both indications of structural features of the paper. While it is known from previous results that TEA ratio analysis cannot differentiate different paper’s sensitivity to cracking, there does seem to be some correlation to the structure that would merit further study.
7. CONCLUSIONS

Three analyses, including tensile stress ratio, consumed energy ratio, and stress intensity factor ratio were developed and used to explore the relative sensitivity of a paper to crack propagation. It is found that tensile stress ratio and stress intensity factor ratio differentiated all papers’ sensitivity to cracking while the energy method (TEA and energy release rate) can only differentiate extreme cases (for example a paper towel).

The work given in Appendix A, considers paper as an inherent flawed fiber web and assumes that the stress intensity factor is constant. The resulting modified LEFM introduces a fracture process zone, \( d \), with a structural component, \( d_s \), and successfully proved an adequate fit for the results of many papers. The magnitude of \( d \) is thus a parameter that can be used to characterize the relative sensitivity of a paper to fracture; the larger the magnitude of \( d \), the lower the sensitivity. While using LEFM to calculate stress intensity factors seems to differentiate papers from one another, it is not justifiable and should not be used.

SEM images and paper properties including grammage and density were used in an attempt to characterize the fiber web’s level of heterogeneity. Paper with a more open surface as shown in the SEM image and lower grammage and density were considered to have less homogenous structure. These results showed that as inhomogeneity increased sensitivity to fracture decreased. Grammage itself did not correlate with the fracture process zone, but rather density is more likely an indicator of the ability of the structure to concentrate load.

The combination of experimental testing and analysis suggests that paper structure plays an important role on relative fracture toughness. Paper towels, as the least homogenous material, were the least sensitive to notches; whereas, a film taken to be the most homogenous exhibited the greatest sensitivity. In addition, the lack of sensitivity to small cracks illustrates that the heterogenous nature of paper influences
fracture. The same papers exhibited more sensitivity in the MD than the CD showing that structure, processing, and resulting material plasticity all play a role in fracture sensitivity.

The fracture analyses further establish the impact of size-scale of paper on fracture toughness. It is concluded that heterogeneity decreases the relative sensitivity of a paper to cracks.

Future work should be done to further investigate the energy analysis, which did not yield fruitful results in this study. One would need a method to extract stored energy at the time of fracture. Given the results presented in Appendix A, it may be more of scientific purposes to pursue the energy analysis rather than to develop a reasonable prediction.

The increase in $\sigma_2$ for larger notches is also of interest and could be tied to structure of the paper. Further investigation of this should be done.
8. REFERENCES


Appendix A
UTILIZATION OF MODIFIED LINEAR ELASTIC FRACTURE MECHANICS TO CHARACTERIZE THE FRACTURE RESISTANCE OF PAPER

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ABSTRACT

Linear elastic fracture mechanics modified to account for an effective fracture process zone is sufficient to characterize and predict fracture resistance for a wide range of papers. The simplicity of the method, which only requires the tensile strength and a measure of the effective fracture process zone length, gives it great advantage over other existing approaches. The results presented here show that for a wide range of commercial papers, samples widths as narrow as 50 mm are sufficient to determine the effective process zone length, and that scaling holds well enough to allow prediction for fracture of wide webs. The results indicate that the tensile strength of paper is a result of a fracture process where the defect is most typically induced from cutting the network structure along the edges. As a consequence, the inherent tensile strength of the network can be significantly larger than the measured tensile strength. The effective fracture process zone length parameter is taken as a measure of the inability for the paper to concentrate load near the crack tip. This ability for network structures to concentrate load has significant impact on the fracture resistance of the sheet relative to its tensile strength.
1 INTRODUCTION

Imperfection limits the strength of paper, but this defect sensitivity is tempered by dissipative material and structural processes and features that limit the ability of the sheet to concentrate load. A substantial amount of literature has been devoted to understanding the fracture resistance of paper; see the reviews of Kortschot [1], Mäkelä [2], and Niskanen [3]. A recent account of the requirements, advantages, disadvantages, and applicability of Linear Elastic Fracture Mechanics [LEFM], nonlinear fracture mechanics, cohesive zone modeling and damage mechanics was provided by Östlund and Mäkelä [4]. It is clear that, LEFM in a strict sense cannot be applied to paper. Uesaka et al. [5] first showed that the J-integral method was better suited for characterization of fracture in paper compared to the stress-intensity factor. At the 10th FRS, Niskanen [3] concluded that LEFM cannot generally be applied to paper. He concludes that LEFM is reasonable only for cases where the crack sizes are sufficiently large to render the plastic fracture process zone negligible and this requirement results in unreasonably large test specimen sizes. Östlund et al. [6] point out that LEFM even with a plastic zone correction is not self-consistent, because a measure of rupture energy over predicts the stress intensity factor and a measure of stress intensity under predicts rupture energy. Despite this fact, there is still evidence in the literature [7–9] that LEFM methods are useful for characterization of fracture resistance.

Andersson and Falk [10] used a Griffith-Irwin type fracture criteria to account for the undefined fracture process zone (FPZ) that precedes the well-defined crack [11]. They did not correct for the finite-width of their samples (15 mm), which is likely too small to fully capture the behavior [12] and it would seem that they under-predict the FPZ (0.6 mm for handsheets). They also conducted constant load fracture tests on larger width samples, and the resulting FPZ seems to be approximately four times larger (on the order of 2.5 mm for handsheets.) Seth and Page [7] utilized LEFM to study the fracture behavior of paper and conclude that for LEFM to be applied to paper, the samples must be of sufficient sample width and crack length. Swinehart and Broek [8] showed that LEFM equations can predict fracture loads of large webs with large cracks with no modification for the FPZ.

Donner [9] continued in the same vein as previous work [7, 10] and separated the FPZ into a structural and material component. By conducting tensile tests on newsprint samples in a cryogenic environment, a very brittle and linear-elastic response was obtained. Fracture tests in the cryogenic environment yielded a FPZ of about 0.5 mm for MD and 1.1 mm for CD. At room temperature, the FPZ was 1.5 mm for MD and 3.7 mm for CD.

Kortshot and Trakas [13] took a similar approach, point stress criteria (PSC), to describe the fracture resistance, utilizing both centered holes of various
Utilization of Modified Linear Elastic Fracture Mechanics

diameters and centered slits for newsprint, bond paper, and a copy paper. For holes they found the characteristic FPZ length to be in the range of 1.2 to 1.7 mm for MD and 3.3 to 3.5 mm for CD. For slits, the characteristic length in MD was smaller, 0.8 to 1.0 mm, which could have been affected by the expression they chose to use for the stress-distribution near the slit. Inspection of their results shows that the strength of a sheet with a 1 mm hole was not significantly different than the tensile strength indicating that the small hole did not affect the ability to the paper to effectively carry load; indicative of Donner’s [9] structural FPZ. Considine et al. [14] utilized the PSC and an average stress criteria (ASC) along with LEFM equations for the stresses near a hole for an orthotropic material. These authors do not report the FPZ, but do report inherent flaw sizes ranging from 0 to 0.88 mm for MD and 0 to 1.55 mm for CD for a range of papers.

The attractiveness of LEFM is the simplicity of its application; an argument proffered by Swinehart and Broek [8] for favoring stress intensity factor over the J-integral method. An abundance of explicit equations are available for LEFM, and if applicable one could apply these equations to characterize paper materials and predict behavior with increased size-scaling with relative ease. Implementing nonlinear fracture mechanics is complex; requiring a description of the constitutive behavior, a library of stored geometric correction factors, and numerical evaluation for each point of interest. To be useful the LEFM method should be capable of predicting the behavior of large samples from measurements made on small samples.

Using nonlinear fracture mechanics, Mäkelä, Nordhagen, and Gregersen [15] demonstrated that they could predict the behavior of wide samples (800–1000 mm) based on fracture toughness measurements of narrow samples (50 mm). Expanding on the approach of Swinehart and Broek [8] for a J-integral method, Mäkelä and Fellers [16] and Mäkelä [17] have presented procedures and explicit equations that can be used for prediction of fracture resistance. While it eliminates the need to complete a finite-element analysis, it still requires a library of correction values, and numerical inversion for each prediction.

Östlund and Mäkelä [4] state the following: “Many fracture mechanics models can be applied to paper materials and products depending on the problem and objectives of the analysis, but is it best to use the simplest possible model that has predictive capability.” The simplicity of LEFM is too attractive to completely dismiss and despite its reported shortcomings it may still be valuable as a predictive tool that can be implemented with minimal testing and little computational difficulty. Although LEFM is stated to be adequate for large sheets with large cracks, it remains to explore its applicability for smaller cracks and predictions based on independent measures. Reporting the results of such an inquiry is one of the main purposes of this contribution.
Douglas W. Coffin, Kun Li and Ji Li

In the following, the LEFM equations are modified with the addition of an inherent FPZ and a normalization to the tensile strength. The previous literature is considered in light of this approach, and the predictive capability using the data from [15] is shown to be just as adequate as the nonlinear fracture mechanics approaches used in [15–17]. By assuming that tensile strength is governed by the same fracture process, the singularity for small cracks sizes is removed, or rather a finite length crack is always present. The interpretation of the FPZ presented here is as a measure of the inability of the paper to concentrate load at the crack-tip. The larger the FPZ the less ability the structure and/or material has to concentrate load at the tip and the higher the fracture toughness relative to the tensile strength. Results are provided to demonstrate how the relative defect sensitivity of papers can be assessed. This development provides a simple method that can be utilized to characterize the fracture toughness of materials and predict the behavior in large webs; at least with small cracks.

2 ANALYSIS

2.1 Modified LEFM

Because paper is a discrete network structure of fibers, it is inherently flawed. There is a scale level below which the assumptions of continuity are invalid. The discrete nature of paper is smoothed because it is stochastic and continuum models can be applied to great success as long as the dimensions of interest are relatively large. With regards to fracture, previous fracture studies [9, 13, and 14] indicate that this inherent flaw is on the order of a few millimeters. The governing equations for an ideal elastic continuum, allow for singular stresses, but in real materials they are limited to some finite maximum. For a brittle response, the stress is limited by this minimum structural scale, and if the failure mechanism is a fracture process, we can cut-off the singularity by normalizing the fracture loads to the strength based on a minimum allowable crack size.

Most papers exhibit sensitivity to cracks or notches, but tend to be relatively tough materials. Notch sensitivity is typically attributed to a concentration of stresses near the tip of the notch. There is a zone around the notch where the material has yielded and/or undergone partial failure. At some level of loading, the material fails globally; typically starting near the notch tip. The maximum load could correspond to the point when the notch length begins to increase or shortly after that event. Inside the zone of influence near the crack tip, a multitude of mechanisms could be occurring to diminish the stress concentrations. Plasticity will limit magnification of stresses. Cohesive failure of the structure will allow
Utilization of Modified Linear Elastic Fracture Mechanics

reduction of stress levels. The inherent structural inhomogeneity will create some scale level below which stresses cannot concentrate. The literature includes successful application of theories that account for one of these aspects while ignoring others; for example see [15] for plasticity and [18] for material heterogeneity. In these models, some parameter is utilized to account for the effective behavior of the material regardless of the actual contribution from various different effects.

In-plane, fracture tests geometries are typically conducted as either a center-notched test (CNT), a double edge-notched test (DENT), or a single edge-notched test (SENT) with specimens as shown in Figure 1. The geometries are defined for each test such that the ligament length to sample width ratio is \(1-a/w\) and that for small cracks the SENT and the DENT would converge to the same fracture strength for the same magnitude of \(a\). Note, this requires that for SENT the width, \(W\), is defined as \(W=2w\) instead of \(2w\). For the cases considered here the notch or crack, \(a\), is considered a slit, with the tip as sharp as the minimum discrete size scale permissible in the structure. The sample is loaded in tension with a load \(F\). Force equilibrium requires that the net force on the remaining ligament must still equal \(F\), but if stresses are higher at the notch tips the failure load reached in fracture will be reduced more so than the reduction in ligament length.

Figure 1. Typical geometries for in-plane fracture tests with paper.
Douglas W. Coffin, Kun Li and Ji Li

Notch sensitivity can be assessed by comparing the ratio of ultimate load of the specimen, \( F \), to that of the un-notched specimen, \( F_0 \). The criterion is

\[
\frac{F}{F_0} < 1 - \frac{a}{w}
\]

implies specimen is notch sensitive. (1)

An equality sign in Equation (1) would imply no sensitivity to the notch, and a greater than inequality would imply the notched specimen is effectively stronger than a specimen whose width equals the ligament length. For a material, whose strength is determined by defects one would expect the load ratio to exceed one as the ligament length approaches \( 2*FPZ \) because if stresses are elevated at a crack tip, the inherent strength must be greater than the bulk tensile strength.

Consider a linear elastic material. Following classic LEFM, the stress intensity factor can be expressed as [19]

\[
K_I = \sigma \sqrt{\pi a} f(a/w)
\]

where \( \sigma \) is the far field stress and \( f(a/w) \) is a correction factor for finite width samples. In Equation (2), the length is assumed to be sufficiently long as to not influence the correction factor. Expressions for the correction factors for the three geometries given in Figure 1, reported to be valid for all \( x<1 \) [19] are:

for CNT: \( f(x) = \left[ 1 - 0.025x^2 + 0.06x^4 \right] \sqrt{\sec(\pi x/2)} \)

for DENT: \( f(x) = \left[ 1 + 0.122\cos^4(\pi x/2) \right] \sqrt{2\tan(\pi x/2)/\pi x} \)

for SENT: \( f(x) = \left[ 0.752 + 2.02x + 0.37\left(1 - \sin(\pi x/2)\right)^3 \right] \sqrt{2\tan(\pi x/2)/\pi x} \)

Note the ratio of center to edge notched correction factor for \( x \) going to zero is 1.122 (The precision reported here is the same as given in [19], experimental significance is accounted for in the determination of \( d \)). If we assume that failure occurs when the stresses at the tip reach some failure level, it implies that the stress intensity factor, \( K_I \), is constant for all crack sizes and Equation (2) can be inverted for \( a>0 \). Now we assume that paper has an inherent characteristic fracture process zone length, \( d \), such that the un-notched limit load can be obtained from Equation (2) as
Utilization of Modified Linear Elastic Fracture Mechanics

\[ F_0 = \frac{K_W t}{f(d/w)\sqrt{\pi d}} \]  \hspace{1cm} (4)

where \( t \) is the thickness. Then the limit load ratio for at any notch \( a>d \) can be written as

\[ \frac{F}{F_0} = \frac{f(d/w)}{f((a+d)/w)} \sqrt{\frac{d}{(a+d)}} \quad \forall \ a < w - 2d. \]  \hspace{1cm} (5)

The length \( d \) is assumed to be composed of both a structural component and a material component \( d = d_s + d_m \). Consider the structural component a result of the discrete nature of the fiber network structure. The length \( d_s \) could be treated as an inherent “flaw”. In the presence of these flaws, edge failure would be more likely in a tensile test because load transfer structure is open at the edges. In addition, comparison of Equation (3) for CNT, DENT, and SENT shows that for small notches edge-notched specimens fail at a lower load then center-notched samples. In addition, both edges would have these flaws so the tensile strength should be similar to the DENT geometry except with small flaws.

For cracks \( a<d_s \), the fracture load should remain equal to \( F_0 \). Thus, equation (5) can be modified to be written as

\[ \frac{F}{F_0} = \begin{cases} 1 & \forall a \leq d_s \\ \frac{f(d/w)}{f((a+d-d_s)/w)} \sqrt{\frac{d}{(a+d-d_s)}} & \forall d_s < a < w - 2d \end{cases} \]  \hspace{1cm} (6)

The load ratio is equivalent to the average far field stress ratio and thus equation (6) provides a prediction for the fracture resistance of a paper. For edge cracked samples, the far-field fracture stress, \( \sigma_f \) can be predicted from the tensile strength, \( TS \), the characteristic fracture process zone length, \( d \) and the structural limit, \( d_s \) as

\[ \sigma_f = TS \frac{1}{f((a+d-d_s)/w)} \sqrt{\frac{d}{(a+d-d_s)}} \quad \forall d_s < a < w - 2d \]  \hspace{1cm} (7)

If center-notched specimens are used, the limiting ratio of failure stress to tensile strength at zero-notch length would be 1.122 because the un-notched specimen is more likely to fail at an edge rather than the center.

Figure 2 illustrates the effect of \( d \) on the load ratio using equation (6). Figure 2a is for the case where the characteristic fracture zone length is \( d=d_s \), and Figure 2b illustrates the case where \( d_s=0 \). Any combination between the two sets of curves can be obtained by adjusting the proportion of \( d_s \) to \( d \).
By re-casting the LEFM equation as shown in Equation (7), two important features missing from classic LEFM are gained. First, instead of relying on a measure of released and consumed energy as a measure of fracture toughness, Equation (7) relies on the tensile strength of the sample for magnification and a determination of \( FPZ = d \) for the relative sensitivity of the material to defects. Second, the stress singularity at the crack tip is removed or rather irrelevant. The assumption of a nonzero \( d \) and constant stress intensity factor ensures that the predicted load will converge to the tensile strength as the crack length goes to zero.

Equation (6) or (7) is a modified LEFM model that can be used to characterize and predict the fracture sensitivity of paper. The \( FPZ \) parameter \( d \) provides a measure of fracture sensitivity relative to the strength of the material. For specimen widths sufficiently larger than \( 2d \), \( f(d/w) \) is approximately \( f(0) \) and the stress intensity factor can be defined as

\[
K_f = 1.122TS \sqrt{\pi d}
\]  
\[\text{(8)}\]

The corresponding elastic energy release rate for an isotropic material is

\[
R = \frac{K_f^2}{E} = 1.259 \pi \frac{TS^2 d}{E}
\]  
\[\text{(9)}\]

For an orthotropic material, the effective modulus can be taken as \( E = E_{11}^{3/4} E_{22}^{-1/4} \), where \( E_{11} \) is the elastic modulus in the load direction and \( E_{22} \) is the elastic modulus in the direction perpendicular to the loading [6].

Figure 2. Fracture load ratio as a function of ligament length for various characteristic fracture lengths for DENT. (a) All structural \( d = d_s \), (b) all material \( d_s = 0 \).
2.2 Fracture of a Flawed Elastic Lattice

To demonstrate the effect of a discrete structure on the fracture sensitivity, a lattice model was developed using MATLAB. The model is shown to the left in Figure 3. The elements were assumed to be linear springs, but large deformations were accounted for with a quasi-static time step updating the length and orientation of the elements with each incremental loading. The lattice is composed of nodes arranged in a square array with the characteristic length $c$. The springs are arranged to be horizontal, vertical, and diagonal. The diagonal elements are not connected where they cross. The stiffnesses are chosen to give an initial isotropic response. Along the top edge the nodes are displaced with a uniform vertical displacement and free to move in the horizontal direction.

Along the bottom horizontal axis of symmetry, the last node is not connected to the line of symmetry. This is the initial flaw and renders the model a DENT. Additional crack lengths of length $a$ are given by releasing the nodes to the right of $a$ from being held to the line of symmetry. The vertical displacement along the top is incrementally increased until the reaction force at the crack tip (node furthest to the right being held to the horizontal line of symmetry) reaches a specified value. The model can easily be adjusted to be orthotropic, allow for plasticity of the elements, or a cohesive release of the nodes fixed to the horizontal line of symmetry. The deformed lattice shown to the right in Figure 3 corresponds to 120 steps to reach 10% effective strain, with $a/w=0.4$.

The initial state is taken with just the right corner node released. The applied force is obtained by summing the reaction forces, which are vertical, along the top edge. Then the model is re-run, but the next node to the left is released, effectively doubling the crack size. The load ratio is then determined for each crack size. If one imagines the crack size to be continuous, then the results of load ratio versus crack length will give a stair-step function. The load remains constant for all crack lengths between two nodes, and has a step discontinuity at a crack length corresponding to a nodal location.

Figure 4 provides the results of two simulations, which illustrate the behavior of the model. Figure 4a corresponds to a lattice with 5 unit cells in the half-width and 25 unit cells in the half-length so that the characteristic length ratio is $c/w=0.2$. Figure 4(b) corresponds to a model with a characteristic length ratio of $c/w=0.1$ by using 10 unit cells in the width direction and 50 in the half-length direction. In Figure 4a, the stair step response of the lattice model is shown. The circle markers represent the model result. The square markers represent the average of the two values at a node. The curves are representative of Equation (6) with three proportions of $d_s$ to $d$. The upper solid curve represents $d_s=0$, the bottom dashed curve represents $d_s=c$, and the middle dash-dot curve represents $d_s=c/2$. For all three lines, the characteristic FPZ is $d=c$. 
Figure 3. Flawed lattice model (1/4 of DENT specimen). (Right: deformed lattice)

Figure 4. Load ratio versus crack length for flawed lattice model. Black squares represent the average load ratio that occurs before and after the release of a node at a crack length of $a$. (a) is for a lattice with $w=5c$ and $L=50c$, and (b) is for $w=10c$ and $L=100c$. 
Utilization of Modified Linear Elastic Fracture Mechanics

Figure 4b shows only the average points from the lattice model, but the three corresponding curves from Equation (6) for \( d_s = c, \, d_s = c/2, \, d_s = c \) are given. As the ratio \( c/w \) decreases, the three curves will converge and the sensitivity to cracking will increase. Equation (6) with \( d = 2d_s = c \) provides an excellent fit of the numerical results. A comparison of the two curves is given in Figure 5.

The length of \( c \) relative to the width determines the ability of the structure to concentrate load. Figure 6 compares the stress distribution for the case when the crack length is \( a/w = 0.4 \) for the lattice models having \( c/w = 0.05, 0.1, \) and \( 0.2 \). With the smaller lattice structure, the stresses can concentrate closer to the tip. This is why the smaller lattice exhibits more sensitivity to cracks.

Thus, for an elastic material the parameter \( d \) represents a measure of the inability of the structure to concentrate load. Plasticity would further limit stress concentrations and increase the effective FPZ, \( d \).

![Figure 5](image.png)

**Figure 5.** Effect of characteristic lattice structure size on sensitivity to fracture.
It remains to be seen if Equations (6) or equivalently (7) are useful to characterize the fracture sensitivity of a paper and is capable of predicting the fracture of large webs based on values obtained from small scale testing. It is worth re-examining the data in the literature to evaluate Equation (6) or (7). For most of these comparisons, the term $d_s$ is set to zero because not enough information is available to distinguish it from $d$. This means that the data from the literature is typically fit with two parameters, tensile strength ($TS$), and the effective FPZ ($d$).

Östlund et al. [6] determine that LEFM could not be used to predict fracture. To make their argument, they used DENT specimens of a copy paper with 2, 4, and 6 mm notches and a sample width of either 100 or 50 mm. They report both the critical fracture stresses and the tensile strength of the samples. This fracture load to tensile strength ratio data is shown in Figure 7 along with a fit of Equation (7) where the parameter $d$ is the fitting parameter. For MD was found to be $d=2\text{mm}$ and for CD $d=5\text{mm}$. Östlund et al. [6] calculated stress intensity factors from two methods. Directly from the fracture strength and from the fracture energy determined from a short span tensile test. They then determined values of

**Figure 6.** Stress distribution along ligament for three characteristic lattice sizes with $a/w=0.4$. 

3 RELEVANCE TO THE LITERATURE

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Utilization of Modified Linear Elastic Fracture Mechanics

$d$ to minimize the error in the fracture stress calculation. A comparison of the current fit to that found in [6] is given in Table 1.

Figure 7 illustrates that Equation (7) provides reasonable fit for the fracture strength. The fact that the same $d$ fits for both the 50 and 100 mm lengths in CD suggests Equation (7) can be used when scaling to larger samples. In other words, for these crack lengths the shape factor is approximately constant as

![Graph showing comparison between Equation (7) and DENT fracture tests of a copy paper [6]]

Figure 7. DENT fracture tests of a copy paper [6] compared to Equation (7).

Table 1. Comparison of parameters from Equation (7) and reference [6]

<table>
<thead>
<tr>
<th></th>
<th>$d$, mm</th>
<th>$K_I$, MPa m$^{1/2}$</th>
<th>$R$, kJ/m$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MD</td>
<td>2</td>
<td>8.0</td>
<td>4.8</td>
</tr>
<tr>
<td>CD</td>
<td>5</td>
<td>11.7</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>Eq (9)</td>
<td>Ref. [6] energy</td>
<td></td>
</tr>
</tbody>
</table>
Douglas W. Coffin, Kun Li and Ji Li

Sample width increases beyond 50 mm. The fact that the tensile strength (zero notch) fits with the other fracture data suggests that one does not need a separate measure of fracture energy. The stress intensity factors determined from the short-span tensile test are large. The values of $K_I = 4.8$ and $3.1 \text{ MPa m}^{1/2}$ from the current analysis are very similar to the results given by Swinhart and Broek [7] for papers with similar tensile strength in MD and CD. Clearly, using the fracture energy calculated from a short-span tensile test causes an over prediction of fracture resistance from LEFM methods for short cracks. The equivalent elastic fracture energy determined from Equation (9) is about 40% of that reported in [4]. There is no reason to expect the LEFM methods, which utilize an effective FPZ, to match the actual energy release, which consumes energy to drive the plastic front. One would expect it to be less than such a measured value. Also, the short span measurement would be valid for deep notches and this could require higher energy than that required to propagate a small crack. This fit shown in Figure 7 does not require an input of fracture energy to be predictive. It requires the tensile strength and at least one fracture test to determine the value of $d$. The results shown in Figure (7) support the assumption that the tensile strength of an un-notched sample is also a result of fracture and that this measure gives us the necessary magnification factors to scale the load ratio factor. Thus, Equations (7), (8) and (9) have validity. Donner [9] also found that the tensile strength was aligned with the fracture data.

Seth and Page [7] attributed the low energy calculated in the work of Anderson and Falk [10] to the small sample width. The one example of fracture load versus notch depth given by Anderson and Falk [10] was in their Figure 4, which is re-plotted in Figure (8). Anderson and Falk [10] plotted the stress squared as a function of $1/a$, which should give a line. Anderson and Falk did not correct for the finite width of their samples. The circle markers in Figure 8 represent corrected values based on Equation (6). Figure 9 shows the stress versus notch length as well as the average ligament stress versus notch data. The stress is normalized to the fracture stress for the smallest notch. Figure 9 reveals that the average ligament stress for the two largest notches exceeds the average stress of the smallest notch. Because of the narrow samples used by Anderson and Falk [10] one might expect that the two deepest notches give results that are not indicative of the fracture resistance, but rather significant yielding across the entire ligament length would allow a higher load to be reached before failure. Anderson and Falk [10] reported a $d=2.5 \text{ mm}$ for handsheets tested with larger width. If this value is used in Equation (6), the two deepest notches are excluded, and the slope is adjusted to pass through $(0, 0)$, the square of the stress intensity factor doubles (see Figure 8). This would then give a fracture energy at about 40% of that reported by Seth and Page [7]. This is similar to the percentage differences reported by Östlund et al. [6].
Utilization of Modified Linear Elastic Fracture Mechanics

Figure 8. Fracture results from Fig 4 of Ref. [10] and Equation (6) with d=2.5 mm.

Figure 9. Relative far field stress level and average ligament stress for data from [10].
Seth and Page [7] reported a match in energy release rates from fracture tests and short-span notched tensile tests for large width DENT, having an aspect ratio \( a/w=0.4 \). In the short webs, they had a length to width ratio of three, but the ratio was one for the larger webs. Given that they tested in MD, the aspect ratio of unity could give a sufficiently different response than the aspect ratio of three. It does not appear that they used a correction factor for length to width to account for the fact that a uniform far-field stress may not be obtained with the length to width ratio of one. Thus, the agreement that Seth and Page observed may not hold for sufficiently long sample lengths.

Swinehart and Broek [20] present failure load versus crack length data from CNT for two papers in their Figure 4. This figure is recreated in Figure 10, where the load has been scaled to the tensile strength and the crack to half-width ratio is used. The dashed lines represent the LEFM fit from [20] and the solid lines represent the prediction from Equation (7). The values of \( d \) given in Figure 10 were calculated from Equation (8) using the stress concentration factors and the tensile strengths reported in [20]. Equation (7) fits the data as well or better at all points compared to the LEFM fit. The largest improvement is for small cracks, where Equation (7) converges to a value of 1.122TS. This comparison shows that the modified LEFM can improve the ability to describe the fracture data at small crack lengths.

![Figure 10](image_url)
Utilization of Modified Linear Elastic Fracture Mechanics

The data of Mäkelä, Nordhagen, and Gregersen [15] can be used to determine if Equation (7) has predictive capabilities. Tensile and fracture test results for these papers were reported in [15, 16, and 17]. The fracture test was on a 50 mm wide sample with a center notched crack, with \(a/w=0.4\). Table 2 provides this data along with the value of \(d\) determined from Equation (7) and the fracture toughness index \(J_{Ic}\) reported in [17].

The measure of \(d\) given in Table 2 was determined by setting \(F/F_0=1.122F/(gr\cdot TI\cdot W)\) and using the solver in Excel to determine the value of \(d\), which satisfied the equality. Mäkelä et al. [15] completed SENT fracture tests on large webs with widths of \(W=800–1000\) mm for the papers listed in Table 2. Figure 11 provides the predictions from Equation (6) compared to the data [15]. An additional prediction using the approach outlined in Mäkelä [17] provides four points, \(a/W=0.005, 0.01, 0.015,\) and 0.025, of a curve based on nonlinear fracture mechanics analysis. To determine these points, the tabled-factors given in [16, 17] along with the material properties shown in the corresponding graph of Figure (11), were used to determine the load ratio. For each point, the Excel solver was utilized to determine the load factor. Inspection of the graphs shows that the prediction of the modified LEFM equation is as good as the prediction from nonlinear fracture mechanics. Inspection of the predictions from nonlinear fracture mechanic combined with finite element analysis obtained in the original work [15] shows that the prediction from LEFM prediction is just as adequate.

Mäkelä [17] provided predictions for CD fracture tests for the same papers shown in Figure 11. The value of \(d\) was determined using the same process described above. For all the papers, the MD fluting had the lowest value of \(d=1.13\) mm and the CD fluting had the largest value of \(d=8.8\) mm. Figures 12 and 13 provide comparisons of the prediction from Equation (7) (vertical axis) and that using the equations of Mäkelä [17] (horizontal axis) for SENT specimens. The various markers represent different width webs, each with \(a/W=0.005, 0.01, 0.015,\) and 0.025. The unrealistic web of \(W=100\) meters is given to demonstrate

<p>| Table 2. MD Properties of papers from [16 and 17] and prediction of (d) from Equation (6) |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|</p>
<table>
<thead>
<tr>
<th><strong>Fluting</strong></th>
<th><strong>Sack</strong></th>
<th><strong>News</strong></th>
<th><strong>Liner</strong></th>
<th><strong>MWC</strong></th>
<th><strong>SC</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>(TI) kNm/kg</td>
<td>124</td>
<td>107</td>
<td>66</td>
<td>61.4</td>
<td>54.5</td>
</tr>
<tr>
<td>(F), N</td>
<td>218</td>
<td>184</td>
<td>61.3</td>
<td>135</td>
<td>106</td>
</tr>
<tr>
<td>(d) mm</td>
<td>1.13</td>
<td>2.63</td>
<td>2.24</td>
<td>2.71</td>
<td>2.57</td>
</tr>
<tr>
<td>(J_{Ic}) Jm/kg</td>
<td>6.1</td>
<td>13.4</td>
<td>3.43</td>
<td>5.3</td>
<td>3.98</td>
</tr>
</tbody>
</table>

Fundamental Research Symposium Information
Figure 11. Comparison of LEFM results to experimental results from large webs [15], and the nonlinear fracture mechanics approach in [17].

that as the web width goes to infinity both solutions converge to that predicted from straight LEFM, which is shown as the dark dashed line. Once the web width gets small, the nonlinear fracture mechanics solution [17] diverges because the singularity at zero crack length remains in the solution. The current solution converges to the tensile strength for zero crack length.

Comparison of Figures 12 and 13 shows that the CD predictions is just as good as that for MD, even with a very large FPZ of $d=8.8$ mm. Figures 12 and 13 demonstrate that the modified LEFM theory is much better than classic LEFM, and because it converges to the tensile strength is likely a better fit than the nonlinear fracture mechanics solution for small sample widths or small crack lengths.
The comparisons made in this section indicate that Equation (7) can be quite useful for characterization and prediction of the fracture sensitivity of papers. For the comparisons made here, only two parameters were needed, tensile strength, and the effective FPZ, $d$. The tensile strength is easily obtained from a standard tensile test, and $d$ can be obtained from one fracture test. It appears that a 50 mm wide sample is sufficient for MD and CD at least up to a an FPZ of $d=9$ mm. Equation (7) has several advantages

- simplicity over methods of nonlinear fracture mechanics
- convergence to the tensile strength for small cracks
- predictive capabilities for a variety of commercial papers.

Figure 12. Comparison of Equation (6) with that from Reference [17] for MD Fluting.
4 EXPERIMENTAL

4.1 DENT Testing

A series of DENT fracture tests were conducted to further elucidate the modified LEFM model. The testing was completed on an Instron model 3344 universal tester, with pneumatic clamping. The grips were 76.2 mm wide and had serrated faces. The constant rate of displacement was 25.4 mm/min.

For samples that showed a tendency to break at the clamps, masking tape was used to reinforce the paper under the grips.

Figure 13. Comparison of Equation (6) with that from Reference [17] for CD Fluting.
Sample dimensions varied but the typical test reported here used a width of 76.2 mm and a gage length of 180 mm. In reflection, MD sample lengths should probably be larger to ensure that the far field stress is more uniform, but conclusions remain the same. Samples were cut with both rotary and guillotine cutters with no significant differences found. Notches were cut prior to mounting with the use of either sharp scissors or a razor blade. Minimal differences in peak loads were found with different methods of sample preparation. For samples with small crack or ligament lengths, the size of the cut crack length was measured after the test.

All testing was conducted under constant environmental conditions of 50% Relative humidity and 22°C.

4.2 Materials

A variety of commercial papers, a polymer film, and several handsheets were tested for fracture resistance. All samples were conditioned to 50% relative humidity and a temperature of 22°C prior to testing. Properties of the commercial sheets are listed in Table 3. The papers represent a wide range of properties that one might expect from different grades. The grammage ranges from 22 to 200 g/m², the breaking length varies from 0.18 to 13.7 km, and the density varies from 170 to 1000 kg/m³.

The stress-strain curves for the materials listed in Table 3 are given in Figures 14 and 15. The stress is normalized with the elastic modulus. The normalized stress was determined by dividing load by the maximum slope evaluated from the load versus strain curve. Strain was determined as change in length divided by original length.

Figure 14 shows that the MD and CD curves for both Newsprint and the Copy paper are essentially the same, except MD is more brittle than CD. The copy paper is more ductile than the newsprint and shows more yielding. The MD

<table>
<thead>
<tr>
<th>Table 3. Physical Properties of Commercial sheets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grammage, g/m²</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Copy paper</td>
</tr>
<tr>
<td>Newsprint</td>
</tr>
<tr>
<td>Paperboard</td>
</tr>
<tr>
<td>Tissue Paper</td>
</tr>
<tr>
<td>Polypropylene</td>
</tr>
</tbody>
</table>

Fundamental Research Symposium Information
Figure 14. MD and CD Stress-strain curves for commercial papers. Stress is normalized to Elastic modulus.

paperboard has significantly less yielding and reaches a higher strength relative to its modulus than in the CD direction.

The tissue paper has high stretch (15%), and a very linear initial loading path for both MD and CD. This is because the tissue paper is in a bond-dominated regime with a very low breaking length and an equally low modulus. The polypropylene film has a well-defined yield point, followed by very slow strain hardening. The film is also quite ductile with a stretch of 150%. The CD paperboard curve is shown both in Figures 14 and 15 for reference.

Handsheets were produced on a 305 mm square Noble and Wood handsheet former. The pulp was NIST reference pulp 8495 (Northern Bleached Kraft Pulp). Beating was carried out in a Valley beater. Sheets were produced to three grammages, 25, 50, and 100 g/m². Pressing was carried out with a benchtop nip press and the sheets were dried on a drum dryer utilizing a tensioned fabric for restraint. Properties of the handsheets are given in Table 4. The focus of the handsheet investigation was to further investigate fracture resistance with large fracture process zones from structure; the emphasis was on no refining, low grammage, and low pressing.
Utilization of Modified Linear Elastic Fracture Mechanics

Figure 15. Stress-strain curves for tissue paper and polypropylene film. Stress is normalized to Elastic modulus.

Table 4. Properties of handsheets

<table>
<thead>
<tr>
<th>CSF</th>
<th>Grammage, g/m²</th>
<th>Density, kg/m³</th>
<th>Tensile Index, kNm/kg</th>
<th>pressing</th>
</tr>
</thead>
<tbody>
<tr>
<td>465</td>
<td>50</td>
<td>712</td>
<td>93</td>
<td>low</td>
</tr>
<tr>
<td>160</td>
<td>50</td>
<td>725</td>
<td>99</td>
<td>low</td>
</tr>
<tr>
<td>705</td>
<td>50</td>
<td>588</td>
<td>36</td>
<td>low</td>
</tr>
<tr>
<td>705</td>
<td>100</td>
<td>634</td>
<td>34</td>
<td>low</td>
</tr>
<tr>
<td>705</td>
<td>25</td>
<td>638</td>
<td>25</td>
<td>low</td>
</tr>
<tr>
<td>705</td>
<td>50</td>
<td>535</td>
<td>14</td>
<td>none</td>
</tr>
</tbody>
</table>

The stress-strain curves for the handsheets are given in Figure 16. Except of the CSF 160 and CSF 465 sheets, the sheets give a response where the efficiency of load transfer is so low that the scaled curves do not superimpose as well as one might expect [21].
5 RESULTS AND DISCUSSION

The focus of the DENT experiments was to determine if Equation (6) could be utilized to characterize the fracture behavior of a wide range of paper material responses, and if the structural contribution of the fracture process zone was necessary to explain the data. Figures 17–20 provide the results of DENT testing as well as fits using Equation (6). The parameters for the FPZ are given in Table 5. If the relative fracture resistance is due to inherent structure, one might expect \( d = 2d_s \), so the ratio is provided in Table 5.

Figures 17 and 18 show that Equation (6) well represents the fracture sensitivity for both MD and CD for a large range of crack sizes for both Newsprint and Copy paper. Equation (6) is a better fit for the Newsprint than the copy paper. For the copy paper, equation (6) under predicts the load ratio for deep notches or small ligament lengths as represented in the figures as small values of \( 1-a/w \). The newsprint appears to have a large contribution from the inherent structure rather than plasticity as observed by the large ratio of \( 2d_s/d \).
Utilization of Modified Linear Elastic Fracture Mechanics

Table 5. Effective Fracture process zone, $d$, and structural zone length $d_s$ for commercial sheets

<table>
<thead>
<tr>
<th>Material</th>
<th>MD</th>
<th>CD</th>
<th>MD</th>
<th>CD</th>
<th>(2$d_s$)/$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copy paper</td>
<td>2.3</td>
<td>5.0</td>
<td>0.25</td>
<td>0.25</td>
<td>0.22</td>
</tr>
<tr>
<td>Newsprint</td>
<td>1.8</td>
<td>4.0</td>
<td>0.8</td>
<td>2.0</td>
<td>0.89</td>
</tr>
<tr>
<td>Paperboard</td>
<td>2.6</td>
<td>6.5</td>
<td>0.4</td>
<td>0.4</td>
<td>0.31</td>
</tr>
<tr>
<td>Tissue Paper</td>
<td>4.2</td>
<td>7.0</td>
<td>1.7</td>
<td>2.5</td>
<td>0.81</td>
</tr>
<tr>
<td>Polypropylene</td>
<td>0.23</td>
<td>6.5</td>
<td>0</td>
<td>0.4</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Figure 17. DENT results of load ratio versus relative ligament length for copy paper. Solid lines represent Equation (6). Dash-dot line represents notch-insensitive response $F/F_0=1-a/w$. 

Fundamental Research Symposium Information 25
Figure 19 shows that for MD DENT Equation (6) provides a reasonable representation of the behavior of the papers as well as the polypropylene film. For these materials, data was not collected for ligament length ratios less than 0.5. As one might expect, even though the polypropylene is by far the most ductile material tested, it has the most significant sensitivity to fracture. The value of $d=0.23$ mm suggests that the film can easily concentrate load and failure occurs at low far-field stresses. The tissue on the other hand has the least relative sensitivity to fracture and it is likely not due to plasticity of the fibers but the structure of the sheet as observed by the relatively high ratio of $2d/d$.

As shown in Figure 20 for CD (deep notches were not tested for CD), the fit of Equation (6) is reasonable, expect perhaps for the paperboard, for which the data forms a curve that cannot be fit well with Equation (6). Perhaps the cohesive failure mechanism for this paperboard in CD is much more dominate.
and the FPZ increases rather fast with crack size. Despite this poor fit, given the simplicity of the representation of Equation (6) it might be acceptable for practical considerations.

When one compares the MD and CD results, the clear trend is that MD is more sensitive to fracture than CD as indicted by the lower values of \( d_s \). In addition, the inclusion of \( d_s \) is more important for CD compared to MD. For larger cracks sizes \( d_s \) can be ignored.

Bither and Waterhouse [22] showed that handsheets produced from unbeaten pulps showed little fracture sensitivity, but as the pulp was beaten, the sensitivity to fracture increased. Their sample width was rather small, 25.4 mm, so the fracture process zone could have been too large for the unbeaten pulps. Seth and Page [21] effectively demonstrated that beating and wet pressing increased the efficiency in load transfer in the sheet. Low bonding leads to inefficiency. Conversely, as the efficiency in which the sheet carries load increases, the ability of the sheet to concentrate load would also likely increase. Therefore, relative fracture sensitivity would also likely increase. So even though fracture toughness might increase with beating, the fracture process zone would likely decrease.

Figure 19. DENT results of load ratio versus relative ligament length for various papers in MD direction. Solid lines represent Equation (6). Dash-dot line represents notch-insensitive response \( F/F_0 = 1 - a/w \).
Therefore, for unbeaten sheets the fracture sensitivity relative to the tensile strength would be lower.

The DENT results for the handsheets, as shown in Figures 21 to 23 reinforce the concept that the loss of efficiency increases the area of paper activated in a fracture process, and thus, the relative sensitivity to fracture decreases. For well bonded sheets, the FPZ is smaller and the relative sensitivity increases. Figure 21 shows that the two sheet made from beaten pulp have small effective FPZ of about 2 mm, while the unbeaten pulp has an effective FPZ of 5 mm. If one considers the stress-strain curves previously given in Figure 16, the two beaten pulps have much better developed stress-strain curves, and represent good transfer of load to fibers. Following the same line of reasoning, Figure 22 demonstrates that un-pressed sheets further increases FPZ and the structural contribution $d_s$.

Figure 23 demonstrates that lower grammage sheets have increased relative fracture resistance. One would expect that with low grammage sheets, coverage is low, the bonded area is low, surface fibers make up a significant portion of the sheet, and thus, load transfer is impeded. This increases the FPZ and thus increases relative fracture resistance.

Figure 20. DENT results of load ratio versus relative ligament length for various papers in CD direction. Solid lines represent Equation (6). Dash-dot line represents notch-insensitive response $F/F_0 = 1-a/w$. 
Utilization of Modified Linear Elastic Fracture Mechanics

Figure 21. Effect of beating on DENT fracture sensitivity for handsheets, 50 g/m².

Figure 22. Effect of pressing on fracture sensitivity of handsheets made from unbeaten pulp.
The results given above suggest that the effective fracture process zone, \( d \), can be used as an indication of the relative fracture sensitivity of the sheet. As the load transfer efficiency is increased by means of improved bonding through beating and pressing, the stress-strain curve can be developed but relative fracture resistance decreases. The decrease in fracture process zone is indicative of an increase in the sheet’s ability to concentrate load. The stress intensity factor would be affected by both the tensile strength and the magnitude of the fracture process zone.

Swinehart and Broek [8] showed that sample scaling with LEFM held at least for wide webs and large cracks. The results from [6] as shown in Figure 7 suggest that scaling holds for narrow widths too. The results given in Figure 11 demonstrated that calculating the effective process zone from a 50 mm wide sample was sufficient to predict fracture loads for small cracks in large webs. The ability of a narrow width samples to provide an estimate of the fracture process zone depends on the magnitude of the FPZ. If the zone is small, say \( d=1.0 \) mm, then even a sample width of 15 mm should be adequate for cracks up to \( a=4 \) mm, a width of 25 mm should be valid for cracks up to \( a=10 \) mm. For a large fracture process zone, \( d=10 \) mm, a sample width of 50 mm should be valid for cuts up to \( a=8 \) mm.

![Figure 23. Influence of grammage on fracture sensitivity of handsheets from unbeaten pulp.](image)
Figure 24 shows results for MD specimens of copy paper with widths of 25.4, 50.8, and 76.2 mm width along with the curves given by Equation (6). These results demonstrate that with $d=2.3$ mm, the scaling predicted from Equation (6) is reasonable, and this should hold for larger webs. Figure 25 shows the results for CD. With $d=5$ mm, Equation (6) does not hold as well for the 25.4 mm width, but it is adequate for the two larger widths. Even for the 25.4 mm wide web, Equation (6) is reasonable for cracks less than 6 mm. Figure 17 shows that this fit is adequate for the 76.2 mm wide web for cuts up to $a=34$ mm or a ligament length of about 8 mm. The actual CD fracture load is larger than that predicted by Equation (6) for these deep cracks. For larger webs, it is likely that the prediction from Equation (6) would be valid for deep cuts and would most likely be a conservative under-estimate of the fracture strength.

For DENT samples where the ligament length is in the range of $d$ to $3d$, the average ligament stress likely exceeds the tensile strength of the material as demonstrated by the results of Tanaka and Yamauchi [23]. The plastic zone length...
determined by Tanaka and Yamauchi [23] from DENT tests with the ligament length one third the sample width can be recalculated to give the ratio of average ligament stress to tensile strength. They varied the width from 3 to 63 mm and their results show that the average ligament strength can exceed the tensile strength by an additional sixty percent. For example, for newsprint a ligament length of 2 mm gave a ligament stress that was 1.3 the tensile strength in MD and 1.5 the tensile strength in CD. This indicates that the intrinsic strength of the sample is higher than the measured tensile strength and that tensile strength is limited by fracture due to the cutting of the structure at the edges. With a DENT sample, fibers crossing the ligament form a path for load transfer. The same fiber cut at the edge of a sample would lose much of its ability to carry load. For smaller ligament lengths, the fracture process zones superimpose, stress concentrations are lower and the measure of fracture load is a better estimate of intrinsic tensile strength of the network.

Figure 26 provides the ratio of fracture ligament stress to tensile strength for four sample types of MD copy paper. Three of the samples are tensile strips ($a=0$) with three widths, 25.4, 75.2, and 2.5 mm. The fourth sample is a DENT with a ligament length of 2.5 mm. The DENT sample has a strength that is 47% larger than the tensile strength of the sample. This suggests that without a notch, the

![Diagram](image-url)
Utilization of Modified Linear Elastic Fracture Mechanics

Fundamental Research Symposium Information 33

sample fractures at the edges because of inherent flaws in the structure, which are opened up when the edges are cut. The structure in the ligament of the DENT sample is intact and can carry significantly more load.

6 CONCLUSIONS

Contrary to statements in the literature, it was found that a modified linear elastic fracture mechanics (LEFM) model can be applied to paper for both materials characterization and prediction. By using a ratio of fracture loads from LEFM equations, fracture resistance can be determined from the tensile strength and an effective fracture process zone, $d$. The fracture process zone can further be split to a structural, $d_s$, and a materials component, although this separation is not needed.
for the majority of cases to obtain reasonable predictions. Equation (6) proved useful in characterizing a wide range of papers from tissue to paperboard for both MD and CD. For most papers, a 50 mm sample width should be sufficient to characterize the materials fracture sensitivity. Results from small samples should scale to large webs at least until the crack depth is quite deep. Simplicity of application is the great advantage offered by the modified LEFM compared to other available methods.

The current results support the previous work of Donner [8] linking the tensile strength directly to the fracture behavior and suggesting that the inherent network structure of paper contributes to fracture toughness. As sheet efficiency decreases, tensile strength decreases, but the effective fracture process zones increases, thus the relative fracture toughness increases. In many cases the actual fracture toughness would decrease because the loss of strength exceeds the gains from an increased fracture process zone.

For a wide range of commercial papers, the effective fracture process zone was in the range of 1 to 3 mm for MD and only larger, about 4 mm, for tissue papers. In CD, the fracture process zone was found to be in the range of 4 to 9 mm for all papers investigated. For tissue papers, which tend to be low grammage and bond-strength dominated, the fracture toughness appears to be structural, a result of a large fracture process zone resulting from poor transfer of load. For newsprint, structure also appears to dominate the fracture toughness as indicated by the ratio of $2d_s/d$ near unity. For other papers, plasticity of the fibers probably plays a larger role in fracture toughness.

Although material plasticity plays an important role in fracture toughness, the material with the largest sensitivity to fracture was the polymer film with a stretch of 150%. That is because the ability of the sheet to concentrate load plays an even greater role in determining fracture toughness. The polymer film can concentrate load much better than paper’s fiber network, and thus when a crack is introduced in the film, the stresses near the tip reach failure loads when the far field load is still quite low. In paper, the network structure impedes the ability of the sheet to concentrate stress and as a result the relative resistance to fracture is much higher. The effective fracture process zone can be considered as an indicator of how well the sheet can concentrate load. The smaller the value of $d$, the better the sheet can concentrate load. Even if the material were perfectly elastic and brittle, increasing the characteristic length of the structure would improve the relative fracture resistance.

The edges of a paper are inherently flawed because the structure is disrupted by cutting fibers that cross the edge. The tensile strength is then a result of fracture resulting from concentrated loads as some point where the edge flaw is largest (This assumes that no larger defects like a large shive or a hole are in the interior of the sheet). The notches or cracks introduced in a DENT cause the average
Utilization of Modified Linear Elastic Fracture Mechanics

stress over the ligament to be high and failure initiates at one or both of the notch tips. The process of cutting a slit induces little damage to the network structure remaining in the ligament. Thus the inherent strength of the sheet can be determined from deeply notch specimens and can easily be 50% greater than the tensile strength.

The second advantage to using the modified LEFM equation presented here, Equation (7), is that the singularity at small crack lengths is eliminated. The reason that LEFM was dismissed is that it over predicts the fracture strength for small cracks as evidenced by the literature where LEFM does a better job of predicting CD compared to MD even though CD has more plasticity associated with it. The modification presented here ratios the load to the tensile strength, which is determined from the effective fracture zone and thus insures reasonable convergence for small crack lengths. This actually provides a better estimate then other models that include plasticity but leave the singularity at zero-crack length. It is important to note that the current LEFM modification does not make use of the yield stress but rather assumes the tensile strength is also a result of fracture. The comparison to experimental results supports this assumption.

Finally, we conclude that by embracing the use of LEFM to describe the fracture resistance of paper, one can obtain new insights into the role of materials and structure to the observed mechanical behavior of paper.

ACKNOWLEDGEMENT

The authors thank Petri Mäkelä for kindly providing the experimental data from [15], which was utilized in Figure 11 and his personal communication on limitations of existing models due to stress singularities.

REFERENCES

Douglas W. Coffin, Kun Li and Ji Li


Session ?: “Session Title”
Appendix B
Fig. B1 Stress vs. Strain Curve of Hand Sheet csf705 upressed Fracture Process

Fig. B2 Stress vs. Strain Curve of Hand Sheet csf705 unpressed Fracture Process
Fig. B3 Stress vs. Strain Curve of Hand Sheet 100 Fracture Process

Fig. B4 Stress vs. Strain Curve of Hand Sheet 100 Fracture Process
Fig. B5 Stress vs. Strain Curve of Hand Sheet 50 Fracture Process

Fig. B6 Stress vs. Strain Curve of Hand Sheet 50 Fracture Process
Fig. B7 Stress vs. Strain Curve of Hand Sheet 25 Fracture Process

Fig. B8 Stress vs. Strain Curve of Hand Sheet 25 Fracture Process
Fig. B9 Stress vs. Strain Curve of Hand Sheet csf705 Fracture Process

Fig. B10 Stress vs. Strain Curve of Hand Sheet csf705 Fracture Process
Fig. B11 Stress vs. Strain Curve of Hand Sheet csf465 Fracture Process

Fig. B12 Stress vs. Strain Curve of Hand Sheet csf465 Fracture Process
Fig. B13 Stress vs. Strain Curve of Hand Sheet csf160 Fracture Process

Fig. B14 Stress vs. Strain Curve of Hand Sheet csf160 Fracture Process
Fig. B15 Stress vs. Strain Curve of Paper Board CD Fracture Process

Fig. B16 Stress vs. Strain Curve of Paper Board CD Fracture Process
Fig. B17 Stress vs. Strain Curve of Paper Board MD Fracture Process

Fig. B18 Stress vs. Strain Curve of Paper Board MD Fracture Process
Fig. B19 Stress vs. Strain Curve of Copy Paper MD Fracture Process

Fig. B20 Stress vs. Strain Curve of Copy Paper MD Fracture Process
Fig. B21 Stress vs. Strain Curve of Copy Paper CD Fracture Process

Fig. B22 Stress vs. Strain Curve of Copy Paper CD Fracture Process
Fig. B23 Stress vs. Strain Curve of News Print Fracture Process

Fig. B24 Stress vs. Strain Curve of News Print Fracture Process
Fig. B25 Stress vs. Strain Curve of Paper Towel MD Fracture Process

Fig. B26 Stress vs. Strain Curve of Paper Towel MD Fracture Process