CASE STUDIES LISTENING TO STUDENTS USING KINESTHETIC MOVEMENT WHILE LEARNING TO GRAPH LINEAR FUNCTIONS.

A dissertation submitted to the Kent State University College of Education, Health, and Human Services in partial fulfillment of the requirements for the degree Doctor of Philosophy

By

Melissa A. Novak

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A dissertation written by

Melissa A. Novak

B.S., University of Dayton, 1990
M.Ed., Kent State University, 1995
Ph.D. Kent State University, 2017

Approved by

__________________________, Director, Doctoral Dissertation Committee
Joanne Caniglia

__________________________, Member, Doctoral Dissertation Committee
Steven Turner

__________________________, Member, Doctoral Dissertation Committee
Marianne Martens

__________________________, Member, Doctoral Dissertation Committee
Walter Gershon

Accepted by

__________________________, Director, School of Teaching, Learning, and Curriculum Studies
Alexa Sandmann

__________________________, Dean, College of Education, Health Services
James C. Hannon
The purpose of this qualitative practitioner research study was to describe middle school algebra students’ experiences of learning linear functions through kinesthetic movement. Participants were comprised of 8th grade algebra students. Practitioner research was used because I wanted to improve my teaching so students will have more success in learning mathematics.

Since this research focused on the mental constructions made by students as they attempted to make sense of mathematics kinesthetically, it is grounded in the philosophical tenants of constructivism (Piaget & Vygotsky), math representation theory, and kinesthetic movement.

This study utilized multiple data sources which included pre-and post-teacher-made assessments with state standardized problems, audio and video transcriptions of class, small group activities, individual discussions, learning style inventory, and attitude survey on kinesthetic learning.

Data was collected and analyzed through triangulation. The results of this study have important curricular implications for math educators to understand how students can learn through kinesthetic movements. Educators can support their students learning by incorporating movement into their classrooms. Recommendations for future research based on unanticipated findings are suggested.
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“Ideas are like stars, you cannot succeed in touching them with your hands, but like the seafaring man in the desert, use them as your guide and you will reach your destiny.”

Carl Schurz
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>ix</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xi</td>
</tr>
<tr>
<td><strong>CHAPTER</strong></td>
<td></td>
</tr>
<tr>
<td><strong>I. INTRODUCTION</strong></td>
<td>1</td>
</tr>
<tr>
<td>Background of the Study</td>
<td>3</td>
</tr>
<tr>
<td>Assumptions</td>
<td>8</td>
</tr>
<tr>
<td>Purpose Statement</td>
<td>8</td>
</tr>
<tr>
<td>Practitioner Research</td>
<td>9</td>
</tr>
<tr>
<td>Research Questions</td>
<td>12</td>
</tr>
<tr>
<td>Significance</td>
<td>12</td>
</tr>
<tr>
<td>Theoretical Framework</td>
<td>13</td>
</tr>
<tr>
<td>Constructivism</td>
<td>14</td>
</tr>
<tr>
<td>Lesh’s Multiple Representation Translation Model</td>
<td>18</td>
</tr>
<tr>
<td>Kinesthetic Movement</td>
<td>20</td>
</tr>
<tr>
<td>Limitations of the Study</td>
<td>21</td>
</tr>
<tr>
<td>Definitions of Terms</td>
<td>22</td>
</tr>
<tr>
<td>Summary</td>
<td>24</td>
</tr>
<tr>
<td><strong>II. REVIEW OF THE LITERATURE</strong></td>
<td>25</td>
</tr>
<tr>
<td>Current Trends in Mathematics Education</td>
<td>25</td>
</tr>
<tr>
<td>Constructivism</td>
<td>30</td>
</tr>
<tr>
<td>Cognitive Constructivism</td>
<td>32</td>
</tr>
<tr>
<td>Social Constructivism</td>
<td>34</td>
</tr>
<tr>
<td>Lesh’s Translation Model Between Mathematical Representations</td>
<td>36</td>
</tr>
<tr>
<td>Kinesthetic Movement</td>
<td>39</td>
</tr>
<tr>
<td>Gestures and Embodied Cognition</td>
<td>39</td>
</tr>
<tr>
<td>Gesturing and Communication</td>
<td>40</td>
</tr>
<tr>
<td>Gesturing as a Mental Model</td>
<td>42</td>
</tr>
<tr>
<td>Linked Representations</td>
<td>44</td>
</tr>
<tr>
<td>Other Forms of Gesturing</td>
<td>47</td>
</tr>
<tr>
<td>Summary of Gesturing Literature</td>
<td>47</td>
</tr>
<tr>
<td>Linear Functions</td>
<td>48</td>
</tr>
<tr>
<td>Students’ Difficulty With Learning Algebra</td>
<td>49</td>
</tr>
<tr>
<td>Difficulty Learning Linear Equations</td>
<td>52</td>
</tr>
</tbody>
</table>
IV. ANALYSIS OF THE FINDINGS ................................................................. 79

Background Data: Learning Style Inventory and Pre-Test ........................................ 80
Learning Style Inventory .................................................................................... 80
Pre-Test.................................................................................................................. 81

Research Question 1 ......................................................................................... 85
Theme 1: Students Used Their Bodies as a Communication Tool ......................... 85
Theme 2: Students Felt Energized and Enjoyed Using Kinesthetic Movement ........ 98

Theme 3: Students Used Kinesthetic Movement as a Tool to Recall and Remember ........................................................................................................... 104

Theme 4: Kinesthetic Movement Was a Useful Visualization Tool ..................... 106
Theme 5: Students Used Their Bodies Subconsciously ....................................... 107

Research Question 2 ......................................................................................... 111
Theme 6: Kinesthetic Movement Can Be Used as a Tool for Developing Conceptual Understanding .......................................................... 116
Theme 7: Kinesthetic Movement Helped Students Form Authentic Vocabulary .................................................................................. 118
Theme 8: Kinesthetic Movement Helped Students Translate Among Representations ................................................................. 119
Major Findings ........................................................................................................................................................................... 123
Summary of Themes ..................................................................................................................................................................... 125

V. DISCUSSION AND IMPLICATIONS ........................................................................................................................................ 127
Discussion of Themes: Research Question One ......................................................................................................................... 129
Students Used Their Bodies as a Communication Tool ........................................................................................................ 130
Students Feel Energized and Enjoy Kinesthetic Movement .................................................................................................. 131
Students Used Kinesthetic Movement as a Tool to Recall and Remember ............................................................................... 132
Kinesthetic Movement Was a Useful Visualization Tool ......................................................................................................... 133
Students Used Their Bodies Subconsciously .......................................................................................................................... 134
Discussion of Themes: Research Question Two ......................................................................................................................... 135
Kinesthetic Movement Can Be Used as A Tool for Developing Conceptual Understanding ....................................................... 135
Kinesthetic Movement Helped Students Construct Authentic Vocabulary .................................................................................. 136
Kinesthetic Movement Helped Students Translate Among Representations ................................................................................ 137
Recommendations ........................................................................................................................................................................ 139
Recommendations for Instruction ........................................................................................................................................... 139
Recommendations for Assessment ............................................................................................................................................ 141
Recommendations for Future Research ..................................................................................................................................... 144
Reflection Through Both the Lens of Researcher and Teacher .................................................................................................. 146
Summary of Chapter 5 .................................................................................................................................................................... 151

APPENDICES .................................................................................................................................................................................... 152
APPENDIX A. LETTER OF CONSENT ....................................................................................................................................... 153
APPENDIX B. LEARNING STYLE INVENTORY .............................................................................................................................. 159
APPENDIX C. PRE-TEST AND POST-TEST ...................................................................................................................................... 164
APPENDIX D. ATTITUDE SURVEY ............................................................................................................................................... 166
APPENDIX E. INSTRUCTIONAL ACTIVITIES ............................................................................................................................... 168
APPENDIX F. RUBRIC FOR PRESENTATIONS ............................................................................................................................. 176
APPENDIX G. ASSESSMENT OF DIFFERENT REPRESENTATIONS ................................................................................................ 178
APPENDIX H. QUIZ ........................................................................................................................................................................... 180

REFERENCES ..................................................................................................................................................................................... 182
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Images of kinesthetic depictions of graphs</td>
<td>3</td>
</tr>
<tr>
<td>2. The Vital Practitioner Research Cycle (EdFutures, 2013)</td>
<td>11</td>
</tr>
<tr>
<td>3. Lesh, Post, and Behr’s (1987) Model of Translation</td>
<td>19</td>
</tr>
<tr>
<td>4. Tower of Hanoi (2017)</td>
<td>41</td>
</tr>
<tr>
<td>5. Learning style inventory results</td>
<td>81</td>
</tr>
<tr>
<td>6. Student Body Movement Demonstrated Graph of ( y = -4 )</td>
<td>86</td>
</tr>
<tr>
<td>7. Group 2 poster for presentation</td>
<td>89</td>
</tr>
<tr>
<td>8. Group 3 poster representing three ways function can be represented</td>
<td>90</td>
</tr>
<tr>
<td>9. Group 3 representing the three different graphs with their own heights</td>
<td>90</td>
</tr>
<tr>
<td>10. Student used their bodies to show what a zero slope looks like</td>
<td>93</td>
</tr>
<tr>
<td>11. Graph images of functions students were representing with their bodies</td>
<td>94</td>
</tr>
<tr>
<td>12. Student body movements representing parts of the graph</td>
<td>96</td>
</tr>
<tr>
<td>13. Student body movements representing a graph that is steeper</td>
<td>98</td>
</tr>
<tr>
<td>14. Students explaining the Graph of ( y = 2x -3 )</td>
<td>101</td>
</tr>
<tr>
<td>15. Does kinesthetic movement help you learn and why</td>
<td>107</td>
</tr>
<tr>
<td>16. Student responses on whether they used their mind and/or their bodies when working on the summative assessment</td>
<td>108</td>
</tr>
<tr>
<td>17. Data representing students’ form of communication during presentations</td>
<td>113</td>
</tr>
<tr>
<td>18. Data representing students’ form of communication during one-on-one interviews</td>
<td>114</td>
</tr>
<tr>
<td>19. Data representing students’ form of communication after assessment interviews</td>
<td>114</td>
</tr>
</tbody>
</table>
20. Data representing students’ form of communication during presentations ....... 115
21. Data representing students’ form of communication during exit interviews...... 115
22. Student A represented a $y = x$ a positive function. Student B represented a $y = -x$ a negative function ................................................................. 117
23. Student A represented $x = 8$; Student moved away from origin to (8,0) on the graph. Student B represented $y = 8$. Student moved back to the origin (0,0) ........................................................................................................ 118
24. Example of students using step-by-step process........................................... 121
25. What is the most beneficial when learning linear functions............................ 124
26. Does kinesthetic movement help you learn and why..................................... 125
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Behaviorist Classroom Compared to a Constructivist Classroom (Taylor, 2015)</td>
<td>51</td>
</tr>
<tr>
<td>2. Practitioner Research Cycle for This Study</td>
<td>59</td>
</tr>
<tr>
<td>3. Outline of Data Sources Answering Which Research Question</td>
<td>64</td>
</tr>
<tr>
<td>4. Student Responses and What Vocabulary They Should Have Used</td>
<td>82</td>
</tr>
<tr>
<td>5. Student Responses Answering How Equations are Similar and/or Different?</td>
<td>84</td>
</tr>
<tr>
<td>6. Examples of Student Comments to What They Felt When Using Kinesthetic Movement in Class</td>
<td>103</td>
</tr>
<tr>
<td>7. Student Translation Between and Among the Different Representations of Linear Functions</td>
<td>122</td>
</tr>
<tr>
<td>8. Themes That Evolved From Data Collection</td>
<td>126</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

Experiences in the classroom can stay with individuals throughout their lifetime. Some are beneficial while others are not; some are educational, and some are socially rewarding. In spite of our shared classroom experiences, there are a variety of solutions of how best to teach and learn. Politicians, superintendents, curriculum specialists, principals, teachers, and parents are some whose voices are heard in a classroom. With so many stakeholders, students’ perspectives can be marginalized.

Numerous studies document effective ways for students to learn and succeed in mathematics from theory and research on teaching (Ball & Hoover, 2014; Boaler, 2000, 2010, 2016; Borman, 2005; Cooney, 1988; Dunson, 2000; Kleibard, 2004; Lester, 2007). Through teachers’ experiences and policymaker’s recommendations, such as state and national leaders, there is ample research on how best to teach and how children learn mathematics (Ball, 1988; Cobb & Yackel, 1996; Kamii, 1989, 2015; Kilpatrick, 1992; Lester, 2007).

There are several qualitative studies on how students learn from students’ perspectives. These studies have investigated when students enjoy a class, their feelings on effective instruction, the level of difficulty of the class, if they felt that the class was valuable, their views ranging from improving the school environment to bullying to classroom setting (Gunter & Thomson, 2007; McCarter & Woolner, 2011). In mathematics, there are many studies on how students learn from their perspective focusing on what a student is thinking when they are trying to solve a problem or explain

However, there are particular topics in mathematics education that have not been investigated from a students’ perspective. One such area is using kinesthetic movement to learn linear functions. The majority of student research with kinesthetic movement occurs in special education and primary classroom settings (Batt, 2009; Beaudoin & Johnston, 2011; Goldin-Meadow, 2001; Griss, 1994). My review of the literature has found a gap and an opportunity to study what students’ perceptions were when learning linear functions when being taught through using their body.

This dissertation examined how a class of students learned to graph linear functions through a kinesthetic activity. In the activity, students used their bodies to create graphs as shown in Figure 1. By teaching mathematics through multiple representations, including kinesthetic movements, this study does contribute to the literature on how to learn mathematics in a conceptual and meaningful way.
Figure 1. Images of kinesthetic depictions of graphs

Background of the Study

Mathematics is not the learning of rote procedures, or memorizing of basic facts. According to The National Council of Teachers of Mathematics (NCTM), success in mathematics is the development of mathematical power. “Mathematical power is the ability to explore, conjecture and reason logically; to solve non-routine problems; to communicate about and through mathematics; and connect ideas within mathematics and between mathematics and other intellectual activities” (NCTM, 1989, p. 1). Many math educators in today’s classrooms are trying to guide their students to this “mathematical power,” through implementing best practices and research (NCTM, 1989, p. 1).
In order to deepen understanding of mathematics, a teacher should emphasize problem solving and provide engaging activities for students that promote realistic application and higher order thinking that is meaningful to them (NCTM, 2000, 2006). NCTM believes the best way for all students to develop “mathematical power” is through creating a curriculum in a classroom setting that is different from students just looking for the right answer and staying at a procedural level of learning. Teachers can encourage a conceptual level of understanding by having classroom experiences for them that relate activities to one another (NCTM, 2000). In addition to this important component of learning, having a focused curriculum also helps students move to a higher conceptual level. In 2006, NCTM created the *Curriculum Focal Points* to help guide teachers in determining what areas of mathematics to emphasize. The goal of the *Focal Points* was for teachers to “focus on a small number of key areas of emphasis, so that students [will] gain extended experience with core concepts and skills. Such experience[s] can facilitate deep understanding, mathematical fluency, and an ability to generalize” (NCTM, 2006, p. 5).

Individuals who can understand mathematics and can successfully complete mathematical tasks improve their chances of advancing in society (NCTM, 2000). “All students should have the opportunity and support necessary to learn significant mathematics with depth and understanding” (NCTM, 2000, p. 50). The *Curriculum Focal Points* give “descriptions of the most significant mathematical concepts and skills at each grade level” (NCTM, 2006, p. 1). The Common Core State Standards of Mathematics (CCSSM) include the Standards for Mathematical Practice that describe
expectations for each grade level that are needed for students to be successful in mathematics. The CCSSM standards have their background in the NCTM process standards and the National Research Council’s (NRC, 2001) report *Adding It Up*. The CCSSM incorporates the following process standards from NCTM: problem solving, reasoning and proof, communication, representation, and connections. The NRC integrates the strands of mathematical proficiency, by including adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (National Governors Association, 2010; NCTM, 2000). The CCSM Practices are:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning. (National Governors Association, 2010, p. 1)

One perspective of teaching mathematics is having teachers create a classroom where “students engage in purposeful activities that grow out of problem situations, requiring reasoning and creative thinking, gathering and applying information, discovering, inventing and communicating ideas, and testing those ideas through critical
reflection and argumentation” (Thompson, 1992, p. 128). For this study, students were engaged in purposeful activities and had the opportunity to use graphing calculators, visual representations with graph paper, written math equations, charts, environments where conversations were encouraged, and the tools to recreate the linear graphs with their bodies using kinesthetic movement. Kinesthetic movement for this particular study is being defined as movement where there is academic understanding and purpose. This activity specifically created opportunities for students to be reflective about the meaning of linear functions; students completed kinesthetic movement exercises by creating graphs with their bodies. Figure 1 is an illustration of how a student used their body kinesthetically to create the linear graphs.

Students represented the functions given; they had the opportunity to engage in reflection, and share their learning process. For example, when the equation was changed from $y = 2x + 3$ to $y = 2x + 4$, the goal was for students to demonstrate how the graphs differed by using their bodies, pictures, or words. By students comparing their graphs kinesthetically, verbally, or pictorially to others, this study examined how students gained feedback, reflected on their graphs, and participated in mathematical conversations.

When we look at some classrooms, we see students sitting at desks, staring out windows, doodling, and not on task. There seems to be little engagement with students. The teachers are disseminating information; students are writing it down (S. Wilson, 2003). Do we ever think why our students are not remembering what they learned the day before? Some teachers call our current curriculum “a mile high and an inch deep” (NRC, 1989, p. 15). This study created a learning environment that was conducive to
middle school students’ active bodies with a teaching method where students were “constructors” of their own knowledge. Research by Carolyn Kieran (1993, 1996) found that students graph linear functions by rote memorization: “plugging the numbers” into a formal chart, then plotting the points on the graph. If students use technology, the student “plugs” the function into the graphing calculator and a graph appears. The student magically gets an answer with no explanation. The National Research Council (1989) finds that most students do not have much success by listening and imitating. The lack of students’ success is based upon the fact that they are not relating to how the material is being taught and they are not actively involved in constructing their own knowledge (NRC, 1989, p. 6). When a student learns concepts with understanding, they gain a foundation for new knowledge to be processed so that they will have the tools to solve future problems that are unfamiliar (NRC, 2001).

By including kinesthetic movement within the mathematics classroom, students are engaging in multiple forms of representation that allow for multiple forms of expression. Mathematics is a “collection of languages” (Kaput, 1989, p. 167); these languages include symbols, graphs, abstractions, rules, definitions, and charts. With so many different “languages,” it is no wonder that students have difficulty expressing them. Both NCTM standards and CCSSM have included the use of representations. The NCTM (2000) representation standard states:

Instructional programs from pre-kindergarten through grade 12 should enable all students to create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical
representations to solve problems; and to use representations to model and interpret physical, social, and mathematical phenomena. (p. 67)

In the CCSSM the emphasis is for students to make connections between the multiple representations of mathematical concepts by communicating and justifying their answers (National Governors Association, 2010). Therefore, math classes should be moving from having only one answer that is right or wrong to students having conceptual understanding through the use of different representations to show their understanding of a concept.

**Assumptions**

In keeping with current mathematics education research and the NCTM standards, a major assumption of this study is that children have varied learning styles, strengths, experiences, and perspectives; and these should be considered when making educational decisions. According to Adams and LaFramenta (2014), educators should not categorize children by their intelligence, but rather educators should provide as many different learning opportunities that incorporate all varied learning styles. Another assumption is that students want to make meaning of their learning in the classroom whether it is through their experiences or social interactions with others in the classroom.

**Purpose Statement**

The purpose of this research is to describe middle school algebra students’ experiences of learning linear functions through kinesthetic movement. Kinesthetic movement for this particular study is being defined as movement where there is academic understanding and purpose. This is known as purposeful movement (Beaudoin &
Unrelated movement is defined as movement performed by students not linked to the educational content being taught. Some examples of unrelated movement are a student jumping rope while doing math facts, or a student standing and solving a problem on the black board.

Because this research focused on the mental constructions made by students as they attempted to make sense of mathematics kinesthetically, it was grounded in the philosophical tenets of cognitive constructivism of Piaget, sociocultural constructivism of Vygotsky, kinesthetic movement, and Lesh’s theory of Multiple Representation Translation Model in mathematics with respect to linear functions. The research utilized a case study design to find common themes as students engaged in activities. Because I was both teacher and researcher, practitioner research methods were utilized.

**Practitioner Research**

As both the researcher and teacher I tried to synthesize commonalities and differences shared by groups in an attempt to understand how students constructed meaning. By using practitioner research, I hoped to enhance my teaching and have better outcomes for my students. As Kathy Richardson (2014) wrote for the Math Perspective Teacher Development Center,

> Those of us who find teaching a rewarding and challenging profession know we will never have it all figured out. We always have something to ponder as we work to create the best educational experiences we can for our students. (para. 4)

Practitioner research is meant to provide a contribution to practice rather than theory (Campbell, 2013). Its purpose is to improve teaching experiences for teachers and
foster better outcomes for their students (Zinskie & Rea, 2016). Practitioner research has been represented in different ways. Some of these terms are action research, teacher research, practitioner inquiry, and practitioner research. The following quotations are different definitions of practitioner research in education. J. McLeod (1999) defined practitioner research as “research carried out by practitioners for the purpose of advancing their own practice” (p. 8). Cochran-Smith and Lytle (1990) originally referred to it as teacher research defined as “systematic and intentionally inquiry by teachers” (p. 3). Later they broadened their definition “to encompass all forms of practitioner inquiry that involve systematic, intentional, and self-critical inquiry about one’s work” (1999, p. 22). They further stated practitioner-researchers are researchers who “in addition to documenting classroom practice and students’ learning, they also systematically document from the inside perspective their own questions, interpretive frameworks, changes in views over time, dilemmas, and recurring themes” (Cochran-Smith & Lytle, 2009, p. 44). Drennon (1994) wrote,

Practitioner inquiry is not field-testing the ideas of others, nor is it simply implementing a new strategy that one is already convinced with work. Instead, it is a process of generating ideas through reflection and examination of practice, and exploring the implications of those ideas within the practitioner’s setting. (p. 3)

Figure 2 shows the process of practitioner research.
Practitioner research allows a teacher to take an outside-in view of their profession. Practitioner research differs from other research because it is shared, invites peer review, and leads to professional knowledge base. This analysis should lead to better outcomes for the students and will foster continued improvement in the teacher’s knowledge and other educators for more successful outcomes. Practitioner research was used because this study will be shared and peer reviewed, and will contribute to the professional knowledge base of my teaching and others. Practitioner research is meant to answer everyday teacher questions that arise out of everyday practice in the classroom through a systematic and intentional inquiry (Cochran-Smith & Lytle, 1999).
The findings are presented as a case study. Yin (2003a) explained that there is a distinctive need for case studies in the classroom because teachers have the desire to understand what is happening in the classroom and the ability to contribute to the research. Through listening to students and examining how they communicated using a sequence of instructional tasks designed intentionally to evoke kinesthetic movement, I observed, collected data, and elaborated on their perceptions of their understanding of learning to graph linear functions.

**Research Questions**

To gain insight from students the following two questions guided the research and enabled common themes to emerge:

- What are students’ perceptions about learning linear functions with kinesthetic movement?
- How do students (if they do) translate between and among mathematical representations through kinesthetic movement (kinesthetic-verbal, kinesthetic-symbols, kinesthetic-real life, kinesthetic-graphical, kinesthetic-oral)?

**Significance**

Linear functions are important foci for all students who are learning algebra. According to the Ohio Academic Standards for Mathematics (ODE, 2014), every student is required to take Algebra II or an equivalent course to earn a high school diploma. Accordingly, as part of this coursework all students will have learned linear functions by the time they graduate. Within the algebra curriculum, students need a deep
understanding of linear equations and functions (National Governors Association, 2010). Although there is a significant amount of research on linear functions and potentially effective interventions (Kieran, 1996; K. Richardson, 2014; van Ameron, 2002), there is limited research on kinesthetic movement in the middle grades and even less research on student perception and understanding with kinesthetic movement while learning to graph linear functions (Abrahamson, 2004; Batt, 2009; Beaudoin & Johnston, 2011; Gross, 2011; Montgomery, 2007; Saylor, 2004). Information gained from this study could be transferable to other topics and strategies.

Because the topic of linear functions is such an integral topic in grades 6–12 mathematics curriculum, how best to teach this topic is always under review within this dissertation. Answers to the research questions will provide a better understanding of how students learn to graph linear functions and the possible use of multiple representations. From a different perspective, the study results will hopefully illuminate students’ perceptions on kinesthetic movement in the middle grades. My goal is to inform practice as understanding of how students learn, and I hope that this study contributes to the current literature on best practices in algebra. The results will also add to the research on ways to teach graphing linear functions.

Theoretical Framework

The theoretical framework for this study included the following three constructs: constructivism (Piaget & Vygotsky), kinesthetic movement, and Lesh’s Multiple Representation Translation Model. The following section gives the rationale for why each is included in this study.
Constructivism

Two of the components for my theoretical framework include Piagetian Cognitive Theory and Vygotsky’s Socialcultural Theory.

Piagetian cognitive theory. The first philosophical foundation of this study was informed by the work of Jean Piaget. Piaget’s theory emphasized the role of memory in cognitive development. An important part to children remembering was through the use of their experiences that surrounded the child. He believed students’ learning took place through their use of symbols and signs (Piaget & Inhelder, 1969). Because the purpose of this study was examining how students constructed knowledge through the use of movement, I found his theory related to my investigation and would be useful as a theoretical framework to guide this study. His theory allows us to understand the cognitive development of students’ learning.

Piaget’s theory of student learning is based on adaptation. Each of us forms mental structures defined as schema. These schemas are mental models that reflect what we know. Through processes known as assimilation and accommodation, these schemas may change based on our experiences and maturity (D. McLeod, 1992). Assimilation is the filtering or modification of the input, and it is the interpretation of events in terms of the existing schemas. On the contrary, accommodation is the modification and changing of the internal schemas to fit reality (Piaget, 1972). A key principle for Piaget was equilibrium. Everyone tries to make sense of their own experiences by matching new information with their own existing ideas. Accommodation is the modification and amendment of thought. However, when a concept does not fit with what we know or
within any of our existing schemas our mind is now in a state of disequilibrium; our cognitive structures are reflecting the information on how to fit this new concept into existing schema. Piaget believed that in order to have learning take place, there must be disequilibrium (Cobb, Yackel, & Woods, 1992; Steffe & Kieran, 1994).

Piaget also believed that “children think differently than adults” (Tizard & Hughes, 1984, p. 101). What we see is not what children see; their interpretations of the world are different than ours. Because student experiences are not the same as adults, teachers need to be mindful of this developmental gap. Papert (1999) explained this concept by quoting a story from Piaget’s research.

Piaget: What makes the wind?

Julia (age 5): The trees.

Piaget: How do you know?

Julia: I saw them waving their arms.

Piaget: How does that make the wind?

Julia: Like this (waving her hand in front of Piaget’s face). Only they are bigger and there are lots of trees.

Piaget: What makes the wind on the ocean?

Julia: It blows there from the land. No, it’s the waves.

“When children are given material, they relate it to something they know to reach understanding, however if the concept is brought to children too quickly, they cannot fully learn and accommodate the new information” (Papert, 1999, para. 7). One important generalization of Piagetian theory is the role of the teacher. Teachers are a
critical element in allowing students to have the freedom to explore intellectually. Teachers provide the proper environment for explorations, self-guided discovery, and communication of their knowledge. By teachers providing these opportunities, students will experience situations in which they can assimilate and accommodate new information. This study used the tenets of Piaget’s theory. Disequilibrium was introduced in this study through kinesthetic movement. This study examined if students revised their schema as they assimilated and accommodated their new knowledge of linear functions.

**Vygotsky’s sociocultural theory.** Throughout this study, students and teacher were interacting and working together in a classroom setting. Vygotsky’s social constructivism is relevant because he believes children learn through social factors that contribute to their knowing. He believed learning is a social process, and human intelligence is created in society and or culture. His major theme of social interaction is the key component in the development of cognition (Vygotsky, 1978). There are two major aspects to his theory. First, interaction with others:

> Every function in the child’s cultural development appears twice: first, on the social level, and later, on the individual level; between people (inter-psychological) and then inside the child (intra-psychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relationships between individuals. (Vygotsky, 1978, p. 57)
The second aspect of Vygotsky’s theory is the “zone of proximal development” (ZPD). Vygotsky explained this as “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers” (p. 86). He further explained the ZPD as “what a child can do with assistance today she will be able to do by herself tomorrow” (p. 87). Vygotsky believed this is all possible because of the social interaction happening while students learn from their peers and teachers and the concept being intertwined with their previous experiences.

An example of Vygotsky’s ZPD (1978) is a student may know a formula in math; however, they do not know how to apply it. With the assistance of the teacher or peers, the student learns to apply the formula.

Vygotsky believed students make sense of their learning experiences in the social setting of a classroom. Teachers play a key role by facilitating and having opportunities for students to be active participants. Through the social interactions in the classroom, students construct concrete understanding. For this study, Vygotsky’s theory (1978) is important in understanding how the interaction with students, peers, and teachers develop the translations between the different representations of linear functions. I want to know if and what social situations helped the students translate between and among the different representations of linear functions. Did students move to an abstract understanding by classroom negotiation? Did watching their classmates perform the kinesthetic movement have an impact on their social learning? Did the interaction with
A student’s understanding cannot just be looked at alone; we have to include the external social world. In summarizing, Vygotsky believed that our social experiences shape our thinking and interpretation of the world. Individual cognition occurs in a social situation. They cannot be separated; the group helps achieve construction of knowledge.

**Lesh’s Multiple Representation Translation Model**

Because this study investigated the role of kinesthetic movement, Lesh’s translation model is a useful theoretical line of study for researching the translations among representations. The term mathematics representation refers to both the process and product; in other words, the act of capturing a mathematical concept or relationship in some form and to the form itself (Lesh, Post, & Behr, 1987). Representations apply to processes and products that are observable externally, as well as to those that occur internally in the minds of people doing mathematics (NCTM, 2000). “Representations are powerful tools, which connect with both the image and word. In mathematics, students can engage and use a variety of representations in order to demonstrate and share their mathematical thinking” (Totolo, 2014, para. 1).

Lesh’s Mathematical Representational Translation Model with learning linear functions is based on “built in” relationships among static pictures, manipulatives, written symbols, real life events, and spoken language. Lesh’s model (Figure 3) shows how math topics can be represented in different modes. Students who can represent mathematical ideas in multiple ways, and have the ability to make connections among the
different embodiments will gain deep understanding of the math topic being taught (Cramer, 2003). Figure 3 shows how the five representations are interconnected (Lesh et al., 1987).

Figure 3. Lesh, Post, and Behr’s (1987) Model of Translation

Transformation occurs when the students see the different representations as connected rather than as their own entity. The five individual representations help the learners understand a concept in one view (Lesh et al., 1987). When the learner is given the opportunity to relate the one representation to another representation, their learning evolves into their own understanding and they make translations and transformation to the different representations. When students are given continual use of different
representations, the student becomes more flexible with all the representations, and when the student is given a particular situation the student will choose the most convenient representation for the task at hand (Lesh et al., 1987).

Bransford, Brown, and Cocking (2000) also found the importance of relating the different representations. They found that teachers who use different strategies to model concepts help with student understanding. When the abstract thought can be presented with different representations, students have more success transferring their current knowledge to the next level because students making connections between the different representations.

**Kinesthetic Movement**

The third construct of the theoretical framework is kinesthetic movement. This study investigated kinesthetic movements by examining student gestures and arm movements when students were communicating their understanding of linear functions. Research among neuroscientists postulate “that mental functions cannot be fully understood without reference to the physical body and the environment in which they are experienced” (Schmalzl, Crane-Godreau, & Payne, 2014).

Roth (2002) noted, “There exists very little educational research concerned with the role of gestures in learning and teaching, particularly in the subject areas that have been characterized as dealing with abstract matters such as science and mathematics” (p. 365). Gesture researchers are suggesting there is a connection between the mind and body movements when learning (Alibali & Nathan, 2012). Goldin-Meadow (1999), a leading researcher with gesture movements, found students who have special needs or
who cannot communicate verbally used their body when communicating. There has been very little research with kinesthetic movement in a middle school math classroom (Roth, 2002).

Because this study was interested in students’ use of their body movements and gestures to understand the translations among the different representations of linear functions, data were collected in a variety of ways for kinesthetic movements. Data collected were derived from noting when gestures matched the verbal communication, and when the words and body movements were inconsistent. Data were also collected that showed when students solely used their bodies to communicate with and without verbal expression.

**Limitations of the Study**

This study is designed as practitioner research because I saw a need for improving student learning with linear functions. Because of my dual role as the researcher and teacher, there may be conflicts of interest. I was both teacher and researcher, which may be perceived as a limitation. As the teacher, it is my passion for student success that drives this study; as the researcher for this study, I observed, listened, understood, and interpreted student learning with kinesthetic movement when graphing linear functions to help not only improve my teaching but further the field. Glesne (2006) posited this as the “researcher as the learner” (p. 46).

Because the students may or may not want to please the researcher, there may be some interview questions that may need to be reviewed. The motivation of the students may also play a factor in determining the students’ level of success because of the
random selection of students in the classroom. Students’ desire to please the
teacher/researcher may cause some discrepancies with the amount of trust reported by
students in their conversations. Students may also feel pressure to be a part of the study.
Methodological triangulation was used in collecting data from the students with the
purpose of avoiding bias.

Another limitation is researcher bias. I believe kinesthetic movement plays a role
in learning. By identifying my assumptions, and offering many methods of data
collection with member checking, I can eliminate bias. Through triangulation of method
used, member checking by students involved in the study, and rigorous analysis and
synthesis of data, I ensure that the results collected reflected what the students expressed
about learning linear functions as opposed to my own beliefs.

Definitions of Terms

The following operational definitions were used in this study.

**Embodied cognition:** Learning is a result of social and physical interactions with
our environment, with the meaning derived from these actions (Gibbs, 2006).

**External representations:** The way by which mathematical ideas could be
communicated and they are presented as physical objects, pictures, spoken language, or
written symbols in translating knowledge (Lesh et al., 1987).

**Gestures:** Hand movements used during talk (Goldin-Meadow & Alibali, 2013a);
spontaneous communication of hand movements that accompany speech (McNeil, 1992);
and a movement of part of the body, especially an arm, hand, or the head, to express an
idea or meaning. The gestures for this study were looking at hand and arm movements when using the body to communicate the graph.

**Internal representation (Mathematical):** An idea, image, or cognitive process formed in one’s mind—cannot be seen or observed by another (Krause, 2001).

“Cognitive or mental models, schemas, concepts, conceptions, and mental objects which are illusive and not directly observed” (Janvier, Girardon, & Morand, 1993, p. 81).

**Kinesthetic movement:** Movement with the body during class time when completing given tasks by the teacher. Kinesthetic movement can be purposeful movement or unrelated movement (Beaudoin & Johnston, 2011).

**Linear equation:** An algebraic equation of the form \( y = mx + b \), involving only a constant and a first order (linear) term, where \( m \) is the slope and \( b \) is the \( y \)-intercept. The above form is aptly known as slope intercept form; alternatively, linear functions can be written in a number of other forms including standard form, intercept form, and so forth. Occasionally, the above is called a “linear equation of two variables, “where \( y \) and \( x \) are the variables such as \( x = 3 \) and \( y = 2 \) are linear equations of a single variable, and \( 2x + 4y + 5z = 6 = 0 \) is an example of linear equation with three variables (Stover, 2015).

**Practitioner research:** Research in this context is defined as any form of systematic enquiry whose design, methods, analysis, and interpretation are open to peer review (Cochran-Smith & Lytle, 1990). (Teacher research and action research are terms that other researchers use that correspond to practitioner research.)
**Purposeful movement:** Kinesthetic movement that is related to the learning at hand, for example, describing a square by showing what a square looks like with your hands (Beaudoin & Johnston, 2011).

**Translation:** The movement between different representations (Lesh et al., 1987).

**Unrelated movement:** Kinesthetic movement that does not pertain to the learning at hand; for example, jumping rope while reciting multiplication tables (Beaudoin & Johnston, 2011).

**Summary**

This chapter introduced the reader to the purpose of the study, its significance, and a brief background of kinesthetic movement in the classroom and how it can relate to the study of linear functions. Essential to this study is the theoretical framework of cognitive and social constructivism, multiple representations, and kinesthetic movement. Above all, this study delves into listening to the students’ verbal and nonverbal communication of their understanding of linear functions.
CHAPTER II

REVIEW OF THE LITERATURE

This study focused on listening to students as they described their understanding of graphing functions kinesthetically and to describe the process they went through. Whereas there is no lack of research in mathematics education, teaching, learning, kinesthetic movement, and linear functions, there is a lack of research on the integration of these concepts: graphing linear functions with different modalities including kinesthetic movement (Nemirovsky, Tierney, & Wright, 1998).

This chapter is divided into five major sections. The first section gives the background of current trends in mathematics education and gives a historical overview of the NCTM Standards and CCSSM that impact the content and processes of teaching linear functions. The second and third sections provide research on constructivism and Lesh’s Model of Translation with mathematical representations of linear functions. Because the field of constructivism is so large, I only incorporate the aspects of constructivism that relate to my research questions: both Piaget’s theory focusing on the mental construction of knowledge by the individual and Vygotsky’s theory of learning with the emphasis on the social interaction of student learning. The fourth section draws upon literature from the field of kinesthetic movement and gestures when referring to linear functions. The fifth section summarizes research on solving linear functions.

Current Trends in Mathematics Education

Working to understand mathematical topics in a classroom setting is a common experience to which almost everyone can relate. Students find themselves in classrooms
where teachers review homework and then ask if students have any questions. The teacher answers the questions followed by going through new material and examples, and finally homework is assigned. If there is class time left, students start their homework (S. Wilson, 2003). The question becomes, “Is this the most effective way to teach mathematics?” Where is the understanding of the concept being taught? Where are connections between new ideas and student experiences encouraged?

Within the behaviorist tradition, the learner is presented with a procedure, which tells the student how to perform certain operations on figures, and when done correctly, a proper result would be achieved (Skinner, 1984). However, no rationale was given for algorithms, so that when the learner had obtained the result, they would not necessarily have understood the answer or how it could be used. To discover such detail, the student was obliged to rely entirely on “the book,” or more frequently on the teacher. “As he [student] began in the dark, so he continued; and the results of his calculation seemed to be obtained by some magical operation rather than by inductions of reason” (S. Wilson, 2003, p. 9). The Behaviorist tradition perpetuated an overemphasis on procedural knowledge (Battista, 1993; Davis, 1989; S. Wilson, 2003).

Professional organizations are making a call for abandoning curricula that promote thinking about mathematics as a rigid system of externally dictated rules governed by standards of accuracy, speed, and memory. A mathematics curriculum that emphasizes computation and algorithms is like a writing curriculum that emphasizes grammar and proper
spelling; both put the cart before the horse. There is no place in a proper

curriculum for mindless mimicry mathematics. (NRC, 1989, p. 44)

The Curriculum Focal Points (NCTM, 2006) advocate for procedural knowledge and conceptual understanding to be intertwined. The procedures need to be more than something that is memorized; the algorithm needs meaning behind it. The CCSSM (2010) indicates that algorithms, no matter at what level of mathematics, need to be used for efficiency, and students should have the understanding of how and why the algorithm works. In Adding it Up, the National Research Council (2001) cites research claiming that the study of algebra is more difficult because students are executing operations rather than seeing relationships being represented. Current reform in mathematics classrooms focuses on teachers giving their students the experiences where concepts are interrelated. Through these experiences students will be able to solve complex problems through reading, writing, and discussion. Students will learn to value mathematics whether it is in enterprise, human affairs, or habits of mind (National Governors Association, 2010). Students then can feel comfortable to be able to explore, guess, make errors, and learn from those errors in order to gain the confidence they need in a more guided setting (NCTM, 2000).

Today’s students live in a technological world where mathematics plays a significant role in their daily lives. Individuals who can understand mathematics and can successfully complete mathematical tasks improve their chances of advancing in society (NCTM, 2000). “All students should have the opportunity and support necessary to learn significant mathematics with depth and understanding” (NCTM, 2000, p. 50). The
NCTM established the *Principles and Standards for School Mathematics* in the 1989 NCTM Standards, which included 13 curriculum standards addressing both content and process. One theme common to the NCTM Standards and to the recent changes in mathematics education is that students can make sense of mathematics if given the reasoning behind the concepts and if they can see relationships. A second document released by the National Council of Teachers of Mathematics was the *Professional Standards for Teaching Mathematics* (1991). This set of standards presented a vision of what teaching should entail to support the changes in curriculum set out in the *Curriculum and Evaluation Standards*. This document spells out what “teachers need to know to teach toward new goals for mathematics education and how teaching should be evaluated for the purpose of improvement” (NCTM, 1991, p. vii). In 1995, NCTM released the *Assessment Standards for Teaching Mathematics*. The purpose of assessments was to support student learning. The document claimed that assessments needed to provide useful information for teachers, students, parents, and community members. Through analysis of formative and summative assessments, changes would be made to instruction that would benefit students’ learning and the vision of NCTM’s vision of school mathematics (NCTM, 1995). In April 2000, NCTM released its *Principles and Standards for School Mathematics*. This document updated the 1989 Curriculum and Evaluation Standards and included 10 standards. The first five standards are content standards whereas the last five are process standards. The five process standards are:

- Problem Solving
Reasoning and Proof
Communication
Connections
Representations

NCTM supports the implementation of the Common Core State Standards, which were released on June 2, 2010. The CCSSM is not a curriculum; it is a guide, with expectations and shared goals for teachers and school systems to implement and focus on K–12 education. The focus is to spend more time on certain concepts for mastery rather than spending a little time on a multitude of topics and not attain mastery (National Governors Association, 2010). The Common Core State Standards for Mathematics (CCSSM) is organized into 2 sections: Mathematical Practices and Mathematical Content Standards. For this study the focus involves all of the Mathematical Practices and the Content Standards specifically addressing linear equations.

The CCSSM Practices were designed based on the NCTM process standards and the strands of mathematical proficiency published in Adding It Up by the NRC (2001). The following is a list of CCSSM Mathematical Practices:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
McCallum (2012), a co-writer of the CCSSM, described the practices as “various ways in which proficient practitioners carry out their work” (p. 7). Through the practices, teachers and students are given descriptions of mathematical “habits of mind” to increase their math readiness for college and careers. The CCSSM (2012) is a set of standards to help guide teachers to establish some curricula for students to provide deeper conceptual understanding of ideas by using different representations of the same idea (National Governors Association, 2010).

The philosophical foundations for the Mathematical Practices and Content Standards are based on the tenets of constructivism, particularly social constructivism. Because this study is seeking to document students’ experiences of learning linear equations from their perspective, constructivism is most compatible with this goal.

**Constructivism**

The philosophy of education that not only involves listening to students but also requires teachers to facilitate the experiences that will enable students to express their understanding is constructivism (Kilpatrick, 1987; NRC, 1989; Skemp, 1987). Constructivism is a theory of cognition and is the basic underlying principle of how students learn (Confrey, 2004; Davis, 2004; von Glasersfeld, 1989). The function of cognition, according to constructivists, is organizing the concepts at hand through adaptation. Through making sense of their world, students can build connections that move them through complex thought than they currently possess (Confrey, 2004; Davis,
By providing students opportunities, teachers facilitate the construction of meaning through various experiences. Ernst von Glasersfeld wrote of two basic tenets of constructivism:

1. Knowledge is not passively received either through the senses or by way of communication, but is actively built by the cognizing subject.

2. The function of cognition is adaptive and serves the subject’s organization of the experiential world, not the discovery of an objective ontological reality.

(von Glasersfeld, 1988, p. 83)

“Piaget, Vygotsky and von Glasersfeld are [some of] the greatest influences in mathematics education,” because of their theories on constructivism (Patrick, 2013, p. 1). Two branches of constructivism are cognitive and social, both foundational to this study. Cognitive constructivists believe that knowledge is constructed within the individual (Piaget, 1955), whereas social constructivists believe it is the social interactions that create our knowing (Vygotsky, 1978).

Vygotsky and Piaget never met or discussed their theories before Vygotsky died. When Piaget was asked to write on Vygotsky’s theory compared to his own, Piaget stated, “certain points I find myself more in agreement with Vygotsky than I would have been in 1934, while on other points I believe I now have better arguments for [our differences]” (Piaget, 1962, para 2). The difference in many constructivist theories lies in how the knowledge is constructed.

Because of the nature of the research questions and the appropriate methodology, cognitive constructivism and social constructivism both serve as essential components of
the theoretical foundation for this study. In this study, constructivist theory was used because students had the opportunity to construct their own understanding of using kinesthetic movement when learning linear functions. The classroom served as an ideal setting for group work, presentations, and assessments to build their knowledge from previous experiences as described by Vygotsky; students also had opportunities to reflect on the meaningful activities used in this study and adapt their schemas to include new information as suggested by Piaget.

**Cognitive Constructivism**

Constructivists following the lead of Piaget contend that all knowledge is constructed. Piaget believed that we cannot merely give children information to learn, but rather they must construct their own knowledge (Piaget, 1955); within children’s minds we build different structures or schema to hold information. By making sense of their world, students can build connections that move them through complex thought: from the knowledge that they currently possess to new levels of understanding (Piaget, 1953, 1970a, 1971a). As teachers provide students with learning opportunities, students are able to construct meaning through various experiences guided by the teacher; the students in turn will gain relational understanding versus instrumental (procedural) knowledge (Skemp, 1977). For cognitive constructivists, learning is an adaptive process in which mental models accommodate new experiences. Constructivism not only emphasizes the role of how students process new information, but it also brings awareness of their constructions and the ability to modify them through conscious reflection (Confrey, 2004; Davis, 2004; Loveless, 2001: von Glasersfeld, 1996). Piaget’s
focus is on the individual and how the individual gains knowledge through an individual process (Applefield, Huber, & Moallem, 2000). Piaget’s cognitive theory expresses that students cannot be given information, which they immediately know and understand; students must construct their own knowledge through their experiences (Piaget, 1923). Piaget viewed children as “little philosophers,” which he called “tiny thought-sacks” and scientists building their own knowledge (Cohen, 2010, p. 86).

Piaget believed that incoming information is filtered in the child’s mind by assimilation and accommodation, leading to cognitive change within the child (Piaget, 1972). Assimilation occurs when the child receives new information, whereas accommodation occurs when a child adapts the new information along with his or her existing knowledge. As this process occurs, a state of disequilibrium is reached in which the new information does not simply assimilate into his or her prior thoughts, but rather an adjustment is made that leads to a better understanding of how everything fits together (Piaget, 1972). Piaget (1955) did not believe that children should just memorize; instead he believed an internalization process is required. Piaget viewed “

Children as builders of their own cognitive tools, as well as of their external realities. Knowledge and the world are both constructed and constantly reconstructed through personal experience. Knowledge is not merely a commodity to be transmitted, encoded, retained and reapplied, but a personal experience to be constructed. (Piaget, 1923)
Social Constructivism

Social constructivism emphasizes construction of knowledge through social interaction. Social constructivism comes from conversations with others and it involves both an individual and a social interaction (Vygotsky, 1978). Language helps the learner build his or her knowledge through conversations with others. Lev Vygotsky is known as the father of social constructivism. His beliefs are that all learning happens through social interaction combining with personal critical thinking. “Human learning presupposed specific nature and a process by which children grow into intellectual life of those around them” (Vygotsky, 1978, p. 88). These interactions “involve sharing, comparing, and debating among learners and mentors (Applefield et al., 2000, p. 7). Vygotsky believed that higher mental processes are developed because of social interaction linking culture and community for learning. Vygotsky (1962) believed that development cannot be separated from social and cultural contexts. When learning, we go through social stages before understanding happens as an internal mental function (Vygotsky, 1962). In Mind in Society, Vygotsky introduced the concept of the zone of proximal development (ZPD). ZPD is “the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (p. 86).

There are two stages to ZPD. The first stage is where the child currently is without assistance, and the second stage is where the child potentially reaches his or her full potential with the help of others. Cognition is explained as the product of social
interaction. A child could not reach stage two of ZPD without the collaborative nature of cultural and social context.

Vygotsky (1978) gave an example of ZPD with gesture movements. When pointing a finger, we see this as a meaningless motion; however, as we react to the gesture, it becomes a movement that has meaning. The pointed gesture represents an interpersonal connection between individuals.

Social constructivism values peer-to-peer interactions and the classroom as a community for learning to take place. Vygotsky explained that a child develops his or her intellect through internalizing concepts based on his or her own interpretation of an activity that occurs in a social setting. Accordingly, social learning precedes development (Vygotsky, 1978). Social interaction is the main tenet for the development of cognition.

In summary “all knowledge is constructed” from a constructivist perspective as opposed to the psychological theory of behaviorism where it is believed that the learner is a blank slate with no previous experiences (von Glasersfeld, 1987). Noddings (2004) described constructivism as an attempt to explicate nature through the workings of the human mind. Constructivist methods in the classroom are meant to cause transformations in teaching and learning (Confrey, 1990). It forms an understanding of how students come to know a topic. Each of us generates personalized mental models used to make sense of our experiences (Steffe, 1988). In this study, students had the opportunities to work in a social setting (presentations, group work, classroom
discussions) and students had other opportunities (assessments, interviews) to reflect on different representations of linear functions with kinesthetic movement.

**Lesh’s Translation Model Between Mathematical Representations**

For this particular study, the theory of multiple mathematical representations of linear functions is reinforced as the student develops different models to demonstrate and interpret the same concept. By using multiple representations, the student is given the opportunity to make connections among the same idea being expressed (Kaput, 1992; Skemp, 1987). Through translating among the different representations, student learning is enhanced through reasoning and communication about the mathematical concept (Kaput, 1987). Different representations guide students to make adjustments to their thinking. Through the interweaving of representations, students can make a connected web of knowledge between the individual representations with the whole concept and the various representations that mean the same thing—just expressed differently (Kaput, 1987). Lesh et al. (1987) explained that most math ideas are presented in five different modes. When students are presented the information in a variety of modalities their learning is enhanced by interconnections. Please refer to Figure 3 which shows the five different modes and how they are interconnected (Lesh et al., 1987).

Mathematical representations help students piece together concepts of an abstract idea (Lesh et al., 1987). There are two types of mathematical representations: internal and external (Krause, 2001). Internal representations are schemas in our mind that represent the cognitive processes such as memories, perception, frames, thoughts, and current and past experiences in one’s own world, and they are often incomplete. External
representations are what are communicated via picture, graph, chart, words, and equations (Krause, 2001). “The external representation is the communication of our internal representations. Internal representations are the mediators in making connections between the external representations” (Krause, 2001, p. 21). Mathematical representations guide students to make adjustments to their thinking. Students become reflective and guide other students through movement among various representations in other categories by communicating their external representations (Goldin & Shteingold, 2001). Reisberg (1987) suggested that when students are given the chance to use external representations, they are more likely to find errors or gaps in their internal representations. Reisberg continued by suggesting that when externalizing one’s thoughts students reflect and try to make more connections to other representations. Pape and Tchoshonov’s (2001) research with problem solving and representations added to this framework by suggesting that students who are taught with only one representation because they tend to be “stuck” to one specific representation. Whereas students who are given the opportunity to examine a variety of representations and translate among them are more successful in learning and expressing their knowledge; for example, when students are only shown an area model of fractions versus a linear or set model.

Snelson’s (2003) research elaborates further on students making connections with different representations of mathematical topics. Her study investigated students learning quadratic functions. Through students’ usage of external representations of communication describing graphs and video clips, Snelson found students having higher
levels of achievement compared to the students who did not have the multiple representations.

For representations to be found useful they must be connected in meaningful ways. Gardner (1991) argued that “the broad spectrum of students—and perhaps the society as a whole—would be better served if disciplines could be presented in a number of ways and learning could be assessed through a variety of means” (p. 12).

Some disadvantages become apparent when students are presented different representations about a particular topic without prerequisite knowledge. Representations become less effective when students have no knowledge or only a little familiarity about the representation’s concept (Lesh et al., 1987). For example, when students in my study were given a graph to work with, they may understand how to plot points/lines to the graph, but they may also experience difficulties in understanding the labeling of the graph or knowing about the values of the graph’s axis (Lesh et al., 1987).

In summary, representations communicate mathematical ideas and are both internal and external (Snelson, 2003). By providing students with the opportunity to connect a variety representation, their learning can be enriched (Lesh et al., 1987). NCTM (1989, 2000) and CCSM (National Governors Association, 2010) consistently identify the use of multiple representations as an important core value. Mathematics is taught most effectively when teachers understand how important a student’s understanding is when valuing the different representations of a concept and connecting them as one (Goldin & Shteingold, 2001). “A single type of external representation does not guarantee student learning and performance” (Lesser & Tchoshanov, 1987, p. 1).
Kinesthetic Movement

This study involves the interconnection among representations and kinesthetic movement. The following section examines the literature on embodied cognition.

Gestures and Embodied Cognition

Cook (2011) defined embodied cognition as using the affordances of one’s body to affect cognition. Through the use of physical actions learning and thinking are supported. Goldin-Meadow (2009) further stated that embodiment interaction used with learning and reviewing of concepts facilitates students being active in their knowing. Maquene and Ionescu (2011) stated that cognition cannot be taken as just a brain activity, but rather that cognition occurs through our sensory motor system and our learning is embedded through our body movement. Embodied cognition for this study includes learning that is a result of social and physical interactions with the environment in a classroom setting while using kinesthetic movement. Alibali and Nathan (2011) argued that mathematical cognition is embodied because students use their bodies in gestures or kinesthetic movement to express their mathematical ideas.

Goldin-Meadow referred to gesture as a communicative act made with your hands. “Gestures are not only a vehicle for expressing action information, but, because it is itself action, gesture can add action information to the gesturer’s mental representations” (Beilock & Goldin-Meadow, n.d., p. 3). Gestures are not merely hand waving; they represent our thoughts, and as such are another form of communication. Some researchers suggest that gestures help us change our thought patterns and aid in the
transformation of an incorrect idea to conceptual understanding (Chu & Kita, 2011; Goldin-Meadow, 2009; Iverson & Goldin-Meadow, 2001).

Gesturing and Communication

Some ideas cannot be expressed in speech. However, when teachers use perceptual motor representations, students gain more than just when verbal teaching happens. In their research, Cook and Tanenhaus (2009) studied students working with the Tower of Hanoi problem. Fourteen pairs of students in an empirical study worked on solving the problem. The Tower of Hanoi (See Figure 4) is a puzzle-type manipulative in which three pegs are to be transferred to other rings in as few steps as possible. One group used a manipulative, and once they completed their task they gave verbal directions to their partner in another room where there was no apparatus. The other group used a computer animation of the Tower of Hanoi, after listening to the instructions. This study’s findings demonstrated that the students who used the computer did not fully understand the relationship between the movements of the circle to another peg because they were simply dragging the computer mouse left and right. Even though they had the verbal directions, they still had difficulty communicating to their partner how to do the task. On the other hand, the group who had to actually remove the ring and place on another pod understood direction, motion, and depth and communicated that through their gestures.
Children often express ideas in gestures before language (Goldin-Meadow & Alibali, 2013; McNeil, 1992). Young children who do not have verbal skills or are not developmentally ready to use their words express their knowledge through gestures (Iverson & Goldin-Meadow, 2005). Goldin-Meadow and Alibali (2013) situated gestures as the early stages of knowing and communicating without using vocabulary. Gestures that students make may have a causal role in their language development of the task at hand (Rowe & Goldin-Meadow, 2009; Rowe, Özçalıskan, & Goldin-Meadow, 2008). Valenzeno, Alibali, and Klatzky (2003) suggested that gestures aid in the facilitation with the complicated language of mathematics. In mathematics, Lakoff and Nunez (2000) concluded that gestures are important not just for ideas but to aid in communication so that others know what is being said. Research supports students not knowing mathematical terms for a variety of reasons. For example, teachers may not understand

Figure 4. Tower of Hanoi (2017)
mathematical terms correctly because of their background or how they were taught, students may not understand the vocabulary, and lastly, students may not be able to put the mathematical terms coordinating with the diagrams and figures (Cooke & Buchholz, 2005; McCrone, 2005; Moshkovich, 2002; Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008). Some researchers also believe that gestures help convey information more than symbolic words (Tversky, Jamalian, Giardino, Kang, & Kessell, 2013).

Goldin-Meadow (2009) suggested that gestures promote math language by showing a different tool used for communication. Through teacher’s nonverbal actions, teachers reveal their message. When a string of gestures is reflected upon as a whole, students gain an understanding of an abstract idea by capturing the visual, which helps them imagine features of the idea. In this way, gestures are complimentary to speech and support the spoken language. Students’ understanding of teacher’s verbal instruction evidences this when gestures are used (Goldin-Meadow et al., 1999).

**Gesturing as a Mental Model**

There is a growing body of research showing how gestures help form mental representations in mathematics (Goldin-Meadow, 2009). Schwartz and Black (1996) found gestures as the “physically unsaturated mental models” interlocking gears that aid in understanding abstract rules. Schwartz and Black represented this in their quantitative research investigating mental rotations with hand movements. Gestures are additional representational powers that help students remember and they are diagrams in space that use actions (Tversky et al., 2013). Preschoolers have demonstrated similar findings as they have benefitted from speech and gestures when learning symmetry with tasks
involving mental rotations (Ehrlich, Levine, & Goldin-Meadow, 2006; Valenzeno et al., 2003).

When the students used gestures, the movement of their hands guided them to see the mental rotations. The gesture seemed to be part of a transitional thought process used to get to the next step. Gestures help reflection of what one is thinking. However, through the use of gesture one’s thinking may be altered (Goldin-Meadow & Beilock, 2010). Because gestures are a strong visual component, when gestures are used it is transitory and disappears in the air (Goldin-Meadow, Kim, & Singer, 1999, p. 729). Gestures help the abstract idea become a unified picture. When students are solving problems they normally use diagram or gestures to help them understand the problem (Kessell & Tversky, 2006; Jamalian, Giardino, et al, 2013). “Gestures have the potential to serve as a unique bridge between action and abstract thought” (Goldin-Meadow & Beilock, 2010, p. 2).

Morsella and Krausse’s (2004) quantitative research investigated the effectiveness of using gestures. Their study included videotaping 79 university students describing 40 different images. The first group was permitted to use gestures when communicating their knowledge, while the other group had to sit on their hands and were not permitted to use gestures. They found subjects who sat on their hands were dysfluent—the mismatch of gesturing and verbal communication. However, the subjects permitted to use gestures when explaining their understanding had a positive correlation with higher understanding. Through the use of gestures their speech and communication of the topic was enhanced. Kessell and Tversky (2006) suggested that gestures are similar to a
sketchpad. Integrating both memory and knowledge through body movement and gestures help the working memory avoid being overloaded.

In an Intelligence study that was published in Newsweek, 28 teenagers came to a lab at Berlin’s Humboldt University. The research was investigating whether students had the mental ability to apply rules they’ve already learned in new situations. The adolescents were given geometric images and they had to discern the patterns between the geometric images. Goldin-Meadow examined the findings of why students who gestured had more success in discerning the images. She explained gesture helps code the memory for long-term retrieval. Gesture lightens the load of cognition making it easier to recall.

Casille and Giese (2006) studied whether motor movement has an influence on visual recognition with or without a visual stimulus. There were three phases to this study: visual pre-test, nonvisual motor training, and a visual posttest. Subjects were given the task of moving their arms 180°, 225°, and 270°. Subjects could not complete the activity without motor training. The results found motor representation to have a direct and high influence on visual perception without having a visual stimulus.

**Linked Representations**

Research also demonstrates that when two different modalities are used in teaching, students’ cognition may benefit (Batt, 2009; Powell, 2007). Lakoff and Nunez (2000) argued that math concepts are supported with visual and motor representations. Gestures are considered to be part of a non-verbal speech that is redundant in reiterating what one is saying. Combining speech and gestures helps students create connections,
learning, and communication (Jamalian, Giardino, & Tversky, 2013). Redundancy is not necessary for all students; however, some students find it beneficial (Becvar, Hollan, & Hutchins, 2008). When children are learning about equivalence problems \((2 + 6 + 7 = \_+7)\), they are more likely to learn material when presented with speech and gestures compared to just speech (Cook & Goldin-Meadow, 2006; Perry, Berch, & Singleton, 1995; Singer & Goldin-Meadow, 2005). Gestures help with mental operations when performing mathematical tasks.

Jamalian, Giardino, et al. (2013) found that when language and symbols are used together they indirectly create meaning; when gestures were used, they helped the students represent understanding by expressing them in space through action. Cook and Tannehaus (2009) believed that gestures helped students remember abstract concepts. The gesture was using visual working memory, which makes it easier for students to use their verbal memory because they have something to help them remember than just words. When students view lessons with gestures, they learn more by pointing and tracing (Valenzeno et al., 2003).

Cook and Goldin-Meadow (2008) also found that the gestures students see influence the students’ own gestures. M. Wilson and Knoblich (2005) further expanded upon this theory by observing others’ gestures with their observational research involving body movement and imitation. When we observe, or perceive other persons’ movements, our brains process this information; through overt imitation, we come to understand the concepts and reinforce the ideas being shown with the body. They believed students’ overt imitation facilitates their covert imitation. Through the imitation of motor skills,
students recall information faster and with fewer errors in mental imagery. When children replicate teachers’ gestures, they also gain understanding of what the gesture means. Through the use of observing and responding to others’ actions, students reflect their own actions by reacting to the observations of their peers (Hard & Tversky, 2009). Here the gesture is seen as a visual diagram that expands thinking for the embodied representations.

“Gestures arise and represent speakers’ active thoughts, and are actually from embodied thinking and through the use of gestures the speaker makes embodiment visible” (Congdon, Hemani-Lopez, & Goldin-Meadow, 2014, p. 511). Thus, cognitive scientists believe that gestures lighten working memory when explaining math solutions (Cook, Yip & Goldin-Meadow, 2012; Goldin-Meadow & Beilock, 2010). Gesturing frees up cognitive resources that can be conserved and used elsewhere (Goldin-Meadow, Nusbaum, Kelly, & Wagner, 2001).

Pine, Lufkin, and Messer (2004) found students use more gestures than verbal responses when answering questions pertaining to balance. Their quantitative research investigated 140 children demonstrating their answers to balance questions. Wagner, Nusbaum, and Goldin-Meadow (2004) supported that gestures facilitate recall through the use of mental representation and that participants remembered more when gesturing. Goldin-Meadow et al. (1999) found students take more information away from the lesson when gestures are used.
**Other Forms of Gesturing**

In a more recent study, Chu and Kita (2011) used gesture coding when investigating 132 adult students working on mental rotation with paper folding and viewing a rotation on a screen. There were three groups: the gesture encouraged group, the gesture permitted group, and the gesture-prohibited group. They found gesture plays a role in learning and when encouraging students to gesture, there is no positive effect. However, when students used gestures there was an increase in understanding. Their findings suggest that our bodies guide our brains and our brain interprets how our body looks. Our brain uses the knowledge of what our body looks like to compute more accurate information on what the task should really look like when transformed and then changes the body (Beilock & Goldin-Meadow, 2011).

In another type of embodiment study, Emmorey, Tversky, and Taylor (2000) worked with individuals who used sign language. Not all participants in this study were deaf. They found that sign language differed from spoken language in that when using sign language, it expressed words but also demonstrated dimensionality. Tversky (2011a) extended that conclusion in the sense that gestures contribute to sense making and spatial knowledge.

**Summary of Gesturing Literature**

An extensive and expansive body of literature on gestures suggests students benefit in learning and retention of concepts when gestures are incorporated within a learning task (Cook & Goldin-Meadow, 2006; Ehrlich et al., 2006; Tversky, 2011a; Valenzeno et al., 2003). In summarizing the research on gestures, gestures are not
entirely happenstance. Instead, gestures reveal cognitive knowledge that cannot be heard with spoken language. Gesturing assists in communication and when used with other representations (i.e., speech), it is highly effective. Gestures demonstrate direct communication of our thoughts and actions and direct thought and actions of others (Tversky, 2011b).

Important to note is that each of the studies on cognitive science mentioned in the above review are of a quantitative nature and the benefits including achievement gains were empirical. This study differs not only in its methodology, but in its joining of the learning of linear functions with kinesthetic movement.

**Linear Functions**

Because this study is examining the role of kinesthetic movement in the understanding of linear functions, this section focuses on the difficulties students encounter in graphing linear equations and in translating among various representations. Practitioners struggle with students learning and relating linear functions with their corresponding graphs (Kieran, 1996). Because many mathematics classrooms use the “plugging in” method with a direct instruction approach, linear equations become formulaic without meaning (Van Ameron, 2003). In a traditional mathematics classroom students experience functions in separate pieces without an appropriate emphasis on other meanings and representations; therefore the linking process that should exist between them is missing (Kaput, 1989).
**Students’ Difficulty With Learning Algebra**

A review of the literature on algebraic teaching and learning indicates that there is no consensus among researchers in the field, resulting in a number of different approaches. Van Ameron (2002) stated,

The teaching and learning of school algebra has become a worldwide topic of interest over the last few years. Still, one issue most researchers agree on is that students are known to struggle with the structural aspects of algebra. Especially the change from a procedural way of thinking in arithmetic to a structural perspective that causes a rupture in the learner’s development. (p. 32)

Carolyn Kieran (1996), a researcher on algebraic structures and how students comprehend them, found numerous ways in which teaching algebra is changing. One of the changes she described is the shift from school algebra to algebraic thinking. She said, “School algebra is the representations of quantities with symbols and teaching the manipulation of symbols while, algebraic thinking is the competence in the use of multiple representations and the ability to reason within and among numerous representations” (p. 271). Another piece that is difficult for students to understand the different representations is because students do not make the connections between the symbolism and the graphs (Phillip, Martin, & Richgels, 1993; Ryan, 1994). They find that there needs to be more of a link of understanding between and among different representations.

Moss (2005) found that one of the first difficulties students have with learning algebra is linking new symbols, with their meanings and new representations. The flow
among the three is difficult to understand the relationships among them. To alleviate this frustration, it is recommended that learning happens in a constructivist classroom that fosters active learning with students using meaningful activities along with opportunities of multiple representations compared to a traditional lecture classroom (NRC, 2005). Table 1 shows a comparison of a traditional classroom (behaviorist) versus constructivist classroom.

Battista and Borrow’s (1998) research based on students in fifth grade found that for students to meaningfully utilize algebra, it is essential that instructors focus their attention making sense, not on symbol manipulation.

The computer-based instructional tasks they have described to relieve students of the demands of computation encouraged the students to progress through different levels of sophistication in their thinking about such general procedures. These activities encourage the formation of the abstractions and related mental operations required for meaningful algebraic thinking. (Battista & Borrow, 1998, p. 478)
Table 1

*Behaviorist Classroom Compared to a Constructivist Classroom (Taylor, 2015)*

<table>
<thead>
<tr>
<th>Behaviorist Classroom</th>
<th>Constructivist Classroom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher directed (didactic dissemination of information)</td>
<td>Learner-centered. Teacher as facilitator—students construct knowledge through critical thinking, manipulatives, primary resources, and hands-on activities.</td>
</tr>
<tr>
<td>Student works independently.</td>
<td>Student works collaboratively in groups, independently or in partners.</td>
</tr>
<tr>
<td>Small parts first. Big idea at the end. Correct answers are the goal.</td>
<td>Big idea first. All parts support big idea. Thinking and support of thinking are the goals.</td>
</tr>
<tr>
<td>Assessments are tests; separate from learning task.</td>
<td>Assessments are observation, conferences, daily work, portfolios and included in learning tasks.</td>
</tr>
<tr>
<td>Teacher evaluator.</td>
<td>Self-reflection, student evaluator and teacher evaluator</td>
</tr>
<tr>
<td>Teacher makes all the decisions.</td>
<td>Shared responsibility and decision making.</td>
</tr>
<tr>
<td>Product-based learning: All students will learn on demand the same thing at the same time aka. One-size-fits-all approach.</td>
<td>Learners create meaning and context by exploring new ideas and experiences, generating hypotheses, problem solving.</td>
</tr>
<tr>
<td>Individual learners.</td>
<td>Group of learners.</td>
</tr>
<tr>
<td>Teacher talks to (at) students; students expected to listen and absorb knowledge.</td>
<td>Teacher-student dialogue through conferring, questioning, and wondering.</td>
</tr>
<tr>
<td>Students work independently.</td>
<td>Student works collaboratively in groups, independently or in partners.</td>
</tr>
</tbody>
</table>
Difficult Learning Linear Equations

Moshkovich’s (1996) *Making Sense of Linear Equations and Graphs* led her to the conclusion that students do not see the straightforward way that researchers and educators make connections between graphs and equations. Moshkovich’s study suggests that when students are learning to graph linear equations, teachers should create classroom activities that focus on conceptual connections between graphs and equations. If given the opportunity to make a table of ordered pairs generated from an equation, most students have success. The difficulty occurs when students are given an equation, but not a table of ordered pairs because they will struggle with what points they need to plot and what the graph will look like. Students focus on the procedure rather than the concept. Another challenge that the author found is that students have difficulty making connections between seeing a graph and putting its equation form. Her research suggests that the conceptual understanding is not there, because students are not relating the graph, chart, and equation all as an interrelated entity (Moshkovich, 1996).

K. Richardson’s (2014) work on linear functions with student understanding finds that what is obvious to adults is not obvious to children and cannot be easily explained to them. As early as 1986, Hughes’ research further elaborated this point by finding that children may understand concepts but may not relate what they know to the symbols representing those concepts. Students can also learn to deal with symbols without understanding them (Worthington & Carruthers, 2003a).

In another study by Moshkovich (1996), the author found students experiencing difficulty with the y-intercept when given the equation $y = mx + b$. Students had the
A misconception of moving the y-intercept left and right rather than on the y axis. Another finding with students’ conceptual understanding was their communication. Some students could express their knowledge without using mathematical terms; for example instead of using the word slope to describe the rate of change with a line being graphed, the students were using the word steepness to describe the line (Moshkovich, 1996). Even though the correct terminology was not used, students still could express their understanding of graphing a line and comparing the line to another graph.

NCTM (2000) suggested that the study of mathematical change is fundamental to understanding functions, and the higher levels of mathematics that are based on it (for example, calculus). Research indicates that function is not a topic that students typically understand (Chazan, 1996), and that the difficulties students have with the notion of function are due to the abrupt and abstract way they are commonly introduced. Thus, the exploration of functional thinking should be gradual and should occur over a long period of time (Warren & Cooper, 2006, p. 152).

According to Clements and Vaiyavutjamai (2006), traditional teaching and testing in math classes isolate skills by having only one correct response, and thus fail to help students make cognitive connections between representations. Before one attempts to measure the immediate effects on student performance and understanding of lessons on linear functions and linear inequalities, one should be clear on the meaning given to “understanding.” Skemp’s research distinguishes between “instrumental” (or procedural) understanding, and “relational” understanding. According to Skemp (1976) instrumental understanding involves “rules without reason” (p. 20), whereas relational understanding
involves “knowing both what to do and why” (p. 20). Skemp maintained that with relational understanding the means became independent of ends to be reached and the task of building up schema within given areas of knowledge became an intrinsically satisfying goal. As a student’s schema became more pervasive he or she developed the confidence needed to find new ways of solving similar tasks without outside help. As schema developed, awareness of possibilities increased, and the process of inquiry became self-continuing and self-rewarding.

Kalchman and Koedinger (2005) stated, “Functions are all around us, though students do not always realize this” (p. 351). Functions happen in everyday life and students need to be able to see functional relationships, whether it be at a gas station and miles per gallon are used, financial problems, demographics, or even figuring out which cell phone plan is the best to use. “By using different algebraic tools through the use of giving students different representations students can reach success by moving from one representation to the other” (p. 351).

Through making connections among different representations, students can be given the opportunity to learn how to switch and be flexible with lines, ordered pairs, and equations (Moshkovich, Schoenfeld, & Arcavi, 1993). This is especially crucial flexibility when explaining why objects in the domain behave in the manner that they do. For example, consider the following question: Why is the graph of \( y = 3x \) steeper than \( y = 2x \)? There are various ways to answer this question; however the answer involves creating connections across representations.
Students should be encouraged to make their own functional notation and then build notations and meaning for slope and y-intercept (Cramer, 2001). Students need to be given time to consolidate their ideas before they move onto new concepts such as a quadratic equation. The Common Core State Standards in mathematics explains this process through documents entitled, *Progressions* (2017). The Progressions document is a narrative describing mathematics across the curriculum. The Progressions were created to explain why standards are sequenced the way they are: “point out cognitive difficulties and pedagogical solutions and give more detail on particularly knotty areas in mathematics” (Common Core Writing Team, 2011, para. # 2).

Based on the literature with students’ difficulty learning linear functions, conclusions can be made that students have a difficult time with the different representations. Research suggests students have a difficult time seeing that the different graphs, charts, and equations are the same idea that are presented in a different representation (Hughes, 1986; Moshkovich, 1996).

**Summary of Literature Review**

A major difficulty in algebra today is students’ lack of understanding linear functions and the ability to translate among various representations. To explore this difficulty, this study utilized a theoretical framework connecting three constructs: Constructivism as posited by Piaget and Vygotsky, Lesh’s Theory of Multiple Representations, and kinesthetic movement in mathematics education. It is the application of this framework that answers the research questions of this study.
CHAPTER III

METHODOLOGY

This chapter begins with addressing the following areas: the rationale for the study design, practitioner research with case study justification, and the role of the teacher as the researcher. The second part of this chapter includes data collection and analysis methods. The final section of this chapter focuses on how the research gained validity for the study.

Practitioner Research

I see myself as a practitioner who wants to be a life-long learner. Because I am intertwining research with my practice I plan on using practitioner research methodology and plan to recognize my role in the research process. Drake and Heath (2011) stated it is important for the teacher/researcher to have knowledge of “the awareness of the theorist of their unique part in the construction of new knowledge” (p. 75).

Most research in education that focuses on kinesthetic/gestures movement uses a quantitative approach (Goldin-Meadow & Alibali, 2013a). Practitioner research refers to “active professional learning” (Grady, 1998) in which practitioners do research to improve the action of their practice. This practitioner research study used a case study to capture what students had to say when learning linear functions with kinesthetic movement. Anderson (2015) stated, “In teacher research, teachers decide what to study. The research question emerges from a teacher’s nagging or curiosity about some aspect of classroom life” (para. 4). Because my research questions have surfaced within my own classroom and were incorporated from my experiences as a mathematics teacher
over the last 25 years, I felt that practitioner research fit my study. I wanted to improve students learning in areas where I see them struggle. One area I wanted to improve my practice was with students learning linear functions. Another area I felt was troublesome dealt with students being restless in the classroom.

In practitioner research, rich description relying on multiple sources of evidence, collection, and triangulation of data ensures validity. An assumption of this study is that using kinesthetic movement in learning contributes to mathematical knowledge and understanding. There are gaps in the research on kinesthetic movement in middle grades, (Turner, Dominguez, Maldonado, & Empson, 2013). I also found no evidence of kinesthetic movement applied to linear functions. This study is driven by my own experience as a mathematics educator of more than 25 years and what I have observed in my own classroom. The main purpose is to help improve current practice in mathematics classrooms with the hope that this information will add to knowledge of both kinesthetic movement and the use of multiple representations in the mathematics classroom.

Cochran-Smith and Lytle (1990) first defined teacher research as “systematic and intentional inquiry by teachers (p. 3). They defined systematic as “ways of gathering and recording information, documenting experience inside and outside of classrooms, and making some kind of written record.” Intentional is defined as “an activity that is planned rather than spontaneous” (p. 3). Inquiry is defined as study stemming “from or generating questions and reflecting teachers’ desires to make sense of their experiences—to adopt a learning stance or openness toward classroom life” (p. 3). In 1999, Cochran-Smith and Lytle broadened their definition to encompass all forms of
practitioner inquiry that involve systematic, intentional, and self-critical inquiry about one’s work” (p. 22). Today they use the term practitioner research as the “conceptual and linguistic umbrellas to refer to a wide array of educational research” (p. 38).

Drake and Heath (2011) stated “a practitioner researcher [engages] with new knowledge at all stages of the project, from conceptualization, through methodology, methods and empirical work to the [paper]” (p. 2).

Drennon (1994) explained,

Practitioner inquiry is not field-testing the ideas of others, nor is it simply implementing a new strategy that one is already convinced with work. Instead, it is a process of generating ideas through reflection and examination of practice, and exploring the implications of those ideas within the practitioner’s setting. (p. 3)

Focused on the need to improve students learning linear functions, I conducted research on what we know about students learning linear functions and why students struggled in this area. I researched kinesthetic movement in the classroom and its benefits. I developed a research plan and created questions that involved kinesthetic movement and linear functions. I performed the research and reflected on the data. Please refer to Figure 2 which shows the practitioner research cycle that guided this study. The cycle in Figure 2 is not only explained in Table 2, but also applied to my study.
### Table 2

**Practitioner Research Cycle for This Study**

<table>
<thead>
<tr>
<th>Practitioner Cycle</th>
<th>Definition</th>
<th>My Study</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Need</strong></td>
<td>Teachers investigate their own practice in order to improve it (Krainer, 2014). Need comes from the teacher in the classroom.</td>
<td>Saw a need for improving students learning linear functions and translating between different representations of linear functions.</td>
</tr>
<tr>
<td><strong>Find out what we know</strong></td>
<td>What do you know about this situation? What do other people know that is relevant? How have others overcome similar problem? (EdFutures, 2011)</td>
<td>Created a literature review to examine what is known about kinesthetic movement and linear functions. Topics Covered: Kinesthetic Movement, Algebra Linear Functions</td>
</tr>
<tr>
<td><strong>Plan</strong></td>
<td>Create a plan to investigate the need for improvement in your practice.</td>
<td>Created research questions that focused on problems that were connected to students learning linear functions while using kinesthetic movement.</td>
</tr>
<tr>
<td><strong>Do</strong></td>
<td>Put plan into action and monitor and analyze what has happened in the study and make changes when necessary.</td>
<td>Pilot study Permission for study. Data Collection</td>
</tr>
<tr>
<td><strong>Reflect on my practice</strong></td>
<td>Use a systematic review focused on finding and synthesizing data collection. Consider different representations of how others might look at it. Check that data supports the information. (EdFutures, 2011)</td>
<td>While doing study: What is happening? What are the reactions to kinesthetic movement? How do I adjust if needed? After the study: What have I captured? What does it mean to math education and students learning linear functions? How does it change teaching linear functions?</td>
</tr>
</tbody>
</table>

*(table continues)*
Table 2 (continued)

Practitioner Research Cycle for This Study

<table>
<thead>
<tr>
<th>Practitioner Cycle</th>
<th>Definition</th>
<th>My Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share</td>
<td>Findings are communicated to other practitioners. (Does not have to be publicized.) Questions to ask: What are the key messages I want to display? Who is my intended audience? What is the most efficient way to present findings? (EdFutures, 2011)</td>
<td>In sharing I can reach the primary goal of practitioner research: “To teach better so as to improve one’s students’ learning” (Grant 2008). Use the results and share the findings and implications with fellow teachers’ researchers in school and at conferences.</td>
</tr>
<tr>
<td>Peer Review</td>
<td>Engage peers for their insights. Gain ideas from others when sharing.</td>
<td>Be open to listening to how others worked with linear functions and kinesthetic movement.</td>
</tr>
<tr>
<td>Professional knowledge Base</td>
<td>Use the results of the study to add to the professional knowledge base in the following areas:</td>
<td>Add to professional knowledge with using kinesthetic movement in mathematics Add to professional knowledge base on how kinesthetic movement may help students learn and recall linear functions.</td>
</tr>
</tbody>
</table>

Case Study Justification

This study followed Yin’s (2003b) and Merriam’s (2009) definition of case study methodology. “A case study is an in-depth description and analysis of a bounded system” (Merriam, 2009, p. 40). Thus, the process that students develop an understanding of linear functions through movement was a single case and the units of analysis were individual student’s construction of meaning. This was measured through multiple data collection methods of pre- and post-tests, interviews, and classroom tape recordings. Merriam (2009) stated that researchers conducting basic qualitative research
would be primarily interested in “(1) how people interpret their experiences, (2) how they construct their worlds, and (3) what meaning they attribute to their experiences” (p. 23).

Bassey (1981) stated if case studies

Are carried out systematically and critically, if they are aimed at the improvement of education, if they are relatable and if by publication of the findings they extend the boundaries of existing knowledge, they there are valid forms of educational research. (p. 86)

Merriam (1998) believed that case study design is a useful approach for studying educational settings. Yin (1994) believed case studies are useful for their rich description and are beneficial when questions are being asked about situations that the researcher has little or no control over.

I have selected a case study methodology for several reasons. The basis of case study research is grounded in constructivism with reality being constructed by individuals in a social setting (Merriam, 2009). Reality is viewed differently for each individual because of his or her own personal experiences but through a shared social setting combined with previous experiences he or she comes to know (Piaget, 1952).

The second reason case study was an appropriate methodology for this research was the number of data sources. Merriam (2009) stated that case study researchers are looking for descriptive data in a variety of forms whether they are pictures, words, videos, or interviews. Rossman and Rallis (2003) concurred as they stated that a case study’s strength is in the detail, complexity, and use of multiple sources to obtain different perspectives of the same idea.
The bounded system is the process in which students are trying to understand the mathematics. The case was the data from the whole class, followed by smaller units consisting of four students’ artifacts and individual interviews. Through the use of units of analysis more descriptive data may be used to capture the student perceptions when learning linear functions.

**Teacher as the Researcher**

One of the characteristics of this qualitative practitioner research study is the researcher/teacher is “a primary instrument for data collection and data analysis” (Merriam & Simpson, 2000, p. 5). Borg (2010) defined research done by the teacher as an inquiry conducted by teachers in their own professional contexts. He further believed research in the classroom should aim to benefit the educational experience and learning and ideally the broader academic community of best practices, which differentiates practitioner research from action research. Hubbard and Power (1999) stressed that teacher researchers should look for questions that not only answer their research questions, but “involve understanding students and teaching in profound ways” (p. 21). In this study, I am both the teacher and researcher, participant and observer in the classroom that serves as the site for this inquiry.

**Research Design**

This chapter describes the context in which the research was carried out and the methods used to answer the research questions. This chapter begins with a description of the research site followed by an introduction of the participants of the study along with how they were chosen. The third section of this chapter describes the data sources used
in the study. The chapter ends with an explanation of the data analysis procedures incorporated for this study.

**Site**

The setting for this study was a public middle school located in a Northeastern Ohio suburb. The school houses approximately 490 students in seventh and eighth grade. The ages of students at the school range from 12–14 years. The middle school diversity is broken down into percentages: 89% White students, 5% Asian students, 7% Hispanic students, 1% Black students, 3% other students. Ten percent of the students are eligible for free lunch, 4% are eligible for reduced lunch. Median family income is $76,964 and 1.3% of the school system is below poverty level.

**Participants**

All of the students for this study were in my current algebra class designed for advanced eighth grade students. Students are randomly assigned to each of the two algebra classes. To be eligible to take eighth grade algebra, students must score in the 95th percentile for the school system on their IOWA test scores from sixth grade and are given the opportunity to take an Algebra Aptitude test. If the student scores between an 85 to 90% on the Algebra Aptitude test, they have the choice of being placed in a pre-algebra class for seventh grade class or regular seventh grade math. The students who choose to take pre-algebra as a seventh grader with a passing grade of a B or higher, and who are recommended by their teacher are placed in one of the algebra classes in eighth grade.
**Data Sources**

In keeping with practitioner research, I conducted the analysis for this study, acting as the observer, listener, questioner, analyzer of the data, and disseminator of results to the professional community. My background and prior experiences of being a math educator for the last 25 years was added to the process. The data sources were collected and included pre- and post-teacher-made assessments with state standardized problems, audio and video transcriptions of class, small group, and individual discussions, Fleming & Mills Learning Style Inventory, and attitude survey on kinesthetic learning. In addition, I kept a teacher journal and a researcher journal. Table 3 guides the reader to understand the relationship between data sources and which research questions they answered.

**Table 3**

*Outline of Data Sources Answering Which Research Question*

<table>
<thead>
<tr>
<th>Research Question 1</th>
<th>Research Question 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>What do students say about learning linear functions with kinesthetic movement?</td>
<td>How do students (if they do) translate between and among mathematical representations (kinesthetic-verbal, kinesthetic-symbols, kinesthetic-real life, kinesthetic-graphical, kinesthetic-oral)?</td>
</tr>
<tr>
<td>Learning Style Inventory</td>
<td>Video Tapes</td>
</tr>
<tr>
<td>Pre-Test</td>
<td>Audio Tapes</td>
</tr>
<tr>
<td>Video Tapes</td>
<td>Small Group Tapes</td>
</tr>
<tr>
<td>Audio Tapes</td>
<td>One-on-One Tapes</td>
</tr>
<tr>
<td>Post-Test</td>
<td>Post-test</td>
</tr>
<tr>
<td>Teaching Journal</td>
<td>Teaching Journal</td>
</tr>
<tr>
<td>Research Journal</td>
<td>Research Journal</td>
</tr>
<tr>
<td>Attitude Survey</td>
<td>Attitude Survey</td>
</tr>
</tbody>
</table>
**Learning Style Inventory**

A learning style inventory by Fleming and Mills (1992) was used in this study to show the preferred learning style of the participants and to identify whether their learning preference plays a role in this investigation. They suggested that there are four categories or preferences an individual takes in information and puts out information: visual, aural, read/write, or kinesthetic (VARK). I chose to use Fleming and Mill’s (1992) VARK classification system because it incorporated not only input learning preferences but also preferred output methods of their knowledge and is compatible with this research (Appendix B).

**Teacher Journal**

The teacher journal contained information relating to the purpose of teaching the class. The specific types of data the teacher included were: lesson plans, student questions about the lesson, successful strategies, key topics that need emphasis, remediation, and the teacher’s scope and sequence. All information pertained to teaching the class and students’ handouts.

**Researcher Journal**

The researcher’s journal was completed after the teacher journal. This information related to the purpose of the study and research questions. These included important data for later in the study and researchers’ observations. Maykut and Morehouse (2003) stressed these notes “contain what has been said and heard by the researcher, but without interpretation” (p. 73). The researcher journal also included other
information pertaining to the study such as activities, ideas, interview appointments, calendar, deadlines, and decisions that changed.

**Pre-Test/Post-Test**

A pre-questionnaire assessing students’ understanding of both procedural and conceptual questions on linear functions were given. A similar questionnaire was used at the end of the unit to compare results with changes in numerical values. The questionnaire consisted of problems from the Partnership for Assessment of Readiness for College and Careers (PARCC) type tasks and Ohio Resource Center (ORC) examples. These problems were selected because they corresponded with what students should know when they complete the unit on linear functions according to state and Common Core Standards.

**Video Recording**

One source of data for this study was videotaping the whole class, small groups, and one-on-one interviews. The videotape equipment was continuously running throughout the study to record student conversations and observe student reactions. The video recorder was placed in the back of the room and another one in the front set up by the media specialist in the researcher’s school.

Through the use of video recording, the researcher coded the specific gestures and body movements that students performed when they were given graphs to complete with paper and pencil, answering questions verbally or explaining to a peer (Appendix E). By observing and only watching the students’ body movement during class and not listening to their voices, the researcher observed whether students were using their body to create
the graphs to help them answer questions on paper or in conversation when explaining graphs. Observations showed “firsthand encounter with the phenomenon of interest” (Merriam, 2002, p. 13). By using observational data of the videos, a first-hand encounter was represented. By using videotapes of every lesson over the entire unit of graphing linear functions (approximately 10 days), the researcher viewed the instruction as a researcher rather than as the instructor. Classroom instructional activities alternated between small group discussion and whole class discussion for sharing progress of the task at hand. The instructional tasks were designed for students to express their answers, questions, thought process, opinions, voice, misconceptions, and understanding of graphing linear functions through kinesthetic movement. Through viewing videotapes during student testing, surveys, and pre- and post-tests, the researcher noted if students referred to kinesthetic movement to help them answer questions. In group work, the researcher assessed if students used their body to express how graphs looked to their group members when discussing problems they were working on. The researcher was able to view if students changed their kinesthetic graphs by looking and or listening to other students when the other students described or defended their answers.

**Audio Recording**

The audio recordings coincided with the class video observations to ensure accurate reporting. The audio recording was situated in the middle of the room during whole class activities. When students were in groups, there were audio recordings happening of each group. Through the use of audio, the researcher had the opportunity to transcribe data and to hear if the students referred to their body movement when
explaining a graph. In addition, the researcher was able to listen several times to the tapes and effectively listen to what the students said or explained in class or during the individual interviews. The use of tapes was beneficial to the researcher in the analysis stage for accuracy in interpretation of student activity.

Watching the videotapes occurred daily. Merriam (1998) explained that data analysis for case studies happens simultaneously with data collection. He emphasized the “interactive nature of data collection, analysis, and reporting” (p. 153). These ongoing findings affected what types of data to continue to collect and were used as a guide to questions I asked in the one-on-one interviews or in the following day’s lessons.

**Interviews**

Seidman (2006) found that “listening is the most important skill in interviewing” (p. 78). The goal of the one-on-one interviews was to gain a deeper understanding of whether the participants were making translations among representations when students were learning linear functions with kinesthetic movement. Merriam (1998) suggested that an interview log be used to gather information pertaining to students’ feelings, and other factors that could influence the data collected.

Four students were randomly selected from the class using purposeful stratified sampling. Using only the students and the parents who agreed to participate in the one-on-one interviews, I chose four students from different groups. The first student was randomly chosen from the group of students who preferred kinesthetic movement; the second student was randomly chosen from the group who did not feel they would use kinesthetic movement; the third student was randomly chosen from the whole class; and
the fourth student was chosen because he was struggling in math class. The interviews were videotaped and transcribed by the researcher.

During the final interview with students, they were asked to read the transcripts of their own one-on-one interviews and asked to report any inaccuracies of the transcription of their interview. Interviews occurred during the student’s study hall time at school or at a scheduled time deemed appropriate by the student and parent. Students were asked to elaborate on their answers to written test questions. For example, I would ask them, “Explain negative.” “Why is your graph going left to right?” Students were also asked to explain their body movements and what they were thinking when they were doing their body movements. Interviews were used to gain an understanding of student thought processes and perceptions of using kinesthetic movement.

I wanted to delve into what students’ perceptions were with using kinesthetic movement, thus instead of just interviewing a group of four students, I chose to interview every student in the class for an exit interview. This gave me a clearer picture of what students’ perceptions were.

Another interview protocol that I changed was with the one-on-one assessment interviews. After examining which questions were missed on the assessments, I decided to have one-on-one interviews with students who agreed and who missed at least one question on the assessments. These interviews gave me additional data to analyze on students’ content knowledge of linear functions and their ability to translate between and among their different representations.
**Attitude Survey**

The attitude survey (Appendix D) was composed of open sentence completion prompts designed to elicit perceptions of what students think about kinesthetic movement when learning linear functions. It was adapted from the Burke Reading Survey (Goodman, Watson, & Burke, 1987) and correlated with attitudes identified by NCTM (2000). Although students’ written responses to the questions enabled a teacher to understand students’ perceptions of kinesthetic movement, the survey focused on students’ feelings and attitudes toward using kinesthetic movement when learning. The open-ended format provided rich information about what students thought as they used their bodies to learn. The attitude survey gave an additional level to information on what students’ perceptions and understanding were when learning linear functions with kinesthetic movement.

**Data Collection Summary**

With an abundance of data sources, I gained a more complete understanding of what students’ perceptions were about using kinesthetic movement while learning linear functions. I collected data through their conversations, physical movements, and surveys of how students translated among and between the different representations of linear functions with kinesthetic movement.

**Data Analysis**

Merriam (1998) saw the data collection and analysis process in case studies as “recursive and dynamic” (p. 155). Creswell (1998) agreed and suggested using a model that is spiral in nature and found that data being collected repeats and builds on previous
findings. Yin (2003b) maintained that data analysis consists of “examining, categorizing, tabulating, or otherwise using qualitative evidence to address the initial propositions of a study” (p. 109). Yin further stated that “case studies are generalizable to theoretical propositions” and explains that a case study is not a representation of a sample but the main goal is to generalize theories” (p. 10). In my study, the data analysis involved searching for patterns in the data with the propositions having a focus and purpose that guided the data collection and discussion.

A combination of qualitative guidelines proposed by Merriam (1998) and Congdon, Novack, and Golden-Meadow (2016) were used when interpreting the data. The guidelines for video research in education helped analyze and interpret the data from the class videos (The Data and Research Development Center [DRDC], 2015). Following a review of the tapes, I noted and transcribed dialogue and student actions. Notes and transcripts were used during the initial viewing of videos. After the data sources were collected, audiotapes and videotapes of the whole class were transcribed by an outside source after my initial viewing. From the transcription, I then listened to the audiotapes from small groups and watched videos from the whole class again. For accuracy, I compared my transcription notes to professional transcriptions. After the transcriptions were completed coding occurred.

**Coding**

I used a coding system that has been established and validated by researchers who have coded gesture movements (Iverson & Goldin-Meadow, 2005; Perry, Breckenridge, & Goldin-Meadow, 1988). The focus of coding gestures/body movements was to
document the communication of their movements when given information on linear functions. Body gestures and movements that I focused on were arm, hand, vertical and horizontal shifts of the body, eye, and any other body movements. Initial coding was separated into three categories: gesture itself, speech, and a combination of gesture and speech. Three sources were used with the coding gesture itself: gesture form, placement, and communication. The coding of speech alone tabulated students who did not use any part of their body when discussing linear functions. The coding of gesture and speech had two categories: The first category looked at which body movements complemented speech; an example given by Iverson and Goldin-Meadow (2005) is pointing at a flower when saying flower (p. 368). The second category coded gestures that supplemented speech; meaning body movement is different but related to the information. Iverson and Goldin-Meadow used the example of speech as saying “nap” while pointing to a picture of a sleeping bird (p. 368). By applying a systematic coding approach to analyzing and categorizing the data, emphasis was placed on the categories that emerged from the findings. The focus was on the significance of understanding these categories as they evolved into themes. To help organize non-numeric data, the NVIVO software package helped to organize non-numeric data. NVIVO assisted in the management of data because the software provided unlimited “bins” into which data were collected, organized, and then color-coded data into a spreadsheet. In addition to the creation of bins, this program organized the recording of source detail, the time and date of the data collection, storage, and search capabilities. A list of categories/propositions that have emerged from the literature include:
- What is body movement accomplishing? (Goldin-Meadow, 2009b)
- How does student conversation with kinesthetic movement happen? (Goldin-Meadow & Alibali, 2013a)
- When students were taking a test, did they use their body to help guide them to their solution? Video was taken to show if student used their body when graphing the function. (Goldin-Meadow et al., 1999)
- When students were working in groups and discussing their work, did students use their body to explain or express their graph? (Goldin-Meadow, 2001)
- Did students compare their kinesthetic graph to others in the area? (Goldin-Meadow & Beilock, 2010)
- If students compared were they reflecting on their own kinesthetic graph and making changes? (Goldin-Meadow et al., 1999)
- What were student perceptions on kinesthetic movement?
- What were the students saying about the kinesthetic movement?
- Were students translating among the different representations with kinesthetic movement? (Goldin-Meadow & Shteingold, 2001)

Answers to these questions provided a better understanding of what students had to say about kinesthetic movement in the classroom when learning to graph linear functions. According to Yin (2003a), propositions assisted the researcher in collecting and generalizing the data sets.
Validity/Trustworthiness

Rigor for quantitative studies is established through reliability, external and internal validity, whereas in qualitative studies “trustworthiness” is considered the standard (Lincoln & Guba, 1985). Qualitative researchers understand that for a study to qualify as trustworthy a detailed explanation of the data collection, analysis procedures, and outcomes are known throughout the study. The four components of trustworthiness are: dependability, transferability, credibility, and confirmability (Lincoln & Guba, 1985). Creswell (2009) believed for qualitative research to be trustworthy, the researcher needs to use at least two of the eight verification procedures in any given study. These include: persistent observation, triangulation, peer review, negative case analysis, and clarification of researcher bias, member checking, rich thick description and external audits. In this study I used triangulation, member checks, and thick description.

In order to avoid bias because of my teaching relationship with participants, I asked an independent mathematics educator to assess a sample of the attitude survey data. Agreement between coders (myself and independent coder) was measured by classifying gesture and gesture/speech combinations.

Dependability

The main question to be answered for dependability is: “Are the results going to be consistent over time?” By the researcher providing a detailed account of the research process, the readers were able to gain an understanding of the study and the processes used. The reader then was able to ascertain whether the researcher’s conclusions are logical and consistent with the findings (Brantlinger, Jimenez, Klingner, Pugach, &
Richardson, 2005; Merriam, 2002). Before the transcription process was completed, member checks were conducted so that students could comment on whether the researcher had accurately captured their voice of the topic at hand. Member checks were used to validate participants’ words and process feedback. Maxwell (2005) stated member checking as

> The single most important way of ruling out the possibility of misinterpreting the meaning of what participants say and do and the perspective they have on what is going on, as well as being an important way of identifying your own biases and misunderstanding of what you observed. (p. 111)

The member checking process involved students coming to my classroom and reading the transcriptions from interviews and class recordings. Creswell (2007) believed member checking makes the research dependable.

**Transferability**

Transferability is asking the question, “Can the findings of this research relate to other situations outside this study?” Through researcher logs and detailed descriptions of processes used in this study, transferability can be solidified through the readers making connections to their own work (Lincoln & Guba, 1985; Miles & Huberman, 1994). Through rich description with sufficient detail, other teachers may find the context of this study and apply the findings to their own classrooms. Transferability invites the readers of research to make connections between elements of a study to their own experience (Barnes et al., 2012).
Credibility

According to Lincoln and Guba (1985), triangulation is essential for establishing credibility of results because of the variety of sources of data. Through the use of multiple sources of data being collected, the researcher was able to provide the thick rich description (Merriam, 2002, p. 29) of students when they were learning to graph linear functions with kinesthetic movement. Observations from the classroom were checked and compared with survey results, assessments, and interviews. Multiple sources of data were crosschecked. These different sources of data provided information of students’ learning linear functions with kinesthetic movement. In the final stages of analysis, data triangulation of the themes, codes and or patterns were identified to show the importance of conversation, interaction, and social atmosphere for listening to students.

Confirmability

Because I am the researcher/teacher who listened, viewed, read, and interpreted the research, I needed to continue looking at my own bias, subjectivity, and personal assumptions to make sure I reflected and reported on all information that was presented. By stating my beliefs and assumptions about education, students, and my classroom, the reader was made aware of my views. Miles and Huberman (1994) viewed a key criterion for confirmability is the extent to which the researcher admits his or her own predispositions.

Pilot Study

Yin (2002) recommended a pilot study be completed to refine data collection. A pilot study was conducted a year earlier with another eighth grade Algebra class. The
pilot study was used to guide the current study in planning methodological details. The questionnaire was changed to include more Common Core Mathematics questions. The attitude survey was altered to enable students to reflect on their attitudes and feelings of kinesthetic movement in the classroom.

**Ethical Considerations**

Ethical considerations are important to consider prior to engaging in any research involving students; practitioner research is no exception. Researchers who conduct research within their own classroom and submit it for peer review must consider the ethical ramifications of conducting an investigation through the use of their own students. The primary concern was confidentiality (Glesne, 2006). Student confidentiality was observed throughout the research. No student names were used in the documentation of data or in the research paper. All research from data collection is kept in a locked cabinet at the researcher’s house. Student identity in the videos was coded so their real names were not used.

The researcher came to the research knowing that working and researching in the same classroom had the potential to cause some ethical dilemmas. Students may have felt that they wanted to please the researcher, or may have been concerned about their grades and may have performed for the camera. Initially each parent and student signed a consent form to participate in the study and/or the one-on-one interviews. Throughout the study, I had checkpoints and asked if anyone did not want to be taped or interviewed. To reassure students who participated or did not participate in this study, I did not score
their work used during this research project. I also continued to express and reinforce the following:

- The purpose of the study
- There are no correct or incorrect answers to the interview questions
- That all responses are entirely confidential
- That students should inquire immediately if there are any questions or concerns about the study
- That their input about kinesthetic movement when learning linear functions is vital and valued in relation to the study
- Students know/told they may withdraw at any time

**Summary**

This qualitative study utilized practitioner research, because I saw a need to improve teaching linear functions and transmitting those results. I used a case study methodology to gain a more in depth and rich description of students learning linear functions with kinesthetic movements. This study used a variety of data sources and key characteristics of case study research in its design. These included a natural setting, member checks, self-reflection, commitment to the participants’ perceptions, and acknowledging the researcher as part of the study. Data sources included documents, interviews, audiotapes, videotapes, and adequate time collecting the data. Through these many sources, I collected rich descriptions (Merriam, 2009) of students learning linear functions through kinesthetic movement.
CHAPTER IV

ANALYSIS OF THE FINDINGS

The purpose of this chapter is to present the findings from multiple data sources collected in the classroom when students learned to graph linear functions through a kinesthetic approach. These findings could provide teachers with new approaches in teaching algebra especially in particular linear functions. The participants consisted of 26 students from my eighth grade algebra class. A smaller sub-unit of four students from the same algebra class was part of the individual interviews for data collection. The smaller unit added more depth to the data being collected. When referring to particular students in either the classroom or one-on-one interviews, pseudonyms were substituted for the names of each of the participants. Students were referred to as Student A, B, C, and so forth. This chapter is comprised of verbatim examples and illustrations of data collection from this study.

The class that was used in this study was considered to be advanced; students were accelerated one year in mathematics compared to their peers. This study was structured around the following research questions:

- What are students’ perceptions and understanding about learning linear functions with kinesthetic movement?
- How do students (if they do) translate between and among mathematical representations through kinesthetic movement (kinesthetic-verbal, kinesthetic-symbols, kinesthetic-real life, kinesthetic-graphical, kinesthetic-oral)?
This chapter is composed of three sections. The first section summarizes the pre-test and learning style inventory. This section gives an overview with what level of knowledge students had at the beginning of the study. The second section gives a detailed account of the themes that emerged from data collected while answering each research question. The final section summarized themes and sub-themes collected from the data.

**Background Data: Learning Style Inventory and Pre-Test**

The learning style inventory and pre-test were given to the students before the unit on linear functions was taught.

**Learning Style Inventory**

The learning style inventory (Appendix A) came from Fleming and Mill’s (1992) VARK classification system. This inventory was used because it not only incorporated input learning preferences but also preferred output methods of students’ knowledge. This inventory categorized the students into four types of learning styles that either overlapped or stood alone. The four main learning categories were visual, kinesthetic, aural, and reading. Figure 5 illustrates the learning style data collected from 26 students. From the figure, one can see that the majority of students ($n = 17$) used multiple learning modalities.
Figure 5. Learning style inventory results

Pre-Test

The pre-test (Appendix C) was given to inform the teacher/researcher with information on student’s prior knowledge and to document student growth and understanding. According to the Ohio Model Curriculum Mathematics (ODE, 2015), algebra students should have knowledge in their pre-algebra class of how to define, evaluate, and compare functions. Students should also be proficient in modeling relationships using numerical and algebraic quantities. In discussion with their seventh grade pre-algebra teacher, students were taught these concepts. Students were taught graphing linear functions in seventh grade. Students also were taught using the method of creating an x-y chart to graph the equation on a coordinate plane. Students were to select at least 3 numbers to substitute for the x values to find the corresponding y values; then students were to connect 3 ordered pairs on a coordinate plane. Vocabulary words included the words: function, linear equation, slope, horizontal, vertical, zero slope,
undefined slope, positive slope, negative slope, and y-intercept. All students were shown linear equations written in different forms such as slope intercept form $y = mx + b$ and standard form $ax + by = c$.

Student responses to the question, “What is kinesthetic movement?” centered around three statements: “I don’t know,” “movement as a physical force,” and “moving while learning.” In summarizing the data, there were nine students that responded, “I don’t know” or left the question blank. Five students associated kinesthetic movement with learning, whereas 12 students discussed kinesthetic movement as a physical force.

Three questions on the pre-test asked students to define vocabulary words with four other questions requiring responses that should have included vocabulary words. I found through the pre-test data, students had an informal understanding of vocabulary, yet had not acquired the academic language of linear functions. Some examples of the lack of formal terminology are included in Table 4.

Table 4

Student Responses and What Vocabulary They Should Have Used

<table>
<thead>
<tr>
<th>What students said:</th>
<th>Vocabulary term they were referring to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line going up and down</td>
<td>Vertical line</td>
</tr>
<tr>
<td>Line going across</td>
<td>Horizontal line</td>
</tr>
<tr>
<td>Line rising to the right</td>
<td>Positive slope</td>
</tr>
<tr>
<td>Traveling up</td>
<td>Slope increasing</td>
</tr>
<tr>
<td>Steeper, going down</td>
<td>Negative slope</td>
</tr>
<tr>
<td>Traveling down</td>
<td>Undefined slope or vertical line</td>
</tr>
<tr>
<td>Going up and down</td>
<td>Y-intercept</td>
</tr>
<tr>
<td>Start at different places, up and over</td>
<td>Slope</td>
</tr>
</tbody>
</table>

The discrepancy between informal and formal understanding was most noticeable in students’ responses when asked if they could and how they would graph \( y = 2x + 4 \). Eighteen students said “yes” they could graph the equation on a coordinate plane. The next question asked the students how to graph that equation; only five of the 18 explained they “would make an x-y chart then plot the points,” five other students tried to remember shortcuts but confused slope and y-intercept, and the final eight students wrote “IDK” (I don’t know) or left the answer blank. This showed there is a discrepancy between what students think they know and what they actually know.

When students were asked to explain the graph of \( x = -4 \), students had difficulty finding the right words to use. The correct math response would be “it is a vertical line going through \( x = -4 \).” Only eight students wrote \( x = -4 \) is a “vertical line.” Instead of using the word vertical line some students \((n = 4)\) explained the graph as a “line going up and down,” but did not go further stating it went through the -4 on the x-axis. Some students used the words “straight line” \((n = 3)\) to describe vertical. This could be misconstrued as a vertical, horizontal, or a diagonal line.

Despite the fact that all students were given a mnemonic device for “horizontal line” (the horizon) in seventh grade, 12 students did not know what a horizontal line looked like if given an equation similar to \( y = 7 \). However, when students were asked to explain the graph of \( y = 7 \), more students knew the word horizontal compared to the vertical terminology and the graph of the \( x = -4 \). A few students described the graph as “going straight across” or “left to right” instead of the word horizontal. One student explained the graph as “seven units up then go straight across.”
Two questions on the pre-test asked students to explain how two equations were similar or different. Students’ responses were partially correct or totally incorrect. Students would talk about the slope and not the y-intercept. Examples of student responses are given in Table 5 along with the equations they were explaining.

Table 5

*Student Responses Answering How Equations are Similar and/or Different?*

<table>
<thead>
<tr>
<th>Example 1: ( y = \frac{1}{2}x + 3 ) to ( y = \frac{1}{2}x - 4 )</th>
<th>Example 2: ( y = -x ) to ( y = -2x )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Partial Correct Responses</strong></td>
<td><strong>Partial Correct Responses</strong></td>
</tr>
<tr>
<td>Graphs start at different places.</td>
<td>One graph is steeper</td>
</tr>
<tr>
<td>Slope is the same.</td>
<td>Both are negative</td>
</tr>
<tr>
<td>One graph you are adding three and the other you are subtracting 4</td>
<td></td>
</tr>
<tr>
<td><strong>Incorrect Responses</strong></td>
<td><strong>Incorrect Responses</strong></td>
</tr>
<tr>
<td>One is negative the other is positive</td>
<td>Increasing Graphs</td>
</tr>
<tr>
<td>One is increasing the other is decreasing</td>
<td>( x ) values are different</td>
</tr>
</tbody>
</table>

The pre-test showed students struggled with writing linear functions in different forms. An example of this is shown when students were given two equations that were the same graph but written in different forms. Students were asked to compare the following two equations, \( 3x + 4y = 12 \) to \( y = -\frac{3}{4}x + 3 \). The first equation is written in standard form and the second equation is in slope-intercept form. Only three students responded that they were the “same equation.” None of the students explained that the
first equation was written in standard form and the second equation was in slope-intercept form.

The responses obtained through the pre-test questions showed students recalled partial information they should have known from the previous year and often incorrect. Students used informal language to describe concepts while academic language was either not used or used inconsistently.

**Research Question 1**

This section lists the five themes that evolved from answering Research Question 1 followed by a description of each of the data sources and how they related to developing that particular theme.

Research Question 1 (RQ1): What are students’ perceptions and understanding about learning linear functions with kinesthetic movement?

Five themes that emerged from the data answering RQ1 include:

- Students used their bodies as a communication tool.
- Students felt energized and enjoyed using kinesthetic movement.
- Students used kinesthetic movement as a tool to recall and remember.
- Kinesthetic movement was a useful visualization tool.
- Students used their bodies subconsciously.

**Theme 1: Students Used Their Bodies as a Communication Tool**

Students showed what graphs looked like by whole body movements, gesture movements with their hands and arms, and movement with their hands using their pencils. An example of student movement is exemplified in Figure 6 to show an example
of how students used body movement. The stick figure in Figure 6 is a representation of the graph of \( y = -4 \). The horizontal line of \( y = -4 \) is being represented by the arms, and the -4 is being shown by the knees being bent so the arms go horizontally across the y-axis at -4 which is below the x-axis.

*Figure 6. Student Body Movement Demonstrated Graph of \( y = -4 \)*

Usage of their bodies as a communication tool emerged through careful analysis of students’ presentations, small group activities, interviews, and data collection of recordings in class. Throughout videotape data there were other examples that showed students representing graphs other than their full body. These gestures and movements were demonstrated by students who used their fingers, hands, and sometimes pencils to communicate through body movement. This theme was found in each data source of the
research. Because this theme was so dominant in the collection of data I have data sources separated to organize this theme.

**Presentations.** The six groups were given two or three equations to present. Their assignment (Appendix E) and rubric (Appendix F) were given to students prior to the group project. Each group randomly chose their two or three equations to compare and contrast. The project was graded on oral communication, content correctness, visual aid, and completeness of explaining 7 components of the graphs. The components included:

- Identifying slope and relating its qualities,
- Identifying the y-intercept in terms of translations,
- Comparing positive and negative slopes,
- Classifying relations as functions or not,
- Describing differences or similarities of graphs and functions,
- Describing differences or similarities of x-y charts,
- Identifying how x-y tables are different or the same as the equation and graph.

Group members chose the visual aid required for this project. I offered suggestions of using an overhead, poster board, PowerPoint, chalkboard, paper and pencil, glogster, prezi, dry erase mini boards, and video. I also would have accepted the students using their bodies as a visual aid, but I did not give that as a suggestion. I wanted to see if students would ask or present with their bodies when explaining their project to the class. The project had to be completed during class or in their academic assist class (AA). No part of this project was to be completed at home.
All six group presentations showed that students used their bodies to communicate what the linear function looked like when they presented their project. Some groups used whole body movements whereas others used hand gestures. One group used their bodies’ heights to show the difference between where each equation and y–intercepts were located on the graph. They also showed how the lines were parallel with their bodies. Throughout the presentations students subconsciously used their bodies when they discussed the graphs. All six groups presented visual representations of their graphs through using their bodies to represent the graph and show the similarities and differences between the equations they were given. Below is a summary of each group’s presentation and excerpts of their project.

**Group 1.** Students used a PowerPoint to guide their presentation. They were trying to explain what a y-axis was and the first presenter said, “y-axis is the secondary vertical axis of a system of coordinates, points along which a value is zero for all other coordinates.” I asked them to explain in simpler language; they ended up showing me what a y-axis is with their hands making the movement of a vertical line up and down. When the first presenter continued to explain the difference of their slopes, he used the word “steepness” and moved his hand along the two slopes on the PowerPoint to show how their steepness was different. The second presenter explained the difference or similarities of the y-intercept for y = x and y = 2x. He did not use his body but said, “both graphs are mostly the same, they both intersect at the origin and both had positive slopes, but y = 2x is steeper than y = x” (no body movement or pointing). The third presenter compared their charts and he did not use his body in explaining.
Group 2. Students incorporated a poster for their visual aid. The students did not use their bodies to become the graph; however when they presented their poster, students used gestures with their hands and fingers and pointed on the poster the differences and similarities of the two equations: \( y = 2x + 3 \) and \( y = \frac{1}{2}x - 2 \).

![Figure 7. Group 2 poster for presentation](image)

Group 3. Group 3 used a poster for the class to follow along with their project. Each person took a different topic to present. Graphs were displayed on the poster (see Figure 8); however, when students were presenting their part all the presenters used their bodies along with words in their communication of their PowerPoint slide. This group ended with a visualization of imposing three different equations with their bodies. Three of the students in the group are of very different heights. Figure 9 is an example of their bodies’ communication when they compared \( y = x + 4 \), \( y = x + 7 \), and \( y = x - 10 \). One student showed with her hand how 7 is higher than four and -10 was lower than the
others when explaining the differences of y-intercepts. The students, through the use of their bodies, communicated how the graphs were parallel and had the same slope with their arms; the height of the graphs was different because their y-intercepts were different.

Figure 8. Group 3 poster representing three ways function can be represented

Figure 9. Group 3 representing the three different graphs with their own heights
The tallest student is showing \( y = x + 7 \). The second tallest student is \( y = x + 4 \). The shortest student is the graph of \( y = x - 10 \). (She could not get in front of the second student because of room constrictions and another student being in that spot.)

**Group 4.** Group 4 had a difficult time when they presented their material. Their equations were \( y = -2x \), \( y = -x \) and \( y = -1/2x \). Their presentation was hard to follow, because their graphs were drawn incorrectly on the board. They did not discuss what was in their PowerPoint. When their presentation was complete, I felt obligated to lead a class discussion that led to a better clarification of their project.

This group did not use their bodies to communicate their knowledge. Student W had been very adamant about not wanting to use her body when she talked in front of her peers. When she communicated without her body, her vocabulary was not correct. She confused positive and negative slope; she struggled with the steepness of the line. You could see she was trying not to use her body. When group 4 completed their presentation Student K in their group asked if she could try and explain in a different way. She asked if she could use her body to communicate. When she used her body with words there was complete understanding. You could visually see her working and thinking through the steps when she was moving from one graph to the other.

**Group 5.** Group 5 used a PowerPoint to lead their presentation, with pictures of graphs \( y = 3/4x - 2 \) and \( y = 4x - 2 \). Students pointed to their graphs when giving responses to which graph belonged to which equation. One of the presenters showed with her arm movements what a vertical line meant. Another presenter pointed to the y-intercept on the PowerPoint to clarify a question from the class. The vocabulary used
to compare and contrast slope were the words “less steep” or “closer to x-axis,” and “more steep, closer to y-axis.” When students compared the charts, the presenter showed “farther apart” by separating her hands.

**Group 6.** Their presentation started with a PowerPoint with equations $y = x$, $y = -x$, $y = 4$ and $x = 3$. The first presenter explained, “$y = x$ is this.” (He showed me with his body and used words.) “Then it is a line that passes through the origin, it has a slope of 1; the $y$-intercept is 0. It has a positive slope and it is a function.” The second presenter read off the chart and explained the equation $y = 4$. He stated, “the slope is zero and yes, it is a function.” I asked, “How do you do a slope of zero? Explain that more to me.” He said, “it doesn’t go up or down, it just stays straight like this.” (He showed me a horizontal line using his arms.) The third presenter explained the equation $x = 3$ is a vertical line demonstrating with his arms moving up and down in a vertical motion. The group was asked to differentiate between $y = x$ and $y = -x$. One student started talking while another student started moving his body. This was done in unison without them knowing that the other student was using his body while the one student explained with words. The body movement went with the following statement, “$y = x$ is a positive slope like this” (other students showed the correct body graph) “and then – $x$ is like that” (other student showed the correct body graph.)

**Small group investigation activity.** Another data source that gave input to the theme that students used their bodies as a communication tool was found when students were given six tasks within a Graphing Lines Investigation Activity (Appendix E) to be completed by their small groups. Task 1 asked them to graph on their graphing
calculator, $y = 2$, $y = 4$, $y = -3$ and $y = 5$. Students were then asked to describe the lines, sketch the lines on a coordinate plane, tell where the lines crossed the $y$- and $x$-axis, use their bodies to show the graph, and in summarizing make a conjecture about $y = a$ (constant). The recordings of the class showed students were excited and used pointing gestures when they used the graphing calculators that are newer because the lines that are graphed were in different colors compared to earlier models. Five of the six groups used their bodies to show what these graphs looked like when they were completing Task 1. An example happened when a student represented the graph of $y = 6$. A student demonstrated in the group what $y = 6$ looked like; he had his arms straight out horizontally aligned with his shoulders and he went up on his tiptoes to show the $y$-intercept at 6.

In between the six tasks, I had checkpoints and would have whole class discussions over each task before they moved on to the next task. Students were asked to describe what a zero-slope looked like and the function $y = 6$. Students used their bodies to communicate these functions. Figure 120 shows examples of how students used their bodies to communicate. The stick figure has its arms out horizontally to represent a zero slope.

\[\text{Figure 10. Student used their bodies to show what a zero slope looks like}\]
During the class discussion, students agreed that all functions that are \( y = \text{some number} \) are zero slopes. Based upon the \( y = \text{some number} \) the graphs would be different because of the change in \( y \)-intercepts. Students were moving up on their toes when the number was positive and squatting down when the number was negative. Most students used their whole body to show the difference between the graphs. Two students used their hands to show the difference. The graphs of \( y = 3 \), \( y = 6 \) and \( y = -2 \) are shown in Figure 11 to show the differences students were trying to express. All students showed with their bodies how the graphs were different by going up on their tiptoes for \( y = 3 \), higher on their tiptoes for \( y = 6 \), and squatting to show \( y = -2 \). Figure 11 shows what the graphs would look like on a coordinate plane. Students used their bodies to represent the graphs rather than verbally describing them or drawing them.

<table>
<thead>
<tr>
<th>( y = 3 )</th>
<th>( y = 6 )</th>
<th>( y = -2 )</th>
</tr>
</thead>
</table>

*Figure 11. Graph images of functions students were representing with their bodies*
In an open discussion in class, students were asked how they would differentiate between the linear functions $y = 6$ and $y = 10$ using their words, body, or both if they had to teach someone these functions. Six students out of 26 raised their hands and thought it was easier to show the difference between the graphs their bodies. Eight students said they would use both words and their bodies, and the rest of the class was non-committal in raising their hands.

At this point of the study the model of our body as a linear function was incomplete and we needed to have a discussion of the alignment of body with the corresponding parts of the coordinate plane. We then had a discussion that explained the different representations of our bodies. Our arms represented lines of the graph. Our upper chest/collar bone denoted the $y$-intercept and our body from head to toe vertically was the $y$-axis. Because students used their arms to represent the function line, students did not have a body part to show the $x$-axis except when the graph was $y = x$. The $x$-axis was one’s shoulder to shoulder; the origin was their throat and upper chest. Because of how our bodies moved when representing the different graphs that did not go through the origin we needed to have an alternate image in our minds to represent the $x$-axis. As a class, we discussed how you had to recall that your shoulders were the $x$-axis but when the $y$-intercept was higher or lower than the origin, you needed to recall the original $x$-axis as an imaginary replica. This description is shown in the figures located in Figure 12. Once students had an understanding of what body parts correlated to the parts of the coordinate plane, students started to compare and argue about their bodies’ graphs.
Figure 12. Student body movements representing parts of the graph

Task 2 investigated the concept of the y-intercept given the equations, $y = x$, $y = x + 2$, $y = x + 5$, $y = x - 3$ and $y = x - 7$. Groups continued with the same process of students working together in groups graphing equations on their graphing calculators, describing the graphs, sketching the graphs, using their bodies to be the graph, and
making a conjecture about the y-intercept in the equation. Video recordings showed four groups used their bodies to explain the graphs to their peers.

The data from the video recordings documented in the transcription notes found students used their bodies to communicate what the functions look like at task 2 checkpoint when we gathered as a whole class compared to task 1 checkpoint. The video recordings also showed students were more comfortable with using their bodies and comparing their graphs; the recordings showed students were laughing and joking with each other about their bodies’ graphs. Lastly, the data showed students used their bodies more when they had to explain their graphs and when students had difficulty they used their bodies to show what the graphs looked like.

Students worked in their groups on task 3 which featured equations that were focused on slope of the lines for $y = x$, $y = 2x$, $y = 4x$ and $y = 6x$. Students continued to use their bodies to communicate these functions in conversations with their groups. Students demonstrated these graphs by making their arms slant higher or steeper than the $y = x$ graph. Students also communicated with their bodies to clarify the vocabulary term “steeper.” One student used the word “higher” to mean steeper while the other group member was confused by the word “higher.” To clarify the difference between “higher” and “steeper” the students went through arm movements to differentiate between higher and steeper. Figure 13 demonstrates this comparison.
Working on tasks 4 through 6 of the small group investigations, students continued to use their bodies to communicate their understanding of how graphs were similar or different. From observations of the four days, students worked in small group investigations with daily checkpoints. It became apparent that at the beginning of the investigation students used their bodies more to communicate their ideas; however by the end of the investigation students were including gestures and vocabulary together.

**Individual interviews.** Because I needed more in depth description of when students used their bodies to communicate, I requested individual interviews of four students. The video recordings of Students A, B, C, and D showed students communicated with their bodies when they had to compare graphs. Students were given the equations $y = x$ and $y = 2x + 3$. In each of the four interviews, students demonstrated the processes of comparing the graphs by using their bodies. All students did the following steps: They started with the graph of $y = x$ then moved to $y = 2x$ (changing the slope by slanting their arms from left to right). Students then showed $y = 2x + 3$.
(changing the y-intercept by moving up on their toes). I noticed in the data collection, students were first changing the slope then the y-intercept. (Note: this is not what students do on paper. Most students plot the y-intercept first then work on slope.) The main reason for this procedural change is that writing on paper requires erasing if starting with the slope first. The recordings of the individual interviews also added data that students used their bodies to communicate their understanding of linear functions.

**Attitude survey.** When students answered questions on the attitude survey (Appendix D), their responses related to how they used their bodies as a communication tool. A subtheme arose from the attitude survey’s questions #12 and 13. Question #12 in the attitude survey asked: How can kinesthetic movement help you? Students explicitly mentioned that their movement helped them to communicate the mathematics as they were extending their arms. Students not only mentioned that using their bodies helped them to communicate, but also in Question #13, students consistently mentioned that communication was easier. One student stated, “Kinesthetic movement helps me when it’s hard to find the words to describe an equation.” Whereas, another student felt that “kinesthetic movement can help you communicate to others what you are thinking.” Both these students expressed how using their bodies is a form of expressing what they know.

Students also expressed in the attitude survey what they thought was best about using kinesthetic movement. Their comments focused on kinesthetic movement made it easier to communicate and understand and that it was another resource to use when communicating mathematics. Their comments were as follows:
“The best thing about kinesthetic movement is that it’s hands-on and instead of trying to explain a line or graph with words, when you use your body it makes it much easier.”

“I can communicate my ideas without using words and other people can understand what I’m trying to get across.”

“The best thing about kinesthetic movement is another way to explain something besides words and moving around the classroom.”

“That [kinesthetic movement] it makes linear equations simpler, and it gives another tool to help solve and visualize equations. It also gives another tool to teach in class.”

Students summarized the value of kinesthetic movement through their explanation of a problem. In a class discussion with written responses, students were given the equation $y = 2x - 3$. They were asked two questions, the first question, “How would you explain to someone the graph of $y = 2x - 3$?” The second question asked, “How would you explain to someone who is having difficulty understanding the graph of $y = 2x - 3$?” More than 50% of the students would use their bodies to communicate what they know about the equation (See Figure 14).
Theme 2: Students Felt Energized and Enjoyed Using Kinesthetic Movement

The second theme emerged through data collected from video recordings, interviews, and the attitude survey. In each of these data sources, students felt energized and enjoyed using kinesthetic movement. Throughout data collection of video recordings of whole class discussions, I heard students’ laughter and giggles when students performed, and watched each other use body graphs to represent linear functions. This is documented in the transcripts of class discussions. At one point in one of the recordings there was a discussion of a student who looked like a bird and other students along with this student smiled, laughed, and imitated the linear functions with bird movements.

During the interviews students were asked if they liked using kinesthetic movement when learning linear functions. The majority of students ($n = 18$) in the exit interviews said “yes” they liked using kinesthetic movement when learning. In the exit
interviews, I asked the students to explain why they liked kinesthetic movement. Some students expressed it is “hard to sit” in a chair all day; other students thought it helped them to be “more focused” when they had a chance to move around in the classroom. Students also expressed that the kinesthetic movement did not have to be educational; for example, student Q talked about how he/she enjoyed class when “I post answers on the board and they get out of their seat and check their answers, then go back to their seat.” For some students, just the motion of getting out of their seats is enjoyable.

An unexpected outcome occurred when students expressed they felt good getting out of their seats. I wanted to explore this further. In the exit interviews, I noticed the first three students expressed they felt more energy; this made me question what was making them feel more energetic. In the rest of the exit interviews, I then asked students to elaborate on what they physically felt when they used kinesthetic movement. I questioned, did they physically feel different by using kinesthetic movement? The responses were generally informative but brief. From the transcription notes, and NVIVO categorizing, I broke their responses into two categories, one dealing with an energized feeling and the other focusing on enjoyment. Students’ comments centered around re-energizing, and refocusing. These were almost universal comments.
Table 6

Examples of Student Comments to What They Felt When Using Kinesthetic Movement in Class

<table>
<thead>
<tr>
<th>Energy</th>
<th>Enjoyment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refreshed</td>
<td>Wish more teachers included kinesthetic movement because they [students] hate sitting.</td>
</tr>
<tr>
<td>Focused and Awake (4)</td>
<td>It is more fun than just sitting there doing nothing.</td>
</tr>
<tr>
<td>More energized (5)</td>
<td>Liked moving and being out of seat. (3) **</td>
</tr>
<tr>
<td>Re-energized</td>
<td>I don’t really have a problem focusing. It was engaging.</td>
</tr>
<tr>
<td>Because then I wasn’t fidgety or bored, I was – like I could refocus *</td>
<td>Because then I wasn’t fidgety or bored, I was – like I could refocus.</td>
</tr>
<tr>
<td>Re-invigorated, I usually like, twirl my pencil or bounce my leg</td>
<td>Yeah, I feel more awake than I was down there</td>
</tr>
</tbody>
</table>

Four students said they physically did not feel different when using kinesthetic movement. I continued to ask questions about their capability of staying focused in class. All four students felt they have never had an issue with focusing. The following are statements from two of the four students.

Student W: “If I just sit down and I say, I have to do this, I just focus.”

Student B: “I didn’t feel any different, but like getting out of my seat.”

The attitude survey also gave evidence that students liked using kinesthetic movement and they felt more energized. The attitude survey (Appendix D) adapted from the Burke Reading Survey of Attitudes and Perceptions (1987) asked questions about kinesthetic movement. The attitude survey is composed of open sentence completion prompts designed to elicit perceptions of what students think about kinesthetic movement. It was adapted from the Burke Reading Survey (Goodman et al., 1987) and correlated with attitudes identified by NCTM (2000). Students’ written responses to the
questions enabled me to understand students’ perceptions of kinesthetic movement. The survey focused on their perceptions toward using kinesthetic movement when learning. The open-ended format provided rich information about what students thought as they used their bodies to learn. Twenty-six students took the survey; however, not all students answered every question. Throughout the attitude survey students stated they liked moving around and not sitting in class. Students expressed they would like to continue to be able to use kinesthetic movement in other classes. Students’ responses on the attitude survey were consistent with interviews. This consistency showed that whether students were responding anonymously or in a face-to-face setting, the theme was verified and further triangulation was supported in the video recordings. Students enjoyed and felt energized when using kinesthetic movement when learning linear functions.

Students summarized the theme of energy in their written comments on the survey. One student wrote, “We need to move around in class otherwise we are sleepy, bored, confused, and have nothing to refer to when the test comes.” Another student wrote, “I think that you should keep doing this and I wish that more teachers did it because it keeps me up and active all day and ready to participate in class.”

**Theme 3: Students Used Kinesthetic Movement as a Tool to Recall and Remember**

An expected theme that evolved through the data collection was the theme that students used kinesthetic movement as a tool to recall and remember mathematical content. One student wrote, “I think that we should have done even more kinesthetic things because maybe then I would use it more and remember the stuff we learned with it more.” Another student’s response also dealt with kinesthetic movement helping them
remember. “Once you do something with your body and move around I feel it helps people remember it better.”

Question number 14 on the attitude survey (Appendix D) asked the students to write anything else they felt was needed to help me (researcher) to understand their thoughts on kinesthetic movement when learning linear functions. The results for this question fell into three categories: Kinesthetic movement helped students focus (Theme 2 RQ1), students liked moving (Theme 2 RQ1), and kinesthetic movement aided students in remembering and recalling information. One of the written responses stated, “We need to move around in class otherwise we are sleepy, bored, confused, and have nothing to refer to when the test comes.” One student in particular wrote that she felt kinesthetic movement helped her learn more efficiently because the movement helped her to remember.

Question 15 in the attitude survey asked the students to express if there is anything else they felt that I (teacher) should know about using kinesthetic movement in the classroom to teach any concept. Several students commented kinesthetic movement helped them to remember. One student wrote, “Once you do something with your body and move around I feel it helps people remember it better.” Another student wrote that the kinesthetic movement helped her go through steps and think through the equation—“It helps more of a step by step.”

After reading students’ comments multiple times I realized that another theme emerged, I then watched the video recordings again to verify the theme of kinesthetic movement being used to recall and remember that dealt with what the students wrote
about in the attitude survey. The video recordings of students in interviews showed students actually went through a step-by-step process with their fingers of what a linear function looked like before they spoke. The gesture movements of their fingers showed students were recalling what the graph would look like before they spoke. Recordings of assessments also confirmed students recalling and remembering through gestures. During the assessments, some students used their pencils as the graph and showed how it changed from the graph of \( y = x \) to another transformation. Students also showed with their pencils how they went through the different steps before they wrote down their answers on the test.

**Theme 4: Kinesthetic Movement Was a Useful Visualization Tool**

The Attitude Survey (Appendix D) asked the students if they thought kinesthetic movement helped them learn and why. The responses to these questions were categorized into several categories. Two categories showed repeated results. First, kinesthetic movement helped students to remember (Theme 3 RQ1) and visualize the graph. Figure 15 showed that nine students responded that they felt that kinesthetic movement was a visual tool.
Figure 15. Does kinesthetic movement help you learn and why

**Theme 5: Students Used Their Bodies Subconsciously**

After conversations with students and collecting data of student body movement from video recordings and interviews, another theme evolved. Students did not realize they used their bodies to communicate their knowledge of linear functions. Through the course of the study, I wanted to know if students used any visual representation in their mind when they would answer their test questions. I wanted to see if students had moved from an external body representation to an internal one. I asked the students a final question on their formative assessment (Appendix G): “At any time during the formative assessment did you refer to your mind, body or both when taking the assessment?” Figure 16 has the represented data for student responses.
Figure 16. Student responses on whether they used their mind and/or their bodies when working on the summative assessment

Figure 16 shows that the majority of students thought they have used their mind, while the video recordings of their assessment showed 11 of the 18 students who stated they used their mind only also used their bodies during the assessment and the students did not realize they had. Other data collected from the video of students taking the formative assessment showed three of the five students who said “no,” they did not use their bodies during the assessment in fact did. They moved their fingers and created a graph when answering the problems, before they wrote their answers down. These students did use kinesthetic movement as an aid to solve the problem but used it through gestures.

The data from video recordings also showed that every student in this study at one time or another used their bodies to communicate their knowledge of linear functions. In
the one-on-one interviews after an assessment (Appendix H), I had students who missed at least one question review their answers. In one-on-one interviews, I asked the student(s) to explain what they meant when they compared a negative graph to a positive graph. The students who missed this on the assessment could not tell me with words, but every one of them \((n = 8)\) showed me with their bodies in some form. If a student could not tell me with words, they showed me. I would then further question and ask for more detail and their gesture movements corresponded with their words in the final assessments. Some of the students used their arms \((n = 2)\) and some used only their hands \((n = 6)\).

One student in particular had a difficult time and was very unyielding that she did not use her body to communicate. Student W was adamant in one-on-one interviews and class discussions that she did not use her body to communicate. I would show her video of when she used her body to communicate and she would get very frustrated. During the group presentations, when Student W did not use her body, her words were incorrect and she struggled. When Student W did use her body with words she was correct in mathematically communicating her knowledge. Student W was one of the 8 who missed the question on the assessment describing the differences between a positive and negative graph. She continued to be opposed to using her body when discussing linear functions. In the exit interview, she sat on her hands and tried not to move her arms when she would answer questions I asked. When she realized, she was going to use her body or a gesture, she tucked her hands further under her legs. Every time I had a one-on-one interview with her, she used some form of gesture motion. When she realized that she did, she
would stop using her body. Student W stated, “I do not like using my body.” When she would watch a video of herself she was quite bothered that she used her body. I have tried to have a conversation with her about why she does not like using her body. She explained that kinesthetic movement was silly and not mathematical. It was not wrong or right that Student W used her body in this study; it is simply noted that she was adamant that she did not want to use her body.

Other video recordings of an assessment showed students who used their bodies, subconsciously using gestures during a quiz. During the quiz, I noticed that six students used gestures when working out problems. These students used a kinesthetic movement with their pencils, hand, fingers, or arms before they wrote their answers down. All six students would read the question on their paper, followed by one of the above kinesthetic movements, then ending with writing their answer down. Their process of answering problems showed students used a step-by-step movement. The students started at \( y = x \) (parent function), then moved the imaginary line with using information of slope and y-intercept, followed by the movement of up or down depending on what the y-intercept was. Students used their bodies and showed slope first, then moved their bodies to demonstrate where the y-intercept was located. An interesting fact is that most people, when using paper and pencil, mark the y-intercept on the graph paper, then draw the slope.

During the post-test, students were asked if they referred to their bodies during the final assessment. Data showed 15 students believed they recalled body movements in their mind, 10 students believed they did not refer to their bodies or their mind, and one
student said they used both methods. As a further note, no students mentioned that they used their bodies anymore at this point in the unit. However, the video recordings showed that some students continued to use their bodies to represent the graph through small gestures with their fingers and pencils. These gesture movements occurred more often when students were challenged to explain a higher-level problem.

**Research Question 2**

Research Question 2 (RQ2): How do students (if they do) translate between and among mathematical representations (kinesthetic-verbal; kinesthetic-symbols, kinesthetic-real life; kinesthetic-graphical; kinesthetic-oral).

Three themes emerged from the data answering Research Question 2:

- Kinesthetic movement can be used as a tool for developing conceptual understanding.
- Kinesthetic movement helped students form authentic vocabulary.
- Kinesthetic movement helped students translate among representations.

To answer RQ2, I used video recordings of presentations and analyzed how students used their bodies and words in this study. The video recordings used in the data collection process were taken from presentations, group work, and daily video of class. I watched video recordings in several different ways. The first time I viewed the recordings I tabulated the number of times students used their bodies as a communication tool without sound. The video recordings were then watched with sound; then data were tabulated on how often the students used words with body movements, the number of times they used their bodies without words, the number of times they did not use their
bodies when explaining graphs/equations/charts, the number of times their bodies matched the words they were saying, and the number of times their bodies did not match their words. Both students’ talk and gestures were tabulated throughout the unit, as opposed to recording just the voice as was done in conventional transcription, this method captured the interaction of gesture and talk for students as they grappled with solving a task with which they have said or shown to have had difficulties. This analysis allowed me to focus on how gesture was being used without talk being present, as well as in conjunction with talk. It provided an avenue that allowed me to explore whether gesture helped students with the task, or whether the gesture and talk guided students to resolve their confusion or trouble. One of the goals of any mathematics instruction is to have students move toward the abstract. I wanted to see if body movements were increasing or decreasing by the end of the study to determine if students moved towards the abstract. This data collection information helped to form themes that answered RQ2. Figures 19 to 23 show data of how students communicated their understanding of linear functions throughout the study.

Figure 17 shows that students used combinations of words and gestures while presenting their projects. The presentations project was at the beginning of the study and this data suggested students were not as familiar with vocabulary terms.
Figure 17. Data representing students’ form of communication during presentations

Figure 18 shows students used their bodies more frequently during the one-on-one interviews. The one-on-one interviews were used to gain more information to describe students’ experiences of learning linear functions through kinesthetic movement.

The after-assessment interviews were conducted with students who missed questions on the assessment. Figure 19 shows students used their bodies more frequently when students were clarifying and communicating their answers that were incorrect on the assessments.

Figure 20 shows when students were communicating in whole-class setting with their peers, students used their bodies the majority of the time. Figure 21 shows by the end of the study students did not use their body as frequently or exclusively as they did at the beginning of the study. It appears that students used their bodies as a supplement with their words. This showed students have started to move towards the abstract.
Figure 18. Data representing students’ form of communication during one-on-one interviews

Figure 19. Data representing students’ form of communication after assessment interviews
Figure 20. Data representing students’ form of communication during presentations

Figure 21. Data representing students’ form of communication during exit interviews
The data in Figures 17–21 suggested that students used their bodies, gesture movements, and language to help them make connections, communicate, and learn linear functions. The data from the after-assessment interview in Figure 19 showed students used their bodies 21 times to explain their missed answers, whereas the exit interviews documented one student used her or his body one time. This data showed students have moved to abstract thinking with linear functions. The next sections show the data above and how it helped develop the three themes when answering RQ2.

**Theme 6: Kinesthetic Movement Can Be Used as a Tool for Developing Conceptual Understanding**

Once again, I found that not only did students use their bodies to communicate what they knew about linear functions, but they also used kinesthetic movement to aid in their understanding. This was shown in one-on-one interviews when students were asked to clarify their answers they missed on an assessment. I conducted one-on-one interviews with every student who missed at least one question on the formative assessment (Appendix G). Students showed me with their bodies the correct answers in the one-on-one interviews. They just could not put their answers into words with mathematical meaning in written form. Every one of these students who missed problems on the formative assessment showed their understanding of linear functions at an informal level by how they used their bodies to communicate their understanding of linear functions. During the formative assessment, students were permitted to use words, pictures, or any other method they thought would be useful in showing me that they understood the concept of linear functions. These students did not use any of these
representations to answer. An example of a one-on-one interview question was: I asked the student(s) to explain what they meant when they compared a negative graph to a positive graph. The eight students who missed this on the assessment could not tell me with words, but showed me with their bodies in some form. Some of the students used their arms \((n = 2)\) and some used only their hands \((n = 6)\).

![Figure 22](image)

**Figure 22.** Student A represented a \(y = x\) a positive function. Student B represented a \(y = -x\) a negative function

Another example happened during a group discussion. One classmate struggled with understanding the difference between \(x = 8\) and \(y = 8\) and what the graphs transferred to on a coordinate plane. In the group discussion vocabulary words were used, students pointed to the graph, however it was not until another student stood up and physically showed the struggling student what the graphs looked like that he finally understood. First the student moved off the origin on the coordinate plane to the \(x\) coordinate \((8,0)\) the student kept his hands at his sides and showed the vertical line of \(x =\)
8. This is shown in Figure 23 by Student A. The student then stepped back to the y-axis and lifted his arms horizontally and went up on his toes and represented the horizontal line of \( y = 8 \) (Student B in Figure 23).

![Figure 23](image)

*Figure 23.* Student A represented \( x = 8 \); Student moved away from origin to \((8,0)\) on the graph. Student B represented \( y = 8 \). Student moved back to the origin \((0,0)\)

As shown through the data collection of video tapes, one-on-one interviews, and class discussions in this section, students demonstrated they knew how to translate between equations and graphs through their bodies movements. At this point the students in the study did not have the mathematical vocabulary to properly express their understanding. Consequently, this led to the next theme.

**Theme 7: Kinesthetic Movement Helped Students Form Authentic Vocabulary**

This theme evolved through video recordings, interviews, exit surveys, and formative assessments. Video recordings of class discussion showed when students did
not have understanding of vocabulary terms during a class discussion, their understanding of the vocabulary word was enhanced by kinesthetic movement. For example, students were supposed to explain the difference between two graphs that had different y-intercepts. The first graph was \( y = x \) and the second function was \( y = x + 2 \). One student explained to the class that the graph of \( y = x + 2 \) would be “steeper.” When this student was asked to explain “steeper,” he struggled. In the class discussion, other students demonstrated with their bodies what steeper meant. The student then thought for a moment and showed with his body what he meant by steeper. This student came to understand the word he was trying to use was higher on the y-axis and that the word steeper should be used when explaining slope and the rate of change.

**Theme 8: Kinesthetic Movement Helped Students Translate Among Representations**

Kinesthetic movement helped students reflect and construct understanding of linear functions. Examination of the video recordings of one-on-one interviews showed some students referred to their fingers, pencils, and arms before they answered questions, whereas others used only their bodies with no words, and several students used words and bodies at the same time. When interviewing students, I would ask questions in several ways:

“Describe the graphs of \( y = x \) and \( y = -2x \).”

“Can you tell me what they look like?”

“What do you mean by ‘lower down’?”

“What are you showing me right now?”
“Explain for me diagonal.”

“Describe it, you can use any method you want. You can draw it. You can tell me, or you can show me.”

The following is an example of a conversation of a student’s thought processes when asked to compare two functions. This student was asked to compare the functions $y = x$ and $y = -2x$. The student had a difficult time expressing how they were similar or different. The student gave me the wrong answer, paused, then with only using her fingers with no verbal communication she showed me the correct answer. I asked the student, “What made you change your mind there?” The student stated, “I just saw it in my mind.” This student used the gestures/movements of her fingers to represent the graph; then she used verbal communication that demonstrated she could translate between different representations.

Another example was shown in the small group investigation activity. After students completed the first two sections of the packet, I gathered the whole class to discuss the first two sections. In watching the video recordings, some patterns became evident. Students gathered knowledge from their peers when their peers used their bodies. It must be noted that students had to be told it was ok to compare their bodies’ graphs to their peers. Some students thought this was cheating and not permitted. When students noticed that their bodies’ graphs were different than their peers ($n = 6$), they would rethink their bodies’ graph. You could visually see them go through the steps with their bodies. Figure 24 showed the steps the students went through when using their bodies to explain how the graph changed physically.
<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = x</td>
<td>Y = ½ x</td>
<td>Y = ½ x + 3</td>
</tr>
<tr>
<td></td>
<td>“The line became more [shallow] or closer to the x-axis.”</td>
<td>The student went up on their tiptoes to show moving up 3 spaces for the y-intercept at 3.</td>
</tr>
</tbody>
</table>

![Diagram of a student standing on tiptoes]

**Figure 24.** Example of students using step-by-step process

Another example, which showed that students were able to translate between and among the representations, was a progressive activity over three days (Appendix F). Every day the students were given a paper that had the three representations on it. Day 1, students were given the equation and had to complete the chart and the graph that corresponded to the equation. Day 2, students were given the chart and had to fill in the equation and the graph. Day 3, the students were given the graph of the linear function and had to fill in the chart and the equation. The purpose of this activity was to check student understanding and translation among the different representations. Table 7 summarizes 24 student responses to this activity.
Table 7

Student Translation Between and Among the Different Representations of Linear Functions

<table>
<thead>
<tr>
<th></th>
<th>Equation</th>
<th>Function Table</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 1</td>
<td>Given equation:</td>
<td>13 students did</td>
<td>11 students did the</td>
</tr>
<tr>
<td></td>
<td>$f(x) = \frac{1}{2} x + 3$</td>
<td>did table first.</td>
<td>graph first.</td>
</tr>
<tr>
<td>Day 2</td>
<td>8 students used the</td>
<td>22 graphed the function first. Of the</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>chart to make the</td>
<td>equation.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>equation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Given Chart:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
|       | $\begin{array}{ll}
|       | X  & Y \\
|       | -4 & -7 \\
|       | -1 & -1 \\
|       | 0  & 1  \\
|       | 4  & 9  \\
|       | 8  & 17 \\
| Day 3 | 20 students did the | 3 students did the | Given the graph: |
| 1 student absent | equation first. | function table first. | |

Table 7 demonstrated all students could translate between and among the different representations of linear functions. However, the data collected showed students prefer to move from a graph to an equation and vice versa than translating to the table. On day 1 when students were given the equation, they needed to create the chart and the graph from the equation. Students did not really have a preference of which one they chose to do first. Day 2 students were given the table to create a graph and an equation. Data showed most students preferred to graph the equation from the chart rather than making the equation first. Also, 16 students who created the graph then created the equation.
from the graph. They did not use the chart at all to create the equation. All students in
the class could do these translations between different representations. However, the data
showed more students felt comfortable going from a graph to an equation than from a
chart to an equation. This also related to day 3 data. Students were given the graph and
had to create the table and equation. The majority of students chose to do the equation
first. This data suggested that students translate between and among the different
representations; however, most students preferred moving between the graph and the
equation more than the chart.

Major Findings

This chapter presented the findings of my research study based on the results from
many sources of data including presentations, assessments, video recordings, attitude and
exit surveys, and pre- and post-tests. The major findings are as follows.

Analyses of the data collected indicated that when students communicated their
mathematical understanding of linear functions with their bodies, they were better able to
communicate what they knew, and others were learning from their gestures. Results
based on student responses from the exit and attitude survey suggested that, overall,
students liked using kinesthetic movement. Usage of kinesthetic movement aided in
students gaining a better understanding of vocabulary terms that corresponded with linear
function terminology. Finally, the results based on students’ post-test suggested that
kinesthetic movement helped students move from an informal understanding of linear
functions to a higher level of comprehension where they no longer needed to use their
bodies to understand, but they used their bodies to complement their knowledge. This
was also demonstrated in student responses on the attitude survey (Appendix D) to question 5 and 12. In question 5 students were asked: “What do you think was the most beneficial in learning linear functions?” The student responses showed 11 students felt that using their bodies had the most impact on their learning. Figure 25 shows a summary of students’ responses to question 5.

![Bar Chart: Most beneficial in learning linear functions](chart)

**Figure 25.** What is the most beneficial when learning linear functions

However, when students were asked in the attitude survey with question number 12: “Does kinesthetic movement help you learn and why?” only two students felt that kinesthetic movement did not help them learn. The summary of their responses is shown in Figure 26.
Figure 26. Does kinesthetic movement help you learn and why

The results of the attitude survey do show that even though 11 students thought kinesthetic movement was the most beneficial tool in learning linear functions, 24 of the students found value in using kinesthetic movement with linear functions.

**Summary of Themes**

By the end of this study, students translated between and among the different representations of linear functions with kinesthetic movement. Kinesthetic movement guided students from an informal understanding where they used their bodies, gesture movements, written pictures, and pointed fingers to demonstrate the mathematical vocabulary for linear functions. By the end of the study, students showed that they no longer needed to use their bodies and gestures; however, these movements helped their vocabulary become precise. (See Table 8.)
Table 8

*Themes That Evolved From Data Collection*

<table>
<thead>
<tr>
<th>Research Question 1</th>
<th>Research Question 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>What do students’ voices have to say about learning linear functions with kinesthetic movement?</td>
<td>How do students (if they do) translate between and among mathematical representations (kinesthetic-verbal; kinesthetic-symbols, kinesthetic-real life; kinesthetic-graphical; kinesthetic-oral)?</td>
</tr>
<tr>
<td>Students used their bodies as a communication tool.</td>
<td>Kinesthetic movement can be used as a tool for developing conceptual understanding.</td>
</tr>
<tr>
<td>Students feel energized and enjoyed kinesthetic movement.</td>
<td>Kinesthetic movement helped students form authentic vocabulary.</td>
</tr>
<tr>
<td>Students used the kinesthetic movement as a tool to recall and remember.</td>
<td>Kinesthetic movement helped students translate among representations.</td>
</tr>
<tr>
<td>Kinesthetic movement was a useful visualization tool.</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER V
DISCUSSION AND IMPLICATIONS

The purpose of this study was to describe middle school algebra students’ understanding of linear equations through kinesthetic movement. Some of my initial questions dealt with middle school learning experiences and what actions I could take to improve student learning in algebra. One of the main questions I was interested in dealt with answering the question about why middle school students struggle when learning linear functions and translating between different representations of linear functions. Since I wanted to improve my practice and share my findings with other teachers, I realized that the best methodology to employ in my study was practitioner research.

This study took place in my classroom; therefore, I was participating both as the researcher and the teacher. I thought that the theoretical framework based on Piaget’s Cognitive Theory and Vygotsky’s Sociocultural Theory aligned with my research questions, as the purpose of the study, where describing students’ experiences of learning linear functions through kinesthetic movement was essential. I included both social and cognitive learning theories since I have observed that my students’ experiences with peers and their own individual experiences complemented the learning experience in the classroom. Through the use of a theoretical framework, which focused on both the social and cognitive learning theories, I investigated whether kinesthetic movement helped students in the construction of knowledge with learning linear functions and the different mathematical representations of linear functions.
Another construct of the theoretical framework for this study was mathematical representations theory that applied Lesh’s Translation Model (Lesh et al., 1987). This model provided a foundation for translating between and among the different representations of linear functions. Lesh’s model utilized different representations from pictorial, verbal, written, manipulatives, and real life situations of mathematical ideas. His model emphasized the importance of translating between and among the different representations of linear functions. Through the use of the different modes, students gained a conceptual understanding by making connections in the different representations of mathematical knowledge (Lesh et al., 1987).

The final construct of the theoretical framework was kinesthetic movement in learning. This component of the theoretical foundation was based on determining if there was a relationship between learning linear functions and kinesthetic movement. By examining student gestures and arm movements, I investigated whether students were able to learn and translate between and among the different representations of linear functions. I also was seeking to determine the role that gestures and arm movements had on the level of student understanding. Finally, I wanted to discover if embodied learning facilitated cognition.

In this chapter, the findings of the study are related to the literature and recommendations for using kinesthetic movement when learning linear equations. This section, which concludes with several teacher-as researcher reflections, provides recommendations for future research. Data sources for this research included interviews, pre-tests, post-tests, small group work, presentations, video recordings, audio recordings,
exit surveys, attitude surveys, and student artifacts. The following research questions were answered:

1. What are students’ perceptions and understanding about learning linear functions with kinesthetic movement?

2. How do students (if they do) translate between and among mathematical representations through kinesthetic movement (kinesthetic-verbal, kinesthetic-symbols, kinesthetic-real life, kinesthetic-graphical, kinesthetic-oral)?

The results of the data analysis demonstrated that kinesthetic movement played an important part of students’ translations between and among representations when learning linear functions. This section reviews the themes in light of current research and connections to the study of linear functions.

Based upon the frequency that a message or idea was communicated through the data, it became apparent that several major themes arose from the analysis. The study results showed eight themes that consistently emerged as the most prevalent messages or ideas that the participants of the study stated were the major experiences of students using kinesthetic movement when learning linear functions. The triangulation of data strengthened the emerging themes, which were shown through data synthesizing. The following themes emerged from numerous data sources and analysis.

**Discussion of Themes: Research Question One**

Through the use of case study methods and the data collected, I found five themes when answering Research Question One and three themes with Research Question Two.
The themes discussed were seen as being crucial to understanding students’ learning of linear functions while using kinesthetic movement. Although each theme is presented separately, some areas overlap.

**Students Used Their Bodies as a Communication Tool**

Across the study, it appears that students communicated with their bodies. Kinesthetic movement played an important part as a communication tool for students when they were learning vocabulary and linear function concepts. When students could not find the correct vocabulary terms to use at the beginning of the study, they used their bodies to explain and express their understanding. By the end of the study, most students did not use their bodies as a communication tool, but rather as a pictorial image that went with the correct verbal explanation of the concept. At the beginning of the study, gestures and body movement helped with communication and compensated for lack of vocabulary. This correlated with Goldin-Meadow and Alibali (2013a) findings, where “gesture movements give children the opportunity to express ideas that they are not yet able to express in speech” (p. 263). Goldin-Meadow and Alibali further explained that gesture movements are useful when the instructor uses terms that are aligned with the gesture the student is using; this, in turn would help children learn the vocabulary term that is communicated with body movements. As findings suggest throughout the triangulation of data, student gesture movements communicated their understanding of linear functions.
Students Feel Energized and Enjoy Kinesthetic Movement

Qualitative description of students’ responses to the exit and attitude surveys presented that students liked moving when learning, and that movement helped the students stay focused while feeling more energized. The video and audio recordings suggest that students enjoyed using kinesthetic movements. This was evident in the video recordings where students could be seen smiling and laughing when they used their bodies in small settings and in whole class settings. The information gathered from the analysis of responses showed that the majority of students’ responses revolved around the themes of energy and focus. This not only corresponded with my data, but also with other research that showed the benefits of physical activity including improved circulation, increased blood flow to the brain, and raised levels of certain “feel good” hormones, which can improve achievement (Taras, 2005). Research by Ratey and Hagerman (2008) also supported student responses in this study. Ratey and Hagerman’s brain research showed that kinesthetic movement enhanced the learning process for students. His study in Naperville, IL, found that students who used movement before class felt energized and outperformed their peers academically on tests. His belief is that there is a connection between higher performance levels and the hormones dopamine and norepinephrine, and other brain chemicals that are released when movement is incorporated into the classroom. When students move, a hormone is released in the brain and students feel more energized and focused during their test. Griss (1994) found that movement helped students with disruptive energy, which allowed them to reap the benefits of creative movement. When children used body movements, they enjoyed what
they were doing. These movements also helped them to stay focused and on task. Wolfman and Bates (2005) found kinesthetic movement helps kids concentrate more, because students are actively engaged in activities and not just sitting and getting bored. According to Westerhold (1998), “kinesthetic curriculum helps make cognitive connections real and puts under-motivated learners into motion” (p. 18).

**Students Used Kinesthetic Movement as a Tool to Recall and Remember**

The development of this theme came through the triangulation of data from several data collection sources. Video recordings showed that students used their bodies to help think through a question before they wrote an answer on a test. When students presented in their groups, it was evident that students used their bodies to demonstrate what the graph looked like before and after the translations of the functions. This theme was validated with students’ responses on the exit and attitude surveys, where they stated that they used their bodies as a tool to “recall” and “remember” linear functions in different representations. Similar findings also were reported by Day (2008), a researcher at Duke University, who found that dancers remembered movements in terms of words, images, and movement-based cues. When students in the current study worked with linear functions, body movement assisted in their recall of vocabulary and images of graphs. This also correlated with research from neurologist Willis (2006), who found that more regions of the brain that can store data were activated by movement, thereby making it easier for students to recall information. The reason that recall was simpler was that there were more interconnections and related pieces of information from multiple storage spaces in the brain able to be called upon. Goldin-Meadow (2010)
explained this process as not overworking the brain, since memory is stored in more places thereby “lightening the mental load.” Because students encode gestures in different brain locations, recalling information becomes easier. According to Susan Greenfield (2008), a British scientist specializing in the physiology of the brain,

As educationists, we often focus on using words and visual representations to deliver concepts in a classroom, which use only a fraction of the neural networks at our disposal. The motor system occupies a large portion of our brain and include some of the most evolutionary advanced systems. Thus, incorporating movement into the learning process offers a powerful tool for deepening the representation of a concept and improving the chance of its being laid down strongly in memory. (p. 37)

The notion that gestures aided in recall was affirmed by Gilakjani (2012), “By creating a rich, contextual environment, kinesthetic learning constructs memories, connected to time, place and emotions, which we call episodic encoding. Students activate and ingrate physical, emotional and cognitive responses to what they are learning, making learning more meaningful” (p. 2).

**Kinesthetic Movement Was a Useful Visualization Tool**

Students expressed that kinesthetic movement was a useful visualization tool in their mind, or that they used kinesthetic movement with body movements, hand gestures, or pencils in the adapted attitude survey. Although dissimilar in content, yet, similar in body movements, researchers from the University of Chicago, Frank and Barner (2012), have demonstrated the same idea that kinesthetic movement was useful as a visualization
tool when students were learning with an abacus. Their study investigated students who were taught with an abacus to do large math problems. Students then were given similar problems to complete without an abacus to determine how the students compensated, if at all. The students in the study did the problems without an abacus, but used their hands as if an abacus were there in order to create an internal visual representation. The movement was a visualization too that helped them solve their math problems. Frank and Barner also believed that working with the movement of the abacus aided in the students’ visual working memory. This was explained as visual representations becoming encoded in the mind. Other research showed that kinesthetic movement is used when teaching public speakers to present. Research from Toastmasters International, a group who provides instruction on becoming a better public speaker, showed that visual stimuli captured the audience’s attention, and therefore, enhanced the retention of the verbal message being presented. Iverson and Goldin-Meadow’s (1997) research also added credibility to this theme. She explained that gestures represent and give meaning to ideas such as size, shape, movement, and location.

**Students Used Their Bodies Subconsciously**

Throughout data collection, students were asked if they used their bodies when they explained their answers or when they were being assessed. Several students always stated that they did not use their bodies at all. However, video recordings showed that these same students used their bodies or some form of gesturing when they explained their answers. Gestures were also recorded when students took assessments.

Golden-Meadow’s research (2003) found that spontaneous gestures were representations
of one’s internal thoughts. Levy and McNeill (1992) contributed to this theme. They stated that gestures are “actually part of language—not a subsystem” (p. 364). Other researchers also believe that gestures are part of language and go hand in hand with our speech (Alibali, Kita, & Young, 2000; Garber, Alibali, & Goldin-Meadow, 1998).

**Discussion of Themes: Research Question Two**

The next section provides details of the three themes that evolved when answering Research Question Two.

**Kinesthetic Movement Can Be Used as A Tool for Developing Conceptual Understanding**

Through the use of kinesthetic movement, vocabulary comprehension of mathematical terms was maximized and substantial information was retained. Survey responses from students explained how they believed that kinesthetic movement aided in their understanding of vocabulary terms. Their kinesthetic movements assisted them with recalling and maximizing the meaning of the mathematical terms, and helped with longer retention of these terms. Research from Macedonia, Muller, and Friederici (2011) showed similar findings when they investigated students who were learning a new language. They discovered that students who used iconic gestures were more efficient than students who used pictures and enrichment activities. In my conversation with Goldin-Meadow (October 15, 2016, at 10:15 am), she summarized her research findings, which confirmed that there is an impact on conceptual understanding of math concepts when gestures are used. She further stated that two benefits are made with students’ understanding of vocabulary. First, gestures used aid others in understanding, and
second, gestures also aid in helping students move to symbolic representation of one’s own thoughts. Also confirming the correlation between kinesthetic movement and learning was a study by Ehrlich, Levine, and Goldin-Meadow (2006), which found that children who used gestures with mental rotations in mathematics learned more than the children who did not use gestures because gestures created thought.

**Kinesthetic Movement Helped Students Construct Authentic Vocabulary**

Kinesthetic movement represented students’ vocabulary words at the beginning of the study when they were not able to use words to describe what they were trying to say. Through kinesthetic movement, students’ vocabulary became more fluent, and comprehension of the mathematical understanding increased. Kinesthetic movement aided in the students’ abilities to communicate their knowledge. At the end of the study, the retention of vocabulary terms developed to the point where only a little kinesthetic movement was needed to express their ideas. This process of using kinesthetic movement allowed the students to physically demonstrate their mathematical knowledge and more fully understand the concepts. This research provided students with multiple ways to adequately demonstrate their understanding. The progression was from the initial rudimentary level to a more meaningful and fully developed understanding at the conclusion of the study. Similar findings also were found with Goldin-Meadow and Alibali (2013a). Their research suggested,

Gesture could play a causal role in language learning by providing children with the opportunity to practice ideas and communicative devices that underlie the
words and constructions that they are not yet able to express in speech. Repeated practice could then pave the way for later acquisition. (p. 262)

Another comparable study from Goodwyn, Acredolo, and Brown (2000) analyzed symbolic gesturing and infant language development. These researchers evaluated the effect of reinforcing communication through symbolic gesturing on verbal language development in hearing infants. The conclusions from my results matched Goodwyn et al.’s, which showed that verbal language acquisition developed when gestures were used. Sheets-Johnstone’s (2013) research proposed that kinesthetic movements are the pre-cursor for language development, and that corporeal concepts, such as “near or far,” “hard and soft,” and “open and closed,” are rooted in animated body movements. They argue that language is post-kinesthetic.

**Kinesthetic Movement Helped Students Translate Among Representations**

Over the course of the study, students were given graphs, tables, and everyday life situations of linear functions. At the beginning of the study, students appeared to rely on their bodies’ movements when solving problems. Students also did not use academic language but rather employed a social language when they communicated their ideas. By combining the use of social language and kinesthetic movement, students’ understanding translated toward a more formal definition of linear functions. Hostetter and Alibali’s (2008) research, which agreed with these findings, added that the physical interaction of combining motor and perceptual processes was important for forming mental representations of the world. Lindgren and Johnson-Glenberg (2013) found that purposeful, planned movement aided in student understanding. These researchers stated,
“cognitive processes involved in learning, such as conceptual development and comprehension, are built upon a foundation of physical embodiment” (p. 446). Other research further adds that gesture movements have the ability to “act as a bridge between thought and action because it is both kinesthetically close to action and yet also symbolic” (Cartmill, Beilock, & Goldin-Meadow, 2012, Para 2.)

John Dewey (1938) wrote, “We do not learn from experience . . . we learn from reflecting on experience.” Student responses in the exit and attitude survey specified that kinesthetic movement helped their minds coalesce their internal and external understandings. Through reflection, students had the opportunity both to use their past experiences with linear functions and to reflect on their kinesthetic movements in order to further develop a complete conceptual understanding of linear functions in various representations. Students also indicated that when they interpreted their peers’ kinesthetic movements of linear functions, they automatically compared their bodies’ graphs to their peers. This caused them to think more deeply about why their graphs were the same or why their graphs were different. Through this process of internal reflection, they made an interpretation about which graph was the correct interpretation of the function at hand.

Students appeared to move from an external representation of linear functions to an internal one. Cook and Goldin-Meadow (2006) presented findings that were similar. They found that when students used gestures, the gestures encouraged the visual imagery of a problem. The visual image accompanied and solidified their verbal representation, which in turn facilitated students’ learning.
Recommendations

Based on the findings of this study, there are three major recommendations for consideration. The following sections discuss recommendations for instruction, assessment, and future research.

Recommendations for Instruction

The data collected from the pre-test showed that students struggled with linear function vocabulary and its application to graphing. Students were aware of this vocabulary from a previous class and were taught a procedural way of graphing linear functions. I questioned why the students did not remember this material or the vocabulary terms one year later. I found that through the use of kinesthetic movement, students developed an understanding of vocabulary rather than employing a memorization strategy. Through my research and the kinesthetic activities as a backdrop, teachers may enhance student learning in their classrooms. Students may benefit by connecting multiple representations through using kinesthetic movement as an instructional strategy.

Kinesthetic movement has an element of inclusion, as it allows all students to be active learners in the classroom. Through their participation, students may make connections to different representations. When students were given the opportunity to compare and reflect on their kinesthetic movements in the classroom, it was clear that they felt more energized and focused when as they used their bodies as a learning tool. There also is a stronger possibility that students will more fully participate in class discussions when they are actively involved.
Kinesthetic movement was a visual aid in helping students decipher what graphs looked like. Goldin-Meadow and Alibali’s (2013a) research suggested that when teachers used gestures to communicate what they were teaching, students were given another tool, which aided them in comprehending and deciphering the meaning of vocabulary terms. This was viewed as an enhancement of the overall learning process.

It became evident through in-class quizzes, daily videos, partner work, and real-life situations that students preferred to move from a chart to a graph then to the equation. When students were given different representations of charts, graphs, and equations, students easily moved from the equation to a chart or to a graph. When given the graph, they also adeptly moved to the other two representations. However, when students first were given the chart, they moved to the graph before they wrote the equation. Students indicated that they did not feel like they were doing math when making the graph. Was this because students only see number crunching as mathematics? Or was it simply easier or more convenient to move from the graph to the equation because they did not have to calculate the slope using the function table? This finding implies that every teacher should try to teach multiple representations concurrently. Commonly, math educators have always taught students the order of first moving from the table to the graph. Importantly, students need to be able to go between any representation in any order.

Teacher education programs could play an important role by encouraging future teachers to consider kinesthetic learning experiences as strategies to encourage
conceptual understanding. Kinesthetic learning could be viewed as another knowledge and skill to inform good teaching practices.

Educators should create activities and learning experiences that allow students to use kinesthetic movement, as this not only will increase student attention, but also will help students to make sense of their understanding of a new concept.

**Recommendations for Assessment**

In this study, it became apparent that students had a difficult time expressing what they knew about linear functions. Students did not have the necessary vocabulary and sometimes could not fully develop the understanding necessary to express what they knew, without the use of kinesthetic movements. By creating formative assessments during classwork where kinesthetic movements were used, teachers can obtain immediate feedback about which students understand the concept being taught.

This study also showed that students understood more than what was indicated by the grading of their written assignments. It is recommended that teachers review questions with students who were not successful on the written assessment.

By incorporating kinesthetic movements with assessments, teachers allow students to feel more comfortable about missing an answer. Instead of the student being marked wrong, the student can change their bodies’ movements and reflect on how they actually do understand the material, but were simply not able to properly express their knowledge in the answer. Kinesthetic movement could be used to work with students after their written assessment. Using kinesthetic movement as an assessment tool creates opportunities for evaluating the student. Through this study, I found that kinesthetic
learning could be used as an alternative form of assessment, since it was obvious that students knew more than what they were able to put on paper. Based upon conversations with students, teachers should incorporate kinesthetic/gesture movements as an alternative form of assessment.

This research suggested that a variety of assessments would be beneficial to teachers as well as to students. The many data sources provided this teacher/researcher with a well-developed understanding of student knowledge, versus a more traditional comparison to a cross section of student knowledge found in a paper, pencil, and/or computer based approach. The results of this research recommend various opportunities for students to communicate what they know. Through gestures, a student may offer insight into their tacit knowledge as well as thoughts that are not articulated in speech (Garber, Alibali, & Goldin-Meadow, 1998). This qualitative study indicated that students had the knowledge of what a linear function looked like. However, they did not have the vocabulary established at the time of the written formative assessment. It should be recognized that when students do not have the mathematical language to explain their answers, one cannot immediately conclude that they do not understand the material. Rather, this may be an indication that the student does not have the necessary communication tools to describe their responses. In such cases, using their bodies is an effective strategy, as it allows students to physically demonstrate their mathematical knowledge.

Kinesthetic movement could also be used as a diagnostic tool. As Cartmill et al. (2012) found in their research, gesture may ground even abstract ideas in physical
actions. Through viewing students’ gestures, teachers have the ability to give immediate feedback to students, which in turn helps the students to reflect on misconceptions and understandings. Students can clarify what they meant to put on paper through gesture. In addition, the results of this study suggest that students performed kinesthetic movements to express their knowledge when given the opportunity to use their bodies as a communication tool. The kinesthetic movement helped students move through the different representations externally which aided in their internal representations. Through the external representations, teachers may examine students’ mathematical knowledge, understanding, and misconceptions of the mathematical ideas being taught.

This study suggests that students used their bodies to communicate and demonstrate their understanding of linear functions. When students use kinesthetic movement to represent their thoughts, teachers may aid in student understanding without students feeling uncomfortable because they do not know something. As an example of this, when a student does an incorrect body movement, and notices that the movement is different from peers, a quick change is able to be made without anyone else noticing. This beneficial immediate feedback to students’ visual representations may make a more comfortable learning environment.

The results of this research suggest some potential benefits to mathematics education. As in the use of other forms of mathematical representation, kinesthetic movement may be used to address a balance between and among other forms of representation. Greenfield (2008) suggested that movement in the classroom creates
connections between brain neurons. He believed that only using one mathematical representation can be detrimental to the learning process.

**Recommendations for Future Research**

The present study examined students using kinesthetic movement to learn linear functions in an algebra class. Thus far, very few studies have been conducted about the use of kinesthetic movement in the middle school math classroom. It is recommended by the researcher that longitudinal studies on kinesthetic movement take place. The following questions emerge: First, when students solve problems involving graphing of linear functions, do they refer to their bodies when communicating their knowledge? Second, do these students incorporate their bodies when learning other mathematical content areas? Third, did students’ understanding of linear functions transfer to other mathematics?

This study examined an algebra class where the students were one year ahead in mathematics compared to their peers. It is recommended that this study be duplicated but in an average or below average algebra class. Some questions that may be answered are: Will this study elicit the same findings? Will average or below average students benefit more, less, or about the same compared to students who are one year ahead? Will average or below average students communicate more or less with their bodies? A study that duplicates this study could add more information to best practices in mathematics when teaching algebra.

It is recommended by the researcher that further research be conducted with students involving brain function and kinesthetic movement, specifically within the
mathematics classroom. In exit interviews, students explained that they physically felt different when given the opportunity to use kinesthetic movement. They stated they felt more energized, focused, and awake. In a conversation with gesture researcher, Goldin-Meadow (personal communication, October 15, 2016) stated that she has never asked about what students felt when they used gesture.

Current and past research on kinesthetic movement included populations such as special education students, the arts, and elementary school settings. Research with kinesthetic movement in secondary education is sparse. Most of the research in the secondary level centered on special education and kinesthetic learners. Since my research suggests that there is a benefit to students when they are taught with kinesthetic movement, I question whether my qualitative findings can be found in other academic disciplines. Research with middle school and secondary students could contribute to the research on kinesthetic movement and on the understanding in mathematics as well as other subjects.

Another field of study to pursue concerns the learning sequence in problem solving. It appears that not only did students translate between and among representations, but they also performed mathematics in different sequences. This may allow for a more complete comprehension to develop. When I discovered that students transferred their gestures into pictorial representations of functions, but that the sequence of problem solving differed in both representations, the question arose about whether enhanced understanding could be achieved if ordering were different. As a concrete
example, it is noted that students may solve a problem differently using their bodies than when using paper and pencil.

**Reflection Through Both the Lens of Researcher and Teacher**

“I think that you should keep doing this and I wish that more teachers did it because it keeps me up and active all day and ready to participate in class.”

“We need to move around in class otherwise we are sleepy, bored, confused, and have nothing to refer to when the test comes.”

“Once you do something with your body and move around I feel it helps people remember it better.”

“I think that we should have done even more kinesthetic things because maybe then I would use it more and remember the stuff we learned with it more.”

“Helps kids to be engaged and active. Sitting and thinking all day breaks student’s focus, but moving around helps to make them more involved with the lesson.”

“Kinesthetic movement was a fun tool in class, and useful for solving and understanding linear functions and graphs.”

The above excerpts were from students’ exit and attitude surveys. These statements begin to reveal the importance of listening to students. Kinesthetic movement may be incorporated into any classroom with a positive impact. Kinesthetic movement also helped students stay motivated to learn and it could be strategically placed in the form of other activities within lessons.

By advancing our understanding of how students can learn through kinesthetic movement, we may empower teaching and learning by creating connections. This
research contributed to the literature on working with middle school students and their views on learning linear functions with kinesthetic movement. This study helped me to understand that students do not enjoy sitting at their desk all day. They want to move and be active learners. More importantly, it helped me to know that even though students may miss a question on an assignment or test, it does not necessarily mean that the student completely misunderstands the concept. Sometimes students do not have the formal vocabulary to explain their answers. Gesturing allows for the development of more discipline-specific vocabulary. Goldin-Meadow (personal communication, October 15, 2016) stated, “gestures allow[s] you a space for abstraction” and “gestures are tied to your speech it is not separate, it is part of your cognition.”

This research suggests there is value in using kinesthetic movement as a tool for educators to see the benefits of implementing kinesthetic/gesture movement into their grading procedures when checking for student understanding. By educating mathematics instructors about the value of kinesthetic movement in the math classroom, or any classroom, teachers can create an environment where students can be successful and active participants.

Researcher bias was a limitation. I believe that kinesthetic movement played an important part in learning linear functions; however, my research identified much more than I imagined. My role was that of researcher, to report what students had to say about kinesthetic movement and linear functions. Before this research, I believed that small group work was where I would collect all of my data and findings. That was not the case, as the data appeared in many different methods that I originally thought were not that
significant to this research. The adapted attitude survey gave me insights on student perceptions to kinesthetic movement. Students let me know what they liked to do in class. Students liked to move. Movement helped them recall information. Movement was a visual tool and a communication tool.

In my role, as both the researcher and the teacher in this qualitative study, I found that using practitioner research provided some personal advantages for collecting data. As the researcher, I was vested in wanting to know how successful kinesthetic movement was when teaching linear functions. This research showed me that I should be incorporating kinesthetic movement in my classroom more frequently. As the teacher, I wanted my students to be successful and have a conceptual understanding of mathematics. This desire was not solely based upon wanting them to achieve strong state test results, but also since I wanted them to have successful mathematical careers in high school and beyond. If something does not work, I do not want to incorporate it into my classroom. I want to improve my practice so my students can be successful. Mills (2011) believed practitioner research benefits the children and the teacher. Per Hensen (1996), practitioner research:

- Helps teachers develop new knowledge directly related to their classrooms.
- Promotes reflective teaching and thinking.
- Expands teachers’ pedagogical repertoire.
- Puts teachers in charge of their craft.
- Reinforces the link between practice and student achievement.
- Fosters an openness toward new ideas and learning new things.
• Gives teachers’ ownership of effective practices.

Before this study, I thought that kinesthetic movement played an important role in helping students translate between and among different representations through anecdotal evidence. This study provided rigorous support of students translating between and among different representations as videos, paper test answers, attitude surveys, and exit interviews. At the beginning of the unit, students would use their bodies in a step-by-step process to get to the final equations. At the end of the unit, students no longer went through the step-by-step process, but went straight to the final equation. During the formative assessments at the beginning, students used more pictures and fewer words, while on the final assessment students used more words and fewer pictures. This indicated that they had developed the vocabulary necessary to express their understanding of the question being asked.

The students went from an external representation to an internal representation with understanding linear functions. Students created and used external representations as tools for communication and problem solving. Student learning involved students forming connections among the different types of representations: concrete, pictorial, symbolic; verbal and visual; internal and external.

Through interpreting the exit interviews and attitude surveys, students expressed they had a positive experience in learning linear functions with kinesthetic movement. I found that when students enjoyed their lessons they actively participated more in class. Students were stimulated to stay on task through kinesthetic movement.
Kinesthetic movement was a useful tool to enhance information recall. Clark and Paivio (1991) believed there are two systems of representation (verbal and visual) that help students with storing information in their memory. Through the interconnection of verbal and visual coding, students had an easier time retrieving information with their linguistic and nonlinguistic explanations.

The importance of this work that has been completed may change the way math teachers teach linear functions and how they decide to teach different lessons. Educators should listen to what students like to do in class. Movement helped students think through the different graphs. Kinesthetic movement created a memory that connected their understanding of linear functions to a conceptual level by helping students translate between and among the different representations. Goldin-Meadow (2006) explained all of these because “students think with their hands.”

Based on my findings and the work of others in the field of education, I learned a great deal about kinesthetic movement when students were learning linear functions in an eighth grade Algebra class. In conclusion, kinesthetic movement could be a method that enables students to recall details (Golden-Meadow, 2001), create mental representations (Koschmann & LeBaron, 2002), help verbalize their understanding (Alibali, 1999), and retain knowledge (Alibali & Goldin-Meadow, 1993). Student body movements are a direct expression of their internal representations. In essence, the results of the study showed the themes that emerged are consistent with research findings. Kinesthetic movement should not be confined to the gymnasium, since there are many benefits for students in regular classroom settings. The evidence also showed that students benefitted...
from using kinesthetic movement. On an ending note, although I could not see into the minds of my students, I was able to listen to what they said with their bodies.

**Summary of Chapter 5**

This chapter summarized the major findings of this study in light of current research and recommendations for teachers, instruction, and assessment. There was a discussion of the summary of themes that evolved in this study along through numerous data sources followed by future recommendations for teachers, assessments, and further research. The chapter ended with a reflection of myself as the researcher and teacher in the study.
APPENDICES
APPENDIX A

LETTER OF CONSENT
Appendix A

Letter of Consent

Parental Consent Document

Informed Consent to Participate in a Research Study

**Study Title:** Case Studies Listening to Student Voice Using Kinesthetic Movement While Learning To Graph Linear Equations

**Principal Investigator:** Dr. Joanne Caniglia, Kent State University

**Co-Investigator:** Melissa Novak, Center Middle School

Your child is being invited to participate in a research study. This consent form will provide you with information on the research project, what your child will need to do, and the associated risks and benefits of the research. Your child’s participation is voluntary. Please read this form carefully. It is important that you ask questions and fully understand the research in order to make an informed decision. You will receive a copy of this document to take with you.

**Purpose**
To describe the voices of middle school algebra students and their understanding of linear equations through kinesthetic movement.

**Procedures**
Your child’s participation will require him/her to be in class and be a student. Data for the study will include pre and post-tests, surveys, video and audio taped lessons, class observations, and one-on-one interviews with some of the students. I will be looking at student’s body movements when participating in class activities.

**Audio and Video Recording and Photography**
Class will be video and audio taped for the length of the unit on linear equations approximately two weeks. I will be examining the tapes and coding student body movements when in class. I will be watching the tapes to see if students refer to their body when taking test/quizzes. I am looking to see if students communicate their math knowledge by using their bodies. The tapes will only be used for my research project and data analysis collection for this research. Still pictures of student body movements and not their faces may be incorporated into the final data analysis.
Benefits
This research will not benefit you or your child directly. However, your child’s participation in this study will help us to better understand what students have to say about using kinesthetic movement in the math classroom.

Risks and Discomforts
There are no anticipated risks beyond those encountered in everyday life.

Privacy and Confidentiality
Identifying information will not be made available in the publications and/or presentations of the research data. Information obtained from your child or materials (if applicable) that are collected from your child (i.e., class projects, homework assignments, journal entries, etc.) will not contain identifying information about you or your child. Throughout the report that I will write based on my findings, I will use pseudonyms for the community, school, and students. You may elect to have your child not participate in my research.

Your research information may, in certain circumstances, be disclosed to the Institutional Review Board (IRB), which oversees research at Kent State University, or to certain federal agencies. Your child’s confidentiality may not be maintained if there is an indication that he/she may harm themselves or others.

Compensation
Participation or non-participation will have no effect on your child’s grade in the classroom.

Voluntary Participation
Taking part in this research study is entirely up to you and your child. You and/or your child may choose not to participate or may discontinue their participation at any time without penalty or loss of benefits to which he/she is otherwise entitled. You will be informed of any new, relevant information that may affect your child’s health, welfare, or willingness to continue participation in this study.

Contact Information
If you have any questions regarding the research project, please contact me by email at novakm@strongnet.org or by calling the main office at (440)572-7090, where I can return your call at my earliest convenience. If you want further information on my research project, or you have any questions or concerns about this research, you may contact Dr. Joanne Caniglia at Kent State University 330.672.2580. This project has been approved by the Kent State University Institutional Review Board. If you have any questions about your rights as a research participant or complaints about the research, you may call the IRB at 330.672.2704.
Consent Statement and Signature
I have read this consent form and have had the opportunity to have my questions answered to my satisfaction. I voluntarily agree to grant permission for my child to participate in this study. I understand that a copy of this consent will be provided to me for future reference.

Parental Signature _____________________________ Date _____________________________

My completion and return of this (survey, questionnaire, or instrument) will be indicative of my consent to participate in this research study. I have been given a copy of this consent form” or “I may print a copy of this consent statement for future reference.”

Assent Form for One-On-One Interviews

Case Studies Listening to Student Voice Using Kinesthetic Movement While Learning To Graph Linear Equations

Procedure for obtaining assent from children

1. Hi ______ name ________________
2. My name is Melissa Novak, and I am trying to learn more about what students have to say about learning linear equations with kinesthetic movement.
3. I would like you to be a participant in my research.
4. Do you want to do this? ________________
5. Do you have any questions before we start? _____________________________
6. If you want to stop at any time just tell me. _____________________________
Sample Consent Form

Dear Parent or Guardian:

As we begin a new school year, I look forward to working with your son/daughter as his or her mathematics teacher. This school year will be my 26th year at Center Middle School. I eagerly look forward to another great year! In addition to being your son/daughter’s math teacher, I am in my final phase of earning my PhD in Curriculum and Instruction at Kent State University. My research is investigating student voice when learning linear equations with kinesthetic movement. I plan on incorporating students using their bodies when they are learning to graph equations on a coordinate plane.

Data for the study will include pre-and post-tests, surveys, video and audio taped lessons, class observations, and one-on-one interviews with some of the students. I will be looking at student responses to these surveys to see their thoughts on kinesthetic movement when learning linear equations.

There are no risks in participating in this study and student participation (or decision not to participate) will have no impact on student grades.

Throughout the report that I will write based on my findings, I will use pseudonyms for the community, school, and students. You may elect to have your child not participate in my research. In either case, I would appreciate if you would return a signature indicating your decision as soon as possible because I cannot begin the research process without it.

If you have any questions regarding the research project, or about the class in general, please contact me by email at novakm@strongnet.org or by calling the main office at (440)572-7090, where I can return your call at my earliest convenience. If you want further information on my research project, feel free to contact my Kent State University Advisor, Dr. Caniglia, Phone number 1-330-672-0615.

Sincerely, Mrs. Melissa Novak

Please sign and have your child return the following permission slip to Mrs. Novak.

Parent/Guardian Permission

Whole Group Research

I give permission for my child to be a participant in the research project as described by Mrs. Novak

I do not wish for my child to be a participant in Mrs. Novak’s research project.
**One-On-One Interviews**

_______ I give permission for my child to be a participant as a one-on-one interview in the research project as described by Mrs. Novak

_______ I do not wish for my child to be a participant as a one-on-one interview in Mrs. Novak’s research project.

(Parent/Guardian Signature) __________________________ (Date) __________________________

**Student Permission**

**Whole Group Research**

_______ I give my assent to be a participant in the research study as described by Mrs. Novak.

_______ I do not wish to be a participant in the research study as described by Mrs. Novak

**One-On-One Interviews**

_______ I give my consent to be a participant as a one-on-one interview in the research project as described by Mrs. Novak

_______ I do my consent to be a participant as a one-on-one interview in Mrs. Novak’s research project.

(Student Signature) ____________________________ (Date) ____________________
APPENDIX B

LEARNING STYLE INVENTORY
Appendix B

Learning Style Inventory

The VARK Questionnaire for Younger People

How Do I Learn Best?

VARK Questionnaire version 7.1

Choose the answer which best explains your preference and click the box next to it. Please click more than one if a single answer does not match your perception. Leave blank any question that does not apply.

You are about to hook up your parent’s new computer. You would:

☐ Unpack the box and start putting the pieces together.
☐ Follow the diagrams that show how it is done.
☐ Phone, text or email a friend and ask how to do it.
☐ Read the instructions that came with it.

Remember when you learned how to play a new computer or board game. You learned best by:

☐ Listening to somebody explaining it and asking questions.
☐ Watching others do it first.
☐ Reading the instructions.
☐ Clues from the diagrams in the instructions.

You are about to buy a new digital camera or mobile phone. Other than price, what would most influence your decision?

☐ Trying it.
☐ The salesperson telling me about it.
☐ Reading the details about its features.
☐ It is the latest design and looks good.
You have been selected as a tutor or a leader for a holiday program. This is interesting for your friends. You would:

- Start practicing the activities I will be doing in the program.
- Show them the map of where it will be held and diagrams about it.
- Show them the list of activities in the program.
- Describe the activities I will be doing in the program.

You want to plan a surprise party for a friend. You would:

- Talk about it on the phone or text others.
- Make lists of what to do and what to buy for the party.
- Draw a map and make a special design for the invitation.
- Invite friends and just let it happen.

You have to present your ideas to your class. You would:

- Write a few key words and say them again and again.
- Make diagrams or get graphs to help explain my ideas
- Gather examples and stories to make it real and practical.
- Write out my speech and learn it by reading it again and again.

A new movie has arrived in town. What would most influence your decision to go (or not go)?

- You read what others say about it online or in a magazine.
- You see a preview of it.
- Hear friends talking about it.
- It is similar to others you have liked.

After reading a play you need to do a project. Would you prefer to:

- Act out a scene from the play?
- Draw or sketch something that happened in the play?
- Write about the play?
- Read a speech from the play?
You are learning to take photos with your new digital camera or mobile phone. You would like to have:
- Examples of good and poor photos and how to improve them.
- Clear written instructions with lists and bullet points.
- A chance to ask questions and talk about the camera’s features.
- Diagrams showing the camera and how to use it.

You are going to make something special for your family. You would:
- Talk it over with my friends.
- Find written instructions to make it.
- Make something I have made before.
- Decide from pictures in magazines.

You want some feedback about an event, competition or test. You would like to have feedback:
- From somebody who discussed it with me.
- That used examples of what I have done
- That used a written description or table of my results.
- That used graphs showing what I achieved.

Do you prefer a teacher who likes to use?
- An overview diagram, charts, labelled diagrams and maps.
- A textbook and plenty of handouts.
- Class discussions, online discussion, online chat and guest speakers.
- Field trips, case studies, videos, labs and hands-on practical sessions.

You have a problem with your knee. Would you prefer that the doctor?
- Showed you a diagram of what was wrong.
- Describe to you what was wrong.
- Gave you an article or brochure that explained knee injuries.
- Demonstrated what was wrong using a model of a knee.
You need to give directions to go to a house nearby. You would:

- Write down the directions as a list.
- Draw a map on a piece of paper or get a map online.
- Tell them the directions.
- Walk with them.

I like websites that have:

- Audio channels for music, chat and discussion.
- Interesting design and visual effects.
- Interesting information and articles in print.
- Things I can click on and do.

A website has a video showing how to make a special graph. There is a person speaking, some lists and words describing what to do and some diagrams. You would learn most from:

- Seeing the diagrams.
- Watching the actions.
- Reading the words.
- Listening.
APPENDIX C

PRE-TEST AND POST-TEST
### Appendix C

#### Pre-Test and Post-Test

<table>
<thead>
<tr>
<th>What is a linear equation?</th>
<th>What is a function?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can you graph $y = 2x + 4$? Yes or no</td>
<td>If yes, how would you graph $y = 2x + 4$?</td>
</tr>
<tr>
<td>Describe the graph of $y = 2x + 4$.</td>
<td>How is the graph of $y = -x$ differ than $y = -2x$?</td>
</tr>
<tr>
<td>How is the graph of $y = 1/2x + 3$ differ than $y = ½ x - 4$?</td>
<td>Describe what the graph of $y = 7$ looks like.</td>
</tr>
<tr>
<td>Describe what the graph of $x = -4$ looks like.</td>
<td>Looking at the graph what is the equation of the line?</td>
</tr>
<tr>
<td>John is buying a new cell phone plan. He will be charged $30 per month plus $0.02 for each text he sends. If he sends 200 texts. What equation would you use to find his monthly bill?</td>
<td>Make a table for the cell phone plan.</td>
</tr>
<tr>
<td>Compare and contrast the following equations and their graphs. $Y = 3x$ and $y = 1/3 x$</td>
<td>Describe the transformation of the following graphs. $f(x) = 4x$ and $g(x) = -4x$</td>
</tr>
</tbody>
</table>
APPENDIX D

ATTITUDE SURVEY
Appendix D

Attitude Survey

1. What is kinesthetic movement?
2. When you are trying to explain to someone what the graph of $y = 2x - 3$ looks like what do you say or do?
3. If you knew someone was having trouble understanding what the graph of $y = -1/2x + 4$ looks like what would you do to help them? *
4. Do you think kinesthetic movement helped you learn in this past unit? Why or why not?
5. Did you like that kinesthetic movement was used? Why or why not?
6. Which do you think was the most beneficial in your learning linear equations? Graphs, body movement, tables, equations, etc. …
7. Do you ever refer to your body when solving a problem comparing equations? For example, how does $y = 2x + 3$ differ from $y = x$?
8. Do you think kinesthetic movement was beneficial in learning linear equations?
9. Do you think kinesthetic movement is easy when……?*
10. Kinesthetic movement is hard when…..?*
11. How can kinesthetic movement help you? *
12. Best thing about kinesthetic movement. *
13. Write anything else you feel is needed to help me understand your thoughts on kinesthetic movement when learning linear equations. *
14. Write anything else you feel is important to let me know your thoughts on kinesthetic movement being used in the classroom. *
APPENDIX E

INSTRUCTIONAL ACTIVITIES
Appendix E
Instructional Activities for Small Group Investigation

Graphing Lines Investigation

Task 1

1. Graph each of the following equations on your graphing calculator.
   Then sketch each graph on the grid. Label each line with its equation.

   A. \( y = 2 \)   B. \( y = 4 \)   C. \( y = -3 \)   D. \( y = 5 \)

2. Describe the graphs and what you see happening.

3. Sketch the graph of the equation \( y = 7 \) on the same grid. Then graph the same equation on the graphing calculator to check your work? Was your sketch correct?

4. Where do each of the graphs cross (intersect) the \( y \)-axis\[ \] \( x \)-axis\[ \]?

5. Using your body graph \( y = 6 \) Show with your body difference between \( y = x \) and \( y = \) some #

6. Make a conjecture about \( y = \) some number
Task 2

1. Graph each of the following equations on your graphing calculator. Then sketch each graph on the grid. Label each line with its equation.

   A. $y = x$
   B. $y = x + 2$
   C. $y = x + 5$
   D. $y = x - 3$
   E. $y = x - 7$

2. Describe the graphs of these equations.

3. Sketch the graph of the equation $y = x + 6$ on the same grid. Then graph the same equation on the graphing calculator to check your work? Was your sketch correct?

4. Where do each of the graphs cross (intersect) the $y$-axis? the $x$-axis?

5. With your body graph $y = x + 10$

6. Make a conjecture about $y = x$ ± some number.

7. How does adding or subtracting a number to $x$ change the original $y = x$ graph?

8. Show with your body difference between $y = x$ and $y = x$...
Task 3

1. Graph each of the following equations on your graphing calculator. Then sketch each graph on the grid. Label each line with its equation.

   B. \( y = x \)  
   B. \( y = 2x \)  
   C. \( y = 4x \)  
   D. \( y = 6x \)

2. Describe the graphs of these equations.

3. Sketch the graph of the equation \( y = 3x \) on the same grid. Graph the same equation on the graphing calculator to check your work. Was your sketch correct?

4. Where do each of the graphs cross (intersect) the \( y \)-axis and \( x \)-axis?

5. Using your body graph \( y = 7x \)

6. As the number that is multiplied by \( x \) increases, describe what happens to the graph of the equation.

7. How does multiplying \( x \) by a positive number change \( y = x \)?

8. Show with your body \( y = x \) and then \( y = 4x \)
Task 4

1. Graph each of the following equations on your graphing calculator. Then sketch each graph on the grid. Label each line with its equation.

   A. \( y = -x \)        B. \( y = -3x \)        C. \( y = -6x \)        D. \( y = -8x \)

2. Describe the graphs of these equations.

3. Where do each of the graphs cross (intersect) \( y \)-axis____________________\( x \)-axis____________________?

4. Sketch the graph of the equation \( y = -7x \) on the same grid. Graph the same equation on the graphing calculator to check your work. Was your sketch correct?

5. Using your body graph \( y = 8x \)

6. As the number that is multiplied by \( x \) decreases, describe what happens to the graph of the equation?

7. Show with your body \( y = x \) and then \( y = -4x \)
Task 5

1. Graph each of the following equations on your graphing calculator.

Then sketch each graph on the grid. Label each line with its equation.

A. $y = x$  
B. $y = 3x$  
C. $y = \frac{1}{3}x$

2. Describe the graphs of these equations in a complete sentence.

3. What is the difference between using the whole number and a fraction?

4. With your body graph $y = x$, $y = 2x$, $y = \frac{1}{2}x$

5. Write a conjecture about $y = \text{fraction times } x$ and $y = \text{a whole number times } x$. 
**Task 6**

1. Graph each of the following equations on your graphing calculator.  

Then sketch each graph on the grid. Label each line with its equation.

- B. \( y = \frac{1}{3} x \)
- B. \( y = \frac{1}{6} x \)
- C. \( y = 0.5x \)
- D. \( y = 0.25x \)

2. Describe the graphs of these equations.
3. Where do each of the graphs cross (intersect) x-axis________ Y-axis________

4. Sketch the graph of the equation \( y = \frac{1}{10} x \) on the same grid. Then graph the same equation on the graphing calculator to check your work? Was your sketch correct?
5. With your body graph \( y = x \) then \( y = \frac{1}{5} x \)
6. Write a conjecture about \( y = \) fraction times \( x \) and \( y = \) a whole number times \( x \).
7. Show with your body \( y = x \) then \( y = \frac{1}{7} x \)
Task 7

1. Graph each of the following equations on your graphing calculator.

Then sketch each graph on the grid. Label each line with its equation.

A. \(2x + 3\)  
B. \(y = 2x - 3\)  
C. \(y = 2x + 5\)  
D. \(y = 2x - 5\)

2. Describe the graphs of these equations in a complete sentence.

3. Sketch the graph of the equation \(y = 5x + 2\) and \(y = 5x - 3\) on the same grid. Then graph the same equations on the graphing calculator to check your work? Was your sketch correct?

4. With your body graph move from one graph to the next

   A. \(Y = x \Rightarrow y = 2x \Rightarrow y = 2x - 3\)
   
   B. \(Y = x \Rightarrow y = -3x \Rightarrow y = -3x + 4\)
   
   C. \(Y = x \Rightarrow y = \frac{1}{2}x \Rightarrow y = \frac{1}{2}x + 3\)
APPENDIX F

RUBRIC FOR PRESENTATIONS
### Appendix F

**Rubric for Presentations**

**Presentation Comparing Linear Equations Rubric**

You must have at least one visual aid in your presentation.

Some choices; overhead, poster board, PowerPoint, chalk board, paper and pencil, glogster, prezi, dry erase mini boards, video, etc.

<table>
<thead>
<tr>
<th></th>
<th>Excellent (5 pts.)</th>
<th>Good (4 pts.)</th>
<th>Satisfactory (3 pts.)</th>
<th>Needs Improvement (2 pts.)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Oral Communication</strong></td>
<td>Presentation was easy to understand and presented with fluency.</td>
<td>Presentation was easy to understand, with some fluency.</td>
<td>Presentation sometimes was understandable.</td>
<td>Presentation was not fluent or easy to understand.</td>
</tr>
<tr>
<td><strong>Correctness</strong></td>
<td>All information presented is correct</td>
<td>One and or two errors in presentation</td>
<td>Three errors in presentation</td>
<td>More than three errors in presentation</td>
</tr>
<tr>
<td><strong>Visual Aid</strong></td>
<td>Information is neat and easy to read and shows significant effort.</td>
<td>Product is neat and easy to read shows moderate effort.</td>
<td>Product is somewhat neat and minimal effort.</td>
<td>Product is not neat and easy to read shows lack of effort.</td>
</tr>
<tr>
<td><strong>Completeness</strong></td>
<td>Comparison of all 7 components.</td>
<td>Comparison of 5-6 components.</td>
<td>Comparison of 3-4 components.</td>
<td>Comparison of 1-2 components.</td>
</tr>
<tr>
<td><strong>Total points</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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</table>

/20 Points
APPENDIX G

ASSESSMENT OF DIFFERENT REPRESENTATIONS
## Appendix G

**Assessment of Different Representations**

<table>
<thead>
<tr>
<th>f(x) = ( \frac{1}{2} x + 3 )</th>
<th>Make a function table for the equation.</th>
<th>Graph the function</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-7</td>
</tr>
<tr>
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<td>-1</td>
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<tr>
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<td>1</td>
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<td>4</td>
<td>9</td>
</tr>
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<td>8</td>
<td>17</td>
</tr>
</tbody>
</table>

**Graph the function**

**Write an equation for the function table.**

**What is the equation?**

**Make a function table for the graph.**
APPENDIX H

QUIZ
Appendix H

Quiz

Name_______________________________________________________

1. What is a y-intercept?

2. What is the x-intercept?

3. Sketch the graph $y = x$.

4. What happens to the graph of $y = x$ when you add a number to it?
   example: $y = x + 3$

5. What happens to the graph of $y - x$ when you subtract a number? example:
   $y = x - 3$

6. What happens to the graph of $y = x$ when you multiply by a positive number? example: $y = 5x$

7. What happens to the graph of $y = x$ when you multiply it by a positive fraction? example: $y = \frac{1}{2} x$

8. What happens to the graph of $y = x$ when you make it $y = -x$?

9. What would this graph do $y = 2x + 4$?

10. What is the slope of the equation of $y = \frac{3}{4} x - 7$?
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REFERENCES


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