Abstract

Ferromagnetic Resonance Study of Square-Array Antidot Permalloy Thin Films

By Wesley Burgei

Ferromagnetic Resonance spectra were obtained for three square-array antidot patterned Permalloy 40-nm thin films. Measurements were made with the external magnetic field pointing out-of-plane and the AC driving field in-plane. In contrast to a Permalloy sheet film, the patterned films’ FMR spectra show the existence of higher order modes of precession in additional to the uniform precession mode. The energy or frequency of the higher modes depends inversely on the spacing of the holes. We suggest the holes act as lateral boundaries allowing for the excitation of 2-D standing spin waves. A simple theoretical fitting procedure is discussed and is shown have an average deviation of only 10% from the experimental data.
Ferromagnetic Resonance Study of Square-Array Antidot Permalloy Thin Films

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By
Wesley A. Burgei
Miami University
Oxford, Ohio
2003

Advisor:____________________
Prof. Michael J. Pechan
Reader:____________________
Prof. Samir Bali
Reader:____________________
Prof. T. William Houk
# Table of Contents

1 Introduction and Research Incentive 1

2 Background 3

2.1 Review of Fundamental Concepts in Magnetism 3
   2.1.1 The Origin of the Magnetic Field 3
   2.1.2 Magnetic Dipole Moment 3

2.2 Dynamics of a Magnetic Dipole Moment in a Magnetic Field 8

2.3 Classification of Magnetic Dipole Moment 9
   2.3.1 Magnetic Susceptibility 10
   2.3.2 Diamagnetism 10
   2.3.3 Paramagnetism 11
   2.3.4 Ferromagnetism 11
   2.3.5 Antiferromagnetism 14

2.4 Magnetic Anisotropy and the Magnetization Process 14
   2.4.1 Shape Anisotropy 15
   2.4.2 Crystalline Magnetic Anisotropy 16
   2.4.3 Growth Induced Anisotropies and Surface Effects in Magnetic Thin Films 17
   2.4.4 The Magnetization Process and Anisotropy Measurement Techniques 17

2.5 Spin Waves and the Magnon Energy Dispersion Relationship 19

2.6 Ferromagnetic Resonance 25
   2.6.1 Uniform Precession 25
   2.6.2 Spin Wave Resonance 26

3 The Samples 27

3.1 Sample Fabrication 27

3.2 Modification of the Properties with Holes 29

4 The Experimental Apparatus 32

4.1 Apparatus Overview 32

4.2 The Microwave Bridge 32

4.3 Signal Detection 33
## 5 The Results Discussion

- 5.1 The Experimental Data 36
- 5.2 Micromagnetic Simulations 40
- 5.3 Technique for Fitting the Experimental Data 42

## 6 References 49
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Chapter (1): Introduction and Research Incentive

Magnetic thin film fabrication and research began in the mid-1950’s and is still a very active research field in condensed matter physics and other disciplines. The potential application of magnetic thin films in the magnetic recording industry has been the catalyst for intensive study. The search for higher recording density has proven to be a relatively successful venture and is still ongoing, as the magnetic recording density has doubled annually for the past decade.

The magnetic properties of thin films differ greatly from their bulk counterparts due in many cases only to their geometry. Since ferromagnetism results from a strong interaction between neighboring atoms then it is easy to realize that atoms at the boundary of a ferromagnetic material would have a reduced symmetry and therefore different magnetic properties. Although this is true for any magnetic sample, in thin films, the ratio of surface atoms to interior atoms is large enough to affect the macroscopic magnetic properties. Thin film growth also plays a large role in the overall magnetic properties. The film substrate can greatly influence the film by introducing strains and allowing different crystalline lattice structures, which would normally not occur in bulk material. Through the development of refined growing techniques, resulting in better growth control, many magnetic properties can be engineered to a desired state. It is this fact that has allowed the widespread use of magnetic thin films in magnetic recording applications.

Recently the construction of magnetic thin film structures with sub-micron sized lateral resolution has become a realization due to the refinement and use of electron beam and photolithography. The most common types of this patterned media are arrays of separated magnetic dots and arrays of holes referred to as antidots, which have both a fundamental and practical interest. The fundamental interest in patterned media results from new geometrical constraints, which can greatly affect the overall static and dynamical magnetic properties. Careful characterization of these induced properties may lead to even better ways to engineer customized magnetic properties in a thin film. From a practical point of view the patterned media may be the next generation of materials for high-density magnetic recording. For the magnetic dot arrays, the idea is that each
separated dot, which is typically smaller than a normal magnetic domain, will be one bit of information. Antidot arrays may also be a candidate for magnetic information storage. It turns out that the introduction of the holes produce a greatly modified domain structure and creates laterally separated domains, which may act as a bit.

The magnetic dot arrays have received the most attention recently. Previous experiments show that in magnetic dot arrays, in contrast to the sheet film, there is an induced in-plane magnetic anisotropy that is strongly dependent on the array symmetry\(^1,2\). Knowledge of how the dot array symmetry affects the in-plane anisotropy will allow the fabrication of a magnetic sample with finely tuned desired anisotropy energy. In contrast to a sheet film, ferromagnetic resonance (FMR) studies show the presence of additional resonance peaks along with the uniform precession mode. This knowledge sheds light on the non-uniform magnetization of the sample and the dipole coupling between neighboring dots.\(^2,3\) Though there is much literature on the properties of magnetic dots, literature on antidot arrays is scarce, especially FMR studies. It is with this in mind that we undertook an investigation of the induced properties of antidot arrays in Permalloy thin films. A previous experiment on this subject shows additional magnetic anisotropy energies and a completely modified domain structure.\(^4\) Our work describes the ferromagnetic resonance spectra of three antidot samples each with square antidot array symmetry but with varying lattice spacing. As with the magnetic dot arrays, the FMR spectra shows peaks in addition to what would be expected for uniform precession. These extra peaks may be attributed to higher order two dimensional spin wave modes that travel parallel to the film plane.
Chapter (2): Background

The following chapter relies heavily on several texts, mainly two that I happened to inherit\textsuperscript{5,6}, and my personal notes from lecture\textsuperscript{7}. Other sources were used on specific topics and are referenced accordingly.

2.1 Review of Fundamental Concepts in Magnetism

2.1.1 The Origin of the Magnetic Field

The origin of the magnetic field is apparent through Ampere’s Law,

\[ \mathbf{B} = \frac{1}{c} \oint \frac{d\mathbf{E}}{dt} \cdot d\mathbf{A}, \]  \hspace{1cm} \text{(cgs units)} \hspace{1cm} (2.1)

where \( \mathbf{B} \) is the magnetic field, \( \mathbf{E} \) is the electric field and \( c \) is the speed of light.

Unlike the electric field there are no charges associated with magnetism but instead the magnetic field is created from an electric field changing in time. Thus fundamentally all magnetism arises from electric currents.

2.1.2 The Magnetic Dipole Moment

We now define the fundamental concept of the magnetic dipole moment. To do this we start with the Lorentz force on a straight current carrying wire due to an external uniform magnetic field, which is given by

\[ \mathbf{F} = \frac{\mathbf{q}}{c} \mathbf{\bar{v}}_d \times \mathbf{B}, \]  \hspace{1cm} \text{(cgs units)} \hspace{1cm} (2.2)

where \( q \) is the charge of whatever is moving (in most cases it’s the charge of the electron), \( \mathbf{\bar{v}}_d \) is the drift velocity of the charge through the wire and \( \mathbf{B} \) is the applied magnetic field. Recall that the average current is defined as

\[ I_{\text{avg}} = \frac{\Delta q}{\Delta t} = \lambda \mathbf{v}_d \]  \hspace{1cm} (2.3)

where \( \lambda = \frac{q}{l} \), the charge per unit length. Combining (2.2) and (2.3) yields

3
\[
\mathbf{dF}_{avg} = \frac{dI}{c} \mathbf{l}_{avg} \times \mathbf{B}. \quad (2.4)
\]

Then, for an arbitrarily shaped path with a constant current
\[
\mathbf{dF} = \frac{I}{c} [\mathbf{d}\mathbf{s} \times \mathbf{B}] = \left( \frac{I}{c} \mathbf{B} \sin \left( \theta_{ds, B} \right) \mathbf{ds} \right) \mathbf{e}_{df}, \quad (2.5)
\]

where \( \theta_{ds, B} \) is, of course the angle between \( \mathbf{B} \) and the infinitesimal portion of the wire \( \mathbf{d}\mathbf{s} \). It can be shown that any closed current path will experience a net force of zero. With this in mind we can look at the torque on a closed circular path due to an applied field. Recall that torque is given as
\[
\vec{\tau} = \mathbf{F} \times \vec{d} \quad \text{or} \quad d\tau = d\mathbf{F} \times \vec{d}
\]
where \( \vec{d} \) is the moment arm. From Figure 1 it’s clear that the magnitude of the moment arm, \( \vec{d} \), is
\[
|\vec{d}| = r \sin \left( \theta \right), \quad (2.6)
\]
where \( r \) is the radius of the current loop. Also, we can see that since \( r \) is always orthogonal to \( \mathbf{d}\mathbf{s} \) then \( \theta \) is in fact always equal to the angle \( \theta_{ds, B} \), the angle between

---

Figure 1: Circular current loop in an external uniform magnetic field experience a net force of zero, however it does experience a torque.
\[
d\mathbf{\tau} = d\mathbf{F} \times \mathbf{a} = d\mathbf{F}(r \sin(\theta))(\sin(\phi_{F,a}))\hat{e}_d, \quad (2.7)
\]
where \((\phi_{F,a} = \phi)\) is the angle between the force on the wire and the moment arm which is also the angle between the unit vector normal to the circular plane \(\mathbf{\hat{n}}\) and \(\mathbf{\tilde{B}}\) or the tilt of the loop. Now substituting in for \(d\mathbf{F}\) from (2.5) and recognizing that \(d\mathbf{s} = r d\phi\),

\[
d\mathbf{\tau} = \frac{1}{c} B(2r^2 \sin^2(\theta))\sin(\phi)\hat{e}_d. \quad (2.8)
\]

Integrating over the circular path we get,

\[
\mathbf{\tau} = \frac{1}{c} \int_0^{2\pi} B(2r^2 \sin^2(\theta))\sin(\phi)\hat{e}_d d\phi
\]

\[
= \frac{1}{c} B r^2 \sin(\phi) \int_0^{2\pi} \sin^2(\theta) d\phi \hat{e}_d
\]

\[
= \frac{1}{c} B r^2 \pi \sin(\phi) \hat{e}_d
\]

\[
\mathbf{\tau} = \frac{1}{c} r^2 \pi \mathbf{\hat{n}} \times \mathbf{\tilde{B}}. \quad (2.9)
\]

Now, recognizing that \((\pi r^2)\) = area we final arrive at the well known result

\[
\mathbf{\tau} = \frac{1}{c} A \mathbf{\hat{n}} \times \mathbf{\tilde{B}}. \quad (2.10)
\]

We now can define the magnetic dipole moment \(\mathbf{\mu}\).

\[
\mathbf{\mu} = \frac{1}{c} A \mathbf{\hat{n}} \quad (2.11)
\]

Although we have derived this result for a circular current loop, it is fact true of any closed current loop.\(^8\) This can be shown heuristically, assuming that any shaped can be modeled as a collection of small circular loops with \(d\mathbf{\mu} = (d\mathbf{l} d\mathbf{A} \mathbf{\hat{n}})\). Now we integrate,
Figure 2: Any arbitrarily shaped current loop can be modeled as a collection of small circular current loops. We can see that all the current on the inside of the loop will cancel, and summing the small (dA) yields the area of the loop.

\[ \oint \frac{d\Phi}{dl} \]

\[ I_{\text{sum}} \]

\[ \nabla \times \mathbf{E} = \mathbf{0} \]

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{d\mathbf{E}}{dt} \]

\[ \mathbf{E} = \mathbf{E}_0 \exp \left( i \left( \omega t - k \cdot \mathbf{r} \right) \right) \]

\[ \mathbf{B} = \mathbf{B}_0 \exp \left( i \left( \omega t - k \cdot \mathbf{r} \right) \right) \]

\[ \mu = \frac{I}{c} \oint d\mathbf{A} \mathbf{\hat{n}} \]

noticing that all the small current on the inside will cancel with each other giving

\[ \mu = \frac{I_{\text{sum}}}{c} \int d\mathbf{A} \mathbf{\hat{n}} = I_{\text{sum}} \mathbf{A} \mathbf{\hat{n}}. \]

Now we look at a circular current loop containing one electron circulating with a frequency (f). In this case the current is

\[ I = \frac{ef}{c} = \frac{e\omega}{2\pi}, \quad (2.12) \]

where (e) is the electron charge and \( \omega \) is the angular frequency. The magnetic moment in this case is

\[ \mu = \frac{e\omega}{2\pi} \mathbf{\hat{n}} = \frac{e\omega}{2} \mathbf{\hat{n}}. \]

The angular momentum, \( \mathbf{L} \), of the electron moving in a circular path is given as,

\[ \mathbf{L} = mr^2 \omega \mathbf{\hat{\omega}}, \quad (2.13) \]

where m is the electron mass and \( \mathbf{\hat{\omega}} = \mathbf{\hat{v}} \times \mathbf{\hat{r}} \) implying \( \mathbf{\hat{v}} \parallel \mathbf{\hat{n}} \).

We can now link angular momentum to the dipole magnetic moment,
\[
\mu = -\frac{e}{2m} \mathbf{L}.
\]  

(2.14)

There are two things to note; first, \( \mu \parallel \mathbf{L} \) and the negative sign implies that \( \mu \) and \( \mathbf{L} \) point in opposite directions, second the constant relating the \( \mu \) to \( \mathbf{L} \) is known as the gyromagnetic ratio, \( \gamma = -\frac{e}{2m} \).

It is well known that electrons in an atom have quantized orbital and intrinsic spin angular moment. Therefore atoms would have a magnetic dipole moment relating to both their orbital path and the intrinsic spin. For reasons only fully realized in relativistic quantum mechanics, the gyromagnetic ratio for the electron spin is \( \gamma_{\text{spin}} = -\frac{e}{m} \).

Apparently the spin is 2-times more “effective” at creating a magnetic moment than the orbital angular momentum. For this reason we introduce the Landé or spectroscopic splitting factor, \( g \), where

\[
g_{\text{spin}} = 2 \quad \text{&} \quad g_{\text{orbit}} = 1.
\]  

(2.15)

We then write the magnetic moment as,

\[
\tilde{\mu} = -g\gamma (\hat{\mathbf{L}} \text{or} \mathbf{S}).
\]  

(2.16)

It should be noted for clarity that often \( \tilde{\mu} \) is expressed as,

\[
\tilde{\mu} = -g\mu_B \mathbf{S}
\]  

(2.17)

where \( \mu_B \) is the Bohr magneton, the fundamental quanta of magnetic moments and is equivalent to

\[
\mu_B = \frac{e\hbar}{2m}.
\]  

(2.18)

However, quantum mechanically Eq. (2.17) should be stated as the expectation value of \( \tilde{\mu} \), or \( \langle \mu \rangle \). This may seem like a trivial technicality but some may wonder why an \( \hbar \) appears all of a sudden.

For systems that include both an orbital and spin angular momentum (which is often the case), we write the total angular momentum, \( \mathbf{J} \), as \( \mathbf{J} = \mathbf{S} + \mathbf{L} \). It is apparent that \( \tilde{\mu}_{\text{total}} \) is not perfectly parallel to \( \mathbf{J} \) due to the fact that \( g_{\text{spin}} = 2 \). However, we can write the average \( \tilde{\mu} \) as,
\[
\bar{\mu}_{\text{avg}} = -g_{\text{total}} \gamma \vec{J},
\]  
(2.19)
or
\[
\frac{d\vec{\mu}}{dt} = \left( \frac{e}{2m} \right) \vec{B} \times \vec{\mu},
\]
(2.22)

Notice that the change in the dipole magnetic moment vector in time is always in a direction that is perpendicular to both \( \vec{B} \) and \( \vec{\mu} \). This in effect causes the magnetic moment to precess about the magnetic field \( \vec{B} \) and is known as Larmor precession. The rate at which the moment precesses \( \omega_L \) is known as the Larmor precession frequency, which can be shown to be
\[
\omega_L = \frac{e}{2m} B.
\]
(2.23)

The magnetic moment in fact never gets any closer to the applied field but maintains the same precession angle. The energy interaction responsible for this motion is the dipole-field interaction which, like its name suggests, is an interaction between the magnetic moment and the applied field, given as
\[
E = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \phi,
\]
(2.24)

Notice that this energy does not change with time due to the fact that the precession angle remains the same. This last statement may seem to contradict the experience of anyone who has ever played with bar magnets. Since all bar magnets are made up of tiny dipole moments, then why does say a compass needle tend to align with the earth’s magnetic field and not precess about it? The key is that all the tiny dipoles inside the bar magnet interact with each other and undergo many collisions that tend to damp out the precession causing alignment. The spin damping or spin relaxation is really the cause of many known effects in macroscopic magnetism. In fact it is because of spin relaxation that macroscopic magnetic materials can become magnetized.

2.3 Classification of Magnetic Materials

In this section I will review the different classifications of magnetic materials beginning with a discussion of magnetic susceptibility. Although important I will only briefly discuss diamagnetism, paramagnetism and antiferromagnetism and spend most the effort on ferromagnetism which is most relevant to this experiment.
2.3.1 Magnetic Susceptibility

As discussed above, when a macroscopic magnetic material is placed in an external magnetic field, due to relaxation, the individual magnetic moments begin to align with the external field. The amount of net magnetization $M$ a sample has at a given external magnetic field $H$ is known as the magnetic susceptibility, $\chi$, expressed as

$$\chi = \frac{M}{H},$$

or more generally,

$$\vec{M} = \chi \vec{H}.$$  \hspace{1cm} (2.26)

Due to hysteresis, Eq. 2.26 is ill defined for ferro- and antiferromagnetic materials and really only applies to para- and diamagnetic materials. It is the functional form of $\chi$ (or lack thereof for ferro- and antiferromagnetic materials) that defines the different magnetic materials. In general $\chi$ is a tensor operator that relates the external field vector $\vec{H}$ to the magnetization vector $\vec{M}$ but we will see that for some materials $\chi$ can be represented by a constant.

2.3.2 Diamagnetism

As discussed above, every atom has intrinsic magnetic moments due to the spin and orbital angular momentum. Hund’s rules state (based on the assumption of the Pauli Exclusion Principle) that the electrons in an atom fill the orbital shells in such a way as to minimize the total angular momentum and consequently, it minimizes the net magnetic moment. When an atom has completely filled shells it will have zero net moment. With this said, the classical description of diamagnetism is based on Lenz’s Law in which currents are set up within an atom that oppose an external magnetic field. It is true that all atoms have a diamagnetic response to an external magnetic field however, materials are deemed diamagnetic when the atom only has a net moment when in an external field and in fact, this moment is in the opposite direction of the applied magnetic field. For this reason, we see that the magnetic susceptibility $\chi$ of a diamagnetic material is negative and $|\chi| << 1$. 
2.3.3 **Paramagnetism**

Atoms that do not have completely filled electron shells have a net angular momentum and hence have a net magnetic moment. As shown in Eq. (2.24), it is energetically favorable for a magnetic moment to align with an external magnetic field. This implies that paramagnetic materials will have a positive magnetic susceptibility, $\chi$. However, if this were the only energy interaction involved with the moment, then even the slightest external field would cause all the moments within a sample to align thus leading to magnetic saturation. This is not the case in real systems. Thermal effects within the material tend to randomize the moment if thermal energy ($kT$) is much larger than the dipole-field interaction ($\mu B$). The randomization would of course disrupt the magnetic ordering. With this competition between the dipole-field potential and the thermal effects it is easy to see that the magnetic susceptibility of paramagnetic material is inversely dependent on temperature, which is in contrast to the largely temperature independent nature of diamagnetism. Since $\chi$ is positive, then as the applied field is increased the magnetization will also increase until the magnetization equals $M_{SAT}$. It should be noted that for paramagnetic materials a very large external field is required to obtain $M_{SAT}$. Keep this in mind during the discussion of ferromagnetic materials. Now, if we remove the field then the thermal effects will completely randomize the moments in the sample leaving no net moment in the sample.

2.3.4 **Ferromagnetism**

Ferromagnetism is characterized by the occurrence of spontaneous magnetization at finite temperatures, which requires the existence of a net magnetic moment alignment in the absence of an applied magnetic field. From the above discussion then it becomes apparent that there must be another energy interaction involved which is not present in paramagnetic materials. Pierre Weiss offered the first good explanation of this by attributing this phenomenon to the existence of a “molecular field” within the material which was proportional to the magnetization, or

$$H_{\text{molecular}} = \lambda M$$

(2.27)
where $\lambda$ is the proportionality constant between the magnetization, $M$, and the molecular field, $H_{\text{molecular}}$. This assumption seems to be a good model to get a physical picture of what is occurring within the material however experimental measurements of this field show it to be a very large value, on the order of $10^7$ gauss. It is hard to imagine the origin of such a “field” within the material. The origin of the “field” is quantum mechanical in nature and is in fact not a field but an exchange energy interaction between neighboring spins, expressed as

$$E_x = -2J \sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1},$$

(2.28)

where $J$ is known as the exchange constant or the exchange integral. Looking at the exchange energy expression one can see the importance of the sign of the exchange constant, $J$. If $J$ is positive then the lowest exchange energy occurs when $\vec{S}_p$ and $\vec{S}_{p+1}$ point in the same direction, however if $J$ is negative then the lowest energy configuration is when $\vec{S}_p$ and $\vec{S}_{p+1}$ point in the opposite direction. Among materials with exchange interactions, it is the sign of the exchange constant that determines the type of material and the overall phenomenological properties.

Ferromagnetic materials have a positive exchange constant while materials with a negative exchange constant are antiferromagnetic and will be discussed briefly in the next section. The most common ferromagnetic materials are Ni, Fe and Co, which have a very strong exchange interaction that tends to align neighboring spins within the material. As most people may know, a chunk of iron is not always magnetized which may seem to contradict the above statement. When all the dipole moments align they effectively create a bar magnet with a north and south pole having curling magnetic field lines that connect the poles. Since any real sample has width and height then some of the field lines connecting the poles, pass through the material. With this occurring, there is now a large dipole-field potential, as shown in Eq. (2.24), that causes a competition between the dipole potential and the exchange interaction. The two competing energies are compromised creating a lower energy state by forming domains. Within a domain all of the dipole moments are aligned but within the whole of the sample the domains are randomly arranged causing a zero net magnetization overall. (The existence of domain formation was actually first predicted by Weiss, within the framework of his molecular
field theory.) Now one may wonder about the large interaction energy between adjacent
domains. The dipole moments, always wanting to be at the lowest energy available, get
around this problem by having a gradual transition between neighboring domains. These
are known as domain walls and the thicker the wall the lower the interaction energy
between the domains.

Now there is another problem. If the thicker the wall, the lower the interaction
ergie then the walls would actually take up the whole domain and there
would be zero net magnetization.
There must be another interaction.
The culprit this time is the crystalline magnetic anisotropy, which will be
discussed in more detail in the following sections. It turns out that in
certain preferred directions along the crystalline structure it is
energetically favorable for the magnetic moments to align. Alignment in other directions
would cause the moments to be at a higher potential. The former is known as an easy
direction while the latter is known as the hard direction and the energy difference
between the two is known as the anisotropy energy. So any
domain wall would tend to limit the number of moments pointing
in the hard directions. Therefore, the competition between the exchange energy and the
anisotropy energy creates a domain wall that minimizes the overall energy of the sample.

The temperature effects on a ferromagnetic material must also be mentioned. As
with paramagnetic materials, the net alignment of the moments within the material is
opposed by thermal effects. If \( kT \) is on the order of the exchange interaction energy
then the ferromagnetic material begins to act like a paramagnetic material. In fact all ferromagnetic materials become paramagnetic at some temperature (assuming they do not melt first). Other materials that are paramagnetic at room temperature can become ferromagnetic at lower temperatures.

2.3.5 Antiferromagnetism

Antiferromagnetism, like its name suggests, is closely related in origin to ferromagnetism. Antiferromagnetic materials have a net magnetic moment and also have a very strong exchange interaction. However, the exchange constant for antiferromagnetic materials is negative which makes for the lowest energy alignment to be anti-parallel. As would be expected, the demagnetizing field for antiferromagnetic materials is not a factor since there is no net magnetization.

![Adjacent moments in an antiferromagnetic material lie antiparallel to each other.](image)

2.4 Magnetic Anisotropy and the Magnetization Process

Magnetic anisotropy is a general term for the orientation dependence of the magnetic properties of a material. A direction that requires the least amount of external field to fully magnetize a sample is known as the easy-direction and conversely a direction that requires the largest external field to magnetize the sample, is called the hard-direction. The difference in the external field required for saturation magnetization between the easy and hard directions is known as the anisotropy field. As with the Weiss molecular field, all the magnetic anisotropies can be modeled as effective fields within the sample to get a physical feel for what is going on. However, in most cases it is mathematically easier and more fundamentally sound to model the anisotropies as energy interactions. The magnetic anisotropy energy is defined as the magnetic potential
difference between an easy and hard direction or in other words the amount of work required to pull the magnetization away from the easy direction.

The following sections briefly describe the different types or origins of a magnetic anisotropy including effects related to geometrical constraints or shape anisotropy, crystalline symmetries and growth induced anisotropies found in magnetic thin films. In this particular experiment only shape anisotropy is really relevant as Permalloy has a very small crystalline anisotropy. However, the other magnetic anisotropies are worth mentioning briefly. The microscopic calculations or calculations from first principles of the crystalline anisotropies are beyond the scope of this thesis, however, the calculation methods can be found in more detailed text books. There is also a brief description of the magnetization process and how magnetic anisotropy energies are measured.

2.4.1 Shape Anisotropy

As we discussed above, the demagnetizing field created by the magnetization of the sample can greatly affect the overall energy potential within. It follows directly that the physical geometry of a sample can affect the demagnetizing field. A sample that is geometrically isotropic, i.e. a spherical sample will also have an isotropic demagnetizing field and thus will have no magnetic anisotropy due to its shape. Now imagine a thin wire that is magnetized along its long axis. Let’s say it has a demagnetizing field of magnitude A in the opposite direction. If the wire is then magnetized in a direction

![Diagram](image-url)  

Figure 2.5: (a) ferromagnetic rod magnetized along its long axis has a minimal demagnetizing field, (b) in this direction the demagnetizing field is maximized.
perpendicular to the axis there will be a much larger field line density within the sample in this orientation and therefore will have a demagnetizing field much greater than $A$. We say that the long axis of the wire is the easy direction and the direction perpendicular is called the hard direction since it would require a greater external magnetic field to overcome the larger demagnetizing field in the direction perpendicular to the wire’s long axis. Therefore the work required to magnetically saturate the wire in the hard direction with respect to the easy direction is the magnetic anisotropy energy.

The shape anisotropy for magnetic thin films is obviously very important to this experiment. Thin film geometry induces a large demagnetizing field when the magnetization points out of plane and relatively no demagnetizing field when the magnetization lies in the plane. It can be shown that the shape anisotropy energy for a thin film sample is $2\pi M^2$ and is minimized when the magnetization lies in-plane.\(^\text{10}\) Also in patterned media there are obviously further geometrical constraints and it is not surprising that there is different anisotropy symmetries.

2.4.2 Crystalline Magnetic Anisotropy

Crystalline magnetic anisotropy or magneto-crystalline anisotropy refers the orientation dependence of the dipole moment with respect to the crystalline lattice structure. Since ferromagnetism originates from the spin moments in the conduction band, which are weakly bound to the lattice, and the exchange interaction between adjacent spins is isotropic with respect to rigid rotation we can see that magneto-crystalline anisotropy has to have its origin in some other interaction. In fact the origin of magneto-crystalline anisotropy is the spin-orbit coupling interaction. Unlike the spin moments the orbital moments are strongly coupled to the nucleus and thus the lattice structure. Therefore, due to spin-orbit coupling the spin moments and thus the overall magnetization properties are indirectly dependent on the lattice structure. In certain preferred directions magnetic saturation via an external field is relatively easy to achieve, these are called “easy directions”. In all other directions it takes a greater external field must be applied to overcome the spin-orbit coupling interaction, and the direction where this is maximized is called the “hard direction”.

16
Phenomenologically it can be realized that different crystalline lattices would, of
course, have different anisotropy symmetries. For example hexagonal-closed-pack (hcp)
Co has uniaxial anisotropy parallel to the hcp c-axis, which implies that there are two
easy directions that lie antiparallel to each other along the axis. On the other hand, Fe
having body-centered cubic symmetry has three easy axes.

2.4.3 Growth Induced Anisotropies and Surface Effects in Magnetic Thin Films

The crystalline structure of a substrate that a film is grown on can greatly
influence the film’s magnetic properties. The substrate’s lattice structure can act as a
template for the film growth allowing crystalline structures, which would not normally
occur in a bulk material. This is known as epitaxial growth and it can drastically affect
the sample’s crystalline anisotropy. Furthermore if there is a large mismatch between the
atomic spacing in the substrate and the growth material there can be a lattice strain in the
film, which would also induce a magnetic anisotropy.

As mentioned earlier the thickness of the sample can also affect the overall
magnetic anisotropy. Since atoms at the surface of the material would see a reduced
symmetry it is easy to realize that they would have different lattice interactions and thus
different crystalline anisotropy symmetries. If the sample is grown thin enough this
surface anisotropy can overcome the “bulk” crystalline anisotropy. Although this may
seem to be more of a geometrical effect and thus should be included in the shape
anisotropy section, this anisotropy energy has its origin in the exchange interaction
between neighboring moments. Thus it would be more at home in the crystalline
anisotropy section. However, since this effect only becomes relevant in very thin
samples, I think that it is makes the most sense to include it in the growth induced
section.

2.4.4 The Magnetization Process and Anisotropy Measurement Techniques

Let us suppose that we magnetize a sample with large crystalline uniaxial anisotropy
along the easy direction. We find that it takes very little external field to bring the sample
to magnetic saturation. We see the Magnetization vs. Applied Field curve does not
follow a linear path but resembles something more complex. This is not too surprising because we know in general that the magnetic susceptibility $\chi$ is not a scalar constant. Now we remove the external magnetic field and see that the sample has retained almost all of its magnetization. We conclude that the anisotropy energy within the sample acts like an internal field resulting in the samples remnant magnetization. Now we begin to

slowly apply an external field in the opposite direction. At first the sample remains magnetized in the original direction but as the field is increased we find that the sample’s magnetization abruptly switched directions. We conclude that since it is just as energetically favorable to point in the positive $x$-direction as in the negative $x$-direction that at some point the applied field will overcome the effective anisotropy field and coerce the magnetization to point in the opposite direction. We call the amount of external field required to coerce the magnetization to switch directions the coercive field. We can obviously see that this coercive field is directly related to the magnetic anisotropy within the sample. Now as we again remove the field we see the sample remains

Figure 2.6: Hysteresis loop of a Permalloy thin film with the applied magnetic field pointing along the sample’s easy direction. First a large positive magnetic field is applied to the sample causing the sample’s magnetization to saturate. The field is then slowly decreased to zero. At this point the Permalloy has retained nearly all of its magnetization. A negative field is then applied and slowing increased causing the sample’s magnetization to eventually abruptly switch directions. The negative field is increased until the sample has reached saturation magnetization in the negative direction. The process in then repeated for a large negative field, ultimately creating a loop.
magnetized in the negative x-direction and as we then apply a positive directed field the sample’s magnetization once again is coerced to switch direction to lie parallel to the external field thus creating a closed loop on the M vs. H plot. This is known as a magnetic hysteresis loop.

Now let’s look at the hysteresis loop in the sample’s hard direction. Again we apply a field in the positive x-direction and we find that it requires much more applied field to bring the sample to saturation magnetization. We conclude that in this direction we have to overcome the large effective anisotropy field that points along the easy direction. We then slowly remove the external field and find that in this situation there is no or very little remnant magnetization along the hard direction. In this case there is no effective anisotropy field to “pin” the magnetization along this direction. We continue to map out the M vs. H hysteresis loop and discover that this loop is very narrow compared to the loop in the easy direction or in other words there is little or no coercive field.

By mapping the remnant and coercive field of the hysteresis loop as a function of angular position we can obtain information on the magnitude and symmetry of the anisotropy energy. There are several common techniques to obtain the magnetic hysteresis loops including the vibrating sample magnetometer or VSM and the magneto-optic Kerr effect apparatus or MOKE. Other techniques for measuring magnetic anisotropy energy include ferromagnetic resonance which do not measure a hysteresis loop but directly measures the effective internal field of the sample. Also there is the torque method which measures the mechanical torque required to pull the magnetization away from the easy direction and is probably the most direct measurement of the anisotropy energy.

2.5 Spin Waves and the Magnon Energy Dispersion Relationship

A spin wave is the propagation of a disturbance in the relative alignment of a spin moment in a system of spins. Analogous to phonons, the quantized quasi-particle of lattice vibrations, magnons are quantized quasi-particles of spin waves. The excitation of magnons corresponds to the reversal of a spin within the system resulting in a decrease in the net magnetization by $\mu$. The following discussion of magnons relies heavily on the
text “Solid State Physics” by Charles Kittel. For this section, I felt it necessary to fill in the steps left out in the text to serve as a more thorough reference for future students.

We begin by looking at the ground state condition of the exchange energy interaction for a one-dimensional ferromagnet (implying $J > 0$) considering only nearest neighbor interaction, Eq. 2.28,

$$E_x = -2J \sum_{p=1}^{N} \vec{S}_p \cdot \vec{S}_{p+1}.$$ 

By inspection, we see that the energy is minimized when all the spins are aligned and we can calculate the ground state energy for a simple system. Consider a system of five spins, $N=5$ as shown in Figure (2.7a) that are aligned in the $+\hat{z}$ direction. Treating the spins as classical vectors with magnitude $S$ the energy is therefore

$$U_0 = -2J(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot \vec{S}_3 + \vec{S}_3 \cdot \vec{S}_4 + \vec{S}_4 \cdot \vec{S}_5),$$

where $\vec{S}_i \cdot \vec{S}_j = +S^2$ for alignment and hence

$$U_0 = -8JS^2,$$

or generally

$$U_0 = -2JS^2(N-1).$$

We can now calculate the energy of the system when one spin is aligned in the $-\hat{z}$ direction while the others remain in the $+\hat{z}$ direction. Looking at Figure (2.7b) it is clear that this time we still get

$$U_1 = -2J(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot \vec{S}_3 + \vec{S}_3 \cdot \vec{S}_4 + \vec{S}_4 \cdot \vec{S}_5).$$

Figure 2.7: (a) Ground state configuration of spins considering the exchange energy. (b) This configuration reduces the net magnetization of this system by $\mu$ but is actually not the first excited state. It is energetically favorable for the flip to be shared by all the spins minimizing the exchange energy.
but in this situation
\[ \vec{S}_1 \cdot \vec{S}_2, \vec{S}_2 \cdot \vec{S}_3 = -S^2 \]
making
\[ U_1 = -2J(-S^2 - S^2 + S^2 + S^2) = 0. \]
Apparently flipping one spin raises the energy by $8JS^2$. Therefore, the question is
whether this situation is the first excited state meaning it contains one magnon. It turns
out, not too surprisingly, that is energetically favorable to “share” the flip among the spin
system thus effectively reducing the magnetization by $\mu$ but minimizing the large
exchange interaction. We now can calculate the classical dispersion between the energy
and the wave number $k$.

We begin by calculating the exchange energy of the nearest neighbors in a one-
dimensional system for the $p^{th}$ spin,
\[ E_x = -2J(\vec{S}_p \cdot \vec{S}_{p+1} + \vec{S}_p \cdot \vec{S}_{p-1}) \]  \hspace{1cm} (2.29)
Knowing
\[ \vec{\mu}_p = -g\mu_B \vec{S}_p, \]
we can make a substitution into (2.29) yielding
\[ E_x = \frac{2J}{g\mu_B} \vec{\mu}_p \cdot (\vec{S}_{p+1} + \vec{S}_{p-1}). \]  \hspace{1cm} (2.30)
Much like the magnetic anisotropy effective fields we notice that the exchange
interaction can be modeled as an interaction between a moment and an effective field.
Recall this is what Weiss originally did with ferromagnets to explain the spontaneous
magnetization. We define the effective exchange field, $B_p$, as
\[ \vec{B}_p \equiv -\frac{2J}{g\mu_B} (\vec{S}_{p+1} + \vec{S}_{p-1}). \]  \hspace{1cm} (2.31)
Now we have the form of a dipole field interaction
\[ E_x = -\vec{\mu}_p \cdot \vec{B}_p. \]
The equation of motion for this interaction is then
\[
\frac{1}{\gamma} \frac{\partial \vec{\mu}_p}{\partial t} = \vec{\mu}_p \times \vec{B}_p.
\]

Making the substitution for \( B_p \) into the equation of motion along with
\[
\vec{\mu}_p = -g \mu_B \vec{s}_p \quad \text{and} \quad \gamma = \frac{g \mu_B}{\hbar},
\]
yields
\[
\hbar \frac{\partial \vec{s}_p}{\partial t} = -g \mu_B \vec{s}_p \times \left[ \frac{-2J}{g \mu_B} (\vec{s}_{p-1} + \vec{s}_{p+1}) \right],
\] (2.31)

\[
\frac{\partial \vec{s}_p}{\partial t} = \frac{2J}{\hbar} \left[ (\vec{s}_p \times \vec{s}_{p-1}) + (\vec{s}_p \times \vec{s}_{p+1}) \right].
\] (2.32)

The equation of motion can be separated into its components with the \( \hat{x} \)-direction
\[
\frac{\partial S_p^{(x)}}{\partial t} = \left( \frac{2J}{\hbar} \right) \left[ S_p^{(x)} (S_{p-1}^{(x)} + S_{p+1}^{(x)}) - S_p^{(x)} (S_{p+1}^{(x)} + S_{p-1}^{(x)}) \right],
\] (2.33)

Similarly,
\[
\frac{\partial S_p^{(y)}}{\partial t} = \left( \frac{2J}{\hbar} \right) \left[ S_p^{(y)} (S_{p-1}^{(y)} + S_{p+1}^{(y)}) - S_p^{(y)} (S_{p+1}^{(y)} + S_{p-1}^{(y)}) \right],
\] (2.34)

\[
\frac{\partial S_p^{(z)}}{\partial t} = \left( \frac{2J}{\hbar} \right) \left[ S_p^{(z)} (S_{p-1}^{(z)} + S_{p+1}^{(z)}) - S_p^{(z)} (S_{p+1}^{(z)} + S_{p-1}^{(z)}) \right].
\] (2.35)

We now make the long wavelength approximation, which is in the limit where \( kT \ll E_x \) a very reasonable approximation. For long wavelengths deviations of the spin \( S \) from the \( \hat{z} \)-direction are very small which implies that \( S_x, S_y \) are very small allowing us to neglect terms quadratic in \( S_x, S_y \). Furthermore, we may substitute \( S_p^{(z)} \rightarrow S \) and realize then that
\[
\frac{\partial S_p^{(z)}}{\partial t} = 0.
\]
Therefore we have,
\[
\frac{\partial S_p^{(x)}}{\partial t} = \left( \frac{2J}{\hbar} \right) \left[ S_p^{(z)} (2S) - S (S_{p+1}^{(y)} + S_{p-1}^{(y)}) \right]
\] (2.36)
Similarly,

\[ \frac{\partial S_p^{(y)}}{\partial t} = \left( \frac{2JS}{h} \right) \left[ 2S_p^{(x)} - S_{p+1}^{(x)} - S_{p-1}^{(x)} \right] \]  

(2.37)

and

\[ \frac{\partial S_p^{(z)}}{\partial t} = 0. \]

(2.38)

Looking for traveling wave solutions for \( S_p^{(x)} \) and \( S_p^{(y)} \) we start with,

\[ S_p^{(x)} = u e^{i(pka-\alpha\ell)} , \quad S_p^{(y)} = ve^{i(pka-\alpha\ell)} , \]

(2.39)

where (a) is the lattice spacing, (p) is an integer referring to the p\(^{th}\) spin, (k) is the wave number and (u,v) are constants. Therefore,

\[ \frac{\partial S_p^{(x)}}{\partial t} = -iu\omega e^{i(pka-\alpha\ell)} \]

\[ = \left( \frac{2JS}{h} \right) \left[ 2ve^{i(pka-\ell\alpha)} - ve^{i[(p+1)ka-\ell\alpha]} - ve^{i[(p-1)ka-\ell\alpha]} \right] \]

(2.40)

Dividing both sides by \( e^{i(pka-\alpha\ell)} \) and using the Euler relation,

\[ = \left( \frac{2JS}{h} \right) \left[ 2 - e^{ika} - e^{-ika} \right] = \left( \frac{4JS}{h} \right) v(1-\cos(ka)). \]

Therefore we have

\[ iu\omega + \left( \frac{4JS}{h} \right) v(1-\cos(ka)) = 0. \]

(2.41)

Performing similar operations on \( S_p^{(y)} \) yields,

\[ iv\omega + \left( \frac{4JS}{h} \right) u(1-\cos(ka)) = 0. \]

(2.42)

We can write the system of equations in matrix as

\[
\begin{pmatrix}
  i\omega & \frac{4JS}{h} (\cos(ka)) \\
-\frac{4JS}{h} (\cos(ka)) & i\omega
\end{pmatrix}
\begin{pmatrix}
  u \\
  v
\end{pmatrix}
= 0
\]

(2.43)

To obtain a non-trivial solution for this system we require the matrix of the coefficients, call [M], to be unitary implying that the inverse, \([M]^{-1}\), does not exist. The inverse of [M] does not exist when \( \det[M]=0 \). Therefore we have,
\[
\det[M] = -\omega^2 + 16 \left( \frac{JS}{\hbar} \right)^2 (1 - \cos(ka))^2 = 0 \quad (2.44)
\]
\[
\omega^2 \hbar^2 = 16 J^2 S^2 (1 - \cos(ka))^2
\quad (2.45)
\]
Taking the root of both sides
\[
\hbar \omega = 4 JS (1 - \cos(ka)) \quad (2.46)
\]
This is the 1-dimensional magnon dispersion relationship from the exchange energy counting only nearest neighbor interactions. Now we again use the long wavelength approximation, \( a \ll L \), implying \( ka = \frac{n\pi a}{L} \ll 1 \). Therefore we can use the small angle approximation
\[
\cos(ka) \equiv \left( 1 - \frac{(ka)^2}{2!} \right).
\]
Substituting this into (2.46) yields,
\[
\hbar \omega = (2JSa^2)k^2, \quad (2.47)
\]
the famous quadratic dispersion relationship. Now knowing \( \omega = \frac{4 JS}{\hbar} (1 - \cos(ka)) \) we can rewrite our matrix \([M]\) as
\[
\begin{pmatrix}
  i\omega & \omega \\
  -\omega & i\omega
\end{pmatrix}
\begin{pmatrix}
  u \\
  v
\end{pmatrix} = 0,
\quad (2.48)
\]
implying
\[
v = -iu. \quad (2.49)
\]
Then again using the Euler relation,
\[
S_p^{(x)} = u[\cos(ka - \omega t) + i \sin(ka - \omega t)];
\]
\[
S_p^{(y)} = -u[i \cos(ka - \omega t) + \sin(ka - \omega t)].
\]
Taking the real part of each,
\[
S_p^{(x)} = uc\cos(ka - \omega t);
\]
\[
S_p^{(y)} = -us\sin(ka - \omega t)
\quad (2.50)
\]
which shows the circular precession of the spin about the \( \hat{z} \)-direction with frequency
\[
\omega = 2 \frac{JSa^2}{\hbar}k^2.
\]
2.6 Ferromagnetic Resonance

2.6.1 Uniform Precession

Ferromagnetic resonance or FMR is the resonance of an external driving field and the precession of the net magnetization vector. Since the net magnetization of material is made up of individual magnetic dipole moments, which are a result of the electron spin, then fundamentally FMR is electron spin resonance or ESR. However, the physics of nuclear magnetic resonance or NMR, which is used for magnetic resonance imaging or MRI, is very similar. Some ESR experiments are also referred to as electron paramagnetic resonance but again it works exactly the same.

To explain the resonance phenomenon we start with Eq. (2.22), which describes the dynamics of a non-interacting dipole moment in an external field. Now if we assume that we apply an external field large enough to magnetically saturate the sample then we can rewrite Eq. (2.22) as

\[
\frac{d\vec{M}_{\text{sat}}}{dt} = \gamma \vec{M}_{\text{sat}} \times \vec{H}_{\text{applied}}.
\]  

That is, if the magnetization is saturated then we can treat the whole magnetization vector as a single non-interacting dipole moment. With this realization we conclude the magnetization vector undergo a uniform precession at a rate given by the Larmor frequency in Eq. (2.23),

\[
\omega = \gamma H_{\text{applied}}.
\]

As mentioned above there is always some sort of anisotropy field also present within the sample in which case we may replace the \( H_{\text{applied}} \) with an effective field \( H_{\text{effective}} \). In thin films with magnetization lying out-of-plane and neglecting crystalline anisotropy we know the \( H_{\text{effective}} \) is actually

\[
\vec{H}_{\text{effective}} = \vec{H}_{\text{applied}} - 4\pi \vec{M}_{\text{sat}},
\]

where \( 4\pi M \) is the demagnetizing field or the effective shape anisotropy field. Therefore,

\[
\omega = \gamma (H_{\text{applied}} - 4\pi M_{\text{sat}}).
\]  

In any real system the magnetization vector is not isolated and therefore precessional damping will occur. In a static external magnetic field the magnetization
vector will spiral to alignment with the external field. Small thermal excitation will pull the magnetization away from its ground state and the magnetization will again go through a precession and damping until it is again in its ground state. An AC magnetic field with frequency \( \omega \) applied in a direction perpendicular to the static field can couple to the precession of the magnetization vector and resonance can occur. In this case the AC field would tend to open up the precession angle. This in effect makes the system a sort of damped driven harmonic oscillator.

Now if we slowly sweep the external DC field and notice when the sample has absorbed the AC field then we would know at that certain DC field the sample’s magnetization vector was precessing at frequency \( \omega \). We call the DC field required to bring the sample to resonance with the AC field the resonance field. Again looking at Eq. 2.21 we could then measure the effective field that the sample “sees.” Subtracting the externally applied field from the effective field yields information on the sample’s effective internal field. Thus we can measure the anisotropy “fields” which can always be related to the anisotropy energies by

\[
\vec{V}_M E = \vec{H}
\]

where \( \vec{V}_M E \) is the magnetization gradient of the involved anisotropy energies. For a more detail mathematical description see previous thesis works.\textsuperscript{11,12,13,14}

2.6.2 Spin Wave Resonance

Kittel showed that long wavelength spin wave odd numbered modes can be excited by a uniform AC field in a magnetic thin film given there is sufficient surface anisotropy to pin the moments at the interface.\textsuperscript{15} Looking at the effective field created by an excited spin wave mode (2.31) and (2.47), it clear that under the right conditions this system can couple to the AC field at discrete applied fields. With the external field applied perpendicular to a thin film’s plane it can be shown that\textsuperscript{5},

\[
\omega_n = \gamma (H_{\text{applied}} - 4\pi M) + Dk^2
\]

where \( D = \frac{2Jsa^2}{\hbar} \) and \( k = \frac{n\pi}{L} \).
Chapter (3): The Samples

This chapter will describe the sample and briefly discuss the main fabrication processes including dc-magnetron sputtering, photolithography and ion-milling\textsuperscript{4,16}. There is also a brief discussion of the modified domain structure and induced anisotropy. In this experiment we used three antidot patterned Permalloy films with square lattice spacings of 3\(\mu\text{m}\times3\mu\text{m}\), 4\(\mu\text{m}\times4\mu\text{m}\), and 5\(\mu\text{m}\times5\mu\text{m}\). We also analyzed the properties of the unpatterned sheet film as a control sample.

3.1 Sample Fabrication

These antidot samples were created at the University of Alabama via photolithography and ion-milling technique. However, first the sheet film had to be fabricated. A 40 nm 80Ni/20Fe Permalloy film was deposited on a native Si oxide 001 substrate via dc-magnetron sputtering. The sputtering process uses ionized argon gas which is attracted to a high voltage cathode where a bulk piece of Permalloy is positioned. The gas is accelerated so much that it collides with the Permalloy with enough momentum to vaporize the iron and nickel atoms. The substrate is placed at another location in the chamber where the atoms randomly and evenly settle. Peizzo-electric crystals monitor the sample’s thickness by measuring the change in the natural vibrational frequency as the sample accretes mass. The sample is patterned by a photolithography and ion-milling technique. First, the sample is evenly covered with a chemical that when exposed to \(\mu\nu\)-light has changes chemical properties known as photo-resist. A Cr\(_2\) mask with a pre-made pattern of holes is then place on top and used as the mold for the sample’s design. The \(\mu\nu\)-light is then applied, chemically modifying the photo-resist only where the mask does not protect. The mask is detached and a chemical etchant is used to remove the photo-resist, which was not exposed to the \(\mu\nu\)-light source.
Figure 3.1(a): Permalloy film is first coated with photo-resist. Then Cr₂ mask is used as a template for patterned film.

Figure 3.1(b): Sample is then exposed to a µv-light source. Places where light is incident on the photo-resist layer the substance is chemically modified.

Figure 3.1(c): Mask is removed and a chemical etchant is used which only removes the un-exposed photo-resist. A method similar to sputtering is then used to erode photo-resist and Permalloy.

Figure 3.1(d): Another etchant is used to remove the remaining photo-resist leaving the finish patterned sample.
This now leaves the Permalloy sheet film exposed through the holes in the photo-resist layer. The ion-milling technique is very similar to what happens in dc sputtering where ionized gas is greatly accelerated towards a high-voltage cathode plate. The high-energy gas collides with the sample and erodes atoms from both the photo-resist and the Permalloy film that is exposed thus creating the holes in the sheet film. The remaining photo-resist is then chemically etched away with some other etchant leaving the finish product.

Atomic force microscopy images were taken of the three samples here at Miami University with the assistance of Dr. John Rakovan, of the Geology Department. An image analyzing software package was used to verify the sample hole size and array symmetry. The AFM images showed that the holes are not perfectly circular and on average have a diameter of about 2µm. We also found that the hole-lattice array symmetry is square with spacings that are nominally ×3µm, 4µm×4µm, and 5µm×5µm on average.

3.2 Modification of the Magnetic Properties with Holes

The presence of the holes in the film greatly modifies of domain structure of the Permalloy. As shown is Figure 3.3 the magnetic domains of the antidot sample are 1-2µm across which are an order of magnitude smaller than the sheet film and are well defined and periodic. The fact that the domain structure is greatly modified is evidence that the internal fields (or the “effective fields”) present in this sample is drastically different than in the sheet film. It is the modified domain structure that makes the antidot patterned films a possible candidate for the next generation of high-density magnetic recording devices. It is really no surprise that the holes induce a magnetic anisotropy that is not found in the sheet film and which is dependent on the array symmetry. The hole array creates new hard axes along directions connecting nearest neighboring holes.²
Figure 3.2 (a): AFM image of the 3µm×3µm Permalloy antidot sample. Notice the holes are not entirely circular.

Figure 3.2 (b): AFM image of the 4µm×4µm Permalloy antidot sample.

Figure 3.2 (c): AFM image of the 5µm×5µm Permalloy antidot sample.

Figure 3.2 (d): This is an image of a hexagonal antidot sample not used in this experiment. The image analysis software package is able to take AFM information and create rotated images to get a better idea of the sample’s topography.
This work describes the existence of excited spin wave modes in the antidot samples which are not present in the sheet film via ferromagnetic resonance. Since we believe that the holes are acting as nodes in which standing spin wave modes can form we would obviously want to look at the dispersion of the multiple peaks as a function of hole array symmetry and lattice spacing. To keep things relatively simple for this study we consider only antidot samples with square lattice arrays and varying lattice spacing.

Figure 3.3: Magnetic Force Microscopy image of antidot sample. Visible here is the greatly modified domain structure.
Chapter (4): The Experimental Apparatus

4.1 Apparatus Overview

This chapter will describe the main features of the FMR apparatus used in this experiment. In this system we use a cavity system and thus a fixed microwave frequency while sweeping the DC applied field via an electromagnetic. The magnet is controlled by a programmable power supply, which is run by a LabVIEW program via a GPIB connection. The customized LabVIEW program is also responsible for the data acquisition and analysis. The signal detection system involves modulating the DC field which puts a “signature” on the data allowing a lock-in amplifier to filter the signal. The cavity and signal detection systems will be discussed in greater detail in the following sections. A discussion of the cryogenic aspects of this system is not relevant to this experiment, however discussions of the cryogenic aspects of this FMR can be found in previous theses.

4.2 The Microwave Bridge

The microwave bridge system, which includes the cavity, waveguide, circulator, microwave source and detector, provides several desirable purposes in any EPR experiment. The cavity and waveguides are made of highly conductive metal whose cross sectional dimensions are on the order of the wavelength of the microwave radiation. As mentioned before, any oscillations and precession inside the sample quickly damp and thus we require a driving field to excite such motion. The waveguides and cavity provide direct pathway for the microwave radiation to the sample’s location and back to a detector. The geometrical constraints of the cavity only allow for certain wavelengths and modes of the standing microwave radiation. We say that this is the cavity resonance. This provides a uniform and predictable driving field for the sample and thus reduces the experimental noise greatly. A cavity system also provides a practical way of controlling small deviations of the cavity resonance via a controllable microwave source and a lock-in amplifier.
In this experiment we use a cavity that supports a transverse electric 102 mode or TE$_{102}$ mode microwaves, which are nominally 10GHz. The sample is placed on a sample rod and inserted into the middle of the cavity where the magnetic field of the microwave radiation is spatially uniform. The standing wave is reflected back out of the cavity to an circulator which deciphers incoming and reflected waves and directs the reflected waves to a photodiode detector. When the sample undergoes resonance it will absorb the microwave radiation resulting in a drop in the reflected radiation incident of the detector.

### 4.3 Signal Detection

We want to detect the sample undergoing resonance with the standing microwave inside the cavity by looking for a decrease in reflected microwave power. We should find the usual sort of Gaussian resonance curve as a function of applied DC magnetic field. However, this signal is very small and most likely much smaller than the noise of the system. Instead we use a field modulation technique which puts a time signature on the
signal of interest, the absorption of microwave power, and which can be filtered by a standard lock-in amplifier. The lock-in provides a 15-kHz AC signal to the modulation coils located within the cavity. This provides an AC magnetic field at the sample location of approximately ±10 gauss. The reflected power signal from the photodiode is fed into the lock-in inputs where the signal is filtered leaving only the component oscillating at the frequency of the AC modulation. The lock-in then sends the rms value of the signal to LabVIEW, which plots this as a function of applied DC field. In doing so we are actually plotting the derivative of the microwave power reflection with respect to the applied field.
Figure 4.3: a) Actual theoretical absorption curve. b) Derivative of the absorption curve that is a result of the way the signal is measured.
Chapter (5): Results and Discussion

5.1 The Experimental Data

Out-of-plane ferromagnetic resonance spectra were obtained at room temperature for (3) antidot-patterned Permalloy thin films with varying square lattice spacing. Data for a Permalloy sheet film was also taken as a control sample. In contrast to the sheet film the antidot films show the presence of quantized higher energy modes of oscillations, which are apparent as additional resonance peaks. Recall that both the sheet and the patterned film are made of the same material and were grown to the same thickness and the only difference between the sheet film and the patterned film is the presence of the holes. Then the obvious conclusion is that holes are somehow the cause of the higher order modes of oscillation. Furthermore the spacing between adjacent peaks is not uniform for all the samples, which lead to the conclusion that the additional resonance peaks depends either directly or indirectly on the spacing of the holes. Figure 5.1 shows the raw FMR spectrum for each sample. It is assumed that the largest intensity peak corresponds to the uniform precession mode. Although each sample has the same thickness, the uniform precession mode resonance occurs at different fields. Qualitatively this can be attributed to the relative difference in each sample of the amount of magnetic material for a given area. Of course, the 3µm×3µm sample has relatively less material per given area than the 4µm×4µm and 5µm×5µm samples resulting in a smaller out-of-plane demagnetizing field. A smaller demagnetizing field corresponds to a lower resonance field. Although no quantitative explanation has been formulated for this thesis the actual difference in the resonance field for the uniform precession in the 5µm×5µm and 4µm×4µm samples as compared to the 3µm×3µm sample is 105 Oe and 20 Oe respectively. Figure 5.2 is a modified spectrum in which 20 Oe and 105 Oe was subtracted from the absolute
Figure 5.1: Raw FMR spectra. (a) is from the 3μm×3μm sample, (b) is from the 4μm×4μm sample and (c) is from the 5μm×5μm sample. The different positions of the uniform precession modes, shows the difference between the demagnetizing field for the three samples. Qualitatively the 3μm×3μm has the least amount of material per area, which means that its lowest magnetization reducing the amount of demagnetizing field lines passing through the material.

Figure 5.2: (a) is from the 3μm×3μm sample, (b) is from the 4μm×4μm sample and (c) is from the 5μm×5μm sample. In this plot the peaks were modified so that the uniform precession mode of the 5μm×5μm and the 4μm×4μm samples overlaps with the 3μm×3μm sample. The plot focuses on the first additional higher energy peak of each sample. Notice that the field separation of the first peak from the uniform precession increases with decrease hole spacing. The second peak also demonstrates this. It must be noted that the first additional resonance peak is not necessarily the first excited spin wave mode.
resonance field for the 4μm×4μm and 5μm×5μm samples respectively to illustrate the inverse relationship between the hole spacing and the separation of the first additional peak. Note that the first additional peak for the 3μm×3μm sample occurs at a noticeably lower field than for the 4μm×4μm sample, which occurs at a noticeably lower field than the 5μm×5μm sample. To demonstrate the complexity of the spectrum I included a zoomed-in plot of the all three sample spectrums, see Figure 5.3. Figure 5.4. Table 5.1 lists the resonance fields of the additional peaks as well as the ΔH values, which I define as the field spacing between the higher order peaks and the uniform precession peak. The resonance fields for the all three samples were fit using a LabVIEW peak-fit program containing derivative Lorentzian curves that were matched to the experimental curves “by eye”.

![Figure 5.3(a): Zoomed-in plot of experimental resonance spectrum of the 3μm×3μm sample.](image-url)
Figure 5.3(b): Zoomed-in plot of the experimental resonance spectrum of the 4μm×4μm sample.

Figure 5.3(c): Zoomed-in plot of the experimental resonance spectrum of the 5μm×5μm sample.
<table>
<thead>
<tr>
<th>$5\mu m\times5\mu m$</th>
<th>$\Delta H$ (Oe)</th>
<th>$4\mu m\times4\mu m$</th>
<th>$\Delta H$ (Oe)</th>
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Table 5.1: Numeric list of the resonance field of all the peaks in each sample. Also listed are the $\Delta H$ values which are the field spacing between the additional peaks and the uniform precession mode.

5.2 Micromagnetic Simulations

Micromagnetic simulations were performed for a ferromagnetic element containing an array of 16 holes with square symmetry and a lattice spacing of 3μm using the public domain software OOMMF\textsuperscript{20}, see Figure 5.5. The simulation integrates the equation of motion of the magnetization vector exposed to a large out-of–plane applied magnetic field. The magnetization vector is tipped slightly from the ground state alignment with the external field and left to precess freely. The simulation shows that the area confined by the holes exhibits behavior that seems to “see” some sort of a lateral boundary. With this in mind, we can try to treat the material in the center of four holes as a single square element. Notice that the square element has a length of 3μm, along the diagonal meaning the sides have a length of $\sqrt{3}\mu m$. The simulation of the 16 hole element is very tedious, taking several weeks to complete. Although the simulation is more accurate with more holes, we also ran a simulation of an element containing four holes. In this simulation we were able to track the motion of the y-component of the magnetization for
Figure 5.4(a): Micromagnetic simulation of a 16 hole element done with the public domain program OOMMF. The shading indicates relative magnitude of the y-component of the magnetization at one instant in time.

Figure 5.4(b): The black circles represent the position of the holes and the square highlights the isolated behavior of the material between holes. In the data fit we treat the square as an isolated magnetic element.

Figure 5.5: Simulation of resonance spectrum for an element containing four holes with a spacing of $3\mu m \times 3\mu m$. Though the uniform precession mode peak occurs at a field 125 Oe higher than the experimental data, the excited peak separation in the simulation agrees with the experimental data with an average deviation of 15%.
a longer period of time. Doing this allowed us to take the Fourier transform of the oscillations of the y-component of the magnetization. This yielded the discrete frequencies (at the field parameter of the simulation), which were converted into discrete resonance fields, see Figure 5.6. Although this simulation is probably an oversimplification of the actual experimental conditions the field separation of the discrete resonance fields has only a 15% average deviation from the experimental data. It should be noted that I have compared the first three additional peaks of simulation and the experimental data. As will be stated later, we are not completely sure which spin wave mode the additional peaks actually represent. We could be comparing “apples and oranges.”

As a suggestion for further study, I believe that more simulations are needed. Simulations of a 16 hole square array with both 5\(\mu\)m and 4\(\mu\)m sides would be very useful in determining whether or not the isolated spin behavior observed in the 3\(\mu\)m\(\times\)3\(\mu\)m simulation is also exists in these samples.

5.3 Technique for Fitting Experimental Data

As discussed in Chapter 2, it has long been known that the spatial confinement of magnetic material allows for the quantization of higher order modes of precession about an applied magnetic field called spin waves or magnons. Kittel worked out a more detailed description including the boundary and experimental conditions which was not discussed in Chapter 2. He showed that a uniform AC magnetic field, applied perpendicular to a DC field which is pointing in the out-of-plane, could excite quantized spin wave modes in a thin film sample with the right thickness. In this case the waves travel parallel to the DC field and are bounded by the sample’s surfaces. Seavey and Tannenwald\(^1\) confirmed Kittel’s calculation with direct observation of spin wave modes in Permalloy thin films of thickness ranging from 2380 to 4250\(\AA\) via ferromagnetic resonance.
In the case of the antidots we believe that the holes act as the lateral confinements leading to spin wave quantization much like the surfaces of the thin film. The question is, can we use this simple model to describe the effects found in the antidot samples? We will take a naïve approach and assume the material between the four holes is effectively isolated from the rest of the sample which is partially supported by the OOMMF simulation. Like Kittel’s approach, we assume that the spin wave modes are only a result of the exchange energy interaction. This obviously simplifies system greatly but it may not be oversimplified. A previous characterization of spin wave modes in Permalloy disks attributed the extra peaks to dipolar and bulk magneto-static interaction and it was reported that the frequencies of these modes were found to be on both higher and lower than the uniform precession mode. We see no such lower frequency modes, which would correspond, in our case to a resonance peak located at a higher field than the uniform precession mode. Spin waves resulting from the exchange interaction would only be at a higher frequency than the uniform precession and thus would resonate with the AC field at a lower applied field. However, this does not of course rule out all other energy interactions which may or may not be relevant these samples. It is simply not known at this time what other energy interactions are involved.

Recall the one dimensional spin wave dispersion relationship from Chapter 2 given as,

\[ h_\Theta = (2JSa^2)k^2. \]

The experimental higher order modes seem not to follow the one dimensional dispersion relation especially the 3µm×3µm sample which seems to show three or four discrete pairs of resonance peaks. We can, however, try to fit the data to a two dimensional wave model and try to find a pattern. Kittel generalizes the dispersion relationship for exchange interaction spin waves to three dimensions given as
\[ \omega_{\text{spin wave}} = \frac{2A\gamma}{M} \pi^2 \left( \frac{k^2}{L_x^2} + \frac{m^2}{L_y^2} + \frac{\ell^2}{L_z^2} \right) \]  

(5.1)

where \( k, m, \) and \( \ell \) are all constant pertaining to excited modes along the \( x, y, \) and \( z \) directions respectively, \( M \) is the saturation magnetization, \( L_x, L_y, \) and \( L_z \) are lengths between boundaries in the three dimensional element, \( \gamma \) is the gyromagnetic ratio and \( (A) \) is the exchange “stiffness” constant. In our case, since the film thickness is only 40nm we can neglect the terms in the \( z \)-direction. At this thickness we would have very short wavelength, which would result in very large exchange energies. It is unlikely that we can excite these high-energy waves. Our assumption to neglect the \( z \)-component is also supported by the fact that the additional resonance peaks are not present in the sheet film. So apparently, the spin wave modes only exist in the \( x \) and \( y \)-directions. Therefore we have an expression of the form,

\[ \frac{\omega_{k,m}}{\gamma} = H_{k,m} = \alpha \left( \frac{k^2}{L_x^2} + \frac{m^2}{L_y^2} \right) \]  

(5.2)

where \( \alpha \) is a proportionality constant that classically is \( \alpha = \frac{2A}{M} \pi^2 \) and \( H_{k,m} \) is the effective field created by the spin wave mode. For our square arrays we have \( L_x= L_y \) giving,

\[ \frac{\omega_{k,m}}{\gamma} = H_{k,m} = \alpha \left( \frac{k^2}{L^2} + \frac{m^2}{L_y^2} \right) \]  

(5.3)

We can then define,

\[ \Delta H_{k,m} = H_0 - H_{k,m}, \]  

(5.4)

where \( H_0 \) is the resonance field of the uniform precession mode and of course, \( \Delta H_{k,m} = \Delta H_{m,k} \). We therefore seek to find a pattern that predicts the \( \Delta H_{k,m} \) values for each sample.

There are two ways I fit the data. The first is to solve for \( \alpha \) by assuming that the first addition peak corresponds to some \( k \) and \( m \) value and predicting the field spacing for the subsequent excited modes. An obvious choice for this normalization is the 5\( \mu \)m\( \times \)5\( \mu \)m sample, which showed the excitation of many higher energy spin wave modes. See Table 5.2. By doing so we can rule out, assuming of course this model is correct, that the first excited peak in the 5\( \mu \)m\( \times \)5\( \mu \)m sample corresponds to either \( H_{1,0} \) or \( H_{1,1} \). As is evident in Table 5.2, the successive mode calculations do not predict very
accurately the experimental peak spacings. For example there is no value to account for 
\( \Delta H=724 \text{ Oe} \) peak in either calculation. However, if we assume that the first additional 
peak of the 5\( \mu \text{m} \times 5\mu \text{m} \) spectrum corresponds to \( H_{2,0} \) then we get a fit that only has an 
average deviation from the experiment by only 3%. Assuming this is correct we must 
somehow account for the reason we do not see \( H_{2,1}, H_{3,0}, H_{3,2}, H_{4,1}, H_{3,3} \) or \( H_{5,1} \). So the 
question is, is this really a good fit or is merely due to the fact that as you go to higher 
and higher energy, the spacing between peaks will become smaller and smaller and at 
some point we can fit anything? Looking at Figure 5.3(c), we can make an argument that 
between the more intense resonance peaks there are lower intensity peaks which may 
correspond to the expected \( H_{2,1}, H_{3,0}, H_{3,2}, H_{4,1}, H_{3,3} \) and \( H_{5,1} \) modes. It may be that the 
excitation of the spin wave modes that we see depends on the direction of the driving AC 
field. So by rotating our samples in the cavity, keeping the direction of the DC field out 
of plane, we may see the excitation of other modes of oscillation. This may explain why 
we do not see some of the expected modes. Obviously this can be easily verified by 
experiment and is a suggestion for future students working on this problem. Also using 
the method outlined by Kittel, it is my opinion that the mode excitation selection rules for 
our experimental conditions should be calculated. Unlike Kittel, we should not assume 
that the square element sees a rigid boundary where the moments are effectively pinned. 
Also we have to use an AC field perturbation that has components along the direction of 
wave propagation. It is my suspicion that the direction of the AC field perturbation 
greatly affects the selection rules for excitation of higher energy modes of oscillations.

One-way to test the validity of the calculated \( \alpha \) is to use this value to fit resonance 
spectrum of the 3\( \mu \text{m} \times 3\mu \text{m} \) and 4\( \mu \text{m} \times 4\mu \text{m} \) samples. In the case of the last fit, we 
obtained \( \alpha=3.19\times10^{-10} \text{ (Oe)-m}^2 \). Table 5.2(b) shows the predicted values for the 
resonance peak spacing for the 3\( \mu \text{m} \times 3\mu \text{m} \) and 4\( \mu \text{m} \times 4\mu \text{m} \) using this value of \( \alpha \). The fit 
for 4\( \mu \text{m} \times 4\mu \text{m} \) sample was not very convincing, jumping from \( \Delta H=358 \text{ Oe} \) to \( \Delta H=518 \text{ Oe} \) 
completely skipping the experimental \( \Delta H=458 \text{ Oe} \). For the 3\( \mu \text{m} \times 3\mu \text{m} \) sample we 
obtain a fit that has an average deviation 10\% from the experimental data. This also 
assumes that for some reason \( H_{2,0} \) and \( H_{2,1} \) modes are not excited.

Attempting to obtain a better fit, I recalculated \( \alpha \) from the 5\( \mu \text{m} \times 5\mu \text{m} \) data 
this time assuming the first additional peak corresponds to the \( H_{2,1} \) mode instead of the 
\( H_{2,0} \) mode, see Table 5.3(a). The theoretical fit for 4\( \mu \text{m} \times 4\mu \text{m} \) sample this time only had
an average deviation of 7% from the experimental data and 10% for the 3µm×3µm sample. Again this assumes that for some reason the AC field does not excite certain modes. It is somewhat surprising that such a simple model can predict the experimental values with an average deviation of only 10%. The other way in which I fit the data was to use the α value from Seavey and Tannenwald since they also used

**ΔH<sub>k,m</sub> for 5µm×5µm sample:**

<table>
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<th>Assuming ΔH&lt;sub&gt;1,0&lt;/sub&gt;=102 (Oe)</th>
<th>Assuming ΔH&lt;sub&gt;1,1&lt;/sub&gt;=102 (Oe)</th>
<th>Assuming ΔH&lt;sub&gt;2,0&lt;/sub&gt;=102* (Oe)</th>
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<td>ΔH&lt;sub&gt;4,3&lt;/sub&gt;=459 (Oe)</td>
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Table 5.2(a): Theoretical resonance field values for higher order spin wave modes. The values with asterisks are ones that compare best with the experimental values. Assuming ΔH<sub>1,0</sub>=102 (Oe), we can get a fit with an average deviation of 3% However, we have to somehow account for extra peaks not seen in the experiment.

**ΔH<sub>k,m</sub> for 4µm×4µm sample**

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<th>ΔH&lt;sub&gt;1,0&lt;/sub&gt;=40 (Oe)</th>
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Table 5.2(b): Theoretical resonance field values for higher order spin wave modes using α=3.19×10⁻¹⁰ (Oe)·m². The values with asterisks are ones that compare best with the experimental values. Again we have to somehow account for extra peaks not seen in the experiment.
Table 5.3(a): Theoretical resonance field values for higher order spin wave modes. The values with asterisks are ones that compare best with the experimental values. This time assuming $\Delta H_{1,1}=102$ (Oe), we can get a fit with an average deviation of 2%.

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<td>$\Delta H_{4,1}$=541 (Oe)</td>
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Table 5.3(b): Theoretical resonance field values for higher order spin wave modes using $g=2.55\times10^{-10}$ (Oe)·m$^2$. The values with asterisks are ones that compare best with the experimental values.

Figure 5.6: Plot of the sum of the squares of the mode numbers vs. $H g/L^2$. The line is the theoretical plot, diamonds are data from the 3$\mu$m×3$\mu$m sample, circles are data from the 4$\mu$m×4$\mu$m sample and triangles are data from the 5$\mu$m×5$\mu$m sample. Error bars are ±10%.
Permalloy films. Now since they have a damping term that is proportional to the inverse square of the wave mode, I calculated $\alpha$ from the higher order terms. I found $\alpha = 1.2 \times 10^{12} \text{ (Oe)} \cdot \text{m}^2$. This is more than 100 times smaller than the $\alpha$ value we fit to the 5$\mu$m×5$\mu$m sample. Using this $\alpha$ value would require that the first excited mode of the 5$\mu$m×5$\mu$m sample to be about $H_{23,23}$. In this case the energy spacing would be very close together and we would not be able to resolve peaks. Unless of course, with the calculation of the selection rules, we find that many of the modes will not be excited. More likely, I think, is the possibility that $\alpha$ for our samples is greatly modified from that of classical theory, the reason being unknown at this time. Not to rule anything out, we may also assume that the $\alpha$ value for our samples is the same as the classical theory and it is actually the element size that we do not know. In this case we would be expecting the element size to be on the order of $1/2$-$\mu$m.

To conclude I must stress that the explanation of these additional spin wave modes is incomplete. Although our simple model can give a decent fit to the data it must be determined whether or not our model is valid for our samples. It may be that we must consider additional energy terms in the calculation of the dispersion relationship. However, before this is done, it is my recommendation that the selection rules for the excitation of exchange energy spin waves be worked out for this situation beginning with the classical equation of motion found in Kittel’s paper. I also recommend that FMR spectra be recorded with the AC field being at several different orientations with respect to the hole-array symmetry. It is my suspicion that the observed resonance spectrum depends on the direction of the AC perturbation.
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