This paper examines the combination of adaptive signal design and orthogonal frequency division multiplexed (OFDM) pulses in order to improve their detection performance and automatic target recognition for multiple varieties of single target scenarios. This adaptive design will be performed through exploiting the target’s reflectivity function. OFDM’s performance for radar systems is discussed and analyzed, as well as its potential in adaptive design. The place of adaptive signal design in developing radar systems is examined, and methods are detailed. Estimators for determining the target’s reflectivity function (TRF) are derived and implemented. Finally, performance results for TRF identification and automatic target recognition are discussed.
ADAPTIVE OFDM RADAR SIGNAL DESIGN

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Chapter 1

Introduction

Detection via echo reflection is an integral part of our daily lives, and an ancient concept. Nature, as usual, was the first to utilize this ingenuity. From the dolphin using high-pitched sound waves in echolocation to hunt for prey in murky waters, to bats flying swiftly in pitch blackness, echo detection has been a critical part of the survival for many animal species. Mankind has since adapted the concept of detection via waveform transmission to his own purposes, with the some of the simplest examples including the visually-handicapped tapping canes on the ground to the sirens employed by emergency vehicles. However, since World War II and moving into future[1][2], there has developed a keen interest in another kind of echo detection – radar.

Radar, or Radio Detection And Ranging, first drew attention when Christian Hulsmeyer, in 1904, used what he called a “telemobiloscope” to detect the presence of ships through dense fog with the intent of collision prevention [3]. Unlike echolocation, radar relies on radio waves for detection – not sound waves. Surprisingly, radar did not see active military interest until World War II, when Robert Watson-Watt determined that detection of incoming bombers via radio waves was a real possibility. The first systems were operational by 1937, and often credited with being critical in winning the Battle of Britain for the allies. While these early systems were an improvement upon Hulsmeyer’s. That is, they could perform detection as well as range imaging, it was not until later that other information started to be extracted from these radio waves by radar systems.

Since then, the radar field has expanded rapidly, filling thousands of papers, many books, dozens of publications and numerous conferences. Advances in key components, such as digital to analog converters and in other signal processing components have allowed the field to make large gains in performance, efficiency and reliability. Clever manipulation of data and signal processing has allowed for increased radar system flexibility. This flexibility of radar has encouraged its use to spread through many diverse
fields, spanning a spectrum from the more exotic topics of advanced weather monitoring, position and navigation management and ground penetrating detection to the more mundane and typical roles, such as high resolution ground mapping, automatic target identification and target tracking[4]. These topics and roles cover many of the military’s interests as well [1].

In weather monitoring, researchers have used radar data to detect if thunderstorms have the potential to form tornadoes by examining the Doppler signature of the storm [5]. To avoid reliance on GPS systems, signal processing methods have been used to adapt radar systems for potential use as a navigation sensor [6]. Radar systems have even been used in ground probing situations, such as being utilized to map shallow depths of terrain in oil prospecting assistance [7] and to detect tunnels hidden underground [8]. The latter especially emphasizes the driving inspiration of the military in radar research, as discovering hidden tunnels is critical in combating insurgency operations abroad, throughout several key war zones. Military demands for radar systems include passive systems, identification capabilities, multi-mission capable and networked as well as low power consumption[2].

Part of radar’s allure is its robust nature compared to many other imaging alternatives. Rain, clouds and other weather conditions along with foliage and other terrain can hamper traditional imaging techniques. Radar systems, however, can design their signals specifically to penetrate these kinds of blocking terrain. This capability of radar systems has been exploited often, from uses in autonomous man-portable robots for overcoming adverse weather conditions [9] to foliage penetration, a great interest of the military in the Vietnam era and since[10]. Further, these systems can be tuned to reflect on specific types of clutter that other signals try to ignore. For example weather radars are tuned to detect precipitation concentration. Since radar is an active system that propagates the waves the system uses for measurement and detection, it is not limited to daytime operation. Furthermore, radar can be used in imaging scenarios to map a bird’s eye view of a target scene, even from miles away, sometimes allowing aircraft to avoid unfavorable conditions entirely. This flexibility and robustness has secured radar’s place as an important detection and imaging technology in both civilian and military operations.

While radar has become an integral part of operations for organizations both military and civilian,
adaptive radar signal design and processing has taken longer to spread in popularity. The earliest mention of it appears in 1963, with Weiss predicting that new systems would allow “the adaptive use of ... optimum modulation waveforms, offers possibilities for more efficient utilization of power and time” within a decade [11]. However, adaptive signal design developed slower than expected, since much of the research has been done only in the last thirty years, as radar systems have faced less favorable channel conditions. While his prediction may have been premature, his ideals for adaptive radar design is still the mantra today – more efficient power and time utilization.

Adaptive radar has become a field of keen interest in the airborne and spaceborne surveillance scenarios. While non-adaptive radar systems have proven their worth time and time again, many systems are now required to detect small targets in unfavorable conditions. Large background interference, such as heavy backscatter can cause a non-adaptive radar system to suffer from severe performance degradation. Additionally, there are many sources of undesired signals, from clutter to noise to incidental and intentional transmissions at the radar’s frequency. All of these channel conditions and others can significantly degrade a radar system’s performance. While different types of channel interference and target scene noise require different adaption, adaptive signal processing and design in these scenarios can lead to performance improvements in excess of 40 dB for the radar’s signal-to-interference ratio [12].

Orthogonal frequency division multiplexing (OFDM) is a signal modulation scheme employed in many newer radio wave based systems. Until recently, however, OFDM has largely seen use as a commercial communication modulation scheme. OFDM is a highly spectrum efficient signal, making it an attractive choice for communication systems. This limited interest expanded however, when in 2002 the Federal Communications Commission authorized the use of Ultra-Wideband in new ranges, greatly increasing the interest in OFDM for radar scenarios.

![OFDM Frequency Domain sinc pulses](image)

Figure 1.2: OFDM frequency domain sinc pulses

While many radar systems design their pulses in the time domain, OFDM starts by designing the
signal in the frequency domain. By using sinc functions in the frequency domain, each sub-carrier only requires a small amount of the entire bandwidth. The properties of the sinc function and proper spacing cause all other sub-carriers to be zero when one sub-carrier peaks, which means that the sub-carriers are orthogonal towards each other. This orthogonality can be seen in figure (1.2), where each sinc function peaks and the other sinc functions are zero at that same frequency value. Also note that this effect continues perpetually, at a regular interval spaced a part based on the period of the sinc functions. Each of the sub-carriers is assigned power independently (referred to as the signal’s ‘weights’). This individual assignment of power between the sub-carriers makes the OFDM signal easily adjusted for adaptive signal design. When converted to the time domain, the OFDM signal looks like noise. This feature makes it difficult to discern the OFDM signal from the channel noise, making it harder to detect and jam by hostile forces.

The radar system adapts its signal via adjustment of the ‘weights’ of the signal. For OFDM signals, these weights are a matrix representing the power distributed to each individual sub-carrier that compromises a component of the composite signal. By intelligently re-distributing the power between these sub-carriers, the signal can improve its power and time efficiency, as well as adjust to channel conditions. As radar systems face harsher channel conditions, such as severe backscatter from target scene clutter, adaptive signal design becomes a more important method in ensuring the performance of radar systems. These adaptive designs also permit the radar system to follow changing channel conditions and target scenes with more optimal waveforms. Some adaptive signal design techniques can also adjust for jamming and interference, which is another important benefit for military applications in hostile environments.

Many approaches have been developed for adaptive radar systems. Space-time Adaptive Processing (STAP) is specific adaptive signal processing technique that applies adaptive detection methods to space-time echo data received by a pulse Doppler radar [13]. This method in particular is designed for use where complicated, varying interference such as ground clutter is an issue [14], and can be utilized to obtain an order-of-magnitude improvement in target detection. These scenarios are common problems faced by airborne and spaceborne radar systems, specifically developed to minimize the effects of external noise, noise jamming and Doppler spectrum spreading on detection of low closing-rate targets[1].

Other approaches include ultra-low sidelobe beamforming and Displaced Phase Center Antenna (DCPA). Sidelobe beamforming is a technique that is particularly useful in jamming scenarios and improving antenna performance [15]. This makes it a method of interest especially for military applications, including both airborne and spaceborne radars. DCPA is a method to compensate for the motion of the platform which allows for attenuation of stationary scatterers’ returns but still permits moving targets to be detected [16]. This method is often employed for satellite based radar systems that are used for imaging and detection of ground-based targets.
OFDM has seen increased attention in recent years, in both radar and electronic communications, and is the focus of this thesis. In particular, Nehorai has performed significant research in the field of adaptive OFDM radar systems, from using adaptive algorithms to improve performance of the improved wideband ambiguity function (WAF) [17], enhanced target detection in clutter [18] and in multipath scenarios [19]. By improving the WAF, the OFDM signal provided a better auto-correlation function, which in turn improves range resolution. This was done by devising a procedure to assign the subcarrier power in such a way that the volume of its WAF best approximated the volume of the desired ambiguity function. To improve target detection in clutter, an adaptive algorithm was developed that modifies its pulse to maximize the probability of detection vs probability of false alarm. Detection of a moving target in urban scenarios is improved through exploitation of different Doppler shifts inherent to different paths.

While Nehorai’s work is certainly related and of interest, there are significant differences between his work and this work. Specifically, this work focuses on utilization of the target’s reflectivity function to improve performance with regards to several performance metrics, including estimation of the target’s reflectivity function and automatic target recognition. Both sets of work, however, take advantage of OFDM’s unique characteristics to improve performance and take advantage of its ease of adaptation, which make it much more suited to the task of adaptive signal processing than more traditional radar signals, such as linear frequency modulated ‘chirps’ and continuous waveforms.

Automatic target recognition (ATR) is an algorithm used by airborne radar systems to identify targets automatically through digital processing, rather than requiring the use of observers and interpreters. It is essentially a pattern recognition program. Data processing efficiency has been of particular interest to the military with regards to processing aerial photographs, as the sheer amount of data makes human processing impractical. This interest was first pursued in the 1960’s with Smillie’s paper [20] discussing the issues that such approaches would face. These algorithms can vastly improve the processing time of large sets of data as compared to the slower processing of human interpreters.

Many problems exist with this approach, however, as minor distortions or deterioration that may not impede a human interpreter could significantly deteriorate the performance of an automatic target recognition algorithm. Further, small changes in the target, such as a shift in orientation are easily discerned by an interpreter, but require the algorithm to have either another object in its set to match the target to, or have some complexity in the algorithm in order to handle the variation in the target. This has become a field in and of itself, with several algorithms existing to help deal with varieties of this problem, such as moving targets in clutter[21] to utilizing probabilistic principal component analysis to improve performance of ATR in situations with poor SNR[22].

We propose to utilize adaptive signal design with an OFDM waveform in order to take advantage of a target’s reflectivity function. Since OFDM signals are generated in the frequency domain, we can
manipulate the signal’s design to exploit the stronger returns of the target’s reflectivity function. The goal of the adaptive signal design is to improve efficiency as well as increase performance measured by our metrics proposed later. Chapter 2 details the background of adaptive radar design. This includes a brief discussion of alternative adaptive techniques. Chapter 3 explains the process for developing the target’s reflectivity function. The least squared estimator is also detailed. Chapter 4 covers the method chosen for adaptive signal design in this paper, and details how it applies to the OFDM waveform. The target scene and effects of the channel are discussed in chapter 5. In Chapter 6 the results of the simulation are analyzed and compared to the benchmark signals. Finally, in Chapter 7, suggestions for future directions are proposed.
Chapter 2

Estimating Target Reflectivity Functions

Figure 2.1: Target scene, with the target of interest among random clutter.

The scenario for this thesis is a static target with unknown, but non-uniform, deterministic, omni-directional (i.e. the target appears the same regardless of aspect angle) TRF model and unknown position. This scenario is seen visually in figure (2.1), where the target is displayed amongst clutter, which will be discussed later. A TRF is a function that maps frequencies to a scaling factor that either amplifies or attenuates the energy of any signal reflected from the target at that specific frequency. While an omni-directional, deterministic target reflectivity function (TRF) model is unrealistic in many scenarios, it allows us to easily compare our benchmark signals to the adapted signal, as a changing TRF could skew results. It also allows us to utilize the least squared estimator. In order to develop a more accurate model, a non-deterministic target would be required to compensate for the angle between
the platform and the target. However, for a non-deterministic target, the minimum mean squared error estimator would be required, which is beyond the scope of this work.

This thesis focuses on improving the energy reflected of the OFDM radar signal and the estimation accuracy of the target’s TRF through utilization of the target’s reflectivity function in order to more efficiently assign power between the individual subcarriers of the OFDM signal. The TRF defines how much energy is scattered from the target for each specific frequency once it is illuminated by the radar pulse. Thus, we can use this information to allocate more energy to subcarrier frequencies that will have greater returns from the target, and this is turn will increase the energy reflected to the radar receiver.

The algorithm developed within this work will have its performance measured through consideration of two metrics. These metrics are to accurately estimate the TRF to improve reflected energy from the target and to identify the target more accurately via automatic target recognition (ATR).

For the purposes of testing this algorithm, we simulate an OFDM radar illuminating a specific target scene repeatedly. The simulation was performed in MATLAB, utilizing the RedHawk cluster. This target scene has a randomly positioned target within a given range. Randomly generated and randomly placed clutter is added to the target scene. Additive White Gaussian noise (AWGN) is added to the signal as well.

Our chosen waveform is modulation is orthogonal frequency-division multiplexing (OFDM). A general form for this technique is displayed below, as $y_n(t)$:

$$y_n(t) = \sum_{k=1}^{2N+1} \tilde{x}_k e^{-j2\pi(k-1)F_s t}, n = 1...2N + 1$$

(2.1)

Where $\tilde{x}_k$ are the subcarrier weights, $N$ is the number of subcarriers and $F_s$ is the sampling frequency.

The form for the received signal model in the general case is $y_r(t)$, such that:

$$y_r(t) = \sum_{k=1}^{2N+1} \hat{a}_k \tilde{x}_k e^{-j2\pi(k-1)F_s t}, n = 1...2N + 1$$

(2.2)

Where $\hat{a}_k$ is the scattering coefficients of the target.

Additive white Gaussian noise (AWGN) is our assumed environment noise, which means that every pair of samples will have independent and uncorrelated values. AWGN serves as a model for our problems, as it is the noise model that most closely approximates the noise that most radar systems encounter. Further, it serves often as a worst case scenario, being more difficult to remove from the signal than many other noise models. This follows a statistical normal distribution such that each value is created from:
\[ f(x) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]  

(2.3)

where the area under the curve \( f(x) \) represents the probability the value \( x \) is selected, \( \sigma \) is the standard deviation of the distribution, \( \mu \) is the mean of the distribution, and \( x \) is all real values from \(-\infty \) to \( \infty \).

For our case, equation (2.3) simplifies to the following, after setting \( \sigma \) to 1 and \( \mu \) to 0:

\[ f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \]  

(2.4)

which is the standard normal distribution. From here, the values from the distribution are modified (either amplified or attenuated) according to the desired signal to noise ratio (SNR), which is a measure comparing the power of the signal to that of the noise, which we calculate as:

\[ \text{SNR}_{dB} = 20 \log_{10}\left( \frac{A_{signal}}{A_{noise}} \right) \]  

(2.5)

Where \( A_{signal} \) is the root mean square voltage of the signal and \( A_{noise} \) is the root mean square voltage of the noise. Noise is not the only perturbation of our signal. We also consider clutter, which has unknown, randomly generated TRF with unknown position, much like our target of interest. This clutter adds the following, \( y_c(t) \), to the signal:

\[ y_c(t) = \sum_{K=1}^{2N+1} \tau_k \tilde{x}_k e^{-j2\pi(k-1)F_s t}, n = 1...2N + 1 \]  

(2.6)

Where \( \tau_k \) is the scattering coefficients of the clutter.

Once the reflection from the target, the TRF, \( \hat{a}_k \), can be extracted from the received signal, due to the fact that the TRF characteristics are embedded within the received signal, as seen in equation (2.2). The closed form for this calculation at each sample point, \( t \), would be the following:

\[ \hat{a}_k = \frac{y_n(t)}{x_k e^{-j2\pi(k-1)F_s t}}, k = 1...2N + 1 \]  

(2.7)

Calculating this provides us with the TRF estimate of the target, which we do through methods detailed later. We need information about the target of interest in order to adapt our signal and therefore improve its performance. We focus on the target reflectivity function. The TRF is a function that defines what fraction of the power is scattered by the target for a given frequency, i.e. a frequency-domain reflectivity pattern.

The TRF is critical to the adaptive process implemented in this paper, as the OFDM pulse can easily be adapted to take advantage of the more beneficial returns identified by the TRF. The basic
concept for adaptive design is to take advantage of the target’s return characteristics in order to increase the radar signal’s power or time efficiency, or both. The technique of adapting a signal based on the target’s reflectivity has been a popular method, for both more traditional radar signals in improving radar performance for detection and tracking[23][24], and has even been examined for OFDM in some cases[17]. Additionally, this adaptivity can permit the signal to adjust to unexpected target or channel characteristics, while traditional static radar systems would see their performance suffer greatly. These performance degradations can come from severe backscatter from clutter, different varieties of jamming or adverse channel characteristics. In some airborne and spaceborne scenarios, the channel conditions can even change drastically with time. This is another important scenario where adaptive signal design plays a role, as it allows the radar system to adjust for these changing conditions. However, this alternative is not pursued in this paper. While in this paper the TRF will not be derived in a closed form, we estimate it using the least mean squared estimator. The least mean squared estimator minimizes the squared difference between the measured return of the signal from the target and the ideal model of the target return.

We utilize the OFDM waveform for its large number of closely-spaced orthogonal sub-carriers, which not only will make estimation of the target reflectivity function (TRF) easier, but also make adaptive signal design simpler as well. OFDM is not without its weaknesses, however. It has a high peak to average power ratio (PAPR), that many try to address, such as through genetic algorithms[25] or random phase updating[26]. Another significant disadvantage is OFDM’s sensitivity to Doppler shift, as its efficiency relies on orthogonality in the frequency domain. Approaches to this problem vary, from antenna schemes[27] to exploiting the Doppler sensitivity to solve other problems, such as Doppler ambiguity[28].

Adjusting the OFDM pulse is done by re-distributing the power assigned to the subcarriers of the signal. Since the OFDM pulse is designed in the frequency domain, this power re-distribution is easily performed. As the TRF defines the target’s expected power return at each of these specific frequencies, we now know which of these frequencies will provide a better return in future transmissions, and thus can assign more power to the more efficient sub carriers. This TRF is estimated as discussed previously in this thesis, and an example of these results are provided in figure (2.2). Without this information, no intelligent allocation of power could be applied, therefore the estimation of the TRF is critical in optimizing the power allocation in this adaptive system. By allocating the power to the frequencies of the target that have the highest return, we should generate a return with more power than the original training signal was able to gather.

Specifically, in this work we utilize an ultra-wideband (UWB) signal. This means that our signal will be spread over a bandwidth exceeding 500 MHz, taking advantage of new available operating ranges[29].
Figure 2.2: Training Pulse TRF Estimation with 6 subcarriers. The blue x’s are the estimation of the actual trf, the red o’s. This was done with a time domain estimation.

Due to our operating range being below 10 GHz, atmospheric attenuation would have relatively minor effect for an airborne radar system[1] and will not be considered.

### 2.1 Time Domain Estimation

#### 2.1.1 Received OFDM Radar Pulse Modeling

In order to utilize the least mean squared estimator, a model of the expected target return needs to be derived, so that the estimator can minimize with respect to the model. In our target scenario, that model will represent the combination of the OFDM transmitted signal along with white Gaussian noise, randomized clutter in the target scene, and the time delay from signal propagation, which will depend on the velocity of the platform. For an OFDM signal, we first begin by considering each of the subcarriers, $s_k$, in the following form:

$$s_k = \tilde{x}_k e^{-j \frac{2\pi(k-1)(n-1)}{2N+1}} \quad (2.8)$$

Where $\tilde{x}_k$ represents the subcarrier weight. An OFDM waveform is made up of a series of these subcarriers, of which there are $2N+1$. For our case, the complete original transmitted OFDM signal takes the following form in the time domain:

$$y_n = \frac{1}{2N+1} \sum_{K=1}^{2N+1} \tilde{x}_k e^{-j \frac{2\pi(k-1)(n-1)}{2N+1}}, n = 1..2N+1 \quad (2.9)$$

Where $n = 1, 2, \ldots, 2N+1$ and $k$ corresponds to the subcarriers. Graphically, this corresponds to the following figure (2.3) in the time domain. Note that the waveform looks like noise in the time domain, and the peak in the time domain can be suppressed by removing the near DC frequency components.
Also note that there are sixty-four subcarriers (i.e. sixty-four frequencies in the frequency domain) for which the signal has the potential to assign energy which are spaced evenly, required for orthogonality. The wide bandwidth created in the frequency domain forces a corresponding short duration time domain pulse in an inverse relationship.

Figure 2.3: An OFDM waveform in time and frequency domain.

After signal propagation, each of the signal’s subcarriers is multiplied by the target’s reflectivity function $A(k)$. White Gaussian noise $e_n$ is also added, representative of the noise expected from the channel transmission. Clutter, present in the target scene, also needs to be represented in the signal model, with scattering coefficients of the clutter being represented by $\tau_k$. The time delay from the signal’s propagation becomes $D_m$, which is dependent on the velocity of the platform such that:

$$D_m = \frac{2R_t}{c + v_m}$$  \hspace{1cm} (2.10)

Where

$$v_m = v_{platform} - v_{target}$$  \hspace{1cm} (2.11)

with $R_t$ is the distance to the target, $c$ is the speed of light, $v_{platform}$ is the radial velocity of the platform and $v_{target}$ is the radial velocity of the target. This makes the received signal model, $\hat{y}_n$, the following:

$$\hat{y}_n = \frac{1}{2N+1} \sum_{k=1}^{2N+1} \hat{a}_k \tilde{x}_k e^{-j\frac{2\pi\left(k-1\right)F_s\left(\frac{n-1}{2N+1}-D_m\right)}} + \frac{1}{2N+1} \sum_{k=1}^{2N+1} \tau_k \tilde{x}_k e^{-j\frac{2\pi\left(k-1\right)F_s\left(\frac{n-1}{2N+1}-D_c\right)}} + e_n,$$  \hspace{1cm} (2.12)

where $\hat{a}_k$ are the scatter coefficients of the target of interest, $F_s$ is the sampling frequency and $D_c$
is the delay from the clutter, calculated with the variables of equation (2.12) replaced with velocity and distance related to the clutter instead of the target. However, this equation cannot be estimated via the least squares approach, as we cannot consider distributions (such as the noise) as a part of our signal model without examining the solution from an minimum mean squared error approach. This is because when utilizing the least squares estimation method, no probabilistic assumptions can be made about the observed data[31]. So instead, our received signal model is the following:

\[ \hat{y}_n = \frac{1}{2N+1} \sum_{K=1}^{2N+1} \hat{a}_k \hat{x}_k e^{-j \frac{2\pi (k-1)F_s (n-1 - D_m)}{2N+1}} + \frac{1}{2N+1} \sum_{K=1}^{2N+1} \tau_k \hat{x}_k e^{-j \frac{2\pi (k-1)F_s (n-1 - D_c)}{2N+1}}, \]  

This equation can be challenging to understand, so figure (2.4) provides a visual representation of the received signal. It is important to remember that while we have a more limited model that we are using for the received signal, this graph contains all deteriments to the signal, including noise. The amplitudes of each frequency subcarrier have also changed due to the noise, clutter and target reflectivity present in the channel. These factors serve to destroy the original symmetry of the transmitted signal that was once present in the frequency domain.

![Filtered Received Signal in Time Domain](time_d.png)

![Received Signal in Freq Domain](freq_d.png)

(a) Time domain  
(b) Frequency domain

Figure 2.4: Received OFDM Signal.

### 2.1.2 Least Mean Squared Estimator

Once the received signal model has been developed, the least squares estimator can be employed. When using the least squares estimator, one is trying to minimize the squared difference between the measured sample and the signal model. We utilize the least squares estimator as it is a unbiased estimator with
minimum variance[31], attempting to make our observed data \( \hat{y}_n[n] \) resemble our purely deterministic original signal \( y_n[n] \). This makes our function to minimize \( J(A(k)) \), in the following form:

\[
J(\hat{\mathbf{A}}) = (\hat{y}_n[n] - y_n[n])^2
\]  

(2.14)

Where \( \hat{y}_n[n] \) is the measured sample, and \( A(k) \) is our parameter of interest, the TRF. Note that \( \hat{\mathbf{A}} \) is a vector of length \( 2N+1 \), with each value corresponding to an estimation for the respective subcarrier.

There are \( 2N \) parameters of interest, the TRF at each subcarrier frequency with the center frequency thrown out, as it corresponds to the DC frequency and thus any estimated return is just a result of noise from the channel. Specifically, we are only interested in the magnitude of the TRF, thus, the TRF is non-negative as well as constrained to being real, since we are not examining the effects of the TRF on phase. Additionally, the target is assumed to have a non-zero reflection for every frequency, making the TRF strictly positive. This results in:

\[
0 < \hat{A} < 1, k = 1, \ldots, 2N + 1
\]  

(2.15)

From here, the actual value for the target’s reflectivity function can be determined through minimization functions. The method we employ will be grid searching, which is a common algorithm that performs an exhaustive search through the parameter space. This minimization technique seeks to find the minimum through brute force, by calculating numerous equidistant points along the function’s domain. The lowest of these is taken for an estimation of the minimum. While this will not select the actual minimum of the function, it will be close enough for us to select the appropriate estimation for the TRF of interest from the set of possible TRF’s for the target.

The estimate of \( \hat{\mathbf{A}} \) is the value at which the function achieves its minimum over the grid. Figure (2.5) shows a series of these minimizations, with the lowest point being chosen. Step size must be chosen small enough in order to 'dive' into this valley while still running in a reasonable amount of time. This requires a bit of practice and testing in order to acquire the appropriate step size such that the valley is found. As noted from the figure, the valleys can be rather narrow.
Each of these grid searches performed will be attempting to estimate a value for a specific \( k \), that is, to approximate one point of \( A(k) \). This means that we will have to perform a grid search for each subcarrier that the original signal contained, to generate \( A(k) \) for each frequency. The grid search attempts to minimize the function by calculating the value of the function at several points. Oftentimes, a function being minimized in this manner will have a valley. The number of points necessary depends on the sharpness of this valley; the greater the slope of the valley dictates a greater number of points for accurate function minimization. While grid searching is a simple method for function minimization and computationally light, it can have issues with minimizing properly when slopes are too sharp. With this limitation in mind, other methods of function of minimization will be examined, potentially in the future works section. Once the estimation for the TRF is completed, adaptive signal design can begin.

Another point of interest is that an estimation for each subcarrier is derived from only a single sample, meaning that an estimation for the entire TRF can be generated from this single sample. This is because each subcarrier of an OFDM carrier is represented in every time domain sample. However, this estimation is not as accurate as the estimation from the averaging of estimations across the entire pulse.

Further complication for the LSE is it has a burdensome computation time, even for a single sample of the received signal. Use of the LSE requires that an error needs to be calculated for each possible value of the TRF for the target. For signal processing computation time, we have \( n \) possible estimates for a single value of the TRF, and \( m \) subcarriers. For the LSE, we have to calculate the received signal model for each estimate of the TRF, resulting in \( n^*(2m + 1) \) estimations for each sample. Each of these estimations has \( 2m+1 \) multiplications, followed by \( 2m+1 \) additions, representing the summing of the component contributed by each subcarrier. This method is displayed in figure (2.6). This provides a representation of how increasing either the number of subcarriers or the number of possible TRF
values increases the computation time of the processing significantly. The method provides the following representation for our signal processing time:

\[ n^m + (2m + 1) + (2m + 1) \]  

(2.16)

To clarify this further, we use common big O notation, which is a method to describe the limiting behavior of an algorithm with regards to input size. Characterizing the growth rate of this problem yields:

\[ O(n^m) \]  

(2.17)

This is a result of the exponential term growing much faster than the polynomial terms, thus removing their need for consideration. This problem quickly grows computationally infeasible in complex scenarios such as 64 subcarriers and 10 estimations for the TRF (yielding \(10^{64}\) calculations).

Figure (2.7) below is a completed estimation for the TRF of a target, estimated using the least squares method. The scenario was created without noise or clutter, in order to demonstrate the effectiveness of the method in the simplest scenario (i.e. that with no random factors nor interference, the method could accurately estimate the target’s reflectivity function). Thus, this shows that the estimation method chosen has the ability to perform in a simpler case, acting as a milestone before further simulation was done.
2.2 Frequency Domain Estimation

Once this signal is received, the return from the clutter needs to be removed for further processing. This is done through calculating the power of each of the returns, and then discarding the smaller returns of the clutter. After the signal has been cleaned, the Fast Fourier Transform (FFT) is performed across the entire vector of 2N+1 samples, generating a result like the above graph. The FFT is an algorithm to calculate the Discrete Fourier Transform (DFT) more efficiently. The DFT is defined as the following:

\[
y_f = \sum_{n=0}^{N-1} y_t e^{-i2\pi nk \frac{2}{N}}, \quad k = 0, \ldots, N - 1.
\]  

MATLAB, the software used to simulate this thesis, calculates the DFT using the common FFT Cooley-Turkey algorithm[32]. The resultant vector can then be used to calculate the initial estimations for the TRF. This initial estimation for each subcarrier of the TRF, \( \bar{a}_k \), is calculated through the following
where \( \hat{x} \) is the original subcarrier weights and \( \hat{x} \) is the received subcarrier values. Note that while the original OFDM signal was designed with symmetrical frequency components across the DC point, the up-converting of the OFDM allows for estimation of both the lower and upper half of the frequency range of the signal. This is because the TRF will reflect the lower and upper half of the frequency components of the OFDM signal with different amplitudes.

Thus, we generate \( 2N \) points of the TRF from the original \( N \) subcarriers. This algorithm corresponds to the block diagram of figure (2.9), where \( x \) represents the original values for the specific subcarriers received, \( sc \) and each \( \text{est} \) block corresponds to a signal estimate we generate. Note that the up-converted point corresponding to the DC value is thrown out, since this value is originally set to zero. Any corresponding value for the TRF is a result from additive noise, instead of an actual return from the target.

A simple demonstration of this estimation method is provided in figure (2.10). There, the scenario was made intentionally very simple - no clutter, few subcarriers and little noise. This scenario was created to generate an example of the results of the frequency domain estimation method and prove its ability in a simple case. The red circles represent the actual TRF, the blue x’s showing the estimation results.
The computational time of frequency domain estimation is different than the time domain estimation, as instead of relying on the slower grid search, the frequency domain estimation instead is limited only by the computational speed of the Fast Fourier Transform (FFT). A further advantage to this method is that the FFT is a subject of near-constant research, with many attempting to optimize its runtime even further[33][34].

\[ n\log(n) + (2m + 1) + (2m + 1) \]  

Characterizing the growth rate of this problem yields:

\[ O(n\log(n)) \]  

The development of the FFT is crucial to the improvement in computation time for this algorithm, as the Fourier transform itself has a run time of \( n^2 \), which would be a significant degradation in performance as the scale of the problem increases.

### 2.3 Adjusting OFDM Pulse

#### 2.3.1 Reflectivity Function

#### 2.3.2 Allocation of Power Across Signal Subcarriers

The method that this thesis will utilize is the water-filling algorithm, which is more typically used with OFDM communication systems, but is optimal for power allocation for OFDM radar as well[35]. The basic idea behind this method is to assign power to the most efficient sub-carriers first. The algorithm starts by selecting the sub-carrier with the best target reflectivity. It assigns the maximum amount of power to this sub-carrier, and then subtracts that power from the total remaining power. The algorithm then repeats this process for the next best target reflectivity, until all power has been assigned. or mathematically:

\[
\bar{x}_n = (\rho | a_n > a_t) \cup \bar{x}_n = (0 | a_n > a_t)
\]  

Where \( n=1...2N+1 \), and \( a_t \) is a threshold value, used to determine if the subcarrier is assigned power. In this case, optimal subcarriers are defined as subcarriers in which the target scatters back the most power to the receiver. This adaption is performed for a six subcarrier example in figure (2.11), where noise was included in the channel. The result of the frequency domain adaption via the water-filling
algorithm is shown in the frequency domain figure. The time domain figure shows the resultant adapted signal once converted back to the time domain after signal processing. Finally, the TRF figure displays the estimated TRF that was used for the adaption of the signal in the frequency domain.

However, it is important to note that in other scenarios the definition for optimality in subcarriers weight assignment may vary. For example, in the case of using adaptive waveform design to improve ATR performance, the algorithm could instead choose to assign more power to the subcarriers that most distinctly differentiate the targets of interest. This, however, would require some knowledge about the possible targets of interest ahead of time.

Figure 2.11: Adapted OFDM Waveform, time- and frequency-domain, with some subcarriers provided maximum power (1) while others have been set to the minimum (.1). Note that those set to 1 correspond to the highest values of the estimated TRF(c), seen on the right as the blue x’s.

Once the signal has been adapted, the pulse is again propagated to illuminate the target scene. This adapted signal is then sent through the same steps that the training pulses were put through – reflection from the target and clutter, added noise, added delay from propagation time, interpolation – and then processed when the return is received. This processing performed will be used to extract the parameters for performance.

2.4 Automatic Target Recognition

2.4.1 Method Derivation

Advances in technologies associated with radar systems, both analog and digital, have made for many new functionalities of radar systems. However, due to the huge amount of data radar systems can now gather, there is an acute need for automation of these functionalities, and algorithms are being developed and refined to solve these problems. Among these is ATR, which is a digital signal processing that attempts to align the estimated TRF with the nearest known TRF.

Once the estimation for the TRF has been generated, ATR can be performed. This is a digital
technique used to calculate which of the known TRF's the target’s estimated TRF most closely resembles. By identifying targets through digital techniques, radar systems can provide real time identification of targets, in addition to the more traditional information, such as range and velocity.

In this thesis, ATR is calculated through the LSE of the difference between the estimated value for the TRF compared to a set of targets with known TRF’s in the following equation:

\[ E_s = \sum_{k=-N}^{N} (\hat{a}_k - \bar{a}_k)^2 \]  

where \( \hat{a}_k \) is a one-dimensional vector corresponding to the TRF of the \( s^{th} \) target. \( E_s \) is the total error term corresponding to the \( s^{th} \) target, which is the term for which the LSE is being minimized. Whichever target has the lowest total error term, \( E_s \), is provided as the target by the ATR process. Again, it is important to note that the DC component, where \( k = 0 \), is thrown out, due to that value simply being the result of the noise. As the algorithm attempts to provide a specific identification, each time the ATR is performed results in either a correct or false identification. Algorithms could be made to learn, if prior results are saved. However, since in our case, each target is randomly generated, this is not a possibility. The figure (2.12) demonstrates this process, with \( TRF_r \) corresponding to the received TRF and \( TRF_1 \) through \( TRF_x \) corresponding to the known TRF’s that the system is attempting to choose between. As shown in the figure, the known TRF that has the minimum error between it and the estimated TRF will be chosen as the TRF for the target of interest.

![Figure 2.12: ATR block diagram.](image)
2.5 Minimum Mean Square Error

An alternative method to using the least squares estimation across a single training pulse is to employ the minimum mean square error (MMSE) method across several training pulses at once, which seeks to minimize the mean square error (MSE). This has the advantage statistically of providing a more accurate estimation for the TRF, at the direct cost of processing time as well as any other cost of additional training pulses. Changing from a single training pulse to a series allows us to consider distributions for the channel, interference and the target(s). This makes the target model more realistic, although it increases processing time. In our case, we will consider AWGN for the noise model and Swerling targets for our target models.

As with many other cases of the MSE method, it is not possible to determine a closed form solution, and instead we employ it as an estimator by attempting to minimize the function it provides. However, just as with the least squares estimator, minimizing the function of the MSE provides an acceptably close estimation. The equation of the MSE looks similar to that of the least squares, as seen below.

\[
\text{MSE} = (\hat{Y}_n[n] - Y_n[n])^2
\]  

(2.24)

Note that while \( \hat{Y}_n \) is measured, it is now a function of measured samples instead of a single sample like in the least squares method. In this work, the function is comprised of the multiple training pulses used in simulation.

2.5.1 Swerling Target Models

For most tests in this work, we have assumed a target model in which the target reflects the exact same amount of energy towards the transmitter, varying only on frequency and ignoring power, angle and time of the transmitted pulse. Swerling models are a more complex and descriptive way to model the RCS of a target, which are based off of a special case of the chi-squared target models[36]. This target model is closest to the Swerling type V, where the RCS of a target is constant even with respect to frequency.

When we employ multiple training pulses, however, and the thus the MMSE, we can incorporate a more complicated model for the target’s RCS, of which the Swerling models provide five variations. For our purposes, we will use Swerling type I, which is used to represent a set of several closely-spaced reflective scatterers of approximately equal size, which can be used to model something similar to an aircraft. We chose this model specifically because most experimental data on aircraft and other airborne targets exhibit behavior similar to this model, although it is important that other models exist for other cases[37]. These other models exist for objects such as balloons and other near-spherical reflectors.
Swerling type I is modeled in following probability density function (PDF), \( f(x, \bar{x}) \) such that:

\[
f(x, \bar{x}) = \frac{1}{\bar{x}} e^{-\frac{x}{\bar{x}}}
\]  

(2.25)

where \( x \) is the input signal-to-noise power ratio, and \( \bar{x} \) is the average of \( x \) over all target fluctuations. Note that while \( \bar{x} \) seems a simple average, in practice it can be difficult to calculate, due to radar systems not able to view a target from all angles. In many cases, this means that an estimated value must be used instead of the actual value. The plot of this PDF is seen in figure (2.13).

Figure 2.13: PDF of a Swerling type III model.
Chapter 3

Results

3.1 Illuminating the Target Scene

For the purposes of testing this algorithm, we simulate an OFDM radar illuminating a specific target scene repeatedly. The simulation was performed in MATLAB, utilizing the RedHawk cluster. This target scene has a randomly positioned target within a given range. Randomly generated and randomly placed clutter is added to the target scene. Additive White Gaussian noise (AWGN) is added to the signal as well.

Sampling rate is another component that introduces error into the received signal. Because the received signal must be sampled, interpolation must be performed in simulation to generate an appropriate representation for the signal. Since this sampling will not be at the same points used to generate the transmitted signal, there will be some error introduced as the interpolation estimates the shape of the signal with the points it samples. This interpolation is calculated as follows:

\[
s = \left( (X_c/c)/(1/F_s) - \text{floor}\left( (X_c/c)/(1/F_s) \right) \right) \times (1/F_s) \tag{3.1}
\]

\[
m = \frac{(y_{t+1} - y_t)}{((1/F_s) \times (t) - (1/F_s) \times (t - 1))} \tag{3.2}
\]

\[
\hat{y}_{t+1} = m \times s + y_{t+1} \tag{3.3}
\]

Where \( m \) is the slope estimation calculated for the interpolation, \( F_s \) is the sampling frequency, \( X_c \) is the distance to the target, \( t \) is the index and \( y_t \) and \( \hat{y}_t \) are the signal before and after interpolation, respectively. The effects of this interpolation are seen below in figure (3.1). It is important to note how the interpolation process "rounds" the original signal, by lowering its peaks and valleys.

In each scenario, we illuminate the entire target scene with multiple signals. First, the training pulse
Figure 3.1: Effects of interpolation. The interpolated signal in blue cuts off the extreme values of the original signal, seen in red.

for the adaptive signal is simulated, via propagation to the target and then back to the radar system. After the TRF has been generated, the adapted signal is then transmitted and received. Finally, for comparison purposes, an LFM pulse is transmitted as well, to act as a benchmark for the OFDM adapted signal. This process is seen in figure(3.2). Now, while not realistic, we keep the target scene identical from transmission to transmission, i.e. we apply identical noise, clutter and other interference to each signal over its pulse duration. In a more realistic scenario, noise, return from clutter and the like would vary from signal pulse to pulse. However, in order to more accurately compare the effectiveness of the algorithm, we make this simplification to remove variables.

Figure 3.2: The process for simulation. Note that while the adapted signal process requires the training pulse, the LFM pulse is completely separate process that could be performed either before or after the training pulse and adapted signal.
For performance benchmarks, the results from the adapted signal are compared both to the training pulses and the results from the LFM pulse. These two are used as benchmarks to ensure that the adapted signal is increasing its performance of power efficiency and target identification.

A linear frequency modulated (LFM) chirp is employed to provide a simple benchmark for the OFDM waveforms, both training and adapted. It is a common waveform used in many radar applications, in which the signal is a simple sinusoid that has an increasing frequency versus time. This increase in frequency, over total time, $t_{\text{total}}$ is defined as the chirp rate, $k$, such that:

$$k = \frac{f_{\text{final}} - f_{\text{original}}}{t_{\text{total}}} \quad (3.4)$$

Which leads to the frequency at time $t$ defined as:

$$f(t) = f_{\text{original}} + kt \quad (3.5)$$

The LFM signal is given by the following equation, with the graph seen below in figure (3.3). Note how the LFM signal compresses as time increases, which is a result of its increasing frequency.

$$y(t) = \sin[2\pi \int_0^t f(t')dt'] \quad (3.6)$$

Figure 3.3: LFM chirp.

### 3.2 Target Reflectivity Function Estimation

The estimation of the target’s reflectivity function was the first performance metric chosen to analyze the performance of the adapted signal versus the training pulse. This estimation was done with the assumption that the TRF had no imaginary components and was non-zero for all frequency values.
In order to gather an idea of how effective our estimation methods were, we calculate error. More specifically, we look at calculating the difference between our known TRF and what our estimated TRF was from the returned signal. For the ATR case later, our error is much simpler. It is just whether the correct target was chosen with the appropriate TRF or not. For error in estimating the TRF, we start with our returned signal expression for each case. In the frequency domain estimation method, our returned signal expression is the following:

\[
\hat{x} = \bar{a}_k \tilde{x}, \quad k = -N, \ldots, N
\]  

(3.7)

Where \( \tilde{x} \) is the original subcarrier weights and \( \hat{x} \) is the received subcarrier values. Using frequency domain values of the returned signal as \( \hat{x} \), we can then solve for \( \bar{a}_k \), which will provide our estimation for the TRF via the following expression:

\[
\bar{a}_k = \frac{\hat{x}}{\tilde{x}}, \quad k = -N, \ldots, N
\]  

(3.8)

For our time domain estimation, our returned signal expression is instead:

\[
y_n = \frac{1}{2N + 1} \sum_{K=1}^{2N+1} a_k \tilde{x}_k e^{-j 2\pi (k-1) \frac{n - D_m}{2N+1}} + \frac{1}{2N + 1} \sum_{K=1}^{2N+1} \tau_k \tilde{x}_k e^{-j 2\pi (k-1) \frac{n - D_c}{2N+1}},
\]  

(3.9)

Where \( y_n \) is the expected value for each time point, \( N \) is the number of subcarriers, \( a_k \) is a vector containing the TRF of the target, \( \tilde{x}_k \) is a vector containing the original subcarrier weight, \( F_s \) is the sampling frequency, \( D_m \) is the delay from the target, \( \tau_k \) is the vector containing the TRF of the clutter and \( D_c \) is the delay from the clutter. Once this time domain expression is generated, the least squares estimation method is performed to estimate the TRF, which is then used for error calculation. The least squares estimation process is detailed in depth earlier in this paper.

Once these equations have been generated, the simulated results can be compared to the expected results. For measuring performance, we calculated the RMS error of the estimated TRF versus the actual TRF. Since our original signal has only real components, and we assume that the TRF also is real, we ignore any imaginary component as a result of channel effects. The RMS error was calculated through the following equation:

\[
\text{Error}_{RMS} = \sqrt{E((y_{\text{recorded}} - y_{\text{actual}})^2)}
\]  

(3.10)

Where \( y_{\text{recorded}} \) is the estimated value for the TRF at a specific frequency and \( y_{\text{actual}} \) is the actual value for the TRF at that frequency. However, RMS error by itself can be misleading. In an effort to
provide a more consistent error metric, we perform an additional step; we normalize the RMS error via the following:

\[ \text{Error}_{NRMS} = \frac{\text{Error}_{RMS}}{(y_{max} - y_{min})} \]  

(3.11)

Where \( y_{max} \) is the maximum value of the estimated TRF from the received signal and \( y_{min} \) is the minimum value of the received signal.

For the LFM chirp, used as a benchmark signal, a similar process is employed in order to calculate its normalized RMS error. First, we start with the received signal model, which is the following:

\[ y(t) = \sin[2\pi \int_0^t \hat{a}_t * f(t') dt'] + e_n + \sin[2\pi \int_0^t \tau_t * f(t') dt'] \]  

(3.12)

Where \( e_n \) is the return from the AWGN, \( \hat{a}_t \) is the FFT of the TRF of the target and \( \tau_t \) is the FFT of the TRF of the clutter. It is important to note that \(*\) represents convolution in the equation (3.12), not multiplication. Taking the FFT allows us to compare each frequency component, such that:

\[ \tilde{a}_k = \hat{x}_k, \quad k = -N, \ldots, N \]  

(3.13)

With \( \hat{x} \) is the modified frequency weights, given from the FFT performed on the previous equation (3.12) and \( \tilde{x} \) is the original frequency weights. Once we have the equation above, we can then solve for \( \tilde{a}_k \), which will then be compared to our known TRF and RMS error will be calculated via the following expression:

\[ \text{Error}_{RMS} = \sqrt{E((y_{\text{recorded}} - y_{\text{actual}})^2)} \]  

(3.14)

And then normalized with:

\[ \text{Error}_{NRMS} = \frac{\text{Error}_{RMS}}{(y_{max} - y_{min})} \]  

(3.15)

The two methods of estimation, time and frequency domain, were used on several target scenarios each with various parameters. First, we will consider the frequency domain estimation. We began with four subcarriers, and increased them from there. Due to limitations in our simulation software, the maximum allowable subcarriers for the time domain estimation is nine. Eight was used instead for the maximum, as eight subcarriers created a signal more easily calculated via Fast Fourier transform.
Figure 3.4: SNR vs RMS Error, 4 subcarriers. The red dotted line is the error from the training pulse, while the solid blue line is the adapted signal error.

3.2.1 Frequency Domain Estimation

As seen in figure (3.4), the accuracy of estimating the TRF for the actual target for both signals improves as SNR for the radar system increases, however, the adapted signal performs better for all values of SNR in the range of interest, [-20 dB, 10 dB], with a performance increase of about 25%. Below in figure (3.5) are the results for both the 6 subcarrier and 8 subcarrier signals. The performance changes based on the subcarriers, however, for each quantity of subcarriers the adapted still outperforms the training signal, on average.

Figure 3.5: SNR vs RMS Error, Frequency domain estimation. The red dotted line is the error from the training pulse, while the solid blue line is the adapted signal error.

As the number of subcarriers increases, the error decreases as well, just as it did for the time domain estimation method for a similar number of subcarriers. This trend is seen once the error has been standardized for the number of subcarriers. This is seen in the plot in figure (3.6). It is important to
note that because in this figure the error is not averaged by the number of subcarriers, both the training and adapted signal have more error because of more subcarriers generating error from the estimation of the target’s TRF.

Figure 3.6: SNR vs RMS Error, Frequency domain estimation. Green is 8 subcarriers, red is 6, blue is 4.

This trend continues throughout increasing number of subcarriers as well, again, once the error has been averaged for the number of subcarriers. Note that for the following graph in figure (3.7), the total power across the entire signal is the same. This is done to keep comparisons between differing numbers of subcarriers more relevant. Since an increasing number of subcarriers requires more points, and thus a faster sampling rate, to generate, the signal’s power remains the same, while more noise points are generated, increasing the power of the noise. This is why the curves corresponding to the higher number of subcarriers do not continue into the higher values of SNR and also why the error is standardized to the number of subcarrier channels, instead of summed across the entire signal.

Note also that this performance testing was only done for the frequency domain estimation method. Due to constraints within MATLAB and overwhelming increase in processing time, the time domain estimation method was infeasible to simulate for a significant amount of trials to provide comparative results.

### 3.2.2 Time Domain Estimation

The above figure (3.8) shows that, as expected, the accuracy for both signals in estimating the actual target of interest’s TRF improves as SNR for the radar system increases, however, the adapted signal performs better for many values of SNR in the range of interest, [-20 dB, 10 dB]. The perceived curve was approaching a more linear result as the number of simulation runs increased and was expected to continue to do so. The high number of variables compared to the number of simulation runs is likely the cause of this, and similar curves in other results. Below are the results for the 6 subcarrier. The
Figure 3.7: SNR vs RMS Error, Frequency domain estimation. Red is 16 subcarriers, magenta 32, blue 64, cyan 128, green 256.

Figure 3.8: SNR vs RMS Error, 4 subcarriers. The red dotted line is the error from the training pulse, while the solid blue line is the adapted signal error.

Performance changes based on the subcarriers, however, for each quantity of subcarriers the adapted still outperforms the training signal, on average.

Figure 3.9: SNR vs RMS Error, Time domain estimation. The red dotted line is the error from the training pulse, while the solid blue line is the adapted signal error.
As the number of subcarriers increases, the error decreases as well, once the error has been standardized for the number of subcarriers. This is seen in the plot below in figure (3.10).

Figure 3.10: SNR vs RMS Error, Time domain estimation. Red is 6 subcarriers, blue is 4.

3.2.3 LFM Benchmark

The LFM waveform is a well known signal used in radar systems, and its performance will be used as a benchmark for our work. However, while being a sufficient signal for many other purposes, its performance in this scenario leaves much to be desired. Not only do both the training pulse and adapted signal outperform the LFM waveform, but they often do so by an order of magnitude. This significant difference in performance is demonstrated through our entire range of interest for SNR, [-20dB 10dB] and every subcarrier number we simulated. The LFM waveform’s performance does improve as SNR increases, and further, its error does decrease per subcarrier as the total number of subcarriers increases, just as the training and adapted signals do. However, the LFM waveform is still significantly behind the training and adapted signals from the frequency domain estimation method, seen below in figure (3.11) and even further behind the time domain estimation method. Since the LFM signal does not intelligently assign its power according to the characteristics of the target, this is expected. However, it is still a relevant comparison since many systems utilize the LFM chirp, or other signals with worse performance characteristics than the LFM chirp.
3.3 Automatic Target Recognition

Automatic target recognition was included as a second metric of performance, which was an attempt by the algorithm to automatically pick the target out of a set of known targets. The radar system was provided with several known TRFs, once of which was the actual TRF of the target, and was provided the task of determining which target was in the scene.

The radar system uses an algorithm to determine which of the targets from the list is most likely the target that it has collected the return from in the previous pulse. It does this through calculating the least squares error, a common statistical algorithm used and explained in more depth previously in this thesis, of each known TRF with the estimated TRF of the target, and then returns the closest known TRF as the identity of the target. This algorithm follows the form such that:

\[ J(\hat{X}) = (\hat{x}[n] - x_a[n])^2 \]  

(3.16)

Where n is the subcarrier number, a is the known TRF being compared to the target, \( \hat{x}[n] \) is the estimated value for the TRF of the target at the frequency value of n, \( x_a[n] \) is the known value for the TRF of the possible target at the frequency value of n and \( J(\hat{X}) \) is the summed squared difference of each of the target’s returns compared to the known TRF of \( x_a \). Once each of these error totals is summed for each TRF, the TRF with the lowest calculated value of \( J(\hat{X}) \) is returned as the identity of the target.

These known TRFs were intentionally chosen to be close to actual TRF of the target, with the same constraints provided for the actual TRF. Many of the TRFs only varied in the return for a specific single frequency. While TRFs can vary widely, significantly different TRFs can provide for a trivial ATR process unless the environment interference is particularly harsh. For our target scenarios that implemented ATR, each radar system was provided with 5 possible TRFS for the target, one of which...
was actually the TRF of the target. For reference, the TRFs can be seen in figure (3.12)

Figure 3.12: Possible TRFs, with the actual TRF represented by the red x’s. All other color and symbol combinations represent the possible incorrect answers.

As expected, the performance of the system’s ATR degrades with the worsening of SNR, or more specifically, the poorer the estimation of the target’s reflectivity function. This is seen in figure(3.13). Since the algorithm is comparing the estimated target’s reflectivity function versus its cache of collected targets, the poorer estimation results in a higher likelihood of an incorrect target choice. Further, the shape of the graph is likely a product of the ATR test methods used. Since the possible incorrect targets were selected to be close to the actual target, but with an increased or decreased return over a few subcarriers, a small amount of error would cause the ATR to gravitate heavily towards these incorrect choices. As the SNR increases, thus increasing the error, the system would essentially end up guessing, improving results again. In future work, once the threshold for performance below guess average is determined, the system could be adjusted for those scenarios.

Figure 3.13: ATR performance for adapted vs LFM, 4 and 6 subcarriers, frequency domain estimation

These performance trends hold true as well for the time domain estimation method when using ATR as a performance metric, as seen below. Note that the time domain estimation method outperforms the
frequency domain estimation method for ATR, shown in figure (3.14), just as it did for the previous performance metric.

![Figure 3.14: ATR performance for adapted vs LFM, 4 subcarriers, time domain estimation](image)

**3.4 Minimum Mean Square Error**

MMSE was included as a final variation of testing for the adaptive algorithm to calculate its ability to take advantage of additional training pulses. Considering each training pulse adds a considerable amount of simulation time to the process, we limited the number of additional training pulses to two, bringing the total to three. Adding the additional training pulses simply multiples the prior big O simulation time by a constant. This constant is ignored in big O notation, thus, the simulation time in big O notation is still the same as in equation (2.1.2). However, adding additional training pulses a very real amount of time to these simulations, thus making them take significantly longer to run.

As expected in the MMSE case, the adaptive algorithm outperforms the LFM benchmark with both the time domain and frequency estimation methods. This is seen in figure (3.15). Note that while the MMSE adapted error is significantly less than that of the LFM pulse, it is worse than that of the LSE adapted case. This is due to the added accuracy in target modeling, specifically considering Swerling type I models instead of targets with constant TRFs. The increasing error total with regards to subcarriers is due to the total power of the signal being limited to the same total, regardless of number of subcarriers. This results in more error in TRF estimation, since smaller fluctuations can then have a greater effect.
3.5 Comparison of Domain Estimation Methods

Time domain estimation has several advantages over frequency domain estimation. Primarily, it can be done over a single sample, versus frequency domain estimation, which requires all of the samples to perform the FFT. This means that the signal processing can begin earlier for the time domain estimation. Further, time domain estimation is done on a per subcarrier basis, rather than over the entire signal as a whole, meaning that time domain estimation can focus on a few specific subcarriers of interest. It also does not require an FFT device in the receiver for its hardware implementation, although since most systems will have this implemented on the receiver end, this is of minor note.

However, time domain estimation has a significant burden, and that is its processing time, which follows the form of:
\[ t_{\text{comp}} \approx m^n \] (3.17)

Compared to the frequency domain estimation’s processing time equation of:

\[ t_{\text{comp}} \approx n \log(n) \] (3.18)

Where \( t_{\text{comp}} \) is the total computational time, \( m \) is the number of possible estimation values and \( n \) is the number of subcarriers. Note that \( t \) is not measured in seconds, as it is relative to the processing time of the system used to run the simulation. While there are other factors that define the computation time, they are minor and as a general practice ignored, since the exponential and linear factors will quickly outgrow them. The result of these processing equations is seen in figure (3.17). Frequency domain estimation is limited only by the speed of the FFT, which has seen much optimization work in the recent past. Time domain estimation, however, requires much lengthier processing, having exponential growth as shown above. Even just a small change in the number of subcarriers, such as four to eight, provides a drastic increase in processing time. Similarly, minor changes in precision for the estimation in time domain require significantly more processing time.

![Figure 3.17: Processing Time for Time Domain Estimation](image)

Note that a small increase in subcarriers (simply going from 5 to 6 subcarriers) creates a dramatic increase in the processing time of the time domain estimation, by an entire scale of magnitude. This is a sharp contrast to the runtime of the frequency domain estimation, which has a near negligible runtime, several orders of magnitude lower than the time domain estimation, which can be seen as the red line in figure (3.17). This tendency continues as the number of subcarriers grows. Also important to note is that the processing time of the time domain estimation also increases with the desired accuracy of estimation for the TRF.
### 3.5.1 Comparative Results

<table>
<thead>
<tr>
<th>Subcarriers</th>
<th>Domain</th>
<th>Signal Type</th>
<th>Error Avg</th>
<th>Error StdDev</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>freq</td>
<td>training</td>
<td>1.4919</td>
<td>1.1642</td>
</tr>
<tr>
<td>4</td>
<td>freq</td>
<td>adapted</td>
<td>1.0138</td>
<td>0.5857</td>
</tr>
<tr>
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<td>freq</td>
<td>LFM</td>
<td>12.4858</td>
<td>9.9906</td>
</tr>
<tr>
<td>6</td>
<td>freq</td>
<td>training</td>
<td>2.0759</td>
<td>1.5617</td>
</tr>
<tr>
<td>6</td>
<td>freq</td>
<td>adapted</td>
<td>1.3553</td>
<td>0.8693</td>
</tr>
<tr>
<td>6</td>
<td>freq</td>
<td>LFM</td>
<td>23.90248</td>
<td>22.52305</td>
</tr>
<tr>
<td>8</td>
<td>freq</td>
<td>training</td>
<td>2.8941</td>
<td>2.3844</td>
</tr>
<tr>
<td>8</td>
<td>freq</td>
<td>adapted</td>
<td>1.7035</td>
<td>1.0943</td>
</tr>
<tr>
<td>8</td>
<td>freq</td>
<td>LFM</td>
<td>23.1616</td>
<td>19.4415</td>
</tr>
<tr>
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<td>training</td>
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</tr>
<tr>
<td>4</td>
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<td>adapted</td>
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</tr>
<tr>
<td>4</td>
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<tr>
<td>6</td>
<td>time</td>
<td>LFM</td>
<td>23.79894</td>
<td>21.49151</td>
</tr>
</tbody>
</table>

This table catalogues the RMS error average and standard deviation for both the time and frequency domain estimation methods, containing the results for the training pulse, adapted signal and LFM benchmark. It is interesting to note that while the time domain estimation takes significantly longer runtime, it performs roughly 3 times better than the frequency domain estimation for the same number of subcarriers for both the training pulse and the adapted signal. Both OFDM estimation methods, however, improve performance over the standard LFM waveform by an order of magnitude.

Another important fact is that the standard deviation for the time domain estimation is exceedingly small compared to the standard deviation of the frequency domain estimation. This means that not only is the time domain estimation more accurate, but it is significantly more consistent in its performance.

The reason for this difference between the time domain and frequency domain estimations is the interference from the noise. In the time domain estimation, the noise perturbs every point, but every point has an individual estimation. Thus, while large spikes in amplitude from the noise may exist, since they only effect a single point, these estimations are then averaged out by the process being applied to every subcarrier. In the frequency domain estimation, however, a small change to a single time domain point reflects on every subcarrier in the frequency domain. Several changes in amplitude, even minor ones, can make for drastic changes in the frequency domain representation, and thus the subsequent
estimation. This vulnerability to noise causes the frequency domain to have more error, on average, as well as a greater standard deviation in its error, especially as SNR worsens.

Finally, once the error average is standardized by the number of subcarriers, average error decreases as the number of subcarrier goes up, making the signal more resistant to negative channel effects.
Chapter 4

Conclusions

In this paper, an adaptive algorithm for OFDM waveforms was developed in order to improve their performance in both target reflectivity function and ATR performance, as well as power efficiency. The focal point of the algorithm is to utilize the target reflectivity function to more optimally assign power between the subcarriers of the transmitted signal, thus generating a greater return from the target. This estimation was performed through use of the LSE method, which was also shown. Two different methods of estimation were derived, one in the time domain and another in the frequency domain. These estimation methods were compared, not only in performance but in processing time as well. A water-filling algorithm, also shown, was utilized to distribute the power between the subcarriers, finishing the signal processing component of the algorithm.

It was shown that the adapted signal design consistently decreases the RMS error in target reflectivity function estimation, as well as improve ATR for nearly all values of SNR compared to the training pulse. This holds true both for the frequency and time domain estimations. In the time domain estimation, it was shown that, on average, the adapted signal provided roughly a 24% more accurate estimation over the training signal, regardless of the number of subcarriers. This adapted signal vastly outperformed the LFM chirp, by more than an order of magnitude, with roughly a 97% more accurate estimation. Even in the MMSE estimation scenario, the adapted algorithm still vastly improved performance, despite a more challenging Swerling target model to estimate.

For the frequency domain, it was shown that the adapted algorithm provided roughly a 32% improvement in accuracy for estimation. Similarly, the frequency domain estimation adapted algorithm provided a drastic increase in accuracy over the standard LFM chirp at 89%. This performance increase is not as drastic as for the time domain estimation, however.

For all signals involved, however, performance improved in all cases as the number of subcarriers
increase, despite the average power of the signal per subcarrier remaining the same.

It was shown that the time domain estimation provides a more accurate, consistent result, as it had less error (63% increase in accuracy) and a lower standard deviation (roughly 72%, as shown). However, the time domain estimation is significantly more burdensome computationally, with the frequency domain estimation provides a 98% increase in runtime for just four subcarriers, and scales better with regards to number of subcarriers. Choice of estimation method, then, comes down to a balance between processing time and accuracy.
Chapter 5

Future Work

5.1 Iterative Training Pulses

This work utilizes either a single training pulse or a single estimation average between several training pulses in order to generate its estimation for the target’s reflectivity function, and the subsequent optimization of the OFDM waveform. A more complicated, but perhaps more efficient alternative would be to use iterative training pulses for the estimation of the target’s reflectivity function. Essentially, this would treat every adapted pulse as a training pulse for all following, using the subsequent pulses to hone in on a more precise estimation. This would allow for a more precise estimation of the function, but would require a longer time and more processing power.

5.2 Real-Time Adaptability

While adaptive signal design is useful for radar systems, no guarantee for processing time has been provided with this algorithm. This may be insufficient for many uses, such as in some airborne radars. Thus, a future consideration is to adjust the algorithm to be more time efficient, in order for radars with time constraints to be able to take advantage.

5.3 Target Imaging with SAR

This paper specifically examines the detection scenario for a radar system. However, also of interest is the imaging scenario, in particular via the use of SAR. In future efforts we would like to examine the efficiency of this adaptive signal design for the imaging scenario.
Bibliography


