IMPLEMENTATION OF AUTHENTIC INVESTIGATIVE ACTIVITIES IN RATIO AND PROPORTION TO ADULT LEARNERS: A CASE STUDY

A dissertation submitted to the Kent State University College of Education, Health and Human Services in partial fulfillment of the requirements for the degree of Doctor of Philosophy

By

Cynthia Reeder Brennan

May 2015
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Cynthia Reeder Brennan

B.S., Ohio State University, 1969

M.B.A., John Carroll University, 1992

Ph.D., Kent State University, 2015

Approved by

_________________________, Director, Doctoral Dissertation Committee
Joanne C. Caniglia

_________________________, Member, Doctoral Dissertation Committee
Karl W. Kosko

_________________________, Member, Doctoral Dissertation Committee
Susan V. Iverson

Accepted by

_________________________, Director, School of Teaching, Learning and Curriculum Studies
Alexa L. Sandmann

_________________________, Dean, College of Education, Health and Human Services
Daniel F. Mahony
Using a constructivist paradigm, the purpose of this qualitative case study was to examine what helps or hinders adult students to learn ratio and proportion when the topic is not the central focus of the mathematics course. This was of interest as there was a lack of scholarly research and literature on adult learners’ understanding of ratio, proportions and proportional reasoning. Further, this study tied theory to educational practice in mathematics education by providing a model of authentic investigative activities that can be used in an IT or seated mathematics classroom.

Through a case study approach, four diverse adult learners from a 7 week IT finite mathematics class, required in a degree completion program, were asked to complete a pre-test questionnaire on ratio/proportion and attitudes, watch authentic investigative activities and then complete a post-test questionnaire on ratio and proportion and attitudes. Document analysis, interviews and observations were conducted to determine if their mathematical thinking and attitudes were impacted by the videos and supporting materials.

Three theoretical frameworks structured this study: Lamon’s seminal theories on rational numbers and proportional reasoning, Ben-Chaim’s theory on the implementation of authentic investigative activities to adult learners, and Lesh’s theory on representations and translations. Because this study focused on adult learners, the discussion of the
results was organized to reflect the three major components that form and construct adult numeracy: context, content, and cognition and affect. Each one of these components was impacted positively by the implementation of the authentic investigative activities on ratio and proportion.
ACKNOWLEDGEMENTS

This incredible journey could not have been accomplished without the support of many people. First, I want to thank my committee, especially my dissertation chair, Joanne, for her inspiration, insight, and strength. Karl, thank you for stepping in on a moment’s notice. And to Susan, who persistently nudged me to get back in the saddle when I hit a major roadblock, I can’t thank you enough. It has been a pleasure to work with each one of you.

To my family, I want to thank Kate for reminding me that while this journey has spanned nearly a decade, I had “a few” other things to take care of along the way. John, I appreciated your technical perspective and advice to have a computer crash plan. And to little Cecelia, I look forward to having much more time to read Harold and the Purple Crayon and count with your abacus.

Finally, I dedicate my dissertation to my grandfather, Ward G. Reeder, who earned his doctorate in education at the University of Chicago in 1921. And to my grandmother, Vivian Scott Reeder, a graduate of Indiana State Normal School and a teacher in a one-room classroom for many years.

Thank you everyone!
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CHAPTER I

INTRODUCTION

Good evening! Attached are the Diagnostic Questionnaire on Ratio and Proportion and Attitude that I have filled out. My answers are bolded under each question. I'm not sure how I did or if my answers are correct, but it was a nice departure from what we are doing in class. It has been such a long time since I have had a math class that it has been a slightly difficult adjustment getting back into the thought process. I hope that my answers help in the study in some way. Enjoy the rest of your night!

Anonymous from a participant in this case study

Mathematical literacy is a focal point in the efforts of the United States government to remain competitive in a global economy. According to a 2007 assessment, 15 year olds in the United States ranked 25th among their peers in 30 developed nations in math literacy and problem solving (National Mathematics Advisory Panel, 2008). The report, commissioned by former Department of Education Secretary, Margaret Spellings states, “…the Panel’s focus should be on the preparation of students for entry into and success in algebra, which itself is a foundation for higher mathematics” (NMAP, 2008, p. 8).

These findings are important for colleges as pre-algebra and elementary algebra are levels in which many college students are placed (Saxon & Boylan, 2008). Thus, the ability of mathematics programs that include pre-algebra and elementary algebra, to prepare students for college-level math is a key to success for students that begin college without the requisite math preparation and skills.
The challenge to increasing mathematics skills is further compounded by the fact that students who test into basic college-level math courses are largely minority and first-generation college students; both characteristics of at-risk students (Bailey, Jenkins, & Leinbach, 2005). The goals of equity and quality in mathematics education are complementary, as success in educating a broader base of citizens in mathematics will create more who understand and support the importance of math in social, economic pursuits (Confrey & Lachance, 2000).

**Statement of Problem**

The curriculum of a college level finite mathematics course, sometimes named survey of mathematics, includes the pre-algebra and elementary algebra topics of ratio and proportion. An understanding of rational number concepts, as well as the procedural skill to accurately execute their operation, leads to the development of proportional reasoning. This process is protracted in development, difficult to teach, mathematically complex, cognitively challenging, and essential to success in higher-level mathematics and science (Lamon, 2007). Ratio and proportion are considered middle school topics. Piaget’s (1972) theory suggests that children should achieve understanding of these topics during adolescence. However, when the concepts of ratio, proportion, and proportional reasoning are viewed in a larger context, their tight connections with fractions and multiplicative thinking, suggest a body of knowledge that spans the entire curriculum (Steffe, 1988, 1992, 2001, 2003; Steffe & Cobb, 1988; Steffe, von Glasersfeld, Richards, & Cobb, 1983) from elementary school through university-level. Thus, a lack of conceptual understanding and procedural skill of rational numbers and the
development of proportional reasoning may act as an effective barrier to the development of understanding of the larger more encompassing concept of proportionality that is used in higher-level mathematics and science programs (Lamon, 2005).

Moreover, rational numbers have been identified as a necessary foundation for algebra, and at present seem to be under-developed in students. In particular, fractions are identified as one of three Critical Foundations of Algebra in a recent report (NMAP, 2008). The ACT test at this mid western university places nearly 85% of incoming students in a college-level finite mathematics course (Online Cohort Profile, 2013). Rational numbers are a component of the test. A lack of procedural and conceptual knowledge of rational numbers may contribute to the estimation that 90% of incoming college freshman at one university do not understand proportional reasoning (Lamon, 2007).

Difficulty with rational numbers (including fractions, decimals and percent's) is pervasive and a major obstacle to further progress in mathematics, including algebra. A nationally representative sample of teachers of Algebra I, surveyed for (NMAP, 2008), rated students as having very poor preparation in rational numbers and procedural operations involving fractions and decimals. A conceptual understanding of fractions and decimals and the operational procedures for using them are mutually reinforcing and necessary. Thus, the curriculum at a college should afford more than one lesson to ensure acquisition of conceptual and procedural knowledge of rational numbers and the development of proportional reasoning. And, both must be accomplished before a proficiency in proportionality is attained (Lamon, 2007).
Foundations of the Research Study

Philosophical Assumptions

Philosophical assumptions consist of a basic set of assumptions or beliefs that guide inquiry (Guba & Lincoln, 2005). There were four possible philosophical assumptions or worldviews that I considered to inform my research: 1) Post-positivist; 2) Constructivist; 3) Participatory; and 4) Pragmatist. Each of these worldviews has common elements but take different stances on the elements with an explanation that the worldviews differ in their ontology or nature of reality, epistemology or how we gain knowledge, axiology or the roles values play in research, methodology or the process of research, and rhetoric or the language used in the research (Creswell, 2007, p. 17). My qualitative case study will follow the worldview of constructivism.

Constructivist worldview. Constructivism is both a cognitive position and a methodological perspective (Noddings, 2006). As a methodological perspective, constructivism assumes that human beings are knowing subjects and their behavior is purposeful with a highly developed capacity for organizing knowledge.

As a cognitive position, constructivism holds that the individual constructs knowledge and that the cognitive structures are themselves products of developmental construction (Piaget, 1953, 1970, 1971, 1972). According to Nesser (1967), all mental processes are constructive, so the line between perception and cognition is blurred. This includes seeing, hearing, and remembering which is often regarded as passive. Von Glasersfeld (1987) says, “Perceiving, from a constructivist point of view, is always an active making rather than a passive receiving . . .” (p. 217).
Moreover, Piaget followed Kantian philosophy by distinguishing between knowledge of the contingent world and knowledge of necessary truths. Piaget explained cognitive structures using the concept of reflective abstraction. Reflective abstraction is a process of interiorizing our physical actions on objects. As students move sets of objects, they are interiorizing the properties of mathematical operations rather than objects. The process enables students to acquire implicit understandings of communativity, associativity, and reversibility. Thus, there is an essential connection between purposive activity and the development of cognitive structures. And, objects or models play a role in reflective abstraction as students and objects are linked in the operational events that develop cognitive structures (Noddings, 2006).

Furthermore, constructivism views the learner as an active knowing organism that knows through continued construction. This active construction indicates there is a base structure from which to begin the construction or the process of assimilation and a process of transformation that is construction. This continual process guides the revision of the structure or accommodation. This need to identify and describe the various cognitive structures in all phases of the construction suggests methods, such as clinical interviews and prolonged observations, to make inferences about the structures that underlie behavior (Noddings, 2006).

Also, Noddings (2006) explains that cognitive constructivism implies pedagogical constructivism. The acceptance of constructivist premises about knowledge and an active knower implies a way of teaching that acknowledges a learner as an active knower. Goldin (2006) notes that clearly a teacher can embrace the pedagogical methods of
constructivists without accepting constructivist premises. Conversely, a convinced philosophical constructivist need not employ only constructivist pedagogical methods (Noddings, 2006).

This may imply, that in today’s reality of a rigid organizational culture in colleges and many adult learners’ perspectives about how mathematics should be taught, there may be a challenge but not necessarily an obstacle to both philosophical and methodological constructivism. Further, The Office of Vocational and Adult Education (OVAE), a subdivision of the United States Department of Education, wrote in 2005 that it did not find scientifically based evidence to support new instructional practices or emerging pedagogical themes as anything more than “promising but unproven” instructional strategies for adult learners. This, again, emphasizes the challenges of utilizing any new teaching strategies in teaching college mathematics.

However, a constructivist perspective such as mine must advocate the new forms of pedagogical practice of the National Council of Teachers of Mathematics, (NCTM) and National Research Council (NRC) as appropriate strategies for a blend of procedural skill and conceptual understanding. I embrace constructivism, as both a cognitive position and a methodological perspective and believe that it is consistent with teaching for understanding rational numbers and proportional reasoning to adult learners. However, the tension created by dichotomous perspectives on instructional practices in mathematics, is the everyday reality of teaching and learning in a finite mathematics course.
Theoretical Foundations

In colloquial terms, proportional reasoning is reasoning up and down in situations in which there exists an invariant (constant) relationship between two quantities that are linked and varying together (Lamon, 2008). It requires explanation beyond the procedural thinking of $\frac{a}{b} = \frac{c}{d}$ (Lamon, 2008). Ratio and proportion are defined as two problem types that are similar to fraction comparison and equivalence problems: comparison and missing value problems (Lamon, 2008) and both require the appropriate use of multiplicative skills. The development of proportional reasoning involves a distinction and clarification between the uses of procedural additive and multiplicative skills (Lamon, 2008).

Lamon (2007) suggests that in order to facilitate rational number understanding and proportional reasoning, there needs to be an assumption that content knowledge in a complex domain such as rational numbers does not involve the target concepts and procedural operations in the domain as much as the central cognitive structures. These central cognitive structures compose the conceptual or structural understandings of many multiplicative topics dealing with rational numbers that are foundational to the development of proportional reasoning.

Also, adult learners must move beyond additive thinking and develop a clear understanding of the Multiplicative Conceptual Field (MCF) (Vergnaud, 1983) before they are able to develop proportional reasoning and ultimately understand proportionality. Understanding the larger concept of proportionality will come about
later, through interaction with mathematical and scientific systems that involve the invariance of a ratio or a product (Lamon, 2007). However, few studies have focused specifically on the development of proportional reasoning in adult learners.

Additionally, adult learners in a college level mathematics course are daunted by the procedures and concepts involved in ratio and proportion because they are unfamiliar with the critical components of proportional reasoning. Many adult learners are on a short time frame to complete the coursework for their programs. The mandate from college administrators is to shorten the amount of time that students spend in mathematics courses. Also, alternative approaches to teaching mathematics, such as the ones advocated by the Council of Teachers of Mathematics (NCTM, 2000), National Research Council (NRC, 2001) are continually being examined to determine the best practices for adult learning (Drago-Severson, 2009; Merriam, Cfferella & Baumgartner 2007; Merriam & Bierema, 2013; Schmitt, Steinback, & Donovan, 2004; Smith & Taylor 2010).

Likewise, Ben-Chaim et al. (2004, 2012) examined and assessed the impact of a model using authentic investigative activities for teaching ratio and proportion in pre-service teacher education. The model followed their pilot studies investigating the change in mathematical and pedagogical knowledge of pre- and in- service mathematics teachers due to experience in authentic proportional reasoning activities. The conclusion of the studies was that application of the model, incorporating theory and practice leads to a dramatic positive change in the pre-service teachers’ content and pedagogical knowledge. In addition, an improvement occurred in their attitudes and beliefs toward
learning and teaching mathematics in general, and ratio & proportion in particular (Ben-Chaim et al., 2004 pp. 81).

In conclusion, proportional reasoning is a complex phenomenon, both in terms of the mathematical relationships and the experiences that promote the mathematics. The phenomenon has been developing since the students’ preschool years along a path that is too complex to retrace (Lamon, 1993, 1995, 2007, 2008; Ben-Chaim et al., 2004, 2012). While proportional reasoning can be recognized when it occurs, there is little knowledge about how to facilitate its development in adult learners. However, Ben-Chaim et al., (2004, 2012) provides a model using authentic investigative activities for teaching ratio and proportion in pre-service teacher education. By incorporating theory and practice, a dramatic positive change was evidenced in the pre-service teachers' content and pedagogical knowledge.

**Standards Based Mathematical Processes**

The difficulties and problems in developing proportional reasoning that Lamon, and Ben-Chaim found may be traced to a lack of conceptual or structural thinking. These structures go beyond manipulation and procedural knowledge to include the five mathematical processes that the National Council of Teachers of Mathematics (NCTM, 2000) advocates to be included in each lesson: problem solving, reasoning and proof, communication, connections, and representation. By utilizing this integrated approach to mathematics content, adult learners may be able to understand problem solving with rational numbers and proportionality when presented in a problem solving, realistic
context (Brookfield, 1995; Schmidt et al., 2004). An example is converting a recipe for two people to the same recipe for six people.

Also, The National Council of Teachers of Mathematics (NCTM), *Principles and Standards for School Mathematics*, revised in 2000, calls for an increased emphasis on the development of meaningful contemporary application, the use of appropriate technologies, as well as activity based and collaborative learning. This approach to the content knowledge of rational numbers and the development of proportional reasoning is also consistent with overcoming the barriers that adult learners face in seeking postsecondary credentials and degrees (National Center for Educational Statistics, 2007; Kazis & Callahan, 2007). While classroom lessons are planned to teach rational numbers and proportional reasoning, the students are also presented with problems and questions that develop conceptual understanding. Reflective problem solvers, like adult learners, are then ready to join a workforce with an understanding of rational numbers and necessary problem-solving and communication skills (Brookfield, 1995).

Furthermore, *Principles and Standards for School Mathematics* (2000) states that “reasoning mathematically is a habit of mind, and like all habits, it must be developed through consistent use in many contexts” (p. 56). Students are encouraged by being challenged with “why” and “how” questions. Adult learners, who have been educated only on the procedures of solving rational number problems, may begin to recognize that it is not enough to be able to solve a problem, that they must reason out the concepts of the underlying mathematics, make conjectures and hypothesis, and communicate solutions and strategies to others (Ben-Chaim et al., 2004, 2012).
Another important point in adult learning, is that in order for adults to validate their thinking or to convince another person that their thinking is accurate, they need to communicate verbally and in writing (Schmitt et al., 2004). Adults, like other students, organize and consolidate their mathematical thinking through communication (Vygotsky, 1978). As they learn to communicate their mathematical thinking precisely and coherently, they also learn to analyze and evaluate the mathematical thinking and strategies of others.

Also, teaching adult learners involves identifying and connecting mathematical ideas to new and previously learned mathematics they know and understand as well as their unique life experiences. This may include finding the one concept or context that the student does understand and presenting a task to facilitate them in relating it to the concept being discussed to build a coherent whole in the domain of rational numbers and proportional reasoning. Connections of mathematics and life experiences often illuminate a point of understanding (Knowles, 1973 and Schmitt et al., 2004). Further, learning models or representations and their translations, such as experience, manipulatives, pictures, spoken language, and written symbols also serve as starting points to understanding concepts for adult learners (Lesh, Post & Behr, 1987). By using representations to model and interpret physical, social, and mathematical phenomena, adult learners can begin to relate the concepts involved in proportional reasoning to a real-world phenomena. Thus, adult learning needs to fit into the context of contemporary society as well as previously learned mathematics (Merriam et al., 2007; Schmitt et al., 2004).
In conclusion, an integrated approach to teaching the mathematical content of rational numbers, proportional reasoning and proportionality is supported by NCTM (2000). Further, the NCTM (2000) mathematical processes are compatible with principles of adult learning (Ginsburg, Manly & Schmitt, 2006). Both are central to understanding how adults develop proportional reasoning.

**Procedural Thinking vs. Conceptual Understanding**

A recent Community College Research Center (CRCC) (2011) working paper on mathematics in colleges identified two major patterns in U.S school mathematics teaching and learning as having the most value:

- *Procedural thinking* (also known as skill or computational efficiency; instrumental understanding), defined as the “accurate, smooth, and rapid execution of mathematical procedures.”

- *Conceptual understanding* (also known as structural thinking or relational understanding), defined as the mental connections among mathematical facts, procedures and ideas. (Skemp, 1976/2006)

Further, abstract mathematical notions can be conceived in two ways: structurally (concepts) or operationally (skills) (Sfard, 1991). Sfard (1991) claims that the shift from a skill conception to a concept conception is not a quick or easy shift for students. This is a critical and significant point in the pedagogy of rational numbers and the development of proportional reasoning in adult learners.

Often, adult learners need to be engaged in the learning process (Knowles, 1973; Mezirow, 1991) and then to learn how to learn (Brookfield, 1995). This process of
learning to learn is challenging because their educational experiences may have been based on procedural skill alone. Former Education Secretary Margaret Spellings reiterates: “to prepare students for algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency and problem solving skills” (NMAP, 2008).

Accordingly, Schoenfeld (1985) provides four categories of knowledge and skills needed to be successful in higher education mathematics: 1) Resources - proposition and procedural knowledge of mathematics, 2) Heuristics - strategies and techniques for problem solving such as working backwards, or drawing figures, 3) Control - decisions about when and what resources and strategies to use, and 4) Beliefs - a mathematical "world view" that determines how someone approaches a problem. These four components comprise Schoenfeld’s (1985) meta-cognitive view of “learning to think mathematically” and the development of problem solving skills that are essential for adult learners.

Therefore, while there is broad agreement in colleges of the importance of both procedural fluency and conceptual understanding, the issues of what skills and concepts should be taught and how they should be taught have not been resolved. The tension caused by the debate on procedural fluency (including what procedures should be taught, in what order, and by whom) and how to foster conceptual understanding continue to have implications for adults learning proportionality (Schmitt et al., 2004; Ginsburg et al., 2006).
And to further complicate the problem, college textbooks and labs, including those used in this IT course in mathematics, replicate and promote the pedagogical approaches of K-12 (NMAC, 2008). For example, in building and simplifying rational numbers the lecture shows only the procedure with little or no discussion about the underlying concepts involved. This is the same traditional curriculum and pedagogy that has not served students well in the past; thus compounding the problem of a lack of balance between procedural skill and conceptual understanding in a complex domain, such as rational numbers and proportional reasoning (NMAC, 2008).

In order to improve the quality of student’s mathematical education in ratio and proportion and proportional reasoning, new forms of pedagogical practice, such as those advocated by NCTM, NRC, and NMAC need to be implemented. These pedagogical practices in adult learning include models of authentic investigative activities that are compatible with a constructivist perspective (Merriman, Caffarella, & Baumgartner, 2007).

Consequently, by using qualitative case study methodology I describe the implementation of authentic investigative activities for teaching ratio and proportion to adults in two sections of an information technology (IT) finite mathematics course. In this dissertation, information technology is used as an umbrella term to represent communication and computing tools (National Science Foundation, 1996). These activities were aligned with the context theories of ben-Chaim et al. (2004, 2012), the content theories of Lamon (2007, 2008), and the NCTM mathematical process with special emphasis of representations. There were two phases in this study: the first phase

**Purpose of the Study**

Thus the purpose of this case study was to examine what helps or hinders adult students to learn ratio and proportion when the topic is not the central focus of the mathematics course. More specifically, the research questions were:

- How did the use of authentic investigative activities, aligned with Lamon’s and Ben-Chaim’s content theories, impact adult learning of ratio and proportion?
- What characteristics of the investigations were most helpful for adult learners to grasp the variety of dimensions of procedural thinking and conceptual understanding of rates, ratios, scale and proportional reasoning?
- How did the videos help to develop a strategy of mathematical thinking and problem solving in adults understanding of ratio and proportion?

**Significance of the Study**

My study extends the scholarly research and literature in adult learners’ understanding of ratio, proportions and proportional reasoning by focusing specifically on the development of proportional reasoning in adults using authentic investigations and by using qualitative methods to provide a deeper understanding of the processes that adults use in learning ratio and proportion. Further, this study can advance knowledge as it ties theory to educational practice in mathematics education by providing a model of authentic investigative activities that can be used in an IT or seated mathematics
classroom. My research goal is that the results of this study will be of interest to learners, instructors and college administrators.

**Definitions**

**Didactical Phenomenology** consists of the content-related components of proportional reasoning: relationship, partitioning, unitizing, absolute and relative thinking, covariance and invariance, and ratio appropriateness.

**Finite Mathematics** applies mathematical techniques to solve real-world problems and involves the study of topics including linear models, systems of equations, financial math, set theory, logic, probability, and statistics.

**Proportions** are relations that exist in many daily, practical applications beyond

$$\frac{a}{b} \equiv \frac{c}{d}.$$ For instance, sales tax is proportional to the cost of the item you are buying or the distance between two cities is proportional to that distance on a map (Lamon, 2005).

**Proportional Reasoning.** “In colloquial terms, proportional reasoning is *reasoning up and down* in situations in which there exists and invariant (constant) relationship between two quantities that are linked and varying together. As the word *reasoning* implies, it requires argumentation and explanation beyond use of symbols

$$\frac{a}{b} \equiv \frac{c}{d}.”$$ (Lamon, 2005, p. 3)
Summary

In this chapter, I presented my worldview of constructivism as the cognitive position and methodological perspective for this qualitative case study. I proposed the theories of Lamon (2007, 2008) and Ben-Chaim et al., (2004, 2012) as the framework for adult learning of ratio and proportion and proportional reasoning along with the representation theory of Lesh, Post, and Behr (1987). Further, I proffered the National Council of Teachers of Mathematics (NCTM), *Principles and Standards for School Mathematics* advocating an increased emphasis on the development of meaningful contemporary application, as well as authentic investigative activities. Also, the pedagogical approach of this study is consistent with overcoming the barriers that many adult learners face in developing procedural skill and conceptual understanding of ratio and proportion.

Chapter II presents an overview of the literature related to this case study of the implementation of authentic investigative activities to adult learners on ratio and proportion. It begins with the theories and models of Lamon and Ben-Chaim. Then, learning models and representations are presented. Finally, the literature on adult learners and numeracy will be presented.
CHAPTER II

LITERATURE REVIEW

Introduction

Chapter II presents the literature that is germane to this study on the implementation of authentic investigative activities on ratio and proportion to adult learners. To this purpose, I present three research agendas: Lamon’s seminal theories on rational numbers and proportional reasoning and Ben-Chaim’s theory and model of the implementation of authentic investigative activities to adult learners, student teachers while utilizing the mathematical processes with special emphasis of Lesh’s multiple representations. In addition, I present research on adult learners and numeracy.

Lamon’s Theories of Rational Numbers and Proportional Reasoning

Over the last ten years, 90 percent of the eighteen- and nineteen-year-old students in my mathematics classes [at Marquette University] have been unable to answer 50 percent of the questions (found on the next page) . . . [which illustrate the broad base of meaning that is associated with fraction symbols.] We can only conclude that they have not yet had enough experience to understand rational numbers. It is difficult to imagine that they will gain that experience by taking courses that assume knowledge of rational numbers” (Lamon, 2001).

Lamon’s Questions Associated with Fraction Symbols

1. Does the shaded area blow show a) 1 (3/8 pie)? b) 3 (1/8 pies)? c) 1 1/2 (1/4 pies)? Does it matter?
2. You have 16 candies. You divide them into 4 groups, select one group, and make it three times its size. What single operation would have accomplished the same result?

3. You have taken only one drink of juice, represented by the unshaded area in the figure below. How much of your day's supply, consisting of two bottles of juice, do you have left?

4. If it takes 9 people 1 1/2 hours to do a job, how long will it take 6 people to do it?

5. Without using common denominators, name three fractions between 7/9 and 7/8.

6. Yesterday Alicia jogged 2 laps around the track in 5 minutes, and today she jogged 3 laps around the track in 8 minutes. On her faster day, assuming that she could maintain her pace, how long would it have taken her to do 5 laps?

7. Here are the dimensions of some photos: a) 9 cm x 10 cm, b) 10 cm x 12 cm, c) 6 cm x 8 cm, d) 5 cm x 6 cm, and e) 8 cm x 9 cm. Which one of them might be an enlargement of which other one? (Lamon, 2001).

Lamon (2001) then posed the question: Which sub-construct: part-whole, quotient, ratio, measure, or operator is the most effective primary interpretation for students to use in building a comprehensive understanding of rational number topics?

Lamon’s theories were developed after longitudinal research that focused on middle school students.

Subsequently, Lamon (2007) presented a framework based on the assumption that pedagogical content knowledge (Shulman, 1986, 1987) in a complex domain such as
Rational numbers does not address the target concepts and operations in the domain. Pedagogical content knowledge must address the central cognitive structures, that are composed of concepts, ways of thinking and the mechanisms for growth that are fundamental to the range of multiplicative topics. She explains this as looking for the cognitive processes, focal points, and big ideas that form the basis of thought and knowledge in rational numbers and related multiplicative topics, including the following:

**Reasoning beyond Mechanization**

Lamon (2007) defines proportional reasoning as “detecting, expressing, analyzing, explaining, and providing evidence in support of assertions about proportional relationships” (p. 367). The word reasoning suggests common sense and a thoughtful approach to problem solving rather than applying rules and operations. Reasoning is not associated with mechanization or procedural skill but with mental, free flowing process that require understanding of the relationships among quantities. When a student understands a concept, they know what a proportion is not and when it does apply. This is not achieved by procedural skill alone.

**Covariance and Invariance**

Reasoning is moving beyond mechanization in order for students to know how and if two quantities are correctly linked (Lamon, 1993, 2005, 2007). Lamon (2007) suggests the development of proportional reasoning includes two major types of variance. The first is the covariance of the ratio of the two quantities in a mathematical structure (e.g., \( y \) directly proportional to \( x \)). The second type is the invariance of the product of the two quantities (e.g., In a work problem when more workers are added, the time that it
takes to complete the work decreases). There are multiple layers of development before students have the ability to detect the invariant quantity when two quantities are changing together to form a third quantity (Kaput, Luke, Poholsky & Sayer, 1986).

**Multiplicative Reasoning**

Another facet of proportional reasoning is multiplicative reasoning. Multiplicative reasoning requires some degree of mathematical reasoning for students to have the ability to understand the difference between adding and multiplying. Inhelder and Piaget (1958) suspected that children who reason additively employ additive transformations. An additive transformation does not preserve a ratio. A student using additive transformations is unable to analyze a proportional situation. However, additive reasoning may be an invariant stage in the development of proportional reasoning. Thus, it may also not be a positive transformative stage from additive to multiplicative reasoning.

Further, Lamon (2007) states that the critical scalar and functional structural relationship of rational numbers is best illustrated by Vergnaud’s (1983) model for analyzing multiplicative structures. Vergnaud’s model shows the functional relationship between corresponding elements of measure spaces. However, these multiplicative explanations are not proportional reasoning but can provide a framework stressing the functional structural relationship.

**Abstraction: Beyond Observation and Direct Measurement**

Lamon (2007) further suggests that multiplicative reasoning moves beyond concrete operations and into formal reasoning and abstractions. Piaget considers it the
hallmark of formal reasoning. Reasoning relies on students’ work in their head, rather than written computations; then logical reasoning patterns can develop along with the ability to draw conclusions or inferences (Vergnaud, 1988). Relative or comparative thinking, as opposed to absolute thinking, is needed to move beyond sense data in the understanding of intense quantities or the relationship between two quantities, such as ratios. Abstraction imposes a relational structure or a classification scheme apart from the object in which it is exemplified (Lamon, 2007).

Relative thinking activities provide students the opportunity to expand their understanding of certain words that have been associated with additive concepts (Pimm, 1987). An example is the word “more” which most students are familiar with as a signal word to add or subtract. It can also have a proportional or relational meaning.

**Ratio Appropriateness**

Further, understanding intensive quantities sometimes requires more than intuition and experience. Knowledge of the content also plays a role as it determines if a ratio is appropriate (Lamon, 2007).

**Didactical Phenomenology Model**

Lamon (1993) had uncovered a number of learning sites from which children gain insights into the critical features of proportional reasoning. Freudenthal (1983) named these learning sites the “didactical phenomenology” of the domain. These points are where a student may enter the learning process and reconstruct an important mathematical idea that is organized by the mathematics, such as ratio and proportion. Rather than starting from the definition of ratio and building, the material looks at
phenomena that may develop conceptual understanding to mathematize a situation using ratio and proportion.

Figure 1. Didactical Phenomenology.

The model is comprised of the components or entry points in the development of proportional reasoning. On the first or initial level of understanding there are three dimensions. One dimension, of key importance, is additive versus multiplicative thinking or the relationship between and within rational numbers. The second dimension is the importance of unit recognition, which is included with partitioning (the third dimension) and equivalence as part of the “basic thinking tools for understanding rational numbers” (Behr, Lesh, Post, & Silver, 1983, p. 109). As Lamon (1993) notes, of particular
importance to reasoning proportionally is “the ability to construct a reference unit or a unit whole, and then to reinterpret a situation in terms of that unit” (p. 133). Lamon also describes how “the process of norming can achieve yet another level of sophistication” (p. 137), whereby an independent unit may be selected as a basis for comparison.

Additionally, there are three content-related components of proportional reasoning in the figure: absolute and relative thinking, ratio sense or appropriateness and covariance and invariance (Lamon, 1993). Conceptual understanding of absolute and relative thinking, covariance and invariance, and ratio appropriateness are critical learning sites for several levels of abstraction, from visual to verbal, manipulative to pictorial that lead to proportional reasoning. While these three components are necessary to understand proportional relationships, they are not enough to produce proportional reasoning. Proportional reasoning is a whole much greater than the sum of its parts whether the students are middle school learners or adult learners (Lamon, 2007, 2008; Ben-Chaim et al., 2004, 2012).

Consequently, many educational researchers perceive that proportional reasoning is then a consequence of understanding rational numbers (Lamon, 2007, 2008; Ben-Chaim et al., 2004, 2012; Lobato & Ellis, 2010). Traditionally, ratio and proportion have been defined as two problem types that are similar to fraction comparison and equivalence problems and comparison and missing value problems (Lamon, 2008). This implicit definition of ratio and proportion by these two problem types has caused some confusion of the terms proportional reasoning and proportionality (Lamon, 2007).
In *Teaching Fractions and Ratios for Understanding*, Lamon (2008) explains that proportional reasoning signifies the attainment of a certain level of mathematical maturity that consolidates many elementary ideas and opens the door to more advanced math and science thinking. Therefore, if a student cannot reason proportionally, then the student does not understand rational numbers.

Therefore, when students reason proportionally, their cognition is marked by most of these conceptual understanding characteristics (Lamon, p. 108).

- They understand equivalence and the concept of *same relative amount*.
- Proportional thinkers do not think solely in terms of 1-units, such as 26 miles per gallon of gas. They think in terms of complex units, such as 3-units or 10-units, or composite units depending on the context. Instead of opening packs of gum and counting the individual sticks, they can think in terms of packs. When possible, they reason without reducing rates, such as 52 miles per gallon of gas.
- They exhibit greater efficiency in problem solving because of their thinking of composite units. An example is where unit prices produce a non-terminating decimal. Apples are priced at 3 for $.68. It is more efficient to think of the price of 12 as $.68 x (4 groups of 3) = $2.72.
- Proportional thinkers can look at a unit displayed in an array and see how many objects are in 1/3, 1/2, 1/5.
- They can flexibly interpret quantities. For example, 3 apples for 24 cents can be interpreted as 8 cents per apple or as 1/8 apple per 1 cent. They can unitize several times without losing track of the unit.
• Proportional thinkers are not afraid of decimals and fractions. Often, students replace the fractions and decimals with whole numbers to help them think about a problem. Proportional thinkers move around flexibly in the world of fractions and decimals.

• They have developed strategies – sometimes unique - for dealing with problems such as finding fractions between two given fractions.

• They are able to mentally use exact divisors to their best advantage and can quickly compute 3/8 if they know 1/8, 80% if they know 10 or 20%.

• They have a sense of co-variant and invariant situations. They can analyze quantities that are changing together and in opposite directions, talk about direction of change and rate of change, and determine relationships that remain unchanged.

• Proportional thinkers can identify everyday contexts in which proportions are or are not useful. Proportions are not just mathematical objects or situations to which they know how to apply an algorithm. They can distinguish proportional from non-proportional situations, and will not blindly apply an algorithm if the situation does not involve proportional relationships.

• They have developed a vocabulary for explaining their thinking in proportional situations.
• Proportional thinkers are adept at using scaling strategies. They can reason up and down in missing value and comparison problems, whether quantities are expressed as fractions, decimals or percent.

Essential Content Knowledge and Instructional Strategies

Instruction can facilitate the joint development of rational number understanding and proportional reasoning. This is an important aspect in my study of adults understanding ratio and proportion and ultimately proportional reasoning. Furthermore, building on the adult’s informal knowledge by relating the learners’ knowledge and experience to rational numbers and proportional reasoning, may make the concept more understandable.

The constant of proportionality. Lamon (2008) explains the mathematical model for proportional reasoning as a linear function of the form \( y = kx \), where \( k \) is the constant of proportionality. Therefore, \( y \) is a constant multiple of \( x \). Also, two quantities are proportional when they vary in such a way as to maintain a constant ratio \( \frac{y}{x} = k \).

The constant \( k \) plays an essential role in understanding proportionality.

Lamon names \( k \) a “slippery character” as it changes its guise in each particular context and representation. At times \( k \) does not appear explicitly in the problem but as a structure beneath the obvious details. In symbols, it is a constant. In a graph, it is slope. In a table, it may be the difference between any entry and the one before it or it may be the rate at which a quantity changes with respect to another expressed as a unit rate.

In rates, it is the constant rate. In reading maps, it is the scale. In shrinking/enlarging context or similar figures, it is the scale factor. If \( k \) can be a percentage as in
sales tax or a theoretical probability when you are rolling dice. Lamon (2008) identifies
the following critical mathematical components of proportional reasoning:

**Critical Mathematical Components of Rational Numbers and Proportions**

**Invariance and covariance.** Lamon (2008) explains that proportional
relationships involve some of the simplest forms of co variation. Two quantities are
linked to each other in such a way that when one changes, the other changes in exactly
the same way as the first.

**Units defined implicitly.** Deciding on the unit in a fraction problem is not a
personal interpretation (Lamon, 2008). In fraction construction, the meaning of fractions
derives from the context in which they are used and each context should define the unit
either implicitly or explicitly. By using units that are defined implicitly, students are able
to reason up and down, preferably doing this work mentally and encouraged to reason out
loud. An example: Frank ate 12 pieces of pizza and Dave ate 15 pieces. “I ate ¼ more,
said Dave. “No! I ate 1/5 less,” said Frank. Why the argument? (Lamon, p. 73)

**Units and unitizing.** Unitizing refers to the process of constructing chunks in
terms of a unit commodity. Lamon (2008) explains this as a normal way to think about a
quantity and should be encouraged in instruction, such as 24 eggs = 2 (dozen) = 4(6
packs).

Depending on the context, chunking a quantity one-way may be more
advantageous than chunking it in another way. Flexible thinkers can chose or anticipate
the best way to do something.
Part-whole fractions. Lamon (2008) defines a part-whole comparison as a designated number of equal parts of unit out of a total number of equal parts into which the unit is divided. Many different fractional names designate the same amount. Part-whole comparisons with unitizing mean that the process of unitizing is used to name equivalent fractions.

Ratios. A ratio is a comparative index that conveys the abstract idea of relative magnitude (Lamon, 1993). Research in the development of critical ideas for proportional reasoning adopts the theoretical perspective that ratio is a composite unit, formed by comparing units that are a result of multiple compositions of other units (Lamon, 1993). Then, more sophisticated reasoning develops with unitizing or the process of building increasingly complex unit structures. Proportional reasoners are adept at building and using composite units and are able to make decisions about which units to use when choices are available, for example five pieces of gum or a pack of gum (Lamon, 1993).

Relative and absolute change. One of the most important types of thinking that is required for proportional reasoning is the ability to analyze change in both absolute and relative terms (Lamon, 1993). It is essential that students are able to understand change from both perspectives. A ratio is a comparative index; it always makes a statement about one measurement in relation to another. The ratio asks the student to think multiplicatively. Again, it is difficult for both children and adult learners to move away from additive thinking with which they are familiar and begin to think relatively (Lamon, 1993).
**Ratio sense or appropriateness.** Students also must be able to distinguish between situations that are appropriately organized by ratios and those that are not (Lamon, 1993). As students realize what it means for a relationship to exist between two quantities, they also realize that it should always be true. They should see that quantities of two types exist, and within each type the second quantity is a multiple of the first. Then, they should realize that given any quantity of the first type and knowing the way the quantities are related, they can find the quantity of the second type. By informally analyzing a variety of proportional and non-proportional relationships, they can develop an intuitive sense and discuss the contexts and mathematical relationships associated with proportions.

**Critical Components of Proportional Reasoning**

Because of the prolonged time needed to develop proportional reasoning and the web-like organization of this body of knowledge that defies linear organization, Lamon (2007) suggests that there is no substitute for longitudinal research that includes the full complexity of classrooms, students’ experiential background and intuitive knowledge and the complexity of the mathematics. Thus, she presents her longitudinal study using a 7-point model for building mathematics instruction, grounding students in one of the sub-constructs of rational numbers as well as the other six ideas that are central to multiplicative thinking. Her model refers to Case’s (1992) six topics as the construct of *central multiplicative structures*. He defines this as a critical system built-in and subject to developmental ceilings, and partly nurtured through instruction. As these systems built up, by a complex interaction of knowledge and experience over a long period of time and
become sufficiently mature and connected, the student develops control structures so that they are under their command. With the students’ reasoning conscious and deliberate, they are able to achieve higher order thinking than was previously impossible, and affecting knowledge and performance across a variety of domains.

Lamon’s (2007, 2008) model of the seven central structures of measurement, quantities and co-variation, relative thinking, unitizing and norming, sharing and comparing, reasoning up and down and rational number interpretations provides alternatives to the traditional part-whole fraction instruction. As the students’ knowledge builds in each of the “nodes,” the “nodes” also grew together. The result was students demonstrating a usable knowledge of multiple rational number interpretations, not only knowledge in the one in which the students’ instruction was grounded. Their thinking was flexible enough to support proportional reasoning in a problem-solving situation. Lamon (2008) states that it is a compelling task for researches to facilitate the joint development of rational number understanding and proportional reasoning. She offers central or core ideas that need to be included in instruction. These concepts, contexts, representations, operations and ways of thinking are highly interrelated. They are shown in the following web-like diagram, rather than in a linear order as shown previously in the Didactical Phenomenology Model (see Figure 1, p. 23).

Some of the nodes in the diagram do not occur spontaneously for children and must be facilitated by instruction. Lamon (2008) describes these nodes as the central structures because they are so critical to mathematical thinking in general and support a much bigger system of proportional reasoning rather than just rational numbers.
At the end of the study, students exposed to all seven constructs had developed a deeper understanding of rational numbers, as measured by the number of sub-constructs they were using. The number of students using proportional reasoning, as well as computational achievement, increased and the time-honored principle of transferability was strongly evident. Lamon (2007) concluded that with proper attention to the central multiplicative structures, students naturally develop an understanding of multiplicative interpretations. Furthermore, the students have the capacity to create ingenious solutions when they are challenged and are not required to follow rules. Findings in the particular sub constructs include the following:

**Part-whole comparisons with unitizing.** Students developed a strong idea of the unit when unitizing was added to the traditional part-whole interpretation. It promoted understanding of equivalent fractions that facilitated the operations of additions and
subtraction. This interpretation has strong and natural connections to measure, ratio and operation.

**Quotients.** Quotients share a natural connection with ratios and rates. This connection is helpful in the sharing and comparing context.

**Operators.** The operator interpretation provided a useful context for multiplication and division, scaling and general fraction sense. However, it did not lead to easy addition and subtraction. Rational numbers have a dual nature and axioms. One is for addition and one is for multiplication.

**Measures.** Students developed a strong sense of unit, subintervals, and equivalence, along with the order and density of rational numbers and the operations of additions and subtraction.

**Ratios.** The greatest difference between ratios and the other interpretations is the way they combine through arithmetic operations. The students, that studied ratios and rates as their primary interpretation of rational numbers, developed a strong understanding of equivalence and of proportional reasoning in general. They moved between ratio and part-whole comparison and few had problems with fraction addition and subtraction. Most students developed their own ways of reasoning about multiplication and division. They also developed a good working knowledge of ratio, part whole, and operators.

The conclusion in the Lamon (2007) study is that the deep connectivity of the rational number sub-constructs makes teaching only one interpretation impossible. The measure sub-construct seems to be the strongest because it most naturally connects to the
other sub-constructs. Not all of the sub-constructs (measure, quotient, ratio, operator, and part-whole) are good starting points for the teaching of rational numbers. Operators and quotients are less powerful than measures, ratios and part-whole sub-constructs.

All of the interpretations taught together may provide a deep rational number understanding, but no single interpretation is a panacea to development of proportional reasoning. However, as Lamon (2007) notes, the interpretations of the seven sub-constructs of rational numbers are tightly intertwined, and with the proper attention to the central multiplicative structures, students will naturally develop an understanding of multiple interpretations. Further, with the interaction of knowledge and experience over time, a foundation of mathematics will develop that allows students to work in the recurrent and increasingly complexity of mathematics and science.

**Summary of Lamon’s Work**

“Proportional reasoning is used to describe sophisticated mathematical ways of thinking that emerge sometime in the late elementary or middle school years and continue to grow in depth and sophistication throughout high school and college years. It signifies the attainment of a certain level of mathematical maturity and consolidates many elementary ideas and opens the door to a more advanced mathematical and scientific thinking” (p. 8). Students build up a complex interaction of knowledge and experience over a long period of time that are central to their understanding of the ideas, processes and representations that continue to be recurrent, recursive and increasingly complex across mathematics and scientific domains (Lamon, 2008).
Lamon’s work addresses the need for curriculum materials, as illustrated in Figure 2, that cross traditional boundaries of rational number instruction to develop proportional reasoning in the late elementary and middle school years. Her seminal work is of central importance to the development of proportional reasoning in students of all ages. Lamon’s insights and models provided me with the generative synthesis of content ideas on ratio and proportion in this study. Furthermore, her work has strongly influenced others in their development of learning models of proportional reasoning.

The following body of literature extends Lamon’s theories on ratio, proportion and proportional reasoning to adult learners, specifically pre and in-service teachers with the use of authentic investigative activities.

**Ben-Chaim’s Model of Authentic Investigative Activities for Adults**

Ben-Chaim, Keret, and Ilany (2004, 2012) examined and assessed the impact of a model using authentic investigative activities for teaching ratio and proportion in pre-service teacher education. The model was developed after investigating the change in mathematical and pedagogical knowledge due to experience in authentic proportional reasoning activities. The authors had previously studied proportional reasoning in seventh grade students (Ben-Chaim, Fey, Fitzgerald, Benedetto & Miller, 1998).

The (2012) study asserts that worldwide there are gaps in the content knowledge of pre-service and in-service elementary and middle school mathematics teachers in ratio and proportion. Their existing knowledge was “technical, schematic, unconnected, and incoherent” (p. 81). This lack of conceptual understanding of ratio and proportion led to
the feeling that as teachers, they were incapable of coping with and teaching the topic (Ben-Chaim et al., 2012).

Ben-Chaim et al. (2012) developed 19 activities to access the influence of exposing pre-service mathematics elementary teachers to authentic investigative proportional reasoning activities. The activities included mathematical tasks that required quantitative and qualitative numerical comparisons between ratio and finding a missing value. The tasks included small and large integer numbers, fractions, decimals, and percents. These activities established the understanding of many concepts related to ratio and proportion and focused on the three main categories of proportional reasoning problems: Rate and Density, Ratio, and Scaling (p. 82).

The studies concluded that pre-service teachers were more successful in solving ratio and proportion problems after their exposure to the authentic ratio and proportion investigative activities. This was exhibited by their use of different problem solving activities and better capability of providing a good quality of written and oral explanations to their work (during interviews). Also, their attitude towards mathematics improved in general as well as all aspects of ratio and proportion problems. The study was replicated at 3 different colleges with similar findings. As a result, a special model for teaching ratios and proportions in mathematics teacher education was developed.
Figure 3: A Model Using Authentic Investigative Activities for Teaching Ratios and Proportions.

The model is comprised of four components with interaction between them. The core of the model includes the authentic investigative activities: Introductive activities, Investigative activities dealing with ratio, dealing with rate, dealing with scaling, and dealing with indirect proportion. For each activity, there is a base part, to build the basic knowledge. Then, there is an extension or Investigative Problem to further develop understanding. An example is provided below (see Figure 4).

The second component includes the structure of the activities. These activities are structured as authentic investigative problems related to content and context familiar to pre-service teachers and elementary and middle school students. The activities include
Figure 2a: Investigative Problem 1: Preference of Cola

**BOLA-COLA OR COLA-NOLA?**

The following results are related to surveys of preference between BOLA-COLA and COLA-NOLA:

A. The ratio of those who preferred BOLA-COLA than COLA-NOLA is 3 to 2.
B. The numbers of those who preferred BOLA-COLA than COLA-NOLA are in ratio of 17,139 to 11,426.
C. 5,713 more participants preferred BOLA-COLA than COLA-NOLA.

1. Decide if the above three statements are necessarily extracted from the same survey? Explain!
2. Which statement describes most accurately the results of comparison between BOLA-COLA and COLA-NOLA? Explain!
3. If you need to advertise the results, which statement seems to be the most effective for advertisement? Why?
4. Suggest other possible ways for comparison between the popularity results of the two kinds of cola.

(Ben-Chaim et al., 2012)

*Figure 4.* Investigative Problem 1: Preference of Cola

Tasks deemed appropriate in professional literature for assessing proportional development: 1.) Missing value problems; 2.) Numerical rate and ratio comparison problems; and 3.) Qualitative prediction and comparison problems requiring comparisons but not dependent on specific numerical values.

The third component includes the structure of the didactical unit. It includes a unit around each concept, such as ratio. The structure of the didactical unit includes:

1) Working by groups; 2) Discussion of results with the whole class; 3) Mathematical and didactical summary; and 4) Homework.
The fourth component includes the evaluation unit with several types of assessment instruments: 1) An instrument to evaluate the student’s mathematical knowledge, and mathematical didactical knowledge; 2) Attitude questionnaire; 3) Portfolio; and 4) An instrument for assessing research report.

The study concluded that the application of the model that incorporates theory and practice led to a positive change in the teacher’s content and pedagogical knowledge. Also, there was an improvement in their attitudes and beliefs towards learning and teaching mathematics in general, and ratio and proportion in particular (p. 81).

**Mathematical Perspective on Ratio and Proportion**

Ben-Chaim et al. (2012) stressed the many uses of ratio in mathematics and other areas of knowledge by quoting Lamon (2007):

> Of all the topics in the school curriculum, fractions, ratios, and proportions arguably hold the distinction of being the most protracted in terms of development, the most difficult to teach, the most mathematically complex, the most cognitively challenging, the most essential to success in higher mathematics and science, and one of the most compelling research sites. (p. 629)

Further, Ben-Chaim et al. (2004, 2012) defined ratio as the quantification of a multiplicative relationship that is calculated by dividing (or multiplying) one quantity by another. For example, if there is twice the number of hours of instruction in an advanced course as compared to a basic course, then the ratio 2:1. This is the quantification of the multiplicative relationships between the two units. Further, ratio can be used explicitly in many ways. An example is a bouquet of flowers where the ratio of tulips to daisies is
1:3. For every 4 flowers in the bouquet, 1 is a tulip and 3 daisies; or, of all the flowers in the bouquet, 1/4 are tulips and 3/4 are daisies. If there are 3 tulips in the bouquet, then there will be 9 daisies.

The authors also pointed out that there may be instances where the ratio is not explicitly obvious, and prerequisite knowledge is required to understand that the concept is actually based on a ratio between two terms. For example, velocity can be defined as the ratio between the distances that a car travels to the time it takes to travel that distance.

Corresponding to Lamon’s theories, Ben-Chaim et al. (2004, 2012) stressed that the concept of ratio infers a multiplicative relationship between two values. In other words, additive thinking leads to addition and subtraction and multiplicative thinking leads to multiplication and division. If the goal is to find how much a certain quantity is larger or smaller than the other, addition or subtraction is used. Comparison by addition or subtraction is the first method encountered by pupils in primary school and for many this concept continues to dominate any situation that requires comparative thinking. However, the types of problems in which addition or subtraction will not be effective are many, and a multiplicative strategy that involves understanding the concept of ratio is required.

Additionally, Lamon (2007, 2008) conveyed the work of (Freudenthal, 1983) and didactical phenomenology as points where students may enter the phenomena of learning ratio and proportion. Ben-Chaim et al. (2012) also utilized his theories in their model with the following descriptions of rate, ratio and scaling:
• Comparing magnitudes of different quantities with a connection, as in miles per hour or unit price. These comparisons are called rates or densities.

• Comparing two parts of a single whole, as in the ratio of girls to boys in a class is 15 to 10.

• Comparing magnitudes of two quantities that are conceptually related, but are not naturally considered as parts of a common whole, as in ‘the ratio of sides of two triangles is 2 to 1.’ Such comparisons are often referred to as scaling, and they include problems of stretching or shrinking in similarity transformations.

Also, these categories illustrate the multiplicative relationships that produce ratio.

The concept of proportion is often used in solving problems in mathematics and other fields. Proportional problems involve situations in which the mathematical relationships are multiplicative (as opposed to additive) in nature and allow the formation of two equal ratios between them. The ability to solve such problems indicates the existence of proportional reasoning, which leads to abstract thinking. In mathematics, pupils in middle school are beginning to use “proportion” to solve a wide range of problems. In grades later grades, they learn to solve problems in algebra (such as dividing quantities into unequal parts, pricing, profit and investment, percentages, motion, and energy). These same rules and properties of proportion are often used to calculate probability, acceleration, and equilibrium. They also play a part in statistical calculations, cartography (map drawing to scale), and profit and loss.
Ben-Chaim et al. (2012) also used the mathematical definition of \( \frac{a}{b} = \frac{c}{d} \) for a direct proportion; where the quotient of the two parts of the ratio, \( a \) and \( b \), is constantly equal to that of \( c \) and \( d \). They use this definition, \( a \times b = c \times d \) for an indirect proportion; where the product of the two parts of the ratio, \( a \) and \( b \), is constantly equal to that of \( c \) and \( d \).

According to Ben-Chaim et al. (2012), *The Collins Dictionary of Mathematics* also adds another aspect to the definition of proportion. It states that proportion is a direct or indirect linear relationship between two variable quantities. This means that corresponding elements of two sets are in proportion when there is a constant ratio (either direct or indirect) between them. For example, according to the gas laws, pressure is directly proportional to temperature: the quotient derived from pressure (numerator) and temperature (denominator) will be constant; however, pressure is inversely proportional to volume, meaning that the product between volume and pressure will be constant (p. 34).

Further, Ben-Chaim et al. (2004, 2012) explain that students will be able to recognize that there is a proportional connection between two or more of the variables in a problem when they are able to identify two criteria that identify a problem as one of proportion: 1.) There must be a multiplicative relationship (a ratio) between the two values; and 2.). The multiplicative relationship must be constant, either in the same (direct proportion), or opposite (inverse proportion) direction. Then, they will be able to discover a solution based on proportional reasoning.
Ben-Chaim et al. (2004, 2012) studied investigative strategies used for solving problems of ratio and proportion and have discovered a wide range of strategies being used. In the process of the students learning the proportional scheme, it appears that the older an individual, the higher the level of sophistication of the chosen strategy; progressing from pre-formal strategies, such as yielding qualitative solutions up to formal ones, such as eliciting a mathematical-quantitative solution (p. 35).

The pre-formal strategies to solve ratio and proportion problems are typical of children in elementary school and may include 1) Intuitive strategies that are suitable for very simple proportional problems and demonstrate an intuitive grasp of multiplicative relationships; 2) Additive strategies that are typical of young children with additive reasoning skills (generally in the lower grades of elementary school). Here, the focus is on the quantitative differences between the values in the problem rather than the ratio between them; 3) Division by ratio that require that the student to be aware of the given ratio, and appreciates the multiplicative relationship that exists between the values given in the problem; 4) Finding the unit (sometimes using a corresponding table). In both this strategy and finding the part from the whole). While the student defines the ratio as the unit, or a part of the whole, in order to calculate the entire amount or the amount of each portion, and then builds his solution on that; 5) Determining the part from the whole; and 6) Missing Value Problems.

When students use formal strategies, they may use the proportion formula or \( \frac{a}{b} = \frac{c}{d} \). This is typical of adolescents and adults and indicates the existence of proportional
reasoning and abstract thinking. At this stage the student will be capable of using algebraic symbols to represent proportion, and will succeed at finding the correct quantitative answer to a problem by using the rules and properties of algebra. Or students may use application of the rules and properties of proportion, such as finding the fourth proportional value in proportional equations. If a proportional relationship exists between 4 variables, and 3 of the variables are known, the fourth proportional may be found by using the rules and properties of proportions. In missing value problems there is a proportion between 4 variables given, where the values of three are known, and the fourth must be found. In order to solve this problem, the ratios must be compared. Finally, two ratios may be compared using the property that enables expansion and reduction of the ratio in order to allow the comparison of two ratios.

Proportional reasoning is the ability to make use of an effective form of the proportional scheme. This ability has a central role in the development of mathematical thinking, and is frequently described as a concept that, on the one hand, is a cornerstone of higher mathematics, and, on the other hand, is the peak of the basic tenets of mathematics (Lesh, Post, & Behr, 1987). According to the NCTM Curriculum and Evaluation Standards (1989), “the ability to reason proportionally develops in students throughout grades 5–8. It is of such great importance that it merits whatever time and effort that must be expended to assure its careful development “(p. 82).

Ben-Chaim et al. (2012) state further that according to Lamon (2007), proportional reasoning along with the concepts of ratio and proportion are widely regarded as a critical bridge between the numerical, concrete mathematics of arithmetic
and the abstraction that follows in algebra and higher mathematics. Further, Fischbein (1995) refers to three basic components of a productive mathematical reasoning as a human activity: the intuitive, the algorithmic, and the formal.

1. Intuitive component. This includes intuitive cognition, intuitive understanding, and intuitive solution. These reflect the ideas and confidence regarding mathematical entities, and the mental images that we use to represent mathematical ideas. Regarding proportion, it also includes the ability to recognize a proportional relationship, either direct or indirect, between the variables.

2. Algorithmic component. This includes the procedures, properties, and methods that are used for the calculations. Regarding proportion, it includes the ability to use algorithms and mathematical techniques to find a quantitative mathematical solution for a proportional problem.

3. Formal component. This aspect includes knowing the axioms, basic principles, definitions, propositions, and proofs connected to the relevant concept. Regarding proportion, this knowledge includes the ability to express the multiplicative relationship of the proportion with mathematical models. This ability is expressed by “quantifying” the first relationship of the ratio and comparing it to the second ratio, and then, according to the type of proportion exhibited in the situation (direct or indirect), correctly representing the four proportional variables in the “proportion formula.”

Acquiring and being able to use and combine the knowledge of these three components leads to conceptualization of the concepts of ratio and proportion and allows...
the user to correctly solve various types of problems: quantitative, with given variables, and quantitative, requiring qualitative comparison. Conversely, Fischbein (1995) claims that the inability to combine intuitive and formal knowledge may result in, among other things, invalid perceptions of the problems to be solved, cognitive conflicts, and incorrect use of algorithms.

Ben-Chaim et al. (2012) again cites Lamon (2007) who shares the view of many scholars by stating that proportional reasoning is a long-term developmental process in which the understanding at one level forms a foundation for higher levels of understanding (p. 637). Further, Lamon (1993) also claimed that the domain represents a critical juncture at which many types of mathematical knowledge are called into play and a point beyond which a student's understanding in the mathematical sciences will be greatly hampered if the conceptual coordination of all the contributing domains is not attained (p.90).

Ben-Chaim et al. (2012) also utilized Inhelder and Piaget (1958) as the first researchers to show that the proportional scheme develops in three stages: intuitive, concrete and formal. They conclude that in most students, the proportional scheme, like other operational schemes, matures naturally at adolescence, when formal thinking processes develop. Then, it may be used as a strategy for solving problems and deriving general, abstract conclusions. Their theory suggests that proportional reasoning is a main indicator of operational development, during the stages of formal development, (p. 50). Thus, the proportional scheme will be chosen spontaneously when problems having a
proportional relationship are encountered which leads to a rational, logical, intelligent plan of action.

However, the Ben-Chaim et al. (2004, 2012) studies show that many adult pre and in-service teachers, struggle with problems on ratio and proportion and do not attain operational development of proportional reasoning. Studies carried out in various colleges in Israel to test the mathematical knowledge of pre-service teachers pointed to many problems stemming from the mathematics education that these pre-service teachers had received. Evidence showed that their knowledge of and experience in mathematics were not rich enough along with negative experiences from their student days (p. 61).

Generally, as pupils mature, they learn new skills in school that enable them to solve increasingly challenging problems. This learning is contingent on appropriate practice that stimulates the development of this potential proportional scheme at maturity. Moreover, some students at the concrete stage have a latent potential proportional scheme. Through a learning process that includes appropriate explanations, practice, and exercises, can develop into an effective cognitive tool to be used in the solution strategy (Ben-Chaim et al., 2012).

Based on their studies, Ben-Chaim et al. (2012) believe that understanding informal methods for solving ratio-and-proportion problems will strengthen the intuitive foundation of the proportional scheme and will encourage students to solve problems using informal strategies before formal instruction is given. Realistic knowledge or experiences that students acquire out of the classroom also have impact on the development of proportional reasoning.
Ben-Chaim et al. (2012) advocate the use of three types of problems for evaluating proportional reasoning: 1) Missing value problems, where three pieces of information are given and the task is to find the fourth or missing piece of information; 2) Numerical comparison problems, where rates/ratios are given and they are to be compared to find if they are equal, greater, or smaller; and 3) Estimation problems that require comparisons not dependent on specific numerical values, such as: Dana uses less juice concentrate and more water in her lemonade, will the lemonade be stronger, weaker, or the same, or is there not enough information to decide.

Further, adolescents and adults, with formal abstract thinking, will use a formal strategy, such as the proportional formula \( \frac{a}{b} = \frac{c}{d} \). All methods rely on multiplicative strategies. Some other multiplicative computational strategies are derived outside the classroom. These strategies are content-dependent and of a non-formal character. Often, these have been successfully invented by students and are based on multiplicative reasoning (Lamon, 1994; Ben-Chaim et al., 2012).

According to Ben-Chaim et al. (2012), the main sources of difficulty in solving ratio-and-proportion problems for children, adolescents and adults alike, are cognitive. First, the proportional scheme is a second-order operational scheme, requiring an operation to be carried out after another operation has been done. Inhelder and Piaget (1958) emphasize that this difficulty is due to the fact that understanding proportion requires the ability to compare two ratios in which the variables must be coordinated into
a relation. In addition, it requires the ability to recognize whether the problem involves direct or indirect proportion (p. 56).

Second, they explain that ratio-and-proportion problems require a multiplicative pattern, which is more complicated than the additive one that children use in elementary school (Vergnaud, 1994). Additive reasoning develops at a young age, but then may present a hindrance to passing into the realm of multiplicative reasoning. Third, when the ratio is an intensive value (a ratio of the “rate” type or category) that forms a new dimensional unit, such as speed, power, etc. The new unit demands understanding of the rules and properties relevant to the subject in question in addition to the mathematical understanding connected to proportion. And, fourth, the requirement to intuitively understand what type of ratio-and-proportion problem is difficult in the case of indirect proportion, which is more complicated for child and adult alike.

Further, Ben-Chaim et al. (2012) posit that there may be other variables that influence the process of adults developing an understanding of proportional reasoning. Once a student has identified the problem as directly or indirectly proportional and has surmounted any cognitive difficulties, they must solve the problem in practice. During the performance process, the student may run into additional variables that may add to the difficulty of finding the correct solution to the problem (p. 57).

There are two types of variables that influence the student and lead to difficulties in the solution process. The first stems from the nature of the assignment. This involves the verbal content of the problem, including the nature of the question and the realm of knowledge described by it. It also involves the nature and structure of the numbers
presented in the problem, such as their dimension (very large or small), or type (problems with rational or discrete values are easier to solve than those with continuous values). These two variables essentially determine the level of difficulty of the problem (Lamon, 1993).

The second type of variable is the personal characteristics of the student: age, stage of development in his proportional scheme, gender, IQ, etc. (Ben-Chaim et al., 2012).

Ben-Chaim et al. (2012) provide two additional sources of difficulty that a student might have in the problem solving process, the first is the curriculum, and the second is the teaching-learning process in the classroom. Traditional classroom curriculum and instruction tends to use problem solving as a tool for practicing technique, and not for developing mathematical thinking or appreciating the connection between mathematics and the real world. As a result, students mainly get procedural experience, without developing a significant understanding of the concepts of ratio and proportion, leading to many difficulties in knowing how to solve ratio and proportion problems.

An effective solution would be the addition of a wide range of more authentic (real life) investigative activities, combined with a rich learning environment. The solution should also include opportunity for the adult learner to reflect on different aspects of each problem. The discourse could be verbal or written in a journal as used in the Ben-Chaim studies. The authentic, investigative activities are familiar from real life and require adult learners to use judgment and knowledge to solve the problems in a tangible way. Authentic activities are complex and not unambiguous, have several stages, and do not necessarily have one specific, correct answer. The activities demand judgment in
deciding on the appropriate knowledge required and its application, skill in deciding on the order of priority, and organization of the stages needed to understand and solve it (Ben-Chaim et al., 2012). They suggest further that when evaluating the ability of the student, it is important to follow the solution and thought processes that guide each student, such as a case study approach to research.

**Summary of Ben-Chaim et al. (2004, 2012) Theories**

Ben-Chaim et al. (2004, 2012) examined and assessed the impact of a model using authentic investigative activities for teaching ratio and proportion in pre-service teacher education. The model was developed after investigating the change in mathematical and pedagogical knowledge due to experience in authentic proportional reasoning activities.

It is important to acknowledge the similarity of the Ben-Chaim theories to those of Lamon by noting the citations to the major themes of her theories on ratio, proportion and proportional reasoning within his work. Therefore, the Ben-Chaim model of authentic investigative activities or context attuned to adult learners could be viewed as a continuation of Lamon’s seminal content work.

The next section discusses how learning models/representations help to develop the complex interaction of knowledge and experience in understanding of proportional reasoning.
Learning Models/Representations

Lesh, Post, and Behr (1987) Representations

Lesh, Post, & Behr (1987) describe the roles that representations and translations between representations play in mathematics learning and problem solving. They define the term representations as external or observable embodiments of students’ internal conceptualizations. Five distinct types of representations occur in mathematics learning and problem solving:

1. Experience-based scripts: knowledge is organized around “real world” events that serve as general contexts for interpreting and solving other problems.
2. Manipulative models, such as Cuisenaire rods, blocks, bars, number lines, etc. These models have little meaning but the “built-in” relationships and operations simulate everyday situations.
3. Pictures or diagrams: static figures that can be internalized as “images.”
4. Spoken language: including sub languages-like those related to domains, such as logic.
5. Written symbols, such as spoken language that can involve specialized sentences and phrases A’U B’ as well as normal English sentences and phrases.

Lesh, Post, and Behr (1987) Translations

As important as the distinct types of representations, are the transformations among them and within them. Students establish a relationship (mapping) of one representational system to another, preserving the structural meaning as in translating
from one language to another (Lesh, Post, & Behr, 1987). Further, between system mappings or translations differ from within system operations or transformations.

A conclusion is that students that have seriously deficient understandings of word problems in “paper and pencil” computations may have equally deficient understandings about the models and languages needed to describe, illustrate and manipulate ideas. Both significantly influence mathematical learning and problem solving performance.

Lesh, Post, & Behr (1987) use the example that when a student is said to understand an idea like the ratio 1/2, they can recognize the idea in a variety of qualitatively different representational systems, such as the decimal 0.5, and flexibly manipulate the idea within the given representational systems and accurately translate the idea from one system to another. Teachers can use the student’s concept of a given idea and the related underlying transformation/translation networks to paraphrase the concept into familiar situations, such as those in the authentic investigative activities. Or, when a student is having learning difficulties, teachers can develop a variety of questions by presenting an idea in one representational mode and asking the student to describe or represent the situation in another mode. A diagnostic question may show that a student does not understand in one mode; then the teacher can rephrase the question in another.

The translation process also corresponds to some of the most important modeling processes that are needed to use these ideas in everyday, real life situations. Essential features of modeling include: 1) Simplifying the original situation by ignoring irrelevant characteristics in order to focus on the more relevant factors; 2) Establishing a mapping between the original situation and the model; 3) Investigating the properties of the model
to generate predictions about the original situation; 4) Translating or mapping the predictions back into the original situation; and 5) Checking to see whether the translated prediction is useful and makes sense (Lesh, Post, & Behr, 1987).

Thus, representing tends to be plural, unstable, and evolving. These attributes also make it possible for concepts and representations to evolve during the course of problem solving sessions. Students tend to work in more than one representation, either in a series or parallel. For example, a student may move from a picture or diagram representing $\frac{1}{2}$ of a pie to a written symbol of $\frac{1}{2}$. Each representation depicts only a portion of the problem. Also, many realistic problems in mathematics are multimodal, such as a two pizza problem where each pizza is each cut into a different number of slices (one into 5 pieces and one into 8 pieces). Each of the problems is a pizza word problem in which one of the student's tasks is to translate the two givens into a homogeneous representation mode so that combining them makes sense (Lesh, Post, & Behr, 1987).

Problems of the preceding type occur naturally in a multimodal form but solution paths often weave back and forth among several representational systems, each of which are typically well suited for representing some parts of the situation but is not suited for representing others. For example, in the previous problem, a student may think about the static quantities, such as two pieces of pizza in a concrete way, maybe in a picture, but may switch to written symbols to carry out the dynamic "combining" actions (Lesh, Landau, & Hamilton, 1983).

Proficient problem solvers tend to be sufficiently flexible in their use of a variety of relevant representational systems so that they instinctively switch to the most
convenient representation at any given point in the solution process. The following figure (Lesh, Post, & Behr, 1987) suggests one way that the act of representation tends to be plural. Solutions are often characterized by several partial mappings from parts of the given situation to parts of several partially incompatible representations. Each partial mapping represents a part of the problem situation, using only part of the available representational system. It is not a mapping from the whole "given" situation to only a single representational system (see Figure 5).

(Reprinted from Lesh et al., 1987)

Figure 5. Representations Tend to be Plural

Another aspect of representational plurality is that a given representational system often appears to be related to several distinct clusters of mathematical ideas. The authors used this example to illustrate:
“The Million Dollar Problem: Imagine that you are watching "The A Team" on television. In the first scene, you see a crook running out of a bank carrying a bag over his shoulder, and you are told that he has stolen one million dollars in small bills. Could this really have been the case?”

One student who solved this problem began by using sheets of typewriter paper to represent several dollar bills. Then, he used a box of typewriter paper to find how many $1 bills such a box would hold. He was thinking about how large (i.e., volume) a box would be needed to hold one million $1 bills. Next, however, holding the box of typewriter paper reminded him to think about weight rather than volume. So, he switched his representation from using a box of typewriter paper to using a book of about the same weight. By lifting a stack of books, he soon concluded that, if each bill was worth no more than $10, then such a bag would be far too large and heavy for a single person to carry (Lesh et al., 1987).

From the preceding solution, the first representation involved a sheet of paper, which was quickly absorbed into a second representation based on the size or volume of boxes. This played a role in switching from conceptualizations based on volume to a conceptualization focused on weight. The meaning(s) associated with each of these representations were plural in nature; and they evolved during the solution process. The unstable nature of the representations was reflected in the fact that when attention was focused on "the whole situation" (or representation), details that previously were thought to be important were not. Or, when attention was focused on one detail, the student often temporarily ignored the others (Lesh et al., 1987).
In summary, Lesh et al. (1987) stress the inherent plural, unstable and evolving nature of the act of representation and translation in both mathematics learning and problem solving.

**Adult Learners**

In order to facilitate the process of learning for adults, it is important to know who adult learners are, how social context shapes their learning and why adults are involved in learning activities (Merriam, Caffarella, & Baumgartner, 2007). Further, how adults learn and how aging affects learning abilities are important to the understanding and facilitation of adult learning.

**Defining Adult Learners**

The stereotyped image of the college student as one who is 18-23 years old in a residential, full-time study program is being challenged by a new reality. The U.S. economy is now information-driven and a college degree has become an increasingly important credential in the marketplace, both for new entrants into the labor force and those already employed. Working adults who want to succeed in the present economic climate are pursuing a college education in increasing numbers, and they are creating a new majority among undergraduates at college campuses across the country. Adult learners are loosely identified with a larger group characterized as "non-traditional. "While definitions vary, the U.S. Department of Education, National Center for Education Statistics (NCES) (2007) has come up with seven characteristics that typically define non-traditional students. According to the NCES (2007), adult learners often:
• Have delayed enrollment into postsecondary education;
• Attend part-time;
• Are financially independent of parents;
• Work full-time while enrolled;
• Have dependents other than a spouse;
• Are a single parent, and
• Lack a standard high school diploma.

By using one or a combination of these criteria, NCES (2007) estimates that over 60 percent of students in U.S. higher education can be characterized as non-traditional.

Using the simpler and more common criterion of age to define "adult learner," we know that some 43 percent (or 14 million) of students in U.S. higher education are 25 or older. And, an estimated 65 percent increase in enrollments of students 35 years of age and older, from 1.7 million to 2.9 million, occurred between 1985 and 1996 (NCES, 2007).

There are three additional factors that are characteristic of American culture that affect what adults want to learn (Merriam et al., 2007). The first factor is the change in the population demographic base as adults outnumber students less than eighteen years of age. Furthermore, the percentage of the population over age sixty-five continues to grow. Second, as a whole, the American population is better educated with more cultural and ethnic diversity. An third, globalization and technology are inter-related; having a
tremendous impact on the U.S. economy. Thus, adults now find that they must continue learning past 12th grade to function at work, home, and in their communities.

Having defined both the demographics of adult learners and the characteristics of American culture that have contributed to the increased number of adult learners engaged in higher education, the literature on models of adult learners is presented.

Models of Adult Learning

**Knowles’ andragogy.** There are a number of frameworks or models that contribute to understanding of adults as learners (Merriam et al., 2007). The best known is andragogy introduced by Malcolm Knowles from Europe in 1968.

Andragogy is a learning strategy focused on adults. It is often interpreted as the process of engaging adult learners within the structure of learning experience. Originally used by Alexander Kapp in 1833, andragogy was developed into a theory of adult education by Malcolm Knowles in 1973. Knowles asserted that andragogy (Greek for “man leading”) should be distinguished from pedagogy (Greek for “child leading.”)

The following assumptions relate to motivation in adult learning (Knowles, 1980).

- Adults need to know the reason for learning something. (Need to know)
- Experience, including error, provides the basis for learning activities (Foundation).
- Adults need to be responsible for their decisions on education - involvement in the planning and evaluation of their instruction (Self-concept).
- Adults are most interested in learning subjects having immediate relevance to their work and/or personal lives (Readiness).
Adult learning is problem-centered rather than content-oriented (Orientation).

Adults respond better to internal versus external motivators (Motivation).

Knowles saw these assumptions about adult learners’ motivations as the foundation for designing programs for adults. He has suggested that the classroom climate should be “adult” both physically and psychologically causing the “adults to feel accepted, respected and supported” with “a spirit of mutuality between teachers and students as joint inquirers” (1980, p. 47).

**Discussion of Knowles model.** Other adult learning theorists have questioned Knowles’ theories. Hartree (1984) proposed that Knowles did not present a theory of adult learning that was different from a child’s theory of learning. Rather, Hartree maintained that Knowles only presented a set of principles of good practice in teaching adults.

Brookfield (1986) has also raised questions concerning andragogy as a “proven theory” of teaching. He argues that three of Knowles assumptions are problematic in the practice of teaching. The first is the assumption of adult self-direction. Brookfield maintained that self-direction might only be a desired outcome rather than a given condition in adult learning. He raised further concerns about the relationship of learning tied to particular social roles, such as single parenthood and focusing on immediate application.

Moreover, Brookfield (1986) finds only Knowles’ experience assumption to be well grounded. However, Merriam et al. (2007) question even this assumption. They contend that although adults have lived longer, it does not necessarily mean that their
experiences are quality experiences and a resource for learning. The experiences may function as a barrier to learning. An example might be an adult’s sole memories of math being their 4th grade teacher making them repeatedly write multiplication tables on the board.

However, Merriam et al. (2007) agree with Knowles that adults may be more internally motivated than children as their level of commitment depends on their reason for learning. However, if the adults’ learning situation is mandated by the government or socially mandated as in a condition for parole, it may not be an internal motivation and consequently not a purposeful or powerful motivation. Knowles’ last assumption is that adults need to know why they need to learn something. However, some adults may learn for the sheer enjoyment of learning (Merriam et al., 2007).

Finally, Knowles wrote in his autobiography in 1989 that he prefers to think of andragogy as a model of assumptions about learning or a conceptual framework that serves as a basis for an emergent theory. Merriam et al. (2007) argue that andragogy appears to be situation specific and not unique to adults. They maintain that andragogy represents a continuum ranging from teacher directed to student directed learning and it is an approach that is appropriate for children and adults. Recent criticism has centered on Knowles’ focus on the individual learner, rather than the socio-historical context in which the learning takes place (Grace, 1996). Andragogy shows little awareness of how the learner is socially situated or the product of the socio-historical and cultural context. Further, Merriam et al. (2007) contend that social institutions and structures may define the learning without even taking into consideration the individual adult learner.
In summary, Knowles’ andragogy continues to be widely accepted by adult teacher practitioners as a helpful rubric rather than a panacea for better understanding adult learners (Merriam et al., 2007). It is especially useful when used as a guide or as a set of assumptions (St.Clair, 2002). While it does not provide the complete picture of adult learning, it is one piece of the mosaic of adult learning (Merriam et al., 2007).

**McClusky’s Theory of Margin.** McClusky’s (1963) theory is based on adulthood as a time of growth, change, and integration in which one continually seeks balance between the amount of energy needed and the amount available. Hiemstra (1993) expounds on McClusky’s theory with an explanation of external factors in adulthood, such as career and family. The life expectations developed by adults, such as aspirations and future expectations are internal factors. Power to an adult is a combination of resources, such as family support, social abilities and economic abilities. It also includes the acquired skills and experiences for effective performance, such as resilience, coping skill and personality. If the adult can maintain balance in their lives, they are better positioned to take a risk and more likely to learn. Consequently, many adult learners need to be adept at juggling multiple responsibilities and demands on their time.

Further, learning in adulthood is often a function of changing roles and responsibilities and physical and mental development (Merriam et al., 2007). McClusky’s (1963) theory has appeal as it focuses on the everyday and life transitions that all adults face. Merriam et al. (2007) suggest that “overloaded” adults will do all they can, regardless of their situation if they think the subject matter is essential and worthwhile and the learning method is convenient.
Jarvis Learning Process. The Jarvis (1987, 2006) model begins with the adult’s life situation or experience. At the start of any learning process there is a disjuncture between the life experiences of the adult to this point and an incident that the adult is unable to handle. The adult learner is viewed as more than a cognitive machine. Learners are whole people who come to a learning situation with a history, a biography that interacts in individual ways with their experience that generates the nature of the learning (Jarvis, 2006). As the whole person encounters an experience in their own social context, such as the loss of a job due to lack of skill or knowledge, they may find that they cannot automatically accommodate or assimilate that creates a disjuncture or a state of unease that can trigger learning.

In the Jarvis model, learning then moves through levels of thinking, doing, and feeling as the adult learner is affected by the learning. This represents the continuous nature of learning with the changed person in their social world continually encountering new experiences that stimulates further learning. The model situates learning in a social context, as an interactive phenomena, not an isolated internal process (Merriam et al., 2007).

In conclusion, Merriam et al. (2007) posit that while Knowles andragogy is widely known and accepted, his theory has not totally captured the field of study on adult learning. However, all of these models have enhanced the understanding of adult learners’ behavior, and at the same time, provoked my further inquiry and reflection into adult learning theory. Thus, I will focus on some aspects of each in my study of adult learners developing proportional reasoning.
Stephen Brookfield. In the previous review of the theories of Lamon and Ben-Chaim, it has been established that proportional reasoning begins in the elementary grades and develops through adolescence and adulthood. Brookfield (1995) claimed, “Adult learning is frequently spoken of by adult educators as if it were a discretely separate domain, having little connection to learning in childhood or adolescence” (p. 5). He examined this claim by exploring four major research areas: self-directed learning, critical reflection, experiential learning and learning to learn. Each of these areas has been proposed as representing unique and exclusive adult learning processes.

**Self-Directed Learning.** Self-directed learning focuses on the process by which adults take control of their own learning, how they set their own learning goals, locate appropriate resources, decide on which learning methods to use and evaluate their progress. Further, Brookfield (1995) maintains the need to understand how periods of self-directedness alternate with more traditional forms of educational participation in adult, thus denying the importance of collective action, common interests and their basic interdependence in favor of an obsessive focus on self.

**Critical reflection.** Also, Brookfield (1995) proposes that critical reflection is probably the idea of the decade for many adult educators who have long been searching for a form and process of learning that could be claimed to be distinctively adult. Moreover, developmental psychology describes how adults use reflection to think contextually and critically (Brookfield, 1987, 1991). Critical reflection focuses on three interrelated processes: 1) The process of questioning and then replacing or reframing an assumption that up to that point has been uncritically accepted as representing
commonsense wisdom; 2) The process of taking alternative perspectives on previously taken for granted ideas, actions, forms of reasoning and ideologies: and 3) The process of coming to recognize the hegemonic aspects of dominant cultural values and to understand how self-evident renderings of the 'natural' state of the world actually bolster the power and self-interest of unrepresentative minorities (Brookfield, 1995).

**Experiential learning.** Further, the belief that adult teaching should be grounded in adults' experiences, and that these experiences represent a valuable resource has prompted theoretical work among researchers of adult learning (Jarvis, 1987). Adult education practice affirms the importance of experiential methods such as games, simulations, case studies, psychodrama, role-play and internships. Brookfield (1995) states that because of the habitual ways that adults draw meaning from their experiences, these experiences can become evidence for the self-fulfilling prophecies that stand in the way of critical insight, “Uncritically affirming people's histories, stories and experiences risks idealizing and romanticizing them. Experiences are neither innocent nor free from the cultural contradictions that inform them” (p. 11).

**Learning to learn.** Brookfield (1995) concludes “the ability of adults to learn how to learn or to become skilled at learning in a range of different situations and through a range of different styles has often been proposed as an overarching purpose for those educators who work with adults” (p. 11). Kitchener and King (1990) propose the concepts of epistemic cognition and reflective judgment as they emphasize that learning how to learn involves an epistemological awareness deeper than simply knowing how one scores on a cognitive style inventory, or what is one's typical or preferred pattern of
learning. Rather, it means that adults possess a self-conscious awareness of how it is they come to know what they know; an awareness of the reasoning, assumptions, evidence and justifications that underlie our beliefs that something is true (Brookfield, 1995).

In conclusion, there is not a universal understanding of adult learning. Brookfield (1995) addresses this by challenging the myths that a good educational practice always meets the needs of adult learners, and that both the adult learning process and the adult form of teaching are unique. Brookfield (1995) argues that learning should be studied across the lifespan as the variables of culture; ethnicity, personality and politics have a greater significance in the explanation of learning than the variable of chronological age. My study will examine how the implementation of authentic investigative activities impact an adult’s understanding of proportional reasoning and focus on their lifetime variables rather than their age.

**Mezirow.** Mezirow (1991) draws on the writings of Habermas and proposes a theory of transformative learning "that can explain how adult learners make sense or meaning of their experiences, the nature of the structures that influence the way they make meaning of their experience, the dynamics involved in modifying meanings, and the way the structures of meaning themselves undergo changes when learners find them to be dysfunctional" (p. xii). His theory defines the change process that transforms the adult’s frames of reference as "the structures of assumptions through which we understand our experiences that selectively shape and delimit expectations, perceptions, cognition, and feelings" (p. 5). According to this view, "actions and behaviors” will be changed based on the changed perspective (Cranton, 1994, p. 730).
As adult learners are confronted with a disorienting dilemma, such as the authentic investigative activities in this study, a discrepancy between what a person has always assumed to be true is exposed. This can contribute to a readiness for change. Cranton (2002) describes this as a "catalyst for transformation" (p. 66). Further, critical reflection is the means that learners use to work through beliefs and assumptions, assessing their validity in the light of new experiences or knowledge, considering their sources, and examining underlying premises (Cranton, 2002, p. 65). Further, Cranton (1994) explains, "Transformative learning theory leads us to view learning as a process of becoming aware of one's assumptions and revising these assumptions" (p. 730). And, Cranton (1994) simply states, "If basic assumptions are not challenged, change will not take place" (p. 739). However, Mezirow (1997) cautions, "learners need practice in recognizing frames of reference and using their imaginations to redefine problems from a different perspective" (p. 10). Consequently, these authentic investigative activities are meant to be the catalyst for transformation and critical reflection.

In addition to critical reflection that challenges assumptions, transformative learning calls for a trusting, social context for the dialogue referred to as reflective discourse (Mezirow, 2000). This implies the need for classroom discussion on the authentic investigative activities on ratio and proportion. Mezirow (1997) also suggests that the instructor serve as a facilitator, in order to foster the self-direction and control needed for transformative learning. This is a characteristic of an teaching model (National Education Association, 2011).
In conclusion, several theorists have expanded on Mesirow’s theory in order to address his emphasis on the rational and linear aspects of transformation. Grabove (1997) further emphasizes the potential for integration of self and other, renewal and rebirth as themes indicative of the non-rational dimensions of transformative learning. The transformative learner "moves in and out of the cognitive and the intuitive, of the rational and the imaginative, of the subjective and the objective, of the personal and the social" (Gabrove, 1997, p. 95).

**Conclusions Regarding Adult Learning**

Merriam et al. (2007) posit that while Knowles andragogy is widely known and accepted, his theory has not totally captured the field of study on adult learning. However, all of these models have enhanced the understanding of adult learners’ behavior, and at the same time, provoked my further inquiry and reflection into adult learning theory.

Further, there is not a universal understanding of adult learning. Brookfield (1995) addresses this by challenging the myths that a good educational practice always meets the needs of all adult learners, and that both the adult learning process and the adult form of teaching are unique. Brookfield (1995) argues that learning should be studied across the lifespan as the variables of culture; ethnicity, personality and politics have a greater significance in the explanation of learning than the variable of chronological age. My study will examine how the implementations of authentic investigative activities impact an adult’s understanding of proportional reasoning based on the importance of lifespan variables rather than age. Further, it has been established with the theories of
Lamon and Ben-Chaim that the understanding of proportional reasoning covers the span of childhood through adulthood and may be influenced by the same lifespan variables.

Finally, Mezirow (1991) draws on the writings of Habermas and proposes a theory of transformative learning that may explain how adult learners make sense or meaning of their experiences and the way the structures of meaning themselves undergo changes when learners find them to be dysfunctional, such as the implementation of the authentic investigative activities in my study.

**Adult Numeracy**

Adult numeracy extends the significance of how class, culture and ethnicity along with adult learners’ cognitive and learning styles affects how adults use mathematics in specific contexts, such as at work or home, and how these contexts affect adults' mathematical behavior. By looking at the use of mathematics inside and outside of school, researchers can help teachers and adult education programs build on what adult learners already know and value. Rather than treating mathematics as a decontextualized skill, researchers and teachers working within the adult numeracy framework are able to see mathematics as a social practice grounded in the larger social and cultural contexts (California Adult Education Research on Adult Numeracy, 2006). The following literature is based on that assumption.

**California Adult Education Research on Adult Numeracy**

Arriola (2000) focuses on the development of learning-to-learn skills in an Adult Basic Education (ABE) math class based on the National Council of Teachers of Mathematics (NCTM) standards. The instructor investigated ways to change learners'
beliefs about math knowledge and help them develop independence and self-monitoring skills while exploring fractions using several approaches and a variety of contexts. Working in pairs and groups, students made manipulatives, invented games, made charts, drew diagrams, constructed word problems, and compared solutions. The instructor learned to reframe questions to allow for problem solving by students. However, at the conclusion of the study, the adult learners remained dependent on the teacher, without gaining much independence in their reasoning with fractions. The research also suggests that development of abstract mathematical thinking is a long, slow process, even for learners who are able to think abstractly in other contexts.

Coben, (2003) provides an overview of current thinking about adult development and adult numeracy. The authors conducted an analysis of adult numeracy research from around the world. Key findings include: 1) Adult numeracy is fast-developing but remains under-researched and under-theorized; 2) Teachers' inadequate subject knowledge is a continuing concern; and 3) Teaching that connects math topics with the world beyond the classroom is associated with improvements in students' attitudes and skill attainment. Coben’s synopsis of current thinking correlates with the purpose and critical aspects of my study. There is a need for research and theory in adult learning of the essential topic of proportional reasoning. Finally, teacher’s knowledge in ratio and proportion and pedagogy in adult education is underdeveloped. The implementation of the authentic investigative activities may promote adult learners’ attitude and development of proportional reasoning.
Further, Costanzo and Paxton (1999) and The Adult Multiple Intelligences (AMI) study explored ways that multiple intelligences (MI) theory (Gardner, 1983) can support instruction and assessment in adult learning. MI-inspired instruction encourages teachers to analyze their own instructional practice to provide students with a range of learning opportunities based on the students' own strengths and interests. As part of the study, teachers created AMI profiles to help learners in an adult secondary education class reflect on the best ways to approach math problems or questions using their own intelligences. The teachers also created MI-related activities for the classroom. In "Learning about MI," teachers explained unfamiliar, nontraditional, MI-informed activities. In "Learning about Ourselves," students increased awareness of their own strengths and developed self-efficacy. In "Learning about Our Ways of Learning," students worked on finding learning strategies that fit their strengths and interests. By creating open-ended assignments that took into account the multiple intelligences of groups of students, the researchers were then able to teach math skills in a variety of ways that provided authentic learning.

EMPower (2004)

Schmitt, Steinback, & Donovan (2004) developed the EMPower Key curriculum on adult numeracy that changes four critical aspects of instruction: content, sequence, pedagogy, and teacher support. The content is different as it is viewed through the lens of the mathematics that adults need to be able to do in contemporary society; where flexible, fluent, and accurate command of numerical, algebraic, statistical and geometric understandings are critical. Furthermore, Schmitt et al., (2004) states that these four
conceptual strands have been the focus of NCTM standards and much research in learning styles.

Schmitt et al. (2004) sequenced instruction differently than traditional math instruction. Algebraic, statistical, and geometrical ideas are developed along with, and sometimes before, numerical ones. This sequence of instruction in the authentic investigative activities will be discussed further in Chapter III. In this way, the developmental progression has a solid footing in all four strands. The pedagogy is changed as classrooms are viewed as learning communities, where participants share strategies and results of mathematical investigations. Further, the teaching experience is different as teachers are supported by elements of the Teacher Book, an essential component of the EMPower program.

In Keeping Things in Proportion, Schmitt et al. (2004) begins by building on students’ intuitive knowledge and the multiplicative relationships that are at the heart of proportionality. The hands-on lessons connect the central ideas of proportion across mathematics. Students work with rates and ratios in shopping, graphic design and sampling situations. As students progress from concrete experiences with ratios to more challenging problems, they develop tools and strategies to solve proportional problems and to examine the relationships within and between ratios.

Some of the tools are the rule of equal fractions, tables, graphs, unit rates, and cross-multiplication. Students always are asked to use two-solution methods to arrive at an answer. Non-proportional problems are considered also. To facilitate conceptual development, numbers start out 'friendly' and turn 'messier' as the unit progresses. The
numbers, however, prove less daunting to students as they apply their secure knowledge about proportion. Formal proportional reasoning evolves over time, and the lessons in this unit ensure that students are able to make proportional predictions and adjustments using a variety of tools effectively.

**Focus on Basics Connecting Research and Practice (2008)**

Steinke (2008) studied “Does “Part-Whole Concept” understanding correlate with success in basic math classes.” She used the model of children's part-whole understanding developed by Steffe & Cobb with von Glasersfeld, 1988. The concept was tested with a group of volunteer community college students. Four of the original eleven adult volunteers (36 percent) showed the same behaviors as the children who lacked part-whole understanding in the Steffe and Cobb's study. In a previous study, eight out of twelve students (67 percent) in a community college GED class also exhibited the same lack of part-whole understanding (Steinke, 2008). Steinke (2008) concludes that many adults lack part-whole thinking.

Moreover, even the adult - student-friendly text (*EMP*ower *Math*) waits until fractions are introduced to mention parts and whole. Steinke (2008) uses the example that they do not physically change when someone uses a different name for them. The key phrase is "different name, same person. "With this connection to personal experience, students almost immediately see the equal sign as a relationship symbol rather than as an action or agent of change. From this new understanding of "equal," Steinke proceeds to "different names" for the same number.
Steinke (2008) explains that tying a new understanding to a personal, concrete experience (like "different names") is the key to helping learners make new conceptual connections. Steffe and Cobb (1988) pointed out that young children learn math starting from personal experiences, then move on to concrete objects (perceptual stage), followed by representations of quantities (figurative stage), and eventually to dealing with numbers as abstractions (abstract stage). Adults go through the same perceptual then figurative then abstract sequence in developing new understandings in math. Because they have more life experiences to connect ideas to than children do, adults are likely to move through the sequence faster than children. However, new concepts have meaning only when tied to the known and familiar. Consequently, Steinke uses the critical missing understanding in adults, part-whole coexistence, with that most personal experience, the human face.

Steinke’ lesson proceeds from pictures of soda six-packs and other everyday groups with a part missing, to simple word problems about money. Using everyday, real-life experiences appeals to learners' interests and allows them to apply their existing skills to math problems. Questions begin with making change. Calculating correct change is difficult for many adults. Making change is a straightforward "find the part" problem. Making change is then contrasted to "How much do I owe at the checkout counter?" which is a "find the whole" problem.

In conclusion, both the instructional strategies of Schmitt et al. (2004) and Steinke (2008) provide a pedagogy that is consistent with adult learning, adult numeracy, NCTM
standards and process and constructivism. Moreover, their paradigm and pedagogy inform and support my research study.

**Some Conclusions of Literature on Adult Learners**

Some of the overarching themes relating to my study involving mathematics follow:

1) In adults, the learning of natural numbers has been facilitated by their daily activities. However, adults have a limited amount of previous knowledge of rational numbers and this continues after their formal schooling concludes.

2) Adults use any number of procedural strategies to solve rational number problems. These strategies may or may not rely on proportional reasoning.

3) The strategies used by adults to solve rational number problems are based on procedural skill rather than conceptual understanding of rational numbers.

4) Adults are more adept at solving rational number problems if they are familiar with the problems in context.

5) Adults have differing lifespan variables that may have impacted their mathematics learning in general and specifically their understanding of ratio and proportion.

**Summary**

Chapter II provided a review of the literature related to this study examining what helps or hinders adult students to learn ratio and proportion when the topic is not the central focus of the mathematics course. The supporting literature included the theories of Lamon and ben-Chaim, learning models and representations, adult learners, and adult numeracy.
This literature review provides the background for this qualitative case study methodology that is discussed in Chapter III.
CHAPTER III

RESEARCH METHODOLOGY

Introduction

Lamon (2008) reminds me …"For too long, proportional reasoning has been an umbrella term. A catch-all phrase that refers to a certain facility with rational number concepts and contexts. The term is ill defined and researchers have been better at determining when a student or an adult does not reason proportionally than at defining the characteristics of one who does” (p. 3). She continues by stating that elementary and middle school mathematics curricula provide only a cursory treatment of rational number ideas and the understanding of proportional reasoning is left to chance. “Yet, the fact that most adults do not reason proportionally – my estimate exceeds 90% - presents compelling evidence that this reasoning process entails more than developmental processes and that instruction must play an active role in its emergence” (Lamon, 2008, p. 3).

While Lamon illuminated the problem of adults reasoning proportionally, her research focused on elementary and middle school students. Ben-Chaim et al., (2004, 2012) extended the literature with their model using authentic investigative activities for teaching ratio and proportion to adults; specifically pre-service and in-service teacher.

Their literature, like Lamon’s, characterized proportional reasoning as being at the heart of mathematics in upper elementary grades and middle school and that in principle, the mathematical relationships in proportions are multiplicative rather than additive.
Additionally, Ben-Chaim et al. (2004) cite NCTM (1989) Standards that state that proportional reasoning is of great importance and thus merits whatever time and effort it takes to assure its careful development (p. 82). Ben-Chaim et al. (2004) concluded that, “the topics of ratio and proportion should not only have a central part in the mathematics curriculum for children in school as well as for pre-service mathematics teacher education” (p. 82).

The literature reviewed in Chapter II consistently showed that the topics of ratio and proportion are problematic for individuals, ranging from middle school students through a large segment of adults in society (Ben-Chaim et al., 2004, 2012; Lamon, 2007, 2008). However, a gap remained in the research and literature on adults developing an understanding of proportional reasoning.

Accordingly, in this chapter, I present case study methodology that examines what helps or hinders adult students to learn ratio and proportion when the topic is not the central focus of the mathematics course. The authentic investigative activities used in my study were aligned with the content theories of Ben-Chaim et al. (2004, 2012) and Lamon (2007, 2008). There were two phases in this qualitative case study: the first phase took place in Spring Term 1, January 13, 2014 – March 2, 2014 and the second phase during Spring Term 2, March 10, 2014 – April 27, 2014. The site for both phases is the same and will be discussed later in detail.

Again, the purpose of this qualitative case study was to examine what helps or hinders adult students to learn ratio and proportion when the topic is not the central focus of the mathematics course. Specifically, the research questions were:
• How did the use of authentic investigative activities, aligned with Lamon’s and Ben-Chaim’s content theories, impact adult learning of ratio and proportion?

• What characteristics of the investigations were most helpful for adult learners to grasp the variety of dimensions of procedural thinking and conceptual understanding of rates, ratios, scale and proportional reasoning?

• How did the videos help to develop a strategy of mathematical thinking and problem solving in adults understanding of ratio and proportion?

**Situating Myself as a Researcher**

The evolution of my worldview or paradigm manifested itself in my approach to ontology, epistemology, and methodology as an IT classroom instructor of mathematics, researcher, and as an adult learner. This evolution began in my first career in corporate training and development where I instructed adult learners, employees, on the basics of the conversions of fractions, decimals, and percentages. At that time, my post positivist paradigm was supported by, not only the corporate and social culture of the 1980s, but by my own undergraduate and graduate education in business.

My paradigm shift (Kuhn, 1962) began as I sensed the dichotomy of how and what I was teaching and the adult learners’ comprehension. I arrived at two premises 1) The adult learner’s prior mathematics education in school was not properly grounded in the procedures and concepts of rational numbers; and 2) My teaching method of lecture was not effective in the adult learners’ mathematics current education.

At the same time, I began to realize that an adult learner’s reality is multiple and socially constructed (Eisner, 1994). Further, students, whether children or adults, do not
learn as passive receptors of knowledge imparted on them - only to be regurgitated out at a later date. Freire (1985) described this as the “banking system of education.”

My career shifted to the classroom, lecture-based community college and regional campus mathematics education in the 1990s and finally in 2009, to IT mathematics classroom teaching at a private, independent mid western university of 6900 students. Of the 6,900 total students, 2,500 are exclusively IT students. At that time, my paradigm shifted and evolved to constructivism. Today, I maintain the philosophical assumptions of constructivism in mathematics education with the work of Inhelder and Piaget (1958), the social constructivism of Vygotsky (1978) and the radical constructivism of von Glaserfeld (1987).

Accordingly, Piaget’s (1953, 1964) cognitive developmental theories signify the adult learners’ need to identify and describe the development of intelligence in all phases of construction of phenomena, such as the development of proportional reasoning. An adult learner may begin this construction in any one of the components of proportional reasoning, for example rate, ratio or scale or indirect proportion (Ben-Chaim et al., 2004, 2012; Lamon 2007, 2008).

And, as an adult learner interacts in the IT environment, interactions among constructs within the individual occur as part of assimilation and accommodation. In other words, adult learner environmental interactions with the mathematics content may engender interactions within the individual that lead to modifications of the interacting, developing constructs in proportional reasoning or of relationships among them. These modifications, in turn, can influence subsequent subject-environment interactions, which
can engender further modifications of the individual’s interacting constructs (Steffe, Thompson, & von Glasersfeld, 2000).

Further, Vygotsky’s (1978) theory of social interaction states: "Every function in the child's cultural development appears twice, first on the social level between people, and later on the individual level inside the person.” Also, radical constructivist, von Glasersfeld (1987) emphasizes the critical need to know what is going on in each individual student’s head. Another critical interaction in an IT mathematics course is student interaction with the instructor. I was the facilitator of the adult’s learning, as I continually interpreted and helped the learner to develop their own understanding of ratio and proportion.

Steffe et al. (2000) state that reflexivity between the two domains of interaction (mine and the adult learners) is fundamental to what radical constructivists mean by “inter-subjective construction of knowledge in social interaction” (p. 269). They explain further that as two individuals - whether student interactions with the instructor or a peer - engage in social interaction, inter-subjective knowledge is established. As this occurs, each individual reciprocally assimilates the language and actions of the other. This reciprocal process of assimilation continues until no further accommodations of the conceptual structures are needed and the two individuals reach a state of mutual agreement about the meaning of the results of their interactions. However, this does not imply that the interacting individuals (myself and the adult learners) will end up with identical understanding of ratio and proportion. Thereby, the theories of Piaget (1953, 1964), Vygotsky (1978), and von Glaserfeld (1987) provided me with a compatible and
coherent constructivist model of ontology, epistemology and methodology for student learning and research.

Ontologically, I saw the adult learner’s mathematical realities change in my IT classroom, as they became more sophisticated in their thinking (Guba & Lincoln, 1994, p. 109). By recognizing their prior knowledge and experiences with ratio and proportion, I could collaboratively help them to continue to shape newly constructed knowledge to understand proportional reasoning. I also knew that the adult learners prior knowledge and experience, with not only ratio and proportion but also mathematics in general, provided different methods of solving problems. This might also lead to different interpretation of the solutions to the problems because of their individual perspectives and multiple realities. Therefore, these differing interpretations created the need for thick, rich descriptions in the participant’s own words in the natural setting of the IT classroom within this qualitative case study to fully explain their expanding understanding of proportional reasoning (Creswell, 2007).

Epistemologically, I was linked interactively to the participants of the study so that the findings were literally created as the study proceeded (Lincoln & Guba, 1994, p. 111). The process was iterative between the participants and myself as we co-constructed their understanding of proportional reasoning. In the IT classroom, we constructed this knowledge together through our interactions in MyMathLab, Moodle and email. The participants were active and involved questioners on the authentic investigative activities within Math Snacks on ratio and proportion. They were also willing to tell their stories
and encourage others to do the same through the Discussion in the Moodle forum, class projects, interviews and observations.

Methodologically, as a constructivist, I used a hermeneutical and dialectical approach (Lincoln & Guba, 1994, p. 109). The participants and I listened to each other and interpreted in order to arrive at a shared meaning, thus co-constructing meaning and understanding. As we went back and forth in their interviews and observations, we refined our understanding of proportional reasoning and credibility was created (Lincoln & Guba, 1985).

Thus, this study was conducted using a constructivist theory of learning. It was designed and conducted with an intention of focusing on the adult’s construction of proportional reasoning. As the researcher, I closely monitored the students’ thinking and understanding of ratio and proportion with the conjuncture that each student would have a unique way of accomplishing this.

**Researcher as Instrument**

I am an IT classroom mathematics instructor with a constructivist worldview. I am also the finite mathematics course content developer as well as lead teacher for the course. After extensive research of available textbooks with companion *MyMathLabs*, I noted and acknowledged that the teaching paradigm was traditional, basically using lectures and practice problems. I also noted that ratio and proportion was glossed over without any relationship developed between the other rational number critical components or a relatable context to adult learners (Ben-Chaim et al., 2004, 2012; Lamon, 2007, 2008). The rote and algorithmic problems were either missing value or
equivalence. My choice of this particular textbook and MyMathLab was basically because of its somewhat real life or authentic activities. This is an important aspect of a curriculum focused on adult learning.

However, as I continued to reassess my teaching in this IT course, it was clear that MyMathLab stressed procedural skill and not conceptual understanding. This paradigm reinforced a positivist view of mathematics that had not been successful in the past. However, within Moodle and MyMathLab, I had some constructivist opportunities to intervene in the instruction. The implementation of the authentic (real life) investigative activities in Math Snacks focused on developing conceptual knowledge in ratio and proportion. Further, the Moodle discussions served as a focus group for the adult learners to discuss with their peers and instructor the concepts involved with ratio and proportion. At the same time, the class project was designed to provide the adult learners further opportunity to individually reflect and abstract the concepts.

**Research Design and Methodology**

The purpose of this qualitative case study was to examine what helps or hinders adult students to learn ratio and proportion when the topic is not the central focus of the mathematics course. To answer the research questions, I chose a qualitative case study design and methodology. My decision to use this method was based on my research focus, types of problems best suited for the design, discipline background, unit of analysis, data collection forms, data analysis strategies and written reports (Creswell, 2007).
In this study, the distinctive need for case study research arose out of a desire to understand, in depth, adult learners developing an understanding of ratio and proportion with authentic investigative activities. A case study starts with an outcome (Creswell, 2007). In this case study, it was the participant’s incremental change in their conceptual understanding of ratio and proportion after the implementation of the authentic investigative activities in *Math Snacks*. Additionally, Creswell provided four other factors to consider: the audience, background, scholarly literature, and personal approach. The majority of previous studies on ratio and proportion with adult learners, including the qualitative phase of Ben-Chaim et al. (2004) were case studies. Thus, I hoped and anticipated to fill the gap in the literature by using this accepted approach.

In addition, Creswell defined a case study as one in which the investigator explores a bounded system or multiple bounded systems over time. The adult learners were each a single case as they completed the pre- and post-tests on ratio and proportion and attitude, viewed the authentic investigative activities and individually completed the accompanying activities, class project and discussions in Moodle. Finally, each case focused this study on the individual while retaining a holistic and real-world perspective (Yin, 2014, p. 4). Then, some overriding themes on what characteristics of the authentic investigative activities were most helpful to the adult learner’s understanding of ratio and proportion were developed. Consequently, this information could guide my future pedagogical approaches in this IT finite mathematics course. Thus, I identified and defined the case study approach as appropriate, cited studies that employ it, and will follow the procedures outlined in the approach (p. 45).
My rationale for adopting a case study approach also coincided with both Stake’s and Yin’s parameters of the case study approach. Stake’s (2000) specifications were that a case study be defined by an analytic focus on an individual event, activity, episode, or a specific phenomenon, not by the methods. Again, this case study explored a “bounded system,” identifiable with time and circumstance. In this study, adult learners developing an understanding of ratio and proportion were the individual cases and satisfy Yin’s (2014) three conditions for case study: the type of research question posed, the extent of control a researcher has over actual behavioral events, and the degree of focus on contemporary as opposed to historical events (p. 9).

However, “The first and the most important condition for differentiating among the various research strategies was to identify the type of research question being asked” (Yin, 1989, p. 19). One qualitative research question in this study focused on “what” characteristics of the authentic investigative activities in Math Snacks were most helpful to the adult learners, as well as the “how” and “why” questions that are common to case studies, Yin (2004) explains that “what” questions do pertain to case studies as well. The “what” research question in this study sought further information to explain the particular reasons the authentic tasks in Math Snacks seemed to support other IT material in MyMathLab (Yin, 2006). This required extensive and “in-depth” description in the individual cases.

In summary, interviews and observations were conducted with the adult learners, and various other sources of information including documents, discussion, and audio/visual materials were gathered as well. This case study required “extensive material
from multiple sources of information to provide an in-depth picture of the case” (Creswell, 2007, p. 96). In the constructivist paradigm this study assumed an emergent design and was context dependent (Creswell, 2007). In addition, Creswell defined a case study as one in which “the investigator explores a bounded system or multiple bounded systems over time, through detailed, in-depth data collection involving multiple sources of information, and reports a case description and case-based themes” (p. 73).

**Method**

**Setting**

This case study was bound in time and setting:

Phase 1: Spring Term 1, January 13, 2014 – March 2, 2014

Phase 2: Spring Term 2, March 10, 2014 - April 27, 2014

The setting in both phases was an IT finite mathematics course at a university, with student enrollment of 6,900 and IT enrollment of 2,500, located in rural Ohio. It was established in 1888 as a private, independent institution, offering the following nationally accredited professionally-focused graduate and undergraduate degrees; including Associate of Arts, Associate of Business Administration, Associate of Criminal Justice, Bachelor of Arts, Bachelor of Business Administration, Bachelor of Criminal Justice, Master of Business Administration, Master of Science in Criminal Justice and Master of Humanities.

The university represents a new kind of institution, that the administration terms a professional university, where the career objectives of traditional college-age students and adult students are optimized through professionally-focused undergraduate and
graduate programs that have a broad general education foundation. Many of the students are the first generation in their families to attend college. The university prides itself on access and opportunity for individuals and facilitates their preparation for successful careers and for productive and satisfying lives. While many of the students are from Ohio, students are recruited globally.

The 7-week IT finite mathematics course, which is the classroom setting, is a required 3-credit college level course for all undergraduate degrees. 85% of students place into this course. The finite mathematics course is a prerequisite for the required elementary statistics course.

Finite Mathematics (sometimes termed Survey of Mathematics) applies mathematical techniques to solve real-world problems and involves the study of topics including linear models, systems of equations, financial math, set theory, logic, probability, and statistics. The course was asynchronous or a student-centered teaching method that used IT learning resources to facilitate information sharing outside the constraints of time and place among a network of people.

The Learning Management System was Moodle. It contained the class project and Moodle Discussion Forum that were part of both phases of this study. Both are discussed in detail in this section. The course curriculum was in MyMathLab, which was a companion to the course textbook. The lectures, homework, quizzes and tests were completed and automatically scored in MyMathLab. There were extensive resources for students in MyMathLab, including step by step instructions on how to solve problems and
a feature known as “Ask my Professor” where an email can be sent directly to me if the student needs some additional help.

The lectures in MyMathLab were taught by a video of an instructor at a whiteboard, depicting a traditional classroom. The topic of ratio and proportion occurred in week one of the course. It was one lesson lasting 5 minutes and covering the traditional missing part and equivalence problems of ratio and proportion. There was also a brief tutorial prior to the weekly quiz reviewing ratio and proportion using the same methods.

There were 21 students in the course in both phases, which was full capacity. The students were an average age of 33, 67% Female, 33% Male, and ethnicity- African American 32%, Hispanic 2%, Pacific Islander 1%, Other 9%, White 51%, Unknown 5%.

Thus, the U.S. Department of Education, National Center for Education Statistics (2007) (NCES) seven characteristics (discussed in Chapter I) that typically define non-traditional students were compatible with the student demographics in this IT finite mathematics course. I chose this site because of the adult learners and the classroom mathematics curriculum and pedagogy.

**Participants**

My request for approval to study the adult learners in my IT finite mathematics course began with the Program Chair of Mathematics and Science. I decided to approach this gatekeeper in Spring term 2013 as I was unfamiliar with the culture of this university and the time needed to gain approval and trust (Creswell, 2007, p. 125). I was asked to provide information on what the participants and the university would gain from the study (Bogdan & Biklen, 1992). After a discussion of the benefits to adult learning and
IT pedagogy, I was referred to the Institutional Review Board (IRB) of the university. I began the process of IRB approval with both the university and my dissertation university in October 2013. This process involved submitting to both boards a proposal that details the procedures in the project (Creswell, 2007). Approval from both boards was received in January 2014.

At that time I advised the Program Chair and the Dean of the School that I would begin the study in Spring 1 term 2014. I advised them of the nature of the study, what I planned to do in the classroom, the duration of the study and who would be involved. I also requested that a mathematics department colleague observe my pedagogy during the term.

The recruitment of participants from the 21 students in the class began with an email to all members of the class during the first week and was repeated in the second week (see Appendix A – C for recruitment materials). The recruitment materials included information on the benefits of the study: 25 extra credit points for completing both the pre and post-test on ratio and proportion and attitude and a $10 gift card for participating in the interviews and observations. Those interested were asked to advise me by email and I offered to telephone or email them to answer any questions or concerns that they might have.

Because I wanted to discover, understand, and gain insight into each adult learners developing an understanding of ratio and proportion, I used purposeful sampling to select 3-5 participants from which the most could be learned (Creswell, 2007; Merriam, 2009). This purposeful sample of adult learners was meant to reflect “the
average adult learner” and their experiences with developing an understanding of ratios and proportions. Additionally, Merriam (2009) suggested a criterion-based selection using a list of attributes essential to the study and then find an adult learner matching the list.

The first criterion was that the participants be adult learners, as described by NCES (2007), since this was the focus of this study. Then, I looked for participants with the greatest change and positive direction of change from the pre to the post-test on both the Diagnostic Questionnaire on Ratio and Proportion and Attitude Survey.

I also used the Rating Form for the Diagnostic Questionnaire in Ratio and Proportion (Ben-Chaim et al., 2012). This document provided a detailed summary of the participants’ work. It detailed correct answers for each question along with three subcategories of: 1) Correct answer only; 2) Correct support work; 3) Incorrect support work. It also included incorrect answers for each question along with the three subcategories of: 1) Incorrect answer only; 2) Correct thinking but wrong conclusions; 3) Incorrect thinking (see Appendix D). There was also an option of “no answer” also. Finally, because I preferred to interview and observe in person, rather than by Facetime or email, I looked for students who could feasibly travel to the university campus or other reasonable location, such as a public library.

**Data Collection**

Following a case study approach, evidence came from a variety of sources including data from 1) The student’s detailed computations on the pre-test Questionnaire on Ratio and Proportion and the Attitude Survey as they provided data on the student’s
understanding of ratio and proportion as well as their attitude on mathematics in general and ratio and proportion specifically; 2) The student’s detailed work on the authentic investigative activities, the *Math Snacks* supporting materials, compatible with *A Model Using Authentic Investigative Activities for Teaching Ratio and Proportion* developed by Ben-Chaim et al. (2004, 2012); 3) The transcripts of the class discussions of these authentic investigative activities in the Moodle Discussion Forum; 4) The student’s detailed computations on the post-test, *Diagnostic Questionnaire of Ratio and Proportion and Questionnaire on Attitude towards Ratio and Proportion* as a summative evaluation of the student’s understanding of ratio and proportion and attitudes; 5) The *Rating Form for the Diagnostic Questionnaire in Ratio and Proportion* (Ben-Chaim, 2012) as a summary and analysis of the questionnaires; 6) Class project; and 7) Interviews and observations. I followed Yin’s (2014) suggestion that by using multiple sources of information or triangulation, case studies are of higher overall quality than those that rely on a single source of information (p. 119). Patton (2002) also advocated the use of data triangulation when evaluating the quality of a case study.

After participants were initially recruited, they were sent copies of the Diagnostic Questionnaires to complete and return by email. I then assigned pseudonyms on the participant’s questionnaires to protect their identity.

The following is a discussion of the instruments used in the data collection process. The data collection process is also outlined by week in a timeline (see Appendix E).
Instruments

**Diagnostic questionnaire in ratio and proportion.** The *Diagnostic Questionnaire in Ratio and Proportion*, developed by Ben-Chaim et al. (2004, 2012), was used as both the pre and post-test (see Appendix F). This is a measuring instrument and was only administered to the participants in the study in week 1 as the pre-test and at the end of week 5 as the post-test. In order to maintain the reliability of the instrument, the questions in the post-test were the same with only the numbers altered.

Ben-Chaim et al. (2004, 2012) assert “According to the NCTM Assessment Standards for School Mathematics (NCTM, 1989), assessment is defined as a process of gathering evidence about a student's knowledge of, ability to use, and disposition towards mathematics and of making inferences from that evidence for a variety of educational purposes” (p. 221). Accordingly, this study will use the Ben-Chaim et al. (2012) questionnaire that was grounded in their teaching model “Using Authentic Investigative Activities for Teaching Ratio and Proportion” discussed in Chapter II.

This diagnostic questionnaire on content knowledge was designed to evaluate the knowledge level of students prior to the administration of the treatment and then, as a post-course test, to evaluate progress made. The two-part questionnaire included a set of brief investigative questions regarding the topic of ratio and proportion and a set of fraction problems (Ben-Chaim et al., 2012). The investigative problems were divided into three sub-topics:
1) Five rate and density problems. The first two dealt with unit prices, one involving numerical comparison and the second a missing value. The third and the fourth dealt with numerical comparison of proportional relations between distance, time and velocity, the main difference between them being their numerical structure—one using only integers and the other using fractions and decimals. The fifth problem on population density used numerical comparison, but the numbers were larger. In all the numerical comparison problems, the ratios were not equal and thus could be considered more difficult than those with equal ratios (Karplus, Pulos & Stage, 1983).

2) Five ratio problems. For the first two, there was no need to solve the problems numerically—the task was to specify ways to find the ratio between the data under study. The third problem compared two ratios, while the fourth compared different ways of representing ratios. In the fifth problem, the whole value must be determined from a given ratio.

3) Five scaling problems. All five required calculation of a ratio and its application with respect to enlarging or reducing the dimensions of pictures. Each problem introduced a different aspect of scaling: the first asked to find the scaling factor; the second asked to compare two ratios; the third one was a missing-value problem; the fourth involved quadratic enlargement (areas); and the fifth concerned a situation with two consecutive scaling steps.

The fraction problems were to assess how students solved simple fraction problems that were not based on verbally phrased questions. There were six fraction
exercises covering a range of methods, from simple numbers and fractions to decimals. These exercises represented the mathematical proficiency that would be required to solve the rate, ratio and scaling problems presented in the first parts of the diagnostic questionnaire and thus could be used to diagnose sources of difficulties students might have with computations or understanding of the verbal parts of the problems (Ben-Chaim et al., 2012, p. 237-238).

In each exercise, students were asked to provide detailed, written solutions in order to begin monitoring their understanding of ratio and proportion. The assessment questions come from familiar situations such as buying soft drinks, riding a bicycle, and population density. Moreover, the problems and the situations were different from any of the investigative activities given during the course (Ben-Chaim, 2012, p. 238).

The reliability and validity of the questionnaires have been established previously in research reports by Ben-Chaim et al. (1998, 2004) and discussed in Chapter II.

**Questionnaire on attitude toward ratio and proportion.** An attitude questionnaire, *Questionnaire: Attitude toward Ratio and Proportion* (see Appendix G) was administered before the pre-test and the post-test Diagnostic Questionnaire on Ratio and Proportion in week 1 and again at the end of week 5 with the post-test. The instrument, adapted from Ben-Chaim et al. (2012), provided both students and instructors a better understanding and appreciation of the students’ attitude towards ratio and proportion in particular and mathematics in general (Ben-Chaim et al., 2012). The first part of the questionnaire, the statements of attitude, was measured along a Likert scale of 1-5 (1 indicating total disagreement and 5 indicating total agreement). The items were
broken down into four categories as follows: 1) Attitude toward mathematics in general (4 items: #1, 3, 5, 11); 2) Confidence in ability to deal with ratio and proportion (6: #2, 4, 6, 9, 13, 14); 3) Attitude toward the importance of ratio and proportion (5: #7, 8, 10, 12, 15) (Ben-Chaim et al., 2012, p. 228).

An analysis of the results of the three categories of the attitude questionnaire (administered both pre and post) was conducted in pilot studies during one-semester proportional reasoning courses in Israeli teacher colleges (Ben-Chaim et al., 2012). The results from two groups of pre-service teachers (n = 49 and n = 15) served as a basis for comparison and reference to tests conducted in this study.

In addition to the reliability and validity of the pre and post-test, the administration of the attitude questionnaire to the adult learners served as an indicator of how the course, by providing instruction on ratio and proportion, primarily through the use of authentic investigative activities in Math Snacks, influenced their attitudes towards ratio and proportion.

**Rating form for questionnaires.** This form summarized the participant’s work. It individually detailed correct and incorrect answers for each question along with three subcategories of: 1) Correct answer only; 2) Correct support work; and 3) Incorrect support work. It also detailed incorrect answers for each question along with the three subcategories of: 1) Incorrect answer only; 2) Correct thinking but wrong conclusions; 3) Incorrect thinking. It included an option of “no answer” also.

**Math Snacks’ Videos and Learners Guides as authentic investigative activities.** The participant’s completed Learner’s Guides were used as a formative evaluation of the
activities on ratio and proportion. A discussion of authentic investigative activities including the definition, learning objectives, didactic explanations based on Ben-Chaim et al. (2004, 2012) and Lamon (2007, 2008) and connectivity to other rational number topics can be found in the Appendices section (see Appendix H). Since, these activities and worksheets were the focus of the class project, all students in the course completed them. They were corrected and returned to the students at the end of the course. The activities and detailed worksheets were the basis for the class project, a critical reflection paper on what characteristics of the authentic investigative activities were most helpful in understanding proportional reasoning. The directions and rubric for the class project are included in the appendix section. (See appendix I).

**Moodle discussion forum.** The Moodle Discussion Forum served as the site for the class focus group discussions. In addition to the adult learners critical reflection on the authentic investigative activities and their developing understanding of ratio and proportion, the discussion was useful in addressing many of the difficulties that students have in learning to reason proportionally. I was able to identify those who used additive reasoning (i.e., using addition, subtraction and/or differences to compare proportional values) instead of multiplicative reasoning (using multiplication, division, ratio, percentages); or identify any misconceptions present (due to erroneous previous knowledge, or from inaccurate definitions of mathematical concepts) (Lamon, 2008, Ben-Chaim et al., 2012). Additionally, during the discussion many questions arose about the intrinsic importance of the topic or ratio and proportion another criteria of adult learning.
As the facilitator of the group discussion on what characteristic of *Math Snacks*, authentic investigative activities were most helpful in developing an understanding of proportional reasoning, I deliberately tried to surface the views of each person in the group so that each adult learner’s perspective and story was told (Krueger and Casey, 2009). Thus, the group discussion was an interview as well as a group discussion (Patton, 2002). Hearing what another student had to say helped to stimulate the conversation as well as trigger a response or story. Fontana and Frey (1994) termed this an exploratory discussion. The initial semi-structured focus group discussion questions were: What characteristics of the authentic (real life) investigative activities were helpful (or not) in understanding ratio and proportion? How would you describe your experience in viewing these videos and completing the worksheets to students in another IT mathematics course? Describe your attitude when you were asked to complete this class project on ratio and proportion?

The Moodle Discussion Forum provided me with a written transcript of all posts and responses. Additionally, the mathematics faculty colleague provided me with detailed notes on the facilitation, discussion, and suggestions. I used their field notes as well as my own to find contradictions and similar experiences in these discussions to use in the individual interviews.

**Interviews.** Each interview began with initial rapport building questions before I initiated the formal interview questions. I reminded the participants of my role as a researcher and interviewer (not their instructor at the time) and repeated the purpose of the research. I again thanked them for their participation. I also reminded them that their
confidentiality was assured and they could withdraw from the study at any time. The interviews lasted approximately one hour in either a reserved conference room in the university library or at a public library.

The interviews were scheduled after the focus group discussions in Moodle to enable the participants to feel more at ease as rapport had been established. Also, the participants had some time to build student/teacher relationships in the classroom. The interviews were grounded in their questionnaires, *Math Snacks*’ Learners Guides, class projects and Moodle discussions and focused on the participant’s procedures and changing behavior in working with ratio and proportion. The interviews were aimed at in depth examination of adult learners’ current mathematical understanding of ratio and proportion (Ben-Chaim et al., 2004, 2012). I attempted to verbally find out what is “in and on someone else’s mind” (Patton, 2002, p. 341). Thus, the Interview Protocol (see Appendix J) involved open-ended questions with ratio and proportion problems to be solved by the participants. The interviews were less structured so the participants could define their changing mathematical understanding of rates, ratios, and scale (Ben-Chaim et al., 2004, 2012). I would later develop inferences based on these interviews reflecting the participants’ cognitive processes. The main advantage of this type of interview was that it allowed me to intervene when the adult learner needed encouragement and to allow them to elaborate on their statements (Singh, 2000). It also allowed the adult learner to be task-involved during the interview.

By using this method, I was able to visualize an “emic concept” or a concept in the participants’ own language and thought. The adult learners thought and talked about
their perspective on ratio and proportion, rather than one that I brought to or developed during my research (Creswell, 2007). This was particularly important in this case study, as adult learners often do not have a grasp of mathematical vocabulary meanings.

All interviews were audio and video taped as the Creswell (2007) approach to data collection involved systematically gathering information and recording it so that it can be preserved and analyzed by the researcher. The Interview Protocol included the questions to be asked and space to record the responses. It also included time, date and place of the interview. I transcribed the information derived from the interviews within 24 hours and evaluated its effectiveness (Maxwell, 2005). By following the interview protocol, I stayed organized and had a backup of the data.

Further, the questions on the Interview Protocol were based on the Rubin and Rubin (2005) model of three question types: 1) The purpose of the main questions was to structure the interview to answer the qualitative research questions; they were prepared in advance and were worded to match the participant’s mathematical experience; 2) The purpose of the follow-up questions was to get depth, richness and nuances helping to ensure thoroughness and credibility by exploring relevant procedures, concepts and themes. These questions were intended to explore the participant’s building of basic knowledge of ratio and proportion and then extend this to further develop understanding of proportional reasoning. Further questions were designed in response to comments and ideas by the participant and to reflect prior answers. These questions were asked in this interview; and 3) The probe questions were used to manage the conversation. The probes
asked for elaboration and detail and keep the conversation on track as the researcher asks for clarification with examples (Rubin & Rubin, 2005, p. 115).

Yin (2014) also states that one of the most important sources of case study evidence is the interview. He states further that the interviews will appear to be guided conversations. Thus, I followed my line of inquiry and asked my interview questions in an unbiased manner that also served the needs of my inquiry (p. 110). The interviews were conducted using Rubin and Rubin’s (2012) “Responsive Interview” model. As Rubin and Rubin (2012) explain:

> When using in-depth qualitative interviewing, one of the key naturalistic research methods, researcher talk to those who have knowledge of or experience with the problem of interest. Through such interviews, researchers explore and detail the experiences, motives, and opinions of others and learn to see the world from perspectives other than their own” (p. 3).

Thus, adult learners wanted to draw on their experiences with the understanding that it is important and appreciated. Rubin and Rubin (2005) explained further that responsive interviewing aimed at solid, deep understanding rather than breadth. In particular, they assert that “Depth is achieved by going after context; dealing with the complexity of multiple, overlapping, and sometimes conflicting themes; and paying attention to the specifics of meanings, situations, and history” (p. 35). For this reason, the participants were requested to demonstrate detailed mathematical computation on their Questionnaires and Worksheets to provide a basis for the interviews. I was looking for a depth of understanding in the participants that could lead to context interpretation. As we
went through the process as “conversational partners”, a relationship became established. I realized that my personality and emotions might affect the conversation with gender, ethnicity and social issues also being important factors (Peshkin, 1988).

Thus, I used the Rubin and Rubin (2012) “Responsive Interview” model because, through conversations, I was able to construct meanings with specific concrete examples and experiences. At the conclusion of the interview, I thanked the participants for their time and sharing of their experiences. I asked if I might solicit their views if I needed to clarify a response by email or Facetime. I also asked I could email them the transcripts of their interview for their review of accuracy. This technique of member checking was a critical technique for establishing credibility (Lincoln & Guba, 1985).

**Observations.** Observation of the participants actually working through problems from the additional problems section helped produce not only physical artifacts but also gave me the ability to view their mathematical reality (Yin, 2014). The observation took place at the same time as the interview. As both the interview and observation were audio/video taped together, I transcribed them together. Again, these transcripts were emailed to each participant for their review of accuracy. All four participants approved the content of the transcripts via return email.

The Observational Protocol (see Appendix K) was also based on the data from the Questionnaires, Learner’s Guides (worksheets), class projects and Moodle Discussion Forum. I took field notes, during the 30-minute observation, on the description of events and processes used by the students to complete the worksheets from *Math Snacks* and information on the characteristics students found helpful. Maxwell (2005) also provided
“Guidelines for Field notes” on observations that suggested initially an “orienting description” of the setting. My notes provided a specific, detailed chronological description of general patterns that were concrete, with interpretation added separately. I rewrote these notes, within a 24-hour period and add information from my memory. This helped me to think and reflect about what I observed. Both theoretical and personal notes were incorporated. The notes finished with a paragraph about my general reflective thoughts, including ideas to check out and any methodological comments. I was particularly interested in information that was a breakthrough to new ways of thinking about prior assumptions, such as events that make the familiar strange. I then wrote additional reflective notes on emerging codes, themes and concerns that arose in the observations.

Yin (2014) stated the four principles of data collection are: using multiple sources of evidence; creating a case study database; maintaining a chain of evidence and exercising care when using data from electronic sources. I have focused on the first of these principles, using multiple sources of data in this section and I now discuss the last three.

**Data Management**

As I worked through the data collection process of the participant’s pre and post-questionnaires on ratio and proportion and attitude, worksheets on the authentic investigative activities in *Math Snacks*, Moodle Discussion Forum posts, class projects, interviews, and observations, I created a data collection matrix to organize each participant’s data. This helped me to document the progress of the study. Stake (1995)
suggested a matrix to keep track of tasks completed and what needs to be accomplished. My matrix also allowed others to trace my steps to arrive at a similar perspective so that the data becomes transparent. At the same time, it communicated the depth and multiple forms of data (see Appendix L).

Further, Stake (1995) emphasized the need to include the approximate amount of time spent on each data source’s tasks and issues so that the data was not skewed. In order to further manage the data, I had a checkbox by each task. Another checkbox, by the task, was checked when the data was transcribed or I had written an analytic memo.

Additionally, during Phase One when I was performing a self-study of my pedagogical approach, I wrote daily in a journal (Creswell, 2007). This included a detailed log of outside resources, announcements, chats, etc. that I performed on a daily basis within the course. I wrote about each activity and the student response to the activity. Then, I was able to critically reflect and make adjustments in the terms ahead.

Following Yin’s (2014) second principle of data management, I created a case study database to include field notes, case study documents, tabular materials, and new narrative compilations. My field notes included the transcribed postings from Moodle and my participant interviews and observations. The files were kept both on my computer in PDF format and in a paper file in a folder labeled with each participant’s name. They also served as a backup to the audio and/or video recordings.

Each participant’s file contained the following:

- Pre and Post questionnaires on ratio and proportion,
- Pre and post-questionnaire on attitude,
Moodle Discussion Forum postings,
Math Snacks Learner’s Guides,
Class Projects,
Transcripts of Interviews and Observations,
Observations of math problems.

Tabular materials, such as the quantitative data in the Ben-Chaim et al. (2004, 2012) studies were organized and stored for later retrieval during data analysis. I also compiled my own new narrative material, such as an adult learner’s biography and cross-references (Yin, 2014). The participant’s individual folder then became the starting point for the within-case analysis.

Data Analysis

Yin (2014) suggested a strategy for data analysis was to rely on theoretical propositions. The Lamon (2007, 2008) and ben-Chaim et al. (2004, 2012) studies on developing understanding of ratio and proportion and ultimately proportional reasoning were reflected in the research questions, review of literature, data collection plan and now the analytic priorities. Their theories organized the entire analysis, pointing to relevant contextual conditions to be described as well as explanations to be examined (p.136). Further, Yin (2014) suggested that a pattern matching technique was desirable when an empirically based pattern, such as the scores on the questionnaires on ratio and proportion and attitude were compared with predicted ones, such as the findings from the Ben-Chaim et al. (2004, 2012) studies on adult learners. As the empirical and predicted
patterns in this case study appeared to be similar, the results helped to strengthen the case study’s internal validity (p. 143).

The case study then transitioned from the field to systematic analysis. While in the Moodle Discussion Forum, some ideas were generated about how and what was happening with the adult’s understanding of the characteristics of the authentic investigative activities. I began to put these early ideas under scrutiny by trying to prove them wrong with negative case analysis, peer debriefing and member checking (Lincoln & Guba, 1994).

In the interview and observation, I inquired about the participants’ views that were expressed in the Moodle Discussion Forum by asking questions that might provide negative or disconfirming evidence (Patton, 1980). Peer debriefing was continuously conducted with another a mathematics faculty peer at the university who agreed to review the transcripts of the interviews and observations. As I stated previously, member checking was conducted by email after the interviews and observations with the intent of understanding the participants’ views of my written analysis in the transcripts as well as what they thought was missing.

I read each transcript numerous times and made analytic memos in order to begin data reduction. Miles and Huberman (2013), describe data condensation, “refer to the process of selecting, focusing, simplifying, abstracting, and/or transforming data that appear in the full corpus (body) of written field notes, interview transcripts, documents, and other empirical materials. By condensing, we make data stronger” (p. 12).
Then, I looked for patterns to emerge that aided in identifying codes. I approached this task both deductively and inductively. Stake (1995) explained that, “It will be useful to use pre-established codes but to go through the data separately looking for new ones” (p. 79). Thus, my pre-established codes came from my research questions; deductively having initial codes for them. Additionally, I was open to inductive emergent patterns. Following the process outlined in the Data Collection process, I began with the questionnaires, worksheets, class projects, and Moodle Discussion posts and combined the patterns to form broader themes and kept them to a manageable number of 5-6 Creswell (2007). I then worked from those codes and themes, continually returning to the original documents as I considered other data sources (Saldana, 2012).

I then moved to the transcribed interviews and observations and used Rubin and Rubin’s “Responsive Interview” model (2012) for their data analysis. Consequently, I continued to search for themes by highlighting similar themes with the same color, such as green for the simple nature of the videos and red for the repetition of topics, to emerge during the interviews. In the process of moving from raw data to evidence-based interpretations (Rubin & Rubin, 2012), I wrote analytic memos to summarize the interview and reflect on how I felt the interviews progressed. Interview data was continually reexamined in order to find coherent and consistent themes with data that supported those themes. It was an iterative process as I refined what I learned from the participants, using the cyclical process of analyzing, collecting data, analyzing, and collecting additional data (Saldana, 2012).
Next, I reread my written notes from the interview transcripts and looked for anything that confirmed or disconfirmed what had already been said, such as, a participant contradicting what they wrote in their class project in their interview. I then went back and color-coded each response according to the theme it came under or I created a new theme. Continuous revisits to the data also helped refine the names of each theme and provided a reference to the data source. I also developed a detailed legend to explain the themes and codes by participant, such as Sarah #1 = Sarah Interview (Appendix N). Each participant’s case was initially analyzed separately and then across the other participants’ cases as I looked for overarching themes between the participants.

These documents were analyzed individually and then collectively:

- Pre-test Questionnaires: Overall scores, categories on Rating Form.
- Pre-Attitude Survey: Ratings of each category using the Likert scale.
- Post-test Questionnaires: Overall scores, categories on Rating Form.
- Post-Attitude Survey: Ratings of each category using the Likert scale.
- Moodle Discussion Forum Posts: Posts of participants compared with other participants on usefulness, characteristics and implementation of authentic investigative activities.
- Math Snacks Authentic Investigative Activities: Scores, areas of understanding and lack of understanding, characteristics of videos.
- Interviews: Usefulness of activities, particular characteristics of videos, and implementation of videos as a teaching method.
- Observations: Participant detailed work on ratio and proportion.
- Class Projects: Participant’s attitudes toward their learning of ratio and proportion.

By using these multiple sources of data and the theories of ben-Chaim and Lamon, I triangulated the corroborating evidence to illuminate themes and perspectives (Creswell, 2007).

All documents were read through numerous times to see if they answered the research questions and/or confirmed or disconfirmed findings from the interviews. Categorical aggregation was used to determine if the total of coded data did help to develop patterns emerge as well as confirm themes (Stake, 1995). I reviewed each participant’s file and looked at all items in it and checked for consistent themes and data that supported those themes. Then, I looked across cases to make comparisons to see if the themes held true. Further, Stake (1995) referred to cross case analysis as similarities of all of the participants’ work. The final representation presented an in-depth account of the cases in tables and figures. I created a table that listed the themes mentioned by each participant to report the number of times supporting data was found for each theme across the students’ data (see Appendix M). Finally, I developed naturalistic generalizations, “that people can learn from the case either for themselves or apply to a population of cases” (Creswell, 2007, p. 163).

**Trustworthiness and Validity**

Creswell (2007) defined validation as “an attempt to assess the ‘accuracy’ of the findings, as best described by the researcher and the participants” (pp. 206-207). The
types of validation I employed were: triangulation, disconfirming evidence, member checking, peer review, researcher reflexivity, and thick, rich description. Yin (2014) cited four kinds of triangulation: data sources, investigator, theory, and methodological triangulation. I used multiple sources of data derived from the questionnaires on ratio and proportion and attitude, Moodle Discussion Forum posts, class projects, worksheets on the authentic investigative activities, interviews and observations. This allowed themes to be verified from multiple respondents and sources. Methodological triangulation also allowed comparisons from the worksheets and interview transcripts and documents from observations to provide corroborating evidence. I looked for disconfirming or negative evidence (Miles & Huberman, 2013). I also used cross-case (participant) analysis to search for consistent themes and for disconfirming evidence. Theory triangulation or different perspectives on the same data set were established with the studies of Lamon (2007, 2008) and ben-Chaim et al. (2004, 2012). Additionally, as I discussed in the data collection section, I used member checking and peer review to ensure the trustworthiness and validity of my study.

Creswell (2007) advocates researcher reflexivity or self-disclosure of assumptions, beliefs, and biases. This was accomplished in the section “Situating Myself as a Researcher” where I disclosed my ontological, epistemological, and methodological beliefs and therefore position myself as a constructivist. In the introduction of this chapter, I also discussed the ethical challenges of being the teacher and researcher. This allows any reader of this study to see my potential biases that may influence my interpretations. I also established credibility by describing the setting, the participants,
Limitations of Study

One limitation of this study might be researcher bias. I have situated myself as an instructor who has seen and worked with adults whose ability to reason proportionally has had an effect on their schooling, careers and personal lives. I clearly believe in the theories of Lamon and ben-Chaim and their ability to instruct students of all ages on ratio and proportion. This was revealed in the section “Situating Myself as a Researcher.” Further, I believe that even though this case study focuses on adults, there is still the possibility of the students viewing this as a power relationship.

The most pervasive issue of concern in this study is the ethical issue of the researcher also being the course instructor. Drake and Heath (2011) affirm that insider research depends upon the researcher having a professional perspective. They state further that ethical research procedures are designed so that researchers anticipate ethical issues arising from their research before the project begins (p. 53). In this study, I have de-identified students and assigned pseudonyms. Confidential information was locked in a file cabinet or a computer separate from the course. The data collected in the study was opened after the course was over and the grades were submitted. I am the only person who knows the identity of the participants. All grades, other than the Class Project are calculated within MyMathLab. The class project was graded using a college-established rubric. I remained committed to utilizing the principle of self-reflexivity (Steier, 1991) and applying the principles of the methodology first and foremost to ones research.
Summary

The purpose of this qualitative case study was to examine what helps or hinders adult students to learn ratio and proportion when the topic is not the central focus of the mathematics course. The adult learners provided valuable first-hand, descriptive, and rich information from a variety of sources gathered in this case study. In Chapter IV, I present the individual cases of the adult learners and the research findings.
CHAPTER IV

RESEARCH FINDINGS

Introduction

The purpose of this qualitative case study was to examine what helps or hinders adult students to learn ratio and proportion when the topic is not the central focus of the mathematics course. More specifically, the research questions were:

- How did the use of authentic investigative activities, aligned with Lamon’s and Ben-Chaim’s content theories, impact adult learning of ratio and proportion?
- What characteristics of the investigations were most helpful for adult learners to grasp the variety of dimensions of procedural thinking and conceptual understanding of rates, ratios, scale and proportional reasoning?
- How did the videos help to develop a strategy of mathematical thinking and problem solving in adults understanding of ratio and proportion?

Four students from a 7-week IT mathematics course, required in a degree completion program, were asked to complete a pre-test questionnaire on ratio/proportion and attitudes, watch authentic (Math Snacks) investigative activities and then complete a post-test questionnaire on ratio and proportion and attitudes. Document analysis, interviews and observations were conducted to determine if their mathematical thinking and attitudes were impacted by the videos and supporting materials.
The Mathematics Behind the *Math Snacks* Videos

The National Science Foundation (NSF) funded project *Using Innovative Media to Fill Conceptual Gaps in Mathematics* is a collaboration at New Mexico State University between the College of Education, the College of Arts and Sciences, and the Learning Games Lab, the *Math Snacks* videos are short animations designed to present mathematics in a very different manner. The videos are not intended to look like traditional (rote and algorithmic) instruction. *Math Snacks* provides students, especially those that do not particularly like mathematics, another way to look at mathematical concepts. The animations are designed to provide a different way to access important mathematical concepts, such as ratio and proportion that students need to know to move into higher-level math. The availability of the *Snacks* on the Internet, iPhone and iPod, makes it possible for students to enjoy the animations at their convenience. The accompanying worksheets can also assist learners in applying their conceptual understanding to additional math problems and activities (New Mexico State University Board of Regents, 2013).

The *Math Snacks* principles present the core mathematics concepts that students should know and be able to do in grades 6, 7, and 8. They reflect research on gaps in mathematical understanding and patterns of mistakes on topics that students don’t usually understand and teachers have a difficult time conveying to students. The research found that most students struggle with the properties of numbers, estimation, fractions, and conversions between fractions, decimals, percentages and whole numbers (NMSU Board of Regents, 2013). Students also had problems with reasoning and explaining their
thinking about problems, with understanding and creating patterns, and with algebraic thinking and problem solving. Thus, the *Math Snacks* project addresses the mathematical learning needs of students from a 4th-8th grade curriculum standard. It situates problem areas in mathematics in a totally different context than those in which students have previously experienced only failure. Further, the mathematics is situated in a real world context that has been determined to be of interest to students.

Moreover, the learning objectives in *Math Snacks* are consistent with the *Standards for Introductory College Mathematics*, the American Mathematical Association for Two Year Colleges (AMATYC, 1995) Standards. These standards provide a vision whereby students develop intellectually by learning central mathematical concepts in settings that provide a rich variety of instructional strategies, such as *Math Snacks*. The *Standards for Content* provides guidelines for content that should be taught in introductory college courses. In particular, Standard C-1: Number Sense states the following:

Students will perform arithmetic operations, as well as reason and draw conclusions from numerical information. Number sense includes the ability to perform arithmetic operations, to estimate reliably, to judge the reasonableness of numerical results, to understand orders of magnitude, and to think proportionally. Suggested topics include pattern recognition, date representation and interpretation, estimation, proportionality, and comparison. (p. 116)

Following the general guidelines for teachers in *Math Snacks*, I implemented the videos, as snacks not full meals. They were intended to be an intervention in the IT math
course taught in MyMathLab to specifically teach the important concepts of ratio and proportion that students often have difficulty. The students were advised to watch the videos multiple times, at their convenience and to work through the learners’ guides (worksheets) completely. This allowed me to see where they became frustrated or confused on the math content as well as the multiple ways they approached and solved the problems. The three Math Snacks animations used were Ratey the Math Cat, Bad Date and Scale Ella. A discussion of the mathematical content of each follows.

**Ratey the Math Cat**

*Ratey the Math Cat* addresses number and operations standards as well as the process standard, as established by NCTM. It supports students in:

- Understanding and using ratios and proportions to represent quantitative relationships.
- Building new mathematical knowledge through problem solving.
- Solving problems that arise in mathematics and in other contexts.

**Learning Objectives:**

- The importance of units
- Rates and Unit Rates
- Proportions as multiplicative situations
- Patterns
- Translating unit rates to a table and a graph

**Vocabulary:** Rate, unit rate, per, dependent variable, independent variables.
**Bad Date**

What do you do when someone asks you if you listen to country music backwards, but won’t give you a chance to answer? In this humorous animation, who speaks how many words determine if a date goes well or not. *Bad Date* addresses number and operation standards and the process standard, as established by NCTM. It guides students in:

- Understanding and using ratio and proportion to describe quantitative relationships.
- Developing, analyzing and explaining methods for solving problems involving proportions, such as scaling and finding equivalent ratios.
- Solving problems that arise in mathematics and other contexts.

**Learning Objectives:**

- Ratios can represent part-whole or part-part relationships.
- Ratios can be extended into patterns using proportional relationships.
- 1:1 ratios can be found in everyday situations.

**Vocabulary:** Proportional relationships, ratios, part-whole, part-part, one-to-one.

**Scale Ella**

The evil Scaleo has escaped from prison and is transforming the length, width, and height of objects until they become useless... or dangerous. Who can put things right?

Our heroine, Scale Ella, uses the power of scale factor to foil the villain.
Using factors, multiples and prime factorization and relatively prime factors to solve problems.

Developing, analyzing and explaining methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Learning Objectives:

- There is a number (the scale factor) that creates the relationship between two items that are being compared to one another.
- Multiplication and division are inverse operations.
- If the scale factor is less than one, the size of an object or a number is being decreased.
- If the scale factor is greater than one, the size of an object or a number is being increased.
- The scale factor can be represented as a decimal, whole number or fraction.

Vocabulary: scale factor, fraction

**Cases of Adult Learners’ Approach to Ratio and Proportion**

In the following four sections, I describe the adult learners who participated in the study. Their profiles include background information, mathematics background and data collected in the study. The data came from interviews, observation, document analysis and a focus group and was arranged by the case of each adult learner. In order to answer the research questions, I have included excerpts from the data to indicate how the *Math Snacks* videos impacted their learning.
Sarah, A Case Study

Hi everyone! I'm the mother of 5 kids ranging in age from 18 to 5. I also work full time as the Dementia Unit Coordinator at a local nursing home. I am in the process of earning my Bachelor Degree in Health Care Administration. To be honest, math is probably my weakest area. I haven't taken any formal math class since 1996 when I was earning my Associate Degree in Nursing. I had limited math in my nursing classes and that was also in to late '90s. I'm taking this class because I have to for my degree, but I'm also hoping that it will refresh what I have learned so long ago so that I can better help my kids with their math. I hope that during this course I can remain calm and not get so frustrated with the math that I get overwhelmed. *Introductory Post in Moodle by Sarah (January 26, 2014)*

Math background. Sarah is a 33 year-old Caucasian female. She went to the local community college from 1995-1999 for her nursing degree (STNA). After graduation, she worked as an STNA so she didn’t go to school from 1999 to 2013. She started the degree completion program with a major in Health Care Administration in 2013 and is currently a second semester freshman.

She had three years of math in high school: algebra 1, geometry, and algebra 2 - trigonometry. In her senior year in 1994, her algebra 2 - trigonometry teachers met her at 7 am for extra help. Now, when she gets frustrated she tries to remember the tutoring on equations and inequality. She may not recall the math details but she does remember the recommendation that she learn to pace mathematics.
Most of the math courses at the community college were not considered advanced enough to transfer to the degree completion program. She asserted in her interview that at the community college, she always felt frustrated and overwhelmed with math. Further Sarah stated, “I don’t do well enough in math; it is my toughest subject. I try to rationalize (understand) it and relate it to real life. I have kids and they see me doing math and I don’t want them to see me overwhelmed” (Sarah, February 23, 2014).

Sarah continued that when she started this particular *MyMathLab* course, she was really nervous. And she repeated that she doesn’t do well in math and her grades (87.5%) on the first homework(s) on equations and inequalities showed that (*MyMathLab*, January 26, 2014). She said that she kept trying to find different ways to understand the math. She didn’t want her kids to see her struggle but they did. Now that the course is almost over she doesn’t feel that it was so difficult or bad as she had thought. Sometimes, she recalled her algebra 2 - trigonometry teacher telling her to pace herself. “I just had to walk away from it – do a load of laundry or something.” She has paced herself in the course by doing one assignment a day. Sarah stated that only 3 times in the 7-week course did she do 2 assignments in one day. This was the recommended time line for the course. She explained that this course was the prerequisite for statistics; another required course for her major.

**Sarah’s mathematical thinking prior to watching the videos.** Prior to watching the *Math Snacks* videos on rate, ratio and scale, Sarah indicated on her Pre-test Questionnaire on Attitude that she was not confident in her current ability to understand the topic of ratio and proportion. She indicated that mathematics was not one of her
favorite subjects in school. However, she considered the learning of ratio and proportion to be relevant. Further, ratio and proportion should be taught in every mathematics course because it is important for student development.

On the other hand, Sarah believed she had the mathematical knowledge to understand ratio and proportion. She said that it appeared to be easy and complicated at the same time. But, the theoretical knowledge that she currently had of ratio and proportion was not sufficient to understand the topic. She realized that she needed to know ratio and proportion for her future career but she did not need to calculate or convert drug dosages as this was done for her at the nursing home. Sarah felt that many students had difficulty understanding ratio and proportion.

Sarah provided the following answers to rate, ratio, scaling and fraction problems on the Pre-test Questionnaire on Ratio and Proportion. The numbers of the questions correspond to the numbers on the pre-test questionnaire. Sarah’s answers are in italics:

**Rate – from questionnaire on rate and proportion.** Question 3. After the trip, Sima and Alex decided to see who could ride the fastest back home. Sima rode 5 km in 20 minutes. Alex rode 7 km in 25 minutes. Who rode the fastest? How do you know?

*Alex rode faster because it only took him 5 more minutes to ride 2km more.*

Sarah’s answer is correct but she does not provide any explanation. She appears to be using her intuition and an additive strategy (confirmed by Sarah in interview). This problem deals with numerical comparison of proportional relationships between distance and time. The numerical structure in this problem is decimals.
Question 4. The next weekend, Max and Alice rode their bikes to the park, taking the long route around the lake. The route is 30 km and it took them 1.5 hours. They ate lunch, and then rode back the short way, which is only 20 km. This took them 3/4 of an hour. In which segment of their trip did they ride the fastest? How do you know?

They rode faster going home because if you double their time and distance they would ride 40 km in 1.5 hours.

Sarah’s answer is correct. She is using a factor of change strategy by doubling both time and distance and then numerically comparing the two rates.

**Ratio – from questionnaire on rate and proportion.** Question 2. In Ms. Shula’s class, 18 students come by bus, and 12 arrive on their own. In what different ways could you compare the numbers of bus riders and those who come on their own in Ms. Shula’s class to those in Mr. Erez’s class? Explain your reasoning in detail.

Mr. Erez has 2 more students arriving by bus and 3 more arriving on their own. Ms. Shula has more 6 more students arriving by bus than on their own while Mr. Erez only has 5 more bus riders than those that arrive on their own. There is a 3:2 ratio in Ms. Shula’s class and a 5:3 ratio in Mr. Erez’s class.

There is no need to solve the problem numerically—the task is to specify ways to find the ratio between the data under study. However, Sarah solves the problem initially using an additive strategy. Then, she uses a factor of change strategy to get the correct answer of a 3:2 ratio in Ms. Shula’s class. Using a factor of change of 6 to simplify the ratio of 18:12.
Question 4. Of the 400 students in the school, 240 arrive by school bus daily. Is the ratio between the numbers of pupils that arrive by bus to the number that arrive on their own in Ms.Shula’s class the same as that of the whole school? How do you know?

Yes because \( \frac{240}{400} = \frac{18}{30} = 6:10 \)

Sarah’s answer is incorrect because she is not using 160 or the number of students to arrive on their own that gives a 3:2 ratio. The problem was intended to compare different ways of representing ratios.

Scale – from questionnaire on rate and proportion. Question 1. A customer asks Fran to enlarge a 3 inch by 2 inch photograph to 18 inches by 12 inches. Can this be done without cutting or distorting the picture? How do you know?

If the picture is sharpened up as it is resized it is possible to enlarge to that size.
Depending on the quality of the picture most can be doubled in size.

Sarah’s answer is incorrect. She does not provide an explanation for the change in scale. This problem requires calculation of a ratio and its application with respect to enlarging or reducing the dimensions of pictures by finding the scaling factor.

Question 3. Phil took a photo of a man and a tree and their shadows. The real-life man is 1.75 meters tall. In the photo, the man is 3 cm tall, his shadow is 1.2 cm long, and the shadow of the tree is 4.5 cm long. If the photo included the entire tree, how tall would the image of the tree be? How tall is the real-life tree in meters? Show your work.

\[
\frac{4.5}{1.2} = 3.75 \\
3.75 \times 3 = 11.25 \text{ cm in the picture} \\
1.75 \text{ meters} = 175 \text{ cm}
\]
\[
\frac{175}{3} = 58 \text{ (roughly)}
\]

\[
11.25 \times 58 = \text{roughly } 652.5 = \text{roughly } 6.5 \text{ meters in real life}
\]

Sarah’s answer is incorrect but she seems to be intuitively reasoning her way to an answer. She is attempting metric conversions as well as setting up the ratios of the shadow of the tree to the shadow of the man in this missing value problem.

**Fractions – from questionnaire on rate and proportion.** Sarah’s answers were correct. She used a factor of change strategy and/or multiplication to find the missing value in the equivalent fractions. She used a range of methods, from simple numbers and fractions to decimals. It appeared from these exercises that Sarah had the mathematical proficiency that would be required to solve the rate, ratio and scaling problems presented in the first parts of the diagnostic questionnaire.

**Examples of ratio – from questionnaire on rate and proportion.** Sarah provided an example of ratio in staffing at work.

**Sarah’s mathematical thinking after watching the videos.** After watching the videos, Sarah was able to describe a situation in detail related to ratio. “I use ratio when staffing my unit at work. I have to staff according to a 6:1 ratio, 6 patients to 1 caregiver” (Sarah, February 22, 2014). She now felt confident that she had the theoretical knowledge of the topic of ratio and proportion to understand it. She believed strongly that students should have a broad range of mathematical knowledge beyond what is learned in school. Also, ratio and proportions should be included within each mathematics course because it was important for student development.
Sarah confirmed that she now somewhat enjoys mathematics and it was no longer her least favorite course. Ratio and proportion seemed easy to her. But she still thought many students had difficulty with the topic and it needed to be taught in every mathematics course.

Sarah provided the following answers to rate, ratio, scaling and fraction problems on the Post-test Questionnaire on Ratio and Proportion (Sarah’s answers are in italics):

**Rate – from questionnaire on ratio and proportion.** Question 3. After the trip, Sima and Alex decided to see who could ride the fastest back home. Sima rode 5 km in 25 minutes. Alex rode 7 km in 20 minutes. Who rode the fastest? How do you know?

*Alex rode fastest, he rode farther in a shorter time.*

In the interview Sarah explained that she knew this by solving for a unit rate and comparing.

Question 4. The next weekend, Max and Alice rode their bikes to the park, taking the long route around the lake. The route is 30 km and it took them 1.5 hours. They ate lunch, and then rode back the short way, which is only 25 km. This took them 3/4 of an hour. In which segment of their trip did they ride the fastest? How do you know?

\[
\frac{30\text{ km}}{90\text{ min}} = 0.33\text{ km/min}
\]

\[
\frac{25\text{ km}}{45\text{ min}} = 0.56\text{ km/min}
\]

*They rode faster going back home.*

Sarah’s answer was correct. She used the strategy of unit rate and then compared.

**Ratio – from questionnaire on rate and proportion.** Question 2. In Ms. Shula’s class, 16 students come by bus, and 14 arrive on their own. In what different ways could
you compare the numbers of bus riders and those who come on their own in Ms. Shula’s class to those in Mr. Erez’s class? Explain your reasoning in detail.

*The bus rider ratio is 5:4. Mr. Erez has more students riding the bus than Ms. Shula. The ratio of students arriving on their own in 5:7. Ms. Shula has more students that arrive on their own than Mr. Erez.*

Sarah’s answer was incorrect. Shula ratio is 8:7. She was verbally explaining the difference without the correct ratios. However, She used the correct ratio in 4.

Question 4. Of the 400 students in the school, 260 arrive by school bus daily. Is the ratio between the numbers of pupils that arrive by bus to the number that arrive on their own in Ms. Shula’s class the same as that of the whole school? How do you know?

*The ratio for the entire school is 20:13. This ratio is different than the 8:7 ratio that Ms. Shula has.*

Sarah’s answer was correct with incorrect use of the numbers in the ratio. She was using the ratio of the total students to the bus riders not the bus riders to arrive on their own. 260:140 = 13:7

*Scaled – from questionnaire on rate and proportion.* Question 1. A customer asks Fran to enlarge a 3 inch by 2 inch photograph to 15 inches by 10 inches. Can this be done without cutting or distorting the picture? How do you know?

*Enlarging by 5 inches all around can be done without cutting. It may distort if not careful.*

Sarah’s answer was correct. She used a factor of change strategy.
Question 5. Phil took a photo of a man and a tree and their shadows. The real-life man is 6 feet tall. In the photo, the man is 3 inches tall, his shadow is 2 inches long, and the shadow of the tree is 4 inches long. If the photo included the entire tree, how tall would the image of the tree be? How tall is the real-life tree in feet? Show your work.

_The tree in the picture would be 5 inches because the shadows are 1 inch shorter than the objects in the picture. The tree is 10 foot high in real life._

Sarah’s answer was numerically incorrect because of her misunderstanding of what ratios she was using in the missing value problem. She needed to set up proportions for each using the shadows: photos and then the shadows to the real life. However, her intuition and additive strategy had her reasoning correctly.

*Fractions – from questionnaire on rate and proportion.* Sarah continued to use a factor of change strategy and/or multiplication to find the missing value in the equivalent fractions. Thus, she demonstrated the ability to use a variety of methods to correctly compute.

*Example – from questionnaire on rate and proportion.* Sarah provided an example of the use of ratio.

*The mathematics that Sarah saw in the Math Snacks videos.* Sarah’s understanding of the mathematics was based on the following:

*Ratey the Math Cat.* Sarah understood and used ratios and proportions to represent quantitative relationships. She built new mathematical knowledge through problem solving. She solved problems that arise in mathematics and in other contexts.
Further, she understood the importance of units, rates, and unit rates. She understood proportions as multiplicative situations and patterns.

**Bad Date.** Sarah understood and used ratio and proportion to describe quantitative relationships. She was able to develop, analyze and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios. She was also able to solve problems that arise in mathematics and other contexts. Furthermore, she understood that ratios can represent part-whole or part-part relationships, ratios can be extended into patterns using proportional relationships, and 1:1 ratios can be found in everyday situations.

**Scale Ella.** Sarah can use factors, multiples and prime factorization and relatively prime factors to solve problems. She was able to develop, analyze and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios. Further, she understood that there was a number (the scale factor) that created the relationship between two items that were being compared to one another. She knew that multiplication and division are inverse operation. And, if the scale factor was less than one, the size of an object or a number was being decreased and if the scale factor was greater than one, the size of an object or a number was being increased. Also, Sarah understood that the scale factor can be represented as a decimal, whole number or fraction.

**Sarah’s perceptions of the Math Snacks videos.** Sarah felt that watching the Math Snacks videos positively impacted her understanding of ratio and proportion. She believed that students benefit from adding videos to what teachers go over in classrooms,
### Table 1. **Summary of Sarah’s Mathematical Thinking**

<table>
<thead>
<tr>
<th>Rate</th>
<th>Pre-Test Strategy/ Correctness</th>
<th>Post-Test Strategy/ Correctness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sarah’s answer was correct but she did not provide any explanation. She appeared to be using her intuition and an additive strategy (confirmed by Sarah in interview). This problem deals with numerical comparison of proportional relationships between distance and time. The numerical structure in this problem is decimals.</td>
<td>Sarah’s answers were correct. She used a strategy of unit rate and comparison.</td>
</tr>
<tr>
<td>Ratio</td>
<td>There was no need to solve the problem numerically—the task was to specify ways to find the ratio between the data under study. However, Sarah solved the problem initially using an additive strategy. Then, she used a factor of change strategy to get the correct answer and simplify it. Sarah’s second answer was incorrect because she misunderstood the verbal content of the question and compared ratios of part to whole rather than part to part. The problem was intended to compare different ways of representing ratios.</td>
<td>Sarah’s answers were incorrect due to misunderstanding of the verbal content. Her strategy was correct to create ratios and compare.</td>
</tr>
<tr>
<td>Scale</td>
<td>Sarah’s first answer was incorrect. She did not provide an explanation for the change in scale. This problem required calculation of a ratio and its application with respect to enlarging or reducing the dimensions of pictures by finding the scaling factor. Sarah’s second answer was incorrect but she seems to intuitively reason her way to an answer. She was attempting metric conversions as well as setting up the ratios in this missing value problem.</td>
<td>Sarah’s first answer was correct. She used a factor of change (scale) strategy. Sarah’s second answer was numerically incorrect because of her misunderstanding of what ratios she was using in the missing value problem. However, her intuition and additive strategy had her reasoning correctly.</td>
</tr>
<tr>
<td>Fractions</td>
<td>Sarah’s answers were correct. She used a factor of change strategy and/or multiplying to find the missing value in the equivalent fractions. She used a range of methods, from simple numbers and fractions to decimals. It appeared from these exercises that Sarah had the mathematical proficiency that would be required to solve the rate, ratio and scaling problems presented.</td>
<td>Sarah used a factor of change strategy and/or multiplication to find the missing value in the equivalent fractions. Thus, she demonstrated the ability to use a variety of methods to correctly compute.</td>
</tr>
</tbody>
</table>
whether the classes were seated or IT and regardless of the student’s age, grade or achievement level in math. Sarah asserted that, “My own children have benefited from Internet sites to increase their understanding in their class work. My children and I have a hard time with math and so these videos were especially helpful to us” (Sarah, February 23, 2014).

As we continued in the interview, I asked Sarah to explain, “What characteristics of the three Math Snacks were most helpful to her to grasp understanding and computations of rates, ratios, scale and proportional reasoning?

We started with Ratey the Math Cat. Sarah commented on the animation and how Ratey popping up “when you least expect him” had helped her to see the everyday uses and relations of rate in her daily life. That made it easier for her to understand and compute rate and to ultimately understand proportional reasoning. She compared the teaching method of animation in Ratey, coupled with everyday relatable scenarios, to the teaching methods of MyMathLab. In the lesson and tutorial on ratio and proportion in MyMathLab, a teacher lectured and wrote the steps to solution of a problem on a whiteboard. She went on to say, “I watched that several times, but I didn’t get how to solve the problem until I watched Math Snacks.” (Sarah, February 23, 2014).

Sarah’s most memorable and favorite video was Bad Date and its use of animation, representations and realism. She commented, “I cracked up when I saw it. My daughter, who is 8, came downstairs because she heard me. She watched it and cracked up too.” When, Sarah asked her daughter if she understood, her daughter said, “she talked and he didn’t and it took a long time until the end when someone talked as
much as she did. Then the bars went up the same.” She explained to her daughter that this was ratio and why. Sarah couldn’t help thinking that this was so understandable to her and her children that she book marked Bad Date to go back to later. Sarah’s final comment was, “I don’t know why they don’t teach like this in their schools.” (Sarah, February 23, 2014).

Sarah continued talking about Bad Date and how she and her daughter learned to use ratio in their cooking.

I can't seem to get the Bad Date video out of my head. I'm not even sure why. Maybe, because my daughter loved it so much that we have been turning our dinners into ratio learning sessions. She (8 year old daughter) likes to cook with me. I feed 6-7 people at meals. Most recipes are for 4 and I like to figure 8 portions. So, I need to double it. I asked my daughter how and she said, “like the video- so like the girl talked once and guy twice.” I asked my daughter how many eggs I needed (the recipe called for 1 egg) and she put 2 eggs down. She is only in 2nd grade but she understood ratio with the demonstration in real life. (Sarah, February 23, 2014)

I asked how the worksheets (Learner’s Guides) in Bad Date helped in her understanding and computations of ratio and proportion. Sarah said that they had helped her practice what she has seen in the video.

I asked Sarah why she did not draw, she said because I can see it. I asked what she meant and she repeated, “I can just see it.” Paper and pencils were provided in the observation but she declined to draw there also (see Figure 6).
Figure 6. Sarah’s *Math Snacks Bad Date* Student Learner Guide

In her class project, Sarah had mentioned that the videos had helped with computations because she could relate it to scheduling of staff at her job (Sarah, February 5, 2014). Consequently, in her interview, I asked her to explain. She replied that, the videos had helped her to understand how important ratio and proportion were in her workplace. She said, “I always thought that basic math like addition, subtraction, multiplication and division were all that I needed in my nursing career. With the steps broken down in the video, I tried to equate it (the problem) to something in life. In my case I am a floor supervisor and I have to do daily staffing. The staff to patient ratio is 6 to 1. I can understand and do the computations by using my own experience rather than a
generic word problem.” In her interview, she goes on to explain that in her present position:

I am in charge of staffing the dementia unit in a long-term care facility. I never thought that I would use ratio and proportion to do staffing. I am required to follow the staffing rules issued by state of Ohio for staffing STNA’s for each day. The guidelines tell me how many STNA’s must work to care for a certain number of patients. Without realizing it, I had been using a ratio provided by the state to aid in staffing my unit. When making monthly schedules, I have to consider how many patients we have in the building and how many STNA’s I need to care for them. The number of patients can change in a day so I have to schedule on the high side of the ratio or higher than the state required 6:1 ratio.

In her observation, I asked Sarah to show me how she uses ratio in staffing. She provided me with the following example (see Figure 7). I noticed that she was using a strategy of reducing by increments (see Figure 8).

I asked her if there was an easier way to do this. She said by dividing by the largest number possible. She said that she had learned this from the Bad Date video and worksheet. However, in staffing, when she had to include part and full time RNs and STNAs it made sense to compute by increments.

I asked if there were other areas in her nursing career, where she believed Bad Date helped her understand and compute ratio and proportion. She had explained previously in her class project (Sarah, February 5, 2014) that she had learned to calculate a “drip” in nursing school. She said that she hasn’t had to actually figure one (a drip)
Figure 7. Sarah’s example of ratio in staffing.

Figure 8. Sarah’s strategy of reducing by increments.
in a long time because this was done in the pharmacy. However, often their calculations
and the instruments on the floor did not coordinate for the patient needs. In her
interview, Sarah continued to explain that in her role as a nurse she still needs to
understand the metric system and conversions to equal 1 cc. Then, she knows when to
ask questions of the pharmacy if their calculations do not seem “right” (Sarah, February
23, 2014).

Also, in her interview, Sarah continued to elaborate on her class project by
providing additional examples of how she used proportional reasoning and hadn’t
realized it prior to watching the videos. Sarah provided,

Now when I go to the grocery store to shop for the week, I use proportional
reasoning to decide how much chicken, turkey and how big a pot roast to buy
depending on the number of servings I need. Also, when I know how many
lunches I will need to pack for the week, I know how much food to buy.
She summarized her thinking by saying,

I no longer think that math beyond the basics is something that I will never use in
daily life. I can extend this to my children and teach them how important it will be
in their everyday lives. I can use something as simple as grocery shopping to
teach them, by my example, how important proportional reasoning is. I intend to
start showing them math’s extreme importance immediately. (Sarah, February
23, 2014)

In her class project, Sarah had also written that she liked the “bubbles” in Bad
Date. In her interview, I asked her why. She explained that the talk bubbles provided a
visual and I am a visual person (learner). For example, she said 1 word and he said 4. She wants a 1:1 conversation. She talked about this in her conversation with her friend. And with the animation and the cuteness of this relatable situation, it stands out. It is simpler to understand than reading or lecture. Further, Sarah explained that as she could see and hear the two people talking, each word was represented in a bubble, the bar rose and then the ratio was illustrated in numbers.

Additionally, all of the representations get your attention and you realize that you need to pay attention. In a lecture, the teacher could tell you this is important or bold it in color. With the animation, it says you need to focus. It also keeps you more interested than a lecture because anyone can relate to a time when someone is dominating the conversation. The media presents this in a humorous way. Sarah pointed out the following frames from the *Bad Date* video (see Figure 9).

In a post in the Moodle discussion group, Sarah had commented on how this video reminded her of the Venn diagrams that we were studying in the *MyMathLab* course (Sarah, February 18, 2014). When I asked her to explain in her interview, she could not provide her reasoning (Sarah, February 23, 2014).

The predominant characteristic of *Scale Ella* that Sarah cited in her class project and interview was the humorous use of animation in the form of a superhero to teach scale. Her most memorable image from the *Scale Ella* video (see Figure 10)
Figure 9. Representations of ratio from Bad Date.
Sarah said “If I plug that video into what I am learning and apply it to a story problem, it makes it more fun and then I want to do it. Math is not so difficult like the notion I had in my head. I thought that it was hard but now I plug information into different scenarios.” Sarah provided another comparative example from the MyMathLab course. “I didn’t think that the statistics part was difficult. But, the graphs would have been easier if Scale Ella taught them.” She indicated that the table representations on the work sheets were helpful to her understanding and computations of scale (see Figure 11).
Figure 11. Table representations of scale on Scale Ella Learners Guide.

Finally, I asked Sarah if she thought the time she spent on the videos was worthwhile to her learning of mathematics. She answered that all three videos helped with her time management in the seven-week course because they made the concepts easier to understand. “It didn’t take 2-3 days to learn one section of a textbook. The videos were easier to understand, so I got it the first or second time. It was easier to comprehend.”

In summary of how the videos impacted her understanding of ratio and proportion, Sarah explained,

The understanding of the individual parts (rate, ratio, and scale) of the concept of proportional reasoning helps my mind to work. For instance, scale is not the only way to do a problem. You need to understand ratio but you also need to know scale to understand. So you need to understand them all. They complement each
other and were all equally important to my understanding of proportional reasoning.” (Sarah, February 23, 2014)

Jessica, A Case Study

Good evening everyone!

I am taking this course as a pre-requisite for a Statistics class within my cohort program that begins in March. My past experiences with math courses were a long time ago and I have not had to take a math class since my junior year of high school. I was very fortunate to have a friend that helped tutor me in high school when the level of math changed to Algebra II and Trigonometry to help me maintain my grade point average. I think this course will help me improve my math ability and I feel hopeful already about this class since completing the first homework assignment (100%) yesterday. The Math Lab videos are quite helpful in understanding the step-by-step process of solving a problem. I do enjoy working with some math problems but I must admit that math has never been a subject that I excelled in. Likewise, I do feel very anxious when dealing with math-related problems and I would rank my comfort level when I am faced with a math-related problem at a medium level. I hope after completing this course that I will be able to feel more confident working with math-related situations.

Introductory Post in Moodle by Jessica (January 26, 2014)

Math background. Jessica is a 38-year old Caucasian female. She had three years of math in high school. She completed Algebra 2-trigonometry 20 years ago in her
junior year of high school. That was the last mathematics course that she had taken until her current enrollment in this finite math course.

She returned to college in 2009 and earned an Associate’s degree in Paralegal Studies in 2011. Jessica did not have to take any mathematics courses for that degree. She then decided to complete her Bachelor of Science degree in Criminal Justice. She will graduate in 2014. She currently has a 4.0 GPA and told me in her interview that she considers a B in a course a failure (Jessica, 2/23/2014).

After completing her Bachelor’s Degree, Jessica hopes to further her career in the criminal justice field by working for the Department of Homeland Security. She also plans to pursue a Master’s Degree after graduation.

In her class project, Jessica explained that:

Each year that I had to complete a math class in high school, all of our teachers said ‘You need math for everything’ and they were correct. It has been several years since I have had to take a math course and I must admit that I am definitely out of practice in this area. Still, the information that a person can gain from mathematics, from basic math to even statistics, is necessary in all careers. The knowledge of mathematics is necessary for a career in criminal justice, whether a person is a patrol officer, crime scene investigator, police chief, or an advanced criminologist. (Jessica, February 16, 2014)

Jessica then provided examples, such as a police officer investigating a traffic accident and their need to understand measurement and angles to calculate speed and
distance from the point of collision. Also, at crime scenes ballistics are employed to
determine trajectory and distance of a bullet (Jessica, February 16, 2014).

**Jessica’s mathematical thinking prior to watching the videos.** Prior to
watching the *Math Snacks* videos on rate, ratio and scale, Jessica indicated on her Pre-test
Questionnaire on Attitude that she was not confident in her ability to understand ratio and
proportion as she lacked the mathematical knowledge she needed to understand it. The
topic seemed very complicated and difficult. Math was not one of her favorite or
enjoyable subjects.

However, she considered the learning of ratio and proportion to be relevant and felt
that she was capable of understanding the topic. She did not think that ratio and
proportions needed to be included in each mathematics course. However, it needed to be
taught because the topic is difficult for many students (Jessica, January 22, 2014).

Jessica provided the following answers to rate, ratio, scaling and fraction
problems on the Pre-test Questionnaire on Ratio and Proportion (Jessica, January 22,
2014):

**Rate – from questionnaire on rate and proportion.** Question 3. After the trip,
Sima and Alex decided to see who could ride the fastest back home. Sima rode 5 km in
20 minutes. Alex rode 7 km in 25 minutes. Who rode the fastest? How do you know?
(see Figure 12).
Figure 12. Jessica's use of algorithm.

This problem deals with numerical comparison of proportional relationships between distance and time. Jessica sets up the problem with the formula \( d = r \times t \). In the interview I asked her why she choose this method. She answered that she needed to use this formula and procedure to set up the rate properly. I then asked her what the .25 represented. She answered that it was miles (not km) per minute. The numerical structure in this problem is decimals. (Jessica February 23, 2014).

Question 4. The next weekend, Max and Alice rode their bikes to the park, taking the long route around the lake. The route is 30 km and it took them 1.5 hours. They ate lunch, and then rode back the short way, which is only 20 km. This took them \( \frac{3}{4} \) of an
hour. In which segment of their trip did they ride the fastest? How do you know? (see Figure 13).

\[ d = r \times t \]

\[ a) \text{ Long route: } 30 \text{ km} = r \times 1.5 \text{ hours} \]
\[ \frac{30}{1.5} = \frac{1.5r}{1.5} \]
\[ r = 20 \text{ mph} \]

\[ b) \text{ Short route: } 20 \text{ km} = r \times \frac{2}{3} \text{ hr} = \frac{.75}{.75} \]
\[ \frac{20}{.75} = \frac{.75r}{.75} \]
\[ r = 26.6 \text{ round up: } 27 \text{ mph} \]

\[ c) \text{ they rode the fastest on the short route. Because their rate of speed was 27 mph} \]

*Figure 13. Jessica's strategy of algorithm.*

Jessica used a strategy of the distance formula to set up the rates. She then compared the two rates to determine her answer. Her mathematical reasoning is correct.

**Ratio – from questionnaire on rate and proportion.** Question 2. In Ms. Shula's class, 16 students come by bus, and 14 arrive on their own. In what different ways could you compare the numbers of bus riders and those who come on their own in Ms. Shula's class to those in Mr. Erez's class? Explain your reasoning in detail. (see Figure 14).

Jessica attempted to set up equations and used different variables to represent bus riders and students who came on their own to a ratio of the total students in the school. Jessica provided in her interview that she developed a system of equations based on the current topic in *MyMathLab* (Jessica, 2/23/2014). She did not make ratios for Ms. Schula and Mr. Erez’s classes and then compare.
Figure 14. Jessica's strategy of a system of equations.

Question 3. Is the ratio of bus riders to those who come on their own in Ms. Shula’s class the same as in Mr. Erez’s class? Explain your reasoning in detail. (see Figure 15).

Figure 15. Jessica's use of intuition and comparison.
Jessica’s answer was correct. She was using her intuition and comparing total students in the ratio. However, she did not show her work in developing the ratios and comparing them.

**Scale – from questionnaire on rate and proportion.** Question 1. A customer asks Fran to enlarge a 3 inch by 2 inch photograph to 18 inches by 12 inches. Can this be done without cutting or distorting the picture? How do you know? (see Figure 16).

![Image](image1.png)

*Figure 16. Jessica's use of intuition.*

Jessica’s answer was correct based on her intuition of scale. However, she did not provide an explanation. This problem required calculation of a ratio and its application with respect to enlarging or reducing the dimensions of pictures by finding the scaling factor.

Question 3. Phil took a photo of a man and a tree and their shadows. The real-life man is 1.75 meters tall. In the photo, the man is 3 cm tall, his shadow is 1.2 cm long, and the shadow of the tree is 4.5 cm long. If the photo included the entire tree, how tall would the image of the tree be? How tall is the real-life tree in meters? Show your work (see Figure 17).
Jessica first attempted to convert centimeters to meters. She did this incorrectly. She incorrectly divided (instead of multiplying by 0.01). Thus, her conversion was 300 meters or a larger number of meters than centimeters. Her flawed procedural skill negatively influenced her solution process.

In her interview she confirmed that her lack of knowledge of the metric system and conversions was an obstacle to finding a solution (Jessica, 2/23/2014).

**Fractions – from questionnaire on rate and proportion.** Jessica attempted a number of strategies to solve these missing value problems. Her first answer (a) was correct. She used a factor of change strategy to find the missing value in the equivalent fractions. In (b) she attempted to solve by cross products but her answer had the
numerator and denominator inverted. In (c) she converted the fraction to a decimal and then multiplied .4 times 5. Jessica followed a procedure but did not understand the concept of equivalence to be able to visualize that this again produced an improper fraction not equivalent to the first fraction in the pair (see Figure 18).

![Problems with Fractions]

*Figure 18.* Jessica's strategies on missing value questions.

It appeared from these exercises that Jessica may not have the mathematical proficiency required to solve rate, ratio and scaling problems.

The second question asked students to determine the smaller fraction in each pair.

Jessica used a variety of methods including converting fractions to decimals and simplifying fractions and comparing to arrive at the correct answers (see Figure 19).
Examples of rate and ratio – from questionnaire on rate and proportion. Jessica provided three examples of using rate and ratio in everyday life.

Jessica’s mathematical thinking after watching the Math Snacks videos. Jessica was now confident in her ability to understand ratio and proportion, as she now possessed the needed mathematical knowledge. She still thought that ratio and proportion were complicated and difficult. However, she somewhat enjoyed math and it was no longer her least favorite subject in school. She was convinced that ratio and proportion were important topics that needed to be taught in every math class as the topic was difficult for many students.

The following are Jessica’s answers (Jessica February 14, 2014).

Rate – from questionnaire on rate and proportion: Question 3. After the trip, Sima and Alex decided to see who could ride the fastest back home. Sima rode 5 km in
25 minutes. Alex rode 7 km in 20 minutes. Who rode the fastest? How do you know? (see Figure 20).

\[
\frac{d}{5} = \frac{r \times t}{25} \\
5 = \frac{r \times 25}{5} \\
r = 5 \text{ mph} - \text{Sima}
\]

\[
d = r \times t \\
\frac{7\text{km}}{r \times 20} \\
\frac{7\text{km}}{20} = \frac{7}{t} \\
r = 2.8571 \text{ m/h} \left(\text{round up}\right) = 3 \text{ mph} - \text{Alex}
\]

Sima rode the fastest at 5 mph versus Alex who only rode 3 mph.

*Figure 20.* Jessica continued to use algorithms as her strategy.

Jessica continued to use the formula \(d = r \times t\) to set up the rate but she does not solve the equation correctly. She reverses the divisor and dividend. She misinterpreted the unit and thus the unit rate as miles per hour. She did not demonstrate understanding of rate in this problem, let alone the procedural skill or number sense involved.

Question 4. The next weekend, Max and Alice rode their bikes to the park, taking the long route around the lake. The route is 30 km and it took them 1.5 hours. They ate lunch and then rode back the short way, which is only 20 km. This took them \(\frac{3}{4}\) of an hour. In which segment of their trip did they ride the fastest? How do you know? (see Figure 21).
In her interview, Jessica explained, “This problem threw me for a loop” (Jessica, February 23, 2014). She again cited her lack of knowledge of the metric system and the decimal representations of the unit hours. Jessica did acknowledge that she sees this problem as a unit rate, but continued to need to use the distance formula to set up the rates and computations.

In her observation (February 23, 2014), I asked Jessica if it was only in problems concerning distance that she needed to use a formula to set up a rate or ratio. She stated that she always needed a formula. So I gave her this problem with area to work on:

The length of the side of square A is 36 cm and the length of the side of square B is 60 cm. What is the ratio between the areas of square A and square B? (Ben Chaim et al, 2012 p. 195) (see Figure 22).
Figure 22. Jessica's use of an algorithm to solve an area problem.

Jessica was initially unable to set up the problem correctly. She then tried to use the formula $A = L \times W$, rather than $A = S^2$ to produce a ratio expressed as a fraction not a decimal (Jessica, February 23, 2014). Thus, her mathematical reasoning was correct but her procedure was flawed.

**Ratio – from questionnaire on rate and proportion.** Question 2. In Ms. Shula's class, 16 students come by bus, and 14 arrive on their own. In what different ways could you compare the numbers of bus riders and those who come on their own in Ms. Shula's class to those in Mr. Erez's class? Explain your reasoning in detail (see Figure 23).
Figure 23. Jessica's comparisons for Ms. Shula's class.

Jessica set up the ratios for Ms. Shula’s class correctly based on total students. She then compared those ratios to Mr. Erez’s total class. She had computed his ratios in an earlier problem. She did not provide other methods of comparison, such as $16/14 = 8/7$ for Ms. Shula’s class.

Question 3. Is the ratio of bus riders to those who come on their own in Ms. Shula’s class the same as in Mr. Erez’s class? Explain your reasoning in detail (see Figure 24).
Jessica misinterprets the verbal content and again refers to total students in the class, rather than a ratio representing students that arrive on the bus to students that arrive on their own. In her interview, I asked her if there was another way to answer this problem. She continued to follow her initial interpretation of total students. However, she added that it could be that the value (of total students in the class) just looks different and she should have started with the total of 400 students in the school and then compared this to his class (Jessica, February 23, 2014). She appears to understand ratios of part to whole but not part to part.

**Scale - from questionnaire on rate and proportion.** Question 1. A customer asks Fran to enlarge a 3 inch by 2 inch photograph to 15 inches by 10 inches. Can this be done without cutting or distorting the picture? How do you know? (see Figure 25).

Yes, it can be done. The scale factor is 5. You begin by asking what $\times 3 = 15$? The answer is 5. Once you know this you can enlarge the photo by 5 inches in height and width to get 15 x 10 size.

**Figure 24.** Jessica's answers for Ms. Shula's class.

**Figure 25.** Jessica using a scale factor of change strategy.
Jessica’s answer was correct. She was using a scale factor of change strategy.

Question 5. Phil took a photo of a man and a tree and their shadows. The real-life man is 6 feet tall. In the photo, the man is 3 inches tall, his shadow is 2 inches long, and the shadow of the tree is 4 inches long. If the photo included the entire tree, how tall would the image of the tree be? How tall is the real-life tree in feet? Show your work (see Figure 26).

\[
\begin{align*}
\text{Real life man} &= 6 \, \text{feet} \\
\text{Photo} &= \text{man is} \ 3 \, \text{inches} \times 2 \, \text{inches} = 6 \\
\text{Tree} &= 4 \, \text{inches} \times \frac{2}{\text{inch}} = 8 \, \text{inches}
\end{align*}
\]

The real life tree = \( \frac{8 \times 12}{12 \, \text{inches in a foot}} = \frac{96 \, \text{inches}}{\text{foot}} \) = 8 feet tall

Figure 26. Jessica’s intuition and change of scale strategy.

Jessica’s answer was numerically incorrect because of her misunderstanding of what ratios she was using in the missing value problem. She needed to set up proportions for each using the shadows: photos and then the shadows to the real life. However, her intuition and change of scale strategy had her mathematical reasoning correct.

Fractions – from questionnaire on ratio and proportion. Jessica’s work (see Figure 27).

Jessica’s answers were correct. She used a factor of change strategy and/or multiplying to find the missing value in the equivalent fractions. She used a range of methods, from simple numbers and fractions to decimals. It appeared from these
exercises that Jessica now had the mathematical proficiency that would be required to solve the rate, ratio and scaling problems.

\[
1. \quad \frac{2}{3} = \frac{x}{18} = \frac{3x}{3} = \frac{3\cdot 6}{3} \quad x = 12
\]

\[
\frac{8}{5} = \frac{20}{x} = \frac{8x}{8} = \frac{100}{8} \quad x = 12.5
\]

\[
\frac{3}{5} = \frac{2.4}{x} = \frac{3x}{3} = \frac{12}{3} \quad x = 4
\]

*Figure 27. Jessica’s strategies on missing value problems.*

*Examples of rate and ratio - from questionnaire on rate and proportion.*

Jessica did not provide any examples.

**The mathematics that Jessica saw in the Math Snacks videos.** Jessica’s understanding of the mathematics based on the videos.

**Ratey the Math Cat.** Jessica did not demonstrate an understanding of ratios and proportions to represent quantitative relationships on the post-test. She continued to rely on formulas to illustrate the relationships of the numbers. This may also reflect her
misunderstanding of the importance of units. She did appear to understand that rates might be used to solve problems that arise in mathematics and in other contexts. Jessica did view proportions as multiplicative situations and she could see patterns.

**Bad Date.** Jessica did begin to demonstrate understanding and the use of ratio and proportion to describe quantitative relationships. On the post-test, she still did not clearly understand that ratios could represent part-whole or part-part relationships. On the Learners Guide, she demonstrated that 1:1 ratios could be found in everyday situations. She was able to develop, analyze and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios. She could see patterns and use tables and graphs. And, she could solve problems that arise in mathematics and other contexts.

**Scale Ella.** Jessica could use factors and multiples to solve problems. She could develop, analyze and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios. She demonstrated her understanding of the scale factor that created the relationship between two items that were being compared to one another. Mathematically, she could distinguish that multiplication and division were inverse operations. If the scale factor was less than one, the size of an object or a number was being decreased. If the scale factor was greater than one, the size of an object or a number was being increased. The scale factor can be represented as a decimal, whole number or fraction.
Table 2. *Summary of Jessica’s Mathematical Thinking*

<table>
<thead>
<tr>
<th></th>
<th>Pre-Test Strategy/ Correctness</th>
<th>Post-Test Strategy/Correctness</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rate</strong></td>
<td>These problem deals with numerical comparison of proportional relationships between distance and time. Jessica set up the problem with the formula ( d = r \times t ). She felt that she needed to use this formula and procedure to set up the rate properly. The numerical structure in the first problem was decimals. This was a challenge to Jessica. Although her answer was numerically incorrect Jessica used productive mathematical reasoning and intuition in her solution. In the second question, Jessica again used a strategy of the distance formula to set up the rates. She then compared the two rates to determine her answer. Her mathematical reasoning is correct.</td>
<td>Jessica continued to use the formula ( d = r \times t ) to set up the rate but she did not solve the equation correctly. She reverses the divisor and dividend. She also misinterpreted the unit, did not demonstrate understanding of rate in this problem, let alone the procedural skill or number sense involved. She felt that her lack of knowledge of the metric system and the decimal representations were overwhelming in her attempt at solving this problem. She felt that she needed to use a formula to set up any type of rate or ratio problem; including area. Her mathematical reasoning was correct after she set up the rates but her procedure was flawed.</td>
</tr>
<tr>
<td><strong>Ratio</strong></td>
<td>Jessica attempted to set up equations and used different variables to represent bus riders and students who came on their own to a ratio of the total students in the school. She used this solution based on the topic of study in <em>MyMathLab</em>. She did not demonstrate the procedural skill or mathematical reasoning to solve this problem. Jessica’s answer was correct. She was using her intuition and comparing total students in the ratio. However, she did not show her work in developing the ratios and comparing them.</td>
<td>Jessica set up the ratios in the first problem correctly and compared them. She did not provide other methods of comparison as the problem requested. In the second question, Jessica misinterprets the verbal content and refers to total students in the class, rather than a part to part ratio representing students that arrive on the bus to students that arrive on their own.</td>
</tr>
</tbody>
</table>
| Scale | Jessica’s answer was correct based on her intuition of scale. However, she did not provide an explanation. This problem required calculation of a ratio and its application with respect to enlarging or reducing the dimensions of pictures by finding the scaling factor.  

Jessica first attempted to convert centimeters to meters. She did this incorrectly. Her flawed procedural skill negatively influenced her solution process. | Jessica’s answer was correct. She was using a scale factor of change strategy. In the second problem, Jessica’s answer was numerically incorrect because of her misunderstanding of what ratios she was using in the missing value problem. However, her intuition and change of scale strategy had her mathematical reasoning correct. |
| Fractions | Jessica attempted a number of strategies to solve these missing value problems. Her first answer (a) was correct. She used a factor of change strategy to find the missing value in the equivalent fractions. In (b) she attempted to solve by cross products but her answer had the numerator and denominator inverted. In (c) she converted the fraction to a decimal and then multiplied .4 times 5. Jessica followed a procedure but did not understand the concept of equivalence to be able to visualize that this again produced an improper fraction not equivalent to the first fraction in the pair.  

It appeared from these exercises that Jessica may not have the mathematical proficiency required to solve rate, ratio and scaling problems. | Jessica’s answers were correct. She used a factor of change strategy and/or multiplying to find the missing value in the equivalent fractions. She used a range of methods, from simple numbers and fractions to decimals. It appeared from these exercises that Jessica now had the mathematical proficiency that would be required to solve the rate, ratio and scaling problems. |
Jessica's perception of the *Math Snacks* videos. Jessica had indicated to me in an email that she initially preferred the teaching method used in *MyMathLab* (traditional lecture and step by step examples) to that of *Math Snacks*. I wanted to have her clarify her comment, so in her interview, I asked Jessica to explain further. She responded that the *Math Snacks* videos were a great way to reinforce the process taught in *MyMathLab*. She also expressed this view in her response to another classmate in a Moodle forum post on February 9, 2014. I then asked her, “What characteristics of the three *Math Snacks* were most helpful to you to grasp understanding and computations of rates, ratios, scale and proportional reasoning?

I started the interview with a conversation about *Ratey the Math Cat*. Jessica had posted the following in the Moodle forum.

> My nephew was in the room when I played the video and he was stopped and came over to watch them with me because he liked the cartoons! He wasn't interested once I mentioned the word homework though! The games were a great way for me to get comfortable with the concepts we covered. I agree with you that these help build anyone's proficiency and repeating the process over and over is a great way to reinforce it. (Jessica, February 9, 2014)

*Ratey the Math Cat*’s animation, along with the games and repetition had hooked Jessica’s interest and increased her comfort level with mathematics (Jessica, February 23, 2014).

We continued the interview with a discussion of *Bad Date*. Jessica explained that she now understands the concepts and how she used ratio in “day to day things” and how
much she used it. In the Moodle forum, Jessica had posted, “I used proportional reasoning today...I was reducing a recipe that was for 18 people down to 6 people. I really think watching the videos this week made this process a lot easier!” (Jessica, February 16, 2014) Jessica continued, “To reduce the recipe I used proportional reasoning. As my grandmother’s family grew, we doubled and then tripled recipes. So when I was making it for 6, I knew I just had to divide all of the ingredients by 3.” (Jessica, February 23, 2014)

Jessica had demonstrated her understanding of ratio, along with 1:1 relationships in real life and their computations on the Bad Date Learner’s Guides (see Figure 28).

![Image](image.png)

*(Math Snacks Learner Guide, 2013)*

**Figure 28.** Jessica’s understanding of 1:1 ratio.

In the interview, Jessica indicated that she liked to develop tables so she could see the patterns of scaling and proportions develop. She was not interested in drawing to help
with her understanding. I offered her paper and pencil in the interview also (Jessica
February 23, 2014) (see Figure 29).

![Image](image_url)

Figure 29. Jessica’s use of tables to understand patterns in ratios.

Jessica commented that the animations and talk bubbles kept her attention and
Illustrated the concepts as she performed the computations (Jessica, February 23, 2014)
(see Figure 30).
Figure 30. Jessica’s work on ratios with animations and talk bubbles.

Jessica continued, “Bad Date was the best to tie things together, not only with ratio and proportion but also with MyMathLab. Jessica was referring to the concepts of rate, ratio and scale. In MyMathLab I don’t think I could do the worksheets without the videos. We cover so much in such a short amount of time.” (Jessica, February 23, 2014) She indicated that she now understands ratio because it is like fractions. It represents a relationship. In the Moodle forum on February 9, 2014, Jessica provided:

Good afternoon! I was asked to help my niece with an assignment for her advanced math class. At first I told her that I may not be the best person to ask until she told me the topic was using ratios. I got really excited! I shared with her the video from our class project and she loved it! Her assignment was to interview each of her classmates and come up with a ratio problem. From her information, we were able to figure out that my niece is the only left-handed student out of class of ten students. So, we created the ratio 9:1. She called me today and said
that her teacher was thrilled with her example and the creativity in her presentation!

Jessica considered *Scale Ella* the most helpful video for her particular learning style. She explained in her class project that *Scale Ella* was helpful because, “…it creates a scenario, such as, Scaleo (the villain) who would increase or decrease a particular item and then Scale Ella would find the scale factor.” Additionally, she described the video’s teaching method as, “…an instructor explaining the process after presenting the information, either by video or written format, which really does provide me with the best way to learn each concept, allows me the opportunity to solve a problem and then confirm my answer or realize my answer is incorrect and try again.” (Jessica, February 16, 2014) The following is part of Jessica’s work from the *Scale Ella* Learners Guide (see Figure 31).

![Scale Ella](image)

*(Jessica, February 16, 2014)*

**Figure 31.** Jessica’s work on a scale factor.
In her interview, Jessica commented further on the bed problem, “I like the comparisons to get the scale factor and the increase and then decrease in an actual examples; like the bed. It is something to relate to.” (Jessica, February 23, 2014) She continued to explain that she watched all three videos several times and then looked over the Learner’s Guides to see what she was asked to do. Then, she watched the videos again because now she knew what she was looking for. Jessica perceived that Scale Ella had more instruction, as well as more illustration of solutions. She says, “This helped me to see how to get to the end result.” She also thought that the games were good practice after she understood the problem by watching the video repeatedly. Jessica summarized how the Math Snack videos helped her understanding and computations of ratio and proportion by saying, ”I have to have communication along with reading, so watching the animations, along with reading their words and it all came together. I need a person next to me to answer my questions. The videos were like having someone to ask questions.” (Jessica, February 23, 2014).

I asked Jessica to tell me more about how the animations helped her learning. She responded, “The animation was different. It was cute. The audio and video got my attention. It was fine with the instructional steps. Anyone I share them with, like my niece and nephew say they got their attention and loved it. I am more technical. I just want the steps. But, I like them because they are unique.” (Jessica, February 23, 2014).

Jessica made a final post in Moodle forum on February 26, 2016:

I used the scale factor concept from the Scale Ella video over the weekend with
my boyfriend at Home Depot. He wants to build a coffee table and really liked these wood crates that we found. However, he was not sure how many he would need to buy. I explained to him that if we knew the height of the crate, which ended up being 8 inches and he wanted the table to be 32 inches, the difference was a scale factor of 4, so he would need to purchase at least 4 crates.

Finally, I asked Jessica to summarize her feelings about the time management of including the Math Snacks videos along with MyMathLab in a 7-week course. She said that the Math Snacks videos were a nice break from MyMathLab and a good transition and learning tool. The two mediums worked together.

Tamara, A Case Study

Hi everyone, my name is Tamara. I am a senior. I wish I could learn to like math, however, I have not done well in math. When I saw this class began with chapter 6 my heart sank. I will do my best and I wish everyone the best. I like the My Lab. It is awesome. I believe I can do well because of the help given. I am a grandmother and great grandmother. I like to read and swim and I write poetry sometimes. *Introductory Post in Moodle by Tamara (March 13, 2014).*

Math background. Tamara is a 50-year old African-American female. She is the mother of 6 children, 2 grandchildren and one great grandchild. She has glaucoma and had successful eye surgery during the course.

She graduated from a community college several years ago where she completed basic math/pre-algebra and elementary algebra courses. The distance education course and program in my study originally held seated classes at this community college.
Tamara is currently a senior with 5 classes to complete, including this finite math course, before she graduates with a BS in Business. She also needs to take statistics and economics. She told me, “I have tried to pass this course a number of times” (Tamara, April 22, 2014). At the midterm of the 7-week course her MyMathLab average was 26.5%.

**Tamara’s mathematical thinking prior to watching the videos.** Mathematics was not one of Tamara’s favorite or enjoyable courses in school. However, she considered the topic of ratio and proportion to be relevant. She also felt strongly that it needed to be taught. She believed that she had the mathematical knowledge to understand ratio and proportion but was not confident in her ability to understand it because it was very complicated. Tamara also did not feel that she had the theoretical knowledge to understand the topic, so it was important for her to study it. She considered herself capable of understanding ratio and proportion. She believed that many students have difficulty understanding the topic and that it should be studied in every mathematics course. Tamara could not provide any examples of ratio and proportion.

Tamara provided the following answers on the questionnaire on ratio and proportion 4/2/2014:

**Rate – from questionnaire on rate and proportion.** Question 1. Max and Alice had to buy the beverages. They saw that cherry soda cost $2 for 16 single-serving boxes. Grapefruit juice cost $1.60 for 12. They decided to buy the grapefruit juice. Was this the best choice economically? Show in detail all the calculations and thought processes with which you arrived at your answer.
\[
\frac{16}{\$2} = 8 \text{ cent} \quad \text{cherry soda per single serving} \quad \frac{\$1.60}{12} = 13 \text{ cent grapefruit juice per single serving}
\]

No the grapefruit juice cost more.

Tamara should have divided 2 by 16 to get the unit rate or cost of the cherry soda. This flawed procedural error was not repeated on the unit rate of the grapefruit juice. She compared the unit rates to draw a conclusion and demonstrate productive mathematical reasoning.

Question 5. In the field near their homes, Max and Alice noticed a number of stray cats. They made a number of phone calls and discovered that in their town there are about 1000 stray cats, and in the neighboring town there are about 1500. Their town has an area of 60 square kilometers, and the neighboring town is 100 square kilometers. Assuming that the cats are evenly scattered over the areas, in which town is there a higher likelihood of seeing a stray? Explain your reasoning.

Their town cats \(\frac{1000}{60} \text{ sq. kilometers} = 16.66 \text{ or 17 cats scattered in their town.}\)

Neighboring town cats \(\frac{1500}{100} \text{ sq. kilometer} = 15 \text{ cats scattered in the neighboring town.}\)

Since the number of cats is greater in a smaller area a person would more likely see a stray in their town.

Tamara used a unit rate strategy where she demonstrated the correct procedural skill. She then compared the unit rates to draw a correct conclusion and explain her mathematical reasoning.

Ratio – from questionnaire on rate and proportion. Question 3. Is the ratio of bus riders to those who come on their own in Ms. Shula’s class the same as in Mr. Erez’s class? Explain your reasoning in detail.
Ms. Shula’s 18/12  Mr. Erez’s 20/15  20/15 = 4/3 No they are not equal since
3/2 does not equal 4/3

Tamara created the ratios correctly and then compares them to conclude they are not equivalent. Both her procedural skill and mathematical reasoning are correct.

Question 5. Of the 400 students in the school, 240 arrive by school bus daily. Is the ratio between the numbers of pupils that arrive by bus to the number that arrive on their own in Ms. Shula’s class the same as that of the whole school? How do you know?

Students in school and on the school bus 400/240 = 5/3 and the students of Ms. Shula’s Class is 18/12 = 3/2 so they are not the same.

Tamara misinterpreted the verbal content of the question. Thus, she did not set up the ratio to reflect the pupils that arrive by bus to pupils that arrive on their own. The correct ratio was 3/2 or equal to Ms. Shula’s class ratio. She did compare the two ratios in order to establish equivalency. Her conclusion was incorrect due to the incorrect ratio of the total students. However, her strategy and mathematical reasoning were correct.

Scale – from questionnaire on rate and proportion. Question 1. A customer asks Fran to enlarge a 3 inch by 2 inch photograph to 18 inches by 12 inches. Can this be done without cutting or distorting the picture? How do you know?

I think it can be done since 18/12 = 3/2

Tamara used an equivalency strategy of the two ratios to draw a correct conclusion. However, she did not explain her mathematical reasoning by using a scale factor.

Question 3. If the photo included the entire tree, how tall would the image of the
tree be? How tall is the real-life tree in meters? Show your work.

*I do not know*

Tamara did not know how to work this problem.

**Fractions – from questionnaire on rate and proportion.**

Tamara did not attempt the missing value problems. In the comparison problems, Tamara converted the fractions to decimals and compared. Her answers were correct.

**Tamara’s mathematical thinking after watching the videos.** Tamara continued to feel strongly that math was not one of her favorite or enjoyable subjects. She was confident that she had the theoretical knowledge and ability to understand ratio and proportions. However, it might take a little longer for her to understand proportion.

She believed that ratio and proportion were important for a student’s mathematical development and needed to be taught. Further, learning the topic of ratio and proportion was relevant. And, students should have a wide range of mathematical knowledge beyond what is learned in school.

**Examples of ratio and proportion – from questionnaire on rate and proportion.**

*Example of ratio: I am the mother of six children. The ratio is 6:1.*

*Example of proportion: My body proportion is off. I am 5’7” and my waist is 39.*

*My body is out of proportion.*

**Rate – from questionnaire on rate and proportion.** Question 1. Max and Alice had to buy the beverages. They saw that cherry soda cost $2 for 14 single-serving boxes. Grapefruit juice cost $1.60 for 12. They decided to buy the grapefruit juice. Was this the
best choice economically? Show in detail all the calculations and thought processes with which you arrived at your answer (see Figure 32).

![Figure 32. Tamara’s work on unit rates.](image)

Tamara divided correctly to develop unit rates, where she could compare and correctly determine that the grapefruit juice was the better buy.

See Figure 33 for Tamara's answer to question 5 showing her use of intuition as a strategy.

5) In the field near their homes, Max and Alice noticed a number of stray cats. They made a number of phone calls and discovered that in their town there are about 1000 stray cats, and in the neighboring town there are about 1500. Their town has an area of 60 square kilometers, and the neighboring town is 125 square kilometers. Assuming that the cats are evenly scattered over the areas, in which town is there a higher likelihood of seeing a stray? Explain your reasoning.

![Figure 33. Tamara's use of intuition as a strategy.](image)
Tamara did not compute the unit rates. However, her intuition was correct. On the pre-test, she had correctly developed unit rates and then compared the number of cats in the two towns. On the post-test, her answer was also correct but she did not provide a mathematical explanation. In the interview, she explained that the metric system confused her (Tamara, April 22, 2014).

**Ratio – from questionnaire on rate and proportion.** Question 1. In Mr. Erezl's class, 25 students came by bus and 10 arrive on their own. In what different ways could you compare the number of students that arrive by bus to the number that arrive on their own to school? Explain your reasoning in detail (see Figure 34).

![Tamara's representation of rate.](image)

*Figure 34.* Tamara’s representation of rate.

Tamara expressed the ratio correctly. She then simplified the ratio and represented it as a fraction. However, her third representation of a 2:1 ratio was incorrect.

In her interview, she could not explain her mathematical reasoning of that representation (Tamara, April 22, 2014). (See Figure 35).
4) Of the 400 students in the school, 260 arrive by school bus daily. Is the ratio between the numbers of pupils that arrive by bus to the number that arrive on their own in Ms. Shula’s class the same as that of the whole school? How do you know? They are not the same. Ms. Shula’s class ratio is 8/7. The school is 13/7.

\[ \frac{260}{140} = \frac{13}{7} \]

**Figure 36.** Tamara’s ratio for Ms. Shula’s class.

Tamara did not understand the verbal content of the question so the ratio for the school is incorrect. It should be \( \frac{260}{140} = \frac{13}{7} \) to reflect students arriving by bus to students arriving on their own. The ratio for Ms. Shula’s class was 8/7. However, she used an equivalency strategy to compare the ratios and demonstrate productive mathematical reasoning.

**Scale – from questionnaire on rate and proportion.** Tamara made this comment prior to starting the post-test questionnaire (Tamara, April 17, 2014) (see Figure 36).

"I still do not understand how to descale so I am guessing."

**Figure 36.** Tamara’s comment.
Question 1 Tamara used her intuition (see Figure 37).

1) A customer asks Fran to enlarge a 3 inch by 2 inch photograph to 15 inches by 10 inches. Can this be done without cutting or distorting the picture? How do you know?

\[
\frac{3}{2} = \frac{15}{10} \quad \text{Yes since} \quad \frac{15}{10} = \frac{3}{2} \quad \text{The measurement would be in proportion}
\]

Figure 37. Tamara’s use of intuition.

Tamara’s intuition and strategy to find the ratios equivalent was correct. However, she did not use a scale factor strategy.

Question 6, Tamara answered (see Figure 38).

If the photo included the entire tree, how tall would the image of the tree be? How tall is the real-life tree in feet? Show your work.

\[
\text{Shadow is } \frac{1}{10} \text{ in for the tree} \quad \frac{8 \times 12}{10} = 96 \text{ inches} \quad \text{The tree is about 8 feet tall}
\]

Figure 38. Tamara’s mathematical reasoning.

Tamara realized that she should be developing ratios of shadows to actual heights and tried to guess. This demonstrated some productive mathematical reasoning. She could have used a missing values strategy to solve the problem but the following fraction computations indicate that she did not have that procedural skill.
Fractions – from questionnaire on rate and proportion.  Question 1 (see Figure 39).

1) **What is the number that can replace the “?” in each of the following problems.** Explain your reasoning and show your work for each.

   a) \( \frac{2}{3} = \frac{?}{18} \)  \( \frac{2}{3} \) \( \frac{12}{18} \) \( \text{Divide each numerator by} \)

   b) \( \frac{8}{5} = \frac{20}{?} \)

   c) \( \frac{3}{5} = \frac{2.4}{?} \)

*Figure 39.* Tamara’s work on fractions.

Theresa still did not attempt the missing value problems. In the comparison problems, Tamara converted the fractions to decimals and compared. Her answers were correct. It appeared from these exercises that Tamara might not have all the mathematical proficiency that would be required to solve the rate, ratio and scaling problems.

**The mathematics that Tamara saw in the Math Snacks videos.** Tamara's understanding of the mathematics based on the videos.

**Ratey the Math Cat.** Tamara understood and used ratios and proportions to represent quantitative relationships. She built new mathematical knowledge through problem solving in mathematics and other contexts. She successfully completed the learning objectives of rates and unit rates, seeing proportions as multiplicative, patterns and translating unit rates to a table or graph.

**Bad Date.** Tamara did not consistently demonstrate understanding and the use of ratio and proportion to describe quantitative relationships on either the post-test or the
Bad Date Learner’s Guide. Thus, she did not always understand or use ratio and proportion correctly to describe quantitative relationships (see Figure 40).

![Image](image.png)

(Math Snacks Learner Guide, 2013)

Figure 40. Tamara’s work on ratios.

While Tamara solved problems using equivalent ratios, she was not able to explain her methods. Tamara misinterpreted ratios on several problems but she was able to represent part-whole or part-part relationships, patterns and 1:1 ratios in everyday situations.

Scale Ella. Tamara used factors, multiples and prime factorization and relatively prime factors to solve problems. She used equivalent ratios and some scale factors but was still unable to explain her methods clearly. She understands that multiplication and division are inverse operations and that a scale factor of less than one decreases the size of an object or number and a scale factor of greater than one increases it. She did not demonstrate a consistent ability to use scale factor as a procedural skill or part of productive mathematical reasoning.
Table 3. *Summary of Tamara’s Mathematical Thinking*

<table>
<thead>
<tr>
<th></th>
<th>Pre-Test Strategy/ Correctness</th>
<th>Post-Test Strategy/Correctness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate</td>
<td>On the first question, Tamara reversed the dividend and the divisor. Thus the unit rate on the cherry soda was incorrect. This flawed procedural error was not repeated on the unit rate of the grapefruit juice. She compared the unit rates to draw a conclusion and demonstrate productive mathematical reasoning. On the second question, Tamara used a unit rate strategy where she demonstrated the correct procedural skill. She then compared the unit rates to draw a correct conclusion and explain her mathematical reasoning.</td>
<td>On the first question, Tamara divided correctly to develop unit rates, where she could compare and correctly determine that the grapefruit juice was the better buy. On the second question, Tamara did not compute the unit rates. However, her intuition was correct. She did not provide a mathematical explanation. She also commented that the metric system confused her. In the first question,</td>
</tr>
<tr>
<td>Ratio</td>
<td>In the first question, Tamara created the ratios correctly and then compares them to conclude they were not equivalent. Both her procedural skill and mathematical reasoning are correct. In the second question, Tamara misinterpreted the verbal content of the question. Thus, she did not set up the ratio to reflect the pupils that arrive by bus to pupils that arrive on their own (part to part) ratio. She did compare the two ratios in order to establish equivalency. However, her strategy and mathematical reasoning although based on the incorrect ratios were correct.</td>
<td>In the first question, Tamara expressed the ratio correctly. She then simplified the ratio and represented it as a fraction. However, her third representation of a 2:1 ratio was incorrect. In the second question, Tamara continued to misinterpret the verbal content of the question. Thus, she did not set up the ratio to reflect the pupils that arrive by bus to pupils that arrive on their own (part to part) ratio. She did compare the two ratios in order to establish equivalency. However, her strategy and mathematical reasoning although based on the incorrect ratios were correct.</td>
</tr>
<tr>
<td>Scale</td>
<td>In the first question, Tamara used an equivalency strategy of the two ratios to draw a correct conclusion. However, she did not explain her mathematical reasoning by using a</td>
<td>In the first question, Tamara’s intuition and strategy to find the ratios equivalent was correct. However, she did not use a scale factor strategy.</td>
</tr>
</tbody>
</table>
Tamara was unable to solve the second problem.

In the second question, Tamara realized how she should be developing the ratios and she tried to guess. Thus, demonstrating some productive mathematical reasoning. She could have used a missing values strategy to solve the problem but the following fraction computations indicate that she did not have that procedural skill to accomplish this.

| Fractions | Tamara did not attempt the missing value problems. In the comparison problems, Tamara converted the fractions to decimals and compared. Her answers were correct. It appeared from these exercises that Tamara might not have all the mathematical proficiency that would be required to solve the rate, ratio and scaling problems. | Tamara still did not attempt the missing value problems. In the comparison problems, Tamara converted the fractions to decimals and compared. Her answers were correct. It appeared from these exercises that Tamara might have some of the mathematical proficiency that would be required to solve the rate, ratio and scaling problems. |

Tamara’s Perceptions of the Math Snacks Videos. Tamara’s anxiety with mathematics continued to be pervasive throughout the study. She posted the following on Moodle on April 9, 2014:

The fact that animation was used in the class project made the problems less stressful. I was able to learn something about math without the anxiety that I usually have when presented with problems from books. I can think that the animations made math fun instead of scary. I happened to think that more information was needed in order for me to get a complete understanding.

Tamara began her interview with general comments about the Math Snacks videos and how they impacted her learning. She asserted, “I liked the cartoons. Note: she is referring to the animation. I don’t feel stressed when I watch them. I am relieved.
Before I came here I looked at the book and final and I started feeling anxiety. I am thinking about talking to a psychiatrist about this anxiety about math” (Tamara, April 22, 2014). Again, Tamara was sincere about her anxiety with math.

She continued and explained specifically that the characteristic of animation made her think of fun and she said, “even though the math is difficult, it is not as difficult as the written word. The cartoons make me think I might actually enjoy it”. She explained that the animations helped to relieve her anxiety, “When I wasn’t anxious I could sit and watch and learn from it. When I go to class, I usually get anxious and block it out” (Tamara, April 22, 2014).

In her class project (April 16,2014), Tamara had mentioned that she had used Kahn Academy:

I am sorry but out of -all the videos the ones on proportion is [sic] the one I did not understand at all. In real life I can see proportion for ex. If a person’s head is too big, I can recognize that. In addition, when a child is walking with a parent I can see it clearly the difference in size. Once it is on paper, I am totally lost. All of the videos did one special thing for me, the videos removed the stress factor and I was able to relax and enjoy learning about math. Most of the time, I get a headache, when I know I have to do math problems. I also cry which does not make any since [sic] to me but it could be because do not expect to succeed. I do not feel that I did well on any part of this but I did go to khan’s [sic] academy to get some understanding. The videos did not have enough explanation for me to
understand completely and when I went to khan’s [sic] academy I had to watch the video more than three times to get any understanding.

Class Project by Tamara (April 16, 2014)

She expanded on this in her interview and said that she liked Kahn Academy very much because he (the instructor) doesn’t show a lot of unnecessary stuff. She conveyed, “Like with the animations, I can watch it over and over again at my leisure. I learned more at Kahn Academy because he broke it down step by step. Then I realized that the animation did the same. But Kahn Academy did more examples” (Tamara, April 22, 2014).

Then, I asked Tamara what specific characteristics of Ratey the Math Cat were helpful to her understanding and strategies in solving rates. Tamara answered that she did not remember Ratey, but Bad Date and Scale Ella stuck in her mind. All she could remember about Ratey was that he was funny. In her class project, Tamara had written that her first impression of Ratey was that it was not helpful to her. She commented:

I did not see the humor or understand rates. However, when I watched the video again, I realized that stress must have kept me from seeing how much information was in the video. I now see that Ratey was the easiest video to learn the math concept. Ratey was doing the math before my eyes; he gave the rate then gave an answer to the problem. Ratey told how many gallons of water were used while the man was showering. He also told how much the fish was per pound and he made it easy to understand how a person would get an answer (Tamara, April 17, 2014).
Tamara explained further that the tables and graphs also helped her to understand and compute unit rates. The following is an excerpt from her completed Learner’s Guide (see Figure 41)

![Table and Graph Example]

Figure 41. Tamara’s tables and graphs.

As the interview continued, Tamara shared with me that she understands ratio because of the bars on the side screen in Bad Date. She could see how many words she spoke and he spoke. She commented that it was interesting and funny and she could
relate to it because, “I am older and I have dated”. On the completed Learner’s Guide, Tamara referred to dating in the figure that she drew. She commented that her “doodling” helped to relax her and to understand the concept (see Figure 42).

![Figure 42. Tamara’s doodling to help her anxiety.](image)

I asked her how the bars and the words spoken by the characters helped her to understand and compute ratios? Tamara responded, “she is talking and at the same time you see the bars. It made my attention stay by noticing the sides with the graphs and hearing what they are saying.”
She continued to discuss this characteristic in *Bad Date* and how it impacted her problem solving of ratios on the questionnaires and the worksheet. She explained:

I knew what it was supposed to look like. I knew what I was supposed to look for comparisons. Then, I went to Kahn Academy and went over their series on ratios in order to get an understanding, then, I watched both the cartoon and Kahn Academy twice more. Rates and ratios are similar and I saw that in *Ratey* and *Bad Date*. You get them the same way.

In both her interview and class project, Tamara mentioned the realism in *Bad Date* and how it reminded her of the use of ratio and proportion in her life. She provided the following examples:

There are many ways that proportional knowledge can be useful in real life. One area that proportion is required is in baking. When I want to make biscuits, I have to know what percentage of flour, salt, baking powder, Crisco and water to use so that the biscuits can taste the way they should. Before the class project, I just baked. I know how essential it is to know the exact amounts to put in a receipt; I never understood that I was doing math or proportions.

Shopping is another area that knowing proportions would be helpful. When grocery shopping, a person would need to know if they brought a large box of cereal for a set amount of money would it be cheaper to buy the box that was on sale. At a clothing store one would need to know if the blouse that was on sale today was a good deal. A person could tell if the price was marked up, then a percentage taken off and the blouse was still the same price. Going shopping is
where the understanding of proportions will be beneficial to me since I was never able to do ratios and calculate if I was losing money or getting a deal.

Also, when I am dieting it is important to eat smaller proportions in order to lose weight. When dieting you have to measure and weigh the food on a scale. That means I have been dealing with ratios, rate, scales and proportions without knowing it (Tamara, Class Project p. 3).

Tamara had indicated her understanding of the concept of ratio and how she used it in her everyday life. Then, I asked her what in particular had helped with her computations. She replied, “Bad Date sticks in my mind to help with computations … the conversation with the friend on the phone. She keeps saying it over and over repeating that every date is always the same. That is how we learn by seeing it over and over” (Tamara, April 22, 2014).

As her interview progressed, we discussed Scale Ella. I commented that the Scale Ella Learner’s Guides appeared to challenge her (see Figure 43). Tamara answered, “I might have studied scale before, but I felt like this was the first time I had seen it. I can see the proportion but I don’t know how to get the answer.” (Tamara, April 22, 2014).

I asked if there was anything in Scale Ella that helped or at least moved her thinking towards solving the problems. Tamara answered that she could see the proportion by sight, “I liked where she scales up her thumbs and then the thumbs were bigger than the keys.” She pointed out the following scene (see Figure 44).
The regular size of a twin bed is 39" wide, 75" long and 24" high. Scaleo has scaled your bed to this size: 13" wide, 25" long and 8" high.

1. What can Scale Ella do so that you can sleep comfortably tonight?

She could scale me down.

3. You have been given Scale Ella’s powers, but before you scale items you have to practice by scaling numbers. Pick a scale factor that will increase the numbers and enter it into box 1. Pick a scale factor that will decrease the numbers and enter it into box 2. Once you pick your scale factors, complete the table by applying the scale factors to increase and decrease the numbers.

<table>
<thead>
<tr>
<th>Numbers</th>
<th>Scale Up By</th>
<th>Scale Down By</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>25</td>
<td>.50</td>
</tr>
<tr>
<td>1/2</td>
<td>75</td>
<td>-3</td>
</tr>
<tr>
<td>7</td>
<td>125</td>
<td>3</td>
</tr>
<tr>
<td>13</td>
<td>125</td>
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<td>25</td>
<td>125</td>
<td>15</td>
</tr>
<tr>
<td>102</td>
<td>510</td>
<td>93</td>
</tr>
</tbody>
</table>

(Math Snacks Learners Guide, 2013)

Figure 43. Tamara’s challenges with scale.
Figure 44. Illustration from Scale Ella.

Tamara indicated that she now understands the concept of ratio and proportion. Then, I asked her, “What in particular in the videos helped with the computations?” Tamara explained:

I am starting to do these in my head. It must be that crazy video Scale Ella. First I can’t do it and then I try. I think about a problem in Scale Ella and that helps. I like the thumbs. I did more scale than ratio in Scale Ella and then I looked at Kahn Academy and back to Scale Ella.

Tamara also indicated that seeing the same theme of scale repeated in different scenarios was helpful to her understanding of ratio and proportion.
After discussing how the *Math Snacks* videos and their characteristics had aided her understanding and computations of ratios and proportions, we went over some of her questionnaires to get more information about her mathematical knowledge and strategies in problem solving. Tamara was concerned that there wasn’t a calculator. I advised her that I was more interested in her steps to solve than the answer. Tamara’s anxiety with math appeared at times to confuse her so that she was unable to answer the same questions that she had answered correctly on the questionnaires, such as those with metric units and conversions from hours to minutes. She was also uncertain of how to set up some rates and ratios, particularly in part to part ratios, such as #4 in ratios. When I asked her why she was confused, she said that it was the way the problem was written.

We continued Tamara’s interview and observation with a conversation about fraction computation with question 1b from the questionnaire (see Figure 45). I asked her to explain her strategy to solve the problem. Tamara replied, “I kept multiplying to get a larger number. You can’t multiply anything by 8 to get 20. So I kept multiplying and then reduced the work”.

![Figure 45. Tamara’s fraction strategy](image)

(Tamara, April 22, 2014)
When I prompted her to show me how she figured out the fraction problem #2, she responded, “The larger the number (pointing to the denominator), the smaller the pieces 3 more pieces than 7 - the larger numbers, the smaller fractions. I remember from elementary school the pizza problems.”

Next, we discussed the strategy Tamara used on the scale problems. On #1, she had correctly identified that the ratio 3 to 2 as proportional to 15 to 10. She explained, “5 goes into 15 and 5 into 20 so I divide. I could also multiply the little one (smaller fraction) so they are proportional.” On scale problem #3, Tamara indicated that she had guessed, she indicated that, “The tree is bigger than the man so I multiplied by inches.”

(Tamara, April 22, 2014)

I also needed to clarify some of her responses on the Attitude Questionnaire, such as why she still didn’t like math. Tamara responded, “Before I came here I looked at the book and I got my anxiety. When I open the book it gets big and I get small. I closed the book and I looked at Kahn Academy” (Tamara, April 22, 2014).

Tamara also indicated that learning ratio and proportion was important. She expounded by saying that you might get cheated and provided an example of buying land and comparing size and cost.

Finally, I asked her about the teaching method in Math Snacks versus MyMathLab to get her opinion on adding a project on the topic of ratio and proportion to a 7-week course. Tamara responded that it was a benefit because she learned ratio and proportion so “she got something out of it”. Additionally, she did not feel that the time she spent on
watching the videos and writing the class project took away from her studying the course material.

When we concluded the interview, Tamara said, “I am wondering now instead of feeling like a failure and as if I could never understand math, I should feel optimistic and confident that I have the ability to learn and perform math. I am going to leave here today feeling good about myself” (Tamara, April 22, 2014).

**Gregory, A Case Study**

My Name is Gregory. I happen to live in our Nation’s Capitol Washington, DC at this time. I am taking Math 174 to fulfill my final requirement in order to graduate with a Bachelors of Arts Degree in Criminal Justice Administration. I would like to use my degree in creating a business dealing with reforming people with a criminal past. I think I could be very successful in transforming human lives providing that the will is present in the person that has fallen. Suffice it to say, that I have always found Math to be very challenging and have not ever done well in Math or Algebra. But I intend to turn this all around. I want to be very successful in his course thus; I expect to work extremely hard at this. I have studied countless hours in preparation for this course. Therefore, it is my desire to master this. Several years ago I promised my late-great Mother that I would get my Bachelors Degree and I fully intend to do so. I expect my mathematic ability to be enhanced by taking this course but that is not saying much because my ability is zilch, inexistent or nada presently. My comfort level is low and my anxiety is high when it comes to math. However, as stated I expect this to change
forthwith. I think this course will be helpful inasmuch as I expect to use the mathematical process to further organize my thinking and to assist me in structuring logical arguments.

*Introductory Post on Moodle by Gregory (March 13, 2014)*

**Math background.** Gregory is a 54-year old African American male. Gregory described his mathematical background:

I have failed this course many times. It is the only course I need for graduation. I lost interest in math in the 4th grade with long division. I remember a teacher who helped me. He showed every problem in 5 steps and connected the concept to the procedure and explained each step. The videos were like that.

Gregory continued that he was a criminal justice major and was interested in inmate reform. He felt he had a poor grounding in all disciples in education, including mathematics. His grade at mid term in *MyMathLab* is 2.6%. He had advised me that he was working with a tutor in Washington, D.C.

Gregory had taken the overnight Am Track train from Washington, D.C., where he resides, to Toledo, Ohio where we met in the Toledo Library, Downtown Branch. He currently works as a security guard at a university in Washington.

**Gregory’s mathematical thinking prior to watching the videos.** Mathematics was one of Gregory’s least favorite and enjoyable subjects in school. He was not confident in his ability to understand the topic of ratio and proportion because he did not have an adequate mathematical or theoretical knowledge. The topic of ratio and proportions appeared to him to be very complicated but relevant.
However, he believed that students should have a wide range of mathematical knowledge beyond what is learned in school. Further, he considered ratio and proportion important topics for a student’s mathematical development. Consequently, he understood that he needed to study the topics and felt that it should be included in each mathematics course. Overall, the ratio and proportion did not seem easy to him and he realizes that many students had difficulty with the topic.

**Rate – from questionnaire on rate and proportion.** Question 1. Max and Alice had to buy the beverages. They saw that cherry soda cost $2 for 16 single-serving boxes. Grapefruit juice cost $1.60 for 12. They decided to buy the grapefruit juice. Was this the best choice economically? Show in detail all the calculations and thought processes with which you arrived at your answer (see Figure 46).

\[
\text{I divided 16 by } \$2.00 = 8 \text{ by } 100 = 4 \text{ by } 50 = 2 \text{ by } 25 = 12.5
\]
\[
\text{And 12 by 160 = 6 by 80 = 3 by 40 = 1.5 by 20 = 75} \quad 13.3333 = 0.08
\]

*Figure 46. Gregory’s misunderstanding of division*

Gregory’s answer was incorrect due to his flawed procedural skill in developing a rate due to his misunderstanding in division that the divisor has to be smaller than the dividend. However, he appears to be attempting to create unit rates and compare the two.

He repeated this incorrect procedure of dividing the number of grapefruit juices by 160 (dollars converted to cents). He again attempted to simplify to create a rate.

This error in creating the unit rate was repeated in the post-test questionnaire so I discussed and attempted to clarify his misunderstanding in his interview. Gregory
explained his problems with division as starting in the 4th grade with the introduction to long division. His confusion with this variable continued to influence his performance processes throughout the study (Gregory, April 24, 2014).

Question 3. After the trip, Sima and Alex decided to see who could ride the fastest back home. Sima rode 5 km in 20 minutes. Alex rode 7 km in 25 minutes. Who rode the fastest? How do you know? (see Figure 47).

\[
20 \text{ divided by } 5 = 4 \quad 25 \text{ divided by } 7 = 3.57143 \quad \text{Alex was the fastest.}
\]

Figure 47. Gregory’s work on rates.

Gregory appears again to have the correct intuition of developing unit rates and comparison but his procedural skill is flawed. Again, he thinks that the divisor has to be smaller than the dividend.

**Ratio – from questionnaire on rate and proportion.** Question 1. In Mr. Erez’s class, 20 students come by bus and 15 arrive on their own. In what different ways could you compare the number of students that arrive by bus to the number that arrive on their own to school? Explain your reasoning in detail (see Figure 48).

\[
15 \text{ to } 20 \text{ is a ratio of } \frac{3}{4} \text{ therefore } \frac{1}{4} \text{ more come by bus.}
\]

Figure 48. Gregory’s use of intuition.

Gregory’s intuition that \(\frac{1}{4}\) more students come by bus was correct. However, the verbal content misled him and he created an incorrect ratio.
Question 4. Of the 400 students in the school, 240 arrive by school bus daily. Is the ratio between the numbers of pupils that arrive by bus to the number that arrive on their own in Ms. Shula’s class the same as that of the whole school? How do you know? (see Figure 49).

No the percentage of the whole school is 100% also 400 – 240 = 160
and 400 divided by 160 is .4

Figure 49. Gregory’s work on ratio.

Gregory’s answer is incorrect. He has misunderstood the verbal content to create a ratio that does not represent the number of students that arrive by bus to the number that arrive on their own.

Scale – from questionnaire on rate and proportion. Question 1. A customer asks Fran to enlarge a 3 inch by 2 inch photograph to 18 inches by 12 inches. Can this be done without cutting or distorting the picture? How do you know? (see Figure 50).

Yes because both equations are factors of 6.

Figure 50. Gregory’s use of intuition.

Gregory’s answer was correct based on his mathematical intuition and use of a multiplicative strategy.

Question 4. To help prevent damage to photos that will be displayed on the wall, Phil laminates the prints with a special sealant. If he needs 400 grams of sealant to laminate a 10 cm by 15 cm print, how much will he need for a 20 cm by 30 cm print? (see Figure 51).
Again, Gregory’s answer was correct based on his mathematical intuition and use of a multiplicative strategy.

**Fractions – from questionnaire on rate and proportion.** Question 1. What is the number that can replace the “?” in each of the following problems. Explain your reasoning and show your work for each. (see Figure 52).

- **a)** $\frac{5}{6} = ? / 18$  
  $\frac{5}{6} = \frac{10}{18}$  
  2 times each number

- **b)** $\frac{8}{5} = \frac{20}{?}$  
  $\frac{8}{5} = \frac{20}{17}$  
  12 is added to both sides.

- **c)** $\frac{2}{5} = 2.4 / ?$  
  $2.4$        
  .4 is added to both sides

2) Circle the smaller fraction in each pair. If they are equal, circle them both. Explain your reasoning and show your work for each.

- **a)** $\frac{5}{7} \ 8/10$  
  The larger the number the smaller the part.

- **b)** $\frac{3}{2} \ 18/12$  
  equal factors of six

- **c)** $\frac{5}{20} \ 7/25$

It appeared again from these exercises that Gregory might not have all the mathematical proficiency that would be required to solve the rate, ratio and scaling problems. His errors included some faulty procedural skill in determining the scale factor and use of an additive strategy.
Gregory’s mathematical thinking after watching the videos. Mathematics was one of Gregory’s least favorite and enjoyable subjects in school. He was not confident in his ability to understand the topic of ratio and proportion because he did not have adequate mathematical or theoretical knowledge. The topic of ratio and proportions seemed very complicated but relevant.

However, he believed that students need a wide range of mathematical knowledge beyond what is learned in school. Ratio and proportion are important topics for a student’s mathematical development. Further, he understood that he needed to study ratio and proportion and felt that it should be included in each mathematics course. Overall, the topic did not seem easy to him and many students had difficulty with it.

Rate – from questionnaire on rate and proportion. For Questions 1 and 3 see Figure 53. On both questions, Gregory’s intuition that he should be creating unit rates and comparing them was correct. His flawed procedural skill and misunderstanding of division continued to influence his performance.

1) Max and Alice had to buy the beverages. They saw that cherry soda cost $2 for 14 single-serving boxes. Grapefruit juice cost $1.60 for 12. They decided to buy the grapefruit juice. Was this the best choice economically? Show in detail all the calculations and thought processes with which you arrived at your answer.

\[
\frac{200}{14} = 14.2857, \quad \frac{160}{12} = 13.3333
\]

9/5

3) After the trip, Sima and Alex decided to see who could ride the fastest back home. Sima rode 5 km in 25 minutes. Alex rode 7 km in 20 minutes. Who rode the fastest? How do you know? \( \frac{20}{7} = 2.85714 \) and \( \frac{25}{5} = 5 \)

Figure 53. Gregory’s continued misunderstanding of division.
**Ratio – from questionnaire on rate and proportion.** See Figure 54 for Gregory's answers to questions 1 and 4. (see Figure 54).

1. In Mr. Erez's class, 25 students come by bus and 10 arrive on their own. In what different ways could you compare the number of students that arrive by bus to the number that arrive on their own to school? Explain your reasoning in detail.

   3 out of 5 come by bus

4. Of the 400 students in the school, 260 arrive by school bus daily. Is the ratio between the numbers of pupils that arrive by bus to the number that arrive on their own in Ms. Shula’s class the same as that of the whole school? How do you know? 3:2

**Figure 54.** Gregory’s challenges with verbal content and division.

On both questions, Gregory understood that ratios needed to be created. However, he was confused by the verbal content as well as the necessary procedural skills.

**Scale – from questionnaire on rate and proportion.** See Figure 55 for Gregory's answer to question 1.

1. A customer asks Fran to enlarge a 3 inch by 2 inch photograph to 15 inches by 10 inches. Can this be done without cutting or distorting the picture? How do you know? Yes, because it can be scaled up by a factor of 5.

**Figure 55.** Gregory’s use of a scale factor strategy.

Gregory used a scale factor strategy and answered the question correctly (after some thought that this question was different than the pre-test).

On the pre-test, Gregory did not attempt question #3 (see Figure 56).

In his interview Gregory confirmed that he used a ruler and measured to answer this question (Gregory, April 24, 2014).
If the photo included the entire tree, how tall would the image of the tree be? How the real-life tree in feet? Show your work.

\[
\begin{align*}
&4 \times 1.5 = 6 \\
&6 \times 2 = 12
\end{align*}
\]

Figure 56. Gregory’s work on scale.

**Fractions – from questionnaire on rate and proportion.**  See Figure 57 for Gregory's work with fractions.

Gregory demonstrated use of a scale factor strategy on question #1 and some understanding of fraction comparison in #2. His flawed procedural skill in division continued. He still considers more units mean smaller parts.

PROBLEMS WITH FRACTIONS

1) What is the number that can replace the “?” in each of the following problems. Explain your reasoning and show your work for each.

a) \(\frac{2}{3} = ?/18\)
   \[= \frac{12}{18} \]
   \[= \frac{6}{3} = 18\]

b) \(8/5 = 20/?\)
   \[= \frac{8 \times 2.5}{5 \times 2.5} = \frac{20}{12.5}\]
   \[\checkmark\]

c) \(3/5 = 2.4/?\)
   \[= \frac{3 \times 8 = 24}{5 \times 8 = 40}\]

2) Circle the smaller fraction in each pair. If they are equal, circle them both. Explain your reasoning and show your work for each.

a) \(\frac{5}{7} > \frac{7}{10}\)

b) \(\frac{3}{2} < \frac{5}{10}\)

c) \(\frac{5}{25} = \frac{7}{30}\)

Figure 57. Gregory’s work with fractions.
The mathematics that Gregory saw in the *Math Snacks* videos. After Gregory’s interview and observation on April 24, 2014, I reflected that at times he was difficult to follow. It was apparent that he generally understood the material in the videos but lacked some prerequisite mathematical knowledge that was not covered in the *Math Snacks* videos or in *MyMathLab*. But he answered my questions and tried to solve the problems. He was open and not afraid to make a mistake. He contributed that he consulted many other sources, for instance *PurpleMath* and other textbooks. Throughout, the interview and observation, I needed to clarify some of his wording. It was a challenge for me to hold back and not do the problem for him as he struggled. Gregory was willing to clarify so that I understood what he was saying. He later reviewed and agreed with the transcripts that I provided for him.

The following were the mathematics that Gregory saw in each of the *Math Snacks* videos:

*Ratey the Math Cat.* Gregory understood and used ratios and proportions to represent quantitative relationships. He built new mathematical knowledge through problem solving in mathematics. He understood the importance of rates and unit rates. He saw proportions as multiplicative situations, patterns and was able to translating unit rates to a table and a graph.

*Bad Date.* Gregory understood and used ratios to describe quantitative relationships. He had difficulty solving problems due to his flawed procedures in division and lack of comprehension of verbal content.
**Scale Ella.** Gregory developed methods for solving problems involving proportions, such as scaling and finding equivalent ratios. He understood that there was a number (the scale factor) that creates the relationship between two items that were being compared to one another. He understood that multiplication and division were inverse operations. (See Table 4).

**Gregory’s perceptions of the Math Snacks videos.** Gregory wrote in his class project, “Proportional reasoning is the hallmark in basic core competency of mathematical concepts. Often it is taken in the fold with a general curriculum and not fully expanded as a basic construct in problem solving…overall all the videos were effective (in my opinion). This is mainly because the animated format was educational and informative.” (Gregory, April 19, 2014) He proceeded to relate his personal goal of prison reform for non-violent inmates to his study of proportional reasoning:

…Proportional Reasoning has a practical [sic] as well. I can use the concepts to form charts, graphs, historical information and future projections to a subject such as Prison Reform for Nonviolent Inmates. This informative [sic] can be used to support my thesis that Prison Reform is not a choice but rather a directive going forward in America… Sending a person to prison in the atmosphere is like sending a person to college but the difference is that in the prisons negative and destructive behavior is the curriculum…Studying Ratios and Proportional Reasoning will be of vital help in helping me to develop concise data in charts and graphs in order to layout background information in order to secure the
Table 4. Summary of Gregory’s Mathematical Thinking

<table>
<thead>
<tr>
<th></th>
<th>Pre-Test Strategy/ Correctness</th>
<th>Post-Test Strategy/Correctness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate</td>
<td>In the first question, Gregory’s answer was incorrect due to his flawed procedural skill in developing a rate. This was due to his misunderstanding in division that the divisor has to be smaller than the dividend. However, he appears to be attempting to create unit rates and compare the two. In the second question, Gregory again appeared to have the correct intuition of developing unit rates and comparison but his procedural skill was flawed. Again, he thinks that the divisor has to be smaller than the dividend.</td>
<td>On both questions, Gregory’s intuition that he should be creating unit rates and comparing them was correct. His flawed procedural skill and misunderstanding of division continued to influence his performance.</td>
</tr>
<tr>
<td>Ratio</td>
<td>In the first question, Gregory’s intuition that ¼ more students come by bus was correct. However, the verbal content misled him and he created an incorrect ratio. In the second question, Gregory’s answer is incorrect. He has misunderstood the verbal content to create a ratio that does not represent the number of students that arrive by bus to the number that arrive on their own (part to part ratio).</td>
<td>On both questions, Gregory understood that ratios needed to be created. However, he was confused by the verbal content as well as the necessary procedural skills.</td>
</tr>
<tr>
<td>Scale</td>
<td>In the first question, Gregory’s answer was correct based on his mathematical intuition and use of a multiplicative strategy. He did not attempt the second question.</td>
<td>Gregory used a scale factor strategy and answered the question correctly. In the second question, Gregory confirmed that he used a ruler and measured to answer this question.</td>
</tr>
<tr>
<td>Fractions</td>
<td>It appeared from these exercises that Gregory might not have all the mathematical proficiency that would be required to solve the rate, ratio and scaling problems. His errors included some faulty procedural skill in determining the scale factor and use of an additive strategy.</td>
<td>Gregory demonstrated use of a scale factor strategy on question #1 and some understanding of fraction comparison in #2. His flawed procedural skill in division continued. He still considers more units mean smaller parts.</td>
</tr>
</tbody>
</table>
funding necessary to take on such an enormous task. Proportional Reasoning also involves finding solutions. Understanding Ratio, Rate, and Inverse Variation will help me to make the case that this endeavor is well conceived and worthy of massive funding. I will also be able to further my appeal for funding by presenting inferences and projections on the negative alternatives of procrastination or avoidance.

… Many prisoners are very skilled and produce gigantic profits for many but nearly [sic] for themselves. Using Proportional Reasoning to state the case for funding will allow forward thinking investors a chance that could make them lots of money while satisfying their social responsibilities. Understanding ratios is paramount to structure futures. The recidivism rate is called that for a reason. Scale becomes important when re-education initiatives begin. What size building will we need? How much oversight is needed per person? These are pertinent questions that can be furthered to solution with the use of Inverse Variation.

… many in the prison population lack the foundation of understanding in the concept of proportional reasoning within themselves. This perpetuates the continuity of a burgeoning population without the creative force of fundamental mathematical development. We are talking about “rate of return, profit over time, expressed as a proportion of the original investment” Does this sound familiar? (Gregory, April 19, 2014).
In his interview, Gregory added that in addition to ingraining the concepts, all the videos showed him how to “get there”. He appreciated that they showed him different solution methods (Gregory, April 24, 2014).

Gregory then provided specific characteristics of the individual *Math Snacks* videos that aided his understanding and computations of the components of proportional reasoning. In his class project, Gregory made this observation about the concept of rate in *Ratey the Math Cat*, “This concept is paramount is dealing with issues of equity because it breaks down and details or pinpoints the degree to which things that are interrelated differ. The work [sic] ‘Per’ can describe many formulas like per minute, per pound, per lawn, per person, per commercial or per gallon of milk. Exponential [sic] concepts are essential to understanding and documenting numerical data. *Ratey* makes us realize that these concepts beginning with the word per are calculations. It appears that these concepts were easily digested due to the animation format. This made the concept of learning fun rather than a chore” (Gregory, April 19, 2014). In his interview, he added that he did not need to think of *Ratey* in detail. I asked him to explain this. Gregory responded that, “Even when it came to days and meals eaten, I had to realize that what was being asked was rate. I just wasn’t used to calculating things like that. In the store, I see servings all the time, so I can relate easier with products of foodstuffs. Or I can get 4 washes per machine load. I do better with things if it is something I know instead of something abstract or I don’t use. It took me a while to get there. I am not used to calculating things like that that I don’t relate to.” (Gregory, April 24, 2014)
Gregory indicated the utility of the following tables and graphs on the Learner’s Guide from *Ratey* (see Figure 58).

![Table and Graph](image)

*Figure 58. Gregory’s use of tables and graphs on rate.*
We continued Gregory’s interview with a discussion of the *Bad Date* video.

Gregory described the impact of the characteristics of the video to his understanding of ratio:

> In BD, for instance, the format of the words that he said and the ratio to what she said. And beyond math if you monopolize a conversation, you can’t learn what the other person is saying. Mathematically speaking, it made me realize that the direct relationship between and beyond numbers is obvious. For instance 45/10 is a ratio and it can be calculated as well and many times. It is the ratio of one to the other. It gives a better understanding of whole and the ratio of part to whole” (Gregory, April 24, 2014).

Then, Gregory described how *Bad Date* had helped him compute ratios; “I got a better grip on the abstract and word problems. I don’t get what they are getting at or what things represent in abstract word problems. You don’t understand math unless you understand the relationships, interrelationships or differences. How numbers are associated. I got a better and broader understanding” (Gregory, April 24, 2014).

In his class project, Gregory had indicated the importance of the representations of comparison of numbers to both his understanding and computations of ratio:

> This point was driven home by the first conversation between the male and female in which the male spoke 175 words to the females 25. Then these numbers were divided and shown to be reflective of a 7:1 ratio. The video then showed the female speaking 36 words to his 6 which was a 6:1 ratio. Then there was a conversation that took place with 57 words being spoken by both parties
and it was pointed out that this is a 1:1 ratio. Aside from these ratios being words in a conversation it demonstrated how a disproportioned conversation always one of the persons frustrated. So, the moral would be that unequal interactions leaves the lesser frustrated and these formulas can help us to see inequities that can be addressed. So the numbers themselves imply or suggest their own solutions.

(Gregory, April 21, 2014)

He also felt that the Learner’s Guides for Bad Date were helpful in providing him an opportunity to practice. However, he did not draw on the Learner’s Guide and he further declined my offer to do so in his interview (see Figure 59).

Figure 59. Gregory’s work on ratio.

(Math Snacks Learners Guide, 2013)
I pointed out to Gregory that I had noticed that he wrote 6 divided by 12 = 2. He had solved the problem correctly but the divisor and dividend had been reversed. He answered, “I have the same issue on the computer when I punch it in. I did this because I am not looking at the concept of parts of whole. The greater number has to be the whole. But if you think of 1 pie for 4 people then you can do it the other way.” He added that there was a relationship between multiplication and division.

Gregory used a strategy of missing value to solve problems on this Learner’s Guide (see Figure 60).

![Image of a math problem](image)

*(Math Snacks Learners Guide, 2013)*

**Figure 60.** Gregory’s work on missing value problems.

In Gregory’s interview, he also indicated the visual characteristics of animation and humor in the *Scale Ella* video that were helpful:

It was kind of funny that the thumbs got larger when it is common sense to make the thumbs smaller. So you can see that you can scale up or down. You want a
ratio of one to the other not the same size. Normally you think size not up and
down...you need to use the right math to bring the thumbs back to proper size.
(Gregory, April 21, 2014)

In his class project, Gregory also mentioned that the animated format in *Scale Ella* was educational and informative. (Gregory, April 21, 2014) He provided in his interview, examples from his Learner’s Guide where *Scale Ella* demonstrated, in a humorous way, scale (the size) working in concert with other variables to directly affect the functional or dysfunctional operation of routine tasks (see Figure 61).
Figure 6.1. Gregory’s work on scale.
Although his flawed procedural skill continued, Gregory demonstrated some productive mathematical reasoning on the Learner’s Guide. (Gregory, April 21, 2014).

Further Gregory made this informative post in the Moodle forum:

I think that the use of the worksheets were extremely effective in re-enforcing the concepts of Proportional Reasoning. The animated format was nonthreatening and really fun. This method took the complication away by:

1). Not bombarding us with tons of work but rather asking pertinent questions that when answered gave us a key on how to address similar or closely related inquiries.

2). Giving graphic illustrations that helped to conceptualize what was being asked for example the bed and the dimensions.

3). By asking questions in a clear and concise way that even a elementary student could understand.

4). By relating the questions on the worksheet directly to the subject matter of the videos there was no room for misunderstanding. (Gregory, April 3, 2014)

In summary, Gregory’s difficulty with the prerequisite knowledge involved with division limited his ability to develop more productive mathematical reasoning. His flawed procedural skill influenced his performance process and understanding of ratio and proportion.

The cases of the four students who participated in the study have been presented. Information was included on their individual mathematical backgrounds, mathematical thinking and strategies prior to watching the *Math Snacks* videos, mathematics including
the characteristics and problem solving strategies they believed to impact their understanding and computations of ratio and proportion in the videos, and their mathematical thinking and strategies after watching the *Math Snacks* videos.

**Findings**

Next, I probed the collected data to understand adult learning of ratio and proportion. I used the information from all sources to answer the research questions of this study; therefore the remainder of this chapter has been arranged by research questions along with the themes that emerged from the data. Themes began to develop as soon as I read through the interview transcripts, documents and forum posts of the students. The individual students had approved the transcriptions of their interviews and observation before I began to analyze. As well, the findings continued to be monitored by my faculty colleague to avoid any possible bias on my part. As I worked through the analytic process I looked for disconfirming evidence, made numerous analytic notes and took time to be reflective of my own ethics (Saldana, 2008).

**Research Question One**

How does the use of authentic investigative activities, aligned with Lamon’s and Ben-Chaim’s content theories, impact adult learning of ratio and proportion? Two themes emerged from the data to answer the first research question: Improved Attitude and Deepened Understanding. Each theme is presented with supporting documentation from the data.

**Improved attitude.** The four students’ initial beliefs and challenges towards mathematics in general were initially communicated in their Introductory Posts in
Moodle and again in interviews. Further information was gleamed from their answers to the *Pre-Questionnaire: Summary of Attitudes on Ratio and Proportion*, which divided the statements into three categories: attitude towards math in general, confidence in ability to deal with ratio and proportion, and attitude towards the importance of ratio and proportion. Summaries of the information provided by the students follow:

Sarah introduced herself to the class in Moodle with the statement that, “math is probably my weakest area” (Sarah, January, 2014). She continued with an explanation that she had limited mathematics in her nursing classes at a community college in the late 90’s. Furthermore, she was only taking this course, as it was a requirement for her degree. However, and hopefully, it would be a refresher for her and would help her with her children’s mathematic challenges. She concluded the Moodle post with, “I hope that during this course I can remain calm and not get so frustrated with the math that I get overwhelmed” (Sarah, January 26, 2014).

Sarah asserted these feelings again in her interview as she stated, “I don’t do well enough in math; it is my toughest subject. I try to rationalize (understand) it and relate it to real life. I have kids and they see me doing math and I don’t want them to see me overwhelmed” (Sarah, February 23, 2014).

Furthermore, in her questionnaire on attitude, Sarah’s ranking of the statements confirmed these expressions of her initial beliefs and challenges in mathematics. However, she could provide examples of ratio and proportion and concepts, words, and subjects related to both.
In Jessica’s Introductory Post in Moodle, she provided, “I do enjoy working with some math problems but I must admit that math has never been a subject that I excelled in” (Jessica, January 26, 2014). She explained that she felt very anxious when dealing with math-related problems and would rank her comfort level as medium. She expressed hope that after completing this course, she would feel more confident working with mathematics (Jessica, January 26, 2014).

In her interview, Jessica confirmed that she had three years of math in high school over 20 years ago. Further, those were the last mathematics course that she had taken until her current enrollment in this finite math course. She knew that she had to pass this course and “Ace” it to maintain her 4.0 GPA and go on to be successful in her last mathematics course, statistics (Jessica, February 23, 2014).

Further, in her questionnaire on attitude, Jessica’s ranking of the statements confirmed these expressions of her initial beliefs and challenges in mathematics. She could provide examples of ratio and proportion and concepts, words, and subjects related to both.

Tamara wrote in her Introductory Post, “I wish I could learn to like math, however, I have not done well in math” (Tamara, March 13, 2014). In her interview, Tamara described herself as very anxious about mathematics in general. While she understood the importance of mathematics in general and ratio and proportion specifically, she felt overwhelmed when faced with any mathematics in a classroom. Tamara described her challenge with mathematics by admitting, “I have tried to pass this course a number of times” (Tamara, April 22, 2014).
In her questionnaire on attitude, Tamara’s ranking of the statements confirmed these expressions of her initial beliefs and challenges in mathematics. Further, she could provide examples of ratio and proportion and concepts, words, and subjects related to both.

Gregory introduced himself in Moodle by posting, “Suffice it to say, that I have always found Math to be very challenging and have not ever done well in Math or Algebra” (Gregory, March 13, 2014). He stated that he intended to turn this all around. He expressed his desire to be successful in the course by working extremely hard. He asserted that he had studied countless hours in preparation for this course.

He continued to explain in his interview that, “This is the only course I need for graduation” (Gregory, April 24, 2014). Gregory expanded on his challenges with mathematics by explaining that he lost interest in the 4th grade with long division. We had discussed this previously in phone conversations and emails as I attempted to find him either IT resources or tutoring to fill in the “gaps” in his prerequisite mathematical knowledge.

Gregory’s ranking of the statements on the questionnaire on attitude, confirmed these expressions of his initial beliefs and challenges in mathematics. He could provide examples of ratio and proportion and concepts, words, and subjects related to both.

Thus, the initial beliefs and challenges of the students indicated a negative attitude towards mathematics in general and a lack of confidence in dealing with ratio and proportion. Each student exhibited some understanding of the importance of the topics by providing examples of ratio and proportion. Additionally, they all mentioned how long it
had been since they had taken any mathematics course and their current lack of proficiency in the subject. In two cases, this course was the last course or prerequisite course to be taken.

After watching the Math Snacks videos and completing the Learner’s Guides, the students provided the following perceptions in their class projects, Moodle discussion posts and interviews. Further information was gleamed from their answers to the Post Questionnaire: Summary of Attitudes on Ratio and Proportion. Summaries of that information follow:

Sarah asserted in both her class project and interview that watching the Math Snacks videos positively impacted her confidence in math, ability to deal with ratio and proportion and the importance of ratio and proportion. In her class project, Sarah communicated her feeling that students benefit from adding videos to what teachers go over in classrooms, whether the classes were seated or IT and regardless of the student’s age, grade or achievement level in math (Sarah, February 3, 2014). Sarah confirmed this in her interview and further asserted that, “My own children have benefited from Internet sites to increase their understanding in their class work. My children and I have a hard time with math and so these videos were especially helpful to us” (Sarah, February 23, 2014).

In the Moodle discussion forum, Sarah had posted, “My daughter and I were making treats for her Valentine's Party at school and we needed a 2:1 ratio per student. My daughter is the one who brought up the Bad Date video because she remembered the talk bubbles showing the ratio clearly as well as the meters on the sides”
(Sarah, February 15, 2014). Again, on February 19, 2014, Sarah posted that she had been turning dinners into ratio learning lessons with recipe conversion. Both of these posts were indicative of Sarah’s attitude toward the importance of ratio and proportion.

The information on Sarah’s attitude towards math in general and her attitude towards the importance of ratio and proportion were confirmed on her Post Questionnaire Summary on Attitude. Interestingly, while Sarah demonstrated an improvement in her attitude toward math in general and attitude towards the importance of ratio and proportion, she did not indicate any greater confidence in her ability to deal with ratio and proportion. In her interview, she offered that the videos had increased her confidence because they were a good refresher on mathematics in general and ratio and proportion specifically. She felt this was very helpful to an adult returning to college after a number of years. Also, as a mother of school age children, she appreciated their enjoyment and increased understanding with the videos (Sarah, February 23, 2014).

Jessica had indicated to me in an email (February 3, 2014), at the beginning of the course, that she thought the teaching style in MyMathLab was more helpful to her understanding and computations than the Math Snacks videos. However, Jessica posted in a Moodle discussion at the end of the course that the videos and Learner’s Guides were a great way for her to learn the concepts and become more confident in ratio and proportion (Jessica, February 19, 2014).

Jessica confirmed this change in attitude in her interview. She also confirmed this in her Post Questionnaire on Attitude by indicating improvement in her attitude towards math in general, confidence in her ability to deal with ratio and proportion and attitude
towards the importance of ratio and performance. She also offered in her interview, that overall she felt more confident and ready to “take on” statistics the following term (Jessica, February 23, 2014).

My impression of Tamara as a mathematics student was that her anxiety with mathematics was strong and pervasive. This attitude continued throughout the study. She posted the following in the Moodle discussion forum, “The fact that animation was used in the class project made the problems less stressful. I was able to learn something about math without the anxiety that I usually have when presented with problems from books” (Tamara, April 9, 2014). She reiterated that she had learned something from the videos without her usual high level of anxiety again in her class project (Tamara, May 2, 2014).

Further, Tamara confirmed this information in her interview with every problem that we discussed. It was evident that she thought that ratio and proportion were important topics but she continued to be very concerned that her understanding and calculations were correct. She needed my constant reassurance. However, when we concluded the interview, Tamara indicated that she could understand and felt confident in her ability with mathematics (Tamara, April 22, 2014).

Tamara had improved her attitude towards math in general. However, on her Post Questionnaire on Attitude, Tamara indicated positive improvement in all but her confidence in her ability to deal with ratio and proportion. Further, in her interview, Tamara indicated that her confidence in dealing with ratio and proportion was lower after watching the videos. She explained that she previously had learned other methods for solving rate, ratio and proportion problems and now she realized how much (conceptual)
knowledge she did not have. She had also needed to use Kahn Academy for more clarifying information on the topic (Tamara, April 22, 2014).

In his interview, Gregory’s difficulty with prerequisite knowledge involving division and other possible topics in mathematics limited his ability to solve problem. As he attempted to show me how to work a problem, his flawed procedural skills influenced his performance process. He appeared to be able to visualize where he should be going in the problem solving process but was unable to perform the computations. Further, he explained that the videos did not address his “gaps” in mathematics knowledge. He did not feel differently about the importance of ratio and proportion because he already knew of their importance. He stressed the significance of mathematics to him personally and professionally and provided examples (Gregory, April 24, 2014). Additionally, in both his class project and Moodle discussions, Gregory’s attitude was positive as he indicated an understanding of the issues involved in ratio and proportion but avoided any computations (Gregory, April 9, 2014).

Gregory’s positivity was confirmed on his Post Questionnaire on Attitude by an improvement in his attitude towards math in general. However, he could not indicate an improvement in his confidence in dealing with ratio and proportion due to his continued inability to perform computations. His attitude towards the importance of ratio and proportion was also unchanged from the Pre Questionnaire on Attitude.

The Questionnaire: Attitude on Ratio and Proportion, Appendix G, used the statements of attitude and measured them along a Likert scale of 1-5 (1 indicating total disagreement and 5 indicating total agreement). The statements were broken down into
three categories as follows: 1) attitude toward mathematics in general; 2) confidence in ability to deal with ratio and proportion; and 3) attitude toward the importance of ratio and proportion).

Table 5 summarizes the Pre and Post Questionnaires. The mean data is compared for each student on the three categories of attitude and then for all the students as a group.

Table 5. **Summary of Attitudes on Ratio and Proportion**

<table>
<thead>
<tr>
<th>Student</th>
<th>Attitude toward math in general</th>
<th>Mean Before Math Snacks Video</th>
<th>Mean After Math Snacks Video</th>
<th>Confidence in Ability to deal with Ratio and Proportion</th>
<th>Mean Before Math Snacks Video</th>
<th>Mean After Math Snacks Video</th>
<th>Attitude Towards the importance of Ratio and Proportion</th>
<th>Mean Before Math Snacks Video</th>
<th>Mean After Math Snacks Video</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sarah</td>
<td></td>
<td>2</td>
<td>3.25</td>
<td>2.17</td>
<td>2.17</td>
<td>2</td>
<td>2.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jessica</td>
<td></td>
<td>2</td>
<td>2.5</td>
<td>1.7</td>
<td>2.57</td>
<td>1.8</td>
<td>2.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tamara</td>
<td>2.75</td>
<td>3.25</td>
<td>2.33</td>
<td>2</td>
<td>1.8</td>
<td>2</td>
<td>2.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gregory</td>
<td>2.25</td>
<td>2.33</td>
<td>2.33</td>
<td>2</td>
<td>2.4</td>
<td>2.4</td>
<td>2.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grand Mean</td>
<td>2.79</td>
<td>2.81</td>
<td>2.12</td>
<td>2.29</td>
<td>2</td>
<td>2.45</td>
<td>2.45</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Summary.** While the individual student’s data varied, the data indicated that as a group there was improved attitude in math in general, improved confidence in the students’ ability to deal with ratio and proportion and improved attitude towards the importance of ratio and proportion.

**Deepened understandings.** The students saw how the Math Snacks videos and Learner’s Guides impacted their learning of ratio and proportion in a variety of ways. The individual student’s responses to select questions on rate, ratio, scale and fractions from the Pre and Post Diagnostic Questionnaires on Ratio and Proportion, Appendix F were presented in each student’s case. The tables indicated both the strategy and correctness of the student’s responses. I choose these questions as representational of individual
students mathematical thinking before and then after watching the *Math Snacks* videos and completing the Learner’s Guides. The impact of the use of the *Math Snacks* videos by student was varied for many possible reasons.

Summary table 6 based on the scores of all the students as a group is based on all 15 questions on rate, ratio and scale included on the *Pre* and *Post Questionnaires*.

Table 6. *Summary of Students’ Pre and Post Questionnaire Scores on Ratio and Proportion by Percentages*

<table>
<thead>
<tr>
<th></th>
<th>Correct answer</th>
<th>Incorrect answer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Correct only</td>
<td>Correct support</td>
</tr>
<tr>
<td><strong>Rate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Pre</em></td>
<td>10</td>
<td>45</td>
</tr>
<tr>
<td><em>Post</em></td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td><strong>Ratio</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Pre</em></td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td><em>Post</em></td>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td><strong>Scale</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Pre</em></td>
<td>35</td>
<td>15</td>
</tr>
<tr>
<td><em>Post</em></td>
<td>25</td>
<td>50</td>
</tr>
</tbody>
</table>

It can be seen that before the *Math Snacks* videos and Learners Guides, 45%, 50% and 15% (Rate, Ratio and Scaling, respectively) of the students responded correctly with correct support work. After the course, they responded correctly with correct support work to 50%, 50% and 50% of the problems. Only their performance on the ratio problems remained unchanged; with an improvement in rate and a significant improvement in scale. Furthermore, before experience with the proportional reasoning
activities, 50% of the students could not even attempt the scale problems or used incorrect thinking, whereas, after the videos 25% could not solve the problems.

While the data collected from the Pre and Post Diagnostic Questionnaires on Ratio and Proportion was a primary data source confirming this theme, the student’s individual worksheets and discussions in our interviews and observations as well Moodle forum provided multidimensionality and alternative sources of insight into their deeper understanding. It is important to note that in some cases, they were actually a more significant representation of the student’s understanding of the topic. The misunderstanding of the verbal content of the questions on the Questionnaire as well as lack of familiarity with the metric system were cited as reasons for incorrect answers. For instance, Jessica demonstrated on the Bad Date Learner’s Guide that ratios could be found in everyday situations while she was challenged with ratio on the questionnaire (Jessica, February 16, 2014). Tamara demonstrated a stronger ability to compute rate and ratio on the Ratey and Bad Date Learner’s Guides as they lead her through the process step by step rather than on the Questionnaire where she had to determine the correct problem solving process (Tamara, April 16, 2014). Gregory also demonstrated computational skill on all the Learner’s Guides than he could on the Questionnaires (Gregory, Learners Guides, April 16, 2014). Both Tamara and Gregory applied productive mathematical reasoning on the Learner’s Guides. Further, the step-by-step process mostly eliminated Gregory’s computational issues and allowed him to focus on the concept at hand.
Thus, the following information represented the essential elements and confirmation of how the use of the videos deepened the understanding and impacted the students’ mathematical thinking.

**Math Snacks’ alignment with the mathematics background of adult learners.** The *Math Snacks* videos were not intended to look like traditional (rote and algorithmic) mathematics as they provided students, who don’t particularly like math, another way to look at mathematical concepts. The mathematics backgrounds of the four students (adult learners) aligned with the principles of *Math Snacks*. It was presented previously, that each student indicated they had not done well or enjoyed previous mathematics courses.

**Math Snacks filled in gaps in understanding.** Further, the mathematics that each adult learner saw in the *Math Snacks* videos varied so they were able to reflect on their individual gaps in mathematical understanding and patterns of mistakes. This type of individual intervention is challenging to any classroom instructor as well as any IT curriculum, such as *MyMathLab*. Sarah just needed a refresher on the concepts and computations of ratio and proportion and she achieved this with the *Math Snacks* videos on rate, ratio and scale. She felt that the components complemented each other and were all equally important to her understanding of proportional reasoning (Sarah, February 23, 2014).

Jessica also needed a refresher as she admitted that she was definitely out of practice in mathematics (Jessica, February 16, 2014). She also felt that the Math Snacks videos were a “great way” to reinforce what was taught in *MyMathLab* (Jessica, February 9, 2014). Tamara indicated that *Math Snacks*, along with Kahn Academy, impacted her
understanding and computations of ratio and proportion (Tamara, April 16, 2014). Gregory’s needs were in prerequisite knowledge in division, and the videos did not address this. However, on the Learner’s Guides he used the step-by-step process to reinforce his understanding of ratio and proportions (Gregory, April 24, 2014).

\textit{Math Snacks managed learning time by providing snacks not meals.} Management of time by teaching the concepts as “snacks not meals” further impacted and deepened understanding. Sarah commented that all three videos helped with her time management in the seven-week course because they made the concepts easier to understand. “It didn’t take 2-3 days to learn one section of a textbook. The videos were easier to understand, so I got it the first or second time. It was easier to comprehend” (Sarah, Interview, February 23, 2014). Jessica appreciated that the videos connected ratio and proportion with the topics in MyMathLab. “We cover so much in such a short amount of time. She said that the Math Snacks videos were a nice break from MyMathLab and a good transition and learning tool. The two mediums worked together” (Jessica, February 23, 2014). Tamara indicated that including the Math Snacks videos were a benefit because she learned ratio and proportion so “she got something out of it”. Additionally, she did not feel that the time she spent on watching the videos and writing the class project took away from her studying the course material (Tamara, April 22, 2014). Gregory felt the Math Snacks videos removed complications in learning by not “bombarding” him with a great deal of work, but by asking pertinent questions and so that he could relate it to similar situations (Gregory, April 3, 2014).
Summary. The theme of deepened understanding was supported by information in the essential elements of student mathematical background, gaps of understanding, and snacks not meals. On the Pre and Post Questionnaire on Ratio and Proportion only the students’ performance on the ratio problems remained unchanged; with an improvement in rate and a significant increase in scale.

Research Question Two

What characteristics of the investigations are most helpful for adult learners to grasp the variety of dimensions of procedural thinking and conceptual understanding of rates, ratios, scale and proportional reasoning? Collectively, the Math Snacks videos’ non-traditional and varied instructional strategies provided students with additional ways to grasp the variety of dimensions of the topics. The students confirmed this with the following information:

Sarah asserted that, “My own children have benefited from Internet sites to increase their understanding in their class work. My children and I have a hard time with math and so these videos were especially helpful to us” (Sarah, February 23, 2014). Jessica described the video’s instructional strategies; “…it was like an instructor explaining the process after presenting the information. I have to have communication along with reading, so watching the animations, along with reading their words and it all came together. I need a person next to me to answer my questions. The videos were like having someone to ask questions” (Jessica, February 23, 2014). Tamara continued to be focused on her mathematics anxiety and how the videos made her laugh and relax so she could learn (Tamara, April 22, 2014). However, she also indicated in her class project,
that the videos did not have enough explanation for her to understand completely and when she went to Kahn Academy she had to watch the video more than three times to get any understanding (Tamara, April 16, 2014). In his interview, Gregory commented that in addition to ingraining the concepts, all the videos showed him how to “get there”. He appreciated that they showed him different solution methods as well as visual examples (Gregory, April 24, 2014). Further, all of the students indicated that they appreciated the availability of the Math Snacks videos on the Internet to repeatedly view the videos and complete the Learner’s Guides.

Due to the non-traditional and varied instructional strategies in the videos, many themes developed to answer the research question. However, three characteristics were the most cogent. Those themes along with the supporting information follow:

**Animations.** Discernably, the Math Snacks videos’ use of animation created a memorable impact on all of the adult learners. The animations were a different way to access and then focus solely on an important mathematical concept like rate, ratio and scale. Additionally, the animations were situated in contexts that were different from the ones the students have seen before in mathematics education.

The students provided the following general information on the use of animation. Sarah commented that the animations told her to focus on the topic in a humorous manner and held her interest better than a lecture (Sarah, February 3, 2014). Jessica provided that the animation was different and cute. She also commented that the audio and video got her attention. However, she said she was more technically oriented and just wanted the steps. But, she liked the animation because it made the Math Snacks videos
unique (Jessica, February 23, 2014). Tamara enjoyed the animations because they relieved her anxiety about mathematics and made her laugh (Tamara, April 22, 2014). Finally, Gregory felt the animations were educational and fun (Gregory, April 24, 2014).

Specifically, Sarah commented on the animation in the videos by making an analogy to a teacher in a lecture telling the class that a topic was important or bolding it in color. The eye catching animated instruction helped her to understand as well as compute rate and to begin to understand proportional reasoning. She compared the teaching method of animation, coupled with everyday relatable scenarios, such as Ratey explaining miles per hour, Bad Date’s comparison of words in a conversation and Scale Ella augmented thumbs, to the traditional teaching methods of MyMathLab. Sarah continued, “I watched MathLabs several times, but I didn’t get how to solve the problem until I watched Math Snacks” (Sarah, February 23, 2014).

She further pointed out in her class project and interview the humorous use of animation in superhero, Scale Ella, by saying, “If I plug that video into what I am learning and apply it to a story problem, it makes it more fun and then I want to do it. Math is not so difficult like the notion I had in my head. I thought that it was hard but now I plug information into different scenarios” (Sarah, February 23, 2014).

Jessica had originally told me that she preferred a traditional teaching method. However, her opinion changed. The animation had hooked Jessica’s interest and increased her comfort level with mathematics (Jessica, February 23, 2014). I asked Jessica to tell me more about how the animations helped her to grasp both procedural thinking and the conceptual understanding. She responded, “The animation was different.
It was cute. The audio and video got my attention. It had enough instructional steps. For instance, in *Scale Ella* there was more instruction, as well as more illustration of solutions. She said, “This helped me to see how to get to the end result. Anyone I shared them with, like my niece and nephew said the animations got their attention and they loved it. I am more technical. I just want the steps. But, I like them because they are unique” (Jessica, February 23, 2014). She later posted these same impressions of the videos in the Moodle Discussion Forum (Jessica, April 9, 2014).

Tamara explained that the characteristic of animation made her think of fun and, “even though the math is difficult, it is not as difficult as the written word. The animations make me think I might actually enjoy it”. Further, she explained that, “When I wasn’t anxious I could sit and watch and learn from it. When I go to class, I usually get anxious and block it out” (Tamara, April 22, 2014). She initially thought that more information was needed in order for her to get a complete understanding (Tamara, April 16, 2014). Consequently, she had gone to Kahn Academy but she then realized she could watch the *Math Snacks* videos and get the same step-by-step instruction (Tamara, April 22, 2014).

Tamara had written in her class project that her first impression of *Ratey* was that it was not helpful to her. However, She later commented, “I now see that *Ratey* was the easiest video to learn the math concept. *Ratey* was doing the math before my eyes; he gave the rate then gave an answer to the problem”. She also thought the animations in *Bad Date* and *Scale Ella*, along with instruction in Kahn Academy were helpful to her understanding and computations of ratio and proportions (Tamara, April 17, 2014).
Gregory commented on *Ratey* by explaining that the animations taught him that concepts beginning with the word “per” are calculations. He considered the topics of ratio and scale presented in *Bad Date* and *Scale Ella* to be “easily digested” due to the animation format. This made the concept of learning fun rather than a chore (Gregory, April 19, 2014).

**Realism and real world context.** The context of the videos’ scenarios was different than the ones where the students had previously seen failure. Thus, *Ratey the Cat* as a back seat driver requesting the driver to speed up their miles per hour; the dating and conversations in *Bad Date*; and the prison break in *Scale Ella* conspicuously drew attention to the procedures and concepts of the topics. The unique settings of each of the videos afforded a variety of instructional strategies so the students could focus and developed intellectually to learn the central mathematical concepts and computations.

Additionally, by situating the mathematics in a relatable context to adults, the students became engaged in their learning and could envision and transfer the use of ratio and proportion to their day-to-day lives. Sarah had mentioned that the videos had helped her with computations because she could relate it to scheduling of staff at her job (Sarah, February 3, 2014). With the steps broken down in the video, she could equate it (the problem) to something in life. She could understand and do computations by using her own experience rather than a generic word problem (Sarah, February 23, 2014).

Jessica expressed that she now understands ratio and proportion and how often she used ratio in her daily routine. In the Moodle forum, Jessica had posted, “I used proportional reasoning today…I was reducing a recipe that was for 18 people down to 6
people. I really think watching the videos this week made this process a lot easier!” Jessica continued, “To reduce the recipe I used proportional reasoning (Jessica, February 16, 2014). In her interview, Jessica commented on the bed size problem on the Scale Ella Learner’s Guide, “I like the comparisons to get the scale factor and the increase and then decrease in an actual examples; like the bed. It is something to relate to” (Jessica, February 23, 2014).

The videos had also engaged Tamara with their realism. She commented that she now thought of how she used ratio everyday (Tamara, April 16, 2014). In her interview, she also commented on how she related to dating and the conversations in Bad Date by saying, “I am older and I have dated” (Tamara, April 22, 2014). She also indicated that the relatable conversations in Bad Date had helped her with computations of ratio. “Bad Date sticks in my mind to help with computations … the conversation with the friend on the phone. She keeps saying it over and over repeating that every date is always the same. That is how we learn by seeing it over and over” (Tamara, April 22, 2014).

Gregory also related to Bad Date, “anyone can relate to a time when someone is dominating a conversation”. He continued by explaining that this engaged him and then kept him more interested than a lecture using traditional examples and strategies (Gregory, April 24, 2014). Gregory added that he did not need to think of Ratey in detail for the same reason. Referring to the relatable context, he said, “I do better with things if it is something I know instead of something abstract or I don’t use. It took me a while to get there. I am not used to calculating things that I don’t relate to” (Gregory, April 24, 2014).
**Representations.** The representations in the videos were necessary for student understanding of the mathematical concepts and relationships as well as their grasp of procedural thinking and computations. This was especially evident to the students in *Bad Date*. Sarah provided an example of the talk bubbles and related this to her visual learning style. Further, Sarah explained that as she could see and hear the two people talking, each word was represented in a bubble, the bar rose, fell and then the ratio was illustrated in numbers. She felt the representation, along with the animation and cuteness of this relatable situation, made the concept of ratio easier to understand than reading or a lecture (Sarah, February 23, 2014).

Jessica commented that in addition to the animations, the talk bubbles in *Bad Date* kept her attention and illustrated the concepts as she learned to perform the computations. Jessica, felt that by far, there was more authentic instruction by representations in *Scale Ella*. The scenarios of the evil Scaleo’s high jinx illustrated both the concept and computation of scale completely (Jessica, February 23, 2014).

Tamara also shared with me that she now understood ratio because of seeing the visual bars as she heard the number of words repeated in *Bad Date*. Additionally, Tamara explained that the tables and graphs in *Ratey* helped her to understand and compute unit rates. Tamara indicated that she could see proportion in *Scale Ella* by sight, “I liked where she scales up her thumbs and then the thumbs were bigger than the keys.” Tamara thinks that the thumb representations helped her to understand and compute scale. She also indicated that seeing the same theme of scale repeated in different scenarios was helpful to her understanding of ratio and proportion (Tamara, April 22, 2014).
In his Class Project, Gregory had indicated that the comparison of numbers and ratios with the talk bubbles, graphs and numbers in *Bad Date* impacted his understanding and computations (Gregory, April 19, 2014). He reiterated this in a post in the Moodle forum with a specific example from *Bad Date* (Gregory, April 21, 2014).

In Gregory’s interview, he also indicated the visual representation of scaling the thumbs up and down in *Scale Ella* was helpful by stating, “Normally you think size not up and down…you need to use the right math to bring the thumbs back to proper size” (Gregory, April 21, 2014).

Finally, Sarah provided another comparative example with the *MyMathLab* course. “I didn’t think that the statistics part (in *MyMathLab*) was difficult. But, the graphs would have been easier if *Scale Ella* taught them” (Sarah, February 23, 2014).

Each *Math Snacks* video had an accompanying Learner’s Guide intended to assist students in applying their conceptual understanding and procedural thinking to additional math problems and activities. Sarah said that the representations and realism in the Learner’s Guides had helped her practice what she has seen in the video (Sarah, February 23, 2014). She also indicated that the table representations in both the *Bad Date* and *Scale Ella* Learner’s Guide were helpful to her understanding and computations of scale. She did not have any interest in sketching in her interview and observation.

In her interview, Jessica explained that she watched all three videos repeatedly and then looked over the Learner’s Guides to see what she was asked to do. Then, she watched the videos again because now she knew what she was looking for. Jessica also liked to develop tables so she could see the patterns of scaling and proportions develop.
However, she was not interested in drawing any type of figure to help with her understanding when I offered her paper and pencil in the interview (Jessica February 23, 2014).

Tamara thought the Learner’s Guides with their tables and graphs helped to reinforce the concept and procedures in each video. She was the only student who drew on the Bad Date Learner’s Guide. Tamara referred to dating in the figure that she drew and commented that her “doodling” helped to relax her and to understand the concept (Tamara, April 22, 2014).

In his class project, Gregory described the Learner’s Guides as a method that took the complication of learning mathematics away by providing graphic illustrations that helped to conceptualize what was being asked; for example the bed and dimensions in Scale Ella. This related the questions on the Learner’s Guides directly to the subject matter of the videos so there was no room for misunderstanding. While he completed all the work, he did not draw on the Learner’s Guides (Gregory, April 3, 2014). Again, Gregory’s passion was inmate reform and he provided examples of how he could use tables and graphs in his research and presentations in his interview (Gregory, April 24, 2014).

Summary. The three themes of animation, realism and relatable contexts and representations in both the videos and Learner’s Guides were the most helpful characteristic for adults learning the procedural thinking and conceptual understanding of rate, ratio, scale and proportional reasoning. The supporting information confirmed these themes. Any evidence that appeared disconfirming was clarified in the student’s
interview or observation. While, there were several instances of student’s initial reluctance to the *Math Snacks* animation and teaching strategies, by the end of the study, all were engaged in the process of learning of rate, ratio, scale, and proportional reasoning.

**Research Question Three**

How do the videos help to develop a strategy of mathematical thinking and problem solving in adults understanding of ratio and proportion? While research has found that most students struggle with the properties of ratio and proportion, the adult learners in this study also had problems with reasoning and explaining their thinking about problems, understanding and creating patterns, and problem solving.

**Non-traditional teaching strategies.** The non-traditional instructional strategies in *Math Snacks*, initiated a contact with the students that implied it would be helpful or beneficial to them. The videos’ instructional strategies also appealed to the students’ individual learning styles with multiple audio, video and textual animations and representations in both the videos and Learner’s Guides.

The summary of each student’s mathematical thinking and strategies of problem solving along with the mathematics they saw in the videos provided essential elements for this theme. This information was presented earlier in each of the student’s cases in this chapter. Further, the students contributed their own perception of how the videos helped them to develop a strategy of mathematical thinking and problem solving of ratio and proportion.
Sarah had mentioned, in her class project, that the videos helped with computations because she could relate ratio to scheduling of staff at her job (Sarah, February 5, 2014). Consequently, in her interview, I asked her to explain. Sarah replied, that she always thought that basic math was all she needed. But seeing the steps broken down in the video, she could equate the problems to something in her every day life. I can understand and do the computations by using my own experience rather than a generic word problem.” In her observation, Sarah demonstrated the process. While, at times she appeared to be using an additive strategy she explained that this was necessary because of part and full time staff so it made sense to maintain the ratio by adding increments. Sarah had developed a workable strategy to accommodate this by performing the computations on the Bad Date Learner’s Guide (Sarah, February 23, 2014).

In her interview, Sarah also explained that in her role as a nurse she needed to understand the metric system and conversions. She had this ability now that she had developed an understanding of ratio and proportions from the videos. She indicated that she now understood ratio because it is like fractions - they both represent a relationship. She attributed this to the combination of audio, visual, and simplified textual characteristics of the Math Snacks. Sarah was now able to reason mathematically and use correct strategies and could verbally explain all of it (Sarah, February 23, 2014).

Jessica’s mathematical reasoning and strategies on ratios had improved on both the questionnaires and the Learner’s Guides after watching Bad Date. Jessica indicated that the bars illustrated along with the words spoken by each character helped her to understand and compute ratios. The combination of audio and visual kept her attention
by seeing the sides with the graphs and hearing what they were saying. She continued to explain that then seeing the ratios simplified in *Bad Date* combined the concept with the procedure, “I knew what it was supposed to look like. I knew what I was supposed to look for—comparisons” (Jessica, February 16, 2014).

In the Moodle forum on February 9, 2014, Jessica provided that she considered *Scale Ella* the most helpful video for her particular learning style. She explained in her class project that *Scale Ella* was helpful because, “…it creates a scenario, such as, Scaleo (the villain) who would increase or decrease a particular item and then Scale Ella would find the scale factor.” Additionally, she described the video’s teaching method as, “…an instructor explaining the process after presenting the information, either by video or written format, which really does provide me with the best way to learn each concept, allows me the opportunity to solve a problem and then confirm my answer or realize my answer is incorrect and try again” (Jessica, February 16, 2014).

Tamara mentioned several times that she did not feel that the videos had enough information for her so she had benefited from combining them with Kahn Academy. She had gone to Kahn Academy and gone over their series on ratios in order to get an understanding, then, she watched both the videos and Kahn Academy twice more. She provided this explanation of her mathematical reasoning and problem solving in her interview, “Rates and ratios are similar and I saw that in *Ratey* and *Bad Date*. You get them the same way” (Tamara, April 22, 2014).

In her interview, I commented that the *Scale Ella* Learner’s Guides appeared to challenge her. Tamara answered, “I might have studied scale before, but I felt like this
was the first time I had seen it. I can see the proportion but I don’t know how to get the answer.” However, later in the interview, she shared, “I am starting to do these in my head. It must be that crazy video Scale Ella. First I can’t do it and then I try. She also indicated that seeing the same theme of scale repeated in different scenarios was helpful to her understanding of ratio and proportion. Tamara could understand the topics and she added that the audio and visual components were helpful but not the textual aspects of either Math Snacks or Kahn Academy (Tamara, April 22, 2014).

Gregory’s focus was on his passion of criminal justice. Mathematics had been a longtime barrier to receiving his B.S. degree. His lack of prerequisite knowledge was likely more extensive than division and algebra. However, he was able to understand the mathematical concepts of ratio and proportion after watching the videos. As many adults, he had found his own methods for solving ratio and proportion problems and appreciated that the videos demonstrated alternative methods also. However, his methods, such as an additive strategy, did not lead to correct solutions.

Gregory understood, based on the audio and visual characteristics of the videos, how the study of ratio and proportions could help him to achieve his goals. Further, he understood that ratios and proportions could be used to graphically illustrate the problem and to find solutions in criminal justice. In his class project, he provided examples of ratio and scale with inmate recidivism and prison size (Gregory, April 19, 2014).

In his interview, Gregory described how Bad Date helped him compute ratios; “I got a better grip on the abstract and word problems. I don’t get what they are getting at or what things represent in abstract word problems. You don’t understand math unless you
understand the relationships, interrelationships or differences. How numbers are associated. I got a better and broader understanding” (Gregory, April 24, 2014).

Gregory was able to compute some problems but this was limited due to his lack of prerequisite skills. However, on the Scale Ella Learner’s Guide, he demonstrated the use of a missing value strategy. In his interview and observation, he could not explain correct mathematical reasoning or problem solving strategy. However, he indicated the audio and visual but not textual aspects of the videos were most helpful (Gregory, April 24, 2014).

**Summary.** The non-traditional instructional strategies in Math Snacks reached out to the students’ individual learning styles with multiple audio, video and textual animations and representations in both the videos and Learner’s Guides. Two of the students cited the audio, visual, and textual animations and representations as accommodating to their learning styles. The other two students only indicated the audio and visual animations and representations as helpful to their mathematical reasoning and problem solving.

**Summary**

There were two main sections in Chapter IV. The first section developed the cases of adult learners approaches to ratio and proportion. The cases included their mathematical background, mathematical thinking prior to the Math Snacks videos and mathematical thinking after the videos as well as artifacts from numerous sources.

The second section provided themes to answer the three research questions based on information from interviews, document analysis and discussion group. It was shown
that overall the students benefited from the videos with improved attitude and deepened understanding of ratio and proportion. The three most helpful characteristics of the videos were animation, realism and relatable context, and representation in both the videos and Learner’s Guides. Finally, the non-traditional teaching strategies, with the multiple audio, visual and textual animations and representations in Math Snacks were the prevailing theme to answer how the videos helped to develop a strategy of mathematical thinking and problem solving in adults understanding of ratio and proportion. The supporting evidence was varied based on the student’s individual level of mathematical thinking and the mathematics they were then able to see in the Math Snacks video.

In the next chapter, discussions and implications of the findings are related to the literature and recommendations for the future use of non-traditional teaching strategies for adult learners in mathematics. Additionally, recommendations for future research and practice are made.
CHAPTER V

DISCUSSION AND IMPLICATIONS

Introduction

Using a constructivist paradigm, my study examined what helps or hinders adult students to learn ratio and proportion when the topic is not the central focus of the mathematics course. More specifically, the research questions were:

- How did the use of authentic investigative activities, aligned with Lamon’s and Ben-Chaim’s content theories, impact adult learning of ratio and proportion?
- What characteristics of the investigations were most helpful for adult learners to grasp the variety of dimensions of procedural thinking and conceptual understanding of rates, ratios, scale and proportional reasoning?
- How did the videos help to develop a strategy of mathematical thinking and problem solving in adults understanding of ratio and proportion?

In this chapter, I will relate the major themes of Chapter IV to the literature that served as the study’s framework as well as discuss implications for research in adult numeracy. Three research theories structured this study: Lamon’s seminal theories on rational numbers and proportional reasoning; Ben-Chaim’s theory on the implementation of authentic investigative activities to adult learners; and Lesh’s model of representations and translations. Because this study focused on adult learners, the discussion section in Chapter V is organized to reflect the three major components that form and construct
numeracy: context, content, and cognition and affect. The second section embodies the implications of the findings.

**Discussion**

**Context**

Ben-Chaim et al. (2012) and Ginsburg, Manly, and Schmitt (2006) asserted that context in numeracy is the use and purpose for which an adult takes on a task with mathematical demands. Based on analysis of the data from interviews, document analysis, and a discussion group, my study showed that the adult learners concurred that the characteristics of realistic and relatable context, animation, and representations in both the videos and supporting materials were helpful to their understanding of ratio and proportion.

**Realistic and relatable context.** Ginsburg et al. (2006) specified that placing mathematical content and processes in scenarios that were realistic, relatable and different than the ones where adults have previously failed was helpful to their understanding of the mathematical content. The relatable, problem-solving scenarios in *Ratey the Cat, Bad Date*, and *Scale Ella* focused each of the adult learners’ attention on the central mathematical concepts and computations of rate, ratio and scale. Thus, as Ben-Chaim et al. (2004, 2012); Ginsburg et al. (2006); Knowles (1980); and Merriam et al. (2007) had stated, the adults in my study developed intellectually as they became engaged in their learning and could envision and relate the use of ratio and proportion to their day-to-day lives.
Further, studies conducted by Ball and Cohen (1999); Even and Ball (2009); Leinhardt, Young, and Merriman (1995) on the professional development of mathematics teachers examined the importance of theoretical and practical contexts to learning. The data in my study revealed that the authentic or realistic scenarios in *Math Snacks* related to the adult learner’s family and personal roles, so that theoretical and practical knowledge were merged and used as a conceptual framework. For instance, Sarah related as a parent, Jessica and Tamara related as adult family members, and Gregory related to his work with non-violent inmates. Also, all of the adult learners associated the authentic scenarios and mathematical content with their interest in cooking and shopping.

Again, confirming the research of Ben-Chaim et al. (2004, 2012) and Ginsburg et al. (2006) was the understanding of the adults of their use of ratio and proportion in their respective careers. Each of the students provided detailed examples, such as, Sarah scheduled nurses on her floor and checked dosages received from the pharmacy. Jessica investigated accidents and crimes as she took on additional employment responsibilities. Tamara was compelled to overcome her math anxiety to ensure that her business decisions were correct. Gregory needed to maintain his credibility to investigate incidents in his work as a security guard.

Also, each of the adult learners discussed ratio and proportion within a framework of community/citizen issues. Sarah and Tamara wanted to see their children’s mathematics education improved. Jessica and Gregory had ambitions to work for social justice in crime and politics. Each of the adult learners had a compelling desire for further learning; as this finite mathematics course was a prerequisite for statistics and graduation.
Consequently, the societal contexts in adults stood in contrast to school age based frameworks. The context is in service to the knowledge goal of understanding the content of rate, ratio, and scale for the requisite numeracy skills of adult tasks. For the adults in this study, the context of the *Math Snacks* was inspiration and motivation to learn as they were exposed to rate, ratio and scale while developing their awareness of proportional reasoning abilities.

The authentic investigative activities in *Math Snacks* were somewhere in between abstract and real with the animations placed in a relatable context as Ginsburg et al. (2006) had noted in their study on numeracy. Further, Ben-Chaim et al. (2004, 2012) makes a point that the problems on the *Questionnaire on Ratio and Proportion* were not like standardized test items; they were meant to be realistic and familiar to the adult learners. By contextualizing both the activities and assessments, the adult learners were engaged in the tasks based on their individual familiarity and ability to identify with the context. Hence, *Bad Date* engaged Sarah, *Scale Ella* motivated Jessica and Gregory, and Tamara related and relaxed to the simplicity of the scenarios in *Ratey*. The adult learners could see the connections between the mathematics and the real world. Thereby, further evidencing the mathematical practices of the adults in my study to the theories of Ben-Chaim and Ginsberg.

**Animations.** LaViola (2005) stated that animations are more than an intuitive aid in visualization of the initial formulation of the mathematical relationships of the variables. Lesh, Post, and Behr (1987) emphasized the importance of representations and translations in their studies. In my study, *Math Snacks*’ use of animation created a
memorable impact on all of the adult learners as a unique, but realistic, humorous method to focus them solely on rate, ratio and scale. These animations created a lasting image that each of the adult learners held on to and used in their daily lives and problem solving. Further, while *Math Snacks*’ animations were created for 6-8th graders, the adult learners found nothing “childlike” in the animations. Rather, they all opined that the animation transcended any age or school based factors. Schmitt (2004) had expressed the importance of this in her work on numeracy. Thus, as Ben-Chaim et al. (2004, 2012) and Ginsburg et al. (2006) had noted in their theories, the eye catching and realistic problem solving animation helped the adult learners to understand, as well as compute, ratio and proportion and begin to understand proportional reasoning.

Finally, Ben-Chaim et al. (2004, 2012); Piaget (1970, 1971, 1972); Steffe (1992); Steffe, Cobb, & vonGlasersfeld (1988) noted the importance of reflective abstraction in learning. As *Math Snack’s* animations and realistic and relatable context extended the adult learners connections of the mathematics to the real world, it provided them opportunity for reflective abstraction. The adults in my study were engaged and motivated to reflect and connect the content they were learning thereby making their own meanings.

**Representations.** Lesh et al. (1987) examined representations and translations of representations in mathematical learning and problem solving. The representations and translations between representations in *Math Snacks* played an important role for the adults in my study. The external and observable representations in *Ratey the Cat, Bad Date* and *Scale Ella*, helped the adult learners’ internal conceptualizations. Additionally,
the talk bubbles and bars in *Bad Date*, and the thumbs in *Scale Ella* served as animated manipulative models that had little meaning on their own. But, the adults could see “built in” relationships and operations with everyday situations. Furthermore, the animations were not static; they were internalized as “images”. Finally, the written and spoken language, especially evident in Bad Date, provided a subtle logic as Lesh et al. (1987) had noted.

For instance, all of the adult learners revealed that in *Bad Date* they heard the two people talking and saw the number of words represented in a bubble as the side bar rose and fell with the ratio illustrated in numbers. They felt the representations made the concept and computations of ratio easier to understand than reading or a lecture. Tamara also commented that the tables and graphs in *Ratey* helped her to understand and compute unit rates and she could visualize proportions in *Scale Ella* with the thumbs. Gregory concurred with Tamara and added, “…you think size up and down and you need to use the right math to bring the thumbs back to proper size”. These observations of the adult learners further confirm and extend the theories of Ben-Chaim et al. (2004, 2012) and Lesh et al. (1987).

Further, data from my study confirmed Lesh et al. (1987) by documenting the adult learners’ agreement that the accompanying supporting materials in *Math Snacks* helped their understanding and computations of ratio and proportion. Sarah, Jessica and Tamara commented that the tables, graphs, and pictorial representations in the Learner’s Guides reinforced the concepts and procedures from each video. Jessica also indicated that developing table representations in both the *Bad Date* and *Scale Ella* Learner’s
Guides were helpful to her understanding of patterns and scale. Gregory described the Learner’s Guides as a method that took the complication of learning mathematics away by providing graphic illustrations that helped to conceptualize what was being asked; for example the bed and dimensions in *Scale Ella*.

The adult learners were divided on drawing or sketching their ideas from the videos on the Learner’s Guides. Sarah, Jessica, and Gregory commented that the animations provided the “lasting image” for them so they did not need to draw their own. However, Tamara referred to dating in the figure that she drew on the *Bad Date* Learner’s Guide and commented that her “doodling” helped to relax her and to understand what she had seen in the video. Gregory completed the Learner’s Guides and provided examples of how he could use tables and graphs in his research and presentations.

As the adult learner’s established their own meanings and understanding with their initial, informal methods of problem solving, they used the realistic and relatable context, animations, and representations in *Math Snacks* to strengthen their intuitive foundations of the proportional scheme and solve problems. Thereby, connecting their own personal knowledge and experiences to their development of proportional reasoning. *Math Snacks* made use of the learners’ initial understanding of the concepts and the related underlying connections to paraphrase the concept into additional familiar situations. Steinke (2008) had expressed the importance of adult learners with previous learning difficulties in ratio and proportion developing the ability to translate from one representation to another. My study supported and extended this literature when the adult learners were presented with an idea in one representational mode and were then asked to
visualize it in another mode, such as the thumbs in *Scale Ella*. As they accomplished this, they could understand the concept and make further connections.

Further, the translation process in *Math Snacks* corresponded to some of the most important modeling processes Lesh identified that are needed for adult learners to use ratio and proportion in their everyday lives:

1. Simplifying ratio and proportion by ignoring irrelevant characteristics to focus on the more relevant factors. All of the adult learners identified this characteristic of *Math Snacks* as invaluable to their understanding of ratio and proportion.

2. Establishing a mapping between the ratio and proportion and a ‘model. Sarah felt that the “talk bubbles” were a helpful model to understand ratio.

3. Investigating the properties of the model to generate predictions about the original situation. Sarah could go back to this model to understand the mathematics of the original problem, such as the state mandated ratio of nurse to patient used in scheduling.

4. Translating or mapping the predictions back into the original situation. Sarah could thereby directly map her predictions back to the staffing ratio.

5. Checking to see whether the translated prediction is useful and makes sense.

Sarah could now see how ratio and proportion were important and made sense in her job.

Lesh, Landau, and Hamilton (1983) explained that representations tend to be plural, unstable, and evolving. In my study the adult learners each worked in more than one representation, either in a series or parallel. Their solution paths wove back and forth
among several representational systems, as they repeatedly watched the videos and worked on parts of the Learners Guides. For instance, Tamara explained with a pizza problem, going back and forth between an image of the pizza and its fractional representation. Representational plurality was also evident as the adult learners related to several distinct clusters of mathematical ideas, such as rate, ratio and scale that were ultimately interwoven into proportional reasoning. Finally, as all of the adult learners evolved into more proficient problem solvers, they became sufficiently flexible in their use of a variety of relevant representational systems so that they instinctively switched to the most convenient representation at any given point in the solution process, such as Sarah’s use of tables in scheduling. Lesh et al. (1987) viewed this attribute as the goal of representations.

Thus, realistic and relatable contexts, animation, and representations in both the videos and supporting materials were helpful to the adult learners’ understanding of rate, ratio and scale. Additionally, Gardner (2003) examined multiple intelligences. Each of the adult learners stressed that the Math Snacks were supportive of their individual learning styles. All agreed that the brief snacks, rather than meals, bolstered their understanding of ratio and proportion in a course where the topics were not the focus of the curriculum but necessary for success in the course and future mathematics. Thus, my study’s findings on context confirm and extend the literature of Ben-Chaim et al. (2004, 2012); Gardener (2003); Ginsburg et al. (2006); Lesh et al. (1987), and Schmitt (2000) not only by introducing adult subjects but also by specifically introducing authentic tasks that serve to support understanding in multiple ways.
Deepened understandings. Ginsburg et al. (2006) indicated that the content component of numeracy includes both the depth of mathematical knowledge that is necessary and the kinds of tasks that adult learners face in their lives. In my study, Lamon’s (1993, 1995, 2001, 2005, 2007, 2008) theories provided the essential concepts and critical components of proportional reasoning. The mathematics found in Lamon’s theories, although focused on a grades 6-8 curriculum, helped the adult learners form a flexible knowledge that could be used in a context of requisite numeracy skills for adult tasks. Further, as Ben-Chaim et al. (2004, 2012); Schmitt et al. (2004); Ginsburg et al. (2006) noted the nature of the realities that these adult learners face in their lives has clearly changed since their last mathematics course.

As explained in Chapter II, Lamon (1993, 1995) uncovered a number of learning sites from which students gained insights into the critical components of proportional reasoning. Freudenthal (1983) named these learning sites the “didactical phenomenology” of the proportional reasoning. These points are where a student may enter the learning process and reconstruct an important mathematical idea that is organized by the mathematics, such as ratio and proportion (see Figure 62).
In Lamon’s model, the first or initial level of understanding includes three components: additive versus multiplicative thinking or the relationship between and within rational numbers, unitizing, and partitioning. Ben-Chaim et al. (2004, 2012) produced a theoretically similar model, *Teaching Model Using Authentic Investigative Activities for Teaching Ratio and Proportion (Figure 3)*, for adults, specifically student teachers.
Further, the literature of Fischbein (1995) and Inhelder and Piaget (1958) was confirmed in my study as the adult learners started with intuitive cognition, intuitive understanding, and intuitive solutions of ratios and proportions or the algorithmic or concrete operational stage of development. Thus, they entered Lamon’s model on the first or initial level. However, their individual entry point and their need for understanding of the interconnectivity with the other dimensions on the first level were unique to each student. Ben-Chaim (2004, 2012); Lamon (2007, 2008); Schmitt et al. (2004) each wrote of the importance of building on the adult learner’s intuitive knowledge. The presentation of the content in *Math Snacks* with the inherent interconnectivity of the components and levels of the model allowed instruction to begin based on the needs of the individual learners.

Initially, Sarah understood unitizing and partitioning but not the relationship component. She achieved moving from additive to multiplicative thinking in most instances, although she would revert to additive thinking if the situation made it more reasonable, such as the need for flexibility in scheduling. She was then able to see the interconnectivity of the components and understand covariance and invariance and ratio appropriateness. At the end of the study, Sarah could verbalize and demonstrate her understanding of proportional reasoning by stating that the components were interconnected and that their sum was greater than their parts.

Jessica started out and completed the study using multiplicative thinking, understood partitioning but was challenged with the unitizing component. She saw the interconnectivity of these components in the learning process. By the end of the study,
she understood and unitized correctly. Although, she resisted changing her rule based approach, she understood covariance and invariance and ratio appropriateness. She could proficiently reason proportionally.

Tamara began the study with a basic understanding of the relationships, unitizing, and partitioning components. She could verbalize and demonstrate multiplicative thinking, covariance and invariance and ratio appropriateness through examples and drawings with the confidence that she developed during the study. She was then able to see the interconnectivity of all of the components and to reason proportionally.

Gregory’s lack of number and operation sense limited his multiplicative thinking so that he would revert to additive thinking. This also limited his ability to unitize and partition. Inhelder and Piaget (1958) suspected that children who reason additively employ additive transformations. An additive transformation does not preserve a ratio. A student, such as Gregory, using additive transformations was unable to analyze a proportional situation. Thus, Gregory’s additive reasoning might be an invariant stage in the development of proportional reasoning or it might be a positive transformative stage from additive to multiplicative reasoning as Inhelder and Piaget suggested. However, at the end of the study, Gregory did use multiplicative reasoning and understood the interconnectivity of all components of proportional reasoning.

Lamon’s (1993, 1995, 2007) components of proportional reasoning were exemplified by each of the adult learners demonstrated conceptual understanding of absolute and relative thinking, covariance and invariance, and ratio appropriateness as components of abstraction in examples of their use in daily tasks. They could
demonstrate an increase in their level of abstraction from visual to verbal and manipulative to pictorial. This would indicate Piaget’s formal operational stage had been attained.

Further, Lamon (2007) had explained that one characteristic of proportional reasoners was the ability to use common sense and a thoughtful approach to problem solving. This was confirmed in my study by each of the adults demonstrating the mental, free flowing processes that are required for understanding and abstraction of relationships. Thus, each student knew what a proportion was not and when it did not apply. However, this was not achieved by the students’ procedural skills alone. Thus, even Tamara and Gregory, who were challenged by computations, developed a deepened understanding of relative thinking, covariance and invariance, and ratio appropriateness.

Lamon (2007) and Ben-Chaim et al. (2004, 2012) suggested that multiplicative reasoning also moves beyond concrete operations and into formal reasoning and abstractions. Piaget (1953) considered it the hallmark of formal reasoning. As reasoning relies on students’ work in their head, rather than written computations, the logical reasoning patterns develop along with the ability to draw conclusions or inferences (Vergnaud, 1988). At the end of my study, each of the adult learners demonstrated and articulated an adequate level of abstraction and hypothesis making to indicate that they were now in the formal operational or reasoning stage of development. Ben-Chaim et al. (2004, 2012) also suggested that proportional reasoning was a main indicator of operational development, during the stages of formal development, (p. 50). Sarah, Jessica, and Tamara spontaneously chose a proportional scheme when problems having a
proportional relationship were encountered and this led to a rational, logical, intelligent plan of action. Gregory struggled but was very close, again hindered by his computational problems. Additionally, Lamon (2008) explained when students reason proportionally, their cognition is marked by most of twelve conceptual understanding characteristics, listed in Chapter II. At the conclusion of this study, each of the adult learners demonstrated many of these characteristics. These characteristics were not noticeably evident prior to their use of the authentic investigative activities.

Further, on the pre and post Diagnostic Questionnaires on Ratio and Proportion developed by Ben-Chiam (2004, 2012), the adult learners encountered two types of variables that influenced their solution process. The nature of the problems included verbal content, the metric system that the adults were not familiar with. It also involved a nature and structure of the numbers presented in the problem, such as their dimension (very large or small), or type (problems with rational or discrete values are easier to solve than those with continuous values). These two variables essentially determined the level of difficulty of the problem and challenged the adult learners in my study. However, all the adult learners appreciated the opportunity to reflect on these different variables in the problems in the Questionnaire. This discourse was verbal in their interviews and written in their class projects and discussion posts.

In conclusion, Table 6 (p. 225) provides a summary of the adult learners’ scores on the entire pre and post Questionnaires on Ratio and Proportion. In my study, the adult learners’ performance on the ratio problems remained unchanged with an improvement in rate and a significant improvement in scale. This summary was modeled from the
student teachers’ summary in Ben-Chaim (2004, 2012) study. My study generally confirmed the Ben-Chaim (2004, 2012) studies that implementation of authentic investigative activities to adult learners incorporating theory and practice can lead to a qualitative change from additive to multiplicative thinking. Further, it attests to the use of the Questionnaire on Ratio and Proportion as an assessment of pre and post knowledge in this study.

Thus, my study confirmed and extended Lamon’s model of the components of proportional reasoning. Further, Ben-Chaim’s work extended Lamon’s model to the types of tasks suitable for adults (mathematics student teachers). My study offers these two theories that allow teachers to investigate students’ progress and ability and to model various proportionality tasks.

Cognitive and Affective Component

The cognitive and affective component of mathematical proficiency includes five processes identified as conceptual understanding, adaptive reasoning, strategic competence, procedural fluency, and productive disposition (National Research Council, 2001). These processes are crucial for the adult learners as well K-12 students. In this study, as in Ginsburg et al. (2006), these processes were not subsumed under context or content but were the mechanisms that enable the linkage between context and content. Confirming this, each of the adult learners demonstrated degrees of the first four processes on both the pre and post Questionnaires on Ratio and Proportion. This information was presented in the Summaries of Student Mathematical Thinking under each of the case studies in Chapter IV.
For instance, conceptual understanding was evident in the integrated and functional grasp of work by Sarah, Jessica and Tamara. However, Gregory did not have a level of conceptual understanding to produce reasonable estimates to catch his computational errors in division. Initially, Sarah, Jessica, Tamara, and Gregory all struggled with recognition of logical mathematical connections needed for adaptive reasoning. By the end of the study they connected the individual elements of rate, ratio and scale and made generalizations about their relationships. Perhaps, the strongest aspect of adaptive reasoning demonstrated by all of the adult learners was their improved ability to follow a logical path of reasoning based on the basic ideas and components of rate, ratio, and scale. The students attributed this to the persistent images of *Ratey, Bad Date* and *Scale Ella* remaining with them. Both Ginsburg et al. (2006) and Lesh et al. (1987) had indicated the importance of images. The elements of logic in their communications in discussion posts, class projects, interviews and observations brought the mathematical and real world context together for all the students. However, only Sarah and Jessica could communicate using mathematical terms and vocabulary.

Further, Kilpatrick, Swafford, and Findell (2001) and Mezirow (2000) stressed ongoing sense making and reflection. This was confirmed in my study with evidence in all of the student’s communication as it became interwoven with mathematical proficiencies. Kilpatrick claimed that formulating mathematical problems, representing them, and then solving them demonstrate strategic competence. Sarah, Tamara, and Gregory showed flexibility in their strategic competence by formulating mathematical problems, representing them, and then solving them. Jessica’s inflexibility in not moving
beyond using rules or formulas to set up rates and ratios presented a disadvantage. Thus, she was unable to use her mathematical knowledge of ratio and proportion strategically as a tool for understanding.

All of the students developed an understanding of how to mathematize a situation and organize a problem into mathematical form to see the underlying mathematical structure in the problem. Sarah did this in her scheduling to maintain a 6:1 ratio of staff to patient with diagrams. Jessica and Tamara looked for patterns to follow in recipe conversions. Gregory could envision the use of tables and graphs to show the mathematical structure of prison reform. Each student understood that they needed an insider’s contextual knowledge to solve these problems (Ben-Chaim et al., 2004, 2012; Ginsburg et al., 2006).

Sarah, Jessica, and Tamara had procedural fluency. They could use mental mathematics, estimation and calculators as needed. Gregory could not use any of these well due to his flawed procedural skills in division. He, as well as the other three adult learners, could understand what the operations do, how they are related to each other and when to use them (Ginsburg et al., 2006).

Further confirming Ginsberg et al. (2006), all of the adult learners used alternative strategies that were based on sound mathematics flexibility and fluency. When Tamara and Gregory were confronted with a scale problem that they couldn’t figure out, they used a ruler and compared the measurements. Sarah had to use part and full time RNs and LPN’s from different floors in her scheduling so at times she used an additive strategy to maintain a ratio. Jessica would go back to a rule, such as \( d = r \times t \) when she could not
figure out how to set up a ratio. Further, Sarah, Jessica, and Tamara could use mental mathematics to integrate the elements of conceptual understanding, adaptive reasoning, and problem solving as well as calculators to construct simpler situations out of complex ones.

The productive dispositions of the adult learners at the conclusion of the study, further confirmed and extended Ben-Chaim et al. (2004, 2012) and Knowles (1980) theories and literature to adult learners in general. In my study, there was an improvement in all of the adult learners’ attitudes, beliefs and emotions towards learning mathematics in general, and ratio and proportion in particular. Additional data collected and analyzed from the discussion forum, interviews, and documents explained why their confidence in their ability to deal with ratio and proportion was neutral. The *Summary of Attitude on Ratio and Proportion* (table 5, p.223) was modeled from a document used in Ben-Chaim et al. (2004, 2012) study with similar results, confirming and explaining this study’s results and reliability.

Consequently, the process of productive disposition was affected by the authentic investigative activities and instructional strategies. The four adult learners credited their engagement to *Math Snacks*’ with the alignment to their individual mathematics background, addressing gaps in their understanding, and providing snacks not full course meals on rate, ratio and scale. The brevity of the snacks was cited by all of the adult learners as a helpful process that went directly to their issues with the topic. Tamara also mentioned that she appreciated this in other videos, such as Kahn’s Academy.
Knowles (1980) stated that andragogy was the process of engaging adult learners within the structure of learning experience. His assumptions were compatible with each of the adult learners reflections on the *Math Snacks*’ videos and learner’s guides. For example:

- The relatable context satisfied the adult learners’ need to know why they were learning ratio and proportion. Sarah initially stated that she didn’t know where she would use ratio and proportion in her career as a nurse until she saw *Bad Date*.
- The adult learners felt an involvement in their instruction with the authentic investigative activities. Tamara felt that the humor in *Ratey* helped her to relax and learn the concept.
- The adult learners felt an immediate relevance to their work and personal lives making them ready to learn. Jessica was ready to learn when she completed the Learner’s Guide in *Scale Ella*.
- Adult learning is problem centered, rather than content-oriented. Thus, the relatable problem-solving scenarios were oriented towards the adults. Gregory related to *Scale Ella* solving the crimes of Scaleo.
- The adult learners responded to the internal motivation of the videos. The animations caught their attention and held it. Both Sara and Jessica indicated the animations in *Bad Date* motivated them.
Finally, the adult learners did not come to this course with a “childlike clean slate”. They each had to counter existing, entrenched negative belief, attitudes and emotions. Each of the adult learners indicated previous failures in math.

Ginsberg et al. (2006) indicated that common negative attitudes of all the adult learners were a dislike of word problems, general feelings of anxiety about algebra and discomfort in asking the teacher questions. This was confirmed in my study by Sarah’s general anxiety with algebra as it was out of her “comfort zone” of arithmetic. She needed to see the mathematics presented in a realistic context that she could relate to her career and personal life. Jessica felt that the Math Snacks’ instructional methods were like having a tutor sit beside her so she didn’t need to ask the instructor a question. Tamara disliked word problems, was anxious about all mathematics and didn’t like to ask questions. Gregory said that he lost contact with mathematics in the 4th grade with long division and had not been able to function successfully in mathematics course since then.

Again, this information from my study confirmed and extended the literature of the Ginsburg et al. (2006) and Knowles (1980).

These adult learners had established counterproductive attitudes, beliefs, and emotions as a by-product of earlier school and other experiences as Tobias (1978) has noted in her numerous studies on mathematics anxiety. Their avoidance was linked to math anxiety, self-perceptions of incompetence, and feelings of a lack of self-control. The Math Snacks experience had consequences of positive attitude, beliefs, and emotions towards numeracy for the adult learners. As noted by Ginsberg et al. (2006), the adults completed the study with feelings of productive engagement, the expectation that
mathematics should and will make sense to them, and a commitment to problem solving and persistence when encountering false starts and other frustrations.

In general, research on the context, content, and cognitive and affective components of numeracy with adult learners studying ratio and proportion through authentic investigative activities has been limited or lacking. Studies have either involved middle school students using elementary textbooks and materials, such as workbooks, or a specific classification of adult learner (mathematics student teachers) using authentic investigative activities. These studies have generally shown that students’ attitudes and understandings were positively impacted by the introduction of activities specifically designed to teach ratio and proportion.

My study brought brief authentic investigative activities or snacks of ratio and proportion to the general population of adult learners as defined by NCES (2007). These adult learners were not preparing for the test of the General Educational Development (GED); they were enrolled in a degree completion program to extend their current careers and personal development. Thus, my study was situated to not only confirm and add to research and literature but also have implications for practitioners, education, and future research in two-year colleges or four-year colleges with remedial mathematics topics.

**Implications of the Work**

In order to facilitate the process of learning for adults, it is important to first know who the adult learners are, how social context shaped their learning and why adults are involved in learning activities. The purpose of this case study was to examine what helps or hinders adult students to learn ratio and proportion when the topic is not the
central focus of the mathematics course. These adult learners were a diverse group, each with a story based on the social context that shaped their learning and reasons for their passion to complete their college degrees and programs.

Thus, it was important to explore and extend the current assumptions and research beyond the theories of Lamon’s content, Ben-Chaim’s context, and Lesh’s representations and translations to adult learners. This study has extended the field of proportional reasoning to adults that require additional support in their mathematical background. By including brief, authentic investigative activities to the college algebra curriculum, the adult learners in this study have deepened their understanding of not only the content, but improved their attitudes toward ratio and proportion and mathematics in general. They are now able to apply ratio and proportion to the everyday tasks required in personal and professional lives. Ginsburg et al. (2006) and Schmitt et al. (2004) presented three components of context, content, and cognition and affect as the foundation of numeracy. These components are intertwined with class, culture and ethnicity along with adult learners’ cognitive and learning styles that affect how adults use mathematics in specific contexts, such as at work or home, and how these contexts affect adults' mathematical behavior.

Schmitt (2000) proposed the potential of adult development. My study will hopefully add to the research claiming that current adult education in the form of adult basic education (ABE) and the GED with its incremental and modular workbooks focused on procedural skills, needs to be challenged and changed. These benign processes followed by word problems are not relatable to adult learners and seem to just
be more practice of the algorithms. Rather, the potential of adult development in students seeking to extend their careers and personal lives now needs to move beyond the basics toward a more realistic, flexible, and adult-centered mathematics curriculum. Supporting Schmitt’s proposals, the adult learners in my study all wanted to understand the mathematics in proportional reasoning, not just the “steps”.

**Implications for Practice**

Adult learners have moved beyond the GED and competency, they now require adult development for success in the future mathematical demands of the world. Their diverse characteristics as well as their usable skills and understandings need to be taken into account in curriculum and instruction development. This can be accomplished by removing the myths that rule-based mathematics is most important, that all adults learn the same way and that learning “happens” by transmission.

Specifically, with the authentic animated investigative activities, based on theoretical knowledge and tailored to adult learners, we are taking into account what we know about the mathematical demands on adults as well as what we know about the development of proportional reasoning. Thus, theory moves to practice with the goal that adult learners can manage a situation or solve a problem in a real context. Further, they are able to respond to information about mathematical ideas that may be represented in a variety of ways. They are then able to activate a range of enabling knowledge, behaviors, and processes as essential components of adult learning.
As practitioners, we need to view numeracy as a bridge that links mathematics to the real world. Good teachers have always attempted to tailor instruction to the needs of their students and to build on the learners existing knowledge and experience. However, effective supportive resources on ratio and proportion for adults, or any topic that is requisite knowledge for success in a course but not included in the curriculum, have not been easy for instructors to locate and implement. My study confirms that in addition to the mathematical content, the resources need to be relatable to the adult learners and support their affective and cognitive needs. The resources need to be brief, focus solely on the topic as well as “reach out” to the adult learners’ individual needs and learning styles.

Additionally, my study extends the literature of numeracy to adult learners taking mathematics at the collegiate level. I believe, based upon the current study, the following provide further insight and support improvements to numeracy in context, content, and cognition and affect:

1. Adult educators and their learners would benefit from a diagnostic questionnaire, such as the one used in this study, which focuses on activities that adult learners are familiar with and could relate to the context. In other words, questions that are realistic or authentic, not standardized test questions or based on rote and algorithms.

2. Practitioners include in their course, even if it is not part of the curriculum, brief resources that are “math in context”, only better to support “gaps” in knowledge. When the curriculum is realistic and relatable, the adult learners begin with a
familiar context, with many representations, and learn problem solving and procedures in service of solving similar realistic and relatable problems. They are able to consider and decide on alternatives. Thereby, addressing the mathematics needed to manage demands of family, workplace, community and further education.

3. The scope and sequence of mathematical content needs to be reassessed as adults have many possible gaps in their education that are best addressed as snacks rather than full meals. As illustrated in my study, adults may need a snippet on division that is aligned with their mathematical backgrounds. While the adults showed that they could work around many of these gaps in knowledge, eventually, as in multiplicative thinking in proportional reasoning, it causes developmental problems. These learning gaps can be included as a resource accessible to all learners. While practicing arithmetic procedures is important, time and attention must be paid to develop the adult learners conceptual understanding. For instance, adults as well as all students need to understand what the operation of division means, what is a sensible answer in a problem and how the numbers might look with other representations.

4. Authentic investigative activities can offer non-traditional instructional strategies. Each of the adult learners commented on how these strategies complemented their personal learning modes and needs. Students learn in ways that are identifiably distinctive. The broad spectrum of adult learners would be better served if
mathematics could be presented by non-traditional teaching strategies in video, audio, and textual modalities.

5. Adult learners’ existing and developing sense of themselves needs to be supported to help them manage frustration during problem solving as well as to develop confidence to better manage real-life numeracy.

6. With technology at their fingertips, students can explore various avenues of writing by engaging in blogs or exchanging ideas with others through email or a discussion forum. In doing so, it would allow students to go back and read their own and their peer’s posts and reflect on the mathematical concepts and problem solving solution (Hrina-Treharn, 2011).

7. Profession development for practitioners should also include acknowledging adult learners’ feelings and their need to be in charge of their learning while learning about themselves as learners. Further, practitioners need to develop an understanding of the importance of adult learners’ engagement.

**Implications for Research**

Adult education needs to draw on the theories of development of children's mathematical thinking and extend that research to address the development of adults' numeracy thinking and practice. Learning is a student responsibility; however the learning environment is also the responsibility of faculty (supported by the institution). Future research should suggest a goal that all students be successful in mathematics and seek to apply sound cognitive research to create conditions that support all students. Essentially, these new approaches should seek to eliminate any correlation between
student characteristics and learning outcomes, to provide equally high chances of success for all students. My study provided insight into what helps or hinders adult students to learn ratio and proportion when the topic is not the central focus of the mathematics course. Thus, a number of valuable studies could naturally flow from this study.

1. How do learners’ prior experiences with these particular mathematics ideas impact their new learning?
2. How do students who have experienced authentic investigative activities that utilize non-traditional teaching strategies seek out and use the same genre of support in other fields of mathematics? How can instructors support this process?
3. How do learners develop competence within new contexts of ratio and proportion?
4. How should teachers use different teaching strategies for different content, such as statistics? How can professional development courses be initiated?
5. How do teachers shift from focusing only on procedural skill to address the cognitive and affective component?
6. Additionally, a longitudinal study might compare the mean scores of another general population adult learners, using authentic investigative activities in both distance education and seated classroom with only mathematics education students.
7. How can researchers develop innovative materials that are focused on a specific topic and “reach out” to adult learners?
8. How can practitioners infuse these innovative materials into their course to complement the curriculum?
These are all examples of research questions to further understanding of how we can help adults build numeracy skills. Further, with continuation of collaborations among practitioners and researchers, bringing theoretical perspectives and practice in numeracy together, the field of ratio and proportions, as well as mathematics in general, will benefit.

**Summary**

In this chapter a discussion of the findings was related and confirmed by the theories of Lamon, Ben-Chaim and Lesh. Further, the findings were corroborated by the theories on adult learners and numeracy by Schmidt and Ginsberg.

Implications for practice provided insight and support for improvements in context, content, and cognition and affect in adult learning of ratio and proportion. Additionally, examples of research questions were presented to further understanding of how we can help adults build these functional numeracy skills.
APPENDICES
APPENDIX A

RECRUITMENT EMAIL
APPENDIX A
RECRUITMENT EMAIL

Study Title: Implementation of Authentic Investigative Activities In Ratio and Proportion to Adult Learners

Investigator: Cynthia Brennan, Doctoral Candidate

Dear MAT 174 Students,

I am writing today to invite you to participate in my study entitled: Implementation of Authentic investigative activities In Ratio and Proportion to Adult Learners.

I am hoping to recruit participants who are enrolled during spring I term 2014 of MAT 174. The study will be conducted over the course of the spring I term 2014.

Phase 1: Participants need to be willing and available to complete both 1.) pre and post-test on ratio and proportion and 2.) attitude towards the topics.

Phase 2: Participants will take part in 2 interviews, each of 30 minutes and one observation lasting 30 minutes.

In total, I would be asking for 3 hours of your time over the duration of the course.

I would like to make it clear that these tests of your pre and post knowledge are not part of your grade for this course. For detailed information about the study and participant requirements, please see the attached consent form. If you are selected to participate in the study I will explain the consent form to you by email or phone and you will have the opportunity to withdraw from the research at any time during the study. Again, participation in this project will not impact your course grade in any way and you are under no obligation to take part.

If you have any questions please do not hesitate to contact me using the details below.

Many thanks for your consideration.

Cynthia

brennancr@tiffin.edu
330-995-1503
APPENDIX B

INFORMED CONSENT TO PARTICIPATE IN A RESEARCH STUDY
APPENDIX B

INFORMED CONSENT TO PARTICIPATE IN A RESEARCH STUDY

**Study Title:** Implementation of Authentic investigative activities in Ratio and Proportion to Adult Learners

**Principal Investigator:** Joanne Caniglia, PhD

**Co-Principal Investigator:** Cynthia Brennan

You are being asked to participate in a research study. This consent form will provide you with information about the research study, what you will need to do and the associated risks and benefits of the research. Your participation is voluntary. Please read the form carefully. It is important that you ask questions and fully understand the research in order to make an informed decision. You will receive a copy of this document to take with you.

**Purpose:**

The goal of this research is to access and understand the impact of implementation of authentic investigative activities in ratio and proportion to adult learners in an IT mathematics course. The research will utilize IT (already created) activities and simulations that require quantitative and qualitative numerical comparisons in ratios to find a missing value. The activities establish the understanding of many concepts related to ratio and proportion topics and are focused on the three main categories of proportional reasoning problems: rate and density, ratio and scaling.

**Procedures:**

If you chose to take part in this study at Tiffin University’s School of Graduate and Distance Education, you will complete the pre and post-test on ratio and proportion and attitude towards the topic. As part of your Case Study for the course you will view the activities and complete the worksheets and then write about your reflections on the learning activity. Each part of the study should take 20-30 minutes. The second phase of the study involves interviews and observations, if you are willing.

**Audio and Video Recording and Photographs:**

Interviews and observations will be recorded with names and identification removed.
Benefits:

The research will benefit you directly. The five additions to the IT course will offer support to the topics of the course. Your participation in the study will help us to understand how authentic activities (real life) help students understand ratio and proportion. If you chose to participate in the pre and post-tests, you will receive 25 extra credit points to your final course grade. Students who chose not to participate can earn the points by completing 25 additional homework problems. If you agree to the interviews, you will receive a $10 gift card.

Risks and Discomforts:

There are no anticipated risks beyond those encountered in everyday life. If you chose not to participate, your course grade will not be negatively impacted. If you chose to participate, your responses will be coded so that no identification will be linked to your grade.

Privacy and Confidentiality:

Your study related information will be kept confidential within the limits of the law. Any identifying information will be stored in a secure location and only the researchers will have access to the data. Research participants will not be identified in any publication or presentations of the results, only aggregate data will be used.

Your research information, may in certain circumstances, be disclosed to the Institutional Review Board (IRB) at Kent State University, Tiffin University, or to certain federal agencies. Confidentiality may not be maintained if you indicate that you may do harm to yourself or others.

Voluntary Participation:

Taking part in this research study is entirely up to you. You may chose not to participate or to discontinue your participation at any time without penalty or loss of benefits to which you are otherwise entitled. You will be informed of any new or relevant information that may affect your health, welfare, or willingness to continue your study participation.

Contact Information:

If you have any questions or concerns about this research, you may contact Joanne Canniglia at jcaniglia1@kent.edu. This project has been approved by both Kent State University and Tiffin University Institutional Review Boards. If you have any questions about your rights as a research participant or complaints about the research, you should
contact Kent State University IRB at 330-672-2704 or Tiffin University Dr. Jonathan Appel, Director, Institutional Review Board (IRB), Tiffin University (Tel. 419.448.3285 or email appelj@tiffin.edu).

**Consent Statement and Signature:**

I have read this consent form and have had the opportunity to have my questions answered to my satisfaction. I voluntarily agree to participate in this study. I understand that a copy of this consent will be provided to me for future reference.

___________________________  ______________________
Participant Signature  Date
APPENDIX C

AUDIOTAPE/VIDEO CONSENT FORM
APPENDIX C

AUDIOTAPE/VIDEO CONSENT FORM

Implementation of Authentic investigative activities in Ratio and Proportion to Adult Learners

Joanne Caniglia, PhD and Cynthia Brennan

I agree to participate in audio-taped/video-taped interviews about authentic investigative activities during this IT class as part of a research project and for the purpose of data collection. I agree that Joanne Caniglia or Cynthia Brennan may audio-tape or video-tape this interview and observation. The date, time and place will be mutually decided upon.

________________________________             ________________________
Signature             Date

I have been told that I have the right to listen to the recordings of the interviews and observation before they are used. I have decided that I:

_____want to listen to the recordings _____do not want to listen to the recordings

Sign now below if you do not want to listen to the recordings. If you want to listen to the recordings, you will be asked to sign after listening to them.

____________________may / may not (circle one) use the audio-tapes made of me. The original tapes or copies may be used for:

_____this research project _____publication _____presentation at professional meetings

________________________________             ________________________
Signature             Date

Address:
APPENDIX D

RATING FORM
APPENDIX D

RATING FORM

DIAGNOSTIC QUESTIONNAIRE IN RATIO AND PROPORTION: RATING FORM

<table>
<thead>
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<th>School Group</th>
<th>Type of and number of problems: Rate</th>
<th>Ratio</th>
<th>Scaling</th>
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<td>Incorrect answer</td>
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<tr>
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<td>Correct answer only</td>
<td>Correct support work</td>
<td>Incorrect thinking</td>
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<tr>
<td></td>
<td>Correct support work</td>
<td>Incorrect answer only</td>
<td>Correct thinking but wrong conclusion</td>
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<tr>
<td></td>
<td>Incorrect thinking</td>
<td>Incorrect thinking</td>
<td></td>
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<td>Overall total</td>
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APPENDIX E

IMPLEMENTATION OF AUTHENTIC INVESTIGATIVE ACTIVITIES IN RATIO AND PROPORTION TO ADULT LEARNERS: CASE STUDY
APPENDIX E

IMPLEMENTATION OF AUTHENTIC INVESTIGATIVE ACTIVITIES IN RATIO AND PROPORTION TO ADULT LEARNERS: A CASE STUDY

<table>
<thead>
<tr>
<th>Week</th>
<th>Instructor Administration &amp; Permissions</th>
<th>Class Activities</th>
<th>Participant Activities 3 -5</th>
<th>Instruments (Instructor in Italics)</th>
</tr>
</thead>
</table>
| 1    | 1. Present class with an overview of the study and invite all class members to participate.  
2. Participants sign student consent form (Appendix A) and return it to me.  
3. Forms are stored on auxiliary computer  
4. Assess and analyze Questionnaire: Attitude on Ratio and Proportion-complete Rating Form (Appendix E).  
5. Assess and analyze Diagnostic Questionnaire in Ratio and Proportion: Rating Form (Appendix F) | Present class with an overview of the study and invite all class members to participate. | 1. Sign and return consent form.  
2. Complete Questionnaire: Attitude on Ratio and Proportion  
3. Complete Diagnostic Questionnaire on Ratio and Proportion  
25 extra credit points for completion | Student Consent Form (Appendix A)  
Questionnaire: Attitude on Ratio and Proportion (Appendix C) aka Pre-test  
Diagnostic Questionnaire on Ratio and Proportion (Appendix D) aka Pre-test  
Questionnaire: Attitude on Ratio and Proportion: Rating Form (Appendix E)  
Diagnostic Questionnaire in Ratio and Proportion: Rating Form (Appendix F) |
<p>| 2    | Repeat Week 1 as needed | Repeat Week 1 | Repeat Week 1 | Repeat Week 1 |
| 4 | Engage students in discussion of videos in Moodle Chat Room. | Discuss helpfulness of videos in understanding ratio and proportion in Moodle Chat Room. | Discuss helpfulness of videos in understanding ratio and proportion in Moodle Chat Room. |
| 5 | Grade worksheets for class project. Set up interviews and observations with participants. | Assess and Analyze worksheets Complete Questionnaire: Attitude on Ratio and Proportion (Appendix C) aka Post-test Diagnostic Questionnaire on Ratio and Proportion (Appendix D) aka Post-test Questionnaire: Attitude on Ratio and Proportion: Rating Form (Appendix E) | Questionnaire: Attitude on Ratio and Proportion (Appendix C) aka Post-test Diagnostic Questionnaire on Ratio and Proportion (Appendix D) aka Post-test Questionnaire: Attitude on Ratio and Proportion: Rating Form (Appendix E) |</p>
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<th></th>
<th>Use Interview Protocol (Appendix G) and Observation Protocol (Appendix H) to collect data</th>
<th>Manage Data.</th>
<th>Use Interview Protocol (Appendix G) and Observation Protocol (Appendix H) to collect data. Manage Data</th>
<th>Begin Data Analysis</th>
<th>Thank class and participants as this is the conclusion of the course</th>
<th>Interviews and Observations</th>
<th>Interview Protocol (Appendix G) Observation Protocol (Appendix H)</th>
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<td>Use Interview Protocol (Appendix G) and Observation Protocol (Appendix H) to collect data</td>
<td>Manage Data.</td>
<td>Interviews and Observations</td>
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<td>Use Interview Protocol (Appendix G) and Observation Protocol (Appendix H) to collect data. Manage Data</td>
<td>Begin Data Analysis</td>
<td>Interviews and Observations</td>
<td>Interview Protocol (Appendix G) Observation Protocol (Appendix H)</td>
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</tr>
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</table>
APPENDIX F

DIAGNOSTIC QUESTIONNAIRE IN RATIO AND PROPORTION
APPENDIX F
DIAGNOSTIC QUESTIONNAIRE IN RATIO AND PROPORTION

Student Number

Date: _____________________

Important: Answer all questions in full, providing detailed explanations of the problem-solving process.

RATE PROBLEMS

A Trip to the Zoo

Max, Alice, Alex, and Sima planned a class bicycle trip to the zoo. The pupils gathered in the parking lot of the school and rode their bikes along the bike path leading to the zoo. They watched the animals for a few hours, and then met by the lake for some snacks and cold drinks before riding back to the school.

1) Max and Alice had to buy the beverages. They saw that cherry soda cost $2 for 16 single-serving boxes. Grapefruit juice cost $1.60 for 12. They decided to buy the grapefruit juice. Was this the best choice economically? Show in detail all the calculations and thought processes with which you arrived at your answer.

2) Sima and Alex organized the purchase of chocolate bars and apples, but they lost their receipts. They remembered, though, that the chocolate cost $2.60 for 8 bars, and the apples were 6 for $1.95.
   a) How much did they pay for 20 chocolate bars? Show your work.
   b) How much did they pay for 20 apples? Show your work.

3) After the trip, Sima and Alex decided to see who could ride the fastest back home. Sima rode 5 km in 20 minutes. Alex rode 7 km in 25 minutes. Who rode the fastest? How do you know?

4) The next weekend, Max and Alice rode their bikes to the park, taking the long route around the lake. The route is 30 km and it took them 1.5 hours.
   They ate lunch, and then rode back the short way, which is only 20 km. This took them 3/4 of an hour. In which segment of their trip did they ride the fastest? How do you know?
5) In the field near their homes, Max and Alice noticed a number of stray cats. They made a number of phone calls and discovered that in their town there are about 1000 stray cats, and in the neighboring town there are about 1500. Their town has an area of 60 square kilometers, and the neighboring town is 100 square kilometers. Assuming that the cats are evenly scattered over the areas, in which town is there a higher likelihood of seeing a stray? Explain your reasoning.

RATIO PROBLEMS

Getting to School

At City Elementary School there are 400 pupils. Some get to school by school bus, and the others get to school on their own, either by walking, riding their bikes, or getting a lift with parents.

1) In Mr. Erez’s class, 20 students come by bus and 15 arrive on their own. In what different ways could you compare the number of students that arrive by bus to the number that arrive on their own to school? Explain your reasoning in detail.

2) In Ms. Shula’s class, 18 students come by bus, and 12 arrive on their own. In what different ways could you compare the numbers of bus riders and those who come on their own in Ms. Shula’s class to those in Mr. Erez’s class? Explain your reasoning in detail.

3) Is the ratio of bus riders to those who come on their own in Ms. Shula’s class the same as in Mr. Erez’s class? Explain your reasoning in detail.

4) Of the 400 students in the school, 240 arrive by school bus daily. Is the ratio between the numbers of pupils that arrive by bus to the number that arrive on their own in Ms. Shula’s class the same as that of the whole school? How do you know?

5) In Ms. Nancy’s class, 25 students arrive by school bus and 15 arrive on their own. Ms. Nancy claims that the ratio between the numbers of pupils who arrive by bus to those that arrive on their own is 5:3. Is she correct? Explain your reasoning in detail.
In the Photography Store

SCALING PROBLEMS

Phil and Fran are photographers who develop their own pictures and also restore old photographs. They have an enlarging and reducing machine that can change the size of photographs as shown below.

Figure 1: Photographs

1) A customer asks Fran to enlarge a 3 inch by 2 inch photograph to 18 inches by 12 inches. Can this be done without cutting or distorting the picture? How do you know?

2) A customer brings in an old photograph to be restored and enlarged. She asks Fran if it can be enlarged from the size on the left to the one on the right. What appears to be the enlargement factor here? How do you know?
3) Phil took a photo of a man and a tree and their shadows. The real-life man is 1.75 meters tall. In the photo, the man is 3 cm tall, his shadow is 1.2 cm long, and the shadow of the tree is 4.5 cm long.

Figure 3: Question 3

If the photo included the entire tree, how tall would the image of the tree be? How tall is the real-life tree in meters? Show your work.

4) To help prevent damage to photos that will be displayed on the wall, Phil laminates
the prints with a special sealant. If he needs 400 grams of sealant to laminate a 10 cm by 15 cm print, how much will he need for a 20 cm by 30 cm print?

Figure 4: Question 4

5) If Fran were to put a photograph in the machine and enlarge it once, then put that enlargement in the machine and enlarge it by the same amount again, which series of lines below shows what could happen to an image of a line in the photograph? Explain why.
PROBLEMS WITH FRACTIONS

1) What is the number that can replace the “?” in each of the following problems. Explain your reasoning and show your work for each.
   a) \( \frac{5}{6} = ?/18 \)
   b) \( \frac{8}{5} = 20/? \)
   c) \( \frac{2}{5} = 2.4/? \)

2) Circle the smaller fraction in each pair. If they are equal, circle them both. Explain your reasoning and show your work for each.
   a) \( \frac{5}{7} \) \( 8/10 \)
   b) \( \frac{3}{2} \) \( 18/12 \)
   c) \( \frac{5}{20} \) \( 7/25 \)
APPENDIX G

QUESTIONNAIRE: ATTITUDE TOWARD RATIO AND PROPORTION
APPENDIX G

QUESTIONNAIRE: ATTITUDE TOWARD RATIO AND PROPORTION

Student Number ___________________________ Date ___________________________

Thank you in advance for completing this questionnaire.

Section A: Please place an X in the space that represents you attitude.

<table>
<thead>
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<th></th>
<th>Totally Disagree</th>
<th>Agree</th>
<th>Totally Agree</th>
</tr>
</thead>
<tbody>
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<td>5</td>
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</tbody>
</table>

1. I am confident in my ability to understand the Topic of ratio and proportion.
2. I feel that I have the mathematical knowledge to understand ratio and proportion.
3. Students should have a wide range of mathematical knowledge (beyond that learned in school).
4. The topic of ratio and proportion seems to me to be very complicated.
5. Among the subjects taught in school, mathematics is one of my favorites.
6. The theoretical knowledge I have of the topic of ratio or proportion is not sufficient to understand it.
7. It is important for me to study ratio and proportion.
8. Learning the topic of ratio and proportion is not relevant.
9. I feel that I am capable of understanding ratio and proportion.
10. Ratio and proportion are important for a student’s mathematical development.
11. I do not enjoy mathematics.
12. Ratio and proportions should be included within each mathematics course.
13. Many students have difficulty understanding ratio and proportion.
14. Ratio and proportion seems easy to me.
15. There is no need to teach ratio and proportion.

Section B: Answer the following questions.

1. Please describe a situation/problem related to ratio.
2. Please describe a situation/problem related to proportion.
3. Please indicate concepts/words/subjects related to ratio and proportion.
APPENDIX H

TOPIC VARIABLE
APPENDIX H

TOPIC VARIABLE

(Authentic Investigative Activity)

Introduction:  http://www.youtube.com/watch?v=FtYjUv2x65g

Teaching Objectives:

- Introduce and increase adult learners’ motivation to explore the topic of Ratio and Proportion.
- Test the intuitive knowledge of the adult learners.
- Crystallize adult learners first understanding of the concept of ratio and proportion.
- Use both common sense and proportional reasoning skills, in order to make qualitative and quantitative comparisons efficiently.  (Ben-Chaim, 2012).
Didactic Explanation:

<table>
<thead>
<tr>
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<td>In colloquial terms, proportional reasoning is <em>reasoning up and down</em> in situations in which there is a constant relationship between two quantities that are linked and varying together. It requires explanation beyond the use of symbols of $a/b = c/d$ (p. 3).</td>
<td>In authentic investigative activities, the topic of ratio and proportion is presented across the mathematical and psychological-didactical spectrum. Students learn to solve proportional problems, recognizing the situation as multiplicative, not additive, and to use and compare the information given (ratios, measurements, fraction, percentages, scale, tables, functions, etc.) in a knowledgeable way to find the correct quantitative mathematical answers. Developing these concepts and the ability to use them intelligently is the heart and soul of proportional reasoning. From a mathematical standpoint, the tasks presented in the activities are varied and include problems comparing qualitative data to predict a qualitative answer; problems comparing quantitative values to determine which value is larger and by how much; and problems with missing values, in which three values of a ratio are known and the fourth must be found (p. 75).</td>
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</table>

Connectivity to other topics: fractions, percentages, graphs, and comparison tables.
Rate

http://mathsnacks.com/ratey.php

Definition: A rate is an extended ratio, a ratio that applies not just to the situation at hand, but to a wide range of situations in which two quantities are related in the same way (Lamon, 2008, p. 192).

Learning Objectives:

- Develop the concept of rate in its varied forms, such as miles per hour, price per unit.
- Practice quantitative comparisons and finding missing values in a given proportion by using the rules and properties of ratio and proportion.
- Find qualitative answers for problems that do not require quantitative solutions (Lamon, 2008, p. 192-193).
- Increase interest and motivation in learning rates by including an actual CNN Broadcast on a contemporary issue relating to rate.
Didactic Explanation:

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<td>Rate can be thought of as an extended ratio. It is a ratio that applies not only to the situation at hand, but also to a wide range of situations in which two quantities are related in the same way. They are identified by the use of the word per in their names and they can be reduced or divided (p. 192).</td>
<td>The formation of the rate concept is achieved through problems wherein values of different variables with a connection are compared, such as kilometers (or miles) per hour, price per unit, kilometers per liter (or miles per gallon), grains per volume, density (individuals per square kilometer or square mile), kilograms per cubic meter, and more. Mathematically, the activities in this group aid in developing the concept of rate in its varied forms, and allow practice in quantitative comparisons and finding missing values in a given proportion by using the rules and properties of ratio and proportion. Additionally, they serve to enhance the ability to find qualitative answers for problems that do not require quantitative solutions.</td>
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Connectivity to other topics: Fractions, comparing simple fractions and decimal numbers.

**Ratio** http://mathsnacks.com/badDate.html

**Definition:** Ratio is a comparative index that conveys the abstract idea of relative magnitude (Lamon, 1993). Research in the development of critical ideas for proportional reasoning adopts the theoretical perspective that ratio is a composite unit, formed by comparing units that are a result of multiple compositions of other units (Lamon, 1992, 1993, 1994a, 1994b).

**Learning Objectives:**

- Develop an understanding of the concept of ratio from different viewpoints (Comparisons can be two parts of a whole, or two amounts that are related conceptually, but are not part of a common whole).
- Carry out qualitative comparisons.
- Find missing values (quantitative) in a given proportion by using the rules and properties of ratio and proportion.
- Supplemental information on *Math Snacks* is available at http://mathsnacks.com/about.php

**Didactic Explanation:**

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<tr>
<td>Ratios have many nuances because of the significance of their individual context. It is important to consider what sort of information one can take from ratio problems, whether implicit or explicit. Also, there are implications for other topics, such as sampling, statistics and probability.</td>
<td>The relationship in ratios involves comparison of values or amounts (in the numerator and denominator of a fraction) that have the same unit of measurement. Such comparisons can be two parts of a whole, or two amounts that are related conceptually, but are not part of a common whole. Solving the activities in this group will aid in developing a mathematical concept of ratio from different viewpoints, and will teach the use of the concept to both carry out qualitative comparisons and to find missing values (quantitative) in a given proportion by using the rules and properties of ratio and proportion.</td>
</tr>
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</table>

**Connectivity to other topics:** Fractions—comparing simple fractions, expansion and reduction of fractions, the fraction as an operator-part of.
Scale  http://mathsnacks.com/scaleella.html

Definition: A scale factor shrinks or enlarges all dimensions simultaneously through the operation of multiplication.

Learning Objectives:

- Calculate the enlargement/reduction factor, or a “scale” of a given diagram/figure.
- Find and use a ratio created by enlarging or reducing a picture; linear (1st degree) stretching or shrinking; quadratic (2nd degree) stretching or shrinking of area; and cubic (3rd degree) stretching or shrinking of volumes (Ben-Chaim, 2012).
- Calculate real distance according to a given scale.
- Use scale to compare sizes of objects.
- Supplemental information on Math Snacks is available at http://mathsnacks.com/about.php

Didactic explanations:

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<tr>
<td>A ratio may be extendible or reducible, depending on the sense it makes.</td>
<td>Scaling is a type of ratio that is created when one needs to enlarge or reduce linear (one-dimensional), two-dimensional, or three-dimensional objects. This activity shows how to calculate the enlargement/reduction factor, or a “scale” of a given diagram/figure. It includes finding and using a ratio created by enlarging or reducing a picture; linear (1st degree) stretching or shrinking; quadratic (2nd degree) stretching or shrinking of area; and cubic (3rd degree) stretching or shrinking of volumes; calculation of real</td>
</tr>
<tr>
<td>However, as an abstract mathematical object, both are possible and may be divided into families or classes. Each class contains all of the extensions and reductions of a particular ratio, called an equivalence class. This is seen in graphs.</td>
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</table>
Connectivity to other topics: Fractions, measurements, perimeters and areas of shapes and volumes of bodies.

**Indirect Proportion**

1. [http://www.bing.com/videos/search?q=tour+de+france+vido&FORM=VIRE2#view=detail&mid=8DDBC00CE5B4D9B531398DDBC00CE5B4D9B53139](http://www.bing.com/videos/search?q=tour+de+france+vido&FORM=VIRE2#view=detail&mid=8DDBC00CE5B4D9B531398DDBC00CE5B4D9B53139)


Definition: Indirect proportion or an inversely proportional relationship occurs when one quantity varies with the other, but in the opposite direction (Lamon, 2008, p. 107).

**Learning Objectives:**

- Conceptualize the concept of indirect proportion in a real world example (Ben-Chaim, 2012).

**Didactic Explanations:**

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<tr>
<td>These problems ask students to compare ratios. The connections between equivalence classes, unit rates, and slope, etc. are helpful in understanding the inverse relationships.</td>
<td>This activity portrays situations in which the proportional relationship is one of inverse or indirect proportion. Briefly, inverse proportion between two positive variables occurs when their product is constant. In other words, if one of the variables changes in one direction (e.g., increases), then the other must change in the opposite direction (i.e., must decrease).</td>
</tr>
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</table>
The activity is presented to complete the development of the concept of proportion. Even though the literature has shown that indirect proportion problems are difficult, completing the conceptualization of the proportional concept requires experience with these types of problems, and they should not be ignored.

Connectivity to other topics: Rates, ratios, scale and direct proportions.

Note: A pilot test of these activities was conducted in Fall II term in the same finite mathematics course.

The dependent variables are the individual adult learner’s score and responses on the post-test, “Diagnostic Questionnaire on Ratio and Proportion” and “Questionnaire on Attitude”. The student’s scores on rate, ratio, scaling, total and attitude are the criterion variables or outcomes presumed to be caused by or influenced by the independent variables and any other independent variables (Creswell, 2009). The outcomes measure the direction of change, the amount of change and the ease with which the participant’s scores change from the pre to the post-test (Rosenthal & Rosnow, 1991).
APPENDIX I

CLASS PROJECT
APPENDIX I

CLASS PROJECT

An understanding of ratio and proportion promotes proportional reasoning. A lack of understanding of proportional reasoning has been identified as one of the barriers to students learning higher-level mathematics. The development of proportional reasoning is necessary to fully understand many of the topics in this course. You first encountered this topic in section 6.2 with a brief tutorial. However, moving beyond the solution of linear equations, you will continue to use this concept. For instance in finance, you will study rate of return, which is the profit on an investment over a period of time, expressed as a proportion of the original investment.

Ilany, Keret, and ben-Chaim (2012) developed a model of using authentic investigative activities based on a decade of previous research on adult learning of proportional reasoning. The activities are based on the components of proportional reasoning: rate, ratio, scale and indirect proportion.

Purpose: The purpose of this project is to help you develop an understanding of ratio and proportion using authentic (real life), investigative activities.

Procedure:
1. View each of the five videos and complete the student learning worksheets. There are three worksheets one for each of the Math Snacks videos. Each work sheet should show your answer and detailed work. These work sheets are in PDF format so you can either type or download and write on the document. Please place in the drop box associated with this assignment or email it to me at brennancr@tiffin.edu

Introduction  http://www.youtube.com/watch?v=FtYjUv2x65g

Rate  http://mathsnacks.com/ratey.php

Ratio  http://mathsnacks.com/badDate.html

Scale  http://mathsnacks.com/scaleella.html
Indirect Proportion

1. http://www.bing.com/videos/search?q=tour+de+france+vidio&FORM=VIRE2#view=detail&mid=8DDBC00CE5B4D9B531398DDBC00CE5B4D9B53139


2. Write your own personal reflection on which activities and characteristics of the video were most helpful to your understanding of proportional reasoning and why. For example, was it the instruction or the media that was most helpful and why.

3. Chose a topic we are studying or something in your career or life where an understanding of proportional reasoning will be useful and tell how you will use this to help with the computations and understanding of the concepts.

The entire study should be 3-5 pages, excluding the worksheets.
APPENDIX J

INTERVIEW PROTOCOL
APPENDIX J
INTERVIEW PROTOCOL

Time of Interview:

Date:

Place:

Interviewer:

Interviewee:

Position of Interviewee:

Qualitative Research Question

What characteristics of the investigations are most helpful for adult learners to grasp the variety of dimensions of procedural thinking and conceptual understanding of rational numbers, proportional reasoning and proportions?

1. What characteristics of the authentic, activities did you find helpful in understanding rate and proportion?

2. In Bad Date. What was the topic?
   a. Can you explain?

3. How did that or those characteristic help your understanding and computations of ratio and proportions?
   a. How did you know that?
   b. Please show me. Draw a picture if you want. Is there another way to do that?

4. Why do you think that characteristic was helpful?
   a. Why did you divide circumference by ratio or vice versa?
   b. Is your answer possible?
c. Can you show me where you made the mistake?

d. You seemed to do that in your head. How did you do that?

5. How do you think the characteristics of the activities, such as animation or video, were significant or do you think the presentation didn’t matter only the content?

6. If, for instance, the video was on scale, such as “Scale Ella”. Did that help you to understand ratio and proportion? Please show me. Draw a picture if you want to.

7. Now try this problem: The length of the side of square A is 36 cm and the length of the side of square B is 60 cm. What is the ratio between the areas of square A and square B?

   a. What are you thinking?

   b. How did you get that?

   c. Is there another way you could do that? Please show me.

8. If the students actions and descriptions indicate, these questions will be added:

   a. Seven workers are given a job to do. Four fell sick, not being able even to start, and the remaining three finished up the job. What is the ratio between the numbers of days that the entire group should have taken to the number of days that the smaller group needed?

   b. Two cars are traveling the same route. The first car travels at 60 km/h, and the second at 80 km/h. What is the ratio between the times that the first and second car needs to complete the distance.

Thank you for participating in this interview. I will call you to schedule a follow-up interview.

I want to assure you of the confidentiality of your responses.

Adapted from Creswell (2007, p. 136); Singh (2000)
APPENDIX K

OBSERVATIONAL PROTOCOL
APPENDIX K

OBSERVATIONAL PROTOCOL

Name of Observer:
Place of Observation:
Date:
Time Observation Began:
Time Observation Ended:
Description of Setting:
Portrait of Subjects:
Reconstruction of Dialogues:
Artifacts:
Descriptive Notes:
Reflective on Analysis:
Reflection on Method:
Reflection on Ethics:
Reflection on the Observer:
Points to Clarify:

Adapted from Creswell (2007, p. 137).
APPENDIX L

DATA COLLECTION MATRIX
APPENDIX L

DATA COLLECTION MATRIX

Data Collection Matrix: Type of information by source and frequency

<table>
<thead>
<tr>
<th>Participant</th>
<th>Questionnaire on Ratio and Proportion</th>
<th>Questionnaire on Attitude</th>
<th>Worksheets</th>
<th>Moodle Chat Room Posts</th>
<th>Class Project</th>
<th>Consent Forms</th>
<th>Post-test Questionnaire</th>
<th>Post-test Attitude</th>
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APPENDIX M

CATEGORICAL AGGREGATION OF THEMES
APPENDIX M

CATEGORICAL AGGREGATION OF THEMES

<table>
<thead>
<tr>
<th>Participants</th>
<th>Theme</th>
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</table>
APPENDIX N

LEGEND OF DOCUMENTS
CODE DATA SOURCES
APPENDIX N

LEGEND OF DOCUMENTS
CODE DATA SOURCES

1. Pre Questionnaire on Attitude
2. Post Questionnaire on Attitude
3. Pre Questionnaire on Ratio and Proportion
4. Post Questionnaire on Ratio and Proportion
5. Ratey Learner’s Guide
6. Bad Date Learner’s Guide
7. Scale Ella Learner’s Guide
8. Observation
9. Interview Transcript
10. Moodle Forum
11. Class Introductions

Color codes
Attitude = green
Deepened Understanding = yellow
Characteristics = purple

Codes for poster:

Sarah

1. Pre Questionnaire on Attitude
2. Post Questionnaire on Attitude
3. Pre Questionnaire on Ratio and Proportion
4. Post Questionnaire on Ratio and Proportion
5. Ratey Learner’s Guide
6. Bad Date Learner’s Guide
7. Scale Ella Learner’s Guide
8. Observation
9. Interview Transcript
10. Moodle Forum
11. Class Introductions
Jessica

1. Pre Questionnaire on Attitude
2. Post Questionnaire on Attitude
3. Pre Questionnaire on Ratio and Proportion
4. Post Questionnaire on Ratio and Proportion
5. Ratey Learner’s Guide
6. Bad Date Learner’s Guide
7. Scale Ella Learner’s Guide
8. Observation
9. Interview Transcript
10. Moodle Forum
11. Class Introductions

Tamara

1. Pre Questionnaire on Attitude
2. Post Questionnaire on Attitude
3. Pre Questionnaire on Ratio and Proportion
4. Post Questionnaire on Ratio and Proportion
5. Ratey Learner’s Guide
6. Bad Date Learner’s Guide
7. Scale Ella Learner’s Guide
8. Observation
9. Interview Transcript
10. Moodle Forum
11. Class Introductions

Gregory

1. Pre Questionnaire on Attitude
2. Post Questionnaire on Attitude
3. Pre Questionnaire on Ratio and Proportion
4. Post Questionnaire on Ratio and Proportion
5. Ratey Learner’s Guide
6. Bad Date Learner’s Guide
7. Scale Ella Learner’s Guide
8. Observation
9. Interview Transcript
10. Moodle Forum
11. Class Introductions

Jes 1 = Jessica Pre Questionnaire on Attitude
Jes 2 = Jessica Post Questionnaire on Attitude
REFERENCES
REFERENCES


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