ABSTRACT

CHARACTERIZATION OF ELECTROMAGNETIC INDUCTION DAMPER

By Willis O. Agutu

The usual magnetorheological (MR) fluid dampers get energy externally to effectively damp unwanted motion. Recent research focuses on self-powered systems that get their energy internally from the vibrations. We have characterized the electromagnetic induction (E.M.I.) damper by two methods. First, a magnet freely traverses a coil of wire like in an Atwood machine and secondly, a magnet is driven sinusoidally in a coil of wire placed on a force sensor. In the first method, we look into how best to model the conversion of mechanical energy into electrical energy. Secondly, damping force is measured in relation to induced voltage and linear velocity. Coefficient of damping of different coils is measured and compared with different wire thickness and magnet length to coil width ratio. An E.M.I damping model is introduced and used in both cases to explain the phenomena of E.M.I. damping.
CHARACTERIZATION OF ELECTROMAGNETIC INDUCTION DAMPER

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1 INTRODUCTION

Vibrations at times can be unwanted. It is well known that dampers are used to remove unwanted vibrations for example; an automobile has shock absorbers that ensure a smooth ride over rough road surfaces. To make shock absorbers work better, luxurious cars like Audi [1] have their magnetorheological shock absorber embedded with soft iron particles. When the electric current from the battery is applied, the iron particles get polarized in one direction resulting in higher viscosity hence more damping. When a coil of wire is fitted below the shock absorber and a magnet made to vibrate in the coil, the system can generate its own electric current to be fed into the damper thereby eliminating the need for a battery. The damping system then uses its own electrical energy which it gets form itself making it self energizing. This has been observed by Young-Tai Choi [2] where electrical energy to is harvested from the vibrations in the environment. The same principle has been observed in trucks where Yoshihiro Suda et al [2] have found that such self-powered active control systems is effective for heavy trucks. To maintain good energy supply, the self-energizing system should generate more energy than it consumes. Nakano, et al [3] confirms such a possibility where a single actuator realizes active control and energy regeneration. In June 2000, the millennium footbridge in London was closed due to hazardous deck motions causing resonance of the deck [5]. Part of the solution was to add fluid damping to the bridge. Probably, magnetorheological fluid dampers would do better. During the process of damping, some mechanical energy is always lost in form of heat energy. This paper looks into converting the energy being lost into electrical energy. Damper characteristics are to ensure that from the smallest available form of vibration, maximum possible electrical energy is derived. For example, a car can have four dampers each generating energy. The energy generated can be used stored and or used for other purposes like lighting, air conditioning or playing music hence reduced reliance on the car battery.

With our magnet made to vibrate in a coil, to generate electrical energy, there is need to investigate more about the coil’s characteristics such that from the coil available,
maximum electrical energy can be generated. We therefore start by characterizing an electromagnetic induction damper. This is done in two parts: first, part deals with energy conversion. The second part characterizes damping force and maximizes power.
1.1 Background

1.1.1. Conventional Dampers

Almost all oscillating systems experience forces that tend to remove energy from them. For example, a pendulum is observed to slow down in amplitude with time. The pendulum experiences drag force that dissipates energy away from it. The motion is therefore said to be damped. Another example where damped motion exists includes shock absorber of an automobile. It ensures that the automobile undergoes a smooth ride over rough road surfaces. [6] A damper can therefore be defined as a device that decreases the amplitude oscillations of a system by ensuring that the energy of oscillation dissipates away. This research focuses on mechanical dampers of which several commonly used examples are shown below:

![Figure 1.1 Mechanical dampers used in automobile](image)

Figure 1.1 shows a damper commonly used in automobiles to give a smooth ride on bumpy road surfaces.

In general, mechanical dampers exhibit damped oscillator motion of the type

\[ X(t) = A(t) \cos \omega t \]

where \( t \) is time and \( \omega \) is the frequency and \( A \) is the amplitude of oscillation. The time dependent amplitude is given as

\[ A(t) = A_0 e^{-bt/2m} \]  \hspace{1cm} (1.1)

Where

- \( A(t) \) is amplitude of oscillation.
- \( b \) is damping constant
- \( t \) is time

---

- $m$ is mass of the oscillating object
- $A_o$ is the initial amplitude of oscillation

Initially, at time $t = 0$, the amplitude $A(t)$ is maximum, $A_o$. As $t$ increases, the amplitude $A(t)$ decreases exponentially. When $b = 0$, energy is conserved, hence no damping. The bigger the damping constant $b$, the faster the oscillation is damped [2].

### 1.1.2 Magnetorheological (M.R.) fluid dampers

M.R. fluid damper is by made by dispersing a soft magnetic material like iron powder in the oil used in the usual shock absorber [1]. Figure 1.2 below shows a cross-section of the usual shock absorber containing oil with soft iron particles embedded in it.

![Iron powder dispersed in oil](image1.jpg)

Figure 1.2: Iron powder dispersed in oil

The purpose of embedding iron particle in the oil is to vary the oil viscosity to control damping force [1]. The oil in the shock absorber has particles a few microns in size which get polarized when magnetic field is applied. Figure 1.3 below shows a sequential order of the iron powder in an MR-fluid damper when electric field is applied.

![Iron powder polarized in MR-fluid damper](image2.jpg)

Figure 1.3: Iron powder polarized in an MR-fluid damper.
From Figure 1.3 above, the oil viscosity changes depending on the strength of the magnetic field applied resulting in more damping. This principle has been applied in luxury cars like Audi A8, Lancia Thesis and Opel /Vauxhall Astra [1]. The source of electrical energy is the car battery.

In this paper, we do not want to rely on the battery to power the MR-fluid damper. Instead, the paper looks into ways of generating electrical energy from vibrations of the shock absorber when a car is in motion and feed the energy into the damper. A magnet is allowed to vibrate in a coil of wire fitted below the shock absorber. During the vibration, electrical energy gets generated then fed into the MR-fluid damper. The system therefore is no longer getting its energy from the battery but instead from the vibrations. Figure 1.4 below shows a schematic diagram of an electromagnetic induction (E.M.I) coil used in the characterization process. A coil of wire was wound on a polyvinyl chloride (P.V.C.) cylinder inside which a magnet is made to vibrate.

![Figure 1.4: Schematic diagram of an E.M.I coil](image)

### 1.2 Theory

#### 1.2.1 Induced Voltage

It is well known that when relative motion exists between a conductor and a magnet then voltage is induced and hence an electric current exists along the conductor. This is according Faraday’s law of electromagnetic induction which gives induced voltage as

$$ V_{\text{induced}} = -N \frac{d\phi}{dt} \quad 1.2 $$

Where N is the number of turns in the coil and \( \phi \) is the magnetic flux. The negative sign in equation 1.2 represents Lenz’s law which gives the direction of induced current. The induced current at the same time is given from Ohm’s law as
\[ I_{\text{induced}} = \frac{V_{\text{induced}}}{R} \]  
1.3

Where \( R \) is the resistance of the circuit.

It is clear from equation 1.3 above that a smaller circuit resistance results in a bigger induced electric current to generate a bigger magnetic field to increase the fluid’s viscosity. Also to be considered is the wire thickness. It is well known that

\[ R = \frac{\rho l}{A} \]  
1.4

Where
- \( R \) is the resistance of the wire.
- \( A \) is cross sectional area of the wire (\( A = \pi r^2 \))
- \( \rho \) is the resistivity of the wire.
- \( l \) is the length of the wire.

From equation 1.3 and 1.4 one can write

\[ I = \frac{V\pi}{\rho l} r^2 \]  
1.5

From equation 1.5, it is observed that for a given voltage, induced current is directly proportional to cross sectional area or thickness of the wire. So thicker wires should be suitable for bigger induced current. The experiments in section 2 deal with the wire thickness.

During the relative motion between the magnet and the coil, induced current exists and charges begin to flow along the coil. This current has an associated magnetism within itself which interacts with the changing magnetic field gradient of the magnet to cause resistance the relative motion. As a result, the magnet is damped. This is electromagnetic induction (E.M.I) damping. At this point the magnet’s acceleration results in change of kinetic energy as well.

1.2.2. Energy

Electromagnetic induction damping process in this paper involves a magnet in motion with a coil in fixed position. There are two cases where E.M.I. damping is investigated in this paper. In the first case, the magnet is let to traverse freely a coil of \( N \) turns placed with its axis of symmetry vertical. The kinetic energy due to the magnet’s motion gets converted into electrical energy. In the second case, a magnet is put to vibrate horizontally in a coil placed with its axis of symmetry horizontal on a force sensor. As
the magnet vibrates at fixed frequencies, it induces current in the coil. The induced current has its associated magnetism which interacts with the magnetic field gradient to damp the magnet motion. The frequencies are adjusted and then damping force is studied with velocity, and wire size configuration.

To start with the first case, a magnet is let to freely traverse a coil and then voltage is induced in the coil hence electrical energy is generated. The magnet in motion has kinetic energy given as

\[ K.E. = \frac{1}{2}mv^2 \] \hspace{1cm} 1.6

Where \( m \) is the mass of magnet and \( v \) is its velocity. The electrical energy generated is given as

\[ Energy = R \int I^2 dt \] \hspace{1cm} 1.7

The set up in Figure 2.1 was used to investigate magnitudes of kinetic energy converted to electrical energy at different situations. The sketch in Figure 1.5 shows a graph of kinetic energy (K.E.) with respect to displacement for a magnet that moves with a constant velocity traversing a coil of wire.

![Figure 1.5: Magnet at constant velocity interacting with the coil.](image)

It can be seen that due to the magnet’s interaction with the coil, there is a drop in the kinetic energy, which Fox and Reiber [12] observe as heat loss due to joule heating effect. If the magnet’s length is greater than the coil width, the profile in Figure 1.5 above changes as the one shown in Figure 1.6 below.
In the above figure, initial kinetic energy, $K.E_i$, is constant before the magnet interacts with the coil and becomes constant later, $K.E_f$, after the interaction. The glitch in Figure 1.6 above represents the time in which the magnet is at the center of the coil. Just before the magnet reaches the center of the coil, induced current in the coil has an associated magnetism. According to Lenz’s law, this current opposes the flux change which causes it to set a magnetic field which interacts with the changing magnetic field gradient of the upper pole of the magnet to repel the magnet. The magnet’s velocity is reduced [10]. When the magnet reaches the center of the coil, the induced current approaches zero. The coil’s interaction with the magnet is kept minimum but only for a very short time. As the magnet emerges from the coil, induced current begins to increase but now in the opposite direction. Again by Lenz’s law, the magnetism associated with the induced current begins to oppose the change causing it thereby attracting the magnet. Again the magnet reduces in velocity. After sometime the magnet moves off the coil with constant velocity and kinetic energy again becomes constant. The situation presented in Figure 1.6 above is where the magnet is not subject to gravitational force. We assume it moves at constant velocity which is why its kinetic energy is initially constant before interacting with the magnet and again remains constant after interacting with the magnet. We now
look at a situation where a magnet is undergoing free fall. As it moves towards the coil, its kinetic energy versus position would resemble the sketch in Figure 1.7 below. Due to the its mass $m$ at a height $h$ and velocity $v$, its kinetic energy would be equated to potential energy as shown below for sections OA and DE in the figure.

\[
\frac{1}{2}mv^2 = mgh \quad \text{and hence} \quad \frac{1}{2}v^2 = gh
\]

![Figure 1.7: Magnet’s encounter with the coil](image)

From: - O to A, the magnet is moving toward the coil.
- A to B, the magnet is entering into the coil.
- B to C, the magnet is in the center of the coil.
- C to D, the magnet is leaving the coil.
- D to E, the magnet is moving very far away from the coil.

Our system is an Atwood type of set up as shown in Figure 2.1 where there is an identical piece of bronze connected to the magnet by an inextensible string via a pulley. This is similar to the set used by Lowell T. Wood [10] in the investigation of conservation of energy. Equation 1.8 changes as

\[
v^2 = 2as
\]

Where $a$ is acceleration and $s$ is displacement. In this set up we now let mass of the magnet be $m_2$, mass of the identical piece of bronze be $m_1$. From equation 1.9, a graph of the square the square of velocity versus displacement $s$ would be a straight line whose
slope is $2a$ resembling part OA before the magnet interacts with the coil and part DE after the magnet interacts with the coil. From point O just before the magnet reaches point A, the system experiences only force of gravity $F_G$ given by

$$F_G = (m_1 - m_2)g$$  \hspace{1cm} 1.10

By applying the work-energy theorem, the net work done is seen as a change in the kinetic energy. So we have

$$\int F_{net}(y)dy = K.E_f - K.E_i$$  \hspace{1cm} 1.11

Where $K.E_f$ is final kinetic energy of the magnet and $K.E_i$ is the initial kinetic energy which is zero because the magnet starts to move from rest. As the magnet interacts with the coil, the system now experiences another force due to the coil’s interaction. Equating the forces before and magnet’s interaction gives

$$\int[(m_1 - m_2)g + F_M(y)]dy = \frac{1}{2}(m_1 + m_2)[V_f^2 - V_i^2]$$  \hspace{1cm} 1.12

But the initial velocity $V_i^2$ is zero. So equation 1.12 changes to

$$(m_1 - m_2)gy + \int F_M(y)dy = \frac{1}{2}(m_1 + m_2)[V_f^2]$$  \hspace{1cm} 1.13

So work done by the magnetic force is given from equation 1.13 as

$$\int F_M(y)dy = \frac{1}{2}(m_1 + m_2)V_f^2 - (m_1 - m_2)gy$$  \hspace{1cm} 1.14

Let us call the part OA before the coil interacts with the magnet as open circuit where there is no magnetic force. Equation 1.14 becomes

$$\frac{1}{2}(m_1 + m_2)V_o^2 = (m_1 - m_2)gy$$  \hspace{1cm} 1.15

After the magnet interacts with the coil there is magnetic force and we now call it closed circuit. Equation 1.14 becomes

$$\int F_M(y)dy = \frac{1}{2}(m_1 + m_2)V_c^2 + (m_1 - m_2)gy$$  \hspace{1cm} 1.16

In the regions where magnetic force $F_M \to 0$, substitute equation 1.15 into 1.16 to get

$$\int F_M(y)dy = \frac{1}{2}(m_1 + m_2)[V_c^2 - V_o^2]$$  \hspace{1cm} 1.17
Equation 1.17 is a measure of kinetic energy change and hence the magnetic damping as the magnet interacts with the coil.

![Figure 1.8: Square of velocity versus displacement](image)

Part OA extrapolated to H represents open circuit. DE represents closed circuit. For the measurement of change in kinetic energy, every point on the displacement axis has two image points on the square of velocity axis. For example, a point d on the displacement axis above has its $V^2_o = G$ and its $V^2_c = H$ on the square of velocity axis. So kinetic energy change can be averaged over several points along the position axis. The average is then substituted into equation 1.17 to get the change in kinetic energy which is understood as energy lost into electrical energy. The converted electrical energy is to be found from the induced current versus time graph as shown in part (i) of Figure 1.9. In both cases as shown in Figure 1.6 and Figure 1.9 below, electrical energy is to be measured simultaneously as kinetic energy changes. This is given as

$$\Delta E = \int P dt = RI^2 dt$$  \hspace{1cm} 1.18

Where P is power, R is resistance and i is the current. For the induced current against time, the curve sketch in Figure 1.9 part (i) is expected and for calculations of power, the curve sketch in part (ii) is expected for the square of induced current versus time graph.
As the magnet goes through the coil, change in energy is proportional to the area under the $i^2$ versus time curve. This area representing electrical energy generated is to be compared with the change in kinetic energy as given by equation 1.17. To summarize our expectation in vibration energy harvesting, we expect results from equations 1.17 and 1.18 to be equal in magnitude.

### 1.2.3. Other factors
From the knowledge of basic electricity and magnetism, the following factors will also be considered.

- a. – ratio of magnet length to coil width
- b. – thickness of the wire
- c. - presence of external resistor

#### 1.2.3.1. Ratio of magnet length to coil width
Generally, when a magnet is in relative motion inside a solenoid of infinite length, there is no induced voltage.

![Diagram of magnet inside coil](image)
This is because when a magnet moves back and forth inside a long solenoid, the parts of the solenoid near the magnet marked as C and D above, induce voltages that are opposite in sign thereby canceling. When the solenoid length is comparable to magnet length, all the parts of the solenoid interact with the magnet at the same time ending up with a net voltage. The same observation has been made in [10] where Lowell T. Wood explains the situation that flux change is too small when a magnet is in a very long solenoid. We will investigate the above effect of the ratio of magnet length to coil width on induced current and consequently the percentage of energy converted. The below figure illustrates coil length and coil width.

![Figure 1. 11: Magnet length and Coil width](image)

### 1.2.3.2. Thickness of the wire.

From Ohm’s law in equations 1.4 and 1.5 shown earlier, it is correct to write

\[ I = \frac{V \pi r^2}{\rho l} \]  

Where \( I \) is induced current, \( r \) is the radius of the wire, \( V \) is voltage, \( l \) is the length and \( \rho \) is the resistivity of the conductor. With other factors constant, we expect the induced current to be proportional to the wire radius. We therefore expect thicker wires to induce more current and hence convert more energy than thinner wires.

### 1.2.3.3. External resistor

From Ohm’s law, resistance is inversely proportional to current in a circuit. So we expect that the presence of external resistor in a circuit to reduce induced current. Because E.M.I
damping depends on the induced current, we further expect less energy change as external resistance increases.

### 1.3 Damping Force

From our expectation of force versus displacement as in Figure 1.12, it is observed that force to increases and decreases as the magnet changes position. From Figure 1.6, change in energy is negative hence energy is being lost. The force acting to remove this energy can be obtained from the partial negative derivative of energy with respect to displacement as shown below:

\[ F = -\frac{\partial U}{\partial X} \] 1.20

Where \( F \) is force, \( U \) is energy, \( X \) is displacement. We now expect force versus displacement profile also to change as shown below in Figure 1.12.

![Figure 1.12: Energy profile on displacement domain](image)

The upper part of Figure 1.12 is from Figure 1.6. The lower part shows how damping force changes with position as the magnet’s kinetic energy also changes with position. When the magnet undergoes kinetic energy change as has been shown above, it experiences a
force that retards its motion and hence electromagnetic induction damping. We will show the fundamental origins of this force.

1.4. Damping force model

From electricity and magnetism, a conductor carrying electric current in a magnetic field experiences a force \( \vec{F} \) given by

\[
\vec{F} = I \vec{l} \times \vec{B}
\]

\( \vec{F} \) is the force on conductor of length \( \vec{l} \) carrying current \( I \) in a magnetic field \( \vec{B} \). It has been observed here that as the magnet approaches a coil (conductor), it experiences force and gets damped. It is necessary to understand this force.

Saslow [8] develops a treatment for determining damping force on a conducting loop in a flaring magnetic field. Consider a flaring magnetic field at an angle \( \theta \) and a single conducting loop as shown below. We assume the field from a monopole for simplicity.

![Diagram of a conducting loop in a flaring magnetic field](image)

If \( \theta \) is the flaring angle of the field, \( (dF) \) is the force on one element \( (dS) \) in a field \( B \).

The component of \( (dF_i) \) a long the x-axis is given as:

\[
(dF_i)_x = I(dS_i)BSin\theta
\]

Summing up along the x-axis gives equation 1.22 as

\[
F_x = \sum_i (dF_i)_x
\]
\[ F_{\text{loop}} = I(2\pi a)BSin\theta \]  

1. 23

Where “a” is the coil radius and $\theta$ is the flaring angle.

Equation 1.23 represents force on a single loop or turn of wire in a flaring magnetic field. However, for N turns of a coil, the total sum of the force is given as

\[ F = NI(2\pi a)BSin\theta \]  

1. 24

The force varies with $x$ because it comes from the interaction between the induced current in the coil and the changing magnetic field gradient of the permanent magnet. The $x$-displacement is changing and that means the equation 1.24 changes as follows:

\[ F(x) = N2\pi aI(x)B_{\text{rad}}(x) \]  

1. 25

One would expect the force in equation 1.24 to be, maximum at $x = 0$ since the radial magnetic field is greatest there. At the same time induced current in the loop is expected to be approaching zero then switch its sign. Since magnetic field B will be switching sign as well, both current and magnetic field are negative or positive at the same time. In essence, force which is now a product of either two negative values or two positive values ends up positive throughout. Therefore, the force versus displacement graph takes the shape shown below.

![Figure 1. 14: Sketch of force versus displacement](image)

The two peaks above correspond to the peaks expected in the induced current versus time graph. That means equation 1.24 has a maximum value given by
Equation 1.26 is the maximum force experienced by a coil of wire of N turns in a flaring magnetic field as a magnet approaches a coil of wire. By substituting current, I from equation 1.3, equation 1.26 changes as follows:

\[ F_{\text{PEAK}} = \frac{2\pi aNB}{R} \]

\[ = 2\pi a \frac{NB_{\text{PEAK}}}{R} \left( N \frac{d\phi}{dt} \right) \]

\[ = 2\pi a \frac{N^2 B_{\text{PEAK}}}{R} \left( \frac{d\phi}{dx} \right) \left( \frac{dx}{dt} \right) \]

Equation 1.27 has a coefficient of velocity term on the right hand side. So letting the coefficient to be a constant makes the equation become

\[ F_{\text{PEAK}} = cv \]

Where \( v \) is velocity and \( c \) is a constant of proportionality between maximum force and velocity. The constant \( c \) is derived from Saslow’s equation 1.27. While force \( F \) and velocity \( v \) are used extensively in most Physics and Engineering text books, it is often phenomenologically based. We therefore expect a linear relation between damping force and velocity when a magnet is put to vibrate at fixed frequencies in a coil of wire.

**1.5 Forced Motion**

Various frequencies are to be chosen by adjusting the motor’s revolutions per minute. The coils are to be placed on a force sensor to measure damping force. Damping force is to be studied at different frequencies or velocities. However, before we look at forced motion deeply, we need to study an E.M.I model. The model which comes from Saslow Wayne [8] is to help us understand the phenomena of E.M.I in both free motion case and forced damping case.

**1.5.1. Damping force versus velocity**

The E.M.I coil is to be placed with its axis of rotation horizontal. Just like in section 1.2.2, the magnet is expected to be damped; the same is expected in this case. Also, as the frequency of vibration increases, the magnet is expected to be damped higher because
velocity and frequency are directly proportional as shown below by equations 1.29, 1.30, and 1.31 below. Induced voltage and velocity are directly proportional since at higher magnet velocity time rate change of flux also goes higher. It is known that

\[ v = 2\pi rf \]  \hspace{1cm} (1.29)

Where \( v \) - is magnet’s linear velocity
\( r \) - is the crank radius
\( f \) - is the frequency of vibration

\[ F = c(2\pi rf) \]  \hspace{1cm} (1.30)

Where \( F \) is damping force. From equation 1.30, it is expected that a plot of damping force versus frequency gives a constant number. The constant number is expected to be unique to every coil.

### 1.5.2. Damping force and power

The double and triple run coils as shown in Figure 2. are expected to give different induced currents and voltages depending on whether they connected in either parallel or in series. It is expected that parallel connections give less resistance to induced current as shown. Let us examine power output

\[ P = i^2 R_{eq} = \frac{V^2}{R_{eq}} = \frac{N^2 \left( \frac{d\phi}{dt} \right)^2}{R_{eq}} \]  \hspace{1cm} (1.31)

Where \( P \) is power, \( i \) is current, \( R \) is the equivalent resistance, \( N \) is number of turns, \( \rho \) is the resistivity of the wire. Also we know from Physics that three coils of equal turns \( N \) give effective turn as

**Series** \( N_{3s} = 3N_1 \) and for parallel \( N_{3p} = N_1 \)  \hspace{1cm} (1.32)

Based on the above, power in parallel and series become:

\[ P_{\text{series}} = \frac{N_{3s}^2}{R_{eq}} \left( \frac{d\phi}{dt} \right)^2 = \frac{3 \left( N^2 \frac{d\phi}{dt} \right)^2}{3R} = 3P \]  \hspace{1cm} (1.33)

And \[ P_{\text{parallel}} = \frac{N_{1}^2}{R_{eq}} \left( \frac{d\phi}{dt} \right)^2 = \frac{3 \left( N^2 \frac{d\phi}{dt} \right)^2}{3} = 3P \]  \hspace{1cm} (1.34)
Power P in equations 1.34 and 1.35 are from equation 1.32. Both equations indicate that power generated by the coils on either parallel or series connections are equal. We will investigate this.

1.5.3. Damping force and wire thickness

From equation 1.4 and 1.5, \( I = \left( \frac{V}{\rho l} \right) r^2 \) where \( r \) is wire radius. It is expected that thicker wires induce more current than thinner wires. Since E.M.I. damping comes from the interaction between magnetic field gradient and magnetism associated with induced current, it is expected that thicker wires to have more damping force because they induce more current than thinner wires.

1.6 Forced Motion – (M.T.S. Machine)

The previous section deals with the magnet driven at fixed frequencies in the coils. It is expected that the same coils give the same result when installed with a prototype E.M.I. damper and a large force equivalent to weight of a car is applied. Due to the damping force expected by equation 1.26 and Figure 1, we expect to a plot of damping force versus magnet’s displacement as shown below when the coils are installed onto the materials testing (M.T.S) machine. This is similar to the observation made by B. F. Spencer Jr. et al in [14].
We expect magnet’s displacement to be highest when the damper is powered by an external power source and lowest when the coil is let open. When the coil is short, we expect the damper force to be in between the open case and the externally powered case. The line marked A from the displacement axis in Figure 1.15 shows the expected force when the coil is in off-state. This is open circuit case. The line marked B represents expected force when the coil is in short circuit and the line marked C represents the expected force when we use external power supply. We should note that the force corresponding to the line marked C is just to guide us. Our centre of focus in this thesis is the difference in force between the line marked B and the line marked C. Other details will be discussed at the end of the thesis.
2 EXPERIMENTAL DETAILS

2.1 Coils used
From the previous chapter, we have seen how E.M.I damping results in a change of kinetic energy which ends in electrical energy. To begin with, we start with the change in kinetic energy into electrical energy by free fall motion. The table below gives a summary of coil parameters used.

<table>
<thead>
<tr>
<th>Coil label</th>
<th>Description /Type</th>
<th>Wire radius, r (mm) ±0.001</th>
<th>Cross-sectional area. (mm²) (4d.c.)</th>
<th>Coil Radius (average) ±0.5mm</th>
<th>Resistance ±0.1(Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 200G22sw single</td>
<td>0.345</td>
<td>0.3217</td>
<td>20</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>1 x 200G22Lw “</td>
<td>0.345</td>
<td>0.3217</td>
<td>22</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>2 x 200G22s double</td>
<td>0.345</td>
<td>0.3217</td>
<td>20</td>
<td>3.6</td>
<td></td>
</tr>
<tr>
<td>“ p</td>
<td>0.345</td>
<td>0.3217</td>
<td>20</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>3 x 200G22s triple</td>
<td>0.345</td>
<td>0.3217</td>
<td>22</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>3 x 200G22p “</td>
<td>0.345</td>
<td>0.3217</td>
<td>22</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>1 x 200G30sw single</td>
<td>0.135</td>
<td>0.0380</td>
<td>25</td>
<td>8.7</td>
<td></td>
</tr>
<tr>
<td>“ Lw “</td>
<td>0.135</td>
<td>0.0380</td>
<td>17</td>
<td>8.5</td>
<td></td>
</tr>
<tr>
<td>2 x 200G30s double</td>
<td>0.135</td>
<td>0.0380</td>
<td>18</td>
<td>17.4</td>
<td></td>
</tr>
<tr>
<td>“ p double</td>
<td>0.135</td>
<td>0.0380</td>
<td>18</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td>3 x 200G30s triple</td>
<td>0.135</td>
<td>0.0380</td>
<td>18</td>
<td>25.5</td>
<td></td>
</tr>
<tr>
<td>“ p triple</td>
<td>0.135</td>
<td>0.0380</td>
<td>18</td>
<td>3.1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Energy conversion</th>
<th>Width (±0.5mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G22Lw single</td>
<td>25</td>
</tr>
<tr>
<td>G22sw “</td>
<td>11</td>
</tr>
<tr>
<td>G22ssw “</td>
<td>8</td>
</tr>
<tr>
<td>G26 “</td>
<td>8</td>
</tr>
<tr>
<td>G30 “</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 2. 1: Parameters of the coils used.
**Experiment 2.1 – Energy conversion.**

A cylindrical neodymium magnet of length 26mm, internal diameter 13mm, external diameter 26mm and mass 89g was used. An identical non-magnetic piece of bronze of mass of 91g was used to form an Atwood machine set up. The two pieces were designated as $m_2$ for the magnet and the piece of bronze solid as $m_1$. Both were connected by an inextensible thread going round a pulley forming an Atwood machine set up as shown below. The magnet was let to go through each coil of 200 turns. The coil is made of copper wire wound on a P.V.C. The pulley used was a Pasco Rotary Motion Sensor (PS-2120). It measures magnet’s placement and velocity with time and position. Induced current is measured via Passport voltage-current sensor(PS-2115). Both the voltage-current sensor and rotary motion sensor were connected to computer interface. The data acquisition system (DAQ) used was Data studio 1.9.7r12. Together with this DAQ software, the following could be measured at the same time for one run:

- Induced current with respect to time and magnet’s displacement.
- Magnet’s velocity with respect to displacement and time.

![Schematic diagram](Image)

1. Figure 2.1 Schematic diagram for the set up with a freely moving magnet

Induced current was collected with the coils shorted to the ammeter. The table below gives a summary of the E.M.I. coil parameters used.

---

1 Details of energy conversion are in appendix A.
<table>
<thead>
<tr>
<th><strong>Coil label</strong></th>
<th><strong>Coil width ± 0.5(mm)</strong></th>
<th><strong>Wire thickness ± 0.001(mm)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>22Lw</td>
<td>25</td>
<td>0.345</td>
</tr>
<tr>
<td>22sw</td>
<td>11</td>
<td>0.345</td>
</tr>
<tr>
<td>22ssw</td>
<td>8</td>
<td>0.345</td>
</tr>
<tr>
<td>30ssw</td>
<td>8</td>
<td>0.135</td>
</tr>
</tbody>
</table>

Table 2.2: Summary of coil parameters.

Shorting the coils to the ammeter was geared towards getting the coil that converts most percentage of the lost kinetic energy into electrical energy. The other data sets that follow were considering other factors like the effect of coil width and wire thickness on converting kinetic energy to into electrical energy.

**2.1.1. Other factors**

In the previous work [10] uses only one type of wire. We investigated other factors to investigate their effect on the percentage of energy converted. The factors investigated include:

a) External load resistor.

b) Ratio of magnet length to coil width.

c) Thickness of the wire.

**2.1.1.1. External resistor**

By considering coil resistance $R_C = (2.8\pm 0.1\Omega)$ different resistances were selected as follows:

a) $R_L = 2.8\Omega \ (R_L = R_C)$

b) $R_L = 10\ \Omega \ (R_L > R_C)$

External resistances of $(2.8\pm0.1)\ \Omega$ and $(10\pm0.1)\ \Omega$ were connected at different times in series with the coil of width $11\text{mm}$ and wire radius $0.345\text{mm}$. The magnet was let to traverse just like it was in the first case for short circuit. The same procedure for experiment in section 2.1 was used to arrive at the results in section 3.1.
2.1.1.2. Ratio of magnet length to coil width
Two coils of widths 25±0.5mm and 11±0.5mm had their readings taken at different times with the same procedure as in the first case. Both were short through the ammeter.

2.1.1.3. Wire thickness
Two coils of wire radii 0.345±0.001mm and 0.135±0.001mm had their readings taken when short through the ammeter.

In all of the above investigations except in 2.1.1(b) for 10Ω external resistor, measurements were done thrice then averaged.

2.2 Damping Force measurement
Figure 2. 2 below shows an experimental set up that was used for the investigation of damping force. E.M.I. damping was investigated and quantified in terms of:

(i) wire thickness
(ii) ratio of magnet length to coil width and
(iii) coil connection types.

We designed different types of E.M.I. coils as shown in Figure 2. 2 below. They can be described as triple and double run coils respectively. The schematic diagram previously seen in Figure1.4 refers to single run.

![Image of E.M.I coil set ups used](image)

Figure 2. 2: E.M.I coil set ups used

During the investigation, damping force was measured against velocity for different coils.
Experiment 2.2
The magnet attached to an adjustable motor was set to vibrate back and forth inside an E.M.I. coil placed with its axis of symmetry vertical on a Pasco Force sensor (CI-6537). Force was collected via a Pasco analog adapter (PS-2158). Induced current was collected from the pick coil via a Pasco voltage-current sensor (PS-2115) to the DAQ. Figure 2.3 below shows a circuit diagram used and Figure 2.4 shows a picture of the same setup.

\[ \text{Figure 2.3: Schematic diagram for investigating damping force.} \]

The Multimeter (Fluke 76) was used for setting up frequency of the motor by connecting it to the pick up coil to read the frequency of the output signal from the coil. The motor was adjusted for various integral frequencies from 2 Hz to 8 Hz. For each frequency, damping force and induced current was measured simultaneously for each E.M.I. coil.

\[ 1 \text{ Other details of Experiment 2.2 are in Appendix B.} \]
Coils were grouped into two major categories: Single run coils in the first and then double and triple runs in the other. Several coils as shown in Table 3.1 were investigated.

### 2.3 Damping Force Model

The experimental setup in Figure 2.1 was modified to take care of experimental verification of E.M.I damping force.

**Experiment 2.3**

The whole set of apparatus from experiment 2.1 is brought to this experiment. A new coil of 100 turns on a light P.V.C. was used unlike other coils in experiment 2.1 which had very heavy coils. This experiment has a Hall probe to measure the radial magnetic field as the magnet moves upwards. The right hand side of equation 1.26 is now completely measured. The second modification as shown in Figure 2.5 is a P.V.C. tube to constrain the magnet in one path as it moves upwards. This is because from equation 1.26, the radial magnetic field varies strongly with the Hall probe position. There is need to ensure that the magnet stays in one vertical position through out. So that means that the radial field has to be averaged over thickness of the coil. To experimentally complete equation
1.26, the strain gauge comes in to measure force due to E.M.I. damping. The strain gauge used is resistance type strain gauge. As can be seen of Figure 2.5, the coil rests between support A on one end and the strain gauge on the other. It moves in the direction Y shown in response to damping force in equation 1.26.

Figure 2.5: Schematic diagram for Saslow’s force measurement

The magnet was initially let to rise in order to get the range of force in which to calibrate the strain gauge. This was done by letting the magnet run then the current and magnetic flux densities measured directly are substituted in equation 1.26. The resistive strain gauge measures voltage but we want force. Several small pieces of metal whose weight is less than the maximum force calculated were gently dropped on the gauge. For each metal dropped, an equivalent voltage corresponding to the weight of the metal was recorded as show in Table 4.1. From the table force was plotted against voltage. The best

1 Details of the measurement are in appendix C
line of fitting curve which converts voltage signals from the strain gauge into force was then applied to our readings of the strain gauge.

During the time damping force was measured, we also considered induced voltage for the same coil set ups when a large force equivalent to weight of a car is applied. This was done by using a materials testing system (M.T.S) machine in the Mechanical engineering laboratory. As it will be seen in section 3.2, different coil characteristics had different observations.

2.4 **Forced Motion - driven magnet (M.T.S. Machine)**

The materials testing system (M.T.S) machine uses Testware SX data acquisition software different from DAQ used in the previous experiments. A prototype of an MR-damper was mounted on M.T.S machine and E.M.I coils fitted below it as shown below.

![Figure2.6: Single large width E.M.I coil installed](image)

![Figure2.7: EMI setup with triple coil installed](image)

Figure2.6 and Figure2.7 above show two different types of E.M.I. coils installed on to an M.T.S machine. Part of the study in this paper is energy maximization; different types of
E.M.I. coils were installed. The induced voltage was studied with respect to the following factors.
   (i) coil widths
   (ii) connection types.
   (iii) wire thicknesses.
Damping force was studied at three stages:
   a) Off–state
   b) E.M.I. coil short
   c) External power.
In (a), each coil was left open and the magnet set to vibrate. Data was recorded for force versus displacement. In (b), the coil was short. Likewise, force and displacement data was recorded. This is when eddy current damping is maximum. In (c), external power supply was used to supply a D.C. voltage of 4.0 V. At the same time electric current was recorded as 0.2 A.
The machine was set to vibrate from frequencies of 0.5 Hz to 2 Hz due to mechanical limitations. The results are discussed in section 3.3.
3 RESULTS AND DISCUSSION

3.1 Energy Conversion

It was observed that as the magnet rises through the coil, it drags at a point below and above the coil. This confirms observation made by R. Kingman et al [12] that maximum induced voltage occurs at half the radius of the coil both below and above the coil. When in the middle of the coil the magnet tends to restore its previous motion. The graph shown in Figure 3.1 (coil labeled 22sw) below represents typical results from one of the coils. It is similar to our expectation as in Figure 1.7 and Figure 1.8.

![Square of Linear Velocity on displacement](image)

Figure 3.1: Square of Linear velocity against displacement

Part (A) – The magnet reaches the lower characteristic point. The repulsive force due to an associated magnetism with induced current in the coil and the changing magnetic field gradient of the magnet reaches maximum. The rising magnet decreases in acceleration.

Part (C) – The magnet reaches the middle of the coil where it undergoes free fall type of motion but only for a very short time just before the magnet reaches the upper characteristic point.

Part (B) – Is the upper characteristic point where the magnet experiences a force of attraction due to associated magnetism of the induced current in the coil and the changing magnetic field gradient.
The kinetic energy change is then found by substituting into equation 1.17 to give:

\[ \Delta K.E. = \frac{1}{2} (m_1 + m_2) \times \sum \frac{1}{n} \left( V_c^2 - V_o^2 \right) \]

\[ = \frac{1}{2} (0.091\text{kg} + 0.089\text{kg}) \times (0.02907) \text{ J} \]

\[ = 2.61 \text{ millijoules of energy} \]

Where \( n = 110 \), is the number of points averaged along the position axis. The converted electrical energy is measured from the graph of the square of induced current against time as shown in Figure 3.3.

![Induced current versus time](image)

Figure 3.2: Induced current versus time

When the current in Figure 3.2 is squared and plotted on time as Figure 1.9 we get the graph in Figure 3.3. It can be seen that the expected results concur with the experimental results. Also, the expected curves in Figure 1.9 concur with the results in Figure 3.2 and Figure 3.3.
By using equation 1.18, 1 electrical energy is measured as

\[ \text{Energy} = R \times \int I^2 \, dt \]

However, the integration is done by summation of the square of current for the same time interval as:

\[ \text{Energy} = R \times \sum_{0}^{2.0} I^2 \, dt \]

\[ = 3.8 \, \Omega \times 0.205 \, A^2 \times 0.004 \, \text{seconds.} \]

\[ = 3.11 \, \text{millijoules} \]

Where \( I \) is the induced current, \( R \) is the total resistance of the 2 ammeter and the coil and \( dt = 0.004s \) the time step of the measurement.

From equations 3.1 and 3.3, it can be seen that within experimental errors the electrical energy generated is about 119% of the available kinetic energy. This conversion is not 100% as expected in section 1.3.3.2. This is mainly because we are not accounting for mechanical energy transferred to coil during the magnet-coil interaction. This energy is sizeable as observed during forced oscillations and while making force measurements with an Atwood machine set up. We observe kinetic energy converted into electrical energy and is greater with lower resistance wires. The graphs below show the available kinetic energy and their corresponding electrical energies.

---

1 Details of electric energy measurement are in Appendix C.
2 Resistance of the ammeter was measured as \( 1 \pm 0.1 \Omega \).
Figure 3.4: Energy change for G22Lw
Figure 3.5: Energy change for G22sw

Figure 3.6: Energy change for G22sw
Table 3.1 gives a summary of the change in kinetic energies observed as electrical energies for different coils. The column of percentage error was obtained by expressing the standard deviation as a percentage of the percent of the lost kinetic energy seen as electrical energy.

\[
%\text{error} = \frac{\text{Deviation}}{\%\text{of K.E. converted}} \times 100
\]
<table>
<thead>
<tr>
<th>Coil label and wire radius ±0.001mm</th>
<th>Coil width ±0.5mm</th>
<th>Resistance ± 0.1 Ω</th>
<th>Δ K.E. (mJ)</th>
<th>Electrical energy (mJ)</th>
<th>% of K.E. lost &amp; observed as Elect. energy</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>G22ssw</td>
<td>0.345</td>
<td>7</td>
<td>2.8</td>
<td>2.4459</td>
<td>2.9866</td>
<td>122</td>
</tr>
<tr>
<td>G30ssw</td>
<td>0.135</td>
<td>7</td>
<td>13</td>
<td>1.8453</td>
<td>1.6833</td>
<td>92</td>
</tr>
<tr>
<td>G22sw</td>
<td>0.345</td>
<td>11</td>
<td>2.6</td>
<td>2.2860</td>
<td>3.1500</td>
<td>137</td>
</tr>
<tr>
<td>G22Lw</td>
<td>0.345</td>
<td>25</td>
<td>2.3</td>
<td>2.5479</td>
<td>3.3643</td>
<td>132</td>
</tr>
</tbody>
</table>

Note: The above resistances exclude resistance of the ammeter (1 ± 0.1Ω)

Table 3.1: Details of the coils investigated.

Although we can not account for part of mechanical energy, Table 3.1 above shows that thicker wires are better than thinner wires in changes in kinetic energy within the experimental uncertainties. This is because their resistances are lower than for thinner wires. Lastly, the slope of OA and BD of Figure 3.1 is theoretically given by substitution into equations 1.9 and 1.16 as

\[
2a = 2 \times \left( \frac{91g - 89g}{91g + 89g} \right) \times 9.8 = 0.2177 \text{m/s}^2
\]

3.4

For the particular coil used in this sample the slopes were fitted to be 0.2047 and 0.2098 for after and before damping respectively. The two slopes deviate by 4.8% from the slope in equation 3.4.
3.2. Other factors considered
The results are given in three groups:

(i) External load resistor.
(ii) Ratio of magnet length to coil width.
(iii) Thickness of the wire.

3.2.1. External resistance.
The thicker wire of radius 0.345mm with the smallest coil width of 7mm was connected to external resistors as:

(i) \( R_L = 2.8 \pm 0.1 \Omega \) (\( R_L = R_C \))
(ii) \( R_L = 10 \pm 0.1 \Omega \) (\( R_L > R_C \))

The results as shown in Figure 3.8 below show induced current versus time for different external resistances.

![Induced Current and external load](image)

It can be seen that induced current is maximum for no external resistance and minimum for the biggest external resistor. This is because from Ohm’s law, resistance is inversely proportional to current. Table 3.1 below showing percentages of kinetic energy converted as electrical energy:
The summary of the change in kinetic energy converted into electrical energy is summarized in the table below

---

1 Coil labeled 22ssw was used in this investigation.
<table>
<thead>
<tr>
<th>External resistance</th>
<th>$\Delta K.E.$</th>
<th>Elect. energy</th>
<th>% of K.E. converted into Elect. energy</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.4459</td>
<td>2.9866</td>
<td>122</td>
<td>8</td>
</tr>
<tr>
<td>2.8</td>
<td>2.2149</td>
<td>1.8866</td>
<td>88</td>
<td>21</td>
</tr>
<tr>
<td>10</td>
<td>1.4764</td>
<td>0.915</td>
<td>62</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 3.2: Effect of external resistance

From the table, it can be observed that as the external resistance increases, the percentage of kinetic energy converted into electrical energy reduces. This is because as the external resistance increases, the induced current also reduces as observed in Figure 3.8 above. Kinetic energy change comes as a result of damping that comes due to the magnitude of the induced current. Therefore if the induced current is higher, the energy change should also be higher. The table shows how external resistance affects energy change.

![Square of linear velocity versus position for different external loads.](image)

Figure 3.9: Kinetic energy change for different external loads.

From Figure 3.8, it can be seen that as external load increases, induced current decreases. Therefore there is less E.M.I. damping, resulting in almost no kinetic energy change as shown in the curve of 10 $\Omega$ in Figure 3.9 above. Also observed is that as the external load increases the square of velocity graph approaches an open circuit situation where
resistance is maximum. According to [10], Power losses due to Joule heating effect are calculated as

$$ P = \frac{V^2}{R} $$

Where V and R are induced voltage and resistance respectively. From equation 3.4 in conjunction with the observation in Figure 3.9, it can be seen that as the external resistance increases, percentage of kinetic energy made available as electrical energy decreases. The bigger the difference between the square of velocities as in equation 3.1, the more the kinetic energy becomes available to be converted into electrical energy. Therefore, a good vibration energy harvester should have the smallest possible external resistance. In general, percentage of kinetic energy converted to electrical is low due large uncertainties at low change in kinetic energy and to a smaller fraction of the initial kinetic energy transferred to mechanical energy in the coil.

### 3.2.2. Ratio of magnet length to coil width

In this paper, we investigated the above effect by having a fixed length 25mm of the magnet with two different coil widths as shown in Figure 1.11 while magnet widths were 25mm, and 7mm. The results for the effect of this ratio as already seen from the last two rows in the column of kinetic energy change of Table 3.1 which indicate that larger coil widths convert more kinetic energy than smaller coil widths. This contradicts the expectation made earlier in section 1.2.3.1. However, drawing conclusion in the coil width from this investigation is hindered with the fact that there is a possibility of an unaccounted for mechanical energy going into the coil but does not appear as electrical energy.

### 3.2.3. Thickness of the wire.

There wire radii investigated here were of 0.135mm and 0.345mm. The results in Figure 3.10 below indicates that a thicker wire induces more current also converts more energy as had been seen in Table 3.1.
From Table 3.1, it can be seen that the thicker wire has more kinetic energy change and also coverts most. This comes because the induced current in thicker wire is more than the thinner wire when the same voltage is applied. So the thicker wire causes more E.M.I damping than the thinner wire.

### 3.3 Damping force – Atwood’s machine

With reference to Figure 2.5, it was observed that as the magnet rises, damping force is so large that one observes the coil being lifted upwards. Figure 3.12 below shows the measured force and also the calculated force when the Gauss probe was put both inside and outside the coil. The force equation derived earlier is shown below.

\[ F(x) = 2\pi aNI(x)B_{rad}(x) \]

From the force equation, the radial magnetic field versus time graph for both probe inside and outside the coil is shown below.

---

**Figure 3.10: Induced and wire thickness for the same width**
The induced current in the coil gives rise to the radial magnetic field detected by the Gauss meter. Because magnetic field is inversely proportional to the cube of distance, the field detected when the Gauss probe is outside the coil is less than the field detected when the Gauss probe inside the coil at any time. This can be observed in Figure 3.11 above. The assumption in the force equation is that the Gauss probe should be directly inside the coil. This is not very possible. That is why the probe is put both inside and outside the coil. The calculated values for both the probe in and out are then compared with the measured value from the strain gauge which does not depend on the position of the Gauss probe. The figure below shows the results of the theoretical and the experimental measurement of the damping force.
The results in Figure 3.12 above are consistent with the expectations from the energy diagrams in Figure 1.12 and Saslow’s model. Also, it can be seen that the experimentally measured force concurs with the calculated force within the limits of the resolution of the instruments. Next, note that when the probe is outside the coil, it senses a smaller radial magnetic field compared to when the probe is inside the coil. That explains why the calculated force with probe inside is higher than when the probe is out.

The result in Figure 3.12 portrays a lot of success in modeling of the damping force. It shows clearly that when one uses the formula to calculate the damping force, the results may not have much uncertainty. This is a very big help especially when one wants to design a damper. A complete analytical modeling of the damping force would involve the relation

$$F(y) = \frac{2\pi a N^2}{R} B(y) \left( \frac{d\phi(y)}{dy} \right) V(y)$$  \hspace{1cm} 3.6$$

Where $\left( \frac{d\phi(y)}{dy} \right)$ is the change of magnetic flux with position and $V(y)$ is the velocity.

The first significant step is already done with results in Figure 3.12. The next step should have measurements of flux change with respect to displacement as the magnet gets damped. When peak value $B(y)$ is substituted in equation 3.6, the coefficient of the
velocity term on the right hand side of the equation should give coefficient of damping of a coil as discussed in the next section. Finally, one could use equation 3.6 to model the force entirely with computational techniques.

**3.4 Damping force-Driven oscillations**

**3.4.1. Damping force and velocity.**

The force experienced by the magnet is proportional to its velocity as had been discussed according equation 1.28. Plots of damping force against velocity for all the coils yield straight line graphs whose slopes is are constant numbers called coefficient of damping [9]. It has been found and confirmed that damping coefficient is unique for each coil as had been expected. A typical graph of this kind is in the Figure 3. 13 below. The other coefficients for other coils are tabled in Table 3.3.

![Damping Force and Velocity for c.w.= 11mm](image)

Figure 3. 13: Damping force and linear velocity
From the above table it can be seen that the thicker wire has more damping coefficient than the thinner wire in all the three categories. This is because the thicker has less resistance than the thinner wire. So induced current is more in the thicker wire resulting in more damping force.

### 3.5 M.T.S Machine

In this section of M.T.S. machine, induced voltage was investigated in relation to:

i. Coil widths

ii. Connection types

When a large force as that of a car was applied to a prototype of an E.M.I damper.

#### 3.5.1 Induced voltage and coil width.

The magnet of length 26mm was set to vibrate at a frequency of 1 Hz with amplitude of 12.69mm. Recall from section 1.2.3.1 that when the coil width is very large, induced voltage is reduced. This is observed to be consistent with the results shown in Figure 3.
below. It can be seen that a smaller coil width induces more voltage than the larger coil width.

![Induced voltage with coil width for \( r = 0.135 \text{mm} \)](image)

\[ f = 1 \text{ Hz}, \text{amplitude} = 12.69 \text{mm} \]

Time (s)

Induced Voltage (V)

Figure 3.14: Coil width factor for thinner wire in narrow and wider widths.

### 3.5.2 Induced voltage and connection type

A magnet of length 26mm was set to vibrate at a frequency of 1 Hz with 12.69mm amplitude in a three run coil of wire radius 0.345mm. The induced voltage was collected when the three runs were in series and when in parallel. A plot of the induced voltage versus time for the two types of connections as shown in Figure 3.15.

![Induced voltage for parallel and series connection types (triple run)](image)

Time (s)

Induced voltage (V)

Figure 3.15: Induced voltage with connection type at 0.5 Hz frequency

It was observed as in that series type of connection gives more induced voltage than its parallel counterpart. From Ohm’s law, induced current is inversely proportional to
resistance when other factors like voltage are kept constant. We have total resistance as
\[ R_T = 3R \] \( \Omega \) and \( R_T = \frac{R}{3} \) \( \Omega \) for series and parallel circuit connections respectively where \( R \) is resistance of each run of the coil. It can then be seen that the series connection type has a bigger resistance hence more induced voltage than the parallel type of connection.

3.6 Power – Vibrating magnet

Power generated is given as \( P = i^2R \) where \( I \) is the induced current and \( R \) is the resistance. Power generated was plotted against frequency with the results in the graphs below.

![Figure 3.16: Power for single run coils](image)

![Figure 3.17: Power for double run coils](image)
In all the three sections above power generated increases in non-linearly with increase in frequency. As frequency increases, velocity increases, flux change increases and hence the induced current. Since Power is proportional to the square of the current a non-linear relation is observed above.

According to our anticipation by equation 1.34, power generated by the double and triple runs should not depend of the type of connection. This is however not observed in all the cases at all the frequencies. At low frequencies, the anticipation is confirmed. As frequency increases the discrepancy increases in all the coils.

### 3.7 Power – M.T.S Machine

The following results were observed for different coils when damper force was plotted versus displacement. Figure 3. 19 shows the results of a wire of radius 0.345mm in a wider coil width of 25mm when the magnet was set to vibrate at a frequency of 0.5 Hz with amplitude of 0.5 inches (12.69mm).
It is very clear as had been expected that the force when the damper is powered externally is larger than the force when the damper is powered internally. To assess the essence of this thesis, we need to observe force when the damper is supplied with the energy from its vibrations. Therefore, when the above is zoomed, we get the below figure.

From the zoomed graph, we observe that there is no clear cut difference in damper force between off-state and coil short states at a frequency of 0.5 Hz. Because damping force had been observed to be proportional to induced current which is proportional to frequency, a plot of another coil at a frequency of 2 Hz is shown below.
It can be observed that at a higher frequency of 2 Hz, and a short coil width, there is still no very clear cut difference between damper output force in off-state and short coil state. From the vibrating magnet results of the coils for example in Figure 3.18, the average power output is 30mW. We observe a bigger damper output force as in Figure 3.19 when the external power supplied to the damper as

\[ P_{\text{external}} = 4.0V \times 0.2A = 800mW \]  

We know that \( P = \frac{V^2}{R} \). We assume the voltage of the coil used with the damper equals the voltage of the same coil when short.

\[ V_{\text{coil,MRI}} = V_{\text{coil,short}} \]  

So the power which the coil feeds into the damper in the M.T.S machine is given as

\[ P_{\text{MRI}} = P_{\text{short}} \times \left( \frac{R_s}{R_{\text{am}}} \right) = \frac{30mW}{\sqrt{2}} \left( \frac{3.8\Omega}{20\Omega + 2.8\Omega} \right) = 3.53mW \]  

\(^1\text{Where 20 } \Omega \text{ is the resistance of the damper, 2.8}\,\Omega \text{ is the coil’s resistance and 3.8 } \Omega \text{ is the total resistance of the coil and the ammeter used in the vibrating magnet set up. Comparing the two power values in equation 3.7 and 3.9 gives us}

\[ \sqrt{2} \]  

\(^1\text{ is used to convert peak current used earlier to rms current.}\)
From equation 3.10, the power of the E.M.I. coil is about 0.4\% of the external power supply from our results. We can see that our coils supply less than 1\% of the power that the external power would supply. Therefore, for us to record the expected damper force output, we need to increase the coil-magnet set up for more power output. Probably, the damper used in this investigation requires higher induced current than the induced current our E.M.I. coils could give out.

\[
\frac{P_{\text{coil}}}{P_{\text{external}}} = \frac{3.53mW}{800mW} = 0.004
\]
4 SUMMARY AND FUTURE WORK

Our investigations have been very successful especially on the damping force modeling. The theoretical calculation clearly concurs with the measured force values despite the two different positions of the Gauss meter. Actually, the radial magnetic field graphs with induced current which causes E.M.I damping has come out so neatly to explain the measured damping is never negative. It is well observed with the different Using our findings, dampers can be designed

Based on the observations made in this paper within the limits of the resolution of the instruments, a good electromagnetic induction damper should have:

i. Its coil made of thick wires
ii. Minimum external resistance
iii. Series type of connection with the coils when there is more than one run.

Looking further on what is done on this paper, we have worked in high damping regime. That is why at some point the electrical energy observed is more than the available kinetic energy change which causes it. This shows there is some sizeable mechanical energy we can not account for. For example, during the magnet-coil’s interaction the coil itself gets some part of the energy transferred to it and some part goes through the coil wire to be observed as electrical energy. In forced oscillations, the coil is well observed to be trying to move off.

Therefore there is need for more work to be done to:

i. Account for the mechanical energy
ii. Mount the coil more rigidly
iii. To optimize the system to get more of the kinetic energy converted to electrical energy.

For better results damper output force in the M.T.S machine, probably using a damper with less power input requirement can yield better results. Conversely, a higher coil magnet configuration would do better.
5 Reference:


8. Saslow Wayne: Electricity, Magnetism and Light. (Thomson Learning, 2002)


6 APPENDICES

6.1 Appendix A: Energy Change
A cylindrical hollow magnet and a similar piece of bronze were connected by a string that goes over the biggest pulley of a rotary force sensor. The coil radius of the coil of wire used in this initial investigation was chosen according to [12] so that a significant deviation of the induced voltage and current could be realized. The difference in mass between the piece of bronze $m_2$ and the magnet $m_1$ was minimum so that the system is kept at almost constant acceleration.

Measurements in Equations 3.1 and 3.3
The equations deal with kinetic energy change as the rising magnet interacts with the coil of wire. The magnet was is set to start from rest at a fixed point below the coil. At the same time, the data acquisition software was started. It was set to automatically stop after a fixed time like 3.0 seconds. The following measurements were taken at the same time.

i. Induced current versus time

ii. Linear velocity versus displacement

The graph in Figure 4.1 below represents the paths traced by the magnet.

![Graph showing square of linear velocity vs. displacement](image)

Figure 4.1: Kinetic energy change

Part OA represents magnet’s motion just before it begins to interact with the coil. This part is extrapolated to E to represent the square velocity in the open circuit ($V_o^2$). Part BD
which is ideally parallel to OE represents the magnet’s motion after interacting with the coil. It represents the square of velocity in the closed circuit ($V_{C}^2$). For each point the displacement domain ($d_1, d_2, d_3, \ldots, d_{110}$), there are two corresponding square of velocities in the open and closed circuit paths. The difference in $V_{O}^2$ and $V_{C}^2$ for each $d_i$ is then averaged over $n$, the number of possible points along the domain. In this case $n$ was 110 points. The kinetic energy equation in equation 3.1 above neglects rotational kinetic energy of the pulley and the kinetic energy of the connecting string. These are:

(i) Rotational K.E. of the pulley is given as:

$$K.E_{rot} = \frac{1}{2} \times (I_p \times \omega^2_p) \text{ Joules.}$$

But $\omega^2 = \left(\frac{V}{r}\right)^2 = \frac{0.02907}{0.0241^2}$ where 0.02907 (m/s)$^2$ is taken from equation 3.1, $r$ is the pulley radius.

$$K.E_{rot} = \frac{1}{2} \times 1.35 \times 10^{-6} \text{ kg.m}^2/\text{s} \times \frac{0.02907 \text{(m/s)}^2}{0.0241^2 \text{ m}^2}$$

$$= 3.37 \times 10^{-2} \text{ millijoules.}$$

Where $I_p$ and $\omega_p$ are moment of inertia and angular velocity of the pulley respectively.

(ii) Kinetic energy of the connecting string is given as:

$$K.E_{string} = \left(\frac{1}{2}\right) \times \lambda \times L \times V^2$$

$$= \left(\frac{1}{2}\right) \times 5.3 \times 10^{-5} \text{ kg/m} \times 1.21 \text{m} \times 0.02907 \text{(m/s)}^2$$

$$= 9.32 \times 10^{-4} \text{ millijoules.}$$

Where $\lambda$ and $L$ are linear mass density and length of the string respectively. Due to the small magnitude of the energies above, they can be neglected.

**Caution** – The choice of number of points along the displacement axis should be as close as possible to point B so as to minimize the effects of gravitational pull.

---

1 The moment of inertia of the pulley is given by the manufacturer of the rotary force sensor.
**Electrical energy measurement.**

From Figure 3.3 integration was done by summing the square of induced current and then performing calculation as in equation 3.3.

Current, G22 W = 7 mm, run 1

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Current (A)</th>
<th>$I^2$</th>
<th>$\sum I^2$</th>
<th>$\int dt$</th>
<th>Rc</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.00054</td>
<td>2.86E-07</td>
<td>0.20468618</td>
<td>0.004 secs.</td>
<td>3.8 $\Omega$</td>
<td>0.00311 Joules</td>
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<tr>
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</table>

Table 4.1: Electrical energy calculation

**6.2 Appendix B: Damping Force**

The objective in this investigation is to study how electromagnetic induction damping behaves in different coil set ups when a magnet in this case is set to vibrate horizontally. The magnet was set to vibrate at specific frequencies for each E.M.I. coil. The procedure below the diagram illustrates more.
Preliminary testing.

a) Balancing the coil on the force sensor.

b) Set the data acquisition software to read induced current and force each on time domain.

c) Place the coil on the force sensor.

d) Set the motor to rotate at low frequency of about 2 Hz. Close S2. Start the data acquisition software. Study carefully the induced current curve on the computer interface. It should come out resembling a sine curve.

e) Adjust the position of the coil forward and backward until a symmetrical curve comes out for induced current. When the current is symmetrical, the coil is balanced.

f) Short the coil to observe a change in the force graph.

g) Stop the motor.
Taking data.

(i) Start the motor at low frequency. With $S_2$ open, close $S_1$ to read the frequency of the induced current signal. Adjust the motor speed for a particular frequency like $f = 2$ Hz to start.

(ii) Open $S_1$. Start the data acquisition system. Let the motor run for about 5 seconds. Observe that induced current graph is reading zero while force graph shows a non-zero reading.

(iii) Close $S_2$. Let the motor run for about 5 seconds more. Observe that the induced current graph this time shows induced current reading. Force graph shows bigger amplitude. Stop the motor, stop the data acquisition system.

(iv) Extract tables of current and force versus time then copy and paste to excel page.

(v) Repeat steps (i) to (iv) for $f = 3$Hz, 4Hz …8Hz.

(vi) For each frequency, there is background force before $S_2$ is closed when current is still zero. After $S_2$ is closed, induced current begins to exist, the magnet begins to experience damping and the force sensor detects a larger force as the figures below illustrate.

![Figure 4.2: Measurement of damping force](image-url)
(vii) Using suitable calculations, determine the root mean square values of force when \( S_2 \) is open and when it is closed for each frequency. The difference is the force due electromagnetic induction damping for each frequency.

(viii) Make a table showing force when \( S_2 \) is open, when closed, net force and root mean square current for each frequency. For each frequency, determine also root mean square current.

6.3 Appendix C: Damping force model

This section deals with experimental measurement of force due to electromagnetic induction damping. A light coil with one hundred turns of wire was mounted on a support A and strain gauge on one side as shown in Figure 2.5. The coil is well observed going up and down as the magnet passes. The magnet was allowed to traverse the coil just like in section 2.1. However, this time the data acquisition system was made to measure the following with time:

a) Induced current - \( I \)

b) Magnetic field intensity – \( B \)

c) Strain signals from the strain gauge.

For each run of the magnet, all the above quantities were measured at the same time.
Procedure used
The magnet was initially let to run through the coil. Induced current and magnetic field was recorded with Gauss probe inside the coil and in the second run, Gauss meter probe was put outside the coil.

Strain gauge calibration.
By using equation 1.26, force was calculated to determine the range of the expected magnitudes of force.

Several masses whose weights were less than the maximum force from the calculation were then used to calibrate the strain gauge. The calibration scale used is shown below in Table 4.2

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<thead>
<tr>
<th>Mass</th>
<th>Voltage</th>
<th>Force (N)</th>
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<td>0.75</td>
<td>0.0009</td>
<td>0.0075</td>
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<tr>
<td>1.01</td>
<td>0.0013</td>
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<td>0.0479</td>
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</table>

Table 4.2: Strain gauge calibration

The best line of fit that was used to fit voltage readings to force was

\[
Force = 6.0322V + 0.005, \quad R^2 = 0.8575
\]

Where V is the voltage reading.
Caution

The strain gauge requires calibration at any time of measurement because it is sensitive to strain.