ABSTRACT

AN APPLICATION OF N-BODY SIMULATION TO THE ROTATIONAL MOTION
OF SOLAR SYSTEM BODIES

by Tiandan Wu

We made computational simulations of N-body rotational dynamics system to investigate either long term or short term behavior of specific objects in the solar system. The HITS method is used to do both the simultaneous orbital and rotational integrations. Simulated results showed that the large asteroids Ceres and Vesta are experiencing a stable long term periodic change of their obliquity. The obliquity of Ceres fluctuates between 0° ~22°, with an average period of ~23 Kyr. The obliquity of Vesta fluctuates by about 25°, with an average period of ~50 Kyr. The high eccentricity of Neptune’s moon Nereid has an important effect on its rotational motion. Nereid’s obliquity is clearly perturbed when Nereid is at peri-Neptune position.
AN APPLICATION OF N-BODY SIMULATION
TO THE ROTATIONAL MOTION OF SOLAR SYSTEM BODIES

A Thesis

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CHAPTER 1
INTRODUCTION

1.1 Rotational motion

As stated in Murray and Dermott’s book (1999), “our solar system is a highly structured assembly of orbiting bodies. However, the structure is not as simple as Kepler’s geometrical model nor as crude as that implied by the Titius-Bode ‘law’.” One could modify this statement slightly by saying “our solar system is a highly structured assembly of orbiting and rotating bodies.” For the last few decades, gravitational N-body simulations have been widely used in the study of solar system dynamics. Most of these studies only consider the orbital motion of bodies and treat the bodies as point masses. We know that most of the major natural planetary bodies in our solar system are observed to be rotating. For non-spherical bodies, the gravitational torques from other planetary bodies will perturb this rotational motion. The most complete combined orbital and rotational evolution studies of planetary bodies have been done by Jacques Laskar (Laskar, 1993; Laskar & Robutel, 1993; Laskar et al., 2004). Also, the obliquity variations of Earth and Mars were studied by Ward (1973, 1979) and Borderies (1980).

1.2 Rotation of Asteroids and Moons

In our solar system, there are many other objects, besides the planets, that exhibit a wide range of rotational behavior. For example, a major group of bodies in the solar system are the asteroids. Most of the known asteroids are in the main asteroid belt, which is between Mars and Jupiter. The sizes of asteroids are quite different, with diameters from a few hundreds of kilometers to just tens of metres. Some of the largest asteroids are roughly spherical and are orbiting the Sun in relatively low eccentricity orbits, which are very much like miniature planets. Also, similar to the Moon, most planetary satellites are experiencing both orbital and rotational perturbations.
1.3 Thesis

At the present time, the rotational motion of most asteroids and some planetary satellites in our solar system are still poorly understood and many interesting problems are worthy of investigation.

In this thesis, we propose to study the rotational motion of bodies from two different classes: the large asteroids, Ceres and Vesta; and a moon of Neptune, Nereid. Ceres and Vesta should be significantly perturbed by Jupiter, while Nereid is unique in that it is in a highly eccentric orbit, and thus experiences extremely variable torques.

Chapter 2 presents the details of our computational technique, a gravitational N-body simulation that simultaneously integrates for both the orbital and rotational motions. Our results for Ceres and Vesta are given in Chapter 3, while those for Nereid are in Chapter 4.
2.1 Rotational dynamics

We consider an N-body rotational dynamics system as shown in Figure 1. The (X, Y, Z) coordinate refers to the heliocentric coordinate system. Body coordinate (x', y', z') refers to the rotating body $m_i$. All the other bodies $m_j$ and the Sun can exert a gravitational force and torque on the rotating body $m_i$. Then the acceleration, $\ddot{a}_i$, and jerk, $\dot{\ddot{a}}_i$, of $m_i$ in the N-body system will be (Abel, 2001)

![Geometry of Rotational N-body Problem](image)

**Figure 2.1:** Geometry of Rotational N-body Problem
\[
\ddot{a}_i = \sum_{j \neq i} Gm_j \frac{\ddot{r}_{ij}}{r_{ij}^3},
\]  
(2.1)
\[
\dot{a}_i = \sum_{j \neq i} Gm_j \left[ \frac{\ddot{v}_{ij}}{r_{ij}^3} + \frac{3(\ddot{v}_{ij} \cdot \ddot{r}_{ij})\ddot{r}_{ij}}{r_{ij}^5} \right],
\]  
(2.2)

where
\[
\ddot{r}_{ij} = \ddot{x}_j - \ddot{x}_i, \quad (2.3)
\]
\[
\ddot{v}_{ij} = \ddot{v}_j - \ddot{v}_i, \quad (2.4)
\]

and \(G\) is the gravitation constant.

We use the 3-2-1 Euler angle convention as in Hughes (1986) to orient the body fixed frame \((x', y', z')\) relative to the inertial frame \((X, Y, Z)\). For this convention, we rotate by angle \(\phi\) about the \(z'\)-axis, then by angle \(\theta\) about the \(y'\)-axis, and finally by angle \(\psi\) about the \(x'\)-axis.

For our geometry, we solve for the orientation of \((x', y', z')\) with respect to the \((X, Y, Z)\) system, which then tells us the angle between the orbital and spin angular momentum vectors, or the obliquity.

The transformation matrix of the relative position is then given by
\[
\bar{r}' = \begin{pmatrix}
\cos \theta \cos \phi & \sin \theta \sin \psi \cos \phi + \cos \psi \sin \phi & -\sin \theta \cos \psi \cos \phi + \sin \psi \sin \phi \\
-\cos \theta \sin \phi & -\sin \theta \sin \psi \sin \phi + \cos \psi \cos \phi & \sin \theta \cos \psi \sin \phi + \sin \psi \cos \phi \\
\sin \theta & -\cos \theta \sin \psi & \cos \theta \cos \psi
\end{pmatrix} \bar{r} = \bar{T} \bar{r},
\]  
(2.5)

and the transformation of the relative velocity vector is
\[
\bar{v}' = \bar{T} \bar{v} + \bar{T} \dot{\bar{r}},
\]  
(2.6)

where the elements of the matrix \(\bar{T}\) are the time derivatives of the matrix elements of \(\bar{T}\).

Murray and Dermott (1999) give the components of the torque about the body principal axes acting on rigid body \(m_i\) from point mass \(m_j\):
\[
N_x' = 3Gm_j(C - B)y'z'/r^5, \quad (2.7)
\]
\[
N_y' = 3Gm_j(A - C)z'x'/r^5, \quad (2.8)
\]
\[
N_z' = 3Gm_j(B - A)x'y'/r^5, \quad (2.9)
\]
where $A$, $B$, and $C$ are the principle moments of inertia of $m_i$ about its body axes $(x', y', z')$, respectively. If the components of $\vec{\omega}$ are the angular velocity of the rotation about the body principal axes, then the Euler equations are

$$A\dot{\omega}_x - (B - C)\omega_y \omega_z = N_x' ,$$  

$$B\dot{\omega}_y - (C - A)\omega_z \omega_x = N_y' ,$$  

$$C\dot{\omega}_z - (A - B)\omega_x \omega_y = N_z' .$$  

Putting the components of the torque into the Euler equations gives

$$\dot{\omega}_x = I_{bac} \omega_y \omega_z - 3Gm_j I_{bca} y' z' / r^5 ,$$  

$$\dot{\omega}_y = I_{cab} \omega_z \omega_x - 3Gm_j I_{cab} z' x' / r^5 ,$$  

$$\dot{\omega}_z = I_{abc} \omega_x \omega_y - 3Gm_j I_{abc} x' y' / r^5 ,$$

where $I_{bca} = (B - C) / A$, etc.

### 2.2 Computational methods (HITS)

The Hermite Individual Timestep Scheme (HITS) method is widely used in N-body simulation studies (Makino and Aarseth, 1992; Kokubo & Ida, 1995, 1996; Lecar & Aarseth, 1986; Alexander & Agnor, 1998). In our N-body solar system, each body has its own time and timestep. Bodies which experience large perturbations will have small timesteps and bodies that experience small perturbations will have large timesteps. Instead of using the shortest timestep for each body, this HITS technique calculates different timesteps for different bodies. As a result, it will not waste computer time on doing unnecessary calculations.

To use the HITS method for $m_j$ bodies (except $m_i$), we must know $\tilde{x}_j(t_j)$, $\tilde{v}_j(t_j)$, $\tilde{a}_j(t_j)$, $\tilde{\xi}_j(t_j)$, and $\Delta t_j$, where $t_j$ is the body’s time. Next, we choose body $m_i$ with minimum $t_i + \Delta t_i$ and set $t = t_i + \Delta t_i$.

Now use Taylor’s series to predict positions $\tilde{x}_j$ and velocities $\tilde{v}_j$ of all bodies at time $t$

$$\tilde{x}_{j1}(t) = \tilde{x}_j(t_j) + \tilde{v}_j(t_j)(t - t_j) + \frac{1}{2} \tilde{a}_j(t_j)(t - t_j)^2 + \frac{1}{6} \tilde{\xi}_j(t_j)(t - t_j)^3 ,$$  

(2.16)
Then similar to equation (2.1) to (2.4), \( \bar{a} \) and \( \hat{a} \) for the update body \( m_i \) can be found at the predicted time \( t \) as

\[
\bar{a}_{i\alpha} = \sum_{j \neq i} Gm_j \frac{\vec{r}_{ij}}{r_{ij}^3}, \tag{2.18}
\]

\[
\hat{a}_{i\alpha} = \sum_{j \neq i} Gm_j \left[ \frac{\vec{v}_{ij}}{r_{ij}^3} + \frac{3(\vec{v}_{ij} \cdot \vec{r}_{ij})\vec{r}_{ij}}{r_{ij}^5} \right], \tag{2.19}
\]

where

\[
\vec{r}_{ij} = \vec{x}_{ji} - \vec{x}_{i\alpha}, \tag{2.20}
\]

\[
\vec{v}_{ij} = \vec{v}_{ji} - \vec{v}_{i\alpha}. \tag{2.21}
\]

Now, for body \( m_i \), we have \( \bar{a} \) and \( \hat{a} \) at two times, i.e. \( \bar{a}_i(t_i) \), \( \hat{a}_i(t_i) \) and \( \bar{a}_{i\alpha}(t) \), \( \hat{a}_{i\alpha}(t) \), where \( t \) is the predicted time. The Taylor expansions for \( \bar{a} \) and \( \hat{a} \) are

\[
\bar{a}_{i\alpha}(t) = \bar{a}_i(t_i) + \hat{a}_i(t-t_i) + \frac{1}{2} \ddot{a}_i(t-t_i)^2 + \frac{1}{6} \dddot{a}_i(t-t_i)^3, \tag{2.22}
\]

\[
\hat{a}_{i\alpha}(t) = \hat{a}_i(t_i) + \ddot{a}_i(t-t_i) + \frac{1}{2} \dddot{a}_i(t-t_i)^2. \tag{2.23}
\]

Here we know \( \bar{a} \) and \( \hat{a} \) at both times, so we solve these two equations for \( \ddot{a}_i \) and \( \dddot{a}_i \):

\[
\ddot{a}_i = \frac{-6(\bar{a}_i - \bar{a}_{i\alpha}) - \Delta t_i (4\ddot{a}_i + 2\dddot{a}_{i\alpha})}{\Delta t_i^3}, \tag{2.24}
\]

\[
\dddot{a}_i = \frac{12(\bar{a}_i - \bar{a}_{i\alpha}) + 6\Delta t_i (\ddot{a}_i + \dddot{a}_{i\alpha})}{\Delta t_i^5}, \tag{2.25}
\]

where

\[
\Delta t_i = t - t_i. \tag{2.26}
\]

Now, use \( \ddot{a}_i \) and \( \dddot{a}_i \) to correct the position and velocity of body \( m_i \) to fifth order:

\[
\bar{x}_i(t) = \bar{x}_{i\alpha}(t) + \dddot{a}_i \frac{\Delta t_i^4}{24} + \dddot{a}_i \frac{\Delta t_i^5}{120}, \tag{2.27}
\]

\[
\bar{v}_i(t) = \bar{v}_{i\alpha}(t) + \dddot{a}_i \frac{\Delta t_i^3}{6} + \dddot{a}_i \frac{\Delta t_i^4}{24}. \tag{2.28}
\]
Now, we can advance body $m_i$ to time $t$ (i.e. $t_i = t$) and calculate a new timestep for body $m_i$:

$$
\Delta t = \sqrt{\frac{\eta}{\frac{1}{a} \left| \dot{a} \right|^2 + \frac{1}{\dot{a}_i} \left| \ddot{a}_i \right|^2}} ,
$$

where $\eta$ is an adjustable parameter to ensure energy conservation. (Makino and Aarseth, 1992)

After finding the timestep for body $m_i$, the program goes back and uses the same method to update the position etc. of the body with the minimum update time $t + \Delta t$.

### 2.3 Initial Euler angles

One problem that is considered during the setup of our program is the initial Euler angles of celestial bodies. Initial Euler angles give the initial orientation of rotating bodies in our simulation. In our computational technique, the Euler equations are expressed in terms of the Euler angles, and their derivatives. As a result, we must determine the initial Euler angles from the known obliquity and orbital elements.

From our definition of the Euler angles, we rotate the body frame by angle $\phi$ about the $z'$-axis. Because of the body’s rotational motion about the $z'$-axis and orbital motion about the $Z$-axis, the angle $\phi$ does not affect the calculation of other angles and could be an arbitrary value. Also, in our solar system, the inclination ($I$) of a planetary body is defined as the angle between the plane of the body’s orbit and the ecliptic. The longitude of the ascending node ($\Omega$) is defined as the angle from the origin of longitude to the ascending node, measured in a reference plane. For a heliocentric orbit, such as Ceres and Vesta, the ecliptic is the reference plane, and the First Point of Aries is the origin of longitude. The angle is measured counterclockwise (as seen from north of the ecliptic) from the First Point of Aries to the node. (Murray & Dermott, 1999)

The equation that contains Euler angles $\theta$ and $\psi$, obliquity, inclination and Longitude of the ascending node is:

$$
\cos \epsilon = \sin \Omega \sin I \sin \theta + \cos \Omega \sin I \cos \theta \sin \psi + \cos I \cos \theta \cos \psi .
$$

From
\[
\sin^2 \theta = 1 - \cos^2 \theta, \\
\sin^2 \psi = 1 - \cos^2 \psi,
\]
we have
\[
\cos \varepsilon = \sin \Omega \sin I \sqrt{1 - \cos^2 \theta} + \cos \Omega \sin I \cos \theta \sqrt{1 - \cos^2 \psi} + \cos I \cos \theta \cos \psi.
\]
Then the initial Euler angles \( \theta \) and \( \psi \) can be calculated from the initial values of obliquity (\( \varepsilon \)), inclination (I) and longitude of ascending node (\( \Omega \)).

Since \( \theta \) and \( \psi \) are the unknown values, we can write equation (2.33) as
\[
A = EC \sqrt{1 - X^2} + DCX \sqrt{1 - Y^2} + BXY,
\]
where
\[
X = \cos \theta, \\
Y = \cos \psi, \\
A = \cos \varepsilon, \\
B = \cos I, \\
C = \sin I, \\
D = \cos \Omega, \\
E = \sin \Omega.
\]
Then the initial Euler angle problem reduces to solving equation (2.34) for \( X \) and \( Y \).

A program was written to solve equation (2.34). When the obliquity (\( \varepsilon \)), inclination (I) and longitude of ascending node (\( \Omega \)) are specified, the program calculates solution pairs of \( \theta \) and \( \phi \). Since \( X \) and \( Y \) are the cosine function of the Euler angles, they must be within the range \([0, 1]\). In this program, \( Y \) changes from 0 to 1 with a small step size (~0.00001). At the same time, if there is a solution of \( X \) within \([0, 1]\), the solution of \( X \) is solved for each \( Y \) value. Then \( \theta \) and \( \phi \) can be easily calculated by the equations below:
\[
\theta = \cos^{-1} X, \\
\psi = \cos^{-1} Y.
\]

When we get all of the initial conditions that are needed in our simulation, the program can be set up and is ready to run.
CHAPTER 3
CERES AND VESTA

3.1 Introduction

Ceres is the largest object in the asteroid belt in the solar system with a diameter of about 1,000 km. Vesta is the second-most massive body and the brightest asteroid in the asteroid belt with approximate radial dimensions of 604km × 539km × 467km (Harris, Warner & Pravec, 2006). Much work has been done to observe and investigate the characteristics of these bodies; however, we still have limited knowledge of them. Currently, both the oblateness and obliquity of Ceres and Vesta are somewhat uncertain. Ceres and Vesta are somewhat tri-axial bodies, with the percentage difference of the equatorial radii unequal by ~5% for Ceres and ~10% for Vesta. The obliquity of Ceres today is estimated to be ~3° (Thomas, 2005); however, the obliquity of Vesta is unknown to within 10°, with neither body’s spin orientation being precisely known.

The DAWN mission will answer a variety of questions about our solar system. One of the questions that the DAWN mission should answer is the shape and orientation of the asteroid belt’s two largest members, Ceres and Vesta. At the same time, the mission will enable us to understand the evolutionary histories of the two objects and determine the factors controlling their evolution.

Based on previous work on the physical and orbital characteristics of Ceres and Vesta, we conclude that both of them should experience long-term obliquity fluctuations. Now, we adopt an N-body simulation to investigate both the orbital and rotational behavior of Ceres and Vesta during a long time period. In our simulation, the HITS technique is used to integrate the orbital equations of motion, solving for the position and derivatives. Also, the Euler equations are integrated by using the expression for the gravitational torque exerted on a non-spherical body. We ran each simulation for 10 million years to get obliquity evolution for both Ceres and Vesta. At the same time, it also gives us their orbital evolution during their long period revolutions.
3.2 Initial conditions

3.2.1 Physical and orbital properties

We need both physical properties and orbital properties of Ceres and Vesta. Since it is hard to know the true conditions 10 Myr ago, we use the current conditions as the initial conditions of our N-body solar system and set initial values as the accepted current values, which are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Ceres</th>
<th>Vesta</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-major axis (AU)</td>
<td>2.767</td>
<td>2.361</td>
<td>(Murray &amp; Dermott, 1999)</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.097</td>
<td>0.089</td>
<td></td>
</tr>
<tr>
<td>Inclination (degree)</td>
<td>9.73</td>
<td>7.133</td>
<td></td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>9.43E+20</td>
<td>2.70E+20</td>
<td></td>
</tr>
<tr>
<td>Longitude of ascending node (degree)</td>
<td>80.41</td>
<td>103.926</td>
<td>(Thomas, 2005)</td>
</tr>
<tr>
<td>Mean anomaly (degree)</td>
<td>262.19</td>
<td>205.652</td>
<td></td>
</tr>
<tr>
<td>Spin rate (hours)</td>
<td>9.076</td>
<td>5.43</td>
<td>(Thomas, 1997)</td>
</tr>
<tr>
<td>Obliquity (degree)</td>
<td>3</td>
<td>30 ± 10</td>
<td>(Drummond, 1998)</td>
</tr>
<tr>
<td>Dimension a (km)</td>
<td>1018</td>
<td>604</td>
<td>(Harris, Warner &amp; Pravec, 2006)</td>
</tr>
<tr>
<td>Dimension b (km)</td>
<td>945</td>
<td>539</td>
<td></td>
</tr>
<tr>
<td>Dimension c (km)</td>
<td>888</td>
<td>467</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: Physical and Orbital Properties of Ceres and Vesta

3.2.2 Initial Euler angles

From the known physical and orbital properties of Ceres and Vesta above, we can find the initial Euler angles as shown in Chapter 2. The obliquity ($\varepsilon$), inclination ($I$) and longitude of ascending node ($\Omega$) are used here. Since the obliquity of Vesta is still unknown to within 10° today, we did 3 simulations for Vesta, using different initial obliquities 20°, 30° and 40°, respectively.
### Table 3.2: Initial Euler Angles of Ceres and Vesta

<table>
<thead>
<tr>
<th></th>
<th>Obliquity (degree)</th>
<th>Inclination (degree)</th>
<th>Longitude of ascending node (degree)</th>
<th>theta (rad)</th>
<th>psi (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ceres</td>
<td>3</td>
<td>9.73</td>
<td>80.41</td>
<td>0.2003</td>
<td>0.0524</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>7.133</td>
<td>103.926</td>
<td>0.3176</td>
<td>0.2653</td>
</tr>
<tr>
<td>Vesta</td>
<td>30</td>
<td>103.926</td>
<td>0.5548</td>
<td>0.5548</td>
<td>0.5548</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 3.3 Ceres

For Ceres, we first considered the 10-body system in the solar system. This system includes Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto and Ceres. Figure 3.1 shows the obliquity and inclination evolution of Ceres with an initial obliquity of 3°. The data for both plots come from the same simulation. The obliquity fluctuates between 0° ~22°, with an average period of ~23 Kyr. The inclination oscillations are much smaller than the obliquity fluctuations. This means that changes in the obliquity are due to changes in the spin-axis orientation and not due to changes in the orientation of the orbital plane.
Figure 3.1: Results of a 1 Myr and 5 Myr N-body integration of the obliquity and inclination evolution of Ceres, for an initial obliquity of 3°.
Since Jupiter is the largest planet in the solar system (~2.5 times more massive than all the other planets in our solar system combined) and is near the asteroid belt, it is likely that Jupiter is the most important planet, perturbing Ceres. To test this idea, we considered the obliquity oscillations of Ceres over 10 million years in a 2-body system (only Ceres and the Sun) and a 3-body system (Ceres, Jupiter and the Sun). Comparing these results with the one we got in 10-body system simulation, may give us more insight about Ceres’ motion and how it interacts with other planets in the solar system.

We did the simulation of Ceres in the 2-body system and 3-body system for 1 Myr and the results are plotted in Figure 3.2, where only the Sun (straight line) and Jupiter & the Sun are considered. Comparing these to the results of the Ceres in the 10-body system, the obliquity evolution of Ceres in the 3-body system is quite similar to the plot of Ceres in the 10-body system, although there is a tiny difference of the overall pattern. This difference should be explained by the existence of the other planets. This result shows that the changes in obliquity are due primarily to the gravitational torques exerted on Ceres by Jupiter.

![Ceres Obliquity, Effects of Jupiter & Sun](image)

**Figure 3.2:** Results of a 1 Myr N-body integration of the obliquity of Ceres, where only the Sun (green) and Jupiter and the Sun (red) are considered.
We should demonstrate that our simulated solar system evolves in a similar manner to other published works. Jupiter and Saturn are two important planets in the 10-body system simulation. Figure 3.3 shows the inclination of Jupiter and Saturn for 200 thousand years from the data of the 10 Myr Ceres integration. Figure 3.4 shows the eccentricity of Jupiter and Saturn for 200 thousand years from the data of the 10 Myr Ceres integration. These results are very similar to the accepted results, which can be found in Murray & Dermott (1999). The small oscillations seen in Figure 3.4 do make sense since the calculations in Murray & Dermott (1999) ignore short period perturbations.

**Figure 3.3:** Results of a 200 thousand years N-body integration of the Inclination of Jupiter and Saturn from the data of Ceres integration.
3.4 Vesta

Vesta is a typical non-spherical body. As mentioned before, the obliquity of Vesta is still unknown to within 10° today. As a result, for Vesta, we did three 10 Myr integrations of the 10-body solar system, using different initial obliquities 20°, 30° and 40°, respectively. Similar to the 10-body system for Ceres, the 10-body system of Vesta includes Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto and Vesta.

Figure 3.5 shows the obliquity and inclination evolution of Vesta, for an initial obliquity of 30°. The obliquity fluctuates between 24° ~ 48°, with an average period of ~48 Kyr. Figure 3.6 shows the results for an initial obliquity of 20°. Here, the obliquity fluctuates between 11° ~ 36°, with an average period of ~50 Kyr. Last, figure 3.7 shows the results for an initial obliquity of 40°. Here, the obliquity fluctuates between 33° ~ 56°, with an average period of ~46 Kyr.

These three graphs are quite similar to each other, although they used different initial obliquities to begin their integrations. We find similar amplitudes of about +/- 12°.
for initial obliquities of 20°, 30°, and 40°. In addition, for all of them, the inclination oscillations are much smaller than the obliquity fluctuations. This means that the changes in obliquity are due to changes in the spin-axis orientation and not due to changes in the orientation of the orbital plane.
Figure 3.5: Results of a 1 Myr and 5 Myr N-body integration of the obliquity and inclination evolution of Vesta, for an initial obliquity of 40°.
Figure 3.6: Results of a 1 Myr and 5 Myr N-body integration of the obliquity and inclination evolution of Vesta, for an initial obliquity of 30°.
Figure 3.7: Results of a 1 Myr and 5 Myr N-body integration of the obliquity and inclination evolution of Vesta, for an initial obliquity of 20°.
3.5 Conclusion

In conclusion, our simulations for Ceres and Vesta display long term behaviors in the obliquity. The orbital results agree with the accepted results in other works. From our simulations, the obliquity of Ceres fluctuates between $0^\circ$ ~$22^\circ$, with an average period of ~23 Kyr. In all the three conditions of Vesta, the obliquity fluctuates by about $25^\circ$, with an average period of ~50 Kyr. Based on our orbital and rotational results, it is clear that changes in the spin-axis orientation plays an important role in affecting the changes in obliquity of both Ceres and Vesta.
CHAPTER 4
NEREID

4.1 Introduction

Nereid is the third largest moon in the Neptune system. The eccentricity of Nereid’s orbit is about 0.75, which is the highest eccentricity of any known regular satellite in our solar system (wikipedia). Schaefer et al. (2008) have described Nereid’s light variability in great detail. The light curve information is used to show that Nereid likely displays various types of rotational behavior. And it seems that there is no significant periodicity of Nereid’s motion in these 20 years. (Schaefer et al., 2008)

We want to simulate the motion of Nereid using the same computational technique that was used for Ceres and Vesta. By comparing our simulation results with the known light curve of Nereid, we hope to confirm our simulation and then predict the obliquity behavior of Nereid over short time periods.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nereid</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-major axis (AU)</td>
<td>0.03685</td>
<td>(Jacobson, Riedel, &amp; Taylor, 1991)</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.7512</td>
<td></td>
</tr>
<tr>
<td>Inclination (degree)</td>
<td>7.23</td>
<td></td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>3.10E+19</td>
<td></td>
</tr>
<tr>
<td>Longitude of ascending node (degree)</td>
<td>333.979</td>
<td></td>
</tr>
<tr>
<td>Mean anomaly (degree)</td>
<td>359.34</td>
<td></td>
</tr>
<tr>
<td>Mean radius (km)</td>
<td>170 ± 25</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Nereid’s known physical and orbital properties.

4.2 Neptune system

Although Nereid is the third largest moon in the Neptune system, it is still a small moon. Its orbital and rotational motions are affected by Neptune and Triton, the largest moon in the system. All of the other moons of Neptune are either too small or too far away from Nereid to perturb it. As a result, we set up a Neptune system that just includes
Neptune, Triton and Nereid. Triton is also unique in the solar system in that its orbit is retrograde. Thus it travels in the opposite direction of Nereid.

![Figure 4.1: Neptune system: Neptune, Triton and Nereid.](image)

The orbital motions of Nereid and Triton are integrated in this Neptune centered system in a similar manner to Ceres and Vesta in a Sun centered system. Before we include the rotational motion of Nereid, we should establish that our simulation gives a stable orbit for Nereid. Figure 4.2 shows Nereid’s orbital elements, the semi-major axis, eccentricity, and inclination for a one thousand year integration without Triton. All these elements are quite constant. Triton, due to its large mass and retrograde orbit, may perturb Nereid’s orbit. Figure 4.3 shows a one thousand year integration of Nereid’s orbit with Triton included. Again, we see no significant changes to Nereid’s orbit.
Figure 4.2: Nereid’s semi-major axis, eccentricity and inclination behavior in Neptune-Nereid (2-body) system for 1000 years.

Figure 4.3: Nereid’s semi-major axis, eccentricity and inclination behavior in Neptune-Nereid (2-body) system for 1000 years.
Figures 4.2 and 4.3 tell us that our results from these two systems are quite similar, which means that the perturbation on Nereid’s orbital motion from Triton is very small and almost negligible.

4.3 Precession period case

As discussed in Schaefer et al. (2008), we have very limited knowledge regarding the dimensions and the rotational state of Nereid. We do know the characteristics of the orbit quite well, i.e. the eccentricity is 0.7512 and the orbital period is 360.13 day. Schaefer et al. (2008) also show that if the observed brightness variations are caused by a rotating axisymmetric body, the precession period is

$$P_{\text{prec}} = \frac{2(P_{\text{orb}}^2)(\frac{C}{A-C})(1-e^2)^{3/2}}{3\cos(\varepsilon)},$$

where $e$ is the eccentricity, $\varepsilon$ is the obliquity, $A$ and $C$ are the moments of inertia, and $P_{\text{orb}}$ and $P_{\text{spin}}$ are the orbital and spin periods, respectively.

Due to the uncertainty in Nereid’s rotational state, we have set up six combinations to study. These are listed in Table 4.2 where equation (4.1) has been used to calculate the precession period.

<table>
<thead>
<tr>
<th></th>
<th>Eccentricity</th>
<th>Orbit period (day)</th>
<th>Spin period (day)</th>
<th>Obliquity (degree)</th>
<th>A/C</th>
<th>Precession period (day)</th>
<th>Precession period (year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7512</td>
<td>360.13</td>
<td>0.48</td>
<td>30</td>
<td>2.4</td>
<td>42727.42</td>
<td>117.06</td>
</tr>
<tr>
<td>2</td>
<td>0.7512</td>
<td>360.13</td>
<td>0.48</td>
<td>20</td>
<td>2.4</td>
<td>39377.8</td>
<td>107.88</td>
</tr>
<tr>
<td>3</td>
<td>0.7512</td>
<td>360.13</td>
<td>3</td>
<td>30</td>
<td>2.4</td>
<td>6836.39</td>
<td>18.73</td>
</tr>
<tr>
<td>4</td>
<td>0.7512</td>
<td>360.13</td>
<td>3</td>
<td>20</td>
<td>2.4</td>
<td>6300.45</td>
<td>17.26</td>
</tr>
<tr>
<td>5</td>
<td>0.7512</td>
<td>360.13</td>
<td>0.48</td>
<td>30</td>
<td>3</td>
<td>29909.19</td>
<td>81.94</td>
</tr>
<tr>
<td>6</td>
<td>0.7512</td>
<td>360.13</td>
<td>0.48</td>
<td>20</td>
<td>3</td>
<td>27564.46</td>
<td>75.52</td>
</tr>
</tbody>
</table>

**Table 4.2:** Calculated precession period of Nereid

In our simulation, the precession rate as calculated in equation (4.1) is not used. Instead, the rotational motion is calculated subject to any torques, and the precession angle at any time is determined from the orientation of the body. It is important that our
simulations agree with the estimated precession rate. We set up six simulations for the conditions shown in Table 4.2 and integrated both the orbit and rotation of Nereid for 1000 years. These simulations include the gravitational effects of both Neptune and Triton.

Figure 4.4 shows the precession angle for case 1 in Table 4.2. The precession rate is not constant, but varies only slightly. An average over one cycle gives a rate of $116.5 \pm 0.5$ days in good agreement with equation (4.1).

![Figure 4.4: Nereid’s precession angle behavior for 1000 years, with spin period = 11.52 hours, initial obliquity = 30° and moment of inertia ratio A/C = 2.4.](image)

Table 4.3 shows the precession rate obtained by our simulation for the six cases in Table 4.2. All of them are in good agreement with those rates calculated with equation (4.1).
Table 4.3: Simulated precession period of Nereid

<table>
<thead>
<tr>
<th></th>
<th>Eccentricity</th>
<th>Orbit period (day)</th>
<th>Spin period (day)</th>
<th>Obliquity (degree)</th>
<th>A/C</th>
<th>Precession period (year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7512</td>
<td>360.13</td>
<td>0.48</td>
<td>30</td>
<td>2.4</td>
<td>116.5 ± 0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.7512</td>
<td>360.13</td>
<td>0.48</td>
<td>20</td>
<td>2.4</td>
<td>107.5 ± 0.5</td>
</tr>
<tr>
<td>3</td>
<td>0.7512</td>
<td>360.13</td>
<td>3</td>
<td>30</td>
<td>2.4</td>
<td>18.5 ± 0.5</td>
</tr>
<tr>
<td>4</td>
<td>0.7512</td>
<td>360.13</td>
<td>3</td>
<td>20</td>
<td>2.4</td>
<td>17.5 ± 0.5</td>
</tr>
<tr>
<td>5</td>
<td>0.7512</td>
<td>360.13</td>
<td>0.48</td>
<td>30</td>
<td>3</td>
<td>81.5 ± 0.5</td>
</tr>
<tr>
<td>6</td>
<td>0.7512</td>
<td>360.13</td>
<td>0.48</td>
<td>20</td>
<td>3</td>
<td>75.5 ± 0.5</td>
</tr>
</tbody>
</table>

4.4 Obliquity evolution

We saw in chapter 3 that solar system objects such as Ceres and Vesta have obliquity angles that evolve over time. This is caused by gravitational torques from other bodies. In the case of Ceres and Vesta, the dominant perturber is Jupiter. Nereid is subject to torques from Neptune and perhaps Triton. Both of these torques are highly variable because of Nereid’s orbital eccentricity and the relative motion of Nereid and Triton. In this section, we examine the possible evolution of Nereid’s obliquity.

Due to uncertainties in the physical properties of Nereid and its rotational state, we set up for four different simulations. Schaefer et al. (2008) argue that Nereid’s spin period should be between 11.52 hours and 3 days while its obliquity is completely unknown. Therefore, we chose initial obliquity angles of 20° and 30°, and our four simulations are for a fast rotator (spin period = 11.52 hours, obliquity = 20° and 30°) and a slow rotator (spin period = 3 days, obliquity = 20° and 30°). For these simulations, we fixed the moment of inertia ratio to be A/C = 2.4. Each case was run as a three-body simulation (Neptune, Nereid and Triton) for 100 years.

4.4.1 Actual orbit

First, we can estimate the relative sizes of the perturbations on Nereid caused by Neptune and Triton. We consider the orbital and rotational motion of Nereid in its original orbit. The torques on Nereid from Neptune and Triton are
where \( \tau \) is torque, \( M \) is mass and \( r \) is distance from Nereid.

Also, the change in Nereid’s angular momentum is

\[
\Delta L \approx \tau \Delta t ,
\]

where \( L \) is angular momentum, and \( \Delta t \) is the time over which the torque acts.

Then the perturbation ratio from Neptune and Triton on Nereid is

\[
\frac{\Delta L_{\text{Neptune}}}{\Delta L_{\text{Triton}}} = \left( \frac{M_{\text{Neptune}}}{M_{\text{Triton}}} \right) \left( \frac{r_{\text{Triton}}}{r_{\text{Neptune}}} \right)^3 \left( \frac{\Delta t_{\text{Neptune}}}{\Delta t_{\text{Triton}}} \right) .
\]

Since the time is related to the speeds of objects,

\[
\Delta t_{\text{Neptune}} \propto v_p ,
\]

\[
\Delta t_{\text{Triton}} \propto |v_p - v_T| ,
\]

where \( v_p \) is the peri-Neptune speed of Nereid and \( v_T \) is the speed of Triton.

As a result, the perturbation ratio from Neptune and Triton on Nereid can be calculated by

\[
\frac{\Delta L_{\text{Neptune}}}{\Delta L_{\text{Triton}}} = \left( \frac{M_{\text{Neptune}}}{M_{\text{Triton}}} \right) \left( \frac{r_{\text{Triton}}}{r_{\text{Neptune}}} \right)^3 \left( \frac{|v_p - v_T|}{v_p} \right) .
\]

Further, the velocity of Neptune and Triton may be calculated by

\[
v_p = 2.957 km/s ,
\]

\[
v_T = 4.387 km/s .
\]

By using the parameters of Neptune and Triton when Nereid is at peri-Neptune position and equation 4.7, the perturbation ratio from Neptune and Triton on Nereid is about 40. So we conclude that in the real condition, the perturbation from Triton on Nereid should be quite small compared to that from Neptune.

We made four different simulations for Nereid in its original highly eccentric orbit for 100 years, using the different initial conditions mentioned above.
Figure 4.5: Nereid’s obliquity and distance from Neptune for 10 years, with eccentricity = 0.7512, initial obliquity = 20°, and spin period = 11.52 hours (fast rotator). (a) 3-body system. (b) 2-body system.
Figure 4.6: Nereid’s obliquity and distance from Neptune for 10 years, with eccentricity = 0.7512, initial obliquity = 30°, and spin period = 11.52 hours (fast rotator).

Figure 4.5 shows our results in (a) 3-body system (Neptune, Nereid and Triton) and (b) 2-body system (Neptune and Triton). Both systems give the same obliquity evolutions. It confirms our estimated result that the perturbation on Nereid’s rotational motion from Triton is quite small and negligible compared to the perturbation from Neptune.

Figure 4.5 and 4.6 show the obliquity behavior of Nereid as a fast rotator with initial obliquities of 20° and 30° for 10 years, respectively. Since the eccentricity of Nereid is very irregular, the figures also show the periodic change of distance between Nereid and Neptune. In both conditions, obliquity changes in a very small range for less than 0.5°. It is clear that for each orbit period, the obliquity of Nereid changes quickly in about 0.5° at each peri-Neptune area and then becomes relatively stable for the rest time. Neptune gives Nereid a large torque when they are close to each other.
Nereid's Obliquity and Distance from Neptune

\[ e = 0.75, \text{ initial obliquity} = 20 \text{ degree, spin period} = 3 \text{ days} \]

(a)

(b)
Figure 4.7: Nereid’s obliquity and distance from Neptune, with eccentricity = 0.7512, initial obliquity = 20°, and spin period = 3 days (slow rotator). (a) Results for 100 years. (b) Results for 10 years. (c) Results for 1 year.
Figure 4.8: Nereid’s obliquity and distance from Neptune, with eccentricity = 0.7512, initial obliquity = 30°, and spin period = 3 days (slow rotator). (a) Results for 100 years. (b) Results for 10 years. (c) Results for 1 year.
Figures 4.7 and 4.8 show the obliquity behavior of Nereid as a slow rotator with initial obliquities of 20° and 30°, respectively, and the periodic change of distance between Nereid and Neptune. In both conditions, obliquity changes in a range for about 8°. At each peri-Neptune area, Neptune gives Nereid a large torque which makes the obliquity of Nereid change quickly for about 8°. For the slow rotator, the obliquity of Nereid still changes quickly in a relatively small range for about 5°. This does make sense because when Nereid rotates slowly, its motion is more easily affected by the environment than fast rotator conditions. In figure 4.7(a) and 4.8(a), the long time obliquity behavior also gives a clear periodic shape.

4.4.2 Circular orbit

To compare and check how the highly eccentric orbit affects Nereid’s rotational motion, we made four different simulations for Nereid in a circular orbit for 100 years, using different initial conditions mentioned above. We expect a quite stable rotational motion of Nereid in this ideal condition since the torques on Nereid should change only slightly during its orbit.

![Nereid's Obliquity and Distance from Neptune](image)

*Figure 4.9:* Nereid’s obliquity and distance from Neptune for 10 years, with eccentricity = 0, initial obliquity = 20°, and spin period = 11.52 hours (fast rotator).
Figure 4.9 displays the obliquity behavior of Nereid in an ideal circular orbit for 10 years, with eccentricity = 0, initial obliquity = 20°, and spin period = 11.52 hours (fast rotator). In this condition, Nereid’s obliquity is quite stable with a periodic change within 0.05°. For the other 3 cases, we got similar stable results for the evolution of Nereid’s obliquity. There is no big difference between the fast rotator and the slow rotator conditions except that the small wobbles in the slow rotator condition are somewhat larger (within 0.5°) than that in the fast rotator case.

4.4.3 Testing orbit

In addition to the ideal circular orbit, we set up another imaginary orbit with eccentricity of 0.4, which is neither highly eccentric nor near circular. We made four different simulations for Nereid in this relatively normal orbit for 100 years, using the different initial conditions as before. If the eccentricity plays an important role in the rotational motion of Nereid, this orbit should give obliquity results between the other two simulations above.

Figure 4.10: Nereid’s obliquity and distance from Neptune for 10 years, with eccentricity = 0.4, initial obliquity = 30°, and spin period = 11.52 hours (fast rotator).
Figure 4.10 shows the obliquity behavior of Nereid in an imaginary orbit with eccentricity of 0.4 for 10 years, beginning from initial obliquity = 30°, and spin period = 11.52 hours. For the other three cases, we get quite similar result to this figure. In all of the conditions, Nereid’s obliquity is changing within 1°. Similar to the original orbit, when Nereid is in the peri-Neptune area, its obliquity is strongly affected by Neptune. However, when we take Nereid as a slow rotator, the change of obliquity is much smaller and stable compared to the results from its original orbit.

4.5 Conclusion

In conclusion, our Neptune system simulation gives reasonable results for Nereid’s orbital and rotational motion. The Neptune system is an acceptable environment to investigate Nereid’s orbital and rotational behavior within a short time period. Also, the precession period results from our simulation agree with the results from the work of others. Then by using the same computational technique as investigating obliquities of Ceres and Vesta, our simulations predict the obliquity behavior of Nereid in different conditions. It shows that the high eccentricity of Nereid’s orbit plays a very important role in Nereid’s orbit and rotational motions. The results give us a direction of how to investigate those still unknown properties of Nereid in the future.
CHAPTER 5
CONCLUSION AND FUTURE WORK

5.1 Conclusion

For an N-body rotational dynamics system, our simulation works well to show either long term or short term behavior of a specific body. The HITS computational technique can efficiently integrate both the orbital and rotational equations.

The long term simulations of Ceres and Vesta display their orbital and rotational evolution. Their periodic obliquity behavior gives a simulated result for their long evolutionary histories. The DAWN mission will provide more accurate data on both the physical properties and the rotational states of Ceres and Vesta. These can be used to make better determination of their future rotational motion.

Also, since we still have limited knowledge about Nereid, our simulations agree with other results so far and predict its unknown motion. The irregular situation of Nereid gives it an interesting behavior. The high eccentricity of Nereid’s orbit makes a big difference during its motion. Our simulated results can be used to help later investigations of Nereid by using different methods.

5.2 Future work

Since our computational techniques work well for the simulations of Nereid, more work about this moon could be done later. First of all, a new complete system could be set for Nereid. In this work, we only considered its motion in the Neptune system. We could do a complete simulation for Nereid in the solar system with Neptune orbiting the Sun and Nereid orbiting Neptune. Some dynamics transformation may be done before setting the simulation in this condition.

Also, we have limited knowledge about Nereid so far. The real shape of Nereid is still unknown. In our simulation, we took Nereid as an axis symmetric body. However, we may need to consider the tri-axial situation for Nereid and do similar simulations. The difference in its shape may affect the perturbation on it and may give some different
results for Nereid’s rotational motion. If the tri-axial body simulation works, more simulations for other tri-axial bodies could also be done in the future.
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