ELEMENTARY TEACHERS’ EVOLVING INTERPRETATIONS
OF THE STANDARDS FOR MATHEMATICAL PRACTICE
IN THE COMMON CORE STATE STANDARDS:
A MULTI-CASE STUDY

A dissertation submitted to the
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by
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The purpose of this study was to explore elementary teachers’ interpretations of three of the Standards for Mathematical Practice in the Common Core State Standards. The research followed how these interpretations evolved during three types of professional learning experiences. The study also explored teachers’ beliefs about the supports that would be necessary to enact these standards successfully in a classroom, school, and district.

A teacher development experiment was utilized for this qualitative study. Three teachers (two from grade five and one from grade one) were recruited from the same school district, and they participated in three individual interviews, two group discussions, and two videotaped lessons as they read and discussed the three chosen standards. Teachers’ comments were analyzed via an interpretive approach reflecting hermeneutic philosophy. Each teacher was considered a case; thus, the analysis focused on each teacher’s thinking as well as the similarities and differences among the teachers’ interpretations. The teachers were able to respond to tentative findings of the study, and adjustments to the analyses were made when appropriate.

Findings indicated that teachers often interpret the text and intent of standards in
very unique ways. These interpretations are influenced by past personal and professional experiences, opportunities to read and discuss standards with others, expectations set forth and support provided by administrators, and observations of student learning. Interpretations do not change quickly or without catalyst; rather, thinking evolves over extended periods of time when opportunities for professional learning and reflection are provided on a regular basis.
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Although I cannot name them here, I am indebted to the three teachers who participated in this study with me, as I learned a great deal from them throughout our conversations and written communication. It was a joy to work with each of them, and I hope that I can continue to collaborate with them in the future.

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CHAPTER I
INTRODUCTION

Policy As a Tool to Support Change in the Educational System

In September 2003, the National Council of Teachers of Mathematics (NCTM) held a Research Catalyst Conference, where many leading mathematics educators and mathematicians from across the United States met to discuss research agendas for the coming years in mathematics education. In one panel discussion, Judith Ramaley of the National Science Foundation commented that one major question for the field was, “What induces change in our educational system?” (Floden, 2004, p. 101) This admittedly broad question seems to encapsulate the goal of many researchers – to find ways to make change for the better to support student learning in mathematics.

In trying to answer this question and induce change for the better, the United States government has become more involved in schooling during the last several decades (Darling-Hammond & Wise, 1981). Courts have handed down decisions that legislators have translated into policy and thus have changed the policies that schools have been operating under, to greater or lesser degrees. Equal opportunity for students has been a driving force behind this movement, and this has come to be understood as equal opportunity of outcome, not just input. This has meant the development of assessments and other indicators that determine whether schools are meeting what has become a legal obligation to provide equal opportunity of outcome. The Coleman Report, for instance, was a survey done as a result of the Civil Rights Act in 1964 that
directed its focus at student outcomes. The Elementary and Secondary Education Act (ESEA) in 1965 initiated the provision of funding to schools for helping to support students who had some sort of disadvantage educationally, and by the end of the decade, the national government was requiring positive educational outcomes based on the use of this funding.

Elmore & McLaughlin (1988) note that a major difficulty with making policy changes at the governmental level is that these changes are proposed and initiated on “electoral time” but implemented in schools in “administrative time” and “practice time.” Cohen and Barnes (1993a) observe that little has been written about how policy educates those who are charged with implementing it, but they point out that policy must educate in order to be effective because it is written to change something about what is happening in a system – that is, the actors in the system must learn if the change is to take place. However, policymakers often lack the understanding that this learning is a crucial part of enacting policy, and policies may fail as a result. Perhaps if policymakers were more explicit about the time and resources necessary to enact a given policy, more policies would have a chance to be implemented well.

Some reforms of the past may have been easier to implement than others because of how they were written – the ideas in them were easier to learn. The so-called “back to basics” reform movement in mathematics of the 1970s and early 1980s, which developed as a public reaction to the “New Math” programs of the 1960s (Wilson, 2003), was not especially challenging to enact because it fit the traditional notions of schooling that most people (including teachers) held already (Cohen & Barnes, 1993a). Enough professional
support was provided to make it seem quite successful, for what it was: skills-driven, low-complexity teaching and learning. Desired behaviors were stated in curriculum materials in simple, understandable terms; instruction was broken into pieces, and many people and institutions across the United States already promoted this view of teaching and learning. However, when policies are not so straightforward, implementing them successfully on a large scale often becomes more problematic.

**Curriculum Policy, Expressed in Standards Documents, as a Tool for Change**

Schwille et al. (1983) write that teachers in the early 1980s had a great deal of liberty in choosing content for their elementary mathematics classes. No content standards were in place on any large scale, and in general, the public did not place a lot of emphasis on standardized test results. These researchers found that the teachers in their study did make changes in content from year to year that depended on the following factors: district content and/or textbook policy, their own perceptions of students’ strengths and weaknesses, traditional content in each grade, and personal experience in previous years. The teachers also usually decided how content would be taught, and to whom, within any given class. (One might ask whether this practice has changed significantly since the 1980s.)

With the rise of governmental involvement in schooling in the United States throughout the 1960s and 1970s, and particularly with the strong call for higher student achievement and more coordination of educational practices in the publication *A Nation at Risk* (National Commission on Excellence in Education, 1983), standards of various types have been used as tools for education reform since the 1980s (Weiss, 2002). In the
1980s and 1990s, there were national pushes in the United States for “hard content” (Porter, Archbald, & Tyree, 1991, p. 11) for all students. The National Research Council, the American Association for the Advancement of Science, and NCTM all published reports echoing this call. This was very different than earlier reforms that had pushed for this type of content for only some students or basic skills for all students (as in the 1970s), and it raised persistent questions about what knowledge was worthwhile for whom, as well as what it meant to teach effectively (Porter, 1989). As a result, research into the influence of standards has been occurring since the 1980s, but Floden and Wilson (2004) point out that at least in most documented research, content standards have been shown to have had more effects on written policy and local expectations for content than on actual pedagogy or student achievement. In other words, standards have affected local policy (as written and/or expected) more than actual teaching or learning.

Tate (2004) observes that there are many different interpretations of the word “standard,” and when using it within research or policy, it must be defined. This definition can also impact how people interpret any given standards document. When standards are created, Tate writes, the authors may intend them to be one or more of the following:

- A vision of ideal practice
- Essential knowledge in a field
- Descriptors of student performance
- Guides to align system components
- Measurable goals for student learning
• Curricular goals for teachers
• Guides in accountability systems
• Descriptors of system inputs
• Mechanisms to discuss prerequisites for opportunity-to-learn and conditions for systemic success

Weiss (2002) also points out that standards can have influence on teaching and learning through their inclusion in curriculum, their use as a basis for teacher development (an element of this study), and their serving as a foundation for mandated assessments that hold schools accountable for student performance relative to the standards.

**The Creation and Implementation of Mathematics Standards**

The National Council of Teachers of Mathematics (NCTM) has published two major sets of recommendations for mathematics standards in pre-kindergarten through grade twelve. In the first, the *Curriculum and Evaluation Standards for School Mathematics* (1989), NCTM articulated a vision for change (not a measurement or assessment framework) (Tate, 2004). The document recommended topics that should be taught in each grade, topics that ought to be de-emphasized (for instance, rote memorization of basic facts and algorithms), and certain general recommendations about practice (for instance, that calculators should be readily available tools in mathematics classrooms in all grades). This document initiated substantial public outcry because many people, both inside and outside of schools, interpreted the recommendations to mean that basic facts and procedures were no longer to be taught at all. Further, since the document said little about how teaching practices would need to change to support these
new goals for student learning, many teachers and schools found it very difficult to support students’ progress toward these goals. In fact, the Video Study that was part of the Third International Mathematics and Science Study (Hiebert et al., 2003) showed quite clearly that in a random sample of eighth-grade classrooms across the United States, the goals set forth in the 1989 NCTM standards were essentially being ignored, even when teachers believed that they were teaching in ways that reflected the standards. This series of outcomes stemming from the release of the 1989 NCTM standards may be the first, or at least most significant, example in mathematics of the need for a relatively common understanding of a set of standards that is meant to guide the development of mathematics programs on a large scale.

In 2000, NCTM released the *Principles and Standards for School Mathematics* (PSSM), which was written to clarify and extend the goals of the 1989 document. This publication contains much more detail about the recommendations for pedagogy in each grade band (Pre-K-2, 3-5, 6-8, 9-12) and much more explanation about what is and is not meant by each standard. Moreover, this document includes more explicit connections among the Content Standards and the Process Standards, which, respectively, are akin to NCTM’s recommendations for the “what” and the “how” of mathematics instruction across the grades. Ironically, some critics of the 1989 NCTM standards called the PSSM a reversal of the 1989 document, and others called it simply an extension of the same (Wilson, 2003). However, this point in itself indicates that even people with somewhat similar perspectives can interpret the same document in very different ways, let alone people who had very different opinions about the two documents.
It is impossible to know if everyone who was extremely vocal about either NCTM standards document had actually read it. Both publications are quite lengthy, and it is unlikely that very many people, whether outside or inside the profession, have literally read every page. So, it is likely that many people were drawing conclusions about the standards (and acting on these conclusions) with minimal information. Even those who did thoroughly and carefully read parts or all of the standards certainly would have had a multitude of different experiences that they brought to this reading, so the recommendations could have resonated very differently in different readers’ minds.

The Common Core State Standards (hereafter referred to as “CCSS”) in Mathematics and English Language Arts were developed over a two-year period under the leadership of the Council of Chief State School Officers and the National Governors’ Association and with mathematics professor William McCallum as the lead writer. Approximately forty educators and leaders from across the country were identified to serve on the mathematics writing group, and three authors were chosen to coordinate the work. The writing group developed the kindergarten through twelfth-grade standards using information from international comparisons of content standards, comparisons of current state standards, research in mathematics education about the teaching and learning of various topics, and the College and Career Readiness Standards (for students graduating from high school) that the same authors had published in the fall of 2009. They also sought feedback on portions of the standards from specialists within specific mathematical fields. A public draft of the K-12 mathematics standards was released online in March 2010, with three weeks allowed for public comment. The final standards
document was published online on June 2, 2010 and adopted in Ohio by the State Board of Education on June 7, 2010. The standards can be found online at the website http://www.corestandards.org/the-standards/mathematics.

In Ohio, the timeline for full implementation of the CCSS ends in the spring of 2015, when districts and schools will be held accountable for students’ learning relative to these standards as demonstrated on new achievement tests, which have yet to be developed. Ohio is participating in two national consortia that are generating various ways to approach large-scale assessment for the CCSS, but as yet, the format of the new assessments has not been determined. Thus, districts and schools will have approximately four years to transition to the CCSS and away from the Ohio Academic Content Standards (OACS), which were adopted in December 2001. However, during these four years, students in grades 3 through 8 will still take the Ohio Achievement Assessments in mathematics (and reading), so teachers will need support in preparing students for the Achievement Assessments while also beginning to prepare them for the new assessments to come.

In 2010 and 2011, the Ohio Department of Education (ODE) generated a number of documents to help districts, schools, and teachers to prepare for the transition to the CCSS. One of the documents currently posted online, called the Crosswalk, is intended to help educators align the OACS with the CCSS and to understand the similarities and differences among the goals of the two sets of standards. Other documents elaborate on the Critical Areas (important skills and ideas) in the K-8 mathematics standards so that teachers and others can better understand the intended learning outcomes for students.
The Model Curriculum, which was posted in its initial form in April 2011, is a series of commentaries that are meant to guide teachers in thinking about and planning for instruction relative to the standards. These commentaries will be expanded and revised over time. They provide expectations for student learning, examples of common student misconceptions, possible instructional strategies and resources, and other information that is helpful in understanding how diverse groups of students may think about different mathematical ideas. In other words, these documents will be very important tools in helping teachers to form interpretations of the CCSS that are consistent with the intent of the authors and the policies of the state of Ohio. All of the aforementioned documents can be found at the following website:

http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEDetail.aspx?page=3&TopicRelationID=1704&ContentID=83475.

Moreover, ODE and other organizations are planning a number of professional development opportunities for teachers across the state in order to help teachers and school leaders become familiar with the standards and begin to collaborate about pedagogical strategies and student learning in general. However, these professional events will certainly not reach every educator in the state, so schools and districts will also need to provide ongoing learning and collaboration opportunities for teachers so that a collective understanding of the standards will begin to emerge each school. In addition, institutions of teacher preparation will need to make the CCSS an explicit focus in their work with practicing and pre-service teachers so that teachers leave university settings more prepared to enact the new standards.
Policy Research as a Means of Supporting the Implementation of Mathematics Standards

Even just given these few examples of mathematics standards that have been published during the last three decades, it is clear that policies have accumulated in schools over the last several decades, with little having been taken away (Cohen & Barnes, 1993a). A major dilemma that faces professional educators today is to determine how schools and teachers are to work with all of these policies in a way that makes sense to support student learning.

In 2000, a group of mathematics, science, and technology educators convened to discuss and create a framework for doing research on the influence of standards in these fields (Weiss, 2002). The framework has two major guiding questions:

1) How has the system responded to the introduction of nationally developed mathematics, science, and technology standards?

2) What are the consequences for student learning?

Porter, Archbald, and Tyree (1991) echo the foundation of this framework, as they write that if the goals for education continue to shift over time, we need to know more about:

- Types of policy instruments that can serve the empowerment strategy [where teachers are given more authority to make decisions] and their effects on practice
- Connections between policy and classroom practice
- Effects of policy on students
- Connections among policies at various levels of the hierarchy (effective alignments of policies at different levels)
Talbert and McLaughlin (1993) also suggest that we need to know what conditions in the many contexts of teaching make it more or less likely that a teacher will become able to “teach for understanding” (p. 167). We need to know this in order to foster and design these contexts. Further, they write that policy research must be sensitive to context so that it can inform future reform efforts. It also needs to help practitioners understand how change can really occur in various types of real settings – not the ideal settings that perhaps have been used too often in research. They conclude: “Policy research thus can play a strategic role in supporting systemic change in practice by describing, interpreting, and broadcasting contextualized examples of the teaching and learning activities reformers pursue” (p. 194).

Two additional research questions that were raised at the NCTM Research Catalyst Conference in 2003 were: “How can we enhance the penetration of Standards [specifically, the NCTM Standards] into practice?” and “How are standards implemented in various contexts and settings?” (Floden, 2004) As Weiss (2002) writes, national standards such as those published by NCTM set forth a vision, but it is important to know how contexts and other forces influence what happens in schools and whether that vision is achieved (and, if so, how). Stein and Shields (2004) write that one overarching question addressing these issues was stated at the Research Catalyst Conference as, “What are the formal and informal structures in local settings that lie currently opaque (i.e., that obscure both the standards and the tools that are designed to assist in their enactment), and what mediational perspectives enable local innovators to implement standards-based tools as intended by their developers?” (p. 142)
Studying Policy from the Perspective of the Teacher

When education policy research began decades ago, researchers primarily studied the processes by which policies were adopted and initially enacted. In the 1970s, people began to recognize the importance of knowing how teachers and others responded to policies and what happened in schools and classrooms; this became known as the “implementation perspective” (Stein, 2004, p. 84). As a result of this more recent research, policies not reaching most classrooms and the “educational core” (Stein, 2004, p. 92) have been well documented (Elmore, 1996). Further, Stein (2004) writes that while policy researchers have often focused on how policy and organizational factors have affected reform efforts, mathematics education researchers have often focused on how teachers respond to policy and what the enactment looks like in classrooms. She argues that we need more research that takes into account all of these perspectives.

As early as 1981, Darling-Hammond and Wise were proposing questions they might study about teachers’ perceptions of their own roles and of the purposes of schooling/teaching. They wondered whether the teachers’ views of schools would be so different than bureaucrats’ views that it would be useful to try to create a new conceptual framework to characterize schooling from the perspective of the teacher. They note that this would be important in designing future policies that were directed at the activities of teachers. Working toward this same goal, the Center for Research on the Context of Secondary School Teaching was established in 1987 to study, among other things, teachers’ perspectives on all elements of the teaching process in secondary schools. Talbert and McLaughlin (1993) note that this work differs from research that looks in
from the outside, through pre-established frameworks and theories. They use constructivism as a theoretical framework (and a way of illustrating teaching for understanding) as they explore the realities of teachers, and they maintain that research from the teacher’s perspective allows us to gain a clearer view of the multiple factors that influence practice and policy enactment. Further, doing research with teachers in a variety of contexts helps to show that the context can extensively impact outcomes in ways that vary from setting to setting because of other factors that are also a part of these contexts. That is, one kind of influence does not always lead to the same outcome in different settings, an idea that is very difficult for anyone wishing to operate entirely from a rational (purely logical) perspective to understand.

Porter, Archbald, and Tyree (1991) write that teachers must be seen (but have not generally been seen) as an important part of curriculum reform. However, training teachers to effectively enact elements of reform is very challenging, both philosophically and practically, for the reasons cited above. To ease this challenge over time, Stein (2004) notes, some researchers have been more closely examining how teachers and others interpret, or make sense of, policy; she cites Spillane and Jennings (1997) and Jennings (1996) as examples. Often, researchers have found that teachers and other local actors adopt the surface features of reforms (such as language, or tools) but not the true intent of these reforms in terms of changing teaching practice and the nature of student learning in mathematics. Also, teachers, who often work in fragmented professional environments, do not have much chance to make sense of new policies or standards
together, so their own individual interpretations often tend to shape their practice relative to these policies.

**Research on the Professional Development of Teachers in Standards-Based Systems**

Romberg and Collins (2000) write that in a standards-based curriculum, we need to give students the opportunity to learn more mathematics, which, instead of meaning learning more facts and procedures, means learning more about relationships among mathematical ideas and representations, spending more time reflecting and asking questions, and exploring fewer topics at a given time to allow for in-depth learning. A consequence of this expectation for school mathematics would be that teachers also learn to view mathematics learning in this way and learn how to structure classroom experiences so that this learning can take place. Research has the potential to support this goal, but this is a relatively new issue within the field of mathematics education research.

Lesh and Lovitts (2000) argue that there should be “an enormous return from giving greater attention to the development of knowledge projects whose objectives are the development of materials, programs, and teachers” (p. 59). They point out that standards-based reform in mathematics and science has been much more successful in areas of these disciplines where the most research about student learning has been done. They argue that teachers ought to know how mathematical ideas develop in individual students, how mathematical ideas have developed historically, and how these ideas can develop in instruction, as well as how these ideas are used in real-world contexts. However, currently, research literature does not offer much insight into how teachers develop these understandings.
Romberg and Collins offer the following questions to orient research related to teacher development in a standards-based educational system (2000, p. 83):

• How is learning for understanding in both school mathematics and school science best characterized?

• What are the important ideas in both school mathematics and school science that we expect students to understand?

• What are the critical instructional features in classrooms that promote understanding for all students?

• What is the appropriate role for teachers in such classrooms? (How can they be helped to assume their appropriate role effectively? How can changes in teachers’ beliefs and practices be made self-sustaining?)

• What organizational capacity – in the school and larger community – is required to support and sustain the development of classrooms that promote understanding?

• What strategies are effective in providing both information and support to policymakers, school administrators, and teachers so that they utilize the findings to create and support classrooms that promote understanding in mathematics and science?

This study helped to provide data and findings relative to each of the questions above, as I will explain further below.
Studying Teachers’ Interpretations of Standards

“Teachers determine what is taught in school.” (Porter, Floden, Freeman, Schmidt, & Schwille, 1988, p. 96) They make decisions about time, topics, which students are taught which content, when in and what order topics are taught, and to what standards of achievement topics are taught. In other words, they create the opportunity to learn. Darling-Hammond and Wise (1981) also observe that teachers are the real arbiters of whatever decisions are made for any given student and the experiences that the student is able to have – teachers make the decisions, and we cannot be sure what might inform those decisions. They note that Jackson (1968) writes that rather than being rationalistic, teachers often make decisions in ways that allow them to fit in with the social structures in schools and in their own classrooms; so, content choices (at least in the past) may have been based more on social acceptance by peers than on what would have been best for student learning. Further, Lortie (1975) found that most teachers in the 1970s derived their beliefs about best practice from their own experiences – as students, and as professionals in their own classrooms. Indeed, Porter et al. (1988) write that in the 1980s, research had indicated that teachers’ choices for content often did not reflect the advice of professional mathematics educators or principals. Elmore (1983) also notes that administrators have little influence over teachers’ decisions about practice and content on a daily basis, and he maintains that as a result, teachers need to be involved in planning for the decisions that they will make. Further, Floden and Wilson (2004) report that teachers often put together a curriculum for their students based on a variety of messages, resources, and their own beliefs about what is best for student
learning. However, teachers’ ongoing learning (thus, their decision-making) may be constrained by the contexts in which they work and the opportunities available to them (Weiss, 2002). Porter et al. (1988) conclude that teachers hear many different messages about content choice and have their own beliefs as well, so persuasion to change seems to be more effective than mandating change. That is, teachers make choices that align with what they think is most effective for students until they are convinced to make a change.

Moreover, Weiss (2002) reports that from the conference of mathematics, science, and technology educators in 2000, the following four questions were written to guide inquiry into the influence of standards. These can be applied to various policies, practices, and programs within the educational system (at any level) and to outside influences that may affect this system, but Weiss also specifies how these questions would apply to research on teachers’ perspectives:

1) How are nationally developed standards being received and interpreted? (and, How have teachers received and interpreted the standards?)
2) What actions have been taken? (and, What actions have they taken in response?)
3) What has changed as a result? (and, What, if anything, about their classroom practice has changed?)
4) Who has been affected and how?

This study focused very directly on a local instance of the first of these questions. Weiss further elaborates on this question in the following way:
Because the vision expressed in the standards for student learning, teaching practice, and system behavior is conveyed through broadly framed statements, it is subject to interpretation. Accordingly, individuals throughout the system will necessarily engage in various forms of sense-making, drawing on prior beliefs, knowledge, and priorities, as they give educational and operational meaning to the standards (Spillane and Callahan, 2000). Thus, to understand anything about the influence of standards, answers to this first central question are needed. The answers will reveal much about how expectations embedded in nationally developed standards are understood, and whether they are accepted, rejected, or altered in that interpretive process. (p. 36)

Moreover, Fullan (1991) writes:

Acting on change is an exercise in pursuing meaning. Selected educational reform that takes individual meaning and development seriously not only stands a better chance of being implemented; it also offers some hope for combating the stagnation, burnout, and cynicism of those in schools – which in the long run will lead to the desiccation of all promising change. (p. 351)

Fullan goes on to argue that anyone working toward a goal of educational change needs to be aware of the phenomenology of teachers (and others in the local context) before any change process can begin. False clarity can occur when people think they have made change and have really not – i.e., when they believe that they understand the goals and
processes related to the change but actually do not. So, change cannot occur unless the meaning for the change is shared among participants. If this is true, in schools today, local actors must understand the Common Core State Standards in similar ways, even though implementation may still play out somewhat differently in individual schools and classrooms.

**Studying Teachers’ Interpretations of One Portion of the Common Core State Standards**

Floden and Wilson (2004) maintain that future research about standards in mathematics education needs to focus on fairly specific aspects of standards and should try to determine the influence of these aspects in specific contexts – rather than trying to do very broad research on an entire set of standards. They suggest targeting a subsection of the *Principles and Standards for School Mathematics* (NCTM, 2000), and this study did target a portion of the CCSS. Floden and Wilson suggest that interviews, surveys, and observations would be effective means of examining how standards are influencing teaching and learning in particular settings.

At the NCTM Research Presession in Indianapolis on April 13, 2011, Fran Arbaugh commented (in a session entitled “Research Opportunities Arising from the Standards for Mathematical Practice”) that the mathematics education community must develop a “common understanding” of the CCSS. She proposed that one way to both explore and shape current understandings would be to conduct research around professional development that focuses on teacher knowledge of the CCSS, specifically, the Standards for Mathematical Practice (SFMP), which are a set of eight practices that
the authors of the standards feel both students and teachers should be developing throughout their years in school. Arbaugh focused on the idea of studying teachers’ own use of these Practices in their own learning and instruction, but she also mentioned teachers’ “own orientations toward the Standards for Mathematical Practice” – which seems to speak to teachers’ understandings of the ideas and intentions behind these standards.

In this same NCTM session, Valerie Mills commented that researchers could study the use and knowledge of the SFMP, as well as to design activities and tools that support, enhance, and/or extend some aspect of their practice. She said that we need to know how teachers might use the document to inform instruction and deepen students’ understanding of content, and that it is crucial for us to know now (as states are planning for eventual implementation) what learning activities will enhance teachers’ knowledge and own use of the SFMP. Dan Heck, also a panelist in this session, echoed this point; we need to be able to provide guidance for teachers who are implementing the CCSS and the SFMP within them. Mills also observed that in order for professional development to be effective on a large scale, we in the mathematics education community need to have a collective understanding of the SFMP – that is, the Practices need to be explicated and related to each other and to the content standards.

**Changing the System**

Ferrini-Mundy (2004) suggests that educators tend to believe that elements of an educational system are aligned when they share a “common curricular framework” (p. 27), and she writes that mathematics education researchers need to be looking at what
this common framework looks like in practice. She suggests that mathematics education researchers need to work with policymakers to design analytic frameworks for alignment that involve both content and pedagogical recommendations (i.e., standards for what is to be taught and learned as well as how it is to be taught), since missing from most proposed reform is a clear illustration of what teaching would look like under the new policy and how to teach in this way. However, we are starting to see more examples in research literature of how policies could emphasize both types of ideas (Cohen & Barnes, 1993b). Magdalene Lampert, in her plenary presentation at the NCTM Research Presession in Indianapolis on April 11, 2011 (entitled “Maintaining Ambitious Teaching: Constraints, Affordances of Schools, Professional Education, Policy”), also noted that the mathematics education community should emphasize teaching to the big ideas and should work to raise the “overall level of consciousness” about what it means to do this. As well as being helpful to educators in general, this would be especially helpful to district leaders who are responsible for the enactment of policies and reforms. As noted by Paul Cobb and Kara Jackson in their plenary presentation “Toward an Empirically Grounded Theory of Action for Improving Mathematics Teaching Quality at Scale” on April 13, 2011 at the same NCTM conference, little research exists to guide administrative leadership in mathematics education.

At the NCTM Research Presession, Valerie Mills (session cited earlier) noted that we must frame research questions in a way that is ultimately relevant and productive for any teacher by asking what experiences are needed to support the development of practices that are aligned with the standards. Kelly Edenfield, another presenter at
NCTM on April 13, 2011, shared in a session called “Studying Higher-Order Thinking During State Curriculum Reform Implementation” that in her study of a small sample of middle school teachers in Georgia, she found that teachers needed to understand statements within standards documents in order to enact the standards effectively for students. She cited Hargreaves (1994), who writes that we need to know what teachers think, believe, and understand in order to help them to make particular changes (or moves) in instructional practice. Moreover, Shulman (1983) writes that above all, we must prepare and support teachers well so that they may ultimately serve students in the way that the system would hope.

Based on the recommendations of the many researchers and authors cited above, this study explored teachers’ evolving interpretations of the overall goals and the Standards for Mathematical Practice in the Common Core State Standards in light of the teachers’ own grade level content. The primary goal in doing so was to provide information to the field about how to support teachers in learning from standards documents and from each other, so that they may make the necessary shifts in their own practice to best support the type of student learning that is described in the standards.

**Research Questions**

This study was based on two main research questions, the first composed of three parts. The research questions were:

1a) How do elementary teachers interpret the overall goals of the Common Core State Standards in Mathematics and the Standards for Mathematical Practice relative to students at their own grade levels?
1b) How do elementary teachers envision these goals and Practices developing in their own schools and classrooms?

1c) What do elementary teachers believe will be necessary supports for schools and teachers as the Standards and Practices are enacted?

2) How do elementary teachers’ interpretations and beliefs about the Standards and Practices evolve throughout the following professional experiences: conversations with other teachers and the researcher, and reflection on lessons in which they attempt to enact content from the Common Core State Standards and Standards for Mathematical Practice 1, 3, and/or 7?
CHAPTER II
REVIEW OF THE LITERATURE

Introduction

Since this study explored elementary teachers’ interpretations of standards and how these changed during collaborative discussions, conversations with the researcher, and reflections on classroom lessons, this literature review includes the following major areas of research:

• General issues of policy enactment
• Teachers’ and others’ general responses to mathematics standards
• Factors that influence teachers’ interpretations of standards (particularly including beliefs)
• Means by which teachers’ interpretations of standards can substantially change
• A hermeneutic perspective on mathematics education

The literature for this review was generated via multiple searches beginning in January 2011 in the Kent State and OhioLink library databases and in Academic Search Complete, a comprehensive online database of research journals. Initial search term combinations included:

• interpretation, mathematics, standards
• decisions, mathematics, education
• mathematics, education, policy
• elementary, mathematics, policy

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• hermeneutics, mathematics, education
• interpret, NCTM, standards (very few results were returned)
• Common Core, standards, mathematics (no results were returned)

I also located many sources that were cited in articles and books that I reviewed and followed through with reading these sources until the topics became removed from those pertinent to my study.

Throughout the process of reviewing literature, it became very clear that few researchers have focused specifically on the interpretations that teachers form of mathematics standards (existing research focus mostly on implementation), and minimal research has focused on mathematics education policy, especially at the elementary level. As a consequence, this study certainly can be informative to the field, particularly as the Common Core State Standards are being enacted.

**General Issues of Policy Enactment**

McDonnell and Elmore (1987) write that state policies have influence on local actors and organizations by providing governmental resources for the enactment of said policies and asserting political authority to assure their enactment. These authors suggest four different types of policy instruments (p. 134):

*Mandates:* These are rules meant to bring people and institutions into compliance within a reasonable degree of variation. The rules assume that *everyone*, however, is capable of meeting them if he/she chooses to do so. Mandates also assume that the policies in them are appropriate for those
affected by them. If mandates fail, they may fail for lack of knowledge or competence on the part of local actors, rather than refusal to comply. 

*Inducements*: These involve the transfer of financial resources in exchange for certain outcomes. They may fail for similar reasons as mandates. 

*Capacity-building policies*: These programs invest in material, human, or intellectual resources. The intended outcomes are not as clear in this case, partially because this type of investment is more long-term; thus, these policies need a great deal of political support in order to succeed because desired results are not obvious to those outside the actors directly involved. 

*System-changing policies*: These redistribute power among actors and institutions in order to change the way that goods or services are delivered. They also need a great deal of political support in order to succeed because of the change in the structure of power among state and local actors. 

Weiss (1990) posits “ideas” as a fifth policy instrument, writing that ideas can help people to envision what could be, to consider alternatives to current practices, and to see social issues from new perspectives. He notes that ideas can be policy instruments in themselves when they catalyze change in behavior even when other, more tangible, incentives to change behavior have not been altered. In education, teachers and others can gain ideas through professional learning and networking and may be persuaded to
change their actions with little or no additional monetary investment on the part of the school district or other motivations to change.

Governments that initiate mandates (or that are responsible to see that they are carried out) can highly influence the results by changing the degree of human and material resources that they utilize to see that the mandates are followed. If policy makers wish to enforce minimum standards, they often use mandates. If they wish to move behaviors beyond certain minimum standards, they often use inducements to try to do so. Porter (1989) writes that it appears that excellence can be a goal set in educational standards, but minimum standards are easier to spell out and mandate within a standards document.

McDonnell and Elmore maintain that it is important to understand the influence of local context on policy implementation. This is particularly true when considering capacity-building and system-changing policies. They note that this is a prime goal for policy research. Darling-Hammond (1990) writes that policy makers must consider how new policy will conflict with or blend with older policies and make this an explicit aspect of crafting new policies. Further, policy must be well communicated if it is to be interpreted as intended, and meaningful discussion among local actors must occur on an ongoing basis in order for it to have lasting, positive effects. Further, policy cannot be expected to take hold immediately after being established; time must be provided along with support.

McDonnell and Elmore explored why people in local contexts respond in certain ways to certain policy instruments and under what circumstances these instruments tend
to have the desired effects, as well as what costs and benefits accrue with certain effects.

One of their main conclusions was that districts tended respond at a surface level to mandates but seemed more likely to accomplish greater change with capacity-building and system-changing policies. However, most policies are mandates or inducements because financially and strategically, they are simpler to implement.

Firestone (1989) describes four general types of reactions to state policy, along two dimensions: “propensity to action” on the part of stakeholders and “perceived utility” of the policy (p. 157). Table 1 is a reproduction of the matrix which he uses to represent these reactions:

Table 1

<table>
<thead>
<tr>
<th>General Types of Reactions to State Policy</th>
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</thead>
<tbody>
<tr>
<td>Perceived utility</td>
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<tr>
<td>Propensity to action</td>
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<tr>
<td>High</td>
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<tr>
<td>Low</td>
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</tbody>
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Firestone notes that active use of a policy, the response for which policy makers would hope, involves eliminating or redefining typical tasks that have been carried out in the past. This can be difficult for local agencies that are entrenched in past practices. Moreover, when policies fail, it is sometimes because the initiating governments have not considered carefully the financial resources needed to support the policies. They may
have simply elected to ask the local agencies to assume most of the cost, which may not always be feasible locally.

**Issues of Enacting New Educational Policy**

Cohen and Ball (1990) point out that it does not cost much money, relatively, to write new policies that demand new instructional practices that focus on engaging students in mathematical activity. However, the successful implementation of such policies depends on the commitment of far more time and resources than the policy generally implies. Also, state education departments often struggle to build capacity for change in school districts because they lack the human and material resources necessary to develop these sorts of resources in local contexts (McDonnell & McLaughlin, 1980).

Local actors who have influence on local policy highly determine the response of a school or district to state policy (Firestone, 1989). This can depend on how these actors interpret the policy and how they view its implications for the school or district. These interpretations can depend on special interests and other aspects of the experience of these actors. However, as aforementioned, most districts comply with mandates most of the time. Inducements encourage districts to experiment, which also often succeeds when districts are willing to make this effort.

Given the literature related to mandates, inducements, capacity-building policies, and system-changing policies, it is interesting to raise the question of whether the CCSS are mandates, or whether they will be interpreted as such by state governments, local districts, administrators, or teachers. In Ohio, they will provide the content for state testing in mathematics, and schools and even (eventually) school staff can be penalized
for long-term failure to achieve current state standards as measured by performance on these tests. In this sense, perhaps they could be viewed as mandates because compliance (or non-compliance) must necessarily be an outcome of a mandate. On the other hand, changes in pedagogy seem to be intended by the authors of the standards and by the Ohio Department of Education (2011), according to statements in the Model Curriculum for grade 5 such as the following:

- “Because students have used various models and strategies to solve problems involving multiplication with whole numbers, they should be able to transition to using standard algorithms effectively. With guidance from the teacher, they should understand the connection between the standard algorithm and their strategies.”

- “Allow students to add and subtract fractions using different strategies such as number lines, area models, fraction bars or strips. Have students share their strategies and discuss commonalities in them.”

- “Present problem situations and have students use models and equations to solve the problem. It is important for students to develop understanding of multiplication and division of fractions through contextual situations.”

These statements seem to imply the need to change practice over a long period of time, which might allow one to argue that the CCSS and supporting resources in Ohio are more accurately classified as capacity-building policies – in which case, it would be more difficult to determine whether the standards have been achieved in practice because intended results are less clear in this type of policy.
New policies always necessarily build on the foundations of old policies in practice – people do not generally just stop doing what they were doing before and start something new. New ways of thinking about teaching and learning – and then enacting them – must be pursued jointly by teachers and students (Cohen & Ball, 1990). Change in schools ultimately depends on the actions of teachers (Fullan, 1982), though district-level policy makers (perhaps influenced by others such as teachers, parents, and principals) determine how the district as a whole will respond to state policy. Further, Porter (1989) writes that people generally either believe that teachers should have relative autonomy in choosing teaching methods and (to some degree) content, or that content and perhaps methods should be given to them and required of them. The question to consider is whether one of these, or some blend of them, results in the highest quality teaching and learning.

**Teachers' and Others' General Responses to Mathematics Standards**

Brown (2001) describes the varying perspectives of mathematics that exist in the United Kingdom and in other places around the world. He discusses the fact that people with different relationships to the educational system view mathematics very differently and see it as serving different purposes within society. In particular, he names teachers, policy makers (whose agendas may change given the current political/economic/social circumstances), university researchers in mathematics, university mathematics educators, and those who utilize mathematics in the business world. He also notes that children in current generations understand the world in quite different ways than adults, and this will only continue to increase as time goes on. He describes two main perspectives that are
held by people in these groups: the phenomenological perspective (i.e., that mathematics is created by and experienced by each person differently) and the “official” perspective (p. 267) (i.e., that mathematics is a pre-determined set of ideas to be passed along from experts to novices). He suggests that within this very complex state of affairs, teachers must be able to position themselves so that they understand their own perspectives (and how they change) while working to serve the needs of other perspectives as well, which can be quite difficult when often policy changes more quickly than the educational system can handle. Another challenging element of this for many teachers is the fact that they have not been entirely comfortable with mathematics since their own schooling years, and so they must work to alter their own conceptions and increase their own knowledge while still trying to serve the desires of others.

According to Ferrini-Mundy (2004), at the NCTM Research Catalyst Conference mentioned earlier, Joe Merlino and Mari Muri also discussed the importance of understanding the various perspectives of people who respond in one way or another to mathematics education reform and standards. They both noted the challenges of addressing all of the different concerns that these groups have. Both Hill (2001) and Porter and Smithson (2001) comment on the idea that standards can be interpreted differently in local contexts, so local actors may entirely miss the intent of a set of standards (Ferrini-Mundy, 2004). In addition, Stein and Shields (2004) write that national standards come to local actors through tools (e.g., curricula, professional development, accountability policies, state assessments) but are mediated through social structures by people with many different perspectives, as described by Lisa Rosen at the
conference. The five “dimensions of mediating structures” (Stein & Shields, 2004, p. 146) that Rosen suggested are:

- Technical dimension (human and technical capacity at a basic level)
- Political dimension (interest groups, relations among them, their relative influence on leaders, various political conflicts, distribution of resources, past memories of reform)
- Normative/cultural dimension (attitudes, beliefs, values of groups involved in implementation)
- Socio-relational dimension (relationships among adults involved, social/professional networks)
- Structural dimension (formal rules or structures that define the environments in which the adults work)

These social dimensions influence the enactment of policy by shaping the learning of new practices, the use of new tools, the interpretations of goals and the intentions of leaders, and any assessments of what local actors know relative to the policy.

When faced with a new reform, teachers judge (whether they realize it or not) the potential merit and chances for success of the reform based on whether it will make the work of teaching more manageable and increase the positive outcomes of teaching more so than creating more challenge. They also consider whether students are likely to learn more effectively from the reform than they have in the past. If teachers are doubtful about one or more of these, the reform is unlikely to succeed (Chazan, Callis, & Lehman, 2008). One study that explored teacher reactions to new standards was conducted with
66 fourth-grade teachers around Michigan State University (Floden, Porter, Schmidt, Freeman, & Schwille, 1981). These authors describe a research technique called “policy capturing” (p. 130), which entails presenting participants with a series of statements about hypothetical situations in which they need to make decisions related to policy and observing how they respond. The teachers were told to imagine that they had changed schools and that they were being subjected to various pressures from principals, parents, and other sources of influence on their decisions. The researchers found that teachers generally were willing to conform to policy as best they could and yet were hesitant to omit topics that had been taught once they had been teaching these topics for several years.

Another study that examined teachers’ perceptions about the effects of standards on their professional experience was conducted with 195 high school mathematics and social studies teachers in 12 schools in urban districts in California, New York, and Florida in 1989-1990 (Archbald & Porter, 1994). The goal of the study was to determine whether teachers in districts with more centralized control over curriculum policies felt more constrained in making decisions related to content and pedagogy than those in districts where control was not centralized. In general, the researchers found that in the districts with high centralized control, the teachers felt that external tests and curriculum guides were a strong influence on the content they chose, significantly more so than in the districts with medium and low levels of centralized control. However, teachers in all types of districts felt similarly that they themselves exerted high influence on their teaching methods via their beliefs and individual decisions. Further, although the
influence of tests and guides was stronger on content in the higher control districts, the influence of textbooks on content was not statistically different in these districts.

Brown (2001) maintains that often university training for teachers and future teachers focuses on affective, phenomenological perspectives of mathematics learning, whereas policy discourse generally focuses on more structural perspectives (i.e., the “what” and the “how” of teaching). He suggests that universities may need to be more intentional about preparing students to encounter multiple perspectives of stakeholders when they enter the workplace and to understand how these perspectives arise and what implications they have for practice. One could argue that this recommendation would also hold for practicing teachers; they ought to be familiar with different perspectives of mathematics since they must somehow respond to each of them in practice.

When we study how standards are interpreted, we must recognize, as Floden and Wilson (2004) point out, that a great deal of variation exists in the way that teachers, schools, districts, and states respond to standards; further, the within-school differences are often greater than the between-school differences. This means that we must look carefully at the characteristics of each context and/or individual in order to better predict what might happen given the introduction of new standards. In order to know how most teachers respond to standards (and to help most of them do so), we have to study and change local and organizational contexts in order to create environments in which teachers can learn to enact new ways of teaching (Stein, 2004). Stein also suggests that much more research needs to be done to study how teachers can learn about the intent and pedagogy associated with mathematics education reform and how school districts can
create environments for this learning (and must do so for reform to be effective). We need to start with the end result that is desired for student learning and work backwards, up through the system, to determine what local support and what more broad policies would support this sort of learning. Clune (1991) echoes the idea that we should try to use research to inform the design of coherent systems of instruction at the school and classroom level, the inclusion of higher-order skills, and the development of systems that maximize participation, excitement, and ownership among teachers. Further, we need to determine whether policy can truly impact classroom practice, and we need to better understand the degree to which conflicts between central and local control (or local autonomy) impact student learning.

Factors that Influence Educators’ Interpretations of Standards

Policy makers tend to believe that once a policy is established, it will be enacted, but the time needed for implementation is generally far longer than the timelines written into the policy (Darling-Hammond, 1990). In education, this is often because teachers (the ultimate enactors of policy) have little opportunity engage directly with the language in the policy and to consider on an ongoing basis what it means for their teaching. Instead, they hear messages about the policy from administrators, professional developers, parents, the media, etc., and they must decide how to implement the policy amidst all of these influences and their own beliefs and experiences. Further, districts are not often prepared to help teachers sort through these often mixed messages and may leave teachers to accomplish this task on their own.
Porter (1989) notes that a great deal of diversity exists among teachers in the United States in terms of knowledge and experience, and he wonders whether any set of external standards can sufficiently inform all of these teachers in order to guarantee good teaching and genuine learning in all classrooms. Further, McDonnell and Elmore (1987) write: “Standards, even when they are clear, are limited in the degree to which they can significantly change behavior.” (p. 142) The authors offer speed limits as an example (i.e., the fact that many people do not follow them), and they point out that academic standards are much less easy to understand, even at a surface level, than simple laws like speed limits. Cohen and Ball (1990) add that language in policy documents is often so vague as to be open to a great many interpretations, and thus the true spirit of the reform is often lost as people either do not change practice at all, change only subtly, or change in ways that do not at all mirror the purpose of the reform.

Cohen and Ball note that ultimately, instructional policy does have an effect on practice, but often this effect is very different from one classroom to another due to teachers’ interpretations of the policy. They also argue that those who are concerned about standards tending to push instruction in one particular direction need not be concerned because teachers base their practices on their own understandings of standards, which, as the authors and others have shown, vary widely due to many influences on the messages that accompany the standards from the public, government, and teachers’ own belief systems and experiences. This was also demonstrated in the United Kingdom with the advent of curricular reforms in the 1990s. Millett and Johnson (1998) note that prior to a study they conducted of interpretations of the new mathematics framework by
educators at various levels (described below), anecdotal evidence had suggested a variety of interpretations of the national mathematics framework by teachers, administrators, inspectors, and policy makers, thus creating a rationale for studying how these interpretations were affecting evaluations of teaching practices and subsequent reports on schools’ compliance with policy. They also write that other studies had shown that curricular materials had been interpreted in different ways (Bowe, Ball, & Gold, 1992; Knip & van der Vegt, 1991).

**Individual Interpretations of Written Text**

When an author creates a text, he/she is trying to make this text understandable for a particular subject (Taylor, 1971). Presumably, this is true in the instance of the CCSS, but questions could be raised regarding who the subjects are in this case (e.g., teachers, administrators, policy makers, citizens). Gadamer (1960; cited in Gallagher, 1992) writes that even though the reader and a text may share a common tradition, the text must present itself as being in need of interpretation (something unfamiliar). When a reader interprets a text, the very traditions which have surrounded them change in the process. The original meaning of a text is transcended when a reader reads and interprets it. There is not universal agreement about the nature of language and how it affects interpretation (Gallagher, 1992), but Gadamer writes that language limits but also empowers our understanding. Gallagher claims that language is a necessary condition for human existence and that we interpret reality by interacting with it through language (though we might question whether it is the only way through which humans interact and learn, and this issue will be discussed later in the context of mathematics teaching and
learning). Still, Dewey (1958) agrees; he writes that we give meaning to parts of our world through language.

We do not entirely control our own processes of interpretation because we are caught up in them constantly (Gallagher, 1992). Our practical interests affect the way that we understand and interpret our world and what we tend to focus on in our own experience (Nicholson, 1984). Dewey (1916/1966) feels that we tend to allow the biases of our social groups to influence the way that we interpret our worlds. Also, he agrees that our practical interests bias our interpretations. Gallagher writes that the reader is “conditioned” (p. 5) by his/her culture, beliefs, experiences, education, linguistic ability, familiarity, interests, etc. The text is “conditioned” by its age, the culture in which it was written, the author’s purpose, and the author’s background. Meaning does not reside entirely in either and is created continually as a text is read and interpreted. In hermeneutics, researchers often explore the factors that affect the process of interpretation. Moreover, the object of interpretation is never entirely familiar, but it is never entirely unfamiliar. Educational experience should open us to the unfamiliar but also raise questions about the familiar. Hermeneutic inquiry can help us to bridge the gap between these.

One key element of interpreting texts is conversing with others about these texts and the readers’ relationships to them. “Conversation forms and transforms minds,” (Fairfield, 2011, p. 89) so it should not be seen as an optional part of education for adults. Dialogue helps them to examine their own ideas and those of others and to understand points of agreement as well as dissimilarity. It is important that participants have
sufficient background knowledge in order to engage in conversation that is truly educative for those involved; for teachers discussing new standards, this would mean having classroom experience and knowledge of prior standards. Further, as participants work to develop their own interpretations, they must be able to accept that someone else’s ideas may be valid and that their own may be wrong. Nevertheless, sometimes we “resist” our education that seeks to broaden our thinking and judgment because it is uncomfortable, so we revert to using more familiar vocabulary and more familiar ways of interpreting the world and our relationship to it (Ramsey, 2011, p. 98).

Teacher Beliefs in Mathematics

“However the only general conclusion so far about belief research is that no general conclusions should be expected in belief research.” (Skott, 2001)

The quote above aptly characterizes one poignant observation that can be offered after an analysis of the research on teacher beliefs about teaching and learning mathematics and about the discipline itself. Essentially all of the existing studies have served to highlight the complexity of beliefs and belief systems, and authors have raised at least as many new questions as they have provided answers.

The formal study of mathematics teacher beliefs began in the 1980s, though many authors cite Green and his chapter on teacher beliefs in his text *The Activities of Teaching* (1971). Green suggests that beliefs may be “primary” (foundational) or “derivative” (based on primary beliefs), and he also suggests that beliefs may be “central” (highly important to the believer) or “peripheral” (not as important). He writes that it is very difficult to challenge primary and central beliefs, which often may be the beliefs that
guide teachers’ decisions. In addition, Green notes that two central beliefs may actually conflict with each other but that the teacher may not recognize this because the beliefs seem to be unrelated, at least in terms of the teacher’s own experience. Thus, Green observes that beliefs can be inconsistent with each other. Interestingly, however, Green appears to be writing simply based on his own observation because he cites no work that would support his assertions.

**External conceptions of mathematics and teacher-directed pedagogy.** Many researchers have documented the fact that teachers and pre-service teachers, on the whole, tend to view mathematics as a static body of skill-based knowledge that is complete and unchanging (Collier, 1972; Steffe, 1990; Wilson, 1994; Gregg, 1995; Philipp et al., 2007; Agudelo-Valderrama, 2008). They also tend to believe that the teacher’s responsibility is to clearly explain these pre-existing rules and procedures so that students can learn them through listening and practicing. In addition, teachers who believe that their job is to “tell” students information often experience dramatic decreases in self-efficacy (the belief that they can affect student learning) when they first attempt to move away from “telling” to more constructivism-oriented practice (Smith, 1996).

Approaching the topic from a different vantage point, Simon, Tzur, Heinz, Kinzel, and Smith (2000) propose two perspectives of mathematics that pre-service teachers (or veteran teachers) may hold: a perception-based perspective and a conception-based perspective. Teachers with a perception-based perspective may or may not believe in teacher-directed instruction, but they do believe that mathematics exists outside the mind and that the role of a teacher is to help students perceive and make sense of the existing
relationships among mathematical objects. Those with conception-based perspectives believe that mathematical concepts and relationships are constructed within the mind of the student and that each student may conceive of these differently; hence, the role of the teacher is to try to understand what students are thinking and to provide experiences that will help students to construct mathematical ideas for themselves. This is a subtle yet important distinction that could inform professional development for teachers who do not feel that they should be the authority in the classroom but are still not thinking particularly about how students are making sense of ideas and relationships in mathematics.

Generally, researchers believe that mathematics educators must endeavor to convince teachers to change their beliefs in favor of more student-centered pedagogy and internal conceptions of mathematics, but this can be very difficult because most school mathematics cultures are still very traditional (i.e., teacher-directed) (Gregg, 1995). Also, many teachers are not trained to implement this type of pedagogy on their own (Beswick, 2007). However, many researchers are motivated to help teachers alter their beliefs because they have found that teachers with more internal, conception-based perspectives of mathematics see increases in student learning (Wood & Sellers, 1997; Simon, Tzur, Heinz, Kinzel, & Smith, 2000).

Factors that can cause changes in teacher beliefs. One of the major influences on teachers’ beliefs about learning is perceived student success, so when teachers start to see increases in student learning after altering their instructional practices to employ constructivist principles, changes in beliefs often occur (Cobb, Wood, & Yackel, 1990;
Cobb et al., 1991; Wood & Sellers, 1997; Senger, 1998; Middleton, 1999). Similarly, when teachers learn to use knowledge of students’ thinking to guide their instruction, they often come to believe in the classroom as a mathematical community in which the sole authority does not lie with them (Fennema, Franke, Carpenter, & Carey, 1993; Fennema et al., 1996). Also, teachers feel more confident and comfortable with new, student-centered pedagogy when they have participated in ongoing professional development that helps them to understand constructivism, both from a theoretical and practical standpoint (Even, 1999), and when they have received collegial support from coaches in their own classrooms (Simon & Schifter, 1991).

Teachers also feel that they need time for training and collaboration within their own school settings, as well as resources designed for student-centered instruction, to move toward this type of practice (Senk, Beckman, & Thompson, 1997; Clarke, 1997; Collopy, 2003). In addition, researchers and teachers have stressed the value of reflection as a necessary component in teacher change (Senger, 1998; Collopy, 2003; Watson & De Geest, 2005). Professional development can also have this effect when immersing the teachers in a constructivist environment as learners themselves so that they can experience what students will feel in such a classroom (Moreira & Noss, 1995). Still, even with the opportunities and resources described above, not all teachers’ beliefs will change, depending on the degree to which they choose to avail themselves of the opportunities and resources and to engage in reflection (Moreira & Noss, 1995; Collopy, 2003). Teachers also need to feel that they have the flexibility to try new methods in their classrooms (Watson & De Geest, 2005).
Overall, this research demonstrates that teachers’ beliefs are highly dependent on their perception of students’ learning, and their beliefs can change when they have ongoing support for new methods of instruction and the time and resources necessary to learn and enact these new methods. However, if a teacher is not reflective by nature and does not feel that any aspect of his/her practice is problematic (i.e., potentially subject to examination), he/she may not be as willing to try new methods even with various types of support. Further, if the teacher perceives that students are not responding well and/or does not have support to enact student-centered pedagogy, beliefs about instruction can become more traditional.

**Context and culture as influences on teacher beliefs.** Many studies have demonstrated that teacher beliefs are influenced not only by previous experiences but also by the context and/or culture in which they work and live. For instance, in the United States, schools and mathematics textbooks often tend to value knowledge that is discrete and skill-based as opposed to holistic and theoretical, so teachers often tend to view mathematics (and teach it) in a manner that reflects these values (Popkewitz, 1988; Gregg, 1995). Also, teachers are often seen as managers in their classrooms, and students are seen as those who should quietly follow all rules that the teacher sets (Popkewitz, 1988). Further, most school mathematics programs are organized in traditional sequences that only provide access to higher-level mathematics to students who have traditionally been high-achievers in procedure-oriented classes; thus, teachers are socialized to believe that only the students in these classes have the innate ability that is necessary for success, and their expectations of students perpetuate this cycle (Gregg,
Similarly, Agudelo-Valderrama (2008) learned that several Colombian secondary teachers’ beliefs about mathematics education were based on their perceptions of what was expected by the school leaders and by students’ parents. In addition, when school leaders attempt to require a new curriculum and/or new practices but do not provide teachers with time and structures for collaboration, teachers may react differently to the new program than they would have with more support (Ponte, Matos, Guimaraes, Leal, & Canavarro, 1994). Highlighting the complexity of beliefs, Skott (2001) found that a new teacher’s beliefs about teaching and learning interacted with his beliefs about effective classroom management styles that suited various purposes, so his beliefs were dependent on the context of the classroom at various times.

International studies can provide interesting insights into teacher beliefs. Whitman and Lai (1990) found that middle school teachers in Japan are encouraged to teach in a way that reaches the greatest number of students but that Hawaiian teachers are encouraged to reach each student individually. Also, the Japanese classes in the study were grouped to include a mix of abilities, whereas the Hawaiian classes were grouped homogeneously. So, the Japanese teachers spent most of their class time on whole-group activities and discussion and believed in allowing students to explore concepts for themselves, whereas the Hawaiian teachers spent most of their class time on reviewing homework or guiding individual students with the current night’s homework and believed in ensuring that students saw mathematically correct ideas in class and setting clear expectations for class behavior. Whitman and Lai felt that the differences in belief were a result of broader cultural norms in each location. In another study, Yang and Cobb
(1995) found that teacher beliefs about key elements in students’ learning of place value varied depending on whether they taught in Taiwan or in Indiana. The Chinese teachers appeared to be exhibiting the cultural tendency to teach young children about number by developing an understanding of groups of tens and ones (and believed that this was most appropriate), whereas the American teachers appeared to reflect the tendency to teach children about number through counting only (and also believed that this was appropriate). Moreover, Andrews and Hatch (2000) found that Hungarian teachers tended to view mathematics education as a procedure-oriented discipline, while teachers in England tended to view it from a more student-centered perspective. These authors found these beliefs to be in line with the general national culture in each country (very pragmatic versus more humanistic). Hence, both culture (in a broad sense) and context can profoundly influence mathematics teacher beliefs.

**Causes of inconsistency between beliefs and actions.** In some cases, teachers’ actions are consistent with whatever beliefs they hold and profess, but this is not always the case, often due to context or non-reflectiveness, as the following studies illustrate. Thompson (1984) studied the beliefs and practices of three middle school mathematics teachers to examine whether their conceptions of mathematics and mathematics teaching influenced their practice and whether they expressed beliefs that were aligned with their observed practice. She discovered that one teacher who had had some very positive and very negative experiences in school mathematics held two differing views of mathematics (one that emphasized mathematical relationships and one that emphasized discrete rules) but did not recognize this. So, the teacher said that she wanted students to
make connections among ideas and that she valued student discussion but actually did not encourage either of these in her classes. Another teacher valued correct answers and procedures, felt that mathematics was of practical use in real life, and believed that the role of the teacher was to explain information clearly to students. She also said that she believed in good discussion in math class but did not encourage this in practice; yet she did not sense this incongruity. (Perhaps her idea of good discussion differed from Thompson’s.) The third teacher valued mathematical thinking and processes and encouraged problem-solving behavior in her students. She was also considerably more reflective about her practice than the first teacher. Thompson concluded that because the first two teachers were not particularly reflective, they did not sense a conflict between their professed beliefs and their practices.

Raymond (1997) had very similar findings in a case study of a teacher with experiences similar to those of Thompson’s first participant, and Cooney, Shealy, and Arvold (1998) also found that pre-service teachers who were not reflective often taught in ways that did not correspond with their professed beliefs.

Context can also create incongruity between belief and action. Cooney (1985) worked with a new teacher before and after he began teaching and found that although the teacher expressed a belief that problem-solving should be the focal point of the mathematics curriculum, the teacher often resorted to using traditional lessons from a skill-based textbook because of time and because students did not respond well to being asked to solve unfamiliar types of problems. Similarly, Arsac, Balacheff, and Mante (1992) and Raymond (1997) found that a teacher’s actions may not reflect his/her beliefs
when he/she feels constrained by the limited amount of time in class and when he/she wants to be certain that students hear accurate mathematical explanations in class. Further, Skott (2004) concluded that the teacher he was studying may not have always been acting in ways that reflected his mathematics-related beliefs but that other beliefs, such as those related to child development and classroom management, sometimes guided his decision-making in his mathematics classes. Moreover, student teachers (and perhaps experienced teachers) can also practice in ways that differ with their beliefs when their knowledge of mathematics content limits their ability to respond flexibly to students’ questions and ideas (Borko et al., 1992; Arsac, Balacheff, & Mante, 1992). Finally, Knapp and Peterson (1995) concluded that teachers may not act on their constructivist beliefs because they do not have the collegial or expert support to work through questions and challenges that naturally arise when teaching in a student-centered environment.

Overall, it seems unreasonable to expect that teachers will act in accordance with one defined set of beliefs in every classroom situation, but it does seem important to consider the major factors that can cause teachers to stray from their professed beliefs very often in action. Helping teachers not only to recognize these inconsistencies but also to identify ways to resolve them could be a useful element of any professional development program.

Teachers’ and Others’ Interpretations of Standards as Observed in Practice

The literature about teachers’ practice related to standards is similar to that of teachers’ practice related to beliefs, which demonstrates the extent to which
interpretations of standards are influenced by beliefs. For instance, Ball (1990) reports on a case study of a teacher in California who was working to enact the new mathematics framework that had been established at the state level. Ball writes that the teacher wanted to implement the framework well, believed she was doing so, but was actually only implementing minor elements of it into instruction and was still teaching traditionally, primarily. However, interestingly, the teacher had barely read any parts of the framework and was assuming that she knew what it said because she felt it was just a checklist of topics and would not influence the way in which she taught, which was, in fact, somewhat consistent with the framework, but not entirely. That is, her style of teaching had not changed at all based on the publication of the framework. She viewed mathematics in an instrumental way – as a set of tools to be taught by her, learned by students, and applied in real situations.

Remillard (1992) writes of a case study of a fifth-grade teacher who, at the time of the study, had never seen the California Mathematics Framework but believed that it was appropriate and that his instruction was aligned with it. He had learned of the Framework, and believed he had an understanding of it, through messages from teachers, administrators, the media, and school curricular materials. His beliefs about the Framework and about his enactment of it were related to his beliefs about mathematics, teaching, and learning. Like many other teachers, he did in fact teach in some ways aligned with the framework and in other ways that were in conflict with it. For example, he did not see mathematics teaching or learning as complex processes; he viewed mathematics as a set of useful strategies for solving real problems, and he did not see the
need to alter his own beliefs because he did not recognize that they were not entirely reflective of the ideas in the framework. Remillard points out that the teacher had had no opportunity to discuss the ideas in the standards by reading the actual document; instead, his district had decided to introduce the framework into schools by purchasing new textbooks that were aligned with it. The teacher did not feel that there was anything especially different or new in the text that did not already coincide with his practices and beliefs.

In addition, Cohen (1990) describes a case study of a teacher who was eager to learn and try new methods of teaching mathematics in her second-grade classroom during the implementation period of the California Mathematics Framework in the 1980s. Cohen notes, however, that this teacher began to incorporate new ideas and resources into familiar instructional strategies; it was not a drastic change from old to new even though she professed a belief in the goals of the framework. Cohen notes that the framework encouraged and specified some new strategies in general terms but did not provide many examples for what these new practices would look like in classrooms. Thus, the result for students was a mixed, sometimes contradictory set of approaches relative to the nature of mathematics and what it means to learn mathematics. The teacher’s views of mathematics, teaching, and learning really changed very little; she still viewed it as an established body of knowledge and not one that needed to be discussed and debated by teacher or students. So, she did not realize how much she still might potentially change her methods because she did not truly recognize the inconsistency between the policy and
her own practice. As Cohen observes, “She thought that her revolution was over.” (p. 325)

We might raise a question about what it means to say that this teacher thought her revolution was over. If nothing in her practice had really changed, then her language and the way she understood the language in policy documents and textbooks must not have changed, and she must not have seen that the language in the policy documents was used to frame mathematics in a new way, even though this is what the authors surely intended. This must have been because she was still relying on her pre-existing views of mathematics; also, perhaps the language in the policy document was not different enough from language familiar to her that she noticed any difference in intent. This reflects Ball’s (1990) comment that language in standards documents must seem understandable to teachers but must still be different enough to highlight the new ideas contained within.

Ball also points out that it is unlikely that district administrators would notice any inconsistency between teachers’ actions and the goals set forth in a state framework. This aligns with the findings in Millett’s and Johnson’s (1998) report of their study in the United Kingdom. The participants for the study were drawn from 7 schools that had recently been inspected. 14 total school personnel were interviewed along with 4 policy makers and 10 inspectors who had conducted inspections at these schools (it may be noted that no teachers were interviewed). The researchers also collected data from documents produced by school leaders and inspectors. They found that, in general, interpretations of good teaching practices in mathematics were similar among the participants, though there were some differences on specific points; for example, one
inspector seemed to have a more rigid view of estimation than the mathematics coordinator in the school he had inspected. However, the authors do note that they observed a potential difference in mathematical expertise between school mathematics coordinators and inspectors – even those inspectors who were supposed to have specialties in mathematics. This, as they write, could cause future difficulties for discussion and improvement based on the results of inspections.

**Means by which Educators’ Interpretations of Standards Can Substantially Change**

Loucks-Horsley and Stiegelbauer (1991) reference over twenty years of research in suggesting seven stages for teacher concern during change:

1) awareness (I am not concerned about it or involved with it)

2) information (I would like to know more about it and how to use/do it)

3) personal (How will using this affect me?)

4) management (How am I using time and resources to help this succeed?)

5) consequence (How is this affecting the students, and what can I do to have more impact?)

6) collaboration (How can I relate what I am doing to what others are doing?)

7) refocusing (Maybe something else would be even better…)

It is interesting to note that if teachers are primarily working in isolation, as has been typical in American schooling, they will be unlikely to move past the fifth stage listed above, and it might be argued that they will not move past the first stage if school leaders
do not encourage the enactment of reforms (i.e., do not provide a reason for teachers to feel accountable to adults outside of their classrooms).

Similarly, Showers, Joyce, and Bennett (1987) suggest four conditions for teacher change:

1) Understand the reason or theory behind the change
2) See demonstrations in a real classroom
3) Practice the new actions in a safe environment
4) Opportunities for feedback and coaching

Again, it is highly improbable that most teachers, who do most of their work individually, will experience any of these conditions, let alone most or all of them, meaning change will be very unlikely to occur. Some schools are starting to provide specialists and coaches who work with teachers in classrooms and in collaborative groups, which could provide more opportunity for these conditions to be met and for change to be initiated.

Brown (2001) writes that teachers generally recognize that they are responsible for understanding professional practices and terminology as well as policy for which they are to enact (whether or not they actually understand the intent of each practice and policy). Thus, they must consider both practical and theoretical implications of how to serve students in the most effective way possible and, some may conclude that they may need to re-examine and change aspects of their practice in order to do this.

**The Influence of Policy on Teacher Change**

Cohen (1990) writes that policy documents tend to tell teachers how to do things and yet promote the idea that we cannot teach mathematics by telling students what to do.
Cohen contends that it is challenging to change a belief that a person holds about how students learn, let alone a belief about the importance of changing how teachers learn. Ball (1990) suggests that policy documents, in order to be likely to have any noticeable effect on teachers’ practice, must be written in ways that make sense within teachers’ current perspectives but that also push teachers in a new direction. This is a great challenge considering Porter’s (1989) comment, cited earlier, that all teachers are in different places when we consider beliefs about teaching and knowledge of effective methods.

Porter (1989) maintains that standards must be persuasive to teachers in order to have true, long-term effects on teaching and learning. Standards that are enforced simply through sanctions and rewards but that do not seem appropriate to teachers will not be especially motivating to teachers or students and likely will be abandoned quickly once this is possible. This type of standard-setting could still be useful to schools that have been very unsuccessful in helping students to learn, but it is not likely to be beneficial in schools that have had success. Also, setting “minimum” standards assumes that students will be at least as prepared to leave school as they would have been had any other combination of standards been chosen. He suggests that it might be helpful for standards documents to include points about content which does not need to be emphasized so that teachers could feel more comfortable eliminating or reducing time spent on these topics. However, the NCTM 1989 Standards document attempted to do just this, and in the eyes of the public, it was seen as a call to eliminate basic skills from the curriculum (even though this was not at all the intent) and caused great public discontent.
Ball also writes that perhaps textbooks, which teachers are more likely to read than state policy documents, are a place where suggestions for engaging students in mathematical activity can be shared. However, she notes that even doing so is not likely to change teachers’ underlying values and beliefs about what it means to be involved in mathematical activity and what quality teaching and learning look like in mathematics.

Sykes (1990) highlights the idea that teachers who have not had a chance to truly read and reflect on policy, especially policy that is written in somewhat unfamiliar ways, are unlikely to enact it as intended. If this is also true of administrators (which it often is), then there may be little monitoring of whether the policy is being enacted at all. One possible exception to this result is when a strong professional community exists in a school or district and is helpful in supporting teachers as they work toward change. However, Remillard (1992) argues that even with more professional development and engagement with an actual standards document, teachers are still going to form interpretations that are based in large part on their prior beliefs, knowledge, and experiences. So, she questions the role of standards documents in changing and bringing consistency to teaching practice because she posits that these documents are attempting to change teachers’ beliefs, but teachers’ beliefs are the frame through which the standards will be interpreted. From a hermeneutic perspective, a researcher might ask whether a frame (or lens) can be altered by the information coming through it, and authors have posited the idea of a “hermeneutic circle” that can never quite be closed because as we form interpretations through a frame of reference, the frame itself changes in the process (Gallagher, 1992, p. 63).
Thus, Remillard seems to suggest that standards documents can present a vision and a rationale for change but that the change itself can only really occur through long-term, personal engagement on the part of teachers. Sykes observes that policy makers need to be aware of these realities as they write and legislate policy, and they need to provide resources to help teachers and others to implement change successfully.

The Roles of School Leaders in Promoting Teacher and School Change

Firestone (1989) argues that leaders must show their active support of a policy in order for many teachers and others to devote time and energy to enacting it (Corbett, Dawson, & Firestone, 1984). Tasks of leaders include initiating and promoting a vision, providing resources (including support from outside experts) (Louis, 1981), providing encouragement and recognition for a job well done, changing the way things have always been done so that the new policies become part of the way things are done (Huberman & Miles, 1984), continually assessing the results of policy change, and managing challenges that arise during enactment of new policies. Principals who set fairly specific goals for their schools and who interact in positive, supportive ways with their teachers are more likely to be able to achieve goals related to district and state policies (Leithwood & Montgomery, 1982). Pressure and carefully directed support must also come from the district-level administrators, in forms similar to those described above (Firestone, 1989).

It must be made known by school leaders that teachers are seen as important contributors to improvement efforts in a school – thus, that their growth as professionals is also important (Bryk, Sebring, Allensworth, Luppescu, & Easton, 2010). Professional development provided within a school should build on and foster teachers’ beliefs and
experience and provide time for inquiry and reflection. Leaders must provide time and support for teachers to learn to think about their practice and how it relates to policy and goals for student learning (Darling-Hammond, 1990).

**Recognizing Teachers as Contributing, Learning Professionals**

In the history of education in the United States, even through the latter part of the twentieth century, teachers were viewed as mechanisms through which policy would be implemented and not independent actors whose knowledge and competencies mattered in determining whether policy would be effectively carried out (Darling-Hammond, 1990). Teachers were controlled, not supported in growing as learning professionals who needed to make complex decisions in the moment. Porter (1989) maintains that simply telling teachers what to do will not help to reach goals of high quality teaching and learning, nor will leaving them entirely to make decisions on their own. He argues that in order for external standard setting to have a real, lasting effect, teachers must be involved in the setting of standards, and they must be supported in an ongoing way in working toward these standards with students. In other words, they must be persuaded, through their own involvement over time, that the standards are worthwhile and achievable and that they are competent enough to enact the standards effectively for student learning.

School leaders must learn to rely on teachers’ collaborative experience and creativity if they wish external standards to be implemented well. Darling-Hammond adds that teachers need to be able to learn more about the policy as time goes on, and they need to be able to try new strategies and reflect on this work so that they can learn how to best enact the policy in their own settings. Firestone (1989) goes on to write that
it is important to teachers (and in fact administrators) that this work be done efficiently but that we must find ways to include all teachers, because otherwise those who do not participate will not likely implement reform in the same ways that others are (Firestone, 1980).

Ultimately, teachers who have clear goals for student learning can make decisions for content and pedagogy based on these goals (Porter, 1989). Efforts to broaden teachers’ knowledge of content and pedagogy can be one means of better supporting student learning. School-based professional development should help to establish common understandings about effective practice – not practice that looks exactly the same in every classroom all of the time – but practice that is grounded in commonly accepted principles of teaching and learning (Bryk et al., 2010; Newmann, 2002).

**Collaborative Professional Learning Experiences**

Chazan, Callis, and Lehman (2008) conducted a case study of a mathematics department in an urban high school in Michigan as it worked over several years to enact more student-focused pedagogy. One teacher, in a narrative reflection in the book published to report the outcomes of the study, points out that if teachers are going to be able to change the classroom experience for students in high school mathematics, then their own notions of what it means to learn mathematics must change. He credits the ongoing collaboration with his high school department colleagues as a major factor in his shifts in thinking.

McLaughlin (1994) argues, based on her own research as well of that of many others, that the most meaningful professional development for teachers occurs in
discourse communities where learning is always the focus. Successful communities establish a framework of problem solving for all discussions and inquiry – they collaborate to inquire into, and hopefully gain insight into, problems of teaching and learning. They also consult with others, as needed, to provide additional insights and information. She has found that the teachers she has worked with in research also believe that these communities are critical in helping them to work through the most challenging problems of practice and (above all) to keep pursuing better opportunities for students. The significance of professional communities of inquiry is echoed by Chazan, Callis, and Lehman (2008), Lieberman (1994), Cobb and McClain (2006), and many other researchers. MacKinnon and Grunau (1994) report that in the study that they did with a group of student teachers in science education, some of the student teachers observed that they came to learn and believe that students learned through inquiry as they observed this occurring and as they (the student teachers) learned through their own collective inquiry into teaching and learning. Many felt, by the end of the year, that they had learned more from each other than they had from their university professors or their mentor teachers in the classrooms.

In addition, teachers are often persuaded to change based on their observations of increased student learning (Chazan, Callis, & Lehman, 2008). Guskey (1986) agrees; he proposes a model that suggests that beliefs, instead of being a prerequisite to change, only change once teachers see that making the change really does help students (and in turn, helps them to experience a greater degree of self-efficacy). Even this persuasion, though, can take time and much collaborative reflection to truly instantiate a change. As Valli
and Hawley (2002) point out, it takes colleagues, some with more experience and/or expertise, who are willing to focus on this inquiry also, and access to educators outside the school who had knowledge of mathematics teaching practices and research on student learning.

So, Cohen and Ball (1990) contend, teachers need to be given the opportunity to see new practices in action as well as to talk in an ongoing way with colleagues and other specialists about how these new practices are working for students and what they might need to change in their own practice. If this does not happen, Cohen and Ball suggest that we cannot expect teaching to change substantially, but they note that schools and governments have not typically given priority to this type of professional learning.

**Individual Reflective Inquiry into Beliefs, Meanings, and Practice**

Brown (2001) writes about his use of writing in working with pre-service and in-service teachers to help to develop their practice through self-reflection and hermeneutic-style inquiry. Teachers wrote in ongoing journals and used the entries in these journals as “fixed” objects on which to reflect and to which to compare and contrast their current understandings of teaching mathematics and of mathematics itself. He writes that in analyzing their own work, they were able to track changes in their thinking and were able to build on these in the frame of particular questions that they had initially raised about their own practice. That is, they could examine each journal entry in light of the others, and their understandings of their own practice developed as time progressed.

When teachers write about their actual practice, they must make a connection between their practice and ways of describing it. Writing in practitioner research can be
seen as a reflection on past action and a guide for future action. Thus, practice and descriptions of it are related and transformed through a hermeneutic-style inquiry (Brown, 2001). Writing can also be a means of clarifying and reflecting on professional goals. It helps them to question why they make the choices that they do and ultimately helps to empower them with a better understanding of themselves as teachers. Improved practice is then a very feasible goal. When teachers describe aspects of their professional lives and practice, they necessarily are positioning themselves in relation to the aspects of experience that they describe; in other words, they are constructing professional identities, which then can be analyzed and deconstructed in future writing and discussion.

In addition, Brown reports, based on his work, when teachers are able to read and reflect on each other’s writing, it helps them to challenge their own understandings and meanings and to better understand other possible ways of thinking.

Brown notes that practitioner (or action) research, similar to his work described above, became more widespread in the 1990s and later. Brown discusses an “emancipatory interest” (p. 213) in self-reflective inquiry; it can help teachers to understand and get past conventional ways of thinking about content, teaching, and learning. It can also help them to think about how language serves those who create it within the system of education. Going further, Elliott (1993, p. 197; cited in Brown, 2001, p. 213) introduces the idea of an “evolutionary interest” in practitioner research, which describes one purpose of such research as intentional change of one’s own practice. The teacher recognizes that he is ultimately the person who can make change in his practice and intends to work to understand himself better so that he can make these
changes. Teachers in the process of conducting such practitioner inquiry often seek out ways to use language to describe new experiences and also to create experiences that match new professional language that they are encountering.

Ultimately, as Wiercinski (2011) comments, when teachers and others take responsibility for their own education, they also take responsibility for developing relationships with others so that they can better understand their positions in the world and allow these positions to evolve over time. For teachers working to help students to grow as members of a global society, this type of self-reflective inquiry to develop personal foundations of knowledge and ways of relating to the world seems appropriate and indeed necessary.

A Hermeneutic Perspective on Mathematics Education

Education as Seen from a Hermeneutic Viewpoint

Gallagher (1992) describes educational experience as “a complex exchange of interactions in which each interpretation may itself be complex: an interpretation conditioned by and conditioning other interpretations” (p. 39). Learning is an ongoing process – we never learn anything immediately, and we always can develop our learning further based on new experience as well as prior experience. Gallagher’s hypothesis is that educational experience is always hermeneutical experience – it always involves interpretation.

Interpretation is an effort to ascertain a meaning – what is truly learned is meaningful. Even if things do not seem especially meaningful to humans as learners, we impose our own meanings on experiences based on our prior ideas and experiences. We
assimilate or accommodate the unfamiliar (Gallagher, 1992). Frank Smith (1975, p. 1) writes about the notion of a “cognitive structure” and explains his belief that the only way we can learn is to connect new ideas to those already in this structure in our minds. We need to be able to draw on these ideas fairly efficiently in order to be able to connect new ideas to them. Gallagher expands upon this idea by noting that learning involves:

a) Interaction between a person’s fore-structure and the subject matter

b) Interaction between the teacher’s interpretation and his/her presentation of material

c) Exchange of interpretations among teacher and student

The teacher cannot just presume to bring the student to think exactly as he thinks. Rather, the teacher’s goal should be to help the student to move through the hermeneutic circle and to make connections among her fore-structure and the new ideas that are being explored. Thus, this will be an individual process for each student. In addition, the teacher must work to be aware of the power of tradition on students’ thinking – that is, culturally constructed ideas that may lie below the level of social or individual awareness. Gadamer (1970, p. 92; cited in Gallagher, 1992, p. 94) calls this “hermeneutical reflection.”

Dewey (1966), among others, writes of the nature of the reflective thinking process, saying that we have experiences that lead us to ask certain questions and investigate aspects of our experience in certain ways. However, we always bring to these learning processes our prior experience and knowledge, which shape the way that we learn in the current moment. Gadamer (1960) also writes that the question on the part of
the learner truly guides the learning process. “The learner is challenged in a process in which she challenges the subject matter” (Gallagher, 1992, p. 167). This also involves the questioning of traditions, which not only shape interpretation but are also shaped by it. Further, the student questions herself and learns to understand herself better as a result. This type of reflection could certainly occur as teachers study new content standards and consider how the standards will inform their practice.

Studying big questions within hermeneutics can give us insight into related questions in education, and vice versa (Gallagher, 1992). In particular, the questions of reproduction, authority and emancipation (whether educational institutions and practices allow one over the other), and trust versus suspicion in interpretation (should the idea of education as transformation be taken as assumed or deconstructed?) can be explored in both fields to perhaps offer some insight into the other. When hermeneutic principles are applied in educational experience, they inform the study of this experience as well as being opened up for inspection and re-creation. So, we should be able to study learning through the lens of these principles, recognizing that the principles are also dynamic and should be questioned along with the study of educational experience that they support.

**Mathematics Teaching and Learning as Seen from a Hermeneutic Viewpoint**

“Mathematics in schools is highly dependent on how it is presented.” (Brown, 2001, p. 70) Factors that may affect this include the published curriculum, the textbook, the teacher’s philosophy, the resources available, and, according to Damasio (1994; cited in Davis, 1996), the moment-to-moment feelings and emotions of the students. Mathematics “can be viewed as being oriented around personal awarenesses engendered
through specific practices” (Brown, 2001, p. 70). Students of mathematics must take statements about ideas, relate them to their own experience, and determine how appropriate they are for particular purposes. Learning, then, could be seen as “an on-going fitting of language to mathematical activity” (p. 71). When students participate in this work, they make these determinations also on a basis of how they relate to the ideas being studied. Thus, as students engage in mathematical activity, they are also making sense of how they relate to their world through experience and language, and this is how meaning comes to be. Moreover, to engage in mathematical activity, a student or teacher must be able to navigate among the different understandings of, and language associated with, mathematics in different subcultures and contexts: school, professional education, policy, standardized tests, a multitude of workplace environments, etc. In other words, what we know as “mathematics” conventionally may just be a shell of what mathematical activity is really all about. But we may not recognize this without an inquiry into the connections between language and mathematics.

Walkerdine (1988) believes that the main goal of teaching and learning should be to help students learn to “capture their experience in language” (cited in Brown, 2001, p. 85). Similarly, Brown writes, “By accepting a hermeneutic view of mathematical understanding we emphasise [sic] the social qualities of mathematical learning by stressing attempts to hold it in socially derived language” (p. 55). That is, in hermeneutic inquiry related to mathematics, the perspective must focus on the learner continuing to create mathematical ideas over time as opposed to learning an externally existing body of knowledge. For example, in a classroom activity that Brown observed, two young
students were playing a multiplication card game but were focusing at least as much on the rules of the game as the mathematics that the teacher intended. Brown notes that the students were even adapting the rules as they played, and he comments that the “mathematics was inseparable from the social activity that generated it” (p. 102). In this sense, Brown maintains that teacher and students jointly construct a mathematical idea as they both act in ways that cause the learning environment to evolve and react to the environment as it does so. Also, students can affect the contexts for each other’s actions in a learning setting. When they respond to each other, they influence the responses that follow. They change the learning environment in doing so (and perhaps not always in a way that aligns with the teacher’s intent).

According to a hermeneutical perspective, “Communicating is about operating on someone’s knowing” (Brown, 2001, p. 100). Mathematical statements help us to understand, in the moment, people’s understandings and help us to orient our own thinking (as both students and listeners) as it develops over time. Part of the task of mathematical activity is finding the most effective ways to communicate with others about what is being learned. Learning moves between understanding and explanation, and it often involves factors that are not considered by the teacher, as listed earlier. This makes assessing mathematics knowledge difficult because learning is always being shaped by language and by new experience. Moreover, hermeneutical perspectives emphasize how fleeting any particular understanding actually is. Teachers can ask students to describe mathematical ideas in their own words, which helps the students to clarify their thinking. This is a way of initiating conversation about the idea and perhaps
working toward the transformation that hermeneutic inquirers would desire. Teachers, in this process, have to make their own determinations about whether the students’ articulations signify the kind of understanding desired, which, of course, requires interpretation (both of the statements themselves and how they relate to a learning goal). When utilizing “hermeneutic listening” in the classroom, the teacher relinquishes sole authority for teaching and providing “understanding” and works to establish a collective authority among teacher and students (Davis, 1996, p. 261). The class is fully engaged, and the learning in participatory and transformative. Students are no longer viewed as essentially autonomous agents, constructing their own meaning individually. We instead view the class and teacher as constructing meaning together on an ongoing basis.

Brown (2001) raises the question of how we can “hold on to our mathematical thinking so that we can talk about it and understand it more easily” (p. 192). (This is made very difficult by the fact that language is not always adequate to represent all aspects of mathematical experience, and mathematical understanding changes over time for individuals as well as groups. Yet, we cannot analyze mathematical understanding or activity entirely outside of language because the language we use to do so is inherently connected to this understanding.) Brown notes that in mathematics, we do not have physical referents available to us for mathematical concepts – only examples that approximate or represent referents (examples: a square, the number 5, etc.). However, any particular notation or representation of a mathematical idea takes meaning in the mind of the reader; it does not hold any particular meaning in itself. Thus, as we think about mathematical words, symbols, ideas, etc., we should consider the idea that they are
continually evolving as we interact with each other in the contexts of various experiences (Davis, 1996). Students move, in the moment, between different modes of representation (linguistic and otherwise), and as they do so, their personal relationships to the mathematics and their understandings grow (Brown, 2001). Symbolic, efficient representations that come from students can represent the discourse and thinking that has taken place prior to the creation of these representations. When we work to develop some sort of shared mathematical understanding, Brown suggests that orienting objects, such as mathematical statements, specific experiences, given diagrams, etc., can help to guide discussion and the relating of one idea to another. This allows us to preserve mathematics knowledge within a society so that the knowledge can be shared or applied meaningfully, even though individuals will have their own interpretations of it.

Because it is necessary to preserve mathematics knowledge within a society and over time, at some point everyone studying mathematics must learn to work within a pre-existing framework of ways of communicating about mathematical ideas. This generally means that students must learn to exhibit their understandings within this framework, though it is possible that the framework could evolve over time. Brown (2001, p. 49) writes: “Mathematical thinking always takes place in an historical context. Similarly, mathematical thinking revolves around a culturally defined set of ideas.” Within this perspective, mathematical ideas do not exist independently outside human thought; they are instead “cultural artifacts” that have existed for so long that we do not tend to recognize that they were not always there. Mathematical ideas are created through the use of language, rather than the converse. “Mathematical meaning is produced in
discourse,” and it is important from a hermeneutical perspective to consider whether we are “producing or reproducing meaning” in this discourse.

As students move between their own understandings and culturally constructed elements of mathematical activity, they form and use expressions of language. Expressions related to mathematical activity are always reflective – they allow us to look back at what we have seen and done. Also, learning takes place with an idea about what is to be learned – similar to the idea of the fore-structure that Gallagher describes. The learner moves back and forth among interpretations and understandings as learning progresses. What the learner says or writes during this process reflects her current understandings and meanings that are being created, in the context of her own experience. This parallels the notion of the hermeneutic circle. As students of mathematics make statements about their observations and interpretations, they will eventually begin to see relationships among certain statements and are able to understand ideas at a more general, abstract level. However, even this understanding is still continually refined as more experiences and reflection inform it. Further, they might believe that others have the same understanding they do, but they may eventually realize that this is not the case and that though understandings may be similar, they are likely not exactly the same. Mason (1989, p. 3; cited in Brown, 2001, p. 52) suggests a helix model to illustrate the spiral-like development of mathematical ideas in a given individual. He uses the phrases “getting a sense of,” “manipulating,” and “articulating” the problem to describe the parts of this helix that is continually in motion. In this sense, anything that a student does changes the environment in which the idea is being explored. Although
Brown does not specify whether he believes that other people are necessarily involved in
this process, it would seem that they must be in order for the cycle to progress very far.
This relates to Davis’ notion that hermeneutic listening on the part of the teacher is a
critical element of teaching situations.

**Hermeneutics, Mathematics, and This Study**

Brown (2001) writes that in an example lesson he describes, it appears that the
work that the students do depends on how they interpret the teacher’s directions and how
able they are to follow through with them; this seems to parallel the relationships among
teachers and policy. Brown also notes that Habermas (1972) saw mathematical language
as existing outside of everyday language and suggests that we might raise questions about
how language is used in the contexts of mathematics and how it serves the interests of
particular people or groups. This is certainly an excellent question that did arise in this
study – i.e., did the participants feel that their interests, and/or those of someone else,
were being served by the language in the Common Core State Standards, and how did
this influence their interpretations and their potential enactment of them?

Brown posits that perhaps a teacher’s task is to help students develop the
mathematical ideas that are already a part of them (since a teacher cannot simply deliver
mathematical knowledge to a student). In this case, teachers would need to study their
own practice in a reflective way (positioning themselves in their classrooms) to think
about how they could foster this kind of learning. Also, similar to the idea that students
cannot directly gain knowledge from teachers, teachers cannot directly gain knowledge of
effective practices from professional developers but must develop it through conversation
with others, and the CCSS document could be utilized as an “orienting object,” as Brown suggests. So, “[t]he task is one of understanding how different interpretations relate to each other [about a standard] rather than one of pursuing a quest for locating the best possible representation” (2001, p. 52).

Wiercinski (2011) suggests that the goal of education should be to allow students (presumably of any age) to break free of any one system or set of goals within education and take responsibility for their own lives based on terms that, through education, they have come to believe are worthy. Perhaps the corollary relative to this study is that teachers need to be able to interpret standards in ways that allow them to provide opportunities for students and for themselves to become self-aware, to become able to critically examine the world around them, and to continually develop their relationships to it. “Primarily, what is produced in educational experience… is understanding which is self-understanding.” (Gallagher, 1992, p. 143) This results from the kinds of interaction described in the sections above, though for most teachers, the process will occur over many years; it will not conclude after one study. Any learner continues to create new possibilities for herself by means of interaction with more experienced others and with culturally established traditions and beliefs that are questioned on an ongoing basis in the process.
CHAPTER III

METHODOLOGY

A knowledge base is needed that will guide the creation of novel and effective teacher education programs. It must include identification of key aspects of teacher knowledge and skills (the goals of teacher education), useful frameworks to describe how such knowledge and skills develop, and useful models of interventions that can promote such development. (Simon, 2000, p. 335-6)

Research Questions

Given the need described in Simon’s statement above and by others as cited in chapter 1, the research questions for this study were:

1a) How do elementary teachers interpret the overall goals of the Common Core State Standards in Mathematics and the Standards for Mathematical Practice relative to students at their own grade levels?

1b) How do elementary teachers envision these goals and Practices developing in their own schools and classrooms?

1c) What do elementary teachers believe will be necessary supports for schools and teachers as the Standards and Practices are enacted?

2) How do elementary teachers’ interpretations and beliefs about the Standards and Practices evolve throughout the following professional experiences: conversations with other teachers and the researcher, and
reflection on lessons in which they attempt to enact content from the
Common Core State Standards and Standards for Mathematical Practice 1,
3, and/or 7?

Since I approached the data analysis in this multi-case study from a hermeneutic
perspective, I allowed for the possibility that the research questions might shift or be
elaborated during the study, an occurrence that is characteristic of teacher development
experiments (described below). This, however, did not occur in this study; the research
questions remained the same throughout since it became clear that I could gather useful
information about each of the questions through interactions with the teachers.

Considerations in Choosing a Methodology for This Study

When considering the methodology to employ in a study in education, it is
important to consider what theoretical framework(s) and method(s) would be best suited
to addressing the research questions. Some educational studies seek to determine
whether relationships exist among sets of quantitative data. This kind of research derives
in large part from experimental psychology (Cobb, 2007). For instance, a researcher
might wish to study whether time spent in mathematics class in fifth grade is correlated
with higher achievement on standardized tests. In this case, the researcher is looking for
average results over large populations of students – in general, whether the average test
score increases as the average time spent increases. This type of study does not focus on
individual students or individual contexts; in fact, the goal is to try to eliminate issues of
context that might influence the conclusions in some way (Cobb, 2007). So, a study of
this type typically includes data from many schools with many different demographic
categories so that the conclusions of the study apply, in general, to as many educational settings as possible.

A similar type of study involves actually searching for a possible cause-effect relationship among sets of numerical data. Taking the research question from the study above, a researcher could ask a related question: Does additional time spent in mathematics class in fifth grade increase students’ performance on standardized tests? In this case, the researcher is not only looking for a relationship among time spent and test scores but is also trying to determine, if there is a relationship, whether the extra time spent actually causes the increased test scores, or whether other factors could potentially be involved. Sometimes in this type of study, the researcher also includes other factors in the analysis that could possibly increase test scores, such as IQ scores or time spent in preparing for standardized tests. Again, the researcher is looking at large numbers of students and schools and is not attempting to explore what is happening in any single setting. Both of these types of studies can be helpful for educational leaders who are interested in using research to make decisions that apply broadly across many settings (Cobb, 2007). However, it is important for leaders to recognize, in making such a decision, that since the data has come from many different contexts, the results of the decision might play out quite uniquely in any single setting. Further, in both of these types of research, the researcher generally assumes that there is a “truth” – a single reality – that can be discovered through experimentation and analysis (Hatch, 2002).

A third, different type of educational research, often called interpretive research, is used in cases where the researcher is attempting to explore and describe “what is going
on” in a given educational setting, or possibly several individual settings. That is, the researcher wishes to study the lived experiences of real people in real settings, to understand participants’ perspectives on their experiences, and to produce “rich” descriptions of these perspectives and/or experiences (Hatch, 2002). In this research, the researcher assumes that each person “constructs” his/her own version of reality or set of meanings; thus, one version of reality does not exist. A setting might be a school, a classroom, or even the case of a single teacher or student. In this kind of study, the researcher is not intending to use the results to generalize his/her conclusions about larger populations or other contexts; the results are specific to the context in which the study takes place. However, this type of research seeks to inform educators’ thinking about **why** particular things happen in certain settings and **how** circumstances from this context might be created (or avoided) in similar contexts. This type of research assumes that the complexity of any given educational setting is too important to ignore and that researchers must try to understand nuances of the context as well as possible in order to understand and report what is happening and why. As a result, the researcher must be involved in a participatory way in the research rather than standing apart from it (as one would attempt to do in the other types of studies above) because the assumption is that only a human being can understand other human beings. Further, the researcher must spend a reasonably long period of time with the participants in order to better understand their perspectives. This kind of study can be helpful for educators who wish to understand how teachers, students, and other people within a given school setting respond
to various situations and how these responses are related to different elements of the social context.

This study sought to explore how elementary teachers interpreted state standards and how these interpretations evolved through conversations with colleagues and with the researcher. These goals could not be effectively studied via research that looked for “averages” and relationships among numerical data, for several reasons. First, it would be very difficult, if not impossible, to quantify teachers’ “interpretations” of standards. Second, even if a researcher were to gather some sort of discrete data on teachers’ interpretations of standards (perhaps via Likert scale surveys), to then summarize this data with statistics would remove essentially all of the information about the variance in teachers’ thinking and understanding and would provide only an “average” of the interpretations across many teachers (Cobb, 2007). This information would be minimally useful to leaders who were trying to help individual teachers make sense of the standards and enact them effectively in individual classrooms to support student learning. Leaders would instead need to be aware of the many different ways in which teachers were thinking about any given standard in order to start to help teachers develop shared understandings, and again, this information would not be available from the results of this type of large-scale, quantitative research. Further, shared understandings develop most readily in contexts that are shared by social actors (McDonnell & Elmore, 1987; Weiss, 1990), and so this type of research is best done in single schools, districts, or classrooms, as opposed to many schools where contextual factors vary widely. This is not to say that this kind of study does not inform work in other settings; descriptive research helps
people in different contexts to consider elements of their own contexts that might affect peoples’ actions or thinking because readers can often find similarities (or differences) between the researched case and their own situations. It can also help readers to consider elements of their own circumstances that might have previously remained unexplored.

For the reasons explained above, I chose to conduct multiple case studies of elementary teachers who worked in similar settings – within the same school district, and two within the same school. These case studies were situated within a “teacher development experiment” (Simon, 2000, p. 336), described below.

**Case Study Research**

Case study is an appropriate research method when the phenomenon to be studied is essentially inseparable from its context (Yin, 2003). That is, if the participant(s), program, etc. were located in a different place or time, the findings of the study might be quite different. Researchers often choose case studies when they want to explore “how” and “why” questions, when they are studying situations that are, in large part, out of their control (Yin, 2008), or when they need to get very close to the participants in order to gather data needed to answer the research questions (Bromley, 1986). This study involved three elementary teachers who could serve as examples drawn from the much larger population of elementary teachers in Ohio. This did not necessarily mean that the examples were assumed to be representative – in fact, as Flyvbjerg (2006; cited in Merriam, 2009) points out, one of the advantages of case study research is that researchers can study unique situations in order to understand the variation in possible occurrences of these circumstances in other places. In any case study, the researcher’s
goal is to understand, as fully as possible, the phenomenon under study and how the elements of that particular context interact to shape the phenomenon (Merriam, 2009).

Researchers choose to conduct multiple case (multi-case) studies in order to provide both variation in the data and findings and also some generalization across cases, which can increase the credibility of the conclusions (Merriam, 2009). That is, one of the strengths of case study research is to demonstrate variability in a phenomenon across contexts and to be able to identify unusual, yet perhaps quite important, differences in the way that the phenomenon occurs – differences that could inform actions of those involved with the phenomenon in the future. Shields (2007) writes that in case study research, researchers “do not attempt to eliminate what cannot be discounted. They do not attempt to simplify what cannot be simplified.” (p. 13; cited in Merriam, 2009, p. 52)

This generally means that there are many variables to consider within data collection and analysis, and thus researchers are wise to utilize multiple data collection methods to create several sources and types of data so that the eventual analysis and conclusions can be as credible and comprehensive as possible (Yin, 2003).

Case study research can be especially informative for readers because they can often relate descriptions and/or findings from the case to their own sets of circumstances – or, sometimes, they can learn that what they believed was very typical in a certain social situation might actually occur very differently in different contexts. Thus, a reader may actually be able to relate this research to cases of which the researcher was not even aware, thus addressing, in some sense, questions about generalizability of findings from case studies (Merriam, 2009). Similarly, Stake (1981; cited in Merriam, 2009) argues
that case study research is more concrete than other research because it focuses on specific, real examples and can resonate with readers’ personal experience.

There are three predominant types of case studies: exploratory, descriptive, and explanatory studies (Yin, 2003). Exploratory case studies are designed to formulate research questions – to determine what ought to be studied in future research. Descriptive case studies provide in-depth descriptions about the phenomena being studied within given contexts. Explanatory case studies are conducted to find cause-effect relationships – to explain why certain events have occurred. This study was descriptive because the goal was to provide rich descriptions (as complete as possible) of elementary teachers’ interpretations of the Standards for Mathematical Practice (SFMP) in light of their own grade level content in the CCSS.

In a descriptive case study, theory is very important because it helps the researcher to characterize what a complete and appropriate description will include (by the time the study is finished). Researchers must consider where a description “starts,” where it “ends,” (Yin, 2003, p. 23) what it includes and excludes, and where possible, what an “idealized” (p. 25) version of the case would be – that is, what the critical or essential elements of an ideal case are, at least as far as the previous research literature can indicate. Of course, data analysis processes should also include the potential for discovering elements of the case that were not anticipated. Although Yin seems to take a slightly more post-positivist perspective on this issue, I would argue that the researcher must decide, based on his/her ontological and epistemological views, whether there is
truly an “ideal” or whether the phenomenon being studied will continue to evolve over time.

In this study, the phenomenon being researched was teachers’ interpretation of the Standards for Mathematical Practice in the CCSS. The Standards for Mathematical Practice are descriptions of mathematical activity in which the authors intend all students to engage on a regular basis. One very important question to consider for this study was whether it was assumed that there was a “correct” interpretation of any given standard, or of the CCSS as a whole. In light of the moderate hermeneutic perspective taken for this study, I did not assume that only one interpretation of a standard was correct. However, again reflecting this hermeneutic perspective, I aimed to move participants toward a deeper, more elaborated interpretation of each standard as the study progressed, and this involved encouraging them to take into account ideas from their colleagues and other local, state, or national sources of support with which they were familiar. In this way, the participants were learning from a variety of professionals and were able to drawn on this learning as they refined their own interpretations.

Sources of support for teacher learning and reflection at this time included materials that were being produced by the authors of the CCSS, the Ohio Department of Education, and the new national collaborative effort called the Mathematics Common Core Coalition (MC²). This project brought together professional organizations and agencies including the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), the Association of Mathematics Teacher Educators (AMTE), the Association of State Supervisors of
Mathematics (ASSM), the SMARTER Balanced Assessment Consortium, and the Partnership for the Assessment of Readiness for College and Careers (PARCC) to produce frameworks and materials to contribute to individual and collective understandings of the CCSS and to guide broad-scale assessment practices for the CCSS.

The website that this organization has created is:

**The Teacher Development Experiment**

Simon (2000) comments that we do not currently have a good research base for understanding how teacher knowledge that is important for standards-based instruction develops, and he suggests that researchers must find ways to conduct this research while simultaneously providing teacher development for standards-based reform, which is a challenge for the field, to be sure. Working to overcome this challenge, Simon describes his notion of the “teacher development experiment,” or “TDE” (p. 336), which is a method that he and his colleagues have employed to research the development of teacher knowledge in a setting that they shape to foster standards-based instruction. The TDE derives its theoretical grounding from the constructivist teaching experiment, described and utilized by Steffe and others (Steffe and Thompson, 2000) to develop “explanatory models” (Simon, 2000, p. 341) for children’s mathematics learning in specific areas of the discipline. The TDE also builds on the “emergent perspective” on learning that has been adopted by Cobb and his colleagues (Cobb, 2000). This perspective highlights the cognitive development of the individual within a social context for learning, indicating that social interaction and social norms influence the development of ideas within any
one person’s mind. That is, ideas and interpretations may develop individually from
person to person, but this development is shaped by the social activity in which the
person participates as he/she is learning. Further, each person’s interpretations then
continue to shape the knowledge and norms that become “taken-as-shared” (p. 338) in the
learning setting.

Constructivist teaching experiments can be conducted with whole classes of
students, and the TDE is often conducted with groups of teachers. Cobb writes that in
choosing or creating a particular class or group to study, researchers can address issues of
generalizability and trustworthiness by intentionally attending to important features of the
context and framing the findings within this context. In this way, the study becomes an
example upon which researchers or readers can reflect when working in other contexts, if
similarities and differences between the two contexts are considered. Also, within a
TDE, researchers may examine individuals’ development in case studies as well as
collective learning processes, which are then analyzed together to inform future research
and practice related to teacher professional development.

One important additional focus in the TDE that is not present in a constructivist
teaching experiment is the participants’ learning relative to pedagogy as well as their
learning relative to mathematics. The TDE includes both collaborative learning
experiences involving multiple participants and the researcher (who acts as facilitator)
and individual learning experiences in the context of teachers’ own classrooms, as the
researcher works with each participant to reflect on his/her own practice, on students’
learning, and on future plans for instruction. In this way, teachers’ learning about mathematics may shape their development in pedagogy, and vice versa.

In this study, the TDE design was most directly reflected in two group conversations that I facilitated based on questions that participants generated. These conversations involved all of the participants as they discussed the portions of the CCSS and SFMP on which I chose to focus for this study (detailed later). The teachers were from the same school district, and one pair of participants was from the same school, so the context for participants’ learning and for data analysis included these schools and this district, as well as experiences common to all three teachers (as a result of teaching in the same district) that may have influenced how they interpreted the standards. Of course, the study itself was also part of the context for participants’ interpretations. Any interpretations that teachers formed in this setting may have been influenced by their interactions with each other and with the text of the standards as well as their prior experiences and beliefs.

Further, I studied each participant and his/her interpretations of the standards as a case, and I conversed with each one individually during the three interviews. This provided opportunity for individual growth on the part of each teacher relative to his/her understanding of not only the goals of the standards but also how pedagogy in his/her classroom could support these goals (whether or not this was currently the case).

In the TDE as well as in the constructivist teaching experiment, the researcher develops knowledge through a cycle of interaction and reflection. She generates initial hypotheses about how the participants will learn as a result of activities that she facilitates
(designed based on any existing research). As she interacts with the participants, she reflects on how participants’ knowledge is developing and may alter her hypotheses and/or future research plan in order to build upon this learning. This reflection is termed “ongoing analysis” (Simon, 2000, p. 341). As this cycle continues, the researcher not only begins to formulate tentative models for how the participants learn particular aspects of a given mathematical topic, but also (later in the process) to formulate more general models about how learning in this area of mathematics develops; this is known as “retrospective analysis” (p. 341). Later in this chapter, as I describe the data collection and data analysis procedures that I employed in the study, I provide additional detail about initial hypotheses for the participants’ learning as well as how “ongoing” and “retrospective” analysis occurred.

Simon notes that teachers and researchers are to be seen as partners in a TDE; they contribute to each other’s learning by bringing their unique perspectives to the activities and discussion within the research. Romberg and Collins (2000) also maintain that research on teacher development needs to be conducted collaboratively with teachers. The design of this study, detailed below, took into account the need for teachers to actively participate in generating their own questions and considering their own needs for learning.

Theoretical Perspective

At a basic level, the design of this study drew on constructivist and symbolic interactionist perspectives in that I operated under the assumption that humans form different meanings for objects and situations in their lives based on interactions with
these objects/situations and with other people (Hatch, 2002; citing Blumer, 1969). The study also drew on sociocultural theory, which studies the “participation of the individual-in-cultural-practice” (Cobb, 2007, p. 22) and particularly Cobb’s “emergent perspective” (discussed further below), since I was examining how teachers’ interpretations changed and were formed as the teachers interacted with each other, with their students, and with the researcher. The data analysis reflected a hermeneutic lens in that I focused on how interpretations of standards were formed via various types of interaction; this was discussed in chapter 2 and is discussed in more detail later in this chapter. Cobb calls this interweaving of theoretical perspectives “bricolage” (2007, p. 29), noting that we may adapt ideas from different perspectives in order to formulate a framework that is appropriate for a given study.

In reflecting on constructivist and symbolic interactionist views relative to the goals of this study, it is helpful to consider the notion that in order for social actors (in this case, teachers) to establish a set of truly common meanings, they must share a more fundamental (often unspoken) set of assumptions about the world in which they live (Bredo & Feinberg, 1982b). In social research, we must acknowledge (though some researchers do not do so explicitly) that any interpretations made and conclusions drawn are made in light of the underlying assumptions and understandings of the given social institutions (here, schools) in which the research takes place. Outside of this basic framework, the conclusions might have no meaning at all.

Common (or intersubjective) meanings are the basis of community (Taylor, 1982). The language that expresses these meanings is framed in social norms and
behaviors that represent a particular view of reality shared within this group. Words and phrases can be understood quite differently within the social contexts (schools) in which they occur; that is, intersubjective meanings are different (Bredo & Feinberg, 1982b). Further, social behavior can be changed by the adoption of new linguistic patterns that come about as a result of use by certain groups; these patterns eventually supplant the older ones and cause people to create new, non-linguistic tools or ways of acting that align with the language (Rorty, 1989/1995). We might argue that this could occur when new content standards truly become a part of the culture of a given school or even of a small group of teachers.

Paul Cobb (2001; 1994) argues for the usefulness of sociocultural theory in mathematics education research. This perspective was promoted by Vygotsky and other Russian philosophers in the early twentieth century, and the essence of this theory is that students learn as they participate in socially established practices with other more experienced actors – in this case, these actors might be teachers or more advanced peers (Cobb, 1994). Throughout years of research in classrooms and with teachers, Cobb and his colleagues have come to believe that students do learn as a result of participating in classroom discourse, both with students and the teacher, and they feel that this participation is key to students’ inclination to reorganize their own thinking based on interactions with others. Cobb (1998; cited in Brown, 2001) speaks of the idea that mathematical meanings can be socially constructed in classrooms when teachers and students work to reach shared understandings of mathematical topics. In this case, the mathematical knowledge that is “taken-as-shared” (Cobb, 2001, p. 129) comprises the
collective interpretations of ideas that have been negotiated and are now seen as legitimate by all students and the teacher. Cobb (2001) emphasizes that this does not necessarily mean that individual interpretations mirror the intersubjective understanding – rather, that these interpretations are informed by, and inform, the collective body of knowledge.

Many scholars believe that constructivism and sociocultural theory are incompatible because of the focus on the individual mind (cognition) in the former and the focus on social participation in the latter. However, Cobb (1994) argues that both can be useful to researchers depending on the questions that are being asked. He views constructivism as particularly helpful in studying how individuals learn, and sociocultural theory as being helpful when considering the possibilities for creating and structuring learning environments, both in the classroom and in the larger institutional context of schools. Cobb (2000) blends these two perspectives in forming his “emergent perspective,” adopted for this study.

To reiterate: in this study, I drew on Cobb’s emergent perspective as I observed the participants as a group and as I analyzed how their interpretations of the CCSS changed – as a result of their interactions with each other and as a result of their own changing experiences and thoughts. I considered how they negotiated intersubjective meanings for the standards (which would be necessary for teachers working in the same school setting). I also considered my data from a hermeneutic perspective because hermeneutics ultimately is concerned with individuals’ interpretations of text (perhaps informed by social activity), and this seemed appropriate in this context since teachers
generally work independently in classrooms and in their own instructional planning – activities in which they might be most often considering the state standards. For this reason, this study focused on individuals’ interpretations more than meanings formulated by the group, though both were assumed to influence each other.

Method

Participants and Sampling

Sample selection in a qualitative case study is dependent on the research questions (Merriam, 2009). Since probabilistic sampling is associated with generalizing findings to a larger population, it is not generally utilized in qualitative studies because the purpose of these studies is not to generalize; rather, the purpose is to gain specific information about particular people, events, organizations, etc. So, purposeful, non-probabilistic sampling tends to be the typical choice for case studies. When each case is a person, researchers should identify the criteria that participants must have in order to provide useful data, justify that these criteria are important, and then find participants who embody these characteristics. Patton (2002, p. 230; cited in Merriam, 2009, p. 77) writes that this sampling method provides “information-rich” cases from which the researcher can learn a great deal relative to the research questions.

I had planned for my group of participants to include four elementary teachers, two each from first grade and fifth grade; in the end, I had three participants, one from first grade and two from fifth grade. I chose these grades because the content standards in each are significantly more rigorous (though also more focused) than they were in the 2001 Ohio Academic Content Standards, and teachers needed to gain understandings of
their grade level content as well as the Standards for Mathematical Practice. My hope was to find participants who had worked in education for different lengths of time, as the research about professional learning groups indicates that this is helpful for sharing diverse ideas (Valli & Hawley, 2002). I also hoped to have participants of both genders. I was able to meet both of these goals with my group.

Merriam (2009) indicates that there are different means of creating a purposeful sample. Patton (2002; p. 236; cited in Merriam, 2009, p. 78) writes that a “typical sample” is one in which the cases selected are not especially unique or unusual; they reflect the characteristics of other similar cases. Other alternatives include a unique sample (in which cases are extreme or unusual), a maximum variation sample (in which the cases are selected to span a wide range of characteristics), snowball sampling (in which new participants are identified by current participants, thus expanding the sample), and a convenience sample (which is created given parameters related to time, money, location, etc.). This study ultimately included a sample of three “typical” elementary teachers (i.e., not unusual, given the aforementioned criteria). To some degree, the sample was a convenience sample, as I did not have the flexibility to travel far to work with participants. However, as most elementary teachers in Ohio at this time had limited knowledge of the CCSS but similar experiences in actually working with students, the convenience sample should not be seen to limit the credibility of my findings. Also, I provide as much information as possible in Chapters 4 and 5 about the characteristics and backgrounds of my participants so that readers are aware of the contexts for my findings. In the scope of this dissertation study, finding three teachers at two different grade levels,
selected purposefully to reflect the “typical” elementary teacher, seemed a reasonable effort to collect data about teachers with a variety of experiences but who taught in similar contexts – thus, the teachers were able to talk with and learn from each other in the group discussions.

I recruited the participants from a suburban, relatively large school district near the one in which I serve as a mathematics consultant. I used only one district in light of the aforementioned need to explore policy enactment in local contexts (McDonnell & Elmore, 1987; Weiss, 1990). If I were to have recruited teachers from my district, I believe that it would have been difficult for the participants to be entirely open about the latter interview questions (listed in Appendix D), which address the implementation of the CCSS in the school and district. Also, since (as discussed below) I asked the teachers to videorecord and reflect on two lessons (videos which I would also see), teachers in my own district might have had some concerns about my seeing and perhaps evaluating them in a lesson, though evaluation was not the purpose of the video. So, I worked with a neighboring district where I have had only minimal interaction with the teachers in the past.

Although the sample for this study was chosen with a specific purpose, it consisted essentially of volunteers. I asked the curriculum director in the chosen district to forward to teachers an e-mail that I sent, explaining the study and asking for participants (see Appendix A for this letter). I also provided a $75 Amazon gift card to each participant for his/her participation at the conclusion of the study. As an incentive to the school district to participate, I promoted this experience as a means of establishing
a small, multi-school mathematics professional learning community, similar to the one I have established in my own district. I may yet still work with the teachers and administrators to support the growth of this community through plans for ongoing inservices, collegial conversations, co-teaching, and online discussions (all ultimately facilitated by faculty in the district).

**Design and Data Collection**

The study consisted of seven phases. A timeline is presented below, and a description of each phase follows. Generally, sessions in a TDE are recorded on video so that careful analysis of the actions and responses of participants can be conducted at a later time (Simon, 2000). I video-recorded each phase of the study.

1) An initial in-depth, video-recorded interview with each participant, in which I explored his/her interpretations of the overall goals of the CCSS and the SFMP in light of his/her grade level content, after an initial reading done independently, prior to the interview (see Appendix B for these pages from the CCSS, pages 3-8)

2) A video-recorded discussion among all participants about these portions of the CCSS, primarily guided by their own questions and ideas

3) A video of one lesson from each participant, which the participant and I watched independently and upon which we reflected independently (about how one SFMP, chosen by the teacher, was reflected in this lesson)

4) A second in-depth, video-recorded interview with each participant, in which I explored his/her interpretations of the overall goals of the CCSS and the
Standards for Mathematical Practice (SFMP) in light of his/her grade level content and how they might have changed since the first interview

5) A second video of one lesson from each participant, which the participant and I watched independently and upon which we reflected independently (about how one SFMP, chosen by the teacher, was reflected in this lesson)

6) A second video-recorded discussion among all participants about the relevant portions of the CCSS, primarily guided by their own questions and ideas

7) A final in-depth, video-recorded interview with each participant, in which I explored his/her interpretations of the goals of the CCSS and the SFMP in light of his/her grade level content and how the interpretations might have changed as a result of the work in the study

Table 2

*Approximate Timeline for Study*

<table>
<thead>
<tr>
<th>Phase</th>
<th>Approximate date of completion</th>
<th>Method of data collection</th>
<th>Method of data analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recruit participants</td>
<td>December 2011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interview 1 and follow-up with</td>
<td>January 2012</td>
<td>Interview (transcribed) and written reflection of each participant and researcher</td>
<td>Memo writing, rich description of individual interpretations of CCSS</td>
</tr>
<tr>
<td>participants</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group discussion 1 and follow-up</td>
<td>February 2012</td>
<td>Focus group (transcribed) and written reflection of each participant and researcher</td>
<td>Memo writing, rich description of individual and group interpretations of CCSS, description of evolution of interpretations</td>
</tr>
<tr>
<td>with participants</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lesson video 1 and follow-up with</td>
<td>March 2012</td>
<td>Videotape (transcribed) and</td>
<td>Memo writing, rich description of</td>
</tr>
<tr>
<td>participants</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Participants</td>
<td>Written Reflection of Each Participant and Researcher</td>
<td>Evidence (or Lack of Evidence) of SFMP in Lesson</td>
<td></td>
</tr>
<tr>
<td>--------------</td>
<td>------------------------------------------------------</td>
<td>--------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Interview 2 and follow-up with participants</td>
<td>April 2012</td>
<td>Interview (transcribed) and written reflection of each participant and researcher</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Memo writing, rich description of individual interpretations of CCSS, description of evolution of interpretations</td>
<td></td>
</tr>
<tr>
<td>Lesson video 2 and follow-up with participants</td>
<td>May 2012</td>
<td>Videotape (transcribed) and written reflection of each participant and researcher</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Memo writing, rich description of evidence (or lack of evidence) of SFMP in lesson</td>
<td></td>
</tr>
<tr>
<td>Group discussion 2 and follow-up with participants</td>
<td>June 2012</td>
<td>Focus group (transcribed) and written reflection of each participant and researcher</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Memo writing, rich description of individual and group interpretations of CCSS, description of evolution of interpretations</td>
<td></td>
</tr>
<tr>
<td>Interview 3 and follow-up with participants</td>
<td>July 2012</td>
<td>Interview (transcribed) and written reflection of each participant and researcher</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Memo writing, rich description of individual interpretations of CCSS, description of evolution of interpretations</td>
<td></td>
</tr>
<tr>
<td>Further data analysis, initial writing, and follow up with participants</td>
<td>October 2012</td>
<td>Written feedback from participants on initial reports</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Further refinement of rich description of individual and group interpretations of CCSS, rich description of evolution of interpretations</td>
<td></td>
</tr>
<tr>
<td>Final analysis and discussion</td>
<td>January 2013 through December 2013</td>
<td>Written feedback from participants on initial reports</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Further refinement of rich description of individual and group interpretations of</td>
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</table>
Standards as tools for professional learning. As noted in chapter 1, Weiss (2002) writes that standards can be used as a basis for teacher development (among many other applications). In each phase of this study, the participants read and discussed the CCSS for their respective grade levels and three of the SFMP, reflecting on the standards individually and in collaboration with other participants and the researcher. The goal of this reading and discussion was to allow each participant’s interpretations of the standards to evolve and deepen so that the standards could be more effectively enacted in his/her classroom.

Three Standards for Mathematical Practice chosen for the study. At the time of this study, the national organization called the Smarter Balanced Assessment Consortium (SBAC) had published a draft document (Burkhardt & Schoenfeld, 2011) specifying mathematical goals from the Common Core State Standards that were proposed for assessment in standardized testing across the nation. In this document, the authors offered four “claims” describing mathematical skills and understandings that they suggested could and should be assessed in light of the CCSS. In their rationales for these claims, they particularly highlighted SFMP 1, 2, 3, 7, and 8, though the others were mentioned also.

Similarly, the national assessment consortium called the Partnership for Assessment of Readiness for College and Careers (PARCC) had published a draft of Model Content Frameworks (2011) for standardized assessments aligned with the CCSS.
The authors of this document provided examples of how the content and practice standards could be related throughout the grades, and they also highlighted SFMP 1, 2, 3, 7, and 8 in these examples.

So, in order to provide an appropriate level of focus for the proposed study, I considered the five standards listed above as possible focus points for participants, and I chose three of the five for final inclusion. My choices were based on my evaluation of the potential for evolving interpretations of each standard as well as the applicability of each to elementary classrooms. The three SFMP appear on pages 6-8 of the CCSS document and are presented here in full in Appendix B. The title statements are:

*SFMP 1. Make sense of problems, and persevere in solving them.*

Rationale: This may be the standard that on the surface makes the most sense to teachers as they read the title (the sentence above), and it expresses a typical goal that most teachers at all levels have for their students. Still, the description of the standard provides many classroom examples that may evoke other (less immediate) interpretations of the initial sentence. Further, in the NCTM Principles and Standards (2000), problem solving is one of the five Process Standards, and problem solving has always been one prime goal of mathematical activity.

*SFMP 3. Construct viable arguments and critique the reasoning of others.*

Rationale: The mathematical habits expressed in this standard are vitally important to the learning process, particularly according to the emergent perspective adopted for this study (which Cobb also applies to student learning in
classrooms, where the idea of this perspective was formulated initially). Again, in the NCTM Principles and Standards, reasoning is one of the Process Standards, and being able to reason about ideas is really a life skill that is a goal for learning in all content areas. Interestingly, though, this may be a more difficult standard for elementary teachers to consider in light of their own students because traditionally, we have not often asked students to consider others’ reasoning or sometimes even to communicate their own reasoning.

*SFMP 7. Look for and make use of structure.*

Rationale: Mathematics is often defined as the science or study of structure (Sfard, 2003), as it can help us to see relationships among phenomena that seemed to be unrelated. Boaler (2008) discusses the idea that students who are able to see and use structure in mathematics are generally much more successful in learning mathematics than students who simply view and attempt to apply mathematics as a set of disconnected facts and skills. However, of the three standards I chose, this may have been the one that the teachers were most unaccustomed to considering intentionally, so it was interesting to observe how their notions of this standard evolved throughout the study. I hypothesized that some of the teachers might discover that they had often pursued this goal with students without explicit recognition of the ideas in the description of the standard, and this was in fact the case for at least one of the participants.

I did not choose SFMP 2 (*Reason abstractly and quantitatively*) because its description is the only one in which no examples are given; it would be difficult for many
educators at all levels to interpret the text that is provided, let alone what this standard would imply for student learning. However, it should be noted that the mathematical understanding described in this practice is very important because students do need to understand that the properties of numbers do not change in different contexts (or out of context), and that mathematical relationships are generally maintained from one context to another, at least in elementary mathematics.

I did not choose SFMP 8 (Look for and express regularity in repeated reasoning) because it seems to express ideas similar to those in SFMP 7, and there seems to be less room for interpretation as it refers primarily to recognizing and applying repeated procedures and methods in order to solve problems.

**Rationale for choices of statements from Standards for Mathematical Practice for interview questions.** In order to try to understand how the participants interpreted specific portions of SFMP 1, 3, and 7 (a primary goal of this study, and one with important implications for those designing future professional development for teachers enacting the standards), I posed questions about these statements in the interviews (see Appendix D). Below, I present all portions of SFMP 1, 3, and 7 in sequence as they appear in the CCSS document (2010, pages cited below), and I provide a brief rationale for why I did or did not include each portion in a specific interview question. The chosen statements are also listed in Appendix E.

**SFMP 1:** “Make sense of problems and persevere in solving them.” (p. 6)

*Include:* The title is the portion of the standard that teachers will be most likely to read and upon which they are most likely to base their practice.
“Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.” (p. 6)

Include: This is the first sentence of the description of the standard, and as such is presumably important to the authors. It describes what students should do when first approaching a problem.

“They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.” (p. 6)

Include: This describes what students should do as they are solving a problem.

“Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.” (p. 6)

Do not include: These statements generally apply to content beyond grade 5.

“Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.” (p. 6)

Include: These examples apply quite directly to elementary students.
“Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, ‘Does this make sense?’” (p. 6)

*Include:* Re-checking solutions and questioning the sense of each solution are habits that most mathematics teachers work to develop in students (and in fact, the teachers discussed these habits frequently during the study).

“They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.” (p. 6)

*Include:* These are skills that often are challenging for elementary students to develop because they tend to be focused on their own thinking (especially when not asked to consider the thinking of others). Again, this is a point on which the teachers often commented.

SFMP 3: “Construct viable arguments and critique the reasoning of others.” (p. 6)

*Include:* The title is the portion of the standard that teachers will be most likely to read and upon which they are most likely to base their practice.

“Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures.” (p. 6)

*Include:* This portion describes how students should be able to build a rationale given specific information, which is an important skill in all disciplines.
“They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples.” (p. 6)

*Include:* The idea of using examples and counterexamples is a typical instructional practice in mathematics and other disciplines.

“They justify their conclusions, communicate them to others, and respond to the arguments of others.” (p. 6-7)

*Include:* This statement describes several communication skills, which are again important in all disciplines, including mathematics.

“They reason inductively about data, making plausible arguments that take into account the context from which the data arose.” (p. 7)

*Do not include:* Almost no standards related to data are a part of the content in grades 1 and 5.

“Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is.” (p. 7)

*Include:* Once again, being able to determine whether lines of reasoning are logical, and identifying portions that are not logical, is an important skill that is developed in all disciplines.

“Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.” (p. 7)
Include: These statements apply directly to elementary students.

“Later, students learn to determine domains to which an argument applies.

Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.”  
(p. 7)

Do not include: The first sentence in this portion applies to older students. The second sentence reiterates skills that are included in other statements from the standard that I am including in the interview.

SFMP 7: “Look for and make use of structure.” (p. 8)

Include: The title is the portion of the standard that teachers will be most likely to read and upon which they are most likely to base their practice.

“Mathematically proficient students look closely to discern a pattern or structure.”  
(p. 8)

Include: The authors are suggesting that this should take place as students start to solve problems and as they are solving them.

“Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have.”  
(p. 8)

Include: The examples here apply directly to elementary students.

“Later, students will see 7 x 8 equals the well remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property.”  
(p. 8)
Include: I will include this in the interview for grade 5 only, since the example does not apply to grade 1 content.

“In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.” (p. 8)

Do not include: These examples do not apply to elementary students.

“They also can step back for an overview and shift perspective.” (p. 8)

Include: This suggests that students should be able to refocus during problem solving, which can be challenging for elementary students.

“They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.” (p. 8)

Include: Although this sentence includes an example from beyond grade 5, the general idea of seeing mathematical objects as composed of parts versus seeing them holistically is important in students’ mathematical development.

“For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.” (p. 8)

Do not include: This example does not apply to elementary students.

**Hypothetical classroom episodes as tools for professional learning.** Kennedy, Ball, and McDiarmid (1993) led a project in which their goal was to design an instrument that would help to assess teachers’ changing and growing knowledge related to teaching writing and mathematics. Part of the final instrument was a series of hypothetical
classroom situations to which teachers responded during interviews, describing what they would do if they were in each situation. In creating and refining these tasks, the authors utilized three criteria. The tasks were to be: relevant to teaching diverse groups of learners, similar to actual teaching situations, and helpful in understanding the different ideas that influenced teachers’ instructional decisions. Further, they identified seven types of situations that seemed representative (together) of the teacher moves that are most typical and necessary in teaching mathematics and writing. These seven types of actions were:

- Responding to student difficulties with a particular concept
- Responding to student novel ideas regarding a particular concept
- Generating representations of concepts
- Responding to student questions
- Helping students learn concepts and procedures
- Planning a unit
- Evaluating student work

For this study, in order to explore teachers’ evolving thinking about instructional decisions reflecting the CCSS and SFMP, I formulated six hypothetical classroom episodes for first grade and six for fifth grade, and I aligned these with six of the types of teacher moves delineated by Kennedy, Ball, and McDiarmid. I eliminated the “planning a unit” category because this broader kind of teacher decision-making was beyond the scope of this study. For the grade 1 episodes, I drew ideas from the grade 1 Model Curriculum document published online by the Ohio Department of Education (2011). I
was unable to find specific connections among the grade 1 content standards and the SFMP at any of the websites of national organizations mentioned earlier in this document. For the grade 5 episodes, I did take some ideas from the Model Curriculum document, but I also drew on connections between the content standards and the SFMP suggested in the PARCC Model Content Frameworks draft document (2011). In addition, I adapted two tasks from the Mathematical Knowledge for Teaching Measures: Mathematics Released Items (2008), which originated in a project led by Deborah Ball with goals similar to those of the earlier work with Kennedy and McDiarmid.

I posed these episodes to the participants during the three interviews (two during each interview), and I asked each teacher to consider and discuss what he/she might recommend regarding the next “moves” of the teacher in each situation, in light of his/her own interpretations of the goals for students in SFMP 1, 3, and/or 7. This provided an opportunity for participants to consider (directly or indirectly) how the SFMP might appear in their own classroom lessons, at that time or in the future. All of these episodes can be found in Appendix F.

**Interviews.** Van Manen (1990) writes that in conducting interviews, the researcher needs to be clear about the initial research question at the outset. Otherwise, one will either gather too much data of which to make sense or not enough data to create a detailed and explanatory report. Interviews are used in hermeneutic phenomenological research for two reasons: for gathering material that can be used to develop an understanding of a given experience, or for developing a relationship with a participant that allows the two to explore the meaning of an experience for the participant. In this
study, interviews were conducted for the first of these two purposes. Van Manen maintains that in a hermeneutic interview, the researcher must work to not impose meaning and to keep herself and the participant oriented toward the topic in question. This allows more reflection and discussion to occur as the conversation progresses.

Hesse-Biber and Leavy (2006) write that in an in-depth interview, the researcher assumes that the respondent has “unique and important knowledge” (p. 119) that can be communicated through conversation. They note that these interviews are issue-oriented, focusing on a particular topic. In-depth interviews can be structured, where each participant responds to the same questions and no others, semi-structured, where the same questions are asked of each participant but the researcher may follow up in different ways depending on the responses, and open-ended, where the researcher asks just a few broad questions and the conversation goes wherever the participant’s answers take it.

Rubin and Rubin (2004) recommend that when designing interview questions, a researcher should write main questions that will evoke depth, detail, vividness, nuance, and richness. Questions that ask participants for specifics, explanations, limits, and exceptions can help to provide depth, detail, and richness, which then help to offer clarity, evidence, meaning, and potentially new ideas in data interpretation and analysis. Asking participants for specific anecdotes or their emotional responses to particular situations and then including relevant portions of their responses in the final report can help the reader relate to the participant’s experiences, which supports the credibility of the findings. Gaining nuance from participants’ responses involves asking follow-up questions to determine whether participants see particular ideas as universal or whether
there are degrees of difference in their interpretations or experiences, which, again, is helpful in formulating conclusions as a researcher and reporting these conclusions in a way that feels realistic to the reader.

In this study, I used semi-structured interviews, because though I asked each participant the same questions (listed in Appendix D), I also asked for follow up and elaboration as it seemed appropriate during each interview. Further, I operated under the assumption that the interpretations each participant formed relative to the CCSS would be under continual construction during these interviews, as would my own interpretations of the CCSS and of participants’ interpretations.

Before the first interview, I asked each teacher to read the following (no more than three days prior to the interview): pages 3 through 5 of the CCSS document; SFMP 1, 3, and 7; and his/her new grade level content standards. Pages 3-5 contain the overall goals of the CCSS, and the content standards specify the understandings and skills to be learned by students in each grade. I also had all of these pages available for teachers’ reference during each interview.

As aforementioned, in order to gain information about how the teachers were interpreting the SFMP, I asked each participant what he/she thought specific portions of SFMP 1, 3, and 7 meant and what he/she might see students doing in class relative to these ideas. (See Appendix E for the portions that were included.)

Simon (2000) notes that interviews can be used before and after classroom lessons to help teachers reflect on the learning (students’ or their own) that took place during the lesson, and if the interviews happen not to occur in close proximity of a lesson, role-plays
or simulations can be used to help generate discussion about specific classroom situations. I appreciated these ideas and applied them in this study; I used the two classroom videotapes as catalysts for discussion in the latter interviews, and I suggested to participants that classroom examples would be helpful to discuss in the group discussions. (It truly seemed natural for teachers to discuss real classroom situations with each other since these comprised their experience each day.) In each interview, as noted earlier, I provided examples of classroom scenarios around which discussion could be framed (see Appendix F).

Before the second interview, I asked each participant to read pages 3-5, SFMP 1, 3, and 7, and his/her grade level standards (no more than three days prior). During the second interview, I continued to explore each teacher’s interpretations of the CCSS and to examine how they might have changed as a result of the individual and collaborative experiences that had occurred thus far in the study. The interview questions were similar to those in the first interviews (see Appendix D).

At the end of the study, I conducted a third in-depth interview with two of the participants (one was not accessible due to serious, ongoing health difficulties) to explore his/her interpretations of the CCSS at that point in time and to further examine how they might have changed as a result of the individual and collaborative experiences in the study. Again, I asked each person to read the relevant pages of the CCSS document no more than three days in advance. The interview questions were similar to those in the first and second interviews, though a few were new (see Appendix D).
All interviews were videotaped and were conducted in each participant’s school, generally in his/her classroom. For the most part, we were able to complete each interview in one sitting, but two sessions (each) were necessary for the first interviews for two of the teachers. Complete interviews lasted between one and two hours. I took a few notes during each interview to highlight ideas that seemed critical to participants at the time, and I transcribed each interview verbatim for more detailed analysis.

**Collaborative professional discussions.** The second main research question in this study focused on how interpretations of standards might change through a collaborative professional learning experience. As such, it was important to consider elements of effective professional growth opportunities, as evidenced in previous research. Valli and Hawley (2002) highlight some of the professional development design principles that they have derived from analysis of other research:

- Focus on what students should learn and how to address challenges students may have in doing so
- Involve teachers in identifying what they need to learn and choosing/designing the process by which they will learn it
- Provide learning that relates to individual needs but that is organized around collaborative problem solving
- Include support from external resources and specialists
- Provide opportunities to develop a theoretical understanding of what students are to learn
Hatch (2002) writes that focus groups in research consist of participants with shared experiences and are focused on specific topics. The researcher assumes that the interaction among group members creates information and meanings that are less likely to have been created by individuals. Little (2002) adds that teachers are most likely to work well together when they have a shared question or problem that they cannot (or don’t know how to) resolve individually, and since in this case, the participants were all relatively unfamiliar with the CCSS, forming interpretations of the standards was a new “problem” that they had not approached to any extent before this study. Further, teacher networks should be flexible enough to respond to the needs of individuals and the groups as a whole as they arise (Lieberman & Miller, 2002); essentially, the group discussions in this study were guided by participants’ comments and questions for each other.

Valli and Hawley (2002) also cite three points about professional development that are informed by the work with cognition of Alexander and Murphy (1998). First, “…creating effective professional development opportunities requires an understanding of how teachers make connections between what they know and how they learn” (p. 93). Second, adults have a need to be self-directed learners, which, again, was a guiding principle of the design of this collaborative experience. Third, study teams should include teachers with varying types and levels of experience. As noted earlier, I was able to find participants who had different amounts of teaching experience (overall and in mathematics), and I had teachers at two different grade levels.

The design of the group experience for the participants in this study took into account all of the aforementioned considerations. In order to provide a general basis for
conversation in each group discussion, I asked each participant during the first and second interviews to suggest one to two main questions for collaborative discussion that would support his/her own learning relative to the SFMP. I provided these questions to the participants two to three days before each group discussion via e-mail so that they could begin to reflect on others’ thinking as well as their own. During each group discussion, I began the conversation by posing one of the questions that seemed to reflect multiple participants’ thinking, and I remained generally in the role of an observer during the discussion, though I periodically suggested another question from those generated by participants when I felt that the conversation had moved away from addressing the research questions. Also, as noted earlier, I suggested that referring to specific classroom situations (hypothetical or real – perhaps from the videotaped lessons) might be useful in reflecting on and deepening participants’ interpretations of the SFMP. This was definitely the case, and I am certain that the teachers would have shared these examples even without the suggestion because it was generally through these examples that teachers reflected on the standards. Thus, the design of this experience was flexible; participants were able to adjust the trajectory of their conversations based on what they saw as their most pressing questions about the CCSS.

Davis (1996) discusses how mathematical activity can be aided by the inclusion of “toys” and “play” in students’ and teachers’ experiences (p. 224-5). When used in this way, manipulatives and other resources are truly part of the mathematical activity and experience – they shape the way students think and are used in a variety of ways to support understanding and the making of meaning. On the other hand, manipulatives and
resources can also be used as mere illustrations of pre-conceived notions – in this case, they are static objects; students do not choose how to use them, and they are generally thought of as precursors to formulated ideas but not really a part of them (and not relevant once the ideas are formulated). The general point is that play and toys (resources, tools) in hermeneutic inquiry in education are integral elements of the learning experience and not just initial steps that are entirely dispensed with once certain ideas have been developed. Thus, in this study, we utilized the CCSS document as such a tool; participants were asked to read the relevant sections again before each discussion.

Each discussion was videotaped and transcribed verbatim so that I could return to it later for careful analysis, though I also took a few notes during the discussions about what ideas participants highlighted, what questions they had, and what seemed to cause interpretations to shift. It would not necessarily be accurate to say that I acted as a participant observer (Rock, 2001) because I did not explicitly contribute my own interpretations of the CCSS to the discussion, though I asked questions as needed to move the conversation along, and I did answer questions that the teachers had about the district in which I work. Each of the two conversations lasted approximately one hour, and they were conducted in the classroom of one of the fifth grade teachers after school.

**Conversation as a means of informing interpretation.** In hermeneutics, one primary goal is self-awareness and in fact self-emancipation (Wiercinski, 2011). This includes the researcher or teacher and the participants or students; all must acknowledge this as the goal for each and act in ways that support this end. According to the perspective of Nietzsche (as referenced in Rorty, 1989/1995), when we change how we
talk, we change, at least from our own perspective, who we are. This may be true in the
case of teachers learning to converse within the framework of a new set of standards –
they may be redefining themselves as mathematics teachers in the process. Ramsey
(2011, p. 102) writes that “we share and confirm our place in the world through
communication.” Effective conversation among adult learners does not follow some pre-
directed plan; it takes the direction in which the different ideas suggested lead (Fairfield,
2011). Language, storytelling, and rereading are key in constructing and reconstructing
understanding (Davis, 1996). Rorty (1989/1995, p. 114) suggests that one question we
can ask in studying the evolution of language and interpretation is whether the use of
certain words “gets in the way of” the use of other words. He posits that revolutionary
advancements occur when someone (or a group) recognizes that two vocabularies are in
conflict with each other and thus creates a new one to replace both. Perhaps this is one
reason that standards are written in mathematics education – educators and others
recognize that they have actually been trying to achieve different goals but would rather
be working toward a more common goal. Then, once standards are published,
conversations must take place to raise awareness about various interpretations of the
standards and, as needed, to develop common understandings of them.

Crusius (1991, p. 37-8; cited in Davis, 1996, p. 27) writes that in dialogue within
hermeneutic inquiry, the point “‘is not to hold a position against all challengers, but to
listen, to allow one’s opinions to be matured by opening oneself to partners in the
dialogue whose horizons differ from our own.’” I addressed this initially with
participants as they began to work together in the second phase of the study, and I
observed how they allowed this to occur within the collaborative learning experience. I considered whether they truly learned from each other – they did, in fact, and I discuss this in Chapters 4 and 5. Gadamer (1960; cited in Davis, 1996) elaborates on this by noting that it is possible to reach moments of unity in understanding through this type of interaction. We do not know what will occur by the end of the conversation – i.e., what understandings will have developed.

Fairfield (2011) suggests that a facilitator of a conversation must be aware of when to pose questions and when to step back. A consequence of this may be that the dialogue proceeds in a different direction than the facilitator was intending or expecting. In the university setting, participants in dialogue may be faced with the current social tendency to strive for “political correctness” (p. 84), a negative consequence of which may be that ideas are not raised for question and consideration when they are in fact quite pertinent and would well inform the discussion. In this study, I made clear that participants were encouraged and urged to express ideas even if they were concerned about being seen as disagreeing with the general thrust of the dialogue (or one person’s ideas specifically). On the other hand, participants were also aware that they should be able to provide some foundation for the ideas they suggested. I, as facilitator, was also responsible for maintaining these two standards within my participation in the dialogue. Though I generally refrained from offering my opinion during the conversations, I still asked questions such as “What does this mean to you?” or “Can you give an example of that?” to move the conversation along.
Fairfield also notes that not all adults feel comfortable to participate in dialogue as described above equally; some will be more reluctant or shy to share ideas; some will prefer to listen as opposed to talking. This was especially true for one of the fifth grade teachers. Thus, the facilitator must work to allow all participants to share in the conversation without forcing participation indifferently or enabling certain voices to dominate. I found this to be necessary, though fairly simple, as I sometimes redirected questions to the teacher who was more apt to allow her colleague to share his thinking aloud.

**Observations of videotaped lessons from teachers’ classes.** Chazan, Callis, and Lehman (2008) write that when we wish to influence and/or study teachers’ professional growth, focusing on their classroom practice with students’ learning in mind is crucial. Many researchers, including McLaughlin (1994), Lieberman and Miler (2002), and Cobb and McClain (2006) have shown that when teachers have the opportunity to reflect on their practice in connection with what they believe to be true about teaching and learning, their practice is likely to be enhanced. In contrast, Cohen (1990), Ball (1990), and Remillard (1992) demonstrate that teachers can believe that they are teaching in a way that reflects state standards when they are actually not. So, it is very important to examine the relationships among teachers’ understandings and their classroom practices.

In this study, the focus was primarily on how participants interpreted the CCSS and how these interpretations changed with professional discussion. However, given the connection between interpretations of standards and classroom practice, I asked each participant to choose a Standard for Mathematical Practice from the three focus standards
in this study, to videotape one classroom lesson which he/she believed would reflect that Standard, to watch the video, and to journal about how he/she saw each Standard appearing in the lesson. After the second interview, this process was repeated with a second SFMP in a second lesson. The teachers were able to discuss their experiences and reflections in the second group conversation. (The directions for participants in this phase of the study appear in Appendix H.) I also watched each video and, in writing, formulated my own ideas about whether each teacher’s interpretations seemed to play out in practice. For instance, if a teacher chose SFMP 1 (“Make sense of problems, and persevere in solving them”), I particularly looked for evidence of the teacher’s enactment of, and students’ progress toward, this goal in the lesson, along with evidence of the other SFMP as well.

One important consideration for videorecording in classrooms is to ensure that the teacher has permission to videotape students in the class. I wrote a permission letter (which appears in Appendix H) to be signed by parents/guardians, and each teacher collected this from each student who was able to appear on video. I maintained copies of the letters until the study was complete.

Hall (2000) points out that when a classroom lesson is videotaped, the video provides a perspective that none of the individual participants could have had, which is important to consider in making the assumption that videos provide “objective” records of social situations (p. 658). That is, the perspective of the person watching the video is privileged in one sense because he/she can see the action from a different point of view and can rewind the tape, stop it, slow it down, etc. On the other hand, watching a video
also places the observer entirely outside the action in the lesson, which results in losing the opportunity to experience the lesson directly, perhaps missing important points of view of the participants. He also argues that we may make unrecognized assumptions about action on video because of the quality of the video or because of a belief that the video provides an unencumbered view of reality. So, in using video, we must be careful to acknowledge both the advantages and limitations of using this technology and be reflexive in analyzing data derived from video sources.

**Written reflections of participants.** I asked each participant to write a one- to two-paragraph reflection about his/her interpretations of the CCSS after each phase of this study – after all three interviews, each collaborative discussion, and the classroom videos, as well as any other time that he/she wished to add something to the written reflections (perhaps if an idea occurred to him/her after a lesson in the classroom that was not part of this study). This reflects the ideas of Brown (2001) and Elliott (1993) regarding teachers writing about their own practice as a means of reflecting on their understandings of teaching and learning and a means of reflecting on how their work supports students’ growth. I have provided the guiding questions for these reflections in Appendix G.

**Initial hypotheses.** Before I began data collection in this study, it was important for me to formulate hypotheses related to my research questions, as Simon (2000) suggests. I then reflected on these during the study, as I explain below in the section on data analysis. Below are these hypotheses.
1) The participants will be relatively unfamiliar with the goals and specific
statements in the CCSS and SFMP, so as they begin to read portions of the
document, they will formulate questions about words and phrases that are
unfamiliar. This could correspond with the “information” stage of teacher change
that Loucks-Horsley and Stiegelbauer (1991) suggest. They will also relate what
they read, as they understand it, to the actions and thinking of students in their
own classrooms.

2) The initial interview may be the first time that participants have attempted to
articulate in conversation how they understand the text of the CCSS and SFMP
that they have read. My conjecture is that the more that participants have thought
specifically about students and lessons in their own classrooms, the more
questions they will have about the standards and the more complex their
interpretations will be. Also, some participants may find that certain meanings
that they formed initially for specific sentences and phrases change as they
describe (and thus reconstruct) them (Boaler, 2008). My interview questions
(listed in Appendix D) are written to elicit this sort of thinking.

3) Similarly, the first group conversation may be the first time that the teachers
hear others’ thinking about the CCSS and the SFMP, which will likely provide
them with potential new ways of thinking about the standards themselves and thus
more questions as well. I believe that participants’ reflections after this
experience will indicate shifts in their thinking that were influenced by the ideas
of their colleagues. If this is not the case for any given participant, it might be
useful to consider whether he/she was truly engaged in thinking about what others were saying during the discussion. Perhaps some participants will maintain their own ideas regardless of what others say.

4) The experience of planning for, videorecording, and reflecting on a lesson in one’s own classroom can evoke many thoughts, ideas, and questions about how students actually respond to goals that we as educators set for their learning. Especially since the prime goal of most teachers is to help students to learn, the videorecording/reflecting experience will likely provide further impetus for deep reflection about the standards, and again, new ideas that have not surfaced before. At this time, participants might move into the “personal” stage of teacher change (Loucks-Horsley & Stiegelbauer, 1991). On the other hand, it is again possible that a participant might have strong beliefs about the standards and students’ responses to them, and these beliefs may not evolve after a lesson if the subsequent reflection is relatively superficial (as in the cases cited by Ball (1990) and Remillard (1992)).

5) In the second interview, I believe that it will be clear whether participants are reflecting on their experiences to date in the study. If this is indeed the case, I expect that overall, participants’ ideas about the standards will be more complex. They will have more questions about how students will respond to certain standards, and they will have more thoughts about supports that teachers will need as the state transitions to the CCSS.
6) As participants prepare for the second videotaped lesson, I hypothesize that they will think carefully about their experience with the first lesson and work to improve upon elements of their practice that could better support the goals of the CCSS and the SFMP. They may be more intentional about incorporating specific portions of the standards in the lesson, both in what they do in the lesson and what they ask students to do.

7) In the second group discussion, I believe that participants will draw on each other’s ideas more as they work to formulate still deeper interpretations of the standards. I suggest this because they may recognize that the task of interpreting a given standard is not as simple as just reading and understanding it, and they may feel that as colleagues within a school and district, they want to be able to share common understandings of the CCSS and SFMP. I also surmise that at this point in the study, they may start to think more about general instructional practices that will support students’ learning under these standards. So, they may be moving into the “management,” “consequence,” or “collaboration” stages of teacher change (Loucks-Horsley & Stiegelbauer, 1991).

8) In the final interview, I anticipate that most or all of the participants will indicate (explicitly or otherwise) that their first understandings of the CCSS and SFMP were limited. I believe that at this point, they will be thinking more deeply about the meanings of particular words and phrases in the standards, as well as examples cited in the SFMP, and how these relate to students in their own classrooms. Again, I believe that they will be asking more questions about how
the standards will be enacted, and I believe that they will feel that their interpretations will continue to evolve over time. I hypothesize that these changes in thinking will have been influenced by participants’ reflection on their experiences in this study as well as day-to-day experiences in their classrooms between phases of the study.

Data Analysis

**Perspectives on data analysis in this study.** As I conducted this study, I was, to a degree, placing myself in the role of a classroom teacher who was trying both to make sense out of and to shape students’ mathematical understandings; however, my main goal was to try to make sense of these understandings. I attempted to pay careful attention to the language that the participants used, the ways in which they used it, the prior experiences that they discussed during the study, the experiences they had during the study, and the way in which they used various tools in new experiences to revise their understandings, as researchers cited earlier had indicated.

As I looked back at my own notes (from direct observation of the interviews and conversations, as well as from watching the videos), I was aware of Van Manen’s (1990) admonition to researchers beginning to analyze this type of data. He writes that when observing participants, the process is not as simple as gathering field notes (i.e., recording what is happening), but instead the researcher must endeavor to see within the life-world of the participant and see the experience from within. The researcher still must maintain a sense of reflectiveness in order to guard against arriving at interpretations that do not express the experience of, or meaning made by, the participant. So, assuming that
meaning is as much enacted as it is developed from purely linguistic experiences, as Davis (1996) suggests, then in thinking about my participants’ ideas in my study, I needed to consider how their experiences and actions, both in the group and in the classroom, influenced their understandings and interpretations of the CCSS and SFMP.

Bredo and Feinberg (1982a) note that interpretivist researchers focus on the intentions of the actors as they write, speak, etc. and do not especially try to look beyond these texts, unless this naturally arises as a need within data analysis. In essence, they try to focus on what is actually said and not what might have been meant in some subconscious way. However, critical theorists would raise concerns about interpretive research that question the lack of focus on unspoken, perhaps unacknowledged traditions or power structures that influence the texts being created. Further, the act of research itself may serve to reinforce existing power structures (via interpretation or any other part of the method) without acknowledging that this may be occurring. As noted earlier, three questions suggested by Foucault (1980; cited in Gallagher, 1992, p. 334) can help a researcher to consider the effects of tradition on interpretation:

• What are the most immediate traditions at work?

• How do they make possible this interpretation?

• How do the interpretations support or modify the traditions?

I kept these questions close at hand as I analyzed data so that I could reflect on what unspoken assumptions might inform the written and spoken comments of my participants. This was related to the assumption that each participant has a “fore-
structure” for any given idea, based on previous experience, that influences how this person interprets this idea (Gallagher, 1992, p. 63).

Additionally, given critical concerns about the nature of interpretive frameworks, I needed to account for each interpretation that I generated. Silvers (1982b) wrote several questions regarding interpretation for a study that he conducted, and (adapted here), they could apply to any interpretive study:

- What is meant by interpretation?
- What phenomenon does the researcher examine, and how are interpretations formed?
- How are such events described?
- What is to be established in print?
- How does a researcher form interpretations about things that are outside of her own life-world?

Here, I will briefly address each of these questions in light of this study.

In this study, at least two levels of interpretation occurred; the participants interpreted the Common Core State Standards, and I interpreted their interpretations as well as actions and interactions that seemed to cause interpretations of the standards to change. I understood the word “interpretation” to refer to a search for meaning related to the written and spoken texts described above, as well as an attempt to determine what influenced this meaning. This reflects the manner in which Davis (1996) pursues hermeneutic inquiry. In this report, I describe these interpretations and meanings using the words that the participants have used to describe them whenever possible, and I
highlight commonalities and differences among them (again, using participants’ words as
frequently as possible). Below, I discuss the self-reflection that is necessary in
hermeneutic inquiry for the researcher to establish credibility for her conclusions.

Davis (1996) points out that as an inquirer in mathematics education with a
hermeneutic perspective, he must be well aware of, and well immersed in, many types of
professional and collaborative learning activities that allow him to have a broad,
informed perspective on the kinds of conditions that may inform various people’s
interpretations of text within mathematics education. That is, we cannot just, as
researchers, listen and interpret and try to reach a common understanding. Taking this
perspective, as I have conducted this study, I have continued to read professional
literature (particularly regarding the Common Core State Standards) and to participate in
professional activities focused on the CCSS, both at the local and state level.

Interpretive research values the act of validating research results in the contexts
from which they were drawn – they must make sense to those who live the experiences
that are being studied (Feinberg & Soltis, 1992). This means that as a researcher, I was
obliged to acknowledge that my own interpretations and experiences would affect the
way that I interpreted what was occurring in the study. Hesse-Biber and Leavy (2006)
call this reflexivity on the part of the researcher. Hermeneutic researchers would argue
that we need to continue to question what we believe is happening in a given setting, as
well as the interpretations that are coming to light, so that we can learn more about the
situation as a whole and in parts, ultimately helping the participants to learn more about
themselves. Further, formal definitions, theories, and laws in social science must be
applied in the contexts within which they were formulated; otherwise, they will not likely make sense (Bredo & Feinberg, 1982a). More universal theories must be applied carefully in specific contexts, taking into account the ways in which they may “fit” in these contexts and ways that they may not.

Silvers (1982b) posits that as interpretive researchers, we should work to establish a discourse between ourselves and the texts within our study. That is, I may have been studying the interpretations that educators brought to a set of mathematics standards, but I also needed to make a strong effort to understand my own interpretations of the data in the study and how they arose. Silvers writes: “…it is necessary constantly to attempt descriptions and conceptualizations of what I am doing in order to account for my previous interpretations and to inform my future analysis. This is a practice of retrospective accountability calling upon my thinking to be reflective…” (p. 174).

Silvers goes on to write that reflective discourse on the part of the researcher as she writes can help to show the sources of the ideas that are suggested in the analysis. Silvers uses the term “domain of interpretation” to refer to a world separate from his own that he is trying to understand through his own interpretations. In a separate work (Silvers, 1982b), he adds that in sociology, the object of study is actually the researcher’s interpretations of the participants’ understandings and meanings. So, the researcher must work to carefully show how she arrived at each interpretation. We have to be extremely careful not to allow our interpretations to be evaluative relative to our own experience (Silvers, 1982a). That is, what we hear someone say may seem incomplete, insufficient, or otherwise inadequate in our own experience, but in the participant’s experience, it may
seem to be a perfectly acceptable response to the events at hand. We must try to interpret participants’ words and actions within their own spheres of existence. This allows the researcher to remain on an even playing field, so to speak, with the participants and not to privilege the researcher over them.

This self-reflection is akin to the practice in hermeneutic inquiry of continuing to bring into question our own assumptions, traditions, and ideas related to the texts of the study. When Silvers (1982a) writes, he presents his findings in a series of interpretations, trying to build each interpretation from earlier ones. He presents more than one interpretation for a given event in order to demonstrate the way in which his analysis developed. He points out that his interpretation could never be complete (echoing the idea that the hermeneutic circle can never be fully closed). It is important to note that though I studied other educators’ interpretations of standards, I also formed my own interpretations related to the standards and to how professional knowledge developed and evolved as the study progressed. Later in this chapter and in my log in Appendix J, I document how I drew conclusions about meanings that the participants (and I) created, and I offer possible alternate meanings when appropriate.

**Procedures for analysis.** A case study is “an intensive, holistic description and analysis of a single, bounded unit. Conveying an understanding of the case is the paramount consideration in analyzing the data.” (Merriam, 2009, p. 203) Researchers must make decisions about what data should be included in a final report and why, so all data must be well organized and maintained so that the researcher can draw on any data as needed. Multiple case studies involve data from several cases. In this type of study,
the researcher collects and analyzes data from each case individually, creating a rich
description of each, and then formulates a cross-case analysis based on all of the cases.
Data analysis in case studies may include identifying themes, models, or theories, both in
individual cases and in the cross-case analysis. As is detailed in Chapters 4 and 5, in
analyzing the participants’ interpretations of the SFMP, I did find themes
(commonalities) among these ideas, and I was able to draw several conclusions about
how each teacher’s interpretations (or collective meanings) formed and evolved over
time.

Describing considerations for data analysis in a TDE, Simon and Blume (1994)
write of a study they carried out called the Construction of Elementary Mathematics
Project. In this project, they worked with a class of pre-service elementary teachers in a
course focused on multiplicative structures. In the course, the authors found that they
were beginning to focus on norms and practices related to justification that emerged in
the students’ interaction as the course progressed, and so in retrospective analyses, they
focused on two questions:

• How were classroom norms with respect to justification and validation negotiated
  in this classroom community?

• What issues affected the development of these norms?

With these two questions having been formulated as a result of the ongoing analysis, the
researchers focused on these questions as they continued to analyze their data, and they
eventually generated models of justification practices and descriptions of important
interactions that contributed to the development of students’ practices in this area. I was
able to employ similar strategies within my data analysis; I began to formulate observations and questions about teachers’ interpretations of the SFMP and how they evolved while I was collecting data. Then, after the last phase of data collection, I returned to the data to consider what seemed to be the most salient aspects of the teachers’ interpretations (both common and unique), focusing on these ideas for my final report.

Simon and Blume write that in analyzing and interpreting their data, they focused on social norms that became explicit throughout the interactions of participants, ideas related to mathematics and pedagogy that became taken as shared (thus, that no longer required justification), the affect of participants relative to particular aspects of the learning taking place, and participants’ learning of specific skills and ideas within the topic being studied. Though it did not seem wise for me to choose to focus on these four aspects of participants’ learning before I actually collected data, I found that several elements of my coding involved participants’ affect (or emotion) when discussing particular ideas, and participants’ assumptions (spoken and unspoken) about mathematics as a discipline, the teaching of mathematics, and the learning of mathematics.

Simon (2000) also discusses the challenge of working with a substantial amount of teacher self-report data in a TDE. He points out that teachers may unintentionally or intentionally provide reflections that parallel what they think the researchers want to hear, either about their own learning or about the nature of the mathematics or pedagogy that is a focus of the research. So, he tends to only take as primary data comments from teachers that indicate perturbations, struggles, or anything that teachers would probably
not write or say if they were hoping to appear exceptionally competent. The author is careful to point out that this does not mean that if teachers do offer such positive comments, they are not genuine, but that when teachers claim to be learning or changing their practices, this must be confirmed by other sources of data (perhaps classroom videotapes or other interactions with colleagues).

Given the recommendations above from other researchers, the data in this study will be analyzed using an inductive approach, similar to that described by Rubin and Rubin (2004). As I coded data throughout the study, I focused on the questions below, and I reflected on them in an electronic log (Appendix J) after each phase of data collection in order to support the “ongoing analysis” that Simon (2000, p. 341) describes. I have numbered the questions in order to refer to them by these numbers in the log. I recognized, before I began collecting data, that depending on how I answered these questions and what evidence I had for my answers, I might need to revise my interview questions for the later interviews and/or ask questions in the group discussions that focus more specifically on aspects of these questions. The main change that I found to be useful in gaining better information about participants’ interpretations was to present each teacher with the statements from each SFMP separated from each other, rather than in the full paragraphs as the SFMP appear in the CCSS document. I discuss this in Chapters 4 and 5. The questions for reflection were:

1. On what is each teacher focusing as he/she works to interpret the standards? (e.g., text, students, classroom experience, implementation issues, etc.) What evidence do I have for this?
2. What explicit or implicit beliefs, experiences, cultural norms, etc. may be influencing how each teacher interprets the standards? What evidence do I have for this?

3. Do each teacher’s interpretations of the standards seem to be changing? What evidence do I have for this?

4. If a teacher’s interpretations of the standards are changing, what might be influencing this? (specific text from conversation with me or other participants, written reflection, reflections on lessons, other experiences) What evidence do I have for this?

5. If a teacher’s interpretations of the standards do not seem to be changing, why might this be the case? (prior beliefs about students, teaching, learning, and/or mathematics; nature of his/her reflection, etc.) What evidence do I have for this?

6. In what ways, if any, are different teachers’ interpretations of the same standard becoming similar over time? In what ways are their interpretations distinct from each other? What might be causing this? What evidence do I have for this?

7. What is each teacher saying about supports that will be necessary in enacting the standards?

8. What does each teacher seem to believe about the value and/or appropriateness of the standards? What evidence do I have for this?

9. At what stage of teacher change does each teacher seem to be? What evidence do I have for this?
10. How would I answer each of these questions about my own interpretations of the standards and my thoughts about their enactment? How might this affect how I interpret my participants’ comments? How can I separate my ideas from those of my participants?

11. What might I be missing in my analysis? Are there gaps or inconsistencies? If so, how can I try to resolve them?

After the first set of interviews, I transcribed each verbatim. I then read each transcript carefully and began to mark each transcript to identify five aspects of each teacher’s dialogue: all words and phrases that related to the interview question, words/phrases that expressed emotion, words/phrases that were repeated often, words/phrases where the teacher was saying something explicit about his/her process of interpretation of the standards, and all instances where a teacher seemed to be learning something about the standards from what one of the other teachers said. I also asked each participant to write a written reflection on the interview and his/her current interpretations of the standards (see Appendix G for the guiding questions for this and all subsequent reflections). I read and similarly marked text in these reflections. I then copied and pasted every word, phrase, or sentence that was marked for any reason into one document, in which I created sections for each interview question, and I color-coded this document by teacher so that I could begin to compare and contrast the teachers’ responses to each question. Next, I read through this document and made notes about possible themes that were emerging, but I also realized that the three teachers’ responses to each question were typically rather unique. However, I was able to identify main ideas
(and sometimes shifts) in each participant’s response for each question, and these ideas and shifts were the foundation of my writing in Chapter 4. So, themes for each question were not as apparent as overall themes in the three teachers’ responses. These larger themes emerged after re-coding the data from all seven phases of data collection as I identified more general ideas related to the research questions in all of the teachers’ comments, and I discuss these ideas in Chapter 5.

I moved through a similar process for the first collaborative discussion, asking for a reflection from each participant afterward, and coding both the transcript and each reflection.

In phenomenological research, a “theme” can be described as a “capturing” of the point or meaning of an experience or a simplification of the experience (Van Manen, 1990, p. 87). Van Manen writes that we can take at least three approaches to looking for themes in text, which could also be helpful in finding ways to characterize participants’ interpretations of standards and how they change with reflective, collaborative inquiry:

- Writing one sentence or phrase that captures the fundamental meaning of the text as a whole
- Choosing sentences or phrases in the text that seem to particularly highlight aspects of the experience
- Looking at every sentence in the text and trying to determine what it tells us about the experience

I particularly utilized the third of these methods as I examined the data from the interviews, the observations, and participants’ writing. I found this to be most beneficial
because each teacher shared so many ideas about the standards and about classroom practice that it would have been difficult to encapsulate his/her thinking in a few sentences or phrases.

To engage in the “retrospective analysis” suggested by Simon (2000, p. 341), I reflected on the following questions in my log after the last phase of data collection. I have identified these questions with letters in order to refer to them in the log. During this time, I was still able to contact participants for their input on my tentative findings.

A. What influences seemed to be important in each teacher’s interpretations of the standards? Were some more important than others? What evidence do I have for this?

B. What were occurrences that seemed to cause shifts in teachers’ interpretations of the standards? What evidence do I have for this?

C. What seems salient and/or important to teachers as they discuss enactment of the standards in their classrooms and schools? What seems less so? What evidence do I have for this?

D. What might I be missing in my tentative conclusions? Are there gaps or inconsistencies? If so, how can I try to resolve them?

Once I began to get a sense of participants’ interpretations of the CCSS and how they were changing (guided by the questions above), I needed to determine how to most effectively represent these interpretations in writing. Again, Van Manen’s suggestions for identifying themes in phenomenological studies provided some insight into a method for doing so in this study. When we are trying to decide if an idea is essential within an
interpretation or not, we must consider whether the interpretation would still be what it is, fundamentally, if this idea were not included. We can then start writing paragraphs that are more descriptive, based on the themes we believe we may have found. This may be the first draft of the report. Van Manen points out that research and writing in human science are intricately connected because we are continually trying to explain and make sense of human experience. He says that writing:

- Separates us from what we know and unites us more closely with what we know
- Distances us from the lifeworld but also draws us closer
- Decontextualizes thought from practice but returns thought to praxis
- Abstracts our experience of the world yet solidifies our understanding of it
- Objectifies thought in print but subjectifies our understanding of something that engages us

Davis (1996) notes that since we must describe using language, we must write and rewrite as we attempt to relate experiences (i.e., how meanings and understandings develop) to the theories we use to explain them. This is especially significant in hermeneutic inquiry since we are attempting to make sense of interpretations of text, but we can essentially only use text in order to do this.

With the recommendations above in mind, for each teacher I wrote a summary of his/her comments in interview 1 and the first group discussion, as well as in his/her participant’s written reflection for each, indicating what I observed to be points of significance relative to each question and citing quotes from the participant’s comments. I asked for feedback on whether this summary seemed to reflect the participant’s
thoughts. This process is known as member checking (Lincoln & Guba, 1984) and is a key strategy in interpretive research for establishing credibility in findings. I would have revised any parts of the summaries that participants felt did not represent their interpretations of the goals in the standards until participants were satisfied with the descriptions. If there were disagreements about meanings shared by the group, I would have made note of the nature of these disagreements because this could have informed later discussion of how challenging it could be to reach collective understandings of standards. However, I did not receive any feedback from the teachers that indicated their disagreement with the summaries; in fact, I only received feedback from Ray on the summary from interview 1 and the first group discussion, as well as interview 2.

Before I watched the videos of the lessons that were recorded, I asked each teacher to tell me what goal he/she chose from the CCSS toward which to strive in the lesson. I watched and transcribed the video, and I identified each statement in the lesson (from teacher or student) that seemed to align with one or more of the three SFMP, adding comments in my own notes as appropriate – both about the SFMP and the teaching and learning that was occurring. I wrote a summary of my observations of each lesson. Then, I read the teacher’s self-reflection, and I made notes comparing and contrasting my summary and the teacher’s. It should be noted that I received reflections on lesson 1 from all three teachers, but only from Ray for lesson 2.

After the second collaborative discussion, I transcribed the video, asked for a written reflection from each person. I received reflections from Carrie and Ray. I chose not to send another summary of each teacher’s responses at this time, partially because
most of their interpretations had not changed substantially, and partially because I was not receiving much feedback from anyone but Ray when I was sending summaries initially.

Following the final interview with each participant, I coded the transcribed interview and the final reflection as before. Finally, when I was approaching the end of data analysis, I provided a final three-page summary to each participant of my conclusions about my tentative findings from the study (drawn from the main ideas in each section of Chapter 5), and I asked once more for feedback. I received feedback from Ray and Sandy (nearly all in agreement with what I had written), and I included this in Chapter 5. It was very important to try to represent each participant’s thinking as faithfully as possible, without leaving out important points or allowing my own beliefs to resonate too strongly in my reports.

In creating this final report on the study, I needed to make decisions about which findings were most relevant to my research questions and which data would be most helpful in justifying my conclusions. Merriam (2009) writes that it is always challenging to find the appropriate balance between presenting data and interpretations of data, as well as broader conclusions that can be drawn from these interpretations. She suggests that an effective way to achieve this is to weave relatively brief quotes from participants within the researcher’s own descriptions and interpretations so that the reader can, in essence, follow the story of the research and understand how the researcher arrived at the findings. Of course, longer quotes should be included when appropriate. This is the format that I have followed in Chapters 4 and 5. It is also important to include
participants’ own responses to the findings when these help to shed light on the researcher’s interpretations. In general, the researcher’s goal is to establish that his/her findings are plausible, given the data collected.

Merriam adds that in reports of case studies, the researcher has a responsibility to provide rich description of the context for the case(s) so that the reader can, as well as possible, understand the particular circumstances in which the data was gathered. In other words, the reader should be able to feel connected to the case, even if he/she has not experienced what the participants have experienced. As aforementioned, this helps the reader to understand the findings of the study, to believe that they are plausible, and potentially to apply the findings to other situations in the reader’s own experience. Through the background I initially provide about the schools and participants in Chapter 4, as well as their own descriptions of their school cultures, I believe I have been able to provide a reasonable feel for the school contexts from which the data from this study are drawn.

**Credibility of Analyses and Conclusions**

One limitation of the interpretive approach is that not assuming a highly rational perspective means that there is no means of judging which interpretation(s) might be more appropriate in given circumstances than others (Bredo & Feinberg, 1982b). So, we need to align our interpretations with what seems to make the most sense in the context of the social world in which we exist, still accounting for the possibility that our interpretations as represented could be inadequate. In this sense, human science can still be rational, and Van Manen (1990) maintains that it is, because to be rational is to believe
that human life can be understood through reason in a broad sense. “To be a rationalist is to believe in the power of thinking, insight, and dialogue” (p. 16), and this study is grounded in reflection and dialogue. However, human scientists also believe that experience cannot be characterized in one description and that there is always an element of the unexpected and the unpredictable within experience; thus, the hermeneutic circle, again, can never be complete.

Van Manen (1990) argues that “objectivity” in human science research means that the researcher is “oriented” to the object and does her best to remain “true” to it as she describes and interprets it (p. 20). Acknowledging the need for “subjectivity” means that the researcher must be insightful, perceptive, and personally involved with the interpretation of the experience being studied, while also continuing to be reflective and to work to provide viable and reasonable interpretations. Davis (1996) explains that he aims for his reports to be viable, reasonable, relevant, and applicable (acknowledging that there is not one truth to be discovered relative to the interpretations and meanings that are being made within the study). The report should relate the experiences of both the researcher and the participants, being as true as possible to the set of circumstances (and intentions, motives) that foreground the study. He notes that the report is “valued for its transformative potential” (p. 104).

Merriam (2009) cites Maxwell (2005) in pointing out that validity in qualitative research has more to do with whether the findings align with the purposes and contexts of the research, and the data gathered, rather than with a general question of whether the findings are “true.” This again relates to the epistemological and ontological
underpinnings of interpretive qualitative research – that one absolute reality does not exist and that reality is in the experience of the interpreter, be this participant or researcher. Similarly, though reliability in quantitative research is a question of whether the data (and thus, the findings) would occur in the same way in a replication of the same study, this hardly makes sense to consider in qualitative research because the goal is to study particular phenomena in context and to discover what may be different (as well as similar) from context to context and participant to participant. So, Merriam and other qualitative researchers address reliability in terms of whether the findings seem to be consistent with the data that has been collected, and it is the researcher’s responsibility to establish these connections between the data and findings in a clear way in the final report.

Merriam (2009) discusses a number of strategies for addressing issues of validity and reliability (often referred to together as credibility) in qualitative research. One strategy, called triangulation, may involve using multiple sources of data. In this study, I gathered data about teachers’ interpretations of the SFMP and how they evolved over time through interviews, group discussions, participants’ written reflections, and my own observation of videotaped lessons. I also utilized member checking, which involved sharing some of my findings with participants and asking them to respond to the following question: “Do my findings seem to fit with your interpretations of the SFMP and your experiences so far in this study? Is there anything you would add or change in what I have shared with you?” This was done after phases 2, 4, and 7 of the study (listed earlier in this chapter). Another strategy for establishing credibility is being engaged
with the participants and the data collection as long as new data is emerging; this is often referred to as “saturation” (p. 229). This helps to show that enough relevant data has been considered in formulating findings. I worked with my participants through the seven proposed phases of research, and I did feel that my data was sufficient at the end of this time because all three teachers were repeating themselves often by the end of the study. However, I believe that a much longer study would have allowed for more evolution in their understandings of the SFMP.

Further, Merriam (2009) discusses the notion of transferability, which relates to the idea of external validity in quantitative research. Although the goal of a qualitative study is not to generalize to a larger population but to learn something about a particular case, participant, or context, the researcher can help the reader to make connections to other cases and contexts by including sufficient detail about the contexts and participants studied, thus allowing the reader to determine how the findings might transfer to other situations. She also notes that in any specific case, there are elements that apply essentially universally, and there are also elements that are unique to that case. By providing rich description and by presenting multiple cases, the researcher can help the reader to understand which elements may be universal and which may be unique. In this study, I strove to provide this detail by answering the following questions as I analyzed the data:

- What are the demographic characteristics of these schools and this district?
• What do the teachers say about their classrooms, schools, and district that provides insight into the professional and/or social norms in these settings? How does each teacher relate to or participate in these norms?

• For each participant, of what personal and professional characteristics am I aware? How might these affect his/her thinking about the standards?

• During the study, have I become aware of any changes in the professional settings or school-related personal experiences of any of the participants? (For instance, a participant might have a student teacher, or he/she might have missed a significant amount of school due to illness.) How might these affect their thinking about the standards?

I attempted to remain vigilant in requesting and accounting for feedback from each participant in each phase of the study, though as noted above, I was not always successful in these attempts. I wished to try to represent each person’s interpretations of the CCSS as they developed throughout the study, so this member checking was necessary in striving for this ideal. As often as possible, I have used participants’ own words in my reporting so that their voices are directly presented to the reader. Further, my reports progressed through several drafts because I wrote and rewrote summaries of my interpretations in the process of writing my final report, and I was well aware that I needed to maintain a self-reflective stance as I wrote, as many authors, cited earlier, have cautioned interpretive researchers to do. The questions (listed earlier) that I used for ongoing and retrospective analysis helped me to do so. This process is termed
“bracketing” in phenomenological research (Husserl, 1913; cited in Hatch, 2002, p. 86) and is helpful to the researcher in trying to remain true to the participants’ meanings.

When writing about data analysis and findings, researchers in qualitative studies are encouraged to describe their particular perspectives, or biases, about the research questions and the data being collected (Merriam, 2009). This can help the reader to understand the approach of the researcher in conducting the study, choosing data on which to focus, and generating conclusions about the data, which can also be useful to the reader in understanding how the findings might or might not apply to other similar sets of circumstances. I attempt to describe my background and biases in the next section.

Certainly, my doctoral committee served as reviewers of my findings, and they helped to raise questions that caused me to return to the data for alternate interpretations or to further justify conclusions that I had drawn. Further, I have a peer who recently earned a doctoral degree in mathematics education and with whom I shared my tentative analyses toward the end of my data analysis. I provided my peer with the lists of reflective questions for myself that I presented above, and I asked her to react to the following questions relative to the data and findings:

- Do my interpretations of the data seem reasonable, given the questions on which I am focusing? If so, what stands out for you as supporting evidence? If not, where do you feel that other interpretations may be more (or equally) plausible?
- In general, am I presenting enough evidence for my conclusions? If not, where should I provide more?
Merriam also suggests that an “audit trail” (p. 223) can be used to delineate the specific steps taken in data analysis and in drawing conclusions about the data in the course of research. This involves creating a record of any decisions that were made to focus on particular data and in formulating particular interpretations based on specific data. Again, including this record helps to establish that the findings are consistent with the data, and I have included an audit trail (in the form of a log) as Appendix J in this report. I added to my log each time I collected, analyzed, or wrote about data, and this record was guided by the following questions:

- What have I done during this work session?
- What, if any, new interpretations or conclusions emerged as I considered the data during this session?
- If there were changes in my tentative findings during this session, how did they occur?
- What is/are my next step(s)?

Ultimately, the findings of this study should be credible if I have shown that the participants agree that their interpretations are represented as adequately as possible (Hesse-Biber & Leavy, 2006) and if I have shown that the findings will resonate with educators in other similar contexts who are working to enact the CCSS. I believe that the latter criterion has been met because I observed that the teachers’ comments about the logistics of enacting the standards were often highly similar to those of teachers in my own district, and it is quite likely that this would be true for teachers in many other
districts. This reflects the notion of “transformative potential” that Davis (1996, p. 104) suggests is key in reporting findings in hermeneutic inquiry.

**Background and Potential Biases of the Researcher**

I have worked for seventeen years in mathematics education, ten of these specifically in professional development. I am licensed to teach middle and high school mathematics, excelling in my undergraduate mathematics courses, and I taught middle school mathematics for six years. Currently, I work with kindergarten through twelfth-grade mathematics teachers in one school district, both in their classrooms (for weeks at a time) as well as in inservices. I also coordinate the district’s mathematics program, including curriculum guides and assessments at all levels.

My goal in any work with teachers is to support them in their work with students, which I very strongly believe involves helping them to understand the content that they teach and the expectations that the mathematics education community has for student learning, at all grade levels and for all students. I do believe that these expectations tend to shift somewhat over time, but I also have read enough literature on the history of mathematics education to recognize that most mathematics educators have been writing essentially the same things for decades about the prime importance of students’ making sense of mathematical ideas for themselves and students’ ability to apply mathematical thinking in familiar and unfamiliar situations. Further, these are the same major ideas that have been expressed in the NCTM *Principles and Standards for School Mathematics* (2000) and in the CCSS document, both of which are based in part on current research on student thinking and in part on the study of mathematics education in nations around the
world whose students traditionally demonstrate higher mathematical achievement and understanding than American students have demonstrated.

When I read the SFMP practice myself, I am able to consider these statements in light of a wide variety of experiences in many classrooms as well as my knowledge of mathematics education research and my work with other mathematics educators. However, I recognize that most elementary teachers are only able to draw on their own classroom experience with mathematics, both as teachers and as students themselves. My participants had rather different foundations for thinking about the SFMP than I did, but one of the prime goals of this study was to learn more about the way teachers might interpret the standards so that professional development could help teachers to understand what the leaders of the mathematics education community hoped to see happening in classrooms. Still, when I was working with the teacher participants, I was mindful that I might tend to listen for certain ideas or words that I felt best represented the ideas in the CCSS and SFMP, but I truly needed to listen for teachers’ own interpretations and not just what I felt was “correct.” Further, as noted earlier, I have not assumed that there was just one “correct” interpretation of any given standard because there is so much potential variation in mathematics learning when human beings are involved. The strategies described in the previous section of this chapter helped me to listen and to analyze the data reflexively.

It is also important for me to note that there are portions of the CCSS with which I do not agree, particularly in regard to the placement of some of the content (I believe some is placed in grades earlier than will be developmentally appropriate for students)
and the apparent focus on the American standard algorithms as a goal in computation (I am concerned that teachers may miss the development of these algorithms, which is often more implicit in the standards). While working with the teachers, I was cognizant of the ways in which I expressed any of my own feelings about these elements of the CCSS – I did not necessarily feel that I needed to “hide” my opinions, but I generally only shared these types of comments when the teachers themselves had already offered similar thoughts. In any case, I believe that in general, the CCSS provide a helpful structure for learning, centered around “big ideas” (called “clusters”), that will help teachers who have sufficient professional support to structure learning experiences in a way that will support students’ development of these ideas.

**Potential Limitations of the Design**

One limitation of conducting a TDE, at least at the current moment in time, is that little research exists to support the development of research studies related to teacher learning and development (Simon, 2000). In the long term, however, studies such as this project will contribute to a growing body of research in this area.

Simon comments that the role of the “observer” is important in a TDE. This is a person outside of the researcher-participant interactions who can offer alternate observations and theories about what is occurring in each cycle of the experiment. A second limitation of this study may have been that I did not have an “observer” who directly participated in the research process with me. However, my dissertation committee members were able to offer alternate suggestions in response to my tentative findings and thus somewhat served in this role.
A third limitation of the study may have been that we did not explore the entire CCSS document; rather, we examined only a portion of it. It is possible that important connections among texts that we read and those we did not read were lost to the consideration of the participants. In other words, we did not study the whole; we only studied a part, and in interpretation of text, this can be problematic (Fairfield, 2011). However, participants were certainly able to refer to other portions of the CCSS if they wished to do so on their own (I am not sure that any of them did so, as they did not refer to other portions). So, I must be clear about the fact that together, participants studied only the overall goals and the Standards for Mathematical Practice, so their interpretations (and my representations of them) primarily refer only to these portions of the documents. Readers, that is, should not attempt to draw conclusions about how the participants might have interpreted other parts of the document. However, follow-up studies in which interpretations of other portions of the document were studied could certainly be useful to those providing professional support to educators implementing the standards.

Another possible constraint of the design of this study simply involved the number of participants and the manner in which they were selected to participate. Certainly, the results of the study are not generalizable to larger populations. However, the purpose of the study was not to generalize but rather to inform plans for professional support and further research into the most effective ways of supporting teachers and school leaders as the standards are implemented over the next several years. Learning
about a small number of educators’ interpretations is one way to stimulate these types of plans for future work.

**Operational Definitions**

The following are definitions for key terms in my research questions (particularly the second one). These remained constant throughout the study, but I intended to document any shifts in these definitions over time and the causes for such shifts.

- **Elementary teachers** – Teachers who work in kindergarten through fifth grade
- **Interpretation** – A search for meaning related to written or spoken texts, as well as an attempt to determine what influences this meaning
- **Overall goals of the CCSS** – Goals that are presented on pages 3-5 of the document; broad goals for all students, teachers, and schools

**Ethical Considerations**

As I recruited participants, I was open about the goals of my study because I have found that in matters of education, adults are much more willing to be open and involved when they feel that honesty is a key element of the process. In fact, one of my participants also observed this as he was discussing elements of successful change in schools. However, as aforementioned, I did not generally express my own interpretations of the CCSS; I did not want to create a situation where a participant felt that he/she must defend every comment with which I might disagree, or where he/she felt that there was little to be said because I already agreed with his/her point of view. I was well aware from two other interview studies that I had conducted that when participants expressed views that are contrary to mine, I needed to focus on staying open-minded and willing to
listen to their perspectives. I have found that this is easier to do when I remind myself that most teachers are well-meaning in their thinking about education and that their beliefs are in fact based on their own experiences, which are very real to them.

Also, as noted earlier, I remained aware of how participants were interacting during the collaborative conversations, and I set and maintained an expectation that participants were respectful of each other and of others’ ideas; all interpretations had meaning to those who formulated them and were worthy of reflection. This was especially critical as two of the participants worked in the same school, and all of them knew each other – both through district activities and because the two fifth grade teachers had taught the first grade teacher’s own children in class.

There was little or no risk to participants in this study since the data collection consisted of interviews, conversation, videotaping, and written reflection, and any participant was free to decline to participate in one or more phases of the study or to leave the study at any time. Also, all data collected during the study was confidential – no identifying information of participants, schools, or districts is included in this report. Pseudonyms are used to identify people and institutions. For the students in the classroom videos, there was little or no risk since they were simply participating in a regular mathematics lesson and were observed only by their teacher and by me (and I had no direct interaction with students).
**Rationale for, and Potential Implications of, This Study**

**Rationale Revisited**

Stein and Shields (2004, p. 144-145) suggest four priorities in mathematics education research on the impact of standards. These are:

1) Understanding the mediating factors that exist between interventions and their intended targets.

2) Understanding ways in which local actors choose what knowledge is legitimate.

3) What are the crucial leverage points for implementing standards?

4) What are effective strategies for “going to scale”?

This study addresses the first three priorities, to some degree. The second, in particular, provides an argument for working with teachers to understand how they interpret and validate for themselves what is important in published standards. The first and the third priorities involve studying how teachers respond to their own interpretations of standards and how they believe these standards could (or do) play out in their own classrooms and schools. Stein and Shields comment that qualitative research is needed to address the first two priorities, because we need to understand the impact of policy from the standpoint of policymakers as well as from the perspective of teachers. Fullan (1991) also writes that we must understand the different realities of different groups involved in the educational system to consider how change efforts will work (or have worked) in those contexts.
Curriculum theorist Elaine Atkins (1988, p. 444; cited in Gallagher, 1992, p. 25) writes, “Very few writers within the hermeneutic tradition deal with pedagogical issues. And those who do often do not go far enough to offer us guidance.” As mathematics education leaders and researchers look to provide guidance to the field, it is important to examine policy implementation from the perspectives of those who must actually change practice as a result of the policy – in this case, teachers (Darling-Hammond, 1990). This is a newer way of approaching policy research – i.e., paying attention to local contexts, local actors’ understandings of teaching and learning and the policy itself, and the change process. Teachers highly shape the content that is taught in school (Weiss, 2002). They base their decisions on beliefs, values, policies, practices, and resources that aid and guide their work. Bryk, Sebring, Allensworth, Luppescu, and Easton (2010, p. 54) conclude that “…the effectiveness of schooling depends largely on the teachers’ capacity to problem-solve regarding classroom concerns and to coordinate instructional work.” Moreover, teachers’ knowledge of content and pedagogy are strong influences on student success.

Cohen (1990) argues that teachers are those who need to change in order for new instructional policies to take hold. Thus, we need to pay attention to what they believe they are doing and what they believe they have yet to do, as well as what they believe teaching and learning are (policy documents cannot directly impact these beliefs). Policies ought to support the development of teachers’ capacity to evaluate and monitor both current and changing practices. However, policies often tend to assume (if in unspoken ways) that teachers are “the problem” (p. 326) and that they need to be fixed in
order to promote high quality instruction. Cohen questions how states can both assume teachers are “the problem” and still create policies that support their growth as reflective professionals. As Porter, Archbald, and Tyree (1991) write (cited earlier), we need more research that helps us to understand the connection between policy and practice as well as what types of policy instruments (e.g., standards) help to empower teachers to make decisions that effectively support student learning.

An education does not end with a learning of the basic content of the subject; instead, it continues as students, or, in this study, teachers, learn to interpret, critique, and question this content (Fairfield, 2011). Such education should continue throughout a lifetime. However, Wilson and Berne (1999; cited in Weiss, 2002) note that little research literature has explored the connection between professional development and teacher learning and growth in practice. The mathematics education community needs to know more about the impact that teacher development can have on change in teachers’ thinking as well as their practice, and ultimately, student learning. Another specific research question that Weiss (2002, p. 57) raises is: “To what extent have teachers acquired more substantive knowledge of standards-based content and improved skills regarding pedagogy and collegial activity, as called for in the standards?” She also suggests that we should be looking at how school districts interpret the standards as they provide professional opportunities for teachers. Finally, Brown (2001) suggests that mathematics education researchers need to conceive of research as a means of better understanding changes that occur in mathematics teaching and learning at the individual and social level and not simply a way of moving toward an ideal mode of instruction.
Potential Implications

Floden, Porter, Schmidt, Freeman, and Schwille (1981) comment that understanding how teachers make decisions about content will help us to understand differences in student achievement. They also write that since school mathematics is traditionally (though perhaps not ideally) fairly specific to the school setting, any decisions that eliminate particular content will likely result in students’ never encountering this content. We might infer that a similar situation would occur with teachers’ treatment of content and the ways in which students engage in it – students may never engage in content in certain ways if teachers do not facilitate this type of engagement. This study will help to highlight ways that teachers focus their attention on elements of the Common Core State Standards and will explore teachers’ ideas about how they would need to make decisions in their classrooms based on those standards.

Further, Darling-Hammond (1990) writes that we do not know a great deal about how policies actually affect the way that teachers and students work, and we do not know much about how teachers interpret policies in light of their beliefs, experience, and knowledge. She says that more studies of this type will be useful to policy analysts as well as policy makers, especially since more and more policies are being written for schools. “If school reform via state-level policy is to prove constructive for education, research on its school- and classroom-level effects will be vital” (p. 341). Similarly, Cohen (1990) notes that leaders need to know what it would take to provide helpful guidance to teachers when policies change and what can be done to help teachers actually learn from this guidance. Firestone (1989) also argues that we need more research that
explores what causes teachers, schools, and districts to enact policies in an active way, and he writes that this research will be useful to administrators as they plan to enact policies as well as policy makers who may then be able to write and support policies in such a way as to make enactment at the local level more likely. Talbert and McLaughlin (1993), Weiss (2002), and the participants of the 2003 NCTM Research Catalyst Conference echo this point, as noted earlier. Moreover, the findings in this study may help to shed light on how capacity-building and system-changing policies (McDonnell and Elmore, 1987) and ideas as policy instruments (Weiss, 1990) might foster new practices and understandings among teachers in shared contexts. So, the results of this study could be quite useful to those working to enact policies at the local and state levels because these results help to illuminate some interpretations of the CCSS that teachers form and also the types of professional interaction that might help teachers to understand the intent of the CCSS more deeply.

In general, this study provides one window into (and perhaps a deeper understanding of) the learning process for teachers because education involves language, interpretation, texts, communication, and the transmission or critique of traditions (Gallagher, 1992). If a person wishes to make a change in his action, one way to work toward this is to make a change in the language he uses in connection with this action (Brown, 2001). In addition, since a significant responsibility of a teacher is to awaken curiosity and a desire for learning in the minds of students, a teacher who is continually seeking to broaden her own mind – to think, to question, to imagine – is more likely to accomplish this goal with students (Nicholson, 2011). The design of this study allowed
the participants to continue their own education relative to mathematics, teaching, and learning, specifically with regard to better understanding the mathematics that they were being asked to teach and how they positioned themselves with respect to this mathematics. For at least two participants, the study helped to foster a professional “evolutionary interest” – i.e., intentional change of their own practice (Elliott, 1993, p. 197; cited in Brown, 2001, p. 213). In addition, the results of the study can help those who provide professional growth experiences to educators consider how to foster similar professional growth in their own communities of learners. The ultimate goal of this study was to better support student learning in mathematics by supporting the ongoing learning of teachers.
CHAPTER IV
DATA

Introduction: Considerations for Reporting Case Study Research

Yin (2008) writes that reports of case study research can be informative because describing and analyzing one case or more can help readers deepen their understanding of a more general phenomenon. In this study, each teacher represents a case, and the general phenomena being studied are teachers’ interpretations of the Standards for Mathematical Practice as well as their thinking about how these standards will be (or should be) implemented in classrooms and schools. Yin also points out that the researcher should identify the audience for the report and should endeavor to communicate directly with the audience, thus guiding choices about the detail and points of emphasis in the writing. Here, the main audience is the dissertation committee and perhaps other education professionals at colleges and universities, so the report will highlight potential answers to the research questions in light of the literature reviewed and discussed in chapter 2.

According to Yin, the researcher has several options for structuring a full case study report. The standard method is the “linear analytic” method, which reflects the form of this dissertation and includes five major sections (chapters). Another option is the “comparative” method, where the same case study is discussed two or more times, relative to a different conceptual model or a different perspective in each instance. A third possibility is the “chronological” method, used to describe the case over time in order to show that certain aspects of the case may have been caused by others. Still
another choice is the “unsequenced” method, where the sequence of the sections is not important in describing the case. As the linear analytic method is aligned with the structure of this dissertation, this is the organizational style I have chosen. However, I am also using an essentially chronological format to share most of the evidence in each case, often highlighting all seven phases of the study for each teacher, because it is important to illustrate how each teacher’s thinking changed (or did not change) over time in order to answer my second research question.

Further, for determining how to include each case in the report, Yin offers several alternatives. In this study, there are multiple cases, so the single-case report is not an option. One choice a researcher may make is to include several narratives, one for each case, as sections of the report. In this circumstance, the researcher would also then include a cross-case analysis to compare and contrast elements of the cases. A second alternative would be to structure the writing about each case in sections by questions and answers, repeating the same questions for each case. Here, the reader may begin to develop his/her own cross-case analysis before reading any such analysis by the author, and the reader may focus on particular aspects of each case that especially interest him/her. A third option would be for the researcher to present only a cross-case analysis, structured by issues that are significant to all of the cases. For this chapter, I present each teacher as a case individually, but I generally structure the evidence for all three teachers in the same way – with the same section headings, which essentially correspond with the research questions. Thus, I am applying elements of the first two alternatives described above.
Although Yin argues that ideally, the participants in a case study are named in the report so that the context can be described as fully as possible and so that readers can seek out other literature and information on the same subjects, I am choosing to maintain the anonymity of my participants by using pseudonyms for all people and places involved in the study. I do so because the comments of the participants could be controversial if read by school district leaders, colleagues, or parents, and also because my analysis of each teacher’s comments may not always align with the teacher’s own perception of his/her ideas or practices. Yin suggests that controversial matters can be a reason to disguise the identities of the participants, so this seems justified for this report.

Moreover, Yin lists several features of an “exemplary” case study. One of these is that the case is complete, meaning that: the “boundaries” between the case and its context are well defined; every effort was made to collect relevant evidence; and the study was completed without limits to time with participants or to resources. Another feature is that alternate perspectives are considered in analyzing and reporting the evidence. Further, the researcher must endeavor to represent enough of the evidence in the report – both supporting and challenging his/her final conclusions – to allow the reader to draw his/her own conclusions about the case. I will work to demonstrate that these conditions are, as well as possible, satisfied in this study and in this report.

Thus, the remainder of this chapter is structured in three major sections, one presenting the case related to each teacher: Carrie (first grade teacher), Sandy (fifth grade teacher) and Ray (fifth grade teacher). Within each section, I first present a brief description of the teacher’s professional and personal background relevant to the study,
followed by the teacher’s interpretations and thinking about portions of the text in each of
the three SFMP chosen for this study, concluding with the teacher’s thinking about the
implementation of the CCSS and SFMP as a whole and about how this study may
influence his/her practice in the future. The section headings are generally parallel for
the information about each of the three participants, but those related to implementation
of the standards may vary slightly because of the different points of emphasis in each
teacher’s discussion. Also, it should be noted that when a word in dialogue is underlined,
this indicates that the teacher audibly emphasized this word while offering the comment
that is quoted.

**Characteristics of the District and Schools**

Sealon City Schools is a suburban district in Northeast Ohio. The following
demographic information was obtained from the website of the Ohio Department of
The student population is approximately 2,450, with 88.1% of students self-identifying as
white, 4.8% as multiracial, 3.0% as black, 2.8% as Hispanic, and 1.2% as Asian/Pacific
Islander. 23% of all students are identified as economically disadvantaged. 12.1% of
students have identified learning disabilities. 6.7% of students did not remain in the
district for the majority of the school year in 2012-2013. The district’s overall four-year
graduation rate is 94.4%, and the five-year rate is 97.4%. 56.2% of the teachers in the
district have Master’s degrees.

In the summer just before the study began, the district had consolidated its
elementary schools into two buildings from three; kindergarten and first grade had moved
into Chapman Elementary with second grade, and third grade had moved from Chapman into Lincoln Elementary, with fourth and fifth grades.

Chapman Elementary School serves approximately 500 students. 86.1% of the students are self-identified as white, 5.5% as multiracial, 3.9% as black, 3.3% as Hispanic, and 0% as Asian/Pacific Islander. 14.7% of students are identified as economically disadvantaged. 14.2% of students have identified learning disabilities. 54.5% of the teachers have Master’s degrees. Within-year mobility rates for Chapman were low enough to be listed as 0%.

Lincoln Elementary School also serves approximately 500 students. 86.7% of the students are self-identified as white, 5.0% as multiracial, 3.6% as black, 2.7% as Hispanic, and 0% as Asian/Pacific Islander. 25.4% of students are identified as economically disadvantaged. 14.2% of students have identified learning disabilities. 46% of the teachers have Master’s degrees. Within-year mobility rates for Lincoln were also low enough to be listed as 0%.

**Case 1: Carrie Joseph**

**Professional and Personal Background**

Carrie Joseph was a first grade teacher who had taught for ten years at the time of this study. This was her second year teaching first grade, and during the other eight years, she had taught kindergarten and some half-days of Reading Recovery (an early intervention program). This year was her second completion of a “looping” sequence with her students, meaning that she had taught the same students in kindergarten and first grade. Recently, Carrie had finished authoring a book with a local university professor
on how to use classroom conversation to guide student learning, and she often described this practice as the basis of her philosophy for instruction. During the first interview, she remarked, “I am not a math genius. I am not a teaching genius.” However, she believed that students should be learning mathematics differently than she did in school. She explained:

[W]hen I was in school, I memorized it – it didn’t not make sense to me, it didn’t make sense to me. It just was what it was… when I got to algebra, nothing made sense to me, I just did, followed the steps, and I was an honors student. And everyone thought I should go into math in college when I knew beans about math. I just knew how to follow the steps and come up with the right number for x…

In interview 2, she added, “I had no idea WHY – ever!” During the first group discussion, she noted that she had been talking with students’ parents about why she taught mathematics in the way that she did, particularly because of her own school experiences.

Before the study began, Carrie had not studied the introductory pages of the CCSS or the SFMP. She was familiar with the kindergarten and first grade content standards because she had taught the first grade standards during this year and because the kindergarten teachers had worked the year before to create a curriculum map for the standards. She added, “In first grade, we’ve developed a critical thinking rubric, and so some of that really applies to this [the SFMP], as I was reading through this again, so that
piece is kind of being developed right now.” Throughout the study, Carrie did not have additional experiences in exploring the SFMP beyond our research.

One element of this research that was unique to Carrie (among the participants) was reflecting on the SFMP in light of her own children’s mathematics experiences in the same school district; both were teenagers at the time of this study, and both had experienced the *Investigations* and *Connected Mathematics* textbooks during their elementary and middle school years. Carrie appreciated the opportunity to see the mathematics programs from a parent’s perspective, and she felt that the implementation of these programs had, in general, very positively influenced her children’s learning. Of her son, she commented:

[H]e was the first kindergarten class to pilot *Investigations*, so it’s been interesting to watch him – the way he thinks about numbers… Math is hard for him. But it makes sense to him, still… When he has homework and he’s explaining to me what he’s doing, what I would say, oh, well you just take this x and move it here and you do that, he explains it in a far more meaningful way.

She appreciated when teachers in all content areas adjusted their instruction based on students’ needs, rather than unwaveringly following the course map. Also, she referred to the SFMP when she explained how her son “always appreciates… when he has a teacher that lets him talk to somebody… about something that he had trouble with. He finds that to be – to help him understand it more.” On the other hand, Carrie also stressed how frustrated she was with the pressure for fact fluency (and posting students’ progress)
in the intermediate grades: “When my kid was there, you know, I thought that was the most horrific thing I had ever seen in my life.” In addition, she felt that as her children had entered middle school and high school, the teachers’ individual styles of instruction and the aspects of mathematics that they emphasized were often very inconsistent with each other, so “it’s like you’re relearning math every single year.”

From her perspective as a mother and a teacher, Carrie noted that she could see the progression of concepts from kindergarten on up, saying, “[I]t really was interesting to see that as an ending point or where my instruction actually leads…” Further, “I think that they’re doing such a much better job of making problems less abstract and more pertinent to kids in the older grades as well.” In the last interview, she commented on her opportunity to talk with the two fifth grade teachers in this study:

If that could happen more often in different venues, then I think that we would all benefit. Because I had kids go through and could see where things were going, but not everybody has that, and my kids are through now, so things are changing, and I don’t know that I’m always gonna be able to keep up on what we’re doing and how it affects the later grades.

So, it was really interesting to have that conversation.

So, Carrie’s thinking about the CCSS and SFMP were highly influenced by her own experiences with mathematics being taught quite rotely in school, her philosophy of guiding learning through discussion, and her perception of her children’s experiences as they had progressed through school mathematics.
**Pages 3-5 of the CCSS**

**Carrie’s interpretation of the overall idea of pages 3-5.** During interview 1, Carrie began her discussion of pages 3 to 5 by commenting:

I was pretty pleased to get a feel for a scaffolding, constructivist sort of approach [in these pages]… I did get that feel that there was a lot of kind of developmental language in here as far as building skills from one concept to the next… Just really giving kids a lot of time to explore… then guide to be able to make the concepts a little bit more concrete. I see this as true constructivism.

She discussed how she developed the operations of addition and subtraction as opposites with her first graders, and she noted, “In first grade, all of a sudden, when they learn that’s what we’re doing, it’s like, ‘Wow! Isn’t that cool?’” She added, “And the way I’m interpreting this [on pages 3 to 5] is developing meaning for operations.”

In interview 2, Carrie remarked of the introduction (with a somewhat questioning tone), “I think that again we’re just talking a lot about the understanding versus the ‘answer the question’ part?” She described how she was always glad to see students attempting to solve a problem, even in a very inefficient way, as opposed to simply skipping the task. She noted, “And never would I say, ‘Oh, no, don’t do that; just move on.’” Further, she felt that these pages emphasized the importance of “understanding math facts, instead of just knowing them”, adding, “I don’t know that the ‘just knowing them’ part is really what we’re trying to accomplish anymore.”
During the last interview, Carrie reflected, “[T]he new standards are really what our district has been stressing for a long time. That number, number sense, understanding number from kindergarten on. To be able to build that foundation of number sense that will help them later on… I think that this is saying that that’s important.”

So, Carrie’s interpretations of the main idea of pages 3 through 5 included a validation of constructivism as a theory of learning that informed mathematics teaching (allowing students to learn based on what they already knew), developing meaning for the operations, understanding and persevering rather than just finding correct answers quickly, and building number sense in students that would serve them throughout their school careers. Although these are all somewhat distinct ideas, they are certainly related, so Carrie’s overall interpretation of these pages did not seem to change very much during the study.

**Carrie’s interpretation of conceptual understanding of key ideas, and using organizing principles to structure ideas (page 4).** Carrie’s first comment about this section in the first interview was, “I like that ‘organizing principles’.” She continued:

I see that as um, what we’re doing right now, revisiting combinations of 10… [we] made 10 a million different ways, a million different times. But now, we’ve kind of, we did a little subtraction in there, and then, now we’re kind of revisiting totals of 10… We did that exploring, and it made sense to them, but now, they actually, I feel like they have a hold of it.”
Carrie added, “And I wrote here [in her own notes], ‘understanding math facts instead of memorizing them’.” She shared:

When I was explaining to parents at conferences, I said, I think that what we’re doing is gonna make such an impact on these kids in high school. Because they’re understanding why 7 + 1 equals 8; they didn’t just memorize it… with kids here understanding what the equal sign means, that both sides of that equal sign are the same, algebra will make so much more sense to them… understanding versus following the steps.

Carrie reflected on her first videotaped lesson, in which students were exploring groups of tens and ones, during the second interview. She noted, “I think it was really interesting, the place value lesson that we did… I always feel like such a guide when we take those leaps.” She discussed how, during the lesson, she had provided students with some review of what they already knew about tens and ones in numbers through 20 (using base ten blocks), and she commented, “I think that’s what this [page 4] speaks to is that everybody has their own perspective of how to problem solve, and it’s a matter of organizing your thoughts around what you know to be able to apply it to new things.”

During the final interview, Carrie shared the following relative to “using organizing principles to structure ideas”: “I’m seeing this as, uh, that constant cycle of, OK, we’re moving on here, but we have to review these skills in order to be able to understand what this principle means,” saying, “[T]hat is that cycle of instruction.” She used an example: “[I]n first grade, when you get to the operations of addition and subtraction, you have to revisit what counting and number is, of course, to have the
operations make sense.” Another example she provided was, “[G]reater than, less than… again, you would have to revisit number and quantity in order to be able to establish that principle.”

Throughout, Carrie seemed to interpret this text as referring to teachers’ intentionally tying new learning to concepts already learned in order to allow students to “hook” onto the prior content. Further, she felt that understanding, as opposed to rote memorization, was an important element of this process.

Carrie’s interpretation of mathematical understanding (page 4). As Carrie reviewed this text in interview 1, she discussed how she was always working to understand how each student was thinking about a mathematical idea. She said, “[E]very time we throw something new mathematically [students’] way, they’re all at a stage of exploring that concept or internalizing it…” She added, “I wrote down the critical thinking skills here, the whole point of ‘how do you know’,,” saying that she and the students ask this question very frequently in conversation – so much so that students expect her to ask it and wait for her to do so, and they also ask each other this question. She feels that this is her best means of assessing the level and nature of students’ thinking at any given time. Further, “[I]t’s kind of all about expressive language, and the looping has really helped with that – ‘cause we’ve been talking the same language for a year and a half.”

In the second interview, Carrie continued to relate to this text by discussing how teachers could be aware of students’ levels of understanding. She said, “[F]or some of my kids that don’t have the facts memorized, I still know that they understand the
principle.” Moreover, even if she watched students make mistakes in solving problems, she could still assess their problem solving strategies and consider how to support them. She explained, “[W]e’re representing with sticks and cubes or whatever so often that I can see their thinking pretty easily.” However, she noted that students who finished problems quickly and correctly often struggled to represent their thinking visually. “If I say, ‘Go back and represent,’ they’ll be like, ‘What do you want me to DRAW?’ You know, like, ‘Why would I do that? I know that that’s right.’” She remarked that this would probably be a struggle in fifth grade also – for Ray and Sandy and their students – because the content was more abstract.

During interview 3, Carrie reiterated some of the ideas she had mentioned earlier for this section as well as the overall goals of the introduction. These included developing students’ number sense, helping students to “understand” – not just “perform” – operations, helping students understand the meaning of the equal sign and what it means in the context of an equation, and learning through discussion, which she related to constructivism again. She said, in summary, “We’ve done that so long in other areas, literacy and so forth, and maybe we’re getting there now in math too. Being able to really build on that, in a more understanding-based way, and not just a principle way. ‘This is the next concept that you need to learn.’” Perhaps what she meant by “not just a principle way” was that we are no longer to teach each “principle” in isolation; rather, each principle is part of a larger progression of ideas.

The interpretation that Carrie brought to this text, especially the idea of mathematical understanding, throughout the study seemed to focus on the idea that
students think about each mathematical concept in different ways as they are learning it – and each student’s thinking may be unique. She said that students explored ideas, usually in more concrete (visual) ways first, and “internalized” them in their own minds through this exploration and discussion. Also, it was not enough for students to be able to “perform” steps to find answers; they also needed to understand each process, each symbol, and each solution in relation to its context. She felt that teachers needed to teach with these goals in mind so that instruction did not simply involve a series of isolated skills or ideas. Particularly, teachers needed to ask “how do you know” questions and become aware of each student’s thinking through informal assessments so that they could build on this thinking in instruction.

Carrie’s interpretation of standards for a diverse population of learners (page 4). When I asked Carrie what her reaction was to this paragraph during the first interview, she said, “I think of that as differentiated instruction… there are extremes to that.” She shared, “I have a little guy on an IEP who is still counting to 3. So, these [first grade content] standards really don’t do [him] any good. The kindergarten standards barely do… So, I think that that’s an issue with inclusion… and special needs kids when we get to first grade.” She explained, “Kindergarten, with so much of the exploration and counting – you really can make that work with all the manipulatives… But now we’re first grade – we’re getting away from the manipulative use, and really internalizing more of those concepts… So there becomes, there starts to become a gap.” Still, she noted, “With my typical population, the highs and lows and middles, I think what I see is just what we talked about with the last question – that some are in that developing,
experimenting, experimental stage with a concept, and others have really kind of gotten a firm grasp on it.” Carrie described how as they “cycle back” through each major idea throughout the year, more of the students understand it each time, though it sometimes surprises her how many times they explore and discuss an idea before it finally makes sense to certain students.

During interview 2, Carrie noted that it is helpful to be able to use manipulatives to represent concepts in first grade as students are learning new ideas, such as showing “that a ten-stick is equal to 10 cubes”. She said, “[J]ust about anything in math right now that is abstract, I could make concrete somehow… that’s not always gonna be the case, of course.” Further, she discussed how she works to bring each student along in his/her understanding, referring to the first videotaped lesson in the latter part of her comments:

And differentiated instruction, you know, that’s kind of been a buzz word for so long, but I don’t know that it’s used really, all the time every day. Knowing in every lesson that you have, that you’re gonna have, really, 22 different ways of understanding what you’re saying. So you have to restate, and you have to understand that some of them are getting it at this level, and that’s where they’re gonna be, and that’s OK right now. But you have others that are gonna soar, and they can soar and kind of take off on their own, and then you get back to those little ones and kind of pull them along a little bit more. Having looped, I know them so well – during that lesson, you wouldn’t have known that, but the ones that I was calling
up in that beginning part – the ‘known’ – were my little ones that I need to get engaged in the lesson right away.

That is, she as a teacher had to constantly be aware of the nature of each student’s thinking (as fully as possible) so that she could involve each student in each lesson in a way that would support learning. She noted that she continued to encourage each student by saying, “‘You can do this’… and then stretch them, and I know that they didn’t follow me all the way to the end, but then the next day, um, during work baskets in the morning, that’s where we are, and we’re doing that again.” Carrie’s descriptions of her careful consideration of how to question and respond to each student to support his/her learning needs very much reflect the third high-leverage practice (of nineteen) that Deborah Ball and her team at TeachingWorks have proposed: eliciting and interpreting individual students’ thinking (http://www.teachingworks.org/work-of-teaching/high-leverage-practices). Carrie summarized her interpretation of all of the text on page 4 of the CCSS at this point by saying, “I think that’s what this speaks to is that everybody has their own perspective of how to problem solve, and it’s a matter of organizing your thoughts around what you know to be able to apply it to new things.”

Carrie continued to discuss these same ideas about knowing her students (and their diverse ways of thinking) in the third interview. She commented, “[I]ntervention is based on knowing what your kids know, and being able to supplement for those that don’t… but you just really I think have to know them well enough to know what they need… because you can’t jump.” Carrie explained this last remark: “[I]n literacy and in math, it’s just not feasible to say, ‘Oh, they don’t know that; we just have to move on.’
We have to fill in those holes, or else that’s gonna be – cause a real problem later on…”. She then added, “Being a teacher of inclusion… it’s easy for me to remember that about all the kids, because I of course have such diverse learners in the classroom… to be able to kind of gauge who needs what.”

Carrie’s thinking about the diversity of learners in her classroom remained quite consistent during the study. In essence, she believed that students would understand mathematical ideas in different ways at different times and that she as the teacher needed to be well aware of each student’s thinking in order to provide experiences that would best help each student to move forward. However, she did note in the first interview that she felt that there were a few students with severe disabilities who might not benefit very much from being included in the general education classroom because their understandings were too far removed from those of most of the students.

**Carrie’s interpretation of learning progressions (pages 4-5).** In the first interview, Carrie commented that she saw using learning progressions as referring to “guiding [students] through their own process, and they all have their own process”. She noted (interestingly, seemingly contradicting what she had just said), “With my loop, that makes so much sense because we’ve all done the same thing… their learning progression doesn’t have any kind of jogs in it. We’re all consistent… their learning progression would be all the same; it’s my job to guide them through it.” She contrasted this to a situation where students have different kindergarten teachers: “[T]hey’re teaching the same standards, and they’re teaching virtually the same program, but different language and different approaches, so you have all of that to account for… at some point you have
to accommodate different language.” Regarding her own students, Carrie remarked, “I can’t really force the issue… I can only give them what they need when they need it… sometimes again and again in different ways, to hope that they’ll make a connection to it.”

In the second interview, Carrie focused more on this last idea as she considered this text – that students reach developmental milestones at different times. “I think that there’s a lot of things like that that you can see with kids that, um, it just DOES not make sense to them. And there’s no way to force that, so you kind of just have to let them sit on it for a while, and come back to it and see if it makes sense then…” She continued:

I don’t know if it’s maybe a different perspective on it, after, you know, they were trying to think so hard about it; they worked themselves into a hole… Or, if they’re just ready for it a little later. Needed a little bit more practice with numbers in a different way, who knows. But I think that there definitely is something to that, um, brain capacity for some of that learning at different stages.

At this point, Carrie seemed to view a learning progression as a path for learning a given concept, common to all children who had had the same teacher, but on which individuals moved at different rates. She implied (though did not directly suggest) that children with different teachers might have different ways of moving along this path. Particularly, Carrie seemed to refer to how each child eventually makes sense of the concept and develops more sophisticated understandings of it. She felt that this should be an
important consideration in teaching individual learners in the context of a classroom setting.

During interview 3, Carrie related learning progressions to her experience in Reading Recovery. “[I]n Reading Recovery, we always talk about going from known to new… And that’s kind of what I think of that as -- the learning progression…” She continued, “[E]very day, if you could make a little stride, from known to new – but you always have to start at that known part. I think that that’s very true in math as well. You have to go back far enough that the kids are – that you’re touching their known… In order to expand and stretch that a little bit.” (Throughout the latter part of the study, Carrie used the word “stretch” more and more to refer to this idea.) “I always feel like that’s what I’m doing, is everyday, stretching them a little in the direction of the concept that I want to ultimately have them understand… that, to me, is learning progressions.”

So, at the end of the study, Carrie was relating a learning progression to connecting learning to prior learning – she often referred to this as “stretching” from the “known” to the “new”. She appeared to believe that every child was actually on the same “progression” (similar to a single, externally or naturally pre-determined path) to learning a particular concept, but some children moved more quickly along this path than others. Given her comments in the first interview, it seems that she felt that helping students to move along this path was more challenging when students had different learning experiences earlier, as they would in classrooms with different teachers, because the next teacher would have to “account for” the various experiences that students had had while still moving all students forward. It would be interesting to ask Carrie to reflect on
whether the path (progression) actually changed when students had different learning experiences.

**Review of how Carrie’s interpretations of pages 3-5 evolved during the study.**

In actuality, Carrie’s general interpretations of the text on these introductory pages did not evolve very much during the study. She was very strongly grounded in her philosophy of instruction, and this seemed to greatly inform her thinking about each of the ideas in this section of the document. She herself commented during interview 1, referring to her philosophy, “That’s where I live, so maybe I’m a little biased and read things that way.”

**Carrie’s Interpretation of SFMP 1**

**Title:** “Make sense of problems, and persevere in solving them” (p. 6). When we began discussing this standard in the first interview, Carrie shared, “[S]o much of this I compare to reading strategies… [the] district has had ten solid years of focus on reading strategies and Reading Recovery principles, and guided reading… there’s a lot of similarities here, between them… the same perseverance that I’ve seen in my kids in reading – attacking a word, and not giving up, is what I see with them in math.” She continued, “I don’t let them off the hook, and that’s my Reading Recovery background, of being able to wait it out… feeding them little supports here and there… but they know I’ll never do it for them… And I’ll even say sometimes, ‘I feel like you’re waiting for me to do it, but it’s not my math work. It’s your math work.’” Carrie then expressed an idea that she stressed throughout the study: “It kind of puts the responsibility back on them.” On the other hand, she added, “Of course, I would never do that if it’s something that I
question whether they can do.” About her classroom as a whole, Carrie noted, “[W]e talk about the learning a lot… that kind of gives them that pride in their work to want to persevere… we've celebrated from the teeniest accomplishment to a big accomplishment…”. So, she felt that her students were willing to persevere because they knew that they had the support of their teacher and their classmates.

Carrie was very interested to learn from Ray and Sandy what it looked like for a fifth grader to “persevere” through a math problem. So, during the first group discussion, this became a significant topic of conversation. Carrie learned that the fifth grade teachers found it challenging to keep students motivated enough to work through longer problems or even to check their own work. The three teachers together discussed that this might be related to the use of personal technology by these older students to obtain information quickly, which was not such a major part of students’ life experience in first grade. Also, they formed an hypothesis that students might be more willing to persevere in first grade because, as Carrie said, “there’s always a way that they can solve the problem that they can do… they don’t ever have that feeling of ‘I can’t do this; it’s not working.’” This was a new realization for Carrie, on which she reflected later. All three teachers wondered when students started to persevere less, and they discussed whether this might occur in second grade when the teachers really had to start stressing students’ fluency with addition and subtraction facts. That is, if students felt pressured with mastering this major element of school mathematics, perhaps they would feel less confident to pursue more complex tasks.
In Carrie’s first lesson, she began by discussing with students the idea of not “giving up” when facing a new kind of problem in math. She asked, “What happens if math gets hard? Give up?” Students said, “Try!” “You’re not learning.” Carrie added, “And the happiest news of all is if you don’t give up and you do your best and you stick to it, and you figure it out, something magical happened in your brain.” Three times during the lesson, she asked the class as a whole, “Are you feeling super smart?”, and at the end of class, she commented, “You all did bring your super smart brains to math today.” It seemed that she asked this question and shared this last remark because she wanted to encourage students in their thinking about combining groups of tens and ones, which was the relatively new concept that students were exploring that day.

During the second interview, Carrie read this title statement aloud and said, “[W]e work through so much together, as a group, in those, in the family room situations where there’s just so much discussion… you always have somebody that can boost you along. So I think that they scaffold for one another as well.” She added that her students were very patient with each other, and they would generally not say “That’s wrong” to someone else. In addition, Carrie noted that when they start a new problem, to help students “make sense” of it, she will often say something like, “‘[T]he first time I’m gonna read this to you, just listen for the important words…’ So they pull out the numbers right away. ‘So, um, write those down. OK. So what are we gonna listen for now? We have to figure out what we’re gonna do with those numbers.’ So, it kind of gives them a starting place…”. Carrie added that when she is working through the one-on-one district assessment tasks with students, every student at least is able to attempt a
solution without prompting, and “even if they go a wrong direction, at least they’ll try something that they know that pertains somewhat to it.” She felt this was because there was “so much trust [between her and students] in that I wouldn’t stump them… So I think that you just have to find them where they are, and encourage.” She wondered if having these same kinds of conversations consistently with students as they grew older might help to alleviate at least some of the issues with perseverance that students had in fifth grade. In fact, she remarked that it was “huge” for her to hear about these issues from Ray and Sandy – “I think that it makes what we are doing in the, that work ethic that we’re instilling here so much more powerful. Because it’s going to meet a lot of challenges between here and there with technology and whatnot, that this has to be really strong.” Carrie summarized her thoughts at this point by saying, “[I]f you want them to be able topersevere, then they have to know that whatever it is that I gave them is something that I truly believe that they can do.” She also remarked, “[W]e [in first grade] have the grand luxury of kids wanting to please us and wanting to have that feeling,” adding with a smile that perhaps students continued to persevere in her class because otherwise, “I think that they’d be afraid of what I would say…”.

In the second lesson, Carrie used some “I Can” statements in the early part of the discussion (such as “I can add numbers”). She seemed to feel that they aligned with SFMP 1 because they helped to promote perseverance and sense-making. Later in the lesson, she said, “Never say never, until you try, right?”, and to regain a few students’ attention at one point, she asked, “Aren’t you interested to see if it works?” Her language with students was very consistent in regard to this goal in SFMP 1.
During the second group discussion, Carrie shared with Ray and Sandy, “I went back to school [after the first group discussion and shared] this grand aha [slaps hand on table] that I had, talking to you guys, about that focus and that attention when everybody in the world thinks that kindergartners and first graders have no attention span...”. She said that her colleagues were also very surprised to learn that since fifth graders are used to “[e]verything being fast paced, and automatic, and instantaneous [snaps fingers], you know, they want math to be that way too.” Later, as the three were discussing algorithms, Carrie commented, “[T]he algorithm plays into what they want. Just tell me how to do it, and I’ll do it. [So, the new standards] might be initially more frustrating for them to think it through.” Ray and Sandy agreed.

In interview 3, Carrie shared more thoughts about how to encourage perseverance in her class activities. “I think that when you’re presenting them with enough time to explore and have their own, um, curiosity, and initiative, to be able to – they almost create their own problems and solve them.” She referred to the creative and diverse ways that students explored with pattern blocks, maintaining their attention for considerable amounts of time. Carrie also highlighted the issue of time relative to class discussions:

[I]f you cut kids off with something as simple as calendar, in kindergarten or first grade, then you are automatically shooting down their perseverance… So, even at the risk of that uncomfortable anxiety time, um, I give kids just a ton of time. Support if they need it, but it is their problem to solve… I think that the feeling of getting over that is a huge confidence builder that can help for life-long experiences.
Carrie also reiterated many of the main ideas that she had mentioned earlier in the study, related to celebrating many moments of learning, providing challenging but doable tasks, and providing support both as a teacher and as a community of learners in the classroom – all with the goal of building confidence in students so that they were willing to persevere.

Overall, Carrie spent a great deal of time in the study discussing this title statement of SFMP 1 – particularly the notion of students’ perseverance. This was a very important part of her own teaching philosophy, and she had refined many strategies for developing students’ confidence and their ability to approach and work through unfamiliar problems. Although she did not speak very explicitly about “making sense of problems”, she seemed to feel that if students were confident enough to think about and draw on what they already knew to begin work with new tasks, then they could almost always find sensible ways to reach solutions, even if the solutions were not entirely correct. The major change in her thinking occurred in talking with the fifth grade teachers about how the perseverance that she saw in students seemed to nearly disappear by fifth grade, which concerned her and caused her to think about how teachers throughout their district might be able to encourage students more consistently.

“Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution” (p. 6). During interview 1, which took place at the end of January during this school year, Carrie’s first comment about this text was, “[W]e’ve just started talking about efficient ways of solving problems and different ways of solving problems.” She explained that each morning, she
wrote a story problem on the board that was specifically designed for the “student teacher” for the day, who would solve the problem in front of the class. Once this student had solved the problem, she would ask if there were other ways to solve the problem, and students would share their ideas. Then, she would ask which way was most efficient. “So now, they’re really striving to figure that out – the more efficient way of solving a problem… And I’m seeing a lot of number sense showing up.” Carrie’s comments seemed to relate somewhat to the idea of looking for entry points to a problem (the second part of the text), though she may have been thinking more about different ways to solve a problem rather than just ways to understand and approach the problem.

In the first lesson, two particular short exchanges between Carrie and two students seemed to exemplify the goal in the text. During one, Carrie said, “[T]ell me about this group here, right now,” and a student replied, “It has 10 in it.” In the second (unrelated to the first), she asked, “What do we know about that?” A student responded, “[W]e can make combinations.”

Carrie seemed to think a bit more about the meaning of the text during the second interview. She noted, “[T]hey’re always saying, ‘Well, if I know that --’… So they’re really pretty savvy at finding something in every problem that they can do… sometimes they try to apply too many things, because we always talk about, ‘Is there more than one way to solve a problem?’” Carrie elaborated on this last comment by describing how students would sometimes try to find an entirely different set of operations to obtain the same answer, even if the operations did not relate to the problem at hand. (In this case, they actually would not be thinking about the meaning of the problem, so this sense-
making was certainly a habit students were still working to develop.) Further, as Carrie discussed one of the hypothetical classroom situations, she remarked, “I’m always giving [students] the chance to find some starting point. Of their own.” However, she did not explicitly relate this to SFMP 1.

During the second lesson, Carrie asked two questions similar to, “What does this mean?” when students were discussing possible meanings of the fractions 1/2 and 1/4 (as written, with numbers). The lesson focused on students’ finding physical strategies for dividing a region or set of two or four items into halves and fourths.

Carrie returned to her discussion about multiple ways of solving problems during the third interview as she reviewed this text. She commented, “[I]n first grade, we do just a lot of different ways of solving a problem,” and she again said that they are starting to look for the most efficient way. She also remarked, “I think that often times, I don’t look at a problem in a different way… but sometimes somebody [a student] will come at it from a back door [gestures with her arm as if to show a curve] that will just be really interesting!” Finally, Carrie noted, “[W]e appreciate, and have the kids appreciate, different ways too; they learn from each other.”

Thus, Carrie rather consistently described this goal as students’ finding and/or understanding “different” and “efficient” ways to solve a given problem. Although in both lessons, she asked a few questions that seemed to involve the “meaning” of a problem (“What does this mean?”; “What do we know?”) and “entry points” (“What are you thinking?”), she rarely spoke directly about these and did not seem to recognize that these kinds of questions might relate to this text.
“They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary” (p. 6). This is a great deal of text for any teacher to interpret at once, and Carrie, as Ray and Sandy, focused on very specific parts of it during the study. In interview 1, her only comment about this text was, “[W]e’ve talked about simpler forms of the original problem…”. During interview 2, she said, “Boy, I’m trying to think of a … of something that we’ve done… that would require multi steps… ‘Cause this is kind of what this sounds like to me.” She referred to the regrouping that students were exploring with base ten blocks in the first lesson as a multi-step task, noting that some students were able to say, “Well, you could just swap that [ten for ten ones].” When asked what part of the text she felt implied multi-step problems, she responded (reading):

‘A solution pathway, rather than simply jumping into a solution attempt’ – that implies to me that there’s more – there’s… I guess you could have a representation here. If you represent it rather than just an answer… But to me this sounds like something that has, um, more than one step. That you would have to do one thing – a pathway to the next - to the next, to get to something. To get to an answer.

Carrie referred to her students also being able to “swap” coins as they were informally exploring them in their center work, perhaps seeing this as a solution pathway to discover
the total amount of money. However, she then commented, “I’m stumped by this
[statement in SFMP 1], I think… I just don’t throw problems at them. To stump them.”
Carrie continued to reflect: “I guess [the district assessment], maybe, as we go further
along in [that], it might be interesting to see how they approach things that we haven’t so
much concentrated on – concepts that haven’t been taught so much… There’s not a
whole lot of apply – now apply this to this situation.”

Carrie did make two comments in the second lesson that seemed to align with the
text involving considering analogous problems. She said, “If we know how to do 1/2,
then perhaps we know how to do 1/4,” and “We can do it with a square, but can we do it
with a circle?” Again, though, she did not appear to connect these statements to this text
herself.

In the third interview, Carrie focused on students’ use of “if-then” relationships
(she did not use the term “relationships”) in their explanations as an example of how the
goals in the text might be interpreted for her classroom. She said, “[B]y the time they
spend a year with me in kindergarten and get to first grade, they have that lingo down of,
‘Well, I was thinking this.’ ‘Well, if I know that this, then this.’ So they have that
language that’s been modeled for them for an entire year of kindergarten… I see them
really taking ownership, then, in first grade of that language, and I think that that is them
being able to explain their thinking through the problem…” In one of the hypothetical
classroom situations, she described how she would use popsicle sticks (spread out) to try
to “convince” students that they were only 3 sticks tall; she felt that when students
responded that this did not make sense, it would help students think about how to
logically measure the length of a table with the sticks. This seemed to be an “analogous problem” used to help the students “change course”, but she did not mention any connection to the text.

In general, Carrie chose two specific phrases in this text on which to reflect. She referred to “solution pathway, rather than simply jumping into a solution attempt” and “simpler forms of the original problem” during the study. Basically, she seemed to view the main goal in this text as students’ ability to use their experiences with other problems to help solve new problems (“If I know this, then this”). However, Carrie did not discuss the remainder of this text in any detail. I do not believe that she avoided the rest intentionally, unless she simply did not feel that she understood it, but it is unclear how she might have interpreted this other text.

“Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem” (p. 6). Interestingly, even though Carrie had always been a primary teacher, during interview 1 she seemed not to want her students to rely overly on manipulatives, nor did she feel that they really had the need to use them. She said, “I don’t really see them needing objects so much… they don’t depend on that anymore at all; sometimes fingers come up, but really I think that that is just a little extra support…” Later, she added, “I think that sometimes we’ve done so much representing, that I think sometimes they think I want them to – tally, or draw those circles or whatever… that’s when I’m trying to kind of diminish that with this whole talk of efficiency.” In the first lesson, however, since Carrie was introducing the idea of combining groups of tens and ones (to which she referred as a “second grade” skill that
was new in the first grade standards), she provided base ten blocks that all of the students were to use in representing and solving the problems she presented. They continually referred to “10-sticks” and “1-cubes” throughout the lesson, and Carrie often asked students to “reorganize” and “count” to find their totals. As noted earlier, the students would suggest that a 10-stick could be “swapped” for ten 1-cubes. As students explored with and combined the blocks, Carrie and students that she chose would write number sentences on the board and discuss the tens and ones that were part of each task. So, in this case, students were very much using concrete objects to understand and solve problems, as stated in the text.

During interview 2, Carrie commented that at that point in the school year, they were using base ten blocks “every day” to continue to explore place value. She pointed out that when she first introduced any new manipulative, she would simply include it as a tool for exploration in students’ center work, and as they “played” with these tools, by the time they came to use them in class, they had “already created somewhat of a schema in their mind of how they work” (such as that ten 1-cubes were the same length as one 10-stick). She noted, “[Y]ou have to establish that purpose and ownership of that with them. Otherwise, it doesn’t make sense, and so why would you use that?” In analyzing one of the hypothetical classroom episodes where students were having trouble understanding what the number “67” meant when they saw it, Carrie said that she would ask students to count 67 cubes, and when this took a long time, she would ask if there were a faster way that they could do this. She would guide them toward the use of 10-sticks and 1-cubes to help them understand and use groups of 10 and 1. In addition, Carrie added that her class
would begin their measurement unit soon, and the students certainly would be using concrete tools during this unit. She also reiterated that she was glad that her students would use visual representations if they were unsure how to start a problem, and she did want to encourage this in order to develop their perseverance and confidence.

As Carrie’s students explored fractions in halves and fourths during the second lesson, they discussed sketches of these fractions on the board and eventually all used play dough to create wholes that they cut into pieces with plastic knives.

In her final interview, Carrie shared some interesting observations about asking first graders to represent their thinking. She commented that she was very surprised early in the year when, even though she had used the word “represent” throughout kindergarten with her students (in reference to creating visual representations of ideas), students were often unsure what she was asking them to do when she used this term. “I’d say, ‘Draw it?’ And, ‘OH!’” She mused, “Something about that word that needs to be changed, or something, to more kid-language, because they don’t get that ‘represent’.” She even suggested that she might create her own word, and later she thought that she might create an icon that she could include next to tasks on paper when she wanted students to use visual representations. Further, Carrie noted, “They can do it; they just sometimes don’t know what I’m asking them to do… at other times, they overuse it when they don’t need to.” She explained that at first, she was asking students to represent a great deal of their thinking, and later in the year, when she was intentionally requiring less of this with ideas that had been established, students often thought that they should still show their thinking visually, perhaps because their work appeared more complete. On the other hand, Carrie
did feel that some of her students still needed to use visual representations to solve problems while others did not, and she recognized that often this was required on state testing in later grades, so as she remarked, “[I]t’s a catch-22. Sometimes you need it; sometimes you don’t. And how are you to expect these little young ones to know when to and when not to use that?” It is noteworthy that everything that Carrie said during this interview implied that the given problem had already been solved and that students were being asked to represent the solution afterward. The actual text focuses on using objects or pictures during problem solving.

Thus, Carrie’s focus and comments related to this text shifted a bit during the study. At first, she said that she did not want her students to rely on manipulatives, and she felt that students generally did not need these tools. That is, at that point, she seemed to feel that they were crutches in problem solving. However, during both lessons, she used manipulatives with students to explore concepts – granted, both lessons were early in the sequence of teaching and learning these ideas – and she mentioned that she often used manipulatives to introduce new ideas to students (which seems to align with the goals in the text). In the last interview, she shared many thoughts about how to help students understand when a visual representation was helpful and when it was unnecessary, though she seemed to be thinking of creating the representation after solving the problem. Her comment regarding concrete and visual modeling creating a “catch-22” perhaps best summarized her perspective on this issue.

“Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, ‘Does this make sense?”’ (p.
6). Carrie explained in interview 1 that the goal in this text was a habit that she was just beginning to pursue more with her students. She noted, “[W]ith everyone’s different answers [to the word problem of the day], we’re starting to then cross check a little bit… ‘Does it make sense’ has kind of come up from them, which is awesome… ‘That doesn’t make sense because if you know 2 + 2 is 4, then’… that kind of thing.” She shared during the first group discussion that one of the “higher level” parts of the critical thinking rubric that the first grade teachers were using for mathematics assessment was “knows more than one way of solving a problem”.

During the first lesson, Carrie asked numerous times whether students agreed with the solutions that were being shared by others, and she asked students whether they had found other ways of solving the same problems. Most had not, at least at this point, since this was their first day of exploration with adding double-digit numbers.

In the second interview, Carrie noted that her daily practice of “turn and talk” was very valuable for students checking their answers using multiple methods and thinking about whether their answers made sense. She emphasized to them that listening was more important than talking, and she explained, “[W]hen we report out, they’re not allowed to say what they did; they have to report what the other person [shared]… you have to teach them how to do that…” Carrie said that because students now understood this expectation, they would ask each other, “How do you know?” She added: “So, they’re listening to other ways of doing it and trying to figure out – and most interesting is if ever they have different answers – how they’re trying to rationalize their own. But there’s never been any kind of, ‘Well, you’re wrong, and I’m right’ kind of thing – they
really do kind of try and stick to the problem and figure it out.” It should be noted that Carrie’s description does not necessarily suggest that students were accustomed to rethinking their answers independently, but it does suggest that students were learning how to rethink their solutions.

In the second lesson, Carrie commented at one point that dividing the wholes in different ways was “reminding [her] of like solving math problems, that there’s more than one way to do it”, but there was essentially no other evidence that students were assessing the reasonableness of their answers or work during the rest of the lesson. Carrie guided students toward correct answers during the group discussion, and while students were working with the play dough, it was not clear whether they were able to determine whether their thinking was logical. This degree of challenge was confirmed in the last interview, as Carrie began her reflection on this text by saying, “That’s hard too… sometimes, you know, they’ll write [an answer] down, and go [smiles really large]… And if I just look at them, they’ll just kind of go back and realize it doesn’t make sense… that is definitely a first grade, um, skill… that needs to be taught.” Still, Carrie maintained, “I don’t have any doubt in my mind, that when they’re doing a problem, they’re thinking, ‘Well, if I know that…’. Further, “I think that the more that you can internalize that conversation, that language, the more that they’re going to be actually looking at it and making sense and not just completing a problem.”

In general, Carrie felt that students’ development related to the goals in the text was highly dependent on teachers’ expectations of them (e.g., “you have to teach them how to do that”). She had built this rechecking of answers and sense-making into the
culture and daily routines of the class, so she felt that students had developed these habits. She appeared to interpret checking answers “using a different method” to mean understanding someone else’s method (and perhaps trying it), and she felt that students asked whether answers made sense when they were able to share their thinking with someone else and knew that they might report on their partners’ thinking. In the last interview, Carrie discussed how she felt she was teaching students to think about their thinking on their own, so she did realize that this was the ultimate goal relative to this text.

“They can understand the approaches of others to solving complex problems and identify correspondences between different approaches” (p. 6). Once again, Carrie’s interpretations of this text remained very constant throughout the study. During interview 1, Carrie’s first response to this statement referred to the previous sentence in the text (above): “[T]hat’s I think the same thing. When they’re talking back and forth about, ‘Well, I did it this way’… they go up and there and do different – use different approaches… I think that they’re starting to see those relationships.” She added during the first group discussion, “And right now, we’ve been doing so much of this, ‘What do you mean – there’s another way to do it?’ In different colors, I’ll have kids come up and show their way of doing it, and then, “Which way did you do… are they – is one right and the other one wrong? No!” In the second interview, she described how she structured the “turn and talk” moments:

[W]hen they turn and talk like that, they just – it’s just whoever’s sitting by them. I don’t assign who it is or anything. Other times, sometimes I
will put them with different ability levels because I know that they can learn from one another, and sometimes I mean those high kids, you know, that maybe can’t do the representation learn from the ones that still need to do that as well.

Carrie said that she was generally able to predict which students would solve a given problem in a particular way. Interestingly, she remarked, “I think we’ve modeled so, so much of that that most of the conversations are probably very similar… it seems like, then, when we report out, that we only need to hear from a couple because it just becomes redundant.” Moreover, during the final interview, she added, “[A]nother looping benefit is that we all have the exact same experiences… they’re starting to make those connections [between experiences]. And I would say that those connections are the different approaches.” Although it is unclear what Carrie meant by this last comment, her interpretation of this text in general seemed to revolve around students’ seeing multiple correct approaches to solving the same problem and understanding not only that they all were logical but understanding the connections among the steps in each approach. She felt that her students were starting to be able to do this by the end of first grade.

 Carrie’s thoughts about implementation of SFMP 1. As described above, Carrie felt that it was of prime importance for her to help her students to feel (and to be) confident and successful in her classroom. She believed that this would help them to continue to appreciate learning and to persevere through challenges in mathematics and all areas as they progressed through school. In the first lesson, Carrie began the class
with an “I Can” statement, which became “I can add numbers” as she talked with the students. She often told the students that they were demonstrating “super smart” thinking and that they were thinking “like second graders”. Similarly, in her second lesson, she commented at one point, “I always feel like we’re extra smart when we find more than one way to solve a problem; don’t you feel like that? Like we did extra…”. In the second interview, she explained that as she continued to work with students on adding double-digit numbers, particularly with those who struggled at first, “[A]ll the other ones love to practice with it because they feel like it’s hard math because I told them it was, but the little ones that struggled with it now are actually pulling it out on their own because they know that they can do it too.” She added, “And that’s maybe then why I like to engage those little ones that struggle right away, ‘You can do this.’” Again, Carrie believed that her students trusted that she would not ask them to try something that they could not do, which helped them remain willing to persevere through tasks. She noted that in the critical thinking rubric that the first grade teachers were using to assess students in reading and mathematics, two of the elements assessed were whether a student could “tackle” a problem and whether the student could solve it in more than way. Further, Carrie carefully fostered a supportive environment among the students in her class, so that it seemed that they all recognized that they were learning together and that they should encourage each other. She noted in the second interview that when a student was working on a problem at the board, she might say to the class, “Oh, gosh, thanks for being so quiet, because I can see thinking happening here.” Also, Carrie believed that she and the other primary teachers had learned to help students to persevere
and make sense of words and sentences while reading, so now the teachers were becoming able to promote these same habits in mathematics. She often said that as a result, students were becoming able to take “ownership” and “responsibility” for their own learning.

Carrie never offered many comments that explicitly related to the goal of “mak[ing] sense of problems”, the first part of the title of SFMP 1. However, she seemed to believe that students’ willingness to re-read and think about problems without giving up, as well as their ability to use various kinds of representations to show their thinking, would allow them to make sense of a problem and then to solve it. Clearly, her goal was to encourage these habits with all of her students on a daily basis.

Review of how Carrie’s interpretations of SFMP 1 seemed to evolve throughout the study. Two aspects of Carrie’s interpretation of SFMP 1 changed in a noticeable way during the study. First, when she spoke with Ray and Sandy about students’ perseverance in the first group discussion, she began to realize that fifth graders generally did not have the same levels of perseverance as her first graders, which seemed to help her think more about the importance of instilling this habit early on. Although this was not necessarily a change in how she interpreted what perseverance meant, it certainly provided Carrie with a new perspective on this element of mathematics learning. Second, Carrie’s comments about manipulatives and visual representations seemed to favor these tools more toward the end of the study than at the start. Early on, she expressed a desire for her students not to need to use them very often, but at the end of the study, she spoke of instances when she very much wished students to be using
these representations (and other instances when she did not). So, her interpretation of what it meant for students to rely on (or even use) concrete and visual representations seemed to evolve based on discussing how she used them in her own classroom. In general, though, Carrie’s interpretations of SFMP 1 remained quite similar throughout the study.

**Carrie’s Interpretation of SFMP 3**

*Title: “Construct viable arguments and critique the reasoning of others”* (p. 6). Carrie’s first comment about this title sentence in interview 1 was, “I wrote a lot about this one [as I read it] – critical thinking.” She referred to the first grade critical thinking rubric, of which she had spoken previously. She continued, “It is really interesting to hear, even my strugglers, talk about mathematical thinking… They always know there’s gonna be a ‘how’ involved… So they’ve learned, and they’ve modeled for each other in our discussions too – how to do that answering.” She then discussed how she responds when a student seems not to truly understand what he/she is saying:

> Sometimes, you’ll hear them say something that they’ve heard used in an explanation before, but you can tell that they don’t really know what that means. If that happens, then there’s some backing up that needs to happen, rebuilding. I really feel like, especially with math and reading, you can’t leave any holes, so there’s no need to forge ahead - if we have holes in our foundation, we have to back up and fill those in. So if somebody’s expressive language doesn’t match what they’re doing, then I
have to go back and figure out how to fill it in because they have to be able to really understand in order to explain it.

As shown above, Carrie seemed to feel that it was largely her responsibility to guide students toward logical and correct ways of expressing mathematical ideas. During both videotaped lessons, whenever a student said or did something that was incorrect, Carrie was nearly always the person who responded quickly to support the student in finding the correct solution. In the first lesson, she had brought a student to the board to talk about a problem that the students had solved, and though she did give him multiple opportunities to eventually answer her questions correctly, she indicated three times that his answers were incorrect without asking other students to offer their suggestions, and she guided him very directly to count the groups of ten and one that were on the board. In lesson 2, a student was starting to try to shade one fourth of a rectangle but appeared to be starting to shade half of it. Carrie quickly said, “Wait, I don’t want two parts; I want four parts” [pointing to the 4 in \(\frac{1}{4}\)]. After the student erased his work and divided the rectangle into four parts, Carrie said, “Then I need one of the 4 parts.” The student started to color in one part slowly, but Carrie took his hand with the marker and helped him to color it in quickly, saying, “That’s the fast way.” Later in this lesson, when another student was somewhat unsure of how to create fourths in a new square, Carrie asked, “Can I show you?” She went on to explain that we could split two parts each into two parts to create four parts. Interestingly, though, in the second interview, she commented, “[I]f I see somebody that’s going the wrong direction with something, I’ll kind of wait it out a little bit because odds are they’re going to figure it out on their own.
Which is the best-case scenario.” So, although she expressed a belief in “waiting it out” when students hesitated, she demonstrated a contradictory practice several times in these lessons, guiding and leading students instead.

Carrie was also proud that the students were able to respond to and support each other’s thinking. In the first group discussion, she shared that when a student found an incorrect answer in front of others, her students usually did not say, “That’s the wrong answer!” Instead, “They’ll say, ‘But I know that…’ – they’ll kind of go about it that way, disproving, instead of accusing… They are wonderful at teaching each other.” She added, “[W]e’re an inclusion class, so there’s been a ton of sensitivity training for two years… talking about you know, knowing when to help someone and how to help someone so they can still learn. Because otherwise I’d have little mother hens doing for my special needs guy constantly.”

During interview 3, Carrie reiterated that she saw her “daily math problem” as a definite opportunity each day for students to discuss their thinking and solution strategies with each other. She would ask questions and offer comments to compare and contrast different strategies, and she noted, “[W]ith me modeling all of that, then they start to assume that responsibility, and are having conversations with each other as well.” Further, as Carrie discussed one of the hypothetical classroom episodes involving length measurement, she described how she might say to two students, “Well, gosh, those tables look like they’re the same size, but you said this and you said that. Why do you think that you have different answers?” She related this to “the whole arguing issue, and explaining your thinking” that she saw as part of SFMP 3. Carrie then reflected,
speaking almost with a sense of awe she thought about the practice of “turn and talk”:

“[T]hat turn-and-talk thing… is just… probably one of the most powerful things that I’ve ever introduced.”

So, Carrie’s interpretation of this text in SFMP 3 did not especially change during the study. She seemed to view “construct[ing] viable arguments” as explaining one’s own thinking, and “critiqu[ing] the reasoning of others” as asking questions and offering supportive comments that would help a student with an incorrect solution reach the correct solution. However, as with many of the SFMP in the minds of many teachers, it is interesting to consider (based on the examples from lessons 1 and 2 above) what she felt was her responsibility in enacting these SFMP and to what extent students should actually have been able to meet the goals in the standards, as in the case of evaluating others’ arguments. She did note that she saw her students doing this more effectively as the year passed.

“Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures” (p. 6). Throughout the study, Carrie seemed to focus on the idea in this text related to students’ using what they knew to find information that they did not know. This seemed to relate more to the first sentence in the text, but it was connected to the ideas in the second also. For instance, she said that students might say, “Well, if I know that such and such…” She noted that the “doubles plus one” and “doubles minus one” strategies were very “powerful” in the middle of the year for learning addition and
subtraction facts, and she remarked that it was “great that they’re using that, so I hear that a lot when we’re talking about these bigger problems.” Carrie also used this example associated with money: “‘I know that if I can count by 5s – 5, 10, 15, so that’s three nickels’… I hear them making those connections.” She felt that as students learned to make connections to the text in literacy, they were also starting to make these connections in other content areas. “So, if you have that thinking pattern, in all things – in science, we – it happens all the time. You know, ‘That reminds me of the butterfly’… So, all of that connecting applies here too; I never thought about it like this. But that’s probably why they’re so ea -- it’s so easy for them to connect their concepts. In math. Use that, ‘Well, if I know that’…”. Carrie said she had been wondering how students had gotten into the habit of using this phrase in math, because she did not believe that she said it very often. She remarked, “But it sounds really textbooky to me… I never thought about that that is actually the basis of our literacy connections all the time – coming out in math.” During the last interview, Carrie commented, “[W]hen you have to explain your thinking, if you really want to learn something, teach it… when they’re doing that, working together, that’s what’s happening. The challenge is to always build groups together that are going to, um, have both parties benefiting… everybody’s comfortable with each other enough, and they know each other… so they’ll help each other along.” Although these statements did not really seem to speak to the text in SFMP 3, perhaps Carrie was thinking about how her students were able to progress toward these goals when they discussed their thinking with each other.
Overall, Carrie’s interpretation of these statements in SFMP 3 seemed to be derived from a broad view of the text, not so much focusing on individual words or phrases within it. Thus, her thinking did not evolve particularly during the study.

“They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples” (p. 6). The idea of a “counterexample” seemed to be unfamiliar to Carrie, at least in the context of mathematics teaching and learning. In the second interview, Carrie said (referring to the children’s word “flip-flop” for the commutative property of addition), “[T]he flip-flops, they talk about, thinking about counterexamples. Um, many of them have not altogether come to grips with, um, subtraction being the counterexample of addition.” She tried to relate this to text from the introductory pages, saying, “[T]hey haven’t come to that – what was it, learning – the learning… progression. It’s not on their learning progression just yet. And I don’t think I could just tell them – it’s not something that I would just say – well if you know 9 + 9 is 18, then 18 – 9 is 9. I don’t think that that would make any sense to them… I think it’s something they’re gonna have to go, ‘Oh! I see!’” At this point, Carrie appeared to view a counterexample as an inverse. During interview 3, she again had little to say about this text, offering that students might say, “Well, if I know that so-and-so, you know, such-and-such, then that has to be true, or then that can’t be true.” Here, perhaps her inclusion of the last phrase – “then that can’t be true” – was her reference to what she believed to be a counterexample. Thinking about the new group of kindergarten students that she would soon be teaching, Carrie remarked, “I think it’s just the teacher’s responsibility to model all of that language in kindergarten; that’s why kindergarten’s so hard.” So,
Carrie’s interpretation of this text was rather nebulous, varying from interview to interview.

“They justify their conclusions, communicate them to others, and respond to the arguments of others” (p. 6-7). When Carrie read this text in the first interview, she again shared, “We do a lot of turn and talk,” which formed the basis of her overall interpretation of this sentence. She noted that when students shared what they had discussed, they were not allowed to share what they were thinking; they had to share “something that [they] learned from the other person”. This, according to Carrie, “embedded that listening piece into the entire curriculum, not just that language part.” She added that sometimes she paired students with different levels of thinking so that they could learn from each other.

During the first group discussion, Carrie suggested that a way for students to think about disagreeing with each other’s thinking might be to see the discussion as a “proof”, so instead of expressing outright disagreement, a student might say, “[P]rove it otherwise.” She pointed to SFMP 3 as she said this. Carrie made very similar comments about proof being a part of explaining one’s answer in the second group discussion, and she said that students expect to be asked to do this now because it has become such a part of the classroom culture.

In her first and second lessons, Carrie asked students many questions to encourage them to justify their thinking – particularly, “How do you know?”, “What are you thinking?”, and “Why?” Students were clearly accustomed to answering these questions, and when they worked together with the base ten blocks to solve problems,
they were sharing ideas about how to solve the problems (though it was difficult to hear exactly the nature of their discussions).

Carrie pointed out during interview 2 that sometimes students wanted to give more justification than was really needed for a particular mathematical situation – mainly because they were used to giving detailed reasons for their thinking. She realized that some students probably felt that not providing clear rationales was the “easy way out” and that this was a “real contradiction” to what they believed she was asking them to do.

During the second interview, Carrie also shared:

I think that there’s been a nice push [in our district] over the past several years to put some more independence into the kids and let them be… freer, I guess… talk to each other… Collaborate, and discuss… and if we’re setting that foundation here, for them to do it productively, then later on in high school… I think that having them hear from one another, each other, is very powerful.

Overall, Carrie definitely saw this standard as students’ being able to talk to (and listen to) each other about their thinking; she said this multiple times throughout the study. She felt that her students were able to do this effectively, and she did not seem to gain additional insights about this text as the study progressed.

“Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is” (p. 7). Early in the study, Carrie’s comments about this text were very similar to those that
she had offered for the earlier parts of SFMP 3. She spoke of students’ finding “a
different way to get there” (to the solution to a problem) and students’ sharing their
reasoning with each other. These comments seemed to align more with the text:
“compare the effectiveness of two plausible arguments”. In the second interview, Carrie
referred to a classroom situation related to the calendar, where students were asking
questions such as, “How long ago was Easter?” She explained that students often
questioned whether to count “today” as part of this answer, so they had had several
discussions about this issue. They would subtract the date of Easter from the current date
and then check to see if their answer matched the number of days obtained by counting.
Carrie continued, “Does it work? So we do it, and then they count back to check it, so
that’s an example of very practical realistic thinking that they’ve had to test some math
type of… To see if it really will work.” This example appeared to relate to the
second and third goals in the text – distinguishing correct logic or reasoning and
identifying flaws in reasoning.

In the second group discussion, the teachers had a brief conversation about when
students are so convincing in an explanation that they persuade others that they are
correct, even when they are not. Carrie remarked, “[A]ll of a sudden, you’re like
thinking, ‘Well, maybe that’s right!’ You know, ‘That’s kind of making sense now’…”
Then they might convince me that I’m wrong, when I was right to begin with!” The
teachers did not come to consensus about a way to avoid this in class, but given what
Carrie did in both lessons, it is likely that she would step in and offer additional
comments to help students make sense of the situation at hand. This is certainly a practical question that teachers must consider as they enact SFMP 3.

In interview 3, Carrie spoke a bit more about the first part of this text. She said, “I think it’s really interesting when [students] are looking at a problem from two different angles, but they’re both right… And they’re explaining it, and then all of a sudden, they go, ‘Oh!... They’re both right! They’re both the same!’” She added that this was “almost more interesting for that than to find the wrong answer”, because “if they’re both right in their thinking, then, it seems like it’s, um, really opened their mind to a different possibility.” Otherwise, “they really will try to show that theirs is correct, and the other person’s is not.”

Carrie also reflected on the second part of the text in interview 3 when discussing the hypothetical classroom episode involving length measurement (cited before). She maintained that if a teacher can lead a student to make an argument about why an idea is illogical, this will help the student to find the logic in the situation. In this case, to help a student understand why it would not make sense to leave gaps between single units of length measurement, she said, “I think measuring, yeah, and calling [a student only] 3 sticks tall, would be just the right thing!” Further, “[I]f you could get them to say it, why, you know, it’s gonna mean so much more than if I say, ‘Oh, wait, wait, wait’ – I would never do that.”

As the study continued, Carrie seemed to think more carefully about the goals in this text. She commented that her thoughts about the classroom calendar in interview 2 were new connections that she was making to this part of SFMP 3, and in the third
interview she shared more ideas about the first part of the text. Giving Carrie more opportunities to reflect on this sentence, in particular, seemed to help her to interpret each goal stated more specifically in the context of her own classroom.

“Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades” (p. 7). During interview 1, Carrie saw this goal as very similar to the goal in SFMP 1 involving students’ use of visual and concrete representations to represent and solve problems. As a result, her only additional comment about this text from SFMP 3 was, “[W]e can do that; I’m hoping that we’re getting away from it.”

In interview 2, Carrie said a bit more relating the use of concrete representations to the process of starting to think about more sophisticated (abstract) concepts:

I guess that’s just the representing part…? [pauses briefly] Um, I think that when my kids are drawing ten-sticks and ones, probably half of them are recognizing that the 3 in 30 is 3 tens… it’s making sense to them, but I still think they have to have more experience with it. And I think that’s where place value fits into first grade… The experience with it as an introduction…

Carrie added that she hoped that in second grade, this exploration with concrete materials would continue with numbers in the hundreds so that students would continue to have the “aha” moments about what written numbers meant in relation to quantity. She also
commented that it was helpful to have concrete materials available to ask students to model ideas that they were not fully understanding.

In interview 3, Carrie seemed to focus on the portion of this text that deals with students’ exploring ideas that they formalize ideas in later grades. She gave examples of her students using strategies that involved multiplication and division concepts and even just regrouping tens and ones. She remarked, “[S]ometimes it blows me away that that’s what they’re thinking about… their number sense is so strong that they’re able to see those patterns and start to break things down… I think that that exactly speaks to this” [pointing at SFMP 3].

As the responses above demonstrate, Carrie’s focus in interpreting this portion of SFMP 3 shifted throughout the study as she reflected on different portions of the text. The second interview was when she seemed to be thinking about the goal in the text as a whole; in the other interviews, she only spoke about certain aspects of the goal.

Carrie’s thoughts about implementation of SFMP 3. Carrie believed that creating a learning “culture” in a classroom and/or school was vital to the successful enactment of SFMP 3. In the third interview, she said, “[R]eally, this school, everyone is buying into the turn-and-talk, collaborative learning kind of thing, which is what you need to have in order to be able to expect that they’re going to listen to other people, and explain, and argue.” She continued to emphasize that it was important for teachers not only to require students to share their thinking but also to encourage students as they did so in order that students would feel confident as they developed this skill. This would also help teachers to know how students were understanding each idea so that the
teachers could “fill in holes” as needed before moving forward. Further, it was important for teachers to model the language that they expected students to be using, such as: “How do you know that?” “Why?” “What did you learn?” Finally, Carrie noted that the critical thinking rubric was allowing the first grade teachers to assess students’ expressive language in mathematics as well as in language arts, which she felt helped her to focus on whether students were using “appropriate terminology” or whether they needed more support with this. Generally, Carrie’s ideas about implementing SFMP 3 were grounded in her overall philosophy of instruction.

**Review of how Carrie’s interpretations of SFMP 3 seemed to evolve throughout the study.** The only noteworthy way in which Carrie’s ideas about SFMP 3 changed during the study was that some of her ideas and connections to her own classroom became more specific as she had the opportunity to reflect on each portion of the standard during the interviews (not so much the group discussions). It seemed that as she read and considered each part of the text, she thought a bit more on each occasion about how these goals related to her goals for her students. Interestingly, this could be seen as similar to the “connections” that she wanted her students to make as they explored the same ideas more over time.

**Carrie’s Interpretation of SFMP 7**

**Title: “Look for and make use of structure” (p. 8).** Carrie seemed to focus on one of the examples given later in the paragraph for SFMP 7 as she spoke about the title sentence in the first interview. She said, “I wrote “flip flop” on [my own notes] because that’s what we call, the uh… what is it? The property of $7 + 3$ is $3 + 7$? [I respond.]
Commutative property! Is officially referred to as the flip flop in first grade.” She remarked with a smile, “[T]hat is the, um, sophisticated language that we see sometimes in some of this explaining… I never said, ‘Look, it’s just the opposite numbers flipped.’ Flip flop actually came from them.” During interview 2, Carrie’s only comment about this goal was related to this last idea – she appreciated when students made connections on their own and then shared them in class because then this could become a part of other students’ learning also.

In the first lesson, both Carrie and the students were using the structure of the base ten blocks to explore adding double-digit numbers. They counted by tens and ones and “swapped” groups of ten ones for single tens, and Carrie often used the word “reorganize” to help students think about how to efficiently count the tens and ones. Also, a few students explained how to combine smaller numbers, such as in the following example: “[I]f 10 plus 6 is 16, you just add on one [to get 17].”

During interview 2, Carrie seemed to think more carefully about the goal in the title of SFMP 7. She said:

I would say that when we’re talking about problems and solving problems, they’re looking – there’s like three different ways that they’re – , head [as in drawing faces], sticks and cubes, groups – they’re trying to figure out how to apply one of those to that problem. If we’re talking about groups and data and things like that, they definitely are really savvy about sorting and making sense of giving some structure.
She added that if students found that a structure that they were using was not “making sense”, then they would “[m]ake another structure that makes sense in the situation… They’re just always really looking for things to fit together…” In addition, she referred to students’ familiarity with the structure of the 100 chart, saying, “[T]hey’ve had so much experience with the 100s chart that I know that when I say 67, they’re thinking it’s about there” [motions as if to indicate a position on the chart]. As an aside, Carrie also shared, “I wrote that on my [reflection] to you – so reaffirming, that something, that this is coming to the forefront as a priority, and it’s not anything that you can assess and measure… I mean, it does show up in the data, for sure…”. So, Carrie was pleased that mathematics education policy was beginning to more explicitly reflect some of the core values that she espoused as a teacher.

The second lesson involved many elements that related to the title of SFMP 7. Students were exploring halves and fourths in regions and sets of two or four, and essentially the entire lesson, including the whole-group discussion and the individual work with play dough and drawing, involved students’ finding structure in the wholes to create the required fractions. Carrie continually asked questions about how many equal parts were in the whole and how many were “colored”, and she allowed students to show many ways to divide a circle, square, and rectangle, as well as a hot dog shape and a set of four objects, into halves and fourths. They briefly discussed whether four parts were fourths if they were not equal in size. It should be noted that as evidenced earlier, if a student started to demonstrate or talk about an idea that was incorrect, Carrie corrected
him or her fairly quickly. Still, ultimately, her goal was to have students start to understand structures that allowed them to create two or four equal parts of a whole.

Carrie focused on the idea of patterning as related to structure in the third interview. She began by saying:

I think it’s really interesting that patterning has kind of disappeared [in the content standards]… it’s not going to in my classroom this year… it’s that perfect known-to-new piece… we’re gonna be doing some sorting, the first week of school… they can explore with [manipulatives], while they’re doing something that they already know how to do… then, the next obvious progression is patterning.

Carrie then pointed out that asking students to sort and to create patterns was an effective way to differentiate instruction and assess students’ thinking early on because not all incoming kindergarten students had had experiences with sorting and patterning, but other students created rather “sophisticated” patterns. She remarked, “So, that’s structure. And that’s the known piece of structure that we need to start with. And I think it’s really important and nothing to be able to skip over, because then that’s gonna lay the foundation for all of that organizing and grouping that’s to come.”

As Carrie reflected on the hypothetical classroom episodes in interview 3, she spoke to the ideas she had shared about the title of SFMP 7. In the first, where a student was struggling with how to write the number 113, she commented, “Teens are now in kindergarten… that would be the known that I would speak to first.” She continued, thinking as she spoke, “[I]f we know… what 13 looks like… then can we add 13 onto
100… and keep going from there.” Regarding the second situation, where students were discussing how to use single units to measure length, she said, “[T]he whole structure issue of… ‘How can you possibly be 3 sticks long? If I know 3 sticks looks like this… then you’re much taller than that.’” Presumably what she meant in this case is that she would have this question in her mind as she worked with the students, because she had said earlier that she would “never” say this to the students directly; she wanted them to recognize this on their own.

Carrie’s interpretation of the title of SFMP 7 perhaps evolved more than her thinking about almost every other portion of text that was considered for this study. During the first interview, she focused on the commutative property example that was actually provided later in the paragraph for SFMP 7 and did not really speak to the text in the title. She did mention at this time that she was pleased when students made connections among ideas independently and then shared them so that other students could think about them. In the second interview, Carrie reflected somewhat on the first lesson and also thought about other classroom examples of students’ looking for and using structure to solve problems – particularly when they explored various correct methods for solving the same problem and thought about how ideas “fit together”. Interestingly, Carrie did not especially reflect on the second lesson during interview 3, even though this lesson seemed to focus very directly on the goals in SFMP 7. However, she did offer comments about students’ patterning being an example of using structure, and she frequently mentioned the idea of moving from “known” to “new” to help students learn, which she felt was a way to build on the structures of ideas that were established in
students’ minds. In addition, Carrie believed that it was critical for students to make these connections and create these mental “structures” for themselves; if she simply told students what connections to make, it was unlikely that these connections would remain a part of students’ thinking. So, Carrie’s thinking about this text seemed to evolve most as she thought generally about her classroom experience while discussing the text during the interviews. She did not, however, focus on the two videotaped lessons as she reflected even though both lessons involved clear applications of structure. Perhaps this indicates that Carrie could benefit from more opportunities to analyze specific lessons in light of the SFMP.

“Mathematically proficient students look closely to discern a pattern or structure” (p. 8). For this text, Carrie continued to share ideas about students’ finding patterns as well as connections among mathematical ideas and others. This was echoed in a comment she offered during the first group discussion: “I think the more experience you give [students] to mess around with it – they are realizing and finding the patterns themselves.”

During the second interview, Carrie related this text to students’ making connections among their learning in different content areas. “We’ve done a lot of attribute work in a lot of different ways… it all fit together.” She described how her students had realized that the idea of an “attribute” in math and science was very similar, and she even helped them make the connection to adjectives in language arts – these were all ways of describing an object. She added, “Similarities, and differences, and all of that, I think across the curriculum has created kids that want to make sense of things.”
Further, Carrie gave an example of how when her class played a game where one student removed numbers from the 100 chart for other students to find, some students were very deliberate about removing numbers in patterns.

In the second lesson, where students were exploring halves and fourths, students shared ideas throughout the lesson that showed that they were looking for structure. For instance, a few students offered comments about symmetry as they observed how regions could be divided into halves and fourths. Also, a question arose about whether a hot dog shape could only be cut in half in one way, and several students made conjectures about whether this was true; Carrie then used two of their suggestions to demonstrate two different ways to divide the hot dog shape in half (many students were unsure about creating two long, narrow halves). In addition, when the students were working with play dough, they had opportunities to consider various ways to divide wholes into fractional parts, and some were exploring thirds, sixths, and eighths as well as the halves and fourths that were required. During a brief class discussion following the work with play dough, Carrie drew a circle on the board, divided it into fourths, then divided each fourth in half to create eighths. Students immediately started attempting to determine how many parts there were (without counting), and 6, 7, 8, and 9 parts were predicted before Carrie actually numbered them.

During interview 3, Carrie returned to thinking about patterns in relation to this text. She shared, “[A]s soon as I put a calendar up, they – ‘Oh! There’s gonna be a pattern’…”. Her response would be, “Oh! OK, well, how do you know? What do you know about it?” She then added, as she reflected on this kind of conversation with
students, “It’s more than just seeing that there’s a pattern there – ‘What do you think about it?’” Further, “[T]hey start to recognize, on the calendar, too, that when there’s a certain pattern, it makes different diagonals, with the different pictures and things like that – which I think is really interesting. As an adult, I don’t think we see that… the patterns that they recognize that our brains either have gotten too concrete or too… you know, assuming.”

Again, Carrie’s focus in interpreting this text was on students’ patterning and making connections among ideas; this was likely because the word “pattern” is actually in this sentence. Her discussion came full circle throughout the study as she spoke mostly about patterns in the first and last interviews. Although her second lesson also involved many other applications of structure, she never mentioned fractions while discussing this text. She seemed to generate more ideas as she had multiple opportunities to think about her classroom while conversing in interviews during the course of the study.

“Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have” (p. 8). As noted earlier, in interview 1, Carrie talked about this statement when asked about the title of SFMP 7. She also shared the same comments in group discussion 1 regarding students’ “noticing” and “naming” the “flip flop” as the commutative property of addition.

During the second interview, however, Carrie seemed to focus more on the second example in this sentence, and she realized that she was not sure how it related to
the example of the commutative property. She said (reading), “‘Or they may sort a’ – what’s that supposed to mean? I don’t understand the connection there.” I encouraged her to speculate, and she offered, “I think we’re just – keep talking about more than one way of solving a problem. A different perspective about how that looks when they’re having a conversation about, ‘Well, this is the way I did it.’” Carrie added, “[T]hose savvy ones always want to overachieve,” meaning that some students worked to find multiple ways to solve the same problem.

Carrie referred in the third interview to several previous ideas that she shared. She returned to the “flip flop” classroom scenario, saying, “I think it was just the most powerful thing, because it was their discovery… as soon as somebody recognized it, they were all like, ‘Yeah! That is right!’” She mentioned the interdisciplinary nature of “attributes” again and remarked, “[I]t was so nice to be able to see math in science. And see science in math.” Further, she reflected more on the sorting example in the text: “I think that [students] are very… um… surface value, when we give them those attribute blocks to sort; it’s real easy for them to sort them by color, or by shape… it is a little bit of a stretch, then, to say, ‘Can we sort them another way?’” She added, “[T]hey can do it – it’s just opening their mind to that possibility that there’s other ways to do – to sort.”

In her comments, Carrie remained focused on the examples in this sentence in the text. Her ideas about the connection between the two examples were most coherent in the last interview, as she discussed students’ ability to see relationships among ideas and also to find other ways to work with the same mathematical objects, as in sorting
activities. In this case, it once again seemed that taking more time to think about and discuss this text helped Carrie to develop her interpretation of it.

“They also can step back for an overview and shift perspective” (p. 8).

During interview 1, Carrie discussed this statement in light of students’ ability to “shift perspective” as they worked with numbers. She shared, “‘Shift [of] perspective’ – that is happening all the time, when they start to make the connection, I think, from the 100 chart to our math facts… with this silly little routine game that we play – ‘Oh’ – I start to see them looking over there at the 100 chart… ‘Oh, that’s like the next number in the skip count.’” Carrie speculated about the authors’ meaning, saying, “I don’t know if that’s what they’re talking about… But that’s the way I take it because being able to shift perspectives from looking at numbers one way to another way, and they still always work the same way.” This idea could be observed in the first lesson when Carrie was asking students to look carefully at a two-digit number on the board and how many ten-sticks and one-cubes they needed to create that number. One of the students observed (unprompted), “There’s three 10-sticks, and then you make a 3, and there’s 6 cubes, and there’s a 6.”

In interview 2, Carrie discussed how she worked to help students consider what they were learning about an idea to connect it to something else that they already knew. She seemed to relate this to gaining an “overview” of the idea in a broader sense and being able to “shift perspective” to an application of the idea in another context. She explained:
I think that that is when we always have to apply it to something else that they know. Um, that’s the sitting on it part… we did a stretch, and now we’re gonna sit on it for a minute and think why, why do we know that? Or what else do we know about that that agrees? You know, what else do you think about that? Just that conversation. And so often it’ll come from them.

As Carrie reflected on this cycle of “stretch” and “sit” in her first lesson (when students were combining double-digit numbers), she commented, “[W]hen I looked back at that video, too, I felt like that’s what was happening.” She continued, “I think it’s their chance to kind of, um, make whatever concept we are talking about relative to themselves… ‘Cause that’s when the conversation goes to, ‘One day I was playing baseball, and’ – whatever – that kind of thing… We want them to – internalize – we want them to personalize the concepts… I would call that the ‘sitting on it’ part.”

In the second lesson, many comments from students and questions from Carrie related to the ideas she had shared about this text in SFMP 7. Carrie often referred students to discussions they had had previously, at one point echoing a students’ unsolicited connection of “half” to symmetry, and she offered observations such as, “[T]hat looks sort of similar to what we just had a minute ago, doesn’t it?” Students made comments such as, “Like the cupcakes!” “That looks like a pizza.” “That looks like a window.” “It’s the same thing except you just did it like a different way.” They also volunteered many ways of dividing a square into fourths. Further, during the play dough activity, Carrie raised the following question (to students who were listening):
“You cut that pizza into 4 parts. And then you cut a different pizza into 2 parts. Which part would you rather have? 1 of 2, or 1 of 4?” She asked a similar question about 1 of 2 or 4 cookies. Most students who responded said that they wanted 1 of 4 parts. Carrie did ask a student who said 1 of 2 to explain why, and the student said it was “more”. Toward the end of the lesson when she brought the class together, Carrie colored four fourths of a circle and also two halves of a circle on the board. She wanted to guide the class toward understanding that two halves and four fourths were both equal to one whole, but only a few students really seemed able to “shift perspective” and to grasp this idea at this point.

During interview 3, Carrie offered the conjecture that perhaps shifting perspective was more difficult, at least in some cases, for adults than it was for students – similar to her comment about finding different patterns. She also felt that some students were beginning to “learn” from adults that having one way of understanding an idea was enough. She did not address, though, how students might respond if they had not “learned” this “trait”, so in essence she seemed to conclude that this was a challenge for students as well. She said, “[W]henever you say, ‘Is there another way to do it?’ Or, ‘Can somebody else do it a different way?’, the kid’ll go up there and that kid already has it in their mind. But, the rest of the kids, who had it solved in their brain a different way, you know, there’s always skepticism involved initially… ‘Well, I already did it [so, why should I have to look at it again?]’.”

In general, Carrie’s interpretations of this text highlighted the ability to see the same mathematical idea or process in different ways. She spoke of this habit in relation to understanding numbers and basic facts in interview 1, connections between
mathematical ideas and personal experiences or prior knowledge in interview 2, and
different ways to solve the same problem in interview 3. Since these are three fairly
distinct examples of how this text in SFMP 7 might apply in a classroom setting, again
one might conclude that Carrie gained more insight about this standard as she reviewed
and discussed it in the interviews.

“They can see complicated things, such as some algebraic expressions, as
single objects or as being composed of several objects” (p. 8). Carrie focused on the
example in the text (“such as some algebraic expressions”) in interview 1 and, as a result,
offered little about the text in general. She noted, “Little bit beyond where we are in first
grade, I think… I do tell them sometimes, when we’re writing number sentences – ‘You
know, this is algebra.’”

In her first lesson, however, Carrie seemed to strongly emphasize the goal in the
text about seeing objects as “single” or “composed of several objects”, because she
wanted students to use the base ten blocks to explore and discuss how to combine tens
and ones. This often involved a “swap” (students’ word) of ten 1-cubes for a 10-stick,
and it was evident that a few students were still coming to understand that the 10-stick
really was composed of ten ones (because they would count the ones in the ten). At one
point, Carrie was guiding a student at the board to consider how many ones were in two
10-sticks as opposed to one 10-stick and two 1-cubes, but the student still needed to count
by ones to determine this. Most other students, though, did seem to understand this
concept enough to apply it to counting by tens and ones and combining them. Carrie
reflected on this lesson in interview 2, saying, “[It] makes it really nice to be able to have
sticks and cubes and all sorts of manipulatives to be able to address that and show, you know, that a 10-stick is equal to ten cubes.” It is unclear, however, whether Carrie connected this process to this text in SFMP 7.

Because the second lesson involved exploring halves and fourths, students had many opportunities to consider how a single object (or set) – a whole – could be composed of several objects – fractional parts. Early in the lesson, one of the students described “fraction” in this way: “It means like you take something, and then you put a line in the middle, and you make it in two.” Much of the discussion during the lesson centered around dividing a set or region into two or four equal parts and naming a given fraction of the whole. As aforementioned, although Carrie was asking for students’ ideas about how to divide the wholes that she was drawing, she generally led students through the process if they seemed unsure. Still, it was noteworthy that toward the end of the lesson, one student noted (without prompting) that the third of four parts that were colored was a “third fourth”, and then the last part was a “fourth fourth”. This student seemed to have made sense out of the idea that the parts could be seen as objects in themselves while still composing the whole.

During interview 3, Carrie seemed to attempt to focus more carefully on the actual text in this part of SFMP 7, and as she read it again, she remarked, “I don’t really know that I understand the single objects or several objects… I mean, a set? Is that what it’s referring to?” Again, I encouraged her to speculate. She continued, “[Students] pick that up really quickly. The set piece… Which… I didn’t learn until… probably pre-algebra… that word ‘set’, and it didn’t make any sense to me. If somebody would have
just said it means a group…”. As she often did, Carrie tried to interpret the goal in the text in light of her own school mathematics experience, which she felt was lacking in many respects. She continued to refer to “sets” as she discussed this part of the standard. “I think that when we’re referring to the group of buyers and the group of packers already [in class], which one is more or less… we’re already establishing that… when we get to the Counting Jar sorts of activities in Investigations, and we refer to it as a set – ‘Now make a set in your set bag’… they’re able to make that jump.” At this point, Carrie comments, “I don’t know that that’s what this is speaking to [points to the text], but I think that that number sense… when you have a strong enough number sense, you can make the jump to those algebra concepts easier.”

At first, Carrie hardly thought that this portion of SFMP 7 even applied to first grade because she noticed the reference to algebraic expressions. In the last interview, she did generate several examples that could be connected to this goal, but her ideas were limited to students’ creating and thinking about sets and how they were composed of particular objects (or people). However, as shown, she did offer students many opportunities in the two lessons to develop their skills relative to this goal, so perhaps she might simply benefit from some guidance about how to relate the goal to what was already occurring in her classroom.

**Carrie’s thoughts about implementation of SFMP 7.** In thinking about enacting SFMP 7 in her classroom, Carrie shared several ideas for providing students with experiences in patterning and in making connections daily among ideas in mathematics and other content areas. She wanted to maintain sorting and patterning as
regular parts of her students’ routines, to encourage students to find personal connections to the ideas that they learned in order to “internalize” them, and to develop in students the habit of looking for similarities, differences, and relationships among ideas and mathematical objects – which also meant including manipulatives when appropriate.

Including these activities throughout the year meant that she as a teacher needed to return to important concepts often so that students truly did have many opportunities to see relationships among ideas – that is, she felt a great deal of responsibility to ensure that these opportunities were in place, so she believed she needed to reflect continually on what each student had learned and how he/she was thinking. As a result, as she noted in the last interview, “I don’t know that you can always, always determine the order [of instruction]. Or how much time you’re gonna have to spend on something… Which I know is just a huge challenge for the older grades… Because they have so much to do, and they have to cover. I feel like what we have is an appropriate amount of stuff. We can get to it.” Still, she felt that her students were able to meet her goals for them “because of across the content, what we’ve done, in literature, literacy, in kindergarten, so much of the connecting…” Further, Carrie believed that her modeling of the language that helped students to make these connections was useful, such as when she might say: “Oh, so you said this, and when I think about it, I think of this.” Once again, Carrie’s beliefs about best practice in her own classroom corresponded with her ideas about how to effectively enact SFMP 7.

Review of how Carrie’s interpretations of SFMP 7 seemed to evolve throughout the study. As shown in the sections above, Carrie’s thinking about SFMP 7
appeared to evolve most when she was reviewing and discussing the text during the three interviews. In this setting, she seemed to be able to think about a variety of scenarios that occurred in her classroom throughout the year, and she often drew on experiences from the most recent week of school. Particularly, her thinking about the title statement became more refined as time progressed. Based on her understanding of the whole standard, though, she may still have been limited in the examples she was considering because she missed some rather clear opportunities to reflect on the goals of SFMP 7 relative to the two videotaped lessons. This could be a goal for her in future professional development.

Carrie’s Vision of the SFMP Implemented Effectively in a Classroom, School, or District

Carrie’s goals for her students and for herself as a teacher. Carrie certainly enjoyed teaching first grade and helping children to learn. In the first group discussion, she remarked, “I’m just happy where I am!” She felt that the kinds of goals in the new standards made “teaching math so much more fun”. As Carrie described how she allowed the students’ thinking to guide her plan for the next few lessons, she commented, “For the greater good, I know what I’m doing is the right thing… Because, in my heart, I always know, I’m giving them what they need when they need it.” She felt that she was able to teach all required content by the end of first grade. However, she added, “[I]f somebody’s telling me, giving me a map or a pace or a lesson plan,” then she knew she would be quickly frustrated. Carrie did enjoy working with another adult in the room because she appreciated being able to share students’ learning with someone else as it
was happening. She observed, “[H]aving student teachers *always* make me a better teacher.”

Carrie also wanted mathematics to make sense to her students and to be enjoyable for her students, and she believed that these were vitals part of learning mathematics. She noted that if she could succeed with the goal of developing “numberness”, “[k]ids won’t have to just follow - do algebra by the rules. They’ll *understand* why algebra works that way…”. That is, “It’s just not the fact, it’s – there’s a *reason* behind everything.” Carrie pointed out that even students who struggled to complete some math tasks would start to learn more efficient methods as they gained enough experience with inefficient methods – they would realize, especially through discussion, how and why to solve the problems more quickly. She wanted her students to have that “feeling of ‘I can do it’.” Moreover, Carrie wanted to teach mathematics “that’s relevant and purposeful, and not scary.” In the last interview, she shared, “I just think that ultimately, my goal for them in math is for them to have that sense of inquiry.” To encourage this, she often began exploration of a new topic one day and continued it the next day: “I always – in everything that we do – end with them still wanting to try another one.”

Because of the goals that Carrie had for her students, she believed that the SFMP and the goals of the CCSS were worthwhile, and in fact achievable, for all of her students. She commented, “If I’m interpreting them correctly – what I said at the beginning was my perception of what it is saying is accurate – absolutely… I think kids can develop their mathematical brain independently with just some guidance. And
number sense.” Also, “All kids can… participate… in that thinking at some level.”

Carrie remarked, “I don’t think they’ll be afraid of second grade math.”

**Carrie’s thoughts about instructional practices that would support the SFMP.** As she reflected on working with students, Carrie maintained, “[A]s a teacher, with this [the standards], I feel like I can’t give up, and I have to keep finding new ways to present to those kids and find some angle that will make a connection for them.” She also said, “I have to be careful to not ever discredit any of [their thinking]… right now, for me, all are wonderful.” As noted before, this was one strategy for supporting students’ perseverance and confidence. As she discussed this goal, Carrie frequently returned to her foundational practices of providing “discovery time” in centers as well as “mini-lessons” tied to whole-group exploration and discussion. She commented, “I have obviously really strong opinions about community and – classroom community, and kids feeling worthy of discussing, and being a part of a conversation.” In addition, “I want kids to work together; I want kids to collaborate. Discovery time, I find, is a very valuable, um, way to encourage kids to interact… together. Not a waste of time, not just playing around.” Further, “Those conversations [in all areas] are valuable and worthwhile, and this [pointing to the standards] I think speaks to that.” As Ray, Carrie, and Sandy talked about students’ apparent lack of motivation in the upper grades, Carrie suggested, “[I]f we’re talking… and everybody’s understanding it, and it’s more concrete, everybody loves it… So if we try to… continue on beyond first grade with that… then perhaps we won’t lose [students].” She did agree that “rich discussion” required a substantial amount of class time. Still, she maintained, “I think that the more
that you can internalize that conversation, that language, the more that they’re going to be actually looking at it and making sense and not just completing a problem.”

Carrie believed in actively supporting all students as learners. To encourage students who struggled, Carrie would say, “You can do this. You know this.” She explained, “I know that they didn’t follow me all the way to the end, but then the next day, um, during work baskets in the morning, that’s where we are, and we’re doing that again.” Carrie used this time during the day as an opportunity to review, reinforce, and extend students’ learning. Further, in interview 2, Carrie noted, “I think that there definitely is something to that, um, brain capacity for some of that learning at different stages.” She gave an example of moving students from concrete to more abstract thinking as she was discussing a hypothetical situation where a student did not understand the place value in “67”: “I’m not gonna just jump from drawing 67 smiley faces to drawing sticks and dots, so I would say, ‘Could these help you?’ [base ten blocks] And then they would do that. And I would give them another number – do you need these still, or could you just show me?” Carrie continued, “I would never say… ‘you do this and then you do this and then you do this’. Because that doesn’t make it make sense – it just means that they’re following steps.” Also, Carrie’s students trusted “that I’m never gonna leave them hanging… I’m never gonna put them in a situation where they look like a fool…” In the same way, she felt that posting students’ fact fluency scores was “so barbaric”… “after all that we know about education and kids and self-esteem”. She did not believe that speed or competition should be emphasized as students learned their facts – this “seemed to undo the number sense, thinking about
numbers”. In general, Carrie said of a classroom where the SFMP were being implemented well: “I think that that is a classroom where math is not stressful!”

Carrie shared her perspective in another way in interview 1:

What makes this work, is them [the students]. Giving them the freedom and tools and feeding them what they need to always be thinking mathematically… I haven’t planned out exactly where I was going all the time, I just kind of let it proceed. But we’ve hit more things than not… And everything else kind of seems like it’s in the realm of the direction we’re going – maybe just with a little detour here or there… So, in a perfect world, the kids are guiding, and still you’re hitting everything you need to hit.

Carrie felt that in first grade, the content standards were not all that different than the Ohio Academic Content Standards, so the teachers would not have to make a major shift in “what” they were teaching – “we were pretty rigorous [even] before [this shift]”. She noted in January during interview 1, “[T]he goal has been up ‘til now addition, subtraction, tens”, and “money is kind of gone for us, and I always kind of thought that was a challenge, so I don’t mind that.” In addition, exploring “combinations” [pairs of addends with the same sum] to support number sense, operation sense, and fact fluency was an important part of first grade. Carrie shared with Sandy and Ray, “I said [to parents at conferences], ‘This might seem monotonous to you, but we’re doing so much work with combinations of 5 and combinations of 7; but my goal is they don’t memorize the facts, they know them. They understand them.’” During the second group
discussion, she remarked, “[We] talk numbers ‘til we’re blue in the face.” In interview 1, Carrie reflected on the content standards as a whole and observed, “The things that are left that we haven’t really touched on so much are just – it seems like they’re in a natural progression right now. Like we’ll be able to naturally get there without too much of forcing the issue.”

Whenever opportunities existed to integrate mathematics with other content areas or with real situations in the classroom or school, Carrie wanted to utilize these opportunities to help students make connections among ideas and to understand how mathematics was important. She felt that a classroom effectively implementing the SFMP would be one “where kids are able to see math outside of the hour of a day that we’re doing math… We talk about math in science; we talk about math in the calendar…”. As she thought about mathematics in the school as a whole, she commented: “[P]robably we could incorporate mathematical thinking school-wide in different ways… if you have a kid reporting to the secretary how many people are missing and how many people are left in the classroom, they are able to use math in different places…”. However, “[A]ttendance and all of that kind of stuff that we do naturally in our classroom – now it’s done on the computer.” In the third interview, she added, “I think we’re missing out on the kids’ participation at that… I think there’s an ownership piece that’s missing, and some math thinking to start the day that was really – is really practical… That’s something that’s really important to them – how many kids are missing today. Who’s absent?” In interview 1, Carrie did note, “I don’t really feel
like we should be having math problems over the PA, or anything like that.” She seemed to feel that such tasks would be a bit too contrived.

Carrie also shared some interesting thoughts about a discussion she had once had with a principal about teaching everything through “anchor concepts” that supported all of the disciplines. Two of these anchor concepts were “same and different” and sequencing. She discussed the idea that consistent vocabulary to support these concepts could be used throughout the grades and “across the curriculum” and asked, “[W]ouldn’t that be the structure that would make learning so much easier?”

At this school, which included all of the kindergarten through second grade classrooms in the district, Carrie believed that the teachers were all already teaching in ways that would support the enactment of the SFMP. She noted, “[T]hese really, I mean, go along a lot with what we do as far as teaching the foundations of math… All of this critical thinking and exploration and the kids figuring it out… there really isn’t much of a change.” In fact, in the last interview, Carrie commented, “I think that really, the three standards that you’ve picked here to talk about are really in every single thing that I ever talk about in math.”

Carrie’s thoughts about assessment practices that would support the SFMP.
Assessment was one area in which Carrie felt the first grade teachers could develop further in order to gain more valuable information about instruction, though she did not believe in standardized tests for their students. During the first group discussion, Carrie shared her perspective on this: “I feel like I know my kids; I know what every one of them can do; I’ll give you a dissertation on each one’s deficiencies and strengths in math.
It’s not that I don’t know them because there’s no assessment tool… I’m just not driven by an assessment tool.”

During interview 1, she commented, “[W]e still are having conversation about assessment and what is the right sort of thing for that…”. Moreover, “[I]f someone were to… write some good assessments, it’d be great in math. But that is definitely a lacking – I mean, it seems there are just none out there.” She acknowledged, “[T]he [district common assessment] is something… I don’t know if it’s the right thing… it gives us some information and that’s more than we had. So, it’s a starting place…”. Carrie went on to explain some of the history of their common assessments:

[The] first grade team worked for two years to build this assessment library of common assessments. And then new people came, and they tried them out, and they don’t really like everything, so then everything has to be tweaked, and then, this happens [puts hand on standards], so here we go again, really looking at these common assessments, which is not rocket science.

She added, “[W]hat we’re assessing isn’t huge concepts [sic],” and she said that the teachers just wanted assessments that were valid and that were not too “labor” intensive (relative to language) for students who still were learning to read and write. Carrie commented, “I’m kind of feeling my way through it right now,” and she said that it would be helpful for the first grade teachers to have professional time together, as well as some training, to review and refine the common assessments. This was one of the only areas in which Carrie seemed to feel that a trainer from outside the school – or allowing
teachers to visit another district where the assessment was in use – might be helpful. In addition, she noted that it would be helpful to have a set schedule for administering the assessments because “it just gets lost in the shuffle”. So, Carrie appreciated being able to use more holistic, thinking-based tasks like those that were part of the assessments the first grade team was using, even though she felt that these needed further development.

In terms of the SFMP being an important part of the CCSS, Carrie commented during interview 2, “I so appreciate that they’re valuing – that their instilling value to something that cannot show up in the data.” (Carrie likely did not realize that PARCC was in fact working to assess the SFMP directly in assessment items.) As she thought about assessing the SFMP in other ways, she continued, “[I]t’s not all about the answers that the kids are writing. It’s more so about the process of how they’re getting to the answer that they’re writing… we’re using that [district assessment] as the assessment tool right now, and I think that the tool itself is very informative, like as I’m giving it; I learn a lot.” Also, as noted earlier, Carrie felt that the critical thinking rubric that the first grade teachers were using was allowing her to assess the SFMP as students were actually working on mathematics tasks. She remarked, “I am all about formative assessments, and I love to watch kids work and see how they solve problems.” During group discussion 2, Carrie and the fifth grade teachers discussed how the fifth grade teachers could probably use the same rubric while assessing their students – just replacing the first grade tasks with appropriately challenging fifth grade tasks that were relatively unfamiliar to students.
One other issue that Carrie raised related to the district assessment practices was the fact that the teachers were asked to enter scores from the common assessments into an online data management system that would allow students’ data to be maintained and monitored throughout the grades. She felt that the numbers from the district assessment that they used in first grade would be “meaningless” to teachers in later grades because these teachers were not currently trained on what this particular data meant. Carrie remarked that entering the common assessment data, other than perhaps the fluency data related to students’ progress with addition and subtraction facts – which would be easier for other teachers to interpret, was “a giant waste of time”.

In general, Carrie believed that the first grade teachers needed more time and perhaps some external support to better develop their assessments, both to assess content knowledge and the SFMP. She did not, however, support the use of traditional standardized assessments for this purpose.

Carrie’s perspectives on challenges related to enacting the CCSS and SFMP. One potential challenge that Carrie felt some teachers had to manage during certain years was having close to thirty students in the classroom. She said:

[W]ith twenty-two kids in the room, you had kind of a third, a third, a third [at certain levels of performance]… we really can do a pretty good balance in the classrooms… Come January, you still have a couple down there, right? So… you have time to really embrace those couple… If you have twenty-seven kids in the room… at the beginning … there’s such a struggle with all of the chaos of twenty-seven in routines and everything,
that you didn’t have a good foundation to start… so now you still have a third of twenty-seven now, in January, that are struggling… That’s really gonna show.

Another issue that Carrie raised, related to teachers across the grade levels (particularly in middle school and high school), was that she felt that the instructional strategies and expectations were quite different among teachers that had taught her own children. She felt that this would pose a challenge to enacting the SFMP effectively as a district because each teacher seemed to have his/her own interpretation of the current standards, let alone the new standards. Moreover, teachers who were new to teaching or at least to teaching a given course had to gain experience as they were teaching, which could be a hindrance to high quality instruction. Carrie also commented that even for some elementary teachers, “I suppose that if you were of the worksheet/workbook kind of era… you wanted the quiet classroom, and kids working in their space with their own individual kinds of things, that you would find these [standards] hard to swallow.” She noted that a teacher could probably address the content “if you want[ed] to stand at the board and use direct instruction and send the kids back to the table to do a worksheet”; however, this would not be addressing the SFMP very effectively.

In general, however, because Carrie believed that most of her colleagues in the primary grades were already teaching through practices that corresponded with the SFMP, she did not feel that many new challenges would arise with the transition to the new standards.
Carrie’s perspectives on policies that would support or conflict with the CCSS and SFMP. Throughout the study, Carrie highlighted several ongoing district initiatives that she believed supported the philosophy of the CCSS and SFMP. One that she mentioned repeatedly was the fact that the district had been providing professional development and resources for Reading Recovery and related reading instructional strategies for several years. A second was the focus on critical thinking district-wide during this year and the previous years. She described this as “that whole approach of letting the kids take on some of the roles, and be more independent in their work, um, so it’s nice to see that is kind of where the district would like to see that going.” Third, Carrie believed that the long-term use of *Investigations* in the elementary grades and the purchase of the second edition for the coming school year represented the district’s commitment to providing mathematics instruction that aligned with the new standards. She added, “[T]he new standards are really what our district has been stressing for a long time… To be able to build that foundation of number sense that will help them later on.” Fourth, the district had mandated that kindergarten implement the new standards during the previous school year and first grade during this year, with second grade to follow, so that by the time these students reached the grades with state assessments, they would be well prepared for these tests.

As Carrie shared her beliefs in a respectful, trusting, classroom learning community, she also noted that the school had, in the past, created an explicit focus on respect throughout the school, by sponsoring assemblies and developing a common language in the building to help students learn how to be a part of such a community.
She felt that bringing back this focus across the grades might support the goals in the SFMP that involved communicating with others and responding to ideas without overtly criticizing other people – which might also help students to continue to persevere through challenges (reflecting SFMP 1).

In addition, the opportunity that the principal provided Carrie to “loop” with her students (to teach them in both kindergarten and first grade) allowed her to create a natural progression in students’ learning from one year to the next. The supportive culture for learning that she established in her classroom grew stronger over the two years, and Carrie believed that the shared experiences of her students helped them to make more connections together over a longer period of time. Moreover, “[A]nything that is more of a developmental issue, I feel like they just need some time, give them some time – we can pick it up next year… we always end up being so far ahead of the game that I have all this luxury of time now to be able to play around with some of these concepts in a lot of different ways.”

As the three teachers discussed plans for the following year in the second group discussion, Carrie asked Sandy and Ray if they were “doing intervention blocks next year”. When I asked what these were, she said, “I don’t know”, because the administrators had not provided much detail yet on the expectations for teachers and students during these blocks. However, she added, “It’s a common, per grade level time, that everybody’s doing intervention.” Sandy and Ray then said that their principal had also said that these would be required at Lincoln. All three teachers felt that there could be some value in this opportunity related to enacting the CCSS and SFMP since teachers
could collaborate and work with various students. Still, in the last interview, Carrie observed, “[I]ntervention is based on knowing what your kids know, and being able to supplement for those that don’t. And in kindergarten, and in first grade as well, um, the format of the day in centers, and different things like that, allow you to be able to, um, supplement the ones that need the intervention…”. She also argued that teachers truly needed to “know” their students in order to support all of them effectively.

From Carrie’s perspective, however, not all of the district-wide policies supported what she saw as the overall goals of the CCSS. Carrie pointed out that the district was requiring more data to be collected by teachers at all grade levels (to show evidence of student learning), and she shared, “A lot of teachers let that data stuff really stress them out, and – and I don’t… I don’t have a lot of worry about having a lot of kids that aren’t going to make the benchmark, or whatever. Um, but I just really feel for those folks… [pauses for two to three seconds] up there.” This comment referred to her colleagues at Monroe, including Ray and Sandy, who felt so pressured to address so much content during the year. Carrie also felt that this anxiety at her own school seemed to prevent teachers from wanting to have professional conversations about issues of teaching and learning, and she described this situation in this way:

[T]here’s just not time to have those discussions right now. Because all that other stuff is just weighing heavy on everyone. Which unfortunately, that sort of stuff is what makes people crabby, and stressed out, to not be able to be patient and understanding in the classroom… It kind of undoes all of the whole culture that we want to have in the classroom because
there’s so much of that stress with numbers, and electronics, that it’s kind of taken everybody out of their comfort zone. And, um, created anxiety-filled teachers.

That is, Carrie felt that it was “contradictory” of the district – a “mixed signal” – to emphasize the initiatives listed earlier that related to supporting and reflecting on students’ thinking and learning, while also stressing the importance of collecting and using numerical data – some of which might not be all that meaningful to teachers. She said in interview 2, “If we’re reporting data, it needs to be useful data. If it’s not useful data, then let’s – why are we doing this?” Further, as shown before, Carrie was continually frustrated with the emphasis on data related to speed and fact fluency in the intermediate grades. As she described this practice in contrast with developing students’ number sense, she said, “It’s two different things to me.”

Connected to the issue of recording and using meaningful data was Carrie’s frustration with her role (as a science lead teacher) on a district committee that was reviewing grading policies. She remarked, “[T]hat’s really very pertinent to high school, but not at all to us. So I’ve asked, well, can we focus on something different – why am I sitting in a meeting talking about grading procedures? When that doesn’t apply to me.” She felt that her time could be better spent helping her colleagues in first grade to develop assessments, as noted earlier, but she commented, “I just don’t know where… math fits into… the, um, district goals right now.”

Further, as Carrie heard the fifth grade teachers talk about their frustration with the pressure to teach so much content during the year, particularly since they were “not
allowed” to review (a policy that they essentially ignored), she became upset on behalf of Sandy and Ray. She said, referring to administrators, “[I]f you’re telling me that my kids need this lesson on this day, what makes you think that you know that?... I spend all day with them; I know exactly what they need.” Carrie continued: “I worry about the day that somebody’s gonna tell me, and I’ve told [our principal] this, when someone says you have to be doing this lesson on this day… ‘Cause I can’t… first of all, I think it’s zero trust… But if I had to own up on paper and say why I’m doing what I’m doing, somebody very well could say, ‘No, you’re not doing this.’ This is ridiculous.” She even pointed out, with disdain and amazement, that the three of them were naturally starting to whisper as they talked about these frustrations, even though the door was closed and they knew that no one would hear this conversation other than themselves and the researcher; they were truly worried that someone might possibly hear them discussing their response to these content-driven policies. Carrie commented later that “it just seems like at some point, you’d be like, ‘You’ve gotta get this, kid!’ [forcefully] You know, force it!” [sounding dismayed] “Completely undoes all the learning that the kid was even ready to do that day.”

So, in Carrie’s mind, the district had created certain policies and initiatives that seemed very much to support the goals in the CCSS and SFMP but also enforced others that seemed to act in opposition to these goals. She was not sure how to reconcile this in her own mind other than to continue to enact what she felt were the most effective practices in her own classroom and to work to comply with seemingly “contradictory”
district policies as best she could, while not abandoning her own philosophies about teaching and learning.

**Carrie’s perspectives on necessary supports for teachers enacting the CCSS and SFMP.** Carrie believed that many teachers probably needed a more complete understanding of the content standards and the SFMP than they had at the time of this study. She explained that it was helpful for her to know and teach both the kindergarten and first grade standards because she could have “the picture of the ending of first grade… to be able to kind of guide through kindergarten”. Again, she also appreciated being able to see how the content progressed through the grades as her own children grew older. However, Carrie shared, “I don’t know that all kindergarten teachers are realizing how meaty first grade is with the math as well.” In fact, “I think that truly people jump over this [pointing to the intro and SFMP] to get to the standards… ‘So what do I need to teach?’ Not the ‘How do I need to teach it?’ So, if these are going to be really used, then they would have to be forced… [to understand the SFMP also].” Carrie also remarked, “I also don’t think that anybody else in this school has thought about them… a fraction of what we’ve done [in this study]… which I think is probably a real disservice.” She then observed, “So I think that come time for kindergarten PLCs, I’m gonna be saying, ‘Oh, by the way – you know that that speaks to this standard in the new Core Standards’… just to have that grounding effect, of, ‘Well you know, when we’re doing this, we’re doing that… And this is why we’re doing that.’” Further, Carrie offered another interesting reflection about the authors of the CCSS: “[T]hey might be adding more things, more
teaching points on, but the grand ‘they’ [quotes in air] are appreciating how we’re doing it.” In fact, “I think that that is probably a good thing for teachers to realize.”

Carrie spoke a great deal about the need for the district to provide professional development – particularly time for “dialogue between teachers” – and administrative support to help the teachers within and across the grades become consistent in their enactment of the content standards and the SFMP. She felt that having “consistent terminology from grade to grade would make learning so much easier for the kids” and would “create that culture across the entire grade level”. In fact, she noted, “Talking with Ray and Sandy was incredibly insightful.” Beyond this type of dialogue, she commented:

It would take I think some real training, professional development, administrative guidance, modeling, PLCs, kind of thing, to get people to buy into the rest of it. To be able to start implementing more of an inquiry, developing understanding approach… I think that the school, the district, the grade level – whoever – is gonna have to really buckle down and force the issue of the instructional practice piece.

As aforementioned, Carrie had also noted that professional development on using the district’s current common assessments would be helpful to teachers (including herself) who had not initially received this training. Generally, Carrie felt that the teachers simply needed “time to work through the things that you need to work through so that you can be patient and kind and understanding with the kids when they need you to be. And not force feed them.” However, she added, “But there’s a prioritizing issue… because of all
content changing… that, um, it seems like language arts… at our level, kind of drives everything.”

In interview 2, as Carrie continued to think about professional support, she noted, “[T]here’s always so much room for across grade level collaboration… [but] it’s just, uh, such a need, and it’s the time factor.” She continued, “That PLC format is I think so valuable… [but] if you have fifteen minutes to have a PLC, you don’t get to that meaty piece of, ‘Oh, yeah – I saw you were teaching that the other day.’ In fifteen minutes, you just barely are getting through the formality of sitting down… PLCs that are an hour long… that’s when those conversations happen.” Further, “[Y]ou have 6 people that are doing something at a grade level… everybody’s doing it a little differently… if you just take a little bit of time to listen to each other, then that I think is far more beneficial than the money spent on… somebody.” She explained, “[W]e have plenty of people that really know good ways of teaching math. We don’t need to hire someone to come in and teach us how to teach math… [a] guru person to come in and tell you about something that’s really different, because then it’s a whole different shift. Instead of something that you’re already doing, just tweaking it a little.” Carrie then brainstormed an idea for the following year: “I think with these three [SFMP], um, it would be really easy to… have a PLC… building-wide, first day of school… just at a staff meeting, the little professional development piece that we have in each one of those… share how you do this in your room… it would be a fabulous discussion… at a table of mixed grade levels.”

Besides professional development and collaborative time, a second resource that Carrie felt was helpful to teachers was a textbook program that supported the CCSS and
SFMP as well as possible. Carrie commented that the primary teachers appreciated *Investigations*, but they were also hoping to see a new edition that fully aligned with the CCSS content. Still, Carrie did not feel that this was an urgent need at the moment. Also, each teacher had received a full set of manipulatives that aligned with the work in *Investigations* and in the content standards when the program was adopted, so Carrie felt that this was a source of support that was already in place. She was also grateful to have a SMART board; this was the first year that she had had this tool, so she was learning how to make use of it for mathematics lessons. Overall, Carrie did not believe that the teachers needed many more material resources, at least at that time.

Another resource that Carrie felt was important for the teachers in kindergarten and first grade was a method of ensuring that they were including all of the new standards sometime during the year. She did not mention the course maps specifically here, likely because she did not use them, but this would be a useful resource for most of the teachers.

Thus, Carrie’s main recommendations for supports for teachers as they transitioned to the CCSS and SFMP focused on professional development (particularly related to assessment) and sufficient time to collaborate with colleagues about instructional practice. She felt that the greatest benefits would be derived from the administrators’ setting expectations for practices aligned with the standards and then providing these resources to help teachers meet these expectations.
Reflections on the Evolution of Carrie’s Thinking about the Standards and Potential Changes in Her Practice

When asked in interview 1 how Carrie believed her practice would change having learned more about the CCSS and SFMP, she responded, “I don’t think it really will change.” Carrie held to this belief throughout the study because she felt that she had ascribed to the philosophy reflected in these goals throughout her teaching career. She noted: “[T]he respectfulness, the responsibility, and all of those traits of learners working together – is what I read in this… Because when you have that kind of community, then they’re able to work together and collaborate, the way these are suggesting.” As she thought about SFMP 7, in interview 2, she did comment, “I think I’ll define it in my mind more, when it’s happening.” Further, “[I]t’s nice to know what the value is of different things that are just byproducts, I suppose, of the… the lesson… I’ve always thought that there’s – in any one lesson – any content, there’s a hundred objectives that are being met… now I’m just able to verbalize more of what the objectives are that are being met during the lesson.”

Carrie did experience some new realizations throughout the study. One of these was the idea that her first graders might actually be more willing to persevere than fifth graders because of the relative lack of reliance on personal technology for children in first grade. This surprised her because the fifth graders were older and might be expected to have longer attention spans. Another realization for Carrie was that since there was such an emphasis on students’ looking for connections while reading and writing, students naturally transferred this habit to mathematics as they searched for relationships among
ideas; Carrie felt that this applied especially to the goals in SFMP 7. More generally, as Carrie read the text of the SFMP more carefully in interviews 2 and 3, she encountered some terminology with which she was not familiar, and she commented that she was not certain about the authors’ intent in these portions of the text.

In interview 3, while reflecting on the study as a whole, Carrie observed, “I don’t think that I’ve ever thought about things this much as we’ve done!” She continued, “[H]aving gone through [a local university], I always was having to reflect on everything… being done with [the university], having finished my Master’s, I probably would have gotten away from that. So, it forced me to think more about my practice, in terms of, um, formal… statements. And the ‘why I do things’ kind of rationale.” She added, “If you don’t have someone that forces you to talk about it, then, I’m not one - I’ve actually tried to do that reflection at the end of the day, and have a journal, and write things down – I just don’t do it.” Carrie also was very glad to have had the opportunity to talk with Ray and Sandy and felt that this conversation would be very helpful for all teachers if it could occur more frequently. She also appreciated watching and discussing the videos and sharing the “fun” experiences working with students in her classroom. In retrospect, Carrie commented that she had wondered why each interview included the same questions, but she shared, “I can appreciate the delving deeper, and um, validating, and… hearing the same answer more than once really makes it true. And not just something you say. Something you believe.” It seemed clear that Carrie’s beliefs and her deeper understandings of the CCSS and SFMP would continue to guide her as she began the year with new kindergarten students in the fall.
Case 2: Sandy Baxter

Professional Background

Sandy Baxter was a fifth-grade teacher at Lincoln Elementary in Sealon City Schools. At the time of this study, Sandy had taught for twenty-seven years at Lincoln. She had previously served as a math and reading tutor. She had volunteered to teach math again after having not taught it for several years; this study took place during her third or fourth year back. She was not sure exactly how many years had passed since she had started teaching math again.

Before the first interview, Sandy had read part of the CCSS when she and several colleagues (including Ray) attended a county workshop to learn more about the new standards. She and Ray had also talked a bit about the standards on their own at school. She noted early in interview 1 that the “amount of standards is decreasing”. She also commented that the “terminology is changing” and that she saw some distinction between “the content area [or the concept] versus the skill and practice.”

Pages 3-5 of the CCSS

Sandy’s interpretation of the overall idea of pages 3-5. Sandy felt that the authors were introducing the standards by arguing that schools needed to teach fewer topics in each grade and to return to a focus on number. During the first interview, Sandy read the following sentence aloud: “Because the mathematics concepts in the U.S. textbooks are often weak, the presentation becomes more mechanical than is ideal.” She then remarked, “That struck me as a profound sentence.” She added, “In a nutshell to me, it’s less is more… is what I’m getting out of it,” and she was very pleased with the
idea of fewer standards in each grade. During the remainder of the study, Sandy did not augment her response to these pages other than in the second interview, when she said, “The one thing I’m really concerned about is the number sense,” particularly referring to the first quote on page 3 from the National Research Council, which emphasizes a focus on number in mathematics learning. As will be shown, Sandy’s frustration with her students’ lack of number sense was a theme in her discussion.

Sandy’s interpretation of conceptual understanding of key ideas, and using organizing principles to structure ideas (page 4). Sandy appeared to view “key ideas” and “organizing principles” as the overarching concepts and skills in mathematics (especially in number) that would serve students throughout the grades. In the first interview, Sandy commented that “the key ideas concept… has huge potential.” She felt that in response to this phrase, teachers would recognize that certain ideas were very important and would work to emphasize them in instruction. Focusing on two example of organizing principles provided by the authors, she said, “We have to get back to… place value and operations.” During interview 2, Sandy added that although students seemed to be able to use the order of operations (which is how she seemed to interpret “properties of operations” in the actual text) because they could learn the acronym PEMDAS, “Find an area [i.e., a classroom] where place value is being used and see that the skills that I’m working on here – they’ve had in fourth grade; they’ve had in third grade.” Sandy wondered why students did not seem to have internalized these major concepts, and she felt that if teachers were more able to focus on specific big ideas under the CCSS, this might allow students to retain more from year to year.
Sandy’s interpretation of mathematical understanding (page 4). Sandy seemed to view mathematical understanding (as used in the text) as students’ ability to justify their thinking, particularly in more than one way. During interview 1, having read the description of mathematical understanding, Sandy said, “Absolutely I agree with that… if they’re successful at understanding mathematics, they may be able to understand the a + b + c times x and y.” In interview 2, Sandy discussed ways in which students might demonstrate their thinking, and she noted that they were justifying for others as well as for themselves. She said: “I think a lot of it comes down to ‘I know this because.’” Describing what students might do on paper in a solution to a problem, she said, “And just them going through and drawing the arrows – how this goes to this, and this goes to this [moving hand back and forth], and you know, manipulating it in that way, that they’re showing themselves exactly what it is they’re doing.” I am unsure if her example truly showcases understanding if the “arrows” refer to steps in algorithms. She noted that many people have not often been able to think about numbers or mathematics in different ways (instead viewing them as “black or white”), but she observed that students did often have multiple ways of finding solutions to problems. Sandy did not comment particularly on “mathematical understanding” during other phases of the study, but it was clear that in her videotaped lessons, she wanted students to be able to show why their thinking was logical.

Sandy’s interpretation of standards for a diverse population of learners (page 4). Responding to this paragraph in interview 1, Sandy said, “I believe that every child, regardless of what their background is, whether they’re special ed, or have special
needs, is entitled to the best education available to them… When I take on special ed children, they’re part of the class as much as anybody else is.” She added that she and the intervention specialist with whom she worked both felt “ownership” in educating these students. Sandy often used the word “ownership” in this first interview, and this seemed to be very important to her in several respects related to teaching and learning (as will be summarized later). Sandy believed that these students should remain in the regular education classroom as much as possible and that the intervention specialist should pull students out only when necessary to reinforce a particular idea or skill. However, Sandy also noted, “The standards are, I believe, written to average [students] and above.” In the second interview, Sandy maintained that all students should be expected to master the same standards, even if some did so earlier than others. “Just because you have any kind of disability, doesn’t mean I lower the goals for you.” “Everyone won’t meet them at the same time or go down the same path to meet them. But everyone should be expected to meet them.”

**Sandy’s interpretation of learning progressions (pages 4-5).** In discussing the phrase “learning progressions” from the introduction, Sandy commented, “When I think of learning progressions, I’ll go right here and use the example they have on the page… if they understand that, then they have the opportunity to expand their knowledge. So that’s how I’m interpreting learning progressions – as building blocks would be an easy analogy. You need a strong foundation, and from there, you can build and build… opportunities can be unlimited. But with a weak foundation, everything crumbles.” The example on the page seemed to help Sandy to formulate her interpretation of the authors’
use of the term “learning progression”. In interview 2, Sandy added little relative to this interpretation except to say that learning progressions should start “from birth”; she had expressed concerns about children starting school and not being as familiar with typical pre-school skills as students had in the past, so she seemed to feel that learning progressions should involve parents as they provide very children with learning experiences at home.

Review of how Sandy’s interpretations of pages 3-5 evolved during the study.

Sandy’s interpretations of the introduction to the CCSS did not evolve noticeably during the study. Her comments in the first interview were essentially repeated or simply expanded upon in later phases. It may be worth noting that her thoughts generally seemed to reflect her frustration with her students’ struggles to learn the mathematics that she was currently required to teach – much of which she felt was important for students to know. So, she agreed with the authors that change was needed, and she agreed with the goal of fewer standards being taught more effectively in each grade.

Sandy’s Interpretation of SFMP 1

Title: “Make sense of problems, and persevere in solving them” (p. 6). When discussing this sentence throughout the study, Sandy particularly focused on the idea of “perseverance” relative to her concern about students’ lack of “ownership” of their learning. In interview 1, she noted that there are “very few children who work at their highest level and give their highest quality, day in and day out.” She added, “They don’t take the initiative to learn something new or to challenge themselves unless there is something tangible tied to it.” More specifically on the idea of perseverance, Sandy
remarked, “They think they’re so proficient, they don’t need to check – they’ve got the right answer.” In interviews 1 and 2 and in the second group discussion, Sandy observed that when an adult worked with given students one-on-one, often the students did persevere and took “ownership” of their learning, not rushing through work. That is, students “do a completely different quality of work than when nobody’s there.”

In terms of making sense of problems, Sandy shared an experience from before interview 1: “We had a whole lesson on looking at the details, answering what the question’s asking,” and she noted repeatedly (and with frustration) that students “won’t take the initiative to do it themselves.” Her comments throughout the study were similar in nature – most students would only make sense of problems if teachers prompted or guided them to do so.

In Sandy’s first videotaped lesson, the class reviewed customary units of capacity as well as methods to calculate area and perimeter of rectangles and triangles. Most of Sandy’s efforts to help students make sense of ideas (not necessarily problems) involved asking them to identify examples in real life of the measurement units they were discussing. She did also ask some relatively basic questions to help students remember the difference between perimeter and area, the units related to each, and the correct way to apply each formula for a given shape. In this lesson, students did not have a great deal of opportunity to make sense of problems for themselves. Students did, however, seem to be fairly engaged and persevering in small group discussions about how they would teach third graders about units of capacity, and they seemed to be actively solving the
perimeter and area problems that were being created on the board by other students. Still, none of these tasks appeared to be especially unfamiliar or challenging to students.

In Sandy’s second lesson, students had a more substantial opportunity to make sense of a problem situation involving the sum of an increasing sequence. While students were working in pairs, Sandy walked around the room, asking students about their thinking and offering encouraging comments. Eventually, making sense of the problem became a focus of the discussion because students’ solutions differed due to the ways in which they interpreted the problem. Sandy asked students to explain how they interpreted the problem, and at the end of the lesson, she clarified the intent of the question and pointed out that different students’ thinking was logical based on their varying interpretations.

In the first group discussion, Sandy shared her frustration about students’ lack of willingness to face even minor challenges in math class, and Carrie noted that she did not see this in first grade. They both discussed the idea that perhaps this was because, as Sandy said, “They haven’t been exposed to failure… everything’s positive in that grade.”

Thus, Sandy seemed to view perseverance as a student’s willingness to keep working on a problem independently even though it might be challenging, and she seemed to view “mak[ing] sense of problems” as students’ determining what the problems were asking and whether solutions were reasonable based on the questions being asked. She seemed to feel that students’ making sense of problems (and then solving them) was dependent on their willingness to persevere.
“Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution” (p. 6). In interview 1, Sandy’s only comment about the text of this statement was that many of her students might believe they were mathematically proficient, but she would not agree with this. She did not comment on this sentence in the first group discussion. In her first lesson, though, she asked at least five questions to guide students to make sense of and calculate perimeter or area of given polygons, and she offered a few suggestions on doing so after students provided ideas. Again, though these were not especially rich tasks, Sandy was working to help students make meaning and look for entry points.

In the second interview, Sandy described more carefully what she thought this statement meant. Among her responses were the following: “Now to me, they need to be able to explain in detail what it is they were asked to do.” “It really comes down to breaking the question down by themselves – that’s truly finding the meaning of the problem – understanding it.” “At this point, I don’t see children, outside of a handful of them, capable of doing that.” “I work on this day in and day out. Underline what the question is asking. Take the question, and make it part of your answer.” “So many kids, when it comes to math, look for numbers, look for key words, and that’s it. Have – have no idea what to do with the numbers once they get them…”.

In the second videotaped lesson, Sandy really focused on the idea of understanding the meaning of the sequence problem and looking for entry points. Most of her dialogue with students, both as they worked in pairs and as the class discussed their work, centered on having students explain what the problem was asking and how
they decided what strategy to use to solve it. Several times, she said repeated the following idea: “There’s different strategies or techniques that you could use to come up with the same answer. And that’s a new trend in math that we’re trying to teach; we want you to realize that there’s more than one way to come up with the right answer.” It was interesting that she used the word “trend” in her statement, as if this practice might disappear at some point, but perhaps she used this word because she thought students would understand it. There may be a very subtle difference between looking for entry points to solving a problem (e.g., what information is key to choosing a strategy) and actually choosing a strategy for solving it, but these actions seem to be more related than distinct. As noted earlier, many students answered a different question than was asked, so this allowed Sandy to reiterate that making sense of the problem was key to arriving at a logical answer.

Although in the second lesson, Sandy was very intent on helping students to recognize the importance of understanding the meaning of a problem and looking for logical starting points, in the second group discussion as she talked about this lesson, she expressed her continued frustration with students’ actions. “They did not take it the step further, and continue reading the question. They figured out the problem; they knew a T-chart would work really good, but then, they didn’t reread, and rethink, what are they really asking me.” So, although it became very clear by the end of the study that Sandy wanted her students to meet this goal in SFMP 1, she also felt that many lacked the skills and/or perseverance to do so.
“They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary” (p. 6). In general, Sandy seemed to focus on these parts of this text: “rather than simply jumping into a solution attempt” and “evaluate their progress and change course if necessary.” She did not say much about the rest of the text at any time during the study, almost seeming to overlook it. Sandy did not have a great deal of confidence in her students’ ability to meet the goals in this statement, at least currently. In interview 1, she said, “I see most of my students as kids that don’t take time to find the meaning of the solution or plan a pathway…” “They don’t need to dissect it piece by piece, reread, carefully analyze…” “[T]hey jump right in to find the solution.” In the second interview, she remarked, “I mean that is so much packed into one statement – wow.” This perhaps confirms that she had not really even attended to some of the text earlier in the study. She added: “I would just love to have one student in my class that did all that!”

Although there was no notable evidence of Sandy’s attempting to directly enact these goals in the first lesson, in the second lesson, she guided students to focus on the information that was given (though no one used the word “given”). This was partially due to the fact that the worksheet that provided the task included questions to help students identify important information about the problem and to make an estimate for
the solution, both of which seemed to align with the goal of making conjectures about the solution pathway. So, when students explained their thinking at the board, many pointed out elements of the problem that they felt were givens. There was some evidence of some students changing their strategies when they realized that their answers did not seem reasonable, but relative to Sandy’s concern, most students simply solved the problem with whatever strategy they initially chose.

Thus, although it is difficult to tell how, or if, Sandy interpreted most of this text in SFMP 1, she seemed to interpret “simply jumping into a solution attempt” as beginning to work on a problem without making sense of the question and considering possible logical strategies. She seemed to relate the phrase “evaluate their progress and change course if necessary” to reflecting on a solution strategy, either while using it or after using it, and changing the strategy if the solution seemed unreasonable.

“**Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem**” (p. 6). Sandy seemed to interpret this sentence fairly literally throughout the study, focusing on the idea of students using manipulatives or drawing pictures to solve problems. Her response to the sentence changed during the course of the study, particularly based on an experience in her first videotaped lesson. During the first interview, she had a very definitive opinion about the idea of students using manipulatives. She said, “Are they great for kindergarten and pre-school? Absolutely!” She added, “…younger students, even prior to coming to school, should be given the opportunity to have concrete objects and manipulate them, and – so they can
truly understand.” She did not, however, feel that manipulatives and pictures were appropriate for fifth graders.

After the first lesson, though, Sandy said in interview 2 (referring to her reflection that she wrote on the lesson), “Surprisingly enough to me, and I wrote this in my paper, I realized, years ago when I taught math, I had a much larger – or I had a large supply of things that I thought would help the kids. And, with going away from it and coming back to it, I realized, like that in that particular lesson with capacity, all the manipulatives I should have, and don’t.” Further, “I need more of that. I need to realize – it almost is against every grain of my body that they do still need counters…. that to me is absolutely repulsive. But, that’s where we’re at, that’s where we’re at. I have to stop… with the denial.” Clearly, Sandy had very strong emotions about her students using manipulatives, though she did not refer to drawing pictures, particularly.

Later in interview 2, in a discussion about a hypothetical classroom episode involving classifying quadrilaterals, when commenting on what she would recommend to the teacher in the episode, Sandy said, “Actually, as sad as this is… [pauses and shakes head] we gotta get out those manipulatives and we gotta play with them. We gotta touch it; we gotta feel it; we gotta move them into piles that… parallelogram, polygon, irregular polygon, things of this nature… um, I can’t believe I’m saying it! Because I can’t believe we have to do this in fifth grade! This is where I get frustrated.” She added, “I would think that by the time you’ve hit fifth grade, you’ve had so many years of playing with the shapes, and sorting them, and classifying them, and fondling them that you don’t need to do it in fifth grade. It should all be visually cemented in your head by this point.
That’s where I struggle as a math teacher… But, to classify geometric shapes, there’s no other way you can do it.”

After Sandy shared these thoughts, I asked her how she would feel about her students using interactive geometry software to explore and classify shapes. Her response was, “I still would think it was primary. Kindergarten primary. And I don’t know if that’s because… with my own personal nieces and nephews, we taught multiple shapes. We discussed them before we went to pre-school. We talked about a square being a cube. So, when you’re 3, you know a square is a cube. A cube is a square.” So, Sandy was drawing on her own personal experience with her family to argue that if students had enough early experiences with manipulatives, perhaps they would not need them later in school. (It is interesting to note, however, that Sandy’s statements about a “square” and “cube” being essentially the same may indicate that her own ways of conceptualizing these figures might not have been as precise as the CCSS would require. If this were true, she might not have recognized what concrete experiences older students might need to develop these conceptualizations more fully.)

Throughout the rest of the study, Sandy did not comment on this issue again, nor were manipulatives or pictures used in her second lesson. But, it was clear that the experience in her first lesson caused her reluctantly to reconsider her position on fifth graders using manipulatives. Again, she never shared thoughts about students drawing pictures to help solve problems, but I also never saw her or her students using pictures in the two lessons other than when they were drawing shapes and calculating perimeter and area. So, it seemed that in general, Sandy leaned toward relying on numbers and symbols
to solve problems, but perhaps her experience in the first lesson and in studying the
SFMP would cause her to rethink this avoidance of concrete and visual representations.

“Mathematically proficient students check their answers to problems using a
different method, and they continually ask themselves, ‘Does this make sense?’” (p. 6). Sandy strongly resonated with this goal throughout the study, also interpreting it fairly literally. She wanted her students to double-check their answers to problems, and she wanted them to determine whether the answers were reasonable. She also felt that most of her students would not do either of these without prompting, but she continued to emphasize them in class.

In interview 1, after reading the sentence, Sandy immediately said, “That’s exactly what I try to teach them every day.” Also, “[W]ith every process, we use the reverse as a check.” Given her additional comments, I recognized that she used the word “reverse” to mean “inverse operation”. This seemed to be a primary method by which she wanted her students to recheck their answers. She continued, “I explain to them I’m an adult, and I do it this way – you should be doing it this way too.” All of these comments seemed to imply that she still was generally thinking of solutions as step-by-step procedures that could easily be reversed (or verified by re-performing the same sequence of steps), and that she was not necessarily considering more complex, rich problems involving multiple ways of thinking. This held true in the first lesson when she twice asked students to “double-check” their answers, referring specifically to verifying their calculations.
In the second interview, Sandy expanded upon (though did not especially change) her interpretation of the text. She said that she had a few students who would use “reverse operation” to check their answers and that as a group, her students did have multiple ways of solving given problems. However, she referred to a recent class experience where students were working on a multiple-choice task:

They take two numbers [claps her hands together], subtract them, done. And of course that’s one of your options… So, I watched wipeout on that question, and it’s all because there’s no number sense. I said, ‘You know you’re supposed to plug the number in as the variable.’ And they shake their head [nods], but they don’t remember to do that. And even when they do, they’ll read it to you, and it doesn’t occur to them that it makes no sense.

She went on to comment, actually describing an element of her own history with mathematics, “Math was typically thought of as, it’s black and white – this is the answer; this is how you get it. Only recently have we realized that you can come to that answer through multiple ways.” She noted at other times during this interview that she hoped to change this view of mathematics that she had long held. I believe that this was a result of her ongoing work with Ray (the third teacher in this study), whom she highly respected for his beliefs and practices, and as a result of wanting to enact the goals in the CCSS.

In the second lesson, Sandy was already showing evidence of working to change her perception that problems could only be solved in one way. Guided, I believe, by the questions on the activity sheet she had chosen as well as by her desire to cause a shift in
her own thinking, she frequently asked students how they knew that their answers made sense, and she asked them to consider other strategies that might have been useful in solving the problem. She reflected on this during the second group discussion, as she pointed out that students approached the problem in different ways, and she saw that it was really their interpretation of the problem situation – rather than their chosen solution strategies – that changed their answers. So, although her interpretation of the SFMP text did not really change during the study, I believe that her confidence in her ability to support students in meeting the goal in the text increased because she not only felt a need to change her practice, but she also saw students using multiple strategies when she focused on this part of the goal in class.

“They can understand the approaches of others to solving complex problems and identify correspondences between different approaches” (p. 6). In the first interview, Sandy’s first response to this statement was, “I personally don’t feel I have any kids at that level right now.” When I asked her to elaborate, she said, “[I]t may be my class, someone else’s class might be better than mine – I know I have a low group – I don’t see them expanding on taking the opportunity to realize there’s more than one way.” She gave the example of students wanting to add 24 seventeen times instead of multiplying 24 by 17, and she remarked: “It may be because they don’t have a strong relationship with numbers. Their number sense is lacking across the board at fifth grade.”

Sandy’s comments in the second interview were very similar; she said:
Children at 10, 11, have a hard time understanding somebody else can be right also. Not so much that they’re wrong – OK? But that there’s – there could be two different ways – they haven’t been raised on that. They don’t understand that we could come from it at two different angles and end up at the same point, and we’re still reaching the goal that we wanted.

In the second lesson, Sandy made it clear to students that she wanted them to recognize that multiple correct approaches to the problem were possible. At the start of the lesson, she said, “You’re gonna work with your partner, and you’re gonna work through the different strategies to find a solution.” Toward the end of class, after students had shared their solutions, she reiterated, “What’s really important is that my question, my ultimate question is: we see how different people solved the problem, right?” However, it did not appear that any given pair of students tried to approach the problem in more than one way, so the students were not really moving toward this goal yet, at least in this task.

Throughout the study, Sandy focused almost entirely on the first part of this statement, related to students’ understanding the approaches of others. She did want students to be able to do reach this goal, but she seemed doubtful that many students in fifth grade would be able to do so. She did not offer any thoughts about students’ looking for relationships between various approaches. It might be that she interpreted this part of the text as simply understanding that other approaches were possible and logical, but this could also mean that in class, her enactment of this goal might not (at least in the near
future) guide students to find relationships among certain elements of differing problem solving strategies.

**Sandy’s thoughts about implementation of SFMP 1.** Some of Sandy’s interpretations of SFMP 1 were related to her thoughts about how this standard would be implemented in her classroom, school, and/or district.

**Sandy’s goals as a teacher, as related to SFMP 1.** Above all, Sandy wanted her students to be successful in school and ultimately in life. Her beliefs and practices were highly related to this value; she wanted to provide them with “the opportunity to be that successful”. She believed that students could and did persevere in learning mathematics, a major goal of SFMP 1, when they could see that they had the support of adults who cared about their learning. This was evident in the second videotaped lesson when she told students, “What’s important is finding a strategy that works for you to solve the problem,” which related to the text in SFMP 1 about looking for entry points and understanding various strategies. Also, at the end of this lesson, she asked, “Did you see how you made a mistake, if you made a mistake? Did you learn from your mistake?” These questions echoed the goal in SFMP 1 of evaluating one’s own progress and changing course when needed.

Sandy also wanted her students to be able to use efficient methods in solving problems in mathematics. It frustrated her when they chose to use very inefficient strategies, and she continually encouraged them to try other methods, as SFMP 1 suggests, though her ultimate goal was still for them to get correct answers and feel confident about these answers. In the second lesson, she said to her class, “So for
example, you could do multiplication to get an answer, or you could do repeated addition. Now, I’ve told you over and over and over on many occasions that repeated addition is not necessarily the best technique for a fifth grader, but it still comes up with the right answer, right? So as long as it’s coming up with the right answer, that’s what’s most important.” These statements showed her valuing efficiency but valuing students’ success – and perseverance – even more.

**Sandy’s perspectives on mathematics and standards, as related to SFMP 1.** As mentioned earlier, Sandy felt that mathematics had been seen as “black and white” in the past, and she felt that people in general, herself included, were starting to recognize (with the advent of new standards) that mathematics problems could be solved logically in different ways, again, a focus of SFMP 1. Sandy also acknowledged in the second group discussion, as she reflected on her second lesson, that the students were basically using the strategy that she had taught them to use (an x/y table), in the way that she had taught them to use it. Many were not thinking about how to adapt this strategy a bit to solve the problem presented that day. This related to her concern that although she felt a few students whom she currently was teaching would be able to meet the goals in SFMP 1, she felt that perhaps some of the goals were “not age-appropriate”. On the other hand, in the second group discussion, Sandy also said that she wanted to work on the new standards as a teacher and with her students, remarking that over time, perhaps these goals would become easier for students to meet as they gained experience with them.

**Sandy’s perspectives on everyday issues of classroom practice, as related to SFMP 1.** One of Sandy’s greatest frustrations was that she felt her students entered fifth
grade without the “number sense” that they needed to make sense of problems and persevere in solving them. In the second interview, she remarked, “I think they – it all makes sense to them! Because none of it makes sense to them!” In other words, students could not recognize when their thinking was not logical because they had little deep understanding on which to base their reflection. This, of course, was causing daily challenges for students and for Sandy.

Sandy also continued to express concern that her students lacked perseverance because they showed little motivation to learn mathematics. She said in interview 1, “[T]hey don’t take the initiative to learn something new or to challenge themselves unless there is something tangible tied to it.” In the first group discussion, Sandy shared that she was bothered that her students seemed almost to refuse to learn different strategies to solve problems as long as they had one strategy that worked for them, and she asked Carrie and Ray when they thought we started “losing” children, given that Carrie had said that students were excited about learning mathematics in first grade. As a group, they discussed the possibility that some students start feeling less motivated when the concepts become more abstract more quickly and when, as a frequent result, these students start experiencing less success in mathematics than they felt in the primary grades.

Throughout the study, Sandy shared some thoughts about particular instructional practices in relation to SFMP 1. As aforementioned, Sandy did have an established habit of showing students how to solve problems in certain ways, and she still felt that this was valuable because it could provide students with entry points, strategies, or even reminders about the need to truly understand what questions were asking and to double-check
answers. She also was working through her strong objection to fifth graders using manipulatives, probably due to her realization that she had herself used some with students in the past as well as her acknowledgment that if students were not ready to use only symbols to solve problems, they really needed manipulatives. Again, she seemed to pass over the typical intermediate representation – pictures.

In the second interview, Sandy discussed what she thought her math class might look like under the CCSS. She described starting each class with one problem: “And it’s going to be a complicated problem, and we’re going to work on it from every different angle and every different aspect, and we’re gonna talk about it, and we’re gonna share it, and we’re gonna talk about it some more, and we’re gonna share it, and we’re gonna talk about it some more, and we’re gonna share it – versus, 20 problems – do the odds.” She also noted that more of the work related to meaning-making would have to be “written” in word and sentence form, which she said would also connect with the new language arts standards – teachers at Lincoln were being asked to consider opportunities for integration of content. This class format reflected what she saw as key in SFMP 1 – that students were to carefully consider all aspects of a problem situation, consider ways to approach the problem, explore and compare multiple solution methods, and determine whether solutions made sense. Within this context, Sandy added that she might have students “trade partners every day, so your way of thinking and your conceptual process is shared with many.” One point of frustration for her, though, was that she felt she already spent a great deal of time trying to help students understand the givens in a problem and evaluate whether their strategies seemed to be reasonable, yet students still did not often take these
actions independently. On the other hand, it was interesting that in the first interview, she made a comment about one of the hypothetical classroom episodes where two students were said to have solved a task about the volume of a box by naming the unit (not specified in the task itself) as a 2-cubic cm block instead of 1 cubic cm. She said that she would still want students to solve the problem “the way it [was] assigned”, but that she “might even give [the students] extra credit for investigating other ways of doing things like that.” So, although Sandy was intent on enacting some portions of SFMP 1 with her students, there seemed to be other portions that were, at least initially, less easy for her to accept when they involved interpreting problems in multiple ways.

**Review of how Sandy’s interpretations of SFMP 1 seemed to evolve throughout the study.** Generally, Sandy’s interpretations of SFMP 1 specifically did not change a great deal throughout the study. However, she did say in both interviews that she wondered if part of her thinking about the goals in SFMP 1 related to her particular group of students that year, because she said they were a “low group”, presumably meaning low-achieving in mathematics. This raises the question of whether teachers’ interpretations of the standards might change depending on the progress of which they believe their own students are capable. The points on which Sandy did seem to change her thinking during the study were whether fifth graders should be able to use manipulatives in mathematics learning and whether they would ever be able to persevere in more challenging problem situations. The former shift seemed to occur through her own recognition that manipulatives would have been helpful in the first lesson, when students were studying units of capacity, and the latter shift seemed to occur when she
heard Carrie describing the perseverance of first grade students. So, her own classroom experience with her students, as well as talking to a familiar colleague about younger students, helped to move her thinking about the standards in somewhat new directions.

**Sandy’s Interpretation of SFMP 3**

*Title: “Construct viable arguments and critique the reasoning of others” (p. 6).* Throughout the study, Sandy seemed to interpret this title statement as meaning that students should be able to give and defend reasoned explanations for their work and also to offer evaluative comments on others’ explanations. She did not, for most of the study, feel that fifth graders were currently mature or experienced enough to handle the second part of this goal. Further, she did not seem to consider the idea that the two parts of this goal could be related – that is, that students might construct, re-construct, or evaluate their own arguments based on seeing or hearing others’ thinking.

In the first interview, Sandy said, “I see a handful of children who can make connections, who can make the arguments, who can defend their mathematical knowledge,” and “they just don’t quite have the schema to chunk that with what critiquing is, and how to evaluate fairly.” However, “Now with the new standards, a couple years from now, kids may come with that skill very well developed.”

In both of Sandy’s lessons, she fostered both parts of this goal among her students. In the first lesson, she asked students to give “thumbs up” several times to show whether they agreed with what other students were saying. She herself also made a few evaluative comments as the students were sharing calculations for perimeter and area on the board, but actually some of the students did so also, and they were generally
respectful to each other. In the second lesson, both she and the students offered comments as other pairs of students were sharing their thinking at the board, and again, generally the discussion was quite civil. This might demonstrate that Sandy actually encouraged the goals in the text more effectively than she might realize, which reflects her desire to always be supportive of students and to help them to be successful, both individually and as they work together. In the second group discussion, Sandy reflected on this second lesson, saying that she had, in essence, said to some of the students, “This is a really, really great start! But it’s not right…”. She then remarked, “I didn’t say anybody was completely wrong.” She appeared to be recognizing that she did not always have to correct students immediately when their thinking was illogical, and perhaps she saw that she could model this “acceptance” of different kinds of thinking as an example for her students to follow.

Given Sandy’s actions – and her students’ actions – in the second lesson and her comments about this lesson in the second group discussion, I believe that she was perhaps starting to feel that her students could be capable of both part of the text in the title statement of SFMP 3, particularly the goal of critiquing other students’ work, about which she was initially very doubtful.

“Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures” (p. 6). At first, Sandy seemed to focus on the first portion of this statement, particularly on students’ understanding and use of “definitions” and
“previously established results” in solving new problems. She had a rather specific idea about what this might mean for her students, saying, “I think a lot of what I’m doing this year, with the taking notes, and writing out definitions and keeping a math notebook, it’s already focusing in on this… keeping a math notebook makes the children understand the importance of definitions”. She explained that this was the first year that she and her students had together maintained math notebooks to keep records of their work and also to record ideas that they had learned together. In the first lesson, this idea of using references for definitions and established methods was demonstrated in that Sandy asked students to review what was in their notebooks and in their OAA preparation workbooks about perimeter and area, and then to solve problems based on what they read: “What do you see in your notes about area?” “Would somebody please read me what the Buckle Down book says about perimeter?” So, at this point, Sandy’s interpretation of the text in the SFMP was fairly limited.

In the second interview, I asked Sandy to tell me what it would look like for fifth graders to meet the goals in this statement. She said, “They would be able to give… [pauses] definitions… of a subject… in their own words… um, show you reasons why this works… by giving examples, and, um, tell you how it could be connected to the next function.” Her phrases “show you reasons why this works” and “connected to the next function” were the only instances when it appeared that she was considering the second goal in the statement, involving students’ creating a logical progression of statements to explore or defend an idea. She did feel that a few students could meet the goals in the
text in grade 5, giving the following example of students’ making logical connections based on ideas they already knew:

\[
\text{If we solve for area, length times width equals area, but when you go into volume, length times width times depth – how they make the connection that inside of the square [making a square shape with her hands] turns into inside of this cube, and move forward from that. There are a few kids that could do that. Not many. They could explain to you the difference of area being squared, cubed being three-dimensional… a few could do that. Not as many as I would like. And I’m not sure why.}
\]

Sandy was both hesitant and thoughtful in her response, trying to refer carefully to the text, though I believe that she found it challenging to interpret the entire statement because of the terms with which perhaps she was not entirely familiar; she never referred to the terms “assumptions” or “conjectures”. She added, “And they have done a remarkable job using their notebooks, from where we’ve taken notes all year long so they have a tool.” Thus, at this point, she still seemed to be focusing on the idea of students’ using information that had been given to them (or established earlier) to solve new problems.

In the second lesson, a few students actually made statements that reflected the text related to using “stated assumptions” as they presented their work on the task to the class. One said, in answering one of the guiding questions on the activity sheet, “What do we know? One doorbell is plus 2 guests that come.” Another critiqued another pair’s work by saying, “[B]ecause it said after the first doorbell rings, so they should have
added 11 to make it correct.” It is not clear whether Sandy recognized these comments as referring to stated assumptions because she did not offer any comments about them during the lesson or later.

Generally, this portion of SFMP 3 was not one on which Sandy focused during the study, again, perhaps because the statement itself is rather dense, with vocabulary not altogether familiar to her.

“They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples” (p. 6). Sandy had very little to say about this statement during any phase of the study. In interview 1, she said, “Not many [students] break it into parts… They can’t do that because they don’t take the time.” In interview 2, she said, “I just don’t see it happening in fifth grade. Not yet,” adding, “Sorry,” as she smiled somewhat sadly. She made no references to the statement in either group discussion or in either lesson. Given that Sandy really did not attempt to interpret any specific portion of the text, it may be that she simply did not understand what the statement was saying – suggesting why she felt that her students were not capable of meeting this goal.

“They justify their conclusions, communicate them to others, and respond to the arguments of others” (p. 6-7). Since this statement is similar to the title statement of SFMP 3, Sandy offered similar interpretations and thoughts for this text. She felt that this sentence suggests that students should be able to give reasons for their thinking, talk about and show (on paper) these reasons, and respond to other students’ comments about
their work. Her perspective on whether her students were capable of meeting this goal changed somewhat during the study, as will be shown below.

In interview 1, Sandy said, “[T]here’s a handful of children who can do that.” Overall, she felt that her students were not able to meet this goal yet, as noted earlier. However, in the first group discussion, she did indicate that she was trying to work toward this goal with students and that it was a focus for her in her practice that year: “I have, up on the board, and there’s one on my desk, too – “I know this because… I spit that out to them, and I want them to spit it back at me, and it becomes part of our regular routine – that is one of the things I’m personally working on this year.” As shown above, there was some evidence of her desire for students to meet the goal in the text as they shared their calculations of perimeter and area and responded to each other in the first lesson, though no one used the phrase “I know this because” during this lesson.

In the second interview, Sandy further elaborated on her concern that her students, at least currently, were not typically able to do this effectively. She shared, “They will state their conclusion, communicate by telling you that they are right, and respond that everyone else is wrong.”

Despite these doubts, in the second lesson, Sandy continued to facilitate communication and discussion of students’ thinking, both in pairs and in the whole class setting. As aforementioned, students were confident in sharing their thinking and respectful in commenting about classmates’ work, and they generally were not defensive in responding to others’ comments. Sandy reflected on this during the second group
discussion: “They’re all explaining it, defending their answer, so on and so forth… So we were able to discuss that.”

Toward the end of the second group discussion, Sandy shared a personal goal she was establishing for herself: “I’m gonna work on implementing these new standards… and focus on retraining myself and training the kids to properly explore, explain, defend, so on and so forth.” At this point, Sandy seemed to believe that fifth graders could develop the skills included in this part of SFMP 3, and she seemed to feel that she would need to support them in this development by developing these skills herself.

“Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is” (p. 7). In interview 1, Sandy suggested that this statement meant that students should understand that “there’s multiple ways to get to one answer.” She did not address the idea of finding flaws in arguments in this part of the interview. However, in the first lesson, both Sandy and some of her students identified small flaws in thinking that students shared on the board, and the students who were presenting were able to correct their work. In this lesson, though, there were no opportunities for students to compare two different “arguments” or strategies for the same task.

In interview 2, Sandy’s first response after reviewing this text was to shake her head and say, “Nothing.” She added, “Not in my classroom, not at this grade level. Not yet.” She elaborated by saying, “[T]hey have not been taught to justify their answers. To respond to the arguments of others. That, to them, is a verbal confrontation.” She did
acknowledge, “[S]ome of that takes place when you put them into smaller groups, in groups of 4. But it doesn’t come across kindly, respectfully. It’s: ‘You’re wrong, I’m right, you dummy’”.

Once again, although Sandy seemed unconvinced that her students could compare arguments and discuss mistakes in them without feeling threatened, in the second lesson, she provided multiple opportunities for them to do so, and students responded well. Only at one point did Sandy interject among students’ whole-class conversation to prevent frustration for students: “OK, let’s not get – let’s not be analytical of anybody. We’ll figure it out in a minute. Did anybody do it a completely different way?” Three pairs of students offered strategies; these strategies led to three different answers. In the second group discussion, Sandy’s comment about this element of the lesson: “[W]e were able to compare… OK, let’s look at this, compared to this.” As noted earlier, she seemed to realize that perhaps her students were able to discuss problem-solving strategies with each other without becoming defensive.

So, by the end of the study, Sandy was focusing more on the goal in the text of students’ discussing various strategies and determining whether they were logical, rather than students’ simply being aware that multiple strategies were possible. Her interpretation seemed to be that students should be able, when presented with multiple strategies to solve a given problem, to determine which strategy or strategies were logical and why (or why not).

“Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and
be correct, even though they are not generalized or made formal until later grades” (p. 7). In interpreting this text, Sandy focused entirely on the first sentence, not offering any comments about the second during the study. Sandy’s ideas about the first sentence relate to her thoughts about the portion of SFMP 1 that deals with students using manipulatives and pictures. In the first interview, she said (of this part of SFMP 3), “I’ll give you more of this than I would on the one [statement] before, because they do, they do use objects, drawings, diagrams, because they need them as a crutch… at fifth grade, they almost need to have objects, drawings, or diagrams to show their argument.” Given her response, it seems that she was not very satisfied with the idea that students “needed” objects or drawings; rather, she wished that they did not. She offered similar views in the second interview: “…they are able to use [reading from the text] concrete objects or drawings, diagrams… but this construct an argument – we don’t have that yet.” Here, she was delving a bit more specifically into the text; she seemed to be thinking that her students could use visual or concrete representations to solve problems, but she did not feel that they were able to create a way to justify their thinking using these representations – which seemed a bit contradictory. She did not discuss any of this text further in either of the group discussions, nor did she offer manipulatives or a suggestion to use diagrams in either of the lessons. In general, Sandy appeared to feel that this type of work was too primary for her fifth-grade students.

**Sandy’s thoughts about implementation of SFMP 3.** As was the case for SFMP 1, some of Sandy’s interpretations of SFMP 3 were related to her thoughts about how this standard would be implemented in her classroom, school, and/or district. I
asked her how she thought this standard could be effectively implemented in her classroom, and she was very focused on her concern about what it would be like for students to critique each other kindly in class, as well as students communicating and defending their thinking. She noted during interview 2 (in connection with SFMP 3), “And everybody talks about how we need to make it more life-skilled, and these are definitely all life skills. We have not been focusing in this area.” In fact, she felt that students acted as if “somebody’s always got to be better. That’s a societal issue.” Sandy even remarked, “I don’t think children in the United States can [discuss their thinking with each other]. I don’t think they’ve been taught to do that properly. Because they come to school from the very beginning – preschool – ‘I’m right and you’re wrong.’” Then, she observed, “[E]ven when you go down south, the respect that you get from children compared to the respect that you have here… there’s a difference. So even, would we be able to teach that standard better in the south than we can here?” Sandy also commented, “Somewhere we have to change that. Maybe, with the right kids, you could work this out in a fifth grade school year.”

Particularly in relation to SFMP 3, Sandy seemed to feel that the teacher should still be the final authority in the classroom about what lines of reasoning were acceptable, though her perspective on this seemed to shift a bit from the beginning of the study to the end. In the first interview, she said, “And it is appalling to me that, one, they would challenge an adult, two, that they’re so comfortable in being sure that they’re right and I’m wrong.” Perhaps this was because she felt she might lose control in the classroom if she admitted to being wrong or to not understanding a student’s idea. However, in the
second group discussion as she discussed her second lesson, she reflected with an air of sheepishness, “I went up and I said, ‘This is really good. But look. You started at doorbell ding number 1; you started at doorbell ding number 2, and you did it completely different.’ And then I said, point blank, ‘Guess what. Wrong, wrong [making large x motions with hands], right [making large circle with hand].’” She felt that perhaps she should not have simply evaluated each strategy as incorrect or correct so quickly, but she also noted that the class was almost over, and she wanted students to have some sense of what was logical about each line of thinking and what was not before they moved on. This idea of closure is certainly a practical issue that teachers must consider with each lesson.

Although Sandy expressed a great deal of doubt that her students were currently able to master the goals in SFMP 3, she did feel that over time, more success might be possible. She observed, “[I]f we learn this early, how to critique, how to self-evaluate, if it’s a skill that we actually instill in children, it could be a huge advantage… Now with the new standards, a couple years from now, kids may come with that skill very well developed.” In both the second interview and in the second group discussion, she noted that having students work together on problems would help support this goal because they would gain experience in communicating with each other, and she planned to do this more in the future. Sandy also intended to continue (during the next year) to have her students create math notebooks in class to use as their own references. She felt that this helped them understand the importance of definitions and using ideas that they already knew to solve new problems, and she had drawn these ideas from one portion of SFMP 3.
Sandy also wanted to continue emphasizing the idea (and wanted students to believe) that multiple strategies were possible in solving most problems, although some strategies might be more efficient than others. Also, she tended to return to the idea that the goal of problem solving was to find the one correct answer to the question; in the second group discussion, she said, “There’s a variety of ways of getting that answer, but there’s one answer.” Her interpretation of SFMP 3 seemed to be based on this idea.

During interview 2, Sandy said she was not sure how she could help her students to develop the skills in this standard, and she remarked, “I can’t see starting fifth grade with this concept at all.” Discussing how SFMP 3 appeared in her first lesson, she said, “Standard 3 pretty much played out with the strongest kid in the group of four spoke his opinion, and everybody agreed. And the strongest person in the group is usually the person that takes cards, takes the paper and pencil, will be the recorder, and be the responder, and everybody else sits there and does nothing.” But, she continued to think aloud, and she shared the following ideas about students’ critiquing the thinking of others on a given task: “[M]y first thought would be to have everybody write it out – throw it in the math box. And I read half a dozen of them… randomly. And we start with it that way, looking at six different perspectives.” In this way, “they say it to me instead of to each other... And there’s not the uncomfortableness of, “My answer was wrong; I’m not answering.” Then, “as we get more comfortable doing it from a behind-the-scenes perspective, bring it into an open forum in the classroom.” Regardless of the way that she would work to enact this standard, she maintained, “I have to come up with ways that
I can do it and not hurt people’s feelings,” which demonstrated her primary concern for supporting students’ success through encouragement and building their confidence.

**Review of how Sandy’s interpretations of SFMP 3 seemed to evolve throughout the study.** Sandy’s interpretations of SFMP 3 actually did not seem to change a great deal throughout the study. In the first interview, she suggested that perhaps her interpretations, or at least her perspectives on them, could be related to the grade level she taught. “And it may just be that I am at a transitional year, at fifth grade… other grade levels above have much more opportunity to see them communicate it and respond with, ‘I know this is right. I did the math! I know!’” In the second interview, she commented (pointing to SFMP 3) that seeing the SFMP statements written separately rather than in paragraphs was changing the way she was thinking about them: “[N]ow that you’re having me sit here and look at this line by line, because I’ve looked at it like this [points to original copy of SFMP] - but when it’s broken down sentence by sentence, I’m seeing it a little differently now.” In fact, she indicated that seeing the statements written separately, which perhaps focused her attention more on unfamiliar terms in the text, confirmed even more for her that the goals might not be achievable by fifth graders, at least not yet. Thus, her expressed interpretations still did not seem to change.

The one point (not really an interpretation) on which she did seem to shift a bit was the idea that her students could be capable, even in the short-term, of critiquing the thinking of classmates respectfully, and this shift appeared to occur when she saw that they were able to do this in the second videotaped lesson. As noted, she set a goal to
support her students in the development of the skills in SFMP 3 in future years, and she felt that this would become easier as students gained more experience with these skills in earlier grades.

**Sandy’s Interpretation of SFMP 7**

**Title: “Look for and make use of structure” (p. 8).** In general, Sandy seemed to interpret this title statement of SFMP 7 as meaning that students should be able to find patterns in problems that they were solving or in ideas that they were studying. Although she really never offered a specific interpretation of the word “structure”, her comments during interviews and her actions in the two videotaped lessons indicated that this is likely the way she viewed this goal.

In the first interview, Sandy once again felt that most of her students were not able to identify patterns independently. She said, “[T]here will be a handful of kids that will see a pattern… Most of them will sit back and say – wait for somebody else to have the aha moment… and say, ‘Oh, OK, I’ll do that too.’”

In the first lesson, Sandy and her students were discussing the relative sizes of a gallon and a liter, and she was asking them to consider objects in front of them as well as imagined objects to make this comparison. In this sense, it could be argued that Sandy was asking students to make use of structure, though she never referred to this episode as such in later reflections.

During her second interview, Sandy still focused on looking for patterns, or possibly relationships, as the main goal of this text, though she was starting to feel that perhaps more of her students would be able to do this on their own, at least in
uncomplicated situations. She said, with some pauses as she considered this, “[I]f you gave them a variety of algorithms [I think she means equations]… and they were to find, you know, I don’t know… solve for… 3. OK… they could go through, find ones that solve for 3. But if they had to do multiple tasks, it’d be too difficult.”

During the second lesson, Sandy was very intent on having students choose a logical strategy to solve the problem – a task that involved a growing pattern. As she was giving directions, she said, “When I say it has lots of different ways for you to answer, what I mean is… there’s lot of different strategies or patterns that you can use.” So, she asked many questions that focused on how students found and used patterns in the various methods that they chose, and students gave specific explanations. Although most students viewed the pattern in essentially the same way, and although some misinterpreted the problem statement and thus arrived at incorrect answers, they were all working toward a goal of looking for and making use of structure (or patterns).

So, though Sandy’s interpretation of this text remained fairly constant throughout the study, she did seem to feel more confident by the end of the study that students could gain facility with this goal as they had more experience with it.

“Mathematically proficient students look closely to discern a pattern or structure” (p. 8). This text is very similar to the text in the title statement, so Sandy’s interpretations of it, and her concerns about students’ ability to meet the goal stated, were similar as well. Once again, she focused on students’ searching for patterns in problem situations.
During interview 1, Sandy said, “They’re not seeking out patterns, finding structures, breaking it into areas that they do understand… if I don’t have number sense, I can’t pick out the patterns.” By “number sense”, Sandy usually seemed to mean students’ understanding of the relationships among numbers, particularly in the context of recalling basic facts and using the four operations in computation.

In her first lesson, Sandy asked her students several times to look for patterns in customary and metric units of capacity as they discussed how they would teach younger students about these units. Students commented that some units (such as cups) were doubled to create other units (such as pints), and Sandy responded that this was true. However, many students seemed confused (one said this directly) about what patterns they were being asked to find, and this might have been because there are not actually many patterns, per se, in the unit conversions in the customary system. One student offered, “There’s 1000 milliliters in a liter, and 1000 liters in a kiloliter,” and Sandy used this comment to briefly review the idea that metric units all are related by powers of 10. So, perhaps part of the reason that Sandy was unsure about whether her students were able to find “patterns” in mathematical ideas is that she herself may have been unsure about the types of patterns (or structure) that could be useful in understanding such ideas.

In interview 2, Sandy was more optimistic about some of her students’ chances of meeting the goal in this text. She first said, “The mathematically proficient do [look closely to discern a pattern or structure]!” She then referred to an incident from the first lesson about which she had written in her reflection on the lesson: “[J]ust writing that paper yesterday for you, me, I had an aha moment, where [a student] said, ‘Ah! Area is
inside of perimeter!’ Yes!” Although she was glad that this student had made the connection between the ideas of area and perimeter and how they fit together (presumably in a “structure”), she was also frustrated that more of the students had not said this before. I asked if she would characterize this student as “mathematically proficient”, and she said, “More so than others, yes.”

As noted in the above section, in the second lesson, students were definitely looking for patterns and/or a structure in the task presented, and they discussed these in pairs and in the whole group discussion. Sandy noted during the second group discussion that some of the students used “T-charts” to solve the problem, and she commented that most of her students were able to set up and use a “T-chart” – “They’ve got that all down.” She went on to say, though, that many did not use the table correctly to solve the problem because they did not make sense of the question being asked.

So, Sandy mainly interpreted this statement to mean that students should be able to find patterns, or possibly relationships (“connections”) in problem situations, which she felt that only a few of her students were able to do at that point in time.

“Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have” (p. 8). Sandy mainly discussed this idea in the two interviews when I specifically asked her about this text. She initially interpreted this text very literally, focusing almost entirely on the first example in the statement and reflecting on her students’ ability to use properties of the four basic
operations and of equality to operate with basic facts (though she did not use the word “properties”). During the first interview, Sandy said:

[T]his goes back to fact families… I believe in fact families. And I believe that the children need to be taught addition and subtraction together, multiplication and division together, so that they see the reverse. They’re learning it forwards and backwards. And when I say they should know it forwards and backwards, that is literally what I’m talking about! Going forwards and going backwards, it’s the same number. If I turn it upside down, it’s still the same number. No matter what I do to it, it’s the same number. Oh, look! $4 + 4$ is the same as $7 + 1$!

During interview 2, Sandy interpreted the text more in the context of students’ solving equations based on their understanding of inverse operations:

[T]his is not necessarily younger students. Because when we’re solving for y, and it’s, you know, divide 3 or multiply 3, OK, they have a hard time understanding which one you’re supposed to use. Um, they know you have to do something to get y by itself, but usually what they want to do is take the two numbers, plug them in at the beginning of the algorithm, put y at the end, and that’s your answer.

Sandy’s focus in interpreting this part of SFMP 7 was basically on students’ application of inverse operations, commutativity, and equality, though she did not use formal language to identify any of these ideas. She never referred to the geometry example in the text.
“Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property” (p. 8). Once again, the ideas in this text only arose during the two interviews with Sandy, not in either lesson or group discussion. In interview 1, Sandy’s only comment about this sentence was, “That’s exactly what I want students to see… that is why I say it is so important for them to have the number sense.” As earlier noted, “number sense” in Sandy’s mind seemed to mean understanding relationships among numbers in the context of basic facts and computation. During the second interview, Sandy first remarked, “This is possible.” However, she added:

I don’t think, if I handed kids $7 \times 8$, and told – asked them to show me other ways that that could be solved, that they would do anything other than $8 \times 7$. I think they would just flip it; I don’t think anybody would think to break it down and use the distributive manner in that. Now, I could be wrong, and I’ll actually experiment with that… because I’d be curious to see if anybody will come up with breaking the 8 down.

So, Sandy’s second reading and discussion of this statement caused her to feel “curious” about how her students would respond to this task, and she planned to try this with her students. I am not sure whether Sandy did this, but it is noteworthy that her reflection on the text caused her to want to try something new and unfamiliar with her students.

In both interviews, Sandy focused very directly on the example in the text. Although one might not necessarily expect a teacher to carry this idea further into a more general consideration of the distributive property or other properties, it is worth observing
that Sandy did not do so. However, she did speak informally about other properties in her earlier comments.

“**They also can step back for an overview and shift perspective**” (p. 8).

During the first interview, Sandy observed, “At fifth grade, they don’t take too many steps back; they don’t do a lot of overviewing. And, as for shifting perspectives… [shakes head] if they’ve got it down one way...”. It appeared that Sandy was seeing “overviewing” as understanding the general idea behind a given problem, or even the question being asked. I believe that at this time, she saw “shifting perspectives” as students’ being able to solve a problem in different ways, about which she had already commented numerous times, saying that she did not see many fifth graders doing so.

During her first lesson, Sandy asked some questions and offered some sentence starters that caused students to reflect on the overall relationships among units of capacity in the customary and metric systems, though I am not sure that Sandy would have characterized these questions as “overview” questions because she did not speak of them later in reference to this part of SFMP 7. For instance, Sandy began, “[I]f you go smaller and smaller and smaller…” and a student replied, “The measurement of how you get to the next unit gets bigger.” Another example was when Sandy said, “[E]verything is in a form of…”, and a student responded, “10, 100...”.

In the second interview, Sandy remarked, “Well, I think the overview is pretty much the big idea – ‘I know this because’ statement. Uh, shift perspective… that gets a little dicey. Um, to me, I read that as, they can understand other people’s logic. Unfortunately, they don’t have the logic themselves, so how can I ask them to understand
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someone else’s?” She added, “[I]f more of them understood the concept themselves, they
might have an opportunity to perceive someone else’s. I don’t know though.” Here, she
commented that she saw the idea of an “overview” as a student’s understanding and
being able to state the “big idea” that underlay the details of the problem at hand. Sandy
discussed “shifting perspective” as being able to “understand other people’s logic”, but
she reiterated that this was difficult for fifth graders, particularly those who struggled
with thinking logically themselves.
During her second lesson, Sandy alluded very briefly to examining one of the
strategies that students had used from a broader (more generalized) perspective, though
there is no evidence that she thought of this short episode in this way. After two pairs of
students had presented their work in “T-charts” (x/y tables) on the board but had not used
the variables x and y, Sandy tried to introduce the variables x and y in relationship to the
numbers in the table. However, Sandy’s use of the variables did not entirely align with
the work that students had done, and she moved away from this discussion fairly quickly.
Thus, Sandy’s interpretations of the text in the second interview were the most
specific, final comments that she shared about this sentence during the study.
“They can see complicated things, such as some algebraic expressions, as
single objects or as being composed of several objects” (p. 8). During the two
interviews, Sandy interpreted the goal in this text as understanding and working with
numbers and variables in algebraic expressions, rather than recognizing that “algebraic
expressions” were actually an example category (of many types of “complicated things”)


to which the authors referred in this statement. So, she spoke primarily about her students’ learning of algebra.

In interview 1, Sandy said, “We’re just going into algebraic expressions… at this point of fifth grade, without them being taught any algebra, from what I remember in past years, it’s almost foreign. Um, they have a real hard time understanding that a letter represents a number… As a teacher, it becomes challenging to make sure you don’t lose them, because this is where the frustration doubles.” So, Sandy was saying that students struggled with understanding the meanings of variables in algebraic expressions, especially if (she added) they did not know their basic facts well.

Interestingly, in her first lesson, Sandy developed the idea of seeing a mathematical object as “being composed of several objects”, but she did not seem to recognize that this was an example of the goal in this part of SFMP 7. As students were reviewing how to find the area of a right triangle, she drew additional line segments on the board to create a rectangle that was twice the size of the triangle, and she led students through a discussion to help them remember that since the triangle was half of the rectangle, they could divide the area of the rectangle by 2 to find the area of the triangle. This seemed to be a prime example of this goal in SFMP 7, and if Sandy had realized this, it might have helped her to interpret the statement more broadly in the context of her own practice.

During her second interview, Sandy responded to this statement by saying, “I think most of them could do that. Take an algebraic expression, um, and see it as the expression, or break it into parts [motions with hands as if showing parts from left to
right]. Only if it has parentheses.” She described a student’s potential thought process: “I can take out this section of parentheses, and that is breaking it apart, because you do this part first,’ but taking that away, I think it would be challenging.” I believe what Sandy meant, relative to the text, was that if students were presented with a fairly complex expression involving parentheses, they could simplify or rewrite it because they would easily be able to identify the “several objects” in the expression. However, given a similarly complex expression without parentheses, they would not be able to “break it into parts” and operate on “several objects” individually. This frustrated Sandy because she felt that this was a very challenging concept for fifth graders in general, but teachers had to teach it because it would be addressed on the OAA.

Sandy’s thoughts about implementation of SFMP 7. During interview 1, Sandy spoke about how the text of SFMP 7 seemed to relate to what she understood about the adopted textbook program, Investigations, that Sealon had adopted several years before for kindergarten through grade 5. She had already indicated that she had stopped teaching mathematics when the district adopted Investigations. She commented:

[W]hen I first read number 7, my first thought is this is Investigations. And I am not a proponent of Investigations. But, when I look deeper into it, it goes back to what I say about how you need to learn it forwards and backwards, upside down or inside out. So, when I see that, um [pauses]… uh, this one’s kinda… I’m getting a negative taste on it, because of, um, the bad taste I have from Investigations.

I asked her to explain her last comment, and she added:
With number 7, where it says, “Young students might notice,” with *Investigations*, from what I understand, since I was never trained in it, children were to investigate, take as long as they needed until they came up with the correct conclusion. In this day and age, I don’t have from now until next year for them to see that $3 + 7$ is the same as $7 + 3$. I’ll show you, you can flip it around; you can juggle it a little bit, but we need to make that connection quick in fifth grade, and move on.

She then noted that many of her students did not make these connections quickly, and this frustrated and concerned her. It was interesting to see Sandy almost tangibly wrestle with her belief that *Investigations* advocated allowing students to take whatever time they needed to make connections among ideas (not, as she said, having had any training in teaching with this series) and what she was inferring from the text (that students needed to learn ideas about number “forwards and backwards, upside down or inside out”), with which she agreed. Her main obstacle in reconciling these two perspectives seemed to be that she felt in fifth grade, especially given the number of topics that were currently in the Ohio standards, teachers did not have time to allow students to make sense of ideas in their own time, and that students should come to fifth grade already understanding number relations “forwards and backwards, upside down or inside out”.

Also during the first interview, Sandy expressed her concerns with students’ meeting the goals in SFMP 7 when they were struggling with developing number sense – once again, focusing on number and algebra because this is what she saw in the examples in the text. “If you don’t have that number sense, if you don’t have some ownership to
the numbers, to the mathematical thinking process, all right, now we’re going to throw an algebraic expression in here, and we’re going to have variables, and the variables can change?” She explained that when her class was studying algebraic concepts, she typically divided the students into small groups and taught them based on individual needs because that helped the students to “stay with” her.

Although Sandy was unsure that her current students could meet the goals in SFMP 7 easily, by the second interview, she felt that future students would have more opportunity to do so. “This, I think, they have a chance. Um… I really do, uh, think that, with enough **modeling**, they would be able to make sense and share out structures. Um, show – find patterns easily. Break things apart. With enough modeling and enough influence for them to know – to be **aware** of doing such.” She continued:

I’m sure they can do this 7 x 8 [pointing at standard]. If they saw that, one, you should break 8 down into multiple pieces. Or 7 into multiple pieces, or whatever. But if they saw that you should break down one of these numbers into multiple pieces, I think they’d pick up on that real easily, as that was the easy way to do that. But, they haven’t been asked to do that. So that’s why I say they need the modeling and the repetition, because all they know is fact family. 7 x 8, 8 x 7 – they’ll regurgitate it to you.

That is, as she said earlier in this interview, “[M]ath has always been black and white”. Again, she seemed to be saying that teachers had not really asked students, in the past, to
work with numbers in different ways and to find different “structures”; one way had been sufficient, and perhaps even viewed as most correct.

Although Sandy still seemed to be interpreting SFMP 7 primarily in terms of number-related examples, she seemed to be thinking more about the words in the text and feeling that her students could, with more experience, look for “structure” and “patterns” and to “break things apart”. They simply needed to see, in her opinion, that this was possible and perhaps useful in solving problems.

**Review of how Sandy’s interpretations of SFMP 7 seemed to evolve throughout the study.** Sandy actually seemed to see two different main ideas within SFMP 7. The first was that students should look for patterns (or perhaps connections among ideas) and use them in solving problems. This interpretation seemed to be taken from the text of the title and of the first sentence in the paragraph for SFMP 7: “Mathematically proficient students look closely to discern a pattern or structure.” Her interpretation stayed relatively intact throughout the study, though toward the end of the study, she seemed to believe that her students were more capable of finding and using patterns than she thought initially.

The second main idea, which Sandy found in the remaining portion of SFMP 7, was understanding and using structure, particularly within the four operations and properties of these operations, both in numeric and algebraic problem situations. She cited one recent classroom experience that helped her think about this goal (in which a student related area to perimeter – ironically, a relationship more involving geometry than number or algebra). However, her re-reading and discussion of the text itself
appeared to help her to clarify her thinking about the standard and to convince her that her students might in fact be able to develop the skills described. She then planned to test this by trying some whole-number tasks involving the distributive property with her students. So, focusing on specific text over time, and relating the ideas in it to her own students, seemed to help Sandy develop a more refined interpretation of SFMP 7.

**Sandy’s Vision of the SFMP Implemented Effectively in a Classroom, School, or District**

*Sandy’s goals for her students and for herself as a teacher.* Above all, Sandy Baxter loved teaching and loved providing her students with fun and engaging experiences that enhanced their lives as children. She mentioned at various times that she enjoyed celebrating holidays and doing crafts with the students, and she thought her colleagues were wise to enjoy their time with students as well.

Sandy’s desire to support children also meant that she wanted to do whatever she could to help her students learn mathematics. As she said very definitively in the second interview:

I think what influences me the most is my sincere concern for the children. That I believe as the United States society, we are failing children. And we just keep continuing to fail them. And because I believe in that, and I want not to be one of those people who fails children, I will work extra hard to make these mesh. To work on these standards. To teach them to think in that higher, critical zone.
Referring to her students in terms of mathematics learning, she added, “[W]e have some strong leaders, and we have some really weak followers. But, we’ve got to make a difference.” However, in helping students learn to discuss their thinking and respond to each other’s questions and comments (as in SFMP 3), she never wanted to “destroy the child in the process of working on the standard. ‘Cause it’s not worth it – to destroy a child for anything.”

Sandy’s efforts to support students in her classroom included specific practices, such as helping students learn to take notes in their math notebooks, allowing students to teach each other using the classroom audio-visual technology because it excited them, and collaborating with Ray, whom she called her “mentor”, to be sure that she was preparing students well for sixth grade. She also espoused more general beliefs about teaching that moved beyond working toward high state test scores: “I personally would rather lose my job any day based on test scores, knowing that I was teaching to the best of my ability, and I was teaching the kids that needed the help the most.” However, as she somewhat sadly pointed out, it was not always easy for her to know how to best help students succeed academically: “I’ve just sat here and I’ve talked about how it’s disappointing me and … how I don’t know how to change it. ’Cause if I had the answers, God knows I’d do them.”

In considering her instructional practices in mathematics, one important goal that Sandy has for her students is for them to solve problems efficiently. She said in the first group discussion, “We’re preparing for middle school.” Thus, she often says to her students, “You’re right, that way works. But is that the way a fifth grader would do it?”
As Sandy considered the goals in the SFMP related to students’ conceptual understanding and efficiency, she asked Carrie in the second group discussion, “As a mother, and a teacher, when do you think we start finalizing a concept? And we get to the algorithm and the answer?” After some discussion, Carrie replied that she felt that some efficient strategies might not be effectively developed until middle school for some students. Sandy’s belief in general was that students needed efficient strategies earlier so that the mathematics might be clearer for them. She even shared an example from a recent professional workshop on the CCSS that the fifth grade teachers had attended; they were to solve a fraction division task with strategies other than the standard algorithm. She said that they were not even sure, after an hour, which strategies made sense. Sandy wondered how students would cope with this lack of clarity, if this were what the SFMP were suggesting. I believe she was also wondering how, as a teacher, she would reconcile the goals of the SFMP with her goal of students’ becoming efficient in their methods.

Sandy’s perspectives on implementing SFMP 1, 3, 7 as a group. As noted before, Sandy commented at various times during the study that mathematics had traditionally been viewed as “black and white” – that each problem had one correct answer and that there was one correct way to find that answer. She realized that this was in contrast to the goals of the CCSS, and she seemed to accept these goals even though she was, I believe, unsure what they would look like in practice. Earlier in the study, Sandy had shared examples of how she wanted students to solve problems as she had taught them to do. However, with some dismay, she commented in the second group
discussion that she saw that her students were, for the most part, solving problems in exactly the same way that she would. Most explanations that students gave were just reiterations of the methods she had taught, and she wanted this to change in her classroom. This was evidence of a shift in Sandy’s thinking about the teacher as authority that aligned her belief more closely with the goals of the SFMP.

As aforementioned, one of Sandy’s greatest sources of frustration, both under the current Ohio standards and in thinking about enacting the CCSS and SFMP, was that students often entered fifth grade without some of the foundational knowledge that they needed to succeed in mathematics at this level. Sandy shared in the first interview, “I would like to see even more responsibility put on grade levels – by kindergarten, grade 1, you must know this… and actually do something about it.” Sandy went on: “They can be challenged with the littlest things because it’s not second nature… A child should not have to think visualize.” In later interviews and group discussions, Sandy made many similar comments, essentially establishing her grave concern with implementing challenging mathematics standards when students were struggling with concepts and skills that they should have learned in earlier grades. She was well aware that students were studying major topics such as perimeter, area, and place value in third and fourth grade (since she saw this happening in other classes at Lincoln), so she could not understand why students seemed to forget much of what they had studied when they were in fifth grade. Of other teachers, she maintained, “There’s no doubt in my mind, everybody’s doing what they’re supposed to,” and she commented to Carrie in the first group discussion (based on Carrie’s description of her own class) that it was clear that in
first grade, teachers and students were working diligently to develop number sense. She asked, “Why isn’t it clicking? Obviously we’re doing something wrong. We have been teaching in isolation those things. Maybe it will make a difference doing it different next year.” Sandy felt as the CCSS and SFMP were enacted, teachers could take new instructional approaches that allowed them to spend more time with these major concepts and to ensure that students had deep understanding before moving on.

On the other hand, Sandy was also concerned that in some ways, students in fifth grade would struggle even more in mathematics during the next year or two, since they would enter the CCSS in the middle of their school careers and would not be accustomed to learning mathematics in the ways that the SFMP suggested. Further, they might be lacking some of the CCSS content that they should have learned in earlier grades, and she noted that every teacher would have to be aware of this issue and account for it as they taught. She commented during the second interview, “[T]hings are becoming so diversified and multi-tasked… these standards are… real life. We haven’t taught real life. We haven’t taught problem solving. Problem solving was ‘solve for x’… So this is a huge, huge difference. For fifth grade.” Further, during the second group discussion, she observed, “I have always taught the algorithm. And the formula. And the process of getting there. Working through the algorithm. [quietly] Now I won’t do that.” “I’m real concerned that – real, real concerned with – I understand the concept of teaching to the understanding. But I’m real concerned with the fact that I’m in fifth grade, and I don’t know the algorithm.” Still, she wanted to help her students work toward change: “At 10
years old, they’re trained, and they have not been trained on this model. So it’s a year to rework, and remold the clay that we have, in my opinion.”

These comments about the past as contrasted with future led Sandy to broader thoughts about whether the goals in the CCSS and the SFMP were achievable and/or worthwhile for students. As has been cited in earlier sections, Sandy’s opinions on this question seemed to vary depending on the specific portions of the standards being considered. In the first interview, she commented, “I think [SFMP 7] is very good for fifth grade, depending upon the depth.” However, she also said, “I look at these standards; they look very doable… I think anyone, any lay person could look at that and say, ‘Oh, sure, yeah, they could teach that – no problem.’ But… [pauses and grimaces]”. That is, she was unsure that the general public understood how challenging it would be for teachers and students to enact these standards successfully.

By the second interview, Sandy was more optimistic about enacting the SFMP: “I do believe they’re worthwhile. I do believe they are achievable – eventually.” But she still saw obstacles to overcome with students: “I don’t think they’re old enough, with these standards [points to SFMP], to comprehend the variety that you can get to the correct answer… Maybe they can; maybe they can’t. I don’t know… six years from now, the kindergartners – sure. It’d be completely different. But the fifth graders I have next year… don’t have a chance.” She also related the likelihood of students’ struggles to her own challenge of integrating the content standards and the SFMP, saying, “If I’m telling you it’s a challenge for me to interweave these two pages right here, how am I going to do that to 10-year-old kids?” Further, she remarked, “These are life skills; these
are societal skills that are embedded in our old-fashioned math program… We’ve taken our old-fashioned math program and embedded a new way of understanding, a new way of reasoning, a new way of rationing, um, rationalizing…” She added, “We are not ready… by any stretch, to take on these standards and be proficient. Will we all do the best we can? Absolutely. Without training, without materials, everybody’s gonna take a different approach on it, and it doesn’t make any sense.” Ultimately, though, Sandy felt that as time progressed, the “gap” in students’ and teachers’ experience with the CCSS and SFMP would decrease: “[E]ach year, it’ll get a little bit smaller, and a little bit smaller, and a little bit smaller, ‘til the gap has been closed.”

Sandy’s thoughts about instructional practices that would support the SFMP. Sandy recognized that her instructional practices would need to change in order to enact the SFMP effectively. In the first interview, Sandy spoke very positively about the idea that with fewer standards to teach, she and the students would be able to delve more deeply into each concept or skill. She commented, “[T]here’s a real strong possibility with the new set of standards you’ll have more time to focus in on true meaning… wow, it’ll be great… forget the drill and kill. We’ve got to do four problems proficiently.” She also thought it would be a helpful change “for children to have time to take ownership of their learning.” During interview 2, Sandy referred to the opening statement of the CCSS, saying, “This very first line… says it all, pretty much.” This sentence reads: “Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time
devoted to number than to other topics. Mathematical process goals should be integrated in these content areas.” Sandy mused, “[M]y remark about [focusing on one] problem of the day… might not be half bad.” In the second group discussion, she added, “I think the whole concept is fabulous. With making everything richer and more detailed. Slowing down the pace.” Still, she admitted that she was concerned about spending too much time with each concept and not being able to formalize as many ideas with students. As the teachers continued talking about how to reinforce skills that had been developed, Sandy noted that she asked students to complete “bell work” before school started for this purpose and also to get them “in the math mode”, since she taught math first during the day. She felt that she would continue to do this as she implemented the CCSS.

In thinking about the structure of each class session under the new standards, Sandy remarked, “[I]deally you want to have mini-lessons… And I think you should have multiple sets of mini-lessons, break apart sessions, back together… I can’t keep their attention on me for fifty minutes.” Sandy described a workshop-type format in which she would present students with a task, ask them to work on the task individually and/or together, and then return to the whole group for discussion, perhaps even repeating this sequence during the same class period. She had come to appreciate the moments when students found ways to solve problems that she had not taught or perhaps not even seen herself. She also noted that students often learned well from each other, either by working together in pairs or groups or by learning from one student who was “teaching” the class. She reflected, “[O]ften times, when they’re working in pairs, their partner will give them the aha moment that they’re looking for.” Sandy also described, during several
phases of the study, how well her students enjoyed and responded to other students’ teaching the class, which Sandy facilitated on a fairly regular basis. “They all pay attention, or the child will say something that draws their attention, and it clicks… They love it.” She also felt that in this type of class structure, she would be able to work with individual students on a more regular basis to meet their specific learning needs.

One point that Sandy addressed a number of times during the study was that she felt that enacting the CCSS would almost require teachers to integrate lessons more across content areas. Sandy said, “I could see where you’ll be teaching more units than you will be actually in particular standards, and the unit will be a thematic unit that encompasses all curriculums more so than the way we’ve taught in the past.” She particularly felt that writing would need to become more a part of each content area.

Then, between the first videotaped lessons and the second interviews, the fifth grade teachers had learned from school leaders that they would all be self-contained starting in the fall and that they would be expected to integrate content whenever possible; they were implementing all of the new standards in language arts, mathematics, science, and social studies. Understandably, this seemed overwhelming to the teachers, who had all been teaching the current Ohio standards in teams where each teacher taught two or three areas at most. In the second group discussion, Sandy remarked that every teacher would have much to learn before the start of the following year: “Everybody’s new in every subject.” Still, Sandy felt that being self-contained would allow some flexibility in the amount of time that she could spend in each content area, and if she needed to stretch a
mathematics lesson one day (or end early because it seemed appropriate), then she could do so.

When the teachers had learned that they would be self-contained, they also learned that they would be expected to integrate more computer application into their own classrooms, in order to prepare students for the new standardized testing online as well as simply preparing students to live and work in a technology-based world. Sandy recognized and agreed with both of these needs, but she was concerned because she only had a few computers in her room and because students would need time with the computers to work through the more complex, integrated tasks of which she had seen samples at an earlier meeting. She was unsure how she was going to manage this unless she was given more computers.

Although the SFMP refer to students using manipulatives to support mathematics learning, Sandy did not feel that this was appropriate for most fifth graders, with the exception of students with disabilities. It is unclear whether she felt that she would start to use manipulatives more in the future; she did make one comment about finding more manipulatives to use as she reflected on her first lesson.

During the study, Sandy also mentioned other teaching practices that she felt were worthwhile in general, including during the enactment of the CCSS. She believed in incorporating different “modalities”: “They hear it; they see it; they write it.” Also, the teachers at Lincoln were studying a book in which the author suggested using recorded music as “background noise” in class when students were working together so that they would not feel that their voices would “be heard” as they took risks to share their ideas.
Basically, Sandy felt that it was important to give students a “variety of opportunities” to learn.

During the first interview, Sandy also explained that she felt, given the lack of many students’ basic skills, that it was valuable to directly teach students methods to solve particular kinds of problems, because “[i]t might not make sense today. Maybe tomorrow it’ll click; maybe the next day it’ll click; there’s a strong possibility, once they’ve played with it enough, it will click when they have the idea of what they’re looking for.” She added that as a teacher, she found this much more rewarding than watching a student struggle because he/she did not understand a concept. It is interesting to note, however, that Sandy did not discuss this instructional method much more in later parts of the study, which might mean that she started to recognize that the SFMP did not especially support this.

Sandy did not discuss assessment of students’ progress toward the goals in the SFMP a great deal during the study. However, she recognized that this would be a necessary challenge that she would need to consider in the future. During the second group discussion, Carrie shared that the first grade teachers had developed a rubric that they were using to assess critical thinking, and as Sandy and Ray heard what the rubric included, they felt that it would be very effective in assessing fifth graders’ progress on the goals in the SFMP. Sandy asked Carrie if she would share the rubric with them, and Carrie quickly agreed. Still, Sandy felt that her main goal for the next school year would be to enact the standards as best she could and that assessment of the SFMP could
become more of a priority later, once she was more familiar with the standards themselves.

Overall, Sandy realized that she would have to make changes to the practices in her classroom in order to enact the SFMP well: “Have to make a lot of them… from the way the children socially interact, to what I ask of them, to how I ask them to do it, has to change drastically, just with this alone.”

**Sandy’s perspectives on challenges related to enacting the CCSS and SFMP.** As Sandy reflected on what would be necessary (or helpful) for teachers as they enacted the CCSS and SFMP, she observed that with the current Ohio standards, “[W]e’ve been so busy with the quantity… I’m hoping I will be a better teacher because I can give a higher quality than I give today.” In the second group discussion, she shared, “I can’t say that we – we never had rich dialogue, but I can say that we were lacking in it.” Sandy definitely felt that “we’ve got to do something different than what we’re doing” to support the goals in the SFMP. In the second interview, she said, “[W]hat it is I don’t know – but I’m willing to try, and I’m gonna dive in and try it different next year.” Referring to some of her ideas about enacting SFMP 3, she remarked, “And hopefully from there, it will lead me to new ideas, to jump forward with.”

The challenges that Sandy faced as a teacher and in improving her practice, however, were always in the forefront of her mind. She felt a great deal of pressure to teach all of the fifth grade standards effectively before the OAA in early May, saying, “I’m constantly on a treadmill.” (She used the word “treadmill” quite often.) “Even with an hour block a day, it’s not enough to get everything in… I personally don’t feel we’re
teaching anything indepth.” By school leaders, “We’re being instructed that we are not allowed to review previous year’s material; [students] didn’t learn it – they have to learn it on their own.” In fact, “[I]t’s to the point where they check our lesson plans to make sure we are not using any curriculum taught as – in review.” Moreover, Sandy said that school leaders had told the fifth grade teachers that they did not need to worry about students’ computational skills because in sixth grade, students would be allowed to use calculators. This upset her, and she shared, “It’s a real struggle with personally, internally, as well as externally, how to juggle all these balls when you’re not comfortable with what you’re doing.” Sandy reiterated throughout the study that they were “one hundred percent driven by the assessment.” This could be observed in a comment that Sandy made to her students during the second videotaped lesson: “You would get partial credit for having the T-chart done correctly, and full credit for adding up all of the people that came in.” Although Sandy did not mention the OAA here, it was clear that she was referring to how constructed response items are scored. Sandy also shared, “It’s very difficult to work under those pressures… you feel guilty for using some of class to do a craft… We’ve taken the fun out of it.” Sandy was truly hoping that implementing the new standards would help them to “get off the treadmill”.

Regarding other overall challenges to improving her practice, Sandy also felt that it was often difficult to find and use appropriate resources for mathematics instruction, which she felt were not generally provided by the district: “[W]e’re – almost disheartened that from the top down, they’re telling us… [makes anticipatory, amused face], ‘We’ll get you materials, but it won’t be for a long time…’” We’re gonna sit back and watch and
wait? The teachers can’t do that. The administration can, but the teachers can’t.” She commented that the teachers had been told that if they wanted to create or find resources on their own time, they could do so. In addition, Sandy felt that she did not have enough opportunities to collaborate with teachers in fourth and sixth grades (and other grades as well). “We do not have good communication K-5.” She went on to say, “Nobody takes time to look at… ways that we could make a difference. Little things that make a huge difference in the way we taught.” Sadly, she added, “And that’s really a negative attitude, and I’m sorry that I have it, but that just comes from being around here too long. It’s what you really get. And it’s a bad year.” So, Sandy felt that the teachers were not always well supported by the school leadership in terms of instructional needs, and she felt that the leaders did not always recognize what these needs were.

Sandy also felt challenged to work with students who were not always excited about learning mathematics or succeeding particularly well in their learning. In the first interview, she commented, “I’m almost burning my own soul to try to get these kids motivated to learn,” meaning that she was trying to do everything that she could, including offering some extra credit which she would rather not do, to encourage students to participate fully in their learning. She remarked, “And I’m not in a bad district.” In the first group discussion, Sandy responded to Carrie’s comment that first graders were eager to learn: “By fifth grade, they’re starting to shut down… And it’s really, really scary.” Although Sandy felt that her students enjoyed teaching each other and that they were “enjoying the variables because it’s new and intriguing”, it seemed that this excitement was the exception rather than the rule. A comment she made at the end of the
first interview demonstrated her emotion and frustration: “I’m walking out of here sad thinking that I’ve just told people, kids don’t love math. They don’t know it like they should.”

When I asked Sandy whether there were policies currently at Lincoln that would either correspond with or conflict with the goals in the CCSS and SFMP, she reiterated that the lack of support for instruction that she typically felt from the administration would render it difficult to enact the standards well. Also, Sandy briefly discussed the idea that under new Ohio law, half of each teacher evaluation was to be based on the growth shown by her students on the state tests (called value-added data). She felt that this meant she must do her best to teach the new standards because the tests would address these standards. However, this would be especially challenging during the next couple of years for two reasons: first, because both teachers and students would be adjusting to the CCSS, and second, because she suspected that Lincoln teachers would still be expected to teach the current Ohio standards as well as the CCSS for two more years since students would still take the OAA – even though her principal had said that only the CCSS should be taught. She was still doubtful and asked, “How do I balance this?” Further, “[Y]ou’re gonna make it even more… almost threatening… if your students don’t do well, neither will your paycheck.”

One policy that Sandy did feel would align with the SFMP was the fact that the school staff considered Lincoln a “bully-free zone”, “which I think… is important with some of these standards”, particularly SFMP 3 which requires that students discuss their differences in thinking. She noted that “school is a safe environment for everyone” and
that she felt she would need to relate this to the expectations in SFMP 3: “And we’ll have to really, in my opinion, reinforce that when it comes to kids giving their arguments and being asked to give an argument as to why your answer’s correct. Because an argument to a 10-year-old is literally a verbal confrontation.”

Overall, Sandy felt that though the implementation of the CCSS and the SFMP would be challenging for her, she felt more prepared to meet this challenge than she thought most of her colleagues would be because of her participation in this study. However, she felt that implementing the new standards could be especially difficult for new teachers or teachers returning to mathematics after not having taught it for several years. Her reason for suspecting this was that these teachers had not been participating even in the minimal professional development that the current math teachers had been attending, and they would need to learn a great deal very quickly. While holding up one of the pages containing the SFMP, Sandy said, “[U]nfortunately, half of us haven’t even seen this piece of paper!” “So they haven’t even been… have a clue what the state standards are in math… But, with working with you, going to the workshops I have gone to – [chuckles] – I’m far ahead of everybody else.”

Sandy’s perspectives on necessary supports for teachers enacting the CCSS and SFMP. Whether teachers would be new to mathematics or not, Sandy discussed the idea that all teachers’ understanding of the new standards would be critical to their successful enactment of the standards, and she hoped that the district would continue to organize professional development opportunities for the teachers. In the first interview, she said, “I think it will be the math teacher’s responsibility to understand the new
standards,” and she noted that the fifth grade teachers had gone (on their own time) to a locally-sponsored ODE workshop that introduced them to the new content standards – though she recognized that the workshop had not focused on the importance of the SFMP, which now frustrated her, because teaching the new content had seemed relatively simple at the time. During the second interview, Sandy added (pausing often to think), “For a classroom… it’s gonna be hard… to incorporate all of these standards at one time. We don’t have nearly the training… or the supplies… or the understanding… We need to work on – which one is it [looks at standards] – number 1. [reading] ‘Make sense of the problem, and persevering [sic] in solving it.’” That is, she felt that teachers would need to “make sense of” the content and practice standards and “persevere” in enacting them. Sandy added, “[M]eshing these two is a learning curve for me, as an experienced teacher”, because the SFMP did not often explicitly refer to content or vice versa. She referred to the episode during a recent workshop with the county consultant when the teachers themselves were not sure how to justify the answer to a fraction division problem. She said that it would be very helpful if the county consultants would come to help them understand the content and practice standards more deeply, as well as how to integrate them. “What exactly does this mean? What exactly do you want me to do? [pauses] How do you expect me to accomplish this?” In addition, “[W]hen there are eight of us teaching math next year, and eight of us looking at this sheet [holds up one of the sheets with the SFMP on it], and eight of us making different interpretations of what each one of these standards is, there’s eight completely different… processes being taught.” Sandy was concerned about this inconsistency among classrooms. She felt that
it would be very beneficial if the state would train at least one person from each county to help all teachers better understand the CCSS, because she felt that if only a few people were trained as trainers across the state, “By the time it comes through the trickle down effect, it’s so watered down, it’s almost useless.”

Sandy also felt that she would need some new resources to enact the CCSS and SFMP well, but she was not sure what resources she would receive (or when) or even for what to ask since the text in the standards did not tend to specify resources that would be useful for teachers and students. As noted earlier, school leaders had initially said that new textbooks would not be purchased for at least a year, but by the end of the study, the teachers had learned that the district would be purchasing the second edition of the adopted program, *Investigations*. The second edition included resources specific to the CCSS and planning details for teachers enacting the standards. Sandy had said early in the study, “I personally like the structure of a textbook… but to be embedded in a textbook day in and day out would drive me crazy.” She did add, “It’s a nice tool to have” for students to use as a reference, and “It’s extremely beneficial to parents” who were helping with homework. Sandy also believed that the district would provide each teacher with a print copy of the new standards, but she was unsure that many other tangible resources would be provided.

Sandy assumed that she would probably need to spend some time creating or locating other useful materials, and she referred to her students’ math notebooks as one resource that she would continue to use. She noted that the notebooks were “child-friendly; they write down the steps, have examples – a resource that they can use when
they go home”. If Carrie eventually shared the first grade critical thinking rubric with Sandy and Ray, I believe that Sandy would use this to help with assessment of the SFMP as she had suggested, though perhaps only after she was more comfortable with the SFMP. Sandy also pointed out that Ray had written grants for helpful materials and professional development for teachers at Lincoln, and she imagined that he would continue to do so. In addition, she felt that she and other teachers would spend some of their own money to purchase useful resources that were not provided by the district because the district was so limited in terms of funding.

One element of implementation that Sandy felt would be essential was additional professional development related to the standards. As noted earlier, she was glad that she had participated in this study because she felt that she was “ahead” of most of her colleagues in learning about the CCSS and SFMP, and she was frustrated that the ODE workshop that the fifth grade teachers had chose to attend one evening in the fall was focused only on content. She remarked, “That’s what three hours of sitting at that workshop got me. It doesn’t have anything to do with this” [holding up a sheet with the SFMP on it]. She continued, “I mean, these [pointing to content standards] are important… But, we were told [reading from her notes] fractions, multiplying and dividing, two-digit divisors, decimals to the hundredths place, and understanding volume. Ah! Easy! Nooo problem.” Sandy contrasted this with the recent workshop with the county consultant where the fifth grade teachers had spent over an hour thinking about one fraction division problem, which she recognized more closely reflected the instructional practices suggested by the SFMP. She believed that part of her role as a
teacher was to “find research” that could offer guidance for her enactment of the new standards, but she did not feel that all of her colleagues necessarily saw this as part of their role. In fact, she was concerned that some of those new to teaching math in the fall would “buck” what she and Ray and perhaps others would try to initiate in their grade-level collaboration relative to the CCSS because they did not understand the goals in the new standards. She was also unsure whether other grade levels would even try to maintain consistency among all of the teachers in terms of instructional practice.

Believing that consistent professional development was key to successful enactment of the CCSS, Sandy discussed opportunities that the district could provide. She felt that an effective way to help everyone better understand the standards would be to work with teachers from other grade levels to study the standards and to plan for the year. During interview 1, she commented, “[J]ust reading their curriculum versus my curriculum is beneficial, but it’s not nearly as beneficial as taking half a day and talking with them” about the standards and the focus that each grade was placing on each major idea. For instance, “[I]f I had an afternoon where I could sit down with fourth grade math teachers and say, ‘I’ve got really, really big holes in fractions.’ OK? And they say, ‘Oh! We only have it on our map for two weeks. Maybe we need to allocate more time into it’ – things of that nature.” Sandy remarked, “I think that will make the biggest difference in a building, and honestly that would probably be my answer for a district too.” During the second interview, Sandy reiterated these ideas, which were clearly important to her, and she added, “They don’t really give us professional time to do that. Where do we do it? Talk at a meeting here or there, when we’re supposed to be listening
to some speaker…” More generally, Sandy felt that if she had known earlier that she would be self-contained the following year, she would have been seeking out professional development in science and perhaps the other content areas as well, so it could be inferred that she hoped the district would provide more of this also.

Since the teachers at each grade level had been participating in grade level collaboration (that the district called “PLCs”, or “professional learning communities”) for several years, Sandy believed that this would continue, and she believed that her PLC would use a substantial amount of time during the following year to plan together and “build [an] agenda for the year”. She did wish that they had more time than they had typically been allotted for this work, and she wished that they had more opportunity to work together as grade “bands”, not just single grades. She explained during the first group discussion that the fifth grade teachers wrote their common assessments together and then planned their units based on these assessments. However, in the second group discussion, she echoed Ray’s concern that since they had been accustomed to working as a team of four people in fifth grade mathematics and now they would have eight, most of whom were new to teaching math in grade five, it could be more difficult to gain consensus on what should be taught and assessed (and how). She asked, “How can I be using… vocabulary, that’s not the same in somebody’s room that’s never taught it?” “How can we give common assessments – or… anything, without all of us being on the same page?” So, Sandy felt that the PLC collaboration was vital, but she also felt that it would be challenging when many of the teachers were inexperienced with fifth grade mathematics and initially unfamiliar with the new standards.
One other form of professional development that Sandy felt might be useful to her was a workshop on Singapore mathematics. Ray had attended a workshop on the Singapore philosophy and practices that had really excited him and had helped him to understand the goals of the new standards – as well as some instructional strategies that could be effective in meeting these goals – and Sandy was interested in attending a workshop as well. She was not sure when she might be able to do this, and she did not know much about Singapore math other than a few ideas that Ray had shared, but the fact that Ray found the workshop so enlightening caused her to feel that it would be beneficial for her also.

So, as Sandy considered supports that would be helpful for her in implementing the CCSS and SFMP, she focused on learning more about the standards themselves, learning how to integrate the content and practice standards, using provided resources and finding more on her own, participating in professional development workshops, and collaborating with her colleagues weekly in their fifth grade PLC. Though she knew that the first year of implementation would be especially challenging, she felt that she would do the best that she could do with these supports and hope to build on successes in following years.

**Additional Concerns that Sandy Shared During the Study**

During each conversation, Sandy shared many thoughts that were not necessarily directly related to the CCSS or SFMP but that affected how she viewed her role as a teacher of mathematics. Many of these thoughts involved her frustration with what she perceived as students’ lack of “number sense”. In interview 1, she commented,
“Sometimes I think society in general is lacking in number sense.” She added, “I think we’ve failed as a society [in encouraging success in mathematics]. It’s OK not to be good in math. Where I grew up, and that was one of the most important things you wanted to do, was to be good in math. If you were good in math, you could have any job you wanted.” “As a society, we have accepted that there are some people that can do math and some people that can’t.” Sandy felt that families needed to do more at home to change this perception of mathematics and to prepare students to enter school with basic math concepts in place. During the first group discussion, she said, “[T]he biggest disappointment I have coming back as a math teacher again is that when I left math ten years ago, kids’ families still had an importance on math [sic]. And they don’t have that importance today.” Observing that many students seemed reliant on their personal technology devices, Sandy remarked, “[T]hey’re raising these kids with the idea that, ‘As long as you can read, it’s OK. You can always use a calculator. A computer does that for me.’” She added that in the past, “learning your multiplication facts was important as learning to read”, with which the other teachers agreed. Her concern arose during the second interview also: “[A] lot of people in society believe that education doesn’t start until you hit school. And people are not educating… in homes, the way we need to keep up with other people’s standards… Turn off the TV! Roll the dice! Let’s play a board game. Instead of everybody sitting in a different room – or in the same room with their own iPods!”

Even more generally, during the first interview, Sandy expressed her frustration with students’ lack of perseverance. “Kids don’t put that much into anything today… We
don’t make them as adults responsible for the perseverance that they should have… A lot of things that we do in society in general, we might not be doing our children a good service, because too many things are disposable… And nobody takes ownership to it.”

The kinds of concerns cited above arose mainly during Sandy’s first interview, and it may be that she simply had had some recent experience that had caused her to express these thoughts more openly than she did later in the study. Still, given other comments throughout the study, these ideas seemed to be on her mind as she read and reflected on the CCSS and as she considered how she would implement the standards with her students.

**Reflections on the Evolution of Sandy’s Thinking about the Standards and Potential Changes in Her Practice**

It should be noted that Sandy was not able to participate in the final interview for this study because she became gravely ill. After undergoing a major operation, she was out of school on medical leave for nearly an entire school year after the other data was collected. I was unable to establish communication with her again as chapter 4 was being written. However, she did participate fully in the first six of the seven phases of the study, and during the last group discussion, she offered some thoughts about her goals for the following year relative to the SFMP (goals which, unfortunately, she was likely unable to accomplish due to her illness). I am confident that the information gathered during the first six phases of research are representative of her thinking about the standards as a whole, as described below.
During the study, Sandy’s interpretations of the SFMP did not, for the most part, change a great deal. This perhaps was to be expected, given that she and the other teachers participated in just four conversations about the standards and taught two lessons in order to reflect on the SFMP in practice. However, she did offer some ideas about how her thinking did evolve in one way or another. First, she said in the second interview that merely the act of participating in this study had helped her become much more familiar with the SFMP and with what would be expected of her and her students, which had not occurred in other professional development sponsored by her district or by the state. Second, in this interview, she shared that she was able to think about the SFMP more specifically when the sentences in each paragraph were presented separately rather than in paragraph form, and it was also clear that she was using the examples in the text to make sense out of the goals being described. Third, talking to Carrie in the group discussions seemed to encourage Sandy that young students (even first graders) were capable of meeting the goals in the SFMP if the teacher was able to facilitate this development in the classroom. Fourth, when Sandy realized that her students could benefit from the use of manipulatives to understand measurement units, she seemed to become more open to the possibility that manipulatives might be useful in some other situations in fifth grade also. Finally, after Sandy saw her students discussing their thinking during the second lesson, she offered at least two reflections: one, that her students often simply shared strategies that she had taught them versus strategies they had developed themselves (which she realized was not meeting the goals of the SMFP), and two, that her students were in fact able to discuss their thinking respectfully with each
other (which was a goal of the SFMP that she was unsure students could meet).

Similarly, she wondered about how her students would respond if she presented them with a question like one of the examples in SFMP 7.

Overall, then, Sandy’s thinking seemed to evolve most when she was discussing the SFMP and the implementation of them with other professionals, when she was able to study the text in sentences rather than paragraphs, and when she saw that her students and even younger students were able to reach the goals in the SFMP and in fact to benefit from them as mathematics learners. In these situations, her concerns and doubts about many of the goals in the SFMP seemed to lessen considerably. Moreover, since she expressed a desire to change her instructional practices to better reflect the goals of the SFMP, more work with colleagues and with her students would likely help her to move toward this goal.

Case 3: Ray Archer

Professional and Personal Background

Ray Archer had taught for nineteen years at the time this study began. He had taught kindergarten, first grade, second grade, third grade, and, for the most recent eleven years, fifth grade. During the first interview, he commented that “math is easy ‘cause it’s like a universal language. I love it.” However, he added, “[M]y generation, we were great at skills… it fell apart, because we couldn’t apply it. We weren’t taught that…”.

Ray referred to himself a “big thinker”, meaning that he tended to see the “big picture” in planning for instruction and sometimes to be less focused on details of implementation. It was clear, however, that throughout this study, he was thinking a great deal about how
he would enact the CCSS and the SFMP. He shared several times that when he began teaching, he had had a mentor who taught him that it was critical to look back at the content in the previous grade level and to look forward at the content in the following grade so that he would know what the students should understand and be able to do when they left his class. He also felt that his experience teaching the earlier grades helped him to understand students’ thinking on different levels, which was very helpful in working with a wide variety of students.

One early experience that seemed to be very significant to Ray – as he described it on at least three different occasions during the study – was that he remembered being a student who often found unique ways to solve mathematics problems, and teachers were not always very accepting of his thinking. As he said: “I was that strange little kid in the back that the teacher – ‘Well, no, that’s not the way I solved it - who else? Who can do it the right way?’ I never volunteered again.” He added that he wanted to see mathematics teaching move away from this philosophy and from the skill-based instruction that he remembered, and he really appreciated the goals of the CCSS because he felt that these standards would allow mathematics education to move in this direction.

Ray often cited educational research during conversation throughout the study. Particularly, he frequently mentioned the “concrete”, “iconic”, and “symbolic” (or “abstract”) modes of student thinking, presumably referring to the “enactive”, “iconic”, and “symbolic” levels of representation proposed by Bruner (1966), Vygotsky’s Zone of Proximal Development for learning (Vygotsky, 1978), Piaget’s levels of thinking (e.g., “formal operational”) (Piaget & Inhelder, 1969), and the TIMSS video studies (Stigler,
He also referred once or twice in passing to “Marilyn Burns” and “Constance Kamii” and their research on young children’s mathematical thinking. Ray seemed to relate this research fairly seamlessly to the ideas he shared about the standards and about instruction, so his understanding of research seemed to inform his thinking in general about the teaching and learning of mathematics.

Ray and the other fifth grade teachers had gone to a county workshop several weeks before the start of this study to learn about the CCSS. Other than this, the district had not yet really provided any direction for the transition to the new standards. He remarked that even though there were not many changes to the content in grade 5 from the Ohio indicators, “it’s more of a paradigm shift from what we are teaching. And how we’re teaching – that’s the big one.” He added, “I’m getting a little nervous, because I kind of want to jump on it as soon as possible, now that we have it,” explaining that he had understood that this would be the “planning year” and that the following year would be the first year of implementation, but that did not seem to be the message the administration was sending at this point.

Ray and a few teachers from third and fourth grades had also attended a workshop about Singapore math about a week before the first interview. [It should be noted that I do not have specific information about this workshop, particularly who sponsored or presented it. Ray simply referred to it as the “Singapore math” workshop throughout the study, and thus I will do so as well.] Ray felt that the Singapore philosophy was “excellent” and that he saw many parallels between it and the CCSS, particularly the SFMP. He remarked that he was “still on a high” from the “powerful” workshop, adding,
“It was huge for me.” He noted, “Now this [pointing at standards] makes so much more sense to me, which is great.” He shared that the workshop caused him to think about how he taught number-related concepts, how he used vocabulary and manipulatives, and how he helped students to create models for word problems. He shared that he wished he had been taught in this way and said, “I thought as a teacher, I should have been – why haven’t I been doing this?” He continued, “I think we present way too many strategies – that’s what I like about Singapore. They only have like three options.” Ray apparently viewed this in contrast to his own school experience, where only one strategy was accepted; he appreciated the idea of several options, but not too many, which he felt could lead to confusion on the part of students. He did comment about the Singapore philosophy overall: “[I]t’s only one method; [but] it’s a good one.”

During the second interview, Ray shared that he and Sandy had been talking with each other more about the CCSS, and he had also reread the standards and felt less overwhelmed by them than he did about the sheer number of the current Ohio grade 5 mathematics indicators. “But,” he added, “now you really have to think about the depth.” He also reflected on his learning from the Singapore workshop: “I’ve calmed down a bit – but I still am very impressed… to read the introduction [to the CCSS], that’s what they just said at this workshop!” Ray added, “It’s almost you’re teaching ‘til – ad nauseam, [students] can’t take it anymore. And they can do it in their sleep.” He had started using some of the quick warmups in one of the books he had bought, and he and the students were enjoying these and finding them helpful in reviewing mental math skills.
During the final interview, Ray described several day-long workshops with the county mathematics consultant in which the fifth grade teachers had participated. Although he did not particularly comment on the SFMP or the goals of the CCSS as he spoke, he noted that they had spent quite a bit of time exploring the progression of content from grades three to five and becoming familiar with what fifth graders should understand by the end of the year. He felt that this time spent had been valuable and felt that more teachers were beginning to understand the “paradigm shift” that should take place with the enactment of the CCSS in their classrooms. In addition, he continued to share his excitement about using some of the Singapore math elements in instruction “daily” and said that the workshop had been “perfect timing” for him in preparation for the transition to the new standards.

Pages 3-5 of the CCSS

Ray’s interpretation of the overall idea of pages 3-5. When discussing page 3 in the introduction to the CCSS during interview 1, Ray said, “I love – there was a quote, in the introduction that we cannot do what we’ve done before and call it new.” He stressed that he appreciated how it was written, as well as, “I think it goes to the whole big paradigm shift of how in the United States we teach children mathematics.” Ray used the words “paradigm shift” numerous times during the study to describe the change that he felt would have to occur in teaching in order to enact the CCSS successfully. He appreciated the “simplification”, saying there is “no substitute for focused standards.” He added, “just because there’s less, doesn’t mean that you’re not rigorous,” but he also remarked, “I thought that could be subject to interpretation, which is fearful.” I believe,
given his later comments, that he meant that teachers might believe that they had less to
teach because fewer standards were required in each grade. Ray also said that he had
highlighted the text related to number relationships and said that these relationships are
the “crux of mathematics” but that geometry (spatial sense) was important also.

Ray’s comments in the second and third interviews about these pages were quite
similar in nature. During interview 2, he maintained that the authors were saying that
“we need to change the philosophy of teaching mathematics in the United States” and
that the curriculum will be “more comprehensive; it’s not gonna spiral anymore”. He
was concerned, however, that people would overlook these pages and only read the
content standards, and he argued that “people can’t skip the intro because this is where
the authors justify why we need to change”. During interview 3, Ray commented,
“[W]e’re going from our curriculum being a mile long – that’s such an overused term -
but I think it’s true. That we’re teaching for the depth…”. Ray added that though skills
were still important, “we’re not just focusing on skills”, and “the crux is problem-
solving”. Further, “[A]s teachers, we can’t be doing the same old, same old – that’s
paramount in the introduction. I mean, the big theme is, you can’t do what you were
doing.”

Ray’s interpretation of conceptual understanding of key ideas, and using
organizing principles to structure ideas (page 4). Overall, Ray interpreted this text in
terms of supporting students in developing conceptual understanding. In interview 1,
Ray responded to this text by saying, “I think they’re getting to the heart of the skills…
the skills still have to be there, but they have to be done to such an extent where the kids
don’t even have to think about it.” Ray noted that students should be able to use vocabulary correctly, “decompose numbers”, and “put them back together”. During the second interview, Ray said, “I think that they’re really stressing here that you have to use what’s appropriate for each individual child.” (I am unsure where Ray saw this idea in this text.) Ray wished to support students in choosing and using a variety of manipulatives or other tools depending on what their thinking was about a given problem. This seemed to be somewhat related to the statement on page 4 in that he wanted to stress conceptual understanding by using manipulatives. He also noted that high-achieving students may need to be challenged more than others to justify their thinking conceptually because they often tend to rely (successfully) on rote methods for finding solutions to problems. He added, “Study the learners, study what they’re gonna need, make it readily available for them, so that’s kind of how I interpret that.” In the third interview, Ray said, “[T]hey’re stressing that skills are important… however, it needs to be taken to that depth”. He added, “[I]n the principles, it even states – the collaboration. That is still SO important.” That is, Ray wanted his students to see problems from others’ perspectives and to justify their own thinking, and he felt that creating flexible groups of students working on different skills could allow this sharing and proof to occur more readily.

So, Ray seemed to feel that this text indicated that conceptual understanding was tied to depth in instruction and that focusing on individual students’ thinking would help teachers to move all students forward in their understanding. Ray did not specifically
mention “organizing principles” at any time, but he felt that decomposing and recomposing numbers were important in students’ experience.

Ray’s interpretation of mathematical understanding (page 4). In each interview, Ray adopted a slightly different perspective on this text. During the first interview, Ray reflected on this text by saying, “Children not only being able to justify and defend but understand and prove… at an early grade level, to be able to prove their solution… that’s kind of how I interpreted that, as opposed to just memorizing.” He noted, “[T]his is usually the discussion in any workshop approach.” In interview 2, Ray began by talking about the difference between teaching for depth and teaching simply for mastering a skill at a surface level: “I think, again, they’re looking for the richness that we don’t currently have. We have the superficial aspects of mathematics, and then, for God’s sakes, we teach it over and over each year.” In addition, “[T]he quality of your instruction and your problems that you are presenting to them have to be much more rich so it sticks to the deeper [points to his head] – to the, you know, deeper level of memory.” Ray continued, “I see this too that I’m guilty with my top kids. Because I know that they can do it, I think I give them a break… now, it’s just an aha moment reading this again, I’m thinking, you know what, I really need to start looking, would it be appropriate, for those to move them to another grade level – in regards to the [OAA].” Here, Ray seemed to think beyond the text and to consider what he could be doing for students who already understand the concepts and who can justify their thinking. He added, “[W]hen they’re saying mnemonic device [pointing at standards], and that’s good, and I have those too… all those little mnemonic – they’re helpful.” During interview 3,
Ray said of the text, “[T]he big crux here is conceptually – how do you take kids… coming into fifth, they’re not at formal operations yet. How do you get them to go beyond… going to Vygotsky, the zone of proximal development… what do I need to get this child up to the next level of thinking? I think that’s what it really hits a lot.”

From the beginning of the study to the end, Ray’s interpretation of this text seemed to shift from focusing on the need for students to understand concepts at a deep level to the teacher’s role in moving each student to deeper (or higher) levels of understanding, depending on what the student understood at any given time. Perhaps Ray was thinking more about the actual enactment of these standards toward the end of the study and about students who enter fifth grade with different levels of understanding of concepts and skills. To support the development of understanding, Ray felt that teachers should provide “rich” experiences for students at all levels.

Ray’s interpretation of standards for a diverse population of learners (page 4). In responding to this paragraph during all three interviews, Ray very clearly demonstrated his enjoyment of working with students with special needs and his belief that all students should have opportunities to “succeed”. During the first interview, he commented, “I currently have special ed children, you know, autism, Asperger’s; I love them because it teaches the other kids patience… and the world is made up of many different individuals.” He seemed to interpret a “diverse population of learners” as students with learning or cognitive disabilities; he did not refer to any other types of diversity among students. Perhaps this was because he did not tend to work with students
in other population subgroups (e.g., students with physical disabilities or students from other cultures).

In interview 1, he remarked that “we need to look at children as individuals”, and “you do what you need to do to get the job done for all children”. He shared that he felt skills groups were helpful in meeting students’ needs, that he paired students to learn from each other, and that concrete representations of concepts were helpful in supporting students’ learning. Of the text, he said, “I think this is all about the differentiation that we still have to do. And you should never give up on it.” During interview 2, Ray pointed out the authors’ recommendation that the CCSS should include the “widest… range” of students in full participation in the standards. He noted that students at all levels can have good reasoning and can learn from each other, and he felt that his role was to be “prescriptive in the whole range” – meaning that he should provide experiences that supported students at all levels of thinking. Ray added, “I think it’s going into creating the questions that are real world, twenty-first century… normally, when you do that, the interest level’s built right in.” So, he also viewed addressing a diversity of learners as providing engaging activities that would be interesting to students. Finally, he remarked, “I think they’re well written, and that’s why these I think can be more successful.” That is, well-written standards should lead to more success for teachers and students – so he believed the standards were achievable when implementing the practices he described, even for students who had often struggled with mathematics. In the third interview, Ray maintained that the text indicated that all students should have equitable opportunities to learn and that he was excited about being able to provide these opportunities in
collaboration with the intervention specialist who worked for part of each day in his classroom. He reiterated that all students could learn from each other.

In general, Ray spoke more to the implementation of the ideas in this paragraph (i.e., how he supported children with diverse learning needs) than the text itself. Still, he seemed to believe that the authors were indicating that all students could learn mathematics at deeper levels, even if some of the students did not show the same degree of growth that others would during one school year. Also, he seemed to feel that the text implied that teachers were responsible for facilitating this growth. His comments remained quite similar throughout the study.

**Ray’s interpretation of learning progressions (pages 4-5).** Ray’s interpretation of “learning progressions” became more refined between the first and second interviews. During the first interview, Ray offered the following analogy:

Grade K – we’re all going down the road of education – some are on tricycles – some are in sports cars. Learning progression… our job is to try to keep everyone, you know, on the road, not off the road, completely off, going somewhere – but to those who are on tricycles, maybe we can bump it up to a two-speeder. Or a ten-speeder. Or even a Volkswagen – something a little slower, and then to keep those guys in sports cars to use rocket fuel to continue…

At this time, Ray appeared to view the term “learning progression” as an *act* in which *students* were participating. He described the idea that all students were moving toward the same goals (perhaps almost literally “progressing” in their “learning”), but that they
would need different ways to reach these goals in varying amounts of time. He felt that teachers had a responsibility to provide the necessary experiences that would allow students to do so.

However, in the second and third interviews, Ray seemed to view a “learning progression” as a means of organizing content in a way that would best fit with students’ learning needs – taking into account that not all students would learn the content in exactly the same way or at the same time. He noted that he did not believe that the content standards were perfect, and he felt that the authors were also acknowledging that the standards might change somewhat in the future. In interview 2, Ray shared, “I think this is it. So, but we will need to fine tune, and we’re finding gaps, I’m sure, and it’s not permanent, but it’s not set in stone, but it’s a living, breathing document, and I think we need to allow for those variances in developmental issues that we have to deal with as well… We’re gonna have to revise some of them, and tweak, but to me, that’s a great relief!” During the third interview, he added:

I like the fact that [the standards] always tell you where [students are] coming from, and where they’re going, and I consider that part of that learning progression… as a teacher, if your kids are behind, and you can follow… you can even take the concept that you’re teaching, and then just bring in new avenues from there, tweak it here, and that’ll spin everything new for those kids who are really “getting it”.

Thus, Ray was discussing how a teacher could use the sequences of concepts and skills presented in the CCSS to better individualize instruction for students with different needs.

**Review of how Ray’s interpretations of pages 3-5 evolved during the study.**

In general, Ray’s interpretations of the text on pages 3 through 5 did not change a great deal from the beginning of the study to the conclusion. However, one exception to this was his idea of “learning progressions”, which changed as described above. Similarly, his interpretation of mathematical understanding also shifted in focus, from a need for students to develop this deep understanding to a need for teachers to provide experiences that would allow students to develop it. Throughout the study, Ray reflected often on the teacher’s role in enacting these standards with student learning in mind, and his discussion of these introductory pages involved more of a focus on the teacher as the study progressed. This may have been because he was thinking more pragmatically about changes in his teaching that he wished to implement when the new school year began. It is also clear from Ray’s comments that he continued to reflect on the standards on his own when we were not gathering data for this study, and this likely helped him to consider additional implications of the goals therein.

**Ray’s Interpretation of SFMP 1**

**Title: “Make sense of problems, and persevere in solving them” (p. 6).**

Throughout the study, Ray’s comments on this sentence focused almost exclusively on the second portion of it, related to students’ perseverance in problem solving. Although he commented at other times on students’ habits related to “making sense” of problem
situations, he basically did not address this idea in discussing this text. He viewed perseverance as students’ willingness to pursue a solution to a problem.

During the first interview, Ray remarked, “I know that attention span’s gonna be a huge issue…” In the first group discussion, he added, “You know, my struggle at this age, fifth grade, is, you know, they don’t stick to anything.” Interestingly, though, when Carrie shared that she did not see the same difficulty with perseverance in her first graders that Ray and Sandy were describing, all three teachers began to wonder if this was due to fifth graders’ access to (and reliance on) information technology for fast, easy answers to questions. Ray suggested that Carrie’s practice of accepting and demonstrating value for all students’ thinking might help her students to be more willing to persevere, with which Carrie agreed. Ray seemed to be trying to utilize a version of this practice during both videotaped lessons, in which he encouraged students to applaud for each other as they shared answers with the entire class.

During interview 2, Ray continued to consider this idea; he shared that building students’ confidence in their mathematical thinking was important to fostering perseverance. Further, he came to believe, in thinking aloud during the conversation, that students who struggled were often unwilling to persevere because the work required too much time and thought, whereas high-performing students were often unwilling to persevere because when they were done with tasks, they wanted to move on from completed work. Ray also began to consider students’ interest in tasks and their opportunities for collaboration as factors in their perseverance; he commented, “[I]n
order for them to persevere, it’s gotta be: number one, real world, should be done in cooperative groups, if appropriate…”.

During the third interview, Ray brainstormed another strategy for developing perseverance in mathematics, comparing it to building stamina in reading (reading for longer continuous periods of time). He offered: “You [a student] can only stick with a problem that you’re trying to solve with a partner… well, let’s try to make it a more difficult problem – multi-stage. And, if we can integrate it into the content, so now, we can stretch this problem all the way to a half-hour, forty, fifty…” He continued, “[Y]ou want to get them hooked, and then start expanding the time… persevering – I think if it’s meaningful, if you get that meaning in there, and it’s relatable to them…” Further, “[T]he big crux of number 1 is you providing that rich problem… that’s the catalyst to all this other for standard number 1, making sense and persevering.” He concluded, “[I]n regards to the persevering, I think if your workshop is inviting enough, and that you’re allowing them to use the appropriate tools, that freedom to share and be wrong and OK with it, and maybe make a little detour… I think you’ll get the best out of kids.”

So, Ray was focusing on students’ confidence, interest, enjoyment of working with others, and experience in solving progressively more complex problems as means of helping them to learn to persevere with problems until they were solved. Again, as in the interpretation of some of the text in the introductory pages, Ray’s focus seemed to shift during the study from what students could or would do relative to this text to what teachers should do to facilitate student growth.
“Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution” (p. 6). Just as Ray’s discussion of the title of SFMP 1 did not focus on the idea of making sense of problems, Ray also did not generally speak explicitly about students’ “explaining to themselves the meaning of a problem”. He did, however, share many thoughts about the second part of the sentence, involving “looking for entry points” to problems.

In the first interview, Ray’s first response to this sentence was, “I like the word ‘entry points’… Because that allows for so many different views on solving it.” Regarding working with students to meet this goal, he continued, “I think teaching them to slow things down – how can you enter this problem?… That’s the hardest part now and what we’re going to face in the future at this point… By the time they get to fifth, they’re thinking they can do it all in their head… my brighter kids, they try to go too fast, and I’m trying to pull them back… they make these errors, these silly little errors…”. Ray suggested that asking students to use manipulatives would slow them down more, and he also felt that if teachers from kindergarten on were working toward this goal with students, fifth graders would eventually be able to carefully consider entry points more readily.

During Ray’s first lesson, he guided the students through some multi-digit multiplication and division problems; one word problem was from the materials that he had obtained at the Singapore math workshop. He had previously taught students a Singapore math strategy for modeling division problems, and they were using this strategy to solve this problem. Ray asked questions to encourage students to reflect on
the question being asked, each value, and each step. It seemed that Ray was working to help students understand the meaning of the problem and to consider methods for initiating a solution process.

In the second interview, Ray continued to focus on how students enter problems, and he noted that teachers who are well aware of how each student tends to approach certain types of tasks will be better able to support student learning: “[W]hen you’re kid-watching… this is really telling you, developmentally, you can see exactly where a child’s at by how they enter a problem. And I think one of my goals as we’re going through the school year is to see huge changes in how they enter problems.” He used an analogy between reading and mathematics to contrast students who read literally (and may not make much sense of a math problem at first) to students who read inferentially (and may be able to make more sense out of a problem on their own). He gave an example of a task he assigns for homework very early in the year: “I say, ‘The answer is 21. Now show me some equations or some story problem, something that would’ – so that tells me a lot, developmentally, where they’re at.”

In Ray’s second lesson, one of his primary goals was to help students think about how they were entering the task that he presented, and he reiterated this goal of “entering the problem” many times. At the beginning of the lesson, he asked why it was important for students to be able to explain their thinking in words, and one of the students said, “So you don’t just memorize the answer, but you know what the problem means.” Ray’s response was, “Beautiful. You don’t just memorize the answer; you know what the problem means. And you show understanding. You understand the problem.” When he
asked students to work on the task, he simply asked them to think about it for two minutes first, before they began to work on it (some students, as he pointed out, tried to ignore this instruction). Later, when he asked students to present their solutions, he often asked them how they started the problem or where they began. He also pointed out that many students had different strategies for approaching the task.

Finally, during his third interview, Ray particularly spoke of students learning from each other and becoming more efficient as they see each others’ strategies for entering problems: “…even your low ones might have entry points – might be wrong entry points… this is where your collaboration really hits home… where they really learn to see how other people are entering, and which one is more efficient.” He referred to his own experience as a student when at least one teacher would only allow students to solve problems in one way, and he instead wanted his students to feel confident in their thinking as they learned find their own approaches to problems.

Ray seemed to feel that it was very important for students to be able to “enter problems” in a variety of ways, particularly ways that made sense to them. However, he did not offer many interpretations or examples of what it meant to enter a problem, and this action could be quite related to how students explain the problem to themselves, also a phrase about which Ray shared few thoughts. Given his comments throughout the study, it appears that he believed “entering a problem” meant that students should take the time, when encountering a problem, to think about what they were to find out and what strategy or strategies might be most helpful in reaching an answer. He also indicated that teachers could learn about students’ thinking by observing how they
approached problems. Generally, his interpretation of this text remained fairly static during the study.

“They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary” (p. 6). During Interview 1, Ray’s interpretation of this text seemed to focus on the idea of students’ thinking carefully about a problem before beginning to solve it, probably alluding to the analysis, conjecturing, and metacognition that are described in these sentences. One of Ray’s first comments about this text during the interview was, “[T]his is again, slowing them down to think.” He noted that it was helpful for students to transform problems “into ‘simpler forms’ – why make things harder?” Ray went on to say that he felt that having students use manipulatives and decompose/recompose numbers would help them to meet the goals in this part of SFMP 1. He added, “Once they get this, I think they’ll really have that satisfaction of being successful… It’s gonna be a challenge, though.” He noted again that students needed these experiences from kindergarten on so that they could continue to grow in each grade. In addition to these ideas, at least twice later in the interview Ray referred to the mention of analogies in this text; he said that he felt that analogies were useful for students in making connections to previously learned ideas.
In Ray’s first lesson, although he did not refer directly to the text above, he did ask students a few questions while solving a multi-digit division problem that involved the habits of considering what the solution could be before beginning any computation. He also modeled the practice of pausing with each step to determine whether the previous step seemed reasonable (monitoring and evaluating progress).

Ray’s comments in the second interview were generally similar to those in first, concentrating on the notions of slowing students down to think and using analogies to help students connect their thinking to problems solved in the past. He remarked, “[M]y top kids – they’re the ones who will be the most frustrated when you ask them to slow it down and model… they can skip steps, and they don’t think – they don’t sometimes even realize they’re skipping steps.” Apparently speaking of analogies, he added, “I think that’s something that I’m weak in, that I probably need to do more of, um, but, relationships in fifth – because if anything, the whole theme of fifth grade math, or most of math, is patterns, and relationships. That truly is mathematics.” Further, Ray commented specifically on part of the text, but he seemed to be referring more toward teachers’ instructional practice than students’ mathematical practice: “‘monitor and evaluate their progress’ – oh, and it’s constantly. You’ve got to monitor; you gotta make adjustments as you go. Absolutely.”

As aforementioned, during Ray’s second lesson, he was very intentional about asking students to consider their entry in the task he presented before they began to work. While giving instructions, he said, “Some of you are really – and I know who you are – want to rush right into this problem. But sometimes we need to take a step back, yes?”
And we need to think more, all right, what do I know already? And what am I gonna need to know to solve this problem?” He continued, “[G]ood mathematicians, what they’re gonna be doing, is they’re gonna be slowing down and analyzing the problem so that they can look for new patterns and new understanding.” He also asked students to consider whether the task could have more than one correct solution and whether or not the pattern contained in the information presented must begin with the number 1. They discussed students’ ideas about these questions later in the lesson.

Again, in interview 3, Ray noted that high-achieving students tended to rush through tasks and that asking them to model and justify their answers might help to alleviate this concern. He also added a new idea to those he had previously shared, perhaps related to the references in the text to givens, constraints, relationships, and analogous problems: “I always like those problems where you can take that, and then change one little variable, and it’ll rearrange – change everything… You just don’t want one problem, one solution – that’s something that you really want to avoid. So I think that’s what [this text] [pause] kind of [pause] reminds me of.” Moreover, he noted later in the interview, “I think traditionally, what was wrong in the previous courses of study is we rushed… without the concept behind there… the problem solving aspect of it was shut down immediately.”

Overall, as Ray interpreted this text in SFMP 1, he spoke mainly of helping students to be careful and mindful as they began to solve problems, particularly about what strategies might be useful and what might be reasonable in terms of solutions. This involved requiring them to represent and justify their thinking and helping them to make
connections (analogies) to other problems. During the last interview, he also mentioned that it was valuable to change elements of problem situations so that students could consider relationships among problems, and he reflected that in the past, mathematics instruction unfortunately often focused on speed in finding solutions rather than the thought processes involved. Ray referred very little to the actual text of the statements during the study, particularly to the phrase “givens, constraints, relationships, and goals”, though he did speak to most of the other ideas in the text in some way.

“Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem” (p. 6). Ray’s first response to this statement during the first interview was, “Absolutely. And older kids should too.” Reflecting on an idea that was a theme in his comments throughout the study, he continued, “[A]lways start with some kind of manipulative, and then move into usually symbolic or iconic and then finally to the abstract.” He added, “[T]hat’s important because if they don’t have it, then conceptually it’s not there.” Ray commented that he wanted to allow students more universal access to his math manipulatives, and during his responses to the hypothetical classroom episodes, he mentioned several possible uses of manipulatives (geoboards, tiles, cubes) to help students better understand concepts.

During Ray’s first lesson, he guided students through the use of a visual model for solving a whole number division word problem that he had learned at the workshop on Singapore math. As he did so, he asked many questions about what the parts of the diagram represented in relation to the problem, and students seemed to be able to use and
interpret this representation fairly readily. Ray commented during the second interview that he noticed this also.

Ray did not say a great deal about manipulatives or pictures in relation to this text during the second interview, other than to repeat that older students could also benefit from manipulative use, particularly in proving that their thinking was logical, because it helped to solidify the meanings of problems and solutions in their minds. “There’s nothing wrong – like I said, if you want to justify multiplying fractions, ‘Show me.’” Also, “[W]hen you have to step back, and really think about it, it’s a totally different can of worms.” He did discuss the value of manipulatives and diagrams at other times during the interview.

During the final interview, Ray spoke at length about using tiles, arrays, and graph paper to support students who were struggling with whole number division (in response to one of the hypothetical classroom episodes):

I would pose a problem – ‘Now, here is a great resource for you – use these tiles.’ So I would begin with arrays immediately. Physically, with graph paper, maybe some of the kids don’t need this. They can figure it out, but they need to provide a proof. And that’s where I would go with that graph paper is to provide a proof… Once they become fluent, and maybe a couple of days, you’ve worked with this, then you begin to introduce more difficult problems with remainders and such… And then just increasing the numbers… when the tiles become exhaustive, just the
sheer number, then we’re gonna start representation, and there’s a little bit of your algebra right there.

He also described using arrays and 10-by-10 grids to help students learn the concept of a decimal: “I would start with the arrays, um… see, and you’re gonna spend a long time on tenths, hundredths – just tenths… then when you start adding whole numbers with the decimal, color in that hundreds array.”

Clearly, Ray was accustomed to thinking about and using visual representations and manipulatives in his classroom, and his interpretation of this sentence in SFMP 1 seemed to consistently reflect each explicit aspect of the statement throughout the study.

“Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, ‘Does this make sense?’” (p. 6). During interview 1 and the first group discussion, Ray responded to this text primarily by discussing his practice of asking students to check their computation using inverse operations, which he seemed to view as the “different method” mentioned in the sentence. However, he noted, “I’m guilty of this, where I teach an algorithm -- if you add, you check your answer with subtraction… I shouldn’t present it that way.” Seeming to address the second portion of the text, he said that he wanted students to be able to decompose and recompose numbers more in their heads so that they could determine whether answers were sensible. In the first group discussion, Ray commented that this habit was difficult to develop with students who preferred “instantaneous” answers, as fifth graders often did.
In the first lesson, Ray emphasized the practice of using a different strategy to check the answer to a whole number computation problem. He demonstrated examples of decomposing and recomposing numbers mentally and of what he called “sidebar” division, which involved subtracting groups of the divisor from the dividend until 0 remained. Students were observing as Ray talked through these examples. At one point, Ray asked students if an answer made sense.

During interview 2, Ray spoke of students’ frequent reluctance to check their answers using different methods – both high-achieving students, who tended to believe that they were correct and that there was no need to check, and low-achieving students, who tended to be glad just to have single answers. This often occurred on tests, where students struggled even with making reasonable estimates. Still, he described how he tried to model the process of determining whether answers were reasonable: “I do a lot of modeling with that, especially with [fractions] – ‘Does that answer make sense?’” He continued: “Sometimes I’ll put a wrong answer there that is huge, or something off – ‘OK, so did everyone get the answers?’ [smiles and chuckles] And my kids love to correct me. And I encourage it!” Considering possibilities for helping students develop this practice as stated in the text, he offered, “I think if you approached it in a more twenty-first century skill way, say, ‘Now, would your employer be happy with that? You turn that number – that was the answer – they were gonna order that?’… I think they might get it. Why it’s important.”

In the second lesson, there was little direct evidence that Ray was thinking about this text from SFMP 1; the only statement he made (to a student) that seemed related to it
was: “So we’re all done, and you’re perfectly, absolutely positive that each of those are correct.” Beyond this, he did not encourage students to check their answers or to determine whether they made sense, though he did ask to search for additional solutions to the same task. Interestingly, in the second group discussion, Ray suggested that perhaps “at certain grades… we’re just stressing that non-thinking. So they come with no number sense. That, ‘Wait a minute – this doesn’t make sense.’” Although it did not appear that Ray was stressing “non-thinking” in his lesson (quite the opposite, actually), perhaps he was not thinking primarily about helping students to make sense of their solutions. He commented about this issue during the third interview.

In interview 3, Ray’s interpretations of this sentence in SFMP 1 focused generally on asking students to provide proofs for their thinking, which could address either portion of the text; however, he did not refer directly to any of the text in the statement. For example, he felt that he might say to a student, “‘[I]f you show me one way, show me a different way.’” He added, “[T]hat’s something that I need to present more, but now that I have more time, I think that’s a natural progression… the document camera’s great for that. Having the kids up, and show their work, and provide proofs as if they have to defend it at a college.” Ray also noted that he felt that one major theme of the SFMP was that students should be able to solve problems in multiple ways and to support their thinking.

During the study, Ray’s thinking about this text seemed to shift from the practice of using inverse operations (to check answers) to using multiple methods to solve problems in order to check initial solutions. He did also discuss the need for students to
consider whether their answers made “sense”, and he suggested late in the study that asking students to show proofs for their solutions was one way to accomplish the goals in this sentence of SFMP 1. It may be that his latter ideas emerged after thinking and talking more about the goals stated in SFMP 1 and 3 as the study progressed.

“They can understand the approaches of others to solving complex problems and identify correspondences between different approaches” (p. 6). In the first interview, Ray responded to this text by saying that a student should “try to look through someone else’s eyes and see how they solved [a problem]”. He continued, “I remember the original TIMSS studies, when I started, and that was it. They would present one problem… the kids worked on that one problem the whole math class, and then the rest were spent coming up and defending their solution.” As he spoke, Ray referred to some of the text in SFMP 3 regarding students’ responding to others’ critiques. He added that he wanted to create more time during his lessons for the “workshop” approach – for students to “be able to discuss [solution methods], and share those out with others. ‘Cause that’s where the learning’s gonna happen.” He wanted students to be able to see different ways of solving problems and “make connections” between problems (he may have meant solutions). He did note, “I think [students may] struggle a little bit more seeing a different viewpoint, but I think if I introduce it in the very beginning, that I could get them there… I think that’s more on me and it’s something I haven’t really focused on this year, but it’s something with the new standards that I’m gonna have to focus on.” In fact, during his first lesson, there was almost no evidence that this goal was present in
Ray’s mind because all solutions that students presented followed specific sequences of steps that Ray had taught.

During the second interview, Ray mainly focused on the first portion of the sentence in the text (understanding the approaches of others). He commented in an amused way that sometimes when students shared their thinking, he himself did not understand what they were saying, and other students had to explain the strategies to him. He did not seem to feel (nor do I) that this was a question of his content knowledge, but rather that the children were articulating their ideas in ways that made more sense to their peers than to Ray. He also suggested that fostering an atmosphere of cooperation in the classroom was helping in achieving the goal in the text, and this was a goal of his starting during the first week of school each year; students worked together quite often in various groupings for various purposes. Referring to the text, he shared, “I’m anxious to do a lot of this at the beginning of the year… how far can I take them now?”

In his second lesson, Ray referred to different solution strategies for the task at hand six times, either stating that he appreciated the different methods he was seeing or asking which students had used different methods than those that had been presented so far. He discussed this lesson further in the third interview, commenting that some of the typically high-achieving students were not convinced that one student’s thinking made sense until another student clarified the thinking for them. As before, he added that often these students were “the most closed-minded, and they should be coming up with a plethora of ideas”. He also reflected, “I guess as a teacher, sometimes I’m that way too until I hear them, that I do have to accept even partial patterns that they see.” Further, he
also discussed allowing students with disabilities to participate in the regular classroom as often as possible (as was the case in this class) so that they could learn and understand what other students had done to solve problems: “…only being around peers who are all below – that is one of the worst things I think you can do to LD [learning disabled] children… Because then there’s no one to say, ‘Oh!’ No one to provide that feedback. Even if they don’t get it, that it’s there, and sometimes, it does eventually sink in.”

Throughout the study, Ray generally focused on the first portion of this statement in SFMP 1, involving students’ understanding the approaches of others. His interpretation of this seemed fairly consistent, as he discussed having students share their thinking in small groups and in whole-class discussions. It is noteworthy that he did not share many ideas related to how he would help students to understand others’ thinking other than in the discussion that would occur when students offered comments or questions on each other’s work. He also did not refer at all to the term “complex problems” in the text, so it is unclear whether he considered this as he offered his interpretations. Other than a bit in the first interview, Ray did not speak to the second part of the statement in the text, regarding students’ identifying correspondences between approaches. Primarily, as Ray considered this goal, he said that he wanted his students to see problems from different perspectives and to justify their thinking.

**Ray’s thoughts about implementation of SFMP 1.** Ray’s goals as a teacher centered around helping students to be successful and also to feel confident and safe as learners in his classroom. When he was speaking of SFMP 1 during interview 1, he noted, “Once they get this, I think they’ll really have that satisfaction of being
successful.” This was evident during his first lesson as he reminded students that he wanted them to find and use strategies with which they were “comfortable” as long as they found the “right answer”. Nevertheless, during the final interview, Ray also said, “[I]f some developmentally aren’t there yet, I’ll slow it – it’s no problem if they’re getting to the answer, but the goal is as we progress along the school year, we do want them to become mathematically proficient, choosing the correct – not correct, but a more effective and efficient way of solving problems.” Ray seemed to mean that he wanted students to be comfortable with their thinking and processes but also for them to move, when they were able, toward more efficient strategies that could ultimately be useful for them.

Ray was asked to consider how he envisioned the goals in SFMP 1 developing in his own classroom or school. As described earlier, Ray did feel that the goals in SFMP 1 were worthwhile for students (both in terms of mathematics learning and in life outside the classroom), but he also felt that time would be needed during the next few years for students to gain more experience with these skills. He believed that teachers were responsible for planning and implementing classroom activities – particularly “rich problems” – that would allow students to work toward the goals in the text, such as persevering, making sense of problems, and understanding various ways to approach and solve such problems. In addition, Ray felt that students simply enjoyed mathematics more when they were solving problems that were interesting to them, referring to the “MARS” or “Balanced Assessment” tasks (created by the Mathematics Assessment Resource Service) that he often used with students. Ray also noted that teachers must be
aware of each student’s thinking and progress so that they could plan activities that would support students’ development.

In terms of specific classroom structures that would support students’ growth relative to SFMP 1, Ray maintained that he would need to spend more time in class specifically working toward these goals with students. He felt he would be able to do this because the new content standards were more cohesive and more appropriate for fifth grade in general than the former standards. He believed that he needed to help students to take more ownership of their learning; for example, he said that he could do so by “sitting more in the back, with kids having more control over questions being asked”.

Often, Ray mentioned the idea of a “math workshop”, where he would present one “rich problem”, which he sometimes described as a real-world problem that integrated math with another content area, and students would work alone or “in cooperative groups, if appropriate”, to solve it. During interview 1, he shared, “I think down the road, that we’re gonna get away from the maxi-lessons that we’ve done in the past.” He continued, “[Y]ou can’t start them off with a huge major problem that shouldn’t take forty minutes, or fifty minutes, to solve, but you’re gonna build – Singapore’s really nice for that.” One important part of the workshop format that he emphasized was the “sharing” at the end of students’ work time, and he reflected that he needed to be certain to leave time for this, which he felt he had not been able to do very much in past years when he felt pressured to teach as much content as possible before the OAA. This collaboration among students, Ray believed, would help students who were thinking in different ways to learn from each other and would be a natural means of differentiating instruction.
Ray felt that providing students with many experiences with concrete, visual, and abstract representations of mathematical ideas would be highly beneficial for their learning and in alignment with SFMP 1. Throughout the study, he shared that he believed strongly that he should “always start with some kind of manipulative, and then move into usually symbolic or iconic and then finally to the abstract”. As noted above, he felt that not only did this encourage students to slow down to think, but it also helped them move from concrete to abstract understandings. In a similar vein, he discussed that having students “play” with numbers – decomposing and recomposing them – as often as possible fostered students’ number sense and their ability to consider multiple ways to solve problems.

Student collaboration was another element of enacting SFMP 1 that Ray considered vital in his classroom and in every classroom. He said during interview 1 that one might “see children up at the board, maybe in pairs, maybe in trios, solving a problem – not solving it, presenting an argument on how they solved a particular problem.” In the second interview, he remarked, “I’m real big on cooperative groups. My first PD [professional development] workshop I ever went to was Spencer Kagan [a well-known researcher on cooperative learning]… it was the best workshop I think I’ve ever taken!” Ray also discussed his practice of asking students to teach a concept that they had learned to the rest of the class in connection with SFMP 1: “[T]hey have to come up and teach one; other kids have to solve it, so we do a lot with that.”

During the third interview, Ray briefly discussed the issue of reviewing homework in class and possibly grading it; he was thinking about this in relation to
finding time in class to ask students to check their solutions to problems using multiple methods and to consider whether their answers made sense (goals in SFMP 1). He explained: “When you were rushing against the clock, you’re trying to go over their homework – because my philosophy was, you did your homework, then I owe you to go over it fifth – it kind of becomes, ‘Are we getting graded on this?’ Everything becomes, is it about the grade? And I don’t want that to happen… because I think it hinders thinking.”

**Review of how Ray’s interpretations of SFMP 1 seemed to evolve throughout the study.** Ray thought a great deal about students’ perseverance in mathematics learning, and much of his reflection during the study was related to how to help his students continue to work through problems with which they were presented. He believed that the more that students enjoyed their work and found success in it, the more confident they would be, and the more they would be willing to persevere. As noted earlier, during the study, Ray’s thinking seemed to move from what students would do when presented with the goals in SFMP 1 to what teachers could and should do to facilitate students’ progress with these goals. This was likely because the new school year was fast approaching by the end of the study, and Ray was literally in the process of setting up his room when we conducted the final interview. Also, Ray’s idea of students’ checking their answers evolved from using inverse operations (which he had encouraged in his class prior to the study) to using and understanding multiple methods (included in the text). This shift may have occurred as a result of the discussions about SFMP 1 during the study. Finally, in the second and third interviews, Ray began to comment
more about the different sorts of challenges that he thought typically high-achieving and low-achieving students might have with the goals in SFMP 1, which reflected his ongoing goal of meeting students’ individual needs through intentional assessment and instructional planning.

As shown earlier, in many cases Ray focused on specific words and phrases in the text of SFMP 1 and seemed to ignore others. The text on which he commented remained relatively consistent during the study, so it is likely that he did not think much, if at all, about the other portions and how they might impact his interpretations or his classroom practice. This may be an important implication for professional development, particularly since Ray was generally very devoted to his own learning, because he might not even have been aware that he essentially overlooked several parts of this text.

**Ray’s Interpretation of SFMP 3**

**Title: “Construct viable arguments and critique the reasoning of others”** (p. 6). Throughout the study, as Ray discussed this statement, he generally described students’ sharing their thinking with each other, often in whole group situations. He seemed to be focused on the word “critique” and the negative connotation that this might imply for students’ interaction with each other. He did not, however, say as much about the act of actually “construct[ing] a viable argument” – he spoke more of students’ *presenting* their thinking about problems once they had found solutions.

During the first interview, Ray commented that this goal suggested “having kids come up and share how they solved” problems, which he also noted was “probably cut most often” for the sake of time in his class and others. He related this goal to the last
portion of SFMP 1: “They can understand the approaches of others to solving complex 
problems and identify correspondences between different approaches.” He continued,
“[T]his is just getting back to critiquing, and listening to all views... teaching them all to 
be kind during that whole time; you don’t want people flying from the room crying.”
Ray was concerned for some students: “I worry for some kids – I think they’ll be gone. 
At the beginning, we’ll lose them.” However, “When I think they see their peers, and 
they start getting into it... when they start feeling success, then they’re on board... 
Because it’s like, ‘I am good at math. Let me see what he did here. Wow.’” In general, 
Ray felt that teachers would need to support students in thinking reflectively enough to 
meet this goal: “…that you can devote this time to think, slowing them down, and that’s 
gonna be, that’s gonna be hard. But I think it can be doable.”

As in the first interview, during the first group discussion, Ray again expressed a 
desire for his students to be respectful to each other as they responded to others’ thinking. 
He used the phrase “disagree agreeably” to describe the behavior that he wished his 
students to exhibit, and he commented, “I need to do more of that in my classroom – now 
that I see these standards.” He remarked that “disagreeing agreeably” was sometimes 
difficult for fifth graders because they often wanted everyone to follow one set of rules – 
thus, some of them also wanted everyone to follow certain “rules” in solving mathematics 
problems and had trouble accepting different ways of thinking. At this time, though Ray 
was speaking basically about students’ “critiqu[ing] the reasoning of others” and not 
about students’ constructing their own arguments, the latter was implicit since students 
would need to present their arguments in order for others to critique them.
At the beginning of his first lesson, Ray reminded his students that he always expected them to “disagree agreeably”, and he continued: “Well, part of the new standards – a big section of it – is how as classmates and peers do you disagree agreeably? So, any ideas - let’s get some ideas out here. Raise your hand if you have any idea how can you critique someone’s work – especially if you think it might not be correct – without attacking them personally…” One student volunteered, “I think you got that wrong. Here, try it this way.” No others volunteered, so Ray told the class that they were going to “practice this today”, and he added, “[T]he new standard says that you should be able to critique other people’s work without being hostile, or … degrading. You know, where someone at the end feels bad, and then they’ll never come up to the board or share out again.” It is interesting that Ray described this potential reaction, because he had shared in the interviews that he reacted in essentially the same way to a teacher who had not accepted his thinking as a student. When reviewing the homework problems, which involved multi-digit multiplication or division, he asked various students to present their work on the board, and he asked other students to show their thumbs up or down depending on whether they agreed with the work or not. Most students had used the standard U.S. algorithms to find the answers. Ray asked each student several questions to clarify his/her explanation while he/she shared, and he offered his own comments before he asked the rest of the class to indicate their agreement or disagreement. At one point, one student laughed when most of the class disagreed with a solution, and Ray asked this student, “Remember my whole talk about disagreeing agreeably, and not laughing?” The student stopped laughing. After the next
task was presented, Ray said, “Excellent. Thumbs up or thumbs down? Well, especially since I said excellent…”. That is, Ray realized, at least for a moment, that since he had already indicated his own agreement with the solution, the students were likely simply to agree also. The lesson continued in a similar format; Ray presented problems for the students to solve using strategies from Singapore math, and he continued to comment on (or critique) students’ presentations as they shared them, rather than asking other students to evaluate them first. At the end of the lesson, Ray returned to the theme of “disagreeing agreeably” and reminded students that this was important for their classroom interactions.

One of Ray’s first comments about this text in the second interview was a reflection on his first lesson: “[T]he videotape was eye-opening for me. Because the biggest thing was like the whole thumbs up [moving his thumbs up and down alternately] – I thought, ‘Oh, God, this is so pathetic.’ It’s not the depth that it needed to go… you don’t know if they’re understanding.” He added, perhaps to indicate that he wanted to try to involve all of the students more in responding to each other, “I usually just have discussion, but I miss people. And that’s the problem I have with it… Somehow, everyone should be able to – ‘I don’t know if I like that’… They all should be internalizing that as a group, as opposed to a few who are into it… So my goal would be to get more into that.” He did say, though, “But they actually are pretty good about critiquing one another… As long as you model it for them.” I believe that he meant that students were usually respectful and helpful in their comments to each other as long as they understood what he expected. He reiterated that he believed that this was because he stressed this type of interaction from the first week of school and used many structures
for “cooperative learning” in class, though he also remarked that since this was a “nice group”, he might have even been able to “take them farther with…the critiquing”.

Shifting his reflection away from the lesson, Ray gave other examples of how he could address this goal in class. For instance, “I like some of those – ‘Little Bobby had a calculator; he ended up with this answer – can you guys figure out – is it true?’ And some of the actual [OAA] questions are formulated that way.” In addition, when he was responding to one of the hypothetical classroom episodes, he referred to this text on his own, saying, “I think that would go with the standard being able to critique, and then also to step back and look at their own – which goes into what – number 3?” This seemed to indicate that he was becoming familiar enough with the text to relate it to classroom scenarios unprompted. He did offer one thought about how the act of “constructing arguments” about mathematics problems reflected real-life skills: “I’m a real-world person, where in like reading, when we finish a book, I don’t go and make a diorama… I want the kids to talk about it. That’s the thing I finish – ‘Oh, my gosh! What’d you think about’ – that’s how we do it as adults – the same in math. I want that to be realistic.” Ray’s final comments about this text during the second interview were: “That’s the – key, that’s when children feel safe, and they’re not afraid to go into the third standard of critiquing, taking criticism… my kids – they know not to ridicule anyone who makes a mistake – oh, they know, because I don’t tolerate that.”

During Ray’s second lesson, he seemed to be considering his reflections on the first lesson, because he never asked students for “thumbs up or thumbs down”; instead, he simply asked for volunteers who had comments or questions for students who had
presented their work on the task that he had assigned, which involved finding simple patterns in number sequences. The students – at least those who chose to be involved in the discussion or presentations – did seem to feel fairly comfortable with discussing each other’s ideas and their own. During the second group discussion, Ray shared his reflections on this lesson: “I thought, you know, after looking at my second video, I’m like, ‘What was I thinking?’ It was already in place! My kids – we had just natural conversations about mathematics. So [puts head in hands briefly] I don’t know why all of a sudden I thought I had to start something new. With that. That was a big aha for me; I’m like, ‘They’re already doing this!’ Great discussion.” So, Ray realized that he did not have to change as much of his classroom interaction as he had thought he might (based on SFMP 3) because the idea was “already in place”. This could be a valuable implication for professional development.

In Ray’s third interview, he first shared more reflections on the second lesson. He remarked, “I think you saw that in the video, with the locker problem, that we had a variety of kids coming up and showing different… different discussions.” Regarding one typically low-achieving student who shared an idea that many students did not feel was reasonable, he said, “I was sad to hear some of them kind of pooh-pooh her… I don’t think they were mean to her, though… Some of them just said, ‘I disagree.’ But she took it personally.” Thinking more generally about how to enact this standard in his classroom, he continued, “[I]n fifth grade, it’s nice because you can have just natural dialogue with kids about being able to critique… we always do role play, and it’s like what would be appropriate, what wouldn’t be appropriate… me modeling, it’s like, ‘OK,
I see where you were going, but here’s where you lost me.’’ He added, “Try to keep comments as a teacher as neutral as possible… however, praising heavily when someone goes and takes a risk… even if it’s wrong… that builds them up, too… They all want to do well, inherently.” Finally, he observed, “[B]eing able to take a chance, take a risk, learn from it, change it – whether you were right or not… For number 3, I think that’s a big… big part of it.” So, his focus in this interview, as during the rest of the study, was on how students responded to each other – ideally respectfully – while sharing their thinking, not so much on how they went about constructing arguments initially. His major shift in thinking seemed to be after the first lesson, when he found that he could support this goal by continuing to encourage positive discussion among students rather than introducing a new (and perhaps less productive) practice in class.

“Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures” (p. 6). Throughout the study, it was somewhat difficult to interpret Ray’s comments about this text because, for the most part, he spoke very generally about it. In fact, during the first interview, he actually was reading the subsequent portion of SFMP 3 when I was asking him about this text, but I did not realize this until I was transcribing the interview because he was mentioning relationships to various parts of SFMP 1 and 3. So, the first comments that were clearly related to this text were from the second interview.
During interview 2, Ray’s initial response to this text was, “[T]hat’s a tough one… this is so high-level… In regards to thinking.” He continued, “[S]tated assumptions, definition, um, results and tying those into analogies – that’s all doable… But, a logical progression… it’s tough to me…” However, later in the interview, he remarked, “[L]ogical progression… yeah, I think this is very doable. I think we could do that. I need to do better at that.” Ray may have been rethinking his earlier comment, and/or he may have been unsure about what a “logical progression” meant. He also said, “And the simplest form. Is that a true statement?” Presumably, Ray meant that he felt that the simplest form of a logical sequence of ideas was a single statement to be proven true or false. Later, while discussing one of the hypothetical classroom episodes, Ray noted that the students should “define” the “rules” or “characteristics” of the geometric shapes that they were studying, but he did not relate this explicitly to SFMP 3.

During the third interview, Ray spoke mainly about students’ constructing proofs in relation to this statement. He offered, “[W]hen it’s saying ‘logical progressions’… where the kids are developmentally plays a bit part of that… Sometimes it could be a one-step – just going from this point to this point, and solving something, the rest might be all wrong, but let’s focus – ‘That was good, what you did right here… Others can be all the way to the end, and solved it beautifully.” Ray suggested that having students at about the same level of thinking work together and use manipulatives and diagrams would be helpful in addressing this goal.

So, Ray offered very few direct interpretations of any parts of this statement during the three interviews, and he did not refer to the text at other times during the
study. He seemed to be focusing somewhat on the idea of constructing arguments, and in
interview 3, he shared some examples of how a teacher might support students in doing
so – so he was thinking a bit more specifically about this aspect of implementation.
However, he still appeared not to have a very clear sense in his own mind about the ideas
in the text.

“They are able to analyze situations by breaking them into cases, and can
recognize and use counterexamples” (p. 6). During interview 1, Ray read this text and
said, “[A]gain, that goes into decomposing, looking at different representations of solving
the problem… showing their model, how they solved it… accepting critiques…” He
seemed to be alluding to the terms “analyze” and “breaking” as he spoke of
understanding others’ thinking and decomposing (respectively), though the authors’ use
of the term “breaking” was not really referring to numbers. Then, he considered a
classroom situation and suggested, “[W]hat would be cool – what if we doubled that
number? What do you think? Where they could even extend it into other areas would be
so cool.” Again, this seems to be connected to decomposition and analyzing problem
situations. He also said, “[U]nder that whole counterexamples – and again, breaking it
down to analogies, and to previous problems presented…” It was unclear what he meant
by this thought at that time, but in interview 2, it became evident that he viewed
“counterexamples” as familiar problems to which students could relate new problems that
they encountered – perhaps through the use of analogies.

In the second interview, Ray noted, “[C]ounterexamples – the more experiences
we give them, the more they can draw from… I think that’s our jobs as math teachers is
to present a wide variety and array…”. Then, he shared the following, referring to the text: “That’s how I interpret that one – into finding the known and stretching them out slightly – stretching out, but still having those anchors that they can hook upon, counterexamples – and as they progress, it’s almost those common problems that we’ve solved together as a group, and listened to that we can make those connections.” Given these comments, it is interesting to consider what may have been his full interpretation of the text at this time; perhaps he really viewed these “anchors” (“counterexamples”) as “cases” (i.e., types of problems). He added that he might like to present one such task per week for students to solve and then to add visual reminders of these problems around the room, for students’ reference, as the year progressed.

Ray’s interpretation seemed to be similar in the second and third interviews. In interview 3, he shared, “I think that goes to patterns, and previous problems… analogies… ‘Remember that problem we were solving?’ And as teacher, always trying to reach back and make connections… ‘Do you remember when little Sam solved it that way? How would that relate to this problem?’… Being able to take this static problem and stretch it.”

Thus, Ray generally did not seem to interpret this text in the way that it was likely meant by the authors, as his references to various terms and phrases in it appeared to refer more to other elements of mathematics and instructional practice. However, his interpretation did evolve a bit during the study as he thought about how he would address the goals that he inferred from the text in his classroom.
“They justify their conclusions, communicate them to others, and respond to the arguments of others” (p. 6-7). Ray’s first comment in interview 1 about this statement was, “I highlighted this,” indicating that this goal was significant to him from the beginning of the study. “Just being open to hearing other people’s – talk about what you’ve presented, and then for you to present… if kids are given the opportunity, they will be kind. Especially when they know they’re gonna be up too…”. As he considered how he might enact this goal in his class, he said, “If I had to guesstimate – 15 to 20 minutes of your math class is that sharing. Because [the standards] went into about arguments – not arguments, but in a positive way – being able to defend your theory or how you solved it, so I think that – personally I need to put that back, me less talk, they more talk.” So, from the start, Ray seemed to feel that justifying one’s conclusions meant “defend[ing] your theory” and that constructive communication among students (not just with the teacher) was important in addressing this goal as well.

During Ray’s first lesson, he asked various students to present their work on several of the tasks, as earlier mentioned. He told one student who was going to explain her work: “And that way you’re going to defend your answer.” As students shared their strategies, Ray asked questions of them, and they responded to him and also to a few other students who were asking questions also.

In the second interview, Ray primarily shared that he felt cooperative learning opportunities helped students practice the skills mentioned in this text. He noted, “[T]hey have to feel comfortable, they have to feel safe… that’s why I start that whole first week of school with cooperative groups… We come up with classroom rules together, the
consequences together, we have classroom meetings, I demo that…”. He added that students would gain better understandings overall “if we spend time explaining the why”. So, rather than saying much about the text in the statement directly, Ray discussed how he felt he promoted these goals in his classroom by establishing an environment where students felt comfortable to communicate, respond, and justify (presumably). He also reflected on the first lesson: “So for that particular standard – there was some good dialogue, um, toward the long division…”.

In his second lesson, Ray began class by asking, “Why is it important to be able to write your mathematical thinking in writing?” One student, as aforementioned, shared that this would help them to understand the meanings of problems. After students worked individually on the task for a few minutes, Ray asked them to “turn and talk” to neighbors to share their strategies. He walked around while students were talking, and he asked questions like, “How do you know that? Can you prove it?” Then, while students presented their thinking to the class and others asked questions, Ray acted as something of a mediator among students as the dialogue progressed, mainly for purposes of clarification because students were quite respectful to each other. The three goals stated in the text were easily identifiable in this lesson.

During the last interview, Ray continued to stress the idea of “proof” as he felt it related to this text, and he discussed his own goal of incorporating proof into his class much more during the following year. He was considering different ways for students to communicate their thinking and respond to each other, including “quick writing” and even blogs. He suggested that he might pose a question on a blog such as, “Last year, I
had a little kid named Susie. And she solved this problem this way. Can you – what do you guys think about this?” He noted that he could even use this as an assessment, and he added, “I’m kind of excited about that – to get that thinking, and the arguments of others, because with the blog, that’s a natural place.” Ray remarked that sometimes his students were so “sweet” that they were “afraid to disagree” with each other, but he felt that if he modeled how to respond to someone, presumably both in person and online, then students would gain confidence in doing so.

During the study, Ray began by interpreting this text with a focus on students’ sharing their work in whole class discussions and generating dialogue about their thinking, which is also how he facilitated some parts of his first lesson. In the second interview, Ray talked more about students working in cooperative groups in order to gain comfort with communicating with each other, and the structure of the second lesson reflected this since students worked in pairs and then participated in a whole-group discussion. Also during the second lesson, and during the final interview, Ray began to use more language related to “proof” and emphasized that he wanted students to work more on justifying their solution processes as well as critiquing and responding to each other’s thoughts. He also listed several ways in which he might provide these opportunities for students. Thus, Ray’s thinking about this text seemed to move from students’ simply sharing (demonstrating) their thinking to students’ communicating with each other about their thinking, as well as showing how their ideas were logical. This shift seemed to occur as Ray taught the two lessons, discussed the text in the interviews, and planned for the new school year.
“Mathematically proficient students are also able to compare the
effectiveness of two plausible arguments, distinguish correct logic or reasoning from
that which is flawed, and – if there is a flaw in an argument – explain what it is” (p. 7). Ray initially seemed to focus on the first part of this text and particularly the phrase “effectiveness of two”, because he appeared to be interpreting this goal as students’ comparing the effectiveness (or perhaps efficiency) of two *reasonable strategies* for solving a problem (rather than *plausible arguments*). He began his comments about this text in interview 1 by saying, “[T]hat’s the biggest struggle that we have with our little low guys, in regards to, just choosing one way to solve… we want to open them up… I think it’s a good learning opportunity for them to see highly successful math students and how they can look at [problems]…”. He then added a similar comment about high-achieving students: “[I]t’s a struggle for them sometimes at this point too, to look, because sometimes they get in their mind – ‘Ooh, look, I solved it; I’m done. Let me be done with this problem.’” He remarked that many of his students played video games on a regular basis, and he commented that video games generally lead players from one point to another without ever actually having the players return to consider other means of arriving at the same point. He felt that this might affect his students’ willingness to consider alternate strategies in mathematics class.

As noted earlier, in the first lesson, Ray asked students to share and comment on different methods for solving the problems assigned. However, as most students used algorithms to solve the problems, there was not a great deal of opportunity for them to consider logic-based arguments – just steps in the algorithms and whether they were
performed correctly. One might question whether the authors of the SFMP intended this goal to include algorithmic processes or (as perhaps implied by the text) series of ideas that were connected by reasoning.

In the second interview, Ray focused more on the text that involves “distinguish[ing] correct logic or reasoning from that which is flawed” and explaining errors if they are present. He used an example that mirrored tasks that he periodically assigned: “Little Jenny Lou and Bobby added these decimals – someone did it right; someone did it wrong. Who – can you guys figure that out for me?” He gave a second example: “[W]e have document cameras this year. Which are wonderful. I’ll grab some stuff from previous years – ‘Look at what this kid wrote. What do you think about this mathematical thinking?’ Our big thing, our big focus this year, is ‘I know this because.’” He remarked, “I think showing them flaws sometimes is easier than showing them the right… ‘Cause they have to see what they as a learner themselves have been doing wrong.” He said that showing students flaws for them to identify and correct was a “strong tool”. Ray continued:

[H]ere’s a prime example: fifth graders coming in from fourth, they will always add fractions numerator plus numerator, denominator plus denominator… So you have to break that fallacy, you know? That’s sometimes hard. ‘You know, guys’ – I start, ‘Wanna see - most fourth graders – not you guys, but most fourth graders come in – watch how – \( \frac{1}{2} + \frac{1}{2} \) – 1 plus 1 and then 2 plus… [gasps loudly] Let me draw what that looks like’ – and then they’re, ‘Oh, they’re crazy!’
It again appeared that he was seeing a flaw in an argument as an incorrect step in a solution strategy, not necessarily a statement that was illogical based on original assumptions. Ray also commented that he felt he should use more problems like the one above in his class, where he presented hypothetical student work that contained errors, because he felt students would feel “safer” about identifying mistakes when they knew their classmates were not directly affected.

In his second lesson, Ray gave students the opportunity to see each other’s solutions and to respond to each other’s questions and comments; the students approved of most of the solutions and asked questions about those that were not entirely reasonable. This reflected his responses in the second interview.

During the second group discussion, the teachers shared their thoughts about what could happen when students shared incorrect ideas in class, particularly if other students tried to adopt these ideas for themselves because they were not able to distinguish flawed logic from sensible logic. Ray said it was difficult for some students to understand what made sense and might simply believe others because the latter were often correct: “If it’s too open, absolutely!... ‘Oh, oh! He’s a smart kid; he must be right!’” This was definitely a point that teachers must consider as they facilitate class discussion with students sharing many ideas.

Ray’s comments in the third interview were similar to those in the second. He gave a similar example of a task he could assign, related to the goal of finding flaws in thinking or confirming that the thinking is correct: “Little Betty solved it this way. Is that right? Or can you find out what she was thinking that maybe wasn’t so accurate? Or
where she went wrong?” He said he could create these experiences via the blog he discussed earlier, and since he believed that students would see questions like this on the current and new standardized assessments, he felt this work was valuable.

Other than a bit in the first interview, Ray did not say much about the first part of the statement in the text, regarding comparing the effectiveness of two plausible arguments. This may be a relatively unfamiliar practice in fifth grade. However, his thinking about the goals as a whole seemed to expand during the study as he re-read the SFMP and as he considered ways to enact SFMP 3 in his classroom the following year.

“Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades” (p. 7). During interview 1, Ray read this text and said, “[T]his is important too, because so many of the kids are visual learners… not only explaining with numbers, which is abstract, to actually drawing the little figures… even using the document camera to get those … darn manipulatives right there… they still allow the opportunity for all levels… sometimes kids will have a little aha moment, going back to those Unifix cubes.” He referred to the word “elementary” in the text and continued, “I think even the upper – older grades, I think – I don’t think there’s any problem with that… Because there’s times when I have to solve something, I still – I’m a visual learner – I have to sketch it out.” So, even in his own adult experience, he had found that visuals were helpful to solve problems. Further, “So, they say it’s not generalized until later grades… I don’t know if that’s so true… I think that could be extended. Because how great would it be
for someone in Algebra I to be able to do a little seesaw [drawing], with the fulcrum, and go, OK… I think that’s a good thing… And being able to justify it up there, other than just using variables, I don’t see a problem with that. So, I can disagree slightly.” That is, he believed that full generalization (which perhaps he was interpreting as entirely symbolic representation) did not even need to occur in the higher grades; they could still use models when needed.

In the second interview, Ray shared, “I think again, in fifth grade, it’s more of an informal situation, although it can be formalized if you’re gonna do some kind of assessment, like I said – that’d be a great quiz.” He appeared to think of the term “formal” as meaning “official”, as in a formal assessment where students are asked to construct an argument using visuals or manipulatives.

During interview 3, Ray’s first response to this text was, “I think, actually, they could change this, and say can – ‘should’. Should construct arguments.” He reflected on his students and his classroom: “[I]n the past, I know myself, using the manipulatives, other kids would frown about it, and there would be times that I would too, because, ‘Oh, God. It’s gonna take forever with the manips.’” In thinking further about his class for the new year, he said:

Now that I do have more time, and I think that if I show them me – with me using them, that I think – and then having them readily available… a couple times, I’m just gonna have to – ‘You know what? I’m gonna need something. Let me’ – where it’s not taboo… Because the kids don’t want to be embarrassed, to have to get up and use… it takes a strong student,
that will do that, if it’s not easily accessible… if they need it – will just go up and use it.

That is, Ray wanted to ensure that his students felt comfortable using manipulatives, so he planned to model this frequently himself.

So, Ray certainly wanted his students to be using manipulatives and pictures consistently, and he interpreted this text as indicating that this should be the case. Ray did not address the use of “actions” to construct arguments, nor did he actually say very much about what it meant to “construct” an “argument” using these tools; he spoke more of solving problems with them. Further, he did not offer an interpretation of “generalized” arguments. In general, Ray’s perspective on this text seemed not to shift during the study, but it is perhaps noteworthy that in the two videotaped lessons, no manipulatives were used, though some diagrams were used during each lesson – modeled by Ray, and used by the students.

**Ray’s thoughts about implementation of SFMP 3.** Ray believed strongly in the SFMP and in many aspects of his own practice; that is, he felt that the SFMP were worthwhile and that he could clearly justify most elements of his classroom instruction – now, perhaps in terms of the SFMP. During interview 1 when discussing the importance of SFMP 3, Ray remarked, “I sound like I’m on a soap box! Gees!” He felt that he was “strong” in fostering the goals in this text and that the class had to be a “safe environment” in order for students to meet these goals. He added that when students felt successful and confident, then they were even more ready to learn. He reiterated these ideas many times during the second and third interviews. In other words, not only did
Ray feel that SFMP 3 was worthwhile, but he also felt it was achievable for most or all students, in time. During interview 1, he observed, “[I]t’s gonna be a process. It’s gonna take a while.” Yet, as Ray shared his concerns about students’ rushing through their mathematics work, he was still optimistic: “[S]lowing them down – that’s gonna be the struggle… as long as we know that’s gonna be the struggle, then starting in K-1, when I walk by the classrooms, I can see them working together… And they do a lot of that already.”

A common theme for both Ray and Sandy was that many of their students entered fifth grade without a solid foundation of knowledge needed to master fifth-grade content; in addition, some struggled in fifth grade simply because the teachers were pressured to address all of the Ohio indicators before the OAA (they were on a “treadmill”). Ray said that students might have particular challenges with sharing their ideas and evaluating others “when they feel weak in mathematics… they’re less apt to share, they’re turned off.” He continued, “I think you would prevent a lot of problems” [by taking more time with the content and with the goals in SFMP 3].

Throughout the study, Ray shared various means by which he could enact SFMP 3 in his own instructional practice. He was interested in facilitating more “workshop” lessons where he would present one “rich” problem that students would solve and discuss during the session. He felt that the “sharing” time at the end of such a lesson was very important for learning and also for meeting the goals of SFMP 3. To support students’ ability to collaborate about problems – with classmates of all ability levels – he planned to continue his practice of teaching cooperative learning strategies starting during the first
week of school. In addition, he often discussed asking students to use manipulatives and
diagrams more consistently. He also wished to present more problems that asked
students to evaluate (in writing) the thinking of a hypothetical student on a given task,
feeling that this would be a valuable learning experience (even through a blog) as well as
a useful assessment. Finally, he spoke frequently about incorporating more of an
expectation for “proof” in his class – both proof through models and proof in writing.

Overall, Ray’s interpretation of this standard was highly tied to (influenced by)
how he envisioned enacting it in his classroom. Most of his interpretations were
connected directly to examples of scenarios or possibilities in his class. Thus, it is
difficult to separate Ray’s thoughts about the meaning of SFMP 3 from his ideas about
how to address the goals in it with his students.

**Review of how Ray’s interpretations of SFMP 3 seemed to evolve throughout
the study.** Ray’s interpretations of SFMP 3 apparently evolved mainly as he considered
his current practice as well as strategies for helping students to master this standard in his
class. For instance, although his thoughts about the title text did not change noticeably
during the study, he did feel after lesson 1 that he would not have to change as much in
his practice as he originally might have thought in order to enact the goal in the title.

During interview 3, he spoke for the first time about teachers’ helping students to
sequence “logical progression[s]” of ideas by praising steps that made sense and building
from these (also by asking students to collaborate and to use concrete and visual models).

In interviews 2 and 3, he referred to the use of “anchor” problems (his interpretation of
“counterexamples”) to help students think about strategies for solving related problems.
Ray’s ideas about students’ learning to “justify their conclusions, communicate them to others, and respond to the arguments of others” became more defined as he taught and reflected on the lessons and prepared for the new year, as well as when he re-read and discussed the text itself. Further, in the latter two interviews, he shared problems that he could assign in order to give students experience with finding flaws in arguments. Again, Ray’s thinking about this standard was strongly related to how he envisioned enacting it in class, and this is the frame from which he offered most of his comments on SFMP 3.

**Ray’s Interpretation of SFMP 7**

**Title: “Look for and make use of structure” (p. 8).** In interview 1, Ray offered these thoughts about this title statement: “[T]o me, patterns is the whole real world stuff. And making connections – connections mnemonically, connections in regard to analogies, to previous problems you’ve solved – ‘cause that’s how you hook ‘em. Zone of proximal learning.” Further, “I think that human beings naturally want to find patterns… number, color, sound, music patterns, light patterns – I think we’re surrounded by patterns.” However, having said this, during the same interview, he commented, “[T]hose proficient just have an easier time, I guess, in making the connections with the patterns.”

During interview 2, when we had finished discussing SFMP 3, Ray turned the page and waved his hand in air, saying “ah”, as if to indicate his complete agreement with this standard. “[T]hat goes to the whole crux of mathematics. We start with the multiples; we start with perfect square numbers – all of these, you see the patterns and the relationships. Um, we do factor pairing… that’s a great beginning to look at structure.”
Ray appeared to see “structure” particularly as referring to patterns and relationships, and all of these as the “crux” of mathematics, which he repeated several times. He did not, though, say as much about the phrase “make use of structure”.

In the third interview, Ray remarked of this text, “[T]hat’s going with the whole problem-solving approach, looking for patterns…” He added, “[N]ow, we have the gift of time. To look for structures, and not fly… I think for number 7, that really, you have to have that sharing component [in class] in order for them to see structures that they found, but to see other people’s structures…” Interestingly, when he was discussing a hypothetical classroom episode in which students were struggling with whole number division, he said, “[I]t’s baby steps. It’s sequential, and it’s that structure – providing that structure – for them to move on there. And it’s not gonna be [snaps several times]… It’s not gonna be done in three days.” One might ask whether Ray was referring to structure in instruction or in the idea of division, or both intermingled. In either case, given his emphasis of the word “structure”, he seemed to be relating his comments to SFMP 7.

Thus, Ray seemed to interpret the term “structure”, at least at this time, as referring both to patterns in number and to physical representations of mathematical ideas. He discussed uses fraction bars and graph paper to help students find structure but also noted that in fifth grade, students worked with large numbers that might not be easily represented with such tools.

Between interviews 1 and 2, Ray’s thinking about this title statement seemed to shift from focusing on finding and using patterns to finding and using relationships among ideas. Further, in interview 3, he described helping students to understand and
use structures that others were seeing, and he observed that this development would probably not occur quickly. That is, his interpretation of the text seemed to deepen simply from discussing and thinking about it over time.

“Mathematically proficient students look closely to discern a pattern or structure” (p. 8). During interview 1, Ray did not say a great deal about this text beyond what he had said about the first sentence of SFMP 7, perhaps because these statements are so similar. He did note that “you can really integrate the real-world experiences with children in regards to patterning”, and that patterns helped students connect their thinking “to something previously learned”. So, he was again thinking about finding patterns in real life and helping students to use them to consider relationships among ideas.

In interview 2, Ray commented that “the whole theme of fifth grade math, or most of math, is patterns, and relationships. That truly is mathematics… you’re looking for these patterns, and it’s exciting to me! That’s the fun part of math!” He again mentioned that high-achieving students often enjoyed searching for patterns: “[I]t’s more of a challenging puzzle, and they don’t mind spending more time on that.” This may have been an allusion to the phrase “mathematically proficient students” in the text. On the other hand, he added, “Your little low guys, they just can’t see, and to teach kids about patterns, that one’s hard because you almost have to start at the basics. Usually these are the kids who don’t know their basic math facts, so they never see, ‘If you know your multiples of 3, you know your 3s times table, guys.’” He said that teachers tried to help these students “make those connections, but they’re far behind that”. Ray seemed to mean that it could be challenging to help lower performing students look for a pattern or
structure because they had enough gaps in their knowledge or skill to make it difficult to find a pattern or structure; what would be structured mentally for some other students was not yet structured for them. However, he offered some ideas about how he could build this foundation for students in his class, and these will be presented in a later section.

During his second lesson, Ray was focused on students’ search for patterns in an incomplete numbered grid that he presented. When he distributed the task, he said, “[T]his problem is going to give you a chance to complete the missing numbers in some patterns; describe a number pattern in words. Why do you think describing a number pattern in words would be important?” One student offered an idea that was inaudible, and then Ray altered the question, shifting the focus from patterns to expressing mathematical ideas in writing (as described earlier). As he discussed the task, he encouraged students to look for multiple patterns that would make sense with the numbers provided (it turned out that in this task, only one pattern was logical). As students presented their work, they made statements about the rows and columns in the grid such as: “I noticed that the pattern going down was plus 9, and I went off of that.” “The last number went down 5, 4, 3, 2, 1 [pointing to numbers in the last column], and then 4, 3, 2, 1, 0 [pointing at next to last column].” “It’s always plus 1.” “I saw that if you go up the diagonal, it’s going up 10.” “Going from right down to left [points from top right to lower left] is plus 8.” “It went odd, even, odd [pointing horizontally], and down it went odd, even, odd [pointing vertically], and then it would go even, odd, even [pointing at another column].” One student also used the vertical pattern of adding 9 to reason that another student’s potential solution was incorrect. Ray responded favorably
to all of this discussion and encouraged students to share all ideas that they had
considered. So, he certainly seemed to be intent on having students “look closely to
discern a pattern” in this lesson.

During interview 3, Ray reiterated that patterns were the “crux of mathematics”
and that humans were always looking for patterns. He also echoed his earlier comment
about students, that “mostly from their peers, is where they begin to see these structures”.
He then spoke more about ideas for enactment of this standard, which will be described
later.

Throughout the study, Ray primarily used the word “patterns” in referring to this
text. Though he did use the word “structure” from time to time, he never provided a
direct interpretation of it, and most of his examples of patterns or structure involved
numbers. Again, at first, he seemed to be thinking mostly about patterns in the real world
(visual, numeric, etc.). During interview 2, Ray described some examples of patterns that
students might see in mathematical situations in class, and he facilitated a search for
simple numeric patterns in his second lesson. He referred a bit more to the idea of
structure in the third interview, noting that students often came to understand certain
“structures” as their peers identified and described them. Although his interpretation of
the text did not change substantially from the first interview to the final one, he did seem
to be more optimistic from the second interview on that his struggling students could
develop this skill by learning from other students. So, one could argue that at least
implicitly, his sense of the phrase “mathematically proficient students” shifted from
describing only students who were already successful to students who could become successful.

“Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have” (p. 8). In the first interview, Ray spoke entirely about the first example in this text, ignoring the geometry example. He described the discovery of the commutative property as being connected to decomposition of number and that using manipulatives helped students learn to do this: “[I]t again goes into how important getting those manipulatives out, and just playing – you’re not playing – they think they’re playing, but actually decomposing… in the successful – you know – countries, they give them more than enough time – almost ad nauseum – to they don’t even want to do that anymore, because they know this.”

During interview 2, Ray again referred to countries other than the United States, saying, “This is very built into the whole Singapore – or actually Asian philosophy of mathematics, and now our new standards… again, you should be decomposing, recomposing numbers constantly.” Here, he seemed to be generalizing more from the given example to other ways of finding structure in number, and he gave examples from his own class of questions he had recently asked when he was trying to help his students learn to use decomposition to compute mentally. One example was from the first videotaped lesson, and he said that he recognized that he was already supporting his students with decomposition and recomposition – it was not a totally new concept for his practice.
In interview 3, Ray reiterated, “[T]he playing with numbers needs to come back. Just exploring with them!” He added, “I think you need to teach [students] that… to make those aha moments again – that’s what we need… Instead of drilling, and – drill and kill.” Here, Ray focused on the idea of helping students to have fun noticing and exploring ideas in mathematics, particularly so that the students would not feel that math was boring or worse, and so that they could enjoy those moments of discovery.

Thus, this statement in SFMP 7 seemed mainly to bring decomposition and “playing with numbers” to mind for Ray. In the first interview, he related this to building a foundation for students’ future learning. In the final interview, he related it to helping students appreciate and enjoy the act of learning something new and (inferring from his comments) discovering a “structure” that they had not recognized before. Generally, Ray’s thinking about this text remained fairly constant throughout the study.

“Later, students will see 7 x 8 equals the well remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property” (p. 8). Ray’s comments about this statement in interview 1 were: “7 times 8, if you don’t know that, let’s work on – let’s hook some things onto ones you know. So, he seemed to interpret this example as a situation where students would use concepts that they already understood to solve a new problem. Describing how he might use manipulatives, pictures, and eventually symbols to help students develop an understanding of the equivalence of the expressions in the text, he added, “[I]t’s that progression… again, this is just a sequence… Going from here to here… but also giving them the tools that they need.”
During the first lesson, Ray demonstrated several examples of decomposing larger numbers to solve multi-digit multiplication and division problems. For instance, to explain his strategy for finding the product of 46 and 20, he said, “40 and 6! Wouldn’t that have been easy? 20 times 40 and 20 times 6, and add the two together to get your product. You could have done that in your head.” A student also volunteered a slightly different way to use decomposition for this multiplication. However, in general, the students did not seem to be using this method, and it is unclear whether most understood Ray’s explanations of his own thinking.

Ray’s comments during interview 2 actually reflected this statement as well as the previous one – again, he felt that decomposition was important for students of all ages to learn in order to apply strategies such as the one suggested by this example. His comments in the third interview were also similar, but he added, “Verbalizing it… you’ve got to do a lot of that,” and he noted that perhaps “verbalizing” these relationships among numbers was a skill that could be developed more fully in grade 5 than in earlier grades. Further, he remarked, “I think in fifth grade once things are in place, we’ll be doing that with those arrays, and looking at 2-digit, 3-digit multiplication, and 2- and 3-digit division, that you’ll be able to see and just extend that structure that’s already been in place in earlier grade levels.”

Once again, throughout the study, Ray felt that this text reflected the need for students’ experience with decomposition and recomposition of number. He mentioned using manipulatives and other models with students to support these experiences, and during the last interview, he suggested that older students should learn to articulate
relationships among numbers and strategies used for operating with numbers. Generally, Ray interpreted this text in a consistent manner during the study.

“**They also can step back for an overview and shift perspective**” (p. 8). In interview 1, Ray read this statement verbatim aloud and said, “That’s tough… the low guys, again, they’re thrilled to be solving it one way… I think working with other children, and that whole sharing aspect – the final aspect of a good math workshop – then I think that can start to train that perspective…” Although his comments were brief, he appeared to indicate that this statement implied that students should be able to solve problems in multiple ways and that they could learn alternate perspectives on problems by working with other students.

During the first lesson, while Ray was guiding his students through the use of a Singapore math model to solve a division problem, he led them in transitioning from a visual diagram to a symbolic statement. This teaching move seemed to reflect the idea of shifting perspective from SFMP 7, though he never referred to this instance in this way.

Ray’s first comment about this text during interview 2 was, “I think that ties in with number 3 as well, looking at critiquing others, stepping back…”. He continued, “[T]hat’s hard for them at times, though. Because they want to run – they want to go – they just want to go forward, forward. And to stop and make them… that’s my job…” He said that he wanted students to be able to take the time to consider others’ perspectives, and he hoped to “train them hopefully down the road to have it internalized… So they can do it on their own.” Further, in discussing one of the hypothetical classroom episodes, Ray noted that students could learn to “step back and
look at their own work and find errors in it (once they understood a previously misunderstood idea), which seemed to allude to this text.

In the third interview, Ray continued to refer to students’ shifting perspectives in different ways. He commented, “They either shift perspective on their own work” by looking again at it (perhaps after “step[ping] back for an overview”) or by viewing each other’s work around the classroom and/or seeing classmates present/explain their work, which would certainly allow an overview of the thinking of the class. He distinguished this from simply asking during class discussions, “All right, can someone come and show me a different way?” Ray then went on to comment that as the teacher, he could intentionally attempt to cause students to shift perspective by adding a new element to a problem (or by changing an existing one): “Well, what about – what if we didn’t know this number? What would happen then?” “How could you solve this problem if you did not know this number?” He remarked, “And it’s like clay – you just want to take this little piece off here [motioning with hands as if sculpting clay] – now, what can we do? Let’s wad it, and put it here! You know… it’s fun!” Ray clearly enjoyed opening his students’ minds to new mathematical ideas.

During the study, Ray’s ideas about this text did not shift a great deal; he viewed it as meaning that students should be able to understand ideas and solve problems from multiple viewpoints – particularly, that they could shift from their initial perspective on an idea or problem. He only spoke about students’ getting an “overview” once, and this was when he described students observing at all of their classmates’ ideas by walking around the classroom; so, it would be interesting to further explore his idea of what an
“overview” meant in terms of students’ thinking and problem solving. However, he did seem to think more about various ways to help students meet the goal in the text as time progressed.

“They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects” (p. 8). During interview 1 and the first group discussion, all of Ray’s comments about this text had more to do with classroom practice and less with the text itself. His first comment during the first interview about this goal was, “[T]hat’s gonna be a huge paradigm shift for the uppers. Huge!” Perhaps in reference to the example of decomposing algebraic expressions in the text, he suggested, “[S]ince you’re going into a much more complicated area, how cool would that be to prove it with manips? ‘Cause then you’re talking your problem solvers that the real world’s gonna want. People able to take and show others – ‘cause that’s another important thing – working with people and being able to explain your ideas…”.

Based on these comments, he seemed to be thinking about the goal in the text as being able to understand the structure of an object and then (extending from the text) being able to communicate this structure to others, which he felt was a “real world” skill. Also during these first two conversations, Ray provided three examples of student actions that seemed to correspond with the goal in this text, but he did not in any way connect them to SFMP 7 in his comments. So, one implication for professional development might be that teachers’ observations of student thinking or work could be related to the SFMP to help teachers relate the standards to their own classrooms.
In the second interview, Ray read this text and remarked, “That’s a hard one.”

From his next comments, it seemed that he was thinking of “objects” as variables, likely because of the reference to algebraic expressions in the text. He referred to a previous OAA practice task and said, “first one they just had to set up the equation using two variables… 10 and 11 year olds don’t think that way. We don’t think that way… but it says ‘some’ algebraic expressions, so I don’t mind that. Getting them just to use a variable, with practice – that’s one of those skills… But, being able to use several objects, oh, that’s tough at this level. Because it is so abstract.” He added, “I struggle with that one...”. This comment may have meant that he struggles with the goal himself, as he had already noted, and/or the fact that students are expected to meet it. He continued:

“[W]ith the inequalities, there was one – because we use OAAs from the past I know what [the test designers are] thinking – but the kids don’t think that way. And you can do some work to try and teach them to do that, but they’ll never automatically try to think that way, with variables, because I don’t think as adults, we do… I don’t; I know I don’t.” Even though Ray, to this point, seemed doubtful than any fifth grade students could achieve the goal in the text, he then said:

I understand what they’re getting at. And some kids it will. It will come.

I don’t mind single objects, but once we get into double variables, or triple variables – I’ve seen those too – in fifth grade, it’s so confusing. You know, two – because then they struggle with which operation goes where [moving hands around in front of himself]. Because we’re also talking order of operations and parentheses; you have to move things around –
that’s high level. Really high level for 10- and 11-year-olds. And if you go to formal operations, some kids never achieve that, ever, or some, they’re 15, 16, 17 when they hit formal. Because it’s definitely formal operations for that, baby.

So, Ray appeared to be interpreting this text primarily as meaning that students should be able to create and manipulate algebraic expressions with multiple variables.

Ray’s responses to this text in the third interview were similar to those in the other phases of the study. He noted, “[W]hen you’re really going up to that complexity… it goes back to proofs… looking at other people’s thinking.” Then, he offered some observations about the grade 5 content standards that he felt related to this goal: “We can use variables, but we’re not gonna do it in that systematic, ‘algorithmic’ [quotes in air] type of environment… So, it’s something we don’t stress, but let’s start using it, too. If they’re ready for it, and they bring it up, I think you have to… expose them to the different shifts…” He added as he glanced back at the text, “[S]ingle objects, especially with your lower kids, you’re gonna go, maybe linear, in a linear way; with your top, start linear, and then branch off into different aspects…” Based on his first statements, he seemed to mean that in order to help students see complex structures, he should provide experiences in which they had to listen to others’ thinking and work through proofs. He did seem to view “single objects” as variables in “linear” relationships and expressions composed of several objects as nonlinear relationships. He noted that different levels of students could handle different kinds of expressions.
Ray’s interpretation of this text became more specific during the study. At first, his thinking seemed to focus on students’ ability to see structure in an algebraic expression and communicate it to others. From the second interview on, the term “objects” in the text seemed to take on the meaning “variables” for him, and rather than thinking so much about the composition referenced in the text, he seemed to think more about students’ ability to write and interpret expressions with one or more variables. It is not clear why Ray’s perspective on this text shifted during the study; perhaps he simply was considering the text more carefully in the second interview. It is noteworthy that all of his interpretation involved algebraic expressions, most certainly because of the example in the text.

**Ray’s thoughts about implementation of SFMP 7.** Once again, Ray’s belief that students need to feel confident (and not overwhelmed) in order to successfully accomplish the goals in SFMP 7 emerged during many of his comments throughout the study. During interview 1, he remarked, “[Y]our little strugglers – they think, oh my God, there’s billions of facts… one of my first lessons I do with my fifth graders, there is a fact family that we can narrow down to – there’s nine facts you have to memorize.” In the second interview, he noted, “[A]s review, we take the multiplication chart – ‘Look at these patterns – what do you notice?’” He continued, “[T]hen you show them the diagonal is perfect square numbers, [gasps] – they are pulling those out; the patterns are just amazing.” So, Ray wanted students to be able to use structure and patterns to simplify their learning and to understand relationships among ideas.
Ray also discussed examples related to SFMP 7 of how he felt he could support students who came to him with gaps in their understanding of number and other concepts. One strategy from the primary years was to help students navigate among numbers to 100 using the structure of the 100 chart: “[G]etting into some of the – ‘All right, let’s add 10. Start at number 7; let’s add 10. Go back 2 digits’…” When he was discussing a hypothetical classroom episode involving decimal concepts in the third interview, he noted, “[Y]ou’ve gotta make sure the tenths are in place before you even hit hundredths; the hundredths have to be in place before you even hit the thousandths… it’s that whole progression… going back to the structure” [glancing toward the standards]. Ray did feel that students entering his class in future years would be more experienced with looking for and using structure; in the final interview, he remarked that having talked with Carrie during the group discussions, he was “very confident that these kids coming up” would be better prepared to meet the goals of SFMP 7 than students in the past.

Also related to supporting students as they learned to understand more abstract structures – such as algebraic expressions and equations – Ray shared that the fifth grade math teachers felt that they had designed their curriculum map for the year that was ending to effectively scaffold students’ learning. He shared, “[W]e really liked that, moving [the fractions unit] forward, so that by the time we get to that algebra, that algebra just solidifies so much. Because we get the biggest ‘ahas’ here because it’s so much at a higher level… but you needed all those skills before you can get there. Then once you get that, if you start the PEMDAS – order of operations, man, they’re moving.” That is, the teachers seemed to feel that studying fractions early in the year helped
students to be prepared for studying algebra later. He even suggested that with the new standards (particularly SFMP 7) implying so many connections to the real world, it might be worthwhile to “make the kids part of the planning. Constructing the curriculum together.”

In terms of classroom practice, Ray maintained that creating “skills groups” and utilizing different instructional strategies based on students’ individual needs would help students accomplish the goals in SFMP 7. During interview 2, he commented:

[T]he new standards, I think, are set up for that… those little skills groups – you’ve got this group here, ‘You know, guys, let’s do multiples. Let’s sing it…’ I have a keyboard back there. For skip counting fractions, the number line – [to a beat] 1 fourth, 2 fourths, 3 fourths… it’s kind of trivial, I know… When you can incorporate other things into the music, and the arts, physical movement, that’s good too… That just builds into the skills. Not into the deeper thinking, but if it’s up there [points to head], it’s a good thing.

During the first interview, Ray also continued to discuss the idea of relating the standards, through the goals in SFMP 7, to the real world and integrating multiple content areas. “[Y]ou can really integrate the real-world experiences with children in regards to patterning, but it gets them ready for algebra, it gets them ready for so many different things…”. He later added, “I’d love to see kids… doing authentic – authentic math, not contrived… Something like for volume – I always wanted to dig up in the backyard here – I’ve never had permission – but that would be something that they would never forget.”
In addition, Ray definitely felt that considering the concrete, iconic, and symbolic levels in students’ thinking – and planning for learning based on these – would be very beneficial for students relative to SFMP 7. However, he also noted that with larger numbers, manipulatives were not always as simple to use, and depending on students’ level of thinking, this might not be necessary. During interview 1 as he spoke about the hypothetical classroom episode involving multi-digit division, he said, “[A]t that point, manips would be hard. I would think. Base 10, you could do… that natural arrangement… if everything’s in place before then, probably wouldn’t have to. You could do symbolic.” In the third interview, he shared, “[F]or me, more graph paper, I think I want to be able to use… to look for the structure, so I think that will be the main tool for them to look for patterns… tiles would be important too, but in fifth, we’re gonna be working with much larger numbers.” Ray extended this thinking beyond the symbolic to reflect on students’ developing their mental computation and estimation skills by decomposing and recomposing numbers – an activity that he argued was very important for students’ mathematical development in general. He felt that teachers needed to model this practice for students, and he mentioned during interview 1 that he thought it could be helpful for students to use calculators in order to experiment with decomposing and recomposing larger numbers.

Further, Ray continued to argue that students learned a great deal from each other when they had the opportunity to do so. In discussing how students could learn to “step back for an overview and shift perspective”, he described the use of a “gallery” walk in class, where students would leave their work areas temporarily and walk around the room
to observe and learn from what others had done. “It’s that whole STEM philosophy, so it really ties in well with it… ‘Now come back, and see if you need to change your direction.’ To get different perspectives.”

More generally, Ray discussed how to help students learn to problem solve using the goals in SFMP 7 as strategies. During the third interview, he referred to “the old problem solving book”, meaning a text that he used to use that provided common problem solving strategies for students. In this book, one of the specific strategies suggested was looking for patterns to help solve problems. However, “I’m not gonna start at the beginning with that – I’d rather do Singapore because it makes more sense to me… And I think the kids will understand it more… maybe midyear… I think to teach it as a skill… would be a good thing.” Ray’s comments highlight what could be an important point about teaching – teachers are most apt to do what makes sense to them in their own instruction.

Thus, Ray’s thoughts about his enactment of SFMP 7 paralleled his thoughts about the enactment of the other SFMP and the CCSS in general – using multiple instructional strategies and a focus on students’ comfort level and individual needs to facilitate learning.

**Review of how Ray’s interpretations of SFMP 7 seemed to evolve throughout the study.** As would be expected, Ray’s interpretations of this standard became more specified and more related to classroom practice throughout the study. Often it seemed that simply discussing and thinking about the goals in the standard and how he would implement them allowed him to refine his understanding of it. Ray never focused a great
deal on the word “structure” in the text, but he often referred to students’ thinking about patterns and students’ ability to make connections among ideas and ways of solving problems. He also spoke frequently about supporting students’ ability to decompose and recompose numbers, both in writing and mentally, in order to solve problems, and this certainly involves an understanding of the structure of the number system. Perhaps his most narrow interpretation of any part of the text occurred relative to the last statement in the paragraph describing SFMP 7, and it seemed that this was caused by the authors’ use of “algebraic expressions” as an example of a “complicated thing” that could be seen as “single objects” or “several objects”. Still, overall, Ray felt that “patterns” and looking for relationships were the “crux” of mathematics, so he clearly felt that this standard was very important for students’ learning. It would be interesting to study, in future years, whether Ray was able to focus more specifically on the various aspects of SFMP 7 and design learning experiences to support it.

**Ray’s Vision of the SFMP Implemented Effectively in a Classroom, School, or District**

**Ray’s goals for his students and for himself as a teacher.** Ray genuinely and deeply enjoyed his role as a teacher and a mentor of children. During the first interview, he said, “Each child is an individual – you love them for who they are.” In interview 2, he added, “I love working with special ed. It’s like, give them to me. Give them all to me.” In addition, in the third interview, he said that it “was really fun” seeing a typically low-achieving student justify her answer in front of some doubting classmates, and in relation to seeing students in general “discovering” new ideas, he remarked, “[T]hat’s the
fun part of teaching kids this age.” Further, he felt that the ability to be creative in planning lessons and activities was “the fun part of being a teacher”. However, Ray felt that a great deal of focus in his school had shifted from this type of personal support of students to simply preparing them for the OAA. He shared during the first group discussion: “…walking by [a colleague’s] room, and you know what I saw up in the windows? It was a little craft – a St. Patty’s Day craft. And I almost started crying. I was like, remember the days when we were allowing them to be kids?” In interview 3, relative to this concern, Ray remarked, “[Y]ou do not get more learning out of anyone who’s being worn out.” Related to this kind of frustration, he also commented at the start of interview 2, “I have a Plan B, because I have a lot of connections. I don’t want to, because I love the school, and I love the kids. But when the administrators…” [Ray shook his head and stopped talking]. Nevertheless, he concluded this interview by saying, “[A]fter twenty years, I still love it. I love what I do. I’m so lucky.”

Ray also believed strongly in his own practices as a teacher, at the same time feeling that he always had room to grow. Since he had taught fifth grade for many years at this point, he felt that he was “getting better” as a teacher and appreciated being able to teach this grade long enough to become effective in it. As he discussed working with colleagues who were not as comfortable with student-centered methods as he was, he commented, “[I]t’s hard when – ‘cause I’m almost the defender of doing what’s developmentally appropriate as opposed to going, because that’s what [other colleagues] know. And that’s been a struggle for me… Because even [our curriculum director] called me ‘traditional’ at one point, which was an insult [grabs his face with his
hands].” He continued, “But I do have to make concessions… that’s been… hard for me… Because I know what’s – I think I know what’s right, especially with my primary background… it has played a huge difference in the way I teach, you know, than I think than if I had only taught fifth grade… I think it’s a benefit because I know where they start from.” In talking about the transition to the CCSS during interview 2, Ray reflected, “I’m not fearful because I’ve taught math all these years. I know fifth grade pretty – but it’s a big change for me, too, because even though a lot of things are the same, they’re not the same.” He knew that most of his colleagues relied heavily on a textbook, but he noted, “For me, I didn’t spend [years in college and grad school] to follow a book - I think I can do better than that.” He also felt that he was already implementing some practices that supported the development of the goals in the SFMP; during the second group discussion, he offered these comments on his second lesson: “I felt better once I did my second video… No, no thumbs up – I didn’t have to; we had a nice dialogue… the kids were defending, being polite…” He added similar comments during the third interview: “I’m good at that [“discovering” ideas with students] – I’m the ham; I’m the actor up there.” Further, he felt that in the midst of changes in his school, he was able to be “honest” in his questions and comments in discussions: “I think that’s one of my biggest strengths, and National Board certification helped me with that.” On a broader scale, he noted, “I think that’s one of my strengths is self-reflection.” So, overall, Ray was confident in his ability to adapt his practice in support of the CCSS, but he knew that this would still take time and effort on his part.
Ray certainly wished for his students to be successful in mathematics, and a major theme in his comments was that when students felt comfortable (“safe”) in his classroom and confident in their ability to learn mathematics, this would ultimately support their learning. Developing this confidence and comfort was often challenging because students entered fifth grade with previous struggles in mathematics. Ray observed, “[M]ost teachers want all of their kids to succeed… you do what you need to do to get the job done for all children.” He said that he and other teachers were very concerned about their students’ understanding and mastery: “[A]s a teacher, you know, we go home and we worry about this stuff. How do we get this child up here ready to do function tables?” He pointed out that some students came to him with large gaps in their knowledge, but he noted, “[Y]ou try to take them… sometimes it’s shockingly how low, but you know what? You don’t blame the kid.” In discussing decisions about assigning homework, he added, “[T]hey are children as well. We can never lose that insight there.” During the second interview, Ray shared that he and the other fifth grade teachers basically agreed that if students could solve a computation problem using a less efficient strategy than a standard algorithm (for instance, the “sidebar” strategy for multi-digit division), this was acceptable because it was comfortable for the students. However, he also said that their “goal” was for students to use the “most efficient” strategy to solve any given problem, adding that by the end of the year, most students were meeting this goal because they were confident in their use of efficient methods. Interestingly, Ray also reflected on a classroom situation where he had demonstrated efficient strategies for fraction multiplication and division because the students “were fearful” of learning these skills.
However, he realized quickly that “they had no understanding,” and he continued, “That’s not the way I probably should – what I’ll do next year.” Related to this particular reflection, he felt in general that teachers needed to consider whether they needed to make changes to do what’s best for kids… more importantly, what’s best for mathematical thinking.” Further, Ray spoke in each interview about his role in creating a “safe” environment in the classroom for meeting the goals in SFMP 3, especially. In general, Ray greatly enjoyed and appreciated the opportunity to help each of his students to succeed in mathematics.

In Ray’s desire for his students to learn successfully, he also realized that he had taught various “tricks” in the past that probably were not in alignment with the ideas in the CCSS. One example he cited was, “I’m guilty of this, when multiplying by multiples of 10, 100, 1000 – ‘oh, it’s ancient Chinese, take all the zeros off’…” He also mentioned the frequently quoted “Does McDonald’s Sell Cheese Burgers?” mnemonic to help students remember the steps in the standard U.S. long division algorithm (the first letters stand for the words “divide, multiply, subtract, check, bring down”). He commented, “I teach these tricks, and I’m like, WHY? I shouldn’t be.” He added that as he and his colleagues had looked at the new content standards and some had mentioned that the standard algorithm for division was listed, he now felt that he wanted to say to them, “Now that we’ve gone deeper into this, it’s like, no, no, no, that’s not what that means. That is just one strategy. But it should be in place as well. Because we’re looking – for all of them, what’s the most efficient?” So, he felt that the standards indicated that
students should learn and understand multiple strategies and also be able to determine which strategy was most efficient for a given problem.

**Ray’s perspectives on implementing SFMP 1, 3, 7 as a group, in conjunction with the content standards.** Ray pointed out, as did Sandy, that in the current Ohio indicators, many skills were taught for several consecutive years, and there was little or no focus on the goals in the SFMP. He noted that the Singapore math presenter called this design of the standards a “whirlpool” as opposed to a “spiral” because rather than “spiraling” down to greater depth each year with each idea, most programs and teachers created a “whirlpool” where ideas simply were taught quickly and lost from students’ minds before the following year. He remarked, “[W]e walk by the third grade classrooms, and we see their place value, we hear them teaching it… And we’re like, ‘You do teach it!’” He commented that perimeter and area, for instance, seemed to be taught in second grade on up, but some students still struggled with these concepts in fifth grade. During interview 2, Ray shared the following thought about the authors’ intent in the CCSS and SFMP, as contrasted with the design of current standards: “I think, um, again, they’re looking for the richness that we don’t currently have. We have the superficial aspects of mathematics… and this is the whole flaw with the current standards. We’re reteaching everything again.” Further, he noted that not only did typical students tend to forget content from year to year because they did not learn it deeply initially, but teachers were often challenged to enrich instruction for students who did make rich connections on their own and did not need the repetition of material.
Given this lack of depth that Ray saw in former sets of standards, he was very pleased with the content in the CCSS and the goals of the SFMP. In fact, he called the SFMP the “introductory part of the Bible”, and he continued, “These are all the Psalms, that are gonna get you to understand the big part of the Bible!” During interview 1, he said of the CCSS as a whole, “I think it’s very doable,” and “I think it’s headed in the right direction to think mathematically… Very intelligently written.” He said that he was “excited” about the new standards and that they were “absolutely” worthwhile. In interview 2, he added (in reference to the CCSS being well written), “[T]hat’s why these I think can be more successful…”. He went on in interview 3 to comment, “[T]he fact that our nation is doing this – it just blows me away. This will be, I think, the change we need… We don’t need to rush these kids through inappropriate stuff… TIMSS has been saying this for years! We’ve always looked at the Japanese and… the Swedish nations, and Denmark, and we’ve always observed them… we’re a little slow to get started.” He even noted, “[T]eachers can also experience it [apparently meaning learning in the style of the SFMP], if they open themselves up to them.” Ray seemed to feel that the standards were worthwhile and achievable because other countries apparently have had success in enacting similar standards for many years. However, he recognized that many teachers in the U.S. had not been teaching in the way that teachers in these other nations had been.

Ray was also adamant that teachers needed to teach these standards as intended and not simply choose to teach what (or how) they wished, not only to support students’
learning within the school but also as future members of a global society. During interview 1, he said:

> With this, there’s no argument… the course of study is your Bible. This is what the state of Ohio is telling you. But now, this is what the nation is telling you you need to do as a teacher to get these kids prepared – not only for the next grade level but for the world… But now we’ve just gotta follow through… you can’t just pick and choose and superficially go over some things… Because this has been in our face for years, what these other countries are doing, and we’ve done nothing about it. Except pat ourselves on the back about how brilliant we are. Meanwhile, we’ve been slumping and slumping.

During the second interview, Ray maintained, almost as if speaking about the CCSS directly to teachers in earlier grades, “[I]t’s more comprehensive, it’s not gonna spiral anymore… your critical indicators now… [students] have to have that down pat… That is huge. Because as teachers, I can’t make the assumption you did it. You DO have to do that. So that I don’t have to start all over with that… Especially when kids move from district to district.” He mentioned later that teachers needed to “get it together with pacing”. He added, “[W]e’re so lucky with the math that we did go national… even just resource-wise, for the math and the language arts, it’s a win for us… we’re not gonna have six hundred-page documents anymore.” He also felt that the CCSS would need only minor revisions in the future, which he appreciated: “I think this is it… we’re finding gaps, I’m sure, and it’s not permanent… it’s a living, breathing document, and I
think we need to allow for those variances in developmental issues that we have to deal
with… I don’t think we’re gonna have to rewrite these… to me, that’s a great relief!”

Further, “[T]he standards themselves – the clusters, they make sense to me…” Ray did
comment that “it would be nice” to have documents that helped teachers understand what
students should have learned in previous years and what they needed to learn to be
successful in grade 6, but it was likely that he was not aware of the learning progression
documents at that time.

During interview 3, Ray repeated some of the same points and also said of the
CCSS, “So specific, that there’s no questions – it’s a no-brainer… this should end all
arguments [turning a page in the standards]; if everyone follows this, there should be
none of that!… This is your goal!” These were interesting comments given that he had
also noted that many teachers might misinterpret what was intended in the content
standards. Ray also spoke a bit about implementation: “I don’t care what product
[textbook] you use… Those are just tools…[the CCSS] is what the state, and now nation,
expect you to teach from… how you do it is up to you. And that’s always been the case.”

Thus, Ray believed that teachers enjoyed flexibility in how they taught any set of
standards, which might also have influenced the way he thought about the new standards.

In the second and third interviews and the second group discussion, Ray
frequently repeated his belief that the SFMP were the “glue” that would hold together and
ground the CCSS content. In interview 2, he said, “In the mosaic of learning [moves
hands around in air] if we were going to use an analogy… So it’s the mortar bed – that’s
the mortar bed that you put the tiles into.” He also called the SFMP “the vehicle for you
to hit the domains and the clusters”, and he passionately described how he believed teachers needed to respond to the SFMP in their practice:

[I]f you don’t follow the practice standards, you’re not gonna be a successful math teacher. ‘Cause you’re gonna be doing exactly what was done 20 years ago: ‘All right, let’s do this page [turning a page in his binder]. OK, now. Do the odd problems for home’ – you’re gonna go right back to that. Without knowing the depth – and it states right there in the standards, too, just because there’s less, doesn’t mean that you can feel like, ‘Ooh, we’re done in five months. Oh, OK.’ No! ‘Cause you have to go even further.

Ray noted, “I like the fact that they have [the SFMP] on each [grade-level intro] page – a lot of people aren’t gonna notice that.” He was similarly concerned that most professional development would not adequately address the SFMP: “I’m afraid it’s gonna skip over the philosophy and get to the nuts and bolts… I’m afraid you’re missing the biggest part.” He wanted teachers to understand that “[the content standards] are not just check” [makes a checking motion in the air], and he commented that once teachers had begun reading the content standards, they needed “to re-read the standard going through looking at each Standard Practice [sic]. How does this put its sway – what influence does this one have on the standard?” He added, “Even at [the county workshop]… I think [the presenter] assumed people are already familiar with [the SFMP]… [shakes head]. But you can’t assume that. Because without those, everything else is meaningless.” So, Ray viewed the standards (both content and practice) as mutually interdependent, whereas
someone who did not accept or understand his “glue” metaphor might see them as separate.

As noted in earlier parts of this chapter, Ray also believed that the SFMP were skills that students would need to be successful as adults living in a global society. He felt that students needed to be effective problem solvers and that “working with people and being able to explain your ideas” was important. He said that if teachers could develop the SFMP with students, it would eventually allow the United States to “be able to be competitive” with other nations and for each individual to become a “good citizen” of the world.

Ray recognized, however, that the development of the SFMP and content standards in classrooms would take time and that in the short term, both teachers and students would be challenged by this work. During interview 1, Ray pointed out that he had to re-read the SFMP several times to even begin to make sense of them, and other teachers would need to accept that they would also need to do so. Also, he felt that teachers needed to carefully read and reflect on the content standards because although they might have appeared similar to the current Ohio indicators, the goals and focuses were different. In thinking about making changes in his practice to enact the new standards, he said that he would have “work into” these changes. Referring to SFMP 3, Ray commented, “I have to find a better tool… because otherwise, sometimes the little low guys aren’t talking at all. Even when you say turn and talk.” Then, thinking about his students’ ability to meet the goals in the SFMP, he noted, “You do have some developmental… could be struggles in there, developmentally where they’re at. Cause
our population is changing too. We’re getting more lower, socio-economic children, children with more issues and problems, um, our special ed is off the charts next year.”

Still, as he pointed out, “[S]ome [students] will perform better in particular [standards], and that’s OK.” During the second group discussion, the teachers felt that students now entering kindergarten should come to fifth grade with much more experience in the SFMP than current fifth graders. Ray shared, “I think each year it should be getting just a little more sophisticated… that foundational piece is already there, because you guys [motions toward Carrie] are already doing it… I think my thing is, um, is they’re not there yet.” He even remarked that the SFMP were still unfamiliar to teachers: “[N]ext year, we’re all gonna be learning everything together!”, adding, “[N]ext year is so huge.” He continued this line of thinking in the final interview, saying, “[W]e’ve just gotta get our feet wet… I think we have to go easy on ourselves, too, because some of these things might not be doable at this moment… But it will, down the road.”

Throughout the study, Ray shared thoughts about the new content standards. During interview 2, he reflected on his study of the standards so far: “I think when I’m reading the standards… it’s kind of like, OK, face value… They give you several examples, which is good, I like that, and the advantage to these is that they do go into depth… This is what it is; this is what it isn’t.” He also noted in interview 1, “I love how it will go – this is where they’re coming from – this is what they’re gonna learn this year, and then this is where they’re going to head. I think for newer teachers I think it was brilliant.” He made a similar comment in the third interview. (It was interesting, then, that in the second interview, as noted earlier, he said that he wished this sort of
information were provided to teachers in the document.) Ray often emphasized the “critical areas” in fifth grade (“volume, fractions, and then division”), and he noted that division [with whole numbers] should probably be studied for up to ten weeks during the year. Although he recognized that the standards in grade 5 were more cohesive than the current Ohio standards, he felt that fifth grade still included a substantial amount of content that perhaps the authors did not believe was suitable for earlier grades. On the other hand, he did not understand why probability had been left out entirely and why algebra and “three-dimensional shapes” seemed to be absent also. (In the second group discussion, he commented, “We can do expressions, but we can’t do equations!”) In addition, Ray remarked that he was “kind of sad” not to be teaching percents anymore because he felt that students really could understand and use the relationships between common percents and fractions to solve problems. He was also curious about the new content in the primary grades, and in the first group discussion, he asked Carrie about this and how she and other teachers felt about the appropriateness of the content.

Ray was very aware of when the CCSS included standard algorithms and when these methods were left for later grades. He discussed the fact that some standard algorithms were mentioned in grade 5, and he remarked with some concern, “[T]hat’s gonna give ‘traditionalists’ [quoting in air] an easy out… It should be taught as a strategy, but only as such. That’s not the main way that you should be looking at numbers…” Referring to the U.S. long division algorithm, Ray commented, “[I]t can be shown as a strategy, because we’re still gonna have kids, because that’s what their parents know. So, to them, we want to get you a head start… I think I really want to kind of discourage that,
because visually, that makes no sense to kids. It’s just something that they memorized. Until they get those arrays.” He added, “I am not going to bring that up until a student brings that up.” He also said that he appreciated that he would be teaching multiplication and division of fractions again, “[w]hich is so much easier! But to explain it is more difficult.” Still, he added, “[I]t’s gonna be really exciting to see how the fractions play out…”

Clearly, Ray had read the content standards fairly carefully and was thinking about them in the context of the SFMP as he prepared for the following year. In general, he was “excited” that the standards were written to be taught with “depth”, and he said during interview 1, “I’m glad there’s no shockers in here… That it’s kind of just basic mathematics, if you will… It gets rid of the craziness… You teach it, and you can teach it well.”

In the second group discussion, Ray and Sandy shared that they were concerned that they might be expected to teach both the current and new standards the following year (since the students would still take the OAA, which assessed the current standards). Ray felt that the two sets of standards did not align with each other very well in fifth grade, and he was also concerned that the students entering fifth grade would not have any of “those foundational pieces” to understand the new standards. So, he thought that creating a curriculum map that addressed these issues would be challenging.

As noted above, Ray felt that many students who entered fifth grade did not possess the skills needed to succeed in mathematical content or practices at that level. He spoke a great deal about students’ fact fluency in the first interview, saying, “[I]f the kids
are fluent in their math facts, it does make it easier,” and “Our bright kids – they know how to solve it, but they end up getting it wrong because of some calculation error. And that is so frustrating for us.” However, Ray also noted that “[f]act fluency is only one part of it.” He continued, “[S]ince we’ve done the fact fluency [in each grade], I think it’s helping, but I think if it’s just blatant memorization before understanding the concept, we’re still screwed.” Still, he felt that this was a difficult issue to address with the teachers in earlier grades: “[W]e kinda have to… tippy-toe around because [first grade teachers are] passionate about what they think… they think too – I don’t believe – you do not teach memorization until that concept is in place… but you can’t hold them back… that kills us completely.” Further, he shared, “One of the third grade teachers said, ‘Oh, yeah, I have a kid who can do 200 problems in 2 minutes,’ but I’m like, ‘Well, does he understand what that means?’ No way. So, he can’t apply it.” Regardless of these points of concern, though, Ray was quick to praise teachers in the primary grades, especially, for the “hard work” that they were doing with young children; he said often that first grade was the “hardest” grade to teach because students were learning so much (academically and socially) for the first time. He remarked that he was “proud” of Carrie for her classroom practices that she had described in the group discussions, and he hoped that she would share Ray’s and Sandy’s comments about the challenges (and general practices) in fifth grade with her colleagues.

More generally, Ray commented that teachers would need to provide “varying levels in our classroom for differentiation, so that’s the other big issue which I see down the road… That’s always the challenge – to teach them when you have such a wide range
of students, but that’s what keeps it fun, too.” During the first group discussion, he said, “[T]he biggest struggle that I see – when they’re coming into K, you have kids who had four years of pre-school; you have kids that never even saw daylight in pre-school. So you have the biggest – you know, continuum of children [all nod]… I think that’s where the downward spiral starts.” He also talked about other examples of concepts and skills with which students in fifth grade struggled, including reading large numbers, understanding place value, finding perimeter and area, understanding that fractional parts with the same name must be the same size in a given whole, and adding fractions (without simply adding numerators and adding denominators). During interview 2, he remarked, “I think this generation is struggling with – I lovingly call them the ‘learn and dump’ generation. ‘Cause there’s too much information… I just think that they, ‘Oh, not important, dump.’ Wait a minute – you just knew that last week!” Ray faulted the current standards for contributing to this situation because “[w]e’re reteaching everything again”. Reflecting on how to alleviate this situation with the transition to the CCSS, Ray said in the second group discussion that teachers would probably need to assess students before starting to teach each new topic so that they would have a better sense of students’ prior knowledge and where to go with instruction. He noted that students might be missing some concepts or skills not just from fourth grade but from prior grades also. “Then, like [Sandy] said, then the following years, it’s gonna be easier and easier and easier.” Ray reiterated these same thoughts and ideas during the final interview, adding that he would probably need to use “exit tickets” and “skills groups” to address the varying levels of
needs among his students — a practice with which he seemed fairly comfortable already and which he discussed often as he talked about his enactment of the SFMP.

Ray’s thoughts about instructional practices that would support the SFMP.

Ray believed that planning for instruction, both in the short term and long term, was critical to effectively facilitating students’ learning. He commented in interview 2, “[E]ven now, to me that’s the biggest part of teaching… I’m very meticulous about my planning. You can’t just do — fly by the seat of your pants. It never will work.” He also said that “pacing” was important for “the bigger picture”, particularly now that very little content would be repeated from grade to grade — thus, students would need to master it within the year that it was to be taught. Further, as aforementioned, he was especially concerned about pacing during the upcoming year because of the expectation to address both the current and new standards. In an inservice with the county mathematics consultant before the final interview, the teachers had started to locate the CCSS content in the new edition of the textbooks that the district had just purchased, and they found that there were a number of “holes” that they knew they would need to address somehow. Ray remarked that in the company’s alignment of the standards to the text, certain standards were listed as being aligned with very brief, one-time activities, and he said, “We don’t play that game. We teach to the depth.” He did feel that perhaps more of the new content could be taught with some with the additional lessons that the company had provided to align with the CCSS. In addition, he noted, “I’ve got some great supplement — I’m resource boy; that’s my goal in life. I didn’t buy things; I bought books. And that’s been a godsend for me.” Since Ray and some of the other teachers had spent the
professional time finding the new content in the texts, he also felt that he could now advise new colleagues on which lessons in the book were vital and which could be eliminated. He thought that it would be helpful for all of the math teachers to participate in a similar day later in the year to study the content further and to write new assessments. Moreover, Ray explained that in their fifth grade mathematics PLC (professional learning community), the teachers generally chose to plan together and create assessments together “to kind of standardize it”. However, school administrators had told the PLC that the time spent together should be devoted essentially entirely to studying data and making decisions based on data – which Ray said that they did also do as they were planning. He felt this was unfortunate and frustrating, and he commented, “PLC is becoming a dirty word.”

Once a plan for the “bigger picture” was in place for all of the teachers in grade 5, Ray believed, teachers had considerable autonomy to facilitate learning as they wished in their own classrooms. It seemed that he saw both advantages and disadvantages to this situation. “Once doors are shut… you’re gonna just make the assumption they’re doing it.” On the other hand, he noted, “I don’t think people should feel… stigmatized, that you’re in this little pigeonhole, while I think [the CCSS have] done the opposite. I think it’s opened everything up.” As noted earlier, he commented that teachers could use manipulatives that they chose, contexts for learning that they chose, and to some degree, sequences of topics that they chose (though some of this might be dictated more by the curriculum map). Ray believed that this allowed teachers to be “creative” while still teaching the required content.
As has already been shown, Ray was very glad that the CCSS and SFMP encouraged depth in instruction and in students’ understanding, and he believed that he and other teachers needed to follow this mandate from the authors. He definitely believed that this would be a change from the pressure that they had felt to rush through many topics in a superficial way (without the SFMP) before the OAA. In one sense, he felt this could be challenging because it could still be hard to find sufficient time to develop these deep understandings with students and to have them engaged in the SFMP on a regular basis; he was not even sure that an hour for class would be enough time. He knew that he wanted to include fifteen to twenty minutes of students’ sharing their ideas at the end of class; he wanted to review homework each day to respect students’ time spent completing it, and he maintained, “You have to do real-world application,” perhaps “through the context of a problem that needs to be solved”. However, he also said in the first group discussion, “I’m hoping with the [less content], we’ll be able to do more.” He continued, “I think we’ve always lacked on the rich problem solving. So I think next year that’ll be more my role.” Additionally, he wanted students to use manipulatives more and to share and explain their work, and he said that all of these experiences required time and “quality” instruction if students were really expected to learn from them. He commented that if each teacher were self-contained (as, by the end of the study, he knew would be the case in the new year), then he/she could extend lessons on certain days as needed to spend extra time with mathematics or any of the content areas. Moreover, Ray believed that if lessons were taught “to depth” with the elements that he was describing in place, students would enjoy mathematics more because they would be
deeply engaged in it – though he added, “[S]ometimes, with the attention spans, you want to keep it moving.”

Ray talked about the above issue – students’ disposition for mathematics – on many occasions during the study. He was frustrated that many of his students seemed to “hate math”, at least when they entered his class early in the year. During the first group discussion, the teachers were wondering when students’ perceptions became less positive, because as Ray observed (based on Carrie’s comments), in first grade, students had not “learned to hate math yet”. He wondered if this might occur around second grade when fact fluency became an important goal – and a difficult challenge – for students.

Referring to his own students late in the study, though, he reflected with frustration: “[U]sually the kids love [math] when they leave, but I don’t think I’ve been that great of a math teacher… because of the pressure of the [OAA]… and the course of study.”

During the second interview, as he thought about changing this scenario in his own class through enacting the CCSS and SFMP, he suggested that using more “rich problems” and less “busy work” (which he said students recognized immediately) would hopefully increase students’ engagement and appreciation for the content. In the third interview, he also mentioned integrating the arts more into his instruction, continuing to individualize whenever possible, and using fewer graded activities so that students would feel less anxiety in class. Further, as Ray reflected on his two lessons, he felt that he needed to allow students to assume more “control” in the classroom, which he believed would help them to feel more “ownership” of their learning. Ray knew this general issue of appreciation for mathematics extended beyond the classroom, however: “[U]nfortunately,
a lot of people... would be embarrassed to say, ‘Oh, my gosh, I can’t – I’m not good at reading.’ No one would ever say that. But math, everyone says it. So hopefully now, we can start changing that, especially with parents. But I think, too, I think we’re gonna need to get parents on board with this as well.”

As described above and in earlier sections of this chapter, during the study, Ray discussed a variety of instructional practices that he believed would support his enactment of the SFMP. He frequently spoke of “workshop” lessons that would include a teacher-led “mini-lesson”, student work time, and then fifteen to twenty minutes of “sharing”, which he said was critical for addressing the SFMP. Many of his other thoughts about instructional methods are presented below.

A theme in many of Ray’s comments during the study was the use of “concrete”, “iconic”, and “symbolic” representations of mathematical ideas to support students’ learning at different levels of sophistication – a practice that he often discussed in connection with text in SFMP 1 and 3 especially. In the final interview, he remarked, “[E]ach one is important... You’re gonna see all of it in fifth – whatever grade level I think you teach.” Examples of “concrete” materials that he mentioned during the study were: tiles (particularly to create arrays), base ten blocks, geoboards, small plastic animals, and clay. By “iconic,” he seemed to mean anything that the students drew, and “symbolic” seemed to refer to using numbers and mathematical symbols. He noted that these practices were beneficial for all students because even those who could “get it quick” sometimes were not very adept at representing the ideas in a problem, and “in order to use those manipulatives, they’re gonna have to think deeper.” He referred
frequently to using hands-on and visual representations to support students who were
trying to understand an idea, even when he was discussing the hypothetical classroom
episodes throughout the study. This might involve a brief assessment of a given concept,
followed by individual or small group instruction that he could “tailor” to the needs of
each student or group of students. He remarked that it would be helpful to have an
assessment that would be a guide for teachers in choosing an appropriate level of
intervention. Ray believed that this progression of representations was emphasized in the
Singapore math program of study as well – another aspect of the program that he
appreciated.

Perhaps more often than Ray mentioned these three levels of thinking and
learning, he discussed the use of manipulatives in his classroom, truly in connection with
all three SFMP that we studied. As noted before, Ray wanted to provide all of his
students access to a variety of manipulatives at all times to support their meaning making
and problem solving, and he wanted to help students feel comfortable to choose to use
them by modeling with them himself. He shared, “I want kids to realize that it doesn’t
mean that you’re ‘lower in math’ [quotes in air] because you need to use the
manipulative.” He added in interview 3, “[E]ven as an adult, sitting through workshops,
sometimes those little manips help me!” In particular, he mentioned asking students to
model fractions and related problems to develop the “concept[s]” involved. During the
first group discussion, Ray said to Carrie that she should continue to have her first-grade
students “rebuilding” with hands-on materials as much as she was doing because “in that
end, that is an investment that is going to help us”. He noted that he could identify
students whose primary teachers had not allowed them to explore a great deal with manipulatives because the students were often unable to visualize three-dimensional figures in different ways. He commented, however, that sometimes in fifth grade it was (or had been) difficult to find enough time for students to learn from manipulatives before moving to symbols. Further, he pointed out in the second interview that “you can force manipulatives, too, and that’s bad”, so teachers and students should use them “when it’s needed, not superficially”.

Ray also believed strongly in “skills groups” for individualizing instruction. In interview 2, he shared thoughts about this practice: “[Y]ou can really be more prescriptive, and then take [students] to the new level, because you have these skills, and then you have the context they’re gonna have to use the skills.” That is, part of his role would be to ensure that all of his students were moving forward, and he could do this by creating skills groups that were flexible but specifically linked to the CCSS content. He remarked, though, “[T]he key is, it has to be fluid… Fluidity is what’s different from the old, the blackbirds, the blue jays [said with some disdain], so that’s my fear – that some [teachers] might resort back to that…”. So, he said he might use pre-assessments and/or exit tickets to set up (at least) two groups of students so that he could work with one group directly while the other was involved in individual problem solving, review, or even a math game. He noted that when he was fortunate enough to be able to co-teach with an intervention specialist, this teacher could also work with a group – not always the students identified with disabilities because Ray wanted to know all of his students well. The teachers could then use different manipulatives with small groups as needed, provide
challenging extension tasks, ask students to share their thinking and respond to each other’s ideas, and so on. Further, Ray noted that it was beneficial “to mix groups, too! You need LD with your gifted! There’s nothing wrong with that, because there’s something to be gained… That’s another aspect of what I’m seeing there [in the introduction to the standards].” Ray remarked that simply relying on a textbook did not allow this kind of flexibility in instruction, adding, “[Y]ou can’t be cookie cutter, where one thing is gonna be good for everyone.” Moreover, Ray said that he had an “aha moment” during interview 2 as he was reading part of the introduction; he wondered if he should be more intentional about moving students forward into sixth grade content, including arranging for them to take the sixth grade OAA if they were ready to do so, rather than involving them in content that they already understood. He said that this might be appropriate “[i]f I see, this is fifth grade, they can do it frontward, backward, this manipulative, that manipulative, they can show me a model of this, this, and this…”. Overall, it was important to Ray to meet each student’s learning needs in the best way feasible.

During interview 1, Ray commented, “Supposedly with the new assessment system, we’re gonna see performance-based [tasks], and not only that but the integration, which is, wow, changing everything that you almost have to be self-contained.” Ray noted that this is also what presenters from ODE had encouraged at a workshop he had attended. In the first group discussion, he said, “[T]hat’s the true circle of learning, when it’s all connected… Not forced…” By the time the second interviews were conducted, the administrators at Lincoln had told the teachers that they would in fact be teaching
self-contained classrooms the following year and that they would be expected to integrate content when possible. Ray noted that “people are freaking. Because it’s different than when they taught math, if they did self-contained previously.” He was optimistic about the change, though: “[I]t’s kind of nice… because the bigger picture now is how are we going to integrate it… [i]nto these bigger projects, so that’s kind of where my mind’s at now.” The following is an example Ray had considered for a geometry “STEM” project:

[T]hey’re gonna make a quilt pillow. They have to use an isosceles triangle that has an area of… 50 square centimeters. They have to use two parallelograms with an area of – this one has to be this, and this. So have them design it on the computer first, in color – then, to come back – this would probably be toward the end of the unit because they have to have all of those skills in place, and then have the kids critique the designs up here, for balance, so I can bring in some experts here. Number one – art teacher, to talk about design and balance… having some parent experts, because that’s my thing is to bring this expertise in – to actually bring in sewing machines and let’s actually make these little quilt pillows… then in language arts, write an ad for eBay… that’s the exciting part of the integration of all these.

Note that Ray mentioned that students would “critique” each other’s designs, which seems an intentional allusion to SFMP 3. He also shared that he had similar ideas for teaching volume through a project involving plaster casts and introducing work with division by using a children’s literature book. He said, “[W]ith STEM, you gotta show
them techniques, but then they have to do it on the culminating project… you also have to apply the twenty-first century standards…” Further, he wanted to try to bring more parents in to discuss why mathematics was important for the jobs that they did each day. In general, Ray wanted to “make those real world connections through math”, and he felt that students would be more engaged in this type of learning than in traditional textbook-based learning: “[I]f they understand, ‘This is a skill to get you ready for this bigger project we have coming up, guys, OK – that’s why we’re doing all this’ – I think they appreciate that… the interest level’s built right in.” Moreover, he felt students remembered these kinds of activities longer than isolated skills. Ray’s comments and ideas during the third interview were quite similar to those in the second interview, so he was very consistent in his thinking about integrated instruction and how it could support the SFMP.

Ray also offered a few thoughts about computer use related to enacting the new content and practice standards in an integrated manner and preparing for the new state assessments. In the first interview, he observed, “[A]nother big thing internally, is how we’re gonna factor in performance-based on computers. That’s gonna be a sticky wicket.” Each teacher did not have very many classroom computers (Ray had three), and during the second interview, he shared, “[W]e’re losing computer time [in the school lab], so that terrifies me, because if it’s gonna be integrated assessments… then how do we teach the kids…?” Seeming to consider this question, he said, “I think that we’re gonna have to build that into these projects we do… we have to sign out the computer lab once a week – we get a ‘time’… And, it’s OK. I think it will have its part, especially in
the science/social studies part. But math too.” Aside from these concerns, Ray did believe that the new assessment system, with a diagnostic assessment early in the year, a performance-based assessment later (similar to the “rich, real-world” problems), and a final assessment near the end of the year would be “a better way of assessing kids” than the current OAA.

Another element of instruction that Ray felt was integral to enacting the SFMP was collaboration among students. This related not only to the mathematics standards but also to students’ learning to function in society. In the first interview, he said, “[T]he world is made up of many different individuals”, and he wanted students to have “the patience to work with other individuals and get through the same”. He referred to the fact that he often had children with disabilities in his class, and he commented that other students were able to “accept” them, adding, “I love them because it teaches the other kids patience.” He tried to pair and group each student with as many different people as possible throughout the year because he genuinely wanted the students to learn from each other. Ray also noted that sometimes he and Sandy had each of their students choose a topic that he/she would teach to the class (saying that being able to teach something is the “highest form of learning”), and he remarked, “[T]he kids learn best from themselves [each other] – they truly do.” Further, the students really seemed to enjoy this activity. Overall, Ray wanted to relinquish to his students more responsibility for learning and sharing their learning, which he said was a “big epiphany” for him in reflecting on his second lesson.
Though Ray did not emphasize this in his conversations, he did feel that there was still a “need” for teachers to model particular strategies that they wanted students to learn and for students to practice skills that they had learned. He noted that in his lessons based on the Singapore program, “I model it – usually the model is for the next day – it’s one new piece of information… that’s how they set it up… then they’re practicing the previous day what I went over… When it’s modeling and gradual release, that’s fine.” However, he did “fear” that some veteran teachers who were new to math the following year would use direct instruction every day because they were too afraid to try new methods. In terms of practicing skills, during interview 1, Ray said, “Math – it’s one of those disciplines, that the more you practice, the better you get at it.” He noted later that the games in the textbook were helpful for providing this practice, and both he and the students were enjoying the “sprints” (one-minute mental warm-ups on one skill each) that were part of the Singapore math materials that he had purchased. He noted that the students enjoyed these because Ray did not grade them. Throughout the study, Ray reiterated several times, “Skills are important.” He felt that he could create centers for students to practice their skills (and continue to use the sprints) because he was unsure that the textbook provided enough practice. He also believed that homework presented an opportunity for this practice. He noted there was “not a lot of research on homework, but with math, it’s like anything – the skills, you gotta have it there.” He continued, “[W]e try to limit it to a certain amount, you know, you do a hundred, you can do five. That kind of philosophy… we also try to do our homework in regards to what we’re teaching… the extended thinking still needs to be there”. He commented that he
generally reviewed homework in class, but he added, “I’m thinking we’re gonna have to rethink homework” because it did take time to review each day, and he wanted to spend more time in class allowing students to work on “rich problems” and share their work with each other.

Ray also shared thoughts on other strategies for mathematics instruction to support the enactment of the SFMP. On several occasions, Ray discussed the idea that mathematics problems should more often be presented in context in order to better reflect the goals of the SFMP. Specifically, he felt that he needed to work more toward this practice in his classroom, particularly after reflecting on his first lesson, during which many tasks were not presented in context. Similarly, Ray believed that involving students physically in constructing meaning for ideas was engaging and also helped them to remember and connect ideas; for instance, he suggested creating a human Venn diagram in the school gym to learn to classify quadrilaterals, with each student holding a picture of a particular shape. In addition, although he felt that the goals in the SFMP involving students’ understanding multiple strategies for solving problems were quite worthwhile and achievable, he also noted, “[S]ometimes I think we teach way too many strategies, and we don’t tell the kids to pick one… then the kids will confuse them.” He wanted to avoid this in the future by carefully facilitating class discussions so that students would be able to focus on one or two strategies that would “work every time” for them. Finally, Ray often emphasized that teaching any skill at any age without establishing conceptual understanding for students first was “probably the worst thing
you can do” because the skill could be “too abstract” for students to internalize. This idea seemed to reflect the SFMP very directly.

As Ray thought about instructional practices suggested by the SFMP, he sometimes made broad comparisons (or contrasts) between expectations in the United States and in other countries. During interview 1, he remarked, “[T]his has been in our face for years, what these other countries are doing, and we’ve done nothing about it. Except pat ourselves on the back about how brilliant we are. Meanwhile, we’ve been slumping and slumping.” He commented that the CCSS was “based on a country that is doing extremely well. Well, countries, I should say.” Similarly, in the first group discussion, he said, “[T]he new standards were written by Asian – or, it was a heavy influence on the new standards.” Then, in the second interview, he added, “[A]t the standards, first I laughed, and I thought, oh my God, they want all our kids to be Asian.” He went on during interview 1 to say that the CCSS content was “still too much; we’re beyond eleven topics like they do [in some other countries], but I think it’s an improvement by the way they’re written.” Further, he commented that more time was spent on mathematics in other nations: “[M]ost European countries and Asian countries, they do use a lot of time – you all go to, here’s school, now you go to your tutor…” He later added, “[T]hose children, they have the advantage of the parent… the work ethic, it’s the same. We don’t have that here.” Ray also often shared his excitement for the Singapore math methods and felt that based on what he knew of the primary grades from teaching them and from Carrie’s conversation, the primary grades had already been “doing that Singapore, European math, method of teaching math for years.” In the last
interview, as he discussed the CCSS, Ray observed, “[T]raditionally in the U.S., we’re innovative, and that if we have this [puts hand on standards] with innovation, that we can take this to new levels we never thought of before.”

Throughout the study, Ray also mentioned a few other practices not specific to mathematics that he felt were useful in his classroom. During interview 2, he said, “I try to keep it fast paced [snaps twice], try to get them moving up and down.” Also, he believed that students identified with disabilities should stay in the regular classroom as much as possible so that all students could learn from each other. In addition, he felt that integrating experiences to support multiple intelligences was “important” in “all subjects”. In contrast to most of his other comments, he did remark, “[T]his year, with new social studies standards, math, and language arts standards, and going self-contained, with days with no specials, and PLCs moved… I’m gonna use the dang old [spelling] workbooks! It’s not best practice, but… [i]t is what it is, and parents understand that! But I’ll do a tweak on it…”. Ray added that he would probably pre-assess and provide different word lists to students, which aligned with his plans for mathematics. Overall, Ray’s feeling about learning was that “it should be fun… As a teacher, and as a kid.”

**Ray’s thoughts about assessment practices that would support the SFMP.**

During the study, Ray discussed several forms of assessment that he felt he would need to consider as he began to enact the CCSS and SFMP. These were: assessment of the SFMP themselves, frequent informal classroom assessments on content, pre-assessments before beginning new units, and common summative assessments that each fifth grade teacher used (which related to the new standardized state assessments).
In the second group discussion and the third interview (thus, toward the end of the study), Ray began to think about assessing students’ progress toward the goals in the SFMP. The fact that Ray had not seemed to consider this before may indicate that he was still primarily focusing on the meanings of the SFMP. His first question at this point was, “Do we do it?” He reiterated his comment that the SFMP were the “glue” that held the content together, and then he remarked, “I’m thinking as a checklist, all right, I’m gonna actually have to do this.” When Carrie described an assessment that first grade had been using to assess the same kinds of goals for the past year, Ray and Sandy were both very interested in seeing it, at least to see how it was “set up”, and Carrie said that she would send it to them. Ray asked whether it really did help first grade to assess both content and practice through a “really rich” task without an obvious solution, and Carrie said that it did. Ray was also interested in seeing another district-chosen assessment that first grade was using to assess students’ levels of thinking, but Carrie was not sure that it would apply as easily to fifth grade. In considering where this progress might be recorded on the report card, he commented, “[W]e do on our report card have – is the effort outstanding, satisfactory – because that’s the only place I can think… that I could even put the Mathematical Practices.” However, he later said, “[T]hat’s another issue – how do we give grades?” He continued, “When you’re talking these bigger issues, and how do you break these – like perseverance… using structure… that one’s a little easier, though, because when you’re seeing their proofs… you can see all that… [but] to synthesize it into a grade… that’s hard.” He felt that this he and the other teachers would need to ponder this further as the new year began.
Ray also discussed ideas related to assessing students’ understanding of content. When Ray talked about using informal (formative) assessments, he often referred to assigning “exit tickets” or having students solve problems on dry erase boards so that he could get a quick glance at how individual students were thinking about particular concepts and/or skills. This would allow him to plan his next steps in instruction, potentially setting up several groups of students who would work on specific goals related to the assessment.

As aforementioned, Ray also thought that he might need to use pre-assessments more during the following year than ever before because students would be entering fifth grade with no experience in the CCSS content. He believed that he would need to “see where the majority [were] at” and focus his instruction to address pre-requisite skills for the fifth grade content – as well as then teaching this content itself.

In reflecting on the common unit assessments that the fifth grade teachers had been writing and refining in their PLC for the last few years, Ray shared that the teachers had tried to keep the tests “at the same rigor and phrasing” as the OAA items, and for the past year, they had been able to use a computer system to score the multiple choice items and to enter their scores for the two- and four-point items. Then, this program provided them with many analyses of the data, which helped them to focus later instruction and review. However, during interview 2, Ray said, “I think we have to re-look at assessments now, because these [new standardized tests] are much bigger, a lot of them, than just paper-pencil.” Ray was unsure how they should proceed with revising their own assessments to support the CCSS because there were almost no specific guidelines
or sample questions yet for the standardized tests based on the CCSS. During interview 1, he commented, “I want to develop better assessments that truly gauge what the kids know” and that the CCSS would likely be “changing some thoughts on how we teach and assess”. He felt that it would be challenging to find “good assessments that will mirror what we have here” that were “ongoing” and “diagnostic”.

Looking toward the next several months and the initial transition to the CCSS, Ray commented in the third interview, “I think that’s the next step for us is looking at assessments… that big problem solving… and how much is enough assessment, how much – what’s not enough assessment, so that’s gonna be a tough one… that’s something we need to work on.” So, he seemed to feel that learning to enact the standards would be teachers’ first priority (his included), and assessments would be a “next step” as the process of implementation continued.

**Ray’s perspectives on challenges related to enacting the CCSS and SFMP.**

Clearly, Ray was dedicated to continually improving his practice to improve his students’ learning of mathematics, and he was quite reflective about his own practice. In the third interview, he remarked, “I just want to keep learning.” However, as noted in earlier sections, he knew that he would need to overcome various challenges to implement many of his ideas for the transition to the new standards. Some of these challenges were simply elements of his practice that he wished to change. First, although he felt rather negative about teaching students “tricks” for computation, he recognized that he still did so, and he wanted to try to avoid that in the future. Part of his planning related to this concern involved using Singapore math strategies more regularly because he felt that this could
help students understand word problems and computation much more deeply than they had in the past. Second, Ray felt that he needed to learn to use manipulatives more effectively in instruction. Third, he had seen that some students in his class did not share their thinking, even with partners, often because they did not believe that they were correct. Ray hoped to find a way to encourage these students to communicate more with others. Fourth, he believed that he did not usually leave enough time at the end of class for students to share their ideas and for the class to gain some closure on the lesson, and he wanted to include this time as often as possible during the new year. Fifth, though Ray certainly knew that he would always have students in his class at different levels of understanding, he wanted to try to better support all students – including students with deep understanding – in moving forward. Finally, perhaps in relation to several of these goals, Ray remarked during interview 1 that he needed to listen more and to allow students to ask more questions, yet in the first group discussion, he commented, “[This] is hard for me… it is hard because we’re like the director, the conductor”. Moreover, he reflected on his first lesson during the second interview, noting, “I think I was doing too much of the modeling – at this point in the year, they should have full control. So if I were to do that again, it’s like I should just sit and shut up.”

Ray also described other challenges related to his own teaching that were more outside of his sphere of control. Because he valued his class time very highly, he was bothered by “the announcements, and the phone calls, and the interruptions – it’s huge.” In addition, as described earlier, Ray felt that many students were not as interested in learning mathematics as he would have liked them to be, though he also seemed willing
to adapt his practice to help students become more engaged. He was continually frustrated with what he viewed as the lack of initiative on the part of administrators to monitor what teachers in each grade were teaching, because he felt that students were coming to fifth grade with large gaps in understanding that could have been prevented. He also felt that fifth grade teachers (himself included) who expressed their dissatisfaction with this situation were seen as being too negative. On this point, he noted that he was glad that the curriculum director had mandated the use of the recently purchased textbooks for the following year. Ray wanted his administrators to be honest with the teachers and for his colleagues to be honest with each other about the need for various changes within the school or district, and he did not always see this occurring. In general, Ray believed that “education [was] being picked on” by the public at large and by politicians who were making decisions that Ray felt hindered public schools, but he did still say that he loved his job – working with students, particularly.

In many ways during the study, Ray expressed varying degrees of anxiety about the district-wide transition to the CCSS, using the words “panicked” and “overwhelmed” numerous times to describe how he felt. He seemed to feel more concern early on because administrators had not yet announced many of the plans for the following year. During interview 1, Ray remarked, “You gotta do this right from the get-go. Because if you don’t, there’s such big room for disaster.” He believed that more work should have already been done in grades 3 to 5 with piloting some of the new standards and potential new resources for teaching and learning. He shared later in the study that when the fifth grade teachers learned they would all be self-contained in the new year, people were
“freaking” because many of them had not taught more than two of the major content areas for years, if ever. Ray was anxious that some of these teachers would disagree with the goals of the CCSS and simply revert to teacher-directed practices (e.g., “cute activities in a file folder”) with which they were comfortable because “they think they know the right way to do it.” He added that these teachers “aren’t trained in how to develop kids’ thinking”, and many of them had hardly had any professional development in mathematics since they graduated from college. Moreover, Ray shared, “The other math teachers that I currently work and plan and do PLC weekly and do common assessments – all have used workbooks in the past… when push comes to shove, when you don’t know better, you go back to how you were taught.”

As noted above, Ray was especially concerned about teachers’ understanding of the goals of the CCSS and SFMP. He was worried that teachers had not participated in much professional development and were not very well informed about the new standards – even at the primary levels where he thought the standards had been implemented that year. In the second interview, he added, “And that’s another fear for me is misinterpretations of these standards. Because algorithm is mentioned many times. That doesn’t mean – you don’t get that book from 1977, and let’s just start doing that.” On a related note, Ray wondered whether teachers new to math (whether new hires or veteran teachers) were prepared to enact the SFMP, which he believed that even professional development might not sufficiently address; he had already experienced one ODE workshop that was essentially a repeat of an earlier one with a focus on content, even though it was supposed to build upon the first. He maintained, “[I]t’s the philosophy that
has to be embraced,” frequently reiterating this idea. Ray used himself as an example as he described how his interpretation of the standards had evolved over time. During the first interview, he said, “I see the bigger picture clearer now, ‘cause I read it and – ‘Oh, dear God, this is gobbledy gook, ooh’ – but it makes sense.” He was referring to how his experience at the Singapore math workshop had helped him to understand the expectations in the standards. Still, as he pointed out in interview 2, “I’m a great reader - I had to read and reread and reread again… so many people, they can read through the clusters and the domains quickly [flipping through the pages of the standards as he talks], and, ‘Oh, this isn’t that much’ – [shakes head] you don’t see the bigger picture.”

Moreover, “[J]ust because there’s less, doesn’t mean that you’re not rigorous.” He even commented that he was not sure that the standards, “as written”, would help to catalyze change in mathematics instruction in his school because if teachers only read the standards and were not provided opportunities to reflect on them, they would not likely understand the authors’ intent for individual standards and the document as a whole. As he noted in interview 2, “Now, teacher interpretation, and are they willing to make the shift, will determine everything.”

Complicating the matter of teachers’ interpretations of standards was Ray’s observation that not all teachers were especially committed to their own professional learning. Ray noted that no primary teachers from Sealon had attended a series of multi-district meetings a few years earlier when the CCSS were first adopted in Ohio, and he was the only fifth grade teacher from Sealon who participated. Although he was glad to support his colleagues when they had questions, he did not feel that he would have a
great deal of extra time during the upcoming year, and he felt that more of them needed to take more responsibility for their own professional development. In the third interview, he commented, “[A]s teachers, we sell ourselves short… if we profess to be lifelong learners, then we have to prove it… In our keeping up on reading… that’s important… a lot of people forget that… that makes me mad… you want our kids to be – but you’re not willing to read a book over the summer?”

Another early concern Ray shared related to the new mathematics teachers especially, but really all of the teachers, was that the district had not recently made any large purchases of quality resources that were aligned with even the current standards, let alone the CCSS. During the first few months of the study, Ray was clearly worried – based on the frequency and tone of his comments – that the standards would be extremely difficult to implement consistently without an appropriate resource that all teachers were using. By the end of the study, though, Ray was pleased because an updated edition of the textbook, with supplements for the CCSS and new manipulatives for the new math teachers, had been purchased. He felt that this would be very helpful, but he also felt that he and his colleagues would still need to find or create other supporting materials because the new books did not necessarily include the depth in the content that Ray believed was required. He remarked that he was “shocked” that textbook publishers had not produced more new resources aligned with the CCSS “because they’ve had time” to do so – and with the CCSS being nearly national standards, he felt that companies had financial incentives for creating aligned materials.
Ray was trying to alleviate some of his own concerns and those of colleagues by supporting teachers new to math in any way that he could (offering encouragement and sharing resources and suggestions for instruction), even though he was not the math lead teacher any longer. Ray offered an interesting observation in the last interview – namely, that elements of this transition would have to occur within teachers’ “zone of proximal [development]” (ZPD), just as he often cited Vygotsky’s notion [cite] that learning experiences were most beneficial when situated within a student’s ZPD. He noted, “I think in the building, it’ll take time for some people to realize [how the philosophy of the CCSS is different]. Based on their own schema. And their own history of math.” By the end of the study, Ray seemed to feel more prepared and less anxious because the teachers had been involved in learning about the CCSS, studying their new text resources, and creating curriculum maps for the following year.

Another theme in Ray’s comments was the idea that adults in general lacked appreciation for mathematics and for the kind of thinking that students would be asked to do as a result of the CCSS. He noted, “When you go to a party, and they find out that you teach math, everyone up to you – ‘Oh, my God, I’m so bad at math! Ohhh, gee, I just – I hate math!’” He continued, “So we need to change that, not only amongst the kids and the teachers but, you know, with people in the United States… And I think it’ll start with the kids too.” Ray maintained that schools needed to help adults understand “that math is important. It’s a universal, international language.” He even suggested, “This actually will be a defense for us – the standards. ‘I bet you can’t do that. Can you? Give you a problem.’ Then I think it would quiet them up.” Ray felt that the fact that
review and practice were still going to be part of mathematics instruction and the fact that more relevant contexts for learning would also be utilized would help more adults to feel comfortable with the changes in the standards.

Ray also believed that the strong reliance on technology among most adults and children in his school community was an obstacle in trying to help students learn to search for answers for themselves (i.e., to think) and not to expect “immediate gratification”. He felt that the resulting lack of perseverance in many students (referring to SFMP 1) was “a societal issue” – “how do you slow people down to think about this problem?” Another frustration related to SFMP 1 was that some students assumed that calculators were always correct and did not think about whether the answer on the screen made sense relative to the given problem.

Thus, Ray cited many challenges that could influence the enactment of the CCSS and SFMP within his classroom, school, and district, including several local and statewide policies and practices, which are described below.

**Ray’s perspectives on policies that would support or conflict with the CCSS and SFMP.** During the study, Ray spoke often about local policies, as well as some state policies, that would influence Sealon’s transition to the new standards. During the first interview, he noted that ODE was encouraging districts to begin to transition to the CCSS as soon as possible so that the change could occur over a few years, and he hoped that Sealon leaders were planning to move in this direction (of course, by the end of the study, he knew that the district would be enacting all four sets of new content standards during the following year). During the first group discussion, he also commented that it would
be very helpful if the primary classes especially were capped at eighteen students, as apparently they were in the past. He also said, “There’s a lot of *unwritten* policies,” but he did not specify what he meant by this.

In interview 2, Ray emphasized, “working conditions are important to me”, but he commented that administrators were essentially saying to teachers that they were “not with kids enough” – which was why they were eliminating some time for specials, asking teachers to spend more time with students during times like lunch and recess, and even eliminating some school holidays during the year so that students saw their teachers more consistently. He noted that it was difficult for students and teachers to sustain energy for learning and teaching with this kind of schedule. Also in this interview and in the third, he shared that he felt that many teachers who essentially had followed school policies (such as a focus on effective intervention) for years were often “punished” when leaders made decisions to mandate action for a few teachers who had not followed these policies.

During interview 3, Ray said that the fifth grade teachers had learned that they would have ninety minutes each day for math, but he was not sure that this would actually be possible with their schedule. Further, although he appreciated the longer class time, he thought that perhaps an hour and fifteen minutes would be better for fifth graders, but he was also unsure whether the half hour of intervention time each day (also newly mandated) would overlap with some of his math class time. In addition, Ray remarked with frustration, “PLC is becoming a dirty word… when it’s being mandated that we can’t plan and pace… PLC was all about ownership… once administrators took them over, it’s changed everything… inherently, that’s so anti-PLC!” Generally, Ray would
have liked to see “more consideration of teacher time, especially from the superintendent as well [raises eyebrows], because all these mandates are being put into place, without appreciating PD time from us.”

Another expectation – essentially an unwritten policy – of the Sealon leaders was that fifth grade scores should continue to improve on the mathematics OAA. This expectation was so dominant in Ray’s mind that it seemed to underlie almost every part of his discussion about standards, teaching, and assessment throughout the study. Ray commented on several occasions that the principal had told the grade 5 teachers that they were “not allowed to review” because they were to teach only fifth grade content, and the principal monitored teachers’ plans and was critical of a teacher if review was included. This included a mandate for not focusing on computation because in sixth grade, the students would be able to use calculators. However, the teachers reviewed anyway – now without including this in lesson plans – because many students simply had not mastered prerequisite content for the fifth grade indicators. During interview 1, Ray commented with great frustration, “We gotta have a superintendent, board of education that is on board, not fighting us, not just looking at test scores – that doesn’t solve everything… I think the board has to be… I know they’re busy people, but if you’re gonna mandate, ‘Oh, you better get those scores up,’ you need to come down and just kind of look and see what we’re doing.” Ray made similar comments throughout the study. He added, “[W]e’ve been told in the past that we’re the scourge of the district… we were all bawling… we work our tails off, and we’re the only grade level showing AYP growth.”

In the first group discussion, he noted that the test was “the only important thing… I hate
to say it – I never thought we’d get there… we’ve gone from being kid-driven to data-driven… That’s killing me.” In the second group discussion, Ray said twice that he felt “guilty” about being so driven by the test. About the term “data-driven”, he observed, “They’ve turned it into a nasty word… Instead of it being helpful, they’ve turned it into something ugly.” Yet, as Ray pointed out, even with all of the concern about fifth grade math scores, Sealon was considered an “Excellent” district according to the state, based on indicators on which all districts were evaluated.

Ray commented that he and the other teachers were always trying to improve their students’ OAA scores, but he had been more frustrated in recent years than even in the past because the state had not released very many test items from the last couple of years, so it had become harder for teachers to understand why their students were scoring low in certain areas and to prepare them adequately for later tests. He did, however, feel that using the online assessment and data management software that the district had purchased was allowing the teachers to gather and use more data on their classroom assessments than they had in the past, which was helpful. (As noted earlier, the teachers tried to use problems similar to OAA items on these assessments.) On the other hand, Ray also felt that he had seen competition among teachers arise during the year as they were studying assessment data because some teachers’ students had better scores than others’ students, and some colleagues had become defensive or had even intimated that others had inflated their own students’ scores. Further, late in the study, Ray commented that during the following year, teachers were expected to address both current and new standards in order to continue to improve the OAA scores while transitioning to the
Common Core, and he felt that this was going to be very difficult, as described earlier. As he often observed, they were “limited by [the] calendar” with so much content to address in one year. He did feel that in the future, when the transition was more complete, that instruction could be more coherent throughout the year – preparing for the state tests included – because the CCSS were more appropriately structured in each grade level than the Ohio indicators were, and because teaching “to the depth”, as Ray said, should allow for deeper learning and better student performance. Further, he believed that focusing on the goals in the SFMP would help students to succeed with various types of test items, especially including the performance-based items and any items that required reasoning similar to that described in the SFMP.

In addition, Ray shared a few concerns about the new system for evaluating teachers in Ohio, which would also be initiated during the following school year. One concern in particular was the fact that half of a teacher’s evaluation was to be derived from value-added data from state test scores, and as shown earlier, Ray was anxious about how the transition to the CCSS would affect his students’ scores during the following year or two. His more general questions related to how exactly the other half of the evaluation, reflecting observations of classroom practice and other professional practice, would be established and how evaluations would be described in contract language between the teachers’ association and the school board.

Despite Ray’s various concerns about the transition to the CCSS, he did offer thoughts about policies and practices that could be enacted to support the implementation of the new standards, and these are detailed in the next section.
Ray’s perspectives on necessary supports for teachers enacting the CCSS and SFMP. One idea that Ray often repeated related to assisting teachers in the coming years involved positive, yet specific, administrative expectations for the implementation of the CCSS. He felt very strongly about this; in fact, after he discussed this during the first interview, he noted, “You noticed I stressed a lot on that.” He argued:

I think we need administrators that are also very cognizant of how math should be taught – you know, if you don’t have that, you’re not gonna win. You’re gonna shut your door quietly and do what you gotta do… Whether it’s principal level, higher-up level… everyone has to be on board… That’s gonna have an impact, and I hope that the bigger ups in the district level are going to be aware of that and make accommodations.

Ray remarked that administrators needed to be well aware of what content was to be taught – and how – and that they needed to hold teachers accountable for meeting these expectations. He said that the enactment of the SFMP should be “systematic” and should start in kindergarten; he was unsure that most of the primary grades teachers had been teaching math in ways that would support the new standards, perhaps because of the gaps that he observed in students’ knowledge, including (but not limited to) fact fluency. “[T]he accountability’s gotta be there. Whether it’s primary or not – tested or not tested…”. He added, “[T]hat is a strong policy that has to be in place by the school administrator, the building, and then upward… That they’re going around and not… kind of undermining us, what we truly do in mathematics.” Moreover, in order for administrators to learn enough about the CCSS and SFMP to enforce such policies, Ray
maintained that they should be attending professional development workshops with the teachers. He noted, “I know they’re busy, but it does validate the changes, all the changes that we’re having to go through… when a principal only sees things on paper, it doesn’t seem as bad… there are things she needed to hear…”. Also, he felt that “everyone who’s involved with children” – board members, administrators, teachers from other grades, and even school support staff – should be expected to visit classrooms periodically (even to teach a bit) so that they could directly experience the kind of “complexity” that was required in the CCSS. “I think it’s always good to walk a mile in someone else’s shoes… Because I think it might change some views – instead of, you know, nailing us to the cross, they might actually have a little sympathy and try to help us.” Ray did comment that he believed that the curriculum director understood the nature of the changes that would need to occur with the transition to the CCSS, but he was not sure that the superintendent did. When I asked whether he felt he could discuss the SFMP and the goals of the CCSS with his principal, he said that he was not sure because he was not the math lead teacher anymore. In general, Ray commented, “I just wish [administrators] were as informed. Because I think a lot of policy would be different.”

Ray believed that if administrators understood the philosophy behind the CCSS, they would be better able to facilitate the establishment of consistency in how the standards were enacted across the grades. He spoke of this “critical” need for consistency quite often throughout the study. During interview 1, he remarked, “And you’re gonna have people who are out there, your trailblazers, gonna be out there on the limb, and you’re gonna have people in the middle who follow the trailblazers – and then
way back here are people who are like, ‘Oh, no. No, no, no!’” He commented, “[E]veryone should be on the same page in regards to training and professional development, adding, “We need that consistency.” Ray felt that it had been helpful, when the current Ohio standards were implemented, for all of the teachers across a grade band to “unpack” the standards together so that they could see what learning needed to occur in each grade, and he hoped that leaders would provide this opportunity for the CCSS. He even mentioned trying to work with pre-schools in the area to help them understand the kinds of learning experiences students ought to have before entering kindergarten. Without this common understanding of the standards among teachers, Ray commented, “[T]hen you’re back to square one, where we’re gonna get this, this, horrible mish-mosh of missing things.”

Without a doubt, Ray believed that professional development for teachers was the “number one” key to successful implementation of the CCSS and the SFMP – without it, he remarked, “[W]e’re gonna be doing the same crap.” Further, “We need to make sure that elementary and primary teachers understand mathematics themselves. And if you don’t, we better get you up to speed.” He called professional development a way for the district to “invest” in “people and their education”, and he felt that ongoing learning was much more valuable than a one-time, “drive-by workshop”. He commented that this was “the answer of keeping it fresh,” even for a “good teacher”, and he noted, “I need to be stretched a little bit more; how can I become more effective in working with the kids to stretch their thinking out?” As aforementioned, he felt that the work the teachers had done so far with the county consultant had been very valuable, and he hoped that more of
this would take place during the next year. He did recognize, however, that most teachers were going to need training not just in mathematics but in fact in all four major content areas since they were transitioning to all of these at once. So, it was possible that these new teachers, especially, would almost need specific guidelines for beginning the year—“What do I have to teach, and how?” Overall, Ray hoped that professional development would focus “not just on the standards—[but also] how do we achieve this… how do we achieve it efficiently?” He also continually emphasized the need for grounding professional development in the SFMP, and he hoped that teachers would be able to “see examples” of “model” lessons where teachers were focusing on the goals in the SFMP.

In addition, Ray was trying to persuade his principal to send two teachers to a Singapore math workshop during the summer in another state with some money remaining from a grant that he had written and received. He noted that he would feel more “calm” about implementing the CCSS with more people “exposed to Singapore as a model” because it would be a “great piece” to have “everyone at least on board”.

Further, as a former math lead teacher, Ray still felt a great sense of responsibility for supporting his colleagues as best he could, and they also viewed him as a source of assistance. He shared that he was starting to field many questions from new mathematics teachers about how to start the school year, and he planned to share his thoughts about beginning with experiences in cooperative learning for students. Additionally, while his students took the OAA, he actually printed all of the new fifth grade standards (in all four areas) for each of his colleagues and created a binder for each person, and he felt that he would be able to provide new teachers with suggestions for pacing. On the other hand,
Ray noted that his colleagues were not always interested in hearing his views on instructional practice because he tended to be much less teacher-directed in his focus than they were, and he said that it was helpful to have an expert “outsider” come in to share these kinds of ideas – particularly those recommended in the SFMP. (He was glad that Sandy, who was very “vocal”, had participated in this study because they both were now better equipped to facilitate others’ learning relative to the SFMP.) Moreover, as aforementioned, Ray remarked that some colleagues almost tried to rely on him too heavily for instructional recommendations, and even though he “loved” helping others, he simply did not have time to help all of them as much as they might like because of his responsibilities to his own students and his family. He did hope (and believe) that many novice teachers who had graduated recently from college would be more familiar with the ideas in the CCSS and the SFMP and more prepared to enact them – he even said, “The newbies probably will teach us,” though he acknowledged that this preparation would vary from person to person.

Ray believed that one of the best formats for professional development and for teachers to support each other was the grade-level mathematics PLC (professional learning community). This group had existed by mandate for several years, and as described earlier, generally the teachers felt comfortable planning together (in order to teach content at about the same time) as well as using common assessment data to guide this planning. The teachers had developed a shared understanding and language for the various problems and lessons that they used throughout the year. Ray hoped that the PLC would be able to continue to work together effectively because there would now be eight
teachers involved instead of four, and of these, only he and Sandy had taught fifth-grade math during the school year that was ending. He also hoped that administrators would allow more flexibility in their expectations of the PLC during the transition to the CCSS since teachers would have many concerns that were not simply related to assessment data, and teachers had been disheartened when they were told that what they were doing in the PLC was not appropriate. He remarked that “just giving people the time to have dialogues” was extremely worthwhile. As he discussed how he and Sandy could mentor their new colleagues, Ray added, “So I think for each PD meeting, and each follow-up PLC meeting that we’ll have, we’ll have people who’ll say, ‘No, no, you need to kind of step this back a little – let me show you what I’m doing now. This has changed.’ So I think that will be an advantage.” He even suggested that teachers could develop lessons together and then observe each other teaching them: “Almost like the kids, like a workshop approach… here are your mini-lessons, and people work together, that collaboration, cooperative learning – then come back together.” Interestingly, in the second group discussion, Ray noted, “PD, I consider that a big chunk of it, the PLC, but I haven’t figured it out.” This seemed to imply that he was unsure how the PLC could best function with the large turnover in staff for the following year. Still, he said, “That discussion [about the standards] has to keep going – hopefully in our PLC.”

Another support that Ray felt would be very valuable for teachers, students, and parents was a consistently used, high-quality textbook series, even though he had commented early on that he did not like to rely on a book entirely because he felt with his undergraduate and graduate preparation, he could “do better than that”. By the end of the
study, as noted earlier, the district had purchased a new edition of the current textbooks for kindergarten through fifth grade; this purchase included material to address standards unique to the CCSS, embedded games that were useful for reinforcing various skills with students, and manipulatives for teachers who did not already have them. Ray did not feel that the textbook could be the only resource used for instruction because it would not necessarily provide recommendations for supporting every individual student need and because it might not provide as many “rich problems” as teachers would need to enact the standards fully. Still, he believed that the program would help to provide needed “consistency” among teachers and grade levels and would help parents understand what their children were learning. As mentioned before, he also hoped that the district would eventually provide more computers for student use and would continue to provide the online assessment program that they were using.

Ray also believed that teachers would need opportunities to find and create additional material to support instruction in their classrooms. He believed in writing curricular units and starting with the end goals in mind, and their fifth grade PLC had created some units together in the past. He anticipated that they would do more of this in the coming year. During interview 2, he noted, “[I]t’s going back to the richness of the problems that I’m going to have to choose or create and find those resources – that was the big ‘aha’ at the end of that whole unit. How am I gonna get to the mathematical practice standards, you know, because that’s the vehicle for you to hit the domains and the clusters.” Further, he planned to share as many of his own resources as he could with teachers who were new to math. He also wanted to encourage other teachers to try the
Singapore math materials and to attend a future Singapore workshop, and he had found some websites that he thought would be useful for teachers and students. During the third interview, he shared a brief, general set of criteria that he might use to evaluate materials: “[T]hat, as a resource, is what we’re always looking for. You know, real quality, and relatable, and ooh… draw it, show me – now, let’s tweak it, and it’s all about keeping things not static.”

Not only did Ray believe that the district should support teachers during this change – it should also provide “stuff” to support parents as well. “Otherwise, [parents] will undermine everything that we try to establish in the classroom in regards to thinking… mathematically… Parents are gonna be resistant, for the simple reason that it’s not how they were taught.” He did also comment, though, that families were “probably doing the best they can” and that parents “want to do what’s best for the kids.” Ray suggested that the school could hold a family math night, as they had done recently with science and in the past with math when the first edition of the current textbook was adopted. He said that parents and students enjoyed working and learning together. He did comment, though, “I think to move away from the games – because the games are great just for those skills… if we told them, ‘Don’t work the algorithm’, they might understand it better… that’ll take some time.” Relative to his own students, Ray said, “I do a newsletter on the website; we have the homework posted there… I always try to explain to parents, ‘Your child’s working on this one; here’s the formula, BUT, [for example] they can also solve volume by layers.’” Generally, Ray added, he felt that most
of the parents really did trust the teachers, but “you’re gonna have some vocal” parents for whom the district would need to provide additional support.

So, Ray argued that several key elements of support should be in place in order for the transition to the CCSS to be as effective as possible for all concerned. First, administrators needed to be very aware of the expectations set forth in the standards and for what they should hold teachers and students accountable as a result. Second, administrators needed to provide or arrange for ongoing professional development for all teachers so that teachers could deepen their understanding of both the SFMP and the content standards and so that the enactment of the standards could be relatively consistent within and across grade levels. Third, Ray hoped that the fifth grade mathematics PLC would still be a forum for continual teacher learning and collaboration and that he could assist his new colleagues as he was able during the following year. Fourth, the teachers needed quality instructional resources – both from the main textbook and from other sources – that allowed them to teach the content framed by the SFMP. More computer technology was also a great need for most classrooms. Finally, Ray felt that the district and school needed to provide support designed to help parents understand the CCSS and SFMP because the goals therein were so different than much of what adults experienced when they were in school. If these types of support were in place, Ray seemed to feel that the potential for success with the CCSS was high. On the other hand, the converse was also true.
Reflections on the Evolution of Ray’s Thinking about the Standards and Potential Changes in His Practice

As noted from the beginning of this section, Ray felt that the Singapore math workshop that he had attended allowed him to gain a better sense of the goals in the CCSS and SFMP before this study began. He referred to this often during the study, saying that at first when he had read the standards, they had seemed unclear and difficult to understand. He added, “I’m a visual learner. So actually seeing videotapes of Singapore math and things like that – wow.” In fact, he remarked during interview 2, “[I]f I hadn’t had that, I think these interviews would be completely different… it made that much of an impact on me.” Ray also noted that professional workshops in general were useful for his learning, and he appreciated the support that the county consultant had provided to help him and his colleagues “clarify” the expectations in the new standards.

Ray often shared that it was very helpful simply to read and discuss the standards as he considered the changes that he wanted to make in his classroom the following year. One example of this that he shared in the first group discussion was his desire to include more opportunities for students to share their agreement and disagreement with each other’s thinking – in a respectful way, of course. In the second interview, he shared that he had had an “aha” moment in realizing that the SFMP were the “vehicle” by which the content should be taught, and he realized that he needed to create and find lessons that allowed him to teach in this way. Further, during this interview, Ray started to think about allowing for more differentiation for his students who needed enrichment, and he also began to see parallels between the lack of perseverance of many of his low-achieving
and high-achieving students. In the third interview, Ray noted, “I liked the self-reflection [in the study], because you know, I could talk anyone’s ear off. Whether it’s in writing, or orally.” He believed that it was crucial for teachers to recognize their strengths and weaknesses and to be willing to improve.

The videotaped lessons were also very valuable for Ray in terms of his reflection on his teaching. He described the first video as “eye-opening”, and after each video he offered many comments about what he saw in his teaching that he would like to improve in the future – particularly facilitating more student discussion and less direction by him. He noted that he wanted to present more problems in context in the future, and that he needed to find better resources to support this goal. Further, by watching the first video, Ray realized that he was addressing content and using some instructional strategies that he had not thought were present in the lesson, and it encouraged him to see that some elements of the new standards were in fact somewhat in place. He shared, “I felt better once I did my second video,” and he looked forward to starting the following year with some of the ideas that he had generated from reflecting on these lessons. Overall, Ray commented, “Videotaping is always good… for any – anyone. Not even just teachers, but being able to reflect and see how you can do things better, I mean, you have to do that… you have to admit where you’re weak at. You can’t pooh-pooh it off and make change.”

Regarding the conversations with Sandy and Carrie, Ray commented, “I like the group discussions… because they bring up new things, and it’s like, wow… there were some tense times, but there’s always, when you’re talking – I’m one, you do the best you
can, whatever grade level you are with the group you got – traditionally, it’s so easy to point fingers…”’. That is, Ray acknowledged that teachers did not always agree on issues related to learning and instruction, but he also believed that each teacher was doing the best that he or she could with the students he or she was teaching. He added that he, Sandy, and Carrie seemed to feel “comfortable” enough with each other to take the “risk” of discussing controversial issues such as fact fluency. As he had said earlier, he believed that it was important for teachers to understand the expectations for students and teachers in different grades, and he hoped that Carrie would be able to share some of her learning about fifth grade with the other first grade teachers.

Ray also mentioned that he found it helpful to read professional journals to gain new ideas and to learn about new ideas in the field of mathematics education. As aforementioned, Ray often referred to research that he likely had studied in college courses, but it is also possible that he saw references to this research (e.g., Piaget’s levels of thinking) in more recent articles.

Reflecting on his participation in the study more generally, Ray commented, “[Sandy and I] feel like we have a heads up on everyone [relative to the CCSS]… I think actually it’s because of this project – which is one of the reasons I really wanted to do it.” Further, “[T]his is the biggest advantage of me working with you, is [really] looking at [the SFMP], and this is why I wanted to do it, is that you need to speak, talk mathematics intelligently, and realize it’s not an easy fix.” Interestingly, in the third interview, Ray shared, “[T]he fact that you focused just on the Mathematical Practices… ‘Why aren’t we going right to the standards, man?’ But I think that was the biggest aha… Sandy and I,
we’re the first ones [raises hand high], to read strictly and go right to the [content] standards – no! No! You gotta back it up!” He added that since the study had focused on “just the three [SFMP], too – then it made me curious about the others!... [I]t really springboarded for me to understand what the other Practices – other than 1, 3, and 7… because of that, I think I’ve a stronger understanding of mathematics as a whole.”

At the end of the final interview, Ray said, “I just want to thank you!... [B]ecause this has been very helpful for me… I like intellectual dialogue.” He also shared, “I love change. I thrive on change. I think it keeps me fresh.” Finally, a comment from the second interview seemed to characterize Ray’s overall attitude about the transition to the new standards: “I think it’s exciting. It’s an exciting time. I like change. Change is good.”

**Conclusion**

In this chapter, I have worked to follow Yin’s (2008) recommendations for writing a report on case study research. I have presented the interpretations and ideas of each teacher in this study individually, based on the issues uncovered through investigation of the research questions through interviews, group discussions, and videotaped lessons. I have offered observations about how each teacher’s thinking seemed to change (or not) throughout the course of the study and what aspects of the study might have promoted these changes. Each teacher has been described carefully within the context of his/her own classroom environment, school, and district, and consideration has been given to various factors that might influence each teacher’s thinking. Evaluative statements about each teacher and his/her interpretations have
generally been avoided and will be left to chapter 5, particularly to allow the reader to
draw conclusions independently. Moreover, participants were asked for feedback on
summaries of the evidence collected throughout the study and provided virtually no
contradictions to what had been written. In the final chapter, I will present a cross-case
analysis, highlighting similarities and differences in the participants’ interpretations and
ideas, and I will discuss implications of the findings for professional development and
further research related to the content of this study.
CHAPTER V

DISCUSSION

Introduction

In this chapter, I present cross-case analyses of the interpretations of the Standards for Mathematical Practice and related implementation considerations for the three teacher participants in this study: Carrie, Ray, and Sandy. In these analyses, I present issues that were common to all three in one way or another, and I discuss how these issues impacted each teacher in unique ways. I also share comments from Sandy and Ray that they sent as final feedback on my tentative conclusions (essentially the main ideas from each section of this chapter), which I sent to them fifteen months after I had last interviewed Ray and eighteen months after I had last seen Sandy, in a group discussion. I sent the same file to Carrie but did not receive a response. Both Ray and Sandy essentially agreed with all of the final conclusions that I shared, and I include here only their comments in which they elaborated on their agreement. It should be noted that I did not send the participants my more specific, critical analyses of their individual interpretations and teaching practices because I did not believe that these analyses would be helpful to them without much more discussion and reflection.

Structuring this chapter in this way allows me to address my research questions relative to individual participants as well as to the group as a whole. Once again, the questions explored in this study were:
1a) How do elementary teachers interpret the overall goals of the Common Core State Standards in Mathematics and the Standards for Mathematical Practice relative to students at their own grade levels?

1b) How do elementary teachers envision these goals and Practices developing in their own schools and classrooms?

1c) What do elementary teachers believe will be necessary supports for schools and teachers as the Standards and Practices are enacted?

2) How do elementary teachers’ interpretations and beliefs about the Standards and Practices evolve throughout the following professional experiences: conversations with other teachers and the researcher, and reflection on lessons in which they attempt to enact content from the Common Core State Standards and Standards for Mathematical Practice 1, 3, and/or 7?

I first present a series of tables that summarize the teachers’ interpretations of each portion of text considered in this study, followed by the cross-case analyses described above. Then, I suggest implications of the findings in this study for school leaders and policy makers, and I offer comments about future research in the field of mathematics education that could help to provide more information about how best to support teaching in enacting the SFMP and the Common Core State Standards in general. I also discuss the limitations of this study and my own experience as a researcher therein, and I conclude with some reflections on the study as a whole.
Three Unique Individuals

Throughout the course of the study, I came to know Carrie, Ray, and Sandy as very caring, jovial, dedicated teachers who each held strong perspectives about teaching and learning. However, it became apparent after the first two phases of the study that although on a few points there was overlap in the teachers’ thinking, in many cases each teacher had developed his/her own distinct views.

Carrie seemed very serious and thoughtful throughout much of the study, though I also believe she was generally fairly relaxed and did not become ruffled easily. She was the most concise of the three teachers overall in her speech; she typically made a statement (or series of statements) once and then was ready to move on in the conversation.

Ray did by far the most talking throughout the study and in the two group discussions, and I actively worked to encourage Carrie and Sandy to respond more when during these discussions so that Ray would not overly dominate the conversation. During the interviews, Ray repeated himself frequently and wandered in his thinking as he talked, as if he was thinking aloud. When Ray shared the same stories about his past experiences as a teacher and student multiple times, this indicated to me that they were especially significant to him and likely impacted his beliefs and interpretations.

Sandy was not always as articulate as Carrie (or Ray, notwithstanding his often fragmented way of speaking), and her speech contained more grammatical errors than that of the other two teachers. Further, her content knowledge seemed to be somewhat lacking, at least in subtle ways, as shown in Chapter 4. In the first interview, she seemed
very negative about teaching and learning; these feelings emerged clearly in her dialogue. In the group discussions, she mainly listened and often let Ray speak for grade five. Still, by our last meeting (the second group discussion), Sandy had set a goal for herself to become progressively familiar with the new standards and comfortable in enacting practices that supported them, and as previously noted, she may have been the participant with the most potential for growth in her practice.

It is noteworthy that the differences in personality among these teachers could easily influence how various kinds of professional development might impact their beliefs of practice. This issue will be addressed further later in the chapter.

**The Researcher’s Own Interpretations of Each Portion of the SFMP**

This study clearly involved layers of interpretation. In my analysis, I attempted to interpret the teachers’ interpretations of the SFMP, which necessarily meant using my own interpretations of the authors’ intent for the goals in SFMP as bases for comparison and contrast – i.e., as “orienting objects” as described by Brown (2001). So, in order to be as open as possible about my own interpretations, I share them here.

**SFMP 1a.** Understand what a problem is asking us to do, and continue to work on it, without giving up, until it is solved. Encourage others to do so as well.

**SFMP 1b.** State to self what a problem is asking, and consider ways in which to begin solving it.

**SFMP 1c.** Consider the limits or parameters that restrict or help to define possible solutions to the problem. Think carefully about solution strategies before beginning. Bring to mind other similar problems that we have solve previously, and
consider whether solution strategies for those problems would be useful here. Assess the reasonableness of the results of steps taken to solve the problem, and re-evaluate or go back if a given result is not reasonable.

**SFMP 1d.** Elementary students should use concrete or visual representations to represent and understand a problem situation.

**SFMP 1e.** Having solved a problem, solve it in another way and observe whether the solutions agree. If they do not, work to find the error(s) in one or both solutions.

**SFMP 1f.** Understand what others did to solve the problem, and describe how specific steps or features in one strategy are related to steps or features of another strategy.

**SFMP 3a.** Create and present logical series of reasoning to explain a solution strategy, and offer evaluations of the reasoning of others.

**SFMP 3b.** Use information that is already known and understood to reason about the solution to a current problem. Generate ideas about a problem solution, and reason with established information to prove or disprove these ideas.

**SFMP 3c.** Understand that certain problems can be seen as several specific sub-problems, which can be solved or considered individually in pursuit of the larger solution. Know that it may be helpful to show that a possible solution or idea does not correspond with the given problem.

**SFMP 3d.** Show that one’s own thinking is valid by presenting this thinking, talking with others about a variety of ideas, and asking questions of others.
SFMP 3e. When two or more possible solution strategies are presented, discuss the validity of each one, and determine whether there are errors in reasoning.

SFMP 3f. Elementary students use concrete representations to explain and/or prove their own thinking.

SFMP 7a. Find patterns or structure in a problem situation, and use them to learn something about, or solve, the problem.

SFMP 7b. Examine a mathematical or real-world situation to find (not necessarily use) patterns or structure within it.

SFMP 7c. Notice specific structural relationships among numbers, and classify and relate mathematical objects based on identified properties.

SFMP 7d. Understand and use the distributive property.

SFMP 7e. Based on knowledge of patterns or structure in a problem situation, examine the problem in a different way to gain new information about it or find an alternate solution strategy. Or, solve a similar problem by understanding the patterns or structure in the original problem.

SFMP 7f. View many mathematical objects (not just algebraic expressions) as composite structures, comprised of smaller, less complex objects. Understand how the smaller parts fit together to form the whole.

Summary Tables of Teachers’ Interpretations of Standards

The tables below summarize the information presented in Chapter 4 about each teacher’s perspectives on the Common Core State Standards introduction and Standards for Mathematical Practice 1, 3, and 7. For each portion of the introduction and each
portion of each SFMP, I offer several observations about the teachers’ interpretations. In the inner cells in each table, I have included page numbers to refer the reader to the specific portion of Chapter 4 that led to each summary statement.

The categories in each table are as follows: first, I offer an analysis of whether the teacher’s interpretation seemed to agree with the likely intent of the authors or not – or whether the teacher’s interpretation was not observable. Second, I note whether the teacher thought about all of the text in the given portion of the standards or just part of it. Third, based in the ideas that seemed to ground each participant’s comments, I share conclusions about the interpretive framework within which comments about each portion of the standards were offered. Fourth, I provide some general ideas for implementation of this portion of the standard that were suggested by the teacher.

Finally, for each portion of the SFMP studied (not the text on pages 3-5 of the CCSS), I offer an assessment of whether the teacher’s classroom practices seemed to allow for this portion of the standards to be enacted with high, medium, or low fidelity. That is, I analyzed whether the teacher was generally enacting the goals in the text (via actions and/or words) in alignment with the authors’ likely intent (high fidelity), somewhat in alignment with the likely intent (medium fidelity), or not in alignment with the likely intent (low fidelity). The process by which I made this determination began while coding and re-coding during my data analysis, as I coded each teacher’s actions and words in class to indicate whether they aligned with:

1. The *authors’ likely intent* of this portion of the SFMP, as well as the teacher’s own interpretation
2. The teacher’s interpretation of this portion of the SFMP, but not the authors’ likely intent

3. Neither the teacher’s own interpretation, nor the authors’ likely intent (rare)

I then created tables of how many times each of these three codes had occurred for each statement in the three SFMP. If the goal in the text was not observed at all, I entered “n/o” in the table. These tables are included in Appendix I. Next, I calculated how many times the first code above appeared, as a percent of the total number of occurrences of actions/words somehow reflecting the given portion of the SFMP. I defined high, medium, and low fidelity in this way:

High fidelity – the teacher’s actions/words were aligned with the authors’ likely intent for this portion of the SFMP at least five times in the two lessons combined, and this represented fifty percent or more of the occurrences of actions/words reflecting this portion of the SFMP

Medium fidelity – the teacher’s actions/words were aligned with the authors’ likely intent for this portion of the SFMP fewer than five times in the two lessons combined, but this represented fifty percent or more of the occurrences of actions/words reflecting this portion of the SFMP

Low fidelity – the teacher’s actions/words were aligned with the authors’ likely intent for this portion of the SFMP fewer than five times in the two lessons combined, and this represented less than fifty percent of the occurrences of actions/words reflecting this portion of the SFMP
Some observations: first, the teachers were more likely simply to ignore parts of the standards than they were to actually misinterpret them. When they did read the text, they generally had a reasonable sense of what the authors intended for classroom practice. However, there were a few occasions where the text seemed to be so burdensome and unfamiliar that even when the teachers read it, they had no framework for understanding the language used. In these cases, the level of implementation was low, which is not surprising. Second, of the three teachers, Ray seemed to have the most dynamic interpretive framework; a great many ideas – numerous, though generally related – seemed to inform his thinking. However, he did seem to waver between thinking of mathematics as a set of skills and mathematics as a means of understanding the world. Carrie and Sandy had, at least at that time, much more solidified interpretive frameworks that clearly influenced all that they said and did during the study.

Table 3

*Overall Idea of Pages 3-5*

<table>
<thead>
<tr>
<th>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corresponding with authors’ likely intent; some not observable</td>
<td>Misinterpretation, some not observable</td>
<td>Corresponding with authors’ likely intent</td>
<td></td>
</tr>
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<td><strong>Focus: most/all of</strong></td>
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<td>Some text</td>
<td>Most/all text</td>
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<tr>
<td>Interpretive framework</td>
<td>Constructivism</td>
<td>Conception of mathematics as a set of challenging, often rote skills to be learned</td>
<td>Belief that mathematics teaching should develop deep understanding in learners</td>
</tr>
<tr>
<td>------------------------</td>
<td>----------------</td>
<td>--------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>Give students time to explore and develop understanding; help to build students’ number sense</td>
<td>Teach fewer standards in each grade</td>
<td>Teach fewer standards in each grade; focus on problem solving, number relationships, and spatial sense</td>
</tr>
</tbody>
</table>

Table 4

*Conceptual Understanding and Organizing Principles*

These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas.
<table>
<thead>
<tr>
<th>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</th>
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<th>Misinterpretation</th>
<th>Some misinterpretation</th>
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</thead>
<tbody>
<tr>
<td>Focus: most/all of the text, some, or a few words only?</td>
<td>Most/all text</td>
<td>Some text</td>
<td>Some text</td>
</tr>
<tr>
<td>Interpretive framework</td>
<td>Constructivism</td>
<td>Conception of mathematics as a set of challenging, often rote skills to be learned</td>
<td>Conception of mathematics as higher level skills and understanding built on basic skills</td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>Explore the same idea in multiple ways over time; relate new concepts to those previously learned</td>
<td>Focus on these key ideas and principles in instruction</td>
<td>Provide enough practice with skills that students do not have to “think about” them; provide manipulatives; provide for</td>
</tr>
<tr>
<td>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</td>
<td>Carrie</td>
<td>Sandy</td>
<td>Ray</td>
</tr>
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</tr>
<tr>
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<td>Misinterpretation</td>
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</table>

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<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
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<td>Most/all text</td>
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<td>Some text</td>
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</table>

<table>
<thead>
<tr>
<th>Interpretive framework</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher should understand students’ thinking; students should develop conceptual understanding</td>
<td>Conception of mathematics as a set of skills that can be explained</td>
<td>Constructivism as facilitated by the teacher</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ideas related to implementation</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ask “how do you know”; encourage</td>
<td>Ask students to begin explanations</td>
<td>Require students to justify their</td>
<td></td>
</tr>
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Table 6

*Standards for a Diverse Population of Learners*

<table>
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<th>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</th>
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<th>Sandy</th>
<th>Ray</th>
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<td></td>
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<td>Corresponding with authors’ likely intent</td>
<td>Corresponding with authors’ likely intent</td>
</tr>
<tr>
<td>Focus: most/all of the text, some, or a few words only?</td>
<td>Some text</td>
<td>A few words only</td>
<td>Some text</td>
</tr>
<tr>
<td>Interpretive</td>
<td>Constructivism;</td>
<td>Standards are for all</td>
<td>Standards are for all</td>
</tr>
<tr>
<td>framework</td>
<td>teacher understanding each student’s thinking</td>
<td>students: all should have access, and all should be held to the standards</td>
<td>students: all should have access</td>
</tr>
<tr>
<td>---------------</td>
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<td>-------------------------------------------------------------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>Provide multiple approaches for learning new ideas; explore the same idea in multiple ways over time; relate new concepts to those previously learned; provide individualized experiences for students based on needs</td>
<td>Keep all students in regular classrooms whenever possible; do not “lower” the goals for certain students</td>
<td>View students as individuals; set up flexible skills groups; differentiate; help students to learn from each other</td>
</tr>
</tbody>
</table>

Table 7

*Learning Progressions*

<table>
<thead>
<tr>
<th>Interpretation:</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
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<td>Corresponding with</td>
</tr>
<tr>
<td>Focus: most/all of the text, some, or a few words only?</td>
<td>Some text</td>
<td>A few words only</td>
<td>Some text</td>
</tr>
<tr>
<td>--------------------------------------------------------</td>
<td>-----------</td>
<td>-----------------</td>
<td>-----------</td>
</tr>
<tr>
<td>Interpretive framework</td>
<td>Constructivism; Reading Recovery; “known to new”</td>
<td>Conception of mathematics as a set of skills that build on each other</td>
<td>Conception of mathematics as a set of skills that build on each other</td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>“Give [students] what they need when they need it”; “stretch” from “known to new”</td>
<td>Parents should provide early learning experiences</td>
<td>Support students with their individual learning needs, based on the progression of content in the CCSS</td>
</tr>
</tbody>
</table>
Table 8

*SFMP 1a*

<table>
<thead>
<tr>
<th>Interpretation: Corresponding with authors’ likely intent, not observable, or misinterpretation?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Corresponding with authors’ likely intent</td>
<td>Corresponding with authors’ likely intent</td>
<td>Corresponding with authors’ likely intent</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Focus: most/all of the text, some, or a few words only?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Some text</td>
<td>Most/all text</td>
<td>Some text</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interpretive framework</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reading Recovery</td>
<td>Conception of mathematics as a set of skills that can be learned with effort</td>
<td>Conception of mathematics as a set of increasingly higher-level skills that can be learned with effort</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ideas related to implementation</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Celebrate accomplishments; build students’</td>
<td>Encourage students as they work; ask students to explain</td>
<td>Encourage students in class; allow them to encourage each</td>
</tr>
</tbody>
</table>

Make sense of problems and persevere in solving them.
<table>
<thead>
<tr>
<th>Evidence of enactment in lessons: high, medium, or low fidelity?</th>
<th>Medium</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
</table>

**Table 9**  
*SFMP 1b*

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

<table>
<thead>
<tr>
<th>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corresponding with authors’ likely intent</td>
<td>Corresponding with authors’ likely intent</td>
<td>Corresponding with authors’ likely intent</td>
<td></td>
</tr>
</tbody>
</table>

<p>| Focus: most/all of | Some text | Most/all text | Some text |</p>
<table>
<thead>
<tr>
<th>the text, some, or a few words only?</th>
<th>Interpretive framework</th>
<th>Evidence of enactment in lessons: high, medium, or low fidelity?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finding multiple ways to solve problems and/or represent solutions</td>
<td>Conception of mathematics as a set of problem-solving skills that can be learned with effort</td>
<td>High</td>
</tr>
<tr>
<td>Conception of mathematics as a set of problem-solving skills that can be learned with effort</td>
<td>Ask students to think about how to enter a problem</td>
<td>High</td>
</tr>
<tr>
<td>Discuss efficiency and multiple methods for solving problems</td>
<td>Ask students to analyze and explain the steps to solve a problem; encourage multiple solution methods</td>
<td>High</td>
</tr>
<tr>
<td>n/o</td>
<td>High</td>
<td>High</td>
</tr>
</tbody>
</table>

Table 10

*SFMP 1c*

They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution pathway rather than simply jumping into a solution
They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

<table>
<thead>
<tr>
<th>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not observable (for the most part)</td>
<td>Not observable (for the most part)</td>
<td>Not observable (for the most part)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Focus: most/all of the text, some, or a few words only?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>A few words only</td>
<td>A few words only</td>
<td>A few words only</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interpretive framework</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students’ using prior experience to solve new problems</td>
<td>Conception of mathematics as a set of problem-solving skills that can be learned with effort</td>
<td>Conception of mathematics as concepts and connections among ideas</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ideas related to implementation</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Provide multi-step problems; model and encourage “if-then” language</td>
<td>Encourage students to re-read and analyze problems; encourage multiple solution methods</td>
<td>Slow students down to think and be mindful of each step in their solutions; use analogies</td>
<td></td>
</tr>
</tbody>
</table>
Evidence of enactment in lessons: high, medium, or low fidelity?

<table>
<thead>
<tr>
<th>Low</th>
<th>High</th>
<th>Medium</th>
</tr>
</thead>
</table>

Table 11

*SFMP 1d*

Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.

<table>
<thead>
<tr>
<th>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corresponding with authors’ likely intent</td>
<td>Misinterpretation</td>
<td>Corresponding with authors’ likely intent</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Focus: most/all of the text, some, or a few words only?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>All text</td>
<td>Most/all relevant text</td>
<td>Most/all text</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interpretive framework</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students’ developing understanding</td>
<td>Conception of mathematics as primarily skill-</td>
<td>Concrete/iconic/abstract representations as means of learning</td>
<td></td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>Ask students to represent solutions in various ways; use hands-on materials in class for exploration</td>
<td>Use manipulatives in primary grades, and only sparingly otherwise</td>
<td>Manipulatives should be in regular use for all students</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Evidence of enactment in lessons: high, medium, or low fidelity?</td>
<td>Low (Carrie was leading them in all manipulative use)</td>
<td>n/o</td>
<td>Low</td>
</tr>
</tbody>
</table>

Table 12

*SFMP 1e*

Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?”

<table>
<thead>
<tr>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</td>
<td>Corresponding with authors’ likely intent</td>
<td>Corresponding with authors’ likely intent</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Focus: most/all of the text, some, or a few words only?</td>
<td>Most/all text</td>
<td>Most/all text</td>
</tr>
<tr>
<td>Interpretive framework</td>
<td>Socio-constructivism; multiple solution/representation methods are helpful</td>
<td>Conception of mathematics as a set of problem-solving skills that can be learned with effort; multiple solution methods can be valid</td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>“Turn and talk” (students share thinking with each other, and ask questions)</td>
<td>Encourage students to use “reverse” [inverse] operations to check solutions;</td>
</tr>
</tbody>
</table>
Table 13

**SFMP 1f**

They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

<table>
<thead>
<tr>
<th>Evidence of enactment in lessons: high, medium, or low fidelity?</th>
<th>encourage multiple solution methods</th>
<th>encourage multiple solution methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium</td>
<td>Medium</td>
<td>Low</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corresponding with authors’ likely intent</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not observable</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corresponding with authors’ likely intent; some not observable</td>
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<td></td>
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</table>

<table>
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<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
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<td>Some text</td>
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<tr>
<td>Some text</td>
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<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interpretive</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Socio-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conception of</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conception of</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRAMEWORK</td>
<td>CONSTRUCTIVISM; FINDING MULTIPLE WAYS TO SOLVE PROBLEMS AND/OR REPRESENT SOLUTIONS</td>
<td>MATHEMATICS AS A SET OF PROBLEM-SOLVING SKILLS THAT CAN BE LEARNED WITH EFFORT; MULTIPLE SOLUTION METHODS CAN BE VALID</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IDEAS RELATED TO IMPLEMENTATION</th>
<th>“TURN AND TALK” (STUDENTS SHARE THINKING WITH EACH OTHER, AND ASK QUESTIONS)</th>
<th>HAVE STUDENTS SHARE DIFFERENT STRATEGIES FOR SOLVING THE SAME PROBLEM</th>
<th>HAVE STUDENTS SHARE DIFFERENT STRATEGIES FOR SOLVING THE SAME PROBLEM</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>EVIDENCE OF ENACTMENT IN LESSONS: HIGH, MEDIUM, OR LOW FIDELITY?</th>
<th>LOW</th>
<th>MEDIUM</th>
<th>HIGH</th>
</tr>
</thead>
</table>

Table 14

*SFMP 3a*

Construct viable arguments and critique the reasoning of others.

<table>
<thead>
<tr>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERPRETATION:</td>
<td>CORRESPONDING WITH</td>
<td>CORRESPONDING WITH</td>
</tr>
<tr>
<td>corresponding with authors’ likely intent, not observable, or misinterpretation?</td>
<td>authors’ likely intent</td>
<td>authors’ likely intent</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Focus: most/all of the text, some, or a few words only?</td>
<td>Most/all text</td>
<td>Most/all text</td>
</tr>
<tr>
<td>Interpretive framework</td>
<td>Socio-constructivism</td>
<td>Conception of mathematics as a set of skills that can be explained</td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>Support students as they learn to explain their thinking; “turn and talk”</td>
<td>Use “thumbs up, thumbs down” for students to agree or disagree; develop this skill from the early grades on; allow students to comment on each other’s work</td>
</tr>
</tbody>
</table>
Evidence of enactment in lessons: high, medium, or low fidelity?

<table>
<thead>
<tr>
<th></th>
<th>Medium</th>
<th>High</th>
<th>Medium</th>
</tr>
</thead>
</table>

Table 15

*SFMP 3b*

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures.

<table>
<thead>
<tr>
<th></th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</td>
<td>Not observable (for the most part)</td>
<td>Not observable (for the most part)</td>
<td>Not observable (for the most part)</td>
</tr>
<tr>
<td>Focus: most/all of the text, some, or a few words only?</td>
<td>A few words only</td>
<td>Some text</td>
<td>A few words only</td>
</tr>
<tr>
<td>Interpretive framework</td>
<td>Students’ using prior experience to solve new problems</td>
<td>Conception of mathematics as a set of skills that can be</td>
<td>Conception of mathematics as a set of ideas that can be</td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>Encourage “if-then” language; encourage making connections</td>
<td>Guide students in creating a math notebook as a reference for “definitions”; ask students to justify their thinking</td>
<td>Ask students to justify their thinking step by step</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>Evidence of enactment in lessons: high, medium, or low fidelity?</td>
<td>n/o</td>
<td>Low</td>
<td>n/o</td>
</tr>
</tbody>
</table>

Table 16  
*SFMP 3c*

They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples.

<table>
<thead>
<tr>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpretation: corresponding with</td>
<td>Misinterpretation</td>
<td>Not observable</td>
</tr>
<tr>
<td>authors’ likely intent, not observable, or misinterpretation?</td>
<td>Focus: most/all of the text, some, or a few words only?</td>
<td>Interpretive framework</td>
</tr>
<tr>
<td>---------------------------------------------------------------</td>
<td>----------------------------------------------------------</td>
<td>------------------------</td>
</tr>
<tr>
<td></td>
<td>A few words only</td>
<td>Students’ levels of development (learning progressions)</td>
</tr>
<tr>
<td></td>
<td>A few words only</td>
<td>Conception of mathematics as a set of problem-solving skills that can be learned with effort</td>
</tr>
<tr>
<td></td>
<td>A few words only</td>
<td>Conception of mathematics as a set of ideas that can be explained and connected to each other</td>
</tr>
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</table>
enactment in lessons: high, medium, or low fidelity?

<table>
<thead>
<tr>
<th>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corresponding with authors’ likely intent</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Focus: most/all of the text, some, or a few words only?</td>
<td>Most/all text</td>
<td>Most/all text</td>
<td>Most/all text</td>
</tr>
<tr>
<td>Interpretive framework</td>
<td>Socio-constructivism</td>
<td>Conception of mathematics as a set of ideas that can be explained</td>
<td>Conception of mathematics as a set of ideas that can be explained and</td>
</tr>
</tbody>
</table>

Table 17

_SFMP 3d_

They justify their conclusions, communicate them to others, and respond to the arguments of others.
<table>
<thead>
<tr>
<th>Ideas related to implementation</th>
<th>Support students as they learn to explain/justify their thinking; “turn and talk”</th>
<th>Ask students to use “I know this because” when they provide answers; allow students to respond to and question each other</th>
<th>Have students share different strategies for solving the same problem; allow students to respond to and question each other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evidence of enactment in lessons: high, medium, or low fidelity?</td>
<td>Medium</td>
<td>Medium</td>
<td>Medium</td>
</tr>
</tbody>
</table>

Table 18

*SFMP 3e*

Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is.

<table>
<thead>
<tr>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpretation: corresponding with authors’ likely</td>
<td>Corresponding with authors’ likely</td>
<td>Corresponding with authors’ likely</td>
</tr>
<tr>
<td>authors’ likely intent, not observable, or misinterpretation?</td>
<td>intent</td>
<td>intent</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Focus: most/all of the text, some, or a few words only?</td>
<td>Most/all text</td>
<td>Most/all text</td>
</tr>
<tr>
<td>Interpretive framework</td>
<td>Socio-constructivism</td>
<td>Conception of mathematics as a set of skills that can be explained</td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>“Turn and talk”; have students share multiple solution methods</td>
<td>Ask students to explain their work; allow students to respond to and question each other</td>
</tr>
<tr>
<td>Evidence of enactment in lessons:</td>
<td>Medium</td>
<td>High</td>
</tr>
</tbody>
</table>
Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.

<table>
<thead>
<tr>
<th>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corresponding with authors’ likely intent</td>
<td>Corresponding with authors’ likely intent</td>
<td>Corresponding with authors’ likely intent; some not observable</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Focus: most/all of the text, some, or a few words only?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Most/all text</td>
<td>Some text</td>
<td>Some text</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interpretive framework</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constructivism</td>
<td>Conception of mathematics as primarily skill-based</td>
<td>Concrete/iconic/abstract representations as means of learning</td>
<td></td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>Ask students to represent solutions in various ways; use hands-on materials in class for exploration</td>
<td>Use manipulatives in primary grades, and only sparingly otherwise</td>
<td>Manipulatives should be accessible to all students at all times; model the use of manipulatives</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>---------------------------------------------------------------</td>
<td>---------------------------------------------------------------</td>
<td>---------------------------------------------------------------</td>
</tr>
<tr>
<td>Evidence of enactment in lessons: high, medium, or low fidelity?</td>
<td>n/o (Students were not constructing arguments on their own)</td>
<td>n/o</td>
<td>n/o</td>
</tr>
</tbody>
</table>

Table 20

*SFMP 7a*

Look for and make use of structure.

<table>
<thead>
<tr>
<th>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
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<td>Corresponding with authors’ likely intent</td>
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<th>Some text</th>
<th>Most/all text</th>
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</thead>
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<td>the text, some, or a few words only?</td>
<td>Interpretive framework</td>
<td>Constructivism</td>
<td>Conception of mathematics as a set of skills that can be explained and connected to each other</td>
</tr>
<tr>
<td>-------------------------------------</td>
<td>------------------------</td>
<td>----------------</td>
<td>---------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>Provide opportunities with patterns and manipulatives that are meant to show structure</td>
<td>Ask students to look for patterns in what they are studying or solving</td>
<td>Provide experiences with patterns often in instruction; ask students to share their thinking so other students can make connections</td>
</tr>
<tr>
<td>Evidence of enactment in lessons: high, medium, or low fidelity?</td>
<td>Low</td>
<td>Low</td>
<td>Medium</td>
</tr>
</tbody>
</table>
Table 21

**SFMP 7b**

Mathematically proficient students look closely to discern a pattern or structure.

<table>
<thead>
<tr>
<th>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Corresponding with authors’ likely intent</td>
<td>Corresponding with authors’ likely intent; some not observable</td>
<td>Corresponding with authors’ likely intent; some not observable</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Focus: most/all of the text, some, or a few words only?</th>
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<th>Most/all text</th>
<th>Some text</th>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Interpretive framework</th>
<th>Constructivism</th>
<th>Conception of mathematics as a set of skills that can be explained and connected to each other</th>
<th>Conception of mathematics as patterns and relationships among ideas</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Ideas related to implementation</th>
<th>Provide opportunities with patterns and other concrete</th>
<th>Ask students to look for patterns in what they are studying or solving</th>
<th>Provide experiences with patterns often in instruction; ask students to share</th>
</tr>
</thead>
</table>
manifestations of their thinking so other students can make connections

| Evidence of enactment in lessons: high, medium, or low fidelity? | Low | Low | Medium |

Table 22

*SFMP 7c*

Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have.

<table>
<thead>
<tr>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</td>
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<td>Corresponding with authors’ likely intent</td>
</tr>
<tr>
<td>Focus: most/all of the text, some, or a few words only?</td>
<td>Most/all text</td>
<td>Some text</td>
</tr>
<tr>
<td>Interpretive framework</td>
<td>Constructivism</td>
<td>Conception of mathematics as a set of skills that can be explained and connected to each other</td>
</tr>
<tr>
<td>------------------------</td>
<td>----------------</td>
<td>---------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>Using students’ language to refer to connections they have noticed; encourage multiple perspectives in a problem</td>
<td>Teaching the connections between mathematical operations and actions that are related in some way</td>
</tr>
<tr>
<td>Evidence of enactment in lessons: high, medium, or low fidelity?</td>
<td>n/o</td>
<td>Low</td>
</tr>
</tbody>
</table>

Table 23

*SFMP 7d*

Later, students will see 7 x 8 equals the well remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. [*grade 5 only*]
<table>
<thead>
<tr>
<th>Interpretation: corresponding with authors’ likely intent, not observable, or misinterpretation?</th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>n/o</td>
<td>Corresponding with authors’ likely intent</td>
<td>Corresponding with authors’ likely intent</td>
<td></td>
</tr>
<tr>
<td>Focus: most/all of the text, some, or a few words only?</td>
<td>Carrie</td>
<td>Sandy</td>
<td>Ray</td>
</tr>
<tr>
<td>n/o</td>
<td>Most/all text</td>
<td>Most/all text</td>
<td></td>
</tr>
<tr>
<td>Interpretive framework</td>
<td>Carrie</td>
<td>Sandy</td>
<td>Ray</td>
</tr>
<tr>
<td>n/o</td>
<td>Conception of mathematics as a set of skills that can be explained and connected to each other</td>
<td>Conception of mathematics as understanding relationships among numbers</td>
<td></td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>Carrie</td>
<td>Sandy</td>
<td>Ray</td>
</tr>
<tr>
<td>n/o</td>
<td>Give this example to students to see how they think about it</td>
<td>Provide experiences with decomposing/recomposing numbers and verbalizing number relationships</td>
<td></td>
</tr>
<tr>
<td>Evidence of</td>
<td>Carrie</td>
<td>Sandy</td>
<td>Ray</td>
</tr>
<tr>
<td>n/o</td>
<td>n/o</td>
<td>n/o</td>
<td></td>
</tr>
</tbody>
</table>
enactment in lessons: high, medium, or low fidelity?

<table>
<thead>
<tr>
<th></th>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interpretation:</strong></td>
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<td>Corresponding with authors’ likely intent</td>
<td>Corresponding with authors’ likely intent; some not observable</td>
</tr>
<tr>
<td><strong>Focus:</strong></td>
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<td>Most/all text</td>
<td>Some text</td>
</tr>
<tr>
<td><strong>Interpretive framework</strong></td>
<td>Socio-constructivism</td>
<td>Conception of mathematics as a set of skills that can be explained and connected to each other</td>
<td>Conception of mathematics as problem-solving and as a set of ideas that can be explained and</td>
</tr>
<tr>
<td>Ideas related to implementation</td>
<td>“Turn and talk”; move students from “known to new”</td>
<td>Ask students to look for connections among ideas</td>
<td>Provide opportunities for students to see and respond to each other’s work; provide variations on problems for further thinking</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>-----------------------------------------------</td>
<td>-----------------------------------------------</td>
<td>--------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Evidence of enactment in lessons: high, medium, or low fidelity?</td>
<td>High</td>
<td>Low</td>
<td>Medium</td>
</tr>
</tbody>
</table>

Table 25

*SFMP 7f*

They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

<table>
<thead>
<tr>
<th>Carrie</th>
<th>Sandy</th>
<th>Ray</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpretation: corresponding with</td>
<td>Misinterpretation</td>
<td>Misinterpretation</td>
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<tr>
<td>authors’ likely intent, not observable, or misinterpretation?</td>
<td>Focus: most/all of the text, some, or a few words only?</td>
<td>Interpretive framework</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Most/all text</td>
<td>Constructivism</td>
<td>Conception of mathematics as a set of skills that can be explained and connected to each other</td>
</tr>
<tr>
<td>Most/all text</td>
<td>Conception of mathematics as problem solving</td>
<td>Teach students to simplify and rewrite algebraic expressions</td>
</tr>
<tr>
<td>Most/all text</td>
<td></td>
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</tr>
</tbody>
</table>
Influences on Teachers’ Interpretations of the SFMP

A Teacher’s General Philosophy: Influential and Relatively Static

A teacher’s general philosophy about how mathematics teaching can support mathematics learning guides what this teacher thinks, says, and does, and this philosophy does not change easily (Cohen, 1990; Green, 1971). That is, these beliefs and values act as a filter for everything that the teacher hears from a variety of sources and thinks about new policy, and often this filtering is done independently, without discussion with others (Darling-Hammond, 1990). In discussions with each teacher, this idea was exemplified.

Although Carrie did not tend to repeat herself often, she repeated multiple times her belief in creating a community of learners in her classroom and her belief that students learn in their own ways and at their own pace when supported in persevering, both of which guided everything she did in her practice. This belief structure exemplified the conception-based perspective posited by Simon, Tzur, Heinz, Kinzel, and Smith (2000). In the first group discussion, when the fifth grade teachers asked whether first grade focused very much on fact fluency, Carrie handled herself well and spoke confidently of her own experience, whereas other teachers might have felt attacked and might have been defensive. She was definitely comfortable enough with her students, her practice, and the content not to know exactly where the lesson would go from start to finish. The establishment of trust between teacher and student was a key element of Carrie’s philosophy and a vital part of her vision of the SFMP in her classroom. Further,
she wrote in her reflection after the second interview: “I still go back to the connection between critical thinking and SFMP. My hope is that these will become the norm of good teaching.” Carrie was the participant who was consistently most able to articulate specific aspects of her philosophy of teaching and learning. Still, she noted at the end of the study that although she was now more aware of the SFMP, she did not feel that her practice would change due to this awareness. At that time, she was getting ready for her new kindergarten class and was thinking about kindergarten instruction compared and contrasted with first grade instruction. This added a dimension to her comments that had not existed before, yet it did not seem to change her basic interpretations of the SFMP.

Sandy stated in the second interview, “I think what influences me the most is my sincere concern for the children.” This, as noted before, was very clear. “That I believe as the United States society, we are failing children. And we just keep continuing to fail them.” Sandy wanted her students to be successful in school and ultimately in life. Her beliefs and practices were highly related to this value; she wanted to provide them with “the opportunity to be that successful”. So, the basis for Sandy’s overall philosophy was more societal in nature, which then influenced what she did for students in the classroom. But there was a parallel between Sandy and Carrie – they both wanted children to be successful and to learn. Nevertheless, Sandy interpreted much of what she read in the CCSS in the context of traditional teacher-directed instruction that emphasized the use of algorithms and rote procedures, and she seemed to have an external conception of mathematics as suggested by Gregg (1995).
Ray remarked in interview 3, “[O]ur goals here are to build children up.” Sandy stressed this point also – one might speculate whether this was coincidence or whether this was something they had talked about – even a goal within the school. It was certainly a belief that they shared. Ray often reiterated that problem-solving and patterns were the crux of mathematics and the crux of the standards and that the SFMP were the “glue” that must hold the content standards together in instruction. Thus, Ray appeared to have a perception-based perspective of mathematics (Simon, Tzur, Heinz, Kinzel, and Smith, 2000), somewhat similar to Carrie’s view, but differing in the sense that he seemed to feel that mathematics was “held” together outside of individual minds and that his role was to help students understand the relationships among ideas in this pre-existing structure.

By the end of the study, it was clear that both Carrie and Ray wanted students to see and understand relationships among mathematical ideas. It is interesting to question whether Ray and Carrie might possibly have been fairly representative of Sealon elementary teachers or whether they were atypical in their thinking. Although all three teachers certainly agreed that their main goal was to support student learning, Sandy differed somewhat from Carrie and Ray in terms of what this goal implied for classroom activity, at least relative to the grade levels that they taught. However, in her comments on my tentative conclusions, Sandy noted that she now believed that manipulatives were valuable in teaching mathematics, so she was unsure about the idea that a teacher’s philosophy was relatively unchanging over time. One might question whether her underlying philosophy had changed or, rather, whether her views of how to achieve the
goals related to this philosophy had changed. Still, in either case, there is certainly an indication that teachers’ beliefs can change, but perhaps only with significant, sustained reflection on their classroom practice (a theme throughout this chapter).

**Teachers’ Prior Experiences**

Dewey (1966) writes that our prior experiences impel us to think about and reflect on new learning in particular ways. In reflecting on the CCSS, Ray and Carrie often spoke of their own experience in school mathematics. Ray shared his experience as a student in classes where teachers did not teach in ways encouraged by the CCSS (non-existent at the time, of course), which ultimately persuaded him not to participate in these classes since the environments were discouraging. He also pointed out (about his and other students’ learning at that time), “[I]t fell apart, because we couldn’t apply it. We weren’t taught that…”. At one point, Ray passionately questioned, “Why? Why wasn’t I taught [this way]?” He also remarked, “Thank God I got out of fifth grade when I did. Because, man, I just remember fractions in fifth. That was it… and long division.” That is, he felt that the content now was much more challenging for students, particularly with the SFMP interwoven, than the content he learned in fifth grade. For example, Ray shared that he did not think about problem situations using multiple variables, so he was not sure how fifth graders could be expected to do this; this was a reference to his interpretation of part of SFMP, involving “several objects”. Further, he felt that he and all learners often simply needed more time to make sense of concepts than one class period, and he felt that this was an important point in considering the enactment of the SFMP.
Similar to Ray’s comments about students not being able to apply their learning, Carrie shared, “[T]he math that I did in high school… never made sense to me.” In speaking about SFMP 1, she added, “I think that often times, I don’t look at a problem in a different way.” She noted that if she had been able to develop the concept of a “set” (for example), with that vocabulary in place, from primary school on, then it would have much more sense to her in an abstract sense in algebra. So, Ray and Carrie both reflected on their school mathematics experiences in fairly negative ways, which caused them to want to create more positive opportunities for their own students. However, they now both seemed confident in their own mathematical abilities, which would have eased this transition. Interestingly, Sandy never mentioned her own school experiences in any way. This might cause one to wonder whether her experience was so negative that she did not wish to think about it, or whether perhaps her experience was so similar to the way she was now teaching that it did not even occur to her to share it in these discussions.

Another type of experience that some teachers may have had is the opportunity to work with a mentor teacher, which in Ray’s case, was very beneficial. He noted, “I had a great mentor that started me fifteen, seventeen years ago. In the morning, we came and planned.” This memory must have been important to him because he mentioned it frequently – which might raise a question about how important a good mentor is in this process of changing the way a teacher teaches. This may be an example of the socio-relational dimension of a social structure, suggested by Lisa Rosen at the NCTM Research Catalyst Conference (cited in Stein & Shields, 2004) through which teachers come to understand standards documents.
All three teachers, in one way or another, reflected on seeing children in one’s own family encounter school mathematics, an experience which in itself could influence one’s thinking (Dewey, 1966). Carrie often drew on her experience of seeing her children move through the Sealon school system. She shared in interview 2, “I think that they’re doing such a much better job of making problems less abstract and more pertinent to kids in the older grades as well. I see my son in algebra, and everything, and it seems like that is so different from when I took…”. She described how her son appreciated being able to talk to other students about the work in class because it helped him to better understand it. She even used an example from a parent’s perspective about her son’s English teacher and how this teacher had adjusted her plans for a project significantly when she recognized that the entire class was struggling. In contrast, though, Carrie said that when her son was at the intermediate school and teachers were posting fact fluency scores for all to see, “I thought that was the most horrific thing I had ever seen in my life.” At the same time, “[E]verything that we did at [the primary school]… was all number sense-based, and then at [the school] with the second and third graders, number sense-based.” So, this seemed very contradictory to Carrie. Moreover, “[E]very math teacher that my kids have had has been so different. And it’s like you’re relearning math every single year.” Related to this concern, she specifically cited the experience and content knowledge of the teacher and the degree to which teachers differentiated instruction to meet student needs. Still, she believed that the habits described in the SFMP were important for her children and all students to develop, noting that it frustrated
her when her own children had trouble making sense of answers that they had found in homework problems.

Sandy also thought about children in her family while reflecting on the SFMP and the content standards. She believed that middle school students should not be using interactive geometry software to learn geometry because it was too primary, noting, “I don’t know if that’s because… with my own personal nieces and nephews, we taught multiple shapes. We discussed them before we went to pre-school.” Similarly, Ray felt that his principal’s personal experience of watching her daughter learn mathematics in another district was affecting what she believed was right for Sealon, and since Ray did not entirely agree with the philosophy of the program in that district, he was frustrated with his principal’s attitude toward their own program.

Carrie and Ray also had some previous experience with enacting the student-centered mode of instruction that is encouraged in the CCSS, probably Carrie more so than Ray, whereas Sandy did not seem to have this experience. Beswick (2007) also notes that many teachers are not trained to enact student-focused practices. So, Carrie and Ray seemed more comfortable with implementing the new standards than Sandy did, at least in terms of what they had previously done in their classrooms. Carrie noted, “[I]t’s kind of always what we’ve done. You know, it’s just enhancing that discussion, critical thinking piece…” She was comfortable letting students struggle in front of each other, which she felt was part of SFMP 3, especially. Interestingly, although Ray was generally in strong agreement with what he read in the standards, and although he felt that the district’s adoption of Investigations several years before had moved Sealon in the
direction of the goals in the CCSS, he often said that he realized that he needed to improve on actually enacting these instructional practices in his classroom. This may have related to his comment that he was a “big picture” thinker who sometimes did not focus on “details”.

Clearly, the teachers thought about their own experiences -- in school, with their families, and as teachers, as they read and considered the SFMP and the goals in the CCSS. These experiences seemed to have notable influence on their interpretations of the standards, as Remillard (1992) notes when describing obstacles in changing teacher beliefs.

**Beliefs About Important Mathematical Knowledge and Student Learning**

Although there was some similarity between Ray’s and Sandy’s views about what mathematics students should learn and how they should learn it, and also some similarity between Ray’s and Carrie’s views, all three teachers once again had fairly unique perspectives. This echoes Skott’s (2001) observation that beliefs are highly complex and not easily generalizable.

All three participants talked in some way about “number sense”, but they did not often clarify what they believed this term actually meant relative to student thinking. Their comments corresponded with the external (Sandy), perception-based (Ray), or conception-based (Carrie) perspectives of mathematics described earlier. During interview 2, Sandy used the phrase “number sense” in connection with the goals of students’ making sense of problems and solutions and understanding the use of the four basic operations, implying a reference to SFMP 1. However, she also questioned why the
new fifth grade content standards did not allow for the formal development of some standard algorithms; she felt that this was the “cherry” on the “ice cream sundae” of the development of number sense in grade 5. Moreover, Sandy said that school leaders had told the fifth grade teachers that they did not need to worry about students’ computational skills because in sixth grade, students would be allowed to use calculators. This upset Sandy because she highly valued computation as a skill to be mastered by her students, and her administrators were saying it was unimportant. In contrast, during interview 3, Carrie observed, “[W]hen you have a strong enough number sense, you can make the jump to those algebra concepts easier.” She continued, though, “[W]e’re being told [by intermediate grades teachers] [students] didn’t have number sense because they didn’t know their math facts. It’s two different things to me.”

Sandy said that she felt her incoming students had a “weak foundation”, particularly referring to their fact fluency and procedural skills. Ray also expressed this concern many times, saying that students struggled greatly in fifth grade when they were lacking in these skills. He did remark during interview 2 that skills and “deeper thinking” were not synonymous, but he indicated that students’ knowing skills on which to draw was “a good thing”. In response, Carrie essentially said that in first grade, they were not concerned about students knowing facts by the end of the year, but in second grade, the teachers seemed to be more concerned with this goal. Carrie indicated that they were building students’ foundation for facts because students were still developing an understanding of the operations and how numbers could be composed and decomposed. Relative to this idea, Ray agreed with Carrie’s thoughts as he commented in group
discussion 1, “With the Singapore philosophy, young children should be making and decomposing groups of numbers over and over again to the point where they don’t rely on fingers. This is fact fluency!” First grade teachers were also not as concerned about students’ efficiency, according to Carrie, because they wanted students to feel successful regardless of how they solved problems.

Carrie felt that the introduction to the CCSS indicated that “understanding math facts, instead of just knowing them” was key for students. She added (agreeing with the text), “I don’t know that the ‘just knowing them’ part is really what we’re trying to accomplish anymore.” During interview 2, she also said, “[I]t’s not all about the answers that the kids are writing. It’s more so about the process of how they’re getting to the answer that they’re writing.” She was glad to see that the standards now explicitly included mathematical practices, especially including perseverance and communicating about one’s thinking (“expressive language”), that she had valued during her entire teaching career. Carrie appreciated seeing her students make connections among ideas on their own and understand various ways of solving problems, and she saw these abilities as crucial parts of SFMP 7 and 1, respectively. Interestingly, though, Carrie was not sure how to help students understand when extra justification or representations were not necessary; she said that she believed the students felt that showing less work was the “easy way out”. In general, Carrie felt (as she wrote in her reflection after interview 1): “I think that the twenty-first century learning skills are quite evident in the perseverance, multiple ways of problem solving and critical thinking involved in these [SFMP] statements.”
Ray agreed that all of the habits in the SFMP were important; he maintained that students “should know the why” behind what they were learning, and he was concerned that some of the intermediate grades teachers were “stressing that non-thinking”. He also wanted his students to be better able to persevere, to approach tasks in different ways, and to critique others’ thinking “appropriately” by the end of their year with him. It is noteworthy that Ray focused more on multiple “entry points” to problems when considering SFMP 1, whereas Carrie focused more on multiple solution strategies. With a perspective similar to Carrie’s, Ray felt that future employers of his students would want them to have developed these habits. However, he had been very frustrated about many of the former fifth grade indicators, as had Sandy, and he wrote after the second group discussion, “I’ve taught these grades and I know that there are some primary teachers that are vehemently anti-fact and will rush to the ‘developmentally inappropriate’ excuse for anything they didn’t like to teach. Words can’t begin to describe the inappropriate indicators we currently have in fifth, and we don’t have a choice!!!!” So, Ray seemed to feel that although students should learn and be able to explain the “why” and “think” about their learning, they should also be fluent in basic facts before grade 5.

Sandy also seemed to believe that a substantial amount of students’ mathematical ability was developed long before they reach fifth grade (either at home or in earlier grades), so there might not be very much that she could do for students if they did not come to her with certain kinds of skills. Sandy did want students to “understand” mathematics and to “justify” their answers (particularly related to SFMP 1 and 3, respectively), also noting (like Ray and Carrie) that the SFMP were “life skills”, but she
seemed to think of these goals as more procedural than conceptual. For example, in interview 2, she said, “They can tell you how to do it. But let them loose, and they rush through and do that [pointing to hypothetical situation]. And it’s because they don’t make sense of the problems.” She referred several times to her desire for students to use “reverse” (inverse) operations to check their answers. Sandy noted that the first paragraph of the CCSS refers to important overall goals for students, and she remarked that if this was really what students ought to be learning in mathematics, she did not currently see that this was happening. So, Sandy felt that even though it probably was not entirely helpful to teach mathematics in the teacher-centered way that it had been taught in the past, and it was important for students to learn in the way that the new standards suggested, it would be very difficult for many students and teachers to adapt to these new practices. To some degree, this corresponded with her comment in her feedback on the conclusions in the study (“strongly agree”) that teachers seem to agree that “number sense” is very important in students’ mathematical development, but there are differences in how teachers interpret the phrase “number sense”.

When considering the paragraph describing standards for a diverse population of learners, Sandy shared how she felt about the current practices in Ohio and in her school, which, to her, lowered the standards for certain students. In this sense, she seemed to share some agreement with Carrie and Ray because they all felt that all students should have the opportunity to master the same standards. Ray argued that all students had ways of contributing to the group’s learning if they could be engaged in the activities so that they were thinking, and he genuinely enjoyed working with students with special needs.
He remarked that one of his goals was to “build the confidence level of these kids, even if it’s simple tasks”. However, he also commented, “The average kids, they’re moving at the right pace,” which seemed to be more of a reference to a “pace” required to earn a successful score on the OAA. Carrie felt that most of her students were able to master the first grade standards and that the only students who struggled were the students with pronounced cognitive disabilities.

So, the teachers all viewed their roles somewhat differently, depending on their perspectives of important goals for learning mathematics. Ultimately, Ray felt that a large part of his role with the new standards would be to ensure that all of his students were moving forward in their learning from where they began the year. He also felt that he needed to help his students find “awe” in mathematics and enjoy it, as he himself did. Carrie made a similar comment several times – she wanted to know that her students were “understanding” concepts by asking them to show or explain their thinking, and she felt, as did Sandy, that sometimes this goal was more of a challenge for students who arrived at answers quickly than for those who did not. Ray noted that when students did understand and could justify ideas, the teacher needed to provide opportunities for them to continue to learn also. Sandy seemed to feel that her role as a teacher was to model for students so that they could learn to solve problems efficiently, judge the reasonableness of the solutions, and understand that multiple strategies were possible for solving most problems. In relation to this goal, she noted in interview 2, “[M]ath was typically thought of as, it’s black and white – this is the answer; this is how you get it. Only recently have
we realized that you can come to that answer through multiple ways.” By “we”, she seemed to refer to herself as well as her students.

**Beliefs About How Students Learn Mathematics**

During the first interview, all three participants mentioned “giving [students] a variety of opportunities” to learn (borrowing Sandy’s words). Ray and Sandy frequently contrasted the ideas in these conversations with the fast-paced, shallow instruction that they felt they had been compelled to implement for years because of the pressure for students to succeed on the OAA. They felt that teaching and learning had been “rushed”, that concepts had been lost, and that students were “worn out”, which Ray remarked was not helpful for learning. Carrie felt that a “classroom where math is not stressful” would be an effective learning environment. This was quite evident in her classroom culture; she led students in yoga frequently, and her tone and manner math lessons were calm and unhurried. Further, although Carrie believed that students should be challenged to “stretch” from “know to new”, she believed that teachers should carefully facilitate this by allowing students to explore new ideas first and then to think about those ideas together and finally individually. That is, she did not want students to feel anxious about learning mathematics, but she wanted them to recognize that “stretch” and to know when they had learned something new.

The teachers often discussed the issue of students’ failing to understand content because it was perhaps too abstract for them at certain ages and/or because it was being taught in ways that did not help students to make sense of the content. For instance, the three teachers collectively questioned in group discussion 1 why second graders
apparently struggled greatly with understanding place value into the hundreds, concluding that perhaps it did not really make sense to the students in kindergarten and first grade because of the way that some teachers taught place value. This conversation seemed to provide a moment of realization for Ray and Sandy, who often commented that they could not understand why they saw third graders in their building studying concepts that their fifth grade students seemed not to comprehend (at least initially), such as decimals, area, perimeter, and place value into the millions. This “aha” moment was similar to those experienced by student teachers involved in collective inquiry about student learning, described by MacKinnon and Grunau (1994). Ray related this discussion to fifth grade as he shared that the teachers had found that teaching the fractions unit early in the year really seemed to support students’ learning in the algebra unit later. He also commented that it could be challenging to help lower performing students look for a pattern or structure because they had enough gaps in their knowledge or skill to make it difficult to find a pattern or structure; that is, what would be structured mentally for some other students was not yet for them. Still, he felt that he could move students toward this, but he needed to be creative and to persist with students. Although none of the teachers mentioned learning progressions during these discussions, this issue seemed to clearly relate to the idea of helping students learn ideas by experiencing them in increasingly sophisticated ways.

All three teachers felt that they would like to be better able to meet individual students’ needs through increased use of differentiated instruction, and they all spoke about this topic on various occasions. Ray and Carrie each commented several times
during the study about the benefits of grouping students in different ways in class (homogeneously, heterogeneously, randomly, etc.) to foster collaborative learning among them, particularly related to specific content in the CCSS. Sandy often noted that some of her students were so low-achieving that they still needed counters to find answers for basic facts, and she wished that she could find more time and resources to support these students and lessen the gaps in their understanding. However, the teachers all felt that the number of students in a classroom was typically a significant deterrent to providing enough differentiation to effectively meet individual needs, and they were not sure how to realistically manage this kind of work in class. Carrie did discuss her practices of working to know her students well enough to scaffold their thinking in conversations about mathematics and to provide them with individual practice at their own levels, in their own time (“give them a little… let them stretch a little”). This, she felt, was appropriate and sufficient differentiation for first grade because she felt that her students were able to master the required content by the end of the year. She also acknowledged, though, that she did not have the pressures of preparing students for the OAA and feeling as if they must master certain content by a certain time in the year. Both Sandy and Ray, in their feedback on the tentative conclusions for the study, continued to express strong agreement with the need for differentiated instruction, and Ray wrote, “With SLO’s [Specific Learning Objectives] in place, I foresee an IEP [Individualized Education Plan] for all students where differentiation will be mandatory.” Specific Learning Objectives had become part of the state’s teacher evaluation procedures by this time; teachers were writing goals for groups of students and then collecting data on progress toward these
goals. Although IEPs were actually only applicable to students with identified learning disabilities, Ray’s reference to them seemed to indicate that he believed that eventually every student would have his/her own learning goals that teachers would need to support in instruction.

Further, the three participants felt that particular goals in the SFMP suggested useful instructional strategies. The fact that these teachers were able to study the text over time, while considering their classroom practices, contrasts with the situation that Darling-Hammond (1990) highlights: that teachers who have few structured opportunities such as these may not understand the true intent of standards and may not enact them as intended. As Ray noted in his final comments on the study, “[W]ithout SFMP, the [Common Core] standards can not be met.” Sandy discussed her goal of increasingly asking students to explain their thinking so that they could justify their solutions for themselves, for her, and for others (all part of SFMP 3), and certainly this was a goal for Ray and Carrie also; Carrie was probably the most practiced at this at the time of the study, as “turn and talk” was a vital practice in her class. Although Sandy did not say this directly, she seemed to indicate that in order to support students in making sense of problems and persevering (in all of the ways described SFMP 1), she would need to spend more time on one problem and have students work on it and discuss it until understanding had developed, which she did fairly effectively in her second lesson. Ray echoed this idea, saying that they could not just “do these quick snippets – although, sometimes, with the attention spans, you want to keep it moving”.
Moreover, Sandy thought that she could work on the “critiquing” in SFMP 3 with her students by having them submit ideas anonymously and sharing these ideas with the whole class, so that the class could discuss the thinking rather than the person who submitted it and “not hurt people’s feelings”. She realized that in a small group, one student often dominated, so she needed to find ways to prevent this from occurring. Sandy did have questions about how to prevent students from internalizing incorrect or illogical ideas that they heard other students share, and all three of the teachers felt that this was a challenge and one not easily resolved other than through continued classroom discourse, which required time and strong content knowledge (though they did not mention the latter) on the part of the teacher. Ray commented that the Singapore method introduced at most three strategies to students for each type of problem, which he felt was helpful in that students did not have a large set of methods from which to choose (and to confuse with each other). All of the above practices related to SFMP 3 align closely with the hermeneutic perspective on mathematics education described by Brown (2001) and Davis (1996).

In the first group discussion, Sandy and Ray shared their practice of having students teach their classes from time to time, particularly in preparing for the OAA. Ray noted that it was “one of the most effective” means they had found to help students learn. Interestingly, Sandy brought this up while discussing SFMP 3, because she seemed to equate teaching an idea – i.e., being able to explain it fully – with constructing a logical argument and critiquing others’ reasoning. Ray even remarked that teaching is the “highest form of learning”. Perhaps Sandy was more focused on the idea of a student
showing how – or that – he/she knew an idea by teaching it, so this justification was part of SFMP 3. However, then Sandy also discussed how students learned from each other because a student might say something differently than a teacher would, which reflects the idea that language and a student’s current way of thinking about an idea can influence how students learn mathematics. This echoes Gallagher’s (1992) idea that learning is affected by the interaction among a learner’s “fore-structure” and the interpretations that are exchanged among teachers and learners. Cobb’s (2000) “emergent perspective” for learning through both individual and collective experience is reflected here as well. Ray and Sandy also noted several times that they enjoyed being able to provide learning activities that were “fun” for students (as “student teaching” seemed to be) because they found that students were more motivated to learn as a result. Sandy remarked, “[I]t’s amazing… because they want to be the teacher…”.

Carrie also addressed the idea of language as she talked about her classroom and the district’s focus on expressive language in first grade, which was connected to the broader focus on critical thinking. Her belief aligns strongly with Walkerdine’s comment (1988; cited in Brown, 2001) that teachers should help students to “capture their experience in language”. She felt that her role was to model this language so that students could learn to share and justify their thinking in class, as well as to respond to others’ ideas. Carrie related this fairly directly to the ideas in SFMP 3. Ray and Sandy did not talk as much about language supporting students’ learning, though Ray did often discuss the goal of students using language to demonstrate their understanding in connection with SFMP 1 and 3.
In the first group discussion, Carrie and Ray talked about the differences in how to help students learn to “critique” each other in first and fifth grades, very much in reference in SFMP 3. Ray pointed out that students in his class wanted to follow rules and that they were vocal when rules were not being followed; this included mathematical rules, so students might not always be tactful when they saw that a student had not followed a mathematical rule logically in solving a problem. Ray says that the students’ age could be a factor in this way, and he noted that perhaps the better way to approach this in fifth grade would be to explore different entry points – a comment that alludes to SFMP 1, and this likely was Ray’s intent. So, Ray believed that he could move fifth graders toward SFMP 3, but it would take some work and practice. Carrie explained that her students would simply explain what they did in contrast to what they were seeing presented, rather than directly saying that they disagreed or that another student was wrong. Sandy, as noted in chapter 4, was very concerned about her students’ ability to learn to discuss their thinking respectfully, though she did feel that if this habit were developed from kindergarten on, it could be possible in grade 5. Cohen and Ball (1990) write that new ways of teaching and learning mathematics must be developed collaboratively by teachers and students, so it seems appropriate that the students in these three classrooms would be participants in the change process as well; they would learn in conjunction with their teachers, and vice versa.

Ray commented during the second interview that he realized that although he had briefly shown the standard algorithm for fraction division to his students so that they would not be “scared” of it, he would introduce this concept differently in the future,
developing it more conceptually, as the standards required. He added, “[T]he quality of your instruction and your problems that you are presenting to them have to be much more rich so it sticks to the deeper [points to head] – to the, you know, deeper level of memory.” This point is elaborated in Laughbaum (2011). Ray also discussed his own use of mnemonic devices to help him remember certain facts, and he seemed to relate this to his use of analogies to “help kids to make the connections”, as he felt was referenced in SFMP 1 (“analogous problems”). Also, he highlighted the benefit of the class being able to refer to past key problems that they had solved when they encountered new problems; he thought that these were “counterexamples” referenced in SFMP 1. He felt that he modeled the use of these types of tools for students, but he admitted that he was unsure whether students were better able to use them as a result.

Carrie also valued providing opportunities for students to “make connections” and to find “relationships between things”, which she often discussed in connection with SFMP 7. She offered this insight: “[T]he curriculum has created kids that want to make sense of things.” Also related to SFMP 7, Carrie saw the idea of “shift[ing] perspective” as related to her idea of “sitting on it” – learning something new and then waiting until another day to pursue it further, perhaps to give students’ minds a chance to make some sense of the new information and to relate it – in a broader sense, as an “overview” – to what they already knew, to problems they had already solved (as Ray said), or to situations in real life. (Ray noted that teachers could help students to shift their perspectives by allowing opportunities to see other students’ work, both in presentations and in informal “gallery” walks around the classroom.) Based on these ideas, Carrie
often led what she called a “mini-lesson” for math and then provided centers where students could review and refine concepts at their own levels of thinking. In addition, she frequently mentioned that she did not “ever tell” her students what to do, “[b]ecause that doesn’t make it make sense – it just means that they’re following steps.” However, as will be shown later, she may have done this in her teaching more than she realized, and it is unclear whether she believed that students understood concepts when they simply agreed with others’ statements. Still, she intentionally modeled patience while students were thinking in front of others.

Carrie also believed in a “constant cycle” of learning, where students were reviewing previous knowledge in order to understand new ideas – going from “known to new”. This represented her sense of a “learning progression”, which was a bit more general than the authors of the CCSS seemed to intend. Ray’s thinking about learning progressions seemed to align more with that of the authors; he believed that he could help students move from less sophisticated notions to more sophisticated, formalized ideas through experiences with concrete, pictorial, and abstract representations, along with discussion among students and teacher. He viewed this type of strategy as beneficial for developing the “structure” for new ideas that was included in SFMP 7, and he often connected these ideas to what he had learned at the Singapore math workshop, which he felt was so valuable for his own thinking about instruction.

The teachers had fairly defined opinions about how manipulatives should be used in their classrooms. Ray believed that manipulatives were essential for helping students begin to explore new concepts before moving to pictorial and abstract representations,
and he also believed that it was important for all students to be able to “prove” their answers to problems using these materials – “‘Cause then you’re talking your problem solvers that the real world’s gonna want” (meaning that students should be able to represent and communicate their ideas in several ways). He pointed out, however, that using such materials was more challenging when teachers were concerned about pacing, as the fifth grade teachers always had been. As noted in chapter 4, Sandy was initially highly uncomfortable with the use of most manipulatives in grade 5, for most students, though she did believe they were appropriate in the primary grades. She also felt, in reading SFMP 3, that her students were not yet able to use manipulatives to “construct arguments”, not appearing to recognize that it would be unlikely that students would learn to do so if they were not able to use them in class. She did, however, see value in allowing students to use measurement tools and allowing students with special needs to use concrete materials to develop their understanding, and later she said that she had to “stop with the denial” about using concrete materials more with all of her students (“we gotta touch it, we gotta feel it”), which was a “struggle” for her “as a math teacher”. In her final feedback, Sandy reiterated that she did want students to understand how to solve problems in multiple ways, but she “hope[d] that they [would] also understand without the use of manipulatives”. Again as aforementioned, Carrie relied heavily on manipulatives to support her students in exploring new concepts, but she did feel that at a certain point as students internalized a given concept and created a mental “schema” for it, they should not need to use these materials any more.
Other strategies for supporting student learning that the teachers mentioned throughout the study included using visual and then abstract representations (Carrie and Ray), encouraging students to look for patterns (Carrie and Ray), moving from conceptually transparent computational methods to more condensed, efficient methods (Ray), encouraging students to share ideas and questions (Carrie, Sandy, and Ray), teaching traditional (general) problem-solving strategies (Sandy and Ray), using technology (Sandy), relating mathematics to children’s literature (Ray), keeping students moving (Ray), helping students to develop notebooks into references for problem solving (Sandy), and integrating content from multiple areas and from the real world (Carrie and Ray). Overall, Carrie believed that the typical instructional strategies utilized in first grade seemed to be reflected in the SFMP for students of all ages, and she felt that this would benefit all students in time if all teachers enacted them consistently with similar language. Currently, she felt that this was not the case because she felt that her own children had experienced mathematics in very different ways as they had moved from teacher to teacher, and this could certainly be a discussion related to school policy.

Thus, it seemed that the teachers agreed that students should be able to learn mathematics through experiences that were appropriate for the age of the students and the grade level. They all wished to better support students’ individual needs, though Carrie felt she was able to do this fairly effectively already. They all wanted students to be able to learn from each other and to understand and appreciate multiple ways of solving problems. The teachers differed a bit more on how much to “tell” students initially when new content was being learned, and they shared somewhat distinct perspectives on the
use of manipulatives. So, their interpretations of the SFMP and their views about how the standards should be enacted were related to their beliefs about how students learned mathematics most successfully. As noted by Chazan, Callis, and Lehman (2008), teachers evaluate new reforms based on how effective they believe the reforms will be in improving student learning, and they implement these reforms in a way that reflects this evaluation.

**Discussing the Standards for Mathematical Practice and the Introduction to the CCSS**

**Some Responses Unrelated to Questions**

Throughout the study, I found that the teachers often did not actually address the questions that I asked very precisely, especially relative to the text in the CCSS document. I do not believe that this was ever intentional; rather, it seemed to be more the case that they were thinking about day-to-day concerns or particular aspects of instruction relative to questions that I had posed. Also, sometimes the teachers began to talk about a particular passion or interest of theirs that sprang to mind as they read particular text in the standards document, and all of them at one time or another questioned how to assess and even grade students’ progress with the SFMP. At some point, all three also spoke about text in other parts of the SFMP when they were asked to focus on a given sentence. Congruous with my observations, Gallagher (1992) writes that interpretations of text are “conditioned” by our beliefs, experiences, interests, background, education, etc., and the text itself is “conditioned” by these qualities of the authors, along with the authors’ intent. Philipp (2007) echoes the notion that beliefs may be “lenses through which one looks
when interpreting the world”. So, meaning continually evolves through reading and interpretation. Ray wrote in his feedback on the study, “I believe having used the Common Core last year made me realize how interconnected the SFMP are!” Conversation also facilitates this meaning-making (Fairfield, 2011).

For instance, Ray often mentioned Singapore math in connection with certain elements of the introduction or the SFMP, and he talked at various times about his enthusiasm for mathematics and how he wanted to pass that on to students. During two of the interviews, he shared an anecdote about a girl in his class who finally gained confidence in her own ability (thus, also perseverance) in the latter part of the year, which pleased him very much. He also often reflected on aspects of his own practice that he wanted to improve when he read certain goals in the CCSS, but he expressed strong concern that other colleagues might not share the same desire to change their practice, especially if the district leaders did not provide sufficient support in doing so. Overall, Ray was “anxious” about the transition to the new standards when he considered all of the logistical details and resources that would need to be in place for it to be successful, a concern which fits into the “political” and “structural” dimensions of structures for change posited by Rosen (in Stein & Shields, 2004).

Sandy, especially in interview 1, shared her frustration with the lack of importance that she felt most students and members of the public ascribed to mathematics and even to perseverance in general – a reflection of the “normative/cultural” dimension of Rosen’s structures for reform. Her strong feelings about manipulatives emerged particularly in this interview any time that she read a standard statement that mentioned
them. On the other hand, in interview 2, she was thinking much more about the text and tying it to classroom practice than she had been in the first interview, and she noted that seeing the standards broken into sentences was causing her to rethink the text. Still, she did not always refer directly to the text even when she was fairly certainly alluding to it, as when she remarked that she would ask her students in frustration, “Does this make sense?” (SFMP 1)

Carrie, in one written reflection, answered a different set of questions than what I had sent for that particular reflection, though her answers were still thoughtful, articulate, and related to the standards and her practice. She also did not seem to choose just one SFMP on which to focus, which had been my instruction, for either of the lesson videos, as she wrote about all three in her reflections. I believe that she may have just been thinking about her schoolwork and lost sight of these details for the study. It is also possible that because she felt she was already essentially enacting the SFMP, she really did not need to focus on the ideas in the text very carefully. She did use words that seemed to refer to specific parts of the document from time to time, and this did seem intentional on her part (e.g., “check your work”, “persevere”).

One other major topic that led to deviation from talking about the SFMP, whether in individual interviews or in the group discussions, was the mathematical content in the new standards. Perhaps the content became an orienting object in the discussions as it was a common referent that helped the teachers to relate the SFMP to their classrooms. Clearly, this was at the forefront of the teachers’ minds as they were considering their instruction for the following year, especially for Sandy and Ray, who would be teaching
the new content standards in the four major academic areas – an enormous transition for even the most seasoned teacher. As noted earlier, a great concern for Sandy and Ray was students’ fact fluency, along with the fact that students entered fifth grade seeming to have forgotten content that the teachers could literally watch being taught in third and fourth grade classrooms. This was compounded by the fact that for the next year, at least, students would enter their classrooms without some of the prerequisite knowledge that was in the grade 4 CCSS content, and Sandy and Ray were still unsure whether school leaders would expect fifth grade to teach both the old and new content or only the new. However, they were pleased that the new content seemed more focused and achievable for students, which they hoped meant their instruction could be more indepth and of higher “quality”. Still, Ray commented that they would need to “rethink” how they handled homework and practice, both in what was assigned and how it was addressed in class, along with truly their entire class structure. Again, Carrie did not especially struggle personally with any of these issues, and first grade had already been teaching the content standards for a year. However, Ray and Carrie often shared ideas for how they could teach content in the standards, especially as it related to real-life situations; they always seemed to be thinking and planning. In fact, sometimes Ray focused more on these ideas than he did on the SFMP text; he was very familiar with the requirements in the content standards for grade 5.

Moreover, when the teachers were analyzing the hypothetical classroom episodes, I had to ask each one of them a follow-up question in nearly every interview to explain to me how their comments related to the SFMP, even though the question introducing each
situation asked for an analysis “in light of SFMP 1, 3, or 7”. They were generally focusing purely on content and specific instructional strategies for that content and not so much on the habits of mind that were included in the SFMP. Of the three teachers, Ray was the one that referred directly to the SFMP more often than Carrie and Sandy did. Again, they interpreted these fictional experiences in light of their own prior experience (Dewey, 1966).

Interestingly, in the second group discussion, I wanted to hear the teachers’ thoughts about professional development that would be helpful to them and their colleagues in transitioning to the CCSS. Though I asked twice about professional development, the discussion shifted very quickly to resources (textbook and manipulatives) and the content standards – not the SFMP. Carrie did speak briefly about another new practice – a common intervention time – that would involve some common planning among teachers at each grade level. Although this was related to differentiation, which is a goal of the CCSS, Carrie did not mention this, and no SFMP were discussed. I eventually decided to stop asking about professional development, at least during this conversation, because it seemed that the teachers were more apt to discuss other topics that related to the logistical details of implementation. At other times, all three had ideas for professional development, which were described in Chapter 4 and will be discussed again later in this chapter.

So, it became clear that the teachers always had multiple thoughts and ideas in mind even as they were asked to focus on very brief excerpts of text, so it seemed difficult for them to isolate the text without thinking about other aspects of their contexts.
and experiences (even other text) that related to it. Indeed, perhaps it would not even be appropriate or possible for them to try to interpret the text in isolation (Brown, 2001; Davis, 1996; Gallagher, 1992). Still, all of the teachers strongly expressed at the end of the study that the SFMP were very important for all teachers to understand (not to overlook); yet they were all concerned that teachers would in fact overlook them if not “forced” by school leaders to become familiar with them. In Sandy’s ending feedback, she reflected on the school year following the study and wrote, “I think many [teachers] did just that.” This suggests that school leaders should be intentional and persistent about providing support for teachers in understanding the SFMP and not just concerning themselves with students’ content mastery (Darling-Hammond, 1990; Firestone, 1989).

Teachers’ Focus on Certain Words or Phrases in the Text

As described in chapter 3, in interview 1, the teachers were each presented with each SFMP as a whole, with the phrases to be discussed underlined within the paragraph. However, in interviews 2 and 3, they saw each SFMP deconstructed by sentence, which all three teachers said was helpful in interpreting the text. Again, each sentence may have acted as a more condensed “orienting object” (Brown, 2001) than the entire paragraph. Nevertheless, in many cases, each teacher might focus on one portion of a standard statement and apparently ignore the remainder, perhaps because the teachers did not have enough experiences that they were able to relate to this text (Brown, 2001). For example, Ray often spoke about his goal for students to be able to respectfully “critique the reasoning of others”, but he did not talk as much about their “construct[ing] viable arguments” (SFMP 3). He also seemed to miss most of the ideas in SFMP 3 surrounding
the creation of a “logical progression” of ideas because his comments related only to that phrase. In interview 2, Ray’s interpretation of the phrase in SFMP 1 “monitor and evaluate their progress” related to teachers’ doing this for students and not students’ doing this for themselves. In Carrie’s written reflection on the first group discussion, she left out many of the important ideas in each SFMP – she focused on one or two major ideas from each. In her two interviews, Sandy focused on opposite parts of the title statement of SFMP 1, never on both parts, which might indicate that she did not recognize that critiquing an argument might be related to generating and understanding one’s own rationale. She was also very concerned about how students would interpret an “argument” (SFMP 3); she felt that they might think this meant they were actually supposed to argue with each other, and she spent a great deal of time in her responses to SFMP 3 discussing how she could avoid this issue, rather than talking about the overall goals in the standard. None of the teachers said much about the first part of the statement in SFMP 3 having to do with comparing the effectiveness of two plausible arguments, though Ray did at one point in interview 2; they typically talked about finding flaws in arguments and explaining what these errors were. Further, as Sandy discussed the sentence in SFMP 1 about explaining the meaning of a problem and finding entry points, she talked mainly about the first part of the statement and not about looking for entry points.

All three teachers had noticeable difficulty interpreting the following portion of SFMP 1:

They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution pathway rather
than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

Sandy focused on the first portion of this text, particularly on students’ understanding and use of “definitions” and “previously established results” in solving new problems, which she related to her use of notebooks with students to help them create a reference with work they had done during the year. This did not seem to correspond with the more general notion of “definition” in mathematics and with using the results of problems to solve new problems, nor did it demonstrate that students understood the importance of these more general mathematical habits. In the second interview, I asked Sandy what she felt each word in this text meant for fifth graders, her response was rather halting and uncertain, and though she tried to consider each word, she was not able to give a very thorough interpretation of the text. When Carrie reflected on this text in interview 2, she essentially left the first sentence out of her discussion and focused mainly on the idea of finding a solution pathway and changing course if needed. She referred somewhat indirectly to the text involving analogous or simpler problems. Ray, during interview 2, read several of the key words in this text aloud and said, “That’s all doable,” but he did not offer any further clarification of his thinking. During interview 3, he talked more about teachers creating new problems that were similar to others that students had solved (for extra learning or enrichment), but the text really seems to speak more to students’ thinking about the solution for one task. It is possible that separating this portion of the
standard into single sentences would have been useful in supporting teachers’
interpretations further.

When any sort of mathematical example was given in the text, the teachers
reflected almost entirely on that example without thinking more broadly about the goal in
the standard, again reflecting the notion of an “orienting object” on which the teachers
could focus. Sandy spoke at length about place value and operations in the first interview
when she saw these two topics referenced as key principles of mathematics in the
introduction to the CCSS. Both Ray and Sandy commented almost entirely about the
distributive property when talking about the sentence in SFMP 7 that includes an
example application of this property, though Ray did extend a bit further into talking
about decomposition during one interview. Carrie responded similarly to the sentence in
SFMP 7 that references the commutative property for addition (with single-digit
numbers). Further, all three teachers referred to “algebraic expressions” specifically
when interpreting the last sentence of SFMP 7 (actually, Carrie was not sure how to
interpret this sentence at all), even though the phrase that includes this term is clearly set
apart as an example of a much more general goal. It seems unfortunate, especially in this
instance, that the teachers all focused on the example because the idea of “see[ing]
complicated things… as single objects or as being composed of several objects” is highly
applicable at every level of mathematics, and I am not sure that any of them understood
the significance of this goal for students.

The teachers did not seem to say much at all about a statement in the text when
they appeared to feel that the wording was very clear (e.g., “Mathematically proficient
students check their answers to problems using a different method, and they continually ask themselves, ‘Does this make sense?’”; “They justify their conclusions, communicate them to others, and respond to the arguments of others”; “Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem”), or, in contrast, when they seemed lost in attempting to interpret the text (this was actually true for many of the statements, but a prime example was: “They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples”). The former situation relates to Gadamer’s (1960) point that a question on the part of the learner guides learning; thus, if there is no question, no learning is likely to take place. The latter scenario might suggest that no “fore-structure” for the text was present in the mind of the teacher (Gallagher, 1992). When they bypassed most or all of the standard itself, they tended to immediately begin talking about what the standard would look like in practice in their classrooms, as if either there was no real reason to interpret the text because the ideas were self-evident, or as if they felt they could not interpret the text because the language was too unfamiliar. Moreover, when I asked teachers to envision how a given SFMP would be enacted effectively in their classrooms or schools, they all usually referred to the title sentence of the standard and not the detail in the full paragraph, even though they had just discussed the SFMP in greater detail. Perhaps school leaders need to be especially concerned about teachers’ interpretations of the statements that describe each SFMP, at either extreme, if it is the case that teachers may not think much about them at all.
On a few occasions, I was unsure how Ray’s response related at all to a given portion of text. For instance, in Ray’s second interview, he read the paragraph on page 4 of the CCSS, which says: “These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the laws of arithmetic to structure those ideas.” Ray’s comment was, “I think that they’re really stressing here that you have to use what’s appropriate for each individual child.” Sometimes Ray read quickly (perhaps because he was in an interview situation) and misread what was in the statements, then suggesting an interpretation which did not even seem to fit the general intent of what was written. If he held to these interpretations, he might truly misinterpret some parts of the standards, and since he appreciated being able to read professional literature, I believe he would want to avoid this. It is very possible that other teachers might do the same thing, which certainly should be a concern for school leaders and policy makers.

When a teacher did offer an interpretation for the entire sentence in one of the SFMP, the sentence was generally short, containing only words familiar to elementary teachers – for example, “Mathematically proficient students look closely to discern a pattern or structure”, or “Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.” For the latter, Carrie mentioned in interview 2 that her students represented their thinking or solutions with hands-on materials or drawings. She said that with more abstract concepts such as place value, they would need more experience in later grades before the ideas were internalized. Also, certain phrases resonated with all three teachers, perhaps because they recognized
them as statements or thoughts from their own classes: “Does this make sense?” and “…persevere in solving [problems]” (SFMP 1), and they spoke of these quite frequently. All of them also responded directly to the text in SFMP 1: “…understand the approaches of others…”, which was a goal that they all had for their students. This was also true for the portion of SFMP 3 that reads “…critique the reasoning of others”, which they wanted to help their students learn to do respectfully. However, they hardly referred to the second part of the above sentence in SFMP 1, regarding identifying correspondences between approaches, other than Carrie in interview 3, when she spoke of students’ identifying relationships among problems (not approaches).

All three teachers also discussed both parts of the sentence in SFMP 7, “They also can step back for an overview and shift perspective.” Ray tied in both the idea of stepping back to understand someone else’s perspective (by seeing what other students have done), which Sandy focused on, and changing one’s own perspective, which Carrie focused on. He did believe, in contrast to Sandy, that fifth graders could do this, especially when they were essentially required to stop their own work (instead of just working quickly to finish) and consider someone else’s thinking. He used the word “internalized”, which Carrie also used. Interestingly, Ray was talking about having this habit internalized, whereas Carrie talked about having a given concept internalized.

Ironically, as shown here, all three teachers overlooked parts of the SFMP themselves on numerous occasions, even as they were concerned that their colleagues would bypass them altogether; they may have even recognized their own lack of comprehensiveness without saying so. In addition, the teachers had varying ways of
interpreting words like “critique” and “structure”, and other words they tended to overlook entirely (e.g., “conjectures”, “correspondences”, “constraints”). Given the substantial variation in the ways that these teachers read and interpreted parts of the standards document, one could conclude that the more carefully that all teachers could reflect on the meaning of each part of the text, especially in collaboration with others, the more likely they would be to enact the standards as intended in their classrooms (Brown, 2001; Cohen & Ball, 1990; Fairfield, 2011; McLaughlin, 1994). Sandy “strongly agree[d]” with this conclusion in her final comments.

**Contradictions Among Statements About the SFMP**

At times during the study, a noticeable conflict could be observed between different statements that a teacher had made. This can often be the case when teachers are trying to understand a new philosophy of teaching and learning as they read new standards and consider their own philosophies. The following examples are all from the second interview with each teacher.

Carrie referred somewhat indirectly to the text in SFMP 1 involving analogous or simpler problems, noting that her students did not usually have to apply ideas to new situations; yet, she spoke often about students making connections to other things they had learned.

Sandy said that she felt that her students could use visual or concrete representations to **solve** problems, but she did not feel that they were able to create a way to **justify** their thinking using these representations. She did not seem to see that there could be a direct connection between these two abilities, perhaps because (as she had
said) she was not sure how manipulatives could be helpful to fifth grade students in general. Also, as noted earlier, she felt that manipulatives were very useful for primary students but almost not at all for her students. Another example of a possible conflict in values was shown in this statement to students in Sandy’s first lesson: “So for example, you could do multiplication to get an answer, or you could do repeated addition. Now, I’ve told you… on many occasions that repeated addition is not necessarily the best technique for a fifth grader, but it still comes up with the right answer, right?... that’s what’s most important.” Sandy valued both efficiency and students’ success and perseverance, and I believe that this sometimes created a conflict for her about which she valued most, or perhaps for what purpose she valued each most (e.g., OAA scores versus long-term student growth).

Ray first said that creating a logical progression of ideas would be difficult for students, and then he said it was “doable” – though perhaps through reflecting aloud on the goal, his initial reaction changed. In addition, Ray referred to the SFMP and said, “But it was written succinctly, and it had to be written at a very difficult level, to make sure there’s no confusion. ‘Cause there’s not a lot of interpretation here.” At the same time, he remarked that he had to read and re-read the SFMP in order to begin to understand the authors’ intent, and he believed that many teachers might not take this time, though it will be necessary. Perhaps Ray believed that the authors intended to be straightforward but did not succeed in this goal. One might be able to draw a parallel between a proof in very high-level mathematics and Ray’s perception of the SFMP – they
are both written very concisely and precisely, but they both have language that must be carefully studied in order to be fully understood because it is so dense with meaning.

Since the teachers did not seem to recognize the conflicts among the statements that they made in these instances, it might be helpful in the future for a colleague or professional development leader to act as a “hermeneutic listener” for each teacher (Davis, 1996) and ask him/her questions about these issues, with the goal of helping the teacher to sort out what his/her perspectives actually are, at least at that point in time. Of course, ideas continue to evolve, so perhaps what can be observed in these teachers’ statements above are bits of evidence that teachers’ ideas were shifting even during the interviews – reflecting the hermeneutic circle described by Gallagher (1992), or perhaps the related helix model suggested by Mason (1989; cited in Brown, 2001), which implies that the circle is never complete but rather spirals as meanings develop progressively.

**Lesson Video Analysis**

**Model for Analysis**

As I analyzed the teachers’ and students’ actions and language in the lesson videos, my primary goal was to code these actions and dialogue relative to whether at least one of the three SFMP was being enacted effectively. To facilitate this analysis, I created a matrix of possible categories for coding. The codes correspond to the four lower right hand cells in this matrix.

Table 26

**Categories for Analyzing Teachers’ Actions and Words in Lessons**

| Teacher’s action or words… | Aligned with the authors’ | Did not align with the |
Aligned with his/her own interpretation of the SFMP

<table>
<thead>
<tr>
<th>intent for the SFMP</th>
<th>authors’ intent for the SFMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ideal; somewhat frequent)</td>
<td>(frequent)</td>
</tr>
</tbody>
</table>

Did not align with his/her own interpretation of the SFMP

<table>
<thead>
<tr>
<th>intent for the SFMP</th>
<th>authors’ intent for the SFMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(unlikely; rare)</td>
<td>(unlikely; rare)</td>
</tr>
</tbody>
</table>

I created a similar matrix for analyzing students’ actions and language:

**Table 27**

*Categories for Analyzing Students’ Actions and Words in Lessons*

<table>
<thead>
<tr>
<th>Student outcome based on teacher action</th>
<th>Student’s action or words reflected the SFMP</th>
<th>Student’s action or words did not reflect the SFMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher enacted SFMP effectively</td>
<td>(ideal; somewhat frequent)</td>
<td>(infrequent)</td>
</tr>
<tr>
<td>Teacher did not enact SFMP effectively</td>
<td>(unlikely)</td>
<td>(frequent)</td>
</tr>
</tbody>
</table>

**Portions of Videotaped Lessons that Conflicted with the SFMP**

*Teachers’ words and actions that corresponded with their own interpretations.* In analyzing the two lessons from each teacher, I examined the dialogue of teachers and students as well as their actions during each lesson. I found that
it seemed useful to determine whether teachers’ statements and actions corresponded with the own interpretations of the SFMP and/or with the likely intent of the authors in the SFMP (or neither). My assumptions about the authors’ intent of the standards were derived from the learning progressions documents available at http://ime.math.arizona.edu/progressions/ (also written by the authors) as well as the literature cited in chapter 2 and other literature that represents the dominant ideas in mathematics education about students’ learning (many examples listed in chapter 3). I also chose to examine whether students’ statements and actions reflected the SFMP, and I related this to the teachers’ statements and actions: students were applying at least one SFMP due to their teachers’ enactment or in spite of the lack of it, and if students were not applying at least one SFMP, this was either due to their teachers’ lack of enactment or was occurring in spite of the teachers’ attempts.

Very frequently, a teacher’s statement or action reflected his/her own interpretation of one or more of the SFMP but not what the authors likely intended. Similar instances occurred in the studies conducted by Thompson (1984), Ball (1990), Cohen (1990), and Raymond (1997) when teachers were not especially familiar with standards documents and/or when they were not very reflective about their own practice as it related to the standards. Perhaps the most difficult part of coming to understand the SFMP for teachers is to realize when they are enacting the standards, distinguished from when their students are enacting them. Following are several examples of this from each teacher.
Carrie. In the first lesson, the content allowed for many opportunities to apply SFMP 1 and 7, since students were working to make sense of the meanings of the digits in a two-digit number and since they were exploring the physical structure of tens and ones. However, Carrie did not take full advantage of these opportunities. She was talking to a student at the board who was looking at the number 11 and also at a ten-stick and a one-square. “So you already had 10, just count on one more. [student nods] How many 10-sticks are there?” She was definitely trying to support all of the students in seeing what the 1 and 1 meant in 11, but the students were just answering her questions about how many ten-sticks and one-squares there were, and there was no follow-up conversation where she asked students to explain what these meant. A second example occurred when Carrie was trying to help students understand how to physically combine tens and ones with a regrouping process. She suggested to the class, “Let’s take a really close look at those cubes. [points at magnets on board] We know that 10 cubes equals one stick, right? So do you think that we can swap? Can we regroup some of those cubes?” If she had simply asked, “What do you think we could do to find the total?” she would have left more of the thinking to the students; regrouping was not an entirely new concept to them. A third example related to the second; Carrie was guiding a student, Olivia, through combining tens and ones magnets at the board: “Put all the 10-sticks together… [Olivia moves the squares together into a group], and all – [Olivia moves the tens together] there you go… all right, can you count it? [Olivia begins to point and apparently count silently.] Let me hear you.” The goals of SFMP 1 and 7 would suggest
that students should be doing this thinking and work for themselves in order to internalize it.

In Carrie’s written reflection on this lesson, she related the district’s critical thinking goal to SFMP 3 and noted that she felt the students were demonstrating critical thinking or some of the goals in SFMP 3 in the lesson. However, SFMP 3 refers to constructing viable arguments and critiquing those of others, and students only were able to do this minimally during the lesson because Carrie corrected students quickly if they went astray. She also wrote, “I was really pleased with how they seemed to take the lead and we moved easily from known to new concepts.” Students did have a chance to “take the lead” at the beginning of the lesson when she was asking for their ideas and asking other questions based on these ideas, but not so much in the second half of the lesson, when she was leading students more. Further, in a later conversation, Carrie stated:

I don’t ever tell them, EVER, even when I’m introducing something brand new, this is, you know, I always start with something that they know and build from there. I would never say, ‘Here’s two two-digit numbers; let’s do this. This is what you do; you do this and then you do this and then you do this.’ Because that doesn’t make it make sense – it just means that they’re following steps.

Perhaps even Carrie, who strongly valued classroom community and allowing students to make sense of ideas on their own, actually led students more than she realized.

Carrie’s second lesson focused on exploring halves and fourths in several modes of representation (numeric, verbal, set, region, and the linear model somewhat). This was
the first day that fourths (or four equal parts) had been introduced, but students were more familiar with halves. Carrie actually demonstrated a great deal of what she wanted students to think about in this lesson, even though this would have been an excellent opportunity for students to apply SFMP 7 as they studied structure within a whole. When she was working to help students understand and name fractions (fourths) of a square that she had drawn on the board, she led a discussion as follows, addressing one student at first: [colors 1 piece of the square] “So would you say that this is 1/4 of the square? [student nods, with one or two others] Interesting. What about if I color this one? [a couple of students raise hands] Now how many parts did I color?” Carrie was doing nearly all of the talking and sense-making (SFMP 1), and students were responding very basically to her questions. Later, when a student volunteered an (incorrect) idea about another way to divide a square into fourths, rather than allowing the student to try (SFMP 7) and allowing others to offer suggestions (SFMP 3), Carrie said, “That’s what [another student] just did, but there were big parts, and little parts. We have to have all equal parts.” Then, when Carrie was preparing students to go to their tables for individual work with play dough, she essentially modeled every task that she was asking students to explore and record, so students who were listening and watching virtually had no thinking to do on their own. I did notice that during the table activity with the play dough, Carrie was asking some important questions that perhaps all of the students needed to hear, but many were talking. It may be that she planned to ask these questions later. Overall, students could have had many additional experiences with SFMP 1 and 7
in this lesson had Carrie not directly told them as much. Carrie did not send a written reflection for this video.

Because of her comments in her one reflection and in other conversations during the study, I believe that Carrie felt she was teaching in alignment with the SFMP, which were part of the “formal curriculum”. The concept of “formal curriculum”, referring to policy documents or textbooks, was suggested by Gerhke, Knapp, and Sirotnik (1992; cited in Stein, Remillard, & Smith, 2007). Her view of the formal curriculum, in other words, included allowing students to make sense of concepts for themselves, as the SFMP encourage. However, the examples above could imply that Carrie’s distinction between guiding (or modeling certain actions for) students and stating ideas outright might have been rather blurred. Carrie wanted students to understand mathematical tools and types of representation well enough to choose and use them on their own rather than being led to do so, but these two lessons were fairly guided in terms of the tools and representations used. She clearly felt a responsibility to “scaffold” for students, as she said – to “give a little, let them stretch a little” – but in practice perhaps this was more difficult to do (without giving too much) for a full class as opposed to one student. Carrie did also work individually with students, so perhaps she asked them to explain and show more thinking in this format, but in class, she seemed to demonstrate more than might be expected, given her general philosophy of teaching and learning. Thus, Carrie’s “enacted curriculum” (Gerhke, Knapp, and Sirotnik, 1992) in this lesson did not align with her interpretation of the formal curriculum.
**Sandy.** During Sandy’s first lesson, her own comfort level with units of capacity measurement may have been less than sufficient for reviewing these units with students, or at least her language for expressing ideas related to these units was not very precise. She had asked students to brainstorm real situations in which capacity would be considered, which seems to be related to making sense of problem situations (SFMP 1), and several students mentioned measurements in baking. After discussing some of these examples, Sandy said, “So you do understand how capacity is the liquid part that you pour into the baking part of it. All right? Give me another example of how we use capacity.” Her first sentence was rather unclear, as were many other similar statements that she made, and although I believe that she was trying to help students understand capacity, I am not sure that she was successful. In another episode, a student asked a question about the difference between a gallon and a liter. Rather than asking students to share ideas or perhaps to explore this for a few minutes using resources in the room, she complimented the student on the question and found a water bottle that held slightly more than half a liter, then presenting a rather detailed comparison of these two units. Clearly, Sandy was trying to help students understand the relationship between these two units (reflecting SFMP 1 and 7), but she did all of the thinking herself rather than allowing students the opportunity.

In addition, in the last part of the lesson when students were reviewing how to calculate area and perimeter for triangles and quadrilaterals, Sandy asked students to open their OAA workbooks and their own notebooks from class and to find what they had written (or read) before about the formulas for area and perimeter. She asked
students to read the definitions and formulas aloud, and I believe that she was trying to apply the part of SFMP 1 that related to using definitions. However, presumably the authors intended more than for students to simply look up definitions in other references, and the only way that Sandy asked students to apply this information was to use the formulas for calculation.

In her written reflection, Sandy commented, “We had a great discussion about our real life examples.” She did not indicate that she felt she had any trouble expressing the relationships among units and describing the meaning of capacity. She used the word “we” when she wrote about the comparison of a liter to a gallon, as if the entire class had played a role in constructing this comparison. She did write, however, “I realized I should have had multiple manipulatives to explore hands on. The next time, I teach this lesson I will have a variety of bottles to explore and read the labels.” Regarding the notebooks, Sandy wrote, “The journals are new, this year, to my teaching. We started them in August and record the concepts and the rules with examples for the lessons we are learning. This is a resource that I am experimenting with to provide the children with a tool to find information about mathematical practices, since we have no textbooks or a structured math program.” It is interesting that she used the phrase “mathematical practices” in this sentences; it is unclear whether she was referring to the SFMP or simply “procedures” that students had learned. Further, Sandy also noted that two “aha moments” occurred when she was reminding students how to find the area of a triangle by demonstrating the relationship between a right triangle and a rectangle (twice the size of the triangle) on the board. I am not sure that many students had “aha moments”
because they were not asked to restate or explain this relationship, but Sandy appears to have felt that they did.

In Sandy’s second lesson, after she introduced the task that students would be solving, she walked around and talked to pairs of students as they worked. Generally, she simply asked students how they were solving the problem, and she did not offer many comments, but several times, she asked a question (related to a SFMP or problem solving strategy) that she essentially answered also. In one instance, she had asked why it was important to estimate the answer before solving a problem (SFMP 1), and students had said it gave them a “ballpark” for what the answer should be. Sandy replied, “It might give you a ballpark of where the answer should fall into. And it’d give you… it would help with the number sense of it, don’t you think?” [Students nod.] It appeared that Sandy was thinking about number sense and wanted students to do so also, but she just suggested this phrase to them without really talking with them further. Then, she asked another pair, “[W]ould you have used a table if we hadn’t done our algebra unit? When we were doing T-charts, and input-output charts?” The students said they would not have used a table. Sandy may have been attempting to help the students to reflect on their choice of strategy (SFMP 1), and maybe also on what they have learned that they didn’t know before, but again, she did nearly all of the talking in this short conversation.

Finally, at the end of class, after students had shared and discussed several strategies as a class, Sandy reminded students, “[W]hen you think you’re finished, go back and reread; recheck, and find out if your answer is sensible. Once you read, and once you check, you will see if your answer is sensible and logical, all right?” It might have been more
helpful to have some of the students brainstorm these ideas or perhaps write about them in an exit ticket, which would have applied SFMP 1 and 3 (“What could you have done to not make mistakes that you made… what could you do next time?”).

Though I did not receive a reflection from Sandy on this lesson, she did reflect on it during the second group discussion. She did not comment on the episodes described above, which may mean she had not thought about them in this way, but she did say that she was not sure if she had gone too far in essentially telling students at the end of class which strategy produced the most accurate answer (of the three that had been presented). She seemed to be considering what she always would have done and what she should do to better enact the SFMP. Interestingly, I feel that her evaluation of these strategies at the end of class really just summarized what students had already said; she was not offering entirely new critiques.

Sandy’s actions described above, particularly in the first lesson, seem to be excellent examples of when a teacher wants to enact the SFMP but does not always give students the opportunity to do so. This tendency again seems to reflect the distinction between Sandy’s view of the formal curriculum and the “attained” or “experienced” curriculum (what students learn from classroom experiences) (Gerhke, Knapp, and Sirotnik, 1992; cited in Stein, Remillard, & Smith, 2007). This habit, over time, can prevent students from meeting the goals in the SFMP because teachers may not consider who is actually doing what the SFMP require. Sandy truly did allow students much more freedom of thought in the second lesson, so it was clear that she was reflecting on the
meanings of the SFMP and considering how best to implement these standards in her classroom.

*Ray.* In Ray’s first lesson, the main goal in the homework and in the tasks that Ray assigned in class was to review multi-digit multiplication and division. When he began the discussion of the homework, Ray said, “What I liked about your homework last night is that you could kind of self-check it. If that product wasn’t there, you know you did something wrong.” This referred to the fact that the answers that the students were supposed to find were all listed on the sheet in an answer bank (including both correct and incorrect answers). Although he was encouraging the students to check their answers (SFMP 1), the “check” in this case would not necessarily involve any mathematics if the students found that their answers were in the bank. Later, as Ray demonstrated multi-digit division on the board, he talked through the “sidebar” (also sometimes called “partial quotients”) method for an entire problem without asking for any student input or discussion about the steps he was taking, though he did also say, “I know some of you learned it that way, and if you feel comfortable that way, Mr. Archer just cares that you get to the right answer.” So, he wanted students to use a method that made sense to them (SFMP 1), but he was not allowing students to make sense out of the method that he was demonstrating (SFMP 1 and 3). At the end of the lesson, Ray reviewed his comments from the beginning of the lesson that he wanted students to think about how to “disagree agreeably” (SFMP 3) and to use “thumbs up and thumbs down” to help to facilitate this. Interestingly, Ray observed aloud, during the lesson, that he needed to refrain from commenting himself on a student’s work before asking the class to agree or disagree with
it (because he did do this once or twice before this comment). However, the only comment he made at the end, probably due to time constraints, was “OK, so, to wrap it up today, guys, listen. Are we good on the thumbs up, thumbs down?” The students said “yes”, and the lesson ended. As he pointed out in his reflection, this was a rather shallow application of SFMP 3, particularly since he did most of the talking during class.

In Ray’s written reflection, he wrote that his intent in this lesson was to focus on SFMP 3, which was clear in the video. However, he also felt, in retrospect, that the students were not really applying SFMP 3; their answers were very brief and “superficial”, as he wrote. He himself had a great deal of back-and-forth conversation with individual students, so he was trying to help students to make sense (in some way) out of the examples that he was demonstrating, but he was really doing most of the sense-making (SFMP 1) and explaining. There was little appearance of SFMP 7 in this lesson; Ray had not chosen this as his focus, and though it could have been applied much more with the multi-digit division, this did not occur. This could have been done on another day, perhaps. Another point to raise was that, as in the other two teachers’ classes, only a few students were really doing most of the responding when Ray would pose a question to the class, and he did not seem to notice or try to respond to this lack of response except for in one instance.

In lesson 2, there were only two examples where it seemed that Ray was acting in alignment with his own interpretations of the SFMP but not with the intent of the document. He had asked students to consider, as they were searching for patterns in a series of numbers, to consider whether the task could have more than one solution. After
seeing several students present the same solution (though with different perspectives), Ray simply made the comment, “It has one set of answers, actually. Because if everyone got the same answer…” [we can assume there is only one answer]. Perhaps this could have been more of a conversation among students that would have reflected SFMP 1 and 7; further, it might have been wise to point out that other patterns still may have been possible even if no students found them. Later, when a student presented an idea that had not been shared yet (but was still rather elementary because the entire task was rather elementary), Ray said, “Give her a round of applause; that’s a great discovery!” He was likely trying to encourage perseverance and confidence (SFMP 1), but one might question whether the student truly felt the applause was deserved because the discovery was very simplistic. Interestingly, Ray had chosen to focus on SFMP 1 for this lesson because he wanted students to focus on “entering” the problem as they looked for patterns. But this task, the first in a series, apparently, was really more appropriate for grade 3 or 4, so the discussion was brief.

Ray did send a reflection from this lesson, but he did not address either of the episodes described above. Other portions of his reflection will be shared later.

So, Ray did realize that his enactment of SFMP 3 in the first lesson was “superficial”, and he set a goal of improving students’ responses to each other in the future. In the second lesson, his action related to SFMP 1 (above) also seemed superficial, and he offered a comment to students that probably could have been discussed among the class instead. He often remarked in interviews that he talked more than he should in class, and he wanted to turn more of the discussion over to students in
the coming year. Thus, although Ray’s formal and enacted curricula did not align with each other, his reflective nature helped him to recognize this and to work to improve this alignment in future lessons. In this sense, he differed from the less reflective teachers that Thompson (1984), Raymond (1997), and Cooney, Shealy, and Arvold (1998) described.

**Teachers’ words and actions that did not correspond with their own interpretations.** On the other hand, quite often, a teacher’s statement or action reflected neither his/her own expressed interpretation of one or more of the SFMP or what the authors likely intended. Thus, the enacted curriculum did not parallel the teacher’s “intended curriculum” (Gerhke, Knapp, and Sirotnik, 1992) or the formal curriculum. At least one example from each lesson is presented here.

**Carrie.** Because Carrie truly did allow her personal philosophy, which did reflect the SFMP, to guide her teaching, there was only one situation during her first lesson which seemed not to align either with her own interpretation of the standards or the likely intent of the authors. This was a relatively long series of statements and questions that Carrie presented as the students watched her model some regrouping on the board (with physical objects), potentially applying SFMP 1, 3, or 7. Here is a portion of her dialogue:

So I have this group, but I could just mix them up and regroup them

[moves all 10 squares off to the side] so that it makes just this group [puts magnetic strip of 10 on board]. So would you say that this [points to the cluster of 10 on the side] is equal to this [points to strip of 10]? [some students nod] So if it’s equal, then it doesn’t matter which one I have.
[points to cluster], or 10 [points to strip] – it doesn’t matter which 10
group I choose, right? [few students respond] It still is 10.

Notice that students were barely responding during her presentation, and she was really
doing what she said she did not “ever” do, which was to show students how to solve a
problem. She did not ask students to re-explain what she had said afterward, though she
did ask them to work in pairs to solve a similar problem. In her reflection, one comment
that Carrie made about this lesson was, “Sometimes, I have to take the lead and guide
[students] more in one direction or another.”

During Carrie’s second lesson, there were many instances where she simply told
students what she wanted them to observe about the halves and fourths with which they
were working, again very much opposed to what she professed to believing she should do
in class. This seemed to be a bit different than the examples cited earlier, when she was
asking leading questions but still giving students the chance to respond. Also, fairly
frequently when a student shared a thought about how to divide a shape into halves or
fourths, Carrie corrected the student immediately if he/she was incorrect rather than
allowing the class to offer suggestions (in contrast with SFMP 3). In one such instance
after a student had colored two of four parts of a whole, thinking this was one fourth,
Carrie said, “I agree there are four. That’s a half, though, remember? You just did that
for me… there were four, and you colored half of them, so you colored two.” In another
example, she had asked students how to divide a square into fourths, and one student
suggested “four lines”. So, Carrie drew three (not four) vertical lines to separate the
square into four parts, and asked, “Four parts?” This could have been a chance for
students to apply SFMP 7. At another point, she had colored two fourths of a square, and she observed, “And you know what – that looks sort of similar to what we just had a minute ago [with one half], doesn’t it?” She paused, but no students responded. Again, this connection could have been an example of SFMP 7, but it is not clear whether any students actually made this connection.

Carrie made three other interesting comments that, once again, seemed quite clearly opposed to the idea of allowing students to make sense of ideas for themselves. During the exploration of halves and fourths, Carrie noted, “So, I’m starting to understand this ‘one fourth’.” It seemed noteworthy that she said she was starting to understand; admittedly, it is possible that she might argue that she tries to offer thoughts that students might be thinking, but by her own philosophy, perhaps she should allow students to verbalize these thoughts if they are actually genuine. Further, she literally asked a student who was struggling at the board to draw fourths, “Can I show you?” This alone differed from many comments she had made about not showing students how to do something. Finally, toward the end of the demonstration, Carrie said to the class:

All right. I’m gonna tell you the secret of fourths. Are you ready for it?

[a few students respond] Whenever you’re making a fourth, if you want to make something into four parts, all you have to do, because we know that two plus two equals four, all you have to do is first cut it in half [has drawn a circle, and draws a vertical diameter], and then, guess what… cut it in half again [draws a horizontal diameter].
It seems plausible that at least a few students, by this time, could have essentially shared these same ideas with their classmates. It was surprising that Carrie did not ask for observations from students at this point.

Unfortunately, I did not receive a reflection from Carrie on this lesson, so I do wonder how she reacted to her own actions when she watched the video (assuming that she did watch it). Given that she is generally reflective about her practice, I might surmise that she would have noticed how many times during the lesson she simply told students what she wanted them to learn, but it might have been helpful if a trusted colleague could have helped her to identify these instances so that she could avoid this more successfully in future lessons.

_Sandy._ As aforementioned, in Sandy’s first lesson, she seemed to struggle with verbalizing her own thinking about units of measurement, and this caused a real lack of clarity with some statements that she made. For example, she wanted students to see “patterns” in the relationships between units within a gallon (SFMP 7), and she said, “Two cups double into a pint; two pints double into a quart; so you can see that things are multiplying. They’re increasing; the pattern is increasing.” Not only had she said that she wanted students to look for patterns, which she then gave to them, but also her statements were very vague and definitely would have not have qualified as identifying patterns or structure as SFMP 7 requires. In addition, when a student said, “U.S. standard is usually bigger than metric,” Sandy repeated this statement, said it was accurate, and asked for no further clarification. Again, although Sandy had said that she wanted students to understand (“make sense of”) the fifth grade content and apply their learning
in real life, this comparison of the two measurement systems was highly incomplete. In this way, Sandy’s actions were similar to those of one of the teachers in Thompson’s (1984) study because she believed that mathematics was of practical importance to students and that, in reality, her role was to show students how to correctly solve problems. Later in the lesson, when they were reviewing perimeter, Sandy refers to the introduction to this page in the OAA workbook and says:

So, they show you a triangle. In a triangle, you have to add side 1, side 2, side 3. In a square, they’re telling you you can measure one side and multiply by 4. In a rectangle, they’re telling you you can do the length times [sic] width times 2. And in a polygon – I’m sorry, not a polygon – in a parallelogram, you can do the length times the width [sic], or 2 parentheses length times width [sic]. Either way, it’s the same.

So, she read to students what she wanted them to focus on, and the focus was strictly the formulas for calculating perimeter. She gave them no opportunity to do any thinking at all, so this truly does not reflect her interpretation of the SFMP (even at that point in the study) or the presumed intent of the authors. Moreover, as indicated above, Sandy’s restatements of three of these formulas were mathematically inaccurate. It may be that she simply misspoke, but perhaps she was not very certain about these processes herself.

In Sandy’s reflection on this lesson, she wrote of the first part of the lesson (after describing in relative detail what had occurred during this portion), “All went well… lesson complete,” going on to say that she felt students were prepared for this content on the OAA. She evidently did not sense, or at least want to acknowledge, that many points
related to the content were unclear in the discussion. In her final paragraph, however, she did write:

My interpretation of SFMP 1, 3 and 7, has changed very much. The discussions we have had between each other have made me more aware of the SFMPs. After viewing my video of the lesson, I worry about SFMP 3 more now then before. It is so important for the students to talk about mathematical procedures, to argue their reasoning for the procedure, and feel comfortable doing so. Many times, children are frightened to speak out in class because of their fear of being wrong. I pride myself in the children having a comfortable environment to learn and work, however I fear the vocal aspect is missing for some students. This might have been because we were being videoed that day or it might be an eye opener for me as a teacher. This gives me something to observe in my classroom and to think about ways to tweak SFMP 3 as I continue to teach.

Sandy’s reflections from lesson 1 seemed to have a strong influence on how she conducted the second lesson. In contrast with what could be observed in Carrie’s second lesson or Sandy’s first lesson, there were very few instances in Sandy’s second lesson where she seemed to be acting or speaking in opposition to the ideas in the SFMP and her own interpretations of them. So, the enacted curriculum corresponded much more closely with the formal curriculum. Still, during one conversation with a pair of students, she had commented that she thought they had done “a good job”, and she continued, “And the reason I think it’s a really good job is ‘cause it’s the same idea I had!” She
laughed, and it was fairly evident that she was joking, but one might wonder whether the students realized this and what their internal responses might have been if they had also had different ideas. SFMP 1 and 3 would suggest that teachers should not imply that their strategies are the only correct methods. Also, toward the end of the lesson, after three pairs of students had shared their strategies on the board, Sandy asked students about how they could write an equation for the pattern that they had identified in the problem (for the total number of people at a party after a given number of groups had entered). After getting one idea from a student, Sandy said, “x plus 2 equals y. That would be our input-output. [Note: this would only have been true if they were looking recursively at the number of people entering, which they were not.] All right? x plus 2 equals y, so x plus 2 was our pattern to give us our y.” So, not only was Sandy outright telling the students a relationship that she wanted them to see, the relationship was again incorrect (violating SFMP 7 in two ways). This was another instance of where Sandy’s own mathematical understanding may have been an issue in enacting the SFMP effectively.

Overall, however, this lesson was far more in line with the spirit of the SFMP than the first lesson that Sandy videotaped, and although I did not have a written reflection from her about this lesson, it was clear that she had grown in her understanding of the standards.

**Ray.** During his first lesson, Ray modeled a strategy for solving a division problem from the Singapore math workshop that he had attended. As he was doing so, he presented the entire strategy while asking for very little input or explanation from
students about what he was doing, which was, ironically, not all that different than what he said his own teachers had done that frustrated him so much as a student. Here is an excerpt from his presentation of this problem (notice that he even refers to “step 6” at the end, which is from the rather prescribed Singapore method):

So now, what I can do is, I can do 17, now I can do – I’m gonna write it up so you guys can see it. 17 – now we have to do the calculation. We have our unit bars – now I have to keep this – I was gonna make it shorter, to represent one student, but that wouldn’t make sense, would it? Because each of these squares represents a car. So, this is – it’s a little bit bigger than those, but it’s about right. So now, I’m on to step 6. I’m gonna divide.

Students simply followed along and took notes as Ray presented this method. Because Ray himself was explaining each step to make some sense out of it, it was likely that he was trying to apply the goals in SFMP 1 and 3, but even he had said in interviews that teachers should not do this, and he reflected after he had watched the video that he “talked too much”. Also, he was trying to enact SFMP 3 in the lesson by using the “thumbs up/thumbs down” practice for students to agree or disagree with work that had been presented by a student, but he was not utilizing it very effectively, as sometimes only a few students put up their thumbs before he made his own comments about the student’s work and moved on. As noted earlier, Ray also felt, in retrospect, that the “thumbs” strategy was not especially useful in supporting the lesson. He wrote in his reflection, “It seems too trivial for the depth of critiquing others’ mathematical reasoning.”
As noted earlier, in Ray’s second lesson, he really did say much more than he allowed students to say, which allowed relatively few opportunities for the SFMP to be applied by the students, which was quite different than Ray’s goals for his classroom. Students rarely had the opportunity (in whole class discussion) to express a complete thought (in contrast with all three SFMP), and many responses were one word or one number. One instance of this was when a student was presenting a solution to the patterning task, but Ray essentially read the entire solution to the class rather than allowing the student to do so: “OK, so, let’s see what he fills in. [reading what the student was writing on board] 1, 2, 3, 4, 5, all right, jumps to 10, then 14, 19, 23, so now we have 28, 29, 30, 31, 32…”. Similarly, when another student pointed out a pattern that she had seen, Ray explained the entire pattern himself: “So, look – so this would have been zero-five, right? [pointing at top right number] So 5, 4 – let’s look at the ones. The ones place value. So we had 5, 4, 3, 2, 1 [pointing], and in the tens, we had 0, 1, 2, 3, 4. Did you guys notice that in the final column? Well, actually, it’s for every column! Look at – we have 1, 0 – which would represent 10 [inaudible] 10 – back to 9, 8, 7…”.

In his reflection on lesson 2, Ray wrote: “After watching the videotape, I realize that I talk way too much and need to back off and let the kids do the work… I think it’s always a control issue for me. I did notice that I tend to restate what the students say when they’re at the board explaining their thinking… I think I do this so that all the kids can hear what was said. I may have to scale back on this… it can be redundant.” So, he clearly recognized that he needed to focus more on the spirit and goals of the SFMP, particularly by relinquishing some of the “control” over class discussions. Ray seemed very similar to
one of the teachers in the Thompson (1984) study, who said that she believed that students should make connections and participate in discussion, but who did not facilitate these habits in class. Both this teacher and Ray seemed to value relationships among ideas as well as mechanics, and this could cause conflict for both in their teaching.

**Students’ words and actions when teachers did not enact the SFMP.** Not surprisingly, when any of the teachers deviated from the goals of the SFMP in class, this often caused students to do so as well. In these cases, the “experienced” curriculum reflected the enacted curriculum but not the intended or the formal curriculum.

Following are some examples from the six videotaped lessons.

In Carrie’s first lesson, after students had solved a problem in pairs, she asked, “Did anybody else get 32? Did you do it the same way?” Very few students responded. It seemed fairly typical for only a few students to respond to most of Carrie’s questions – probably the same few students each time. One might question whether they the only students who were truly mentally engaged, or perhaps the only ones confident enough to answer. It was unclear that many of the students were applying any of the SFMP when Carrie was leading portions of the lesson in this manner. Similarly, in the second lesson, due to some aspects of Carrie’s questioning style, students often simply answered “yes” or “no”, or they echoed other students’ responses, which did not provide most students with the opportunity to express their own thinking.

Students in Sandy’s class seemed to have trouble articulating mathematical ideas precisely during the first lesson, which likely reflected the fact that Sandy seemed to struggle with this herself at times. This may also have been because Sandy was not in the
habit of encouraging students to share complete thoughts. For instance, one of the students shared the following comment about relationships among metric units: “Um, all of them have numbers that have 1s and 0s in them; none of them are like 56; they’re all like [pauses] 1s and 0s.” Sandy did not ask the student to elaborate; in fact, she acknowledged that this was true and continued on with the discussion. Again, none of the three SFMP that were part of this study were applied effectively here; little sense was made of these relationships, and generally students’ communication of their own thinking was relatively weak. Although this was less prevalent in the second lesson, it was still evident that students were not highly accustomed to talking about their thinking or about reasonable solutions to problems, because one student’s hesitant response to Sandy’s question about why estimating was important was, “It might kind of tell us… what the answer… could be…”, and Sandy then essentially offered her own thought about the value of estimation.

As a student shared the steps in a computational strategy in Ray’s first lesson: “I did 4 times 7; 28; 8, carry the 2…”, Ray reiterated what the student was saying and offered some further explanation for the steps rather than allowing the student to do so. Here, the student simply talked through mechanical steps while the teacher tried to enact SFMP 1 and 3. Interestingly, in Ray’s reflection, he wrote that he realized that students were not given the opportunity to share very many mathematical ideas:

Because of this, in reflection, I think it was the quality of the mathematical questions I was asking of them. We were reviewing two-digit by two-digit multiplication that was reviewed and practiced (in homework) yesterday.
None of the problems I was asking them was in a problem-solving context... until I did the Singapore math test prep which had several components that fed in to the bigger problem to be solved.

As noted earlier, similar examples of Ray’s providing lengthy explanations of students’ thinking occurred in the second lesson, so students had relatively few opportunities to do this on their own. It would be interesting to see, in the future, whether Ray was able to achieve his own goal of allowing all students to participate more actively in class discussion.

On the other hand, there were a few instances during the six lessons when students applied one or more of the SFMP in spite of the teacher’s lack of enactment of them. Of course, this was rather unusual because one would not expect many students to be enacting practices that their teachers were not (at least at that moment). One example was in Carrie’s first lesson, when although Carrie had not referred at all to each piece of a whole divided into four pieces as a “fourth” (meaning that we could name and count these pieces as fourths), one student offered the comment, “It’s a third fourth,” after Carrie colored the third of four equal parts of a whole. Then, the same student said, “This is a fourth...fourth,” when Carrie colored the final piece. The student was truly connecting the names of the pieces to the structure of the fractions in the whole (SFMP 1 and 7), which Carrie had not done herself during the lesson. Another more general example occurred in Sandy’s first lesson, when she essentially “corrected” a student even though his statement was logical. The student said that capacity was “the volume of the container with the liquid”, and Sandy replied, “It’s the capacity [stress on this word] of the container,” as if
to call this measurement “volume” was incorrect. Perhaps a point to be taken from these two examples would be that teachers need to be able to build on students’ comments that are unique and related to the SFMP, which requires teachers to have strong content knowledge and an understanding of the important concepts that lead to deep understanding for students.

Thus, it is clear that when teachers enact the SFMP with only partial fidelity or not at all, students’ learning can be negatively impacted because students lose opportunities to apply the SFMP themselves. In this examples, the hermeneutic principles described by Gallagher (1992), Davis, (1996), and Brown (2001) were violated; language was not a conduit through which mathematical meaning was made and shared. However, this was not always the case in these lessons, as will be shown next.

**Portions of Videotaped Lessons that Aligned with the SFMP**

**Teachers’ words and actions that corresponded with the SFMP.** Due to the teachers’ purposeful efforts to enact the SFMP as well as some of their prior practices that already aligned with the standards, many episodes in the six lessons did exemplify one or more of the SFMP.

**Carrie.** When Carrie asked her students open-ended questions to help them to think on their own in the first lesson, she was operating very much in alignment with the SFMP. In this way, she could be compared to the third teacher in Thompson’s (1984) research, who both believed in student-focused practice and employed it in class. Some examples of these questions were: “Can you come do it?” (when a student had offered a suggestion) “What are you thinking?” (both to individual students and to the class as a
whole) “All right, now can you reorganize that so that we can figure out what the answer is?” (to the class) Also, after asking students to combine the numbers 24 and 12 with base ten blocks, she said, “All right, turn and talk about what you need to do. [Students turned to each other excitedly and began to discuss. The number 36 could be heard repeatedly among them. Students talked for about a minute.] Ryan, what are you thinking?” In Carrie’s reflection on the lesson, she wrote:

They [students] were able to “try out” some problem solving ideas and explain why they thought it would work (#3). The kids are used to the phrase “How do you know?” and “Why?” I ask often for students to explain their thoughts. We used a model – ten sticks and one cubes to create a more concrete bridge from simpler math facts to problems that involved “swapping” – regrouping (#7).

Although Carrie’s second lesson was far more teacher-led than her first lesson, she still asked a number of good questions of students and posed interesting tasks for them to consider. Some of the questions she asked were: “What does it mean, fractions?” “What do you think this means?” (both SFMP 1) “Did somebody say there’s another way?” (SFMP 3) “Why do you have to have 4 of them?” (when talking about fourths) “What’s half of the cookies? What do you think?” “You cut that pizza into four parts. And then you cut a different pizza into two parts. Which part would you rather have? One of two, or one of four?” (all SFMP 7) The challenges she offered to students included: “I wonder if there’s another way to make a square into two parts.” “When you’re all done, see if you can make another pizza and cut it in six.” “[T]ry making a
square, and see if you can cut it into three parts.” (SFMP 7) In addition, at the start of the lesson, she wrote “I can use fractions” on the board; this was a strategy she used each day to encourage confidence and perseverance (SFMP 1). Also, during the demonstration part of the lesson, she asked the students to think about creating fractions of four different shapes as well as of a set; though the fractional pieces can be created similarly, this would not have been obvious to all students, and this is definitely an application of SFMP 7 in terms of looking for and using structure and decomposing an object into multiple objects. Moreover, the last few minutes of this lesson (when Carrie divided a circle on the board into progressively smaller pieces) might have been that “stretch” to which she often referred that benefits those few students who need an additional challenge, addressing the CCSS goal of supporting a diverse group of learners. Certainly, Carrie knew and used effective strategies to foster students’ thinking and their application of the SFMP, when she remembered to do so.

**Sandy.** In Sandy’s first lesson, although the discussion was not always very precise, Sandy did still ask some helpful questions to facilitate students’ sense-making. Examples included: “When are times in real life that we actually use capacity?” (SFMP 1) “So, what can you tell me about that pattern?” (SFMP 7) “Are we talking centimeters? Inches? Feet? Miles?” (when students had solved a problem involving perimeter – SFMP 1) Further, she employed, in a limited way, instructional strategies that supported the SFMP. She told students that she wanted them to listen to each other so that they could agree or disagree with what others were saying, and she did ask them to respond a few times. Also, she randomly chose students to create and solve perimeter
and area problems on the board; although this in itself did not necessarily align directly with any of the SFMP (other than perhaps SFMP 1), it did support the overall goals of the CCSS in that all students had the opportunity to be engaged actively at some point.

Sandy mentioned the discussion of capacity, the search for patterns, and the request for agreement/disagreement in her reflection and felt that these were positive points in the lesson.

Sandy seemed, in her second lesson, to be focusing on the idea that there was more than one way to get to a (single) correct answer in the problem she presented to students. This focus did align with SFMP 3 because she was fostering students’ understanding of multiple logical ways to arrive at a solution. Also, it demonstrated that she wanted students to be successful and comfortable with the strategies that they used.

The activity sheet that included the task did have a fair amount of guidance for students to facilitate their making sense of the problem, but this seemed to be an effective scaffold as much for Sandy, as a teacher learning to enact the SFMP (especially SFMP 1), as it was for the students. Here is her dialogue as she shared the questions that were provided on the sheet:

‘What do I know? What do I want to find out? Do I need more information? Is there information that I don’t need? If so, I will cross it out of the problem.’ And, ‘Can I estimate, or make a prediction?’ And then they list a whole bunch of different strategies at the bottom that you can use… on the back side, they give you an area to show your work, OK? And then they have some questions for you to summarize it. Then, after
you’re done working with your partner, we’re gonna share different strategies, and see if they come up with the right answer.

As students began to work, Sandy walked throughout the classroom with the videocamera, recording her conversations with students. She asked many open-ended questions related to the goals in SFMP 1 to gain information about students’ thinking, including: “What do you think?” “How would an estimate or prediction help you to come up with a good answer?” “Why’d you pick a table?” Later in the lesson, a few minutes before Sandy began calling students to the board to share their solutions, she read some guiding questions from the back of the activity sheet, again emphasizing SFMP 1: “ ‘Does your answer make sense? Does it seem reasonable? Did you estimate or predict? What have you learned?’ And, ‘Could you have used any other strategies? If yes, what might you have picked?’ Because I’m gonna ask you those questions when you go to the board in a minute.”

When Sandy did begin calling on students to share their thinking, she facilitated the discussion so that all students could be involved, without sharing her own comments until the end. When one pair of students commented that they realized their estimate was not especially logical (SFMP 1), she affirmed that this was a helpful realization in the process of problem solving. As other students shared and those in their seats were anxious to share comments about the mathematics, Sandy said to one, “Just a minute… let’s let them put up what they’ve got, and then you can tell me your comment.” Later, after a bit of disagreement, she added, “[L]et’s not be analytical of anybody. We’ll figure it out in a minute.” She was clearly working toward the goals of SFMP 3, where students
were expected to discuss their thinking and critique that of others. Sandy asked many questions during this process to support thinking, including: “Can somebody tell me what the difference in the two charts is?” “Did anybody do it a completely different way?” “Why do you think that?” “Do you know what kind of mistake that was?” (when a student had made an error in reading the original problem) Sandy refrained from commenting or even letting on whether the answers being shared were right or wrong (or close) until the very end of class, when she compared and contrasted the three solutions and related each of them to the wording of the initial task. Since limiting one’s own comments is difficult to do if a teacher is unaccustomed to it, perhaps Sandy did this more than she even realized. At the end of the lesson, Sandy said, “What’s really important is that my question, my ultimate question is: we see how different people solved the problem, right?” So, she reminded students again of her overarching goal, which echoed SFMP 1 and 3.

Although Sandy did not submit a reflection for this lesson, her comments in the second group discussion regarding the lesson indicated that she felt her approach differed somewhat, in a positive way, from the way she would have led this type of discussion in the past. She was, though, still uncertain about how to handle situations where students presented different solutions. She said, “[T]he one that had a correct answer, went outside the box… And how different – that was not an approach that I had taught. But we were able to compare… This is a really, really great start! But it’s not right. So we were able to discuss that… I didn’t say anybody was completely wrong.” She then shared how she had offered some strong comments at a recent meeting about the
importance of the SFMP in instruction. Thus, Sandy was really making an effort to allow her practice to evolve to support the goals of the SFMP.

Ray. Ray also asked several good questions in accord with the SFMP in his first lesson; if he had followed through more on students’ thinking afterward, the lesson could have been even more effective. Some of these questions (really all related to SFMP 1) were: “Did anybody do it a different way?” “How would I have done that in my head?” (On this question, he waited until several hands went up before he called on a student.) “What does the 4 represent?” “Does this make sense?” Also, at the start of the lesson, he discussed SFMP 3 explicitly and said to students, “Raise your hand if you have any idea how can you critique someone’s work – especially if you think it might not be correct – without attacking them personally…” Students then volunteered ideas. Ray addressed this in his reflection since his goal was to focus on SFMP 3, but he did not write about the various questions listed above. Ray does have the potential to facilitate rich learning for his students.

In Ray’s second lesson, he was very explicit with students that his goals for them were to consider different ways to enter the problem and to consider the possibility that more than one correct answer existed. He said, as he gave directions: “Some of you are really – and I know who you are – want to rush right into this problem. But sometimes we need to take a step back, yes? And we need to think more, all right, what do I know already? And what am I gonna need to know to solve this problem? Don’t just jump into the problem.” He then asked, “Why do you think describing a number pattern in words would be important?” After two students had responded to this question, Ray shared:
These timed tests [holds up a stack of papers] are beautiful, but they only do one thing. And one thing only. They just check to see how fast you are at solving these facts, right? That’s the only purpose. But good mathematicians, what they’re gonna be doing, is they’re gonna be slowing down and analyzing the problem so that they can look for new patterns and new understanding.

When Ray asked students to look at the task, he said, “I want you to look at this problem by yourself for about two minutes. I want you to think about, number one, how are you going to what we call enter this problem? How are you going to begin it?” He added, “I want you to see if you can solve it two different ways.” After these two minutes, Ray asked students to work with partners at their tables, and he walked around and talked to students, asking questions, and listening to what students were saying. He posed questions and comments such as: “How do you know that?” “Can you prove it?” “Did you solve it the exact same way?” “Does it always have to start at 1?” “Is that the most important part?” Just knowing the answer? How you got to it. That’s more interesting!”

Ray was always vigilant about reminding students about the essential goals of the CCSS and SFMP, which he did again in his closure for this lesson. Sometimes he used words from the actual text, and sometimes he did not. Although, as Ray observed about himself, he needed to allow students to articulate their own thinking more, he definitely had developed a mental framework for the goals of the new standards and was poised to
accomplish them in connection with his goal of giving students more opportunities to engage in discussion.

In his reflection on this lesson, Ray wrote:

I do like how I had the kids preview, gave them 2 minutes to attempt the problem individually, then turn and talk to partners. I think this allowed the kids to think about how they were going to enter the problem independently and then to follow through with a partner. Because of this, I think I had more examples of different strategies that were used when solving – I’ve got to remember this. They were able to find more complex patterns and relationships with the numbers involved.

It is noteworthy that Ray wrote, “I’ve got to remember this,” because he recognized that this way of setting up the lesson allowed students to apply more of the goals of the SFMP. Moreover, he continued:

I had a major epiphany after this last videotaping. I had thought that I have to “do” the SPMP with students—that I had to teach them. My big “aha” moment was that SFMP is not about me, but importantly it’s what the students will be doing. It’s my job to provide the environment with the situations that encompass the SFMP and let the students do the work. I know that it’s corny and cliché about saying that we teachers “become” the coaches, but it’s true.

If Ray is able to follow through on his thoughts in this paragraph in the future, his enactment of the SFMP will be quite effective.
Teachers’ successful enactment of SFMP also aligned with their own interpretations. As would be expected, I found no instances in the six lessons where teachers’ actions or statements corresponded with the likely intent of the SFMP but not with the teachers’ own interpretations of the standards. If this were the case, it would mean that the enacted curriculum aligned with the formal curriculum but not with the intended curriculum, which seems quite unlikely. If anything, one would expect that the teachers’ actions and words would at least align with their own understanding of the standards, and this indeed appeared to be true in the examples described in the previous paragraphs.

Influences on teachers’ effective enactment of the SFMP. A question to consider may be what caused the teachers to be able to enact the SFMP, at least in part, during these two lessons. For Carrie especially, her established personal philosophy and conception-based perspective of mathematics seemed to affect the way that she conducted class, for the most part, and this was true to a degree for Ray as well, though he may have had slightly less success enacting the standards in the way that he intended. Green (1971) proposes that “primary” and “central” beliefs guide teachers’ decisions, and Porter (1989) writes that teachers who establish clear goals for student learning can make decisions for practice that support these goals. Ray also seemed to become more reflective about his practice after viewing the videos. Sandy, on the other hand, had begun the study with a rather different history in terms of her teaching practice, which was shown in her first lesson. However, by the time she taught the second lesson, she was thinking more about the SFMP, having read and discussed them several times for this study, and she believed that it was important for her to adapt her instruction to be more in alignment with the
standards. So, she was relatively successful in applying them in the later lesson. Sandy’s experience with this evolution in thinking and practice mirrored that of the teachers in the study reported by Chazan, Callis, and Lehman (2008). Ironically, this may have been less true for Carrie in her second lesson, perhaps because she had not been as self-critical about her first lesson or her practice in general during the study, similar to two of Thompson’s (1984) teachers. These observations all suggest implications for professional development, which will be discussed later.

**Students’ words and actions that corresponded with the SFMP.** When teachers were able to implement the SFMP effectively, this often led to students being able to apply these standards independently as well. In these circumstances, the formal, intended, enacted, and attained curricula were all in alignment, and this is certainly a goal for which teachers should be striving. In the examples below, the teachers asked many questions and seemed to become the “hermeneutic listeners” that Davis (1996) maintains they should be, helping students to form their own meanings and understandings through language and experience.

Several times in Carrie’s first lesson, students volunteered comments that were unsolicited and that were clearly reflective of the SFMP. As they were working on combining tens and ones with base ten blocks and in writing with numbers, students shared thoughts such as: “I found another way to find it out.” “Give us another one.” (both SFMP 1) “There’s three 10-sticks, and then you make a 3, and there’s 6 cubes, and there’s a 6.” (SFMP 7) “I was thinking if we added two more on, we could swap.” (SFMP 3 and 7) Also, at one point when students were asked to work in partners to find
a total (which was 26), it was clear that they were talking excitedly about ways to solve the problem, which seemed aligned with SFMP 1.

Carrie’s students did seem fairly accustomed to volunteering ideas, asking questions, and exploring concepts with which they were unfamiliar – although it may have been only a few students who did all of these regularly. During her second lesson, in which SFMP 1 and 7 were especially applicable, when she had asked what students knew about “fractions” (which was a good way to begin an open-ended discussion and learn what students were thinking), one student shared, “It means like you take something, and then you put a line in the middle, and you make it in two.” Carrie reminded them that on the previous day, a student had noted that “when you cut something into half… you make it into two parts, it’s like symmetry.” When Carrie asked about how to divide a square into fourths, another student offered, “You could have a square, and then you would just put like little boxes, because you could have one square and then put like little boxes in it. Four little boxes.” In response to Carrie’s question about the meaning of the fraction 4/4, a student volunteered, “It’s when you make something and you cut it into four, and then you like color all of them.”

Later, during the play dough exploration, a student argued (about the hot dog shape), “You can only cut it in half once,” meaning that it could not be cut from end to end. Some students disagreed, and Carrie modeled this, apparently convincing the first student that this could in fact be done. Further, Carrie asked how to find half of a set of four cookies, and one student suggested, “[I]f you take 2 cookies away, it would be half.” Further, Carrie asked a student if she could cut the pizza shape into six equal parts. The
student cut the pizza in half, then in fourths, then cut a diameter between the other two to create six unequal pieces; she then counted and tried to stick two pieces back together, counting again a couple of times. Though Carrie did not see most of this work, the student was definitely applying SFMP 1 and 7 as she attempted to solve this problem.

During Sandy’s first lesson, students shared several comments that showed that they were thinking in ways encouraged by the SFMP, though these instances were less common in this lesson because Sandy led a great deal of the discussion. During the discussion of applications of capacity in real life (SFMP 1), a student suggested, “[W]hen like the companies fill up like the two-liter Coke bottles.” Another student asked, “I just thought of like, is there a difference between a gallon and a liter?” Later, as they reviewed perimeter and area, one student asked (about one of the shapes presented), “Is that an irregular polygon?” and a different, enthusiastic student remarked, “I wanna draw one so bad.” (also both SFMP 1) In addition, as Sandy modeled the method for deriving the formula for the area of a triangle (by starting with a rectangle), another student said, “So you’re like adding the whole rectangle up together, and then you’re cutting it in half to figure out what the triangle would be.” Although the student’s language was a bit imprecise, he was definitely trying to understand how the relationship between the structures of these shapes helped to produce the formula (SFMP 7).

Sandy’s second lesson allowed more chances for students to enact the goals of the SFMP. When Sandy was talking to students about their estimates before solving the task, one student remarked that 3 would be too low an estimate (in this context) because “Three [people] wouldn’t really be a party.” Another pair found a relationship between
the number of doorbell rings and the number of people who walked in (SFMP 7), but they were not considering the total, apparently. A different pair did find the correct answer, using a logical strategy that gave them the right answer (SFMP 7), which was actually somewhat atypical in this lesson because many students interpreted the problem situation incorrectly.

When students were presenting their work, they generally read the answers to the sense-making questions on the activity sheet verbatim, but again, the questions seemed to be helpful in supporting them to consider various aspects of the problem (the question would be whether they would continue to do this on their own or whether they would need more experience first). One presenter read, “‘What do I want to find out?’ How many people entered once the doorbell rings 10 times.” She also noted, in response to the questions, that she and her partner had made a table and that they believed their answer was reasonable. (all SFMP 1) The students at their seats who were commenting on the presentations were basically respectful and were careful to comment on the mathematics and not the students; most of them were engaged in listening to each other and in replying to what was shared (SFMP 3). One student critiqued a solution by saying, “Um, they should have – like [this pair’s] chart – they should have added ten doorbell rings, because it said after the first doorbell rings, so they should have added 11 to make it correct, and [the first pair’s] – they only missed that, but I like [the first pair’s] better…” Another student disagreed and read from the original task, “‘The first time the doorbell rings, 1 guest enters. Each time the doorbell rings after that’ – ten times.” Toward the end of the lesson after three solutions were shared, a student referred to one of these and remarked,
“I think [her] group is one of the only ones who did it correctly.” A second student agreed: “[I]f you stopped at 19, then where do all of these people that walked in before that, where would they go?”

In Ray’s first lesson, students did not have very many opportunities to offer their own thoughts in relation to the goals in the SFMP, but there were at least two instances when students did so. Both of these were in response to questions that Ray had asked or guidance that the Singapore method had provided, but the students were still applying certain aspects of the SFMP. In one case, the student said that the value they were trying to find was, “How many students are in one car.” In the other case, the student followed the Singapore steps and said, “I determined ‘who’ and ‘what’, which were the custodian and the chairs.”

During the second lesson, students did share their observations about patterns in the number sequence, which reflected SFMP 7 even though the patterns were rather simplistic; one rather unique example was this comment: “[I]t went odd, even, odd [pointing horizontally], and down it went odd, even, odd [pointing vertically], and then it would go even, odd, even [pointing at another column]…”. Another student presented a different solution, without saying very much (Ray spoke for her, primarily, when he saw what she was writing), and it was rather brave for this student to do so because everyone else had agreed with another answer. One student offered, “I see what you did with the 9, 18, 27, 36 – but it doesn’t follow any set rule.” He then went on to explain what he meant by this. A third student started to disagree with one of the shared ideas but then realized that he had an incorrect calculation that caused his disagreement. All of this
conversation echoed the goals in SFMP 3. In Ray’s reflection, he wrote, “I was pleased with the student interactions about disagreeing with a student without being aggressive or condescending… Throughout the year, I’ve stressed creating a learning environment where kids feel safe and willing to take risks… I liked how the discussion flowed… and it was a natural, respectful discussion about mathematics.” He also wrote, “Overall, I was pleased with the demonstration of SFMP 1… kids were able to provide mathematical ‘proofs’ for their patterns” (which actually seems to refer more to SFMP 3). In addition, “I thought the evidence the kids provided was at a high level that used many aspects of this mathematical process.” Again, I would tend to disagree that the thinking was “high level”, but the basic goals of SFMP 3 were still applied.

**Student actions not always aligned with teacher actions.** In spite of teachers’ best intentions and efforts with the SFMP, sometimes students did not enact the SFMP in their words or actions, which certainly will occur as part of the learning process. In Sandy’s first lesson, students sometimes offered very vague comments about units of measurement even when she had asked fairly direct, reasonable questions; one example of this was, “For every unit of measurement, we do problems that equal 1.” Sandy followed up with this student to help her clarify that she meant that they had often considered how certain units could be combined to create other “whole” units. When Ray was asking students to agree or disagree in his first lesson, even though he was doing this fairly consistently, only a few students were showing their agreement or disagreement until he commented that he wanted to see more “thumbs”. In Carrie’s fractions lesson, several students were still seeing the fractional parts as whole pieces
rather than parts of another whole and counting them as such, though this was still early in their work with fractions. Further, when the students were observing others’ presentations during Sandy’s second lesson, Sandy asked what was different about two of the tables on the board (a good question), and one student said that one pair had drawn borders around the rows and columns, and the other pair had not – clearly not thinking about the values in the tables. Sandy responded kindly and said that she was thinking more about the numbers in the tables. In cases such as these, it is important for the teacher to continue to facilitate meaning-making with students by building on students’ fore-structures and fostering interactions that move the hermeneutic cycle forward.

In all six lessons, the majority of the students’ ideas and actions were prompted in some way by the way that the teacher facilitated each portion of the lesson, as demonstrated here. It is perhaps noteworthy that the majority of the instances where one standard was evident were examples of SFMP 1, which perhaps reflects intentionality on the part of the authors for placing this standard first in the list of eight in the CCSS document. There were fewer examples of SFMP 3 and somewhat fewer yet of SFMP 7, though all three standards were obviously visible in the lessons at various times. Further, most of the occurrences of the SFMP paralleled the parts of the text of the standards that the teachers had discussed during the interviews and group discussions (not so much the text that they had avoided). Thus, it is clear that the teachers’ enactment of the SFMP had a strong influence on students’ opportunities to develop the habits described in these standards. This will be discussed further in the next section.
Students’ Opportunities to Learn Related to Teacher Expectations and/or Knowledge of the SFMP

Although it has been difficult to demonstrate that teachers’ expectations of students and knowledge of mathematics standards directly affect students’ learning (Hill, Sleep, Lewis, & Ball, 2007), previous literature does suggest that the expectations teachers have for students’ learning (Hiebert & Grouws, 2007; Philipp, 2007; Staub & Stern, 2002; Sztajn, 2003), as well as teachers’ understanding of standards (Fennema & Franke, 1992; Hill, Sleep, Lewis, & Ball, 2007) influence what and how teachers teach. For example, as noted earlier, all three teachers had incorrect ideas about the meaning of “counterexample”, which would imply that unless this were corrected, their students also would not have the opportunity to consider counterexamples in learning new concepts. Sometimes, it seemed that the teachers had trouble conceptualizing the SFMP for their own grades because the descriptors were written in a way that seemed to illustrate goals for the end of high school, which the teachers might have felt their students could not reach because they were too sophisticated. So, it would be important for all teachers to be able to read and study the standards so that they could carefully consider how to enact them effectively at their own respective grade levels.

Carrie’s limited reflection about her own application of the SFMP may have caused her to overlook aspects of students’ learning processes that could have been enhanced otherwise. For instance, she remarked in interview 2, “[I]t seems like, then, when we report out, that we only need to hear from a couple [of students] because it just becomes redundant.” Perhaps this was an implication that she had guided students too
much (contrary to her philosophy) because they were generally solving problems in the same way. Also, she commented that her students were “all there” in understanding combining tens and ones in the first lesson, which may not have been as true as she believed since she modeled a great deal during the lesson. Further, her interpretation of the word “conjectures” in SFMP 1 and 3 seemed to suggest proving ideas (“If I know this, then…”) rather than generating hypotheses, so it is possible that she would not foster the habit of making conjectures with her students, at least in math lessons. As noted in earlier sections, at times, she was unsure about the meanings of certain words or ideas in the SFMP, e.g., “complicated things” or the pairing of examples about sorting and the commutative property in SFMP 7, which might imply that she would not implement SFMP 7 with fidelity in her classroom. Further, Carrie felt that certain uses of visual “representation” were unnecessary once students understood particular ideas, so potentially she could communicate this thought to students and lead them to believe that visual models were unsophisticated and not to be applied in later grades.

Nevertheless, overall, Carrie strongly believed that all of her students were capable of applying the SFMP, and this was relatively evident in the way that she conducted her lessons and required students to justify their thinking on a daily basis. She felt that SFMP 7 suggested that teachers should be helping students to make connections among ideas and that she needed to foster this purposefully every day. In her reflection from the second interview, she wrote, “Good teaching is student led and teacher guided – if that truly is occurring, then all of the math practices are being implemented.” Also, after the second group discussion, she wrote, “[A]ctively engage in the discussion and
listen to what the children are telling you in their thinking!!” Carrie believed that students were able to meet the goals in the SFMP as long as teachers provided the opportunity to learn from each other and a safe classroom “culture”, because students were able to develop confidence and motivation as they experienced feelings of success. In fact, she felt that her students enjoyed math so much that they would be genuinely upset if they learned that they were not having a math lesson on a given day, and she appreciated being able to use this enthusiasm to hold their interest from one day to the next. Her only concern about some students’ applying the SFMP was that some of her students seemed to have “learned” from parents or other adults that in mathematics, only one method was correct in solving any given problem, and she hoped to change that through her daily practices.

Sandy’s focus on certain aspects of the SFMP seemed to relate to her classroom practice and the expectations that she had for student learning. For instance, her comments about the importance of “definitions” in SFMP related to her use of notebooks with students and how useful she felt these had been as resources for them. She noted, “[K]eeping a math notebook makes the children understand the importance of definitions.” It is unclear that she had evidence of this, but it is possible that students simply knew that Sandy believed the information in these notebooks was important without understanding why. She also did not seem to be able to interpret the word “analyze” in SFMP 1 very precisely during interview 2. In each lesson, especially the first, she was trying to enact the standards, but she was so accustomed to telling students how to solve problems that she did not seem to think very often to ask students to give
explanations, which would probably be expected of a teacher who was not used to doing so. However, after her second lesson, on which she reflected in the second group discussion, she recognized that the students focused on the use of a particular strategy and not so much on making sense of the given problem situation, and she felt that this was due to her emphasis on that strategy in prior class sessions. So, she planned to remember this in future lessons and to try to encourage more independent thinking.

Throughout the study (especially early on), Sandy expressed doubt that in the near future, students would be able to apply the SFMP in grade 5 because of their age and maturity, their lack of a basic “foundation” which would support the understanding of relationships among ideas, their reliance on personal technology, their unwillingness to appear “different” in terms of their thinking, and the lack of support for perseverance (motivation) and respectful conversation at home. In contrast, Carrie was confident that students were already applying the standards in grade 1, which Ray also believed about his students. Perhaps Sandy’s perception partially resulted from the language in the text being so foreign that she did not recognize what these habits of mind might actually look like in fifth grade. She did say that perhaps other students in other schools or classes might be better able to meet the standards than many of her own students. Moreover, Sandy was not sure at all that she agreed with using manipulatives in grade 5, which the SFMP suggested. If Sandy taught her students with all of these thoughts in mind, chances are that she would not provide them with as many rich learning opportunities that incorporated the SFMP.
Sandy also seemed to have some trouble articulating and understanding some mathematical ideas herself. This might perhaps have made her even more reluctant, to some degree, to ask students to explain their own thinking, particularly since she said that it bothered her when students (children) challenged her (an adult). It is also possible that she recognized that she struggled with some of the SFMP, and her opinion of students’ chances of meeting them might have been thus diminished. Still, Sandy said that she enjoyed the days that students taught in preparation for the OAA – as did the students, who even enjoyed simply using instructional technology, and she did feel that the students’ notebooks were valuable throughout the year. She may have just needed more time develop these practices to support the SFMP more effectively, which she felt could “eventually” (not in one year) support students’ achievement of the goals within them. Moreover, after her second lesson, Sandy acknowledged that fifth graders could develop the skills included in the title of SFMP 3, and she recognized that she would need to support them in this development by also developing these skills herself, which reflects Brown’s (2001) point that when teachers have not been comfortable with aspects of mathematics themselves, it can be a challenge to develop their own understanding while developing students’ also.

Ray was quite sure that the SFMP were vital elements of mathematics instruction – in essence, the grounding forces underlying it. He believed that his students could be successful with most of the goals described because they were written well and with intent and because other nations were students typically scored high on mathematics tests had already been teaching with these goals. Ray differentiated between teaching for
depth (with the SFMP) and teaching simply for mastering the content standards at a “superficial” level (without the SFMP). He said in interview 2, “[I]f you don’t follow the practice standards, you’re not gonna be a successful math teacher,” and he added that it was his “job” to help students to slow down and think about how they were solving problems, which was similar to Carrie’s comment about her role in helping students to make connections. When Ray described mathematics teaching as being similar to working with “clay”, it evoked an image of shaping learning in unique ways for each student. He felt, though, that the goals would come more easily to students who were typically “proficient” (using a word from the text) and who had support at home because some of the goals reflected fairly “abstract” thinking and true perseverance. Ray also remarked that students were only able to “keep so much” in their heads because they were so reliant on personal technology. However, he was especially pleased when students suggested strategies that he not considered, and he felt that students “want[ed] to do well, inherently”. His enjoyment of mathematics, and his purposeful “rediscovering” of new mathematical ideas with them each year, would act as supports for students who were not entirely comfortable with the content, a point that Ray recognized and felt was integral in his teaching. That is, Ray wished to foster the “productive disposition” advocated as part of mathematical proficiency by the National Research Council (2001).

On the other hand, Ray’s own disjointed reading of the SFMP may have contributed to incomplete understanding of the goals therein; he often passed over words that were unfamiliar without a great deal of comment. For example, his comment about teachers’ monitoring and evaluating students’ progress (from SFMP 1) was actually to
apply to students’ monitoring their own progress. Also, in SFMP 7, he believed that
“objects” were variables, whereas this term could actually refer to many concepts within
mathematics. Further, Ray noted that students “loved” to correct him when he made
errors in class (intentionally or otherwise), and he seemed to imply that this meant that
students were able to do this in examining their own work, but the evidence for this was
not obvious. In fact, he cited an example task that the students had completed for OAA
practice where many had chosen a very unreasonable answer from the four listed. So,
perhaps this was evidence that many students were not actually learning to look for their
own errors, even though Ray felt he was modeling this habit continually. Nevertheless,
Ray did believe that his students were capable of, and often successful in, offering
feedback to each other and achieving the other goals in the SFMP, so he would
presumably continue to encourage these habits in class.

Sandy stressed that the content and practice standards were not aligned explicitly
in the standards document, and even for her, as an experienced mathematics teacher, it
would take time and work to understand how to weave them together in class. She felt
this would be extremely challenging for teachers who had not taught math in several
years. Cohen and Ball (1990) and Ball (1990) write that standards documents must be
written in ways that are understandable for teachers in order for reforms to be likely to
succeed. Carrie had also made a similar point in the second group discussion, saying that
when teachers were fairly unfamiliar with the content they were teaching (or were
inexperienced in general), students could have vastly different experiences than they had
in classrooms where teachers had had years to develop their courses and their
instructional practices. All three teachers were concerned about this issue since all elementary teachers were moving to the new standards in all four major academic areas in the following year. Ray commented, “[W]e’re all gonna be learning everything together!”, and Sandy asked, “[I]f the adults are this challenged, then how are we expecting our kids to get it?”

Carrie and Ray also believed that students’ motivation to learn (and perseverance) could be enhanced by the practices that they enacted in their classrooms, again echoing the notion of a productive disposition on the part of students. In the third interview, Carrie shared, “The perseverance is just confidence. I think that that is, um, a community issue, a culture issue in the classroom, of always, always finishing something, and giving people time, and then applauding and celebrating accomplishments.” Ray felt that his students would develop more positive dispositions for math when the teachers were able to teach “to depth” with “meaningful” problems and did not have to rush through content. Carrie also talked about using problems based within the school environment to foster students’ interest and not relying on competitive math activities for this purpose. Ray had also noticed that his students appreciated non-graded warm-up activities, and he was considering using a blog to work toward SFMP 3 with students, which he felt would be engaging for them. In interview 3, he commented, “But in regards to the persevering, I think if your workshop is inviting enough, and that you’re allowing them to use the appropriate tools, that freedom to share and be wrong and OK with it, and maybe make a little detour… I think you’ll get the best out of kids.” In contrast to Carrie and Ray, Sandy did not seem to connect students’ motivation with her own practices; her
comments indicated that she believed students were either motivated to learn or were not
– and she felt that most were not. All of the teachers, though, appeared to believe in a
clear causal connection between students’ motivation to learn and what they actually
learned. This relationship may seem self-evident, but it is worth noting.

The importance of the “opportunity to learn” in students’ eventual learning has
been documented by Brophy (1999), Grouws (2004), and the National Research Council
(2001). In general, Carrie, Sandy, and Ray believed that all students should have
opportunities to master the same standards, even if some did so on a different timeline
than others. They all wanted to provide opportunities for students at different levels to
grow as mathematics learners, and they each recognized, though in somewhat different
ways, that they needed to create classroom environments that encouraged the SFMP so
that students could accomplish these goals. In Ray’s final comments, he wrote, “Here’s a
great analogy: The Common Core standards are the destinations on a road map while the
SFMP is [sic] the vehicle you’re driving in.”

Teachers’ Reflections On Issues Related to the CCSS and SFMP

Teachers’ Written Reflections

The teachers’ reflections after different phases of this study very much echoed
what they had said in conversation – particularly ideas that they had reiterated in multiple
conversations. Sometimes, a given teacher would refer to ideas in the SFMP, and at other
times he/she wrote in more general terms, which also echoed the pattern of responses in
the interviews. Generally, my observations about the videotaped lessons corresponded
with what the teachers wrote in their reflections in terms of what SFMP were present
(and, in fact, whether the standards were effectively enacted), though I often noticed additional instances of one or two of the standards that they did not mention. As Senger (1998) and Watson and DeGeest (2005) note, reflection is a useful means of helping teachers to change their practice. Brown (2001) maintains that written reflection allows teachers to monitor their own thinking over time as well as to express descriptions of their own practice in words, which requires them to think about this practice in relation to their goals and beliefs. In this process, they can gain increased self-understanding as teachers and set further goals for improvement. Evidence for this goalsetting can definitely be seen in the many excerpts from the teachers’ reflections that have been included in this chapter.

**Teachers’ Views on Their Thinking (or Others’ Thinking) about the SFMP**

Carrie never felt that her interpretations of the SFMP changed substantially during the study, though she did believe that she was better able to articulate how her practice aligned with the standards by the end of the research. However, I am not sure whether she feels this way because of what she has read in the standards or because of the fact that she has read the standards – the latter is probably more likely because she quickly agreed with what she read, and she believed that kindergarten and first grade had been teaching in alignment with the SFMP for years. Carrie was pleased that the goals in the SFMP were not simply encouraged anymore; they were “expected” of all teachers and students, and the document “affirmed” and “validated” the way that she and her colleagues taught. Brown (2001), citing Habermas (1972), notes that we should consider how language serves the interests of particular people or groups, and Carrie definitely felt that the
SFMP corresponded with her interests as a teacher. Early on, Carrie had said that she thought teachers could teach the new content standards with “worksheets” and without enacting the SFMP, but she changed her position on this later in the study. Carrie did note when she was not sure what some of the words or phrases meant in the SFMP, so this might be a seed for growth in her own thinking. Also, it is noteworthy that she related the SFMP quite often to the strategies used to help students develop their ability to read, so this could be a key idea for professional development on the SFMP in the primary grades.

Sandy’s and Ray’s descriptions of their own thinking about the SFMP were more similar to each other than to Carrie’s. Sandy did not seem to feel comfortable with the SFMP before this study because she had not read or thought about them, and she remarked that math had always been viewed as “black and white”, whereas the view of mathematics in the CCSS was quite different. So, her feelings definitely evolved; as noted earlier, she credited professional activities at school as well as this study – particularly the opportunity to read each standard paragraph sentence by sentence. She believed by the second group discussion that her future practice would need to be more centered on problem solving and reasoning based on real-life, rich problems. Sandy also commented that she realized that in the past, when she had asked students to explain their work in class, they were simply echoing what she had shown them instead of generating and sharing their own thinking. She wanted to change this in the future. It is not clear whether Sandy thought the language of the standards served her interests even at this
point, but she did seem to believe that it served her students’ interests, which aligned with her goals as a teacher (Brown, 2001).

After the first lesson, Ray wrote of the SFMP, “Has my interpretation changed? Heck yes!” He added, “I think this standard is much more difficult to attain from its written form! Having viewed myself teaching SFMP #3, I think I have to make sure that I don’t trivialize or superficially expect students to present viable arguments and critiquing others.” On the other hand, he later wrote, “In my first video tape, I thought I had to do something ‘spectacular’ for SFMP #3, but having seen both videos, there’s a nice flow in the mathematical dialogue without it being forced… I like approaching my workshops like we do as adults – we read a book and we can’t wait to talk about it… in math, I look for that natural intellectual conversation that occurs.” He also wanted to support students in learning to “prove” that their thinking made sense. Ray said that he was not “fearful” about the new standards because he had taught math for many years, and he felt that the Singapore math workshop had helped him to understand the philosophy of the CCSS. He also felt that the content standards could be read at “face value”, and he appreciated the examples provided and the specific details about the understandings and skills expected of students in each grade. In addition, he was glad that we had focused on just three of the SFMP during the study because this had fostered his interest in learning about the other five. Ray certainly felt that the SFMP aligned with his purposes and beliefs (Brown, 2001).

All three teachers felt that they would be able to support their colleagues as they (the other teachers) were learning the SFMP and trying to interpret them for their own
classrooms, because the standards, as written, were not very easy to read and understand. Early in the study, they believed that most teachers in grades 2 and up probably had not even seen the SFMP, even if they had been to workshops on the CCSS because the content was the major focus at these workshops. In fact, Ray was hesitant to believe that many of the kindergarten and first grade teachers, who had been teaching the new content standards for a year, were really enacting the SFMP because they had not had a great deal of professional development for these standards. At the same time, all of the teachers believed that the SFMP and content must be woven together in order for the best teaching and learning to occur, and toward the end of the study, Ray remarked that he thought that his colleagues were beginning to understand the importance of this shift in instruction, though this understanding was still developing. However, he wished that administrators were as “informed” as the teachers about the new requirements because he believed that if they were, “a lot of policy would be different.” Darling-Hammond (1990) and Millett and Johnson (1998) also note that administrators may assume and communicate messages about standards to teachers that differ from the intent of the authors of the policies.

**Individual Teacher Learning and Personal Strengths that Prompted Changes in Beliefs/Practice**

Carrie and her students seemed to benefit from Carrie’s conception-based perspective of mathematics; she highly valued knowing her students and meeting each student where he/she was in order to move his/her thinking forward. She actively worked every day to ensure that students felt successful and confident about their learning and in exploring new ideas without anxiety or fear of failure. Carrie also saw many connections
between instruction and learning goals in reading and mathematics, and since she was especially trained in reading instruction, this could be helpful in her future reflections on mathematics teaching and learning. She commented at one point that she thought she could do more with relating the combination of multiples of ten to combining single digit numbers. She also commented during interview 3, “I like to do inclusion, because I have another adult in the room, and I can bounce things off – having student teachers always make me a better teacher.” That is, she felt that she became more reflective when she was able to work with another teacher in her room on a consistent basis. Further, Carrie noted that having a lead teacher (whether this were herself or someone else) who could support colleagues in the enactment of standards through guidance for quality instruction would be very helpful in enacting the standards well. This point is echoed by Valli and Hawley (2002).

I did wonder what it would take to convince Carrie that a change might be warranted in her instructional practice, but I suppose this would be a realization that students were not understanding something as she thought they were. For instance, her surprise about students’ not understanding the word “represent” caused her to think about a way to alleviate this confusion. Also, she did brainstorm reasonable solutions to classroom “problems” fairly effectively in the hypothetical situations that I posed. Since her students’ learning was of utmost importance to her, it is likely that she would make any changes that she felt were needed to better support this learning. This motivation for change corresponds with the “consequence” stage of teacher change proposed by Loucks-
Horsley and Stiegelbauer (1991), in which teachers are willing to change because they want to have more impact on students.

Ray was reflective and had many engaging ideas for instruction, commenting that he “like[d] change” – “Change is good”. He was interested in mathematics education research and felt that he learned a great deal from reading professional literature and attending workshops. He also felt that the lesson videos were very helpful for reflecting on his own practice and making change because he could not deny what he saw in the video. Ray definitely seemed to be at the consequence, collaboration, or refocusing stages of teacher change (Loucks-Horsley & Stiegelbauer, 1991); he seemed to have a continual desire to learn and grow. I believe that at least some of his ideas were already playing out in his classroom, from what I observed in the videos. Ray saw part of his role in the future as “finding and creating more elaborate, skills-based mathematical challenges in a problem-solving context!” He said that this would not be easy, and he added, “I also would like to see examples of how teachers who are already implementing the standards introduce and use SFMP #3 at grade 5. By having better quality work for students to solve would lend itself to a better quality of this standard.” Ray realized that he wanted his students to be doing more of the work and more of the presenting in class. He also felt that one of his strengths as a teacher was using cooperative learning, and this would be useful in enacting in the SFMP. In addition, Ray recognized that he would have to be careful not to “show” concepts to students instead of having students explore them through various representations and contexts. He wrote after the second group discussion: “I think I have always tried to integrate SFMP in my classroom but I know I’m extremely guilty of
rushing the kids from one activity to the next due to time… I’ve never really allowed the kids to really delve into a problem because I knew that I was teaching to the test [the OAA].”

As aforementioned, Ray was viewed as an informal leader among the math teachers, and since he felt that his own mentor was very helpful more than ten years ago, he hoped that he could support others in the same way. He also believed that teachers must be honest with each other and with themselves in order to make real change – implying that this was true both individually and collectively. If this were the case, it would be likely that he could learn from his colleagues as well; he believed that he could grow as a teacher and continue to help his students to grow.

Sandy cared deeply about her students as people and wanted to do the best for them, and she was reflective and willing to brainstorm and try new ideas related to the SFMP. Sandy wrote after her first lesson: “I pride myself in the children having a comfortable environment to learn and work, however I fear the vocal aspect is missing for some students. This might have been because we were being videoed that day or it might be an eye opener for me as a teacher. This gives me something to observe in my classroom and to think about ways to tweak SFMP 3 as I continue to teach.” Further, she shared during interview 2, “It’s a standard – I’ve got to work on it. I can’t just, you know, think about it occasionally.” I noted that Sandy’s curiosity about how her students would respond to several tasks or situations that we discussed might be a good element of professional development for her (and for other teachers – as research has already shown). She also wanted to incorporate more integrated, real-world problems in her
teaching. Further, reflecting on her own lessons seemed to provide Sandy with insight about what her students could actually accomplish (beyond what she initially expected) and how she could support them in this process. She remarked, with some excitement, “Have to make a lot of [changes]… from the way the children socially interact, to what I ask of them, to how I ask them to do it, has to change drastically…” Sandy definitely had great potential for growth, probably the most of the three teachers, as she demonstrated in these and other comments when she spoke of “retraining” herself and the students. Guskey (1986) writes that when a teacher realizes that her own changes in practice will serve students well, thus increasing the teacher’s own self-efficacy, the teacher is much more likely to initiate this change—though time and reflection will still be needed for the change to take hold.

**Teachers’ Views on Their Own Professional Learning Related to Practice**

The teachers all also felt that common planning and collaborative time across a grade level, on a regular basis, was vital in implementing standards consistently and effectively. This was perhaps the greatest point of consistency among the three teachers throughout the study, and they did not have many alternate ideas for professional development that they seemed to believe would be equally as beneficial. Both Sandy and Ray emphasized this again in their final feedback. The teachers’ feelings are well corroborated in research that demonstrates the value of collaborative professional learning (Chazan, Callis, & Lehman, 2008; Cobb & McClain, 2006; Cohen & Ball, 1990; Lieberman, 1994; McLaughlin, 1994). Sandy specifically spoke of grade-to-grade collaboration as well as within-grade collaboration and said that teachers in their school
typically had little structured opportunity for either. She felt that the teachers themselves would take the initiative (at least in her grade – perhaps not in others) to discuss and share ideas about the SFMP as a grade level. All three teachers mentioned vocabulary as one particular element of teaching and learning that could be developed consistently if teachers were able to discuss this language frequently. They also all felt that they needed more professional development time to focus on methods for assessment of the new content and the SFMP. Because of Ray’s excitement about the Singapore math workshops, Carrie and Sandy both expressed interest in attending one in the future, and Ray hoped that other colleagues would be able to do so as well. Further, Carrie wrote, “I find value in understanding more about the progression of learning from grade level to grade level.” In other words, the teachers did not feel that they knew enough about the teaching and learning in classrooms at other grades. Teachers may make unintentional assumptions simply based on their own experience and a lack of knowledge about other grades, which could lead to misunderstanding and frustration among teachers rather than cooperation.

Ray and Sandy felt that it was not helpful to attend workshops where the focus was only the new content and not on the SFMP. Carrie believed that the district did not need to bring outside presenters in to introduce new ideas to teachers; rather, teachers needed time to share ideas and to build upon the expertise that was already present among the faculty. Moreover, all three felt that they would not necessarily gain much through guidance from their administrators, who were relatively unfamiliar with the details of the CCSS and probably would remain so. It is also worth noting that Sandy and
Ray experienced the same professional development at the same time on the fifth grade content standards and left with two different observations about what had occurred. As hermeneutic principles would suggest, this likely occurred because Ray and Sandy had different backgrounds (fore-structures) for what they were learning and thus interpreted the experience differently (Brown, 2001; Gallagher, 1992). This suggests that teachers have different perspectives on what is useful to them in professional development, and this is a clear implication for those designing such opportunities.

**Teachers at Different Levels of Readiness for Change**

As aforementioned, Carrie felt that the first grade teachers were quite ready for the new standards because their philosophies – in reading, mathematics, and other areas – were already basically aligned with the CCSS and SFMP. If this were indeed the case, these teachers would likely have been at the consequence, collaboration, or refocusing stage of teacher change (Loucks-Horsley & Stiegelbauer, 1991), perhaps because they had experienced at least three of the conditions for teacher change suggested by Showers, Joyce, and Bennett (1987): understanding the rationale for the reform, seeing the reform enacted in a classroom, and implementing the reform in a non-threatening environment.

In contrast, during interview 1, Ray remarked, “[Y]ou’re gonna have people who are out there, your trailblazers, gonna be out there on the limb, and you’re gonna have people in the middle who follow the trailblazers – and then way back here are people who are like, ‘Oh, no. No, no, no!’” He was concerned that some of his colleagues in fifth grade would actually “fight” him and Sandy about what was necessary in enacting the new standards because these teachers were so entrenched in teacher-directed pedagogy
and narrow views of content. He also felt that some of these teachers were not especially committed to their own professional learning (Collopy, 2003; Moreira & Noss, 1995) and resisted district efforts to support them in this growth – instead, teaching in ways that were familiar – which would certainly limit their ability to implement the standards well, as noted by Firestone (1980) and Ramsey (2011). Ray admitted that giving up “control” in his own classroom to allow students more room to explore and discuss ideas would be challenging – even though he fully believed that he should do so.

At first, Sandy believed that the teachers at her school would be easily able to enact the SFMP because they wanted to improve their practice. However, a desire to improve one’s practice and doing this in relation to new standards are not necessarily complementary because of the complex factors that affect what teachers do in their classrooms, as research has shown (Brown, 2001; McDonnell & Elmore, 1987; Porter, 1989). As the study progressed, Sandy frequently remarked that it would be difficult for her, let alone the other teachers who had not taught math for several years, to enact the SFMP well. In addition, all of the teachers were faced with teaching at least two new content areas since they would be self-contained the following year, so this added an extra set of new challenges.

Finally, Sandy and Ray both commented that change in their instruction would take time, and they did not expect that they would be enacting the CCSS highly effectively in a single year or even two or three years – a scenario also described by Darling-Hammond (1990). Professional support would be critical in moving this transition forward more quickly (Corbett, Dawson, & Firestone, 1984; Remillard, 1992).
Teachers Learning from Each Other (What They Would Not Have Learned Otherwise)

One of the interesting aspects of this study was watching and listening to the teachers learning as a result of what other colleagues shared – learning that could not have happened individually. Ray commented at the end of the study that it was “always true” that colleagues learned from each other. This reflects Cobb’s (2000) emergent perspective, and it was also a frequent topic of conversation regarding the teachers’ other colleagues and their collective professional learning. Examples of this among the three teachers that have already been discussed were: discovering that students’ perseverance in grades 1 and 5 seemed to be influenced by very different factors, with different results; exploring possible reasons for students’ lack of place value understanding coming into fifth grade; recognizing that the foundation for the SFMP in the early grades truly does impact students later; learning that the first grade content was designed to be taught for conceptual understanding; learning about the types of assessment that were emphasized in each grade and how some of these could be utilized in other grades.

As the three teachers thought more about their colleagues and how they would all benefit by collaborating in the transition to the CCSS, they all felt that they should take responsibility for sharing some of the insights they had gained in this study with the other teachers. Their comments agreed strongly with previous findings that grade-level collaboration (currently occurring in the form of their PLCs), as well as opportunities for cross-grade collaboration, would be instrumental in ensuring that the standards were implemented well (Chazan, Callis, & Lehman, 2008; Cobb & McClain, 2006; Cohen &
Ball, 1990; Lieberman, 1994; McLaughlin, 1994). Sandy had already volunteered some comments about the importance of the SFMP at a staff meeting and in a recent workshop with the county mathematics consultant. However, Ray also pointed out that the principal’s perspective about the sole purpose of a PLC – examining assessment data – was in conflict with the teachers’ perspectives about the most beneficial uses of this time together, which included planning, discussing standards, and also creating and examining assessments. Porter (1989) writes that dictating goals to teachers without including them in decision-making will not result in effective teaching. Carrie also noted that the current amount of time set aside for their PLCs (about twenty minutes) was insufficient for worthwhile discussion to take place.

Ray had also suggested that teachers as well as administrative and support staff should be able to visit other classrooms periodically so that they could gain a better perspective on the goals and practices at each grade level. This, he felt, would allow the school faculty and staff to support one another with more knowledge of what was actually necessary to facilitate student learning. The lack of familiarity with each other’s teaching became evident in the two group discussions, when the teachers were asking each other questions about their different grade levels and seeming a bit unsure about what questions they could ask without offending each other. However, it was clear that Sandy and Ray trusted each other and worked together well, so they had daily opportunities to learn from one another. In fact, it appeared that they did talk frequently about mathematics instruction because in individual interviews, they often responded with comments that were nearly identical. It is likely that Carrie and one or more of her
first grade colleagues were also able to share ideas daily in this way. On the other hand, it is noteworthy that although Sandy and Ray did share many of the same perspectives about teaching, their thoughts about students and student learning were often quite disparate, as has been shown. This could indicate that even teachers and other school staff who work together closely may still have very different ways of thinking about how students learn mathematics, as noted by Floden and Wilson (2004).

**Genuine, General Frustration That Can Affect Teachers’ Attitudes**

Throughout the study, Sandy and Ray expressed a number of frustrations about situations that affected their teaching in some way even though they were not necessarily related to mathematics. Because these concerns were inseparable from the school context, they also influenced the teachers’ interpretations of the new standards (Gallagher, 1992; Nicholson, 1984). One issue, of which the teachers were aware by the time we conducted the second interviews, was that the school calendar, as well as the daily schedule, were going to be changed fairly significantly for the following year, with the result that teachers would spend more time with students during each school day with a more compacted school year from start to finish. Ray worried that this might, in essence, wear out students and teachers and cause a loss of motivation for both groups during the year. He also remarked that he felt that teachers who tried innovative practices in their classrooms were sometimes “punished” because the principal would make decisions that effectively ended these practices. Sandy and Ray both felt that their administrators were not especially supportive or respectful of teachers, nor did the leaders try to understand what would be most useful in transitioning to the new standards in
terms of professional learning, collaborative time, instructional resources, and general expectations for learning and (especially) OAA scores. They also felt that parents often provided little support for students’ mathematics learning because the parents believed mathematics was unnecessary or simply not interesting.

Interestingly, Carrie did not seem to have these same concerns, which may have been more a function of her relaxed personality than the possibility that these concerns did not affect first grade. She did mention, for instance, that some of her colleagues were often very bothered by the assessment data that they were to collect, and Carrie commented that she just did what she was asked to do, trying to gain something in the process, and not allowing it to upset her. She may have been able to do this because her primary and central beliefs about teaching were quite strong (Green, 1971).

**Resources for Teaching**

In general, all of the teachers felt that they could use more updated textbook resources that were aligned with the CCSS. This is quite typical of teachers who are transitioning to new standards and new ways of teaching (Collopy, 2003; Senk, Beckman, & Thompson, 1997). Carrie, however, felt that first grade could get by without purchasing a new book right away, whereas Sandy and Ray truly hoped that fifth grade would be able to buy a “quality” new text resource and new technology quite soon because the new expectations (both for content and standardized assessment) were so vastly different than the previous Ohio indicators. In the meantime, Sandy planned to continue to have students create journal notebooks throughout the year, and Ray planned to use more of the Singapore materials that he had bought; he did not feel as wedded to
one textbook as other teachers might because he tended to rely on his own creativity as well in order to keep his lessons from becoming “static”. They both felt that they would need to find or create more real-world problems that would involve multiple content areas and allow them to integrate the content more readily than in the past. Sandy also commented that she believed that school leaders would provide teachers with copies of the standards but perhaps little more, at least for the time being. Carrie noted that she felt the best resource that could be provided for first grade would be time for teachers to work with each other regularly to strengthen consistency with standards as well as to maintain similar learning cultures in classrooms. Ray mentioned several times that he felt the district should be thinking about how to provide some resources to parents to help them understand the CCSS and SFMP as well.

Before the third interviews, the district had in fact purchased the newest edition of *Investigations* for kindergarten through fifth grade, as well as new sets of manipulatives for teachers who had not been teaching math. Initially, Ray seemed pleased overall with the program, though he was still concerned about gaps between the books and the CCSS and how the teachers would supplement the texts, particularly with problems that integrated multiple content areas. (Later, after the study, he wrote that he was “not impressed” with the lessons that had been inserted in these texts to align with the CCSS because he felt the “majority of the lessons [were] direct instruction”.) During the study, though, Ray was glad that the district administrators had “mandated” that teachers would use these new books because previously, “some people… just wanted to do what they wanted to do. And that makes an impact.” One might raise the question of whether
mandating that all teachers would utilize the textbooks would guarantee that they would in fact do so, which is another issue related to policy and how it is implemented over time. The purchase of *Investigations* and the mandate for its use as written represented a capacity-building policy (McDonnell & Elmore, 1987) because these actions were long-term investments in material resources. As McDonnell and Elmore point out, this type of policy requires much consistent support in order to succeed because the results will not be as obvious to the general observer as the results of a policy simply requiring compliance would be, and belief in the policy may be more easily abandoned.

**Mandated Assessment and Related Policy: Help or Hindrance**

During each phase of the study, *many* of Ray’s and Sandy’s comments focused on their frustration with rushing through content to support OAA scores. This was not only due to expectations that the district and school leaders placed on them for these scores, but it was also a policy issue because fifth grade teachers were not allowed to review any content. So, both pacing and teaching could be quite difficult as far as the teachers were concerned because students did not always know or remember the material that was taught before grade 5. Because the pace was so regimented, their planning could not allow for much deviation based on what they observed about student understanding or readiness to learn. This deviation from what they believed to be truly appropriate for teaching and learning reflects findings of Cooney (1985), Arsac, Balacheff, and Mante (1992) and Raymond (1997) that showed that teachers may not act in accordance with their beliefs when they feel constrained by time and when they wish to be certain that students see and hear precise mathematical explanations. Sandy and Ray also felt that
they could not take time to do “fun” activities with students anymore. Moreover, Ray commented that some of the fifth grade math teachers were becoming upset with each other when they shared assessment data in their PLC if they saw that another teacher’s students performed better than their own. They were worried that if these pressures were maintained even when the content and standardized tests changed, it could be challenging to enact SFMP 3 (particularly involving communication of ideas among students) and the other SFMP. The superintendent told teachers before interview 2 that he wanted a return to “basics” in the classroom, and it was unclear whether he understood the overall goals of the new standards or whether he expected teachers to teach both the new and old standards for the coming year. Ray wrote at the end of the study that in the spring following the final interviews, the fifth grade OAA scores had dropped again since the OAA was not especially aligned with the CCSS. Ray felt that the fifth grade staff, as a result, were being viewed as “bad teachers”. (Ray’s idea of administrators’ visiting classrooms might help the superintendent to understand what instructional mandates for state assessments might mean in practice.) Ray and Sandy were also wondering whether the district would be able to provide them with more computers in classrooms to help students prepare for the PARCC assessments, which would be taken online. The provision of computers would represent a capacity-building policy as defined by McDonnell and Elmore (1987).

As Sandy and Ray discussed these pressures throughout the study, there were often noticeable tones of desperation, guilt, and decreased self-efficacy in their comments for not being able to do what they felt was appropriate for students, and they even felt
compelled to whisper from time to time when they expressed these emotions even though they knew that no one outside our research group would hear what they were saying. They were also concerned about the new system of teacher evaluations in Ohio, requiring that half of a teacher’s evaluation should be based on value-added data from state test scores. Still, they were hopeful that the fact that the content was becoming a more coherent whole in fifth grade might alleviate enough of the time constraints that they could in fact enact the SFMP without concern that test scores would be negatively affected. Also, they planned to re-design their local assessments to reflect the new content and practice standards as well as possible, though they admitted that this would be challenging until more examples of the PARCC assessment items were released. They hoped that their online grade reporting system would also change to reflect the CCSS.

Carrie felt extremely disheartened for her fifth grade colleagues because she could not imagine teaching with these types of pressures or constraints on lesson planning; she believed that these practices conflicted with the district’s critical thinking initiative as well as all that educators knew about how students learn in different ways. She discussed the current assessments that first grade was using for math as their part of the district’s focus on collecting and using assessment data. Carrie felt that the assessments were helpful to her in knowing her students; she did not explicitly say whether she believed they reflected the SFMP, though she seemed to imply this by the fact that she referenced the assessments in response to my question about district policies related to the SFMP. However, she was doubtful that the actual data from them would be useful to anyone who had not been trained to use these assessments because the scores would have no meaning.
Carrie noted that perhaps eventually, the district would determine what data was actually helpful to collect in each grade.

The fifth grade teachers were interested in hearing how first grade was essentially assessing several of the SFMP with their quarterly assessments involving a critical thinking rubric, and Carrie planned to share this rubric with Sandy and Ray. This is a type of assessment that they all felt would be truly useful to them in supporting student learning (and another example of how teachers’ shared ideas can be beneficial to others). Sharing this assessment would have exemplified capacity building among teachers, even if utilizing the assessment were not a policy per se.

Thus, as Carrie had expressed, the district did seem to have contradictory priorities related to assessment. The fifth grade teachers felt incredible pressure to help students to perform well on the OAA, which seemed to drive nearly everything that they did with students and with each other throughout the school year. Ray expressed additional concern in his final feedback: “At this point, our administrator wants pretests before every unit and topic and I feel we’re testing these kids to death with material they never had before. This is seriously taking up instructional time – not to mention the time needed in grading these pretests which provide us with no data.” In contrast, during the study Carrie felt that the district had begun to establish some helpful assessment practices in first grade, though she noted that these practices needed to be refined in order to be as useful as possible. A possible implication for district leaders could be that they needed to decide how assessment should fit into the overall scheme of district policy, clarify this for teachers, and then apply this policy consistently across all grades. Leithwood and
Montgomery (1982) note that when school leaders set specific goals and who actively support teachers in achieving these goals are more likely to lead their goals to comply with local and state policy as well.

**Policies That Support the SFMP**

The teachers mentioned three district or school policies that they felt did align with the goals of the SFMP. One of these, addressed by all three teachers, was Sealon’s anti-bullying policy, which the teachers believed supported the goals in SFMP 3 particularly because of the emphasis on positive communication and respect for others and others’ ideas. Carrie referenced the Rich Allen work that the district had been emphasizing; this was an external resource that she felt supported the ideas in the CCSS and SFMP, even though it was not specific to mathematics. Also, as noted above, Carrie believed that the assessments that first grade was using supported the development of the SFMP, though she did not feel that they were being utilized as effectively as possible yet. Other than these three ideas, the teachers did not suggest other school policies that, in their minds, corresponded with the goals of the SFMP. However, Huberman and Miles (1984) write that school principals must find ways to change practices within schools to align with new policies so that the new practices support these new policies. The Sealon administrators would be well advised to do so as the transition to the CCSS occurred.

**Implications of the Findings in This Study for Policy**

One of the major goals of this study was to provide the mathematics education field with a perspective on elementary teachers’ experiences in encountering new standards and how these experiences influence teachers’ interpretations in the short and
long term. Given the fact that all three teachers generally interpreted the standards uniquely and often overlooked certain words or phrases in the text of the standards, it could be helpful in professional development to divide some of the sentences in the SFMP into phrases or even just single words to “force” teachers to focus on them – thus, to providing “orienting objects” for discussion and shared learning (Brown, 2001). S. A. Courtney (personal communication, July 7, 2013) also found that pre-service middle school mathematics teachers in the same university course interpreted the SFMP quite differently as they identified them in their own problem solving, even when the standards descriptors were presented as “bullet points” rather than in complete paragraphs. However, in his study, he did not involve the pre-service teachers in discussion with each other about the SFMP after they had solved the problems. In this study, Carrie, Sandy, and Ray all explicitly commented that it was helpful for them to study the standards in this format, and it was when we began to do this (in the second interviews) that all three began to recognize (and perhaps admit) that they were unsure how to interpret particular words or phrases. That is, the teachers were able to “operate on” their own “knowing” in accord with a hermeneutical perspective (Brown, 2001). Continued conversation and reflection would also teachers to be more prepared to enact the individual elements of the SFMP in their classrooms – not just the specific portions on which they had focused, which is what I often observed in the lessons that the teachers videotaped for this study.

Carrie really did draw on her experience as a mother, watching her own children go through the district and experiencing mathematics classes, as she tried to understand mathematics instruction beyond grade 1. She pointed out that not all teachers have this
opportunity. If vertical (cross-grade) communication is not offered regularly within a school district, teachers have little opportunity to understand what does occur in other grades. Thus, it is likely that they form impressions of these grades will based on their own school experiences or perhaps sporadic comments that they might hear from time to time from adults or children associated with other grade levels. Ray’s suggestion of allowing teachers and other school staff to visit various classrooms, as well as Sandy’s and Carrie’s suggestions about cross-grade discussions, would be excellent strategies for providing teachers with more direct, personal experience (Dewey, 1966) to develop more grounded understandings of the current expectations in other grades.

Regarding professional learning, one idea that Carrie, Sandy, and Ray consistently expressed was that they needed more time to work with their grade-level colleagues on a regular basis, as has also been shown by Clarke (1997) and much research cited earlier. All three believed that this was the most important resource that administrators could provide in order to foster successful implementation of the new standards. Ray shared a comment that teachers and his principal had approached him after workshops that he attended and asked him to educate them in what he had learned, even in a very brief amount of time, as if this would make up for the fact that the others had not attended. Ray was rather taken aback by these requests because, as he argued, he could not communicate everything that he had learned in a day-long or multi-day workshop in a few minutes. Related to this issue, Ray noted in his final comments that the current administrators still did not understand the importance of ongoing collaboration within or across grade levels. So, in general, when administrators plan
professional development opportunities for teachers, they would be wise to include all teachers in all relevant experiences and make every effort themselves to participate as well (Firestone, 1980; Firestone, 1989). Further, Ray commented that he would like to see more examples of lessons and practices from teachers’ classrooms where the SFMP were already being enacted, which is indeed helpful to teachers working to change their teaching (Showers, Joyce, & Bennett, 1987). In general, the individual needs of teachers should be considered in planning professional development because (as shown in this chapter) certainly different teachers have different needs for professional learning.

On a different note: on several occasions during the study, Ray shared comments about either education research or current policies or circumstances related to mathematics education that were not entirely accurate, though he seemed to believe that they were. This situation was similar to those described by Ball (1990), Remillard (1992), and Sykes (1990), where teachers had not read new standards very carefully, if at all, and thus made assumptions about them that were inaccurate. Sometimes, Ray made these comments in the group discussions, which meant that Sandy and Carrie might have left these discussions believing these points also. Perhaps this spreading of misinformation can never really be entirely avoided, but more easily accessible sources of information, as well as ongoing, consistent professional development, could help alleviate this issue as the CCSS are implemented.

In order to effectively support teachers in their responsibilities with students, administrators must also know and understand the goals of the CCSS, including the SFMP. Leaders must be aware of whether each teacher is teaching in alignment with the
standards and, if not, how to guide the teacher and provide professional learning opportunities that will help to move this teacher in the direction of the standards in a supportive, rather than threatening, way. This suggests that professional development for administrators alone, as well as for administrators and teachers together, ought to be a vital element of a transition to new standards. Sandy again “strongly agree[d]” with this point in her ending feedback, and Ray wrote, “Yes!!!” However, Ball (1990) points out that this is often not the case, and such administrators are not even meeting the first condition for teacher change that Showers, Joyce, and Bennett (1987) suggest, which is understanding the reason for the change. Sandy and Ray often commented that they felt belittled by school leaders with regard to the OAA scores and that this overshadowed all of their planning each day and throughout the year. So, it seemed that at least in the past, the Sealon administrators had understood the standards primarily through state assessment results and responded to the standards in light of political concerns for achieving high scores, as Stein and Shields (2004) note can easily occur.

Notwithstanding the pressure related to the OAA, since Ray had taught fifth grade for many years, he felt that he was “getting better” as a teacher and appreciated being able to teach this grade long enough to become effective in it. Administrators might be advised to consider this point carefully as they assign teachers’ positions each year; experience can allow teachers to feel, and generally to be, more effective in their teaching. By the time the second interviews were conducted, the administrators at Lincoln had told the teachers that they would be teaching self-contained classrooms during the following year in order to integrate content when appropriate. This would
mean that teachers would spend only a portion of their planning and professional learning
time on mathematics and about this same amount of time in each of the other three content areas. One might question whether this structure would allow any teacher to understand the standards well enough in any of the content areas to enact them effectively. Perhaps assignments where teachers only taught two content areas might have provided more support for teachers to learn their standards and to better facilitate student learning (Fennema & Franke, 1992; Hill, Sleep, Lewis, & Ball, 2007).

Carrie also offered an interesting point about what students see as important in mathematics class when teachers post names on the wall for certain achievements (usually related to math facts). Perhaps if administrators encouraged teachers to post students’ names for success related to the SFMP, students might come to understand that these goals are also extremely vital in their learning. Brown (2001), Cohen and Ball (1990), and Davis (1996) also argue that students and teachers should be joint actors in transforming their understandings of mathematics in classrooms.

All three teachers expressed the opinion that their school administrators could be well served if they asked teachers for input more frequently about the practices and policies currently in place – particularly, what changes might be made to better support teachers and students. The idea of involving teachers in ongoing decision-making (as well as professional learning) is echoed by Bryk, Sebring, Allensworth, Luppescu, and Easton (2010) and Porter (1989).

To an extent, it is frustrating none of the teachers’ perspectives on the SFMP changed a great deal during the study, though small shifts in their thinking were
observed. This probably would only have been different if we had spent a great deal more time discussing the SFMP in connection with episodes in the teachers’ classes, as this seemed to be the primary catalyst for changes in teachers’ interpretations of the standards. (Ray’s final response to the notion of our discussion of scenarios from class being the main catalyst for change was, “Absolutely!!!”) It is clear that teachers’ interpretations of the standards can depend upon the progress of which they believe their own students are capable (Hiebert & Grouws, 2007; Philipp, 2007; Staub & Stern, 2002; Sztajn, 2003); this was particularly evident in the differences between Sandy’s comments about the standards and Carrie’s and Ray’s comments, at least early in the study. Sandy was highly unsure that her students would be able to even approach the goals in the standards in fifth grade, whereas Ray believed that his students could do so, and Carrie believed that her first grade students were already applying many of the goals in their learning. Notably, though, when Sandy made an effort to enact the SFMP in her second lesson, she seemed to recognize that these goals were within reach for her students, and she set a definite goal for herself for the following year to strive toward the SFMP herself as well as to provide opportunities for her students to do so (a result also noted in Chazan, Callis, & Lehman, 2008; Cobb, Wood, & Yackel, 1990; Guskey, 1986; Wood & Sellers, 1997). This provision of opportunities for student learning is key; students are highly unlikely to learn content or habits of mind if they do not have experiences with them (Brophy, 1999; Grouws, 2004; National Research Council, 2001).

In standards documents particularly, it would seem that the language needs to be written to be understandable for teachers while helping them to consider practices that
perhaps they have not utilized in the past (Ball, 1990; Cohen & Ball, 1990; Porter, 1989), which, admittedly, is not a simple task when teachers all have different experiences that inform their reading of such documents. Moreover, using examples within the standards forces authors and policymakers to negotiate a line between being too vague and too specific. If the language is too vague, teachers will not know what is intended; interpretation will be too individual, as has been seen in this study and in other research. If the text is too specific with certain examples provided, teachers may focus on those details and leave out many other important ideas in their interpretation process. Perhaps it would be important for authors of standards documents to include notes such as, “Many other examples related to other mathematical topics can also be applied to this statement,” though the best way for teachers to understand various applications of standards is probably still for them to actually read and discuss them frequently with colleagues and knowledgeable leaders (Darling-Hammond, 1990; Remillard, 1992), as well as to study other resources that are published in connection with the authors to elaborate the standards further. Examples of this for the CCSS and SFMP include the learning progressions documents that are being written at the University of Arizona and the example tasks at http://www.illustrativemathematics.org.

In general, policymakers would be wise to consider how education policy (standards included) should be written and enacted in order to truly affect individual classrooms and students (Darling-Hammond, 1990; Sykes, 1990). Perhaps when teachers’ jobs, or at least accountability measures, depend on their implementation of a policy, this can have an impact, as shown with Sandy and Ray. This would be considered
an “inducement” type of policy instrument (McDonnell & Elmore, 1987). However, it is important to remember, as noted earlier, that expectations for OAA scores did not change Sandy’s and Ray’s beliefs about teaching and learning, only their actions. Still, it has been shown in research cited above that when teachers alter their actions and then see students’ learning change for the better, their beliefs can begin to change, as occurred with Sandy in the latter phases of this study. It may be that an implication is that if policy “requires” teachers to change their actions in ways that truly do align with current research on how students learn mathematics, which would represent a system-changing policy (McDonnell & Elmore, 1987), teachers’ beliefs and expectations can change as they witness their students’ learning develop in ways that the teachers did not believe were possible. School leaders need to be continually supportive of teachers and focused on established goals throughout the change process (Corbett, Dawson, & Firestone, 1984; Firestone, 1989; Louis, 1981).

As a final note on the impact that this study and the enactment of the new standards had on Ray’s teaching, I share his last comment in his feedback on my tentative conclusions:

As a result of having implemented the CC[SS] and SFMP last year, I absolutely LOVE teaching math again!!!! The Standards are doable and I feel like I’m off that treadmill set at 10 (having to “cover” all the old indicators – there were way too many!!). I now teach for understanding and I’m not rushing the kids!!! We are able to share strategies and
agree/disagree agreeably! They do persevere and look for structures!! I feel so much more confident this year as my math instruction improves.

**Suggestions for Future Research**

The findings in this study raise many questions that could be considered in future studies. Any future research should consider the *context* in which teachers interpret and enact standards because as shown in this study, teachers think about standards in the contexts of their day-to-day school environments, not in isolation (Brown, 2001; Stein, 2004). One major question, as noted above, would be: How does education policy need to be written and enacted in order to positively and substantively impact individual classrooms and students? Related, what can administrators do and/or learn to best support teachers in the implementation of new policies and/or standards? What resources are most useful to teachers in understanding the intent of standards, and how are they useful? What influence does administrator understanding of standards have on teachers’ enactment of these standards? How can school leaders be aware of teachers’ instructional practices and support teachers in moving more toward practices that align with the SFMP?

In addition, each of the statements in the SFMP is very dense with potential meaning for teachers. This study could be adapted to focus on any one SFMP and the ideas within it (as an orienting object) to gain more insight into how teachers interpret and think about that standard in relation to their work in their own classrooms with students. Further, a future study could involve more work with teachers in their classrooms, including more videotaping, so that they could reflect more frequently and
deeply about how they and their students were applying the SFMP on a daily basis (Darling-Hammond, 1990; Even, 1999; Showers, Joyce, & Bennett, 1987). This could even involve exploring how students of different ages come to understand and strive toward the goals in the SFMP, particularly in relation to teachers’ thinking and expectations. Moreover, this study, or an adaptation of it, could be replicated with more teachers in other schools to study other local contexts (Hill, 2001; McDonnell & Elmore, 1987; Porter & Smithson, 2001) and eventually adapted to include much larger groups of teachers so that more information could be gained about how teachers in general respond to standards and what helps them to enact standards as intended, a need also suggested by Clune (1991) and Stein (2004).

Further, it might be interesting to assess Carrie’s, Sandy’s and Ray’s long-term memory of each of the three SFMP that we studied – given that at the time of this writing, a full school year has passed since our last interviews – and examine what ideas and practices they now associate with each standard in their own minds. Certainly, this research would also involve learning what professional experiences they had had during the year that might have affected their perspectives on the standards. This might help to provide information about how teachers’ interpretations of standards evolve over time depending on the kinds of learning opportunities in which they have participated. Cohen and Ball (1990) and Remillard (1992) write that professional learning must be ongoing in order to support genuine, lasting change in classrooms, which also reflects the notion that interpretation is never fully concluded (Gallagher, 1992).
Finally, as all of the teachers in this study felt that professional collaboration within (and among) their grade levels was so important in enacting standards effectively, future research should explore how this kind of ongoing collaborative work influences teachers’ actions in their classrooms and, ultimately, how it affects student learning. Cobb and McClain (2006), Chazan, Callis, and Lehman (2008) and many others have reported their findings in this area, and with the CCSS becoming new policy for many teachers in 2013, more research will be useful to current leaders and policymakers.

Limitations of the Findings in This Study

Certainly, since this study only involved three teachers, the findings are not easily generalizable to a larger population of teachers (Hatch, 2002). The reader now has a sense of how Carrie, Sandy, and Ray interpreted the SFMP and the goals of the CCSS during their first encounters with these standards over a seven-month period. Still, it is noteworthy that all three teachers had rather unique interpretations of the text, even though they worked in the same district and though two of them taught the same grade, across the hall from each other. This is one example of how even teachers in very similar professional situations may think about standards quite differently, so it is reasonable to conjecture that teachers in the general population would also have many different thoughts about the SFMP.

Since there were words and phrases in the SFMP that each teacher seemed to avoid – or that he/she simply was unable to interpret due to unfamiliarity – I would like to have returned to the teachers to strictly focus on these words and phrases. Perhaps
their understanding of the standards would have grown, and I might have gathered more
information about how teachers think about text that is new to them.

When Ray and Sandy videotaped their lessons, they each set up the videocamera
and simply let it record, except in Sandy’s second lesson when she carried it with her as
she talked to each pair of students. For Carrie’s lessons, she asked me to come to her
classroom and videotape for her. In all of these cases, there was a limit to what one
videocamera could capture in terms of the teacher’s actions and students’ experiences and
dialogue. Sometimes, particularly in Ray’s and Sandy’s videos, the person speaking was
not shown on camera, and some students were never on camera. So, although
videotaping is very useful for helping teachers to reflect on lessons, it cannot reveal
everything about the experiences of the students and adults involved in the lessons (Hall,
2000).

Although I asked each teacher for very brief responses to two written questions
after each interview, group discussion, and lesson, I only received some of these from
Carrie and Sandy even though I repeated my requests throughout the study. Ray did
provide most of his reflections promptly. This lack of response was also true for the
summaries of the teachers’ comments in interviews and group discussions that I sent to
them for feedback. I do feel, however, that since the teachers’ comments were relatively
consistent throughout the study, any additional thoughts shared in writing would also
have been similar, but it still would have been interesting to see how they expressed these
thoughts in reflections as compared or contrasted with the ways in which they shared
them in conversation.
Related to the issue above, it was rather frustrating that I was not able to conduct Sandy’s last interview in order to explore how she felt her interpretations had evolved throughout the study and especially after the second lesson. Unfortunately, though, she had been struggling with health issues and underwent major surgery during the time that I was conducting Ray’s and Carrie’s third interviews. I contacted Sandy multiple times by e-mail in the months following the surgery, assuring her that I did not want to cause her any unnecessary stress, but saying that I would like to follow up with her in whatever way she felt was most convenient. Although I did receive one e-mail from her early in the school year in which she said that she would like to complete her part in the study, I did not receive any other replies from her to subsequent messages that I sent. The last message I wrote to her before I requested final feedback was in the late spring of the school year (following the study), so I determined at that point that it was probably too late to pursue any additional discussion with her. Ultimately, I do believe that I learned enough about Sandy and her interpretations in the six phases of the study that we completed to communicate these ideas adequately in this report because my observations seemed to be consistent with her own observations about how her thinking changed during the study.

Finally, as the researcher in this study, I have designed the study, conducted my analysis, and written this report from my own perspective. Although I made many efforts to gather data in different ways over seven months and to examine this data with the perspectives of the participants in mind (Van Manen, 1990), it would not be possible for me to entirely understand or convey the thinking of the teachers in the study at any given
moment in time, let alone over a period of time. Thus, I feel it is important for me to discuss my own experience as the researcher in the study, which I do in the section below.

**Researcher’s Own Experiences in This Study**

In conducting this study, I have thought very much about how I interpret each statement in the SFMP, and how my own interpretations have evolved over time, because I recognize that this influences how I view the participants’ thinking, as well as how I compare and contrast their comments with each other. Literature on interpretive research (Davis, 1996; Feinberg & Soltis, 1992; Hesse-Biber & Leavy, 2006) suggests that a researcher in this type of interpretive study must constantly reflect on his/her own perspectives on the research questions and the text. I realize that I cannot simply consider my own interpretations of the SFMP ideals by which I judge the teachers’ interpretations, nor can I assume that I understand the authors’ intent for each standard. (On the other hand, we should also not assume that the authors’ intent is the “correct” interpretation of each standard for every classroom, teacher, and student.) Another type of related issue was simply when I felt that I disagreed with a teacher’s opinion – for instance, when Sandy expressed her strong doubts that the SFMP were achievable in fifth grade, when Ray said that Singapore math seemed well suited to meeting the goals of the CCSS, or when Carrie shared that she felt her practice already was very well aligned with the SFMP. In each case, I simply acknowledged the comment, often smiled, asked a follow-up question if appropriate, and then moved on in the conversation. I cannot be sure whether the teachers realized when I disagreed with them or not, but they never
indicated this if they did. In order to create a record of my thoughts and questions about the data over time, I have maintained a daily log of my activities for the study; this log has doubled as a journal for reflection on the questions that I listed in chapter 3 for monitoring my own thinking and how it might be affecting my analysis of the data (Silvers, 1982a; Silvers, 1982b). This log is included as Appendix J.

In addition, in watching and analyzing the classroom videos for the three teachers, I worked to avoid being overly critical of the teachers’ actions in general, but I was trying to watch very carefully for when the teacher, as contrasted with individual students, as contrasted with all of the students were developing the goals in the SFMP. These actions were related to, but still separate from, teachers’ interpretations; a teacher might be well able to explain what it means for students to enact one of the SFMP and then not facilitate this well in the classroom, as I particularly observed in Carrie’s and Ray’s lessons.

As I am a mathematics specialist in a district near Sealon, working with teachers in kindergarten through high school on a daily basis, I often have found myself thinking about the text and goals of the SFMP in relation to our own teachers and students and how I wished to go about fostering their understanding of the standards. One point that I questioned (with some hesitation), for example, was whether all, or even most, students are truly capable of applying the SFMP in each grade in age-appropriate ways. This question arose not only due to what the teachers were saying but also due to my own work with students. However, for now, I do believe that all students can apply the SFMP in one way or another in each grade; this might simply look a bit different from one
student to another depending on the sophistication in students’ thinking. I also
recognized, for instance, that the study of fractions allows for SFMP 1, 3, and 7 to be
quite effectively enacted simultaneously, and this is not something that I had considered
at all until I watched and reflected on Carrie’s second lesson. Further, I have often
thought that as our district transitions to the new CCSS, I will need to become very
familiar with the content and SFMP in all grade levels, which I will need to do over time
and which I often do as I help to develop the curriculum maps for each grade and course.

I have also thought about our district’s own implementation of the CCSS and
SFMP while I have conducted this study. I have reflected on our classroom practices, our
local formative and summative assessments and how we can collect and use data
efficiently and beneficially, how we develop both number sense and fact fluency in
students, how our teachers respond to requirements to record and submit assessment data,
how we can support parents as they work with students at home, how to help students
stay engaged in mathematics in the upper grades when some of the content may not seem
very meaningful to them, and how we help students of all ages to feel confident,
successful, and excited in math class. Although I am not an administrator per se, I do
work in tandem with our administrators at the school and district level to create and enact
local policy, so the implications for policy that I described above are very much in my
thoughts as I consider future directions for the mathematics program in our district. In
conversation with the teachers in the study, I did offer a comment from time to time
about the work in our district, but I intentionally limited myself in doing so because I did
not wish for my thoughts to overly influence what the participants shared.
Conclusions

The exploration of three teachers’ thinking about the Common Core State Standards and Standards for Mathematical Practice in this study has demonstrated that individual teachers, even those working together closely, may interpret the text and intent of standards in very individual ways. These interpretations are influenced by previous personal and professional experiences, opportunities to read and discuss standards with other professionals, expectations set forth as well as support provided by administrators, and both past and current observations of students in teachers’ own classrooms. Interpretations do not seem to change quickly or without catalyst; rather, thinking evolves over extended periods of time when opportunities for professional learning and reflection are provided on a regular basis. Thus, policymakers and school leaders cannot simply present teachers with new standards and assume that teachers will enact the standards as intended. Instead, leaders must work together with teachers to establish expectations and ongoing support for standards implementation, and these efforts must be based on quality research on teaching and learning and a focus on growth for all teachers and students.
APPENDIX A

LETTER OF RECRUITMENT AND CONSENT FORMS
Appendix A

Letter of recruitment and consent forms

Informed Consent to Participate in a Research Study

**Study Title:** Elementary Teachers’ Evolving Interpretations of the Standards for Mathematical Practice in the Common Core State Standards: A Multi-Case Study

**Principal Investigator:** Kimberly Yoak

You are being invited to participate in a research study. This consent form will provide you with information on the research project, what you will need to do, and the associated risks and benefits of the research. Your participation is voluntary. Please read this form carefully. It is important that you ask questions and fully understand the research in order to make an informed decision. You will receive a copy of this document to take with you.

**Purpose**

The purpose of this study is to explore the interpretations that elementary teachers form of the overall goals of the Common Core State Standards in Mathematics and three of the Standards for Mathematical Practice. The study will attempt to determine how these interpretations may change over time with individual and collaborative reflection on the standards.

**Procedures**

Each participant will have three hour-long individual interviews with the researcher, participate in two hour-long group conversations with the other participants, and videotape
and reflect (individually) on two lessons in his/her own classroom. Each participant will also be asked to write a short reflection on his/her current interpretations of the Standards for Mathematical Practice after each interview, video, and group conversation. All data collection is planned for January through July 2012. The length of time for potential contact with the researcher will be at most two years, since follow-up may be needed after data collection is complete.

**Audio and Video Recording and Photography**

All interviews and group conversations will be videotaped, and the classroom lessons will be videotaped. No one but the researcher will ever see these tapes, and the tapes will be used for transcribing dialogue, as well as for providing a visual record for the researcher of the interaction during each conversation. All video/audio data will be deleted once the study is complete. Each participant may view the video recordings of his/her own interviews and the collaborative discussions upon request, until the recordings are deleted.

**Benefits**

Participants in this study may gain a clearer understanding of the Common Core State Standards in Mathematics, which could allow them to better enact these standards in their classrooms and to support their school colleagues in doing so (thus, students may benefit as well). The educational system at large will gain knowledge regarding the interpretations that educators form about the standards and potentially how to better design professional learning opportunities to support teachers as they enact the standards.
**Risks and Discomforts**

In participating in this study, there are no anticipated risks beyond those encountered in everyday life. However, some of the questions that you will be asked are related to your perceptions about practices at your school, and it is possible that you may feel uncomfortable answering them. You may ask to see the questions before deciding whether or not to participate in the study. If you do not wish to answer a question during the interview, you may skip it and go on to the next question.

**Privacy and Confidentiality**

Data collection will be confidential. Your signed consent form will be kept separate from your study data, and responses will not be linked to you by name. All participants, schools, and school districts will be assigned pseudonyms. However, you will be involved in two group conversations with other participants, so all participants will know who the other participants are. In signing this consent form, you are also agreeing to keep all identifying information and comments of the other participants confidential.

Your study-related information, i.e., the video recordings and any documents that you share, will be kept confidential within the limits of the law. The video recordings will be deleted once the study is complete, and documents will be shredded or deleted electronically. Any identifying information will be kept in a secure location at Kent State University, and only the researcher will have access to the data. Research participants will not be identified in any publication or presentation of research results; only pseudonyms and aggregate data will be used.
Your research information may, in certain circumstances, be disclosed to the Institutional Review Board (IRB), which oversees research at Kent State University, or to certain federal agencies. Confidentiality may not be maintained if you indicate that you may do harm to yourself or others.

**Compensation**

You will receive a $50 Amazon gift card at the completion of the third interview (the last phase of data collection in this study). Also, the researcher has worked with your school district to allow you to earn up to 1 Continuing Education Unit (CEU) for your participation in this study, if you choose to apply for it.

**Voluntary Participation**

Taking part in this research study is entirely up to you. You may choose not to participate or you may discontinue your participation at any time without penalty or loss of benefits to which you are otherwise entitled. You will be informed of any new, relevant information that may affect your health, welfare, or willingness to continue your study participation.

**Contact Information**

If you have any questions or concerns about this research, you may contact Dr. Michael Mikusa at (330) 672-0647. This project has been approved by the Kent State University Institutional Review Board. If you have any questions about your rights as a research participant or complaints about the research, you may call the IRB at 330-672-2704.
**Consent Statement and Signature**

I have read this consent form and have had the opportunity to have my questions answered to my satisfaction. I voluntarily agree to participate in this study and understand that I can withdraw at any time. I understand that a copy of this consent will be provided to me for future reference.

________________________________  ______________________
Participant Signature              Date

(Please see next page for video recording consent form.)
Video Recording Consent

Study Title: Elementary Teachers’ Evolving Interpretations of the Standards for Mathematical Practice in the Common Core State Standards: A Multi-Case Study

Principal Investigator: Kimberly Yoak

I agree to participate in three videotaped interviews, two videotaped group conversations with the other participants, and two videotaped lessons in my own classroom, all involving my interpretations of the Standards for Mathematical Practice, as part of this project and for the purposes of data analysis. I agree that Kimberly Yoak may videotape these conversations and that she may use all of the videos for data analysis. The date, time, and place of the interview will be mutually agreed upon.

__________________________________________  ________________________________
Signature                                           Date

I have been told that I have the right to watch any of these videotapes before they are used. I have decided that I:

_____want to watch the recordings  _____do not want to watch the recordings

Sign now below if you do not want to watch the recordings. If you want to watch the recordings, you will be asked to sign after watching them.
Kimberly Yoak  may / may not  (circle one) use the videotapes made of me. The original tapes or copies may be used for (please check all that apply):

____this research project  ____ publication  ____ presentation at professional meetings

___________________________________________________

Signature                               Date

Address:
APPENDIX B

PAGES 3 THROUGH 8 OF THE COMMON CORE STATE STANDARDS:
OVERALL GOALS AND THE STANDARDS FOR MATHEMATICAL PRACTICE
Appendix B

Pages 3 through 8 of the Common Core State Standards:

Overall goals and the Standards for Mathematical Practice

(beginning on following page)
Introduction

Toward greater focus and coherence

Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics. Mathematical process goals should be integrated in these content areas.

— National Research Council, 2009

The composite standards (of Hong Kong, Korea and Singapore) have a number of features that can inform an international benchmarking process for the development of K-6 mathematics standards in the U.S. First, the composite standards concentrate the early learning of mathematics on the number, measurement, and geometry strands with less emphasis on data analysis and little exposure to algebra. The Hong Kong standards for grades 1-3 devote approximately half the targeted time to numbers and almost all the time remaining to geometry and measurement.

— Ginsburg, Leinwand and Dossey, 2009

Because the mathematics concepts in [U.S.] textbooks are often weak, the presentation becomes more mechanical than is ideal. We looked at both traditional and non-traditional textbooks used in the US and found this conceptual weakness in both.

— Ginsburg et al., 2005

There are many ways to organize curricula. The challenge, now rarely met, is to avoid those that distort mathematics and turn off students.

— Steen, 2007

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is “a mile wide and an inch deep.” These Standards are a substantial answer to that challenge.

It is important to recognize that “fewer standards” are no substitute for focused standards. Achieving “fewer standards” would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity.

Assessing the coherence of a set of standards is more difficult than assessing their focus. William Schmidt and Richard Houang (2002) have said that content standards and curriculum are coherent if they are:

- articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, but also the key ideas that determine how knowledge is organized and generated within that discipline. This implies...
that to be coherent, a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (such as an understanding of the rational number system and its properties). (emphasis added)

These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas.

In addition, the “sequence of topics and performances” that is outlined in a body of mathematics standards must also respect what is known about how students learn. As Carpenter (2007) points out, developing “sequenced obstacles and challenges for students [. . .] absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise.” In recognition of this, the development of these Standards began with research-based learning progressions detailing what is known today about how students’ mathematical knowledge, skill, and understanding develop over time.

Understanding mathematics

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

The Standards begin on page 6 with eight Standards for Mathematical Practice.

Figure 2. Page 4 of the Common Core State Standards in Mathematics.
How to read the grade level standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.

These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally, then, each standard in this document might have been phrased in the form, “Students who already know ... should next come to learn ...” But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity, therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers, and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children but promises we intend to keep.

Figure 3. Page 5 of the Common Core State Standards in Mathematics.
Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,
communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see \(7 \times 8\) equals the well remembered \(7 \times 5 + 7 \times 3\), in preparation for learning about the distributive property. In the expression \(x^2 + 9x + 14\), older students can see the 14 as \(2 \times 7\) and the 9 as \(2 + 7\). They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see \(5 - (x - y)^2\) as \(5\) minus a positive number times a square and use that to realize that its value cannot be more than \(5\) for any real numbers \(x\) and \(y\).

8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing \(26\) by \(11\) that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through \((1, 2)\) with slope \(3\), middle school students might abstract the equation \((y - 2) = 3(x - 1)\) Notice the regularity in the way terms cancel when expanding \((x - 1)(x^2 + x + 1)\) and \((x - 1)(x^3 + x + 1)\) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. Those points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Figure 6. Page 8 of the Common Core State Standards in Mathematics.
APPENDIX C

COMMON CORE STATE STANDARDS:

FIRST GRADE AND FIFTH GRADE
Appendix C

Common Core State Standards:

First grade and fifth grade

(beginning on following page)
Mathematics | Grade 1

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of end solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.1

(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

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1Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Figure 7. Page 13 of the Common Core State Standards in Mathematics.
Grade 1 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.

Number and Operations in Base Ten

- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure lengths indirectly and by iterating length units.
- Tell and write time.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

3. Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)

4. Understand subtraction as an unknown-addend problem. For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.

Add and subtract within 20.

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

8. Determine the unknown whole number in an addition or subtraction equation relating to two whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? - 3, 6 + ? = 8.

Number and Operations in Base Ten

Extend the counting sequence.

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
   a. 10 can be thought of as a bundle of ten ones — called a "ten."
   b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

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Figure 9. Page 15 of the Common Core State Standards in Mathematics.
3. Compare two two-digit numbers based on meanings of the tens and ones
digits, recording the results of comparisons with the symbols >, =, and <.

**Use place value understanding and properties of operations to add**
**and subtract.**

4. Add within 100, including adding a two-digit number and a one-digit
number, and adding a two-digit number and a multiple of 10, using concrete
models or drawings and strategies based on place value, properties of
operations, and/or the relationship between addition and subtraction;
relate the strategy to a written method and explain the reasoning used.
Understand that in adding two-digit numbers, one adds tens and tens, ones
and ones; and sometimes it is necessary to compose a ten.

5. Given a two-digit number, mentally find 10 more or 10 less than the
number, without having to count; explain the reasoning used.

6. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the
range 10–90 (positive or zero differences), using concrete models or
drawings and strategies based on place value, properties of operations,
and/or the relationship between addition and subtraction; relate the
strategy to a written method and explain the reasoning used.

### Measurement and Data 1.MD

#### Measure lengths indirectly and by iterating length units.

1. Order three objects by length; compare the lengths of two objects
indirectly by using a third object.

2. Express the length of an object as a whole number of length units, by
laying multiple copies of a shorter object (the length unit) end to end;
understand that the length measurement of an object is the number
of same-size length units that span it with no gaps or overlaps. Limit to
contexts where the object being measured is spanned by a whole number
of length units with no gaps or overlaps.

#### Tell and write time.

3. Tell and write time in hours and half-hours using analog and digital
clocks.

#### Represent and interpret data.

4. Organize, represent, and interpret data with up to three categories; ask
and answer questions about the total number of data points, how many
in each category, and how many more or less are in one category than in
another.

### Geometry 1.G

#### Reason with shapes and their attributes.

1. Distinguish between defining attributes (e.g., triangles are closed and
three-sided) versus non-defining attributes (e.g., color, orientation,
overall size); build and draw shapes to possess defining attributes.

2. Compose two-dimensional shapes (rectangles, squares, trapezoids,
triangles, half-circles, and quarter-circles) or combine two-dimensional
shapes to create a composite shape, and compose new shapes from
the composite shape.¹

3. Partition circles and rectangles into two and four equal shares, describe
the shares using the words halves, fourths, and quarters, and use the
phrases half of, fourth of, and quarter of. Describe the whole as two of
or four of the shares. Understand for these examples that decomposing
into more equal shares creates smaller shares.

¹Students do not need to learn formal names such as “right rectangular prism.”

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*Figure 10. Page 16 of the Common Core State Standards in Mathematics.*
Mathematics | Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real-world and mathematical problems.
Grade 5 Overview

Operations and Algebraic Thinking
• Write and interpret numerical expressions.
• Analyze patterns and relationships.

Number and Operations in Base Ten
• Understand the place value system.
• Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions
• Use equivalent fractions as a strategy to add and subtract fractions.
• Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data
• Convert like measurement units within a given measurement system.
• Represent and interpret data.
• Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry
• Graph points on the coordinate plane to solve real-world and mathematical problems.
• Classify two-dimensional figures into categories based on their properties.

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Figure 12. Page 34 of the Common Core State Standards in Mathematics.
Operations and Algebraic Thinking 5.OA

**Write and interpret numerical expressions.**

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as 2 × (8 + 7). Recognize that 3 × (8 × 5) is three times as large as 8 × 5, without having to calculate the indicated sum or product.

**Analyze patterns and relationships.**

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Number and Operations in Base Ten 5.NBT

**Understand the place value system.**

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

3. Read, write, and compare decimals to thousandths.

   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.925 = 3 × 100 + 4 × 10 + 7 × 1 + 9 × (1/10) + 2 × (1/100) + 5 × (1/1000).

   b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

4. Use place value understanding to round decimals to any place.

**Perform operations with multi-digit whole numbers and with decimals to hundredths.**

5. Fluently multiply multi-digit whole numbers using the standard algorithm.

6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

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Figure 13. Page 35 of the Common Core State Standards in Mathematics.
Number and Operations—Fractions

**5.NF**

**Use equivalent fractions as a strategy to add and subtract fractions.**

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2/3 + 3/5 = 10/15 + 9/15 = 19/15$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}$.)

2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.

**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3/4$ as the result of dividing $3$ by $4$, noting that $3/4$ multiplied by $4$ equals $3$, and that when $3$ wholes are shared equally among $4$ people each person has a share of size $3/4$. If $9$ people want to share a $50$-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

   a. Interpret the product $(a/b) \times q$ as a part of a partition of $q$ into $b$ equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) = 4/6$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)

   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5. Interpret multiplication as scaling (resizing), by.

   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

   b. Explaining why multiplying a given number by a fraction greater than $1$ results in a product greater than the given number (recognizing multiplication as scaling by whole numbers greater than $1$ as a familiar case); explaining why multiplying a given number by a fraction less than $1$ results in a product smaller than the given number, and relating the principle of fraction equivalence $a/b = (n\times a)/(n\times b)$ to the effect of multiplying a $a/b$ by $1$.

6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

   a. Interpret division of a unit fraction by a non-zero whole number.

*Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.*

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**Figure 14.** Page 36 of the Common Core State Standards in Mathematics.
and compute such quotients. For example, create a story context for \( \frac{1}{3} \div \frac{1}{2} \), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \( \frac{1}{3} \div \frac{1}{2} = \frac{2}{3} \) because \( \frac{1}{2} \times \frac{2}{3} = \frac{1}{3} \).

b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for \( 4 \div \frac{1}{2} \), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \( 4 \div \frac{1}{2} = 8 \) because \( 8 \times \frac{1}{2} = 4 \).

c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 5 people share \( \frac{1}{2} \) lb of chocolate equally? How many \( \frac{1}{2} \)-cup servings are in 3 cups of raisins?

**Measurement and Data**

<table>
<thead>
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<th>E.MD</th>
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**Convert like measurement units within a given measurement system.**

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.

**Represent and Interpret data.**

2. Make a line plot to display a data set of measurements in fractions of a unit (\( \frac{1}{2} \), \( \frac{1}{4} \), \( \frac{1}{8} \)). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers was redistributed equally.

**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
   a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
   b. A solid figure which can be packed without gaps or overlaps using \( n \) unit cubes is said to have a volume of \( n \) cubic units.

4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5. Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.
   a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
   b. Apply the formulas \( V = l \times w \times h \) and \( V = b \times h \) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.
   c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

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**Figure 15.** Page 37 of the Common Core State Standards in Mathematics.
**Geometry 5.G**

**Graph points on the coordinate plane to solve real-world and mathematical problems.**

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

2. Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpreting coordinate values of points in the context of the situation.

**Classify two-dimensional figures into categories based on their properties.**

3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

4. Classify two-dimensional figures in a hierarchy based on properties.

*Figure 16. Page 38 of the Common Core State Standards in Mathematics.*
APPENDIX D

INTERVIEW QUESTIONS
Appendix D

Interview questions

First interview (beginning of study)

What is your current position in education? How long have you served in this position?

What other positions have you held, and for how long?

Have you read and/or discussed pages 3-8 in the CCSS (or any part of these pages) or your new grade level standards prior to this research? If so, in what contexts did this reading/discussion take place?

Have you read or discussed other parts of the CCSS? If so, in what contexts?

What do you feel your role(s) will be in the transition to, and implementation of, the CCSS? What leads you to believe this?

What do you see as the overall idea expressed on page 3? What leads you to believe this?

(provide specific words, phrases, etc. as needed)

How do you interpret the first full paragraph on page 4? (starting with “These standards…”)

How do you interpret the description of “mathematical understanding” in the first paragraph under “Understanding mathematics”? (page 4)

What is your reaction to the next paragraph on page 4, regarding standards for a diverse population of learners?

From the references to “learning progressions” on pages 4-5, what is your interpretation of this phrase and how it applies to teaching/learning?
At this point in each interview, refer to the list of statements from SFMP 1, 3, and 7 presented in Appendix E, and ask the following questions about each. What do you think this portion of this Standard for Mathematical Practice means for students at your grade level? Can you give examples of what you might see students doing? Consider your new grade level content as you respond to this question.

How do you envision SFMP 1 playing out in your classroom and/or the school in which you work, if it is implemented effectively, based on your understanding of it? [ask same questions for SFMP 3 and 7]

At this point in each interview, refer to two of the hypothetical classroom episodes presented in Appendix F, so as to pose all six throughout the three interviews. A teacher at your grade level encounters the following situation in his/her classroom. What would you suggest that he/she do next, in light of SFMP 1, 3, and/or 7?

What do you believe are characteristics of a classroom where the goals on pages 3-5 and in SFMP 1, 3, and 7 can be effectively accomplished for most or all students? [ask same question about a school and a district]

In your setting, what resources could be provided internally to support the accomplishment of these goals? What resources do you believe will be provided internally? What leads you to believe this?

In your setting, what resources would need to be brought in from external sources to support the accomplishment of these goals? What resources do you believe will be brought in? What leads you to believe this?
How, if at all, do you believe your practice will change as a result of what you have read in these standards? Why?

Do you feel that the standards are compelling enough to catalyze substantial change in your classroom and/or school?

Are there other policies that your school follows that will correspond well with the standards? Are there policies that might conflict with the ideas in the standards? If so, how do you believe this conflict will be addressed?

Do you believe the goals on pages 3-5 and in SFMP 1, 3, and 7 are worthwhile for students? Do you believe they are achievable for most or all students? Are there other goals that are equally or more worthwhile and/or achievable?

What do you believe influences the way you read, think about, and interpret these standards?

Is there anything you would like to share that we have not yet discussed?

Please suggest one to two questions related to SFMP 1, 3, and 7 that could be used in the upcoming discussion with the other participants that would help to develop your understanding of these standards.

Second interview

I will ask the same questions as in the first interview (other than the first demographic questions), and the questions below about the classroom videos will be added.

The second and third questions will be worded in this way:

Have you read and/or discussed pages 3-8 in the CCSS (or any part of these pages) or your new grade level standards outside of this research? If so, in what contexts did this
reading/discussion take place, and how has this affected your thinking about the
standards?

Have you read or discussed other parts of the CCSS outside of this research? If so, in
what contexts, and how has this affected your thinking about the standards?

*The following questions will be added:*

In your videotaped lesson, how did you feel that the SFMP that you chose was reflected?

After watching the lesson myself, I want to ask you about the following scenario during
the lesson: [I will choose a portion of the lesson that the teacher did not discuss]. How
did you feel that the SFMP that you chose was reflected in this part of the lesson? What
about SFMP number ___? [I will choose a different SFMP than the one on which the
teacher chose to focus.]

**Third interview (end of study)**

*I will ask the same questions as in the second interview. After these questions, I will ask:*

How do you feel that the work in this study has affected the way that you now read, think
about, and interpret these standards? Try to be specific to the different types of work that
you did.

In what ways, and to what extent, do you feel that the work in this study was useful? Try
to be specific to the different types of work that you did.

Is there anything that you would change about the work in this study, in retrospect?

How could you share your learning from this study with colleagues? Do you plan to do
this?
If you were asked to make recommendations for professional development involving the enactment of the SFMP (for teachers in your setting), what would you suggest?

Is there anything you would like to share that we have not yet discussed?
APPENDIX E

STATEMENTS FROM THE STANDARDS FOR MATHEMATICAL PRACTICE
INCLUDED IN INTERVIEWS
Appendix E

Statements from the Standards for Mathematical Practice included in interviews

For each statement below, ask: *What do you think this portion of this Standard for Mathematical Practice means for students at your grade level? Can you give examples of what you might see students doing? Consider your new grade level content as you respond to this question.*

SFMP 1: “Make sense of problems and persevere in solving them.” (p. 6)

“Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.” (p. 6)

“They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.” (p. 6)

“Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.” (p. 6)

“Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, ‘Does this make sense?’” (p. 6)

“They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.” (p. 6)
SFMP 3: “Construct viable arguments and critique the reasoning of others.” (p. 6)

“Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures.” (p. 6)

“They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples.” (p. 6)

“They justify their conclusions, communicate them to others, and respond to the arguments of others.” (p. 6-7)

“Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is.” (p. 7)

“Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.” (p. 7)

SFMP 7: “Look for and make use of structure.” (p. 8)

“Mathematically proficient students look closely to discern a pattern or structure.” (p. 8)

“Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have.” (p. 8)

[grade 5 only] “Later, students will see 7 x 8 equals the well remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property.” (p. 8)

“They also can step back for an overview and shift perspective.” (p. 8)
“They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.” (p. 8)
APPENDIX F

HYPOTHETICAL CLASSROOM EPISODES USED DURING INTERVIEWS
Appendix F

Hypothetical classroom episodes used during interviews
(two episodes during each interview, in the sequence listed)

For each episode, begin the interview prompt with: A teacher encounters the following situation in his/her classroom. What would you suggest that he/she do next, in light of SFMP 1, 3, and/or 7?

Grade 1 (all situations are assumed to be in first-grade classrooms)

Responding to student difficulties with a particular concept

Several students in Miss Johnson’s class are convinced that when a square is rotated so that the base is not parallel to the horizontal, the shape is no longer a square. [idea taken from “common misconception” presented in grade 1 Ohio Model Curriculum document, p. 22; aligned with CCSS; aligned with CCSS 1.G.1]

Responding to students’ novel ideas regarding a particular concept

Mrs. Miller has asked her students to come up with many different ways to represent the number 43. They are able to use a variety of manipulatives as well as pictures, numbers, and words. One student draws a picture of a person and says that this is his 43-year-old uncle. [idea taken from “instructional strategy” presented in grade 1 Ohio Model Curriculum document, p. 12; aligned with CCSS 1.NBT.2]

Generating representations of concepts

In Mr. Wagner’s class, students are learning to add multiples of 10 to two-digit numbers. He realizes that some of his students are having trouble thinking about and using the
larger numbers with which they are working when they just see the numbers written with numerals in the standard way. [idea taken from “instructional strategy” presented in grade 1 Ohio Model Curriculum document, p. 14; aligned with CCSS 1.NBT.4]

Responding to students’ questions

The students in Mrs. Smith’s class have written number sentences to represent this problem: “Suzie had 10 cookies, and she ate 3 of them. How many cookies does Suzie have left?” Several students have written $3 - 10 = 7$, and they do not understand why other students are saying this does not mean the same thing as $10 - 3 = 7$. [idea taken from “common misconception” presented in grade 1 Ohio Model Curriculum document, p. 5; aligned with CCSS 1.OA.3]

Helping students to learn concepts and procedures

Miss Thomas’ class is learning to count past 100, to 120. Miss Thomas wants to be sure that her students understand the number system, but she is a first-year teacher and is not sure how to best support her students in this learning. [idea taken from “instructional strategy” presented in grade 1 Ohio Model Curriculum document, p. 10; aligned with CCSS 1.NBT.1]

Evaluating student work

Mr. Jones has asked his students to use popsicle sticks to measure the longest side of one of the tables at which they sit in class (all of these tables are the same shape and size). Some students lay the popsicle sticks end to end along the length of the table, while others leave gaps in between their sticks, and one pair of students lays pairs of popsicle
sticks end to end. [idea taken from “common misconception” presented in grade 1 Ohio Model Curriculum document, p. 17; aligned with CCSS 1.MD.2]

**Grade 5**

*Responding to student difficulties with a particular concept*

Ms. Brockton assigned the following problem to her students: “How many 4s are there in 3?” Several students said that this question could not be answered because 4 was larger than 3. [task, excluding the final sentence, taken from Mathematical Knowledge for Teaching Measures, Mathematics Released Items, 2008, p. 18; aligned with CCSS 5.NF.3]

*Responding to students’ novel ideas regarding a particular concept*

Mr. Powell is facilitating a series of activities related to volume with his class. As students are filling various containers with centimeter linking cubes, two students come to him and say that they have decided to link pairs of cubes together to fill each shape, which means all of the volumes are different than what the students are finding. [idea taken from “instructional strategies” presented in grade 5 Ohio Model Curriculum document, p. 18; aligned with CCSS 5.MD.4]

*Generating representations of concepts*

After a lesson on fraction addition, Mrs. Winters gives her students a short exit ticket, in which they are asked to add 2/3 and 4/7. Several students write that the answer is 6/10. The next day, Mrs. Winters would like to facilitate a discussion on how students might prove or disprove this answer. [idea taken from “instructional strategies” presented in grade 5 Ohio Model Curriculum document, p. 10; aligned with CCSS 5.NF.2]
Responding to students’ questions

In Mrs. Nelson’s class, students are learning to classify geometric shapes in multiple ways. Students have questions such as, “Why is a rectangle a parallelogram?” and “Why is this rhombus not a square?” [idea taken from “instructional strategies” presented in grade 5 Ohio Model Curriculum document, p. 22; aligned with CCSS 5.G.3]

Helping students to learn concepts and procedures

The students in Miss Taylor’s class are learning to divide four-digit numbers by two-digit numbers. Miss Taylor wants students to have at least two meaningful strategies for this computation, but all she remembers from school is the typical algorithm used in the United States. [idea taken from PARCC Model Content Frameworks draft document, 2011, p. 13 in “Grade by Grade Standards Analyses”; aligned with CCSS 5.NBT.6]

Evaluating student work

Students in Mr. Hayes’ class have been working on putting decimals in order. Three students – Andy, Clara, and Keisha – presented 1.1, 12, 48, 102, 31.3, .676 as decimals ordered from least to greatest. [task, excluding the final sentence, taken from Mathematical Knowledge for Teaching Measures, Mathematics Released Items, 2008, p. 10; aligned with CCSS 5.NBT.3]
APPENDIX G

GUIDING QUESTIONS FOR PARTICIPANTS’ WRITTEN REFLECTIONS
Appendix G

Guiding questions for participants’ written reflections

*After the first interview:*

1) In general, what stands out for you in SFMP 1, 3, and/or 7 right now?

2) Do you feel that any of your interpretations of SFMP 1, 3, and/or 7 changed during the interview or between the interview and this reflection? If so, describe what changed and why you believe it changed.

*After the first collaborative discussion:*

1) In general, what stands out for you in SFMP 1, 3, and/or 7 right now?

2) Do you feel that any of your interpretations of SFMP 1, 3, and/or 7 have changed since the first interview? If so, describe what changed and why you believe it changed.

*After the first classroom video:*

1) What Standard for Mathematical Practice did you choose to focus on in this lesson?

2) How do you see this Standard (or other goals) appearing in the lesson?

3) Do you feel that any of your interpretations of SFMP 1, 3, and/or 7 have changed since the first collaborative discussion? If so, describe what changed and why you believe it changed.

*After the second interview:*

1) In general, what stands out for you in SFMP 1, 3, and/or 7 right now?

2) Do you feel that any of your interpretations of SFMP 1, 3, and/or 7 have changed since you videotaped and reflected on your lesson? If so, describe what changed and why you
believe it changed.

After the second classroom video:

1) What Standard for Mathematical Practice did you choose to focus on in this lesson?

2) How do you see this Standard (or other goals) appearing in the lesson?

3) Do you feel that any of your interpretations of SFMP 1, 3, and/or 7 have changed since the second interview? If so, describe what changed and why you believe it changed.

After the second collaborative discussion:

1) In general, what stands out for you in SFMP 1, 3, and/or 7 right now?

2) Do you feel that any of your interpretations of SFMP 1, 3, and/or 7 have changed since you videotaped and reflected on your second lesson? If so, describe what changed and why you believe it changed.

After the third interview:

1) In general, what stands out for you in SFMP 1, 3, and/or 7 right now?

2) Do you feel that any of your interpretations of SFMP 1, 3, and/or 7 have changed since the second collaborative discussion? If so, describe what changed and why you believe it changed.

3) How would you describe your professional learning experiences in this study?

4) Do you have anything else that you would like to add?
APPENDIX H

DIRECTIONS FOR PARTICIPANTS FOR VIDEOTAPING LESSONS,
INCLUDING CONSENT FORM FOR STUDENTS
Appendix H

Directions for participants for videotaping lessons,

including consent form for students

1) Choose Standard for Mathematical Practice (SFMP) 1, 3, or 7 from the CCSS.

2) After gaining permission for videotaping students in your classroom (permission letter template below), videotape one lesson which you believe will reflect this SFMP and at least one grade level standard from the CCSS content standards.

3) Watch the video, and journal about how you see the SFMP appearing in the lesson.

(Permission letter for students on next pages)
November 1, 2011

Dear Parent/Guardian:

I, along with other teachers in [school district], am currently participating in a project that is helping teachers to learn more about the new Common Core State Standards in Mathematics. The goal of the project is to learn how to best support teachers in implementing the new standards in their classrooms. The project is directed by Kimberly Yoak, a doctoral candidate in Curriculum and Instruction at Kent State University.

As part of the study, I will be videotaping two lessons in our mathematics class and examining my own practice in light of the Common Core State Standards.

I would like your permission to include your child (along with other students in the class) in these videotapes. No one but Kimberly Yoak and I will ever see the videotapes. Please explain this project to your child; then ask your child if he or she is willing to be included in videotaped mathematics lessons. If you and your child are willing, both of you should sign the included permission form and return it to me as soon as possible. If you and/or your child are not willing, I will seat your child in a location in class so that he/she does not appear on video (for these sessions only).

If you have any questions about this project, please do not hesitate to contact me at the school. You may also contact Kim Yoak at 330-689-5200, ext. 6255, or her doctoral adviser, Dr. Michael Mikusa, at 330-672-0647.

Thank you -

[teacher's signature]
VIDEOTAPE PARTICIPATION CONSENT FORM

Study Title: Elementary Teachers’ Evolving Interpretations of the Standards for Mathematical Practice in the Common Core State Standards: A Multi-Case Study

Principal Investigator: Kimberly Yoak

I agree to be included in videotaped sessions in my mathematics class.

____________________________
Child's Printed Name

____________________________  __________________________
Child's Signature  Date

I agree to let my child be included in videotaped sessions of his or her mathematics class.

____________________________
Parent's Printed Name

____________________________  __________________________
Parent's Signature  Date
APPENDIX I

TABLES OF DATA USED TO ASSESS EACH TEACHER’S FIDELITY
TO THE STANDARDS FOR MATHEMATICAL PRACTICE
Appendix I

Tables of data used to assess each teacher’s fidelity to the Standards for Mathematical Practice

Table 28

Fidelity Data: Carrie

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<th>Carrie</th>
<th>Action/words align with authors’ likely intent and own interpretation of statement</th>
<th>Action/words align with own interpretation of statement but not authors’ likely intent</th>
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Table 29

**Fidelity Data: Sandy**

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Appendix J

Researcher’s log for this study, beginning with data collection

Table 31

Researcher’s Log

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<th>Activity</th>
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<td>E-mailed participants to set up first interviews</td>
<td>Late January 2012</td>
<td>Q1, Q2 Sandy seems very passionate about her teaching and very much wanting to help her students, but she spoke rather negatively about a lot of issues related to education in general (society, students’ work ethic, students’ prior knowledge). At the end of the interview, she even said that she felt sad having said all of these things, because she does enjoy working with students. I noticed that many times when I asked her about text in a SFM, she almost entirely avoided it and started talking about her own concerns and frustrations. She seems to have rather traditional views of teaching and learning (teacher-directed instruction, e.g.).</td>
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<tr>
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<td></td>
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<td>Date</td>
<td>Qs</td>
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<td>Q1, Q2, Q4</td>
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<td>Carrie – interview 1</td>
<td>February 15, 2012</td>
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graders would be able to achieve the goals in the SFMP, at least anytime in the near future. She did acknowledge that in the future, once students had been “raised” with these standards, there might be more of a chance for success with them at this level.

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<tr>
<th>Ray – interview 1 part 2</th>
<th>February 22, 2012</th>
<th>Q1, Q2, Q4, Q8</th>
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<tr>
<td>Ray continued to talk a lot about the Singapore math workshop that he had attended; this clearly had an influence on his recent thinking, and he said it had helped him to understand the new standards much better than he would have otherwise. He also talked (as did Sandy) about an ODE workshop he had attended earlier in the year that really did not touch on the SFMP, and he was frustrated now about this because he realized that they were very important in the framework of the CCSS. Ray has many specific ideas about how to help students learn mathematical concepts and skills, and he seems to think a lot about concept development as well as skill development. He is clearly excited about math and teaching!</td>
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<thead>
<tr>
<th>Transcribing all first interviews</th>
<th>February 23 – March 5, 2012</th>
<th>Q5, Q8, Q9</th>
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<tr>
<td>Of the three teachers, Ray is definitely the hardest to transcribe because he often interjects in his own thoughts and talks about a variety of ideas at the same time. He is definitely always thinking – I think a lot of this thinking is done aloud! Sandy and Carrie are more deliberate in their speaking, pausing more to think as they respond. Carrie seems the most sure, overall, that she is already enacting many or all of the ideas in the SFMP, and Sandy seems the least sure (or even that this is possible in grade 5 right now). Ray seems to feel that it is possible and he believes in what the SFMP say, but he thinks that he can improve on doing some of what the SFMP suggest (to support students’ learning).</td>
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<tr>
<th>Group discussion 1</th>
<th>March 7, 2012</th>
<th>Q1, Q2, Q3, Q4, Q6</th>
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<td>This was a really interesting discussion that definitely could have lasted longer. All of the teachers had some experience with each other because Carrie’s own children had been in class with Ray and Sandy in the past. Of the three teachers, Ray did the most talking, and Sandy did the least, often echoing what Ray said. The teachers spent a good amount of time talking about</td>
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students’ perseverance in first grade vs. fifth grade, and they realized that (contrary to what they might have thought) first graders might actually be more willing to persevere with problems because they were not as used to personal technology that would give them answers immediately. Carrie heard about how Ray and Sandy are so continually frustrated by the pressure of the OAA, and she felt for them and said this should not have to be this way. I think it was helpful for all of them to hear some in-depth perspective from someone at a different grade level, but also to hear that the SFMP could be similarly enacted at each level.

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<tr>
<th>Transcribing group discussion 1</th>
<th>March 9-15, 2012</th>
<th>See notes above</th>
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<tbody>
<tr>
<td>Videotaping Carrie – lesson 1</td>
<td>April 3, 2012</td>
<td>Q1, Q2, Q8</td>
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|                                |                  | It is always so interesting to visit a new classroom – especially in a different district! I had to keep in mind that I was there strictly to operate a camera and not to participate or offer comments, which is a challenge (but I managed). The students, as Carrie had noted, were used to other adults being in the room for various reasons, and they were also used to being videotaped for things that Carrie had done in the past. So they basically ignored me. The goal for the day was to explore double-digit addition with base ten blocks. This was the first time that they had ventured into double-digit addition, so Carrie herself was curious about how the students would respond. Using the base ten blocks seemed to help the students work through the problems fairly well, and I liked how Carrie drew on students’ ideas to create new problems for them to solve. I did wonder, though, whether some of the students who might have been struggling a bit to understand the conversation (and particularly the numeric representations that others were writing on the board) really got much out of the lesson. But this was an introduction, and Carrie says that she then provides more individual attention to students when they do morning work and center work in days following a lesson like this. She did have the students “turn and talk” quite a bit, and she had students show some of their thinking on the board, so her goal of being a
A collaborative classroom was definitely demonstrated here. I saw evidence of all three SFMP in this lesson, and I will be analyzing this more carefully when I am in the data analysis phase of the study.

**Transcribing Carrie lesson 1**

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<tr>
<th>Date</th>
<th>Q2, Q11</th>
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<tr>
<td>April 6-7, 2012</td>
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Beyond what I wrote above, I did notice in transcribing this lesson that a lot of what students said was fairly minimal. Carrie expressed a lot of complete thoughts and questions as she spoke, and students’ answers were often brief (one or two words or numbers). I wonder if this would have been different if this were not an introductory lesson in a new concept – would students have articulated more complete thinking in another activity?

**Watching first lesson videos – Ray and Sandy**

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<tr>
<th>Date</th>
<th>Q2, Q9, Q10, Q11</th>
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<tr>
<td>April 20-22, 2012</td>
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Sandy: This first lesson was fairly teacher-directed and generally not very deep in terms of conceptual development, which didn’t surprise me, given the kinds of comments that Sandy had made in the interviews and discussions. It was a review of their previous work with metric and customary units of capacity and of finding perimeter and area of various polygons. Much of the language (from both Sandy and the students) was imprecise to some degree. I know that Sandy was trying to help students look for “patterns” (SFMP 7) in the measurement units, but this really did not work very well because unfortunately there are not a whole lot of patterns to be noted with customary units, and with metric units there is just one major pattern, which did come out in discussion (powers of 10). Sandy did also ask students to come to the board to create and work through perimeter and area tasks, so this part was a bit more interactive, with students commenting (mostly respectfully) on each other’s work (SFMP 3).

Ray: Ray’s goal in this lesson focused on SFMP 3, so he began by talking to the students about the idea in this standard of offering thoughts about each other’s work – “disagreeing agreeably” – and he used the “thumbs up/down” strategy during the lesson to allow students to do this. I was actually sort of surprised, though, at how teacher-directed the lesson was overall,
considering how Ray talks about wanting his students to be engaged in lessons and sharing their thinking with each other. The homework review involved multi-digit multiplication, and the lesson involved multi-digit division. Students really did not say a whole lot in general; Ray did most of the talking, and he did a good amount of leading to demonstrate a couple of strategies for division. I will be interested to see what his own reflection is for this lesson.

<table>
<thead>
<tr>
<th>Carrie – interview 2</th>
<th>April 23, 2012</th>
<th>Q2, Q4, Q5, Q8</th>
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<tr>
<td>Carrie mentioned in this interview was that it was really interesting to hear from the fifth grade teachers about how many students at that level found it difficult to persevere through a problem. She had shared this with her first grade colleagues, and they were surprised at this as well. She felt that the more that the early grades teachers could instill in students the desire and confidence to persevere, perhaps this would help the students more when they grew older. This confirmed what Carrie already believed about helping students to feel autonomous as learners. Carrie also talked about how she felt that SFMP 7 had a lot to do with students’ making connections among ideas, not only in math but across other content areas also. She felt herself making some new connections as she read the standards again, especially since (as she said) I had separated the paragraphs into sentences or groups of sentences for this interview.</td>
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<tr>
<th>Sandy – interview 2</th>
<th>April 26, 2012</th>
<th>Q2, Q3, Q4, Q7, Q8</th>
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<td>Sandy was again more positive overall in this interview, much more so than in the first session we had together. She felt more confident in talking with her colleagues about the SFMP and had actually brought this topic up in a recent staff meeting (saying that the SFMP were really important and couldn’t be dismissed). She still was not sure what some of the text meant in the standards, and in some cases she was still unsure that fifth graders, at least right now, were capable (in general) of meeting these standards, but she seemed more optimistic that over time, teachers working collaboratively and consistently across grade levels could help students to meet the goals. Sandy noted that the teachers really needed more time to</td>
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collaborate as grade levels and across grade levels in order to provide the best kind of instruction (relative to the CCSS) over time to their students. Sandy also said that it was helpful to have the SFMP presented in portions rather than as paragraphs. I did notice some more imprecise mathematical language that caused me to question some of Sandy’s content knowledge, so it may be that her own lack of total comfort with some of the content is causing her to question whether students can reach the high goals in the CCSS and SFMP.

Ray – interview 2
May 3, 2012
Q1, Q2, Q7, Q8, Q11
Wow, this was a long interview! 2 hours, and I actually had to cut a few questions short in order to fit the conversation into this time frame. Ray is, again, very passionate about his teaching and his role in supporting student learning. He talked about some of his ideas for the following year in terms of integrating the four content areas in real-life, engaging problems that he thought would help to meet instructional goals but that would also help students to want to persevere. Ray definitely feels that it is his role (self-chosen, in some ways) to support his colleagues who will be new to math teaching next year (since they all will be self-contained), and he spoke about the need for professional support in terms of grade level collaboration and sending people to outside workshops (like Singapore math) when there was available funding. He was very “anxious” about his colleagues not understanding the importance of the SFMP or how to enact them. Ray expressed more frustration at what he felt was a general lack of interactive engagement on the part of his administrators in terms of working WITH teachers to solve school-wide issues rather than setting rules that were designed to keep all teachers doing certain (prescribed) things in their classrooms. One other note: Ray again ignored some of the text in the standards and focused only on certain words in many cases. I think I would have to literally ask him about each phrase in order to really get him to think about more of the text carefully.

Videotaping Carrie – lesson 2
May 16, 2012
Q2, Q5, Q9, Q10
This was a very interesting lesson for me because it was focused on fractions – halves and quarters, to be
precise. I am always interested in how students think about fractions. The students had many ideas (mostly correct) about how to divide a circle, square, and rectangle into halves and quarters, and Carrie was asking them to do this on the board as part of a whole group discussion. One thing I found myself thinking, though, was that Carrie was doing quite a bit of modeling herself, or correcting incorrect ideas quickly, rather than letting students talk much about what they were seeing. This was even more true as she really did model what she wanted them to do with Play Dough to create halves and fourths at their tables; they essentially saw her do everything they were supposed to do before they did it. It is always fascinating to me how teachers who are so good at helping children to be independent readers and “citizens” (in first grade) still feel that they have to do more leading with math – and they may not even recognize it.

<table>
<thead>
<tr>
<th>Group discussion 2 (also got lesson video 2s from Ray and Sandy)</th>
<th>May 30, 2012</th>
<th>Q4, Q5, Q7, Q9</th>
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<td>Another interesting conversation… one of my goals in this discussion (based on the questions that the teachers had submitted) was to have them talk about professional support/development related to the CCSS and SFMP. But I found that whenever I asked this question, the talk went almost immediately to resources rather than ideas for human interaction. (not entirely, but mostly) They all did say that they felt within-grade collaboration was really important, and they wished they could talk to teachers from other grades more as well. When I asked about their own personal next steps for the standards, Sandy’s comments showed a lot of desire for growth and learning as she began to enact the standards during the following year. She attributed this to the work in this study and thinking about her own teaching. Ray talked about his ideas for creating more lessons/activities and sharing them with colleagues, and Carrie talked about assessment relative to the SFMP.</td>
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<p>| Transcribing Sandy lesson 1 | June 19-20, 2012 | See notes above (April 20-22) |
| Transcribing Ray lesson 1 | June 22, 2012 | See notes above (April 20-22) |
| Reading/ | June 24- | Q1, Q11 |</p>
<table>
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<tr>
<th>Activity</th>
<th>Date</th>
<th>Notes</th>
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<tbody>
<tr>
<td>Coding each interview 1 (first time)</td>
<td>27, 2012</td>
<td>I am using 4 methods of marking responses – no marking (just recording the response), underlining (emotion in response), bold (teacher said this a lot), italics (teacher saying something explicit about his/her interpretation process) – already a few minor differences from first read-through – am I really noticing all of the things that are said a lot? I think I will notice this more when I start putting the responses in categories. I notice that I didn’t mark nearly as many points of emphasis in Carrie’s interview – maybe because she just was more concise and didn’t tend to repeat much. But there were a few points that came up more than once.</td>
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<tr>
<td>Organizing responses from interview 1 by question (all partic.)</td>
<td>June 28, 2012</td>
<td>This was an interesting process. I found that on some questions, there was a great deal of overlap in what the teachers said, but in many cases, they had their own unique viewpoints. I started to make comments here and there in the document by the time I got to the end… things I was noticing… I started to note when the teachers didn’t really even address the question that I had asked (especially relative to the standard or text in the CCSS).</td>
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<tr>
<td>Categorizing responses from int. 1 by question (all partic.)</td>
<td>June 29, 2012</td>
<td>Q2, Q4, Q6 I started to mark (but need to do more of this when I go back to it) the places where I saw the group learning (or individuals learning) as a result of what others were saying – learning that could not have happened individually. I also made note of where they weren’t really addressing the questions that I was asking (even though the questions came from them!) because they were talking about certain issues that were of importance to them (that these other things seemed to come to mind when I asked the questions referring to the standards). Also, I started to recognize that it seemed to be important to note when the fifth grade teachers said things that were practically verbatim what the other had said in the interviews (even when they didn’t know what had been said in the interviews). I wondered if these were things that they talked about a lot during the school year.</td>
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<tr>
<td>Read through 1: Gp discussion 1 transcript</td>
<td>June 29, 2012</td>
<td>Q1, Q7, Q8, Q10 I noticed that Carrie (Carrie) seemed to talk more quickly this time, particularly when she was talking</td>
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<tr>
<td>Transcribing Carrie’s interview 2 (all)</td>
<td>June 29-30, 2012</td>
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but about 24 minutes toward the beginning) about things in her class. Was this because she was more comfortable with me? I noticed that she seemed to slow down when she was either: a) trying to say something diplomatically about other teachers (without being critical, but really wanting to be), b) not sure of what she was saying or thinking – especially on questions that might not have been what she was expecting), c) when she was saying something that was very important to her – that she was passionate about. I noticed that my questions about policy seem to be very important (for all three participants) because all of the teachers basically end up talking about policy anyway in one way or another, so it would seem to be leaving out an important part of the context if we did not specifically talk about this. I think that many lay people might be surprised to hear even the most committed teachers like Carrie (Carrie) talking about the day-to-day issues that can really constrain their teaching in one way or another, and that teachers can’t just forget about these things. This especially applies to things like new standards – she noted that most teachers will probably skip over the SFMP and just read the content, and interestingly, I will note that she even skipped over a lot of verbage in the SFMP in the first part of the study (case in point). I still think that even Carrie, who really seems to think a lot about classroom community and allowing kids to make sense on their own, may lead them a bit more than she might realize (in math). But she has so many fewer issues with having to rush kids through things than the 5th grade teachers, thus not diminishing the kids’ perseverance because they don’t feel rushed or like they may fail. I will also note that I am thinking so much about Westminster while I do this… our RTI assessments and how to collect and use data efficiently (even hearing her say some of the same things our teachers do about this), how to help kids stay engaged in math in the upper grades when we are so worried about data and getting through this content that really doesn’t have a lot of meaning for kids… how do we keep that inclination to learn intact?

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<tr>
<th>Transcribing 15 minutes of</th>
<th>July 1, 2012</th>
<th>Q2, Q8, Q10</th>
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<td>Carrie is really focused on letting the kids’ thinking</td>
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guide where she takes their instruction, knowing that she has 22 little minds to think about. She seems to be able (whether this is the actual result of not) to spend enough time with the whole group as well as individuals that she can bring kids along who take longer to process ideas. But she has said this is harder when you have a class of 27. She completely believes that students MUST feel they can accomplish what she is asking of them and that she has to build from where they are. This is a luxury that the upper grades don’t have, I think, which I think she is starting to recognize (or maybe has seen all along).

I’m still thinking about this question of how to keep older students engaged, interested, and wanting to persevere. How do we make the content relevant for them? How do we set up the tasks and the classroom as a whole to help them feel that they CAN be successful and that they can accept the challenge?

On a side note, I’m thinking about the idea that one of the benefits of inclusion is that kids who struggle can learn from the thinking of others – some people refer to this as the “spark”. I can relate this to seeing a colleague’s dissertation defense. Now actually having that in my experience and in my memory, I can say to myself, “This is what it looks (and sort of feels) like when you actually finish. I can do this!” It doesn’t seem so foreign and scary now. Is this what kids can feel when they see others achieving something that they are working toward? The trick is that the “end” that is being observed can’t be SO far from where the student actually is that it still seems like too difficult a road. If I saw someone fly the Space Shuttle right now, I’m not sure I would feel that it was achievable for me, though I suppose the more I heard them talk about it, the more I might feel that it was within my realm of potential.

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<tr>
<th>Organizing responses from reflection on Interview 1</th>
<th>July 1, 2012</th>
<th>Q11, Q5</th>
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<td>I do not have a reflection from Sandy for this interview; she says she sent it in an e-mail, but this never got to me. I do know that she was having issues with her school e-mail at the time, so we switched to</td>
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<td>Date</td>
<td>Action and Notes</td>
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| July 2, 2012 | Copying/pasting comments from Gp. Discussion 1 by question  
Q11  
I need to make sure that I keep the original printed document with me when I start writing (or maybe tomorrow when I organize, so I don’t forget) because there were a few written comments of mine at the beginning that I didn’t include in the new file. |
| July 3, 2012 | Organizing and beginning to analyze and comment on comments from Gp. Disc. 1  
Q11, Q6, Q7, Q5, Q2  
At one point in my notes, I wrote that it is helpful to include in the transcript when they all agreed, but then I questioned whether this is really something that can be assessed in a group discussion like this. Nodding or vocalized agreement doesn’t necessarily actually mean that someone agrees with a comment; I would have to interview them individually to determine this (with more certainty). Again, I am noticing that many parts of the discussion do not specifically mention the SFMP (even though I tried to raise the questions in a way that did directly address them – this paralleled some of the answers in the interviews… the teachers really have troubled focusing on just the text of the standards without thinking of all the other daily details that play into how they will be able to enact these SFMP, but maybe that is actually really important for professional development design). I would say that the ideas in the SFMP and the intro do underlie most of the discussion that takes place. All three of the teachers echo or refer to things that they said in the first interview, and Ray/Sandy even make comments that sound like the other person’s comments in that interview, which leads me to believe that since the same language is used, they often have conversations about these things, or at least had one or two recently (during or before the study). Sandy did not say as much overall as Ray or Carrie; she listened and often let Ray speak for grade 5. (not sure if one could avoid this) SO many of Ray’s and |
Sandy’s comments focus on rushing through content to support test scores (due to policy) of students who are unengaged (due to technology and society) and unable to learn as quickly as they should be (due to lack of understanding and mastery of concepts from earlier grades). I thought Carrie handled herself well and just spoke of her own experience, whereas other teachers might have felt attacked and might have been defensive. I noted that the teachers did seem to learn from each other (from the conversation – emergent perspective), things that they could not have come to on their own without hearing the experiences of others.

| Organizing responses from Gp. Disc. 1 reflections | July 3, 2012 | Q5, Q1, Q7  
I do have all three of these, though Carrie wrote about the questions from the discussion and not the reflection guide; however, her answers relatively well mirrored the other questions because the questions also did. All three participants in some way talk about number sense. Ray and Carrie also talk about classroom practice and logistics, and Ray asks a question about students’ disposition that arose in the discussion. I did not see much change in thinking in any of their responses at this point, though I noticed some subtle things of which they did not make much mention. Again, their responses related to interpretation are always couched in larger issues of classroom practice, student learning, and policy. |

| Writing the summary of phases 1 and 2 - Carrie | July 5, 2012 | Q1, Q11  
I wanted to use as much of Carrie’s voice as I could – I think this is very important. I tried to do this especially when her statements seemed like they emphasized points that were important to her (things she said often) or that would resonate with policymakers and other readers who may not be familiar with teachers’ experience. In the summary of the group discussion, I only included things Carrie said that were substantially different than things in the first interview, and I also did this in the reflection from the discussion. I found that I wanted to include more “analysis”, but I ended up shying away from this because I wasn’t sure I wanted her to feel that I was trying to get inside her head too much or that was I was trying to pick apart every word she is saying. This gets back to what one
of my committee members said – that sometimes we don’t want the participants to see everything we write about them. So we have to figure out where that line is. The summary was 11 pages long… I hope that Carrie will send me feedback. I don’t know if that’s too long… I’ll have to think about this as I do later ones.

Writing the first part of the summary of phases 1-2:
Sandy

July 5, 2012
Q1, Q11
My comments here are similar to those for Carrie’s summary above – though I actually wonder if this summary is going to be longer yet! I am not sure why that might be the case other than that perhaps I am using even more of Sandy’s dialogue than I used of Carrie’s. We will see. This summary is a bit more tricky because Sandy is not always as articulate as Carrie, and I am trying to include quotes in such a way that I do not include grammatical errors and such. Further, I am wondering, even though I am essentially quoting Sandy and not really commenting on most of what she said, whether when she reads it, she will feel that I am making her (or trying to make her) sound like she is really negative about teaching and learning. But honestly, in that first interview, that is exactly what I felt because it came out so clearly in her dialogue. I am trying to leave out some of the repetitions of these kinds of comments (though I sometimes mention them because they seemed important to her that day), and I am also trying to add my own comments that she sounded “sad” or “frustrated” as she was speaking.

Writing the second part of the summary of phases 1-2:
Sandy and the entire summary of phases 1-2:
Ray

July 6, 2012
Q1, Q2, Q4, Q5, Q6, Q7, Q8
I have similar comments to those I made yesterday. I am definitely starting to think of my audience as non-educators and particularly policymakers, or also those who design professional development for teachers. I want people to understand what teachers experience when they are encountered with standards (new or otherwise) and how that influences their interpretations in the short- and long-term. One side note: when Ray talked about students in some countries doing the same kind of math task until they know it completely, I wondered if that would “fly” in the U.S. in most schools because parents might start to demand other activities for students who had already gotten it – the
<table>
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<tr>
<th>Reading lesson 1 transcripts – Carrie and Sandy</th>
<th>July 8-9, 2012</th>
<th>Q11, Q1, Q10</th>
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<td>whole issue of “you are teaching to the low end and not the high.” Isn’t that interesting… could we argue that these other countries are doing that and that is why they are so much further ahead in scores? Is that why it is often seen that our advanced students are on par with theirs?? Do we actually focus more on enrichment and such than many other countries?</td>
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<td>I am going to have think about how best to code/analyze these. When I read through them, I marked any instances of SFMP 1, 3, or 7 being demonstrated by the teacher or students. I also noted when I thought the teacher was saying something that she probably should/could have asked the students to think about (so it was a SFMP, but it could have been more of a learning opportunity for the students). I saw instances of some of the other SFMP as well but did not mark these. I’m trying to think about what else I might miss… what else I could look for… things that are not there from the three SFMP? Maybe I need to look more carefully at which parts of each SFMP are really apparent in what is happening. I’m also noticing that it’s often difficult to distinguish SFMP 1 from 7, because often when people are making sense of a problem situation, they are looking for structure in it and/or for connections to other mathematical ideas. Could I also think about suggesting what the teacher could have done to carry the SFMP further? Maybe that’s not necessary here. When I write the summaries of the lessons for the teachers, I will have to be careful about what I say. I think I might make a 3-column table to analyze – one for the text, one for the SFMP that I see, and one for my own comments. Having finished these (now later in the day), I found that the 3 columns work very well, and I think I will be able to write summaries fairly easily from these (using the second column for what I send to the teachers and including the third column for my own analyses). I find that it is hard for me not to “judge” the teachers, so I am including my own evaluative comments as well so that at least they are recorded and I am being open about this. But I am attempting to stick within the frame of the three SFMP. I could analyze every</td>
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element of the lesson much more than I am, but that is
not really the purpose here. Both of the teachers, in
their reflections, commented that they were becoming
more aware of the SFMP by watching their own
videos.

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<tr>
<th>Reading lesson 1 transcript - Ray</th>
<th>July 11, 2012</th>
<th>Q1, Q2, Q3, Q8, Q10</th>
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<tr>
<td>I felt that Ray definitely focused on SFMP 3, which was his intent in this lesson. However, I also agree with his reflection in that he felt, in retrospect, that the students were not really doing much of what SFMP 3 says; their answers were very brief and “superficial”, as he wrote. There was a lot of back-and-forth conversation, so he was trying to get students to (in some way) make sense out of the examples that he was demonstrating, but he was really the one doing most of the sensemaking and explaining. There was little appearance of SFMP 7 in this lesson, which is not a bad comment – just an observation. He had not chosen this as his focus for the video, and though it could have been drawn in much more with the multi-digit division, Ray did not. This could have been done on another day, perhaps (though I’m not sure if it was). One thing that was very clear was that Ray was working to bring his students “along” in his focus on SFMP 3 – he was very explicit with them in his focus and talked about it with them before he even began the lesson. I thought that this was a positive way to initiate the lesson. He did return to it at the end also, briefly (in terms of a reflection). One other observation I made was that, as in the other two teachers’ classes, only a few students were really doing most of the responding when he would pose a question to the class, and he did not seem to notice or try to respond to this lack of response except for in one instance.</td>
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<table>
<thead>
<tr>
<th>Writing summaries of lesson 1 – Carrie and Sandy</th>
<th>July 12, 2012</th>
<th>Q1, Q2, Q3, Q8, Q10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Once again, I found it challenging to write a summary without offering negative evaluative comments (I also tried to stay away from positive evaluative comments). I wanted to summarize when I observed SFMP 1, 3, and 7 in the lesson and relate my observations, at least somewhat, to what each teacher wrote in her reflection. I used some quotes from the lessons but not many… somehow it seemed easier to describe the examples of</td>
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each SFMP as it occurred, and of course the details of
the lesson are not the focus of my study. I did use
quotes when I felt they were needed to clarify my
observations. It seemed easiest to proceed through
the portions of each lesson and describe the SFMP I saw in
each part, though I did not adopt this method until I
started Sandy’s summary, so Carrie’s may seem more
disjointed. Perhaps not… her lesson really had one
focus, whereas Sandy’s (and Ray’s) had multiple parts.
Generally, my observations corresponded with what
the teachers wrote in their reflections in terms of what
SFMP were present (and, in fact, whether the standards
were all that effectively enacted), though I often
noticed additional instances of one or two of the
standards that they did not mention. I think the most
difficult part is for teachers to realize when THEY are
enacting the SFMP versus when their students are
enacting it. I did mention in all three summaries that
most of the students’ responses that reflected any of the
SFMP were offered in response to a teacher’s question,
which underscores the role of the teacher in this
learning process. I suppose ideally we would like for
most students to be able to offer these questions and
comments without prompting, but perhaps in many
ways this is an unrealistic goal – at least for a good deal
of classroom interaction since instruction is
traditionally directed by the teacher in one way or
another (due to the content standards), especially in the
upper grades, which may mean that students won’t be
as naturally engaged as they may be in the primary
years.

<table>
<thead>
<tr>
<th>Transcribing remainder of Carrie’s interview 2 and starting Sandy’s interview 2</th>
<th>July 15, 2012</th>
<th>Q10, Q11, Q4, Q2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not too much to say here… I did start trying (inconsistently) to underline the words that I slightly emphasized when I read the SFMP aloud, but I should go back and do that a bit more (or at least from here on in). It definitely is helping the teachers to focus more on all parts of the statements. With Sandy, I really feel like a lot of what she says is based on the fact that the students are not coming up through the grades with student-centered instruction, nor are they getting very much of it in her class (maybe partially because of limited content knowledge? “reverse operation”).</td>
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<tr>
<td>Task</td>
<td>Date</td>
<td>Questions</td>
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<td>----------------------------------------------------------------------</td>
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<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Transcribing remainder of Sandy’s interview 2 and starting Ray’s interview 2</td>
<td>Last part of July, 2012</td>
<td>Q10, Q11, Q4, Q2</td>
</tr>
<tr>
<td>Ray’s final interview</td>
<td>August 10, 2012</td>
<td>Q2</td>
</tr>
<tr>
<td>Carrie’s final interview</td>
<td>August 13, 2012</td>
<td>Q2, Q1, Q2, Q5, Q8</td>
</tr>
<tr>
<td>Finishing transcription of Ray’s interview 2</td>
<td>August 27 – Sept. 3, 2012</td>
<td>QA, QD</td>
</tr>
</tbody>
</table>
| First reading of second interview with Carrie                       | Sept. 8, 2012             | QD, QC, QA                                                               | This was sort of an isolated reading time-wise… I need to go back and look more at this transcript. [did this later] I started to make note of when she referred (often without a direct reference) to the SFMP. We both had a joint realization (during discussion) that young students might think that the place between ones and hundreds is fifties because 50 is in the middle of l
and 100. Carrie often relates her comments to her own experiences or to her son’s.

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<thead>
<tr>
<th>First reading of second interview with Sandy</th>
<th>Sept. 10-11, 2012</th>
<th>QD</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>I need to go back and look at comments that I made in this transcript. [These are now in the responses by question document.]</td>
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<table>
<thead>
<tr>
<th>First reading of second interview with Ray</th>
<th>Sept. 8 and Oct. 22-24, 2012</th>
<th>QD, QA, QC</th>
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<td>Wow, does he repeat himself a lot. And this interview really just went all over the place; I had to skip several questions – though I skipped the ones I felt he had basically already addressed as he talked and talked. However, in reading the transcript, I did find that I not only continued to identify repeated themes in his comments, but I also started to think more carefully about the implications of these repeated comments and even some specific words or phrases that he used to describe particular situations. (metaphors… I thought of a colleague’s work) Repeated themes that stand out in my mind for Ray: PD, OAA, resources, paradigm shift, everyone has to be on board (and not everyone is, especially some veterans, and some administrators/board members seem not to understand the pressures of the OAA in 5th math), the SFMP are the “glue” that holds the content together. Ray also feels responsible for helping his colleagues. He is very self-reflective and says outright that he likes change and that change is good. I notice that both Carrie and Ray had the wrong idea of a counterexample. I will have to look back at what Sandy said about this.</td>
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<thead>
<tr>
<th>Learned that Sandy is on medical leave and may not teach this year at all</th>
<th>Oct. 22, 2012</th>
<th>QD</th>
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<td></td>
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<td>I am to write her at the start of November, at which time she will know more. She said that she had not forgotten about me and wanted to finish her work for me.</td>
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<thead>
<tr>
<th>Organizing responses from interview 2 by question (Carrie)</th>
<th>Oct. 25, 2012</th>
<th>QA, QC</th>
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<td></td>
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<td>I worked through part of this interview today… through part of the third SFMP. I am making comments as I go and putting them in brackets [] in the document. I am marking some things as “themes” or “big ideas” for her that I hadn’t marked initially, but I think since I now know what she said during the rest of...</td>
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the interview, I see that these were themes for her that day. Two comments: She definitely focuses a lot on the idea of fostering perseverance by allowing students to experience success at whatever level they are. Fifth grade teachers didn’t feel they had a lot of time to do this (and that the standards perhaps were not suited for this because they are often abstract and formal). Also, I noted that Carrie commented that her students often shared the same ideas in whole-group share-out after turn and talk – is this because she has modeled a bit too much, and they only have one way of thinking, essentially? She did not seem to see it that way, because she doesn’t want to present them tasks just to think about totally on their own (unfamiliar ones).

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<tr>
<th>Organizing responses from interview 2 by question (Carrie)</th>
<th>Oct. 26, 2012</th>
<th>QC</th>
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<tbody>
<tr>
<td>A lot of comments in this part of the interview about looping, developing students’ trust in her to foster their perseverance, making sense of ideas and connections among them (important to her in all areas), building on what students know, students’ patience with each other (again, fostered in community over two years). She started to talk about things that are developmentally appropriate for students of various ages.</td>
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<tr>
<th>Organizing responses from interview 2 by question (Carrie and Sandy)</th>
<th>Oct. 27, 2012</th>
<th>QD, QA</th>
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<tr>
<td>I really had a lot of questions, as before, about Sandy’s perception of what students can or cannot do and why. She does seem to believe that teachers can have an effect on student learning, but she also seems to believe that a lot of students’ ability is developed long before they get to fifth grade (either at home or in earlier grades), so there may not be too much she can do if they don’t come to her knowing or being able to do certain things. For instance, some of what she says is in direct contradiction with what Carrie said about what first graders COULD do, so there is definitely something to be said for teachers’ expectations of students and how they establish a learning community in class. Her language indicates some gaps in her content knowledge also. However, on the positive side, she cares deeply about her students as people and wants to do the best for them, and she is reflective and willing to brainstorm new ideas. I noted that her curiosity about how her students would respond to a</td>
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couple of tasks/situations might be a good element of PD for her (and for other teachers – as research has already shown).

Organizing responses from interview 2 by question (Sandy)  
Oct. 28, 2012  
QC  
I finished organizing Sandy’s second interview by question today. I noted that both Sandy and Carrie mentioned the school’s policies and explicit practices to decrease bullying – in relation to the standards implying that students need to be able to talk and listen to each other (without getting upset – the standards don’t say this, of course). Sandy definitely feels that most teachers will want to skip over the SFMP and go right to the content standards, and that the content will seem “easy” when the SFMP are not considered. She also notes that without PD, most teachers may not even recognize that they should spend time with the SFMP, and (my inference from what she said) even if they do, they may not be able to interpret them very effectively. Again, she does seem to believe that over time, both she and her students can learn to master these SFMP, but it will not occur quickly. (Although she says in about two years, it will be second nature to her.)

Organizing responses from interview 2 by question (Ray)  
Oct. 29, 2012  
QD, QA, QC  
I began organizing Ray’s comments in the second interview today. I find that I am starting to think more about questions I could ask the participants as follow up… I also find that I may be able to start summarizing some of their major ideas. I can picture myself doing an outline for each person fairly soon… at least as far as the end of the second interview. Ray talks more about the paradigm shift nationally and the fact that he is worried that many people will try to ignore the SFMP. He says that teaching traditionally will simply not work anymore. He is reflective and has a lot of great ideas. Do these play out in his classroom? I’m sure that at least some of them do, from what I have seen in the videos.

Organizing responses from interview 2 by question (Ray)  
Oct. 30, 2012  
QA, QC  
I noted that Ray repeats himself a lot and that I will have to concentrate on finding overriding themes in what he says. I think that sometimes Ray reads quickly (maybe only because he is with me) and misreads what is in the statements, then coming up with an
interpretation which doesn’t even seem to fit the general intent of what was written. If he sticks to these interpretations, will he be thrown amiss? Would other teachers do the same thing? Also, as I noticed before, he and Sandy are sometimes in direct contrast with what they say about fifth graders’ ability to master these standards (even at a fifth grade level). I do think that teachers in general have trouble sometimes thinking of the standards for their grade… not just the end goal (which they feel their students can’t reach because it’s too sophisticated).

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<tr>
<th>Organizing responses from interview 2 by question (Ray)</th>
<th>Oct. 31, 2012</th>
<th>QA, QC</th>
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<tbody>
<tr>
<td>I continued work with Ray’s second interview today. He likes to cite research fairly frequently, and usually he does this fairly adeptly. Sometimes I think what he reads in front of me is read a bit too quickly because he sometimes reads things that are not really there (and I know he really appreciates being able to read professionally, so I think he would want to avoid this). His profession is important to him as he spends a lot of out-of-school time working with it. I think he sometimes gets a bit too excited as he is thinking about ways to solve problems of practice and loses a bit of sight of what the actual issue is, sometimes in terms of content, others in terms of practice. But all in all, I am always impressed with his reflection and his ability to generate examples of ways to work with students that might better support their learning (music, for instance, with counting fractions).</td>
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<tr>
<th>Organizing responses from interview 2 by question (Ray)</th>
<th>Nov. 2, 2012</th>
<th>QC</th>
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<tr>
<td>Almost done with this interview… I started making some notes tonight in the compilation file about the following: insiders vs. outsiders in PD and making change within a school, interpretation based on feeling vs. “fact”, the use of a textbook that you trust vs. one that you do not (entirely or at all). Ray just hits the PD issue again and again – we must have it, or things will not change. And people around the school must understand also. Kids have to feel safe and engaged. Teachers have to feel supported (not beat down) both in words and materials, as well as PD.</td>
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<tr>
<th>Finished organizing</th>
<th>Nov. 3, 2012</th>
<th>QD, QA, QC</th>
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<tr>
<td>As I am organizing the responses from all three</td>
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responses from Ray’s interview 2; began categorizing/commenting on responses from all three participants and trying to make connections among them, I am finding once again that they all have fairly distinct viewpoints, though there is overlap from time to time in their comments and ideas. I am trying to summarize each section of each person’s response as I go so that it may be easier to write the summaries when I get to that point. I am trying to look for connections to the text of the standards (which often are lacking) and connections among the three teachers’ ideas (which also are often minimal). I am working to interpret from each person’s perspective but also to offer my own comments on their thinking. I did notice that they often jump right to their own practice and experience without working to connect their ideas to the text of the standards, which would likely make it difficult to know whether the standards were really being enacted or not.

<table>
<thead>
<tr>
<th>Categorizing/commenting on responses from all three (int. 2)</th>
<th>Nov. 4, 2012</th>
<th>QA, QB</th>
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<td>I am still making specific comments about the major portions of each response from each person, also trying to highlight comparisons and contrasts among them. I am starting to think about how these could lead to larger implications for PD (for chapter 5), for instance, the fact that teachers clearly focus on different portions of the SFMP and virtually ignore other portions. I almost feel that some of the sentences would have to be broken into phrases or even just single words to “force” teachers to focus on these. I just noticed that I don’t appear to have separated part of SFMP 1 into sentences in this interview… I’m not sure why.</td>
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<th>Categorizing/commenting on responses from all three (int. 2)</th>
<th>Nov. 5, 2012</th>
<th>QD</th>
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<td>Thoughts I have had tonight: 1) I am starting to do better about focusing on how the three are actually interpreting and thinking about each part of the statement from the SFMP. I am making reference to this as I add notes. 2) I wish that I had broken all of the statements into single sentences; some of them are really lengthy to interpret all at once. 3) I am finding that I am thinking much more about how I interpret each statement because this is a basis for comparison/contrast (as well as contrasting their comments with each other’s). This is also what the research literature suggests we do – keep reflecting on</td>
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</table>
our own perspectives on these questions and the text. 4) It is often hard to know when they are thinking about the text (even if they are not referring directly to it) and when they are going another direction in their own minds, while I can still see a connection to the text. 5) We keep returning to the question of whether students can do the things that are stated; even I questioned that at one point tonight, which I don’t like to do. (explore a conjecture using a logical progression) 6) Each of these statements is so deep!! One could do a study of any of the SFMP alone!

| Categorizing/   | Nov. 6,   | QA, QC, QD |
| commenting on   | 2012      |            |
| responses from  |           |            |
| all three (int. 2) |         |            |
| Nov. 8, 2012   | QD, QA    |
| Nov. 8, 2012   |            |            |

Categorizing/ commenting on responses from all three (int. 2) Nov. 6, 2012 QA, QC, QD

One thought: The teachers don’t seem to say much directly about a statement when they appear to feel that the wording is very clear. At this point, they tend to jump right to talking about what this looks like in their classroom, like there is no real reason to interpret because the text is clear. Are these perhaps the statements that we need to be most concerned about because teachers don’t think much about them at all? I suppose, though, that that is just as true for the ones that make no sense to them at first. Also: I have to keep in mind that I really need to focus on teachers’ perceptions of what their students can or cannot do (etc.) because to them, this is reality. They will interpret the standards through this lens. Finally: I would like to go back and ask each of them even about certain words, let alone phrases, that they seem to have missed in their reading and discussion.

Categorizing/commenting on responses from all three (int. 2) Nov. 8, 2012 QD, QA

One thing I have been noticing is that in my own comments, I am writing “she seems to think” or “he seems to say” a lot. Is there really any way to avoid this? Am I going to write this throughout the entire final report? Or is there a way to establish this at the beginning and just say that the report showcases my best attempts at understanding the teachers’ thinking? I saw tonight that the teachers are more likely to analyze each part of the statement when it is brief (and, I suppose, easy to understand in terms of the language). In the summary SFMP questions, teachers focus more on the language in the title than anything else. On another note, they often refer to events in their own
<table>
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<tr>
<th>Categorizing/commenting on responses from all three (int. 2)</th>
<th>Nov. 9, 2012</th>
<th>QA, QC</th>
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<td>When the teachers are discussing the hypothetical situations, they tend to place themselves into the situations rather than talking about the imaginary teacher. They also all jump right to the content and how to help students get past misconceptions from a content perspective; usually, I had to ask them to refer to the SFMP and tell how these were reflected in their suggestions. Sometimes they do mention them first, though, and I would think that a teacher who did this consistently could be said to have internalized the SFMP. All three of the teachers talk about students feeling comfortable to take risks as they share ideas and questions in class, and the teachers say that they must work to establish this kind of community. Both Sandy and Ray point out that it will be a challenge for many teachers (especially those fairly new to math) to enact the SFMP well without a good deal of collaboration and helpful resources. Carrie has mentioned before that many teachers will just skip over the SFMP, and elsewhere she has said that she thinks someone could teach the content without enacting the SFMP if he/she chose to do so.</td>
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<table>
<thead>
<tr>
<th>Categorizing/commenting on responses from all three (int. 2)</th>
<th>Nov. 10, 2012</th>
<th>QD, QB</th>
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<td>As I read the last parts of each interview, I tried to think beyond what each teacher was saying in terms of how their responses related to the SFMP and to other things that they had said or that the other teachers had said. It just occurs to me now that of the three teachers, Sandy could possibly be the one to make the most change as time goes on (in her practice) if she is able to continue these kinds of conversations with her colleagues about the SFMP. Carrie and Ray already do many things that seem to align with the SFMP, Carrie more than anyone, so they may not have as far to go. And Sandy does seem interested/excited about the possibility of moving in that direction. Ray does too; Carrie doesn’t seem to feel that she needs to move at all.</td>
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<tr>
<th>Started summary of</th>
<th>Nov. 10, 2012</th>
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<td>Trying to do this more efficiently/briefly than before by using my comments on the interview responses.</td>
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<tr>
<td>Event Description</td>
<td>Date</td>
<td>Notes</td>
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<tr>
<td>Int. 2 with Carrie</td>
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<td>Working well so far.</td>
</tr>
<tr>
<td>Finished Carrie’s interview 2 summary and did Sandy’s</td>
<td>Nov. 11, 2012</td>
<td>QD</td>
</tr>
<tr>
<td>Compiled the first half of Ray’s interview 2 summary</td>
<td>Nov. 12, 2012</td>
<td>QD</td>
</tr>
<tr>
<td>Met with Dr. Mikusa</td>
<td>Nov. 13, 2012</td>
<td>QD</td>
</tr>
<tr>
<td>Continued compiling Ray’s interview 2 summary</td>
<td>Nov. 14, 2012</td>
<td>QD</td>
</tr>
<tr>
<td>Finished Ray’s interview 2 summary</td>
<td>Nov. 15, 2012</td>
<td>QD</td>
</tr>
<tr>
<td>Sent Ray his summaries and wrote Sandy</td>
<td>Nov. 17, 2012</td>
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<tr>
<td>Date</td>
<td>Event Description</td>
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<td>Nov. 18, 2012</td>
<td>Began transcribing Sandy’s lesson 2 about Skyping (?). This lesson began with Sandy having to tell her class that the father of one of their classmate died the day before. Sandy then gave her students a task to work on in pairs, an interesting algebra-type problem involving a growing pattern (sequence). Students were given a worksheet to guide them through the process of thinking about and solving the problem, and they were able to choose from a variety of strategies to solve it. (This was the main goal of the lesson – to see different strategies that would produce the same answer.)</td>
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<td>Nov. 19, 2012</td>
<td>Continued transcribing Sandy’s lesson 2. During this part of the lesson, Sandy walked around and talked with quite a few pairs of students. However, she really did most of the talking; most of the students said “yes” or nodded or said a few numbers or a sentence. She described for most of them what strategy they used and why it might be a good strategy to use (rather than letting them do so). Interestingly, the task could have been solved fairly easily by 3rd or 4th graders at this point in the year, so I will be interested to see how the discussion goes and whether it is taken to a level that 5th graders should be at by now. I will also be interested to see whether the students are the ones mainly having the discussion or whether Sandy interjects more of the time than not.</td>
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<td>Nov. 21, 2012</td>
<td>Finished transcribing Sandy’s lesson 2 and began transcribing Carrie’s lesson 2. QB, QA I have to commend Sandy for doing what she did later in the lesson, because she had three pairs of students present their thinking, and she did not offer any comments but instead allowed students to comment and question. The students did fairly well with talking to each other without a lot of intervention from Sandy, though one exchange seemed to get a bit “edgy” when there was some disagreement. Sandy did intervene at that point. Interestingly, though, the goal of getting to the same answer with different strategies didn’t entirely play out because only one of the three pairs who presented had the right answer (and the other two were different). But Sandy did point out that minus a reading mistake, as she called it, and then an error in not finishing the problem based on this, there were a</td>
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couple of strategies that would have gotten students to the right answer (table and equation). On the other hand, what she was using as a rule for the problem was only a recursive rule for one variable. But I could not see the board at this point to see what she was writing where because the camera was pointed elsewhere. Carrie started a lesson on fractions, and students had been working with ½ earlier that day. I remember thinking that I was concerned with her continued reference to ½ meaning “1 of 2 [equal] parts” – gets at the issue of kids thinking of the num/denom only as whole numbers.

Transcribed more of Carrie’s lesson
2 Nov. 22, 2012 QA I have to think that Carrie does a lot of modeling, and I’m really not sure that a lot of kids are getting what she’s talking about even though she seems to act like she thinks that they do. But maybe she does more individual assessment when students are working on their own.

Finished transcribing Carrie’s lesson
2 and started transcribing Ray’s lesson
2 Nov. 23, 2012 QA, QD, QC Carrie essentially modeled at the board exactly what she wanted students to do on their activity sheets, so most of the students were able to do it. Would they have been able to had she not done so? What are the pros and cons of this? I’m not sure most students were really thinking; they were following the 2 or 3 students who kept talking and being called on during the carpet discussion (and Carrie also). Ray began his lesson by saying that he wanted the students to spend time thinking about how to start the problem (that they were going to solve) and then to try to come up with more than one answer to the problem. He read through the problem, gave students 2 minutes to think on their own about it, and then asked them to talk to neighbors. (How well do students communicate with neighbors? I know this is important to Ray, and he develops this practice from the start of the year, so maybe they are better in this class than in some.)

Finished transcribing Ray’s lesson
2 Nov. 24, 2012

Made initial notes on SFMP Nov. 25, 2012 QA Ray did a lot of talking in his class – much more than
and other thoughts on Ray’s lesson 2; started this for Sandy’s lesson 2

<table>
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<tr>
<th>Made more initial notes on SFMP and other thoughts on Sandy’s lesson 2</th>
<th>Nov. 26, 2012</th>
<th>QB, QA</th>
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<tr>
<td>I will have to compliment Sandy on her work in this lesson. Although she is in the beginning phases of learning how to instruct in a student-centered way, I am impressed with how she listened to students and how she allowed them to basically discuss with each other as students were presenting. She did not jump in very often, except toward the end of class when I know she wanted to get some closure. The discussion probably could have gone on longer otherwise. I still definitely see a focus on SFMP 3, but SFMP 1 also came out in the discussion (in trying to make sense of the “givens” in the problem – this to decide which solutions made sense or not), as did SFMP 7 because some students used a table to structure their work for the problem.</td>
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<tr>
<th>Finished initial notes on SFMP and other thoughts on Sandy’s lesson 2; started this for Carrie’s lesson 2</th>
<th>Nov. 27, 2012</th>
<th>QD</th>
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<td>It occurred to me today that I wasn’t entirely sure if some of Sandy’s students thought they were actually trying to figure out how many people entered on the 10th ring or whether they knew they were to figure out the total in 10 rings and just got muddled in the process (thus leaving out the last step). I think I made my other main comments yesterday. I started looking at Carrie’s lesson today, and I’m not sure which SFMP she was focusing on (I’m thinking SFMP 1 based on her own language in the lesson,</td>
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about “I can” – getting at confidence and perseverance, but wow, almost every note I have so far is about SFMP 7 because they are talking about what fractions are and creating parts from wholes. What a great application of SFMP 7 that I hadn’t really thought about until now!! (How can I not have seen this?!) I need to share this in Westminster.

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<tr>
<th>Continued initial notes on SFMP and other thoughts on Carrie’s lesson 2</th>
<th>Dec. 1, 2012</th>
<th>QD, QA</th>
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<td>I am almost done with the initial identification of the SFMP in this lesson. It is interesting that this transcript is very long, but maybe this is because there was a lot of back and forth between Carrie and the students during the lesson. I noticed tonight that Carrie had to say very few things related to classroom management, which I guess I found interesting for first grade (I didn’t really think about this for the fifth grade classes, but I don’t think either Ray or Sandy really had to say a lot of this sort of thing either – maybe a few more times). Carrie, again, does quite a bit of modeling, and I do wonder how much the students are thinking on their own and how much they are just following her lead. Would they know what to do with some of these concepts if they were presented in a non-familiar way? Maybe… because I know she also promotes perseverance and thinking on one’s own. Hmm. There was lots of overlap in my notes today between SFMP 1 and 7 as students were making sense of how to draw given fractions, and SFMP 3 also entered in (not as much) when students or Carrie were critiquing what someone else had done or were giving reasons for their own thinking. Sometimes I think Carrie could wait just a bit longer to allow students to comment more on each other’s work, but I know as teachers we also have to watch the clock. This is always a balance.</td>
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<th>Continued other thoughts on Carrie’s lesson 2</th>
<th>Dec. 2, 2012</th>
<th>QA, QD</th>
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<td>I ended up writing a lot of comments that had to do with Carrie’s intervening too soon in student attempts at sensemaking, and also her own modeling of what she wanted them to do without letting them try much of it first. She also seemed to leave out an emphasis on the language of fractions. I’m sure that some of these things would be part of later lessons, but I do wonder if students are really not as independent in her math class</td>
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as she might hope (as she even pointed out earlier, a lot of their explanations end up being very similar). It is often a few students who say a lot and are at the board a lot, even though the rest all are able to try later. Counting and naming… these are perhaps more vital in all of mathematics than I had thought about before… if we are able to name something and count (or measure) it, then we can use it in other situations or at least to solve a given problem. If we can’t do both of these things with a given concept, there will be holes in our understanding. I need to keep thinking about this.

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<tr>
<th>Finished other thoughts on Carrie’s lesson 2; wrote summary of Ray’s lesson 2</th>
<th>Dec. 3, 2012</th>
<th>QA, QD</th>
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<tr>
<td>I didn’t really have many more additional thoughts about Carrie’s lesson than I have already included here, though it did occur to me that the last few minutes of video (where she is cutting a circle into progressively smaller pieces) might have been that “stretch” to which she often refers that benefits those few students who need an additional challenge. I did notice that during the table activity with the Play Dough, she was asking some important questions that I thought all of the students needed to hear, but many were talking. Maybe she planned to ask these questions later. I am not sure that SFMP 1 was enacted by the students as much as she might have thought in this lesson – not a whole lot of independent thinking. But this was early in the process of studying fractions, and I did not see what came next. Ray and I had extremely similar ultimate views on how his lesson went – he did a lot of talking and probably should have let the students do more. However, we had different perspectives on the level of the problem itself; I felt it was far too simple for fifth grade, and he was pleased with the level of thinking that he saw. But, he definitely focused very explicitly on SFMP 1 (and 3 and 7 were also present in how he taught). I think he gave students the opportunity to enact SFMP 1, though not at a level of thinking that I felt was reasonable for grade 5.</td>
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<tr>
<th>Wrote summaries of Carrie’s lesson 2 and Sandy’s</th>
<th>Dec. 5, 2012</th>
<th>QA, QD, QB</th>
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<td>I actually feel that Sandy better enacted the SFMP in this second lesson than Carrie did. Sandy gave her students more opportunity to explore, and I think that</td>
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<td>Lesson 2</td>
<td>Carrie’s students could have handled some more exploring before she showed them how to do what she asked them to do with Play Dough. As always, I find that I have to keep in mind that my own interpretations of the SFMP cannot simply be ideals by which I judge the teachers’ interpretations. I also have to be careful of being overly critical, but I do know that I am trying to watch very carefully for when the teacher and individual students versus all of the students are developing the goals in the SFMP. In some ways, though, this is separate from teachers’ interpretations; a teacher might be well able to explain what it means to have students enacting a SFMP and then not facilitate this well in the classroom. However, I would hope that over time, being able to watch videos and reflect, a teacher would start to recognize what he/she would need to do in order to provide students with more opportunities along these lines. It would be interesting to give my participants a little quiz to see if they now remember which statements are part of which standards. Of course, there are some that I even feel overlap in terms of their thrust, so I’m not sure that this would really give me that much information, and of course I am not expecting them to ever memorize the paragraphs word for word. But, it would still be interesting to at least assess their long-term memory of each of the three SFMP that we have studied and see what they associate with each standard now in their own minds.</td>
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<tr>
<td>Began transcribing Group Discussion 2</td>
<td>Dec. 6, 2012</td>
<td>QC</td>
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women are starting to say more. I know that Sandy was dealing with the death of her student’s father from two days before, and Carrie seemed very serious throughout much of the discussion, though I think she is just generally fairly laid back and low key.

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<th>Continued transcribing Group Discussion 2</th>
<th>Dec. 9, 2012</th>
<th>QC</th>
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<td>During this part of the discussion, the teachers were talking a lot about discussion during math class (“math talk”) and how this could be motivating for students and engaging. Carrie brought up the idea that if language were more consistent from grade to grade, it would make more sense for students. She mentioned that they had talked before about professional development across grade levels focused on this goal. This conversation arose from the issue that many of the fifth graders seemed turned off of math and that their attention spans were relatively short, neither of which was true for the first graders, according to Carrie. She had even gone back after the first group discussion and shared some of the conversation with her colleagues about fifth graders’ lack of perseverance (perhaps due to their familiarity with technology), which was very surprising to the first grade teachers. I need to schedule another meeting with Dr. Mikusa.</td>
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<th>Continued transcribing Group Discussion 2</th>
<th>Dec. 10, 2012</th>
<th>QD, QC</th>
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<td>I think this is going to take me a while because there is a lot of talking at the same time, and I keep having to back up and listen to the same segment several times to catch all of the comments. I think it is really interesting to hear Sandy’s description of her videotaped lesson just after it happened (I think two days later, or even just one), because so far I agree entirely with what she says she saw. We haven’t reached the conclusion yet. Earlier in the discussion, there was still this vibe of not being sure whether to accuse or trust the teachers at the other grade level in terms of whether they were really doing what was best to help students learn – either whether first grade is preparing them well or whether fifth grade is doing things that are beneficial and not harmful. They are asking each other questions but sort of talking around this issue, which I think is OK because at least they are willing to broach the issue; I don’t think there was</td>
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enough familiarity among them to talk extremely candidly about their concerns. During this segment, Sandy also talked about how at their workshop at the county, 11 teachers in a room spent over an hour talking about one fraction division problem and how to model the answer in order to prove that the algorithm worked. She wondered how this would be possible in classrooms with students. This discussion led into her comments about her own classroom because I asked how do teachers handle that…

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<td>Dec. 16, 2012</td>
<td>Continued transcribing Group Discussion 2</td>
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| Dec. 28, 2012    | Continued transcribing Group Discussion 2  
QC, QD  
The teachers were talking about assessment when I picked this up again today, and they talked about the primary critical thinking rubric and how it could be used to assess the SFMP specifically, with few changes, at grade 5. They got into some discussion about PD also (after a question of mine), particularly about how grade 5 PLC will operate with 8 people instead of 4 if they really want to all be teaching essentially the same things at the same time. They also talked a bit about the “intervention blocks” that they had heard were coming but didn’t have much info about yet. Carrie said that she understood that all teachers in the grade would do this at the same time so that in theory, people could switch kids and use flexible groups. It was noteworthy to me that my question about PD was not really answered well at first; I had to come back to it again because the first time the discussion turned almost immediately to practical matters of implementation, somewhat based on the idea of common planning. Is this because we are not used to PD being a part of the school day and year? (PD as opposed to PLCs – which I would argue are not one and the same) |
| Dec. 29, 2012    | Continued transcribing Group Discussion 2  
QC  
The teachers were talking about text resources when I picked this up today, and how Investigations may not be always very helpful in providing the content that is needed for the standards. I even commented on this |
based on what we had found in Westminster. They were just hoping to have some resource that was even fairly closely aligned – and to have it for the following year. Ray and Sandy mentioned the fact that they would have started their own PD earlier if they had known they would be self-contained. Carrie felt that language arts PD would take first priority for grade 1, but she felt that that connected fairly well to the SFMP. I basically gave up on the PD question at this point and asked them each to say what they thought their next steps would be in thinking about the SFMP. Carrie talked about learning how these standards would work in kindergarten; Sandy mentioned that she wanted to work on implementing the standards and retraining herself and her students to think about math in these ways, and Ray mentioned finding good resources to use (which is where I stopped for today). It’s just interesting to me that none of them really had much to say about PD (other than that they would all like to go to a Singapore math workshop) and kept coming back to the resources and how these standards were going to work with students in their classes.

Ray mentions looking for good resources but then really talks more about mapping and trying to figure out how everything will fit into the year, including the old standards that the leaders were saying wouldn’t have to be taught but that the teachers were figuring would have to be in the long run. They discussed bell work for a bit, at which point Carrie had to leave (she had brought this up because the HS did it). I mentioned that this kind of review could also foster discussions about methods (the SFMP), and Sandy agreed because that’s what happened in her second lesson (with which I agree, having seen the video). Ray asked about the old problem solving strategies, which I said I felt fit with the SFMP as long as we were working to get students to choose the strategies for themselves rather than being told which to use. The conversation with Ray and Sandy finished with some logistical discussion about the next/last steps in the study. Once again, my main thought here was that Sandy really did have some good reflection about what
happened in her second video, and Ray does tend to be a bit scattered in jumping from one idea to another. But I think this is partly because he wants to support all of his new colleagues who are coming into math, and there is just a lot to think about in terms of doing that. As I go back through this transcript, I need to especially look for what I think the three teachers learned from each other in having this discussion that they might not have learned in just studying the standards alone.

Read through transcript of Group Discussion 2 and marked for initial categorizing

Jan. 6-8, 2013

QA, QC, QB

I used my typical marking system as I read the transcript. I made a few notes here and there about a larger idea that was beyond the specifics of what was being said… trying to think about what teachers focus on when they have these discussions and what they do not focus on. There was a great deal of talk about the OAA and what administrators wanted (well, that is, from the grade 5 teachers), and a lot of talk from both grades about needing a good, new resource to teach the CCSS. Carrie referred several times to her children’s experiences in math. Ray continued to bring up timelines and who was doing what when. He is not always as correct about details as he might appear to be; I’m not sure if he knows this (and is still talking to appear knowledgeable) or if he really thinks he is right. I found that the fifth grade teachers were very interested in how first grade assesses critical thinking skills, and Carrie was offering some suggestions about how the fifth grade teachers might incorporate the SFMP into their classwork on a daily basis. Once again, even by a comment that Sandy made when I asked about next steps for each teacher, it is clear to me that at this point, the most potential for change in practice lay with Sandy because she actually talked about retraining herself and the students. Ray and Carrie talked more about refining what they already did. I do think that they have probably been closer to the SFMP goals before, but it is interesting that they don’t seem to see as much of a need to reflect on/develop their practice. Well, this is really Carrie – Ray does seem to see a need for improvement in what he does. Is it so bad, though, to know that you are
<table>
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<th>Color coded transcript of Group Disc. 2</th>
<th>Jan. 10-11, 2013</th>
<th>doing well and be able to justify this? I am not sure, though, that Carrie is doing AS well as she might think, as I have written before.</th>
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<tbody>
<tr>
<td>Began to organize comments from Group Disc. 2</td>
<td>Jan. 11-12, 2013</td>
<td>One thing I noticed while doing this is that Ray talked a lot during this discussion. He had far more dialogue than either of the other teachers and probably more than I did. I think this is partially a nervous habit.</td>
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<tr>
<td>Finished organizing comments from GD 2; began to group them into episodes and make more notes</td>
<td>Jan. 13, 2013</td>
<td>QD, QA, QC Since I wasn’t able to ask a whole lot of questions during this discussion, I haven’t done a whole lot with trying to separate the discussion into episodes yet. I have basically just been working to get the relevant dialogue put together in sequence, and I am making a few notes as I go. When I have all of the comments included (I’m about halfway through now), I will go back through, make more notes, and separate the discussion into episodes by topic.</td>
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<td>Finished grouping GD 2 comments by episode; started making more notes, particularly about when the teachers alluded to the</td>
<td>Jan. 15, 2013</td>
<td>QD, QA, QC Which of the three SFMP do the teachers refer or allude to most? And why might this be the case? I spent a good amount of time today making notes about when the teachers alluded to the SFMP. So far, there are almost no references to SFMP 7, and we have the most discussion about SFMP 3. SFMP 1 is referenced every once in a while. However, when I say “referenced”, I really do mean “alluded to” because the teachers virtually never actually use the words...</td>
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“Standard for Mathematical Practice” or refer to the numbers of the standards. So they are taking the ideas from the SFMP and trying to incorporate them into their thinking, but perhaps in a relatively general or holistic way. PD might have to work on helping teachers to think more carefully about each, which we have done some of in Westminster. As I make my notes, I am trying to think about the perspectives of the teachers: what they focus on, what their primary concerns are, what matters to them in terms of everyday implementation and implementation over the course of a year. As might be expected, these vary substantially between Carrie and Ray/Sandy because of the different academic, social, and logistical concerns at each grade level.

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<tr>
<th>SFMP</th>
<th>Jan. 16, 2013</th>
<th>QC</th>
<th>Jan. 18, 2013</th>
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<tr>
<td>Finished making more notes in the comments from GD 2, particularly about when the teachers alluded to the SFMP</td>
<td>QC</td>
<td>Transcribed first part of Carrie’s third interview</td>
<td>QC, QA, QD</td>
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<tr>
<td>Jan. 16, 2013</td>
<td>QC</td>
<td>Jan. 18, 2013</td>
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<td>It definitely seems that the teachers see resources as part of PD – or at least Ray and probably Sandy. Carrie… perhaps. I’m not sure that any of them have experienced enough quality PD in math to have a way of saying what ought to be included in it. I made a lot of notes in the other document, so I don’t think I have much to add here right now.</td>
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<td>Carrie was definitely more concise in her answers during this interview; she was in her classroom getting ready for the school year to start. She was going back to kindergarten from first grade, so she was really thinking about both grades as she was talking. This added a dimension to her comments that hadn’t been there before, though I’m not sure that it really changed her basic interpretations all that much. Her ideas about what the various statements in the CCSS meant really hadn’t changed much since the first interview… which lends credence to the idea that unless these are discussed directly with other people, or perhaps unless something in class causes a teacher to reflect on an idea in text, the teacher may tend to stay with her initial idea about it. However, I do wonder if this might change a bit if I waited a year or two to ask. The time frame for this study was eight months.</td>
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<td>Jan. 19, 2013</td>
<td>Carrie made a point about the word “represent” being hard for children to understand – could this at all be connected to the fact that representing, as a skill in itself, is hard also?? We know that young children often struggle to allow something to represent something else, particularly in math when this means we are getting fairly abstract… hmm… an interesting linguistic circle. Hard to represent, hard to understand the word “represent”. What about different kinds of representations that we need through the various grades? I did bring this up in the interview after Carrie’s comments about not drawing (or elaborating in this way) so much in second grade, and it did look a bit like she had an “aha” when I said this – like the visual representations don’t just go away permanently; we need them with each new idea. Was this something she had not thought about before? I’m not sure. When Carrie was talking about students’ understanding correspondences between approaches, I felt more as if she were talking about kids seeing parallels among types of problem situations, not so much the approaches to solve these problems. I guess in kindergarten especially, I might look more for kids to see the similarity between counting by rote, on paper, or using objects to count to solve a problem.</td>
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<td>Jan. 20, 2013</td>
<td>QA, QD</td>
<td>Do all kids really understand it (and are able to apply it) just because they agree with someone else who says it? I think Carrie gets caught up in thinking this sometimes, as probably all teachers do at one time or another. There is a huge difference between agreeing that what someone says sounds logical and trying to reproduce it either in our own way in the same context or in an entirely different context on our own. I think Carrie knows this, but sometimes I am not sure that she applies it consistently. Maybe she sees and catches more of this when she works with individuals, which I know she does often. Carrie focuses on two specific examples in SFMP 7, in the statement about “Young students”. Sandy did this also, and I can’t remember if Ray did – I think he may</td>
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have. I am not sure why the authors gave these two examples when they basically left examples out of most of the rest of the SFMP. Unfortunately, when we write standards like this, we walk the line between two vague and too specific – too vague, and no one will know what is meant; interpretation will be too individual (as we have already seen in other parts of this study, and as we know from research). Too specific, and people may focus on those details and leave out many other important ideas in their interpretation process. I suppose it would be important for the authors to say something like, “Many other examples related to other mathematical topics can also be applied to this statement,” or something like that.

### Continued transcribing Carrie’s third interview

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<td>Jan. 21, 2013</td>
<td>QA</td>
<td>I loved Carrie’s idea about using students’ height as an avenue toward their understanding of how we must measure consistently with units of measurement. It is totally relevant to them and gets across the point well. She is so good of thinking about things that mean something to first graders. In this segment, she also mentioned that she felt that the three SFMP that I had chosen really fit into basically every lesson.</td>
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### Continued transcribing Carrie’s third interview

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| Jan. 22, 2013 | QC, QD | Carrie feels that math should not be stressful – this would be a factor in implementing the SFMP. She talked about a math magazine for girls that she had seen on the Today show. She brings up the idea of practicing to get better rather than practicing to win. (with math facts, especially) Carrie feels that teachers need time to share and listen to each other – an hour vs. 20 min. She did not mention anything about a protocol or agenda for doing so, and I do wonder if sometimes these conversations are best when they just “happen”, or perhaps when they are prompted by a specific, immediate need. Or, if they are scheduled, then does there need to be an agenda pre-set by the participants so that the discussion does not veer too much? And what happens if it does? How can teachers determine if their conversation has been productive – for the purpose of improving instruction? How much of this can be venting and how much should specifically be about practice? Is the
venting helping to support the disposition for teaching, or is it hindering it? Or does it depend on everyone’s response to the venting (including that of the venter)? Carrie mentioned math announcements as a way of having more math incorporated into the school day, but she used an example of a real problem related to the school (she did not say this, but I think she meant not just a made-up problem that doesn’t apply to the actual kids).

She also noted that she thought it was more helpful when PLCs could work to tweak something they (or a teacher) are/is doing based on each other’s experience rather than trying/starting something totally different that someone from outside suggests.

Facts vs number sense – how do we reconcile this dichotomy… she gave a good example of how we could apply number sense to a fact at the later grades… and I agree with her that knowing facts does not constitute number sense. However, I also believe that in the intermediate grades and beyond, a fairly fluent knowledge of the facts is fairly vital to recognizing multiplicative and proportional relationships, which are the foundation of SO much of what we do in real life with math. Perhaps the emphasis, though, should not be on who learns the facts the most quickly, but who continues to improve. It struck me that she said that kids get their names on the wall for fact knowledge… so (as she said) is this what kids think means being successful and knowing math in those grades? What else could we put kids’ names on the wall for?

Finished transcribing Carrie’s third interview Jan. 24, 2013 QB, QA, QC

What Carrie says here about people of the traditional era is somewhat different than what she said earlier about this. She had said before that she thought teachers could teach the content standards with worksheets. But she is saying here that she doesn’t think the SFMP could be taught in this way. While talking about her initial question about why we were talking about the same questions in all of the interviews, she said she realized, “Hearing the same answer more than once really makes it true.” I think this does hold true to the qualitative methodology… if someone says the same thing over a period of months,
you can start to believe that they really believe this (as she says) and are not just “saying” it. Interestingly, though, this does not cover the idea that beliefs might change… though I suppose you could also see that happening based on what the person said overall. Her beliefs did not especially change, but she said that the study was affirming to her because now “they” who write standards are affirming that what she feels is important in her classroom is important, in fact (at least according to the authors of the CCSS). At the end, we talked about the idea of teaching a few core principles each year that might stay the same throughout the grades – really, this is like the International Baccalaureate (IB) concept. I brought the idea of same and different up, and then she expanded on this, and we talked about how that might look at different levels. I wonder if this is something that all teachers ought to think about… could all teachers at a grade or in a course agree on this list? Maybe not 100%, but they could perhaps agree on a majority of the items on the list that were important to emphasize throughout the course. These might be the SFMP. They might be the IB principles, or they might be the 5 things Carrie and her principal had brainstormed. But a very small set of ideas to guide learning and help students feel that their learning in all areas is connected to the larger world of learning beyond this classroom (as well as being connected within the classroom).

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<th>Began the first read-through of Carrie’s third interview</th>
<th>Jan. 25, 2013</th>
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<td>One thing Carrie mentions several times during this interview is the idea of “known” to “new” from Reading Recovery. In fact, she draws a number of parallels between the way their school teaches reading and the way the new standards say we should be teaching math (which she feels she basically does now). She relates most of what is in the SFMP to language (including turn-and-talk) and how it is a key element of learning in her classroom. She again discusses the idea of perseverance and that this basically comes from a child’s having confidence to move forward because of the teacher providing opportunities appropriately. She did have a few interpretations of some of the SFMP statements that I</td>
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thought were not quite what the authors meant, but of course I am not sure that I am right either! She believes, as I do, that patterns and sorting should still be a part of K and 1.

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<tr>
<th>Date</th>
<th>Notes</th>
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<tbody>
<tr>
<td>Jan. 26, 2013</td>
<td>Finished the first read-through of Carrie’s third interview and started organizing her comments by question</td>
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<tr>
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<td>QD, QA, QC</td>
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<td></td>
<td>As I was reading how being in this study affected Carrie (well, more like how she felt this kind of work would be useful for other teachers, since she does not seem to see a need for change in her practice), I thought that I should try to draw on these questions both directly and more deeply to suggest implications for professional development related to the SFMP. Two things I noticed in particular: Carrie really does draw on her experience as a mother, watching her own children go through the district and learning how they experienced mathematics classes, as she tries to understand what does happen beyond grade 1. She points out that not all teachers have this opportunity. How do they form perceptions of what older grades do if there is little or no vertical communication? (from their own school experience?) Also, she makes an interesting point about what kids see as important when we post names on the wall for certain achievements (usually related to math facts). My question – What if we posted names for other good participation related to the SFMP?</td>
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<th>Date</th>
<th>Notes</th>
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<tr>
<td>Jan. 27, 2013</td>
<td>Started organizing Carrie’s third interview comments by question</td>
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<td>QA, QB</td>
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<td>Carrie speaks very concisely and consistently about her classroom, which definitely makes me believe that she believes very strongly what she says and that she tries to live by her beliefs as she teaches. Again, this makes me wonder what it would take to convince her that a change might be warranted… but I suppose this would be a realization that students weren’t understanding something as she thought they were. For instance, her surprise about students’ not understanding the word “represent” caused her to think about a way to alleviate this confusion. She does brainstorm reasonable solutions to classroom “problems” fairly effectively (like in the hypothetical situations).</td>
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<th>Date</th>
<th>Notes</th>
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<tbody>
<tr>
<td>Jan. 28, 2013</td>
<td>Continued organizing Carrie’s third</td>
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<tr>
<td></td>
<td>QC</td>
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<tr>
<td></td>
<td>I made a lot of notes in the compilation file tonight… tried to think about my 4 research questions as I was</td>
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</table>
interview comments by question

| doing this. The last comment was noteworthy – Carrie called the authors of the standards “the grand ‘they’”, and I thought this was an interesting characterization of the people who often write policy and/or require the enactment of it. How does this affect the actual enactment by teachers and others in schools? |

| Finished organizing Carrie’s third interview comments by question; started transcribing Ray’s third interview | Jan. 29, 2013 | QA, QC, QD | Made some last (initial) comments on Carrie’s interview responses. So much of what she says throughout the study is that students need time to think; they need opportunities to discuss and learn from each other; they need to make connections among ideas. Also, teachers need time to discuss what is happening in their classrooms, relative to standards and other expectations. In Ray’s interview so far, we have talked about a student who presented an idea that was different from everyone else’s in Ray’s second lesson video (and how he really wants his students to be kind and respectful to each other and to listen to each other’s ideas). We have also talked about the problem that Sandy and he shared during the second group discussion… it was a task from an inservice with the county consultant. Teachers really struggled to model a fairly “basic” (seemingly) fraction division task. I really need to go through the new content standards more carefully myself! Teachers keep asking about them, and I don’t always know the answers to their questions, even at a basic level. Soon! |

| Continued transcribing Ray’s third interview | Jan. 30, 2013 | QC | Ray spent a lot of time talking about what he thought his role would be in the implementation of the new standards, and also showed me (I paused the tape) the new Investigations texts that they had gotten, which he talked quite a bit about on tape also (the texts in general, as he has in the past). He also talked about integration and how their times for each content area during the day are changing – another logistical concern for teachers that may not change the way that they teach but will certainly affect how much they can do in a day. He noted again that he didn’t think the CCSS could be implemented without the SFMP. He said something about CC arts standards, but I think he |
meant the new Ohio fine arts standards. Here is another example of how teachers (Ray in particular) take a bit of information they have and translate it into something that they think they know… and then perhaps pass it on… all the while not really knowing the exact facts. I don’t know that this can ever really be entirely avoided, but more easily accessible sources of information, as well as ongoing, consistent PD, will help with this in implementing the CCSS.

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<tr>
<th>Continued transcribing Ray’s third interview</th>
<th>Jan. 31, 2013</th>
<th>QA</th>
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<tbody>
<tr>
<td>Ray does tend to tell a lot of the same stories multiple times – which I know means that they are significant to him. Singapore, growing up the “strange little kid” in math, Spencer Kagan, and beliefs like all kids are important in his classroom and have valid ideas. He sees problem solving as the crux of the standards, and he refers to Singapore math as a way to supplement Investigations for this. I am not sure that the problems in Singapore math (at least that I’ve seen) are any more rich than the problems in Investigations. He also talks more about the three levels of understanding (concrete, visual, and abstract – at least in general terms) and how he feels this links to the way that the standards are designed.</td>
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<tr>
<th>Continued transcribing Ray’s third interview</th>
<th>Feb. 1-2, 2013</th>
<th>QA</th>
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<td>Ray talked a lot about his beliefs about students with disabilities and how he believes that they should be as fully integrated into the learning in the regular classroom as possible. He wants to be able to move these children forward, and he gave an extended example of one student who made progress in his class last year but too late to do well on the OAAs. He believes that learning in general should be as authentic, relevant, and integrated as possible, and he wants to do more of this in the future. He refers to the BAM (Balanced Assessment in Mathematics) tasks as quality, and he talks about wanting his students to do more proofs for their solutions in the future (showing each other and critiquing as well).</td>
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<tr>
<th>Continued transcribing Ray’s third interview</th>
<th>Feb. 3, 2013</th>
<th>QA</th>
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<td>Ray kept coming back to the idea of proof, using various manipulatives to work through problems and prove solutions, and that this is something all levels of</td>
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students should be doing. He mentioned that he would like to try blogs in order to help students learn to share and discuss their work with others, which he says is a skill that has to be modeled for students. He commented that the “overlying theme” in the SFMP is solving problems in more than one way and then proving that these solutions are logical.

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<tr>
<th>Continued transcribing Ray’s third interview</th>
<th>Feb. 5, 2013</th>
<th>QA, QC</th>
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<tr>
<td>In this segment, Ray talked at length about how he wanted to use different manipulatives in his class, saying that he often used them but that he wanted students to have more daily access to them and use them as they needed to do so. He also talked about teaching problem solving strategies, both from Singapore math (which he wanted to do first) and then the oft-used list of strategies that have been taught for years. He felt Singapore would make more sense to kids, and I found this interesting… I wonder if he felt this way because it is more prescriptive. He was looking forward to students coming over the next few years that had grown up with the CCSS and the SFMP. He spoke some about “playing” with numbers and how he thinks we need to do more of this in school, to help students enjoy math. I thought it was interesting how he talked about students shifting perspective either by looking at their own work or at someone else’s thinking. He also pointed out that teachers (he) can push this shift by posing additional questions about the same problem to extend it in various directions. He used an interesting analogy of a given problem being like “clay” that could be reshaped in different ways. I like this! It gives an interesting picture of a problem not being a static object but something that can be acted upon and changed in order to create something new. (and learn something new)</td>
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<tr>
<th>Continued transcribing Ray’s third interview</th>
<th>Feb. 6, 2013</th>
<th>QA</th>
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<td>I’m not sure that what Ray said about the last part of SFMP 7 really fit with the authors’ intent. Ray seems to have a perspective similar to Carrie’s about how to encourage perseverance, noting (among other things) that students should feel they have some ownership and control in their learning. This reminds me of a segment I heard on NPR this morning about how a lot of</td>
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research has shown that companies and businesses can improve their output and quality if they really ask and listen to their workers about what would make the job easier and able to be accomplished more effectively, since they are the ones who are actually doing that work. (This would apply to teachers as well, of course.) Ray sees SFMP 7 as involving helping students to see other students’ structures that have been found as well as their own. I found his comment interesting that teachers have to rediscover ideas every year with their students (at first he just said discover the ideas along with them – and I really agree with this at least as much because we really have to discover what the ideas mean to the kids – not what they mean to us – or at most, rediscover or adjust what they mean to us). Good insight! (though I’m not sure he is thinking about this in such epistemological terms) Ray’s example for the first hypothetical situation resembles Carrie’s ways of thinking as well, because he talks about using a lit book to draw students into an interesting problem context involving the skill/concept at hand – then teaching based on this. This reminds me of Marilyn Burns saying (in “What are you Teaching My Child?”) that a context can be imaginary as long as it is interesting to the student.

| Continued transcribing Ray’s third interview | Feb. 7, 2013 |
| QA, QC |
| Ray continues by describing how in both classroom episodes, he would start with a lit book, use manipulatives, then guide students to use pictorial representations and work with larger numbers as well. He refers to the “structure” again (after both situations) and the “progression” (after the one about decimals). He feels like he has enough resources because he has bought a lot of extras for himself over the years. He is a bit worried about having to help colleagues who come to him this year because he believes they will come for help in math, science (for which he is the lead teacher in fifth grade), and even reading. He notes that this takes time away from his own work, though he loves helping people. In a larger picture, this may be an implication for PD… if only certain people are trained in a given practice, they can help others to a point but can’t really help the others learn it directly. |
And assuming that this can be done may devalue the PD itself. (my thought – though it came from his statement about trying to condense a week-long workshop into 15 minutes)

<table>
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<tr>
<th>Finished transcribing Ray’s third interview</th>
<th>Feb. 8-9, 2013</th>
<th>QC, QB</th>
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<td>I’m not sure that Ray said much of anything new in these last segments of this interview. He did talk about some potential conflict between all of the new schedules and logistics that were being mandated by the administration and trying to enact the new standards well. He felt that the principal and superintendent probably didn’t have a very good handle on what the goals of the new standards really were, or what it would take to enact them. He basically expressed in many ways how he agrees with what is in the CCSS and that he knows he will need to change some of his practices to achieve the goals therein. He talked a bit about how he believes that we need honesty in professional meetings to resolve big issues and that we can’t hide from them (as he felt there was some candor in the group discussions about fact fluency… though probably still some diplomacy also, in my opinion). He reiterated that they would need to look at assessing the content and practice standards and that the Singapore math workshop was really eye-opening for him, at the right time.</td>
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<tr>
<th>Began initial organization of responses from Ray’s third interview</th>
<th>Feb. 10-11, 2013</th>
<th>QA, QB, QC</th>
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<td>On the 10th, I read through most of the 59-page interview and marked as usual – marked any significant comments (most of them in one way or another, excluding some comments about textbooks or other subjects that really had nothing to do with the math CCSS or implementation of them), emotions, comments that showed professional learning or reflection, and comments that had been repeated a lot. I finished this reading and marking on the 11th and began copying and pasting comments from the transcript into the document where I had organized Carrie’s comments by question. I am recognizing that the comments in bold (from all participants) will give me a good place to start my outlines. As always, Ray really repeated himself a lot, so I am getting better at including comments that are repeated in different parts</td>
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of the interview but not comments that are repeated immediately, because I think the immediate repetition is generally just a nervous habit. But if he says these things at different times, I do think they are particularly significant for him.

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<tr>
<th>Continued initial organization of responses from Ray’s third interview</th>
<th>Feb. 12, 2013</th>
<th>QA, QD, QC</th>
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<tr>
<td>It occurred to me this evening that as I spot what seem to be expressions of beliefs about teaching and learning (and/or mathematics) in the teachers’ comments—beliefs that may affect their practice significantly—I should make note of that. So, I’ve started to do this. This would certainly tie into my lit review. Personal experience and state testing are other things that I am noting when they are mentioned—because they are mentioned a lot. Also, the ideas of flexible grouping, students proving their ideas and critiquing each other in a respectful way, and the idea that all students (at all levels) deserve to have quality learning opportunities that (in my words) show that we value them and their learning as much as we do for any other student. We need to give them every opportunity to learn because many of them can succeed at high levels when we do this.</td>
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<th>Continued initial organization of responses from Ray’s third interview</th>
<th>Feb. 13, 2013</th>
<th>QD, QA</th>
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<tr>
<td>I was thinking this evening that I ought to think about what it means when I make two different kinds of marks on the same comment. For instance, if I mark the same comment as bold and an emotion, that means this is an emotion that the teacher has expressed a lot, which is definitely a frequent occurrence in Ray’s case. If I mark a comment as bold and a learning experience or reflection, that means that it is a significant learning experience because the teacher mentioned it multiple times, and it probably has impacted his/her teaching (or least thinking) quite a bit. If I mark a comment as an emotion and a learning experience, this might be pertinent in understanding why or how the experience affected the teacher in the way that it did (if emotion was associated with it). As far as Ray goes… I continue to notice how quickly he can generate a plan for teaching a given topic, and I am also noticing more that some of his comments are similar to those of Carrie and Sandy. (specific comments)</td>
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<td>Task Description</td>
<td>Date(s)</td>
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<tr>
<td>Finished initial organization of responses from Ray’s third interview</td>
<td>Feb. 14, 2013</td>
<td>Not much new to say… looking forward to getting into more analysis and then some outlining.</td>
</tr>
<tr>
<td>Began writing additional notes about Carrie’s and Ray’s third interview comments</td>
<td>Feb. 16-17, 2013</td>
<td>QA, QC, QD I am going back through Carrie’s and Ray’s comments in the third interviews and starting to add notes about how what they said answered my questions (particularly, how they are interpreting the text in the CCSS/SFMP). I add other notes as needed to focus on noteworthy elements of their comments. (I know I will need to go back through eventually and pull out the boldfaced terms and phrases because these are the things that they really talked about a lot, whether or not they seem to directly relate to my interview or research questions.) I always have to think pretty carefully, as I make these notes, about a) how the teacher interpreted the text – and whether I agree with this interpretation or not – knowing that my perspective is not necessarily more correct than his/hers – though it might at least be more informed, b) what parts of the text he/she focused on, c) what parts of the text were ignored, d) examples from the classroom that were used, e) other comments the teacher offered that were not especially related to the text - because they all may be important somehow in understanding the teacher’s perspective and in informing the design of professional development/support related to the CCSS/SFMP.</td>
</tr>
<tr>
<td>Continued writing additional notes about Carrie’s and Ray’s third interview comments</td>
<td>Feb. 18, 2013</td>
<td>QA, QD It is getting easier to do those things that I listed above when I examine each response. Carrie and Ray, by this time, are often saying the things that are most important and central to their ideas about the standards and about teaching, and it is interesting to see how similar many of their comments are. It does make me wonder how representative of the rest of the Sealon staff they are. Ray does seem to be better at bringing the SFMP into his analysis naturally – or at least without my having to ask. I usually had to ask Carrie to relate what she was saying to the SFMP. Is this because the ideas are so ingrained in her that they seem totally second nature now, or is it because she may...</td>
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<td>Event Description</td>
<td>Date</td>
<td>Comments</td>
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<tr>
<td>Finished writing additional notes about Carrie’s and Ray’s third interview comments</td>
<td>Feb. 19, 2013</td>
<td>QD, QC, QA</td>
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<tr>
<td>Began outline of Ch. 4</td>
<td>Feb. 20, 2013</td>
<td>QA, QB, QC</td>
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information from various phases will appear together in
the report (assuming this doesn’t change), because
again, I think that will make more sense to the reader
than having to say again and again, “And I saw more of
the same in phase X.” The point is to provide data to
answer the research questions, not to present
chronologically the information gathered during all 7
phases of data collection.

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<thead>
<tr>
<th>Continued outline of Ch. 4</th>
<th>Feb. 21, 2013</th>
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| QB                        | I got a response from Mike B about putting the
                          comparison/contrast among teachers’ thinking in
chapter 5 (probably) because I will be discussing it
more informally, not with a thorough analysis. I
thought this made sense because I also realized that the
comparison/contrast is not part of my research
questions. This made me think, though, that I probably
do need to address the evolution of teachers’ thinking
fairly carefully in chapter 4, since this will directly
address my second research question, so I worked that
into the outline today. I am a bit puzzled right now
about how to address the changes in teachers’ thinking
over time… but actually, I would say that most of their
interpretations really did not change that much
throughout the study, so for those that did, I could
address this as I talk about each SFMP. If they said
basically the same thing about a given statement
throughout, there is no reason to repeat it three times.
Perhaps I need to say up front that I will not note
changes in thinking except when they occurred.
Tonight, I wrote notes in the outline for each teacher’s
background and prior experience with the
CCSS/SFMP. |

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<tr>
<th>Continued outline of Ch. 4</th>
<th>Feb. 22-24, 2013</th>
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| QA, QB, QC, QD            | I have decided to work on Sandy’s outline in detail,
                          then send this to my committee to get their feedback on
the general format to see if I need to change things
before I work on the other two teachers. The outline is
getting more complex as it goes, but that’s OK (and
expected). I am trying to condense in a coherent and
logical way, which is meaning that I need to work
between the various phases of data for some parts of
the outline. Right now I am finding that I can briefly
comment on how Sandy’s perspectives changed (or did |
not) from one interview to another, but I know that I also need to incorporate what happened between the interviews. Maybe what I need to do is lay out all of Sandy’s data, in chronological order, and examine it that way to draw some conclusions.

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<tr>
<th>Continued outline of Ch. 4</th>
<th>Feb. 25, 2013</th>
<th>QD</th>
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</table>
| Today, I continued putting together the parts of the outline that had to do with Sandy’s interpretations of SFMP 3 and 7 in the interviews. Tomorrow, I really need to go through Sandy’s entire set of data, from start to finish, and just code for all references to SFMP 1, 3, and 7, along with the other topics in the outline that I have not addressed yet. Then, I can complete her part of the outline (at least the initial draft!) and get it sent to the committee for feedback.

<table>
<thead>
<tr>
<th>Began actually coding Sandy’s Int. 1</th>
<th>Feb. 26, 2013</th>
<th>QD, QC</th>
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</table>
| I put together a long list of codes based on the outline that I had created so far, just for Sandy’s section of the chapter. Then, I printed this, and I started at the beginning of Int. 1 and coded every comment she made. I added quite a few codes to the outline as I progressed, but I am finding that I’m needing to add fewer as I get further through the interview. I can use most of the codes that I’ve already created. Even though, since the codes are still organized in an outline, the outline itself would be a bit disorganized right now (not a good progression of topics in some of the sections since I added codes at the ends), I think that I will be able to go back and rearrange the outline to get a good flow of topics. This is helping me to organize all of Sandy’s comments besides just the ones that specifically related to the SFMP, which is what I was doing with the outline through yesterday. I feel pretty good about what I did tonight; it feels like I’m making a step in a good direction. Also, I think that once I finish coding all of at least Sandy’s comments in each large file, I will create a third column (since the file is one large table), number every row in the table, and then sort the table by codes so that I can easily organize the comments and ideas. I should save one copy of the file where I haven’t resorted the rows, just as a backup (even though I could just resort by the row number to get it back to the original form). The sections where I
added the most codes tonight, beyond what I already had in the outline, were the sections having to do with implementation issues and other thoughts that hadn’t been mentioned yet. (all in section F right now, but I might end up separating this into at least two sections) I will finish as much of this as I can before Friday and then send the big outline, at least one file with my coding, and my list of codes to Dr. Mikusa.

<table>
<thead>
<tr>
<th>Continuation of coding: Sandy Int. 1 and Sandy GD 1</th>
<th>Mar. 3, 2013</th>
<th>QD, QC, QA</th>
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<tbody>
<tr>
<td>I added a number of new codes to the list this evening, mainly about all of the “other” issues that Sandy talks about related to her circumstances of teaching mathematics (and her feelings about them). It will be interesting to see how many of these codes overlap with the other teachers’ comments and how many more codes I have to create for the others. It will be interesting to see how the codes come and go throughout the data… what was each teacher really focusing on at different points in the study, and why? I may not be able to answer that second question, but I can perhaps speculate if there is evidence to support my theory.</td>
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<tr>
<th>Met with Dr. Mikusa</th>
<th>Mar. 4, 2013</th>
<th>QD</th>
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<tbody>
<tr>
<td>I didn’t add too many more codes after I met with Mike… I think the list is now fairly solid for Sandy. It will be interesting to see if I need to add many more for Ray and Carrie when I delve into their data.</td>
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<thead>
<tr>
<th>Finished coding Sandy GD 1 and Int. 2</th>
<th>Mar. 5-7, 2013</th>
<th>QA</th>
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<tbody>
<tr>
<td>I am finding that as I code the lesson video transcripts for each of the statements in the SFMP that I studied, there are a few of them that are often repeated and several that are almost never used. This tells me that Sandy really focused more on particular elements of the SFMP (intentionally or otherwise) and is really disregarding some parts of them. This would be good information for PD. Also, some of the codes seem to match or go together (I find I use them a lot together), and I plan to make a list of these correspondences among codes. Once I do this, I think I am ready to start putting together a second draft of the outline for Sandy, now that I have literally gone through (again) all of the</td>
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data I have from her. I am going to organize this section according to what Mike suggested – one major section for each SFMP and then a review at the end of her ideas/comments about the SFMP in general.

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<thead>
<tr>
<th>Date</th>
<th>Description</th>
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<tbody>
<tr>
<td>Mar. 11, 2013</td>
<td>Made a list of codes from the SFMP that align in Sandy’s lessons, listed the few statements that are not enacted in one or both lessons, and organized all codes from Sandy’s interview 1 and group discussion 1</td>
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<tr>
<td>Mar. 12, 2013</td>
<td>Continued organizing codes from Sandy’s data (Int. 2)</td>
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<tr>
<td>Mar. 13, 2013</td>
<td>Continued organizing codes from Sandy’s data (Gp. Disc. 2 and lesson 1)</td>
</tr>
<tr>
<td>Mar. 14, 2013</td>
<td>Continued organizing codes from Sandy’s data (lesson 2)</td>
</tr>
<tr>
<td>Mar. 11, 2013</td>
<td>QD I really feel like this outline is starting to fall into place. One of my next steps (soon) needs to be to put all of the codes for Sandy in a logical order for a new outline. I think I will need to do this once I organize the data by code from the 2nd interview, 2nd group discussion, and the two lessons. I think I will be able to do most of this tomorrow. Once I’ve done that, I should be able to start writing Sandy’s section of Ch. 4. One thing I will need to remember when I am writing is that I have a lot of commentary in the data that did not transfer into the sorted lists of codes because that commentary was not coded.</td>
</tr>
<tr>
<td>Mar. 12, 2013</td>
<td>QD I did what I did yesterday, getting all of Sandy’s interview 2 codes sorted by code. I was hoping to get more done tonight; now I need to do this with her codes from the second group discussion and the two lessons, but I don’t think those will take quite as much time each. Then, after that, I need to put the codes in a logical order, reorganize the Sandy data file in this order, with each code entirely together from each of the 6 phases of my data from her, and start writing.</td>
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<tr>
<td>Mar. 13, 2013</td>
<td>I did what I did the last couple of days, getting all of Sandy’s codes from the second group discussion and the first lesson sorted by code. I need to do the second lesson tomorrow, then put the codes in a logical order, reorganize the Sandy data file in this order, and start writing.</td>
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<tr>
<td>Mar. 14, 2013</td>
<td>QD I organized the codes from the second lesson. I did start to look at reorganizing the codes, but now I’m not sure that I need to do this. Perhaps I just need to think about writing Sandy’s section based on pages 3-5 and the SFMP, but then there are the questions I asked about implementation and policy. So perhaps I could</td>
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<tr>
<td>Mar. 16-17, 2013</td>
<td>I am probably about halfway done with this task, and then I will really be able to start writing.</td>
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<td>Mar. 18, 2013</td>
<td>QA, QD I think I am now 75% done with this task. I am glad that I have the codes organized in this way because I think it will also help me to make some comments in Chapter 5 about what codes were very dominant or tied together and which were rare or disconnected in general. I am also learning that when I organize Ray’s and Carrie’s data, I think I can do some things to make this process easier and more efficient.</td>
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<tr>
<td>Mar. 19, 2013</td>
<td>I am essentially finished with the outline of my section on Sandy for Ch. 4. The only thing I want to do tomorrow is see if I can stick the 3 pages of code that have not been placed somewhere in the rest of the outline. I am definitely going to have to be concise about what I write because the list of codes (in sequence) is about 80 pages long. But I think I can really start writing tomorrow.</td>
</tr>
<tr>
<td>Mar. 20, 2013</td>
<td>QD, QA I wrote a few pages, not much… I am trying to get as much as I can “right” the first time, though I know I will have to refine and edit. I realize already that I will have to go back to the section on pages 3-5 of the CCSS and write more about how I think Sandy is interpreting the text… it’s just that she said so little about the text, overall, that this will be tricky to do. I am in the middle of working on what she said about the title of SFMP 1. Since she said a lot about perseverance (or the lack thereof) and students’ making sense (or not) of problems, I am trying to figure out the best way to write this while still focusing on her</td>
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interpretation over time. I already have one question about whether a discussion with Carrie should go in Ch. 5 or not.

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<tr>
<th>Continued Sandy’s section of chapter 4</th>
<th>Mar. 22, 2013</th>
<th>QA, QD</th>
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<tr>
<td>Tonight I worked on a few more sections of chapter 4 for Sandy. I am remembering to provide a specific interpretation of Sandy’s interpretation of each statement (I am working on SFMP 1 and worked also on pages 3-5). I do need to go back and include some comments from her reflections, though I only have 2 of them. I feel pretty good about what I’ve written so far.</td>
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<tr>
<th>Continued Sandy’s section of chapter 4</th>
<th>Mar. 23-24, 2013</th>
<th>QD</th>
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<tr>
<td>I am moving forward well. I finished the section on SFMP 3 tonight (having finished SFMP 1), and I will plan to do SFMP 7 tomorrow and the ending section on Tuesday. It would be great to get through a lot of the organization of Ray’s data during the rest of this week so that I can begin writing his section starting early next week. I am finding that the sections on implementation related to interpretation are fairly concise, because there will be more to say about implementation in general, not so much about each SFMP individually. I am doing better about focusing on Sandy’s interpretation, as I see it, and not just her reactions to each statement. One thing I am noticing is that some of my own evaluative thoughts about Sandy’s comments seem not to fit in this chapter because this is supposed to be about Sandy. So, those thoughts will probably go in chapter 5 as I discuss implications of the data. I also have to be careful not to generalize and say things about “teachers” when I am really just writing about what Sandy has said.</td>
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<th>Continued Sandy’s section of chapter 4</th>
<th>Mar. 25, 2013</th>
<th>QD, QB</th>
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| Today I wrote the section on SFMP 7 for Sandy and started the section about implementing the three SFMP in general. I hope to finish Sandy’s section either tomorrow or Wednesday and then move on to starting to re-code Ray’s data. I am still working to condense where possible and not repeat myself, and still working to be sure that I am addressing the research questions. In some ways, it is frustrating to me that her perspectives did not seem to change a great deal during the study, but I think this is really true for all of the
participants and probably would only be different if we had spent a great deal more time discussing and working to enact the standards in their classes. At least there are small shifts that I can discuss, and there is evidence of how these shifts occurred.

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<tr>
<th>Continued Sandy’s section of chapter 4</th>
<th>Mar. 26-27, 2013</th>
<th>I have now written nearly the entire draft of Sandy’s section of chapter 4, just having the concluding paragraph or two to write (a review of how her thoughts seemed to evolve, or not, during the study). I have made a couple of notes for chapter 5 that are at the end of the file. I believe that I have addressed essentially every part of Sandy’s data in one place or another; often as I was writing the latter parts of the section, I would look at the data and think, “I already included that” (mainly because many of the statements were coded with multiple codes). I know that I will want to double-check the organization of the section this week and will need to edit this section once Dr. Mikusa has read it, but I think it is very decent shape for a first draft. Tomorrow I will write the concluding paragraphs, check the organization, and then start organizing Ray’s data for coding.</th>
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<tr>
<td>Finished Sandy’s section of Ch. 4 (for now) and began coding/organizing data from Ray interview 1</td>
<td>Mar. 28, 2013</td>
<td>QD The only thing I haven’t included yet are comments from Sandy’s reflections, but I think I actually have included some of her thoughts because she shared them in the conversations also. I have added a number of codes for Ray, but many of the codes that I already was using are applicable. I am coding each line and duplicating it if I want to use multiple codes so that it will be easier to reorganize the data for the outline. Also, I am making sure to add notes in brackets when the dialogue is vague so that I know to what Ray was referring. Further, I am trying to carefully attend to when Ray was talking about each SFMP and when he was not so that I can be sure to report on his interpretations. I did this with Sandy, but I really did it backwards because I had to go back to her original data and check on when she said certain things.</td>
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<tr>
<td>Finished coding Ray’s interview 1 and GD 1 data; sent</td>
<td>Mar. 29, 2013</td>
<td>See above notes. I am also finding it useful to merge lines of data when Ray said several things in sequence that fit under the same code.</td>
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<tr>
<td>Sandy’s section of Ch. 4 to Dr. Mikusa</td>
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<tr>
<td>Coded Ray’s interview 2</td>
<td>Mar. 30, 2013</td>
<td>I kept most of my added comments in the file, and I think this will make it much easier when I am writing because I’ve really already written a lot that I could use in chapter 4. I coded these also so that they will fall with the matching codes when I organize to write. On to his second lesson and GD 2 tomorrow.</td>
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<tr>
<td>Coded Ray’s lesson 2 and Group Discussion 2</td>
<td>Mar. 31, 2013</td>
<td>QD I think in chapter 5 I will want to include a section where I write some comments about the teachers’ lessons that are outside of (different than) what they said in their own reflections.</td>
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<tr>
<td>Coded Ray’s interview 3</td>
<td>Apr. 1, 2013</td>
<td>I coded Ray’s third interview. Will organize and sequence data on Wed. and start writing on Thursday, I believe.</td>
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<tr>
<td>Wrote Ray’s and Carrie’s sections of chapter 4</td>
<td>April-May 2013</td>
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<tr>
<td>Wrote intro and conclusion to Ch. 4 and sent to Dr. Battista</td>
<td>June 1-2, 2013</td>
<td>I used info from the Yin (2008) book on case studies to justify how I was organizing the chapter and what I was focusing on.</td>
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<tr>
<td>Began organizing notes from dissertation log to create a set of topics to address in Ch. 5 (along with some for Ch. 3-4)</td>
<td>June 8, 2013</td>
<td>I feel that using these notes, along with the notes I included in the working copies of the data files, as well as Dr. Mikusa’s comments, will give me a very good set of topics to address in Ch. 5. I will then need to reorganize these to put them in a logical sequence for the last chapter. Today, I got through the log and the first group discussion. I had almost no notes in the compiled copy of the quotes from interview 1, but all of the notes from GD 1 were added to my organization file for Ch. 5. I don’t think I will use them all, but at least it will help to remind me what the major points were that I was noticing as I was analyzing data, so I can use these to organize Ch. 5 eventually. I assume I will mainly organize it by topic, and perhaps within each topic by person if needed. This is supposed to be the cross-case analysis.</td>
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<td>June 9-11, 2013</td>
<td>Finished organizing notes from log and data files for all phases. I got a bit stymied after I started this process because I felt like it wasn’t quite going to be helpful… plus it was taking a lot of time. Then I realized that I could simply cut and paste the tables from my data files into my file for chapter 5 and delete any rows (dialogue) that did not have my comments in them. This was far faster, and today I finished all seven phases, plus adding in the reflections for all of the teachers for each phase (those that were sent to me, anyway). I also made a list of the steps I need to go through for chapter 5, and the next is to put Dr. Mikusa’s comments in my organizational file. I know that in the notes from the data files, there is a lot of overlap with what I wrote in chapter 4, so I will want to avoid saying the same things… the focus has to be to talk about issues, which will mean drawing on specific points and considering them more from the perspective of implications.</td>
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<td>June 12, 2013</td>
<td>Put Dr. Mikusa’s notes into the organizational file and began coding information by cross-case themes. I put Dr. Mikusa’s notes in the organizational file in with the appropriate sections. Then, I labeled all of the notes by the category or phase of the study under which they fell, and I began coding from scratch with the information from Interview 1. I currently have 28 codes, a few of which may eventually be split into two. I finished the notes from Interview 1 and GD 1, and I began Interview 2. I skipped Lesson 1 temporarily because I wanted to think about what I was looking for. After some thought, I sketched out a series of descriptors that are basically parallel to the four levels of curriculum enactment that are in the NCTM Research Handbook (I think MK Stein and/or Peg Smith, perhaps?). Written SFMP, intended or interpreted SFMP, enacted SFMP, and received. I also made some notes about how the latter two levels might be affected by the previous levels. I’m going to look over these descriptors a bit more tonight and hopefully start coding Lesson 1 tomorrow.</td>
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<tr>
<td>June 13, 2013</td>
<td>Thoughts on coding lessons and ch. 4-5 organization. I’m writing at the start of my work today to get some thoughts down… last night, I did look again at the coding I had drafted for the lessons. I ended up changing what I initially thought, realizing that I was really addressing the written and intended/interpreted...</td>
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curriculum in what I had already written in Ch. 4. The enacted and received were really shown in the lessons. So, I came up four possibilities for teacher enactment of the three SFMP: a) it aligns both with the authors’ intent [as I see it] and their own interpretation; b) it aligns with their own interpretation but not the authors’ intent; c) it aligns with the authors’ intent but not their own interpretation (which seems unlikely); d) it aligns with neither the authors’ intent nor their own interpretation (which seems quite unlikely). Then, for students’ application of the SFMP: a) a student applies one or more because the teacher’s enactment supports this; b) a student applies one or more in spite of the teacher’s lack of enactment; c) a student does not apply one or more in spite of the teacher’s quality enactment; d) a student does not apply one or more because of the teacher’s lack of enactment. I think these should work when I am coding the lessons.

Then, as far as the other parts of Ch. 5... I was thinking about the idea of summarizing the teachers’ ideas about each section of each SFMP, or even each SFMP as a whole. However, as I have repeatedly found, their interpretations are unique enough that I’m thinking this would largely be a repeat of Ch. 4, only organized differently. I will say this at the start of Ch. 5, and I think what I need to do as I talk about the cross-case issues is to carefully explain how the teachers’ similar or contrasting views of the SFMP influence their perspectives on these issues.

I finished coding through page 90 in my data file (minus the Lesson 1 pages), and in this I did start coding Lesson 2. I think the codes are working and will be useful, though it does occur to me that I am only coding a sample of the dialogue because I only put in this file what I commented on. But I suppose I had a reason for commenting on certain lines, and I think I will/have been able to say enough about the SFMP in the lessons between Chapters 4-5. I also feel that my codes for the other phases are still holding true... one large issue that I am coding as “F” right now are teachers’ thoughts about students’ chances for success.
with the SFMP. Also, I am a little gray right now on how to code teachers’ inaccurate or incomplete interpretations of the SFMP, at least as I see them. However, I am generally putting these under E or F and will need to tease this out also when I outline the chapter.

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<tr>
<th>Continued coding by issue for Ch. 5</th>
<th>June 14, 2013</th>
<th>QD</th>
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| I am through page 131 in the data file (minus lesson 1) and hope to be done with this coding by Sunday (two more days). I have not added any more codes, and I think the set I have is pretty solid. When I was coding lesson 2 today for all of the teachers, I found that there was a very fine line between CC (teacher’s enactment of SFMP aligns with own interpretation but not authors’) and EE (teacher’s enactment aligns neither with own interpretation nor with authors’). Basically, I went with EE if whatever the teacher was doing did not reflect what he/she said in the interviews, even if it seemed to align with what he/she might have thought was the intent of the standard. (Go with what can be observed.) For Carrie, I decided to use CC when she would ask leading questions without actually telling students something, but EE when she did basically tell them what to think or do. This was a bit different for Sandy because her own interpretation of the SFMP was probably less deep than Carrie’s, so if she did something fairly teacher-directed, it would have aligned with her own interpretation more than if Carrie had done something like this in the lesson.

I did begin to wonder: at what point in a lesson would the authors believe that it is OK for a teacher to actually offer a comment on what is happening? I would think the answer would not be “never”, so how would one judge this?

Also, I will say that all three teachers really seemed to have a good idea, in general, about the overall goals of the SFMP. That is, they seemed to understand, at least broadly, the overall goals. However, the devil is in the details... what are we actually looking for in teachers and students during a lesson that shows that these standards are really being enacted with fidelity?
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<td>June 15, 2013</td>
<td>Continued coding by issue for Ch. 5</td>
<td>I am through page 160 in the data file, and I hope to finish the coding with my own codes tomorrow. (then go back to the lit from Ch. 2 and compare/add) I noted that codes C/S, F/R, V/W, and V/X seemed to correspond (in these pairs). It is often hard to separate one from the other. Also, again, it is still hard to know what to code what one of the teachers thinks a standard is saying about classroom practice… (or even just in general) but I guess this is really what I addressed in Ch. 4 anyway.</td>
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<tr>
<td>June 16, 2013</td>
<td>Finished coding based on my own codes</td>
<td>Code X (among many others) will definitely lead to implications. In Sandy’s lesson 2, it was hard to code some of the student work because they were responding to what she was asking them to do (FF), but it was pretty superficial (seemed like II sometimes). It could be coded as HH also if Sandy’s question at that moment could be viewed as a good question, and the students just responded really superficially (but again, this may be because this is what they were used to). So the code didn’t always seem to depend on whether students were trying…it was more what they actually DID that guided the coding. Seems like the coding from “best to worst” goes: BB, CC, DD, EE (teacher) FF, GG, HH, II (student) Interesting that for Ray with the Singapore method, we may lean closer to EE than he would like. Ray and Carrie certainly seem to be in the flowery middle… Carrie a bit more toward actual enactment of the SFMP than Ray is.</td>
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| June 17, 2013      | Began comparing lit review with themes    | I got through about half of my lit review today… just realized I probably should go back to Ch. 1 also… working to compare the lit with the themes I have identified. The one that seems to be overarching is “policy” (yet I only have two theme related to this specifically because I really feel that many of the rest could then inform policy). I may still need to adjust that. I need to make my file more easy to work with – it’s all on one handwritten sheet right now, which is
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<th>Task Description</th>
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<tr>
<td>Getting quite crowded.</td>
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<tr>
<td>Finished comparing lit review with</td>
<td>June 26, 2013</td>
<td>Today I finished reading through the lit review and making notes about where my themes were addressed in the lit review. Basically, everything in the lit review was addressed at one place or another in my themes. One thing that especially struck me was the idea that teachers’ beliefs about whether their values are expressed in standards documents will affect how they interpret/enact the standards – this totally reflected Carrie’s comment about “the grand they” and how she was glad to see the practices in which she believed finally reflected in policy. So interesting! Now I need to think about how to organize chapter 5 and how to incorporate these connections to the lit, because there are many connections with most of my themes. I am thinking that it will make the most sense to organize the chapter by my themes, as I would have anyway, and then as I write about each theme, cite the literature that reflects it. I sequenced chapter 5 by major idea later in the day, and I began writing – covering the first 5 pages of my data file. I did not include research yet because it took quite a while to write these pages, and I am also unsure of how best to choose which research to cite when.</td>
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<tr>
<td>Continued to draft chapter 5</td>
<td>June 27, 2013</td>
<td>I wrote more today, getting through page 15 in my data file, and I realized that I think the best way for me to include the research that I need to cite is to wait until the chapter is written – then go back and cite a bit of research or two for each point that I am making… the goal will be to include all of the research that I can, but also not to repeat citations unless I absolutely feel it is necessary and appropriate. So, I plan to move up my timeline for writing the parts based on my own findings so that I can spend the last few days just adding in the research where it makes sense.</td>
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<tr>
<td>Continued writing draft of chapter 5</td>
<td>June 28, 2013</td>
<td>I wrote through page 45 in the data file today. This definitely takes more time than writing for chapter 4 because I have to think more carefully about how to integrate the notes from before, as well as what I want to include and what I can eliminate. But I am moving. I do wonder if this will allow me to eliminate bits of chapter 4… though I am doing my best not to repeat points I know I have stressed earlier.</td>
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<td>June 29, 2013</td>
<td>Today I got into the first part of the lesson analysis, which is interesting because it’s not something I was able to write about in chapter 4, and nice because it’s going relatively quickly. I’m through page 67 in the data file.</td>
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<td>June 30, 2013</td>
<td>I continued the lesson analysis today, finishing the entire portion about teachers’ not enacting the SFMP as might be intended, and I started the portion about when they do enact the SFMP at least partially as intended. I feel that the flow of the chapter is fairly good, and I am including comments from teachers’ reflections as I have them. As I continue along, I am finding that I have to say less about their reflections because I have already written a lot about them, so I just include short points as needed. I continually try to “be kind to the reader” while still always tying my writing back to the SFMP. (or larger goals of the CCSS)</td>
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<td>July 1, 2013</td>
<td>QA, QD I wrote about Sandy’s and Ray’s second lessons today and how they did implement some goals from the SFMP (code BB). It is interesting to consider that they both (and Carrie) know what kinds of questions to ask, but it may be the consistency which with they ask them, along with the students of whom they ask them (all, some, or few) that really puts the SFMP into play. It’s like what I’ve always said about engaging 25% of the class really well and leaving the other 75% to watch, listen, and be disengaged. (FOR PLC DAY 4) I also wrote a note that I need to think about what causes teachers to be able to enact these partial goals from the SFMP (code BB) and make some general comments about this before I go on to the next code.</td>
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<td>July 2, 2013</td>
<td>QA I finished the lesson analysis section today and moved on to the related section about teacher expectations and knowledge of the SFMP affecting students’ learning. The section is coming together well; it seems to have a reasonable flow. For each teacher, I am highlighting aspects of their expectations/knowledge that would support the SFMP and aspects that might hinder the enactment of them (and student learning). I am mainly writing about each teacher individually, but I am comparing/contrasting when it is appropriate.</td>
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<tr>
<td>July 3, 2013</td>
<td>Continued writing draft of chapter 5</td>
<td>I finished the sections about teacher expectations, knowledge, and professional learning, and I am getting to the point where I am beginning to talk about policy. I do wonder if I will eventually cut some parts of this chapter (or of chapter 4), because there is some overlap, but I will wait until I get feedback from at least Mike B and probably Mike M also. I am noticing, as I knew would be the case, that even as I am doing the cross-case analysis, I am still often writing basically about one teacher, then another, then the last, because their ideas were different. However, I am making an effort, and I may want to do this more in the editing process, to draw comparisons and contrasts more explicitly among their perspectives.</td>
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<td>July 4, 2013</td>
<td>Continued writing draft of chapter 5</td>
<td>I continued with chapter 5.</td>
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<tr>
<td>July 5, 2013</td>
<td>Finished draft of chapter 5 (based on my own observations) and began including references to literature</td>
<td>I completed the portions of chapter 5 today that were based on my own research, and I began to include the references to earlier research. I became a bit stymied as I tried to do this systematically; my original notes about what research I thought I would include with each section of the document were not very helpful because the detail in the chapter involved ideas from throughout chapter 2. So, I paused later in the day and decided to start again tomorrow.</td>
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<tr>
<td>July 6, 2013</td>
<td>Continued including references to literature in chapter 5</td>
<td>I began this process again, and again I found that it was rather slow going, but at least I did move along and found that I was able to include many references throughout the chapter. I am now nearly finished with reading through the chapter, paragraph by paragraph, to consider where to include references, and tomorrow I will finish this process and then go back through chapter 2 once more to be sure that there are not other references that I would also like to include.</td>
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<td>July 7, 2013</td>
<td>Finished Ch. 5</td>
<td>I sent Ch. 4-5 to Dr. Battista.</td>
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<td>August 22, 2013</td>
<td>Met with Mike M. on Ch. 4-5</td>
<td>QD Mike and I met, and he gave me some comments on Ch. 4-5. Not too many things to change, really, but I need to think about how to “introduce” the material in Ch. 5 a bit more directly.</td>
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<td>September 12, 2013</td>
<td>Phone conference with Mike and Mike</td>
<td>Neither Mike M nor Mike B had too many major comments, but Mike B asked if I could create a table with a summary of the information about each teacher from Ch. 4 with which to introduce Ch. 5. He gave me some suggestions for categories in this table, and Mike M said he thought that would help to introduce Ch. 5 in the way that he was thinking would be useful.</td>
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<tr>
<td>October 5, 2013</td>
<td>Sent refinements of Ch. 4-5 to Mike and Mike, including table summaries of each teacher’s interpretations in Ch. 5</td>
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<tr>
<td>October 19-21, 2013</td>
<td>E-mail discussion with Mike B about how to define fidelity of implementation of SFMP</td>
<td>Mike B would like me to be more precise about how I am characterizing the fidelity of implementation for the SFMP in each of the six lessons. We wrote back and forth to discuss this, and I think what I am going to do is some sort of percentage of the number of times teachers’ actions or dialogue seem to reflect the SFMP.</td>
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<tr>
<td>November 1-5, 2013</td>
<td>Worked on defining fidelity of implementation of SFMP</td>
<td>I went back to the data from each of the six lessons, and I more specifically coded each piece of dialogue (or action without dialogue) from the teacher using the four categories that I listed earlier: a) it aligns both with the authors’ intent [as I see it] and their own interpretation; b) it aligns with their own interpretation but not the authors’ intent; c) it aligns with the authors’ intent but not their own interpretation (which seems unlikely); d) it aligns with neither the authors’ intent nor their own interpretation (which seems quite unlikely). I also noted which portion of each SFMP the dialogue seemed to reflect. Given all of this data, I was then able to use percentages and counts to classify each teacher’s enactment of each portion of each standard as high, medium, or low fidelity (see definitions in Ch. 5).</td>
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<td>November 6-9, 2013</td>
<td>Created a list of my tentative conclusions</td>
<td>I listed three pages of tentative conclusions from the</td>
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<td>Date/Action</td>
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<td>final conclusions in the study, and sent to three teachers for feedback</td>
<td>I basically drew from the main idea in each section in chapter 5. I found in the past that it was not easy to get feedback from the teachers, so I wanted to make this as brief and as reader-friendly as possible. I once again had to eliminate ideas that might have been viewed as at all negative about the teachers’ interpretations, and I basically presented the conclusions that applied to all three teachers (which truly were the main ideas in chapter 5 anyway). I asked for the feedback within a week. I received feedback from Ray and Sandy (was glad that Sandy was back and recovered from her illness), but not Carrie. I also sent the copy of chapter 4 to a peer from my doctoral program for her suggestions and feedback.</td>
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<tr>
<td>Added feedback from Ray, Sandy, and my peer into chapters 4 and 5</td>
<td>November 16-17, 2013 QD, QA, QB, QC It felt worthwhile to include specific feedback from Ray and Sandy in Chapter 5, especially since I had not been able to do the final interview with Sandy. It was interesting to see for what statements they wrote particular comments. Essentially, they both agreed with everything I had written, with one exception: Sandy was not sure about the statement that a teacher’s philosophy is relatively unchanging over time, because she felt her views on using manipulatives had changed noticeably since we began the study (due to the inclusion of manipulatives in the SFMP). My peer offered helpful comments for chapter 4 in terms of being more explicit about a few of my claims, so I am adjusting and revising based on this feedback.</td>
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<td>Added info to chapter 4 about the schools and district</td>
<td>November 29, 2013 QD I had realized that I needed to include some descriptive information about the schools and district in which the study took place, so I found this information on the ODE website for local report cards (a public site) and added it today.</td>
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<tr>
<td>Finalized other details and formatting for pre-defense</td>
<td>November 30 – December 4, 2013</td>
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