LARGE AREA TUNABLE LIQUID CRYSTAL LENS

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by

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DEDICATION

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CHAPTER 1

Introduction

Experts have always seen large area electronic lens as persuasive for many applications including virtual reality systems [1], imagers [2], [3] and adaptive eyeglasses to correct the refractive errors in the human eye [4], [5], [6]. Tunable liquid crystal (LC) lens represent a mature field which is now being spun out into commercial applications [LensVector, PixelOptics, DigiLens, DeepOptics]. Large area LC lens with a diameter of more than 10 mm, however, have still face challenges. This dissertation aims at finding a solution for this challenging issue. The objective is to devise and implement a large aperture tunable LC lens with 20 mm diameter and an optical power range of 0 to about 2 D\(^1\) for mainly two near-to-eye applications, first, focus cue in Virtual Reality (VR)/ Augmented Reality (AR)/ Three Dimensional (3D) displays and second, variable-focus vision glasses. The following sections explain the significance of designing large aperture electronic

\(^1\) D stands for diopter that is the unit of optical power and equals to reciprocal of the focal length (in meters).
lenses for these applications. In addition, an overview of the design criteria and previous works are provided.

![Figure 1-1 Illustration of emulation of a window by a display.](image)

1.1 Large Aperture Electronic Lenses for VR/AR/3D Displays

In the real world, the eyes accommodation (focus) and convergence distances are always equal. In 3D display systems, the eyes converge to the displayed 3D objects which can be behind (far distance) or in front (near distance) of the display but the accommodation of the eyes remains fixed on the (apparent) display distance. The mismatch between accommodation and convergence distances causes an unnatural viewing experience. If the display is far enough, as it is in Cinema Theater, display and 3D object distances are at infinity and this issue is negligibly observed. As the display distance to the eye decreases, for instance in 3D displays or Head Mounted Displays (HMDs) including VR and AR, accommodation-convergence (AC) conflict becomes significant and results in user discomfort, eye strain and sometimes headache. In recent years, researchers have investigated a variety of
approaches to solve AC issue. Light field displays including integral imaging and virtual retinal [7], [8], [9], [10], multifocal [11], [12], and digital holography displays [13] are among the main technologies that address the issue.

Light field displays project multiple perspectives of the virtual objects into different parts of pupil. To elaborate more on the requirement of light field technology one can consider emulation of a window by a display. Considering that the window has patches, below the resolution limit of the eye, that are replaced by pixels of a display where each pixel emits a bundle of rays each with color and intensity corresponding to a patch in a window as illustrated in Figure 1-1.

If several angularly distinct rays for a point in the object space are intercepted by the pupil, the eye lens will focus them to a point on the retina. The further away the point being considered, the more parallel the intercepted rays, and the lower the required eye lens power. Takaki showed that if a display provides enough angular resolution, the eye will focus correctly [14]. He demonstrated a display having 0.23° angular resolution that was shown to provide the accommodation cue. Therefore if a 0.2° angular resolution of the pixels is considered (color and intensity of each pixel can have a different value for a 0.2° change in angle) with a HMD that has a defined eyebox, the number of distinct rays from each pixel can be calculated. For example if the eyebox is 1 cm on a side, and the apparent display distance is 0.5 m, the largest angle the pixel needs to emit will be 0.57° and thus each pixel needs to emit
Therefore the bandwidth of display must be 16 times that of a 2D display with the same pixel count for a HMD.

Love [12] used four field sequential volumetric display (multifocal) system to provide focus cue which requires only 4X increase in bandwidth, but the four-field sequential system is still a stretch for existing technology.

Although recent development in digital holography made compact displays with large fields of view possible, the display etendue, which is the product of field of view and viewing eye box, remains constant [13]. Therefore increasing the field of view results in a decrease in the viewing eyebox. Small eyebox size leads to image disappearance when the eye rotates. Maimone demonstrated a digital holographic display with high image quality presenting focus cue but with about a millimeter eyebox [13]. Thus pupil expansion is required in digital holographic display using additional elements such as beam steering devices.

Accommodation-invariant (AI) computational near-eye display is another approach proposed to solve AC issue [15]. AI approach is the only one-for-all solution to AC problem that has the potential to correct the refractive errors of the users, however, the image resolution is compromised.
Another solution to the AC issue is to use adaptive focus. The Oculus Half Dome prototype is a technology that uses the adaptive focus approach. In this prototype the display distance is mechanically changed to provide focus cue [16].

We propose another adaptive focus approach in which the focusing power of VR lens is electronically changed in combination with an eye tracking system. In this case an electronic lens will be added to the existing VR system. Therefore the AC issue can be solved without reduction in image resolution or the need to increase the system resolution and refresh rate [1]. A recent study by Kramida and Varshney which performed an extensive literature review and provided assessments of benefits and limitations of each proposed solutions for AC problem concluded that eye-tracked varifocal optics with LC lenses have the highest potential for solving the AC problem [17]. To avoid reducing the field of view of VR system, the aperture size of LC lens must be large (preferably as large as the VR lens aperture size). This dissertation targets the design of LC lens with 20 mm diameter, which provides minimum form factor that is acceptable for an eyewear. The next section explains the required optical power of such a lens for resolving the AC issue in HMD and 3D displays.

1.1.1 VR Optics with an Adjustable Focal Length

In a conventional VR system, the lens shown in Figure 1-2 has a fixed focal length, but in the system considered here the VR lens has adjustable power. There
are two conditions that are the starting points for determining the range of adjustment required. First is that the display is focused on the retina:

\[ D_e + D_{VR} = \frac{1}{TDD} + \frac{1}{ED} \]  \hspace{1cm} (1.1)

where \( D_e \) and \( D_{VR} \) are the powers in diopters of the eye lens and the VR lens, TDD and ED are the display distance to the eye lens and the distance from the eye lens to the retina, respectively. And second is that the eye lens will be accommodated to the apparent display distance (ADD).

\[ D_e = \frac{1}{ADD} + \frac{1}{ED} \]  \hspace{1cm} (1.2)

Therefore the power of VR lens is given by:

\[ D_{VR} = \frac{1}{TDD} - \frac{1}{ADD} \]  \hspace{1cm} (1.3)

If we would like to electrically adjust ADD, The range of power of the VR lens is determined by the range in ADD:

\[ \Delta D_{VR} = \left( \frac{1}{TDD} - \frac{1}{ADD_{\text{max}}} \right) - \left( \frac{1}{TDD} - \frac{1}{ADD_{\text{min}}} \right) = \frac{1}{ADD_{\text{min}}} - \frac{1}{ADD_{\text{max}}} \]  \hspace{1cm} (1.4)
Figure 1-2 Illustration of Virtual Reality System. TDD is the true display distance. ADD is the apparent display distance (user eyes focus at this distance by means of VR lens). AOD is the apparent object distance (left and right eye converge at this distance).

Ideally for no accommodation-convergence mismatch, \( ADD = AOD \) where AOD is apparent object distance, so the ideal range of the power of the VR lens is given by the above expression with AOD substituted for ADD.

\[
\Delta D_{VR,\text{min}} = \frac{1}{AOD_{\text{min}}} - \frac{1}{AOD_{\text{max}}} 
\]  

(1.5)

Based on Shibata’s analysis 0.5 diopter or smaller mismatch between eye focus and convergence angle can be still considered in viewer’s eyes “zone of comfort” [18]. Therefore to be within the limit of “zone of comfort”, the power of VR lens may be off by 0.5 diopter thus;

\[
\Delta D_{VR,\text{min}} = \left(\frac{1}{AOD_{\text{min}}} - \frac{1}{2}\right) - \left(\frac{1}{AOD_{\text{max}}} + \frac{1}{2}\right) = \frac{1}{AOD_{\text{min}}} - \frac{1}{AOD_{\text{max}}} - 1
\]  

(1.6)
If we assume $AOD_{\text{min}}$ is 0.5 meters and $AOD_{\text{max}}$ is infinity, then the range of power of the VR lens for perfect accommodation is 2 D (equation (1.5)), and the minimum to stay in the zone of comfort is 1 D (equation (1.6)). To control the power of LC lens, an eye tracking system should be used to determine the distance of the object being considered by the user (AOD). More specifically a camera can locate the “toe-in” of pupils to determine the distance from viewer to virtual object. AOD then can be obtained from $x$, which is the offset distance of the pupil of the eye from its location when looking an object at infinity, user’s interpapillary distance (IPD) and the radius of the eye-ball (ER) as shown in equation (1.7).

$$AOD = IPD \times \frac{ER}{2x}$$

(1.7)

The above indicates that if the range of AOD determined by eye tracking system is limited to be from 0.5 meters to infinity, the accommodation-convergence mismatch problem can be solved completely with a 2 D variable lens, and minimally with a 1 D variable lens. This research aimed to design large area LC lens (diameter of 20 mm) with about 2 D optical power.

1.2 Large Aperture Electronic Lenses for Vision Correction of Presbyopic Eye

A large-aperture electronic lens can also be used in ophthalmic applications for the presbyopic eye with other vision problems. Presbyopia is the failure of the eye
lens to accommodate. The continuous decline of accommodative amplitude begins
during childhood and by the time that individuals reach their 35’s-55’s, the eye lens
ceases accommodation completely [19], [20]. Bi/trifocal and progressive glasses
have been extensively used by presbyopic patients who required prescription
glasses. However these approaches carry with them various well known limitations.
Bi/trifocal glasses provide two/three zones of fixed dioptric power that are
distinguished with a discontinuity known as “ledge” [21]. As the sightline passes
through the ledges, the recognized issue of “image jump” occurs; which is a sudden
change in the image size and location. The other two major drawbacks of bifocal
glasses are first, the prismatic effect of low diopteric power region resulting the
displacement of the object and second, the lack of intermediate vision correction
[22]. In contrast with bi/tri focal, progressive glasses provide a seamless increase in
positive optical power without any ledge. This continuous power progress is
obtained by introducing different amount of surface astigmatism in the horizontal
region of the lens surface. The peripheral undesired surface astigmatism or
cylindrical power increases with dioptric power of progressive lenses [21] which is
perhaps the most serious disadvantage of such glasses. Moreover, in all of bi/tri
focal and progressive glasses, the subjects are required to gaze down to see a sharp
image of an object at a close distance, which is not the most preferred.
The assessment of bi/tri/multifocal glasses has heightened the need for variable focus lenses which allows the full field of view at different focus distance. These multi-focal lenses must meet a certain criteria including extended aperture size (> 15 mm), fast response time (> 1.127 ± .658 \( \text{diopter/sec} \) [23]), compact shape, optimized weight and continuous tunability between .25 D to about 3.5 D. These should be also not hazardous to human vision and provide excellent visual acuity. Besides solving AC issue in VR/AR/3D, the other motivation of this research was to design the LC lens such that it satisfies the requirement for ophthalmic application which are similar to ones demanded for VR/AR/3D application.

1.3 Previous Work on Electronic Lenses

The design of tunable lens has been widely investigated on the basis of both refractive and diffractive optics. Refractive tunable lenses can be acquired by either changing the radius of curvature or the difference in refractive index. Electrowetting and the use of deformable membrane, are among the approaches that have been explored to modulate the radius of curvature. In an electrowetting-driven liquid lens, the shape of the spherical meniscus between two immiscible liquids, one conductive and the other non-conductive, of equal density with different refractive

---

\(^2\) Maximum accommodation speed measured for middle-age participants. The eye focus changed from a far target (4 m) to near target (0.7 m) during the experiment.
index, is changed by the electrowetting effect. The bulky system and high voltage required for switching (typically between 50-100 V) inhibits the use of such lenses for near-to-eye applications [24], [25]. In a deformable membrane liquid lens, a fluid is pumped into a cavity covered by a transparent membrane. The shape of membrane can be tuned using either electrostatic or piezoelectric actuators [4], [26], [27]. The operating voltage of liquid lens is also substantially high (>100 V), which makes them unfavorable for vision industry or VR/AR/3D applications. Previous studies have reported the use of linear (Pockels) and quadratic (Kerr) effects of electrooptic materials such as lead lanthanum zirconate titanate and lithium tantalite to design tunable lens. The fabricated lenses have practical constraints including high operating voltage (in some case >300 V), small aperture (~2mm) and long focal length (> 1 D) that makes them not feasible for near-to-eye applications [28], [29], [30].

A considerable amount of literature has been published on LC-based tunable lenses. A clear benefit of an LC lens is its low operating voltage (a few volts). Nematic LCs are anisotropic elongated rod-like molecules whose time-averaged long axis can be oriented with external voltage thereby the refractive index seen by an optical wave can be tuned accordingly. The design of refractive LC lens by filling a hollow convex or concave substrate and applying uniform voltage to LC medium was first proposed by Berreman and Sato in late seventies [31], [32]. Later, around
1990, Sato et al. designed the first LC microlenses [33], [34]. The fabricated lens based on this approach either had a very slow response time due to the large thickness of the LC layer [32] or in order to reduce the thickness, the aperture size was limited to 1 mm [35].

The first cylindrical lens by the use of several electrodes in the structure was designed in 1982 by Cleverly [36]. Years later in 1998 Naumov proposed a technique to reduce the number of required electrodes to drive the LC lens. In order to create the radial change in voltage across the lens aperture, he suggested to deposit a high resistive layer onto the patterned electrodes thus requiring only one voltage [37, 38, 39].

One of the most successful refractive LC lens designs, which was the base design for a commercial product emPower for ophthalmic application by PixelOptic [40], used cholesteric LCs between two smooth and patterned substrates coated with an optically transparent electrode [6], [41], [42]. In this design, material properties including pitch and birefringence of cholesteric LC and incident wavelength are adjusted as the incoming optical wave parallel to the helix sees only the average refractive index. This design has the advantage of polarization insensitivity in comparison with designs using nematic LCs. However, the exact control of the phase profile with only two interconnections may be difficult and leads to optical artifacts.
Excellent control of the phase profile and diffraction-limited performance was achieved using discrete ring electrode design [43]. However, the aperture size of the designed refractive lens (2.4 mm) is not sufficient for our applications of interest [43]. The use of discrete [44], [45], [46] and resistive [47], [48] electrodes to produce the desired phase profile were studied in many other designs but they were also limited to minimum optical power < 1 D for lens diameters ≥ 10 mm.

A well-established approach to obtain a large aperture lens is to use the Fresnel structure to achieve a cost-effective, light, and compact lens. Numerous studies have attempted to design tunable Fresnel lenses using LCs based on different physics that are collectively called Fresnel LC lenses. There are three types of Fresnel lenses: Fresnel zone plate lenses, which include areas that have a phase difference of a half-wave to the adjacent areas; diffractive Fresnel lenses, which have segmented parabolic phase profiles where the segments are small and can result in significant diffraction; and refractive Fresnel lenses, which have a segmented parabolic profile where the segments are large enough so that diffraction effects are minimized, which are called here Segmented Phase Profile (SPP) lenses.

The first LC Fresnel-zone plate was proposed by Clark in 1989 [49]. After Clark’s work, large and growing body of literature has investigated numerous approaches to design Fresnel zone plate LC lenses [48-64]. The ultraviolet alignment of polyimide to induce a discontinuous distribution of the LC refractive
index [50, 51, 52, 53] and patterned polymer relief were two approaches that have been explored for the design of Fresnel zone plate LC lenses [54, 55]. Dye-doped LC [56, 57, 58] and combinations of polymer/LC [59, 60, 61, 62, 63, 64, 65, 66] are among the other approaches to the design of Fresnel zone plate LC lenses. The most successful design of Fresnel zone plate was based on employing discrete ring electrodes, which resulted in precise control of the phase profile [5, 67, 68].

In 1996 McManamon proposed to use an array of phase shifters also called an Optical Phased Array (OPA) to design diffractive Fresnel LC lenses [69]. Later, Lu used McManamon’s suggested approach to design a diffractive Fresnel LC lens based on linear electrodes [70]. Another diffractive Fresnel LC lens was designed by filling the Fresnel lens substrate with LC as an index matching fluid [71]. Beside significant diffractive issues, in this design maximum diopteric power was obtained at voltage off-state because of the index-matching of LCs which is not desirable for ophthalmic application. Although Fresnel zone plate and diffractive Fresnel lenses can potentially provide a large aperture with a smaller thickness, their strong wavelength dependency inhibits their application.

The ideal variable focus lens design for near-to-eye applications would be achromatic like refractive lenses as well as thin, fast and powerful similar to diffractive lenses. The ultimate aim in the lens design is to generate a parabolic spatial variation of the phase that causes the transmitted beams to converge to a
point. Parabolic phase is acquired primarily by two main approaches. First is to control the phase gradient by the optical path difference \( OPD = \Delta n \cdot d \) where \( \Delta n \) denotes the effective birefringence and \( d \) represents the thickness.) and second method for modulating the incident light is to utilize Pancharatnam phase. All the aforementioned designs control the optical path difference to achieve the parabolic phase profile. Among all designs the method that most successfully controlled the refractive index profile of the LC director is the discrete ring electrode design [5, 6, 43] that showed diffraction limited performance for 2.4 mm aperture size. This dissertation describes the design, fabrication and characterization of LC lens based on discrete ring electrodes with the significantly larger aperture size of 20 mm and an optical power swing of 0 to 1.5 D. To expand the clear aperture without compromising the response time, phase segments with sufficiently larger width are introduced into the phase profile. The diffractive behavior of proposed design, the so-called SPP LC lens, is carefully studied both numerically (Chapter 2) and experimentally (Chapter 3) [72]. To further enhance the optical power to 2.5 D, in Chapter 4 an improved hybrid design based on both optical path difference and Pancharatnam phase concepts is introduced for VR/3D applications [73]. Chapter 5 takes a new look at the application of Pancharatnam phase lens in correcting chromatic aberration of optics with large diopteric power that are commonly used in VR systems [74]. A summary of this dissertation and areas for improvement are
included in Chapter 6. User evaluation tests for both ophthalmic [75] and 3D applications are performed in this research and discussed in Chapter 3 and 6, respectively.
CHAPTER 2

Design and Numerical Simulation of Segmented Phase Profile LC Lens

There is large body of closely related work in LC lenses that spans several decades; Chapter 1 selectively discussed several sets of work. A comprehensive overview of LC lens research can be found in [76]. We draw on much related work. Like Li [5] and Valley [77], we used discrete ring electrodes for precise control on the phase profile. Like Riza [46] and Naumov [37, 38, 39], we used resistor networks to minimize the number of external connections. Like Pishanyak [78] and Asatryan [79], we used a stack of two LC lenses to enhance the off-axis performance. Like Li [43], we used floating electrodes to eliminate the diffraction and thus haze resulted by gap between the electrodes. Finally we have introduced phase segments on the lens profile to obtain large area LC lens with reduced effective thickness and without compromising the response time. Phase segments were designed sufficiently large that the resulting diffraction angle is not observable to human eye. The designed lens is continuously tunable between 0 to 1.5 D and has an active area 20 mm in diameter. The following sections elaborate the design details.
2.1 Optical Path Difference Equation

The LC lens design discussed in this dissertation is based on the principle of GRadient in the INdex of refraction (GRIN) lens.

![Illustration of focusing of parallel rays by LC lens where there is GRadient in the INdex of refraction (GRIN lens)](image)

**Figure 2-1** Illustration of focusing of parallel rays by LC lens where there is GRadient in the INdex of refraction (GRIN lens)

In a positive GRIN lens, the refractive index has its maximum at the center of the lens \((n_{max}) \) [80]. The Optical Path length (OPL) of a ray that passes from the optic axis of GRIN lens equals to \(OPL_c = n_{max}d\) where \(d\) is the thickness of LC cell as shown in Figure 2-1. The OPL of a ray that passes from a point with distance \(r\) from the center is approximately equal to \(OPL_r = n(r)d\). For the light to be converged to point F shown in Figure 2-1, the planar wavefront needs to bend into
a spherical wavefront and thus OPLs from planar to spherical wavefront or vice versa along any direction must be equal [80].

\[ OPL_r + \overline{AB} = OPL_c \] (2.1)

Rewriting equation (2.1) taking into account that \( f \) shown in Figure 2-1 equals to \( \overline{BF} \) results in [80];

\[ n(r)d + \overline{AF} - f = n_{max}d \] (2.2)

Substituting \( \overline{AF} = \sqrt{r^2 + f^2} \) gives;

\[ (n_{max} - n(r))d = \sqrt{r^2 + f^2} - f = f \sqrt{\left(\frac{r^2}{f^2} + 1\right)} - f \] (2.3)

Note that for \( 0 \leq r \leq a \), the change in the refractive index and thus Optical Path Difference (OPD) will follow \( n_{min} \leq n(r) \leq n_{max} \) and \( 0 \leq OPD(r) \leq \Delta n.d \), respectively. For a thin lens (\( f \gg r \)) equation (2.3) is approximated with the Maclaurin series and thus the OPD will be equal to

\[ OPD(r) = f \left(1 + \frac{1}{2} \frac{r^2}{f^2} + \cdots\right) - f = \frac{r^2}{2f} \] (2.4)

Equation (2.4) indicates that if the refractive index reduces quadratically from the center of a GRIN lens, a collimated beam converges to a point [80].
2.2 Speed vs Optical Power Basic Design Consideration

Diffraction-limited performance was obtained with a small aperture (diameter of 2.4 mm) LC lens [43]. Difficulties arise, however, when an attempt is made to expand the aperture size (diameter ≥ 10 mm). To further elaborate, it should be noted that the \( OPD \) of an LC lens is proportional to the cell thickness (d) and the birefringence of the LC material (\( \Delta n \)) as \( OPD = \Delta n \times d \). The parabolic phase profile of positive lens shown in equation (2.4) \( (OPD = \frac{r^2}{2f}) \) indicates that expanding the lens radius (r) changes the requirement for the \( OPD \), meaning that increasing the radius of the lens requires a higher \( OPD \), keeping the optical power \( (\frac{1}{f}) \) constant. On the other hand, for a specific material higher \( OPD \) is achieved by increasing \( d \), which is related to the lens radius squared \( (d \propto r^2) \). It is well-known that the response time \( (\tau) \) of an Electrically Controlled Birefringence (ECB) LC cell, which is the time the material requires to recover its original state, is quadratically dependent on cell thickness \( (\tau \propto d^2) \) as shown in equation (2.5) [81]:

\[
\tau = \frac{\gamma \times d^2}{K_{11} \times \pi^2}
\]

(2.5)

where \( \gamma \) and \( K_{11} \) are rotational viscosity and splay elastic constant of the LC material, respectively. The power-law scaling of response time with the cell
thickness indicates a greater lens radius will result in a dramatic increase of the response time ($\tau \propto r^4$).

### 2.2.1 Addition of Phase Segments

The pragmatic solution that allows a large aperture size for the LC lens without compromising the response time is to introduce phase segments in the parabolic phase profile. As a result, a SPP LC lens is obtained. For example Figure 2-2A shows the desired phase profile for a $\pm 0.375$ D LC lens with radius of 10 mm, where the OPD equals to 35 $\lambda$. The actual thickness of the LC cell in this case then would be about $\frac{35 \times 0.543}{0.27}$ or 70 $\mu$m for LC lens with birefringence value of 0.27. To decrease the effective thickness of the LC cell, consider the introduction of phase segments into the phase profile. The general shape of the OPD profile that is considered for a SPP LC lens is shown in Figure 2-2B. To distinguish the optical effect of using the OPD profile shown in Figure 2-2B as opposed to the one depicted in Figure 2-2A one can compute the difference between the two phase profiles, which is shown in Figure 2-3A. In this Figure each OPD step has a height that can be specified as being the sum of an integer number (n) of wavelengths of light ($\lambda$), plus additional optical path length ($\Phi$), thus $OPD \ Step = n\lambda + \Phi$. 
Figure 2-2A. Ideal parabolic phase B. 2D phase map of the designed LC Fresnel lens with 5 segments.
The most significant effect on optical performance occurs when the value of $\Phi$ is $\frac{\lambda}{2}$. In this case one can compute modulo $2\pi$ of Figure 2-3A to obtain the binary
profile shown in Figure 2-3B. It can be seen from Figure 2-3B that the worst case optical effect will be that of a binary grating with a variable period. The effect of the grating in a local region where the width of an OPD segment is \( \Lambda \), is to diffract light mostly to the first-order beams at an angle \( \theta \), given by equation (2.6).

\[
\theta = \sin^{-1}\left(\frac{\lambda}{2\Lambda}\right)
\]  

(2.6)

This indicates that for the worst-case OPD profile depicted in Figure 2-3B, a ray of light from infinity passing through a lens with the OPD profile shown in Figure 2-2B, will have the effect of the phase profile of Figure 2-2A to deflect the light towards the lens focal point. It also will have the effect of the OPD profile illustrated in Figure 2-3B, which most significantly splits that ray into diffracted beams at an angle of \( +\theta \) and \( -\theta \) from the beam direction set by the OPD profile of Figure 2-3B. Figure 2-4 shows a lens with the OPD profile of Figure 2-2A on the left side, and a lens with the phase profile of Figure 2-2B on the right side, for the

**Figure 2-4** Schematic representation of diffraction occurring at \( \lambda/2 \) phase difference.
The worst case of $\Phi = \frac{\lambda}{2}$. The angle between the two rays equals $2\theta$. The angular resolution of the human eye is about 1 arcmin (0.016°) [80]. To design the lens with a negligible diffraction angle for near-to-eye applications, the minimum grating period for a green wavelength of 543.5 nm, should be (using equation (2.6)) larger than 1 mm.

Certain criteria should be met to obtain an ideal LC lens [43]. Most importantly, the phase profile has to be parabolic in the entire optical power range so that the light distribution pattern at the focal length (Point Spread Function (PSF)) is close to the theoretical value of the spot profile of the diffraction-limited lens. In addition, no chromatic aberration or scattering should be observed in the imaging performance of the high-quality LC lens. It was shown that good control of the refractive index profile of the LC director can be obtained using the discrete ring electrode design [5, 6, 43].

2.3 Discrete Ring Electrode and Resistor Network Design Concepts

Using concentric ring electrodes with equal areas one can obtain a step-wise phase profile of LC lens which can be precisely tuned. Factors that have been identified as being potentially important are discussed below.
2.3.1 Shape of Phase Profile

The parabolic shape of phase profile plays a significant role in the final performance of LC lens. The drop in Strehl ratio can be up to 80% by a defocused aberration as small as $\frac{\lambda}{4}$ [82]. It was shown that the discrete ring electrode design is capable of controlling the electric field distribution and thus the shape of phase profile across the lens aperture [83]. In this approach the parabolic refractive index pattern (equation (2.4)) is created by a radial change in LC molecule’s orientation by an external electric field applied to concentric ring electrodes with equal areas ($A \propto r^2$). The voltage profile is calculated using an LC director simulation considering the electrode location, LC material properties and cell thickness. An overview of this calculation is given in Appendix A.

2.3.2 Number of Electrodes per Wave of OPD

The continuous phase profile in a discrete ring electrode design is sampled with phase steps. The analytical diffraction efficiency of the lens relates to the number of steps per wave ($\vartheta_\lambda$) as shown in equation (2.7) [84].

$$\eta \alpha \frac{\sin^2 \left( \frac{\pi}{\vartheta_\lambda} \right)}{\left( \frac{\pi}{\vartheta_\lambda} \right)^2}$$  \hspace{1cm} (2.7)
The higher the number of steps per wave is, the larger the diffraction efficiency becomes. To reduce the phase aberration, a minimum number of 10 steps per wave is suggested [84]. The resulting 96.8% theoretical diffraction efficiency might slightly increase in the real sample because of the effect of the fringing field on smoothing the phase profile [83]. The number of steps/wave or the sampling rate of the phase profile ($\theta_\lambda$) is decided based on the desired diffraction efficiency. Then, the number of required electrodes ($N$) for a lens with radius $r$ and target focal length of $f$ is determined by equation (2.8) [83]:

$$N = \frac{r^2 \theta_\lambda}{2f\lambda}$$  \hspace{1cm} (2.8)

Equation (2.8) indicates that a lens with a radius of 10 mm and an optical power of 0.375 D assuming 9 steps per wave requires 288 electrodes for green wavelength of 585 nm. The outer radius of each electrode ($r_{outer-i}$) can be found by [83]:

$$r_{outer-i} = \sqrt{\frac{2\lambda \theta_i}{\theta_\lambda}} \quad i = [1: N]$$  \hspace{1cm} (2.9)
where $i$ shows the electrode number. As seen in equation (2.9), the electrode geometry follows a parabolic pattern. Figure 2-5 shows the parabolic geometry of electrodes for an example lens with 20 mm diameter and optical power 0.375D. The electrode width decreases with the increase in the slope of the OPD. For the considered example, the radius of first electrode at the center of the lens is 509 μm whereas the width of the last electrode at the edge is 11.55 μm.

![Figure 2-5](image)

**Figure 2-5** Electrode geometry of the designed lens.

### 2.3.3 Methods to Apply Voltages to Ring Electrodes

With a parabolic electrode geometry, one can obtain the parabolic phase when the phase difference between adjacent electrodes is the same. If the phase is proportional to the applied voltage, a linear change in the voltage across the electrodes (the same difference in voltage between any two electrodes) would yield
a parabolic phase profile. To impose a linear voltage drop over a several electrodes, inter-ring resistors can be used. The resistors between electrodes act as voltage dividers. If there was a linear relation between phase and voltage of LC cell, only two interconnections would be required to drive the lens. However, the slope of OPD versus the voltage curve of a LC material does not remain constant but rather becomes steeper for lower voltage values (Figure 2-6). However the OPD versus voltage curve may be segmented to several linear sections, as shown with cyan rectangles in Figure 2-6. Therefore, when using resistors of the same value, the voltage must be defined by a connection to a programmable voltage source at the end points of the linear segments shown in Figure 2-6. The number of linear sections determines the required number of interconnections and bus lines to drive the lens.

2.3.4 Issue with Haze due to Gaps, and the Floating Electrode Fix

A gap between adjacent electrodes is required for applying different voltages to electrodes and adjusting the optical power. Due to different external field in the spacing compared to the electrode area, the tilt angle of the director and therefore the refractive index is different, which results in phase or index aberration [83]. To mitigate this issue, the space between the ring electrodes should be much smaller than the LC cell thickness. The diffraction efficiency drops as the gap between the electrodes expands, which is represented in equation (2.10) [85].
Highlighted cyan rectangles represent the linear sections of the profile that can be used to determine the number of required interconnections to drive the LC lens. BL in this figure stands for Bus Line (the interconnection)\(^3\).

\[
\eta \propto \left(1 - \frac{A_F}{A}\right)^2
\]  \hspace{1cm} (2.10)

In this equation, \(A_F\) is the area of electrode spacing and \(A\) is the total area. As an example, the analytical value for the diffraction efficiency of a lens with a diameter of 20 mm, a total electrode number of 288 and a 3 \(\mu\)m gap between electrodes equals to 83.46%. In addition to the analytical prediction of diffraction efficiency, the effect of gap between electrodes can be numerically calculated.

\(^3\)Black squares on the curve represent the measured phase of a uniform 20 \(\mu\)m cell filled with MLC 18349. The red curve is calculated by the liquid crystal relaxation method described in Appendix A.
considering electric field distribution. Previous calculation where the effect of fringing field were taken into account shows that gap larger than 1 μm results in noticeable phase bumps [83]. The magnitude of phase bumps changes radially; it increases close to the edge of the lens where the voltage of the electrode is much higher [83]. The phase variation due to the gap between electrodes in discrete ring electrode design decreases the dark level of the image, which is observed as haze due to variable grating periods. The effect of haze induced by electrode gap leads to a drop in the low frequency Modulation Transfer Function (MTF) [83]. For a better image quality the electrode gap should be as small as possible. Separating the odd and even number electrodes in two layers can potentially eliminate the gap between electrodes, however approaches of this kind carry with them various well-known limitations such as difficulties in the alignment of multi layers and the need to address electrodes of every layers [67]. A more suitable approach is to use ring electrodes that are floating above the driven ring electrodes. These “floating” electrodes are not driven by ohmic connection, but are capacitively coupled to the driven electrodes [83]. The floating electrodes are designed to cover half of the area of each of neighboring driven electrodes. The voltage of the floating electrode have intermediate voltage values of two neighboring electrodes as each floating electrode generates two capacitors connected in series with the underneath driven electrodes. Another advantage of using floating electrodes is the increase in sampling rate
which helps smooth the phase profile and enhance the diffraction efficiency. It was shown that implementing the floating electrodes in the LC lens configuration improved not only the Strehl ratio by 10% but also the MTF profile [83].

2.3.5 Issue with Viewing Angle and Fix with Multi-Cell Approach

Another important factor in the LC lens design is the effect of viewing angle. The effective refractive index of an LC material is inherently dependent on the angle between the optic axis and the propagation direction of incident light as shown in equation (2.11) [81].

\[
n_{\text{eff}} = \frac{n_e n_o}{\sqrt{n_e^2 \cos^2 \theta + n_o^2 \sin^2 \theta}}
\]  

(2.11)

Here \( n_e \) and \( n_o \) are the extraordinary and ordinary refractive index of LC and \( \theta \) is the angle between the optic axis and normal to the polarization direction of incident light. Additionally the OPD of LC material defines as

\[
OPD = (n_{\text{eff}} - n_o) \cdot d
\]

(2.12)

where \( d \) is the thickness of LC cell. As seen in equations (2.11) and (2.12), the OPD of LC lens is influenced by the angle of incidence. To improve the viewing angle performance of the LC cell, two cells with opposite rubbing direction will self-compensate each other [78, 79]. Opposite director tilt of the two cells cancel each
other, enhancing the viewing angle performance. Stacking two LC lenses not only improves the viewing angle but also increases the total optical power without any increase in the response time.

2.3.6 Approach for Polarization Independence, If Needed

LC lenses are phase modulator devices which utilize the relative difference in retardation of e-mode rays from different points across the aperture (the o-mode vibrates in the perpendicular to the propagation (k) vector and the optic axis. The e-mode vibrates perpendicular to the o wave and the k vector). In such devices, a polarizer is used to select only the e-mode. The polarization dependency of the OPD of LC is shown in equation (2.11) and (2.12). In order to eliminate the polarization dependency of a dual stack LC lenses (with improved viewing angle), it should be optically coupled with a second dual stack LC lenses that has the orthogonal rubbing direction with respect to the first one.

2.3.7 Reflection and Scattering

Reflection and scattering are the other factors that is important to assess. A 16% reflection loss due to the multi-layer structure in the discrete ring electrode configuration was measured previously [83] which is twice the glass-air reflection in an ideal glass lens. Transmittance of the LC lens, with less than 1% scattering
due to spacers and materials, was measured 83.1% [83]. Therefore scattering and reflection loss are not a significant concern in the configuration of interest.

2.3.8 Pairing LC lens of $\pm \frac{1}{f}$ D with a Fixed Lens of $\frac{1}{f}$ D

The discrete ring electrodes configuration makes the LC lens tunable in the range of $[-\frac{1}{f} \text{D} : +\frac{1}{f} \text{D}]$, which can be changed to $[0 \text{D} : +\frac{2}{f} \text{D}]$ by adding a convex lens with optical power of $+\frac{1}{f}$ D.

2.4 Modeling methods for design optimization

This section explains the numerical calculation methods used to optimize the basic design parameters.

2.4.1 Director modelling

A LC relaxation method is used to numerically calculate the director configuration at equilibrium, through minimizing the free energy for a given set of boundary conditions and external fields. We used this precise calculation to obtain the desired phase profile at near-field. A detailed explanation of this calculation is given in Appendix A. The near-field phase profile is required as an input field in propagation calculation using scalar diffraction discussed below.
2.4.2 PSF/MTF Calculations

Computational modeling and simulations help to predict the optical performance of the lens and therefore tackle design problems prior to fabrication. One of the difficulties in studying SPP LC lens with variable grating period lies in obtaining accurate information about the diffraction behavior of phase segments. Given the near-field phase profile calculated from the director profile, the far-field intensity is obtained from the Rayleigh-Sommerfeld diffraction formula [86].

Assuming monochromatic light from a two-dimensional (2D) source plane with coordinate variables of $\xi$ and $\eta$ and a field distribution of $u_1(\xi, \eta)$ (Figure 2-7), one can use the Rayleigh-Sommerfeld diffraction solution to find the field in an outlying observation plane $u_2(x, y)$ as represented by

$$u_2(x, y) = \frac{z}{j\lambda} \int \int_{\Sigma} u_1(\xi, \eta) \frac{e^{jkr}}{r_{12}^2} d\xi d\eta$$

(2.13)

here $\Sigma$ is the illumination aperture, and $\lambda$ is the wavelength of light, $k$ is the wave number, $z$ is the distance between the source plane and observation plane, and $r$ is the distance between a point $(\xi, \eta)$ on the source plane and a point $(x, y)$ on the observation plane ($r = \sqrt{z^2 + (x - \xi)^2 + (y - \eta)^2}$).
We calculate equation (2.13) using the efficient fast Fourier transform (FFT) method described by Voelz [87]. In our numerical model, an incident point source is considered, and the light propagates through different optical elements in the system. At the plane of each element, the phase profile of that element is taken into account and is added to the light phase. Then a Rayleigh–Sommerfeld transfer function conveys the light from one optical element to the other. Finally, the light intensity at the focal plane (the PSF) is determined. The modulation transfer function (MTF) can be then calculated by taking the Fourier transform of PSF. We have explicitly discussed our numerical model that is used for scalar propagation of light using FFT in Appendix A.
2.5 Lens Design and Modeling Summary

In summary, the discrete electrode design with inter-ring resistors is capable of producing a perfect parabolic lens. Stacking of two cells with opposite rubbing direction not only enhances the optical power but also improves the off-axis performance of the lens. There exist competition between aperture size and response speed of LC lens. Introducing the phase segments in phase profile yields a large aperture lens without compromising the response time. The segment width can be designed sufficiently large that the resulting diffraction is invisible to human eye. The numerical calculation of the Rayleigh-Sommerfeld diffraction formula in Fourier domain can precisely predict the imaging performance of optical system.

2.6 Example Lens

This section is allocated to a tunable focusing system that is designed, modeled, fabricated, and characterized based on the approaches mentioned above.

2.6.1 Specifications

As an example design, consider a 20 mm diameter lens with a tunable optical power range of 0 to +1.5 D. As discussed in section 2.3.5, because of the poor
viewing angle of a single lens, two LC lenses must be optically coupled [88]. Because the ring electrode lens is voltage programmed to be either of positive or negative power, the desired variation in power can be accomplished with two variable LC lenses whose combined power goes from -0.75 D to +0.75 D, and a fixed lens with power +0.75 D. Therefore, in this example system, the power of the three-element lens will be able to change from 0 to +1.5 D. The target optical power range (±0.75 D) with diameter of 20 mm requires a 139 μm thickness (using equation (2.1) and $OPD = \Delta n \cdot d$), assuming the birefringence of LC is 0.27. Optical coupling of the two LC lenses together reduces that value for each lens by a factor of two. As shown in Figure 2-2B and considering the minimum phase segment width of 1 mm defined in Section 2.2.1, five segments in each LC lens phase profile can further reduce the thickness of the pair of lenses. The calculation and experimental results discussed in this dissertation are based on introducing 4.5 segments in the phase profile (Figure 2-8).
Figure 2-8 Phase profile of SPP LC lens with 4.5 segments and optical power of ±0.34 D (2.92 m). Our modeling, fabrication, and characterization results are based on this phase profile.

Considering the effect of the pair of lenses and the phase segments, the thickness of the LC can be reduced up to nine times (4.5 resets × 2) and consequently improves the response speed by a factor of 81. The total optical range of +0.065 D to +1.435 D is obtainable by the use of two LC lenses (the phase profile is shown in Figure 2-8) whose combined power goes from -0.685 D to +0.685 D, and a fixed lens with power +0.75 D. Given the aperture size (20 mm), the optical power of ±0.34 D, and the sampling rate (9 phase steps per wave), and the total number of electrodes (288), the outer radius of each electrode is calculated. (See section 2.3.2 for calculation methods.) To reduce the local variations in the electric field due to the gaps between the electrodes, a small 3 μm gap between each electrode is considered (gap≪cell thickness). The total number of eight bus lines is determined by considering the OPD versus voltage curve. To address the image degradation due
to the gap, we implemented “floating” electrodes that are not driven by ohmic connection, but are capacitively coupled to the driven electrodes [43]. The floating electrodes are designed to cover half of the area of each of neighboring driven electrodes. In chapter 3 we compare the image quality of lens with and without the presence of floating electrodes. In the evaluation of the designed lens we consider an approximate 50% loss in the resolution of the human eye at 10 cyc/deg and that the system should reach the same resolution loss at a higher cyc/deg.

2.6.2 Modeling of Expected Performance

In this section, we use scalar optics for propagation and numerically solve Rayleigh-Sommerfeld diffraction formula in Fourier domain. The scalar propagation model developed in MATLAB is extensively discussed in Appendix A. In a near-to-eye display system there is a magnifier lens that places the image of the display at a distance the eye can accommodate. In our analysis of an auxiliary lens system for the magnifier lens, we will not consider the magnifier lens explicitly, but assume the display is at an apparent distance, $d_5$ of 67 cm.

Problem Definition
Our variable lens system shown in Figure 2-9 consists of two variable LC lenses whose combined power goes from -0.685 D to +0.685 D, and a fixed lens with power +0.75 D. The purpose of this study is to determine the image that can be captured by the human eye while looking through the proposed SPP LC lens. We will consider the eye to be “perfect,” and model it as an ideal lens with a focal length, \( d_1 \), of 20 mm. The eye focus is at infinity in our numerical calculation.

![Schematic diagram of the optical system considered for scalar diffraction calculation. The parameters used in our calculations are: \( d_1 = 20 \) mm, \( d_2 = 20 \) mm, \( d_3 = 2 \) mm, \( d_4 = 1 \) mm and \( d_5 = 67 \) mm.](image)

**Figure 2-9** Schematic diagram of the optical system considered for scalar diffraction calculation. The parameters used in our calculations are: \( d_1 = 20 \) mm, \( d_2 = 20 \) mm, \( d_3 = 2 \) mm, \( d_4 = 1 \) mm and \( d_5 = 67 \) mm.

**On-axis Performance**

The pupil size of the eye varies between 2 and 7 mm according to the illumination condition. For that reason we consider an average area with a 5 mm diameter for our calculation. Note that while the LC lens is 20 mm in diameter, the
eye will collect light only through a small portion of its area. When calculating the optical performance of our lens, we first consider the “on-axis” situation; in other words, the user is looking straight ahead through the lens. In this case, the light from the object that reaches the retina only passes through the center region of the lens. This center region of the lens has a parabolic phase profile, as marked by the red rectangle in Figure 2-8. In this case the modeled PSF is an ideal diffraction-limited profile. The radius of first minimum is observed at $2.50 \times 10^{-6} \text{ m}$ (shown in Figure 2-10), which is in good agreement with the value obtained from a diffraction-limited Airy disk ($q = 1.22 \frac{\lambda f}{d} = 2.58 \times 10^{-6} \text{ m}$, where $f = 0.0194 \text{ m}$ and $d = 5 \text{ mm}$).

![Graph](image.png)

**Figure 2-10** On-axis PSF of the optical system shown in Figure 2-9 with a combined power of 51.435 D (0.0194 m).
Effect of Off-axis Illumination

The concern with our lens is its off-axis performance. This parameter becomes crucial in our designed lens due to the presence of phase segments with decreasing width moving from the center toward the edge. Figure 2-11 schematically represents the “off-axis” situation. The lens with radius \( r_2 \) in this figure is comprised of a positive lens and pairs of LC lenses previously shown in Figure 2-9. The red dashed line in Figure 2-11 is the optic axis that passes through the edge of the designed lens, the center of eye lens, and lies at the center of the retina. Three effects for off-axis performance must be studied: the effect of the different OPD for off-axis incident light; the effect of phase segments on the image formed at retina; and the parallax effect.

Figure 2-11 Schematic representation of real user experience by looking through the lens at an off-axis position. Lens with a radius \( r_2 \) here is the combination of the three lenses shown in Figure 2-9.
Effect of OPD Dependency on the Off-axis Performance

As discussed in section 2.3.5, the birefringence of LC materials is inherently dependent on the angle between the wavefront normal of the light and the optic axis of LC. Assuming the rubbing direction of the LC lens, and thus the azimuthal orientation of the molecules, is along the x-axis and also the transmission axis of the polarizer is along the x-axis, the propagation direction (k vector) of on-axis light will be along the z-axis. For oblique incident light, the propagation vector is in either the y-z (Figure 2-12B) or the x-z planes (Figure 2-12C). In the former case, regardless of the off-axis angle, the relative angle between k vector and LC director is the same as the on-axis case and the phase profile is always parabolic (Figure 2-12B). However, when the propagation direction is in x-z plane (Figure 2-12C), which means that the propagation direction and LC director are co-planar, the relative angle between the k vector and the LC director changes significantly with angle of incidence. This results in deviation of the phase profile from the ideal parabolic one at off-axis incident angle. The LC director has a preferred tilt angle induced by the rubbing direction, which leads to the asymmetry of the phase profile depending on the direction of the incident angle (positive/negative) [89]. As explained in section 2.3.5, the solution to this issue is to have another cell with opposite rubbing direction so that the two cells self-compensate their change in birefringence due to the off-axis illumination [78, 79, 83].
We used the extended Jones matrix method to calculate the effect of 20° off-axis illumination on the phase profile of the designed lens for the case where the propagation direction and LC director are co-planar (Figure 2-12C). It should be noted that for the lens with a 20 mm diameter, the maximum gaze angle of the eye, assuming that the distance of the designed lens and the eye lens is about 20 mm, will be about 15° (angle θ shown in Figure 2-11) and therefore 20° will be a

\[\text{Figure 2-12A. LC lens configuration; B. Cross section of lens in y-z plane; C. Cross section of lens in x-z plane.}\]
sufficient value for this study. Similar to what [89] observed, our results showed that if the two cells are rubbed in the same direction, there will be a significant deviation from the parabolic phase profile. The most dramatic change is observed at the center of each phase segment since the highest change in retardation occurs where the tilt angle is about 45°. The asymmetry of the phase profile with respect to the direction of the incident angle is eliminated for the stack with an opposite rubbing direction. Figure 2-13 summarizes the effect of off-axis incident angle on the phase profile of SPP LC lens for the case when the k vector and LC director are co-planar and the solution to the issue (stacking of two cells with anti-rubbing direction).

![Graph showing phase profile and optical path difference](image)

**Figure 2-13** Effect of viewing angle for the ideal parabolic lens is shown in the black curve (±20°), pink curve shows the result for on-axis illumination ((±20°), green curve represents the stack of...
two designed SPP LC lens with opposite rubbing direction (±20°), blue curve demonstrates the stack of two designed SPP LC lens with same rubbing direction (-20°) and red curve shows the stack of two designed SPP LC lens with same rubbing direction (+20°). The calculation is performed for the off-axis case shown in Figure 2-12C, where the propagation direction of the light and LC optic axis are co-planar.

**Effect of Phase Segment on the Off-axis Performance**

In the actual situation when the light is viewed through the edge of the lens in a near-to-eye application, the lens can be assumed to be tilted by an angle θ (shown in Figure 2-11). However, the effect of the phase segments is modeled as shown by the ray in Figure 2-9. This is because we would like to separate the effect of phase segments from the other off-axis effects that are discussed in section 2.3.5. The phase profile of the modeled region of the lens (with a 5 mm aperture size) is marked with a blue rectangle in Figure 2-8. Since the optical effect of the SPP LC lens is expected to be wavelength dependent, we first numerically calculated the effect of wavelength on the optical performance of the designed lens. To do so, we modelled one SPP LC lens and calculated the spot profile at the focal plane (at 2.92 m).

Considering $OPD\ Step = n\lambda + \varphi$, at the design wavelength the phase difference ($\varphi$) is zero, thus $OPD\ Step = n\lambda_D$. For a non-design wavelength ($\lambda_i$), the OPD Step is unchanged, so $n\lambda_D = n'\lambda_i + \varphi$. There are many values of $\lambda_i$ that will meet the worst-case condition of $\varphi = \frac{\lambda_i}{2}$, as shown in Figure 2-3B. The closest $\lambda_i$ to the $\lambda_D$ that will result in the worst-case condition is when $\lambda_i = \frac{n\lambda_D}{(n + \frac{1}{2})}$. 
To directly observe the effect of the diffraction, the lens profile in Figure 2-8 was used, where $OPD\ Step = 7\lambda_d$ for the case of 543.5 nm light. As presented in Figure 2-14, no diffraction effect is observed at the design wavelength of 543.5 nm (where the value of $\varphi = 0$). This result indicates that the outgoing beams from the lens segments are all in-phase and constructively interfere to form a point at the focal plane. On the other hand, a dramatic effect is observed when the illumination wavelength yields $\varphi = \frac{\lambda_i}{2}$. In this example, the closest wavelength to the design wavelength that meets this condition is $\lambda_i = 507\ nm$. Using the focal length and the spacing between the two spots, we calculated the diffraction angle corresponding to the half wave phase difference shown in Figure 2-14, which is $0.0086^\circ$. This resulting diffraction angle is likely to be unobservable by the human eye because we have chosen the minimum size of the phase segments to meet that condition. The effect of the MTF on the structure shown in Figure 2-9 is shown in Figure 2-15. Here we include two SPP LC lenses so that the value of OPD step and therefore $\varphi$ is twice that for the single lens modeled for Figure 2-14. Figure 2-15 demonstrates the MTF for the design wavelength where $\varphi = 0$ (543.5 nm) and the wavelength where $\varphi = \frac{\lambda_i}{2}$ (563.63 nm).
Figure 2-14 Effect of wavelength on the light distribution at the focal plane of the SPP LC lens for wavelengths that cause the phase segments to be different by a phase difference of a half wave; a quarter wave; 0 (at the design wavelength); and a half wave.

The significance of the computed MTF for the designed lens is quantified by its comparison to the MTF of the human eye. Williams et al. reported the MTF of the human eye is reduced to 0.4 at 10 cyc/deg [90]. Our calculations show the MTF of the designed lens does not reach 0.4 until about 33 cyc/deg.
Effect of Parallax on the Off-axis Performance

We also investigated the effect of parallax on the off-axis performance. As discussed in a previous section, to improve the viewing angle performance of the lens, the stacking of two lenses is required. The stack of two cells does not change the on-axis performance. However, the parallax effect of the phase segments of the two cells for off-axis illumination could be a problem. This parallax effect for the small deviation from on axis should be more dramatic than the larger deviation because, for a smaller deviation, there will be a section with a narrow width that might cause large angle diffraction. To investigate the effect of parallax due to the
phase segments, we considered an 8° angle of incidence, which is approximately the angle that the eye uses for normal scanning through the lens. Two SPP LC lenses are separated by about 1 mm (the thickness of two glass substrates is 0.8 mm). The parallax in the radial direction is therefore 140 μm. By shifting the phase profile of one of the cells by 140 μm, we studied the effect of parallax on the MTF, and found the curve to be the same as the green curve in Figure 2-15. The obtained result is also shown in Figure 2-16.

Figure 2-16 MTF and simulated image of the optical system shown in Figure 2-10, at the designed wavelength λ = 543.5 nm with parallax effect.

2.7 Summary

The active area of LC lens was expanded for near-to-eye applications by introducing large segments into the phase profile with negligible diffraction. An example SPP LC lens with 4.5 segments and ± 0.375 D optical power was designed.
An extended Jones matrix calculation verified that dual stacking of lenses improves the off-axis phase profile. With dual stacking of example lenses, the total optical power increased to ± 0.75D. The stack of SPP LC lenses was combined with a positive lens of +0.75 D to result the optical power range of 0 D to +1.5 D. The diffraction behavior of phase segments in the example SPP LC lens and also in a near-to-eye system was numerically modelled using a scalar diffraction calculation. As analytically expected, numerical modelling confirmed that for SPP LC lens worst case diffraction occurs when the phase difference between the design and incident wavelength reaches $\frac{\lambda}{2}$. The width segment was selected to be sufficiently large to create a diffraction angle smaller than the resolution angle of the human eye (1 arc/min), which was verified numerically by comparing the MTF of near-to-eye system with the MTF of the human eye.
CHAPTER 3

Fabrication and Characterization of Example SPP LC Lens

In this chapter, the fabrication and characterization of an example SPP LC lens with 20 mm diameter are discussed.

3.1 Fabrication Process

The test lens consists of two substrates. On one substrate is a layer of Indium tin oxide (ITO); an insulator layer of Silicon dioxide (SiO₂); a conductive layer of Nickel (Ni); and polyimide (PI). For the example lens with floating electrodes, there is an additional layer of SiO₂ and ITO deposited prior to the polyimide layer. Figure 3.1 schematically represents the fabrication process of the designed lens. The first ITO layer on the first substrate consists of 288 discrete ring electrodes with equal areas that are separated with 3 μm gap (15% ratio of gap/thickness). Between each adjacent electrode, there is a resistor with a 10 μm length and a 5 μm width that is staggered with a 5° spiral pattern across the aperture (shown in close-up of Figure 3.2A). The resistance of the inter-ring resistor is 200 Ω across the lens aperture. The angular separation exists to help control the resistance distribution in the radial
direction and the linear voltage drop between the electrodes. The SiO$_2$ layer is coated using vacuum sputtering. This layer then patterned with rectangular vias that are 20 μm by 10 μm that work as a bridge to connect the driven ITO electrodes on the bottom to the top Ni bus lines. The Ni layer is also deposited using the same method that was used for the insulator layer and is patterned to create eight bus lines for external connections with a 20 μm width. These bus lines are placed

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Cleaning</td>
</tr>
<tr>
<td>2.</td>
<td>ITO Patterning</td>
</tr>
<tr>
<td>3.</td>
<td>SiO$_2$ Deposition</td>
</tr>
<tr>
<td>4.</td>
<td>Via’s Patterning</td>
</tr>
<tr>
<td>5.</td>
<td>Ni Deposition</td>
</tr>
<tr>
<td>6.</td>
<td>Bus line’s Patterning</td>
</tr>
<tr>
<td>7.</td>
<td>2$^{nd}$ SiO$_2$ &amp; ITO Deposition</td>
</tr>
<tr>
<td>8.</td>
<td>2$^{nd}$ ITO Patterning</td>
</tr>
<tr>
<td>9.</td>
<td>PI Coating &amp; Rubbing</td>
</tr>
<tr>
<td>10.</td>
<td>Cell Assembly, Filling &amp; ACF bonding</td>
</tr>
</tbody>
</table>

**Figure 3-1** Fabrication process of SPP LC lens
with a 45° angle spoke pattern shown in Figure 3-2A. Then PI layer is spun coated

![Figure 3-2A. Electrode geometry in the example lens B. Final look of the fabricated lens.](image)

on the patterned plate and cured. The next steps of the process are the rubbing of PI, spraying the 20 μm fiber spacer, assembling the common plate with patterned plate, filling with MLC 18349 [91], and sealing the end, respectively. Then the bus-line electrodes are connected to a flex bonding. The final look of the fabricated lens is shown in Figure 3-2B. In summary, the fabrication process of the designed lens uses processes standard in the display industry. The alignment of vias with the ITO and Ni electrodes and etching the 3 μm gap between the ITO electrodes, however, are the challenging steps of the fabrication. A detailed explanation of the fabrication process is included in Appendix B.
3.2 SPP LC Lens Characterization

In this section, we discuss the metrology steps to assess the optical performance of SPP LC lens for the applications of interest.

![Figure 3-3](image)

Figure 3-3 Red circles show the measured points, green line indicates the calculated points, and blue line shows the ideal parabolic points for positive and negative phases.

3.2.1 Phase Profile

Phase profile measurement of the constructed LC lens is done using a variable compensator. The variable compensator is itself a cell with the same thickness as
the LC lens (20 μm) that is filled with the same LC material (18349 of 0.27 birefringence). The measurement set-up is comprised of white light crossed polarizers with LC lens and variable retarder between them. The optic axis directions of the LC lens and the variable retarder are at an orthogonal position with respect to each other while they are at 45° with respect to the transmission axis of either of the polarizers. The intensity of light passing through the stack of the LC lens and the variable retarder, which are sandwiched between crossed polarizers, can be determined as

\[ \frac{I}{I_0} = \sin^2 \left( \frac{|\Delta \phi_m|}{2} \right) \]

where \( \phi^{LCL} \) and \( \phi^{LCV} \) are the phase of the LC lens and the LC variable retarder, respectively. If the variable retarder phase (\( \phi^{LCL} \)) exactly matches the LC lens phase in a region of the lens (\( \phi^{LCL} \) (\( \Delta \phi_m = 0 \)), that region appears black (See Figure C-4 of Appendix C.). The procedure is to first measure the phase of the variable retarder versus the voltage applied to it (measurement data shown in Figure 2-6). Second, determine the phase of an area of the lens by finding the voltage that must be applied to the variable retarder to make that area black when using crossed polarizers and white light illumination. This method is used to map out the phase
retardation across a diameter of the lens and is shown in Figure 3-3. More details on the measurement procedure are included in Appendix C.

### 3.2.2 Effect of Gap between Electrodes

In this section, we consider the effect of the gaps between the electrodes in the lens without “floating,” and the improvement resulting from adding them. Section 2.3.4 discussed that the gaps between electrodes effectively generate variable grating periods which consequently results in haze. The haze was successfully excluded using “floating” electrodes with additional processing steps [43]. It is essential to evaluate the importance of “floating” electrodes in near-to-eye applications to avoid any un-necessary processing steps.

![Figure 3-4](image)

**Figure 3-4** A. 3D model of holder for SPP LC/Glass stack: I. Holder to secure glass lens; II. Place to slide the SPP LC stack. B. VR VUE head set: I. VR optical lens; II. IPD adjustment; III. Smart phone holder. C. VR VUE with attached SPP LC/Glass stack secured in the 3D printed holder shown in part A.

To evaluate the performance of SPP LC lens without the presence of “floating electrode” in near-to-eye applications we used a VR platform developed by JEM accessories named “VR VUE” with an adjustable InterPupillary Distance (IPD) head mount for smart phones with same basis as of Google Cardboard (Figure 3-
4B). We used a 3D printed holder (Figure 3-4A) to attach the SPP LC/Glass stack to the VR lens. Figure 3-4C shows the VR viewer with attached SPP LC/Glass stacks. A Nokia Lumia 920 cell phone was held in place using grippers. A static high contrast image was displayed on the Nokia phone. Figure 3-5 illustrates the images captured from the display of the VR VUE through the SPP LC/Glass stack (without floating) that was mounted on top of the cardboard lens. To capture these images, we used a 13 MP f/1.9 camera with a 28 mm focal length. Figure 3-6A was taken while the camera was centered with the center of the SPP LC/Glass stack and the cardboard lens as shown in Figure 3-5A. Figure 3-6B, however, shows the image quality when the capturing camera was centered at the periphery of the test lens and therefore “looking” through the lens near its edge as shown in Figure 3-5B.

**Figure 3-5** Schematic presentation of test set-up to evaluate the image quality of SPP LC lens in VR viewer system. Two cases are considered. Center of fast camera lens is aligned with A. center of SPP LC /Glass stack and the center of VR lens. B. edge of SPP LC/G Glass stack and the center of VR lens. 1. f/1.9 with 28 mm focal length cell phone camera (Samsung J7 prime). 2. SPP LC lens. 3. +0.75 D glass lens. 4. VR lens. 5. Nokia Lumia smart phone’s display.

High quality image (Figure 3-6A) was attainable provided that the camera lens aligns with the center of SPP LC/Glass stack as shown in Figure 3-5A. Double
images however were observed when the camera lens, SPP LC/Glass stack and VR lens were aligned as shown in Figure 3-5B. To see the double image at the periphery of the lens we enhanced the brightness, as shown in Figure 3-6C. The appearance of double images toward the periphery of the lens shown in Figure 3-6 can occur because of one or a combination of any of the following reasons: a parallax effect from misalignment of the stack of LC lenses; diffraction from a higher density of phase segments at the lens periphery; or diffraction from the gap between electrodes acting as narrow slits.

To distinguish between these three potential reasons, a monochromatic experiment was carried out. To eliminate the parallax effect in this experiment we only used a single LC lens rather than a stack of two. We also removed the effect of phase segments from this experiment by driving the entire LC lens with the same voltage. Therefore, we isolated the effect of the gap between the electrodes. We illuminated the LC lens mounted on a lateral translation stage with 633 nm laser

**Figure 3-6** Static images are presented in Google cardboard VR set. Stack of SPP LC/Glass (without floating) was mounted on the VR lens. Upon driving the lens, the high-contrast image is captured after A. Camera is adjusted at the center of the stack; and B. Camera is adjusted at the edge of the stack (highlighted area in Figure 2-9). C. Enhanced brightness of B.
light of approximately 1 mm diameter and we recorded the light distribution pattern at a 3.24 m distance. Figures 3-7A and 3-7B demonstrate the light distribution pattern when illuminating the center and periphery of the lens, respectively. For better realization of a diffraction pattern, we enhanced the brightness of Figure 3-7B in Figure 3-7C.

![Figure 3-7](image)

**Figure 3-7** Light distribution pattern after passing through the SPP LC lens without floating electrodes. A. Beams passing through the center of the lens (marked with blue in Figure 2-8). B. Beams passing through the edge of the lens (marked with red in Figure 2-8). C. Contrast of figure B is enhanced.

We measured a 7% efficiency loss because of light diffraction to a 2.033° angle at 2.5 V. This large diffraction angle corresponds to the electrode spacing of about 18 μm within the region illuminated by the laser beam. By adding the floating electrodes into the lens design, the diffraction loss is lowered to 0.6% at the periphery of the lens measured under the same conditions as above. Table 3-1 shows
the results of these measurements. The values shown in the Table 3-1 normalized with measured intensity passing through the central area of the lens at 0 V.

<table>
<thead>
<tr>
<th>Area of laser illumination</th>
<th>0th order</th>
<th>Σ ± 1st order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center WO Floating Electrodes</td>
<td>0.9962</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>Edge WO Floating Electrodes</td>
<td>0.9334</td>
<td>0.013</td>
</tr>
<tr>
<td>Center W Floating Electrodes</td>
<td>0.9959</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>Edge W Floating Electrodes</td>
<td>0.994</td>
<td>0.0057</td>
</tr>
</tbody>
</table>

### 3.2.3 Image Resolution

The imaging resolution of the designed system for both large aperture and near-to-eye applications was investigated. A negative USAF 1951 test target with a backside illumination (fluorescent light box) was used as the object. The images shown in this section were taken with a Canon EOS Rebel XSi/450 D and 50 mm Canon lens. The “SPP LC/Glass stack” used in our measurement consists of a linear polarizer, the two example SPP LC lenses described above with opposite rubbing directions, and a +0.75 D glass lens. The transmission axis of the linear polarizer was along the rubbing direction of SPP LC lenses. The optical power of the two SPP LC lenses in Figure 3-9 and Figure 3-12 goes from -0.628 D to +0.628 D. The +0.75 D glass lens is positioned at 2 mm distance from the SPP LC stack. Therefore, the optical power of the three combined elements goes from +0.12 D to +1.37 D, which is close to the optical power range expected from the lens modeling (+0.065 D to +1.435 D). The difference is likely due to a small difference between the designed and actual LC cell spacing. The optical power range of the two SPP LC lenses in...
Figure 3-10 and 3-11 is the same as calculation. Figure 3-8 shows the setup used for imaging resolution test.

![Experimental setup for imaging resolution test. 1. Fluorescence light 2. USAF 1951 test target 3. SPP LC/Glass stack positioned on XYZ translational stage 4. Canon camera with 5.26/17.85 mm aperture. 5. Ribbon cables going to National Instrument.](image)

**Figure 3-8** Experimental setup for imaging resolution test. 1. Fluorescence light 2. USAF 1951 test target 3. SPP LC/Glass stack positioned on XYZ translational stage 4. Canon camera with 5.26/17.85 mm aperture. 5. Ribbon cables going to National Instrument.

**Near-to-Eye Application**

Human pupil size varies between 2 to 8 mm depending on the amount of light entering the eye. Therefore an average pupil size of 5 mm is considered for near-to-eye including ophthalmic and VR/AR/3D display applications (eye’s average pupil size in VR system is measured 3.73 mm [92]). To evaluate the imaging quality of SPP LC/Glass stack for near-to-eye applications, a small aperture of 5.26 mm (f/9.5, f=50 mm lens) was used, similar to average size of eye’s pupil and our modeling’s assumption. Two cases were studied: 1. center of camera lens and center of SPP LC/Glass stack were aligned. In this case light passed through the center of SPP LC/Glass that has no phase segments (marked with Red in Figure 2-8). 2. SPP LC/Glass stack was translated by 7.5 mm. In this case center of the camera lens was
aligned with the edge of SPP LC/Glass stack where the light passed through the phase segments (marked with Blue in Figure 2-8). Case 2 will determine the effect of phase segments when looking off-axis in near-to-eye application through the SPP LC/Glass stack ignoring other off-axis effects including phase profile deviation and parallax. Figure 3.9, 3.10 and 3.11 show insignificant image degradation through the center of the lens, and also for the light imaged through the phase segments near the edge of the lens when floating electrodes are used. The obtainable resolution with the designed focusing system can be compared to a reference image taken either only with the camera lens (shown in Figure 3-9A) or taken with camera lens at infinity and additional glass lens (Figures 3-10A and 3-11A).

**Large Aperture Imaging Application**

For large aperture imaging applications, to assess the image resolution in an imaging application, pictures of the USAF test pattern were acquired using a camera with a large aperture (17.86 mm (F/2.8)). Specifically, the camera lens was focused at 2.08 m and positioned to capture the light beam straight ahead through the center of the lens. The images shown in Figure 3-12 demonstrate the excellent resolution of the lens. There is some image degradation that can be seen when comparing pictures Figure 3-12C and Figure 3-12D with Figure 3-12A. Some of the degradation, however, could be associated with multiple reflections related to the air gap between optical surfaces added in Figures 3-12C and 3-12D.
Figure 3-9 Pictures taken of a resolution chart A. Camera auto focused at object distance of 54 cm. with no other optical elements in place. B. Camera lens focused at 2.08 m with object at 54 cm. Image is taken with addition of SPP LC/0.75D Glass stack of 1.37 D power when camera is adjusted to capture light through the center of SPP LC lens (marked with red in Figure 2-8). The SPP LC lens used in the stack did not have the floating electrodes. C. Same as B, but camera is adjusted to capture light through the edge of the SPP LC lens (marked with blue in Figure 2-8). D. Same as B, but using SPP LC lens with floating electrodes. E. Same as C, but using SPP LC lens with floating electrodes. Other experimental details are in the text.
Figure 3-10 Camera lens is focused at infinity. A. Image is taken with camera lens at infinity and addition of 1.5 D glass lens. The focus is found when the object is placed at 72 cm. B. Object is at
71.5 cm and the picture is taken with camera lens at infinity. No additional lens is placed. C. SPP LC/0.75 D glass stack at its maximum power (1.43 D) is added to condition mentioned at B. We stop the camera aperture using a 5 mm diameter physical aperture. The center of camera lens, center of aperture and center of SPP LC/0.75D Glass stack are aligned with each other. The best focus was found at 75 cm. D. The same condition as C only we moved the stack by 7.5 mm to capture the image through the outer most edge. A’, B’, C’, D’ images shown on the bottom are the expanded part of groups 0 and 1 of charts shown in A, B, C, D images on the top, respectively. The SPP LC lenses used in this experiment have the floating layer of electrodes.

**Figure 3-11** Camera lens is focused at infinity. A. Image is taken with camera lens at infinity and addition of 0.5 D glass lens. The focus is found when the object is placed at 2 m and 3 cm. B. SPP LC/0.75 D glass stack at its minimum power (0.06 D) is added to condition mentioned at A. The best focus was found at 1 m and 98 cm. C. The same condition as B only the SPP LC/0.75 D Glass stack was translated by 7.5 mm to capture the image through the outer most edge. The SPP LC lenses used in this experiment have the floating layer of electrodes.

### 3.2.4 Measurement of Point Spread Function (PSF)

This section includes the spot profile measurement of the SPP LC/Glass stack at on and off-axis cases. For this measurement, the intensity of expanded linear polarized light illuminated from He-Ne laser with wavelength of 543.5 nm is reduced with Neutral Density (ND) filter before passing through an aperture stop with 5 mm diameter. 5 mm beam then passes through the SPP LC/Glass stack with 1.43 D and the spot profile is measured at focal length using Canon Rebel XSI 450 D.
Figure 3-12A. Camera auto focused at 54 cm. B. Camera lens focused at 2.08 m and image taken at 54 cm. C. Camera lens focused at 2.08 m and image taken with addition of an SPP LC/Glass stack at +1.37 D at 54 cm. The SPP LC lenses do not have the floating layer. D. Camera lens focused at 2.08 m and image taken with addition of an SPP LC/Glass stack at +1.37 D at 54 cm. The floating electrodes are included in the fabrication of the SPP LC lenses.

CCD with 5.2 μm pixel size. The schematic diagram of the PSF experimental set-up is shown in Figure 3-13.

Figure 3-13 Experimental setup for PSF measurement. 1. Laser He-Ne 543 nm 2. Linear Polarizer 3. 10x Beam Expander 4. ND Filter 5. Aperture Stop 6. SPP LC/Glass stack on XY stage 7. CCD camera with 5.2 μm pixel size.
On-axis PSF

For on-axis spot profile shown in Figure 3-14 beam after aperture stop (5) passes through the center of SPP LC/Glass stack (6) and falls at the center of CCD sensor (7). Comparing the PSF of SPP LC/Glass stack (1.43 D) with a glass lens of similar power (1.5 D) shown in Figure 3-14 suggests that the designed lens has performance close to diffraction limited when looking at the center. This experimental result agrees with numerical result calculated using scalar diffraction method shown in chapter 2, section 2.6.2.

Off-axis PSF

For the off-axis case three separate effects are taken into account;

![Figure 3-14](image) PSF of 5 mm diameter at the center of; A1 and A2. Glass lens with 1.5 D. Best focus was found at 0.76 m. A1’ and A1”. PSF plot along the red and blue line shown in A1, respectively. B1 and B2. SPP LC/Glass lens stack with 1.43 D. Best focus was found 0.88 m. B1’ and B1”. PSF plot along the red and blue line shown in B1, respectively. The black curves in A1’, A1”, B1’ and B1” shows the theoretical PSF curve.
1. Effect of viewing angle: As discussed in Chapter 2, section 2.3.5, the phase profile of the SPP LC lens depends on the angle between the wavefront normal and the LC optic axis. Notable deviation from the parabolic phase profile occurs when the propagation direction of incident light and the LC optic axis are co-planar. We have shown numerically that stacking two SPP LC lenses with opposite rubbing direction improves the viewing angle performance. To verify the calculation results, we measured the spot profile of SPP LC/Glass stack that was rotated by 20° angle along the rubbing direction (thus k vector and LC director are co-planar.). During this measurement laser beam passed through the center of SPP LC/Glass stack with no phase segments. Beside 0.75 D glass lens, the stack consisted of two SPP LC lenses (±0.75 D) with opposite rubbing direction. Figures 3-15B1 and B2 indicate that the opposite director tilt of two cells at 20° incident angle considerably compensate the viewing angle issue.

2. Effect of phase segments: to assess this effect spot profile measurement is performed for translated SPP LC/Glass stack by 7.5 mm while the beam passed through the edge of the lens (marked with red in Figure 2-8). We previously explained in chapter 2, section 2.3.5 that worst diffraction effect due to segments occurs when the phase difference between the designed and incident wavelengths reaches to $\frac{\lambda}{2}$ at which we realize the combination of parabolic lens profile and a binary phase grating resulting in a double image. The experimental result shown in
Figure 3-15 C1 and C2 well agrees with the calculation result shown in Figure 2-14 with $\frac{3\lambda}{4}$. The phase difference is very likely due to thickness variation during the fabrication process. Comprehensive study on the cause of phase variation at segment boundaries which leads to diffraction was done and included in Appendix C.

3. Parallax effect: Measuring spot profile at same condition as of 1 but this time SPP LC/Glass stack was translated by 7.5 mm so the laser beam saw the segments (marked with red in Figure 2-8). This measurement enables us to assess the collective effect of both viewing angle and parallax effects. After measuring the spot profile, the PSF was obtained (Figure 3-15) and compared with the numerical one shown in chapter 2, section 2.3.5. In consideration of the results shown in Figure 3-15D1 and D2 with the ones shown in Figure 3-15C1 and C2, one can realize that the parallax effect does not significantly changes the spot profile, which matches with our modeling results shown in Figure 2-16.
Figure 3-15 PSF of 5 mm aperture at; A1 and A2. Edge of a Glass lens with 1.5 D and 22 mm diameter. Best focus was found at 0.76 m. A1’ and A1”’. PSF plot along the red and blue line shown in A1, respectively. B1 and B2. Center of SPP LC/Glass stack with 1.43 D which was rotated by 20° along rub direction of LC lens. Best focus was found 0.88 m. B1’ and B1”’. PSF plot along the red and blue line shown in B1, respectively. C1 and C2. Edge (7.5 mm from the center of lens) of SPP LC/Glass stack with 1.43 D. Best focus was found 0.88 m. C1’ and C1”’. PSF plot along the red and blue line shown in C1, respectively. D1 and D2. Same condition as of C1 and C1 but rotating the SPP LC/Glass stack by 20° along rub direction. D1’ and D1”’. PSF plot along the red and blue line shown in D1, respectively. The black curves in all spot profile charts shows the theoretical PSF curve.

It might be useful to point out that the vertical axis of the PSF graphs shown in this dissertation are normalized by the maximum pixel values. The horizontal axis of the theoretical PSF curve ($x_{th}$) is obtained by
\[ x_{th} = -\frac{x_{CCD}}{2} + dx_{th}: dx_{th} = \frac{x_{CCD}}{2} - d_{x_{th}} \]  

where \( d_{x_{th}} \) is the axis interval and \( x_{CCD} \) is the horizontal length of CCD in micron; which is obtained from horizontal pixel count (\( N_{px} \)) and pixel size (\( PS \)) as \( x_{CCD} = N_{px} \times PS \). The spot profile is processed in MATLAB by collecting the pixel values across an arbitrary lines (red and blue shown in spot profile images). The pixel value on the arbitrary line are adjusted based on the angle between the arbitrary line and x-axis.

### 3.2.5 User Evaluation at Maximum and Minimum Optical Power

Finally, we performed two simple user evaluation experiments on 20 subjects to assess the optical performance of designed SPP LC/Glass stack at its maximum (1.43 D) and minimum (0.06 D) optical power range. To measure the eye resolution at maximum optical power of the SPP LC/Glass stack, we used a back illuminated USAF 1951 test target mounted on an optical rail. The subject position was fixed on a chin rest. The test setup is shown in Figure 3.16A. Initially we measured the subject’s eye resolution without any visual aid. To do so, we asked the subject to adjust the distance of the test target for his/her best acuity. More specifically we asked the subject to focus on the elements of group 0 in the USAF 1951 test target and bring the test target close until he/she cannot focus on one or more elements of group 0. At that point the subject was asked to state the group and element number.
that still appears sharp to him/her. The recorded group and element number then used to calculate the resolution as shown in equation (3.3) [93].

\[ \text{Resolution} \left( \frac{lp}{mm} \right) = 2^{\left( \text{Group} + \left( \frac{\text{element} - 1}{6} \right) \right)} \]  (3.3)

Without any change in the test target distance, the user was handed the SPP LC/Glass stack at the maximum power of 1.43 D and asked to state the element and group number that appears sharp to him/her which was recorded accordingly. The experiment was repeated but this time with a polarized glass lens of 1.5 D optical power. The subject resolution limits without any visual aid, with SPP LC lens and with the glass lens of 1.5 D laminated with a linear polarizer are shown in Figure 3-16B. The majority of subjects resolved in cyc/mm with their own eye half as well as with addition of the polarized glass lens (1.5 D) or SPP LC/Glass stack (1.43 D). In addition, except for subject 9, the resolution increase with the 1.43 D SPP LC/Glass stack and with the 1.5 D glass lens was equal.
Figure 3-16 A. The experimental setup consist of a fluorescence light box, a negative USAF 1951 test target placed on the post mounted on the x-stage and a chin rest. The distance between the user and the test target was adjustable by means of a rail. B. Limiting resolution of the subject eye without any visual aid, with linear polarized 1.5 D glass lens and with SPP LC/Glass stack of 1.43 D vs the subject number. C. SPP LC/Glass stack mounted on holder with Inter Pupillary Distance (IPD) adjustment.

At a minimum optical power of the SPP LC/Glass stack (0.06 D), the negative phase profiles are imposed on the SPP LC lenses. To ensure the imaging quality of negative phase profiles, we run the second user study of testing the visual acuity. Visual acuity is defined as the estimation of the ability of the eye to discriminate between two points. The use of Snellen chart is one of the recognition acuity tests. There are 11 lines of block letters in a Snellen chart starting with one large letter. The number of letters in the subsequent rows increase with smaller size. The normal height of the letter E is 88 mm and the viewing distance is 6 m (~20 ft). The Snellen
fraction also called visual acuity (VA) shown at right side of each rows (Figure 3-17A) is obtained by [94];

\[
VA = \frac{WD}{LS}
\]  

(3.4)

Where WD is the viewing distance and LS is the distance at which the letter subtends an angle of 5 min of arc (letter size). To elaborate on the 5 min of arc, it should be noted that each letter is placed in a square that is divided to 5 by 5 squares. Each component of the letter subtends an angle of 1 min of arc (resolution limit of human eye) at eye lens from a certain distance. Thus each letter subtends an angle of 5 min of arc at eye lens (see Figure 3-17B).

**Figure 3-17** A. Snellen chart used for visual acuity test. B. Principle behind standardized Snellen chart.
We have printed Snellen charts for this study in A4 format. The height of letter E in printed chart was 42 mm and thus the viewing distance was set at 2.8 m ($\frac{42 \text{ mm}}{88 \text{ mm}} \times 6m$). Three Snellen charts were attached to the wall with center to center distance of 57 cm. The lighting condition of the room was adjusted to make sure the user has the best possible chance to see and read the chart. Figure 3-18 illustrates the visual acuity test setup. We first measure the visual acuity of the subject’s eyes one at a time. We asked the subject to cover one eye with a card and reminded him/her to not press on the eye. Then subject was asked to read the Snellen chart from the top and from left to right. The smallest line read is expressed as fraction for example 20/30; the top number is the distance of the Snellen chart to the subject in ft and the bottom is the distance at which a perfect eye can see the same line of the chart in ft. Based on the height of letters in Snellen chart the distance to the subject is adjusted so the Snellen fractions (VA) shown in the chart are always accurate. The smallest line read was defined as the smallest line that the subject can read either with no mistake or with up to two mistakes. After measuring the visual acuity of the subject’s eye. We continue the test with the eye that had larger value of the visual acuity. If both eyes had same visual acuity, we asked the subject to choose one eye. Then the subject was handed with a polarized glass lens of 0.25 D and asked to read the Snellen chart from top and from left to right through the polarized glass lens. Next, the SPP LC/Glass stack with 0.06 D optical power was
given to the subject. First subject was asked to read the Snellen chart in the middle with the same procedure. Then, the subject was asked to read the Snellen chart at the sides, one at the time, without turning his/her head.

![Image of visual acuity test setup](image.png)

**Figure 3-18** Visual acuity test setup. $d_1 = 2.8 \text{ m}$, $d_2 = 0.297 \text{ m}$, $d_3 = 0.21 \text{ m}$, $d_4 = 0.57 \text{ m}$ and $d_5 = 0.088 \text{ m}$.

but rather looking off-axis. If subject visual acuity while reading the Snellen chart at the sides through the SPP LC/Glass lens was not equal, the minimum visual acuity is recorded. The visual acuity in decimal form recorded for subject’s eye, with
polarized glass lens and with SPP LC lens at center and at edge are shown in Figure 3-19.

Figure 3-19 Visual acuity of subject eye without any visual aid, with linear polarized 0.25 D glass lens and with SPP LC/Glass stack of 0.08 D vs the subject number is shown. We have measured the visual acuity of subject looking through SPP LC/Glass stack on axis by central Snellen chart and also off-axis by Snellen charts placed with 57 cm distance from the center. Visual acuity of the polarized glass lens and the subject’s eye was measured only using the central Snellen chart.

The visual acuity measurement (Figure 3-19) shows that 6 out of 20 subjects had less visual acuity looking on axis through SPP LC/Glass stack compared to polarized 0.25 D glass lens. Surprisingly, 2 out of 20 subjects had better visual acuity in this case with SPP LC/Glass stack compared to polarized 0.25 D glass lens. It is perhaps more interesting to know about visual acuity degradation where the line of sight meets the phase segments by looking at Snellen charts that had 57 cm
center to center distance from the central one. In this case 6 subjects out of 20 realize up to 0.25 less visual acuity compared to the on-axis case while looking at the Snellen chart at the center. The measurement results shown in Figures 3-16 and 3-19 suggest that designed refractive LC lens can be a good alternative to progressive glasses.
CHAPTER 4

Optical Power Enhancement Using a Hybrid Design

The straightforward approach to make a tunable lens to solve the AC problem is to electrically adjust the OPD to provide a parabolic phase profile. One of the best approaches to obtain a parabolic phase profile is to use discrete concentric ring electrodes with equal area, as in this case the phase step between adjacent electrodes is a constant. A complicating issue is that for the considered application its aperture must be large. There is a tradeoff between aperture size and response speed and thus designing a LC lens with large aperture and reasonable response time is an uphill task. In chapter 2 and 3 we addressed this issue by introducing segments in the phase profile to increase the effective OPD without sacrificing the response time. We selected the phase segment width sufficiently large thus no observable diffraction was realized. A simple user evaluation study on SPP LC lens for 3D application was performed and is included in Chapter 6. Using 4.5 segments in phase profile and dual stacking we could successfully reduce the response time by 81 times. The obtain response time is still about 720 ms; according to a study on human eye
accommodation response [23], the maximum accommodation rate for young people is in the range of $1.878 \pm 0.625$ diopter/s that implies our obtained response time with SPP LC lens is sufficiently fast to keep pace with eye accommodation. However, a higher power and faster response time is preferred for video over what we have demonstrated in previous chapters.

4.1 Combining SPP LC and Pancharatnam Phase Lenses

It is well known that the response time could be fourfold improved by stacking cells with half of thickness. By stacking four SPP LC lenses with 10 µm thickness, we could obtain continuous focus tunability between 0 to 1.5 D with 180 ms switching speed between extreme optical powers. To enhance the optical power stacking, even more cells are required. Stacking more cells raises questions regarding processing cost. What follows is the alternative solution (here we call hybrid approach) that we can proceed to enhance optical power without need to stack more SPP LC lenses.

4.1.1 Design

One of the component in the hybrid system design is a Pancharatnam Phase lens (PPL). PPLs are polarization-sensitive lenses that can have optical power of different signs depending on the incident polarization state. Therefore by means of a switchable half wave plate, one can switch between orthogonal circular polarization states of light and make the PPL to toggle between positive and
negative optical powers. The basic design of a PPL has been described elsewhere [95, 96, 97, 98, 99], but here will provide a brief review of the structure and operation of these devices. A PPL is a thin film of a birefringent material, where the optical axis of the material is in the plane of the film while making an angle ($\beta$) that is a function of the radius of the lens. The film’s thickness is set by the condition 
\[
\frac{\Delta n \cdot d}{\lambda} = \frac{1}{2}
\]
so that when circularly polarized light that is Right-Handed Circular (RHC) or Left-Handed Circular (LHC) is incident on this device, the light exits as the orthogonal polarization state (either LHC or RHC, respectively). The interesting thing about this structure is that the relative phase of light exiting the aperture from any two points will have a phase difference that is given by $2\Delta \beta$, where $\Delta \beta$ is the difference in the value of $\beta$ between those points. If the propagation direction of light is along the $z$ axis, common phase factor of $\exp(i2\beta(x))$ in transmitted circularly polarized light from a half-wave retarder whose slow retarder axis has an in-plane spiral configuration where the azimuthal angle $\beta$ linearly and continuously rotates can be seen as a result of Jones calculus.

If a right handed circular wave ($E_{in}$) incident on a Pancharatnam film, the transmitted light leaving the half-wave plate ($E_{out}$) is defined as;

\[
E_{out} = R \cdot H \cdot W \cdot R^{-1} \begin{bmatrix} E_{xin} \\ E_{yin} \end{bmatrix}
\]  

(4.1)
where $R$ and $HW$ are rotation matrix and Jones matrix in principle axis of half-wave plate, respectively. Thus:

$$E_{out} = \begin{bmatrix} \cos\beta & \sin\beta \\ \sin\beta & -\cos\beta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{bmatrix} \cdot \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} E_{xin} \\ iE_{xin} \end{bmatrix} \quad (4.2)$$

where $\beta$ is the angle between the slow axis of the half-wave retarder and the x-axis. By multiplying the matrices in equation (4.2) we get:

$$E_{out} = \begin{bmatrix} \cos2\beta & \sin2\beta \\ \sin2\beta & -\cos2\beta \end{bmatrix} \begin{bmatrix} E_{xin} \\ iE_{xin} \end{bmatrix} = \begin{bmatrix} e^{i2\beta}E_{xin} \\ ie^{i2\beta}E_{xin} \end{bmatrix} \quad (4.3)$$

As last term in equation (4.2) suggests the phase of transmitted light leaving the Pancharatnam film can be accurately controlled by the azimuthal angle $\beta$. This type of device has a continuous phase profile (shown in Figure 4-1) and can increase to any value, which yields devices with very high efficiency.

For a lens of focal length $f$, the phase as a function of the lens radius $r$, is given by

$$\Gamma(r) = \frac{\pi r^2}{\lambda f} \quad (4.4)$$

With $\Gamma(r)$ equal to $2\beta(r)$, the value of $\beta(r)$ is given by:

$$\beta(r) = \frac{\pi r^2}{2\lambda f} \quad (4.5)$$
Perhaps the main disadvantage of PPL is the dependency of its focal length \( f \) to the incident wavelength \( \lambda \) as shown in equation (4.6).

\[
f = \frac{\pi r^2}{2\beta(r)\lambda}
\]

(4.6)

where \( r \) and \( \beta(r) \) are the lens radius and azimuthal angle of the optic axis of the half wave retarder, respectively.

**Figure 4-1** Pancharatnam phase concept resulting continuous phase that is defined by the azimuthal angle of the director.

Longitudinal chromatic aberration of PPL causes red and blue light to focus at different point than green light. Related to human perception, we have modeled a test system with scalar diffraction method [100]. In the test system, retina is modeled
as a flat plane, 20 mm behind the eye lens. The eye lens is modeled with parabolic phase profile, optical power of 50 D and 3 mm aperture that provides a diffraction-limited spot on the retina for an object located at infinity. The PPL meets the condition provided in equation (4.6), which yields a parabolic phase profile. The spatial variation of the azimuthal angle of the optic axis $\beta(r)$ is chosen to provide a power ($D_g$) of 0, 0.25, 0.5, 1.5, 2, 4, and 10 D for $\lambda_g$, which corresponds to green light. It is noted that the power will change for red and blue wavelengths, such that $D_r = \frac{\lambda_r}{\lambda_g}D_g$ for red light, and $D_b = \frac{\lambda_b}{\lambda_g}D_g$ for blue light. In the case of green light, a point source on the image plane is expected to yield a diffraction-limited spot on the image plane. Red and blue wavelengths are expected to yield a larger spot size since the focal length is wavelength dependent. Figure 4-2 summarizes the results of the calculations for values of $D_g$, by plotting the cycle/degree value corresponding to an MTF value of 0.8. It is reported that the resolution limit of the human eye is between six and 10 cycles/degree, so it can be concluded from Figure 4-2 that PPLs with optical powers of less than 2 D have the potential to provide a level of image degradation not observable by the human eye.

Y. H. Lee showed that PPL can provide fast response focus for AR/VR application [101]. His proposed design has the potential to switch between two planes. However, we integrate PPL with SPP LC lenses to be able to have a continuous range of focus. Our system goal is to have a lens that is able to vary
power from 0 to 2.5 D but here due to component availability, we construct and evaluate a system that is continuously variable from 0.375 to 2.625 D.

![Graph of cycles/degree corresponding to an MTF value of 0.8 versus PPD lens power.](image)

**Figure 4-2** Graph of cycles/degree corresponding to an MTF value of 0.8 versus PPD lens power.

Figure 4-2 illustrates the designed hybrid system with a continuous optical power range of 2.25 D (from 0.375 to 2.625 D). As mentioned earlier, a PPL can change power from positive to negative by changing the handedness of incident circularly polarized light. In our designed system, we first use a linear polarizer and a quarter wave plate to make the un-polarized light of real world circularly polarized. Next, there exists a LC switchable half wave plate to change the handedness of the incident light if desired. Other optical elements in the system are the PPL with ±0.375 D, glass lens with optical power of +1.5D and lastly a stack of two SPP LC lens with continuous tunability in the range of ±0.75D. With this
system, when the power of the PPL is -0.375D (determined by the state of the switchable half wave plate) the system power can be varied continuously from 0.375 to 1.875D when the SPP LC is varied from -0.75 to +0.75D. And when the power of the PPL is +0.375D the system range goes from 1.125 to 2.625 as the SPP LC is varied over its range. The design, fabrication and application of PPL have been reported by many research groups [95, 96, 97, 98, 99]. The fabrication details of PPL used in this study is included in Appendix D of this dissertation.

![Figure 4-3 Designed hybrid system including I: Light source, II: Glass lens, III: PPL, IV: Switchable half wave plate, V: Quarter wave plate, VI: SPP LC lens, VII: Linear polarizer, VIII: Imaging system (eye/CCD).](image)

**4.2 Characterization**

This section overviews the characterization steps performed on the hybrid system.

**4.2.1 Spot Profile Measurement**

We have captured the spot size of the hybrid system at focal point for minimum and maximum optical powers. Beam size of He/Ne laser of 543 nm wavelength was expanded by means of a 10X beam expander and then sent through an aperture with
5 mm diameter. A Canon Rebel XSi CCD sensor (camera with lens removed) was placed at focal point to capture the spot profile. Figures 4-4A and B show the spot profile at 2.625 D and 0.375 D optical powers, respectively. For each optical power state we presented the spot profile at two exposure levels (maximum optical power (2.625 D): Figures 4-4A.1 and 4-4A.2 & minimum optical power (0.375 D): Figures 4-4B.1 and 4-4B.2). Furthermore, we showed the intensity profiles across the blue and red lines of Figures 4-4A.2 and 4-4B.2 in Figures 4-4A.3, 4-4A.4, 4-4B.3 and 4-4B.4, respectively. Black curves in these images represent the theoretical spot size. The spot size measurement shows that the deviation from theoretical spot size is insignificant and imaging performance of hybrid design is promising.

**Figure 4-4A.1** Spot size of hybrid system at 0.375 D state captured at 1/30th exposure. **A.2** Spot size of hybrid system at 0.375 D state captured at 1/3000th exposure. **A.3** Intensity profile across blue line shown in Figure A.2. **A.4** Intensity profile across red line shown in Figure A.2. **B.1** Spot size of hybrid system at 2.625 D state captured at 1/30th exposure. **B.2** Spot size of hybrid system at 2.625 D state captured at 1/3000th exposure. **B.3** Intensity profile across blue line shown in Figure B.2. **B.4** Intensity profile across red line shown in Figure B.2. The dark curves shown in graph is the theoretical spot size.
We also measured the spot profile at three intermediate power states of hybrid system including 0.75 D, 1.5 D and 2.25 D. In 0.75 D and 1.5 D states, we added a glass lens with 1.5 D and 0.75 D, respectively to make the total optical power of system in all cases 2.25 D which make the comparison much easier. We plotted the obtained results in Figure 4-5 which shows excellent optical performance of the system in different focusing power.

![Figure 4-5](image1.png)

**Figure 4-5** Light passes through 5 mm aperture at the center of hybrid system. Three optical power states of the lens system shown in Figure 4-1 were measured for and optical power of 2.25 D (solid lines), 1.5 D (lines marked with star) and 0.75 D (lines marked with circle). As described in the text auxiliary glass lens were added to make the total optical power of the measured system constant at 2.25 D.

### 4.2.2 Imaging Resolution

We have evaluated the imaging resolution of our design system at its maximum and minimum optical power states using USAF 1951 which is shown in Figures 4-6 and 4-7. The condition of each image is explained in the Figure caption.
Figure 4-6 The camera lens is focused at infinity. The eyechart is placed at 40.5 cm. A.1. Image is taken with 50 mm camera lens. B.1. Image is taken with 2.5 D glass lens plus 50 mm camera lens. C.1. Images is taken with our designed hybrid system at 2.625 D power state plus 50 mm camera lens. The object was moved to 39.5 cm distance to find the best focus. A.2, B.2 and C.2 are expanded image of the highlighted region with red square of image A.2, B.2 and C.2, respectively.

Figure 4-7 The camera lens is focused at infinity. The eyechart is placed at 2 m. A. Image is taken with 50 mm camera lens. B. Image is taken with 0.5 D glass lens plus 50 mm camera lens. C. Images is taken with our designed hybrid system at 0.375 D power state plus 50 mm camera lens. The object was moved to 166 cm distance to find the best focus.
4.3 Discussion

In considering the data shown in Figure 4-5 one can realize that the solid line which indicates the PSF of the 2.25D optical power is wider than the PSF of the other two optical powers of 1.5D (shown with lines marked with star) and 0.75D (lines marked with circle). This is probably because the aperture size during the last experiments was <5mm. In the assessment of data shown in Figures 4-6 and 4-7, no chromatic aberration was observed. This was expected because of the low optical power selected for the PPL. Using our proposed hybrid system we could obtain similar imaging resolution as with a glass lens (Figures 4-6B2 and C2). The imaging resolution of the system at minimum optical power (Figure 4-7) is degraded in comparison with Figure 4-6 which is most probably due to the negative phase profile of SPP LC lens that requires further voltage adjustment to be exactly parabolic. Contrast reduction also known as haze observed in the images captured by the hybrid system is due to the gap between the electrodes used in SPP LC lens. As discussed in Chapter 2, to eliminate the gap between the electrodes and thus the haze, use of “floating” electrodes that are not driven but capacitively coupled to the electrodes beneath are suggested [43]. We have demonstrated and compared the performance of SPP LC lens with and without “floating electrodes” in Chapter 3. For the example hybrid system discussed in this chapter, the SPP LC lens used did not have “floating electrodes,” which results the observed haze. This haze can be
easily eliminated for practical application using the SPP LC lens with “floating” electrodes. The characterization details of hybrid system design are included in Appendix E.

4.4 Conclusion

In this chapter we proposed a hybrid system consist of SPP LC lenses and PPL. The proposed system provides continuous tunability in the range of 0.375 to 2.625 D that can be used to solve AC issue in 3D displays and VR systems. It should be noted that since the 3D and VR displays are polarized the proposed designed do not reduce the transmission level of the system.
CHAPTER 5
A Thin Film LC-based Compensator for the Chromatic Aberration of Optical Lenses

Chromatic aberration is recognized as being one of the critical issues in designing refractive lenses used in polychromatic applications. In a simple optical system, chromatic aberration can be controlled by lens shape or stop position [102]. However, the majority of optical systems require higher precision that is obtained by combining multiple optical elements. In 1733, Chester MoorHall successfully corrected the chromatic aberration of telescope lenses by combining elements made of crown and flint glasses [103]. Since then many attempts have been made to make an achromatic lens [103, 104, 105, 106, 107, 108, 109, 110, 111, 112] [113, 114, 115, 116, 117, 118, 119]. Although the usage of optical elements with differing dispersion enables correcting of chromatic aberration, in practice it suffers a number of pitfalls including high cost, and difficulty in producing large sizes. The aim of theoretical study in this chapter is to suggest a new and more practical achromatic lens system in which only requires an inexpensive thin film coating to be used with a single glass lens. As discussed in Chapter 4, Pancharatnam Phase Lenses (PPLs)
have very high efficiency and a well-defined parabolic phase profile [120, 121, 122, 123]. They can be experimentally fabricated using the approach of polarization holography [124, 125, 126, 95]. This kind of lens is very compact, with the thickness only being limited by glass substrate on which the thin active layer (~2 μm) is coated. We have discussed in Chapter 4 that PPL is a half-wave retarder that has its optical axis in the plane of the film with the azimuthal angle (β), which is spatially varying along the radial direction. More specifically, when circularly polarized light is incident on a PPL, the phase of circularly polarized light exiting it is related to the angle β between the optic axis of a half-wave retarder with respect to a fixed lab axis. If β increases in a continuous manner, the phase Γ increases continuously with Γ = 2β. A PPL can be fabricated by meeting the condition Γ = \( \frac{\pi r^2}{\lambda f} \), which defines a parabolic phase profile for a lens with radius r and focal length f at a designed wavelength λ as shown in Figure 5-3. The condition can be written as \( f = \frac{\pi r^2}{2\beta\lambda} \), where we can see f is dependent of λ when r is fixed. A PPL can be utilized to correct chromatic aberration of a conventional glass lens with large dioptric power because it has a strong chromatic aberration of the opposite sign as that of a conventional glass lens. Using this principle, Lee has shown that the chromatic aberration of a switchable PPL can be compensated by a glass lens [101]. The emphasis of this chapter is the use of a passive thin film layer to compensate the chromatic aberration of a glass lens.
5.1 Problem Definition

Our objective in this chapter is to show that a doublet consisting of a glass lens and a PPL can have a low chromatic aberration. We start with the requirement that the optical power of a doublet in blue, red and green must be equal.

\[ D_{d b} = D_{d g} = D_{d r} \]  

(5.1)

In this chapter, D denotes for optical power in diopter, superscripts are used to define the wavelengths (b for blue, g for green and r for red) and optical elements are designated in subscripts (d for doublet, gl for glass lens with chromatic aberration and ppl for PPL). The optical power of doublet is the sum of optical power of glass lens and PPL:

\[ D_{d b} = D_{gl b} + D_{ppl b} \]  

(5.2)
\[ D^g_d = D^g_{gl} + D^g_{ppl} \]  \hfill (5.3)

\[ D^r_d = D^r_{gl} + D^r_{ppl} \]  \hfill (5.4)

5.2 Parameter definition

The optical power of a thin glass lens in air is defined with the lens maker equation [80] where \( n \) is the refractive index and \( R_1 \) and \( R_2 \) are the radii of curvature.

With \( \rho = \frac{1}{R_1} - \frac{1}{R_2} \):

\[ D^g_{gl} = (n_g - 1)\rho \]  \hfill (5.5)

\[ D^b_{gl} = (n_b - 1)\rho \]  \hfill (5.6)

Writing equation (5.5) based on \( \rho \) and substituting it into equation (5.6) gives;

\[ D^b_{gl} = \frac{n_b - 1}{n_g - 1}D^g_{gl} \]  \hfill (5.7)

Similarly the optical power of the glass lens in red is;

\[ D^r_{gl} = \frac{n_r - 1}{n_g - 1}D^g_{gl} \]  \hfill (5.8)

The optical power of a PPL is represented in the following equations, where \( r \) is the lens radius, \( \lambda \) is the incident wavelength and \( \beta \) is azimuthal angle of the optic axis of the half wave retarder in the plane of the film:

\[ D^g_{ppl} = \frac{2\beta(r)\lambda_g}{\pi r^2} \]  \hfill (5.9)

\[ D^b_{ppl} = \frac{2\beta(r)\lambda_b}{\pi r^2} \]  \hfill (5.10)
Using equation (5.9) to define the value of $\beta(r)$, for a given value of $D_{ppl}^g$, we can express equation (5.10) as the following;

$$\frac{D_b^b}{\lambda_g} = D_{ppl}^g$$  \hspace{1cm} (5.11)

Likewise the optical power of PPL in red can be written as equation (5.12).

$$\frac{D_r^r}{\lambda_g} = D_{ppl}^g$$  \hspace{1cm} (5.12)

**5.3 Solution**

Setting doublet optical power in blue and red equal ($D_d^b = D_d^r = D_{gl}^r + D_{ppl}^r$) and using equation (5.8) and (5.12) results;

$$D_d^b = \frac{n_r - 1}{n_g - 1} D_{gl}^g + \frac{\lambda_r}{\lambda_g} D_{ppl}^g$$  \hspace{1cm} (5.13)

Solving equation (5.13) for $\frac{D_{gl}^g}{D_{ppl}^g}$ provides;

$$\frac{D_{gl}^g}{D_{ppl}^g} = \frac{n_g - 1}{n_r - 1} \left( \frac{D_d^b}{D_{ppl}^g} - \frac{\lambda_r}{\lambda_g} \right)$$  \hspace{1cm} (5.14)

In the above equation replacing $D_d^b$ with $D_{gl}^b + D_{ppl}^b$ and using equations (5.7) and (5.11), gives;

$$\frac{D_{gl}^g}{D_{ppl}^g} = \frac{n_g - 1}{n_g - n_b} \left( \frac{\lambda_r - \lambda_b}{\lambda_g} \right)$$  \hspace{1cm} (5.15)
Term $\frac{n_g - 1}{n_b - n_r}$ is the definition of Abbe number ($V_D$) [127, 128] and so equation (5.15) can be expressed as:

$$D_{ppt}^g = \frac{D_{gt}^g}{V_D \left( \frac{\lambda_g}{\lambda_r - \lambda_b} \right)}$$

(5.16)

Using equation (5.16), the power of a thin film PPL can be specified to produce an achromatic lens system that has a power close to that of the glass lens being chromatically corrected. The reciprocal of the abbe number determines the amount of dispersion. Low dispersion optical glasses normally used for achromatic reasons are crown glasses with Abbe number around 60 and more specifically borosilicate glass Schott BK7 with Abbe number of 64.2 [129].
Figure 5-2 Dioptric change with respect to wavelength for 1. Pancharatnam lens (cyan line) 2. Glass lens made of BK7 (red line); Abbe number value of 64.2 and radius of curvature 20.6 mm. Proposed super achromat system (dark blue line) which is the combination of cyan line and red line.

As an example, if we consider a 25 D glass lens with an abbe number of 64.2 and substitute the wavelength of the Fraunhofer D-, F- and C- spectral line (589.3 nm, 486.1 nm and 656.3 nm, respectively.) Then $\frac{D^{g}}{D^{ppl}}$ is 18.54 and the required optical power of PPL to correct the chromatic aberration of a 25D lens is 1.35 D. as shown in Figure 5-1.
CHAPTER 6

Conclusions and Future Work

Ivan Sutherland, known to be one of the pioneers in the field of VR and the inventor of one of the earliest HMD, Sword of Damocles, describes the ideal display [130];

"The ultimate display would, of course, be a room within which the computer can control the existence of matter. A chair displayed in such a room would be good enough to sit in. Handcuffs displayed in such a room would be confining, and a bullet displayed in such a room would be fatal."

Since 1957 when the first VR system, Sensorama, was built by Morton Heilig [131], there have been increasing amount of efforts to create the virtual world as real as possible. Focus is one of the critical cues to generate a seamless virtual experience, yet inconsistency of accommodation and convergence (the AC problem) is the leading cause of user discomfort in all current HMD and 3D displays. Although extensive research has been carried out to resolve the AC problem, significant advancement in display technology is required before the suggested approaches could be practical.
This dissertation seeks to solve AC issue with LC lens and eye tracking system that does not have practical constraint. The main purpose of this study was to develop a large area LC lens that can be used not only in VR/AR/3D systems but also for ophthalmic applications.

6.1 Conclusions

The concept of eye tracked LC lenses was originally proved by running a simple study on the effect of additional lens “fix” on reducing the eye strain while looking at a 3D scene. To ensure that the only factor under study is AC conflict, all the other factors which might also cause eye strain including wrong calculation in creating 3D scene or the movement of the head were eliminated. Nvidia 120 Hz active LCD glasses based 3D viewing system was used in this analysis. A 3D scene was created that consisted of three objects (wire frame cubes) placed at different distances to the user: 50 cm (object A: on screen), 80 cm (object C: 30 cm behind the screen) and 100 cm (object B: 50 cm behind the screen). The parallax corresponding to the depth of each object was rendered correctly to provide the stereopsis cue. The perspective and occlusion cues were all considered when creating the scene. To appreciate the relative motion cue, a chin rest was used in our set up to fix the user’s head exactly at 50 cm distance to the screen. This prevented the user to move his/her head. The user was asked to sit on the chair at the distance of 50 cm to display and place his/her chin on the chin rest and to wear the active glasses combined with a pair of reading
glasses. The user was not aware of the value of the optical power, but was informed that there existed three conditions 1. diopteric power close to zero (0 D); 2. medium diopteric power (1 D); and 3. high diopteric power (2 D). Figure 6-1 shows the test set-up. First, the user was handed the reading glasses of 0 D and was reminded to only focus on object A for 10 s. Following questions were asked from user during the test. “What is the level of comfort ranging from 0 to 10 in viewing of focused object?” If the user asked us “what do you mean by the level of comfort?, we answered “it implies how clear and without any problem the 3D scene can be seen. If your eye gets tired or you have difficulty focusing on the object, the level of comfort should be low.” In the next steps, the reading glasses with 1 and 2 D were examined, causing the eye to focus as if it was looking at an object at the distance of object C and B, respectively, rather than the distance to the display screen. Subsequently, we repeated the experiment but this time, the user was asked to focus only on object B. The results of this study is shown in Figure 6-2. This figure illustrates the preference of users in selecting the best power state for observing object A and B, and agrees well with the hypothesis that adding optical power to the viewers eyewear, making the focus cue consistent with the convergence cue, is beneficial to the viewing experience.

Eye tracking systems and tunable LC lenses can also be used in vision application for presbyopic patients who suffer from other refractive errors. The
The proposed system can eventually replace the progressive glasses to provide the patient with full field of view (FOV). In this dissertation, we collectively called both applications as near-to-eye. In our approach, the depth of virtual objects is determined by tracking the user’s eye pupils which is used to set the tunable LC lens’ optical power. The desired FOV necessitates large aperture LC lenses which have been overlooked due to the significant increase in the response time of LC lens. Design and fabrication of large aperture tunable lens with reasonable response time and optical power were the goals of this dissertation that we have elaborated in previous chapters.

![Figure 6-1](image)

**Figure 6-1** The experimental setup. Object A. on the screen with 50 cm distance to the user and Object B. behind the screen with 100 cm distance to the user.

The minimum acceptable aperture size for near-to-eye application is 20 mm diameter and the desired optical power range is about 2 D. In chapter 2, we discussed that a parabolic LC lens with 20 mm diameter and optical power range between -
0.375 D to +0.375 D has significant response time due to the large thickness. Solutions to enhance optical power without increase in the thickness and thus the response time include multiple LC lenses stacking and introducing phase segments into phase profile. As surveyed in section 2.3.5, with dual stacking of LC lenses not only twice of the optical power range is obtained but also the off-axis performance is improved. However, even the dual stacking is not sufficient to obtain the desired optical power with 20 mm diameter. Chapter 2 showed that by introducing 4.5 segments into phase profile of the dual stack LC lenses the desired optical power range for 20 mm diameter is achieved with 81 times faster response time.

![Figure 6-2](image)

**Figure 6-2.** Preference of users in optical power states while viewing. A. Object A: on the screen with 0.5 m distance to the user B. Object B: behind the screen with 1 m distance to the user ■ 0 D □ 1 D △ 2 D ▽ 1 or 2 D.

The central question in this dissertation asks what the optical effect of phase segments is. This study answers to this question by considering the worse diffraction effect appearing as double images. This worse case diffraction occurs when there
exists $\frac{\lambda}{2}$ phase difference between the designed and incident wavelengths. This research analytically shows that the width of phase segments can be designed sufficiently large (>1 mm) to make the resulting diffraction angle smaller than 1 arc/min which is the angular resolution of human eye. The designed SPP LC lens has been numerically reexamined with a scalar diffraction calculation for a near-to-eye system in Chapter 2. The calculation results showed that the MTF drop due to the phase segments is not observable to human eye [Chapter 2]. The details of modelling methods used for the design and performance evaluation of LC lens including LC director calculation and scalar propagation of light are included in Appendix A of this dissertation. The designed SSP LC lens was successfully fabricated and full explanation of the fabrication process are given in Appendix B. The full characterization and user evaluation tests showed that SPP LC lens is an excellent solution for both AC conflict in VR/AR/3D displays and limited field of view in progressive/bifocal speckles [Chapter 3]. Furthermore optical power enhancement for VR/3D applications is obtained using a hybrid system which consists of SPP LC and Panchartnam phase lenses [Chapter 4]. Finally, this dissertation proposes a useful application for a Panchartnam phase thin film that can compensate the chromatic aberration of optics with large diopteric power, for instance VR lenses [Chapter 5].
To conclude, the author would like to include the results of the user preference study that was implemented with SPP LC lens. The same setup and procedure as discussed earlier in this chapter was used with only replacing the reading glasses with stack of two SPP LC lenses (with ±0.68 D optical power) and a fixed power glass lens (with +0.75 D optical power) that was added to the active LCD glasses. The optical power range of the stack was from 0.07 to 1.43 D. Similar to the previous study, the user was not aware of the value of optical power, but was informed that there exist two conditions 1. diopteric power close to zero (0.07D), and 2. higher diopteric power (1.42D). For object A and B the two conditions have been examined with exactly same procedure that was discussed previously. Figure 6.3 shows the results of this user preference test. These results, using LC lenses, are in line with the more controlled experiments of Padmanaban [132].

![Figure 6-3 Preference of users in optical power states while viewing. A. Object A: on the screen with 0.5 m distance to the user B. Object B: behind the screen with 1 m distance to the user 0.07 D 1.43 D.](image)

From the beginning of this work, our efforts were focused to design a large aperture LC lens with a reasonable response time. Here the author would like to
comment on the obtained response time by the designed SPP LC lens and also the hybrid system with 20 mm diameter. The LC material used for the designed SPP LC lens was MLC 18349 from Merck with 0.2704 birefringence. The properties of this material are $K_{11} = 14.1 \times 10^{-12} \text{N}$ [133], $K_{22} = 7.1 \times 10^{-12} \text{N}$ [83], $K_{33} = 25 \times 10^{-12} \text{N}$ [133], $n_e = 1.8025 \text{ at } 589.3 \text{ nm and } 20^\circ\text{C}$ [133], $n_o = 1.5321 \text{ at } 589.3 \text{ nm and } 20^\circ\text{C}$ [133], $\varepsilon_\parallel = 22.0 \text{ at } 1\text{kHz and } 20^\circ\text{C}$ [133], $\varepsilon_\perp = 5.5 \text{ at } 1\text{kHz and } 20^\circ\text{C}$ [133]. The viscosity of MLC 18349 was not available in the datasheet neither we have measured it. Therefore to roughly estimate the analytical response time of the designed lens, the rotational viscosity of E7 (250 mPa.s [134]) is used. E7 was chosen due to similar birefringence of 0.223. Using equation 2.5, the response time of the designed SPP LC lens with 20 μm thickness to switch from 1.5 D to 0 is about 0.72 s. The switchable retarder used in the hybrid system discussed in chapter 4 can be designed to switch ranging between 10μs to 25 ms. Thus, the response time of the hybrid system also is defined by the SPP LC lens. Although this response time is faster than the human eye accommodation speed for a static image (>1.12 ± 0.658 D/s) [23], it should probably be faster for higher frequency scenes such as videos. To enhance the response time of the SPP LC lens by four times, we have fabricated and optically coupled four 10 μm SPP LC lens to obtain 180 ms response time which is desirable for VR/AR/3D application [135]. The interferogram images of the fabricated 10 μm SPP LC lens at maximum and
minimum optical powers are shown in Figure 6-4. The image resolution of the stack of four 10 μm LC SPP lenses combined with 0.75 D glass lens at its maximum power (1.43 D) is included in Figure 6-5. For this experiment the camera lens is focused at infinity and USAF 1951 test target is placed at 80 cm.

Figure 6-4 Interferogram images of SPP LC lens with 10 μm thickness at A. +0.17 D. B. -0.17 D.

Figure 6-5A was taken when the center of camera lens was aligned with the center of SPP LC/Glass stack (including four SPP LC lenses of 10 μm). For Figure 6-5B the center of SPP LC/Glass stack was translated by 7.5 mm and aligned with the center of camera lens. For both Figure 6-5A and B a 5 mm physical aperture was used between the camera lens and SPP LC/Glass stack.
Figure 6-5. Image resolution of stack of four SPP LC lens (10μm) and 0.75 D glass lens at maximum optical power of 1.43D when A. Center of camera lens and SPP LC/Glass stack are aligned. B. Center of camera lens and edge of SPP LC/Glass stack are aligned. The camera lens is focused at infinity and the best focus is found when the object was placed at 80 cm.

It is clear that the response time can further be optimized by modifying the LC material with higher birefringence [91] and lower viscosity (materials with 80 mPa.s are commercially available [134]).

6.2 Future Work

Chapter 4 considers a hybrid system consisting of a SPP LC lens and PPL as a solution to AC issue in VR and 3D displays. Although we can use the dual stacking with orthogonal polarization as discussed in section 2.6.3. to make the SPP LC lens polarization independent, the PPL requires a circular polarization state. In VR and 3D displays the light is already polarized thus using the hybrid system does not change the system transmission significantly. However, in the AR system where adding hybrid lens in front of the user’s eye is considered (this excludes the case where the hybrid system is added to the projector side.) and also in vision
applications, a polarized lens is not desirable due to 50% transmission loss. The question is whether we can make the PPL lens polarization independent. Previous works show that this is possible. Based on the Pancharatnam phase concept, H. Cheng propose dual twist optimization to enhance the diffraction efficiency for large steering angles. In this structure the LC director spirals about y both in-plane (function of x position) and out of plane (function of y position) thus referred to as dual twist [136]. Y. Weng also showed the diffraction efficiency enhancement with dual-twist structure but called it Polarization Volume Grating (PVG) [137]. K. Gao used two such Dual-Twist Pancharatnam Phase Deflectors (DTPPD) with equal in-plane periodicity and equal but opposite out of plane twist angle to obtain a beam deflector with 80° FOV [138]. The director of LC/reactive mesogen, in a dual-twist configurations changes with following the equation (6.1) [139].

\[ n(x, y) = [\cos(a(x, y)), \sin(a(x, y)), 0] \]  
\[ \text{where } a(x, y) = \frac{H}{A_x} x + \frac{V}{A_y} y \]  

Depending on the sign of H and V, four dual-twist configurations are possible as shown in Figure 6-6. Simulation results by Beam. Co showed that if two dual-twist configurations of H+V+ and H-V- are combined, polarization in-dependency is achieved as shown in Figure 6-7 [139]. This is a very promising simulation results
which is not yet experimentally obtained. It would be very interesting to fabricate a polarization independent PPL based on the dual-twist configurations suggested in [139] which can find applications in many fields. Making polarization-independent PPL that is also switchable is an interesting topic for research that if would find solution would let us to use the hybrid system proposed in chapter 4 for AR and vision applications.

![Diagram of DTPPD configurations](image)

**Figure 6-6** Four configurations possible for DTPPD.
Another important question that needs to be investigated further is whether we can further expand the aperture size of the SPP LC lens. One might imagine that by adding more phase segments, the problem might be solved. We have done an analytical prediction based on the minimum grating pitch for the case of having 5 and 10 segments in the phase profile as shown in Figure 6-8. Angular resolution of human eye is the reference point which is 1 arc/min (0.016°). The analytical prediction shows that although the diffraction angle resulted from five segments always remains smaller than the angular resolution of the human eye, the diffraction angle generated by adding ten segments into phase profile will be observable to human eye. Therefore the diffraction might be still acceptable for an intermediate number of resets, for instance 7, especially because the diffraction will be at the periphery of the user’s sight for on-axis viewing.
Without adding segments to the phase profile is there any simple way to expand the field of view of SPP LC lens? This is an important question which should be further investigated.

Related to SPP LC lens characterization, some image quality degradation was observed when negative phase was imposed on the SPP LC lens, which requires further phase profile optimization. The SPP LC lens was designed based on a circular ring with equal area and resistor networks. As surveyed in chapter 2, the ring electrodes follow a parabolic pattern and with the help of resistor network, constant voltage drops between adjacent electrodes positioned within two vias are imposed. In order to test the linear voltage change between vias, Conductive Atomic Force Microscopy (C-AFM) will be very informative and is recommended.
Analytical calculation of the diffraction angle of the resets across visible range of spectra for 5 and 10 phase segments.

Based on our design the voltage drop between to vias ($V_B - V_A$) is linear and thus equation (6.3) is satisfied.

\[ V_B = V_A + (V_B - V_A) \times \frac{x}{L} \]  \hspace{1cm} (6.3)

where $V_A$ and $V_B$ are the voltages of first and last electrodes of each segments, respectively and $L$ is the distance between the two vias and $0 \leq x \leq L$. By applying voltage to the first electrode of each segments ($V_A$), and applying $V_B$ calculated from (6.3) to the AFM tip, one can measure the electrical current and thus determine the resistance. If the resistance changes linearly between points A and B, the linear
voltage variation between two vias is obtained [141]. C-AFM enables us to predict the shape of phase profile based on the mask design.

Taken together, the results of our study on the SPP LC lens and hybrid lens designs are significant in at least major two respects; it indicates that LC based lenses are promising for near-to-eye applications and moreover, the combination of theory, modeling, fabrication and characterization details provided in this study has important implication for developing LC lenses. There is abundant room for further research in this topic including user perception studies on the performance of SPP LC lens and hybrid lens in a HMD, and whether we should and could expand the aperture size of SPP LC lens further, also how well we can achieve polarization independency by stacking SPP LC lenses and whether we can design hybrid system to be insensitive to the polarization.
APPENDIX A

Modeling Approaches

A.1. Director Relaxation Calculation

The most important part of simulation is to define the problem. Our main objective to calculate director configuration is to obtain the phase profile. This near field phase profile is used to calculate the phase at far-field using scalar diffraction calculation which is explained in section A.2.

SPP LC lens consists of a liquid crystal layer which is sandwiched between two substrates; the top substrate with continuous ITO electrode and the bottom substrates with stripped electrodes. For simplicity, we assumed that the director orientation is only changing along the normal to the cell (z-axis) and along one arbitrary axis in the plane of the substrates (here along the lens diameter, x-axis). The electrode stripes are placed along y-axis. This assumption will reduce our simulation to a two dimensional model.
Our aim is to obtain spatially variable phase (in radian) across the lens aperture which is defined as follow:

\[ \phi(x) = \frac{OPD(x)}{\lambda} \times 2\pi \]  

(A.1)

here \( \lambda \) is the incident wavelength and \( OPD \) is the liquid crystal retardation which is given by:

\[ OPD(x) = \int_0^d (n_{eff}(x, z) - n_o) dz \]  

(A.2)

where \( d \) is the thickness of liquid crystal cell and \( n_{eff} = \frac{n_o n_e}{\sqrt{n_o^2 \cos^2 \theta(x,z) + n_e^2 \sin^2 \theta(x,z)}} \)

(for normal incident light with linear polarization along x-axis), while \( \theta \) is the angle between the director and normal to the cell (z-axis).

To obtain the change in OPD along x-axis (equation (A.2)) and thus the phase profile (equation (A.1)), we need to calculate the change in polar angle \( \theta(x, z) \) (director orientation). In the following section we describe the director modeling in details.

**A.1.1 Mathematical Equations**

To calculate the equilibrium (or dynamic) state of director configuration in a confined geometry, free energy must be minimized. To form a mathematical description for the free energy, information about all forces acting on the liquid crystal director is necessary. These forces include elastic forces, forces due to the interaction with applied fields and surface forces. The Frank-Oseen free energy
density equation describes the energy density related to elastic forces for a nematic liquid crystal and is given by [142]:

\[
f_{el} = \frac{K_{11}}{2} (\nabla \cdot \vec{n})^2 + \frac{K_{22}}{2} (\vec{n} \cdot \nabla \times \vec{n} + q_0)^2 + \frac{K_{33}}{2} (\vec{n} \times \nabla \times \vec{n})^2
\]  

(A.3)

Here \(K_{11}, K_{22}\) and \(K_{33}\) are splay, twist and bend elastic constant. \(q_0\) represents the chiral property of twist deformation.

Several mathematical representation of elastic free energy have been used. Vector representation is the simplest method. This model assumes that the director points in one direction similar to a vector which is not correct for a nematic LC. However, vector method satisfies all the requirements for the simulation. By careful definition of the initial director configuration, vector method can provide accurate results which agrees with experiment. In this method if the director points in one direction, the value of its elastic free energy density obtained from the model will be different from the value of elastic free energy density of the case when the director turns 180° and points in opposite direction. This does not happen in a real experiment because the molecules do not know which direction they are pointing. To ensure that this lack of symmetry does not limit the vector method, we define the boundary conditions so the simulation starts from the correct topological state.

The other part of free energy relates to the interaction with the applied field. Liquid crystal molecules can be reoriented by applying a magnetic or an electric
field. In SPP LC lens the liquid crystal are interacting with electric field and for that reason we will not discuss the free energy due to the interaction with magnetic field. Liquid crystals are approximately rod-shaped molecules with two dielectric constants, along the long axis ($\epsilon_{ll}$) and along the short axis ($\epsilon_{s}$). The dielectric anisotropy is defined as $\Delta \epsilon = \epsilon_{ll} - \epsilon_{s}$. Electric dipoles are induced under applied electric field to liquid crystal. These dipoles want to align with electric field. This results the liquid crystal director to align parallel (if $\Delta \epsilon > 0$) or perpendicular (if $\Delta \epsilon < 0$) to the electric field. Depending on whether the liquid crystal is connected to a voltage source or not, the contribution of electric energy to free energy is different. If the liquid crystal cell is disconnected from voltage source, the free charge between liquid crystal and electrode is fixed (constant charge). In that case we add $+\frac{1}{2} \vec{E} \cdot \vec{D}$ to equation (A.3) and generally we call the total free energy in this case “Helmholtz”. If the liquid crystal cell is connected to voltage source such that the voltage applied across the cell is fixed (constant potential) which is the case for SPP LC lens, we add $-\frac{1}{2} \vec{E} \cdot \vec{D}$ to equation (A.3). The electric free energy ($f_{E}$) at constant voltage [81] is given by:

$$f_{E} = -\frac{1}{2} \vec{E} \cdot \vec{D} = -\frac{1}{2} \vec{E} \cdot \left[ \epsilon_{0} \epsilon_{s} \vec{E} + \epsilon_{0} \Delta \epsilon (\vec{E} \cdot \vec{n}) \vec{n} \right]$$  \hspace{1cm} (A.4)

Which can be rewritten as:
\[ f_E = -\frac{1}{2} \varepsilon_0 \varepsilon_\perp E^2 - \frac{1}{2} \varepsilon_0 \Delta \varepsilon E_i E_j n_i n_j \quad i, j = x, y, z \]  

(A.5)

The last part of free energy relates to liquid crystal interaction with surfaces (anchoring energy). We assume that the anchoring strength is infinite. To model “infinite” anchoring, the director at the surface is used as a boundary condition. We do not update the director at surfaces and is fixed.

A.1.2 Director Update Formula

With mathematical description of Gibbs free energy \( f_G \) which is the sum of elastic free energy (equation (A.3)) and electric free energy (equation (A.5)), generalized force can be calculated by taking functional Euler-Lagrange derivative of the Gibbs free energy density as follow [143]:

\[
[f_G]_i = -\delta f_G n_i = \frac{\partial f_G}{\partial n_i} - \sum_{j=x,y,z} \frac{d}{dj} \left[ \frac{\partial f_G}{\partial \left( \frac{dn_i}{dj} \right)} \right]_j i = x, y, z
\]  

(A.6)

Generalized force is equal to viscous torque as shown in equation (A.7) [143].

\[
\gamma \frac{dn_i}{dt} = [f_G]_i + \lambda n_i
\]  

(A.7)

Here \( \gamma \) is the viscosity coefficient and \( \lambda \) is the Lagrange multiplier term to keep the unit length of the director. We drop the Lagrange multiplier and renormalize the
director to be of unit length at each iteration. We replace the derivatives in equation (A.7) with finite difference to find the update formula for the director.

\[
\gamma \frac{\Delta n_i}{\Delta t} = [f_G]_{n_i}
\]

\[
\Delta n_i = \frac{\Delta t}{\gamma} [f_G]_{n_i}
\]

\[
n_i^{\text{new}} - n_i^{\text{old}} = \frac{\Delta t}{\gamma} [f_G]_{n_i}
\] (A.8)

The above equation can be used to iterate the director simulation. After the director is updated, we renormalize it back to unit length as follow:

\[
n_i = \frac{n_i}{\sqrt{n_x^2 + n_y^2 + n_z^2}} \quad i = x, y, z
\] (A.9)

**A.1.3 Voltage Update Formula**

To calculate the voltage after each iteration, we use direct solve method. In this method, the electric field is discretized into grids. Electric field is related to potential \((\vec{E} = -\nabla \varphi)\). We start from Gauss’s law and considering that there is no free charge in liquid crystal medium we get:

\[
\nabla \cdot \vec{D} = \nabla \cdot (\hat{\vec{E}} \cdot \vec{E}) = -\nabla \cdot (\hat{\vec{E}} \nabla \varphi) = 0
\] (A.10)

If the medium is isotropic and uniform we will obtain the Laplace equation as
\n\n\[ \n\n\n\]

We solve Laplace equation using finite difference method. We find the potential at each point using the potential of its nearest neighbors. The dielectric constant of liquid crystal in electric field calculation is assumed to be isotropic ($\varepsilon_{LC} = 10$). The voltage go to zero at infinity rather than liquid crystal-glass interface. Therefore in simulation, we first calculate the voltage profile in the glass layer and then in liquid crystal-glass interface. Voltage profile in the glass layer is calculated similar to the voltage profile in liquid crystal layer. Voltage at the liquid crystal-glass interface is calculated using boundary condition for Maxwell’s equation, the continuity of the component of electric displacement perpendicular to the boundary. We set the z-component of the electric displacements equal. The voltage on the common plate is set to 0 V and on patterned substrate the voltage on the ITO electrodes is given by initial voltage profile and on the gap between electrodes is unknown. An imaginary boundary is placed 50 μm away from patterned substrate where the potential is set to zero. The potential in the gaps then is calculated based on their neighboring grid points. Under a given voltage profile, the electric field is calculated and then the steps to calculate the equilibrium state of the director is taken.

To calculate the equilibrium state of liquid crystal director, we first set the initial values into variables. We then set the boundary conditions. With these, the iteration can start. Inside this iteration loop over the calculation grid at each grid point first,
we calculate functional derivatives \([f_\Theta]_n x, [f_\Theta]_n y, [f_\Theta]_n z\) with respect to director components. We then use these values to update the director using equation (A.8) and then we normalize the director using equation (A.9). Next we calculate new value of voltage at current grid point. After calculating these values, we update the variable with the new values for each grid point. This method is called successive displacement which means we update each variable as we calculate it. Next, we update the time forward (+\(\Delta t\)). The iteration continues until the condition \(|f_\Theta^{new} - f_\Theta^{old}| < 10^{-6} \frac{N}{m^2}\) is obtained at which point we output the director configuration and voltage profile.

A.1.4. Implementation

We use finite difference method to estimate the derivatives. In this method the computational domain is divided into rectangular cubes, all with same dimensions. For spatial derivatives we use the centered difference formula given by equation (A.12) and (A.13).

\[
f'(x) = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} \quad (A.12)
\]

\[
f''(x) = \frac{f(x + 2\Delta x) - 2f(x) + f(x - \Delta x)}{\Delta x^2} \quad (A.13)
\]

For time derivative we use forward difference formula given by equation (A.14).
A.1.5. Initial Condition

The two most important parameters that need to be defined at the beginning of program is the initial director configuration and initial voltage profile. For equilibrium calculation, an educated guess about the director configuration can significantly reduce the computational time. Initial voltage profile will also impact the speed and accuracy of the calculation.

We use one dimensional director calculation when the director changes only along the normal to the cell (z-axis) to calculate the initial director configuration and voltage profile. For this one dimensional calculation, we consider that there is no electrode pattern and both electrodes are uniform. We then calculate OPD of liquid crystal layer at various voltages (retardation curve). Also we calculate the ideal parabolic phase profile of desired optical power and thus we calculate the desire OPD at each grid point. There is a voltage corresponding to each OPD in retardation curve (shown in Figure 2-6). Thus the initial voltage profile at the electrode stripes are obtained. Related to retardation curve, we choose the linear section to avoid the two plateau of low and high voltage values.

A.1.6 Boundary Condition
Director and voltage boundary conditions are the two types of boundary conditions required for our simulation. The director boundary conditions include surface and computational boundaries. At the surface, since we are using vector method without any inversion symmetry, definition of boundary condition requires some thought. As discussed earlier, we must define the boundary conditions so they show correct topological state. Specifically in SPP LC lens the boundary condition should represent splay deformation to reserve the topological state of liquid crystal layer. For lateral computational boundaries, we used periodic boundary condition. For boundaries perpendicular to substrates, analytically, electric field goes to zero at infinity. In this work, we used an approximation where electric field is set to zero at an imaginary boundary far away from substrate (z=50μm).

A.1.7 Simulation Parameters

The grid spacing in our simulation is 0.25 μm along the cell normal and 0.5 μm in x-axis to account for electrode width (>11 μm) and gap between electrodes (3 μm).

A.1.8 Voltage Optimization

Once the initial voltages are obtained from 1D director simulation, 2D liquid crystal configuration and lens OPD are calculated by following the steps described above. Next, the calculated OPD profile is compared to ideal OPD profile of the
same optical power. If there is any deviation, the voltage will be updated using retardation curve. Required change in voltage is obtained by measured change in phase between ideal and calculated divided by the slope of the retardation curve. With optimized voltage profile, director configuration and OPD profile are calculated.

**A.2. Scalar Diffraction Calculation**

Our aim here for scalar diffraction calculation is to describe the evolution of an optical field as it propagates from point source to focal plane. The phenomenon of diffraction underlies the behavior of propagating waves. Diffraction is the bending and spreading of waves, when its lateral extent is confined for example around an obstacle. The effect of diffraction is more apparent when the confinement size is in the scale of wavelength of light. Diffraction is used in many optical application for example; diffraction of x-ray is used to make image of small structure such as DNA and diffraction is also used for astronomical imaging.

The propagation behavior of an optical wave is fundamentally governed by Maxwell’s equations. In most cases, electric field $E(E_x, E_y, E_z)$ and magnetic field $H(H_x, H_y, H_z)$ are coupled. There is also coupling between the individual components of the electric field, as well as between the magnetic components. However, in the case that a wave that is propagating in a dielectric medium that is linear (field quantities from separate sources can be summed), isotropic
(independent of the wave polarization), homogenous (permittivity of the medium is independent of position), nondispersive (permittivity is independent of wavelength) and nonmagnetic (magnetic permeability is equal to the vacuum permeability), Maxwell’s vector expressions become decoupled, and the behavior of each component of the electric or magnetic fields can be expressed independently from other components [87]. Propagation behavior of light under this condition is scalar diffraction. Propagation in free-space is one example of scalar diffraction. One can characterize a linear, isotropic and homogeneous system with a single impulse response. A point source at origin of an input plane is considered. This point source produces some field and amplitude on output plane distance z away from the input plane. The output field and amplitude $h(x, y)$ is called impulse response. For an arbitrary coherent input light, we can obtain the output field and amplitude $u_2(x, y)$ by taking impulse response $h(x, y)$ and convolve it with its input field $u_1(x, y)$. In order to understand how the point source propagates and gives impulse response, we need to understand the propagation. Although based on Huygens-Fresnel principle we can guess that $h(x, y)$ must be spherical waves, mathematical proof of impulse response is provided in the following.

A propagating optical wave in any media follows Maxwell’s equation:

$$\nabla \times \mathbf{E} = -\frac{\delta \mathbf{B}}{\delta t}$$  \hspace{1cm} (A.15)
\[ \nabla \times H = J + \frac{\delta D}{\delta t} \]  \hspace{1cm} (A.16)

\[ \nabla \cdot D = \rho \]  \hspace{1cm} (A.17)

\[ \nabla \cdot B = 0 \]  \hspace{1cm} (A.18)

In the above \( J \) is the total electric current density and \( \rho \) is the total electric charge density. The relation between the displacement field \( (D) \) and the electric field \( (E) \) as well as the magnetic flux density \( (B) \) and magnetic field strength \( (H) \) is described by constitutive relations which are

\[ D = \varepsilon E \]  \hspace{1cm} (A.19)

\[ B = \mu H \]  \hspace{1cm} (A.20)

where \( \varepsilon \) is permittivity and \( \mu \) is permeability of the material. In free space the total electric current density \( (J) \) and the total electric charge density \( (\rho) \) are zero and permittivity \( (\varepsilon) \) and permeability \( (\mu) \) are scalar constants. Therefore the Maxwell’s equations in free space using constitutive relations are

\[ \nabla \times E = -\mu \frac{\delta H}{\delta t} \]  \hspace{1cm} (A.21)

\[ \nabla \times H = \varepsilon \frac{\delta E}{\delta t} \]  \hspace{1cm} (A.22)

\[ \nabla \cdot E = 0 \]  \hspace{1cm} (A.23)

\[ \nabla \cdot B = 0 \]  \hspace{1cm} (A.24)
With the time harmonic field assumption (the electric field and magnetic fields are sinusoidal in nature and are oscillating with frequency of $\omega$) one can write the electric and magnetic fields as follow:

$$E(x, y, z; t) = E_0(x, y, z)e^{-j\omega t} \quad (A.25)$$

$$H(x, y, z; t) = H_0(x, y, z)e^{-j\omega t} \quad (A.26)$$

This assumption simplifies the differential equations. Maxwell’s equations in free space and under time harmonic assumption are given by:

$$\nabla \times E_0 = j\omega \mu H_0 \quad (A.27)$$

$$\nabla \times H_0 = -j\omega \varepsilon E_0 \quad (A.28)$$

$$\nabla . E_0 = 0 \quad (A.29)$$

$$\nabla . B_0 = 0 \quad (A.30)$$

Performing curl operation on equation (A.27) gives:

$$\nabla \times \nabla \times E_0 = j\omega \mu \nabla \times H_0 \quad (A.31)$$

Using equation (A.28) gives:

$$\nabla \times \nabla \times E_0 = \omega^2 \mu \varepsilon E_0 \quad (A.32)$$

Using vector identity of $\nabla \times \nabla \times F = \nabla(\nabla . F) - \nabla^2 F$ and equation (A.29), we get:

$$\nabla^2 E_0 = -\omega^2 \mu \varepsilon E_0 \quad (A.33)$$

The partial differential equation shown in equation (A.33) is the electric wave equation. Any electric field in free space and under the time harmonic assumption must obey electric wave equation which can also be shown as follow:
where \( k^2 = \omega^2 \mu \varepsilon \) is referred to the magnitude of propagation vector. In general \( E_0 \) is a vector with x, y and z components and equation (A.34) is called electric wave equation. However, if \( E_0 \) is scalar equation (A.34) is called Helmholtz equation. Propagation behavior of light can be treated by different approaches; first is to solve Maxwell’s vector equations which is the most accurate type of propagation solution and is called vector optics. Second is to consider electric field as scalar which is called scalar optics or Fourier optics. Use of scalar optics causes the loss of polarization concept which requires vector optics treatment. Third is the ray optics which is the least accurate solution for the light propagation. In this dissertation we assume \( E_0 \) in equation (A.34) is scalar. Using the spherical coordinates and discarding \( \delta \right\phi \) and \( \delta \right\theta \) terms equation (A.34) simplifies to

\[
\left[ \frac{1}{r^2} \frac{\delta}{\delta r} r^2 \frac{\delta}{\delta r} + k^2 \right] E_0 = 0 \tag{A.35}
\]

If we let \( E_0 = \frac{F_0}{r} \) then

\[
\frac{\delta^2 F}{\delta r} + k^2 F_0 = 0 \tag{A.36}
\]

The solution to equation (A.36) is clearly

\[
F_0 = C_1 e^{jkr} + C_2 e^{-jkr} \tag{A.37}
\]

Substituting \( E_0 \) gives
\[ E_0 = \frac{C_1}{r} e^{jk}\rho + \frac{C_2}{r} e^{-jk}\rho \]  \hspace{1cm} (A.38)

Equation (A.38) shows that the solution to Helmholtz equation are diverging (radiating source) and converging (sink) spherical waves. Thus the impulse response \( h_z(x, y) \) of the radiating source, located at the origin of an input plane, \( z \) distance away from input plane on the output plane is given by \( (C_2 = 0) \)

\[ h_z(x, y) = E_0 = \frac{C_1}{r} e^{jk}\rho \]  \hspace{1cm} (A.39)

It can be proved that \( C_1 = \frac{1}{j\lambda} \) thus the impulse response is given by [86]

\[ h_z(x, y) = \frac{1}{j\lambda r} e^{jk}\rho \]  \hspace{1cm} (A.40)

where \( r = \sqrt{x^2 + y^2 + z^2} \) is the distance between a point on the output plane to the origin of input plane. In equation (A.40) \( z \), the distance between input and output planes, is the system parameter and \( x \) and \( y \) are the variables of the system. For a coherent light, convolving the input field with the impulse response gives the output field.

\[ u_2(x, y) = u_1(x, y) * h_z(x, y) \]  \hspace{1cm} (A.41)

This convolution gives the so called Rayleigh-Sommerfield diffraction integral. To have general form of the integral, one can consider the monochromatic radiating point source located at \( (\xi, \eta) \) on the input plane with input field distribution of \( u_1(\xi, \eta) \) (Figure A-1), the field distribution in the output plane \( u_2(x, y) \) with
distance $z$ away from the input plane can be obtained with Rayleigh-Sommerfield diffraction integral as shown in equation (A.42) [86].

$$u_2(x, y) = \frac{Z}{j\lambda} \int \int u_1(\xi, \eta) \frac{e^{jkr}}{r^2} d\xi d\eta$$  \hspace{1cm} (A.42)

**Figure A-1** Presentation of coordinate system of Rayleigh-Sommerfeld Diffraction theory.

Here $\Sigma$ shows the illumination aperture. $\lambda$ is the wavelength of light, $k$ is the wave number, $z$ is the distance between the source plane and observation plane, and $r$ is the distance between the source point $(\xi, \eta)$ on the source plane and a point $(x, y)$ on observation plane ($r = \sqrt{z^2 + (x-\xi)^2 + (y-\eta)^2}$). The Rayleigh-Sommerfeld diffraction formula is the most accurate solution for the precise scalar diffraction calculation and propagation of light in an isotropic, homogenous and linear medium that is well founded when the aperture size is much greater than the wavelength of light [86]. As discussed earlier equation (A.42) consists of two main terms: the field
term \((U_1(\xi, \eta))\) and the Rayleigh-Sommerfeld impulse response \(h(x, y) = \frac{1}{j\lambda} \frac{e^{jkr}}{r}\) multiply by obliquity factor \((\chi = \frac{z}{r})\) which forces propagation to be in \(z\) direction.

The term \(\frac{e^{jkr}}{r}\) express Fresnel-Huygen’s principle that each point in the aperture acts as a source of spherical waves that combine to give the diffraction pattern [80].

Besides some specific situations, it is not feasible to analytically solve the Rayleigh-Sommerfeld diffraction formula [144]. Under specific criteria, Fresnel and Fraunhofer approximation of Rayleigh-Sommerfeld integral are used to compute the light diffraction. If the area of the aperture is smaller than the separation between the two planes \(((x - \xi)^2 \ll z^2 \text{ and } (y - \eta)^2 \ll z^2)\) the Fresnel impulse response or the Fresnel point spread function is obtained after some trivial mathematical steps.

\[
h_F(x, y) = \frac{e^{jkz}}{j\lambda} e^{\frac{jk}{2z}(x^2 + y^2)} \tag{A.43}
\]

The propagating wave shown in equation (A.43) is parabolic rather than spherical in the case of Rayleigh-Sommerfeld equation. Fraunhofer approximation is an approximation to Fresnel impulse response (A.43). In this approximation the separation between the two planes are far such that the parabolic waves become plane waves. Although Fresnel and Fraunhofer approximations are very useful in certain regimes yet an accurate assessment of diffraction in both far and near fields can only be achieved if approximations are avoided. In this dissertation to study the
propagation behavior of the light field we numerically solved Rayleigh-Sommerfeld diffraction integral without any approximation.

A number of algorithms are suggested in the literature to numerically solve the Rayleigh-Sommerfeld integral, mainly based on the two methods of Angular Spectrum (AS) and Direct Integration (DI) [145, 146, 147, 148, 149]. For the purpose of our study, the DI approach has been utilize, the approach calculates Rayleigh-Sommerfeld integrals in the spatial domain using numerical integration. In this procedure, an integration is considered as a linear convolution and can be computed by virtue of Fast Fourier Transform (FFT) and the Inverse Fourier Transform (IFFT). The best presentation of computing the diffraction integral by means of linear convolution and FFT was provided by Voelz [87]. Our numerical modeling of the light propagation based on DI Rayleigh-Sommerfeld is built on the Voelz strategy.

According to the DI method, the superposition integral shown in equation (A.42) can be expressed as a convolution integral:

\[ u_2(x, y) = \iint u_1(\xi, \eta) h(x - \xi, y - \eta) d\xi d\eta \]  \hspace{1cm} (A.44)

Taking the Fourier transform of both sides of equation (A.44) and using the convolution theorem (Fourier transform of a convolution is the simple multiplication of Fourier transforms.), one can re-write equation (A.44) as follow:
\[ u_2(x, y) = F^{-1}\{F\{u_1(x, y)\}.F\{h(x, y)\}\} \]  
(A.45)

or we can re-write equation (A.45):

\[ u_2(x, y) = F^{-1}\{F\{u_1(x, y)\}.H(f_x, f_y)\} \]  
(A.46)

where \( H(f_x, f_y) \) is the Rayleigh-Sommerfeld transfer function given by

\[ H(f_x, f_y) = \exp(jkz\sqrt{1 - (\lambda f_x)^2 - (\lambda f_y)^2}) \]  
(A.47)

In equation (A.47) the condition \( \sqrt{f_x^2 + f_y^2} < \frac{1}{\lambda} \) must be always satisfied.

In our numerical model, a radiating point source is considered. The light propagates through different optical elements in the system. At the plane of each element, the phase profile of that element is taken into account and is added to the light phase. Then the Rayleigh-Sommerfeld transfer function conveys the light from one optical element to the other. Lastly, the light intensity at the focal plane (point spread function, PSF) is determined. The Modulation Transfer Function (MTF) can be then calculated by taking the Fourier transform of PSF. More specifically, the out coming field of \( m^{th} \) optical element is determined by the input complex field amplitude (\( u_{in}^m \)) together with the phase associated with that optical element (\( \phi^m \)).

\[ u_{out}^m = u_{in}^m . e^{i\phi^m} \]  
(A.48)

The Fourier transform of the complex field amplitude at exit plane of optical element \( m \) is then calculated.
\[ U_m = F\{u_{out}^m\} \] (A.49)

The field of optical element \( m \) is carried to the next optical element of the system \((m + 1)\) using the Rayleigh-Sommerfeld Optical Transfer Function (OTF) which is the Fourier transform of the Rayleigh-Sommerfeld impulse response in the spatial domain \( H(F_u, F_v) = F\{h(x, y)\} \).

\[ H_m(F_u, F_v) = F\{e^{i k z \sqrt{1 - (\lambda F_u)^2 - (\lambda F_v)^2}} \} \] (A.50)

One can compute the field at observation plane by substituting equation (A.49) and (A.50) in equation (A.46):

\[ u_f = F^{-1}\{U_m(F_u, F_v) . H_m(F_u, F_v)\} \] (A.51)

where \( u_f \) denotes the final field at observation plane. The PSF is the incoherent impulse response \((|h(x, y)|^2)\) which can consequently be determined at the observation plane. In order to evaluate the imaging performance of the optical system of interest, one can calculate the modulus of OTF which is known as the Modulation Transfer Function (MTF) and is given by:

\[ MTF = \left| H_f(F_u, F_v) \right|_{norm} \] (A.52)
APPENDIX B

Fabrication Process for an SPP LC Lens

Appendix B describes the fabrication details of an SPP LC lens. This fabrication process is the standard wet photolithography process used for Liquid Crystal Display (LCD) manufacturing. It consists of deposition and patterning of ITO, SiO$_2$ and Ni, coating of alignment layer, rubbing, cell assembly, filling, sealing and bonding which are explained in following sections.

B.1 Scribing

The first step of the process is to scribe 14” by 16” sheet of ITO coated glass with 80 $\Omega$ sheet resistance into 4” by 4” substrates using a Villa Precision, INC (VPI GS 200-SERIES) scribing machine. The scribing was implemented at the class-10000 clean room of Liquid Crystal Institute (LCI) with 254 mm/second scribing speed. Figure B-1 is a picture of the VPI machine used in our fabrication process.
B.2 Cleaning

Next the glass substrates must be thoroughly cleaned. The cleaning process includes 15 minutes of ultrasonic cleaning with 10 ml detergent in 15 liter DeIonized (DI) water at 65°C degree. The low amount of all-purpose Cavi-Clean liquid detergent used was to improve the wetting and efficiency of ultrasonic cleaning. Detergents well adhere to the surface and therefore their higher concentration during cleaning is not recommended. After 7 minutes of ultrasonic cleaning, the substrates were removed, rubbed with Laundered Polyester Class 10 Wiper and then the remaining ultrasonic cleaning was resumed. After that the substrates were rinsed with DI water for at least 5 min, which was followed by an Isopropanol rinse. Next,
plates were dried at 90 °C for 10 min in oven. Since any humidity will significantly reduce the surface adhesion, prior to the spin coating the substrates were baked at 200 °C for 15 min. The cleaning steps are demonstrated in Figure B-2.

![Figure B-2](image1.png) Different steps of cleaning.

**B.3 ITO Patterning**

Positive photoresist (with photoresist (S1818) to solvent (AZ EBR) ratio of 1:3)) is spun coated for 30 s at 1500 rpm on the substrate. The solvent was then evaporated from plate’s surface using a hot plate at 95 °C for 90 s.

Next a Karl Suss exposure system was used to pattern the ITO film into concentric discrete rings with a 3 μm gap. The polymer chain in positive photoresist under 365 nm UV light with 350 W power and 3 s time breaks down and will be then soluble in low PH carboxylic acid. The exposure set-up used for fabrication of the test device is shown in Figure B-4.
Figure B-3 Spin coating process.

Figure B-4 Exposure and alignment set-up.
The next step is developing. In this step, the exposed area will be removed by low PH carboxylic acid and the non-exposed photoresist that was protected by chrome mask will remain on the plate. After that the plate will be etched with HCL to form discrete and concentric ring electrodes with about 100 nm heights and different widths from about 550 μm to 11 μm. The gap between each electrodes is 3 μm. There are inter-ring resistors with $200 \, \Omega$ resistance (10 μm length and 5 μm width) that are staggered with 5° angular distance. In the next step the photoresist is removed using acetone. The last step for ITO patterning is cleaning the substrates, which includes a water rinse for at least 5 min followed by an iso propanol rinse to remove finger prints of water and drying for 10 min at 90 °C.

![Image]

**Figure B-5** Developing and etching steps of the lens fabrication.

The microscopic images of ITO pattern are depicted in Figure B-6.
Figure B-6 ITO discrete rings A. Different part of the substrate; B. Inter-ring resistor.

B.4 SiO₂ Deposition & Patterning

For all the deposition steps including SiO₂, a high pressure vacuum magnetron sputtering machine was used. The sputtering cathode used in the system is Mu Inset cathode which was developed by Material Research Cooperation (MRC) [150]. Combination of electromagnetic wave as an output of magnetron and plasma ionization process result in the sputtering of target materials. Magnetron are also called cross field systems employing both electric and magnetic fields. Magnets are positioned on the back of the magnetron cathode to produce a constant magnetic field orthogonal to the cathode electric field. Magnetron is a special sputter gun that is used to increase the path length of electron flow via imposed magnetic field. In MRC there is also a sputtering gas, a chemically inert and heavy gas like Argon (for ITO deposition a mixed gas which is argon with some oxygen is used.), under high vacuum and high electric field (to provide enough energy for electron formation in
vacuum) will be ionized and create plasma. MRC is a high vacuum system with a base pressure in the order of $10^{-6}$ torr. The applied electric field can be provided by DC or RF power supply depending on the target material. If a conductive target material like ITO is used, DC power is the better option whereas for dielectric target material like SiO$_2$, RF power is normally used. We can use RF power for all target material deposition but generating RF power is costly. The deposition rate of RF compared to DC and the electron flux on the surface is low and may cause significant heating. After applying voltage, a large number of high energy free electrons inside the plasma with low mean field path bombard the heated cathode which is the target materials (Ni, SiO$_2$ or ITO). High energy electrons, due to the high temperature in cathode, will be released and deposited on the anode surface which is the substrate. The magnetic field behind the cathode trap the electrons and enhance the plasma ionization density near the target surface, which increases the sputtering rate.

MRC performs sputtering of the target material in three steps; 1. High vacuum to ensure that the base pressure of the system is low enough. In this step, the system pumps down to $10^{-6}$ torr within 10 min. 2. Etching, which is a pre-clean step to make the surface receptive to the coating and enhance the adhesion. In this step of the process RF power supply with 500 W is turned on to introduce argon gas with 10 mtorr pressure for 10 second. 3. Deposition is the last step of the coating. The film
thickness is directly proportional to the speed with which the pallet is moving across the chamber and also the number of cycles that the pallet takes. These parameters, together with the power of RF supplier and the pressure of the gas are specified in the deposition step. To obtain 140 nm thickness of SiO₂, we have used 10 cm/min pallet speed with 6 cycles. The RF power and the argon pressure for this deposition were 750 W and 10 mtorr, respectively. 50 second of pre-sputtering was done prior to the film deposition to stabilize the system. Figure B-7 shows the MRC machine used for SPP LC lens fabrication.

Figure B-7 Picture of MRC machine used in our SPP LC lens fabrication process; 1. Domb area to load/unload substrates. 2. DC Power supply 3. Target position: from left to right SiO₂, Nickel and ITO. 4. Manual console control to manually control the valves, pumps etc. The schematic diagram of the machine is shown also in this part that illustrates the position of the valves and thermocouple gates and whether they are closed or opened. If the item is red or green it indicates it is closed or opened, respectively. 5. Switch box: On the top right of the machine circuit needed for switching the power from DC to RF. 6. Main operator interface console to program the machine or type a command. 7. Main power switch. 8. Cryo pump: High vacuum pump on the back of the system which freeze out any hydrogen, nitrogen or water vapor exist. 9. Power switch for pulse
control required during ITO deposition. 10. Vacuum gauge controllers: TC1 is domb pressure. TC2 is main chamber pressure. TC3 is mechanical pump pressure and TC4 is the cryo pressure. 11. Flow controllers: second box on the center top of the machine. Two gas flow controllers were displayed on the box argon (used for Ni and SiO$_2$) and mixed gas (contains oxygen and used for ITO). Read outs on top of each gas show pressure in millitorr and on the bottom show flow rate.

After the SiO$_2$ deposition, the MCC primer 80/20 by Micro Chem which consisted of 20% HDMS and 80% PM Acetate was spun at 1500 rpm for 30s to improve the adhesion of photoresist to the SiO$_2$ layer by producing an interfacial bonding layer for photoresist. The existent solvent in MCC primer which acts as pre-wetting agent is then evaporated on hot-plate at 95 °C for 90s. This step is followed by spin coating of positive photoresist (with photoresist (S1818) to solvent (AZ EBR) ratio of 1:3)) for 30 s at 1500 rpm on the substrate. The solvent was then evaporated from plate’ surface using a hot plate at 95 °C for 90 s.

Prior to the exposure, the SiO$_2$ layer should be aligned with the underneath ITO layer using Ni coated Fiducials using Karl Suss alignment machine. In the next step, the substrate is exposed to 365 nm UV light with 350 W power for 3 s. The substrate is then developed by low PH carboxylic acid. For the SiO$_2$ etching process a solution containing HF and aluminum hydroxide is used which damages the glass substrate. Therefore before the etching process, the non-ITO side of the substrate is spun coat at 1500 rpm for 30 s with photoresist and then dried on the hotplate at 95 °C for 90 s. After calibrating the etching time, the substrate is dipped into the plastic container filled with etchant solution. It must be noted that the entire wet-photolithography steps are carried out under a hood and the operator must be extra
vigilant while handling HF solution. At last the plate is stripped by acetone and rinsed with water followed by IPA and dried in Oven for 10 min at 90 °C. The SiO$_2$ pattern consists of rectangular shape vias that are 20 μm by 10 μm through which the top layer (Ni) is connected to concentric ITO ring electrodes. The purpose of the SiO$_2$ layer is to insulate the ITO ring electrode from the Ni layer except the parts that should not be insulated. There are 37 vias across the designed SPP LC lens: 8 vias in the first four segments and 5 vias in the last segment. The locations of vias based on ITO ring electrode number are listed in Table B-1.

Table B-1 Via location in each phase segments.

<table>
<thead>
<tr>
<th>Bus Line #</th>
<th>Zone 1, Ring #</th>
<th>Zone 2, Ring #</th>
<th>Zone 3, Ring #</th>
<th>Zone 4, Ring #</th>
<th>Zone 5, Ring #</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>65</td>
<td>129</td>
<td>193</td>
<td>257</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>75</td>
<td>139</td>
<td>203</td>
<td>267</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>85</td>
<td>149</td>
<td>213</td>
<td>277</td>
</tr>
<tr>
<td>4</td>
<td>31</td>
<td>95</td>
<td>159</td>
<td>223</td>
<td>284</td>
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<tr>
<td>5</td>
<td>41</td>
<td>105</td>
<td>169</td>
<td>233</td>
<td>288</td>
</tr>
<tr>
<td>6</td>
<td>49</td>
<td>113</td>
<td>177</td>
<td>241</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>57</td>
<td>121</td>
<td>185</td>
<td>249</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>64</td>
<td>128</td>
<td>192</td>
<td>256</td>
<td></td>
</tr>
</tbody>
</table>

B.5 Ni Deposition & Patterning

Next is to deposit Ni using vacuum sputter coating machine. The same parameters as SiO$_2$ layer were used during high vacuum and etching steps of MRC. To obtain 130 nm thickness of Ni for the deposition step of MRC, we have used 20 cm/min pallet speed with 2 cycles. The RF power and the argon pressure for this
deposition were 750 W RF and 10 mtorr, respectively. 50 second of pre-sputtering was done prior to the film deposition to stabilize the system.

We encountered some challenges during Ni deposition and patterning. Following the normal photolithography procedure used for the two underneath layers did not provide acceptable patterns. The Ni pattern consists of 8 bus lines that have wide width on the lens’ periphery but their width reduces to 20 μm on the active area. The bus lines with 20 μm have the same length as the radius of the lens. With the normal wet-photolithography process, we have observed mouse bite, transitional line and pealing defects. These defects can be caused by combination of factors including low adhesion of Ni to SiO₂ layer that can be due to the residual photoresist or any organic contaminant on the surface, low adhesion of photoresist to Ni and/or deposition non-uniformity. In order to overcome the process difficulty of Ni patterning, we performed some optimizations including A, stripping at elevated temperature in SiO₂ patterning to avoid residual of photoresist and also UV/Ozone cleaning for 10 min prior to Ni deposition. B, After Ni deposition, the plates were immediately coated with photoresist for the better adhesion. We have also increased the temperature during solvent evaporation to 110 °C and perform the etching at 35 °C while avoiding extra contact of the plate with etchant (this might be challenging because the Ni etching occurs non-uniformly.) C, increase the number of deposition cycles to 4 and rotate the plates in the chamber so it is not
covered by any inhibitors. Figure B-8 demonstrates the Ni bus line patterning before and after process optimization. After stripping we should avoid any physical contact or ultrasonic cleaning in order to protect the pattern.

Figure B-8 Ni patterning before and after optimization.

Alignment of Ni with underneath vias also plays a significant role: this alignment is done by a Karl Suss alignment layer. Figure B-9. shows an example of the alignment of the bus line with via. The developer and stripper used for patterning of Ni layer was the same as the ones used for ITO and SiO₂ layers. The Ni etchant used in our process was the type TFB purchased from Transene Company, Inc.
Figure B-9 Bus line patterns and bus line alignment with rectangular via underneath.

B.6 Floating Electrode Deposition & Patterning

After ITO layer, we have deposited and patterned a second layer of SiO$_2$ with the same procedure described in section B.4. The role of this insulator layer was to prevent the short between bus lines and the floating electrodes. Then we deposited an ITO layer on to the insulator layer using MRC with the same parameters in the high vacuum and etching steps. To obtain a transparent ITO layer with 120 $\Omega$ during the deposition step of MRC, we have used 50 cm/min pallet speed with 2 cycles. The RF power and the argon pressure for this deposition were 500 W RF and 8 mtorr, respectively. 50 second of pre-sputtering was done prior to the film
deposition to stabilize the system. The ITO was patterned as described in Section B.3.

**B.7 PI coating & Rubbing**

After completing the patterning process, polyimide SE 2177 is spun coated on the plate at 1500 rpm for 30 s. The solvent then is evaporated using hot plate at 80 °C for 5 min. Subsequently, the polyimide is placed in oven for 1 h at 200 °C. Next, the 6” by 6” plates are scribed to 3” by 3” parts. The PI coating and scribing is also performed for the common ITO plate which is patterned following the same procedure described in section B.3. Then both patterned and common plated are rubbed with opposite direction with respect to each other. The rubbing direction of patterned plate is from the electrode on the bonding edge towards the center of the lens. For the rubbing we used a weight covered with cloth and we repeated the rubbing for 10 times. Figure B-10 illustrates the steps involved in PI coating and rubbing of the cell.

**B.8 Cell Assembly**

After the rubbing, 20 μm glass fiber spacers are sprayed on the common plates. The density of spacers is checked on the bright light box and under the microscope. Thermal glue is mixed with 3% 20 μm spherical spacers and is dispensed on the periphery of the cell. The pressure, needle size and height are all adjusted for the
optimized glue width. Careful alignment of the top and bottom substrates is done in this step. Then the cell is placed in the plastic bag and the vacuum is imposed to the bag using the vacuum bagging machine at 175 °F temperature and 20 s pumping.

![Image](image1.png)

**Figure B-10** PI coating and curing, scribing to 3” by 3” and rubbing.

down time. Next, for the cure, the plates are placed in the oven. The oven takes 10 min to reach to 90 °C and stays at that temperature for 5 min. The temperature will be then increased to 150 °C and stays there for 90 min. It is suggested to cool down the plates gradually after curing in order to avoid thermal shock. Figure B-11 demonstrates some steps of assembly process.
Figure B-11 Spacer spraying, checking the spacer density on the microscope, glue mixing with spacer, glue dispensing and glue curing.

**B.9 Filling & End-Sealing**

The 3” by 3” plates are scribed into 1.5” by 1.5” cells. To consume less of LC material prior to filling, the distance between the glue line and glass edge will be filled with UV glue and cured. This leads the LC material to only enter to the active area and not be wasted between the glue line and the glass edge. Then the cells are filled with 18349 LC material that has a high birefringence of 0.27. After the cells are filled, we pressed the spacers with 14 psi pressure and wait for about 5 min for any extra LC material to leave the cell. As the cell is pressed, the filling port is sealed using UV glue and UV gun. After the sealing the bond ledge is cleaned and the PI layer is removed using Plasma pen.
**B.10 Anisotropic Conductive Film Bonding**

Anisotropic Conductive Film (ACF) is the process of creating electrical conductive adhesive bonds between flexible and rigid circuit boards. This step is the last step of our fabrication.
APPENDIX C

SPP LC Lens Characterization Details

C.1 Via Connection

The first step in SPP LC characterization is to make sure all the bus lines are connected to the ITO ring electrodes through vias. Any misalignment or etching issue during fabrication can lead to non-connecting vias. If one or more of vias are not connected the phase profile deviates from a parabola. The vias located at the steeper part of Figure 2-6 have a significant effect on controlling the shape of the phase profile. To check the via-connection, the SPP LC lens was placed between crossed polarizers. The rubbing direction of the SPP LC lens was at a 45° angle with respect to the transmission axis of either of the polarizers. One bus line at the time was driven with 5 V while all the other bus lines were grounded. The image was captured using a 12 MP Canon Rebel XSI with 50 mm canon lens and 10 D close-up lens. The SPP lens was illuminated with green light that was generated by white
light and a green color filter. The image of the experimental set-up is shown in Figure C-1. Every bus lines is connected to several zones. By driving each bus line one at the time, all the ring electrodes connecting to the driven bus line are turned on. Figure C-2 shows the connected vias in one of the example SPP LC lens. As shown in part 6, 7 and 8 of Figure C-2, the bus line 6, 7 and 8 are only connected to four zones while the other bus lines are connected to five zones.

![Figure C-1](image1.jpg)

**Figure C-1** Pictures of experimental set-up used for verification of via connection and interferogram images.

### C.2 Phase Profile

As discussed in Appendix B, there are 8 vias that connect the external bus lines to the ITO rings in each segment. A precise voltage applied to the ITO ring through vias and resistor network between the ITO ring electrodes provide a well-controlled
parabolic shape of the phase profile. This section overviews the procedure to determine the accurate voltage values of the external bus lines.

**Figure C-2** Pictures of SPP LC lens (Cell 11-3) used for checking via connections taken by setup shown in Figure C-1. For each image one bus line was driven with 5 V while others were grounded. The number of driven bus line matches the number of Figures. For example in Figure 1, bus line 1 was driven with 5v and for Figure 8, bus line 8 was driven with 5 v.

Based on target phase profile shown in Figure 2-9, for a 20 μm SPP LC lens the value of phase difference between bus lines 1 and 8 must be $7\lambda$:

$$\Delta \varphi_t = \varphi_1 - \varphi_8 = 6.9\lambda$$

(C. 1)

There are 10 electrodes between bus line 1 and 2, 2 and 3, 3 and 4, 4 and 5 therefore the phase difference between these bus lines are equal to each other and let’s assume their phase value is equal to $A$.

$$\Delta \varphi_1 = \varphi_1 - \varphi_2 = A$$

(C. 2)
\[ \Delta \phi_2 = \phi_2 - \phi_3 = A \quad (C.3) \]
\[ \Delta \phi_3 = \phi_3 - \phi_4 = A \quad (C.4) \]
\[ \Delta \phi_4 = \phi_4 - \phi_5 = A \quad (C.5) \]

There are 8 electrodes between bus line 6 and 5, 7 and 6, therefore the phase difference between these bus lines are equal to each other and equal to \( \frac{8}{10} A \).

\[ \Delta \phi_5 = \phi_5 - \phi_6 = \frac{8}{10} A \quad (C.6) \]
\[ \Delta \phi_6 = \phi_6 - \phi_7 = \frac{8}{10} A \quad (C.7) \]

There are 7 electrodes between bus line 7 and 8, therefore the phase difference between these two bus lines are equal to \( \frac{7}{10} A \).

\[ \Delta \phi_7 = \phi_7 - \phi_8 = \frac{7}{10} A \quad (C.8) \]

Total phase difference is obtained by the sum of equations (C.2) to (C.8).

\[ \Delta \phi_t = 4A + \frac{16}{10} A + \frac{7}{10} A = 6.3 A \quad (C.9) \]

Equating (C.9) and (C.1) gives;

\[ A = \frac{6.9}{6.3} \lambda = 1.095 \lambda \]
Having A, the phase difference between bus lines is determined using equation (C.2 to 8). Knowing the desired phase values on the via location enables us to define the same phase value on the variable retarder and with white light microscopy and crossed polarizers be able to adjust the voltage on SPP LC lens until the phase value of the variable retarder and SPP LC lens are equal. When this happens the electrodes on which there are vias appear dark. Theoretical details of this approach are discussed in Chapter 3. Thus first step the phase vs voltage of the variable retarder is measured. Figure C.3 shows the setup used for this measurement. Table C-1 shows the measured phase value vs voltage for 20 μm variable retarder filled with MLC 18349. With linear interpolation, the voltage values of the desired positive and negative phase on the vias are obtained as shown in Tables C-2 and C-3.
Electro-Optic Measurement (EOM) setup consists of collimated monochromatic source, crossed polarizers, and a photodiode detector. The variable retarder cell is placed between polarizers. The optic axis of the variable retarder is at 45° angle with transmission axes of the polarizers. Using a function generator, voltages are applied to the variable phase and the intensity of the variable retarder (LC cell) between crossed polarizers is measured. Using Jones calculus, the phase of the light is determined from the measured intensity.

**Table C-1** Phase value vs voltage for the 20 μm variable retarder filled with MLC 18349.

<table>
<thead>
<tr>
<th>V(v)</th>
<th>.51</th>
<th>.56</th>
<th>.85</th>
<th>1.05</th>
<th>1.12</th>
<th>1.19</th>
<th>1.28</th>
<th>1.36</th>
<th>1.44</th>
<th>1.51</th>
<th>1.6</th>
<th>1.7</th>
<th>1.82</th>
<th>1.94</th>
<th>2.11</th>
<th>2.31</th>
<th>2.58</th>
<th>2.98</th>
<th>3.74</th>
<th>5.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Φ (λ)</td>
<td>10</td>
<td>9.5</td>
<td>9</td>
<td>8.5</td>
<td>8</td>
<td>7.5</td>
<td>7</td>
<td>6.5</td>
<td>6</td>
<td>5.5</td>
<td>5</td>
<td>5</td>
<td>4.5</td>
<td>4</td>
<td>3.5</td>
<td>3</td>
<td>2.5</td>
<td>2</td>
<td>1.5</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table C-2** Voltage values for defining positive phase on the variable retarder

<table>
<thead>
<tr>
<th>V(v)</th>
<th>1.05</th>
<th>1.21</th>
<th>1.39</th>
<th>1.56</th>
<th>1.79</th>
<th>2.03</th>
<th>2.38</th>
<th>2.90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Φ (λ)</td>
<td>8.5</td>
<td>7.405</td>
<td>6.31</td>
<td>5.21</td>
<td>4.12</td>
<td>3.24</td>
<td>2.37</td>
<td>1.60</td>
</tr>
</tbody>
</table>
Table C-3 Voltage values for defining negative phase on the variable retarder

<table>
<thead>
<tr>
<th>V(ν)</th>
<th>2.98</th>
<th>2.26</th>
<th>1.89</th>
<th>1.64</th>
<th>1.46</th>
<th>1.32</th>
<th>1.17</th>
<th>1.06</th>
</tr>
</thead>
<tbody>
<tr>
<td>Φ (λ)</td>
<td>1.5</td>
<td>2.5952</td>
<td>3.69</td>
<td>4.78</td>
<td>5.88</td>
<td>6.76</td>
<td>7.63</td>
<td>8.399</td>
</tr>
</tbody>
</table>

The last step is to define the desired voltage of vias to the variable retarder by applying voltages listed in Table C-2 and C-3 and adjust the voltage on the SPP LC lens until dark fringe appears as shown in Figures C-4. The voltage values corresponding to Figures C-4 are listed in Tables C-4.

Figure C-4 Microscopic images of the electrodes on which via are located. The electrodes appear dark because the phase of area of SPP LC lens matches the phase defined on the variable retarder.
**Table C-4** Voltage values adjusted using the variable retarder to impose positive/negative parabolic phase profile.

<table>
<thead>
<tr>
<th>Cell #</th>
<th>BL1</th>
<th>BL2</th>
<th>BL3</th>
<th>BL4</th>
<th>BL5</th>
<th>BL6</th>
<th>BL7</th>
<th>BL8</th>
</tr>
</thead>
<tbody>
<tr>
<td>+.34 D</td>
<td>1.12</td>
<td>1.298</td>
<td>1.516</td>
<td>1.692</td>
<td>1.956</td>
<td>2.215</td>
<td>2.592</td>
<td>3.375</td>
</tr>
<tr>
<td>-.34 D</td>
<td>3.37</td>
<td>2.427</td>
<td>2.06</td>
<td>1.796</td>
<td>1.607</td>
<td>1.466</td>
<td>1.273</td>
<td>1.155</td>
</tr>
</tbody>
</table>

Using the set-up shown in Figure C-1, we captured the interferogram images of the SPP LC lens at both maximum and minimum optical power when voltage values listed in Table C-4 were applied (Figure C-5 and Figure C-6).

![Cell 11-4](image)

**Figure C-5** Interferogram images of SPP LC lens at +0.34 D.
Figure C-6 Interferogram images of SPP LC lens at -0.34 D.

The intensity profile across the blue line in Figure C-5 is shown in Figure C-7.

Figure C-7 Intensity modulation across the blue line shown in Figure C-5.
Considering microscopic images in Figure C-4, it can be seen that the phase around the last via is not symmetric. This issue can be seen in the last via of each zone.

Possible reasons:

1. Capacitive coupling between the common and the patterned electrodes might cause this. However if this is the cause, the asymmetry in phase profile should be dependent on the radial location of the vias and it should get worse close to the edge of the SPP LC lens because the width of the electrodes get smaller in comparison with the cell thickness as moving from the center to the edge of the lens. Moreover the capacitive coupling has less influence in the presence of the resistors network. Since the capacitive coupling is frequency dependent, we checked the significance of this effect by changing the frequency. Figure C-8 shows the microscopic images of the edge of the SPP LC lens. Electrode 256 is the last electrode of zone 4 which appears dark in these images. As can be seen from these images the asymmetry of the phase profile is independent to the frequency. Therefore it is very likely that capacitive coupling is not the underlying reason for asymmetry observed in phase profile at zone boundaries (Figure C-7).
Figure C-8 Microscopic images of the edge of the SPP LC lens. Several frequency was tested to assess the significance of capacitive coupling of common and pattern electrodes on the asymmetry of phase profile around the last electrode of zone 4 (electrode# 256).

2. The other reason for asymmetry at segments boundaries might be due to absence of floating electrodes. Figure C-9 shows the microscopic images of the electrodes of SPP LC lens which indicates that at rest boundaries there is no floating electrode. We can verify this hypothesis considering the microscopic image of the samples without floating electrodes (Figure C-10, A). The asymmetry of the phase profile should be less noticeable in the sample without floating electrodes. Figure C-10, B shows the gap-less microscopic image of the sample with floating electrodes that has twice sampling rate compared to the sample that do not have floating electrodes. Based on microscopic observation shown in Figure C-10, although less
significant asymmetry in phase profile is observed in non-floating sample it is still difficult to make a conclusion.

**Figure C-9** Microscopic image of SPP LC lens; at segment boundaries there is no floating electrode.

![Microscopic image of SPP LC lens](image)

**Figure C-10** Microscopic images of SPP LC lens at segment boundary A. Sample without floating electrodes. B. Sample with floating electrodes.
3. Different resistance network between electrodes connected to last bus line in each zone (bus line 8 in zone 1 to 4 and bus line 5 in zone 5) compared to other bus lines might result the asymmetry of the phase profile at the segment boundaries. Except the last bus line in each zones, all the other bus lines are connected to resistors on two sides.

Based on above discussion the asymmetry of the phase profile at segment boundaries is possibly due to different resistor network connected to the last bus lines in comparison with the other bus lines. Phase difference from the designed phase ($7\lambda$) at segment boundaries due to the asymmetry of the profile is about $\frac{3\lambda}{4}$ as shown in Figure C-7. Another reason for this phase difference might be thickness variation during fabrication process. 407 nm thickness variation is sufficient to induce $\frac{3\lambda}{4}$ phase difference. We further verified this thickness variation by checking blue and red interferograms of SPP LC lens (Figures C-11 and C-12).
Figure C-11 Blue interferogram images of SPP LC lens at +0.34 D.

Figure C-12 Red interferogram images of SPP LC lens at +0.34 D.
The intensity modulation across the blue and red interferograms shown in Figure C-13 and C-14 suggests that the difference at segment boundaries is a function of wavelength, which indicates that the thickness of fabricated sample is not multiple integer number of green wave.
In our modeling and calculation we have always considered the designed phase profile with multiple integer of wave at each segment. Our calculation showed by choosing segment larger than 1 mm no diffraction is expected even for the outer most edge at the designed wavelength. Diffraction is worst when the phase shift between the design and incident wavelengths reach half-wave causing double images. Related to human perception, the resulting diffraction angle is smaller than the eye resolution and therefore is not perceivable. Our characterization result confirmed the calculation, double spots measured towards the edge of SPP LC/Glass stack shown in Figure 3-11C and D agrees with modeling result shown in Figure 2-15 when the phase difference is $\frac{3\lambda}{4}$. 
APPENDIX D

Fabrication Process of PPL

What we have done on PPL fabrication is largely based upon empirical studies carried out by Gao [151]. The PPL fabrication process includes two major steps of photoalignment and polymerization of reactive mesogen (RM). Factors found to be influencing the photoalignment step of the process have been explored comprehensively by Wang and McGinty [152]. Humidity control (<40%) during the spin-coating process of Brilliant Yellow (BY) and Nitrogen flow during RM polymerization are the two crucial factors for obtaining high quality PPL [151, 152]. Appendix D describes the general fabrication process of PPL and what we have found to be important during the exposure step to obtain high quality PPL.

A Mach-Zehnder interferometer is used for the optical recording of the spiral configuration required to fulfill the PPL. The holographic set-up consists of 457 nm laser with 3 mm beam size that is expanded to 30 mm by means of a 10x beam expander after reflecting from the mirror. The expanded beam is subsequently
distributed into two arms using a beam splitter (BS) after passing through a 2 cm stop. The two diverged beams become left-handed circularly polarized by quarter wave plates (QWP) and then are merged proceeding through the beam combiner (BC). One of the two arms serves as reference (arm 1) and the other is focused by a template lens (arm 2). The template lens in the set-up is placed before the beam combiner as shown in Figure D-1. The reference path interferes with the template path to generate the spiral configuration on the glass substrate coated with alignment layer (here Brilliant Yellow (BY)).

To acquire the desired recording on BY, the optical elements in the exposure set-up are precisely aligned as the beam coordinates in the x-z plane remained unchanged through entire propagation path. The measured intensity at the sample position is $6.398 \text{ mw cm}^2$ to which path 1 and 2 contribute $3.056 \text{ mw cm}^2$ and $3.342 \text{ mw cm}^2$ respectively. Each path is ensured to be circularly polarized with less than 0.15 intensity variation. Given the desired focal length for PPL at designed wavelength, one can determine the focal length of the PPL at exposed wavelength ($f_{\text{PPL}}$) using equation (D.1). In this equation $\lambda_d$ and $\lambda_e$ are the designed and exposed wavelength, respectively.

$$f_{\text{PPL}} = f_d \times \frac{\lambda_d}{\lambda_e} \quad (D.1)$$
The focal length of template lens should be larger than $f_{PPL}$. To obtain $f_{PPL}$, we placed the BY coated substrate at distance equal to $f_{PPL}$ from the focal point of the template lens. The resulting aperture size of PPL is governed by the size of beam combiner’ side (24 mm by 24 mm in our set-up) as shown in equation (D.2).

$$d = \Delta \times \frac{f_{PPL}}{f_t} \quad \text{(D.2)}$$

Here $d$ is the PPL lens diameter, $\Delta$ is the beam combiner size and $f_t$ shows the focal length of the template lens as shown in Figure D-1.

Followed by [138], prior to spinning the BY layer, the substrate is thoroughly cleaned and the surface is treated with UV/ozone for 10 min. Subsequently 1.5% by weight BY dissolved in dimethylformamide (DMF) is spun at 3000 rpm for 30 s. If the humidity was not below 40% a nitrogen flow was used to control the humidity during BY spin coating [152]. Right after 15 min exposure at the discussed exposure set-up, reactive mesogen (LC monomer) solution is spun at 2000 rpm for 30 s.

![Figure D-1](Image)

**Figure D-1** Schematic representation of the position of BY coated substrate (III) with respect to focal plane of template lens (IV) and beam combiner (II). In this image I represents the template lens, $f_{PPL}$ is the focal length of PPL at the exposed wavelength, $f_t$ is the focal length of template lens, $d$ is the PPL diameter and $\Delta$ is the beam combiner size.
Reactive mesogen (RM) solution contains 10% by weight RM dissolved in toluene plus photoinitiator irgacure with the amount of 5 % of RM weight. After spin coating the substrate is soft baked at 55 °C for 60 s and cured under nitrogen flow with 365 nm fluorescence UV light for 7 min [151]. The RM coating step is repeated until the thickness of the film is equal to \( \frac{\lambda}{2} \).

Figure D-2 Schematic depiction of the optical recording set-up used for BY exposure. I: 457 nm laser, II: mirror, III: 10X beam expander, IV: beam splitter, V & VI: mirror, VII & VIII: quarter wave plate, IX: template lens with optical power of .25 diopter, X: beam combiner, XI: BY coated substrate.

Figure D-3 shows the phase profile of one the PPL that was fabricated during this work.
The most important factors in the exposure was found to be uniform and sufficient intensity and very accurate alignment.

Figure D-3 Phase profile of example 1D PPL fabricated with described set-up
APPENDIX E

Hybrid System Characterization Details

This section includes details on the characterization of hybrid system. This system consists of two SPP LC lenses, a 1.5 D glass lens and a 0.375 D PPL. The SPP LC lenses used in the hybrid system do not have floating electrodes. Hybrid system provides larger optical power without compromising the response time.

E.1 Phase Profile

One of SPP LC lenses used in the hybrid system had some connection issues are observed in bus lines 2 and 3 (Figure E-1). These connection issues cause that the shape of phase profile to deviate from a parabola towards the edge (Figure E-2).
Although the shape of phase profile deviated from parabola toward the edge of the lens, the shape of first three segments are still parabola. The phase profile of the fabricated PPL used in the hybrid design also was not ideal. It was off-centered as shown in Figure E-3. As our application of interest is near-to-eye, 5 mm aperture (average pupil size) at the center of the lens will be sufficient for performance assessment of the hybrid design.

To characterize the hybrid system, we started by characterizing individual parts of the system.
Figure E-2 Interferogram image of the SPP LC lens used for hybrid system at +0.34 D.

Figure E-3 Interferogram image of PPL at A-1: +0.375 D, B-2: -0.375 D. A-2 and B-2 shows A-1 and A-2 with 5 mm physical aperture.
E.2 Spot Profile and PSF

One of the component of the designed hybrid system is a glass lens with 1.5 D. Figure E-4 shows the spot profile and PSF of the glass lens used in the proposed hybrid system. The PSF measurement setup and procedure are the same as what discussed in Appendix C.

![1.5 D glass lens](image)

**Figure E-4** Spot profile and PSF of central 5 mm diameter of the glass lens with 1.5 D used in hybrid system. Best focus was found at 0.76 m.

Next we measured the spot profile of stack of two SPP LC lenses and .75D glass lens (Figure E-5), the maximum optical power of the stack was 1.43 D which can
be easily compared with spot profile of the glass lens of 1.5 D. As mentioned earlier SPP LC lenses did not have the floating electrodes.

**Figure E-5** Spot profile and PSF of central 5 mm diameter of the SPP LC (non-floating)/0.75 D glass lens at 1.43 D. Best focus was found at 0.87 m.

Next we checked the spot profile of SPP LC lenses with 1.5 D glass lens which is shown in Figure E-6.
The spot profile of the entire stack with the PPL, two SPP LC lenses and a 1.5 D glass lens at minimum +0.375 D and maximum +2.625 D are shown in Figure 4-2. The spot profiles at three different optical powers including 2.25 D, 1.5 D and 0.75 D have been also checked for the system. The corresponding optical power of each component is mentioned in Figure E-8.
Figure E-8 Spot profile and PSF of central 5 mm diameter of the 1.5 D glass lens/PPL/SPP LC lenses at top: +2.25 D. Best focus was found at 0.48 m., middle: +1.5 D. Best focus was found at 0.65 m and bottom: +0.75 D. Best focus was found at 1.22 m.

In Figure E-9 we used additional glass lens of 0.75 D and 1.5 D to bring the optical power to 2.25 D. Figure 4-3 in Chapter 4, shows graphs in Figure E-9 in one plot.
Figure E-9 Spot profile and PSF of central 5 mm diameter of the 1.5 D glass lens/PPL/SPP LC lenses at top: +2.25 D. Best focus was found at 0.48 m, middle: +1.5 D in addition to 0.75 D glass lens. Best focus was found at 0.46 m and bottom: +0.75 D in addition to 1.5 D glass lens. Best focus was found at 0.465 m.
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[91] After this work was started, Merck introduced improved materials, MLC 2171 and MLC 2172, that have a birefringence value of 0.29..


[133] Merck, "Technical data sheet- Licristal 18349".


[135] Prof. Gordon Wetzstein from Standford University clarified in OSA Imaging and Applied Optics conference June 2018 that 300 to 350 ms is the desired response time.


[141] Thanks to Prof. Hiroshi Yokoyama for the suggestion.


