SEARCH FOR MAXIMUM NUCLEAR COMPRESSION IN A MODEL OF NUCLEUS-NUCLEUS COLLISIONS

A thesis submitted to
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CHAPTER 1

Introduction

Fundamental discoveries since quarks were proposed in 1963 by physicists Murray Gell-Mann and George Zweig prove that quarks are the most elementary constituents of the nucleus, since there is strong evidence that quarks themselves have no substructure (they are not made of other particles) [1, 2]. The Standard Model is a unified description of matter in terms of quarks, particles in the electron family (leptons), and the fields mediating their interactions. According to the Standard Model, there are three generations of elementary constituents among the quarks and among the leptons, with two quarks and two leptons in each generation. All of the above have spin $\frac{1}{2}$, and are called fermions. Interactions among fermions are mediated by emission and absorption of force field quanta, which have integer spin (bosons). The forces are comprised of the strong nuclear force mediated by gluons, the electromagnetic force mediated by photons, the weak nuclear force mediated by the $W$ and $Z$ bosons, and the gravitational force mediated by gravitons.
The final elementary particle in the Standard Model is the Higgs boson. In 2012, it was discovered at the LHC (Large Hadron Collider) [4, 5]. The Higgs has zero spin, and no electric charge or color charge — see Fig. 1.1. Flavor refers to set a of quantum numbers that are carried by quarks. There are three quarks with an electric charge of $-\frac{1}{3}$ units, and these three have flavors $d$, $s$ and $b$, while the remaining three quark types carry an electric charge of $+\frac{2}{3}$ units, and these three have flavors $u$, $c$ and $t$. Leptons have either electric charge -1, namely $e^-$ (electron), $\mu^-$ (muon) or $\tau^-$ (tau), or zero charge in the case of $\nu_e$ (electron neutrino), $\nu_\mu$ (muon neutrino) or $\nu_\tau$ (tau neutrino). The baryon number $B$ is $+\frac{1}{3}$ for every quark, and is zero for leptons. The lepton number $L$ is $+1$ for leptons and zero for quarks. All of the above fermions have antimatter partner particles with equal mass and opposite sign of quantum
numbers, including electric charge. Quarks are never detected individually; in normal matter, we observe only baryons (three quarks bound together), or antibaryons (three antiquarks) or mesons (quark-antiquark pair). The part of the Standard Model that deals with quarks and their interactions is called Quantum Chromodynamics (QCD).

Experiments at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Lab, on Long Island, New York, study the particles emerging from very energetic collisions between heavy nuclei like gold (each made up of 197 protons and neutrons), and offer unique means for investigating the properties of quarks and the particles associated with the forces between quarks. The most high-profile scientific breakthrough so far at the RHIC accelerator was the discovery of quark-gluon plasma (QGP) [6, 7, 8, 9, 10, 11]. Fig 1.2 shows the locations of the QGP phase in the phase diagram of nuclear matter.
The existence of this new phase of matter had been predicted on theoretical grounds, and it is believed to have existed fleetingly during the first microsecond after the Big Bang. Its discovery was further confirmation of QCD and the Standard Model.

Until recently, the available evidence suggested that the transition between an ordinary matter phase and QGP phase is a continuous transition[13, 14, 15]. To explain phase transitions [16, 17], I will introduce the pressure of QCD matter, since it is a variable that is relevant in this thesis. The pressure $P$ due to QCD matter at
a high temperature \( T \) with zero net baryon density is

\[
P = \frac{E}{3V} = 37\frac{\pi^2}{90}T^4,
\]

\( (1.1) \)

where \( \frac{E}{V} \) is the energy density. The pressure due to a high baryon density with zero temperature is

\[
P = \frac{E}{3V} = \frac{g_q}{24\pi^2}\mu_q^4,
\]

\( (1.2) \)

where \( g_q \) is the degeneracy number of quarks and \( \mu_q \) is the quark Fermi momentum.

The pressure of the QCD matter can be found as combination of these two equations at high temperature and high baryon density. If the first derivative of the energy density with respect to temperature is discontinuous, the phase transformation is called a first-order transition. A familiar example of a discontinuous phase transition occurs when ice changes to liquid water, or when liquid water changes to vapor. However, if the first derivative mentioned above is continuous but the second derivative is not, the phase transition is dubbed a second-order transition or a continuous transition. An example of a continuous transition occurs when a ferromagnet is heated above the Curie temperature where its ferromagnetic properties disappear.

Theorists had anticipated that the normal maximum energy of the RHIC accelerator is too high to observe a first-order phase transition, and they predicted that it might be seen at a lower energy[18, 19, 20, 21]. Without any doubt, we know that the peak net-baryon density increases in nuclear collisions as the beam energy is lowered below the maximum energy of the RHIC accelerator at Brookhaven. When we refer to peak net-baryon density, we recognize that net-baryon density rises and falls during the early stages of a collision, and the peak refers to the maximum value reached during the time evolution of the collision. The above-mentioned beam-energy
dependence of peak net-baryon density happens because most baryons emitted from collisions at high energies come from baryon-antibaryon pairs excited from the vacuum, and thus those baryons contribute zero net-baryon density. However, at lower energies, there are more baryons from the initial colliding nuclei that are slowed-down or “stopped” by the collision process. On the other hand, as the beam energy drops, there might be insufficient energy to reach or cross the yellow transition region depicted in Fig. 1.2. These insights and questions motived scientists at RHIC to conduct a series of experiments known as the Beam Energy Scan (BES)[22, 23, 24]. These BES experiments investigated a very wide range of beam energies: $\sqrt{s_{NN}}$ values are 200 GeV, 130 GeV, 62.4 GeV, 39 GeV, 27 GeV, 19.6 GeV, 14.5 GeV, 11.5 GeV and 7.7 GeV.

The Beam Energy Scan is a major project that requires a lot of resources: a large number of scientists need to put several years of effort into it, and the total cost to operate the RHIC accelerator is over $100 million per year. Detailed theoretical models are essential for interpreting the measurements from the Beam Energy Scan. Nuclear transport models are an important category, and they are implemented as event-generating computer codes using Monte Carlo techniques which simulate the random processes in a relativistic collision between two nuclei. An example of such a model is described in chapter 2. The codes need to be run many thousands of times in order to build up enough model statistics. With a sufficient number of models events, the theory can be tested in comparisons with experimental results.

The transport models that are available at the present time do not include the physics of a phase transition to QGP, but assume that the excited matter of the colliding nuclei stays in the ordinary phase of hadronic matter throughout the collision
process. Although we believe that a phase transition to QGP does indeed happen during the early stage of these collisions, it is still essential to study models without a phase transition. This allows us to understand the “background” hadronic processes, which can be quite complicated, and therefore helps us to identify what features of the experimental measurements cannot be explained unless a phase transition to QGP happens.

One important question that models need to answer relates to stopping in relativistic nuclear collisions. The relevance of nuclear stopping was outlined earlier. In high-energy nucleus-nucleus collisions in a head-on configuration (zero impact parameter), a low amount of nuclear stopping would be consistent with a large fraction of the nucleons from the initial nuclei retaining their original momentum, or close to it, after the collision is finished. Besides revealing information about the reaction mechanism, the level of nuclear stopping offer information about whether there is sufficient excitation of the QCD matter to form quark-gluon plasma.

We know that stopping decreases at the beam energy of relativistic nuclear collisions increases, and decreased stopping in turn leads to less nuclear compression. On the other hand, it is obvious that more energy is available for exciting the colliding nuclear matter as the beam energy increases, and some of this excitation must be in the form of compressional potential energy. Therefore, it has been speculated [25, 22] that there is a particular beam energy, or range of beam energies, where the competition between these two opposing factors might cause a maximum in the net-baryon compression. If so, it is important to estimate the beam energy region of this maximum net-baryon compression. Such an energy region, if it exists, would be an ideal location for the study of phase transition effects. Furthermore, if we know the beam
energy region where the peak net-baryon density is the highest, then in this region, we stay as far as possible to the right in Fig. 1.2 while still exciting the QCD matter sufficiently to reach the quark-gluon plasma phase. In that scenario, we would have the best chance of exploring the region of a first-order phase transition QGP.
CHAPTER 2

UrQMD Model

The UrQMD (Ultra-relativistic Quantum Molecular Dynamics) model [26] is a microscopic transport theory based on phase space description of the reaction [27, 28, 29]. It is suitable for simulating heavy ion collisions over a wide range of energies, from a few GeV to hundreds of GeV. It is intended to describe many aspects of the collision process like rates and spectra of all produced particles, correlations, etc. One of the strengths of UrQMD is that a variety of options and parameters can be changed. In this chapter, I summarize how the UrQMD model initializes the two colliding nuclei, and introduce the assumptions behind the reactions and the collision terms.

First of all, the two incident nuclei are constituted based on the Fermi-gas approach [27]. The wave functions of the nucleons are represented by Gaussian-shaped density distributions. There are many restrictions and constraints that every initialization of a nucleus has to fulfill. The nucleus has to be at rest and centered around zero. That means the summation of the nucleon velocity must be zero and the summation of the distances between nucleons must also be zero. Also, the nucleus has to be in the ground state. The radius of the nucleus is given by

\[ R(A) = r_0 \left( \frac{1}{2} \left[ A + (A^{1/3} - 1)^3 \right] \right)^{1/3}. \]  

(2.1)

A further constraint is that the binding energy of the two nuclei should agree with the binding energy calculated by the Bethe-Weizsäcker formula. The initial momenta
of the nucleons are randomly chosen between zero and the local Thomas-Fermi momentum. The code uses the system clock for each event to generate a random seed for all probabilistic calculations. This ensures that the output result of each collision of two nuclei is different.

In the UrQMD model, the collision between nucleons is based on a non-relativistic density-dependent Skyrme potential plus Yukawa potential and Coulomb potential. The Pauli potential can be optionally added to the formula where a momentum-dependent potential is not included. The model has 55 different baryon species and 32 different meson species, not counting antiparticles. For a particle-particle collision to happen, the distance between the two particles $d_{\text{trans}}$ must be within

$$d_{\text{trans}} \leq d_0 = \sqrt{\sigma_{\text{tot}}/\pi}, \quad \sigma_{\text{tot}} = \sigma(\sqrt{s}, \text{type}),$$  \hspace{1cm} (2.2)

where $\sigma_{\text{tot}}$ is the total cross section for that binary particle interaction. The total cross sections depend on the particle species in the initial and final state of the binary interaction, on the center-of-mass energy, and on isospins. The trajectory and evolution of each individual particle is followed, and some aspects of the collision environment are recalculated every time step, where the interval between time steps is a parameter chosen by the user of the code. A typical interval between time steps is 0.1 fm/c, which corresponds to $3.33 \times 10^{-25}$ seconds.

For the model to be executed properly, a set of parameters and options has to be defined according to the study one wants to conduct. The code will read these parameters and options first, and then starts simulating the collisions. One such example is impact parameter $b$. The impact parameter is defined as the minimum distance between the centers of the two nuclei if they were to pass each other without any interaction (see Fig. 2.1). For exactly head-on collisions, the impact parameter
is $b = 0$.

Fig. 2.1: Left: two heavy nuclei at impact parameter $b$ before they collide. Right: spectators and participants after the two nuclei collide [30]. Note that the left side of this diagram depicts Lorentz-contracted nuclei, whereas on the right, in order to represent the participant zone with better clarity, no Lorentz contraction is applied.

The left side of this figure gives a cartoon-like illustration of two heavy nuclei approaching each other from opposite directions, as in the laboratory frame at a collider. Each nucleus is assumed to have a speed $v$ that is a significant fraction of the speed of light ($c$), and therefore there is a Lorentz contraction effect along the direction of motion by a factor $1/\gamma = \sqrt{1 - v^2/c^2}$ (also shown in Fig. 3.1). On the right side of Fig. 2.1, the Lorentz contraction is not applied, since this allows the details of the excited participant zone to be shown more clearly.
In this chapter, I show the result of my work. I carried out a simulation based on a widely-used model of nucleus-nucleus collisions: UrQMD. As explained in Chapter 1, the main goal of this thesis is to map-out the change in net-baryon compression as the beam energy is varied over the range available to current experiments at RHIC. It is of particular interest to determine if UrQMD predicts that there is a definite beam energy region where the net-baryon compression is a maximum.

The initial nuclei studied in this work are Mercury on Mercury ($^{200}\text{Hg} + ^{200}\text{Hg}$). This stable isotope has 200 nucleons: 80 protons and 120 neutrons. $^{200}\text{Hg}$ and $^{200}\text{Hg}$ are the two most abundant isotopes present in terrestrial ores. The exact number of proton or neutron is not important in the present work. Protons and neutrons can be considered to be the same particle state (the nucleon) as far as the strong interaction is concerned, and they differ only slightly in that their electromagnetic interactions are different. Mercury-200 was chosen because it is close enough to the two most commonly-studied heavy nuclei at accelerators — $^{197}\text{Au}$ (Gold) at most US facilities and $^{208}\text{Pb}$ (Lead) at CERN. For most aspects of high-energy nuclear collisions, changing the number of nucleons up or down by a couple of percent makes almost no difference. The situation in the study of low-energy nuclear structure is the complete opposite, because in such cases, adding or removing a single nucleon from any nucleus can dramatically change the nuclear structure and its internal properties.

In this study, the impact parameter was set to zero ($b = 0$) at all energies, since
I am just interested in investigating the maximum compression. My studied center-of-mass energies per nucleon pair are 2, 3, 5, 10, 20, 40, 80, 160 and 320 GeV, which covers a range that is a little broader than what is available using the RHIC facility at Brookhaven. I have modified the standard UrQMD code, version 3.3 as downloaded from the UrQMD website [26] in 2014, to allow the number of baryons and antibaryons inside three volumes to be counted and recorded as a function of the time step in the UrQMD collision evolution. The baryon and antibaryon numbers are counted inside three spheres, with radii $R_0 = 1.2, 1.5$ and $2.0$ fm. The center of each sphere is always at the mid-point of the line joining the centers of the two colliding nuclei.
Fig. 3.1: A representation of two $^{200}$Hg nuclei at a beam energy of $\sqrt{s_{NN}} = 20$ GeV, before a collision at $b = 0$. The semi-random distribution of nucleons within the nuclear volume can be seen. The plot also indicates the three spheres within which baryons and antibaryons are counted.

At the beginning of the collision, when the nuclei first touch, there are no antibaryons yet, and the baryons counted are nucleons from the relevant volumes of the initial-state nuclei. After a few time steps, the baryon counts increase because the two nuclei are interacting.
Fig. 3.2: Example 1 of a single $^{200}$Hg + $^{200}$Hg collision event. This shows the number of baryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process at a center-of-mass beam energy of 20 GeV per nucleon pair in the UrQMD model, at zero impact parameter. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. This plot describes a single nucleus-nucleus collision (event), and therefore statistical fluctuations can be seen.
Fig. 3.3: Example 1 of a single $^{200}\text{Hg} + ^{200}\text{Hg}$ collision event. This shows the number of antibaryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process at a center-of-mass beam energy of 20 GeV per nucleon pair in the UrQMD model, at zero impact parameter. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. This plot describes a single nucleus-nucleus collision (event), and therefore statistical fluctuations can be seen.
Fig. 3.4: Example 2 of a single $^{200}\text{Hg} + ^{200}\text{Hg}$ collision event. This shows the number of baryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process at a center-of-mass beam energy of 20 GeV per nucleon pair in the UrQMD model, at zero impact parameter. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. This plot is the same as shown in Fig. 3.2 except that a different event, with different random fluctuations, is shown.
Fig. 3.5: Example 2 of a single $^{200}\text{Hg} + ^{200}\text{Hg}$ collision event. This shows the number of antibaryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process at a center-of-mass beam energy of 20 GeV per nucleon pair in the UrQMD model, at zero impact parameter. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. This plot is the same as shown in Fig. 3.3 except that a different event, with different random fluctuations, is shown.

Figs. 3.6 to 3.21 show similar plots as Figs. 3.2 and 3.3, except now the baryon and antibaryon fluctuations are suppressed by taking an average over 100 independent events for each of the nine studied beam energies. There are several conclusions that can be reached on the basis of these plots:

First, the time of the peak in baryon and antibaryon density is almost independent of the radius $R_0$ of the test spheres.
Second, at the time steps of interest, the number of baryons and antibaryons inside each of the three spheres is not proportional to the sphere’s volume, probably because the two larger spheres include some regions of empty space (an example of this can be seen from Fig. 3.1).

Third, as expected, the time of the peak in baryon and antibaryon density shifts earlier as the beam energy increases and the time needed to reach the fully overlapped configuration of the two nuclei becomes shorter.

Fourth, also as expected, the height of the peak in baryon and antibaryon density increases as the beam energy increases. This is mostly caused by the increasing production of baryon-antibaryon pairs at higher beam energies.

Fifth, the shape of the time evolution of the baryon and antibaryon density changes significantly with beam energy, and at later times than the peak, the tail becomes much more stretched-out as the beam energy increases. This is also to be expected, because baryons from baryon-antibaryon pairs produced nearly at rest in the center-of-mass frame become increasingly dominant at the higher beam energies.
Fig. 3.6: Average number of baryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process for $^{200}\text{Hg} + ^{200}\text{Hg}$ in the UrQMD model. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. To suppress statistical fluctuations, the number of baryons has been averaged over 100 events. The center-of-mass energy $\sqrt{s_{NN}}$ for this collision is 2 GeV per nucleon pair, and the impact parameter is zero (central collisions).
Fig. 3.7: Average number of baryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process for $^{200}$Hg + $^{200}$Hg in the UrQMD model. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. This plot is the same as shown in Fig. 3.6 except that the center-of-mass energy $\sqrt{s_{NN}}$ for this collision is 3 GeV per nucleon pair.
Fig. 3.8: Average number of baryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process for $^{200}$Hg + $^{200}$Hg in the UrQMD model. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. This plot is the same as shown in Fig. 3.6 except that the center-of-mass energy $\sqrt{s_{NN}}$ for this collision is 5 GeV per nucleon pair.
Fig. 3.9: Average number of antibaryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process for $^{200}$Hg + $^{200}$Hg in the UrQMD model. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. To suppress statistical fluctuations, the number of antibaryons has been averaged over 100 events. However, quantization and statistical fluctuations still can be seen because of the low probability for production of antibaryons at this center-of-mass energy. The center-of-mass energy $\sqrt{s_{NN}}$ for this collision is 5 GeV per nucleon pair, and the impact parameter is zero (central collisions).
Fig. 3.10: Average number of baryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process for $^{200}\text{Hg} + ^{200}\text{Hg}$ in the UrQMD model. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. This plot is the same as shown in Fig. 3.6 except that the center-of-mass energy $\sqrt{s_{NN}}$ for this collision is 10 GeV per nucleon pair.
Fig. 3.11: Average number of antibaryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process for $^{200}\text{Hg} + ^{200}\text{Hg}$ in the UrQMD model. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. This plot is the same as shown in Fig. 3.9 except that the center-of-mass energy $\sqrt{s_{NN}}$ for this collision is 10 GeV per nucleon pair.
Fig. 3.12: Average number of baryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process for $^{200}$Hg + $^{200}$Hg in the UrQMD model. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. This plot is the same as shown in Fig. 3.6 except that the center-of-mass energy $\sqrt{s_{NN}}$ for this collision is 20 GeV per nucleon pair.
Fig. 3.13: Average number of antibaryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process for $^{200}$Hg + $^{200}$Hg in the UrQMD model. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. This plot is the same as shown in Fig. 3.9 except that the center-of-mass energy $\sqrt{s_{NN}}$ for this collision is 20 GeV per nucleon pair.
Fig. 3.14: Average number of baryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process for $^{200}\text{Hg} + ^{200}\text{Hg}$ in the UrQMD model. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. This plot is the same as shown in Fig. 3.6 except that the center-of-mass energy $\sqrt{s_{NN}}$ for this collision is 40 GeV per nucleon pair.
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Fig. 3.16: Average number of baryons inside a sphere of radius \( R_0 \) as a function of the time step (in units of fm/c) during the collision process for \(^{200}\text{Hg} + ^{200}\text{Hg}\) in the UrQMD model. Three values of the radius \( R_0 \) are shown: 1.2, 1.5 and 2.0 fm. This plot is the same as shown in Fig. 3.6 except that the center-of-mass energy \( \sqrt{s_{\text{NN}}} \) for this collision is 80 GeV per nucleon pair.
Fig. 3.17: Average number of antibaryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process for $^{200}$Hg + $^{200}$Hg in the UrQMD model. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. This plot is the same as shown in Fig. 3.9 except that the center-of-mass energy $\sqrt{s_{NN}}$ for this collision is 80 GeV per nucleon pair.
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Fig. 3.21: Average number of antibaryons inside a sphere of radius $R_0$ as a function of the time step (in units of fm/c) during the collision process for $^{200}$Hg + $^{200}$Hg in the UrQMD model. Three values of the radius $R_0$ are shown: 1.2, 1.5 and 2.0 fm. This plot is the same as shown in Fig. 3.9 except that the center-of-mass energy $\sqrt{s_{NN}}$ for this collision is 320 GeV per nucleon pair.
Fig. 3.22: The maximum number of baryons inside a sphere of radius $R_0$ as a function of beam energy. This maximum refers to the peak vertical axis values (average number of baryons) in Figs. 3.6 through 3.20.

At $\sqrt{s_{NN}} = 5$ GeV, the available energy is well above the threshold for antiproton production. However, 5 GeV is still too low to produce antibaryons except with a low probability, and the initial-state baryons dominate. Antibaryons are created with steeply increasing probability as the beam energy is increased above 5 GeV. Baryon number is always conserved, so the baryons produced from the vacuum are always exactly balanced by equal numbers of antibaryons produced from the vacuum. The only way to build up a high density of net baryons is to slow down the incoming baryons from the initial Hg nuclei and compress them. This is the compression effect
that needs to be exploited in order to move to the right in the QCD phase diagram, as explained in Chapter 1. In particular, we want to find out what is the best beam energy for maximizing the peak net-baryon density, according to the UrQMD model.

Two different methods for calculating the peak net-baryon density have been tested:
In method #1, the number of net baryons is calculated for each time step. Then, the average number of net-baryons is computed based on the sample of 100 events for each beam energy. Finally, the maximum number of net baryons is determined from the result of the prior step.
In method #2, the number of baryons and antibaryons is separately counted for each time step. Then, the average number of baryons and the average number of antibaryons are computed based on the sample of 100 events for each beam energy. Finally, the maximum number of net baryons is determined from the result of the prior step.
Fig. 3.23: The maximum number of net baryons inside a sphere of radius $R_0$ as a function of beam energy. This plot is similar to that shown in Fig. 3.22 except that net baryons, defined as baryons minus antibaryons, are shown here. A large net baryon density combined with sufficient thermal excitation of the QCD medium is needed to explore an important region of the QCD phase diagram (see Chapter 1).

These two methods yield similar results, and lead to net-baryon densities that agree within a few percent. This difference is so small that it can be neglected. The first method has been used in this study.
The Net Baryon Density

<table>
<thead>
<tr>
<th>Beam Energy (GeV)</th>
<th>Net Baryon Density (fm$^{-3}$)</th>
<th>Radius 1.2 fm</th>
<th>Radius 1.5 fm</th>
<th>Radius 2.0 fm</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.8</td>
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<td></td>
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</tr>
<tr>
<td>5</td>
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</tr>
<tr>
<td>10</td>
<td>0.6</td>
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</tr>
<tr>
<td>20</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>40</td>
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</tr>
<tr>
<td>80</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>320</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 3.24: The maximum net baryon density, in units of fm$^{-3}$, computed for a sphere of radius $R_0$, as a function of beam energy. All densities are divided by $\gamma = \sqrt{s_{NN}/2m}$, where $m$ is the nucleon mass. This factor $1/\gamma$ corrects for the apparent compression caused by Lorentz contraction. The error bars plotted here are statistical. There are systematic uncertainties that probably are larger for the larger radii, arising from integrating over a relatively large volume within which the true density may fluctuate as low as zero (see text). This result is based on $^{200}$Hg + $^{200}$Hg collisions in the UrQMD model, at zero impact parameter.

It is possible to carry out a polynomial fit to the data presented in Fig. 3.24 and in this way infer a beam energy associated with the maximum at each of the three $R_0$ values. This has been explored by assigning an integer $x = 1, 2, 3, 4, 5, 6, 7, 8$ and 9 to the nine beam energies, and the data for $R_0 = 1.2, 1.5$ and 2.0 fm are denoted by $(y_1, x_1), (y_2, x_2)$ and $(y_3, x_3)$, respectively. Then a 2nd-order polynomial fit gives
The derivatives of Eqs. (3.1), (3.2) and (3.3) are

\[
\frac{dy_1}{dx_1} = 0.0015x_1^4 - 0.0388x_1^3 + 0.3486x_1^2 - 1.273x_1 + 1.3957 \quad (3.4)
\]

\[
\frac{dy_2}{dx_2} = 0.001x_2^4 - 0.0304x_2^3 + 0.2832x_2^2 - 1.0558x_2 + 1.1538 \quad (3.5)
\]
\[
\frac{dy_3}{dx_3} = 0.0015x_3^4 - 0.0376x_3^3 + 0.3258x_3^2 - 1.1368x_3 + 1.1695 
\] 

(3.6)

From Eqs. (3.4), (3.5) and (3.6), the values of \(x_i\) where the slopes go to zero are

\[x_1 = 1.866 \quad x_2 = 1.813 \quad x_3 = 1.719\]

There is a relationship between beam energy and the integer \(x\) as defined above; it can be described by

\[E = 0.025x_i^5 - 0.375x_i^4 + 2.4583x_i^3 - 7.125x_i^2 + 10.017x_i - 3,\]

and therefore the three inferred beam energies \(E_{\text{peak}}\) associated with the maximum net-baryon density are

\[E_{\text{peak}}^{R_0=1.2\text{fm}} = 2.87 \text{ GeV} \quad E_{\text{peak}}^{R_0=1.5\text{fm}} = 2.83 \text{ GeV} \quad E_{\text{peak}}^{R_0=2.0\text{fm}} = 2.75 \text{ GeV}\]

In Fig. 3.24, the net-baryon density in units of \(\text{fm}^{-3}\) as function of the beam energy shows the peak net-baryon density within each of the three spheres. The uncertainties given by the plotted error bars are from Poisson statistics only. The difference in density for the three different radii are mostly artifacts of averaging over quite large volumes in the case of the two bigger radii, and so the results associated with two larger \(R_0\) values ought to be considered to have relatively large systematic uncertainties. With the larger radii, the true density at different positions within the test sphere may fluctuate as low as zero, which biases the mean density towards lower values. However, the systematic uncertainties when comparing different beam energies at the same \(R_0\) are likely much smaller than the systematic uncertainties when comparing densities at different \(R_0\) values at fixed beam energy. Taking UrQMD at face value, Figs. 3.24 and 3.25 tell us that the maximum net-baryon density occurs just slightly less than 3 GeV, and still remains quite close to the maximum up to a few GeV higher than that energy. In other words, the maximum net-baryon density
lies below the lower end of the Beam Energy Scan (BES) region at RHIC, but still we can infer that the lower end of the BES region is not much below the point of maximum net-baryon density.
CHAPTER 4

Summary and Suggestions for Future Research

This thesis has used the UrQMD nuclear transport model to probe certain questions with a connection to future experiments at the Relativistic Heavy Ion Collider. The UrQMD model has a long track record of at least reasonable qualitative agreement with a wide range of experimental measurements over the range of beam energies under consideration. However, an important caveat is that this model assumes a hadronic phase of matter at all times during the collision, and therefore possible signatures of a phase transition will never be present. The quantities of particular interest here are baryonic compression during the early stages of the collision process. To be even more specific, net-baryon density during the early times of a collision is especially of interest because it is a crucial variable (of course it is a purely theoretical quantity, not accessible via experiment) in describing the phase diagram of Quantum Chromodynamics (QCD).

Results presented in Chapter 3 offer estimates of baryon and antibaryon density at the center of the collision zone in the center of mass frame, for exactly head-on (zero impact parameter) $^{200}$Hg + $^{200}$Hg collisions. A spherical test volume, with three different radii, has been used for calculating densities. Due to Lorentz contraction, which varies widely over the beam energies of interest, a spherical volume might not be ideal, but it is the natural and obvious first step in an investigation such as this one. Future investigations might benefit from using a volume shaped like an oblate ellipsoid with its shortest axis along the beam direction. Much higher model
statistics might also be of benefit, and would allow even smaller test volumes for density calculations.

Generally, most of the overall trends observed in the output from UrQMD conform to normal expectations. The main result of interest is the peak net-baryon density, which happens anywhere from about 0.08 to 28 fm/c after the nuclei begin to touch, depending on the beam energy. Peak baryon densities rise steadily with beam energy. Peak net-baryon densities rise with beam energy up to about 3 GeV per nucleon pair, then drop significantly. If the results presented in Chapter 3 are taken at face value, it suggests that a broad range of beam energies centered near $\sqrt{s_{NN}} \sim 2.82$ GeV is best for maximizing net-baryon density. This conclusion is consistent with our knowledge that stopping decreases significantly, both in experiment [6, 7, 8, 9, 10, 22] and in UrQMD and similar models[18, 21, 27, 28, 29], over the studied beam energy range.
BIBLIOGRAPHY


