Three Essays on Performance Evaluation in Operations and Supply Chain Management

A dissertation submitted to the
Kent State University Graduate School of Management
in partial fulfillment of the requirements
for the degree of Doctor of Philosophy

By
Hongyan Liang

September 2017
Dissertation written by

Hongyan Liang
B.A., Shanxi University, 2010

Master of Financial Economics, Ohio University, 2012

Ph.D., Kent State University, 2017

Approved by

Co-Chair, Doctoral Dissertation Committee

Co-Chair, Doctoral Dissertation Committee

Members, Doctoral Dissertation Committee

Accepted by

Doctoral Director, Graduate School of Management

Dean, Graduate School of Management
Abstract

Three Essays on Performance Evaluation in Operations and Supply Chain Management

Hongyan Liang
Doctor of Philosophy
Department of Management and Information Systems
Kent State University
2017

In today’s globally competitive marketplace, organizations are challenged to increase their levels of customer service while under pressure to simultaneously reduce operating costs and the time to market of products and services. In meeting these challenges, organizations have adopted performance measurement systems to gauge current performance and to set benchmarks for improving future performance. As discussed in Neely et al. (1995), managerial success in improving performance is perquisite on having a formal performance measurement system that provides management with meaningful short term (day-to-day) as well as long term performance goals. Within the operations and supply chain management literatures the importance of integrating performance measurement systems into decision making has been addressed by many researchers (see for example, Ramaa et al., 2013; Martin & Patterson, 2009; Shepherd & Günter, 2006; Gunasekaran et al. 2004). For effective performance measurement, formal quantitative models for performance measurement are needed (Suwignjo et al., 2000; Bititci et al., 2001).

In this dissertation, we examine three different classes of performance evaluation models, which are used by decision makers in operations and supply chain management. The general forms of these three classes of models are: i) learning-based models for continuous improvement, ii) stochastic inventory models for shortages, and iii) cost-volume profit models
for decision analysis. Despite a vast supporting literature for each class of model, there are adaptations of these models that can lead to further contributions that will be of interest to both the academic and practitioner communities.

In the research of improving supply chain delivery performance, Guiffrida & Nagi (2006) developed a cost-based delivery performance model whereby improvement in delivery performance is achieved by reducing the variance of the delivery time distribution using a learning-based function. A limitation of Guiffrida & Nagi (2006) is the failure to include forgetting into the learning process. Given the discrete nature of the delivery process, learning can be lost during the time periods that accrue between deliveries. The first essay extends the research of Guiffrida & Nagi (2006) to include forgetting into the learning based approach for improving supply chain delivery performance.

The literature on green and sustainable inventory management is quite limited and has mainly focused on the carbon footprint resulting with inventory management decisions (Bouchery et al. 2012). An examination of review papers on sustainable inventory models exposes the lack of integration of green and sustainable measures into stochastic inventory models. Of particular interest are stochastic inventory models with stockouts which examine the tradeoffs between backorders and lost sales. A limitation of these models is the failure to address environmental concerns in the stochastic inventory models with stockout decisions. The second essay integrates green and sustainable measures into stochastic inventory models which examine tradeoffs between backorders and lost sales.

A review of stochastic cost-volume-profit (CVP) analysis models indicates that the inputs of the models as well as the resulting profit function are modeling using either the normal or the lognormal distribution. Using normal and lognormal distributions in stochastic
CVP modeling represents a limitation to the general applicability of the models. In the third essay, we employ Mellin Transforms to expand and generalize the stochastic CVP model.
Acknowledgement

I would like to thank the members of my dissertation committee Alfred Guiffrida, Butje Eddy Patuwo, and Michael Hu for their continuous guidance and invaluable advice. I am particularly indebted to my supervisors Dr. Alfred Guiffrida and Dr. Butje Eddy Patuwo for their unconditional support and encouragement during my stay at Kent State University. I am also very grateful to Dr. Michael Hu, whose support and help has been invaluable for my research and job search. Special thanks to the other KSU MIS faculty members and my Ph.D. colleagues for their comments and suggestions.

Finally, I owe much to my parents, and my husband, Zilong Liu, for their tremendous support during my Ph.D. studies.
# Table of Contents

INTRODUCTION ......................................................................................................................... 1

1.1. Background ......................................................................................................................... 1

1.2 Limitations of Existing Models .......................................................................................... 2

1.3 Research Objectives ............................................................................................................ 4

1.4 Organization of the Dissertation ...................................................................................... 4

Supply Chain Delivery Performance under Conditions of .................................................. 5

Learning and Forgetting ........................................................................................................... 5

2.1. Background ......................................................................................................................... 5

2.2. Model Development .......................................................................................................... 10

2.2.1 Penalty cost function ....................................................................................................... 10

2.2.2 Learn-forget curve model ............................................................................................... 12

2.3.2. Managerial neglect in delivery performance improvement with interruption .......... 14

2.4. Numerical Analyses .......................................................................................................... 15

2.5. Summary and Conclusions ............................................................................................. 17

Environmental Consequences of Inventory Stockout Decisions ........................................... 24

3.1 Introduction ......................................................................................................................... 24

3.2 Sustainability and Stockouts ............................................................................................. 21

3.3 Zhang et al. 2003 model ..................................................................................................... 24

3.4 New model .......................................................................................................................... 26

3.4.1 Advanced hybrid inventory system .................................................................................. 26

A Generalized Stochastic Cost-Volume-Profit Model ............................................................. 31

4.1 Introduction .......................................................................................................................... 31

4.2 CVP Model Definition ....................................................................................................... 33

4.3 Literature Review ............................................................................................................... 34

4.4 Model Development .......................................................................................................... 342

4.5 Stochastic CVP Model ....................................................................................................... 349

Summary ................................................................................................................................... 31

REFERENCES ............................................................................................................................. 49
LIST OF FIGURES

FIGURE 2-1: Illustration a delivery window under a truncated normal distribution. 
(Guiffrida & Nagi 2006) ........................................................................................................... 11
FIGURE 2-2: The learning and forgetting effects of the delivery variance ....................... 13
FIGURE 2-3: Cooperation-based improvement ........................................................................ 14
FIGURE 2-4: Managerial neglect of cooperation-based improvement .............................. 15
FIGURE 3-1: The average total cost for τ = 0 to 10 and T= 1, 2, 4, and 6 ......................... 29
LIST OF TABLES

TABLE 2-1: Summary of Literature on Forgetting Models .......................................................... 8
TABLE 2-2: Summary of net present value calculations ................................................................. 16
TABLE 2-3: The effect of neglecting period on managerial neglect and delivery variance when l=80%, R=200 ....... 16
TABLE 2-4: The effect of break length on managerial neglect when l =80%, R=200 ............ 17
TABLE 3-1: Notation and description ............................................................................................. 24
TABLE 4-1: Summary of Literature on Stochastic CVP Models .................................................... 34
TABLE 4-2: Parameter values used in the numerical illustrations. ............................................... 49
TABLE 4-3: Distribution Characteristics of the product of Q and C ............................................. 49
TABLE 4-4: Comparison of true versus estimated probability values for V=QC ....................... 50
CHAPTER 1

Introduction

1.1. Background

In today’s globally competitive marketplace, organizations are challenged to increase their levels of customer service while under pressure to simultaneously reduce operating costs and the time to market of products and services. In meeting these challenges, organizations have adopted performance measurement systems to gauge current performance and to set benchmarks for improving future performance. As discussed in Neely et al. (1995), managerial success in improving performance is perquisite on having a formal performance measurement system that provides management with meaningful short term (day-to-day) as well as long term performance goals. Within the operations and supply chain management literatures the importance of integrating performance measurement systems into decision making has been addressed by many researchers (see for example, Maestrini et al., 2017; Balfaqih et al., 2016; Ramaa et al., 2013; Martin & Patterson, 2009; Shepherd & Günter, 2006; Gunasekaran et al. 2004).

For effective performance measurement, formal quantitative models for performance measurement are needed (Suwignjo et al., 2000; Bititci et al., 2001). Performance measurement metrics are critical to the implementation of a quantitative performance measurement model (Gopal & Thakkar, 2012; Sambasivan et al., 2009; Gunasekaran & Kobe, 2007). A recent trend in the performance measurement literature is the expanding importance of performance measurement in green and sustainable supply chains (Mishra et al., 2017; Ahi & Searcy, 2015; Beske-Janssen et al., 2015; Taticchi et al., 2013).
1.2 Limitations of Existing Models

In this dissertation we examine three different classes of performance models which are used by decision makers in operations and supply chain management. The general forms of these three classes of models are: i) learning-based models for continuous improvement of supply chain delivery performance, ii) stochastic inventory models for shortages, and iii) cost-volume profit models for decision analysis. Despite a vast supporting literature for each class of model, there are adaptations of these models that can lead to further contributions that will be of interest to both the practitioner and academic communities.

Research on improving delivery performance in supply chains has led to the development of cost-based supply chain delivery performance models whereby improvement in delivery performance is achieved by reducing the variance of the delivery time distribution. In the highly cited model of Guiffrida & Nagi (2006), a learning-based function is used for reducing the delivery variance as experience in the delivery process is gained. A limitation of Guiffrida & Nagi (2006) is the failure to include forgetting into the learning process that reduces the variance of the delivery time distribution. Given the discrete nature of the delivery process, learning can be lost during the time periods that accrue between deliveries. The first essay extends the learning-based delivery improvement model of Guiffrida & Nagi (2006) to include forgetting. The extended model that is developed in the first essay captures both learning and forgetting thereby providing a more comprehensive framework for modeling and assessing improvement in supply chain delivery performance.
The literature on green and sustainable inventory management is evolving and continues to focus on the carbon footprint resulting with inventory management decisions (see for example, Marklund and Berling, 2017; Hovelaque and Bironneau, 2015; Jawad et al., 2015; Bouchery et al. 2012). An examination of papers on sustainable inventory models exposes the lack of integration of green and sustainable measures into stochastic inventory models. Of particular interest are stochastic inventory models, which examine the tradeoffs between backorders and lost sales. A limitation of these models is the failure to address environmental concerns in the stochastic inventory models with stockout decisions. The second essay will explore this issue and provide insights on how the inclusion of costs associated with the carbon footprint resulting from managing stockouts impacts inventory decision making.

Cost-volume-profit (CVP analysis) is a decision tool that is used in many facets of managerial decision making. Parameter inputs of the traditional deterministic CVP model have been extend to incorporate uncertainty and as a result stochastic CVP models have been contributed to the literature. A review of this stochastic CVP literature indicates that the stochastic inputs of the model are restricted to either normal or lognormal distributions. The resultant profitability measure of the CVP is then restricted to being modeled as a normal or lognormal random variable which in application is a limitation. In essay three, Mellin Transformations are utilized to develop a generalization of the stochastic CVP model which allows an expanded portfolio of random variables to be used to describe the stochastic inputs to the CVP model thus generalizing the stochastic CVP model.
1.3 Research Objectives

The objectives of this research are as follows:

a. to develop a comprehensive framework for modeling improvement in supply chain delivery performance through the development of a cost-based decision model which includes both experience-based learning and forgetting in the delivery process.

b. to evaluate the impact that environmental consequences have on inventory policies for decisions on how to manage stockouts.

c. to develop a generalized stochastic CVP model.

d. to identify gaps in the research on performance measurement in operations and supply chain management which could stimulate new research agendas.

1.4 Organization of the Dissertation

This dissertation is organized into three essays. The first essay (Chapter 2) expands Guiffrida and Nagi (2006) by introducing forgetting into the learning-based modeling of managerial neglect by using the Learn-forget curve model of Jaber and Bonney (1997a). The model presented herein modifies the Learn-forget curve model to evaluate the early and late delivery deviation in terms of time units. The second essay (Chapter 3) examines the impact of environmental concerns on inventory policy decisions of how to manage inventory stockouts. The third essay (Chapter 4) Mellin Transformations are used as a methodology to generalize the stochastic cost-volume-profit (CVP). Chapter 5 summarizes the contributions of the three essays and provides directions for future research.
CHAPTER 2

Supply Chain Delivery Performance under Conditions of Learning and Forgetting

2.1. Background

In today’s competitive business environment, there is widespread adoption of the supply chain management (SCM) philosophy by firms. Under the supply chain management philosophy firms work in union with each other by sharing information such as sales forecasts, inventory levels and production schedules to insure better planning with the ultimate goal of meeting customer demand in an efficient and cost effective manner. The financial benefit of SCM to organizations has been documented in numerous studies (see from example Wagner et al. 2012; Shi and Yu, 2012; Flynn et al. 2010; Ou et al. 2010).

Supply chain managers rely on performance measurement in meeting both short term day-to-day objectives and long term strategic goals. As such, performance measurement in supply chains plays a key role in operationalizing the SCM philosophy (Ramaa et al. 2013; Martin and Patterson, 2009). Quantitative models for evaluating performance are an integral part of a performance measurement system in a supply chain (Suwignjo et al. 2000; Bititci et al. 2001) and frameworks for supply chain performance measurement have been established (Cuthbertson and Piotrowicz, 2011; Chan et al. 2006). Specific performance metrics for use in evaluating supply chain performance are reviewed in Gopal and Thakkar (2012) and
In this chapter, we focus on one aspect of supply chain performance, the delivery process for making deliveries to the end customer in the supply chain. The delivery process within a supply chain is highly monitored by supply chain managers since delivery performance directly impacts customer satisfaction levels (Chapman et al., 2011; Lockamy and McCormack, 2004; Vachon and Klassen, 2002).

The delivery process is one of five supply chain processes (plan, source, make, deliver, and return) found in the Supply Chain Operations Reference-model (SCOR). Delivery lead time to the end customer in a supply chain is defined to be the elapsed time from the receipt of an order by the supplier to the receipt of the product ordered by the customer and is composed of a series of internal (manufacturing and processing) lead times and external (distribution and transportation) lead times found at the various stages of the supply chain. Within the hierarchy of supply chain performance measures, Gunasekaran et al. (2004) classify delivery performance as a strategic level performance measure. Rao et al. (2011) characterize delivery performance as the most important metric in a supply chain since it serves to integrate and involve the measurement of performance throughout the stages of the supply chain. The importance that customers place on on-time delivery within supply chains has been well documented in the literature (da Silveira and Arkader, 2007; Iyer et al. 2004; Salvador et al. 2001).

When deliveries are not made on time, excess costs enter into the supply chain. As demonstrated in Bushuev and Guiffrida (2012), early deliveries contribute to increased inventory holding costs; late deliveries contribute to production stoppage costs. Delivery performance models for financially evaluating the costs associated with untimely (early and late) delivery performance are reviewed in Guiffrida (2014). As discussed in Soepenberg et al. (2008),
improvement in the reliability of delivery performance requires the accurate measurement of
delivery performance as the first step. As a result of this initial step, the current level of delivery
performance has been signaled to management. Improvement in the reliability of delivery
performance then requires a diagnostic analysis of the delivery process to identify attributes of
the delivery process than can be improved.

Specific actions that can be initiated by supply chain managers to improve delivery
performance have been addressed in Guiffrida & Paul (2010) and Soepenberg et al. (2008). The
implementation of these actions requires financial investment and hence the justification of the
resources. Building on contemporary management theories that advocate the reduction of
variance to improve a process, several researchers have developed cost-based supply chain
delivery performance models whereby improvement in delivery performance is achieved by
reducing the variance of the delivery time distribution (see for example Guiffrida & Jaber, 2008;
contributed to the literature on variance-based improvement in delivery performance by
modeling the opportunity cost of management failing to implement actions to improve delivery
performance by reducing the variance of the delivery time distribution. This managerial
oversight is referred to by Guiffrida & Nagi (2006) as managerial neglect and inherent to their
model is a learning-based function for reducing the delivery variance as experience in the
delivery process is gained.

The learning aspect of the managerial neglect model of Guiffrida & Nagi (2006) is well
established in the literature as evident by the widespread use of learning curve theory found in
the literature (see for example, Jaber, 2011; Anzanello & Fogliatto, 2011; Yelle, 1979). Guiffrida
and Nagi (2006) assumes the variance of delivery time decreases as the number of deliveries
increase. With more deliveries, the cooperation between the supplier and buyer becomes more closer, which decreases the volatility of the delivery time. As a result, the variance of delivery time decrease. However, a limitation of Guiffrida & Nagi (2006) is the failure to include forgetting into the delivery process that impedes the decreases in the variance of the delivery time distribution. Given the discrete nature of the delivery process, learning can be lost during the break between deliveries. The break can be real break between deliveries, and be delivery to different buyers. This loss of learning due to forgetting has been investigated in several studies which are summarized in Table 2-1.

<table>
<thead>
<tr>
<th>Author</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jaber, Givi, &amp; Neumann (2013)</td>
<td>This paper incorporates human fatigue and recovery into the learning-forgetting process. It studies the effects of learning rate, the batch size, the fatigue rate and recovery on productivity.</td>
</tr>
<tr>
<td>Alamri &amp; Balkhi (2007)</td>
<td>This paper studies the effects of learning and forgetting on the production lot size problems for an infinite planning horizon.</td>
</tr>
<tr>
<td>Jaber &amp; Guiffrida (2007)</td>
<td>This paper develops and revises the existing learning models to overcome the two limitations of the economic order quantity (EOQ) model with learning and forgetting.</td>
</tr>
<tr>
<td>Jaber &amp; Bonney (2007)</td>
<td>This paper investigates the effect of lot-size dependent learning and forgetting rates on the lot-size problem by incorporating the dual-phase learning-forgetting model (DPLFM) into the economic manufacture quantity model.</td>
</tr>
<tr>
<td>Jaber &amp; Kher (2004)</td>
<td>This paper attempts to correct the deficiency of the learn-forget curve model (LFCM) which is the assumption that the time for total forgetting is invariant of the experience gained prior to interruption.</td>
</tr>
<tr>
<td>Jaber &amp; Sikström (2004b)</td>
<td>This paper investigates and discusses three potential models; namely, the learn-forget curve model (LFCM), the regency model (RC), and the</td>
</tr>
</tbody>
</table>
power integration diffusion (PID) with their similarities and differences addressed.

<table>
<thead>
<tr>
<th>Jaber, Kher &amp; Davis (2003)</th>
<th>This paper reviews factors that influence worker forgetting in industrial settings, and analyzes the degree to which existing mathematical models conform to observed human forgetting behavior.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jaber &amp; Bonney (2003)</td>
<td>This paper investigates the effects that learning and forgetting in set-ups and product quality have on the economic lot-sizing problem.</td>
</tr>
<tr>
<td>Jaber &amp; Kher (2002)</td>
<td>This paper develops the dual-phase learning-forgetting model (DPLFM) to predict task times for tasks that require some degree of both cognitive and motor skills. In the DPLFM, the learning rates can be adjusted (upwards or downwards) as experience is gained (lost).</td>
</tr>
<tr>
<td>Nembhard &amp; Uzumeri (2000)</td>
<td>A population based approach to measuring workforce learning and retention forgetting is described through empirical industrial examples that include a manual task and a procedural task.</td>
</tr>
<tr>
<td>Jaber &amp; Bonney (1997a)</td>
<td>This paper extends the work of Karwan et al. (1988). It studies the effect of interruptions on the economic manufactured quantity under conditions of learning, and of both learning and forgetting.</td>
</tr>
<tr>
<td>Jaber &amp; Bonney (1997b)</td>
<td>Mathematical models of the forgetting process are reviewed. Three models, the VRIF, VRVF, and LFCM models are compared and their differences and similarities are discussed.</td>
</tr>
<tr>
<td>Jaber &amp; Bonney (1996)</td>
<td>The learn-forget curve model (LFCM) is developed. The forgetting rate is shown to be mathematically dependent on the learning rate, the length of the interruption, the equivalent accumulated output by the point of interruption, and the maximum break to which total forgetting is assumed. The effects of learning and forgetting on both the optimum production quantity and the minimum total inventory system cost are also investigated in this paper.</td>
</tr>
<tr>
<td>Globerson, Levin &amp; Shtub (1989)</td>
<td>This paper describes and analyzes a laboratory experiment designed to investigate the nature of forgetting in a working environment.</td>
</tr>
</tbody>
</table>
In this chapter, we extend Guiffrida & Nagi (2006) by introducing the Learn-forget curve model (LFCM) of Jaber & Bonney (1996) into the cost-based analysis of improving delivery performance through the reduction in the variance of delivery. Our research objective is to create a more robust model for supply chain managers to justify the use of financial resources to improve delivery performance through the reduction of the variance of the delivery time distribution. This chapter is organized as follows. Section 2.2 describes the assumptions and notations used in the model. Section 2.3 develops a modified learn-forget curve model and managerial neglect in delivery performance improvement with interruption. Numerical analyses of the model are presented in Section 2.4. In Section 2.5 we present our conclusions and identify future research directions.

2.2. Model Development

2.2.1 Penalty cost function

Delivery windows are often used to model the expected penalty costs of untimely delivery. Suppliers are given an earliest allowable date to deliver and the latest allowable date to deliver. The time range is defined as the delivery window. Figure 2.1 illustrates a delivery window under a truncated normal distribution. a and b are the truncated earliest allowable date and the latest allowable date. \( c_2 - c_1 \) is the on-time delivery portion of the delivery window. Any deliveries made in \( c_2 - c_1 \) are on-time delivery. Deliveries that are made between \( a \) and \( c_1 \) are early delivery, and deliveries that are made between \( c_2 \) and \( b \) are late delivery. Early delivery increases the inventory holding cost, while the late delivery may cause the stoppage of production. There are penalty cost for both early delivery and late delivery.
Guiffrida and Nagi (2006) studied the expected penalty cost under the condition of a two-stage supply chain. The expected penalty cost per delivery period for untimely delivery, $Y$, is

\[ Y = QH \int_a^{c_1} (c_1 - x) h_X(x) \, dx + K \int_{c_2}^b (x - c_2) h_X(x) \, dx \tag{2-1} \]

where

- $Q$ = constant delivery lot size per cycle
- $H$ = supplier’s inventory holding cost per unit per time
- $K$ = penalty cost per time unit late
- $a, b, c_1, c_2$ = parameters defining the delivery window
- $h_X(x)$ = density function of delivery time
Assume the delivery time $X$ follows a normal distribution with mean $\mu$ and variance $\nu$, the expected penalty cost

$$Y = QH \left\{ \sqrt{\nu} \phi \left( \frac{c_1 - \mu}{\sqrt{\nu}} \right) + (c_1 - \mu) \left[ \Phi \left( \frac{c_1 - \mu}{\sqrt{\nu}} \right) \right] \right\} + K \left\{ \sqrt{\nu} \phi \left( \frac{c_2 - \mu}{\sqrt{\nu}} \right) - (c_2 - \mu) \left[ 1 - \Phi \left( \frac{c_2 - \mu}{\sqrt{\nu}} \right) \right] \right\}$$

$$\text{(2-2)}$$

2.2.2 Learn-forget curve model

Wright’s (1936) learning curve suggests the time to perform a task decreases at a constant rate. Guiffrida and Nagi (2006) introduces the log-linear learning curve model (Yelle, 1979) into the delivery. In this study, we introduce the Learn-forget curve model (LFCM) of Jaber and Bonney (1996). The LFCM model of Bonney & Jaber (1996) is a power-form forgetting model that has been shown to more completely model the characterisitcs of foregetting in industrial settings and produce accurate fits to empirical dataon learning and forgetting (see for example, Jaber & Sikström, 2004b; and Jaber et al. 2003).

The delivery variance during the learning process is defined as

$$\nu(x) = \nu(1)(x^{-l}), \quad \text{(2-3)}$$

where $\nu(1)$ is the initial variance of the delivery distribution, $\nu(x)$ is the variance for $x$th delivery, $l$ is learning slope, and $l = ln(\theta)/ln(2)$, where $\theta$ is learning rate ($0 \leq \theta < 1$).

The delivery variance during the forgetting process is defined as

$$\hat{\nu}(x) = \hat{\nu}(1)x^f, \quad \text{(2-4)}$$

where $\hat{\nu}(1)$ is the intercept of the forgetting curve, $\hat{\nu}(x)$ is the variance for $x$th delivery, $f$ is forgetting slope.
The delivery variance at the end of the first cycle (see Figure 2-2) is equal to the delivery variance at the beginning of the forgetting cycle, that is

\[ v(1)(q^{-l}) = \hat{v}(1)q^f \]  
\[ \text{(2-5)} \]

Solving for \( \hat{v}(1) \) yields

\[ \hat{v}(1) = v(1)q^{-(l+f)} \]  
\[ \text{(2-6)} \]

Figure 2-2. The learning and forgetting effects of the delivery variance

Jaber and Bonney (1996) developed the forgetting exponent at the \( j \)th cycle, \( f_j \), and the equivalent delivery number \( u_j \) at the beginning of \( j \)th cycle after an interruption as

\[ f_j = \frac{(1-l)\log(u_j+q)}{\log(1+R/(u_j+q))}, \]  
\[ \text{(2-7)} \]

\[ u_{j+1} = (u_j + q)^{(1+f_j/l)}(u_j + q + s)^{-f_j/l}, \]  
\[ \text{(2-8)} \]

Where \( u_1 = 0 \). \( q \) is the number of deliveries in a cycle, \( s \) is the equivalent number of unites of
delivery that could have been delivered if delivery interruption did not occur. When \( l=0, f=0 \).

When there is no learning involved, hence there is no opportunity for forgetting.

### 2.3. Managerial neglect in delivery performance improvement with interruption

Guiffrida and Nagi (2006) have used cost-based model to get the present worth of managerial neglect by taking the difference of the expected cost stream over time horizon \( T \) when \( C \geq 1 \) (managerial neglect) and \( C=0 \) (no managerial neglect). In the research herein, the learn-forget curve model is used to determine the delivery variance over time horizon (see Figures 2-2 and 2-3). By taking the difference of the expected cost over time horizon when \( C \geq 1 \) (managerial neglect) and \( C=0 \) (no managerial neglect), the present value of managerial neglect can be determined. Given that there are there are \( n \) cycles and \( q \) units of deliveries occur in each cycle there are a total of \( nq \) deliveries.

![Figure 2-3. Cooperation-based improvement](image-url)
This yields

\[ N_{NPV} = \{Y(v, 1)e^{-ri} + Y(v, 1)e^{-2ri} + \cdots + Y(v, 1)e^{-(C+1)ri} + \cdots + Y(v, m) + C)e^{-mri}\} - \{Y(v, 1)e^{-ri} + Y(v, 2)e^{-2ri} + \cdots + Y(v, m) + C)e^{-mri}\} \]

(2-9)

Equation (2-9) defines the present worth of managerial neglect.

2.4. Numerical Analyses

The following numerical example illustrates the net present value of managerial neglect when the neglect period \( C \) equals 0, 6, 12, 18, 24, 30, and 36 months. For clarity of model presentation, we limit the number of cycles in the numerical illustration to \( n=3 \).

The parameters used are: \( Q = 400, H = $10 \) per day, \( K = $10,000 \) per day, \( T = 3 \) years, \( m = 36, i = 1/12 \) years, \( \mu = 15 \) days, \( |c_1 - \mu| = 1 \) day, \( c_2 - \mu = 1 \) day, \( r = 0.3, \sqrt{v(1)} = 4 \) days and \( \theta = 0.9, 0.8 \) and 0.7. Dutton and Thomas (1984) and Dar-El (2000) have found that the learning rate in practice is in the range 70-90%. Accordingly, we set the learning rate to 70%, 80%, and 90%. Results are presented in Table 2-1, 2-2 and 2-3.
Table 2-2. Summary of net present value calculations

<table>
<thead>
<tr>
<th>Length of the Neglect Period</th>
<th>90% NPV</th>
<th>Neglect</th>
<th>80% NPV</th>
<th>Neglect</th>
<th>70% NPV</th>
<th>Neglect</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Neglect (C=0)</td>
<td>416,092</td>
<td>341,117</td>
<td>275,948</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/2 year Neglect (C=6)</td>
<td>417,454</td>
<td>1362</td>
<td>346,288</td>
<td>5,170</td>
<td>285,989</td>
<td>10,041</td>
</tr>
<tr>
<td>1 year Neglect (C=12)</td>
<td>458,745</td>
<td>42653</td>
<td>421,191</td>
<td>80,074</td>
<td>387,663</td>
<td>111,715</td>
</tr>
<tr>
<td>3/2 year Neglect (C=18)</td>
<td>468,441</td>
<td>52349</td>
<td>440,714</td>
<td>99,597</td>
<td>416,778</td>
<td>140,830</td>
</tr>
<tr>
<td>3 year Neglect (C=36)</td>
<td>499,889</td>
<td>83797</td>
<td>499,889</td>
<td>158,771</td>
<td>499,889</td>
<td>223,941</td>
</tr>
</tbody>
</table>

Average forgetting exponent

|               | 0.1475 | 0.2495 | 0.2853 |

Results presented in Table 2-2 demonstrate that delaying the implementation of an improvement program leads to an unnecessary opportunity costs. Based on the analysis conducted, the higher the learning rate, the smaller the managerial neglect for the same neglect period. For the same learning rate, the longer the neglect period, the larger the managerial neglect. The worst case is when the learning rate is 70% and no improvement implemented, the managerial neglect reaches $223,941, which accounts for 45% of the net present value of total penalty cost. The average forgetting exponent is larger for lower learning rate, which means the smaller the learning rate does not only reduce the delivery variance slower, also forget faster.

Table 2-3. The effect of neglecting period on managerial neglect and delivery variance when \( l = 80\% \), \( R=200 \), \( s=3 \)

<table>
<thead>
<tr>
<th></th>
<th>C=0</th>
<th>C=6</th>
<th>C=12</th>
<th>C=18</th>
<th>C=36</th>
</tr>
</thead>
<tbody>
<tr>
<td>Managerial neglect</td>
<td>5,170</td>
<td>80,074</td>
<td>99,597</td>
<td>158,771</td>
<td></td>
</tr>
<tr>
<td>Average delivery variance</td>
<td>7.24</td>
<td>7.36</td>
<td>10.71</td>
<td>11.76</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 2-3 presents the effect of neglecting period on managerial neglect and delivery variance when \( l = 80\% \), \( R=200 \), and \( s=3 \). For the same learning rate, the longer the neglect
period, the larger the managerial neglect will be. This illustrates that there is an increasing opportunity cost related to longer neglect period. When \( C = 6 \), the managerial neglect is $5,170, and it almost doubles when \( C = 18 \). When no learning is involved, the worst case is $158,771. It is smart to implement improvement early, but is never too late to take action. The average delivery variance is larger for longer neglecting period. This explains the increasing of managerial neglect.

Table 2-4. The effect of break length on managerial neglect when \( l =80\% \), \( R=200 \)

<table>
<thead>
<tr>
<th>Break length</th>
<th>( s=3 )</th>
<th>( C=0 )</th>
<th>( C=6 )</th>
<th>( C=12 )</th>
<th>( C=18 )</th>
<th>( C=36 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s=3 )</td>
<td>5,170</td>
<td>80,074</td>
<td>99,597</td>
<td>158,771</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s=6 )</td>
<td>7,220</td>
<td>76,307</td>
<td>94,888</td>
<td>145,807</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2-4 presents the effect of break length on managerial neglect when \( l =80\% \), \( R=200 \). Break length between each delivery cycle, \( s \) is 3 and 6. For the same learning rate and neglect period, the net present worth of managerial neglect is smaller for longer break time.

In conclusion, the shorter the neglect, the smaller the NPV of managerial neglect will be; the higher the learning rate, the smaller the NPV of managerial neglect will be; to some extent, the longer the break, the smaller the NPV of managerial neglect will be. Supply chain managers can make decision by using this information as an implement.

2.5. Summary and Conclusions

This research expands Guiffrida and Nagi (2006) by introducing forgetting into the learning-based modeling of managerial neglect by using the Learn-forget curve model developed by Jaber and Bonney (1996). The model presented herein provides a more robust framework for supply chain managers to use when justifying resources to improve long-term delivery performance.

There are several aspects of this research that could be improved. First, a simulation
methodology could be used to gain insights on responsiveness of the model in terms of interaction between the delivery unit and the break length in each cycle. Second, the model is confined to measuring delivery performance in serial supply chains. Many supply chains are structured as networks with multiple retailers, manufacturers and raw material suppliers within each echelon of the supply chain. Measuring delivery performance in a network supply chain where delivery time to the final customer can be determined by multiple paths through the supply network would require a modeling strategy that could address this combinatorial aspect in defining total delivery time. Lastly, the model assumes that the total delivery time is the sum of independent upstage activity times. Expanding the model to allow for dependent stage activity times where the activity time of a predecessor stage impacts the activity time of a successor stage would further generalize the model.
CHAPTER 3

Environmental Consequences of Inventory Stockout Decisions

3.1 Introduction

The efficient operation of an inventory control system requires managers to make decisions to the two following questions: (1) how large should an inventory replenishment order be, and (2) when should an inventory replenishment order be placed. Inventory managers must make decisions to these two questions subject to the objective of providing a desired level of customer service at a minimal cost. Since the introduction of the Harris Economic Order Quantity (EOQ) Model in 1913, a vast number of inventory models have been developed to aid managers in making inventory management decisions. As an indication of the scope of this literature, a google scholar search using the key phrase “inventory model” returns over 2,790,000 hits. In recent years researchers have attempted to review the literature on inventory models using various classification schemes (see for example, Bushuev et al., 2015; Drake & Marley, 2014; Glock et al., 2014).

In the years following the introduction of the deterministic Harris EOQ model, the literature on inventory models has been ever expanding in its attempts to answer the fundamental questions of how much to order and when to place an order. Numerous variants and extensions of the EOQ rapidly appeared in the literature. Wilson (1934) introduced the concept of the reorder point and this model represents one of the first attempts to include probabilistic behavior in the parameters of an inventory management model.
Inventory models can be broadly classified as deterministic or stochastic inventory models. Deterministic models assume that the demand and lead time characteristics of the inventory system are known with complete certainty. The typical total cost function includes the costs associated with ordering, holding and purchasing inventory and the optimal order quantity that minimizes the total cost can be easily found. The underlying mathematical structure for defining lead time demand is relatively straightforward for deterministic models and determining how much to order and when to place the replenishment order such that total costs are minimized is easily found.

Stochastic inventory models first appeared in the literature in the early 1950s. In the seminal models developed by Arrow et al. (1951), Dworetzky et al. (1952a, b) and Whitin (1954), uncertainty and variability in lead time demand (which had been previously assumed away in deterministic models) was captured. As a result of incorporating uncertainty in demand and/or lead-time into the modeling of lead-time demand, a more complex mathematical structure was created for determining the order quantity and replenishment timing. The order quantity and replenishment must be determined subject to an inventory system performance measure such as a service level or fill rate.

In today’s competitive business environment customer dissatisfaction resulting from stockouts (or lost sales) is of high concern to inventory and supply chain managers. A customer experiences a stockout when their demand for an item cannot be satisfied by its supplier. A stockout can lead to the supplier experiencing either a loss of the sale outright or having to schedule a backorder. Traditionally, the effect of stockouts are captured in inventory optimization models using a shortage cost term that is based on a penalty cost for the stockout. Stockout penalty costs are commonly structured as a fixed cost per stockout occasion or a
fractional charge per unit short (see for example, Peterson and Silver, 1979). Regardless of the costing policy implemented, the penalty cost assigned to a stockout can be difficult to quantify (Xu, 2017; Liberopoulos et al. 2010; Dhall, 2008; Gardner, 1980). The magnitude of an inventory stockout penalty cost is a function of factors such as the time value of money associated with the product’s revenue/profit margin, added administrative costs incurred to manage the stockout occasion, costs of expedited delivery and loss of customer goodwill. Reviews of the negative impacts that stockouts have on customers are found in Zinn and Liu (2008), Campo et al. (2003), Dion and Banting (1995) and Dion et al. (1991).

3.2 Sustainability and Stockouts

As a result of increased environmental awareness, many firms have adopted and integrated green and sustainable practices into their operations as means to support more effective and efficient resource usage, energy consumption and waste abatement (see for example, Ortiz-de-Mandojana and Bansal, 2016; Beske and Seuring, 2014; Walker et al., 2014; Wellington et al. 2014; Carter and Rogers, 2008). Many firms that have adopted green and sustainable practices into their business operations often operate under the Triple-Bottom-Line (TBL) framework that was developed by Elkington (1998). Under the TBL framework, the traditional financial performance measures of profitability and return on investment are expanded to include both environmental and social dimensions lending the TBL to be thought of as a “three – pillar” approach (profits, planet and people) which integrates economic, environmental and social measures to overall firm performance (Slaper & Hall, 2011).

The quantification of carbon emissions in operations and supply chain management is a highly active research area and a variety of modeling approaches have been developed to audit and measure carbon emission across the various functional areas of a supply chain. Recent
papers by Amui et al. (2017), Fahimnia et al. (2017), Dubey et al. (2017) and Hassini et al. (2012) provide a comprehensive summary of this literature. Within this literature two research streams, green and sustainable inventory management and carbon emissions resulting from motor carrier transportation, are relevant to the research herein.

The adoption of green and sustainable business practices has had a direct impact on inventory management. The literature on green and sustainable inventory management is growing and to date has mainly focused on the carbon footprint resulting from inventory management decisions (see for example, Marklund and Berling, 2017; Hovelaque and Bironneau, 2015; Jawad et al., 2015; Bouchery et al., 2012). Reviews of the green and sustainable aspects of inventory models such as the EOQ (and its variants) and the newsvendor problem are found in Brown et al. (2016), Bushuev et al. (2015), and Bonney and Jaber (2012).

The importance and relevance of sustainability within the logistics function of a supply chain has also been summarized by several review papers (see for example, Qaiser et al. 2017; Herold and Lee, 2017; Dubey and Gunasekaran, 2015; Ellram and Golicic, 2015; Evangelista, 2014). Models for measuring the carbon footprint associated with maritime, air, rail, pipeline, motor carrier and intermodal transportation have been reported in the literature. Of particular interest in this research paper is the carbon emission generated from motor carrier transportation. Carbon emission for motor carrier transport is generally calculated as a function of distance traveled, the weight of the load transported, and the type of vehicle used (Ülkü, 2012; Figliozzi et al. 2011).

An examination of the literature on green and sustainable inventory models exposes the lack of integration of green and sustainable measures into stochastic inventory models. Of particular interest are stochastic inventory models which examine the tradeoffs between
backorders and lost sales. Often times, emergency orders are issued to correct a stockout occurrence with the vast majority of emergency orders involving transportation of the product by motor carriers. Hence, there is a direct sustainability connection between the research streams of inventory management and freight transport by motor carrier (See for example, Demir et al. 2015; Konur, 2014; Konur and Schaefer, 2014).

In this essay we develop a stochastic inventory model for examining the decision of when to backorder or use an emergency order to prevent a stockout. The model establishes a cost optimal critical cut off time for assisting management in deciding whether to use a backorder versus or an emergency order to manage a stockout occasion. Existing models found in the literature do not address the environmental concerns associated with this decision hence a research opportunity exists to reexamine the decision to backorder, stockout or use a mixture of backordering and lost sales. The model herein advances the literature on hybrid inventory models which examine the decision to backorder versus place an emergency order by incorporating sustainability costs due to inventory management and freight transportation into the analysis. Models for examining the decision to backorder as opposed to stockout have appeared in the literature (see for example Zhang et al. 2003; Chu et al. 2001). These models do not address environmental concerns hence a research opportunity exists to reexamine the decision to backorder, stockout or use a mixture of backordering and lost sales exists.

3.3 Model Development

3.3.1 Notation

Table 3-1 gives the notation used in this essay.
### Table 3-1. Notation and description

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{AC}^(*)$</td>
<td>Expected annual cost</td>
<td>$</td>
</tr>
<tr>
<td>$\bar{\text{AC}}^(*)$</td>
<td>Expected annual cost with environment impact</td>
<td>$</td>
</tr>
<tr>
<td>E(T)</td>
<td>Expected cycle time</td>
<td>time period</td>
</tr>
<tr>
<td>E(T~)</td>
<td>Expected cycle time with environment impact</td>
<td>time period</td>
</tr>
<tr>
<td>E(C)</td>
<td>Expected cost</td>
<td>$</td>
</tr>
<tr>
<td>E(C~)</td>
<td>Expected cost with environment impact</td>
<td>$</td>
</tr>
<tr>
<td>E(IT)</td>
<td>Expected inventory per cycle</td>
<td>units</td>
</tr>
<tr>
<td>E(EO)</td>
<td>Expected emergency order per cycle</td>
<td>units</td>
</tr>
<tr>
<td>E(B)</td>
<td>Expected backorders per cycle</td>
<td>units</td>
</tr>
<tr>
<td>E(BT)</td>
<td>Expected time dependent backorders per cycle</td>
<td>unit year</td>
</tr>
<tr>
<td>L</td>
<td>Lead time</td>
<td>time period</td>
</tr>
<tr>
<td>Q</td>
<td>Inventory level after replenishment</td>
<td>units</td>
</tr>
<tr>
<td>r</td>
<td>Reorder inventory level</td>
<td>units</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Demand arrival rate</td>
<td>unit/period</td>
</tr>
<tr>
<td>$\tau$</td>
<td>A cutoff time between emergency order and backorder</td>
<td>time period</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Per unit cost of emergency orders</td>
<td>$</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>Per unit cost of backorders</td>
<td>$</td>
</tr>
<tr>
<td>$\pi'$</td>
<td>Per unit-year cost of backordered demands</td>
<td>$</td>
</tr>
<tr>
<td>K</td>
<td>Fixed cost for each replenishment cycle, including setups, order process, and transportation;</td>
<td>$</td>
</tr>
<tr>
<td>c</td>
<td>Per unit variable cost in producing activities;</td>
<td>$</td>
</tr>
<tr>
<td>h</td>
<td>Per unit cost in holding and handling inventory.</td>
<td>$</td>
</tr>
<tr>
<td>K'</td>
<td>Fixed cost of environmental impact for each replenishment cycle, including setups, order process, and transportation;</td>
<td>$</td>
</tr>
<tr>
<td>c'</td>
<td>Per unit variable cost of environmental impact in producing activities;</td>
<td>$</td>
</tr>
<tr>
<td>h'</td>
<td>Per unit cost of environmental impact in holding and handling inventory.</td>
<td>$</td>
</tr>
<tr>
<td>$E_{\text{CO}_2}$</td>
<td>The CO$_2$ emission</td>
<td>lbs</td>
</tr>
<tr>
<td>$S_{\text{CO}_2}$</td>
<td>The shipping emission factor</td>
<td>lbs/cwt-mile</td>
</tr>
<tr>
<td>w</td>
<td>The weight per unit emergency order</td>
<td>lbs</td>
</tr>
<tr>
<td>v</td>
<td>The average speed of truck</td>
<td>mile per hour</td>
</tr>
<tr>
<td>T</td>
<td>The transport time of emergency order</td>
<td>time period</td>
</tr>
<tr>
<td>m</td>
<td>Per unit cost of CO$_2$ emissions in transporting the emergency orders</td>
<td>$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>The emission cost per lb of CO$_2$ emissions</td>
<td>$/bl</td>
</tr>
</tbody>
</table>

#### 3.3.2 Zhang et al. 2003 model

When a stockout occurs, customers either choose to wait for the next replenishment.
(backorder) or turn to other suppliers (loss sales). A backorder means an unfilled customer order or commitment are satisfied by the next replenishment. According to Kim and Park (1985), the penalty cost of backorder is proportional to the length of time for which the backorder exists. Lost sales would mean permanent loss of goodwill and customer dissatisfaction. In this case, firms are more likely to adopt emergency order rather than loss sale, even through it has the same mathematical format with loss sales. Emergency order means firms buy similar products from competitors to satisfy their customers demand to avoid loss of goodwill. Emergency order is costly, however, loss of goodwill and customers is more costly and more difficult to regain. Managers are challenged to optimize the profit or minimize the cost during stockout. Decisions are needed to make about when and how many products should be backordered.

In Zhang et al. (2003), a cutoff time \( \tau \) is incorporated with the traditional \((Q, r)\) systems. Following Hadley and Whitin (1963), three unit shortage costs are included in the model:

- \( \pi \) = Per unit cost of emergency orders;
- \( \hat{\pi} \) = Per unit cost of backorders;
- \( \pi' \) = Per unit-year cost of backordered demand.

According to Zhang et al. (2003), the expected annual cost \( AC(Q, r, \tau) \) is given by

\[
AC(Q, r, \tau) = \frac{E[C]}{E[T]} \tag{3-1}
\]

Where \( C \) is the random total cost within a cycle and \( T \) is the cycle time.

\[
E(C) = K + cQ + hE(I_T) + \pi E[EO] + \hat{\pi}E[B] + \pi'E[B_T] \tag{3-2}
\]
3.4 New model

3.4.1 Advanced hybrid inventory system

Following Zhang et al. (2003), we consider a continuous review \((Q, r)\) system where the demand process is a stationary Poisson process with arrival rate \(\lambda\). When environmental impact in both holding and transporting inventory is considered, more economic costs are introduced in the model. Many environmental approaches with different assumptions are studied in prior researches (Arslan and Turkay, 2013, Hammami et al. 2015). Bouchery et al. (2010) proposed key performance indicators for transportation subprocess and warehouse subprocess. Five most widely studied approaches are direct accounting, carbon tax, direct cap, cap & trade, and carbon offsets (Arslan and Turkay, 2013). The direct accounting approach treating carbon footprint as additional economic cost is the basic approach. The other four approaches are policy related. Without considering the government policy, the direct accounting approach is the most suitable way to start with.

Following Arslan and Turkay (2013), based on direct accounting approach, let

\(K'\) : fixed cost of environmental impact for each replenishment cycle, including setups, order process, and transportation;

\(c'\) : per unit variable cost of environmental impact in producing activities;

\(h'\) : per unit cost of environmental impact in holding and handling inventory.

When emergency order incurs, products are transported from competitors. Since transportation is not planned, truckload is not optimized. To meet the customer demand, under-optimized truckload is transported. This results in higher cost per unit product transported.

We have two assumptions in the hybrid model:

Assumption I: all backorders are satisfied at the beginning of each cycle
Assumption II: all emergency orders must be completed before the beginning of each cycle.

In this study, we calculate the CO₂ emission as a function of weight and travel distance. Assume CO₂ emission $E_{CO_2}$ is a function of the weight of emergency order and the distance to transport the emergency order. We have

$$E_{CO_2} = Funcion(\text{weight, distance}).$$

More precisely, the emission $E_{CO_2}$ is

$$E_{CO_2} = S_{CO_2} \{w[E[EO]]\} \{vT\},$$

$$0 < T \leq \frac{(Q-r)}{\lambda}; \quad (3-3)$$

where $S_{CO_2}$ is the shipping emission factor, which is defined as the average amount of CO₂ emitted when a type of vehicle carries a hundredweight (i.e. cwt) of product per mile (Ulku 2012). For truck, the emission factor equals to 0.0169 (e.g., www.carbonfund.org) Let $w$ be the weight per unit emergency order, the total weight of emergency order is $w[E[EO]]$. $T$ is the transport time of emergency order. $v$ is the average speed of truck. $vT$ is the travel distance.

Based on Assumption II, emergency orders must complete before $\frac{(Q-r)}{\lambda}$. Hence the farthest distance the trucks can travel to delivery emergency order is $\frac{(Q-r)}{\lambda} v$.

We introduce $m$ as per unit cost of CO₂ emissions in transporting the emergency orders. Assume $\alpha$ being the emission cost per lb of CO₂ emissions,

$$m = \alpha E_{CO_2} \quad (3-4)$$

As for backorder, since no inventory or transportation is incurred, no extra cost of environmental impact is included.

We assume the expected annual cost $\overline{AC} (Q, r, \tau)$ considering environmental impact is given by
\[
\bar{A}\bar{C}(Q, r, \tau) = \frac{E[\bar{C}]}{E[\bar{T}]} \tag{3-5}
\]

Where \(\bar{C}\) is the random total cost within a cycle considering environmental impact and \(\bar{T}\) is the cycle time.

\[
E(\bar{C}) = (K + K') + (c + c')Q + (h + h')E(I_T) + (\pi + m)E[EO] + \hat{n}E[B] + \pi'E[B_T] \tag{3-6}
\]

\(E(I_T), E[EO], E[B], \) and \(E[B_T]\) are derived in Zhang et al. (2003).

### 3.4.2 Numerical illustration

In this section, we present numerical illustrations of the model to explore the impact of environmental costs associated with sustainability inventory management and transportation on the optimal solution to the hybrid inventory model, Following Zhang et al. (2003), we use the following parameters values: \(\lambda = 3, L = 10, r = 8, Q = 26, K = 100, c = 10, h = 3, \pi = 26, \hat{n} = 0, \pi' = 4\). When the environmental effects are considered, we assume that \(K', c', h'\) is 10% of \(K, c, h\). That is \(K' = 10, c' = 1, h' = 0.3\). The shipping emission factor \(S_{CO_2} = 0.0169\) bl/cwt mile (Ülkü, 2012). The cost per lb of CO\(_2\) emissions \(\alpha = 0.0075\) dollars/bl.
When $\tau$ is less than 5, the expected total costs for $T = 1, 2, 4, \text{ and } 6$ are minimal, hence they appear in Figure 1 as a line. This means, since when $\tau$ is less than 5, the stockout is mainly serviced by back order, any cost differences in emergency order are not significant. Examining Figure 1, we note the expected total costs considering the environmental factor are parallel to the expected total costs without environmental factor; however, there is an upward shift in the expected total costs. The upward shift in the cost function is due to the added environmental costs associated with the inventory related costs of ordering, holding, and transporting the emergency order.
When $\tau$ is greater than 5, the differences in the expected total costs increase when the environmental factor is considered. The cost differences between expected cost with the environmental factor and without environmental factor are larger for larger value of $T$ and also the larger value of $\tau$. 
CHAPTER 4

A Generalized Stochastic Cost-Volume-Profit Model

4.1 Introduction

As a part of their overall longterm business strategy, organizations are continually faced with the task of making decisions over competing alternatives. A wide range of decision models exist to aid managers in evaluating and selecting the best alternative from a set of competing alternatives. When the decision under study requires the evaluation of profitability as a function of output, cost-volume-profit (CVP) analysis is an attractive decision tool to use since it enables managers to make decisions under business conditions where costs, revenue and volume are changing (Braun and Tietz, 2013). For situations involving a single product, decision makers can use CVP to determine the sales volume needed to achieve a targeted profit when sales price, variable cost, or fixed cost change. When making decisions on establishing and/or revising a product line, CVP analysis can also be used to identify the most profitable combination of products to include within the product line.

The stochastic CVP model has been studied for several decades. Applications of the model still prevail in practice and the attractiveness and usefulness of the model can be further advanced by generalizing the model to bridge the following gaps. First, as identified in review of the literature of the stochastic CVP model which is found in Table 4-1, the stochastic inputs of the model are restricted to either normal or lognormal distributions thus resulting in profitability also being modeled as a normal or lognormal random variable. In our opinion, normal and
lognormal random variables are assumed only for calculation simplicity. A more generalized modeling of the stochastic CVP model would entail using a wider range of candidate probability distributions for defining the inputs of the model. This would support a potentially expanded scope of application for the model since the model’s resulting profitability distribution which would no longer have to be restricted to being normally or lognormally distributed. This greater flexibility in evaluating the resulting profitability of the stochastic CVP model would enhance the application of the model.

In this research we introduce the Mellin Transform technique as a means to generalize the stochastic modeling of the CVP model. For a gateway into the methodology of Mellin Transforms the reader is referred to Bertrand et al. (2000) and Epstein (1948). Due to the unique properties of the Mellin Transform, a much wider range of non-negative random variables can be used in the formulation and analysis of stochastic CVP models. Second, when multiple stochastic inputs to the stochastic CVP model have been used, limiting assumptions have been used to simplify how the resulting probability distribution representing profitability is defined. The scope of the stochastic CVP model can be further advanced by using model structures that more accurately represent the true underlying form of the model’s profitability distribution when multiple stochastic parameters exist. Using Mellin transforms, we further generalize the stochastic CVP model to include such model structures where more than one input parameter is defined stochastically.

The remaining sections of this chapter are organized as follows. In Section 4.2, we introduce the CVP model. In Section 4.3 we review the literature on stochastic CVP models and identify limitations in the models. In Section 4.4 we introduce models for the stochastic CVP model that overcome the limitations of the model as found in the literature and conduct
supporting numerical analyses to illustrate the models developed. In Section 4.5 we present the summary and conclusions to the chapter.

4. 2 CVP Model Definition

The basic CVP analysis involves the formulation of a total profit function for a single product (see equation 4-1). Total profit is defined as total revenue minus total cost,

\[ Z = Q(P - V) - F \]  (4-1)

where
- \( Z \) = Total Profit
- \( Q \) = Sales (in units)
- \( P \) = Selling price per unit
- \( V \) = Variable cost per unit
- \( F \) = Fixed cost.

The objective is to determine the level of sales where total revenue equals total cost. At this equilibrium, commonly referred to as the break-even point, the break-even quantity is defined as \( Q_{BEP} = \frac{F}{P-V} \). When the sales volume is lower than \( Q_{BEP} \) a loss is incurred; when the sales volume is greater than \( Q_{BEP} \) a profit is generated.

When comparing across two competing alternatives, profit functions for each alternative can be defined and the quantity level where the decision maker is indifferent between the two alternatives can be determined. Given knowledge of this indifference point, the decision maker can then determine for a projected sales level which alternative is most profitable.

The traditional CVP model is based on the following set of assumptions:

1. Costs are only affected by a change in the sales level.
2. Costs (both variable and fixed) are linearly related to the sales level.
3. Revenues are linearly related to the sale level.

4. The inventory level is constant within a given time period.

5. All model parameters are deterministic.

4.3 Literature Review

In reality, uncertainty and risk exists and therefore potentially affects each input parameters of the CVP model. Hence, the deterministic CVP model defined in (4-1) is very limited in practical application. Jaedicke and Robichek (1964) were the first to introduce stochastic parameters into the CVP model thus broadening the appeal and application of the model and establishing the baseline for the literature on stochastic CVP models. Table 4-1 provides a summary of the literature on stochastic CVP models.

Table 4-1. Summary of Literature on Stochastic CVP Models.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jaedicke and Robichek (1964)</td>
<td>This paper introduces risk and uncertainty into the Cost-Volume-Profit analysis. They develop a procedure for approximating the distribution of profit, where unit sales (Q), the unit selling price (P), the fixed cost (F), and the variable cost (V) are independent normally distributed variables. They assume that the product of two independent normally distribution random variables is normal distributed.</td>
</tr>
<tr>
<td>Ferrara, Hayya and Nachman (1972)</td>
<td>This paper identifies that the underlying assumption in Jaedicke and Robichek (1964) that the product of two independent normally distributed random variables, unit sales Q times unit selling price P minus variable cost V, yields a distribution of profits that is normally distributed. Computer simulation is used to define ranges of values for the coefficient of variations for Q and P – V under which the distribution of profit could be considered normally distributed.</td>
</tr>
<tr>
<td>Jarrett (1973)</td>
<td>This paper uses Bayesian Decision Theory to provide guidelines on whether management should make a decision on its own estimates of the parameters or postpone a terminal decision until further research into estimating the parameters for costs, price, and demand are determined.</td>
</tr>
<tr>
<td>Kim (1973)</td>
<td>This paper presents two variants of the stochastic CVP models and illustrates the usefulness and properties of the models in financial analysis.</td>
</tr>
<tr>
<td>Hilliard and Leitch (1975)</td>
<td>In this paper quantity and contribution margin are assumed to be lognormal random variables and fixed costs are deterministic. The model relaxes the commonly held assumption of independence thus allowing dependency between the input random variables.</td>
</tr>
<tr>
<td>Author(s)</td>
<td>Description</td>
</tr>
<tr>
<td>---------------------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Lau and Lau (1976)</td>
<td>They note that the assumption of positive skewness in the model of Hilliard and Leitch (1975) represents a model limitation. They point out that the primary variables in CVP analysis could be skewed either positively or negatively, depending on the major factors influencing the values of these variables. They conclude that the assumption of symmetric distribution is better than the blanket assumption of positive skewness.</td>
</tr>
<tr>
<td>Adar, Barnea and Lev (1977)</td>
<td>A comprehensive CVP analysis under uncertainty which includes the risk preferences of decision makers is conducted. Combining the probability characteristics of variables with risk preference function provides the insights for the study of the effect of fixed cost changes on optimal short-run output.</td>
</tr>
<tr>
<td>Shih (1979)</td>
<td>This paper takes into account the actual sales in the CVP analysis. The more practical assumption enables the management to choose the best products and optimal production quantity to produce.</td>
</tr>
<tr>
<td>Cantrell and Ramsay (1984)</td>
<td>This paper presents maximum likelihood point and interval estimators of long-run and single-period target quantities for a simple C-V-P model containing a semi-variable cost function.</td>
</tr>
<tr>
<td>Kim, Abdolmohammadi, and Klein (1996)</td>
<td>This paper argues that profitability not only affect the level of production, but also the level of investment in risky assets.</td>
</tr>
<tr>
<td>Gonzalez (2001)</td>
<td>This paper applies the CVP analysis into multiproduct situation. The unique non-optimal solutions are provided when production constraints do not exist.</td>
</tr>
<tr>
<td>Yunker (2001)</td>
<td>This paper extents a CVP analysis by introducing the downward-sloping demand curve and the U-shaped average cost curve.</td>
</tr>
</tbody>
</table>

Jaedicke and Robichek (1964) is the first work to integrate uncertainty into the CVP model by defining profitability as a random variable under two different model formats. In the first model, sales volume is represented as a normally distributed random variable while selling price, unit cost and fixed cost remain deterministic; in the second model, all four model inputs are defined as independent normally distributed random variables. In model one expected profit $T$, which is a function of the combination of one normally distributed random variable (sales $Q$) and three constants (selling price $P$, variable cost $V$ and fixed cost $F$), is normally distributed. In model two the function defining expected profit $T$, which is defined by a function involving the difference of two normal random variables, $(V – F)$ and the product of two normal random
variables $Q(P - V)$, is assumed to be normally distributed. While the difference of two independent normally distributed random variables is well known in the literature to be normally distributed, the product of two independent normal random variables is not normally distributed and has relative complicated distribution form (Craig, 1936). To simplify their model, Jaedicke and Robichek (1964) assume that the product of the two independent normal random variables is normal. Under the normality assumption, the probabilities to gain different level of profits, including the probability to reach the break-even level for competing production alternatives, can be easy calculated using the model. This study provides the foundation to the stochastic CVP model and its managerial application for selecting alternatives under conditions of uncertainty.

Ferrara, Hayya and Nachman (1972) point out the limitations in the model of Jaedicke and Robichek (1946) and provide evidence from the literature that challenges the assumption made in Jaedicke and Robichek (1946) that the product of two independent normal variables ($Q$ and $P - V$) is also a normal variable. They argue the assumption that the product of two independent normally distributed random variables results in a normal distributed random variable holds only under certain conditions relating to the magnitude of the coefficients of variation of the random variables being multiplied. Using Monte Carlo simulation, they recommend that if the sum of the coefficients of variation for unit sales $Q$ times unit selling price $P$ minus variable cost $V$ is less than or equal to 12 percent, the assumption that the product of two independent random variables, $Q(P - V)$, is also normally distributed can be accepted at the 0.05 significance level.

Hilliard and Leitch (1975) argue that the use of independent normally distributed random variables normapresent a stochastic CVP model formulation argue that the condition under which the normality assumption cannot be rejected is too rigorous in Jaedicke and Robichek
the independent assumption is lack of practical supporting, since quantity, price, and variable cost are often correlated; and finally the normal assumption renders sales, prices, and variable costs large chance to take a negative value when the standard deviation is large, which is problematic. Instead, they suggest that quantity and contribution margins are bivariate log-normal random variables and that dependent relationships obtain. Since the product of bivariate lognormal variables is known to be lognormal, Hilliard and Leitch (1975) overcome the limitation faced by Jaedicke and Robichek (1946). However, the differencing of lognormal random variables continues to be a problem. To overcome this difficulty, Hilliard and Leitch (1975) assume that the contribution margin as a whole is a lognormal variable and the fixed variable is a constant.

Lau and Lau (1976) argue that the input variables in a stochastic CPV analysis could be right skewed or left skewed, which is determined by some crucial factors. They posit that a symmetry assumption is better then the right skewness assumption, in that symmetry assumption includes the possibility of both right and left skewness. This work, to some extent, lessens the contribution of Hilliard and Leitch (1975).

Jarrett (1973) provides guidance for managers on how to use Bayesian Decision Theory to estimate parameters of the CVP. According to Jarrett (1973), two options exist for parameter estimation: i) use current information to guide all estimation, or ii) postpone estimation until additional study on the parameters are completed. This paper also provides practical suggestions for managers to consider when selecting a given estimation option.
Kim (1973) presents two modified CVP models aimed to help financial analysts to avoid the problem of the difference ex-post and ex-ante costs and prices in sensitivity analysis. Some unexpected statistical and financial properties are displayed in the study.

Adar, Barnea, and Lev (1977) present a comprehensive CVP model, which integrates uncertainty and the risk preference of decision makers. The combination evokes an interesting finding on the effect of fixed costs, which has been ignored in prior literatures.

Shih (1979) points out the deficiency of the tradition CVP model of failing to distinguish sales, demand and production. It argues when production is larger than demand, the profit calculated from tradition CVP model is overestimated, especially when the unsold products are perishable goods or the demand for the unsold products lasts only for a certain period of time. The tradition CVP model only includes the variable costs and fixed cost to produce the goods, but not the costs to dispose the leftovers. The tradition CVP model is limited in its applicability when the sales, demands and productions are different. Shih (1979) modifies the tradition CVP model to take into account the random demand and determined production. The modified model enables managers to make optimal decision under uncertainty.

Yunker (2001) extend the tradition CVP model by incorporating the downward-sloping demand curve and the U-shaped average cost curve, which is more realistic than Shih (1979). It demonstrates that a firm confronting uncertainty will produce a smaller quantity than an equivalent firm under certainty, given a risk-averse manager. While if the manager is risk-neutral, equivalent firms under uncertainty and certainty will produce the same amount.

Cantrell and Ramsay (1984) study the statistical properties of the target quantity estimator. The point estimate of target quantity is found to be biased, possessing no moments,
but consistent. The interval estimate of target quantity is provided by using Maximum likelihood method for short-term and long-term analysis. The procedure minimizes the ambiguity in the stochastic CVP model.

Kim, Abdolmohammadi, and Klein (1996) incorporate the utility function of the decision maker into the CVP model. The authors argue that decision makers (managers) are trying their best to maximize their utility when they make decision on the investment in risky assets. Two main results are found. First, a change in fixed costs affects not only the manager’s decision on production, but also the decision on risky assets. The increasing risk aversion manager tends to invest more (less) in risky assets and less (more) in risk-free assets with increasing fixed costs. Second, for managers displaying constant absolute risk aversion, the optimal combination of risky and risk-free assets is constant regardless of the changes in fixed costs. This study provides some insight information on the managers’ investment behaviors.

Gonzalez (2001) extends the single product CVP model into multiproduct CVP analysis that is designed to be implemented at the enterprise level. The modified model requires the user to formulate a contribution rule that is consistent with the operating characteristics of the business environment and also with the user’s assessment of the degree to which the different products must contribute to recovering costs so as to meet a targeted profitability level.

4.4 Model development

Mathematical transformation methods are frequently used in operation research when solving complex problems. Obtaining solutions by using transformation methods is an indirect approach and often times avoids the complexity of using a direct solution procedure. Three commonly used transformation methods that are used to analyze continuous functions include
the Fourier, Laplace, and Mellin transforms. These transforms are particularly useful in defining and analyzing the characteristic and moment generating functions of continuous random variables. All continuous transforms are based on the Fourier transform. The Fourier, Laplace, and Mellin transforms are interchangeable, with each having advantages in different applications.

The Fourier transform (see equation 4-2) is well-known for deriving the probability densities of sums and differences of random variables. If \( f(t) \) is a function of \( t \), then \( F_t(u) \) is the Fourier transform of \( f(t) \). \( F_t(u) \) and \( f(t) \) form a unique transform pair, because \( F_t(u) \) uniquely defines \( f(t) \) and \( f(t) \) uniquely defines \( F_t(u) \).

\[
F_t(u) = \int_{-\infty}^{\infty} f(t)e^{-iu\,t}dt \tag{4-2}
\]

The Laplace transform is a derivative of the Fourier transform. If \( f(t) \) is a function of \( t \), then \( L(s) \) is the Laplace transform of \( f(t) \) (see equation 4-3). \( L(s) \) and \( f(t) \) form a unique transform pair. The Laplace transform finds its most widespread application in solving differential equations. Laplace transforms is widely used in operation management and engineering, due to the well-developed transform pair tables.

\[
L(s) = \int_{-\infty}^{\infty} f(t)e^{-st}dt \tag{4-3}
\]

If \( f(x) \) is a function of \( x \), the Mellin transform of \( f(x) \) is defined by

\[
F_{M_x}(s) = \int_{0}^{\infty} x^{s-1}f(x)dx \tag{4-4}
\]

The Mellin transform may also be defined for \(-\infty < x < \infty\), but for application purpose, we only focus on the positive part. We note that \( f(x) \) and \( F_{M_x}(s) \) are unique transform pairs. The
Mellin transform is a convenient technique for analyzing products and quotients of continuous random variables.

Let \( x_1 \) and \( x_2 \) be positive-valued continuous random variables with probability density functions (pdf) of \( g(x_1) \) and \( h(x_2) \), respectively. Let \( x = x_1 x_2 \), and let \( f(x) \) be the probability density function of \( x \). Define \( F_{Mx_1}(s) \) and \( F_{Mx_2}(s) \) as the Mellin transforms of \( g(x_1) \) and \( h(x_2) \), respectively. The Mellin transform \( F_{Mx}(s) \) of \( f(x) \) is given by (Schmidt and Davis, 1981)

\[
F_{Mx}(s) = F_{Mx_1}(s)F_{Mx_2}(s)
\]  

(4-5)

See Appendix A for products of two random variables for the cases of the uniform, gamma and normal random variables.

If \( x = x_1/x_2 \), and \( f(x) \) is the probability density function of \( x \), then the Mellin transform \( F_{Mx}(s) \) of \( f(x) \) is given by (Schmidt and Davis, 1981)

\[
F_{Mx}(s) = F_{Mx_1}(s)F_{Mx_2}(2-s)
\]  

(4-6)

Similarly, if \( x = 1/x_1 \), and \( f(x) \) is the probability density function of \( x \), then the Mellin transform \( F_{Mx}(s) \) of \( f(x) \) is given by (Schmidt and Davis, 1981)

\[
F_{Mx}(s) = F_{Mx_1}(2-s)
\]  

(4-7)

Schmidt and Davis (1981) presents the Mellin transform pairs for the most widely used continuous random variables. Hence, the Mellin transforms of the products and quotients of two continuous random variables can be easily obtained, given the Mellin transforms of the two random variables. Since the Mellin transform and the original function are uniquely paired, if we know the Mellin transform of the products or quotients of two random variables, we will find the
probability density function by inversing the Mellin transform function. However, the fact is that the product of two Mellin transforms is rarely any format that we can find the inverse function. So it is a challenge to find the pdf of the products or quotients of two random variables.

Nevertheless, like other continuous transforms, the Mellin transform can be used to compute the moments of random variables. From equation (4-7), we can get

\[ F_{M_x}(s) = E(x^{s-1}). \]  

(4-8)

This unique format of Mellin transform simplifies the process to find the \( n \)th moment of a continuous random variable. Replacing \( s \) with \( n + 1 \) in equation (4-8) gives \( E(x^n) \), the \( n \)th moment of random variable \( x \), which is defined as

\[ m_n = F_{M_x}(n + 1). \]  

(4-9)

The first and second moments allow the definition of the mean and variance of a random variable; the third and fourth moments can be used to determine the skewness and excess kurtosis of the random variable. When it is difficult to find the exact pdf of a random variable, the key attributes of the random variable and its distributional form can be estimated by studying the moments of the random variable.

4.5 Stochastic CVP model

The stochastic CVP model has been studied for several decades. Applications of the model still prevail in practice and the attractiveness and usefulness of the model can be advanced by further generalizing the model to bridge the following gaps. As identified in review of the literature of the stochastic CVP model, the stochastic inputs of the model are restricted to either normal or lognormal distributions thus resulting in profitability being modeled as either a normal
or lognormal random variable. In our opinion, normal and lognormal random variables are assumed only for calculation simplicity. A more generalized distribution of profit in a stochastic CVP model that is not restricted to being normally or lognormally distributed can be obtained by using the Mellin Transform technique. The greater flexibility of selecting a wider range of different random variable types will enhance the application of the stochastic CVP model.

In this section, we apply Mellin Transformation techniques to examine the stochastic CVP model when model inputs and profitability are not restricted to the normal and lognormal distributions. Two cases will be examined. In the first case, the model inputs to the stochastic CVP model will involve the product of two uniform random variables; in the second case, the model inputs will involve the product of two gamma random variables.

The stochastic CVP model takes the general form of

\[ Z = Q(P - V) - F. \]  

(4-10)

Defining the contribution per unit as \( C = P - V \) yields

\[ Z = QC - F. \]  

(4-11)

We note that in (4-11) \( Q \) and \( C \) are assumed to be independent random variables and \( F \) is constant thus defining the stochastic CVP model as the product of two independent random variables. Restating the contribution margin as \( W = QC \), the stochastic CVP model is redefined as

\[ Z = W - F. \]  

(4-12)
Where
Z=Profit
Q=Unit sales
P=Price/Unit
V=Variable Cost/Unit
F=Fixed cost
C=P-V= Contribution/Unit
W= QC=Contribution Margin

Case 1:
Assume Q and C following uniform distribution,

\[ Q \sim \text{Uniform} \ (q_{\min}, q_{\max}) \]
\[ C \sim \text{Uniform} \ (c_{\min}, c_{\max}) \]

Where, \( q_{\min} \) and \( c_{\min} \) are minimum values of random variable Q and C, \( q_{\max} \) and \( c_{\max} \) are maximum value of random variable Q and C. Per Appendix A, the moment generating function of \( W_{\text{uni}} = QC \) is

\[ m_n(W_{\text{uni}}) = \frac{(q_{\max}^{n+1} - q_{\min}^{n+1})(c_{\max}^{n+1} - c_{\min}^{n+1})}{(n+1)^2(q_{\max} - q_{\min})(c_{\max} - c_{\min})} \] (4-13)

Using, (4-13) and setting \( n = 1 \), the first moment is

\[ m_1(W_{\text{uni}}) = \frac{(q_{\max}^2 - q_{\min}^2)(c_{\max}^2 - c_{\min}^2)}{4(q_{\max} - q_{\min})(c_{\max} - c_{\min})} \] (4-14)

which simplifies to,

\[ m_1(W_{\text{uni}}) = \frac{1}{4}(q_{\max} + q_{\min})(c_{\max} + c_{\min}). \] (4-15)

Similarly, for \( n = 2 \) the second moment is

\[ m_2(W_{\text{uni}}) = \frac{(q_{\max}^3 - q_{\min}^3)(c_{\max}^3 - c_{\min}^3)}{9(q_{\max} - q_{\min})(c_{\max} - c_{\min})} \] (4-16)

which simplifies to,
\[ m_2(W_{uni}) = \frac{1}{9}(q_{max}^2 + q_{max}q_{min} + q_{min}^2)(c_{max}^2 + c_{max}c_{min} + c_{min}^2). \]  

(4-17)

Setting \( n = 3 \) and \( n = 4 \), the third and fourth moments are respectively,

\[ m_3(W_{uni}) = \frac{(q_{max}^4 - q_{min}^4)(c_{max}^4 - c_{min}^4)}{25(q_{max} - q_{min})(c_{max} - c_{min})} \]  

(4-18)

and,

\[ m_4(W_{uni}) = \frac{(q_{max}^5 - q_{min}^5)(c_{max}^5 - c_{min}^5)}{36(q_{max} - q_{min})(c_{max} - c_{min})}. \]  

(4-19)

Using equations (4-13 to 4-19), the mean, variance, skewness and excess kurtosis of \( W_{uni} \) are:

mean of \( W_{uni} \) is

\[ Mean(W_{uni}) = m_1(W_{uni}), \]  

(4-20)

variance of \( W_{uni} \) is

\[ Var(W_{uni}) = m_2(W_{uni}) - [m_1(W_{uni})]^2 \]  

(4-21)

skewness, \( \gamma_1 \), of \( W_{uni} \) is

\[ \gamma_1(W_{uni}) = \frac{m_3(W_{uni})}{[m_2(W_{uni})]^{3/2}} \]  

(4-22)

excess kurtosis, \( \gamma_2 \), of \( W_{uni} \) is

\[ \gamma_2(W_{uni}) = \frac{m_4(W_{uni})}{[m_2(W_{uni})]^2} - 3 \]  

(4-23)

We assume fixed cost, \( F \), is a constant number \( f \). With the above distribution characteristics of contribution margin, \( W \), we can get the distribution characteristics of total profit, \( Z \),

mean of \( Z_{uni} \) is

\[ Mean(Z_{uni}) = Mean(W_{uni}) + f, \]  

(4-24)
The cumulative distribution function (CDF) of the product of two uniform random variables (Q and C) where \( a = q_{\min}, b = q_{\max}, c = e_{\min}, \) and \( d = e_{\max} \) when \( ad < bc, \) is

\[
H(v) = \begin{cases} 
0 & -\infty \leq v \leq ac \\
\left(\frac{1}{b-a}\right) \left(\frac{1}{d-c}\right) [v(lnv - lnac - 1) + ac] & ac \leq v \leq ad \\
\left(\frac{1}{b-a}\right) \left(\frac{1}{d-c}\right) [lnb - lnc](v - ad) & ad \leq v \leq bc \\
\left(\frac{1}{b-a}\right) \left(\frac{1}{d-c}\right) [v(lnbd - lnv + 1)] + bc[lnbc - lnbd - 1] & bc \leq v \leq bd \\
1 & v > bd
\end{cases}
\]  

(4-28)

When \( ad = bc \) or when \( ad > bc, \) the process illustrated by equations (4-13) to (4-27) can be used to define the resultant CDF.

**Case 2:**

Assume \( Q \) and \( C \) following Gamma distribution,

\[
Q \sim \text{Gamma} \left( q_{\text{shape}}, q_{\text{scale}} \right) \\
C \sim \text{Gamma} \left( e_{\text{shape}}, e_{\text{scale}} \right)
\]

Where, \( q_{\text{shape}} \) and \( e_{\text{shape}} \) are shape parameters of random variable \( Q \) and \( C, \) \( q_{\text{scale}} \) and \( e_{\text{scale}} \) are scale parameters of random variable \( Q \) and \( C. \) Per Appendix A, the moment generating function of function of \( W_{gam} \) is
\[ m_n(W_{gam}) = \left( \frac{1}{q_{scale}^c scale} \right)^n \frac{r(q_{shape}+n) r(c_{shape}+n)}{r(q_{shape}) r(c_{shape})} \]  \hspace{1cm} (4-29)

Using (4-24), the first moment is

\[ m_1(W_{gam}) = \left( \frac{1}{q_{scale}^c scale} \right)^1 \frac{r(q_{shape}+1) r(c_{shape}+1)}{r(q_{shape}) r(c_{shape})} \]  \hspace{1cm} (4-30)

which simplifies to,

\[ m_1(W_{gam}) = \frac{q_{shape}^c shape}{q_{scale}^c scale} \]  \hspace{1cm} (4-31)

Similarly, the second moment is

\[ m_2(W_{gam}) = \left( \frac{1}{q_{scale}^c scale} \right)^2 \frac{r(q_{shape}+2) r(c_{shape}+2)}{r(q_{shape}) r(c_{shape})} \]  \hspace{1cm} (4-32)

which simplifies to,

\[ m_2(W_{gam}) = \left( \frac{1}{q_{scale}^c scale} \right)^2 q_{shape}^c shape (q_{shape} + 1)c_{shape}^c shape (c_{shape} + 1) \]  \hspace{1cm} (4-33)

The third moment

\[ m_3(W_{gam}) = \left( \frac{1}{q_{scale}^c scale} \right)^3 \frac{r(q_{shape}+3) r(c_{shape}+3)}{r(q_{shape}) r(c_{shape})} \]  \hspace{1cm} (4-34)

simplifies to,

\[ m_3(W_{gam}) = \left( \frac{1}{q_{scale}^c scale} \right)^3 q_{shape}^c shape (q_{shape} + 1)(q_{shape} + 2)c_{shape}^c shape (c_{shape} + 1)(c_{shape} + 2) \]  \hspace{1cm} (4-35)

The forth moment

\[ m_4(W_{gam}) = \left( \frac{1}{q_{scale}^c scale} \right)^4 \frac{r(q_{shape}+4) r(c_{shape}+4)}{r(q_{shape}) r(c_{shape})} \]  \hspace{1cm} (4-36)

simplifies to,

\[ m_4(W_{gam}) = \left( \frac{1}{q_{scale}^c scale} \right)^4 q_{shape}^c shape (q_{shape} + 1)(q_{shape} + 2)(q_{shape} + 3)c_{shape}^c shape (c_{shape} + 1)(c_{shape} + 2)(q_{shape} + 3) \]  \hspace{1cm} (4-37)
Using equations (4-29 to 4-37), the mean, variance, skewness and excess kurtosis of $W_{gam}$ are:

$$\text{Mean}(W_{gam}) = m_1(W_{gam}), \quad (4-38)$$

The variance of $W_{gam}$ is

$$\text{Var}(W_{gam}) = m_2(W_{gam}) - [m_1(W_{gam})]^2 \quad (4-39)$$

The skewness, $\gamma_1$, of $W_{gam}$ is

$$\gamma_1(W_{gam}) = \frac{m_3(W_{gam})}{[m_2(W_{gam})]^{3/2}} \quad (4-40)$$

The excess kurtosis, $\gamma_2$, of $W_{gam}$ is

$$\gamma_2(W_{uni}) = \frac{m_4(W_{uni})}{[m_2(W_{uni})]^2} - 3 \quad (4-41)$$

We assume fixed cost, $F$, is a constant number $f$. With the above distribution characteristics of contribution margin, $W$, we can get the distribution characteristics of total profit, $Z$,

mean of $Z_{gam}$ is

$$\text{Mean}(Z_{gam}) = \text{Mean}(W_{gam}) + f, \quad (4-42)$$

difference of $Z_{gam}$ is

$$\text{Var}(Z_{gam}) = \text{Var}(W_{gam}) \quad (4-43)$$

skewness, $\gamma_1$, of $Z_{gam}$ is

$$\gamma_1(Z_{gam}) = \gamma_1(W_{gam}) \quad (4-44)$$

excess kurtosis, $\gamma_2$, of $Z_{gam}$ is

$$\gamma_2(Z_{gam}) = \gamma_2(W_{gam}) \quad (4-45)$$

No closed form solution for the CDF of the product of two gamma random variables exists.
4.6 Numerical illustration

The following numerical example illustrates the application of the stochastic CVP model for cases I and II. In each case, the mean and the variance of the unit sales are 5000 units and 300 units, respectively. The mean and the variance of the contribution per unit used in each case are $120 and $27, respectively. The specific parameters used in each case are presented in Table 4.2.

<table>
<thead>
<tr>
<th>Table 4.2 Parameter values used in the numerical illustrations.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit sales (Q)</strong></td>
</tr>
<tr>
<td>Minimum value ($q_{\text{min}}$)</td>
</tr>
<tr>
<td>Case 1: Uniform Distribution</td>
</tr>
<tr>
<td>Case 2: Gamma Distribution</td>
</tr>
</tbody>
</table>

The distribution characteristics of the product of $Q$ and $C$ are presented in Table 4.3. The central and dispersion of the product of $Q$ and $C$ depends heavily on the distribution of $Q$ and $C$.

<table>
<thead>
<tr>
<th>Table 4.3 Distribution Characteristics of the product of $Q$ and $C$.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
</tr>
<tr>
<td>Case 1: Uniform Distribution</td>
</tr>
<tr>
<td>Case 2: Gamma Distribution</td>
</tr>
</tbody>
</table>
As denoted in the literature, many researchers assume that the product of Q and C is normally distributed. As demonstrated in the skewness and excess kurtosis values found in Table 4.3, the assumption of using a normal distribution is weakly supported, since a normal distribution would have a skewness of zero and a excess kurtosis of zero. Using the parameter set upon which the results of Tables 4.2 and 4.3 are based, we employ the CDF as defined in equation (4-28) to generate comparative probability calculations in Table 4.4 to further illustrate the inaccuracy of using the normal. Examining the forth column, the percentage error associated with using the normal appreciation ranges from an underestimation of approximately 31% to an overestimation of approximately 5%.

<table>
<thead>
<tr>
<th>Selected values of V=QC</th>
<th>True P(&lt;V=QC) per (4-28)</th>
<th>Estimated P(&lt;V=QC) per normal</th>
<th>Percentage Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>470,000</td>
<td>0.0210</td>
<td>0.0146</td>
<td>-30.7</td>
</tr>
<tr>
<td>540,000</td>
<td>0.4567</td>
<td>0.4796</td>
<td>5.0</td>
</tr>
<tr>
<td>610,000</td>
<td>0.9312</td>
<td>0.9812</td>
<td>5.4</td>
</tr>
</tbody>
</table>
CHAPTER 5

Summary

Performance measurement is key to the success of an organization in today’s globally competitive marketplace. Management requires timely and accurate information on operational performance to ensure that day-to-day as well as long term goals and objectives are being met and to set benchmarks for improving future performance. For effective performance measurement, formal quantitative performance evaluation models with meaningful performance metrics are needed. Organizational decisions are generally made under conditions of uncertainty hence it critical that performance measurement systems acknowledge the stochastic nature of the environment in which decisions are made. This dissertation, which is presented in the form of three essays, has examined three different classes of stochastic performance models in operations and supply chain management. The general forms of these three models are: i) learning-based models for continuous improvement of supply chain delivery performance, ii) stochastic inventory models for shortages, and iii) cost-volume profit models for decision analysis. Collectively the models developed in this dissertation contribute to the literature on performance measurement by providing enhancements to existing performance models on supply chain delivery performance, environmentally conscious policy decision making in inventory management, and cost-volume-profit (CVP) analysis.

The first essay (Chapter 2) expands the delivery performance model of Guiffrida and Nagi (2006) by introducing forgetting into the learning-based modeling of managerial neglect by using the Learn-forget curve model of Jaber and Bonney (1997a). The model presented herein modifies the Learn-forget curve model to evaluate the early and late delivery deviation in terms
of time units. The second essay (Chapter 3) examines the impact of environmental concerns on inventory policy decisions of how to manage inventory stockouts in a stochastic reorder point inventory model. The third essay (Chapter 4) uses Mellin Transformations as a methodology to generalize the stochastic cost-volume-profit (CVP). The three different classes of stochastic performance models in this dissertation lead to further contributions that will be of interest to both the practitioner and academic communities.

Research on improving delivery performance in supply chains has led to the development of cost-based supply chain delivery performance models whereby improvement in delivery performance is achieved by reducing the variance of the delivery time distribution. In the highly cited model of Guiffrida & Nagi (2006), a learning-based function is used for reducing the delivery variance as experience in the delivery process is gained. The first essay addresses the limitation of Guiffrida & Nagi (2006) and includes forgetting into the learning process that reduces the variance of the delivery time distribution. The model presented provides a more robust framework for supply chain managers to use when justifying resources to improve long-term delivery performance.

An examination of papers on sustainable inventory models exposes the lack of integration of green and sustainable measures into stochastic inventory models. Of particular interest are stochastic inventory models, which examine the tradeoffs between backorders and lost sales. The second essay address the limitation of previous models of failure to address environmental concerns in the stochastic inventory models with stockout decisions. The second essay explores this issue and provide insights on how the inclusion of costs associated with the carbon footprint resulting from managing stockouts impacts inventory decision making. A numerical illustration shows the effect of environment cost in decision making. When $\tau$ is less than 5, the stockout is
mainly serviced by back order, any cost differences in emergency order are not significant. We found the expected total costs considering the environmental factor are parallel to the expected total costs without environmental factor; however, there is an upward shift in the expected total costs. When $\tau$ is greater than 5, the differences in the expected total costs increase when the environmental factor is considered. The cost differences between expected cost with the environmental factor and without environmental factor are larger for larger value of $T$ and also the larger value of $\tau$.

Cost-volume-profit (CVP analysis) is a decision tool that is used in many facets of managerial decision making. Parameter inputs of the traditional deterministic CVP model have been extended to incorporate uncertainty and as a result stochastic CVP models have been contributed to the literature. A review of this stochastic CVP literature indicates that the stochastic inputs of the model are restricted to either normal or lognormal distributions. The resultant profitability measure of the CVP is then restricted to being modeled as a normal or lognormal random variable which in application is a limitation. In essay three, Mellin Transformations are utilized to develop a generalization of the stochastic CVP model which allows an expanded portfolio of random variables to be used to describe the stochastic inputs to the CVP model thus generalizing the stochastic CVP model. A numerical illustration is conducted to demonstrate the usage of the model developed in essay three. The model allows a more realistic and flexible application of CVP model in business, which provide more information for decision makers in business.

The models presented in this dissertation provide a foundation for an ongoing research program in performance measurement in operations and supply chain management. There are interesting opportunities to further expand the performance evaluation models presented in each
essay. The Learn-forget delivery performance model of Essay 1 can be advanced by the introduction of stochastic parameter estimates for the penalty costs associated with untimely delivery. The inventory model in Essay 2 can be extended beyond the current carbon footprint to include additional greenhouse gases to capture the long term environmental impact of inventory policy decisions for managing stockouts. Lastly, the stochastic CVP model of Essay 3 can be expanded to the case of multiple products.
REFERENCES


Management, 28(9), 1019-1040.


Ülkü, M. A. (2012). Dare to care: Shipment consolidation reduces not only costs, but also environmental damage. International Journal of Production Economics, 139(2), 438-446.


Appendix:
PDF, CDF and Moment Generating Functions for the Product of Uniform, Gamma and Normal Random Variables.

<table>
<thead>
<tr>
<th>PDFs</th>
<th>PDF or CDF of products of two random variables</th>
<th>Moment generating function</th>
</tr>
</thead>
</table>
| $X \sim \text{Uniform}(a, b)$  
$Y \sim \text{Uniform}(c, d)$  
Let $V = XY$, $h(v)$ is the PDF of $V$. | If $ad < bc$,  
$h(v) =  
\begin{cases}  
\left(\frac{1}{b-a}\right) \left(\frac{1}{d-c}\right) \left[\ln v - \ln ac\right] & ac \leq v \leq ad \\
\left(\frac{1}{b-a}\right) \left(\frac{1}{d-c}\right) \left[\ln d - \ln c\right] & ad \leq v \leq bc \\
\left(\frac{1}{b-a}\right) \left(\frac{1}{d-c}\right) \left[\ln bd - \ln v\right] & bc \leq v \leq bd  
\end{cases}$ | $m_n = \frac{(b^{n+1} - a^{n+1})(d^{n+1} - c^{n+1})}{(n + 1)^2(b - a)(d - c)}$ |
| If $ad = bc$,  
h(v)  
\begin{align*}  
&= \left(\frac{1}{b-a}\right) \left(\frac{1}{d-c}\right) \left[\ln b - \ln a\right] & ac \leq v \leq ad \\
&= \left(\frac{1}{b-a}\right) \left(\frac{1}{d-c}\right) \left[\ln bd - \ln v\right] & ad \leq v \leq bd  
\end{align*}  
| If $ad > bc$,  
h(v)  
\begin{align*}  
&= \left(\frac{1}{b-a}\right) \left(\frac{1}{d-c}\right) \left[\ln b - \ln a\right] & ac \leq v \leq bc \\
&= \left(\frac{1}{b-a}\right) \left(\frac{1}{d-c}\right) \left[\ln b - \ln a\right] & bc \leq v \leq ad \\
&= \left(\frac{1}{b-a}\right) \left(\frac{1}{d-c}\right) \left[\ln bd - \ln v\right] & ad \leq v \leq bd  
\end{align*}  
(Glen, Leemis, & Drew (2004))
\(X \sim \text{Gamma}(a, b)
\)
\(Y \sim \text{Gamma}(c, d)
\)

X and Y are independent.
Let \(V = X\ Y\), \(g(v)\) is the PDF.

\(X \sim \text{Normal}(\mu_x, \sigma_x^2)
\)
\(Y \sim \text{Normal}(\mu_y, \sigma_y^2)
\)

X and Y are independent.
Let \(W = X\ Y\), \(F(W)\) is the cumulative density function of \(W\).

\[
F(W) = \frac{e^{-(\frac{\mu_x^2}{2\sigma_x^2} + \frac{\mu_y^2}{2\sigma_y^2})}}{2\pi\sigma_x\sigma_y} \left[ \int_{0}^{\infty} \Phi(w, x) \frac{dx}{x} \right]
- \int_{-\infty}^{0} \Phi(w, x) \frac{dx}{x}]
\]

Where
\[
\Phi(w, x) = \exp\left[-(\sigma_y^2x^4 - 2\mu_y\sigma_y^2x^3 + 2\mu_y\sigma_y^2wx + \sigma_x^2w^2)/2\sigma_x^2\sigma_y^2x^2 \right]
\]
(Craig (1936))

\[
m_n = \left(\frac{1}{bd}\right)^n \frac{\Gamma(a + n)}{\Gamma(a)} \frac{\Gamma(c + n)}{\Gamma(c)}
\]
(Craig (1936))