THE EFFECTIVENESS OF PROBLEM-SOLVING-BASED ALGEBRA II INSTRUCTION AT WILLIAMSTOWN HIGH SCHOOL IN WILLIAMSTOWN, WV

A Thesis
Presented in Partial Fulfillment of the Requirements for
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Current high school seniors at Williamstown High School are required to take three or four credits of math before graduation. Those students on a professional career path are required to take four credits and those on a technical or entry-level path need to have three credits. If the students take Algebra I their freshman year, then they are required to have at least Algebra II by graduation. This leads to a large percentage of seniors in a course in which they are not interested nor see a need to have. Through this thesis, the researcher investigated whether a problem-solving-based Algebra II curriculum would lead to more interest and higher level thinking in the classroom. The researcher conducted a pre- and post-test study that showed an increase in the ability to solve critical thinking questions that meet the West Virginia Curriculum Standards in Algebra II.
Dedicated to my mother, who showed me

the value of education and the excitement she had for fractions.
ACKNOWLEDGEMENTS

I wish to thank my adviser, Dr. Marybeth Peebles, for intellectual support and encouragement, which made this thesis possible, and for her endurance in correcting both my stylistic and scientific errors.

I thank Ms. Felecia Simms for always being there when I needed assistance or insight while writing this thesis.

I am grateful to all my students, past, present and future, which make me want to find the best teaching strategies for their success.
VITA

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FIELDS OF STUDY

Major Field: Education
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CHAPTER 1

INTRODUCTION

Statement of the Problem

According to Institute of Educational Sciences (IES), in 1984 a mere 14 percent of high school seniors took three credits of math to graduate. In the same year, 31 percent elected to take two credits of math. Presently, at Williamstown High School seniors in the professional career path were required to take three credits and those on the entry-level or technical path were required to have two math credits. Fifty percent of the seniors, who graduated in 1994 took at least three credits and 75 percent took two or more math credits (1996). When the researcher graduated in 1997 from a neighboring school, she was required to take at least three math classes to graduate in the college preparatory program.

Because of the need for so many credits, there are more students taking higher-level math classes. A study by the National Institute for Educational Statistics found that, “In mathematics, for example, between 1982 and 2004, the percentage of graduates who completed a year of geometry increased from 47 to 76 percent, the percentage who completed a semester or more of algebra II increased from 40 to 67 percent, and the percentage who completed a semester or more of analysis/pre-calculus increased from 6 to 28 percent.”

The higher number of credits required means all or most students have to take at least an Algebra or Geometry class in order to graduate. By ninth grade, most students take the Algebra class because they are scheduled for it. After that first year of high
school math, some students find themselves in a couple different situations. One, they may realize their attitude toward math has changed. As they enter the higher courses, a subject that seemed to make sense and come easy to them becomes a difficult subject to understand. Two, they have realized, to an extent, what career path they want to choose. Both of these factors lead to a decreased interest in abstract math such as Algebra 2. The main reason for this decrease in interest is an inability to see how Algebra 2 can be applied to the real world. The students then choose subjects that interest them instead of monitoring the credits they need to graduate.

When their senior year approaches, the students are then faced with the realization that they can not avoid the state requirements. Some managed to take Geometry during their sophomore or junior years. Others did not, and spent their senior year playing catch-up, taking two classes that they have little or no interest in learning.

Most math teachers realize that some of their students do not want to be in their class. They see that this may be a minority of their class population, so they continue to teach the course, as they have taught it previously. This practice more often than not means going through the book, chapter-by-chapter, section-by-section. If they incorporate real world or problem solving math, it’s only because it is an example in the book.

High school students need to be engaged in their own learning. Educators are taught the levels of educational development in undergraduate psychology classes. Any math teacher can tell the reader that up until the age of eleven students need concrete examples. Most teachers can also remind the reader that according to Gardner’s theory of multiple intelligences there are 50 year old men who still need a concrete, relatable
example in order to understand and learn a new skill, because they are tactile learners. Educators must remember that the developmental levels are a guideline, nothing more. Students of all ages must have relatable and concrete examples to learn from.

_Hypothesis:_

The best way to ensure the exiting seniors and juniors can be interested in math and see how they can harness the usefulness of math, is to teach math as a problem-solving-based course. By incorporating this kind of teaching and learning method, a student can construct his own methods of solving problems. Too often seniors are being pushed through rote memorization of formulas and they do not understand anything about the problem or the solution. They may know the steps, but they do not realize the need for a solution.

In problem-solving-based instruction, students will receive a “problem,” not a line of letters and numbers. They will have to read the question and be able to locate the information they need. Then the learning is their job. With all of the various ways to solve problems, the students have a toolbox stocked with the right tools. All the students will not solve the problems the same way, because they have not been force-fed a “right” way of doing it. Instead they will find a way that makes sense to them.

Students can then be presented with a similar problem to see if their method works every time. If it does, then they have solved the problem and they can share their method with the class. For example, a calculus teacher at a high school explained that he would even start calling their method by the student’s name. If a student named Barnes, figures out the Pythagorean theorem, it becomes Barnes theorem for that class. This gives the students ownership in their learning.
The problem-solving-based method also uses high order thinking skills. The students must think critically and transfer what they have learned in the past to this new problem. An example of this would be the inevitable math question involving a farmer with cows and chickens. He knows there are 54 legs and 26 heads in all, but he’s not sure how many of each animal there are. Students know from previous experiences that cows have four legs and chickens have two legs. Although all problems are not as simple as this, past experiences will still be used.

The researcher hypothesizes that through problem-solving-based instruction, the third period Algebra 2 class at Williamstown High School will have a greater increase in test scores than the eighth period Algebra 2 class. The null hypothesis is that the students’ scores in third period will stay the same or increase less than the grades of eighth period. Both classes are a heterogeneous mix of fewer than 10 students with comparable abilities. There are three girls in each class and seven boys. All of the students have the same ability to participate in class and learn the concepts. Third period has a lower test average, 80, than eighth period, 95. However, the averages for third period for in-class activities and quizzes are higher than those of eighth period.
Teaching problem solving is not a new idea. In 1897 John Dewey suggested this strategy. When writing his *Pedagogical Creed* (1897) he stated that the goal of education was to prepare the student for situations he would face in the future. He continued by saying that due to changes in society and technology there was no way to determine an exact situation to expect for the student (1897). Therefore a way of finding solutions to a variety of everyday problems was imperative.

Dewey also suggested a theory of “learning by doing.” He understood that in order to use the processes needed to solve problems, the student would need to practice the problem solving steps. The contrast of solving problems is rote memorization and fact knowing. Such practices were being used at Dewey’s time and continue to be used in the straight-rowed, whole-class, lecture driven classroom of present. Dewey’s learning requires a different classroom, a room designed for working out problems; areas for thinking, calculating, and doing.

Many educators have adhered to Dewey’s thoughts on education to a certain extent. They agree that science students should be doing lab experiments and inquiry learning. They also support the real life environment of vocational education. When the discussion of “doing” arises in math education, however, educators become lenient. Some may say that a fifth or sixth grader should not be allowed to “play” with pattern blocks because he should be memorizing his multiplication facts. These teachers do not realize that in actuality by manipulating the blocks the student has the opportunity to see the definite patterns that emerge in multiplication. If the pattern in internalized, not only will
he know his facts, but he can also recall the pattern and perhaps use it to help with another situation.

When a student is in elementary school, teachers are in complete support of giving the students the chance to “do” math. This is due largely to the cognitive theories of Jean Piaget. In his theory Piaget (1955) outlines a generic list of ages and their corresponding cognitive abilities. According to this guideline students up to the age of seven need concrete examples in order to best understand a concept. Seven year olds are generally in the first or second grade. It is no surprise to see counting blocks, cash registers, carpentry tools, and pattern beads in these classrooms. From seven to eleven years of age the student’s thinking begins to shift more to abstract reasoning, as seen in Piaget’s theory. This age group includes third through fifth grades. Some manipulatives are still present, but become resources that may be tucked away in a cabinet and only used for one or two specific lessons. A classroom visitor would rarely find a cash register or a carpenter’s square when she walked into a fifth grade room.

As the student progresses in school, math becomes a number and operations class. No longer is a student encouraged to work with his peers to figure out a problem. Instead he is required to sit and learn seemingly mindless steps, an algorithm designed to solve one specific set of problems.

In the 1930’s through the 1950’s this changed temporarily because of John Dewey’s ideas and an educational movement called progressive education. Progressive education is, according the Alfie Kohn, an author and lecturer, the following: student-centered, community driven, student-teacher partnered, fair and just, motivation building, purposeful, and active (2008). Each component of progressive education has a particular
purpose. By being student-centered, the student is more motivated to learn a concept he wants to learn about or a concept he is interested in. Community driven education again makes the learning relatable to the student while teaching him about his role in society. If the student learns in a fair and just environment he will understand the importance of justice and citizenship. The student enters the progressive classroom with the motivation needed to approach new problems and work with his peers. He also walks into the room knowing what is expected of him from his teacher and classmates. Lastly, the student is active in his own learning. He talks and collaborates with his peers. He is presented problems and is allowed the time and the tools to solve them successfully.

The progressive education approach to learning is present in the textbooks of the time. *Everyday Algebra: Elementary Course* is a math textbook published in 1951. This textbook does not start like the algebra books of present. *Everyday Algebra* is “The Place of mathematics in Human Affairs.” It discusses how math evolved and the presence of math in everything the student encounters. The introduction explains that math is a universal language “because the world is incurably mathematical”(Betz 1951, p4). The author does not make this statement in order to scare the student, but to empower the student. The proof for empowerment is found in Betz’s previous paragraph. Here math is described as “a truly universal servant of mankind”(p4). Betz encourages the student to see numbers as something he can control and take possession of, and employ.

Each section of *Everyday Algebra* begins with a real-life problem. The author allows the student to solve the problem arithmetically, and then shows him how to solve it algebraically. The first section explains abbreviations seen in day-to-day life. The student is then asked to use symbols such as $h$, $l$, and $w$ to label the sides of a school
building. The last section does not give a specific problem, but states that the concept of quadratic equations will appear in science and in engineering. Further into the lesson it gives an example of using the quadratic equation and its graphed parabola to make headlights and telescope mirrors (Betz 1951). This book is a prime example of the progressive era in education.

As the researcher began the journey to become a teacher in the late 1990’s, the education books of that time also reflected similar practices of “real-life” learning. The article “Constructivist Theory in the Classroom: Internalizing Concepts Through Inquiry Learning” is found in a compilation of articles required for Education students. This article discusses what the authors call the Learning Cycle. The learning cycle consists of three phases: exploration, discussion and new content, and application (Bevevino, Dengel, and Adams 1999). These three steps lead the learner through active thinking and problem solving.

The focus of the article is on history, but the same steps can be applied to math. In mathematics, the students are presented with a problem and encouraged to find a solution. Once a solution is determined they discuss how they arrived at their answers and compare methods and solutions. There is then time for teacher input and scaffolding to introduce the new topic. The students are then given another problem or set of problems to practice with and apply the new concept.

Other articles from the late 1990’s also show similar teaching methods. In 1996, J. Hiebert and others wrote, “students should be allowed to make the subject problematic.” In the same literature the authors ask educators to give the students the opportunity to ask why certain techniques or concepts work and how they are connected to one another. By
having access to this knowledge, the student can add methods and relationships to his repertoire. When faced with a new problem they have two new tools. They can recall the steps and see if the steps work for the new problem. They can also look at the problem and see a similarity to the solution of a previous problem. This would lead to the strategy of solving a simpler problem, which is one of the methods taught in problem solving.

Heibert and the other authors of the article also refer to John Dewey. They discuss his reflective inquiry and his theories on acquiring knowledge (knowing) and application (doing) (1999). Dewey’s reflective inquiry was developed in 1910 and includes identifying a problem, making a hypothesis, collecting data, analyzing and interpreting data, and constructing conclusions (Burden and Byrd 1999).

In 2000, Micheal Lawson and Mohan Chinnappan used Hiebert and his colleagues’ work to focus on problem solving. Lawson and Chinnappan state that the purpose of “mathematics education is to devise ways of encouraging students to take more active roles in the acquiring, experimenting with, and using mathematical ideas and procedures” (2000). Lawson and Chinnappan understood the need for students to “play” with math and find the patterns and relationships it possesses.

Also, they express that better problem solving skills can lead to better understanding of the content and its application. When the students fully understand a concept they can see how it connects with other concepts. For example, if a student graphing absolute value equations understands what each part of the equation does to the graph (moves it left/right, moves it up/down, or opens it further) then he will have a better understanding of how the variables affect the graph of a quadratic equation. The authors call this “knowledge connectedness” and insist that it needs to be learned and
maintained because it is required for problem solving (2000). Besides a lack of knowledge connectedness, they site “persistence” and “ineffective use of clues” as reasons problem solving may not be successful (2000).

Lawson and Chinnappan’s research was conducted in Australia and involved a group of all male students in eleventh grade at a private school. The students participating had volunteered to be part of the research. Lawson and Chinnappan used elaboration tasks, which had the students compare and contrast different geometry theorems used on the same problem. The students created their own connections between the theorems (2000).

The research consisted of a group of high achievers and a group of low achievers. The researchers were looking for differences in content and connectedness. The results of the study showed differences in both aspects. The higher ability students scored higher on all tasks and showed more content knowledge and a higher ability to access that knowledge. The low ability students depended quite a bit on the hints issued by the administrator. The conclusion of the survey as stated by the researchers is that “classroom instruction time should be allocated to display and discussion of the schemas that students develop for topics within their mathematics program” (2000).
CHAPTER 3

METHODS

Participants

The participants of this study included the nine students in Algebra II during third period and the nine students in Algebra II during eighth period at Williamstown High School in Williamstown, West Virginia. The third period class had three girls and seven boys, as did the eighth period class. The students in the classes had comparable abilities. The researcher was the instructor for both classes.

Instruction Conditions

Two convenience groups were identified. A pretest was administered to both groups. This allowed the researcher to have one score to compare with the posttest score. This also allowed the researcher to see if any participant had more previous knowledge about the concepts.

The teacher/researcher continued to teach the predetermined Algebra II chapter in eighth period, the control group, as it had been presented all year. This generally included an introduction tying the new concept to a previously learned concept. Then the teacher did a few examples in a whole class discussion. When the students could talk the teacher through the steps with minimal hints, the assignment for the section was given from the textbook, McDougal-Littell’s Algebra 2. The assignment consisted of approximately ten algebraic questions of varying difficulty and two or three word
problems. The teacher circulated around the room addressing individual questions as they arose and reiterated the steps used on the board. After students were permitted ample time to complete the assignment, the teacher did the word problems with the students on the board, tying the current lesson to the problem. This teaching method showed the dependence of the teacher to make the connections initially for the student.

The other group of students, the experimental group, was presented the same concepts and spent the same amount of time on each concept. With these controls in place, the teacher implemented the treatment. This group was given a word problem to solve that incorporates the new concept. After being given the problem the students worked individually to solve the problem. After five minutes they began to work with a partner. As partners they developed a plan to solve the problem and recorded this plan and their results.

Once each pair had an answer, they presented their results and steps to solve the problem. As a whole class, the students and teacher discussed the answers and compared and contrasted the different methods. The teacher led the students to the method that lent itself to the new concept. They reviewed the method and practiced it on another word problem and a few algebraic problems. An assignment was then given to the experimental group consisting of ten algebraic problems and two or three word problems.

After the predetermined chapter was concluded, the posttest was distributed to both groups. The posttest was twenty-five questions and address the following aspects of the chapter: recognizing terms and vocabulary (Matching), identifying the processes/formulas needed to solve the problem (T/F or short answer), evaluating various
numerical/algebraic problems (Multiple choice/Constructed Response), and applying the knowledge to novel problems (Constructed Response/Short Answer).

Measures

The results of the pretest and posttest were analyzed to determine if there was a positive correlation between the problem-solving-based teaching method and the students’ scores. Individual scores were converted into t-scores and a correlation coefficient was calculated. The researcher completed these calculations, with any additional analysis by a SPSS computer program.

The favorable results for the research was a positive correlation coefficient close to 1, which would show a strong positive relationship between the teaching strategy and the scores. If the research showed that there is little or no correlation between the teaching method and the test scores, the correlation coefficient would be between 0 and –1. A coefficient in this range would mean the strategy had no affect or had an adverse affect on the students.
CHAPTER 4

RESULTS

Pretest

Comparison of Pretests

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</tr>
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<tr>
<td>Standard Deviation</td>
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The pretest for the chapter on quadratic equations was given over a period of two days. This allowed those who were absent the first day to complete the test before the class began the new content. Both classes were instructed to read and attempt every problem. The majority of the students followed these instructions and their previous knowledge was evident on their test. An example of this was one student’s attempt at question 8, which was to solve for the square root of –9. He wrote out 3 times 3 equals 9, and –3 times –3 equals 9. All of his calculations were correct, but the problem required information he had not learned to solve correctly.
Posttest

Comparison of Post-tests

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The posttest was completed in one day because all students were present. The students in the experimental group raised their grade an average on 15.4 points. This was fifty percent higher than the average increase of 10.2 points in the control group.

The correlation coefficients were calculated for both groups of Algebra II students to determine the relationship between the teaching strategies used and their effect on the posttest scores. Both groups showed a positive correlation.

Correlation Coefficients

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<tr>
<td>Correlation Coefficient</td>
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<td>0.19</td>
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CHAPTER 5
DISCUSSION

Analysis of Results:
The pretest scores were another proof that the two classes were comparable. The similar average test scores showed that all the students had a limited understanding of quadratic equations and their applications. The high accuracy of the answers to the multiple choice questions can be attributed to guessing or deduction.

The results of the experimental group’s post-test showed a greater increase in the knowledge and understanding of quadratic equations than the control group, the experimental group’s average score was six points higher. A correlation coefficient of .58 showed a very strong positive correlation between the teaching strategy and the increase of scores. The hands-on problem solving activities used to introduce new concepts worked as they were predicted to work in the hypothesis.

The correlation coefficient of .19 for the control group shows that the text-based teaching method was also effective, but only slightly effective. This group’s scores showed that teaching straight from the book and in the same order as the book was not as effective as problem-solving based instruction. This fact supported the hypothesis stated in the introduction.

Future Implications

The most immediate implication of this data made was a need for the change in teaching strategy for the rest of the school year. There were changes that took place in the classroom environment as a result of the problem based teaching strategy. Prior knowledge was used and encouraged. Students who know about engine sizes started a
word problem about engines with more confidence. The students began working together to solve problems, instead of depending on the teacher as the only expert in the room. The behavior of the students even changed, they listened to instruction and asked more questions. The experimental group looked at the post-test as a challenge to do well and show their newfound interest in Algebra II.

Although the school system required most students to take an Algebra II course, they expected the students to learn at the same pace. A suggestion for future years was to include a specific course titled *Applied Algebra II*. This would give the concrete learners in the group the same, if not a better chance to learn the material. The term *applied* often carries a negative connotation, so a more positive name may be recommended.

The process of changing to problem solving based instruction was not difficult. The teacher just rearranged the order the lesson is presented in the book. Support materials included a hands-on or concrete word problem for most sections. Some work was required for a section or two that did not have an example. A little creative thinking that all teachers do anyways. Textbooks evolved away from application problems; instead they focused on a step-by-step approach that generally only works for one set of given problems. Problem solving encourages learning new concepts and learning how to face new challenges.
REFERENCES


