INVESTIGATING THE ADULT LEARNERS’ EXPERIENCE WHEN SOLVING MATHEMATICAL WORD PROBLEMS

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By
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INVESTIGATING THE ADULT LEARNERS’ EXPERIENCE WHEN SOLVING MATHEMATICAL WORD PROBLEMS (387 pp.)

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The purpose of the study was to understand and describe the experiences adult learners have while solving mathematical word problems. The focus of the study was on how these adult students used prior mathematical knowledge and how their past experiences with mathematics influenced solving of word problems typically found in an algebra course.

The research methodology was multiple case studies. The research sample was comprised of the students taking a Beginning Algebra class at a Midwest community college. Individual interviews with open-ended questions and observations of students solving mathematical word problems were employed to gather information about the individual’s constructions and experiences. The within-case and cross-case data analyses followed the data collection.

The study found that the attitudes, feelings and beliefs that adult students in the study hold toward mathematics are an integral part of their mathematics learning experience. This study also reports on the particular pattern observed within the participants’ attitude toward mathematics education during their schooling years. In addition, the study found that the majority of the participants were not ready to tackle the
traditional word problems because of the lack of necessary cognitive resources/previous knowledge of such concepts as motion and concentration. Finally, the study found that even after learning the topic in class, the participants had difficulties with applying algebraic approaches to word problem solving. The participants mostly relied on the memorization rather than conceptual understanding. In addition, the majority of the participants displayed no transfer of learning between the classroom and everyday activities.
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My darling girls, you are my life! Everything I do is for you. I am nothing without you.

My dearest husband, what would I do without you? Thank you for sharing my life. Thank you for cooking, cleaning, shopping, fixing, and most importantly, for being patient while I was thinking only about this study. You are truly one of a kind.
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CHAPTER I

INTRODUCTION

Because learning transforms who we are and what we can do, it is an experience of identity. It is not just an accumulation of skills and information, but a process of becoming. (Wenger, 2004, p. 215)

The single most important reason to teach mathematics is that it is an ideal discipline for training students how to think. (Schoenfeld, 1982, p. 32)

Difficulties Experienced by Adults Learning Mathematics

I would like to begin with providing some information on the studies in adult mathematics education. These studies found that the numeracy proficiency of 58.6% of U.S. adults was below level 3, the minimum level needed for managing today’s working and living requirements (Statistic Canada and OECD, 2005). Furthermore, the quantitative literacy skills of 55% of U.S. adults are at a Basic or Below Basic level (NCES, 2006). The economic impact of having low numeracy skills has been reported by the Adult Literacy and Lifeskills Survey (ALL). U. S. adults performing at numeracy levels 1 and 2 (the lowest of five levels) are about three times more likely to receive social assistance payments from the state than those who score in levels 3, 4, or 5 (Statistics Canada and OECD, 2005). The Adult Literacy and Lifeskills Survey (OECD, 2005), examined adults numeracy skills in the context of daily life and work across seven countries, including the United State, and showed that those with low numeracy skill
levels are more likely to be unemployed for six months longer that those at higher levels and three times more likely to receive social assistance payments. In 2009, passing rates on the GED mathematics exam were the lowest among the five academic subjects tests (American Council on Education, 2010).

Additionally, about 60 percent of community college students in the United States are referred to take developmental courses since these students are deemed insufficiently prepared to start college-level work (Attewell, Lavin, Domina, & Levey, 2006; Bailey, Jeong, & Cho, 2010). Mathematics classes in particular are a common roadblock for a large proportion of the community college student population (Achieving the Dream, 2006c). Approximately two of three community college students referred to a remedial mathematics sequence do not complete it (Bailey et al., 2010). According to a U.S. Department of Education study (Adelman, 2004), the three courses with the highest rates of failure and withdrawal in postsecondary education are developmental mathematics courses. About 50% of the thousands of individuals interviewed for the National Adult Literacy Survey, including numerous persons holding high school and college credentials, have major difficulties with quantitative literacy (Nesbit, 1996). This data reveals that the adult numeracy issue in the United States is severe, and its negative effects fall far beyond the classroom.
The Importance and Necessity of Teaching Mathematics to Adult Learners

I began working with adults in mathematics education a decade ago. During this time, as a teacher, I experienced both success and failure. My aspiration to help my adult students gain knowledge in mathematics and apply it in their lives influenced my decision to research adult mathematics education.

John Dewey began writing about the concept of adult education in 1916 (FitzSimons, 2001). Since that time, the quickly changing aspects of modern society, the more public existence led by individuals, and the increasing educational demands of employers have elevated the topic’s significance. Educationally, adult learning is seen as a foundation of personal development and growth, a way of enhancing an individual’s life. Economically, adult education is the way to become commercially competitive and to provide sufficiently for one’s life style. Politically, lifelong learning can be seen as a carrier for social strength and integrity.

My personal and professional beliefs and inspirations resonate with the 1996 UNESCO Report Learning: The Treasure Within in which the following observation was made: “Education is at the heart of both personal and community development; its mission is to enable each of us, without exceptions, to develop all our talents to the full and to realize our creative potentials, including responsibility for our own lives and achievement of our personal aims” (FitzSimons, 2001, p.1).

It is imperative that mathematics education should be part of adult education since there is hardly a single human being around the world who is not using quantitative
reasoning in everyday life (Gal, 2000; Ginsburg, Manly, & Schmitt, 2006). Thus, the importance of research in adult math education is growing constantly. Johansen (2002) presented the following grounds for teaching mathematics to adults: (a) to assure the demands of the modern information society, (b) to satisfy the demands of the labor market, (c) to provide individuals with the skills needed to lead an adequate social and private life. All the reasons stated above are clearly connected to the main motive for providing an education to any individual – to contribute to the development of the human society. The United States National Literacy Act of 1991 stated that literacy is an “individual ability to read, write, and speak, and to compute and solve problems at levels of proficiency necessary to function on the job and in society, to achieve one’s goals, and develop one’s knowledge and potential” (Gal, 2000, p. 9).

It is also vital to understand that many adult learners report negative attitudes about learning mathematics that affect their mathematics education (Gal, 2000; Tobias, 1993; Evans, 2000; DeBellis & Goldin, 2006). Those perceptions are generally attributed to the negative prior experiences they have had in a mathematics classroom (Tobias, 1993). As I have seen during my years of teaching adults, such attitudes and beliefs commonly obstruct the development of new math-related skills and severely affect test performance. The negative attitudes also impact the metacognitive habits. Consequently, one of the goals of adult mathematics education is to overcome our students’ math anxiety, unproductive beliefs about the relevance of mathematics to real-life, and negative self-perceptions.
Mathematical Problem Solving

The NCTM Standards (1989, 2000) and the reform movement that followed emphasized that the view of “mathematics as problem solving” should be a key process standard in mathematics education. A number of years ago, a taskforce of the Adult Numeracy Practitioners Network (ANN) in the United States presented a framework describing numeracy skills needed to “equip learners for the future” (Curry, 1996). Building on the NCTM Standards, this project indentified seven themes that should serve as fundamentals or standards for adult numeracy (the term used for adult mathematics education) as well as a consequential need for adults to become confident in their knowledge. One of them is Problem Solving. This idea has also been expressed by the American Mathematical Association of two-Year Colleges (AMATYC), whose Crossroads in Mathematics: Standards for Introductory College Mathematics before Calculus (1995) named Problem Solving as the first standard for intellectual development. Nevertheless, according to Lester (1994):

…It is safe to say that since the publication of the Agenda (An Agenda for Action, NCTM, 1980), problem solving has been the most written about, but possibly the least understood, topic in the mathematics curriculum in the United States (p.661).

Word Problem Solving

Within the general topic of problem solving much recent attention has been paid to the ubiquitous tradition of word problems. A word problem in mathematics teaching
refers to an authentic or possible situation in an everyday context (Wyndhamn & Saljo, 1997). According to Rojano (1996), from its beginning in Babylon to its culmination during the Renaissance, algebra constituted a sophisticated form of solving word problems. During this extensive period, “the problem” and “the equation to solve it” were indistinguishable. The “Palatino” or Greek Anthology contains a group of 46 numerical problems stated in epigrammatic form and compiled in 500 A. D. These problems, alluded to by Plato, are similar to some of those found on the Rhind Papyrus (1650 B. C.) and correspond to the categories which traditionally appear in algebra schoolbooks, that is, problems of “distribution”, “work”, “mixture”, and “age.”

There has been extensive research on word problem solving by school children: Bednarz, Kieran, and Lee (1996); Bednarz, and Janvier (1996); Dewolf, Van Dooren, and Verschaffel (2011); Greer (1997); Pantziara, Gagatsis, and Elia (2009); Schoenfeld (1992); Uesaka, Manalo, and Ichikawa (2007); Vamvakoussi, Van Dooren, and Verschaffel (2012), Verschaffel and De Corte (1997); Verschaffel, Greer, and De Corte (2000); Wyndhamn and Saljo (1997); Yerushalmy (2000). According to the research, word problems in general constitute an important part of the mathematics education. Initially they had an application function, that is, they were used to teach students to apply the formal mathematical knowledge and skills learned at school to real world situations. Later on, word problems were given other meanings as well: they were taught to develop students' general problem-solving ability and make the mathematics lessons more interesting and inspiring (Verschaffel & De Corte, 1997; Dewolf, Van Dooren, &
In spite of this long tradition in educational practice, the international research literature is filled with evidence that word problems do not fulfill these functions well. By the end of elementary school, many students do not see the applicability of their formal mathematical knowledge to real-world situations (Nesher, 1980); they do not have adequate use of heuristic and metacognitive strategies (De Corte, 1992; Greer, 1993); lastly they seem to dislike mathematics in general and word problems in particular (McLeod, 1992). In addition, the stereotyped solutions for many traditional word problems do not stand examination when considered in terms of realistic constraints (Greer, 2001; Sfard, 2005). In summary, even arithmetic and algebraic word problems are supposed to represent the real life context according to the NCTM standards, this objective is not universally met.

The findings of the study by a team of adult mathematics education experts from the Center for Literacy Studies at the University of Tennessee, Rutgers University, and TERC affirmed that there is the strong disconnect in the way that algebra topics are taught to adult learners—the disconnect between teaching algebraic topics abstractly and teaching within authentic contexts that most often appeal to adult learners. The guidelines provided by the team after the study indicated that if mathematical ideas are taught using “real-world” contexts then students’ performance on assessments involving similar “real-world” problems is improved. However, college algebra assessments and courses are often focused on other aspects of mathematics learn, such as computation, simple word problems, and equation solving (NMAP, 2011). Indeed the National Council of Teachers
of Mathematics in their three standard documents (NCTM 1980, 1989, 2000),
demeanorize traditional “word problems” where formulas and tables provide formulaic
solutions. Instead they recommend a well-developed “operational sense” for deciding
which procedures should be applied for practical problem solving.

**Purpose of My Research**

As problem solving is one of the key issues of the present reform in mathematics
education, and, at the same time, is the most difficult concept for many adult learners, I
am interested in studying how adults in pursuit of learning mathematics gain knowledge
of word problem solving. In the words of Alan H. Schoenfeld (1982), one of the major
contributors of the theory of mathematical problem solving,

> All too often we focus on a narrow collection of well-defined tasks and train
students to execute those tasks in a routine, if not algorithmic fashion. Then we
test the students on tasks that are very close to the ones they have been taught. If
they succeed on those problems, we and they congratulate each other on the fact
that they have learned some powerful mathematical techniques. In fact, they may
be able to use such techniques mechanically while lacking some rudimentary
thinking skills. To allow them, and ourselves, to believe that they understand the
mathematics is deceptive and fraudulent. (p. 29)

Despite continuous, based on the NCTM Standards (1989), changes in math education,
those words, written a quarter of a century ago, hold true today.
Teaching mathematics to children and teaching mathematics to adults certainly have a lot in common; nevertheless, they are not identical. Gal (2000) argued that adults do not “solve” situations the way children solve word problems. Adults rather “manage” them. When the requirement arises to solve a real-life numerical problem, adults opt for one of several routes of action based on factors such as personal and situational resources, personal goals, and circumstantial demands. Consequently, instead of traditional computation methods, adults may use estimation, nonstandard, even invented ways of getting the answer.

The purpose of the study is to understand and describe the experiences adult learners have while solving mathematical word problems in order to better understand the diverse beliefs and cognitive meanings of algebraic thinking of adult learners when attempting to solve word problems in order to provide teachers with new approaches to teach algebra in general and solving mathematical word problems in particular.

**Research Questions**

The following questions guided the direction of the study:

1. What attitudes, emotions, and beliefs do adult students hold regarding mathematics education in general and word problem solving in particular?
2. What mathematical content knowledge do adult learners access when solving word problems?
   a) How is this knowledge used? How is it chosen?
b) Why does the solution evolve the way it does?

3. What strategies do adult learners use to solve word problems?
   a) What formal approaches are employed?
   b) What informal approaches are employed?
   c) What are the adult students’ symbolic language and the dynamics of reasoning/thinking?

Definition of Key Terms

This section provides an interpretation of some of the key terminology used in the study.

*Mathematics* is a living subject which seeks to understand patterns that permeate both the world around us and the mind within us. Now much more than arithmetic and geometry, mathematics today is a diverse discipline that deals with data, measurements and observations from science; with inference, deduction and proof; and with mathematical models of natural phenomena, of human behavior, and of social systems. Although the language of mathematics is based on rules that must be learned, it is important for motivation that students move beyond rules to be able to express things in the language of mathematics (National Research Council, 1989; NCTM *Standards*, 1989).

*Adult learner*. The definition revolves around the learner, not the level of mathematics being studied. Knowles (1990) argued that there four definitions of the term
adult: biological, legal, social, and psychological. The last occurs at a point where self-direction comes into function and is the most central from the point of learning. In this study, adults are individuals of 18 years or older and continue their education intentionally. For some of them, it is a continuation of their school experience; for others there may be a break of a few years or more since their last formal mathematics course.

Mathematics education in the study is a field defined by a multiplicity of practices including:

- The teaching and learning of mathematics at all levels in school and college
- Out of school learning of mathematics
- The design, writing and construction of texts and learning material

Mathematical problems can be defined in many different ways. In the study, mathematical problems will consist of problems in which the individual would analyze the situation (s), draw diagrams, pose questions, search for patterns and solutions, use reasoning, and illustrate and interpret results.

Problem solving in the study is defined as “…finding a way where no way is known off-hand, to find a way out of a difficulty, to find a way around an obstacle, to attain a desired end, that is not immediately attainable, by appropriate means” (Polya, 1980, p. 1). Problem solving is a process through which individuals utilize the knowledge they have gained previously and applied it to a new unique situation or condition.
Word problems (or story problems) is the traditional format of application problems intended to develop in students the skills of knowing when and how to apply their mathematics effectively in diverse problem situations encountered in everyday life. Word problems are also defined as verbal descriptions of problem situations wherein one or more questions are raised, the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement (Verschaffel, Greer, & De Corte, 2000).

Numeracy describes an accumulation of skills, knowledge, beliefs, dispositions, communication resources, and problem-solving skills that individuals need in order to separately engage and effectively manage numeracy situations that involve numbers, quantitative or quantifiable information, and visual or textual information, that are based on mathematical ideas or have embedded mathematical elements (Gal, 2000).

Affect is a combination of three dimensions: beliefs, attitudes, and emotions (Evans, 2000).

A complete and coherent verbal reason means one based on a described pattern (Goldin, 1998).

A coherent external representation means a drawing, an equation, or a graph (Goldin, 1998).

Internal cognitive representations include five kinds of mutually interacting systems which are (Goldin, 1987): (a) a verbal/syntactic system (use of language), (b) imagistic systems (visual/spatial, auditory, kinesthetic encoding), (c) formal notational
systems (use of mathematical notation), (d) planning, monitoring, and executive control (use of heuristic strategies), and (e) affective representation (changing moods and emotions during problem solving).

*Symbolic language* includes words, symbols, and notations used (Goldin, 1987).

**Significance of the Study**

This study represents my interest in the research of adult mathematics education. In the 21st century, an increasing number of adults will begin or continue their education. In addition, as society becomes increasingly dependent on technology, continuing mathematics education will be a necessity in every country (Bishop, 2000). Additionally, factors of researching adult mathematics education make it more difficult to research than school mathematics education. In the latter, the learning situation has been thoroughly explored by the literature, the goals are usually made explicit, it takes place at a specified location, the materials are publicly available, and the assessment results are readily accessible. In adult mathematics education, the situation is more complex (Bishop, 2000; Gal, 2000). Thus this is an important area to research, and this study aims to cover some of the open ground in the field. In addition, if solving mathematical word problems by children has been at the focus of research for a number of years, solving word problems by adults is yet to be thoroughly considered. By examining case studies of how adult students solve word problems, a more in-depth understanding can be obtained about mathematics adults need to know in the contexts of enrolling in postsecondary education,
strategies for teaching adults the mathematics they need within these contexts, and the proper training necessary for adult education mathematics instructors.
CHAPTER 2

LITERATURE REVIEW

In developing this chapter, information was sought from authors with background in mathematics education, adult education, problem solving, and word problems. The intention is to address theoretical aspects and classroom realities as well as the quickly developing field of adult education.

The chapter begins with information on Adult Mathematics Education including Review of Research on Adult Learning Mathematics, Teaching Mathematics to Adults, Numeracy and Sociomathematics, and Study of Affect. The chapter proceeds with Mathematical Problem Solving consisting of Polya’s Legacy, Ideas and Theories of Mathematical Problem Solving, Teaching Strategies for Developing Problem Solving Abilities, Modeling and Problem-Solving, and Adult Learners and Problem-Solving Environment. The chapter ends with Solving Word Problems that is divided into Word Problem Solving and Cognition and Word Problem Solving and Informal Real Life Mathematics.

Adult Mathematics Education

Review of Research on Adult Learning Mathematics

This review is limited for reasons of space, to English-language publications, the majority of which relate to studies conducted in North America, Europe, and Australia.

Due to the progress of modern society, more and more attention is focused on adult mathematics education (Ginsburg, Manly, & Schmitt, 2006). At the same time,
overwhelming public opinion of the discipline labels it as exclusive, abstract, inhuman and cold (Cockcroft, 1982). Very often it is viewed as importance- and culture-free. In addition, the subject is perceived as difficult and having significance relevant only to itself.

The method of instruction in the field of adult mathematics education focuses on technique, thus being deficient in connectedness or historicity (Tobias, 1990). Adult math education that is based on rote-learning and drill and practice is unrelated to real world application and to the learners’ experience (Nesbit, 1995). In addition, there is a breach between the application-based, problem-solving approach required on the job and the traditional basic set of skills in educational curriculum (Buckingham, 1997; FitzSimons, 1998; Strasser, 1998).

As regards socio-psychological issues, there are a number of studies (Swindell, 1995; Kasworm, 1990; Gal, 2000) reporting on indirect deficiency of adult learners such as negative prior educational experiences, lack of motivation, and study skills. Kasworm (1990) also reported that chronological age of a learner is not the crucial variable. He stated that rather life experience, previous education, sociocultural context, and attitude are more important. Considering affective domain to be one of the most important variables in the learning process, it is vital to see that at the same time surveys of attitude and beliefs of adult students showed that it may not be possible for many people to ever learn mathematics easily and effectively, or to find it interesting (Davis, 1996; Taylor, 1995; Agar & Knopfmacher, 1995). In addition, Wedege (2003) reported the apparent
contradiction demonstrated by adults having difficulties with mathematics in formal educational settings, but who perform competently in everyday quantitative situations.

**Teaching Mathematics to Adult Learners**

To discuss teaching mathematics to adults, I want to recall that according to Knowles (1990), the term andragogy, which was coined in Europe in 1833, is understood as the “art and science of helping adults learn” (p. 54). Knowles argued that there is a distinction between pedagogy and andragogy. The first one is the theory of teaching children or youth, in which the teachers traditionally would assume full responsibility for making decisions about the learning process. His model of andragogy may include some conjectures from pedagogy, but not vice versa. Knowles (1990) recommended that teachers of adults become aware of their students’ backgrounds, readiness to learn, previous experiences, self-concept, and motivation. Barnes (1994) suggested considering students’ goals, values, and beliefs about the nature and purpose of mathematics. Ginsburg and Gal (1997) argued for the importance of developing students’ reasoning skills, built on their extensive informal knowledge of mathematics and suggested authentic real-life problems to solve.

Baker (2001) presented two main theories of pedagogy being used in the field of adult mathematics education. The first is the traditional model. This model originated with the work of John B. Watson and of Ivan Pavlov. According to Baker (2001), the Behaviorist school of psychology developed this model first for “military personnel, then
for the industrial-style high schools that were sweeping the United States in the first half of the twentieth century” (p.392). Behaviorists defined learning as an acquisition of a new behavior. According to this theory, knowledge exists outside of individuals and can be conveyed from an instructor to a student. In spite of the fact that this model has been in practice for decades, many math educators recognized the flaws in the Behaviorist movement. Behaviorists degraded such attributes of a human mind as uniqueness, perception, and construction. The traditional approach still widely used by educators and publishers, works for only an insignificant portion of the population who easily learn by “abstraction and symbolic manipulation” (Baker, 2001, p. 394). According to Gordon (1993), the traditional lecture approach and assessment practices prevented non-traditional students from learning efficiently. So, the second model, constructivism, came to light.

Constructivism is a philosophy of learning founded on the premise that human beings construct their own understanding by reflecting on their experience (Baker, 2001; Goldin, 1990, 2004; Lerman, 1989). Constructivism can be traced to the eighteenth century and the work of the philosopher Giambattista Vico, who maintained that humans could understand only what they have themselves constructed. A great many philosophers and educationalists have worked with these ideas, but the first major contemporaries to develop a clear scheme of what constructivism involves were Jean Piaget, Lev Vygotsky, and John Dewey. Under Constructivism educators teach their students to analyze, understand, and predict information. Within the paradigm, the accent
is on the learner rather than the teacher. In 1989, The National Council of Teachers of Mathematics (NCTM) took steps towards reform, releasing the new standards in mathematics education for K-12. Such reform was extended in 1995 to adult learners with the publication of *Crossroads in Mathematics: Standards for Introductory Mathematics before Calculus*. This work is based on the modern constructivist theory.

**Numeracy and Sociomathematics**

One of the main terms used in any modern source on adult mathematics education is *numeracy* (Colleran, O’Donoghue, & Murphy, 2002; Gal, 2000; Ginsburg et al., 2006; Johansen, 2002). There are various definitions of numeracy. These definitions, while different in phrasing and accent, recognize that numeracy and traditional mathematics education have a lot in common, but are not the same (Ginsburg et al., 2006). According to the authors, mathematics is based on abstract concepts while numeracy strives toward engagement with real life’s diverse situations and concepts and implies mathematical topics contextually connected to personal life, work, and community. In addition, “unlike pure mathematics, numeracy has a distinct personal element” (p. 1). The traditional *mathematics* is still being used in the German-speaking and Nordic countries, but, at the same time, in Anglo-American publications and public discussions, the term *adult numeracy* appears with increasing frequency.
Wedege (2003) argued that including numeracy into mathematics provides consolidation of the mathematics classroom, society, and adults’ everyday lives. There are a number of definitions of numeracy:

- Numeracy is the ability to process, interpret and communicate numerical, quantitative, spatial, statistical, even mathematical, information, in ways that are appropriate for a variety of contexts and that will enable a typical member of the culture or subculture to participate effectively in activities that they value (Evans, 2000).

- Numeracy consists of functional mathematical skills and understanding that in principle all people need to have. Numeracy changes in time and space along with social change and technological development (Schmitt & Manly, 2006; Wedege, 2003).

- Numeracy encompasses the knowledge and skills required to effectively manage mathematical demands in personal, societal and work situation, in combination with the ability to accommodate and adjust flexibly to new demands in a continuously rapidly changing society that is highly dominated by quantitative information and technology. (Groenestijn, cited in Wedege, 2002)

Therefore, numeracy has been seen as the ability of individuals to satisfactorily perform various activities in modern society. Those activities directly or indirectly include quantitative information. In addition, Wedege (2001) argued that for adults, managing any numerical situation is based on their cultural and academic background.

Context may be defined as the purpose for which an adult undertakes a task with mathematical demands; it may be centered on the individual, the family, the work environment, or the community. Content is defined as the mathematical knowledge that is necessary for the tasks dealt with. It includes (a) Number and Operation Sense, (b) Patterns, Functions and Algebra, (c) Measurement and Shape, (d) Data, Statistics, and Probability.

Cognitive and Affective processes enable an individual to solve problems and thereby link the content and the context. The cognitive and affective component includes:

a) **Conceptual understanding**: an integrated and functional grasp of the mathematical ideas

b) **Adaptive reasoning**: the capacity to think logically about the relationships among the situation and the concepts

c) **Strategic competence**: the ability to formulate a mathematical problem, represent it in meaningful ways, and decide, if necessary, how to manipulate numbers to come to a useful solution

d) **Procedural fluency**: the ability to perform needed precise calculations or make estimates mentally or using a calculator
e) **Productive disposition**: the combination of beliefs, attitudes, and emotions that contribute to the learner’s ability and willingness to engage, solve, and persevere in mathematical thinking and learning.

This is a recursive process, meaning each step should be monitored and reevaluated if needed (Ginsburg, et al., 2006).

An in-depth analysis of the difference between adult mathematical education and numeracy is beyond the scope of the paper. From now on those two terms will be used interchangeably unless stated otherwise.

Another concept that is related to numeracy and learning mathematics is *Sociomathematics*. Wedge (2001) used the term to illustrate relations among people’s lives, mathematics, and education. She argues that the society, culture, and learning are strongly connected.

![Diagram of Sociomathematics by Wedge (2003)](image)

*Figure 1. Definition of Sociomathematics by Wedge (2003)*
According to the diagram, Sociomathematics represents the structure where humans, their society, and learning mathematics are all united. They depend on each other; they function as a whole. Wedege (2003) also explained the difference between Sociomathematics and Ethnomathematics. Ethnomathematics puts emphasis on the links between mathematics and culture, where culture is considered to be the way we comprehend the world around us. Sociomathematics is a field within math education which studies the relationship between mathematics and society, where society is the set of regulations people follow in our social life.

The NCTM Curriculum Standards (NCTM, 1989, 2000) presented the cognitive component of mathematics teaching and learning as developing reasoning skills, problem solving, communication, and connections between knowledge and skills areas. Building on the NCTM documents, a taskforce of the Adult Numeracy Practitioners Network (ANN) in the United States presented foundations or standards for adult numeracy education (Curry & Schmitt, 1996). These are

- Relevance/Connections
- Problem Solving/Reasoning/ Decision Making
- Communication
- Number and Number Sense
- Data, Statistics, and Probability
- Geometry: Spatial Sense and Measurement
- Algebra: Patterns and Functions.
The interaction of these seven themes results in what is currently defined as numeracy.

**Study of Affect**

When we approach the problem of the interrelation between thought and language and other aspects of the mind, the first question that arises is that of intellect and affect. Their separation as subjects of study is a major weakness of traditional psychology. (Vygotsky, 1962, p.10)

Both mathematics educators and cognitive researchers have highlighted the importance of the affective domain in the teaching and learning of mathematics (Dai & Sternberg, 2004; DeBellis & Goldin, 2006; Evans, 2000; Goldin, 2008; Leder & Pehkonen, 2002; Lesh, Hamilton, & Kaput, 2007; Schoenfeld, 1992). The research has focused on the influence of affect and beliefs on problem-solving abilities and on individual expressions of affect during mathematics problem solving.

_Affect_ includes different feelings (conscious as well as unconscious) during mathematical problem solving. McLeod (1992) presented three types of affect: beliefs, attitudes, and emotions. The major finding of the research was that the individual emotional state can enhance cognition (for example, through mathematical curiosity) or impede it (for example, through math anxiety). According to Goldin (2008) the affect of students engaged in problem-solving activity should be regarded as an internal representational system.
In regard to beliefs and dispositions, Schoenfeld (1992) presented the following list of common beliefs that have strong (often negative) influences on mathematical thinking:

- Math problems have one and only one right answer.
- There is only one correct way to solve any mathematics problem—usually the rule the teacher has most recently demonstrated to the class.
- Ordinary students cannot expect to understand mathematics; they expect simply to memorize it and apply what they have learned mechanically without understanding.
- Math is a solitary activity, done by individuals in isolation.
- Students who have understood the mathematics they have studied will be able to solve any assigned problem in three minutes or less.
- The mathematics learned in school has little or nothing to do with the real world.
- Formal proof is irrelevant to the processes of discovery or invention.

Schoenfeld (1992) also argued that the students’ beliefs about formal mathematics and their sense of the discipline are based in large measure on their experiences in the classroom. Those beliefs shape students’ behavior in very powerful ways that have extraordinary consequences.

In addition to the research on affect, DeBellis and Goldin (2006) argued that the concepts of mathematical intimacy and mathematical integrity also affect mathematical problem solving ability. They see human affect as an internal representational system
exchanging information with cognitive systems, mathematical intimacy as a deep emotional engagement with mathematics, and mathematical integrity as an individual commitment to mathematical understanding and mathematical truth. The authors argued that the development of mathematical intimacy and mathematical integrity are vital components of the problem-solving ability in learners.

In relation to affective issues of adult students, Gal (2000) and McLeod (1992) argued that how well a numeracy situation is managed depends not only on the knowledge of mathematical rules and operations and linguistic skills, but also on the students’ beliefs, attitudes, metacognitive habits and skills, self-concept, and feelings about the situation. Adult students’ dispositions may affect the way they approach or react to learning episodes in a classroom as well as numeracy situations in real life. In realistic contexts, adults with a negative mathematical self-concept may decide to avoid a problem with quantitative elements, address only part of it, or subcontract another person to help. Those actions carry negative consequences such as not being able to fully achieve one’s goals.

According to Evans (2000), affect in general, and mathematical anxiety in particular, were considered as relatively established characteristics of an adult learner which have an ongoing effect on mathematical thinking, performance, and participation in mathematics courses. He argued that there is a positive correlation between affect and cognitive outcomes across all students. The students’ beliefs, attitudes and emotions are
context-dependent and may be resultant from previous activities and/or from prevailing beliefs in the culture at large (Carter & Yackel, 1989; Evans, 2000).

Proper metacognitive habits of adult students are imperative as well since we want adults to be able to comprehend and properly handle diverse quantitative situations, to encourage a critical standpoint, and to further invest the mental effort needed to ask acute questions and try to answer them. Without the standpoint, people might accept objectionable arguments and develop an incorrect world view (Gal, 2000).

I would like to end this topic with the words of Evans (2000) who claims that “in much of the research on the use of mathematics by adults, the situation has not changed much from that described in psychology by Vygotsky three generations ago (see the quotation at the beginning of the topic). There is still little or no explicit acknowledgment of the importance of the affective – feelings of anxiety, frustration, pleasure, and/or satisfaction which attend the learning of mathematics and the solution of numerate problems” (p.108). Consequently, one of the basic goals of adult mathematics education is not only to deal with purely cognitive issues, but also with students’ dispositions and beliefs. To work on the latter, numeracy educators should present quantitative reasoning as a viable way to approach life’s challenges as a gate opener instead of a gatekeeper (Benn, 1997; Gal, Ginsburg, & Schau, 1997), be particularly sensitive to their students’ previous experiences of mathematics and their cultural approaches to quantitative situations (Coben, 2000), and help students build their confidence in their ability to do
mathematics (Kloosterman & Gorman, 1990). Some of my research questions address the study of affect.

Summary

As is evident from the literature review above, the field of adult mathematics education is expanding in both practice and research in many countries. It greatly contributes to the empowerment, successful functioning, economic condition, and welfare of people and their communities (Coben, 2000; Gall, 2000; Ginsburg et al., 2006). The nature of numeracy skills is dynamic and relative. There exist multiple perspectives, definitions, and challenges. The need to attend to adults’ quantitative skills is apparent since they have to manage multiple and diverse quantitative situations.

The term *numeracy* is being used along with adult mathematics education. It describes a collection of competence, knowledge, beliefs, problem-solving skills, dispositions, and communication capabilities that individuals need to autonomously engage and effectively manage numeracy situations. Such situations involve quantitative and qualitative information, numbers, textual and visual information that originate in mathematical ideas. Numeracy is connected to literacy and to the individual’s social and communal circumstances (Gal, 2000).

In regard to teaching adult students, the range of skills and dispositions required for effective functioning in most real life situations is much wider and often quite different than that which has been traditionally addressed in K-12 mathematics education.
According to Plaza (1997), there are nine reasons for adults to know mathematics: everyday life, interpretation of information, the world of work, being a consumer, health, dealing with technologies, environment, social justice, life in democracy.

Solving of a numeracy situation depends on mathematical knowledge as well as students’ beliefs, attitudes, metacognitive skills, and emotions. Consequently, mathematics educators should facilitate the development all of those traits.

Some educators believe that traditional modes of math instruction effectively allow students to develop solid basic skills and logic sufficient to manage real-life quantitative situations. Nevertheless, there is a strong voice advocating pedagogy based on constructivism in adult math education (Baker, 2001; Curry et al., 1996; Kieran, 1994). In addition the NCTM Standards (1989) and the reform movement that followed emphasized that the view of “mathematics as problem solving” should be a key process standard in mathematics education. Consequently, the focus of teaching and learning for adults should be on problems and numeracy situations (Gal, 2000; Ginsburg et al., 2006). My research therefore focuses on the manner in which adult students approach and solve mathematical word problems.

Mathematical Problem Solving

Polya’s Legacy

This section of the paper presents an overview of George Polya’s problem solving strategy and the main focus of early (prior to 1990) research on problem solving. I have
elected to treat this topic in a separate section as it remains, despite its obvious shortcomings, the most popular strategy for solving word problems presented in current mathematical texts (at least, for adults).

Polya’s four-stage model of problem solving can be summarized as follows (Polya, 1957; Pressley, Burkell, & Schneider, 1995):

- Understand the problem
- Devise a plan
- Carry out the plan
- Check and estimate the answer.

Montague and Bos (1990) studied the problem solving performance of a group of eighth-grade students of various academic abilities. They concluded that, first, the higher students’ mathematical achievements, the more Polya problem-solving steps were observed during problem solving. Second, the higher the math achievement, the more strategic competence was reflected in the interview data.

Cardelle-Elawar (1990) presented a study of disadvantaged sixth grade students experiencing difficulties in mathematics to implement Polya’s strategy for word problem solving. The study reported on a huge effect of the intervention on a mathematics achievement post-test.

Hembree (1992) when examining almost 500 studies of problem-solving conducted in the twentieth century, reported on very strong associations between problem-solving performance and the use of four Polya strategies. The greatest
association identified by Hembree was between selecting a sound sequence of operations (that is devising a plan) and problem-solving performance. The next great association was between understanding a problem and performance. One of the most interesting outcomes of the study was that the impact of teaching Polya’s strategies increased with increased grade level. While the association between teaching of the Polya approach and achievements was quite modest at the elementary school level, it increased during the middle and the high school years.

Considering the importance of Polya’s ideas, Schoenfeld (1987a) stated, “For mathematics education and for the world of problem solving Polya’s work marked a line of demarcation between two eras, problem solving before and after Polya” (p. 28). Passmore (2000) credited Polya’s work as very radical at the time it was first presented since at that period, drill and practice on basic skills were the norm in teaching mathematics. He believed that some of Polya’s ideas are surprisingly modern, like the distinction between the way mathematics is formally presented and the very different way in which it is actually done, insufficiency of mathematics teacher education, different standards for teaching mathematics to mathematics major and non-specialists in mathematics, and that problems should be designed to be valuable for students, should entail recognizing an essential mathematics concept and require exploration and estimation.

There is also strong criticism of Polya’s heuristics. Schoenfeld (1987) pointed out “…there was at best marginal evidence (if any) of improved problem-solving
performance. Despite all the enthusiasm for approach, there was no clear evidence that the students had actually learned more as a result of their heuristic instruction or that they had learned any general problem-solving skills that transferred to novel situation” (p. 41). In addition, the heuristic strategies were too general to be applied to a specific problem. Schoenfeld (1987) argued that different problems require different approaches. Lester (1994) while summarizing research findings also agreed that just following Polya’s heuristic did not improve problem-solving skills; instead, the learning was taking place in doing, and problem-solving ability would build up gradually over a period of time.

According to Passmore (2003), cognitive research showed that besides training in the strategies of problem solving, students also need training in command and control – in self-regulation and resource distribution throughout the problem-solving process. Lester (1994) suggested that metacognitive instruction should not be pursued as a general approach, but should rather be set in the context of learning particular concepts.

If one considers early problem solving research (prior to 1990) in general, one finds only three major areas of research that appeared in the Journal for Research in Mathematics Education. These are (a) Determinants of problem difficulty; (b) Distinctions between “good” and “poor” problem solvers; (c) Problem-solving instructions (Lesh & Zawojewski, 2007). And these “problem solving characteristics were selected for no special reason other than that they were easily measured…” (p.6). There was no attempt to identify such vital variables relevant to problem solving research as student response variables. That is, the problem solver’s interpretation depends not
only on external factors (so called *task variables*), but even more so on internal factors that define how one interprets, or “sees”, the mathematics problem. Such a situation in mathematical problem solving research seems quite appropriate to the mainly Behaviorist theory of teaching and learning that was the leading pedagogy at that time.

**Ideas and Theories of Mathematical Problem Solving**

…It is safe to say that since the publication of the Agenda (An Agenda for Action, NCTM, 1980), problem solving has been the most written about, but possibly the least understood, topic in the mathematics curriculum in the United States.

(Lester, 1994, p. 661)

In this section, I present the ideas and theories of mathematical problem solving appearing within the last two decades. I begin this section with the work of Alan Schoenfeld because his book *Mathematical Problem Solving* was eye opening for me – influencing tremendously my pedagogical approach to teaching and assessing problem-solving skills.

Schoenfeld (1985) believed that learning to think mathematically engages much more than just having large amounts of subject matter at hand. Thinking mathematically also means applying knowledge of mathematics efficiently, being flexible and inventive within the discipline, and understanding and agreeing to the unspoken rules of mathematics as well. His research indicated that when instruction focuses mainly on mastering of mathematics facts and procedures, students are not likely to develop the
ability to think mathematically. Only when teaching actually makes higher-order skills a focal point of the educational process do students learn effectively.

Schoenfeld (1985) proposed that the knowledge and behavior necessary for adequate problem-solving performance is comprised of four intertwining and interrelated issues:

- **Resources** which are the mathematical knowledge an individual begins with. It includes the person’s “informal and intuitive knowledge about the problem domain; knowledge of facts and definitions; the ability to execute algorithmic procedure; familiarity with routine procedures; the possession of spectrum of relevant competencies; the knowledge about the rules of discourse in the problem domain” (p. 68). Resources also include the way this information is organized and stored.

- **Heuristic** which is a set of extensive strategies for approaching different problems. Some of the strategies are working backwards and exploiting analogies. One of the arguments Schoenfeld (1985) made was that attempts to teach students to use such strategies were much less productive than expected.

- **Control** which deals with choosing and pursuing the right approach, the ability to understand and recover from wrong choices, managing resources, and overseeing the problem-solving process as a whole. When in control, individuals can use their resources efficiently and solve rather challenging problems. The deficiency of control leads to failing to solve even simple problems, which are within the
student’s ability. Schoenfeld mentioned that one of the features of individual control strategies is the person’s capability to maintain an internal dialog while attempting to solve a problem. Social interactions are primary factors in developing this internal cognitive structure.

- **Belief systems** which comprise one’s view of mathematics and the attitude toward mathematical tasks. Beliefs set up the environment within which resources, heuristics, and control operate.

Hung’s (2000) project focused on the importance of establishing conceptual “closures,” or comprehending the inner relationships in the problem statements and solution process. According to Hung, when students reflect on mathematical statements and symbols, they discover more than one mathematical interpretation of the statements and symbols. In this respect, mathematical meanings can be built at different conceptual levels. Hung characterized “closure” in learning at three possible levels of conceptual understanding: (1) **symbol level** (or mere numerical and computational understanding), (2) **problem level** (problem as a whole), (3) **situational level** (relating problems to other problems and mathematical concepts). The conclusion of the study was that the conceptual meanings are not likely to be developed unless there is (a) active exploration and discovery of the problem tasks and conditions, (b) closure in understanding at the problem-level or situational-level, (c) recognition of meaning patterns that can be transferred from one context to another.
According to Lester (1994), Lesh (1982), Silver (1982), and Schoenfeld (1992), metacognitive actions are driving forces in problem solving. The authors argue that (a) effective metacognitive activity during problem solving requires knowing not only what and when to monitor, but also how to monitor, (b) teaching students to be aware of their cognition during problem-solving actions should take place in the context of learning specific mathematics concepts and techniques (general metacognition instruction is likely to be less effective), and (c) the development of healthy metacognitive skills is difficult and often requires “unlearning” inappropriate metacognitive behaviors developed through previous experience.

Studies in mathematical problem solving over several decades also had shown that the skills-based approach to the teaching of algebra did not lead to skilled performance among algebra students (Kieran, 2007; Radford, 2004). The students participating in the studies showed neither the understanding of the ways the variables were used in algebra, nor the structural features of algebraic expressions. On the other hand, the reform-based approach would be giving a great weight to functions and functional situations, consequently leading the solution of real-world problems by methods other than manual symbolic manipulation. Kieran (2007) and Radford (2004) stated that there are three main sources of algebraic meanings: (a) meanings from algebraic structure itself, involving the letter-symbolic forms and from other mathematical representations, including multiple representations (that is the ability to see abstract ideas hidden behind symbols), (b) meaning from the problem context, and, (c)
meaning derived from the external context connected to the mathematics/problem context. The development and mastering of problem solving ability depends on acquiring algebraic meanings from all three sources. According to Kieran (2007), Radford (2004), and Sfard and Linchevski (1994), even the first source that is the easiest to obtain is difficult for students to grasp. Talking about multirepresentations, Kaput (1989) and Bednarz and Janvier (1996) argued that algebraic syntax is extremely difficult for students to master. They also claimed that the ability to create binary representations - graphs, and equations -- is crucial for problem solving as well as the algebraic reasoning.

As can be seen from the work discussed above, Polya’s strategies represent only a component of mathematical problem solving ability. The other elements are cognitive skills and affect. The presence of those is absolutely necessary for effective problem solving. According to Lester (1994), Schoenfeld (1985; 1992), Pressley et al. (1995), those skills can be and should be taught.

**Teaching Strategies for Developing Problem Solving Abilities**

This section presents strategies for encouraging the development of mathematical problem solving abilities.

The current reform in mathematics education is critical of memorization and drill of mathematics facts and procedures, arguing instead for instruction that emphasizes understanding (Cobb et al., 1991; NCTM Standards, 1989). Teaching for understanding
is one of the key problem solving strategies (Pressley, Burkell, & Schneider, 1995). Such understanding includes knowing that mathematics is a way of representing and organizing real experiences, knowing why mathematics is important, knowing the mathematical facts and procedures and how they are related to one another, and understanding how this knowledge relates to information already understood. The following strategies for teaching problem-solving are presented:

- Learning of mathematical problem-solving is best when students are active participants in the process. Although the teachers’ input and support play a significant part in teaching and learning, their role is more to facilitate the process.
- Instruction should emphasize the relationship of mathematics to the real world.
- Word problems should be part of any concept.
- Use of manipulatives helps to concretize teaching abstract mathematical concepts.
- Teachers should model problem solving as well as making obvious that there are alternative solutions to problems.
- As students attempt problems, they should be taught metacognitive skills.
- Small group problem solving should be encouraged, since by working in small groups learners experience diverse methods of problem solving as well as come to see that mathematics is a social and collaborative activity.
- Technology (calculators) helps eliminate the need to attend to lower-order operations, releasing the students’ minds to concentrate on higher-order tasks.
• Students should be encouraged to understand that their mathematical achievement is under their own control.

Presley et al. (1995) argued that the direct explanation of problem solving, accompanied by modeling and followed by scaffolding practice is one of the crucial strategies in problem solving:

“We favor heavy doses of explanation and modeling with respect to mathematics as well, fully aware as we do so that there are those in mathematics education fraternity who would disagree, believing that such explanations prevent children from discovering effective mathematics instruction on their own. The approach they favor is arranging conditions so that problem solution discovery is likely…. When done well, a teacher explanation provides students with good start towards understanding a problem solving procedure.” (p.193)

The direct explanation should include a great deal of information about why the procedure is being taught. Even constructivist mathematical educators would favor students’ discovery methods, Presley et al. (1995) argued that such discovery can be inefficient and misleading at times, while teacher scaffolding will lead to sharpening students’ knowledge and understanding. Another efficient method for learning mathematics is to provide worked examples. Sweller and Cooper (1985) stated that students who provided worked examples processed the topic faster and performed better than students who practiced solving the same problems on their own. Yet, as the study shows, the worked examples strategy worked best with identically structured problems.
There is also much theory and a great deal of evidence to make the case for practice as part of mathematics instruction. Such practice provides the student with a firm foundation of math facts. Siegler (1989) argued that it is vital to know math facts. Students who have mastered them rely less on counting strategies and open up their minds for other things by retrieving answers from long-term memory. Even among some mathematics educators who do not seem to have a favorable view of student practice, equating it with rote learning, there are a number of studies showing practice as one of the key to developing problem solving skills.

Another strategy is to develop automatic recognition of problem types. Mayer (1981) identified about 100 common problem types presented in textbooks. He stated that schematic representations of typical problem types reside in the long-term memories of students thus making them more accomplished problem solvers. Modeling is another approach recommended. It is a flexible, powerful, and engaging tool for problem solving (Ferrucci, Yeap, & Carter, 2003). Their studies demonstrated that the long-established traditional modes of introducing and teaching word problems often leave students looking for key words that hint at an operation instead of drawing on previously acquired mathematical experience to apply. Such experience “induces in pupils a strong tendency to approach word problems in a mindless, superficial, routine-based way to identify the correct arithmetic operation needed to solve a problem” (Verschaffel & De Corte, 1999). Contrary to the approach, modeling uses a pictorial representation of the quantities in a
problem and the relations between those quantities. Thus, solving problems by modeling is intended to help students visualize abstract mathematical concepts.

Several researchers (Booth & Thomas, 2000; Diezmann & English, 2001; Novick & Hurley, 2001; Pantziara, Gagatsis, & Elia, 2009; Presmeg, 2006; Vekiri, 2002) emphasized the importance of using graphical representations in the mathematical problem-solving process. Vekiri (2002) talked about four groups of graphical representations: diagrams, graphs, maps, and charts. In problem solving, a diagram can serve to represent the structure of a problem; thus, it can be a useful tool in the solution of the problem. Nunokava (2004) considered drawing to be one of the important strategies for mathematical problem solving. Drawings may help solvers to understand a problem, to advance the solving process, and to provide critical information that directly leads to the problem solution. Using graphical representations has been identified as one of the most effective strategies to improve mathematical problem solving performance (Hembree, 1992; Uesaka, Manalo & Ichikawa, 2007). In addition, Moyer, Sowder, Threadgill-Sowder, & Moyer (1984) stated that problems presented to students in the drawn format were easier than the ones in verbal and telegraphic formats. The difference was especially significant for students with low reading abilities.

A study presented by Mevarech and Fridkin (2006) examined the effects of IMPROVE, a meta-cognitive instructional method, on students’ mathematical knowledge, mathematical reasoning, and meta-cognition. The founder of research on meta-cognition Flavell defined meta-cognition as “thinking about thinking” (Mevarech &
Fridkin, 2006). Schoenfeld (1985) also reported that college students who were trained to apply self-addressed questions like: “What am I doing right now? Why am I doing this? How does it help me?” improved their mathematics performance. IMPROVE is an acronym for the following teaching steps: Introducing the new concepts, Meta-cognitive questioning, Practicing, Reviewing, Obtaining mastery, Verification, and Enrichment. In IMPROVE, the teacher begins with introducing the new concepts by modeling the meta-cognitive technique which consists of comprehension questions, connection questions, strategic questions, and reflective questions. Following the teacher’s introduction, students practice problem solving using the meta-cognitive questioning technique. The studies showed that IMPROVE students did considerably better than the non-treatment control group on algebra assessment that included numerals, substitution, word problems, and reasoning (Kramarski, Mevarech & Arami, 2002; Mevarech & Kramarski, 1997; Mevarech & Fridkin, 2006). Interestingly, the positive effects of IMPROVE were observed not only on topics presented right before the examination, but also on those introduced six to eight months earlier.

Tanner, Jones, and Davis (2002) presented a study of three different teaching approaches to problem solving. Two of these were unsuccessful. In the first approach, students were given problem-solving tasks and left to think unaided. Most of the students failed the task. In the second, the teachers provided students with an algorithm to follow. The students failed to proceed with similar tasks later, thus indicating that learning had not occurred. A third approach resulted in the students significantly outperforming their
counterparts on tests of problem-solving ability. This approach was referred to as “start-stop-go.” With this approach, the problem-solving tasks begin with students thinking in silence about the problem. They then discuss their ideas and plans in small groups. Those ideas are brainstormed and evaluated within the whole class afterwards. Throughout these phases, the teacher would scaffold mathematical thinking through a range of questions that aimed to focus students’ attention on the structural features of the problem and help to organize their thoughts. The last phrase was peer and self-assessment that led students to reflect on their work.

The strategies offered above present problem-solving approaches going far beyond the traditional Polya’s theory. The “start-stop-go” approach as well as IMPROVE help to develop students’ metacognitive skills which are necessary for effective problem-solving performance. Conscious control of thought process is achieved only after such control has been practiced unconsciously and spontaneously. The reflective discourse generated by teachers, and then supported by students, provides the teacher with opportunities to focus attention on key points and strategic learning. Those strategies are also undoubtedly an example of Vygotsky’s sociocultural theory of learning where scaffolding by teachers and socialization between pupils create a foundation for learning (Vygotsky, 1978).
Modeling in Mathematics Learning and Problem Solving

The issues of modeling and representation have been the focus of a number of problem-solving studies (Doerr & English, 2003; English, 2006; Goldin, 2003, 2008; Janvier, 1987; Lesh & Zawojewski, 2007; Vergnaud, 1998).

A representation is a configuration that represents something else in some way (Goldin, 2008). A representational system further includes ways of combining the elements into permitted configurations. Those configurations are well defined rules such as formulas, lists, or sentences. There are external and internal representational systems for mathematics. External systems are axioms and theorems. Internal systems include language, personal constructs, visual and spatial imagery, problem solving heuristics, affects and so forth (Goldin, 2008; Kaput, 1994). Effective learning takes place when students make inferences about internal representations based on mathematical conceptions and interactions with external representations. Representational systems are not transcribed from outside into the human mind; they rather develop in learners, structured by the presence of prior systems. Goldin (2008) brought the following model to such development: 1) inventive/semiotic stage, in which new internal configurations are constructed and assigned meaning based on previous representations, 2) structural development period, driven by the meanings assigned, during which the higher structure of the new system is built, 3) autonomous stage, in which the new representational system detaches entirely from previous arrangement and functions powerfully with new meanings. Constructivist theories of learning consider the act of representation as a
A model is a specific structure of some kind that symbolizes features of an object, a situation, or a class or situations or phenomena (Goldin, 2008). Modeling refers to the construction of models, which are important structures within one or more representational systems. Problem solving is seen as engaging in modeling activity (Lesh & Doerr, 2003; Kaput, 2007; Thompson & Yoon, 2007). Modeling cycles that are parts of the activity can be observed and studied. Conceptual development is taking place during modeling cycles. Therefore, those ideas suggest that an explicit focus of teaching problem solving should be on model-eliciting activities. Modeling activities are distinct from traditional classroom problem solving (English, 2006; Doerr & English, 2003).

**Adult Learners and Problem-Solving**

The National Council of Teachers of Mathematics (NCTM) released the *Curriculum and Evaluation Standards for School Mathematics* in 1989, a document calling for a reform in math education. This document puts an emphasis on understanding as opposed to instruction-based learning. The American Mathematical Association of Two-Year Colleges (AMATYC) extended the motion in 1995 to adult learners with the publication of *Crossroads in Mathematics: Standards for Introductory Mathematics before Calculus*. This approach is based on Jean Piaget’s theory of constructivism and
Lev Vygotsky’s theory of sociocultural aspects of learning. One of the core standards in both documents is *problem solving.*

Kloosterman, Mohamad-Ali, and Wiest (2000) provided the following suggestions for applying NCTM and AMATYC principles in adult education settings:

- Help learners see that mathematics is more than a set of rules to be memorized
- Build on learners’ previous knowledge of mathematics
- Be a facilitator of learning rather than just a lecturer
- Get learners to work together
- Ask complex questions
- Help learners to become confident in their ability to use mathematics (p.70).

One of the issues in developing problem-solving skills by adult students is the role of *commonsense.* Colleran, O’Donoghue, and Murphy (2002) affirmed that usual, everyday *commonsense* provides a confident basis on which adult learners began the quantitative problem-solving process. The foundation for their project was Bernard Lonergan’s three levels of knowing:

1. *Commonsense knowing* which happens spontaneously in the concrete world
2. *Scientific knowing* which is employed when an individual engages a new situation and the mental processes move from the concrete to the abstract
3. *Critical knowing* which enables learners to solve quantitative problems.
Lonergan’s problem-solving and decision-making program can therefore be visualized not as a two-dimensional cycle of mental activities but as a three-dimensional helix which dynamically connects concrete understanding at the lower, commonsense level to a deeper and more abstract understanding at the intermediate, scientific level and, finally, to an even deeper metacognitive understanding at the top of the helix (Colleran et al., 2002). Thus, they argued, commonsense should be added as a significant resource to the five proposed by Schoenfeld (1992) and Mayer (1982) in the framework of adult mathematical problem solving:

1. Domain specific knowledge
2. Heuristics
3. Metacognition
4. Beliefs
5. Context
6. Commonsense.

Commonsense is a collection of insights accumulated by a community, or individuals within that community, in a socio-historic setting. The context within which it operates is quite specific. It is specialized in the concrete objects of everyday living in terms of their relationships, not to one another, but to the individual. It is bounded by the concerns of human living and by workable solutions to daily tasks (Lonergan, 1957, cited in Colleran, et al., 2002).
Colleran, O’Donoghue, and Murphy (2002) concluded that among adult learners, prior understanding, or commonsense, provides a problem-solving resource with three distinct elements: (a) a store of practical understanding and knowledge, (b) a confident starting point on which to begin the problem-solving process, (c) a base on which to build a formal solution. They put together an educational program aimed at improving the quantitative problem-solving and decision-making skills of adult learners. The theoretical basis for the program was Lonergan’s work. There are four ideas which when put together provide the strong structure for the program. First and primarily, it is authors’ belief that problem-solving skills can be improved through a structured educational program. Second, the quantitative problem situations addressed by the learners must be
relevant, realistic, and meaningful for the learners. Third, a social learning environment should be provided. This environment facilitates discussion and dialogue which are essential to the development of thinking skills. Fourth, the adaptation of Lonergan’s philosophy would allow learners to discover the way they think when they are solving problems. As a result of this study, the authors are convinced that Bernard Lonergan, Canadian theologian and philosopher, offered a sound theoretical framework for a problem-solving process that can be employed successfully by researchers and practitioners in the field of mathematical problem solving.

As a mathematics teacher in an adult basic education (ABE) setting, Arriola (2000) investigated the obstacles to learning that adults face in the NCTM Standards driven mathematics classroom. Implementing the Standards means creating a learning setting in which students are active participants in the learning process and in which the learners’ interest, resourcefulness, risk taking, and reflection play major roles. Arriola stated that, “…from our perspective, it seems reasonable to assume that by simply removing the barriers and judgments that perpetuate passive learning, our adult students will move naturally and with great relief into an active learner role. But, in fact, our ABE students haven’t the slightest idea what it means to learn math in this way” (p. 226). Arriola argued that this gap in experience and understanding is the main problem adult mathematics educators faced in the Standards-based classroom. In other words, our biggest challenge is to change students’ beliefs about doing mathematics and mathematical problem solving.
Summary

Research in mathematics education is relatively young with the earliest work being conducted in the mid-1930s (Lesh & Zawojewski, 2006). Furthermore, the work directed at mathematical problem solving (word problems in particular) began in 1960s. Even considering not such a long period of time, the field of problem solving went through some considerable changes. Polya’s ideas, presenting a whole problem solving strategy a while ago, now became only a part of a much bigger picture. The change in the field in mathematics education as well the change in the perception of problem solving goes parallels the changes in people’s lives in modern society. According to the National Research Council (1999, cited in Lesh & Zawojewski, 2006),

Changing technologies will continue to alter skills and eliminate jobs at a rapid rate. Although skill requirements for some jobs may be reused, the net effects of changing technologies are more likely to raise skill requirements and change them in ways that give greater emphasis to cognitive, communications and interactive.
(p.780)

There is a necessity for people to adapt and create mathematics for use in everyday environment. Consequently, there is a new perspective on problem solving that emerges. There is also evidence growing on de-emphasizing “transfer” of learning but moving instead toward dynamically interrelated and complex intellectual, emotional, and social processes of acquiring quantitative understanding and developing mathematical problem solving skills. Part of it is instinctive thinking skills available to all normal adults. There
is no movement of intellectual resources. It is the individual learner that moves from one intellectual level to another. “We seem to be moving from a hard, rationalistic, scientific and wholly intellectual framework for understanding the transfer of learning to a softer, emotionally charged and individualistic view. The modern scientific view that frames its models on measurable, observable, structured objectivity, which did not concern itself generally with the impact of human individuality and particularly emotions, seems to be broadening out to include the post structural evaluation of the unique individuality and subjectivity of learners” (Colleran et al., 2002, p.80). The environment within which the problem-solving abilities grow is always socially interactive and emotionally charged.

There is also the difference between the mathematics of school and of the workplace that is critical to a new perspective to problem solving (MSEB, 1998; Oakes, Rud, & Gainsburg, 2003; Magajna & Monaghan, 2003, cited in Lesh and Zawojewski). This difference is leading to mathematics modeling consistently emerging as one of the most important types of activities.

Research on Word Problem Solving

Word Problems Solving and Cognition

There are two major types of research on word problems. The first one focuses on individual cognition and competences for decoding textual information into symbols and obtaining correct answers. The second one focuses on the connection of word problem-solving and real-life quantitative situations.
From the cognitive perspective, mathematical word problem-solving processes begin with reading of the problem’s text (Nathan et al., 1992). The next step is forming an object-based or mental model based on prior knowledge and continuing processing of the text elements. Those models may involve real or pictorial external representations that facilitate the development of mental images or internal representations of problem elements and their relationships. Thus, an active model is constructed through active transformation of the text base, activation of problem-type schemas, and integration of the problem elements within those schemas. Success or failure depends largely on the coherence of the mental representation formed (Pape, 2004; Kintsch & Greeno, 1985; Nathan et al., 1992) and the ability to control and change the problem-solving process (Schoenfeld, 1992). In addition, when solving word problems, people are required to move between diverse linguistic and symbolic codes (Wyndhamn & Saljo, 1997). Events articulated in normal language and describing what resembles real-life activities have to be converted to the notational system of mathematics and subjected to formal operations defined in the mathematical domain. The research shows that students experience difficulties with aligning the semantics and syntactic of everyday language with the structure of formal mathematical reasoning (Wyndhamn & Saljo, 1997).

There are several factors (variables) in constituting difficulties students encounter with algebraic and arithmetic word problems and consequently impacting students’ performance. One of them is the classification of word problems or the type of problem situation (De Corte & Verschaffel, 1988; Greeno & Heller, 1983; Greer, 1993; Nesher,
1982; Riley & Fuson, 1992; Vergnaud, 1982; Dewolf, Van Dooren, & Verschaffel, 2011; Verschaffel, 2012). The other factors are the exact phrasing of the problem, the particular numbers used, and the age and instructional background of the students. According to De Corte, Verschaffel, and De Win (1985), linguistic knowledge of a student is one of the variables of the word problem-solving outcome. They, as well as Stern (1993) and Cummins (1991), stated that rewording a problem so that the described relation is made more explicit makes the construction of an appropriate problem representation easier.

Cummins (1991), Okamoto (1996), and Pape (2004) stated that students’ failure to solve certain types of word problems is related to students’ difficulties in comprehending the abstract language of mathematics. Moyer, Sowder, Threadgill and Moyer (1984) reported that readers of high ability in grades 3 through 7, as measured by a reading test, are more successful when solving word problems than low-ability readers. Caldwell and Goldin (1979) compared the relative difficulties for elementary school children of four types of word problems (abstract factual, abstract hypothetical, concrete factual, and concrete hypothetical). The findings of the study confirmed that for elementary school children concrete verbal problems are substantially less difficult than abstract ones, when other relevant variables are controlled. Rile, Greeno, & Heller (1983) found that word problem difficulty is strongly affected by the role (or position) of the unknown quantity within the problem statement. Result-unknown problems, considered arithmetic-level problems, present much less difficulty than start-unknown problems, considered to be algebra-level problems (Nathan & Koedinger, 2000; Dewolf, Van Dooren, & Verschaffel, 2011).
Ballew and Cunningham (1982) did a study that involved sixth graders solving word problems. The profile of each student consisted of the following four scores: computation score, problem interpretation score, reading-problem interpretation, and reading problem-solving score. Ballew and Cunningham (1982) stated that since all four components are vital for successful word problem solving, the lack of at least one leads to a failure. In the study only 19 of the 217 students had all four scores thus having solved the problems correctly.

When studying junior high school students solving algebraic word problems, Koedinger and Nathan (2004) observed that using multiple representations such as tables, graphs, and equations increases the possibility of obtaining the right answer. While doing similar research, Kieran (2007) added that generating an equation to represent the relationships is the area of difficulty for algebra students across the ages. Middle and high school students’ preference for arithmetic reasoning and their difficulties with equations while solving algebraic word problems were reported by Bednarz and Janvier (1996), Cortes (1998), and Swafford and Langrall (2000). Nathan and Koedinger (2000) also reported that high school students’ solving word problems ability was negatively affected by the impression that the story problems are essentially harder than symbolic ones. They also observed that even after a full year of algebra students were particularly challenged by the demands of comprehending the formal symbolic representation of a quantitative relation, consequently attempting translation of word problems into standard equation only five percent of the time.
Bednarz and Janvier (1996) investigated the emergence and development of algebraic thinking by reflecting on the various types of problems that are given to students in introductory algebra courses. The analysis identified three major classes of problems based on the nature of the quantities involved in the relationships between them: (a) problems of unequal sharing, (b) problems involving a magnitude transformation, and (c) problems involving non-homogeneous magnitudes and a rate. The general construction of a problem brings out the quantities, knowns and unknowns, their connection to one another, and the type of connection involved. These connections are given more or less explicitly in the problem statement and must be reconstructed by the student with the help of the known quantities or other mathematical or contextual knowledge prior to solving. The following conclusions about the passage from arithmetic to algebra were made:

- In arithmetic, the problems generally given to students are problems that Bednarz and Janvier (1996) label “connected”. A relationship can easily be established between two known pieces of data, thus leading to the possibility of arithmetic reasoning (from the known to the unknown quantity at the end of the process). Working in this way, there is no need for the student to deal with more than one state at the time. Starting with an initial known state, he/ she can obtain a new state, and the final quantity can be found from these intermediary states. The success rate on those problems was about 80%.

- On the contrary, in algebra the problems that are generally given to students are
labeled “disconnected”: no direct bridging can be established between the known data and the unknown data. The composition of two or more types of relationships involved in some problems, which implies that the student perceives that the same quantity is repeated and included in the new quantity, is causing a great deal of difficulty. The success rate on those problems was between three and 31%.

- The majority of students involved were choosing arithmetic ways of solving problems rather than using the classic equation method.

The authors found that the process of generating a relationship using an unknown appears to be extremely complex for the students. In their actions the students never take on the composition of the two relationships: they go through an intermediary state, the different states being generated by a sequence of two-state actions. In addition, substitution, which requires the passage to a single unknown, appears to be equally difficult for the students. For some students, these difficulties are accompanied by a refusal to operate on the unknown. The symbolism used to present the relationships in the equation creates difficulties as well.

Stacey and MacGregor (2000) propose that a major reason for difficulty is not misunderstanding the logic of solving a problem by algebra, but the deflection from the algebraic path grounded in arithmetic problem solving methods. Students in their study (aged 13-16) comprehended the problems but most did not formulate equations. Instead they tried to use a sequence of calculations to obtain the answer. They reported on the “cognitive gap” between calculating with numbers to operating with the unknown. The
same has been reported by Herskovics and Linchevski (1994). Stacey and MacGregor (2000) also reported on difficulties students have with understanding the concept of ‘an equation” as an equivalent relationship, not just a formula or a procedure (p.151). There is a students’ belief that problems are solved by direct calculation instead of applying analytic methods. This impulse prevents students from looking for, selecting, and naming the appropriate unknown (s) and, consequently, in formulating an equation. Algebraic thinking does not develop spontaneously and students are unlikely to switch from an arithmetic approach unless they are specifically taught (Herskovics & Linchevski, 1994; Stacey & MacGregor, 2000; Verschaffel, 2012). Johanning (2004) when reporting on students’ informal strategies while solving word problems observed systematic guess and check along with unwinding (working backwards).

There is evidence that adult students experience difficulties solving word problems as well (Ginsburg, et al., 2006). Working with science-oriented college students, Clement (1982) reported that a large proportion of the students were unable to solve a very simple kind of algebra word problem. While 99% of a group of freshmen engineering students solved the linear equation 5x = 50 and 6/4=30/x correctly, only 27% of them did ratio/proportion word problem. Data obtained from the group testing indicated that a significant number of college students produce reversal errors in formulating algebraic equations of this kind. The sources for those errors are syntactic, that is, word order matching and semantic that is a static symbolization process. Those results lead to a concern about the extent to which students understand how equations are
used to symbolize meanings.

In addition, Lee (1996) argued that the children’s and adults’ word problems solving techniques are not identical. When looking at the behavior of adults as well as high school students during word problem solving, she concluded that for both groups observed, using the algebraic language was extremely difficult. The qualitative responses of the adult students and those of the high school students were almost identical as well. Nevertheless, it was the interview behavior of the adult students that revealed some differences with their high school counterparts. During the interview, adults showed less concern about putting some algebra down on their papers and did not go off on meaningless algebraic manipulations to the same extent as high school students. Adults demonstrated a similar, if not greater, inability to use algebra and to do generalization. None were able to let $X$ be any number while attempting to solve word problems and were troubled by word associations or misunderstandings.

**Word Problems Solving and Informal Real Life Mathematics**

A second type of study arose from the domain of Ethnomathematics and students’ life and functions outside of the formal mathematical classroom. According to Lave (1988), Saxe (1991), and Schliemann and Acioly (1989), there is evidence that individuals perform at lower levels of their ability when solving formal mathematical word problems rather than solving quantitative problems presented by real life situations.

In a study by Verschaffel, De Corte, and Lazure (1994) seventy five 11 to 12-year-
old students were collectively given a word problem test involving multiplication and division problems in an ordinary mathematics classroom context. Besides standard problems in which the relationship between the situation and the corresponding mathematical operation is simple and straightforward, the test contained parallel versions of these problems in which the mathematical modeling assumptions are problematic. For this latter type of problem, only a small minority of the students was successful. Similar results for 13 to 14-year-old students have been reported by Greer (1993). These findings convincingly demonstrated that considerable experience with traditional school mathematics word problems develops in students a strong inclination to exclude real world knowledge and reasonable contemplation from their solution process. The instructional implication of this finding is that “the impoverished diet of standard word problems currently offered in mathematics classrooms, should be replaced -- or at the least supplemented -- by a wide variety of problems that draw students' attention to realistic modeling, so that they do not implicitly learn that if there are two numbers in the problem, the answer will be found by adding, subtracting, multiplying or dividing these two numbers” (Greer, 1993, p. 292). Verschaffel and de Corte (1997) highlighted the role of being familiar with the game of school word problems -- puzzle like tasks -- as a main factor of the ability to deal successfully with word problems. Nunes, Schliemann, and Carraher (1993) when comparing problem-solving skills of young street vendors in Brazil and corresponding school problems, stated that: "if mathematics education is going to be realistic, problems will have to be sought that respect assumptions about life outside
school” (p.148). Greer (1997) and Verschaffel (2012) also commented on a very widespread tendency of children to answer school mathematics word problems with evident disrespect for the reality of the situations described by the text of these problems. Analysis of this behavior strongly suggests that the reason for it is not a cognitive deficit of the children, but rather in the culture of the classroom where word problems are presented in stereotyped fashion, with an assumption that the application of one or more of the basic arithmetical or algebraic operations to the numbers given is appropriate. With such pressure, the observed behavior of the children may be considered a reasonable response.

To address the concern of inadequate problem solving performance, careful attention should be paid to the variety and complexity of the school mathematical word problems. This means that a wide variety of more authentic and more complex problem situations that activate and stimulate realistic modeling should be included into the instructional process (Verschaffel, de Corte, & Lasure, 1994). When addressing the problem of the lack of connection children make between mathematics in school and mathematics out of school, Greer (1993) pointed out to the commonly observed pattern that the stereotyped nature of word problems means that illusory success can be achieved through superficial instructional methods. He also believed that an alternative conceptualization of word problems as situations calling for mathematical modeling that takes into account real world knowledge is vital.
Summary

There is a wide variety of types of word problems as well as an equally wide variety of strategies that may be employed for solving them. The above research indicates that despite the vast number of strategies, students continue to have difficulties solving all types of mathematical word problems. There is wide variation of explanations for these difficulties. Those include reading comprehension, classification of word problems, type of problem situation, exact phrasing of the problem, particular numbers used, and the age and instructional background of the students (De Corte & Verschaffel, 1988; Dewolf, Van Dooren, & Verschaffel, 2011; Greeno & Heller, 1983; Greer, 1993; Nesher, 1982; Riley & Fuson, 1992; Vergnaud, 1982).

According to Reed (1999), the success of both algebraic and nonalgebraic strategies depends on constructing a correct model of the situation described in the problem. He argued that students are typically able to understand the situation, but fail to quantitatively model the situation to find the value of the unknown. In addition, algebraic approaches present more difficulties than arithmetic ones since such change requires a new way of thinking (Herskovics & Linchevski, 1994; Johanning, 2004; Lee, 1996; Stacey & MacGregor, 2000).

There is also a notion that considerable experience with traditional school word problems cultivates in students the tendency to exclude real-life knowledge from different stages of the solution process, i.e. the original understanding of the problem, the construction of a mathematical model, the computational process, and the interpretation
of the result obtained. Rather than encourage students to apply their common sense, traditional word problems have become unrealistic tasks which are being viewed as completely unrelated to real life (Verschaffel et al, 1994). This observation seems to apply to traditional students as well as adults (Ginsburg, et al. 2006; Nunes, Schliemann, & Carraher, 1993; Saxe, 1988).

Thus, one of the aims of this study is to see how adult students interpret word problems, what kind of difficulty they experience, and what methods, formal and/or informal, they apply to solving mathematical word problems.
CHAPTER III

METHODOLOGY

The purpose of the study is to understand and describe the experiences adult learners have while solving mathematical word problems in order to understand the different cognitive meanings of algebraic thinking of adult learners. This chapter, the Methodology, presents the way I carried out the study.

In this chapter, I describe the study’s design. First, I articulate the Theoretical Framework as rationale for design choices. This includes discussion of Qualitative Research and Mathematics Education, Multiple Case Studies as the Chosen Methodology, and My Philosophical Paradigm. Next, I explicate the study’s Methods, including description of Data Sources, Data Collection, Interview Protocols, Observations, Data Analysis, Validity Issues, Ethical Considerations, and Limitations of Research.

Theoretical Framework

Qualitative Research and Mathematics Education

Qualitative research in general is described by Denzin and Lincoln (1994) as a cross-disciplinary field of inquiry being applied in education, psychology, sociology, political science, and anthropology. Qualitative research focuses on processes, meanings, and the socially constituted nature of individual reality, and provides insights into the phenomena being studied that cannot be revealed by other types of inquiry (Teppo,
1998). It is primarily concerned with human understanding, interpretation, intersubjectivity, and lived truth. Its aim is to record phenomena in terms of participant understanding.

Sociologists and anthropologists have employed qualitative research methods since the turn of the 20th century, but only in the last thirty years has this type of inquiry become viewed as a valid method for research in education (Teppo, 1998). Before that time, the dominant practice of research was quantitative analysis, which is grounded on postpositivist assumptions for developing knowledge (Creswell, 2003). These assumptions rest on the system of beliefs of the scientific method which considers reality as definable and measurable and views the researcher as distant and removed from his or her studies. In quantitative research, the investigator looks for description or explanation among variables that constitute a problem (Creswell, 2002). The focus of the research is on the rather large population sample. When discussing research in education, Denzin and Lincoln (1994) characterized the 1980s as the years of the “paradigm wars”, since during this time a wide range of paradigms, methods, and strategies appeared in the field. Next to quantitative approach, qualitative researchers in education tested perspectives, theories, and methods from a variety of fields. In the end, educational researchers emerged from those methodological debates with the recognition that each paradigm offers a different way to focus research on the complexities of contemporary education (Cizek, 1995).
The research in mathematics education in particular followed progress in general education research, psychology, social and cognitive sciences, and philosophy (Schoenfeld, 1992). Through the 1960s and 1970s the “scientific” study of thinking and learning in mathematics education was dominant. Consequently, these years are mostly presented by correlational, factor-analytic, and statistical “treatment A vs. treatment B” comparison studies. During the mid-1970s, the limitations of scientific studies became obvious. At this time, Kilpatrick (1978, as cited in Schoenfeld, 1992) when comparing U.S. research in the field to the qualitative research conducted in the Soviet Union in the 1970s, called the U.S. approach rigorous, but somewhat sterile. In his opinion, U. S. researchers lost the understanding of a person’s meaningful mathematical behavior while striving for rigor. In contrast, the studies done in the Soviet Union at this time while being decidedly not rigorous focused on such important aspects as behavior, abilities, and mathematical thinking. So the late 1970s and 1980s are characterized by a turn toward process-oriented studies. However, since the 1990s, there is a great deal of diversity in the field on mathematical education research. It has brought together researchers from numerous disciplines presenting different perspectives, broad views, and a great diversity of methods into the field of study. As the result, qualitative methodology developed into one of the main approaches to research in mathematics education. It incorporates vast interest and recognition of the work of Jean Piaget on constructivism, sociocultural theory by Lev Vygotsky, and the ideas of theoretical cognitive structures (Asiala et al., 2004; Teppo, 1998).
According to Creswell (2003), a researcher should consider four questions in
designing a research proposal. These questions are about epistemology, philosophical
stance, methodology, and methods that inform and govern the choice of research. As a
researcher, I assume that meanings are constructed by human beings as they engage with
the world they are interpreting. These meanings are varied and multiple and lead to the
complexity of views rather than expressing only a few ideas. The purpose of my study is
to investigate adult students’ experience when solving mathematical word problems. In
order to investigate the experience, I employed open-ended questions so that participants
were able to express their attitudes and feelings. In addition, I believe that humans’ view
of their world is based on their social and cultural perspective. Consequently, I used the
individual interviews to gather information about each participant in order to focus on the
individual’s constructions and the specific contexts in which the participant lives and
learns. Even though I have worked with adult students for the last twenty years, I have
had neither theory nor hypothesis to guide my research. On the contrary, I inductively
developed a description of the participants’ constructions and experiences at the end of
the study. In addition, I also cleared my biases and let the data emerge. I also decided to
do a qualitative study since the questions of my research are process-oriented and are
focused on interpreting the phenomena of solving word problems in terms of the
meanings adult students bring to the mathematics classroom. The questions of the study
were

1. What attitudes and beliefs do adult students hold about solving word problems?
2. What mathematical content knowledge do adult learners gain access to when solving word problems?
   a) How is this knowledge used? How is it chosen?
   b) How does the solution evolve the way it does?

3. Do adult learners use formal or informal approaches to solve word problems?
   a) When formal approaches are employed, what strategy(s) do adults use?
   b) When informal approaches are employed, what strategies are used?
   c) What are the adult students’ symbolic language and the dynamics of reasoning/thinking?

**Multiple Case Studies as the Chosen Paradigm**

There are several definitions of case study research. Yin (1994) defined case study in terms of the research process. He stated that it is “an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident (p. 13). Stake (1995) focused on the unit of study—the case. Merriam (2009, p.21) defined it in terms of the end product: “A qualitative case study is an intensive, holistic description and analysis of a single instance, phenomenon, or social unit.” Wilson (1979) presented the case study as a process “which tries to describe and analyze some entity in qualitative, complex and comprehensive terms not infrequently as it unfolds over a period of time” (p. 448). Cronbach (1975) differentiated case study from other research designs by calling it
“interpretation in context” (p.123). Additional characteristics of case study research are *object of study or case* (Merriam, 2009), *bounded system* (Smith, 1978), and *integrated system* (Stake, 1995). The purpose of my study is to understand and describe the experience of adult learners solving mathematical word problems in order to better understand the cognitive meanings of algebraic thinking of adult learners. Therefore, I believe that case study is the right perspective of qualitative research that would serve the purpose of the study. The unit or the case of the study is adult students solving mathematical word problems. Merriam (2009) also stated that the qualitative case studies are *particularistic, descriptive, and heuristic*. This study is *particularistic* since it focuses on a particular phenomenon—adult students solving mathematical word problems. This study is *descriptive* since the end product is a “thick” description of the phenomenon under study. The description presents documentation of the events, artifacts, and quotes. This study is *heuristic* due to the fact that it concludes with my understanding of the phenomenon under study. In addition, I offer a cross-case analysis suggesting generalizations about adult students and the process they use to solve word problems from the individual case studies I initially employed. This results in a multiple case study format.

**My Philosophical Paradigm**

The methodology of any study is influenced by the question of the study as well as the researcher’s philosophical paradigm (Creswell, 2003). As a person, educator, and
the researcher, I believe that every human being embraces a different reality depending on his or her social and cultural backgrounds and life experiences. I also believe that the world in which we exist is value-laden, preconceptions are present in our coexistence, and ethics and spirituality are important elements of human investigation.

As a native of the former Soviet Union, it makes me proud to say that I was born in the same part of the world as Lev Semenovich Vygotsky, the famous sociologist and pedagogue. His social explanation of human psychology has been enormously influential on psychology and pedagogy in a universal sense, but also on my experiences as a person, an educator, and a researcher. His immense contributions in both theory and method of education have influenced my teaching techniques, as well as educational theory and method in general. My strong belief in his theory of the socio-historical nature of psychological phenomena has generated an acute vision that learning takes place within a socially supported circumstance. I also believe that individuals actively construct their knowledge based on the meaning of their experience. Those subjective meanings are formed through interaction with others and through social and historical surrounding. Therefore, I look at the world through the lenses of a social constructivist and strongly believe that social and cultural issues have a tremendous impact on any educational matter, including research in the field.
Methods

Data Sources

Qualitative inquiry typically focuses in depth on relatively small samples selected purposefully. The power of purposeful sampling is in selecting information-rich cases for the study (Merriam, 2009). In addition, a characteristic of case study research is the use of multiple data source (Creswell, 2007; Patton, 1990; Yin, 2003). My general considerations for locating and selecting the research participants were age, race, gender, and previous education and credentials. Essential criteria included experiencing the phenomenon, being interested in understanding its nature, and willing to participate in the interviewing process. Thus, the criteria for constructing the sample for my study were adult learners of different age groups and diverse academic and cultural backgrounds in order to acquire and describe the themes that cut across a wide population. Nevertheless, in reality my sample group depended on the students who volunteered for the study. Thus, my sample was purposeful as well as convenient. The size of the sample in my research corresponded to Lincoln and Guba’s (1985) recommendation which states, “In purposeful sampling the size of the sample is determined by informational considerations. If the purpose is to maximize information, the sampling is terminated when no new information is forthcoming from new sampled units; thus redundancy is the primary criterion” (p.202).

I recruited my research sample from the students who were taking a Beginning Algebra class at a Midwestern community college. The choice of choosing community
college students as participants of my study was based on my professional interest and the desire to help adult learners to succeed in learning mathematics. Beginning Algebra was chosen for the study as the course where the topic of solving word problems is being taught. The demographics for the course of my interest presented in the Tables 1 and 2 below.

Table 1. *Enrolment Trends Report for Fall 2011 Age and Gender*

<table>
<thead>
<tr>
<th>Total Number of Students</th>
<th>Average Age</th>
<th>Female</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>876</td>
<td>27.6</td>
<td>543</td>
<td>332</td>
</tr>
</tbody>
</table>

Table 2. *Enrolment Trends Report for Fall 2011 Race*

<table>
<thead>
<tr>
<th>American Indian</th>
<th>Asian</th>
<th>Black</th>
<th>Caucasian</th>
<th>Hispanic</th>
<th>No Response</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>8</td>
<td>499</td>
<td>197</td>
<td>7</td>
<td>148</td>
<td>10</td>
</tr>
</tbody>
</table>

In order to recruit participants for the study, I visited a number of Beginning Algebra sections inviting all students from the sections to participate. Some of these sections were evening sections, some were day sections. During the visits I explained to students in these sections the nature of the research I was to conduct and invited their
participation. After that, students interested in participating called me or contacted me via email. Prior to conducting the research, I met with the students who responded to my invitation individually in my office to further explain the nature and the purpose of the study. At this meeting I invited the students who met my criteria to participate in the study. Among students who agreed to participate, I chose a group that approximately corresponded to the demographics of the college, including gender and racial diversity. I also invited students with at least an adequate to proficient ability to communicate orally and in writing in order to (hopefully) yield highly verbal participants who seemed prepared to articulate their thinking process and to make transparent the steps they make to solve word problems in their head. In addition, I was looking for students with various levels of success in previous mathematics courses. In terms of the size of the sample, I chose a group of sixteen (16) people for the first interview and observation. Thus, considering that some students would withdraw from the course, I intended to have six to eight participants for the second interview and observation.

Table 3. Participants in the Study

<table>
<thead>
<tr>
<th>Name in the Study</th>
<th>Race, Gender</th>
<th>Age</th>
<th>Previous Degree</th>
<th>Prior Math Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liz</td>
<td>BF</td>
<td>42</td>
<td>High School Diploma</td>
<td>Algebra and Geometry</td>
</tr>
<tr>
<td>Mina</td>
<td>WF</td>
<td>30</td>
<td>Licensure in the Massage Therapy</td>
<td>Algebra</td>
</tr>
<tr>
<td>Name</td>
<td>Gender</td>
<td>Age</td>
<td>Education</td>
<td>Subject(s)</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>-----</td>
<td>-----------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>John</td>
<td>BM</td>
<td>42</td>
<td>GED</td>
<td>Business Math</td>
</tr>
<tr>
<td>Ron</td>
<td>BM</td>
<td>46</td>
<td>High School Diploma</td>
<td>Business Math</td>
</tr>
<tr>
<td>Jana</td>
<td>BF</td>
<td>38</td>
<td>Associate Degree in</td>
<td>Algebra</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Fashion Merchandizing</td>
<td></td>
</tr>
<tr>
<td>Ken</td>
<td>WM</td>
<td>50</td>
<td>High School Diploma</td>
<td>Algebra</td>
</tr>
<tr>
<td>Rocky</td>
<td>BF</td>
<td>19</td>
<td>High School Diploma</td>
<td>Algebra, Geometry</td>
</tr>
<tr>
<td>Raul</td>
<td>BM</td>
<td>29</td>
<td>High School Diploma</td>
<td>Algebra, Geometry</td>
</tr>
<tr>
<td>Kate</td>
<td>BF</td>
<td>50</td>
<td>BA Psychology</td>
<td>Calculus (unfinished)</td>
</tr>
<tr>
<td>Martin</td>
<td>WM</td>
<td>26</td>
<td>High School Diploma</td>
<td>Algebra, Geometry</td>
</tr>
<tr>
<td>Tom</td>
<td>WM</td>
<td>31</td>
<td>High School Diploma</td>
<td>Algebra</td>
</tr>
<tr>
<td>Lora</td>
<td>BF</td>
<td>19</td>
<td>High School Diploma</td>
<td>Algebra, Geometry</td>
</tr>
<tr>
<td>Nell</td>
<td>WF</td>
<td>30</td>
<td>LNP</td>
<td>Business Math</td>
</tr>
<tr>
<td>Sarah</td>
<td>WF</td>
<td>19</td>
<td>High School Diploma</td>
<td>Algebra, Geometry</td>
</tr>
<tr>
<td>Tammy</td>
<td>BF</td>
<td>29</td>
<td>Associate in Culinary art</td>
<td>Algebra and Geometry</td>
</tr>
<tr>
<td>Sal</td>
<td>WM</td>
<td>46</td>
<td>High School Diploma</td>
<td>Algebra</td>
</tr>
</tbody>
</table>
Data Collection

Merriam (2009) stated that the case study data collecting “is about asking, watching, and reviewing” (p.69). I pursued the following types of data collection:

- In-depth, semi structured task-based (also called clinical) oral interviews about and during participants’ problem solving activities. These interviews were video/audio tape recorded and then transcribed.

- Participant observation; that is, observing the participants in an actual problem-solving situation.

- A documentary study in which the writings, diagrams, graphs, etc. of the participants were reviewed to derive “meanings” from them. Examination of the artifacts was used in conjunction with the interviews and observations.

The order of the process was:

1. Interviewing individual students to determine their background and past experience with mathematics, and to allow them to express their feelings about mathematics and word problem solving.

2. Observing and interviewing a participant while she or he was solving two traditional word problems individually before learning the topic in class. According to the official course syllabus, students were to learn the word problem solving method in week 5. Thus, by week 4, I interviewed the students about their background and administered the word problems for them to solve.
3. Observing and interviewing a participant about solving traditional word problems after learning the topic in order to see if the student’s thinking process and/or attitude has been changed. This process took place after the week 6 of the semester.

The observations and interviews took about one hour. In every observation and interview, optional objects were provided for external representation, such as paper, pencils, markers, and a hand calculator.

Clinical Interview as the Foundation of the Constructivist Methodology

As I pointed out above, semi structured task-based (also called clinical) oral interviews were used as one of the data collection tools. Clinical interviews are one of the essentials of Piaget’s constructivism that is a fundamental theoretical perspective on learning in mathematics education (Ernest, 1998). During the interview, a participant is asked to perform particular tasks, which are carefully designed by the researcher. Then the researcher is to prompt and explore the interview questions while observing the individual. Piaget’s interview makes a significant input to qualitative research methodology in mathematics education since it provides thorough information about an individual’s thinking and cognitive processing (Ernest, 1998).

Over a period of two decades, mathematics education has advanced to emphasize conceptual understanding and higher-level problem solving processes over the drill and mechanical memorization of the steps to follow (Bednarz, Kieran, & Lee, 1996; Coben,
O’Donoghue, & FitzSimons, 2000; Cummins, Kintsch, Reusser, & Weimer, 1988; Goldin, 2004; Kieran, 1994; Lesh & Zawojewski, 2007; Lester, 1994; Schoenfeld, 1987; Verschaffel & De Corte, 1997). Consequently, one of the important goals for me in the interview is to reveal maximum variation in the ways the phenomenon of solving word problems appears in the adult learners’ experience. Goldin (1998) supported using the clinical task-based individual interviews as one of the major research tools in mathematics education. I was using the interviews to fulfill the following purposes: (a) observing the mathematical behavior of students in an investigative problem solving environment, and (b) drawing conjectures from the observations to allow conclusions about the interviewee’s possible knowledge structures, affect, and cognitive process. During the interviews, as a researcher, I intended to look into the cognitive processes of the participants, into how they think and what they know about solving word problems, into representational structures these students develop, and into the beliefs about mathematics they acquire in the process of word problem solving. My objectives were to observe complex, mathematical word problem-solving behavior in detail and to draw inferences from the observations about the adult learners’ thinking and development. So, using the clinical interviews, I hoped to describe individual word problem-solving experience in as much detail as possible rather than focusing on the standard algorithmic approach (Polya’s four-step process).
In addition, the emergence of social constructivism in research in mathematics education has produced new emphases that are essential to the qualitative research paradigm in general and this study in particular. These include

- attending to the previous constructions that learners bring with them;
- attending to the social contexts of learning;
- questioning the status of knowledge, including mathematical knowledge and logic, and the learner’s subjective knowledge;
- proceeding cautiously with regard to methodological approaches, since there is no “royal road” to knowledge or “truth”;
- attending to the beliefs and conceptions of knowledge of the learner, teacher, and researcher, as well as their cognitions, goals, metacognition, and strategic self-regulatory activity;
- attending to language, discussion, collaboration, negotiation, and shared meanings in the personal construction of knowledge (Ernest, 1998).

Questions pertaining to these issues were an important part of the clinical interview process in the study, since the participants’ replies constituted their experience when solving word problems thus serving the purpose of the study.

**First Interview Protocol**

I introduced the study to the participants by saying the following: “I am trying to study how students like you think about and solve word problems. I am interested in what
you think about while and after you read the problem, how you decide on the approach to solve it, and why you think what you found is correct. Even if you do not get through the whole problem in the time allocated, I am still interested in what you are thinking about.”

The first interview and observation began with the following introductory questions designed to bring forth some of the learner’s affect in relation to mathematical problem solving (Goldin, 1998):

- “Could you think back to the first time you remember doing mathematics? When and where did it happen? What else do you remember? Was it at school?”
- “Do you remember doing mathematics at home with your parents or siblings? Do you remember playing games and/or doing puzzles?”
- “Do you remember doing mathematics with friends?”
- “How old were you at that time when you first remember doing mathematics? Could you tell me more about what happened?”
- “How did you feel when that happened? How do you feel about mathematics now? How do you feel about word problem solving now? What (if anything) helped you to change your mind?”
- “What do you believe makes a person a good problem solver? Are you good at solving problems?”

The meeting proceeded with me observing and interviewing a participant solving the following traditional word problems that were taken from the text currently used in the class:
Two planes leave the same airport at the same time, flying in opposite directions. The rate of the faster plane is 300 miles per hour. The rate of the slower plane is 200 miles per hour. After how many hours will the planes be 1000 miles apart?

and

How many ounces of a 50% alcohol solution must be mixed with 80 ounces of a 20% alcohol solution to make a 40% alcohol solution?

Each participant was given about 20 minutes to solve the problem while being observed. Prior to each interview, I informed each participant that I was mainly interested in their thinking, not in numbers. After that, the participant was asked to explain his or her approach. A series of exploratory questions followed. These were based on the nature of the responses and special emphases were placed on studying the participant’s construction and use of external representations. I was using the following exploratory questions, which are based on the work of Schoenfeld (1987, p. 24) for the second half of the interview (while the participant was solving the word problem):

- “What are we looking for? What is known?”
- “Can you draw a picture/a model to illustrate the condition?”
- “Can you introduce suitable notation?”
- “Could you restate the problem in your own words?”
- “Can you obtain something useful from the given?”
- “Did you use all the information provided?”
- “Can you derive the result differently?”
Depending on the participant’s response, I asked additional exploratory questions:

- “Can you tell me more about this?”
- “Do you think you could explain how you thought about the problem?”
- “Have you ever done a similar problem before? (If yes, when?) What do you remember about it?”

**Second Interview Protocol**

The second observation and interview took place after the participants learned about solving word problems in their Beginning Algebra course. The participants were asked to solve the following two problems taken from the current textbook:

*How many ounces of a 15% alcohol solution must be mixed with 4 ounces of a 20% alcohol solution to make 17% alcohol solution?*

and

*Two cities are 315 miles apart. A car leaves one of the cities traveling toward the second city at 50 miles per hour. At the same time, a bus leaves the second city bound for the first city at 55 miles per hour. How long will it take for them to meet?*

These two problems were similar to the problems used in the first interview. The second interview was completed in a manner similar to the first interview.

**Observation**

For the observations, I followed the steps suggested by Creswell (2002): (a) the site for observation was negotiated with the participants. All the interviews and
observations took place at my office, (b) I observed a student solving a word problem. The time of observation was up to 60 minutes, (c) I followed the observation protocol presented later as a method for recording notes. I included both descriptive and reflective notes (notes about my experiences, intuition, and learning), and (d) I recorded such aspects as behavior and the mood of the informant, the physical setting, particular events and activities, and my own reactions. The summary of data collection is presented in the following table.

Table 4. *Summary of Data Collection*

<table>
<thead>
<tr>
<th>Events</th>
<th>Time</th>
<th>Purpose/Questions</th>
<th>People Involved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation &amp;</td>
<td>January 2012</td>
<td>Introduction to the study; Questions about participants’ attitudes and beliefs in relation to mathematical education; Observing participants solving a traditional word problem and asking them to explain their approach. Collecting artifacts.</td>
<td>Participants and myself</td>
</tr>
<tr>
<td>Interview I</td>
<td>(before the section on problem solving was taught)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observation &amp;</td>
<td>February-April 2012</td>
<td>Procedures are similar to the Observation and Interview I</td>
<td>Same as above</td>
</tr>
<tr>
<td>Interview II</td>
<td>(after the topic was taught)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Data Analysis

Data analysis is the process of making sense out of the data collected during the research. This process involves consolidating, reducing, and interpreting what participants have said and the researcher has observed. In my study, I followed the Merriam’s (2009) outline for data analysis. I began with within-case (a participant solving a word problem) particular descriptive accounts and then followed up with constructing conceptual themes that cut across the prevalence of the data. When working on descriptive accounts, I compressed the raw data and linked it together in a narrative to communicate the meaning I derived from studying the phenomenon. I then proceeded with the general descriptions, cross-case analysis and interpretive summary. The themes and subthemes (codes) were constructed through the constant comparative method of data analysis. These themes and subthemes were derived from the data, but they are not the data itself. These categories and codes will be presented later in the chapter. The cross-case synthesis was conducted using word tables and bar graphs (Appendixes C through G).

Observation Protocol

For any observation there must be concepts or categories to focus on (Denzin, 1978; Patton, 1990; Silverman, 2001). These categories or concepts are derived by the study design and the nature of the questions being considered. The observation concepts fundamental to my research were
- Location: The interviews with concurrent observations were conducted in my office.
- Styles of self-presentation: Was the participant anxious, calm, excited, or else?
- Chronology: Did the participant begin with reading, setting equations, drawing, guessing, or else?
- Reading comprehension: Did the participant appear to understand the problem after reading the text?
- Graphic representations: What graphic representations, if any, were being used (pictures, charts, graphs, or other)?
- Symbolic representations: Were symbolic representations being used when solving the word problems? If yes, which ones: algebraic, numerical, or other?
- Setting up an equation: Did the participant attempt to set up an equation or did he or she use non-algebraic methods? If non-algebraic methods were used, what were they?
- Articulation: Did the participant speak to him/herself when solving a problem? What body language was presented?
- Processes: What types of communication, decision making, questions, technology, and drawings were used?
- Affect: What emotion(s) did the participant show during the observation and interview?
Artifacts Protocol and Analysis

The artifacts gathered in my study were analyzed according to the criteria proposed by Bodner and Goldin (1991) and Silverman (2001) that is:

- What kind of graphic elicitation has been done: pictures, diagrams, equations or else?
- Was there a single graphic elicitation or multiple?
- Whether or not an equation has been used, what do the assigned variables represent?
- Was there one or more variables used?
- What were the notations used?
- Was trial and error used?
- Was the correct answer obtained?
- Was the answer checked?

Using all three types of data collection such as interview, observation, and artifacts provided a multi-method, triangulation approach to data collection, which added to both the validity and the reliability of the research (Patton, 1990).

Coding the Data: Categories (Themes) and Subcategories

In order to analyze the data I gathered, the audio/video taped interviews with the participants in the study were coded with a set of priori categories selected from the research in mathematics education (Goldin, 1998; Pirie, 1998) and other codes emerged
in vivo. Such categories were “estimation,” “translation into expression,” and “setting an equation, solving the equation”. The choice of categories was inevitably subjective, although its bases were in current textbooks and previous research. The categories obtained were broken down into subcategories that were further coded. The complete list of the codes will be presented below. The data through interviews were repeatedly gathered and analyzed, with each subsequent data-collection decision being dependent on the analyses of the previous data collected. One of my goals was to be as open as possible to all interpretations of the categories and subcategories while the other was to be as systematic and theoretically guided as possible during the process.

The following tables 4, 5, and 6 present the codes used to analyze the participants’ experiences with math education and solving word problems. The categories of codes column indicate major themes found in the research. The first letter of each code refers to this major category.

Table 5. Participants’ Experiences with Math Education Data Analyses Codes

<table>
<thead>
<tr>
<th>Categories of Codes</th>
<th>Codes</th>
<th>Meanings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AC</td>
<td>Comfortable in math class</td>
</tr>
<tr>
<td></td>
<td>AL</td>
<td>Likes mathematics</td>
</tr>
<tr>
<td></td>
<td>AD</td>
<td>Dislikes mathematics</td>
</tr>
<tr>
<td></td>
<td>ANRL</td>
<td>Doesn’t understand the importance of math/connection to</td>
</tr>
<tr>
<td>Attitude toward Mathematics</td>
<td>real life</td>
<td></td>
</tr>
<tr>
<td>-----------------------------</td>
<td>----------</td>
<td></td>
</tr>
<tr>
<td>ARL</td>
<td>Understands the importance of math/connection to real life</td>
<td></td>
</tr>
<tr>
<td>ANE</td>
<td>States negative experiences like anxiety, insecurity, and such</td>
<td></td>
</tr>
<tr>
<td>AS</td>
<td>States determination in order to pass the math class</td>
<td></td>
</tr>
<tr>
<td>AM</td>
<td>States mixed attitude toward mathematics</td>
<td></td>
</tr>
<tr>
<td>ANR</td>
<td>No recollection of teaching/learning in math classes</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level of Success in Math</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SA</td>
<td>Average in math</td>
</tr>
<tr>
<td>SG</td>
<td>Good in math</td>
</tr>
<tr>
<td>SDA</td>
<td>Good/average in arithmetic, but struggling with algebra</td>
</tr>
<tr>
<td>SM</td>
<td>Struggling and/or feeling unsuccessful in math class</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Teacher/Teaching Perception</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>TG</td>
<td>Good mathematics teacher</td>
</tr>
<tr>
<td>TP</td>
<td>Poor/Inadequate mathematics teacher</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student Behavior toward Math</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BNI</td>
<td>Lack of interest in studying including mathematics</td>
</tr>
<tr>
<td>BED</td>
<td>Reported being easily distracted by internal factors</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solving Word Problems</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>WPN</td>
<td>Does not remember solving word problems</td>
</tr>
<tr>
<td>WPC</td>
<td>Comfortable with solving word problems</td>
</tr>
<tr>
<td>Mathematics at Home</td>
<td>Codes</td>
</tr>
<tr>
<td>---------------------</td>
<td>-------</td>
</tr>
<tr>
<td>WPU</td>
<td>Uncomfortable with solving word problems</td>
</tr>
<tr>
<td>HS</td>
<td>Family member(s) support studying</td>
</tr>
<tr>
<td>HN</td>
<td>No support with studying</td>
</tr>
<tr>
<td>MI</td>
<td>Informal (games, puzzles, etc.) mathematical activities</td>
</tr>
<tr>
<td>MF</td>
<td>Formal (school homework) mathematical activities</td>
</tr>
</tbody>
</table>

Table 6. Participants’ Experiences with Motion Problem I and II Data Analyses Codes

<table>
<thead>
<tr>
<th>Categories of Codes</th>
<th>Codes</th>
<th>Meanings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding the Problem</td>
<td>Codes</td>
<td>Meanings</td>
</tr>
<tr>
<td>URA</td>
<td>Read the problem aloud</td>
<td></td>
</tr>
<tr>
<td>URC</td>
<td>Rephrased the problem correctly</td>
<td></td>
</tr>
<tr>
<td>URN</td>
<td>Rephrased the problem incorrectly</td>
<td></td>
</tr>
<tr>
<td>UGU</td>
<td>Identified given and unknown</td>
<td></td>
</tr>
<tr>
<td>UIM</td>
<td>Modeled as two independent vehicles by obtaining two values of the time</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Heuristic Used to Solve the Problem</th>
<th>Codes</th>
<th>Meanings</th>
</tr>
</thead>
<tbody>
<tr>
<td>HAC</td>
<td>Used arithmetic approach correctly to solve the problem</td>
<td></td>
</tr>
<tr>
<td>HAI</td>
<td>Used arithmetic approach incorrectly to solve the problem</td>
<td></td>
</tr>
<tr>
<td>HALC</td>
<td>Used algebraic approach correctly to solve the problem</td>
<td></td>
</tr>
<tr>
<td>HALI</td>
<td>Used algebraic approach incorrectly to solve the problem</td>
<td></td>
</tr>
<tr>
<td>HNA</td>
<td>Did not attempt to solve or used no logical approach with</td>
<td></td>
</tr>
<tr>
<td>Code</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>-----------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>HVP</td>
<td>Used a variable properly</td>
<td></td>
</tr>
<tr>
<td>HVI</td>
<td>Used a variable improperly</td>
<td></td>
</tr>
<tr>
<td>HNV</td>
<td>Used no variable</td>
<td></td>
</tr>
<tr>
<td>HCEQ</td>
<td>Wrote a correct algebraic equation</td>
<td></td>
</tr>
<tr>
<td>HIEQ</td>
<td>Wrote an incorrect algebraic equation</td>
<td></td>
</tr>
<tr>
<td>HNEQ</td>
<td>Wrote no algebraic equation</td>
<td></td>
</tr>
<tr>
<td>HCS</td>
<td>Solved the equation correctly</td>
<td></td>
</tr>
<tr>
<td>HIS</td>
<td>Solved the equation incorrectly</td>
<td></td>
</tr>
<tr>
<td>HCE</td>
<td>Explained the equation coherently</td>
<td></td>
</tr>
<tr>
<td>HIE</td>
<td>No explanation or incoherent explanation of the equation</td>
<td></td>
</tr>
<tr>
<td>HCP</td>
<td>Complete picture or drawing</td>
<td></td>
</tr>
<tr>
<td>HIP</td>
<td>Incomplete picture or drawing</td>
<td></td>
</tr>
<tr>
<td>HPR</td>
<td>Picture done after researcher’s request</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Resources</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RCF</td>
<td>Used the formula D=RT correctly</td>
</tr>
<tr>
<td>RIF</td>
<td>Used the formula D=RT incorrectly</td>
</tr>
<tr>
<td>RNF</td>
<td>No explicit use of the formula</td>
</tr>
<tr>
<td>RSF</td>
<td>Stated the formula D=RT correctly</td>
</tr>
<tr>
<td>RSI</td>
<td>Stated the formula D=RT incorrectly</td>
</tr>
<tr>
<td>RC</td>
<td>Used a calculator</td>
</tr>
</tbody>
</table>
Table 7. Participants’ Experiences with Mixture Problems I and II Data Analyses Codes

<table>
<thead>
<tr>
<th>Categories of Codes</th>
<th>Codes</th>
<th>Meanings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding the problem</td>
<td>URA</td>
<td>Read the problem aloud</td>
</tr>
<tr>
<td></td>
<td>URC</td>
<td>Rephrased the problem correctly</td>
</tr>
<tr>
<td></td>
<td>URN</td>
<td>Rephrased the problem incorrectly or vague</td>
</tr>
<tr>
<td></td>
<td>UGU</td>
<td>Identified given and unknown</td>
</tr>
<tr>
<td>Heuristic used to solve the problem</td>
<td>HAC</td>
<td>Used arithmetic approach correctly</td>
</tr>
<tr>
<td></td>
<td>HAI</td>
<td>Used arithmetic approach incorrectly</td>
</tr>
<tr>
<td></td>
<td>HALC</td>
<td>Used algebraic approach correctly</td>
</tr>
<tr>
<td></td>
<td>HALI</td>
<td>Used algebraic approach incorrectly</td>
</tr>
<tr>
<td></td>
<td>HNA</td>
<td>Didn’t attempt to solve the problem or used no logical approach with no answer obtained</td>
</tr>
<tr>
<td></td>
<td>HVP</td>
<td>Used a variable properly</td>
</tr>
<tr>
<td></td>
<td>HVI</td>
<td>Used a variable improperly</td>
</tr>
<tr>
<td></td>
<td>HNV</td>
<td>Used no variable</td>
</tr>
<tr>
<td></td>
<td>HCEQ</td>
<td>Wrote a correct algebraic equation</td>
</tr>
<tr>
<td></td>
<td>HIEQ</td>
<td>Wrote an incorrect algebraic equation</td>
</tr>
<tr>
<td></td>
<td>HNEQ</td>
<td>Wrote no algebraic equation</td>
</tr>
<tr>
<td></td>
<td>HCS</td>
<td>Solved the equation correctly</td>
</tr>
<tr>
<td></td>
<td>HIS</td>
<td>Solved the equation incorrectly</td>
</tr>
<tr>
<td>Code</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>HCE</td>
<td>Explained the equation logically</td>
<td></td>
</tr>
<tr>
<td>HIE</td>
<td>Didn’t explained the equation logically</td>
<td></td>
</tr>
<tr>
<td>HCP</td>
<td>Coherent picture or drawing</td>
<td></td>
</tr>
<tr>
<td>HIP</td>
<td>Incoherent picture or drawing</td>
<td></td>
</tr>
<tr>
<td>HPR</td>
<td>Picture done upon request</td>
<td></td>
</tr>
</tbody>
</table>

### Resources

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RCF</td>
<td>Used the mixture formula/concept (amount of the agent equals the product of the amount of the mixture and the concentration percent) correctly</td>
</tr>
<tr>
<td>RIF</td>
<td>Used the mixture formula/concept incorrectly</td>
</tr>
<tr>
<td>RNF</td>
<td>No use of the formula</td>
</tr>
<tr>
<td>RC</td>
<td>Used a calculator</td>
</tr>
</tbody>
</table>

### Validity

Eisner (1991) used the term *credibility* instead of *validity* while describing qualitative research. His standards are structural corroboration, consensual validation, and referential adequacy. Structural corroboration stands for involving multiple types of data to support or oppose the interpretation. He recommended that to exhibit credibility, the amount of evidence should become convincing. Consensual validation is referred to as “an agreement among competent others that the description, interpretation, and evaluation and thematic of an educational situation are right” (p. 162). Referential
Adequacy refers to the importance of criticism as illuminating the subject matter and bringing more complex human perception to the discussion.

Angen (2000) suggested that within interpretive research validation is “a judgment of trustworthiness or goodness of a piece of research” (p. 387). She believed the consideration of validations is not authoritative as the final word on the topic. In addition, she moved forward with two types of validation: ethical validation and substantive validation. Ethical validation means that all research schemas must question their fundamental moral assumptions, political and ethical allegations, and the fair treatment of diverse voices. Substantive validation means the contribution of self-reflection.

Creswell (2007) summarized the validation process as the following: (a) validation in qualitative research is an attempt to assess the “accuracy” of the findings, as best described by the researcher and the participants. Any report is a representation by the author; (b) validation is a distinct strength of qualitative research in that the account made through extensive time spent in the field, the detailed thick description, and the closeness of the researcher to the participants in the study; (c) the term “validation” is used to emphasize the process rather than “verification” that has quantitative overtones. There are many types of qualitative validation and the authors should choose the types and terms with which they are comfortable; (d) validation should be used regardless of the type of qualitative approach.
To assure the validity of my study, I followed the strategies presented by Eisner (1991), Angen (2000), and Creswell and Miller (2000). These included

a) Assuring structural justification by involving multiple types of data: observation, interview and artifacts;

b) Assuring credibility by keep collecting data till the amount of evidence becomes convincing;

c) Building trust with participants, learning the culture, and checking for misinformation as the results of prolonged engagement and persistent observation in the field;

d) Assuring consensual validation by using peer review of the process and the findings as an external check of the research;

e) Providing fair treatment of diverse voices;

f) Clarifying all my (as a researcher) bias from the outset of study in order to minimize its impact on the inquiry;

g) Creating rich, thick descriptions to allow readers to make decisions regarding transferability.

**Ethical Considerations**

The following steps were taken to respect the participants and the sites for research:

- I conveyed the purpose of the study to the participants prior to data collection.
The results of the study were shared with the participants to ensure reciprocity between the participants and myself as a researcher. The informed consent form (Appendix B) was designed to ensure that participants were notified of the following before they engage in the research (Creswell, 2002): (a) The right to participate voluntarily and the right to withdraw at any moment, so that the individual was not being coerced into participation; (b) The purpose of the study, so that individuals understood the nature of the research and its impact on them; (c) The procedures of the study; and (d) The rights to ask questions, obtain a copy of the results, and have their privacy respected.

I used pseudonyms to protect the anonymity of individuals and places during the data analysis and interpretation.

I will keep the data in my office for a year and then destroy it.

**Limitations**

Despite sound research design and credible findings, several limitations exist. One limiting factor of this study is the type of institution (community college) and the population (adult students) from which I collected my data, potentially contributing to questions about the study’s generalizability. However, by design, I do not intend to generate generalizable conclusions, but rather offer credible interpretation of the adult students’ reasoning and thinking solving mathematical word problems. Future research involving different institutional types and other demographic groups would be beneficial.
Another limitation is embedded in some interview questions, which asked the participants to reflect on their past. Again, this was a methodological choice; however, the passage of time would have had the effect on the participants’ answers. Future research could develop objective measures of pre-college factors.

Additionally, researcher bias is a limitation for this study; the lens through which I view this research risks being clouded by my insider’s perspective. There are benefits to being an insider (i.e. understanding participants’ specialized use of words and terms, their assumptions and viewpoints); yet, insider-researchers also face ethical challenges associated with insider roles, e.g., participants revealing more or less information because of their relationship with the researcher (McGinn, 2005). Thus, as an “insider” to the context under investigation, it was critical that I indentified and employed strategies to attend to any authority and knowledge that could influence my analysis and interpretation.

I utilized several strategies (as described above) to contribute to the trustworthiness of this study. Yet, as noted, some limitations are inherent, and future study (which will be discussed later in this document) is warranted to further explore this phenomenon.

**Access to Data**

The data are stored in a locked cabinet in my office. I am the only one to have access to data during and after the research.
Summary

This chapter delineated the design and methods for my study. This study incorporates characteristics of qualitative research including that the research takes place in the natural setting, makes use of multiple methods of data collection, is emergent rather than predicted, is based on the interpretation of the researcher, is viewed holistically, utilizes both inductive and deductive reasoning processes, and employs a strategy of inquiry (Creswell, 2003). I was studying adult students learning mathematics at a regional community college. Clinical task-based interviews, observations, and artifacts analysis were the methods of data collection. The interviews and observations were audio and video taped. As a researcher, I then analyzed and interpreted the data in order to collect the participants’ meanings.
CHAPTER IV
PRESENTATION OF DATA AND FINDINGS

The purpose of the study is to understand and describe the experiences adult learners have while solving mathematical word problems and to understand the different cognitive meanings of algebraic thinking of adult learners. This could provide teachers with new approaches in teaching algebra in general and word problems in particular. Since this research is process-oriented and is focused on interpreting the phenomena of solving word problems in terms of the meanings adult students bring to mathematics classrooms, I have used qualitative multiple case studies methodology to answer the questions of the study that are about (a) attitudes and beliefs that adult students hold about solving word problems, (b) mathematical content knowledge that adult learners gain access to when solving word problems, and (c) the approaches used by adult learners to solve word problems. In order to gather information about each participant, I have used two semi structured task-based (also called clinical) oral interviews during participants’ problem solving activities. During these two interviews, the participants were asked questions about their experience with mathematics education as well as their attitudes toward solving word problems. In addition, I asked them to solve two word problems (motion and mixture) during the first interview and two word problems (motion and mixture) during the second one.

This chapter presents the data collected and analyzed for the study. The chapter comprises of within-case particular descriptions (verbatim examples) illustrating the
collection of data, within-case general descriptions, and cross-case analysis and interpretations.

Within-Case Particular and General Descriptions of Participants’ Experiences

Mina

Observation and particular description of Mina’s experience with math education. Mina is a 30-year-old student who emigrated from the former Soviet Union about seven years ago. She graduated from the high school there and earned a Bachelor’s Degree in Fashion Merchandise. Mina appeared to be comfortable answering the researcher’s questions; nevertheless, she seemed to be apprehensive speaking English since it is her second language. Mina sounded rather uneasy about her experience with mathematics beginning with her earliest experience. When recalling her elementary school teacher, Mina said, “The teacher was very good, very caring and patient with students. I also remember struggling with mathematics. My grade was a ‘C’ usually”. She described her post-arithmetic experience as “I was born and went to school in the former USSR. All the teachers were very tough, rigid, and when you wouldn’t know the right answer, they would ask you how you can be so stupid and not understand it. I would get very embarrassed by these comments and consequently would try to avoid any dealings with math. She maybe was a good teacher, but her attitude made me feel very uncomfortable in math class. In addition, in Russia, we were asked to go to the board to
solve problems, and if it is not correct, again, she would make a negative comment in front of the whole class about my math abilities. I dreaded this”. Mina’s tone of voice and facial expression got much more relaxed and content when she began talking about the math class she is taking now. She stated, “Now I understand better how to solve it [algebraic problems]. This understanding of what I am doing changed my attitude toward math. I have never liked math in school, but now I really do.” Mina interpreted her change of attitude as follows: “It [change in attitude toward mathematics] happened maybe because the whole attitude toward learning is different in this country. It is not pushy or judgmental. It is very individual and not critical. I do not feel pressured when I ask a question even if everybody in the class is listening to my question.” When asked what she thought about mathematics being relative to real life, Mina replied, “Back in Russia, I was a fashion designer, so I needed math to calculate the amount of material needed to cut the precise pattern. Also, I needed to know the amount of time I spent doing different tasks to meet the deadline. Now, I work as a massage therapist. I do not think I use math now, but maybe will use it later”. When asked about solving word problems, the participant acknowledged that she did remember solving them and that “it always was very difficult for me.” Mina was also asked about doing informal mathematics like games, puzzles with family and/or friends. She replied, “No, there were no puzzles in Russia. It was only formal school mathematics. I have never done any math games at home either.” She also added, “My mom would always help me with my math.”
General description of Mina’s experience with math education. Mina seemed to have a mixed attitude toward her earliest mathematics education experience. She did like her first math teacher, but at the same time she didn’t believe that she was a strong and confident math student. Her attitude toward math became much less positive during her post-arithmetic years. She believed that the reason for such an attitude was her limited math abilities, as well as the teachers’ manners in her old country. Since then Mina’s attitude has changed. It is positive now as a result of her belief that learning in the USA is not judgmental, but rather individual and supportive. Mina remembered solving word problems before and recalled it was not easy for her. There was no evidence if Mina ever did any informal mathematics either at school or at home. During the first part of the interview, Mina appeared to enjoy her new experience in the math classroom, in spite of the difficulties she had while being a student in a Soviet school. She kept smiling during the interview.

Observation and particular description of Mina’s experience solving motion problem I. Mina began reading it silently and then repeated aloud. She at once started looking confused and puzzled. Mina said, “I do not remember how to do it and do not know if I am right or wrong. Thirty minus, no, three hundred miles, now I do not remember if it is minus two hundred miles per hour, divide or multiply.” Mina identified the given as “One plane is faster with three hundred miles per hour”. She explained the term “rate” as “I guess it is like a price, the rate, and the set up of kind of miles per hour the plane has to fly. It is like a cost of a product. So, the rates are two hundred and three
hundred.” She added, reading from the text, “After how many hours will the planes be one thousand miles apart. So, we need to find out in how many hours.” When asked to explain the meaning of 1000 miles, Mina replied after thinking for a while that it is “The whole amount the planes spend in the air. The distance, I think.” When I asked if there is another characteristic of a motion being considered in the problem, Mina replied, “Hours.” When Mina was asked about an attribute of a motion measured in hours, she replied that she didn’t know. She uncertainly identified the unknown as “The distance, no, it is miles, so it is time.” When asked about an equation or a formula that connects three attributes of a motion: distance, rate, and time, Mina’s reply was, “We have a rate that we know, and we have a distance that we know; we need to find out the hours. They will be represented by x”. She attempted to set up an equation containing a plus sign and explained her steps as follows: “I am trying to set up an equation. To get the time, I guess I am trying to get it separate for each plane.” After writing some calculations down, she added, “I guess I took the rate of the first plane that is 300 miles per hour and I added the distance which is 1000 miles and I get 1300. I don’t know if I did it right.” Mina was asked if when solving word problems, she would try to visualize the story. Her reply was, “I only think about the math.” Mina attempted to solve the problem again using a calculator this time, but ended up becoming visibly frustrated and saying, “No, I do not know.”
Figure 3. Mina’s solution of motion problem I.

General description of Mina’s experience solving motion problem I. Mina’s reading comprehension appeared not be an issue since she correctly identified the given information and what the unknown was. When attempting to solve the motion problem, she seemed to be thinking about the needed operation without understanding the context of the problem. When asked about the connection between the attributes of a motion, Mina didn’t provide any answer. There is no evidence of the previous knowledge of the formula D = RT, nor had the formula been used when solving the problem. Her strategy did involve using the variable x as representation of the unknown, but no equation had been set up. The idea of the simultaneous motion of two planes was not been presented in her calculations. Mina presented each plane moving independently from the other one. No picture, graph, or diagram was used. A calculator was used.

Observation and particular description of Mina’s experience solving mixture problem I. Mina began the solution process with reading the problem silently. Her facial expression was of uncertainty and confusion. She proceeded with doing some calculations on a calculator and then said, “In both samples, I think you have to find, I do
not know for sure, but think you have to find how many--don’t know.” When asked to identify the given and the unknown, Mina didn’t provide a sound answer. She explained the meaning of the “50% solution” as “It means half of it is alcohol” and identified the rest of the solution as “probably just water”. Mina confirmed that enough information was given to solve the problem but admitted that she didn’t know how to do it. When she was asked about the total amount of the mixture of twenty ounces of a solution and ten ounces of another solution, she identified the result as the sum of twenty and ten. When asked to approximate the concentration of the mixture of 50% solution and 80% solution, Mina replied, “I would say it would be between 20% and 30%”, but failed to provide a coherent explanation of the number. Mina admitted that the problem is relevant to real life and provided the following example: “To obtain a stronger solution maybe, to have more solution maybe. Chocolates can be mixed, dark with milk chocolate. Water can be mixed with oils. In medicine, ingredients can be mixed together to get a new ingredient.” Mina’s occupation is a massage therapist at the present time. When asked if she uses alcohol in her professional life, Mina replied, “No, I do not. I use other ointments though.”

\[
\begin{align*}
50\% & \quad 50 + 80 \cdot \frac{20}{100} = 0.82 \\
20\% & \quad 0.5 + 0.2
\end{align*}
\]

Figure 4. Mina’s solution of mixture problem I.
General description of Mina’s experience solving mixture problem I. During her attempt to solve the problem, Mina appeared confused and uncertain. She admitted that the problem is relevant to real life and that enough information is given to solve it. Nevertheless, she also admitted that she had no idea how to solve it. Mina appeared to understand the meaning of “50% concentration solution”. However, when asked about the concentration of the mixture of two solutions, she didn’t provide the correct answer. No representations, equations, or pictures were done at this time. No answer(s) were obtained either.

Observation and particular description of Mina’s experience solving motion problem II. Mina began with reading the problem aloud, thinking for a while, and taking notes. She stated, “Ok, there are two cities, 315 miles apart. A car leaves one of the cities moving toward the second city at 50 miles per hour. At the same time, a bus leaves the second city bound for the first city at 55 miles per hour. How long would it take for them to meet? I guess I found it is three hours because the car is going from one city towards the city that bus leaves, and the bus goes toward what the car leaves. I do not know how it is in math, but I know that we need to add in this case to find the hours.” When asked to explain the meaning of the product of 55 and the x, she replied, “55 are miles per hour.” She clarified the answer as “It is 55 miles; we do not know when the car left. I multiply miles by hours--55x [is] miles per hour.” Mina also explained the meaning of the second product (50x) as “Well, if 55 is for a bus, then 50 is the car. But we don’t know what time the car left. Like the car goes 50 miles per hour. Same thing is with the
bus.” When asked about the attribute of a motion that is connected to the units of miles per hour, she replied, “It is how fast. Instead of saying how fast, we say 55 miles per hour.” When asked again what can be measured by miles per hour, Mina said uncertainly, “Speed, hours.” When asked about the attribute measured by hours, she replied anxiously, “Minutes, the distance, the process, the work.” After some hesitation, she added “time.” Mina presented the meaning of the addition sign in her equation as “I had 315 miles apart, because there is 315 miles between these two cities.” Then she explained the meaning of her equation as “My equation says that the bus that travels 55 miles per hour, I add to car that travels 50 miles per hour, and the distance all together in order to find how long before they meet on the road. And I got three hours.” Mina’s explanation of the addition of two products was, “Because we need to find the whole, because of the formula.” When asked to clarify “the formula” further, she stated, “It is $a = b + c$ from the book.” When asked to think about a situation that would require subtraction, Mina responded, “For subtraction, we have to have a bus leave the same city the car does. Or maybe a few hours later, may be traveling with less miles per hour, because we are trying to find the distance. Car and bus coming toward each other and they are going the same direction. I would do the subtraction because I need to find the distance like that example in the book.” Mina also stated that there was enough information to solve the problem and that she actually tried to picture the car and the bus going toward each other. When asked again what attributes of the motion are being added in the equation, she replied, “We are adding their miles per hour which is how fast they are going.” She checked her
answer as, “I added 55 and 50, x is like same thing, and then divided 315 by 105, and got three hours.”

\[ A = b + c, \]
\[ s = \frac{55x + 50x}{105} = \frac{315}{105} \]
\[ x = 3 \text{ hours}. \]

**Figure 5.** Mina’s solution of motion problem II

**General description of Mina’s experience solving motion problem II.** Mina approached the problem with a positive attitude stating that motion problems are easier for her to solve than the mixture ones. Mina applied an algebraic approach to solving the problem. Her representations of the unknown and the equation were correct. The participant presented the graphical representation—the picture—only when I asked her to do so. The picture presented was quite coherent, but incomplete. When asked to explain the meanings of the terms of the equation that are the products of the rate and time, Mina
failed to do so by citing the procedure instead of the meaning. She got confused again, as during her first interview, when she was asked about the units of the attributes of motion.

Observation and particular description of Mina’s experience solving mixture problem II. Mina began with reading the problem aloud and then writing notes. She restated the problem as “We have a percentage that needs to change into a decimal. We need to multiply everything by a hundred. So, it is going to be twenty multiplied by x plus seventeen multiplied by four plus four multiplied by x.” When asked to explain the meaning of her equation and the meaning of the product of the values of twenty and four, Mina replied, “In our problem, it says ‘how many ounces of fifteen percent alcohol solution must be mixed with four ounces of twenty percent alcohol solution,’ so we know that four ounces is 20% alcohol and I have like twenty multiplied by four.” When asked about the value and the units of the product, Mina replied, “Eighty” and then asked me what “units” means. After the explanation, Mina uncertainly added, “I guess it could be ounces.” When asked the meaning of the product once more, Mina failed to respond. Mina identified the meaning of the twenty hundredth in the equation as “I converted percentage into a decimal” and the units of the value as “point twenty.” She then added that she didn’t understand the meaning of the term units. When asked to identify the meaning of the number 80 in the equation, Mina replied, “It is percentage of the solution, of the alcohol solution, of 20 ounces of alcohol solution.” When asked the reason for using addition in her equation, Mina responded, “I am adding this 17% to make a 17% alcohol solution. I am adding the seventeen multiplied by four
ounces plus x because we do not know how many ounces in the fifteen percent solution.”

When asked about the meaning of the sum of four and x, Mina replied, “It is like we are adding both solutions. It is the formula.” She identified the unknown variable x as “x is for --we are trying to find ounces, ounces that are for the fifteen percent alcohol solution. Then we get a linear equation.” Mina identified the meaning of the product of seventeen and the sum of four and the x as “The product of 17 is like an alcohol, so we get how many, how much alcohol solution we need, how much percentage.” She also identified the meaning of the expression on the right side of the equation as “It is 15 multiplied by x. And the x is how many ounces of 15% alcohol solution. We have the percentage, but we don’t know how many ounces.” In addition, Mina identified the meaning of a product of a percentage of the solution by its amount as “I still get the 15x because it is still going to be 15.” When asked to explain the meaning of her equation, she stated, “The equation says that the 20% alcohol solution when mixed with the 17% solution …. It is telling us what we need to do to find the ounces if we have a 15% alcohol solution, so we are adding twenty percent solution with the four ounces of the 17% alcohol solution to find out how much alcohol we need to get if we have fifteen percent of alcohol solution. Because I use the strongest alcohol solution that is the twenty percent and the weakest that is fifteen percent. The mixture of two is seventeen percent solution. I am adding everything.” She then added, “I got one mistake though. At the beginning, instead of four ounces added to x, I am supposed to minus.” When asked to explain the meaning of the subtraction, the participant replied, “I remember the formula every time you get an
amount, if I have a strong solution or a weak solution, it would always be a minus of the mixture.” She also stated that her answer was 4.265 liters; I guess it is ounces, how many ounces we need to have 15% of alcohol solution.”

Figure 6. Mina’s solution of mixture problem II

General description of Mina’s experience solving mixture problem II. During the second interview, Mina again appeared to have a very positive attitude toward the experience in the math class she is taking. She spent a few minutes reading the problem and proceeded writing the solution. When attempting to restate the problem, the participant stated the equation she set up rather than the problem given. Mina used x as a
representation for the unknown and was able to set up and solve the equation. While talking to herself, she concentrated on the procedure of setting the equation rather than her understanding of the situation presented in the problem. When asked to explain the meanings of the products involved and the equation set, Mina restated the procedure (multiplication) without presenting the meaning of the products of the percentage and the amount. When asked about units of the product, she got confused about the concept of the unit and when explained, she appeared hesitant before answering. When asked later about the units again, the participant admitted that she didn’t understand the concept. When asked to explain the meaning of her equation, Mina replied that it is based on the formula given in class. While setting her equation up, the participant confused the solutions to be mixed together and the final result. Therefore, the solution of the equation was incorrect. The approach used seemed to be based on the memorization of the procedure rather than understanding of the process.

John

Observation and particular description of John’s experience with math education. John is a 42-year-old student taking Beginning Algebra for the third time. He dropped out of school, earned his GED, and enrolled in the Marine Corps. His main reason for joining the research project was to get help with the class work. John appeared to be quite talkative and articulate during the interview. He admitted being frustrated about his lack of success in the math class and not knowing what to do to fix it. Before
coming back to school, John revealed having numerous clerical jobs after finishing his military service. When asked about his math experience, John’s reply was, “I guess, in elementary school I was pretty average in math, but as a kid I was very easily distracted. And because I was daydreaming or talking or drawing pictures, I would miss the part of bringing it all together. And so, you know, when it was time to do the homework, I would do it, or my mother would make me do it, and when she wasn’t looking, I simply would not do it. And I would go to school the next day and try to disappear in the classroom, so the teacher would not call on me.” Later on John added, “My math teachers were some of the best teachers that I have ever met. It was never them. It was always me. I went to school in a predominantly black community. And the teachers were genuinely concerned and wanted their students to learn. I wouldn’t do the work, was disruptive in class and the teachers would give up on me.” While continuing his story, John got visibly upset recalling the following about his past-elementary school years: “They [the school personnel] gave me the choice the first day of school. The students had to go and see the guidance counselor so the guidance counselor would help them map out the curriculum. I remember being in line to see the guidance counselor, and it was a girl in front of me, and the guidance counselor comes out with a pen and pad and asked the student, “What do you want to do when you graduate high school?” And the girl in front of me said she wanted to work in a department store. And that is what I said when it was my turn. And so they sent me to Business Math, English, Social Studies, and Black history. When I was asked what I wanted to do, that is when I shot myself in a foot. I know I wanted to be
Sammy Davis Jr. I wanted to be song and dance man, Bing Crosby, and I wanted to be that. So, school had nothing to do with my reality. I didn’t graduate; I dropped out of school. I was eighteen years old in the eleventh grade and I knew for me to continue to the twelfth grade, I would have a full schedule of classes to make up, because I didn’t do anything in the eleventh grade. So I dropped out and joined the Marine Corp. And I got my GED while in the Marine Corp”. The last mathematics course that John took in school was Business Math. When asked about his attitude toward the present math class, John replied, “It makes me kind of nervous. First of all, I don’t test very well. As we cover something in class, I can pretty much wrap my brain around it. But by the time I leave the classroom, I have forgotten a lot. And then in the evening, when I am at home doing my homework, I forget a lot. I am not a problem solver.” When talking about doing mathematics with family and/or friends, John revealed that there was no informal mathematics in his experience. He also added that his mom would always stress out the importance of education to him; she probably would be able to help him with simple math homework when needed, but he would have never asked her because she was a very impatient person. John stated that word problems are relevant to real life but he didn’t remember ever solving them in class.

**General description of John’s experience with mathematics education.** John appeared very eager to discuss his attitude and his achievement in the math class. He disclosed that when he was young his interest in mathematics was average. Nevertheless, his achievements were below average due to his lack of both discipline and
concentration. John complimented his mother who would always be stressing the importance of his school work. He also admitted that she was not able to help him and was easily irritated. He didn’t recall doing informal mathematics, games, puzzles etc. with his family or friends. Since elementary school, math class continues to make John nervous even now through his college years. John believed that he was not a problem solver. In spite of his low achievements in math classes, John admitted that his mathematics teachers were very good. The participant showed his frustration with himself for not working harder and finishing high school. John didn’t remember solving word problems before. The last math class John took before college was Business Math.

Observation and particular description of John’s experience solving motion word problem I. John began solving the problem with silent reading. He proceeded with writing notes. His first comment was, “Ok, I probably didn’t write it out right. It is six hours.” John explained his answer as “I have learned in arithmetic and prealgebra that distance equals rate times time. So, the first plane, distance… they gave me 1000 miles equals 300 miles per hour divided by 60 which is an hour. I got 18000, which is a ridiculous number to me.” After doing some calculations on a calculator, John added, “Break it down in minutes, ok, you divide that 18000 by 60, no, ok, I got a hundred hours here. It takes 60 minutes to take an hour. I got a hundred something and so I am subtracting 60 from that.” When asked about the given and the unknown, John replied that the given is “How fast the planes are going, how miles apart” and the unknown is “How many miles apart they will be after flying at a certain rate? How many hours would
it take?” John was asked to clarify what is measured in hours, and his reply was “Seconds, minutes”. After some hesitation, he added, “Time.” John’s explanation of how he used the formula $D=RT$ was: “300 miles per hour is the rate, the hour is the time, and the distance is the 1000 miles apart. Then I did the same for the second plane and then I subtracted it. Obviously, the first plane will reach the certain point faster than the second plane will. You want to know when they are 1000 miles apart, so I subtracted the information of the second plane from the information of the first plane.” When asked about the answer, John replied, “Actually, I got an hour and 40 minutes. When I subtract 200, that is how long it will take the second plane, from 300, it is how long it will take the first one, I got 100. One hour is 60 minutes. I subtracted that from 100 and it gives me 40 minutes left over. [My answer is] an hour and 40 minutes”. John stated that the problem is relevant to real life and that he didn’t remember solving similar problems before. He also admitted flying in a plane.

![Figure 7. John’s solution of motion problem I](image_url)
General description of John’s experience solving motion word problem I.

When solving the motion problem, John was able to identify the characteristics of motion and to state the D = RT formula properly. After that he began transforming given units into different ones without any coherent explanation. All the notes he did consisted of the transformation mainly. The formula was neither written properly nor used in any way, despite his knowing of the formula. Recognition of what was given and what needs to be found didn’t come easily. Even giving proper answers at times, John appeared to be uncertain. John attempted to solve the problem arithmetically but failed to see the model of simultaneous motion of two planes. It seems that John was looking for a time needed by each plane to cover the distance of 1000 miles. Even doing so his answers were incorrect since he began converting given units into something else. No graphical representation was attempted. No variable(s) and/or equation(s) have been used.

Observation and particular description of John’s experience solving mixture problem I. John began with reading the problem silently. He then restated the problem as “You are trying to mix 50% alcohol with 20% alcohol to make 40 proof.” He continued stating that in real life people would mix ingredients in order to “to create a chemical, a reaction, a cause and an effect, to mix ingredients to do something with it because the properties of two or more ingredients are needed to do something that they [people] want.” When I asked him to explain the meaning of the product of .20 and sixteen, John replied, “Sixteen ounces make a pound, so that is the only thing I know. Sixteen may not be percentages. The percentages were given.” When asked about the meaning of the
twenty hundredth, John replied, “Point twenty is twenty percent.” John’s explanation of the product of the five tenth and sixteen hundredth was, “Point fifty is 50%. And 50% multiplied by point sixteen is eight. Twenty percent multiplied by point sixteen is 3.00. I subtracted 3.00 from 8.00.” When I asked John to identify the three and twenty hundredth, he replied, “Ounces, percentage, I don’t know what it represents.” The same answer was given about the value of eight. When asked about the final answer, John replied, “Nineteen point twenty. I would say it is ounces but I was going to multiply those 19.20 by .80 or 80 ounces.” After some manual calculations, John added that the final answer was 3536 ounces. There was no other coherent explanation of concentration given by the participant. When asked about the amount of the mixture of twenty ounces of one solution and ten ounces of another solution, John properly identified the result as the sum of twenty and ten. When he was asked about the possible concentration of the mixture of 50% solution and 70% solution, John said that it would be 120 %. The participant admitted that the mixture problem is relevant to real life. In addition, John explained the meaning of the “50% solution” as “Half is alcohol, half is water.”
General description of John’s experience solving mixture problem I. John was able to restate the problem in his own words but he didn’t define and/or apply properly the concepts of percentage and concentration. He unsuccessfully attempted some unit conversion without explaining the reason for that. John clearly articulated that he didn’t know the concept and couldn’t solve the problem. He presented some arithmetic calculations based on the given values, but couldn’t explain what he had done. No variables and/or graphic representations were used. At the same time, the participant admitted that the problem is relevant to real life.

Observation and particular description of John’s experience solving motion problem II. John began with reading the problem aloud and then restating it as “Two cities [are] 315 miles apart. One car leaves [the city], [it is] distance equals rate times time, 315 is the distance. We got a rate, distance, two cars. [Let’s] see if x can equal the
time it would take them to meet. So, you got a distance of 315 miles per hour.” The participant proceeded with making a chart. When asked about a reason for the step, John replied, “Because I am using two different vehicles I need to figure out the time one vehicle can reach a certain point and then subtract.” John also added that even though he attempted to make the chart, he didn’t really know how to use it properly. When asked about the source for the chart, John stated, “I was shown it [the chart] in class.” The participant defined units of distance as “miles per hour” and the units of rate as “50 miles per hour, wait a minute, miles.” He defined units of time as “how long, 315, we use miles per hour to measure time.” John stated, “How long is unknown, [it is] time.” He also identified the given as “distance and rate”. John described the situation as a motion of two vehicles, but at the same time he did not sound certain when asked if the vehicles are involved in simultaneous motion. He stated that the vehicles are moving “towards each other in opposite directions. So, x equals time, time equals distance times time. Time equals rate times distance. And the first car is 50 miles per hour. We go 50; wait a minute, 50 miles per hour is 50x. I still got this backwards I think.” The participant stated that the variable x stands for time and 50 represents “Speed [or] rate”. When asked about the meaning of the product of 50 and the variable, John replied, “X is you do not know, how far it is going to get, rate times distance.” When asked to explain the difference between 315 and the x, John replied, “X is the time. If I can figure out how long it takes one car, then I can subtract that from 315. It will give me the time it will take the bus to get there at 55 miles per hour.” At this point, John also stated that the distance apart was
the unknown. When asked about the quotient of 315 and 50, the participant replied, “[It is] a ratio of 50 miles per hour to a distance of 315 miles.” The participant added that the problem is relevant to real life and that he has seen such a problem in his algebra class.

![Figure 9](image)

**Figure 9.** John’s solution of motion problem II

**General description of John’s experience solving motion problem II.** John agreed to come for the second interview; nevertheless he didn’t appear to be in a good mood. When asked to solve the second motion problem, John seemed to be able to restate the problem properly and define the unknown at first, but later on got confused and uncertain. He attempted to use a chart that was shown in class, but failed to finish it. When talking to himself while solving the problem, John mentioned the proper formula that connects attributes of motion. Nevertheless, later on in the interview, he restated it incorrectly. His unhappy mood continued because he was frustrated by his inability to solve the problems. John chose x to represent the unknown time and even set up a product of rate and time as 50x, but when asked to explain the meaning of the product, he
provided a few different and incomprehensible answers. As with the mixture problem, John seemed to start thinking about using a ratio again stating that it might be needed since the problems solved by ratio are in the same section of the textbook. The participant hadn’t obtained any answer to the problem. At the same time, he admitted that the problem looked familiar since it was discussed in class.

**Observation and particular description of John’s experience solving mixture problem II.** John began the solution process with silent reading and proceeded by rereading and restating the problem as follows: “Fifteen percent solution must be mixed with four ounces of twenty percent alcohol solution to make a seventeen percent alcohol solution. Ok, so, how many ounces of alcohol must be mixed with four ounces of twenty percent solution to make a seventeen percent solution?” John also stated that the problem looks familiar and added, “Ok, how many, there is x, x is unknown, how many ounces; I am going to try fifteen percent over twenty to see if I can make it proportional--with x over seventeen, cross-multiply 20x equals fifteen times seventeen equals 255. And I am going divide 255 by 20 to give me x equals 225 divided by 20. So, x is 12.75 ounces, four ounces.” John then clarified his last statement as “If it would be kind of a question that asked me how many ounces of an alcohol solution it would take to make fifteen percent alcohol, if it was proportional, how many ounces it would take to make twenty percent? This might be appropriate, but you don’t know how it is all one question. I think there are too many elements, too many unknowns.” When asked to identify the unknown in the problem, John replied, “Ounces, the ounces they would take to make, how many ounces
of alcohol, how many ounces of fifteen it is going to take, how many ounces of fifteen percent solution must be mixed with four ounces of twenty percent solution to make a seventeen percent solution? [It is] fifteen over twenty.” When I asked John the reason for using a proportion to solve the problem, he replied, “Because it seems appropriate to involve proportion.” John defined a proportion as “How much, how many ingredients are used to make this amount. We need to make the same ingredient except more of it. [It] sounds proportional.” John identified the two numbers he wrote down, fifteen and twenty, as “Fifteen percent alcohol solution and twenty percent alcohol solution.” When asked to explain the meaning of the expression “fifteen percent alcohol solution,” John replied, “Fifteen percent of 100, it is fifteen over a hundred or point one five.” When asked again to identify the meaning of the expression, the participant stated, “Part of a whole.” John compared a fifteen percent alcohol solution and twenty percent alcohol solution as “One is more than the other.” John also clarified the term “more than” as “alcohol solution, twenty percent, more of the alcohol solution.” When asked to identify the unknown used, John replied, “How many solutions it would take to make seventeen percent.” John identified the units of the x as ounces. He also identified the units of the number fifteen as “solution, alcohol solution” and the units of the number seventeen as the solution. When I asked John to approximate the final concentration of the mixture of the fifteen percent solution and the twenty percent solution, he replied that it would be 35%. When asked about the answer he obtained when solving the problem, John replied, “Twelve point seventy five, I am thinking ounces.”
John’s solution of mixture problem II

General description of John’s experience solving mixture problem II. John agreed to come for the second interview; nevertheless he didn’t appear to be in a good mood. After reading the problem, the participant had difficulty restating it without looking at the text. He admitted that the wording of the problem was not clear to him. John seemed to have difficulties with the given information, the condition of the problem, and the unknown quantity. He also admitted having difficulties with the model of mixing solutions since his intention was to solve the problem using ratios and a proportion. When asked about the concept of a proportion and its relevance to the problem, the participant replied that it just sounded proportional to him. In addition, John had difficulties distinguishing the terms alcohol and alcohol solution. It seemed that the participant didn’t differentiate them.
Ron

**Observation and particular description of Ron’s experience with math education.** Ron is 46-year-old retired military personnel. He holds a high school diploma. The last math class he took was Business Math. During the first interview, Ron appeared to be very reserved and not articulate. He kept his answers short. When recollecting his math experience, Ron said, “It is elementary school, basic arithmetic, decimals, fractions, and most things of that nature. I was very confident. In the elementary school, they [the teachers] were excellent. As things progressed [after arithmetic and elementary school], a lot of things happened with the school system, and that is when things got blurry because I was in classes getting credits and not doing any type of work in that class. Algebra was an algebra class but we didn’t have an algebra teacher and geometry was the same thing.” Ron seemed upset about missing opportunities to learn. At the same time, the participant did not express much attitude toward mathematics as a subject. He commented though on his status in the present math class as, “Well, we just started linear equations, and I am a little lost right now.” When asked about his family involvement with his school and informal mathematics activities, Ron didn’t talk about his family, but stated, “Actually it [any mathematics he has done when was young] was based on homework, so it was formal.”

**General description of Ron’s experience with math education.** Ron showed mild emotions toward mathematics and education. He did admit that he felt confident learning arithmetic and complimented his mathematics teacher at that time. He didn’t
recall much about his post-arithmetic years attributing this to the absence of teachers in both his Algebra and Geometry classes. Ron seemed to disapprove of the system which awarded educational credits without learning achievement. He admitted not feeling comfortable in his present math class as the result of having difficulties with algebraic concepts. There was no evidence that Ron was exposed to any informal mathematical activities when he was young and/or was supported and helped by his family members to be successful in a math class. He had vague recollection of solving word problems.

Observation and general description of Ron’s experience solving motion problem I. Ron began by restating it as follows: “Two planes in the same airport at the same time one flying this way and one flying that way (showing opposite directions using his hands). One plane flies at 300 miles per hour, and the other one 200 miles per hour, so the first plane is faster. What time or hours the planes will be 1000 miles apart?” He added in a few minutes, “Taking all this into consideration, it is going to take the slower plane with the rate of travel five hours to get 1000 miles away.” Ron named the attributes of a motion as, “The speed they [the planes] are traveling.” He also identified the units of speed as “miles per hour”. When Ron was asked if the speed was given, he replied, “Yes, actually, no, it is not.” Later on, Ron recalled distance and time as attributes of a motion as well. When asked about a formula connecting these attributes, Ron replied that, yes, they are connected and the connection is “something times time, speed times time would be distance.” Ron also identified units of distance as “miles” and units of time as “hours”. When asked what was given, Ron replied, “The speed is, 300 miles per hour, and the
distance, the miles.” He identified the unknown as “time”. When asked to explain his solution, Ron said, “I just look at the smaller one, and at this rate of travel it would take the slower plane approximately five hours to travel 1000 miles, traveling 200 miles per hour. Oh, I was looking at how long it would take the first plane to reach the same distance. It would be 3.3 hours.” When asked about the answer to the problem, Ron replied, “I would go with five hours because it is slower; it would take this one longer to get there. They are traveling the same distance at a different rate.” When asked if he concentrated on the numbers given or tried to visualize the story given, Ron replied that he was “visualizing the problem and then [doing] the calculations. Ron said that the given problem is relevant to real life and that he has seen something similar “a while ago”.

Figure 11. Ron’s solution of motion problem I

General description of Ron’s experience solving motion problem I. Ron restated the motion problem properly; nevertheless, he began with calculation of the time needed by a slower plane to reach 1000 miles and then proceeded with calculating the time needed by the faster plane to reach the same distance. Ron failed to identify the
problem as a simultaneous motion of two objects moving in opposite directions. When asked to identify attributes of a motion, Ron named distance, speed, and time as the ones. Even though his answer to the questions was correct, Ron seemed not sure of his own answers. He recalled, though just as uncertain as before, that the formula connecting attributes of a motion is distance equals speed times time. No variable and/or equation were used. No graphical representations were made either. Ron acknowledged that the given problem is relevant to real life and recalled seeing similar problems earlier in his life.

Observation and particular description of Ron’s solution of mixture problem

I. Ron restated the problem as, “They [the solutions] are diluted. If this one is 50%, so it must be mixed with 80 ounces to get twenty percent. No, to get 40% you need to dilute it, to break it down. With this one, I have no idea how to work this one.” Ron proceeded by writing down a few numbers. He then indicated that it is as much as he can do. When asked to explain the meaning of 50% alcohol solution, Ron replied, “It is half, half alcohol.” When I asked about the other half, Ron replied uncertainly, “It does not say what it is. I assume it can be water.” Ron identified the ingredients of twenty ounces of 50% alcohol solution as, “Half is alcohol, and half is water. It is 50-50 mix. It is twenty ounces, so it is ten ounces water and ten ounces alcohol.” When asked if the problem is relevant to real life, Ron replied that yes, it is. He provided the following examples of the relativity: “Measurement--it is kind of, you say, cutting grass and you have to use so much gas and so much oil mixture in there to make the solution to run the machine.” Ron
stated that people would mix alcohol solutions in real life “to prepare things, to administer medications, in cooking.” When I asked Ron to explain the meaning of a percent, he replied, “[it is] a portion of, very small portion, point one zero, point zero one, one of a tenth.” When asked about the total amount of the mixture of two solutions, twenty ounces and 30 ounces respectfully, Ron replied that it would be 50 ounces, the sum of the given numbers. When asked to approximate the concentration of the mixture of 50% solution and 80% solution, Ron replied that it would be 130 % as the sum of the given concentrations as well. The participant identified the first one as the weaker solution and the other one as the stronger solution. In addition, he stated that he was not sure if enough information was provided to solve the problem.

![Figure 12](image.png)

**Figure 12.** Ron’s solution of mixture problem I

**General description of Ron’s experience solving mixture problem I.** Ron was able to restate the problem coherently, but admitted right after reading it that he was not able to solve it. He properly identified the 50% concentration alcohol solution as half alcohol and half water. At the same time when asked about the possible concentration of a mixture of two solutions, Ron added the given percentages together. He didn’t seem to be confused by the fact that the final concentration was above 100 %. When asked to define a percent, the participant became hesitant and unsure of the answer. He introduced
neither a variable nor an equation to solve the problem algebraically. An arithmetic solution has not been proposed either.

Observation and general description of Ron’s experience solving motion problem II. Ron began with reading the problem aloud and then restating it as follows:

“A bus and a car both leave a city. Two cities [are] 315 miles apart. The car is traveling to the second city 50 miles per hour. At the same time, the bus leaves the second city down to the first city at 55 miles an hour. How long will it take for them to meet?” Ron proceeded by trying to make a chart. When I asked him to draw a picture, he articulated his picture as “This way 50 miles an hour and the bus is 55 miles per hour.” Ron identified the values of 50 and 55 as “the rate or speed they are traveling” and the given as “the rate and speed and the distance, 315 miles.” When asked to show the 315 miles on his picture, he said that he couldn’t and added, “It is represented by the number because I am stuck”. Ron acknowledged the units of distance as “miles” and presented the unknown as “We multiply rate, it is a formula, rate times time times distance. We are looking for the time. So, I am looking for the time.” When asked about formula that connects the three attributes of a motion, John stated, “Yes, rate times time equals distance. I am looking at it and divided. I divided the miles. They travel 55 miles per hour; it would be six point three and the same thing over here.” Ron divided 315 by 50 and stated that the quotient was “six point three”. When asked to explain the meaning of the answer, Ron replied, “The total time for the trip for this vehicle traveling 50 miles an hour will be six point three hours. And with the bus traveling at faster rate, they will
actually make the trip in five point seven hours. I was looking at averaging them as dividing by half. And I came up with three point fifteen hours. Three hours fifteen minutes actually. And they will meet at the half way point somewhere. I divided them in half because I was looking for a medium point where they would probably pass each other. These two numbers are for the entire trip. This [vehicle] drove that way and this [vehicle] drove that way (pointing to the picture). At some point, they would pass each other. The bus is traveling five miles faster so it would get there faster. So, allowing for that I chose three point fifteen.” When asked why he decided that three and fifteen hundredth would be the answer, but not the two and eighty five hundredth [half of the second quotient], Ron replied, “I was thinking that because of the size of the bus traveling at 55 miles an hour it would more likely to get there faster.” Ron later added that the problem is relevant to real life and that he was given enough information to solve it. He also stated that he was exposed to this type of word problems in his algebra class.
Observation and general description of Ron’s experience solving mixture problem II. Ron appeared to be silently reading and then proceeded to restate the problem as follows: “They start with fifteen percent alcohol. Four ounces of it should be mixed with four ounces of twenty percent alcohol to come up with the solution that has seventeen percent alcohol solution.” He identified the given and the unknown as “fifteen percent alcohol and the twenty percent alcohol. You mix two solutions to get one. We are looking to see how many ounces of the fifteen should be mixed with the twenty to come up with four ounces of seventeen percent.” The participant proceeded with setting up a chart. He stated that the chart was shown in his algebra class. Ron explained his approach as “You already have fifteen, so it is undefined. I am trying to break it down to come up with fifteen. But you are adding the higher percent alcohol to the solution. Ok, twenty to
come up with the seventeen, starting with fifteen, how much fifteen, change percent to a
decimal, fifteen, four ounces that is given the four ounces, fifteen x, point fifteen.” When
asked to make a picture illustrating the story, Ron drew three bottles and then stated, “In
this bottle right here, it is an empty bottle because it is where we are mixing things in.
Actually, it is given. It is the four ounces of twenty percent alcohol.” Ron also stated that
“point two zero” represents the twenty percent alcohol solution. When asked to explain
the meaning of the expression “twenty percent alcohol solution,” Ron replied, “It is
twenty of a hundred. [It is] a little less than five percent. I was breaking it down to two
fives and it takes twenty fives to make a hundred. Twenty--it makes five of them to make
a hundred.” Ron described the contents of four ounces of a twenty percent alcohol
solution as “That is given, four ounces of it. Twenty percent alcohol and I don’t know
what the rest is, may be water.” When asked about the amount of alcohol and the amount
of water in the solution, the participant replied, “Four ounces of twenty percent alcohol.
How much water? I am not sure.” Ron presented the variable x as “I use x to show how
many ounces of this fifteen--this is the unknown.” He then identified the meaning of his
equation as “Yes, fifteen times x, I am trying to figure out because I need to know how
much of this to use, because I know how much of this I have, so how much of this I need
to mix with this, because it is the amount of fifteen ounces taken away from the fifteen. I
am all confused.” When asked why he added the products together, Ron replied, “I am
combining fifteen and the twenty to see how to come up with seventeen, with the
percentages. And x is the unknown. That it is why I did the distribution property and
combined liked terms. And I am coming down with subtraction to isolate x, to get the value x.” When asked why for some values he used whole numbers and for some decimals, Ron replied, “Because I looked at it as percentages and I write them as whole numbers, but I thought of breaking out of percentages. The percentage is written as decimal. So, fifteen percent became point fifteen. Twenty percent became point twenty.” When asked to explain how he obtained a negative value as the answer, Ron stated, “Because I got these two together and I came with fifteen and twenty; it gave me thirty five. The 35x was point thirty five x equals seventeen. To solve for x, I subtracted 35 on this side and the same on that side. I am subtracting seventeen minus 35 and I got negative 0.18. On the other side x [is] equaling 0.18, negative 0.18.” Ron described the contents of twenty ounces of 50% alcohol solution as “This whole thing is 20%. Half of it is 50% of alcohol. So, ten ounces is half of the twenty is alcohol.” He added that the rest was water. When asked to calculate the amount of the mixture of twenty ounces of 50% alcohol solution and 30 ounces of 70% solution, Ron replied that it would be 50 ounces. When asked to estimate the concentration of the same mixture, he said it would be 120. The participant also stated that the problem was relevant to real life and came up with following examples: “It depends on what your profession is. If you are a hair-dresser, you are missing dye and stuff like that, or if you are a chemist. If you work in landscaping, you are mixing gas to oil.”
Figure 14. Ron’s solution of mixture problem II

General description of Ron’s experience solving mixture problem II. During the second interview, Ron seemed calm and open toward answering my questions. Nevertheless, after reading the mixture problem, he became apprehensive. When asked to restate the mixture problem in his own words, Ron confused the solutions being mixed together and the final mixture. In a few minutes he was able to clarify his thinking though. The participant’s intention was to use the chart he had seen in class, but he was not able to repeat the procedure. In addition, he intended to use a variable to solve the
problem, but was not able to set up and solve an equation correctly. When asked to explain the product of a concentration and an amount of a solution, the participant was neither able to explain the meaning nor properly find the value of the product. Nevertheless, later on, when asked about the 50% concentration, he was able to explain that it is half of the amount. The participant drew a pictorial representation only when he was asked to do so. The fact that he obtained a negative value to represent an amount of a solution didn’t seem to puzzle him.

Jana

Observation and particular description of Jana’s experience with math education. Jana is a 39-year-old student who holds an Associate Degree in Fashion Merchandising. She appeared to be very open toward our conversation and was smiling all the time while answering the questions. Jana began the story about her math experience with, “My parents were very scholarly, and my dad was making sure we spoke properly and did things correctly, especially our school stuff. When I was young, I liked math at that point of my life. Going to school, math was fine for me as well.” As time progressed, her attitude toward math education changed. Her account of the change was: “I don’t know at what point I started to dislike math. I don’t know if it was in high school because I just didn’t like school at all. I did not see why I needed it [mathematics]; we have calculators. I went to a Fashion School because my mom is a designer and my sisters are too. We did some math there, but not a lot. We focused more on the glitzy part
of it. I know I went to school and I did math, but it was of those subjects that never
interested me. And the funny thing is that now I have a daughter, it [mathematics] is the
one thing I am making sure that she knows because it is valuable.” When asked about her
experience with mathematics at the present time, Jana stated, “I realized that when I did
math 0910 last semester, we use math in every function of life. And when I had to take a
test, I have to show how you get 30% of something and I didn’t know how to do it, I was
asking myself ‘are you serious?’ This is something I robbed myself of things; I don’t
know how to work it out without a calculator. It is why I think I started to take it more
seriously.” Jana also added that at the present time, “I have a range of emotions in class.
When I go to class, I am excited especially when I understand the homework. I sit in the
front. I want to come to every single class and want to learn. And I am getting it. I feel
better now.” When asked if she remember solving word problems, Jana replied that she
didn’t.

**General description of Jana’s experience with math education.** Jana appeared
to be open toward our conversation and was smiling all the time while answering the
questions. She admitted that while her parents were always stressing the importance of
education, they were mostly interested in their children’s success in English rather than
mathematics. When Jana was young, she felt comfortable with mathematics;
evertheless, her attitude changed when she became older. She mentioned disliking
school in general during her post-elementary school years. Jana also admitted that she
didn’t understand either the significance of math or relativity of it to real life at that time.
This understanding only came to her during last semester when she was taking Pre-algebra at the college. Now, after some schooling and years of working, she appeared to be extremely determined to succeed in learning mathematics and in making sure that her child learns it as well. At the present time, Jana has mixed emotions toward her math class. It would make her feel excited and happy when she understood the material, and not so when she would encounter some difficulties. Jana didn’t remember much about solving word problems before.

**Observation and particular description of Jana’s experience when solving motion problem I.** Jana began solving the problem with reading it silently. Then she said, “I think I have the first one. I doubt though it is right.” Her answer to the problem was correct. When asked to restate the problem, Jana replied, “The first plane was flying at 300 miles per hour, and the second plane was flying at 200 miles per hour. And since they are going in opposite directions, I added them up.” She explained the reason for addition as “Because I need to get this ending part of how many hours when the planes will be 1000 miles apart. So, I knew that I had to add these to figure out the ending result.” When I asked Jana about a situation that would require subtraction rather than addition, she replied, “I don’t know why I didn’t subtract; I think because I knew I needed to get this 1000 figure, and this is the reason I said ok I am pretty sure I have to add, and then possibly divide whatever I got by that number, and this will give the number of hours. That one way I am thinking that you can do the subtraction is if you would have plane one leaves at five pm and plane two left at one [pm], who got there
first. Then you would have to subtract them.” Jana identified the given and the unknown as, “I know that both planes left the same airport. They are flying in opposite directions, one is flying faster than the other one, but I need to figure out how many miles apart they are going to be at, no, how many hours it will take them to be 100 miles apart. So, even if they are going in opposite directions, it is that space between them that I am trying to figure out. Because I am looking at the first plane is over here, and that is 300, and the second one is over here, and it is 200. So, my area is right in the middle here where I need to know the hours for 1000 miles. So, that is what I mean by space.” She also stated that 300 and 200 represent the speed of the planes, and the “speed” means “how fast someone is going or something is going.” Jana identified the units of speed as “miles per hour”. When asked about attributes of a motion besides speed, Jana replied, “Distance and time.” Nevertheless, when she was asked about the formula connecting them, her reply was, “I am sure there is because you have airplanes. But I don’t know what the formula would be. It is probably distance times time equals speed. I have no idea.” She explained her solution as, “I got 500 as their [the rates] total and I wanted to know how many hours it would be before the planes were 1000 miles apart. So, I am dividing my total miles per hour, which were 500 by the expected or given miles per hour which is 1000. When I asked Jana for the reason for the division, she replied, “I guess it is like common sense.” She properly identified the units of distance as “miles, but not miles per hour”. When asked if the problem was relevant to real life, Jana replied, “Somewhat, I mean I would not care knowing where planes are going but if I needed to find out how
long it is going to take me to get from point A to point B by an airplane or a bus, that would be good to know."

*Figure 15.* Jana’s solution of motion problem I.

**General description of Jana’s experience when solving motion problem I.**

Jana restated the problem properly and was able to solve it arithmetically. Neither variable nor equation was used. Jana realized that the story was about two objects moving simultaneously in two opposite directions. She added up the given rates and then obtained the quotient of the distance and the sum of the rates. Even though the answer to the problem was correct, Jana’s explanation of why she added the rates, not subtracted, was not clear. At the same time she stated that if the objects would be moving in the same
direction, she would subtract the rates. Jana was able to identify the given and the unknown without visible difficulty. She correctly identified the attributes of motion, but admitted that even she believed there was a formula connecting them, she didn’t remember it. At the same time, when asked to solve the problem, Jana intuitively divided the distance by the rate to obtain the time. She also properly identified the units of speed and distance.

**Observation and particular description of Jana’s experience solving mixture problem I.** Jana began by reading the problem aloud. Her facial expression seemed to sadden, reflecting the participant’s negativity toward the problem. When asked about the reason for such change in her attitude, Jana replied, “Percent, percent, percent.” She defined “percent” as “to divide by 100” and stated, “I cannot figure this [the given problem] out at all.” When I asked Jana to present her understanding of the term “concentration of a solution”, she replied that if she buys a 50% orange juice solution, it means that it is half orange juice. When asked to identify the other half, she replied, “Whatever, it is unknown if does not say what it is. Half will be the orange juice and half whatever else, juice or water.” Jana stated that the mixture problems are relevant to real life and added that people would mix solutions together in order “to make something else; it can be anything. If you are mixing juices, that is one thing. If you are mixing potions, that is another thing.” When asked what happens to a concentration if two solutions are being mixed together, Jana replied, “It gets stronger. I would think if you are mixing two of anything--I am guessing at this point--you would think that solution
would get either stronger or powerful because you are adding one to the other.” When asked to describe the ingredients of twenty ounces of 50% alcohol solution, Jana replied, “Your cup is twenty ounces, half of it is alcohol, 50%, ten ounces.” The participant admitted that when shopping for drinks for her daughter she checks the labels and gets only 100% juice. Jana identified the total amount of mixture of twenty ounces orange juice solution and 30 ounces of the solution as 50 ounces that is the sum of the given individual amounts. At the same time she estimated the possible concentration of the mixture of 50% juice and 80% juice solutions as, “It will be over a 100%.” She also added that the 80% juice solution would be stronger than 50% juice solution. When I asked Jana to try to solve the problem again, she replied that she couldn’t. She also added, “I think what is getting me is all these numbers, and I don’t know where to start. I know that it has to be some equation to follow it, but all I am thinking of is equations. I am looking for an unknown. I need to put the x to the unknown. But it seems like everything here is known. How many ounces (reading the problem) must be mixed with 80 ounces of a twenty ounce solution to make twenty? You switch from ounces to percent. You have ounces here, ounces of 50% and 80 ounces there, twenty percent solution to make 40% solution. So, this part I know I have to do multiplication of some sort. How many ounces of 50% solution must be mixed with 80 ounces? So, I would do point five zero, because I am changing it to a decimal, times point eight zero equals point two zero times point four. It does not look right. I have twenty percent.” When asked about the meaning of the twenty percent, Jana replied, “The solution of the problem that
is what it should stand for. How many ounces have to be mixed with 80 ounces? It does not make sense.”

\[
\begin{align*}
0.50 \cdot 0.80 &= 0.40 \\
0.4 &= 0.08 \\
2 &= 20\%
\end{align*}
\]

*Figure 16. Jana’s solution of mixture problem I*

**General description of Jana’s experience when solving mixture problem I.**

Jana’s mood completely changed when she was offered to solve a mixture problem. Right away she admitted that she would not be able to solve it. In spite of the latter, Jana described the concept of a percent correctly and portrayed 50% solution of orange juice as half juice and half another ingredient. Jana stated that the problem is related to real life and properly indentified the 80% concentration solution as being stronger than the 50% one, but when she was asked to approximate a concentration of two solutions combined, she added both the amounts and the concentrations. The fact that the resultant concentration would be above 100% didn’t cause any suspicion. At the end of the conversation Jana recalled that while dealing with percent you do multiplication, but failed to do it properly. When attempting to solve the problem, she was doing it arithmetically, not algebraically. No graphical representations, charts, etc. were used.

**Observation and particular description of Jana’s experience solving motion problem II.** Jana began with reading the problem aloud and proceeded with restating it
as follows: “We have two cities 315 miles apart. Car leaves one city traveling toward the second city traveling 50 miles per hour. At the same time the bus leaves, you see, that is what I am saying. At the same time bus leaves the second city at 55 miles per hour. How long will it take for them to meet? Ok, we have two vehicles leaving two cities driving at two different speeds. Two cities are 315 miles apart. We want to know how long it will take them to meet. I understand this one.” Jana then added that she believed there is enough information to solve the problem and that she had seen similar problems in her algebra class before. When asked to identify the given and the unknown, Jana stated, “315, oh, no, I don’t do that. So, my time is unknown. It is probably the wrong way, but I don’t know what my time is. I know what my distance is. I added these [the rates] and I got 105. And I just divided that [the distance by the rate]. And this is how I got the time.” When asked why she added the rates, Jana replied, “The reason I added it is because I am trying to figure out how long combined. So, my thinking was I have to add. I cannot just do one because it is going to give one vehicle. I have to do both of them and then divide by the total amount of time because they were coming together.” When I then asked Jana about a situation with two vehicles that would require subtraction, she responded, “Here we are trying to figure out if they left at the same point. And they are going in opposite directions. I don’t know. They are in opposite directions but to the same central point, versus the other way. This way they are coming this way (pointing to her picture), versus they start at a point and they are going in different directions.” When Jana was asked about the attributes of a motion and the formula connecting them, she said, “Distance
equals rate times time.” She identified rate as “speed” and its units as “miles per hour.” Jana also identified units of time as “hours” and of distance as “miles”. Jana solved the problem arithmetically and was asked if she could do it differently. She replied that she couldn’t solve it algebraically because “I couldn’t remember. I could have had two xs because I could do x and x. This second x has to be little different, right? Because how are you going to differentiate the car from the bus? So, I forgot how one has to be different--either x minus 50 or something.”

Figure 17. Jana’s solution of the motion problem II.

**General description of Jana’s solution of the motion problem II.** Jana solved the motion problem arithmetically similarly to the first interview. She didn’t have visible difficulties restating the problem or making a picture. At the same time, she failed to show the distance in the picture, presenting it instead as a point. She didn’t make this
picture until she was asked to do so. Jana properly identified the three attributes of motion, their units, and the formula connecting them. When asked to solve the problem algebraically, Jana set up the proper variable for time for one of the vehicles, but somehow stated that another variable should be used as time needed by another vehicle since different vehicles would require different time intervals. She failed to develop an equation for the problem.

Observation and particular description of Jana’s experience solving mixture problem II. Jana began the solution process with reading the problem aloud and proceeded with restating it as follows: “How many ounces of fifteen percent alcohol solution must be mixed with four ounces of twenty percent alcohol solution to make a seventeen percent alcohol solution? Ok, we are mixing different solutions, but ultimately we are trying to get a twenty percent. We are trying to get a seventeen percent alcohol solution.” When asked about the number of solutions being mixed together, she replied uncertainly, “I would think two. How many ounces of fifteen percent alcohol solution must be mixed with four ounces of twenty percent? There are three solutions. We are mixing one--fifteen percent mixed with the four [ounces] twenty percent solution. I am guessing we are mixing two [solutions].” Jana identified the given as “You know your percentages and you know your ounces.” She then identified the unknown as “We are looking to find out how many ounces we have to mix to get a seventeen percent alcohol solution.” When asked if the problem appeared familiar, Jana stated, “Yea, we did it for homework like a million years ago. And this problem was one of them, but all I
remember doing was a table like one of this (pointing to the page out of a textbook) and putting knowns and unknowns in it. That is about it.” The participant then attempted to make a picture upon my request. When asked about the difference between the ounces and the percentages, Jana replied, “Four stands for ounces and twenty for the percentage.” When asked to identify the contents of four ounces of twenty percent solution, Jana replied that four ounces is “how much is in it” and that the percentage is “this is the percentage of 100.” She also added that the percentage means “twenty out of 100. It is 80 percent.” When asked to determine the contents of twenty ounces of 50% alcohol solution, Jana replied, “Half would be alcohol solution.” She identified the other half as “water, juice, whatever.” When asked to identify the expression “alcohol solution,” Jana responded, “Just alcohol.” When asked to clarify the meaning of the expression “20% alcohol solution”, Jana replied uncertainly, “Ok, you have your cup. And it is four ounces. We have fifteen percent, twenty percent; the whole thing should be equal to 100 percent. We want to know how many ounces of the solution to get the seventeen percent. It would be twenty out of 100.” The participant added that she attempted to use the chart shown in the tutoring center earlier. When asked about the variable, Jana replied, “The unknown would the x.” When asked about the units of the unknown, she said, “It would be the seventeen, because it is what we are trying to figure out, the amount of ounces.” When asked to illuminate the solution whose amount is unknown, Jana replied, “I think all of them. So, we are looking for two. I don’t know. I think we are throwing in extra jargon just to confuse you. Do I even care about the first
part where it says how many ounces of the fifteen percent alcohol solution? It is not even a part of the problem. Isn’t? I think that’s what irritated me in the first place because they put so much extra stuff in it here that really isn’t necessary and all you are trying to find out is how many ounces you would need to make a twenty percent solution or a seventeen percent solution.” When asked if she encountered any word problems in math classes that had unneeded information before, Jana answered yes. She then added that she couldn’t finish the chart without her class notes and that she was lost. Jana estimated the final concentration of the mixture of the 50% solution and 70% solution as “It will be more than a 100, if you are adding them.” When asked why she used the addition sigh in her picture, she replied, “Just by reading that; the words that they use make you think about adding.”

![Figure 18. Jana’s solution of mixture problem II](image)

Figure 18. Jana’s solution of mixture problem II
General description of Jana’s solution of mixture problem II. When restating the problem, Jana kept confusing the solutions to be mixed. She admitted not remembering how to solve the problem because it had been a while since discussing it in class. Jana mentioned a table that was used by a tutor to solve similar problems, but she couldn’t set one up. She also admitted being confused by the wording of the problem. She believed that some special jargon had been used as well as some unneeded information provided. When asked to clarify what she meant by the unneeded information, Jana pointed out the names of the places given in a motion problem in the textbook. Jana was also asked to explain the difference between the attribute represented by the ounces and the one represented by percentage, but failed to provide a correct answer. She also had a visibly difficult time trying to define the concentration of a solution and identify the given percentage as such. Jana failed to supply either arithmetic or algebraic way to solve the problem. When asked to make a picture, Jana combined all three solutions into one container. The participant seemed to be in a good mood during the interview in spite of her inability to solve the problem. She was laughing at times at her own uncertainty.

Ken

Observation and particular description of Ken’s experience with math education. Ken is a 50-year-old student who revealed that his decision to participate in the study was based completely on the desire to help other students. He holds a high
school diploma. Ken took Algebra and Geometry during his high school years. Presently, Ken works as a painter and is seeking a degree in information technology. During the interview, Ken seemed to be very friendly, open toward the conversation and determined to get a college degree. When asked about his math experience, Ken replied, “We were homeschooled for a few years and also attended a Christian school in Sandusky, Ohio where an education was important. Regardless of what you are going to do, you have to get some math. You come home, the first thing you do is your homework, and mom would check it. It better be right. As far as math, my dad was a pastor, but he also did a lot of painting and papering on a side. He needed to figure out how much paint he would need, so you have to know the basic geometry to figure area of the walls, area of the wallpaper, how to divide it, so you have to have math to get along.” Ken was one of a few participants who didn’t display any apprehension toward mathematics education as his schooling progressed into post-elementary years. When recollecting this time, Ken stated, “[I was] excited, but not happy excited, excited to be challenged. In Consumer Math it made sense to me because you are learning how to balance your checkbook and do things like figuring out your payment, your interest on something. To me …you are supposed to know all this stuff. In Geometry I started the class enjoying the class in the first semester. I need to find the area of the room so I can paint it. As we got later in the year and with algebra I thought I was going to become a minister, so, why do I need this in the field I am going into? I thought my dad is a pastor and he does not use it very often, you know; why learn something I am never going to use. So I would remember just
long enough to pass a test and then move on. I was not concerned about learning, just remember for the test and let it go.” When asked about his math education presently, Ken replied, “Math is not my strong suit and I was more concerned with my core curriculum being completed. Now I just have to finish math.” He also added that even he was in no doubt that mathematics is a part of life; he didn’t like it though. Ken stated that he didn’t remember much about solving word problems.

**General description of Ken’s experience with math education.** Ken seemed to be eager to share his experience. During the interview, Ken didn’t show much emotion toward his arithmetic experience and elementary school mathematics. His parents taught him that education was important, so he took his studies as a matter-of-fact at that time. As time progressed, the participant continued doing whatever was needed to stay afloat at school, but mathematics was not his interest. Ken admitted that when he was young, his main concern in a math class was to do well on a test rather than to actually learn the material. At the same time, Ken was one of a few participants who didn’t display any apprehension toward mathematics education during his middle and/or high school years. As for solving word problems he didn’t remember much. Ken expressed a strong belief that mathematics is a part of life, so it is vital for everybody, especially for someone seeking a degree in information technology, to study the subject. At the same time, even with recognizing the great importance of the subject, he still didn’t favor it.

**Observation and particular description of Ken’s experience solving motion problem I.** Ken began solving the problem with reading it silently. He said, shortly, “I
know the answer but I don’t know how I got there. The answer is two hours. He is going 300 miles one way; he [the other plane] is going 200. In one hour, they will be 500 miles apart. In two hours they will be 1000 miles apart. But how do I make the equation for that? That is where I struggle.” Ken also added, “Plane one will be 300 miles from the starting point in one hour. Plane two will be 200 miles away from the starting point in one hour. So, in one hour, the planes will be 500 miles apart. So, in two hours they will be 1000 miles apart. The miles per hour are given and the total distance is given. And I need to find the time, the total time amount.” Ken identified the attributes of a motion as, “Total time, total distance and speed.” When he was asked about the formula connecting them, the reply was, “For sure there is, but I don’t know it.” Nevertheless, in a while, he recalled that, “speed times time equals my distance.” Ken admitted that when trying to solve the problem, he was thinking about calculations as well as trying to visualize the two moving planes. He stated, “I started to try to make a formula for it and that was not right, let’s just figure this thing out: 200 miles, 300 miles, 500 miles in one hour, 1000 miles in two hours.” Ken stated that the problem is relevant to real life and that he has seen similar word problems before.
General description of Ken’s experience solving motion problem I. When asked to solve the problem, Ken quickly obtained a correct answer using an arithmetic approach. He was able to identify three attributes of motion and their units but failed at first to recall the formula connecting them. His solution was based on the application of the formula, but there was no formal concept presented. Ken correctly identified the given and the unknown in the problem. When asked to try solving it differently, the participant attempted to employ an algebraic approach but was unable to set up a proper equation and, consequently, to obtain the correct answer. Eventually, the participant came up with the proper formula.

Observation and particular description of Ken’s experience solving mixture problem I. Ken began with the silent reading and then said, “I don’t know. That one I have to stop and think about it. I would struggle because I am trying to raise my original 50% of alcohol with 80 ounces of twenty percent alcohol. I want to raise it to 40%, so I
want to raise it by twenty percent but I am using a 50% solution.” At this point of the conversation the participant seemed to be confused. When I asked Ken about the meaning of the 50% solution, he replied, “Percentage is of 100. If it is 50%, it is half--half alcohol, half diluted water, or something else.” Ken presented the twenty percent solution as “the other one is 80% of something else.” In addition, Ken described twenty ounces of 50% orange juice drink as “ten ounces of the juice, ten is water.” His examples of real life relativity of the concept were “salad dressing, vinegar, and oil.” When asked about the total amount of the mixture and possible concentration of twenty ounces of 50% solution and 30 ounces of 80% solution, Ken replied that the amount would be 50 ounces and the concentration “would be somewhere probably in the 65%.” He also calculated correctly that there are sixteen ounces of alcohol in 80 ounces of twenty percent alcohol solution. When I offered to Ken to make another attempt to solve the problem, he stated, “To raise it [the concentration] to 40, I would need approximately probably 32 ounces of 50% solution. In 80 ounces the alcohol is actually sixteen ounces that is 20%. If I want to raise it by twenty percent, I want to raise sixteen ounces, sixteen ounces of alcohol 50% solution means I need 32 ounces of the solution to equal to 16 ounces or 20% to raise that. So, the answer is 32 ounces.”
General description of Ken’s experience solving mixture problem I. When Ken was asked to solve the problem, his attitude visibly changed. He admitted that this type of problems is difficult for him. He was able to define a percent and a concentration of a solution correctly, but had difficulty restating and solving the given problem. When Ken was asked to solve a one-step problem that involved an amount and a concentration of a single solution, he obtained the correct answer quickly. Nevertheless, he failed to introduce a variable and/or set up and solve an equation for the original problem. No graphic representations were done during the solution process either.

Observation and particular description of Ken’s experience solving motion problem II. Ken began by reading the problem aloud and restating it as follows: “Ok, two cities, 350 miles apart. Car leaves one [city] traveling at 50; bus leaves the other one traveling 55 heading to the first. They are going to meet somewhere in the middle. You need to know the time.” After some thinking and writing down notes, Ken stated that the answer is three hours. When asked what the variable used [x] represented and how he obtained the answer, he replied, “Time, time and a rate, and the distance is 315. So, the car is 50 times x. And the bus is 55 times x. So, 105 times the x is 315 and divided. The x is three.” Ken presented the product of the 55 and x as, “The speed of the vehicle times the time it will take to get the distance.” He also added that the distances covered by the vehicles “are going to be close, but not going to be the same.” Ken admitted that he has seen similar problems in his algebra class, stated that the problem is relevant to real life,
and made the following example of the relativity of the problem: “Yes, for example, my daughter lives in Toledo, and I live here. You leave at two o’clock and I leave at two o’clock. Where are we going to meet?”

![Figure 21. Ken’s solution of motion problem II](image)

**General description of Ken’s solution of motion problem II.** Ken was able to restate the motion problem correctly and to identify the unknown and the given without visible difficulties. He then solved the problem algebraically and was able to identify the terms in the equation—the products of rate and time—as distances. The picture made was clear but incomplete.

**Observation and general description of Ken’s experience solving mixture problem II.** Ken began with reading the problem aloud and proceeded by restating it as, “I already know that [there are] four ounces of twenty percent alcohol solution. I want to make a seventeen percent alcohol solution. And I am going to use the fifteen percent alcohol solution with the twenty to get a seventeen percent alcohol solution. How many
ounces do I need of the fifteen percent?” He identified the given as “the four ounces of twenty percent alcohol solution” and the unknown as “the amount of ounces of fifteen percent solution to make unknown ounces of a seventeen percent solution.” He also added, “I need a seventeen percent solution and I have a fifteen percent solution and a twenty, so I have to combine them to make a seventeen percent for whatever the situation is.” Ken made known that the problem sounded similar to the word problems done in his algebra class and that he had no difficulties solving it. Ken began writing notes and soon announced that the answer was six ounces. He solved the problem using a chart similar to the one shown in class and stated, “It is easier for me to work things out. I can picture it, handle it. I am not good with the abstract, so to me a chart helps to visualize what I need to do.” He identified the variable x as, “My unknown is how much of the fifteen percent solution I need. It will give me x plus four which is my seventeen percent solution.” When asked to explain the meaning of his equation, Ken replied, “Fifteen percent or one point five times x plus four times point two or twenty percent equals the seventeen percent which is point one seven times x plus four. Then I just worked it out.” Ken then identified the meaning of the product of the fifteen and the x as “It is fifteen percent times x.” When asked to clarify his response, Ken replied, “I am not sure I follow where you are going. It represents fifteen percent of the alcohol solution.” When asked about the meaning of the product of four ounces and twenty percent and the reason these two were multiplied together, Ken responded, “Because it is the percentage of the ounces. It is twenty percent of four ounces. If you want to get the percentage, it is multiplication.” He
also stated that the meaning of the product of seventeen percent and the sum of the x and four was, “My seventeen is from here and four is from here and again it is my ending solution that I want. And it is the percentage of that total.” When asked if when solving the problem, he was visualizing the situation or thinking about calculations, Ken replied, “Little bit of both. I learn visually and using my hands. In my mind I am picturing how I am going to make a solution. I use the algebraic formula to get there. Otherwise you are going to be doing trial and error testing it.” Ken didn’t make any pictures so I asked him to do so. Ken also stated that students are asked to solve word problems in math classes since the word problems symbolize real life. He added that he needed to do mixing in his professional life since he is a painter and needs to mix paints to obtain particular results. When asked to approximate the concentration of the mixture of the 50% and 70% solutions, Ken replied that it would be approximately 60%.

Figure 22. Ken’s solution of mixture problem II
General description of Ken’s experience solving mixture problem II. Ken correctly identified the unknown and the given. He then recalled solving this type of problems in math class and not experiencing difficulties with the process. He seemed to be quite confident and calm during the interview process and not intimidated by word problems at all. Ken obtained the correct answer to the problem by setting up and solving an algebraic equation. Nevertheless, he failed to identify the product of a percentage and the corresponding amount of the solution as the amount of alcohol in the solution. He presented the product as a multiplication of two factors only, thus citing the procedure instead of the meaning. Ken included a graphical representation—a picture illustrating the story—only when he was asked to. When asked to estimate the concentration of the mixture of two solutions, the participant obtained a coherent approximation instead of just adding the original concentrations.

Liz

Observation and particular description of Liz’s experience with math education. Liz is a 42-year-old student who has been taking classes at the college for three years now. She was one of the first students to contact me about her willingness to participate in the study. Liz appeared in a very good mood at the beginning of the interview, eager to talk to someone about her life experience in general and about mathematics in particular. She came back to school after being unemployed a number of years. Liz earned a high school diploma and took Algebra and Geometry in school. She
recalled her earliest math experience as, “I remember my fourth grade. We were put into learning groups by your abilities and by how fast you are learning. I remember I was pretty good at math, but I was not the best in math. Our teacher cared about her students. She would take her time and help each person individually. She has been one of my favorite teachers.” Liz’s facial expressions changed slightly and became quite apprehensive when she began talking about her post-arithmetic experiences. When recollecting her middle and high school years, Liz stated, “Math became more time consuming…more demanding. As coming to my high school years, it was like ‘I don’t need this math. I know I am good at it, but I don’t need it’. So, during my senior year I took this general math just to graduate. Friends are meeting; friends are going out. It was more important. Anything which was not connected to doing homework, going to a party [was more important]. Math was just too much thinking, so I just really didn’t want to do it.” When asked about her experience with mathematics and her class now, Liz stated that she didn’t see any connection between mathematics and real life when she was younger, but she does see it now. She also said, “I see when math plays part in it—even when I am driving—probability and distance and stuff. It is what I say to my kids—it is like doing math. You have to see the view, the range. But back then it didn’t add up.” Liz admitted that neither numbers nor variables intimidate her in the math class she is taking now. Liz also added, “I can see where all the terms, expressions, equations are important in solving word problems. This is where I was messing up before. I didn’t study vocabulary terms, I didn’t know what an equation was, and I didn’t know it really meant something. When
it gets to the test, and you don’t know the terminology, you cannot solve a problem. Now it all makes sense. I have had a lot of years in my life without doing anything. And I know I am pretty smart and I have wasted a lot of years. And I don’t think it is too late. And I am trying to be an example for the members of my family. I am the first one to actually go to college. I want to show the younger generation that it is not too late and they can do it. I messed up a lot in my life, doing drugs and stuff, and alcohol, and I am better today.” When asked about solving word problems, Liz replied that she didn’t remember much except that it was difficult. She didn’t share any details about doing mathematics with her family and/or friends.

**General description of Liz’s experience with math education.** The participant recalled that her experience in elementary school math class was mostly positive. She believed that mathematics was not her strongest subject at that time, but she did say that she was “pretty good at it”. Liz also complimented her elementary school math teacher for being a nice person and caring and knowledgeable in her profession. Liz admitted that while she was a high school student her attitude and her level of achievement in the math class completely changed. She attributed the change to her immaturity and peer pressure. Presently, she believes that studying mathematics is a necessary part of any education and regretted not taking it seriously when she was younger. Liz also stated that mathematics is relevant to real life. At the same time, she confessed that she was having difficulties solving the mathematical word problems.
Observation and particular description of Liz’s experience solving motion word problem I. Liz began by reading the text silently, writing notes, and then said, “I don’t know.” She restated the problem by saying, “Ok, the faster plane is 300 miles per hour. The rate of the other plane is 200 miles per hour. After how many hours will the planes will be 1000 miles apart? So, 1000 is gone in one hour. I am thinking of the first car going 300 miles per hour in a first hour. And the slower plane has only gone 200 miles per hour for the second hour. And I need to count how much it is going to take each plane to get into a 1000. I am thinking that it is going to be 200 per hour, so it is going to be five hours for the slower plane to reach a 1000. So, 200 times five is a 1000, right? And the other one is going to be 3.3 hours, right?” At this point, Liz got visibly confused and frustrated not knowing if the problem has been solved correctly. She added, “Then I have to figure out how to [calculate it] before they get a 1000 miles apart. Liz properly defined the given as the rates of two planes, one is faster, and the other one is slower. When asked about the meaning of the term “rate”, she replied, “I don’t know. It is like a price, like an addition problem.” Liz correctly identified units of rate as “miles per hour”. When asked about the attributes of a motion, her reply was, “The rate of a slower plane, time, speed, and stops”. Liz identified the difference between the terms “rate” and “speed” as, “There is a difference because it says one is faster. They are not the same.” When asked if she tried to visualize the problem rather than concentrating on calculations, Liz replied that she visualized two planes. At the end of the process, Liz said, “I need to know how long it would take them to be a 1000 apart. So, we need to
subtract something.” Liz identified the “how long” unknown as time. When asked about a formula connecting the attributes of a motion, she replied, “Rate times time equals distance or something like that.” Liz stated that the problem did have a solution, but admitted that she didn’t know how to find it.

Figure 23. Liz’s solution of motion problem I

General description of Liz’s experience solving motion word problem I.

When solving the problem, Liz visibly became frustrated by her uncertainty of how to solve it. She used the formula D=RT at the very beginning of the solution process without formally identifying it. She just divided 1000 miles by the rate of each plane in order to get the time. Liz failed to recognize the problem as a simultaneous motion of two objects and calculated the time needed by each plane to reach 1000 miles only. Liz properly identified the given information. She also identified time as the unknown. When she was asked about the formula that connects the attributes of a motion later, she sounded very uncertain, saying that the product of rate and time is distance “or
something”. Liz seemed to be able to recall the attributes of a motion and their units after some thinking and hesitation. She mentioned “speed” as one of the attributes of a motion, but saw it as something different from the term “rate”. During the discussion Liz appeared confused and hesitant. She didn’t attempt to use a variable and/or an equation when solving the problem. The participant produced a graphical representation only when she was asked to do so. The correct answer to the problem was not obtained.

**Observation and particular description of Liz’s experience solving motion problem II.** Liz began with reading the problem silently and then restated it as follows:

“There are two cities, and I am going to put them all in two--east and west of each other. They are 315 miles apart. There is a car leaving city number one, traveling toward the second city. And it is traveling 50 miles per hour. At the same time, there is a bus. They left at the same time; one is going to get there sooner than the other because it is going five miles per hour faster toward the first city. So, we have to know how long it will take for them to meet. I am having a problem setting it up, so I did the picture. My bus is the big one. The other is the car.” When asked to show the graphical representation of the distance between the cities, Liz replied, “[It is] in the middle. It is so many miles apart. I needed to know the distance here for each one.” When asked about the units of the value of 315, Liz said, “Fives, tens.” When asked what units would be used to measure the length of a room in a house, Liz replied, “Length times height. I don’t understand what you are asking me. What are the units? Are they numbers?” Liz explained her solution as, “I wanted to know the hours, so I know one goes 55 miles per hour, the other one is 50
miles per hour. I add these two and it is how much time they would have to in one hour—105. [It is] miles per hour that they travel together. It is their total in one hour. And then I added another 55 and 50 the second hour. This would give me another 105. I added it to 105 and came up with 210. And in order for me to get the third one, I added 105 additional. After that I didn’t need to add 105 because I got 315 in one, two, three hours.”

When asked to identify the attributes of a motion, Liz replied, “Speed and rate and time.” She stated that the formula that connects the attributes is “Distance equals rate times time.” Liz also stated that the term “rate” is similar to the term “speed” or “how fast we are going.” The participant stated that she was given enough information to solve the problem and that the problem was relevant to real life. She also added that she drives on a regular basis and she had seen similar problems in her algebra class. When asked if she could solve the problem differently, Liz stated, yes, by using her picture. She attempted to solve the problem using the formula \( D=RT \) and obtained 63 as her answer. When asked how she got it, Liz replied, “Because I set the distance 315 miles apart, right? I got a rate of 55. And I don’t know the time. It is 63.” When asked to explain the reason she divided both sides of the equation by five, Liz replied, “I don’t know. This is what I got.”

After another attempt to solve the problem, Liz got five and point seventy two hundredths as the answer, and after that she got six and three tenths. At this point, she became upset and uncertain. She explained the last answer as “the time needed by one car to meet.” When asked if there are two different answers to the problem, she replied, “Yea, because one is going faster than another one. Car number one is going to take a little bit longer or
more time to get to meet the other one.” She stated again that the answer was “five point seventy-two” for the car and “six point three hours” for the bus. Liz clarified the two solutions as “For this particular problem I like how I did it this way, the way with the drawing [the first one]. Even though I know how to use the formula, but right now it doesn’t look right to me.”

![Figure 24. Liz’s solution of motion II problem](image)

**General description of Liz’s experience solving motion problem II.** The participant was able to restate the problem and identify the given and the unknown without difficulties. She proceeded with solving the problem arithmetically without using algebraic representations and explained her trial and error approach coherently. When asked about the attributes of a motion and their connection, Liz was able to recall distance, rate, and time and the formula connecting them. She also attempted to solve the problem algebraically, but tried to find two separate values for time seeing the model as
the motion of two independent vehicles. She also had difficulties defining and identifying the concept of a unit. The graphical representation of the problem was partially adequate with showing two vehicles moving toward each other but not showing the initial distance between the cities properly. Liz introduced the variable properly, but failed to set up a correct equation.

**Observation and particular description of Liz’s experience solving mixture problem II.** Liz began with reading the problem silently. Shortly after that her facial impression became quite hesitant and embarrassed. She proceeded by saying, “I don’t know if I am doing it right. Can I check the answer?” After I informed her that the answer was not correct, Liz became visibly upset. She restated the problem as “How many ounces, which is something I don’t know, of a fifteen percent solution, so, I took the x, and it represents the unknown ounces of the fifteen percent solution. So, I multiply the fifteen x, point one five, I changed it to a decimal, must be mixed, sounds to me like I have to add, four ounces which is four times point two zero, gives me a seventeen percent solution, which is equal point seventeen.” Since Liz didn’t produce a picture representing the problem, I asked her to do so. Liz drew two partially shaded bottles. She described the first bottle as “fifteen percent alcohol solution” and the second one as “the second mixed bottle of alcohol, but I don’t know how to make it--four ounces of twenty percent alcohol solution. It is going to be a little bit more.” When asked about the number of solutions involved in the problem, Liz responded that there are three solutions. When asked why she only showed two bottles, Liz replied, “Because they already gave
me the third one.” Liz identified the meaning of the product of the variable x and the fifteen hundredth as, “Because of the word ‘of’. There is no other reason.” Liz identified the variable x as “the unknown ounces of fifteen percent alcohol solution.” When asked about the reason for and the meaning of the second product–four times .20, Liz responded, “Same thing, because of the word ‘of’”. Liz identified the meaning of the right side of the equation as “seventeen percent.” When asked to explain the meaning of her equation, Liz replied, “It is saying that after I mix this unknown amount of ounces of a fifteen percent solution with the four ounces of a twenty percent solution, it should make a seventeen percent. I want to know how much should I mix, how many ounces must be mixed. Is it eleven percent? I multiplied fifteen percent by x, or point fifteen by x. Then I distributed four and point twenty and got zero point eight. It is equal to point one seven. Then I tried solving the equation and get x by itself. So, I subtracted point zero eight from both sides. Point fifteen x is equal to point zero nine, divided to get x by itself. I got x equals zero point six.” Liz also added she believed that enough information was given to solve the problem, “but I am not figuring it out. I haven’t figured out how to come up with that one number to get to seventeen percent, because seventeen percent, x is equal to seventeen percent, so I might have to take this x number, replace back here, multiply by the fifteen percent.” When asked to identify the meaning of eighty hundredth, the participant replied, “The percentage of it [is] the solution that I needed to mix two solutions.” Liz also added that the problem was relevant to real life and that she “can see where it is relevant, especially if you are a cook or a hairdresser. Probably in science you
would mix chemicals.” She admitted that she has seen similar problems in her math class and revealed that she experienced difficulties solving them in class as well as now. When asked to describe the contents of twenty ounces of the 50% alcohol solution, Liz replied, “It is going to be twenty ounces, 50% of something.” She identified the meaning of the expression ‘50%’ as “It means point five o. You might have to subtract from twenty ounces, 50% from twenty ounces.” She added, “They [twenty ounces] are mixed with something. Fifty percent of it was mixed with something else. There is ten ounces of something. I don’t know what it is, solution or alcohol.” When asked why there is a difference between these two terms: the solution and alcohol, Liz replied, “Well, in the problem it says ‘alcohol solution’, so yea. There are two solutions in there. One is alcohol, one is something else.” When asked to calculate the amount and estimate the concentration of the mixture of twenty ounces of the 50% solution and thirty ounces of the 70% alcohol solution, Liz responded that the final amount would be “fifty percent, I mean fifty ounces, and the concentration would be 130 ounces. I added them together, the 70 and the 50.”
Figure 25. Liz’ solution of mixture problem II

General description of Liz’s experience solving mixture problem II. Liz seemed to be in a good mood and eager to attempt solving problems at the beginning of the second interview. This participant appeared very task oriented and persistent in obtaining the correct answers. Nevertheless, after she read the mixture problem, Liz became visibly hesitant and upset. Liz restated the mixture problem without difficulty. She also presented the given and the unknown correctly. Liz didn’t attempt to make any graphical representation and was asked to make a picture. She did, but her picture consisted of only two bottles instead of three. When asked why, Liz replied that the third bottle is given. She attempted solving the mixture problems algebraically by using x as representation for the unknown and setting up an equation. The right side of the equation has only the concentration rather than the product of it and the amount of the solution. Consequently, the answer obtained was incorrect. When asked to discuss the equation
and the meaning of each term on the left side, Liz failed to do so. There is no evidence that the participant comprehended the difference between the concepts of alcohol and alcohol solution since when asked about it, she has failed to provide clear answers. When asked about the mixture of two solutions, Liz added the amounts of the initial ingredients and their concentrations in order to obtain the mixture.

Rocky

Observation and particular description of Rocky’s experience with math education. Rocky is an 18-year-old full time student. She graduated from high school last year. The participant took Algebra, Geometry, and Trigonometry back in high school. Rocky appeared to be relaxed and maintained a positive attitude toward the conversation. She kept smiling, even laughing at times, eager to answer the questions inquired. Rocky recollected her earliest math experience as, “I remember elementary school. Well, at that time math was easy. I got it quickly. I also remember it being boring because most of the things they were teaching, I already knew. Especially with addition, subtraction, and stuff like that. I remember having a little problem with multiplication. I remember them [the math teachers] being really nice and patient.” When asked about her post-arithmetic experience with mathematics, Rocky stated that at that point in her life, “No [math did not make sense]. It was only with the groceries, but everything else was not.” She commented on her attitude toward mathematics at that time as, “I was never nervous in class. It was only when I would get an equation wrong, so people would start talking
about it.” Rocky described her family involvement with her math work as follows: “I remember doing time tables, finding areas of squares with my older brother. My mom and my dad did not know how to do anything.” When asked about her attitude toward mathematics now and if math is relevant to real life, Rocky uncertainly replied, “I think it is relevant to different fields like buying a house, renting an apartment. If you are going to sign a contract, you are counting how much money you are going to spend, mortgage, insurance.” When asked if she remembered solving word problems, Rocky said, “Yes [about remembering solving word problems]. Some of them were about how much money somebody would spend on a car or what the interest on the car was, and about buying a house, calculating the mortgage. [They would be solved] with equations. Some of them were x equals d times t.” Her attitude toward solving word problems was described as, “If I really read it, it comes easy. And if I only scan it without paying attention, it comes out wrong.”

**General description of Rocky’s experience with math education.** Rocky did not display any negative emotions towards mathematics at the time of the interview. She was smiling, recalling her school experience with mathematics. Her perception of the elementary and middle school math teachers was very positive. Rocky believed that she was being a good math student during her school years and was quite content in class; nevertheless, mathematics didn’t seem relevant to real life at that point. Rocky stated that her vision of mathematics has changed and she believes now that it [mathematics] is relevant to almost any aspect of everyday life. The participant didn’t express any
particular emotions toward her present math class; at the same time she sounded
confident and content when talking about it. She mentioned remembering solving word
problems using equations and believed that if she would read the problem carefully, she
would be able to solve it. Rocky stated that reading comprehension is the key to solving
word problems.

Observation and particular description of Rocky’s experience solving motion

problem I. When asked to solve the problem, Rocky began reading it and taking to
writing the notes shortly. She did not read the problem aloud; instead she silently kept
writing her calculations. Rocky identified the given as “the rates of the planes and this
thousand.” When Rocky was asked to clarify the latter term, she replied, “How far apart
they are and the distance they are flying each way.” When asked about the unknown, her
reply was, “the hours, time.” Rocky stated that the formula connecting the attributes of
motion is “Distance equals time times rate”. When asked to explain the meaning of the
term “rate”, Rocky replied, “In this problem it is speed or how fast each plane is going.”
Rocky identified the units of speed as “speed, cube, square” and the units of distance as
“kilo something, kilometers maybe”. Rocky explained her approach to solve the problem
as, “We add two rates together and divide it by it by distance.” When I asked her why the
two rates were added, Rocky replied, “Because the formula has only one rate, so I added
them to make it one.” When she was asked to clarify her answer, Rocky replied that, “I
do not know. It just happened”. The obtained answer was 500 hours. In addition, Rocky
stated that when solving the problem she had been thinking about the numbers only rather than visualizing the story.

Figure 26. Rocky’s solution of motion problem I.

**General description of Rocky’s experience solving motion problem I.** Rocky rewrote the problem and underlined key attributes in the word problem. She also identified these without visible difficulties and stated the distance formula as well. She was having problems when asked about units of rate and distance though. Rocky attempted to solve the problem algebraically by setting up and solving an equation. She properly realized that the rates should be added together but failed to explain the reason for such addition. She failed to solve the equation properly afterword. No graphical representations were done.
Observation and particular description of Rocky’s experience solving mixture problem I. Rocky began with silent reading and continued with restating the problem as follows: “The percent of the alcohol solution is given. And it has so many ounces. I look for other ounces. It is fifty percent alcohol solution.” Rocky also stated the problem is relevant to real life and when asked about the situation requiring mixing solutions together, she replied, “It would be science. Alcohol can be mixed with something. I would mix to make a cake, make lemonade, sugar, and water.” When I asked Rocky to explain the meaning of fifty percent alcohol solution, she replied, “Half of it is fifty percent.” Rocky was asked to describe the ingredients of twenty ounces of 50% alcohol solution. She replied, “It is fifty percent alcohol.” When asked about using ounces as a measurement, Rocky replied that, “I only know about cups. It is an amount.” She added that twenty ounces of 50% solution is “the total amount. At least half of it is alcohol.” Rocky identified the rest of the solution as “it is mixed with twenty percent alcohol solution.” Rocky was asked to compare two sugar solutions – 50% and 80%. Her reply, though very uncertain, was, “It does not have the same thing, ounces. 80% would be more sugary taste.” After spending some time talking quietly, Rocky stated with obvious hesitation that there are ten ounces of alcohol in twenty ounces of 50 % alcohol solution. She also added the rest would be “something else”. When the participant was asked to calculate the total amount and approximate the possible concentration of the mixture of twenty ounces of 50 % alcohol solution and ten ounces of 80% alcohol solution, she replied that the amount is 30 ounces and added, “I cannot answer [about the
concentration] because I do not know the ounces.” I then asked her if it is possible to end up with a twenty percent solution, and she said yes. When asked about 130% solution, she replied that it would be impossible, but failed to provide the explanation.

**General description of Rocky’s experience solving mixture problem I.** Rocky restated the problem unclearly and seemed unsure about what to do. She defined 50% concentration solution as half but failed to find an amount of an initial substance if the amount and the concentration of a solution were given. The participant was able to answer the question when asked again shortly after. She had visible difficulty identifying the units of the amount of a solution. Rocky hadn’t made any attempt to solve the problem looking confused and lost at the end. When asked about mixing two solutions together, Rocky properly replied that the concentration over 100% is not appropriate, but failed to explain her answer. She properly determined that the total amount of the mixture would be the sum of the original amounts. Rocky also stated that the mixture problem is relevant to real life and brought up coherent examples of the matter.

**Observation and general description of Rocky’s experience solving motion problem II.** Rocky began the solution process with silent reading, thinking, and then writing her calculations. Shortly, she announced that she was done. Rocky also stated that for her motion problems are more difficult to solve than the other types “because you got to remember what to do if they move in the same direction or not the same direction. If it is opposite, then you add, and if it is the same you subtract.” When asked to explain the connection between the directions of the motion given and the corresponding operations,
she replied, “Because if it is same, you got to subtract, because you have to find the distance in between the two. My teacher has told us that.” The participant used a chart when solving the problem. When asked to explain the chart, Rocky stated, “The chart tells you how to set an equation. The faster [object] is first. Because it is what the teacher said.” She used letter t to represent the unknown and when asked what it stood for, Rocky replied, “It stands for rate.” She also came up with the term “speed” or “how fast something is going” as synonyms for rate. When I asked Rocky why she placed the attributes of a motion in a particular order when doing the chart, she replied, “Because it is how the equation is: R times T equals D.” Rocky identified the units of time as “hours” and the units of rate as “miles per hour”. When asked what the unknown was and what the variable in her equation stood for, Rocky replied, “To take the place of the time, so I know it.” When asked to explain the meaning of the product of 55 and the x, Rocky responded, “It is the speed times the time and then it is added to the 50x which is also rate times time and we get distance. The distance is 315 and 55x represents the rate and the time.” When asked how she solved the equation, Rocky stated, “I combined like terms and divided 105 into 315”. She also was able to check the answer by substitution. Rocky added, though slightly uncertain, that the problem was relevant to real life.
General description of Rocky’s experience solving motion problem II. Rocky appeared to be calm and content again during this interview. She was all smiles and eager to solve the problems. She began solving the problem by stating that she considered this type of problem to be more difficult than the mixture ones. The participant added that it was difficult for her to remember the fact that when two vehicles move in opposite directions, she was supposed to add, and when in the same—to subtract. When asked to explain the reason for such operations, Rocky replied that she was told so by her teacher. She attempted to solve the problem using a chart and when asked why she was using it, she replied again that the teacher said to. No more explanation was given on the matter. Rocky was able to properly identify the given and the unknown. She came up with the proper formula relating distance, rate, and time and their units. The participant used a variable to represent the unknown time and solved the problem algebraically. She was able to solve the equation properly as well. The participant properly identified the right side of her equation as the total distance and the other side as the sum of the distances.
covered by the vehicles given. When asked once more about the reason she added the rates, Rocky replied that she was not sure.

**Observation and particular description Rocky’s experience solving mixture problem II.** Rocky began with reading the problem silently and restated it as, “It is trying to figure out how much of one solution needs to be added to another solution to get the overall solution.” She then attempted to solve the problem using a chart. When asked about the letters s, w, and m used in the chart, the participant replied, “S is the strongest, w is weak, and m is the mixture.” Rocky identified the term “the strongest solution” as “the larger percent.” She then failed to identify the term percent correctly. Rocky also added that starting the chart with the weaker solution would make no difference in terms of the answer since it was an addition. When asked to describe the meaning of her equation, the participant hesitantly replied, “It is that twenty percent times four plus fifteen times x equals seventeen times four plus x.” Rocky properly identified the units of the percentages and the amounts. When asked to describe the meaning of the product on the right side of the equation, Rocky replied, “Because it is what the formula says, it is the amount of the seventeen percent.” When asked to identify the meaning of the product of twenty percent and four ounces, she cautiously replied, “The total amount, amount of the strong percent solution. It is what they use in the mixture. I don’t know how to explain it.” When asked to explain the meaning of the product 15 and the x in her equation, the participant replied, “This x would be how much of the fifteen percent is used in the mixture.” Rocky also stated that she added the two products together so she
“can get the amount of the seventeen percent solution. It is an overall product.” When asked about the units of the product of the twenty hundredth and four, Rocky replied “ounces and percentage” and stated that there are two units. Rocky identified the units of the second product as percent and ounces as well. When asked about the meaning of a percent, she replied, “It is something out of a hundred.” When I asked Rocky to describe the contents of one gallon of 50% orange juice drink, she replied, “Half is orange juice and another half is something else.” When I asked Rocky to approximate the concentration of the mixture of 50% orange drink and 80% orange drink, she said hesitantly, “It will be like 104%.” When asked to clarify the value, Rocky said that it is 140%, not 104%. She admitted that the problem is relevant to real life and that mixing is being used for making “cakes and pies for baking.” Rocky also admitted that now she has quite a positive attitude toward solving word problems and the change in her attitude happened when she realized how to differentiate the types of word problems from each other.

![Figure 28. Rocky’s solution of mixture problem II](image-url)
General description of Rocky’s experience solving mixture problem II.

Rocky appeared to be calm and content again during this interview. She was all smiles and eager to solve the problem. The participant was able to restate it without visible difficulty. When asked to explain the meaning of the concentration of a solution, she replied that she didn’t know. Rocky attempted to solve the problem using an algebraic equation and was able to obtain a correct answer. At the same time, when she was asked to explain the meaning of her equation, she replied that it was a sum of two products. She was not able to explain the meanings of the terms “amount” and “concentration” either. When asked about the product of the amount of a solution by its concentration and the product’s units, Rocky replied that she was not sure. Nevertheless, when asked to identify the contents of a given amount of a 50% solution, Rocky was able to answer correctly. Still she estimated the concentration of the mixture of the 50% solution and 80% solution as a 140% concentration solution thus adding the given values. She admitted that she felt better about solving word problems now than she did before. The reason presented was her ability to recognize the type of problem and then follow the appropriate steps. Rocky didn’t include any graphical representation with her solution of the problems.

Raul

Observation and particular description of Raul’s experience with math education. Raul is a 28-year-old student who holds a high school diploma. Raul took Algebra, Geometry, and Trigonometry in high school. He is a full time student at the
present time. He seemed very relaxed, content, and confident during the interview. When asked about his earliest experience with mathematics, Raul stated, “I remember going through my first grade and doing math, like addition or subtraction, and second grade, doing division and multiplication. I always felt really comfortable with math; unlike other things I am not sure about, with math whenever I am unsure, I just ask a few questions. So, for the most part I kind of get math; it’s probably one of my best subjects”. When I asked Raul about his math experience after arithmetic and elementary school, he stated that he has never felt uncomfortable with the subject, but at the same time he added, “I definitely felt like math was relevant to real life. No [never felt uncomfortable], the only time I got a bad grade in math it was just all my doing with not showing up to class, being immature. But that is the only time I ever felt uncomfortable with math.” When asked about his attitude toward the present math class, Raul replied, “I feel fine now. Math is the only one I am taking in person [not online] though. The thing I like about math is that there is almost only one right answer. There are sometimes multiple right answers, but typically one right answer, and only one formula to get to the right answer.” Raul also stated that the general memory about his middle and high school math teachers was definitely good. When asked if his family members and/or friends were helping him with his math studies. Raul replied, “Yeah, my father was always really good at math. He was an accountant for a long time; it was easy for him to help me with my homework.” Raul was also asked if when young he had done puzzles, math games or some other informal math activities. The reply was: “No. We did those [flashcards] in class and I was
always pretty sharp with math.” When recollecting his experience with word problems, Raul stated, “I don’t know; some people just get intimidated by numbers. I kind of understand how people get intimidated by word problems. If you don’t pay attention to every part of the word problem, it will change the way you put the word problem together. For instance, today in class we were going over one of the word problems and the professor brought up a point that was good to know. He said the word *and* in a word problem doesn’t mean to combine things. It does not necessarily mean plus or minus or anything. And it is like three and a number would be 3x. If you didn’t see it that way, you would put 3 + x and get the wrong answer. They [word problems] are definitely relevant [to real life]. I just gave you the example with football. If you would take a job in statistics, you would need to know that if you were tracking the game, you would need to know how many yards that team made. It takes a really complicated one [word problem] to intimidate me. Usually I am just reading it too fast, or incorporating words the wrong way. If you go over the notes that we take in class, we go over words like ‘difference’, ‘product’, ‘quotient’, etc.”

**General description of Raul’s experience with math education.** Raul seemed very relaxed, content and confident toward his math experience. He admitted not remembering much about his earliest math experience, but at the same time stated that he always has been good at math. Raul acknowledged that he prefers mathematics to other subjects because in math one can ask a question and get the answer. Raul recalled that his father was quite good at math since he [the father] was an accountant. They would do
school work together as necessary, but Raul hasn’t been exposed to any informal math activity. The participant stated that even when he was comparatively young, he was confident that mathematics was relevant to everyday life. At some point, Raul recalled an instance when he was being uncomfortable with mathematics, but explained this as being immature and not making any effort. His attitude toward the present class is quite positive. Raul complimented his professor about doing a good job with teaching word problem solving. When asked if he remember doing word problems before, Raul stated that he did it in Algebra and Trigonometry. He doesn’t feel intimidated by word problems and strongly believes that the key to success is to read them carefully and to know the key words.

**Observation and particular description of Raul’s experience solving motion problem I.** Raul restated the problem as follows: “One plane was going one third faster than the other plane because one was going 300 miles per hour and the other was going 200 miles per hour. The question was at what point are the planes going to be 1000 miles apart?” He added, “I tried to put it in the graph, but that was harder. What I ended up finding out was that I needed to take 300 times two to check my answer, which gave me 600, so that gave me 400. Then 200 times two going that way is 400, and 600 plus 400 would be 1000, and it would take four hours for them to get there.” When asked what was given, Raul replied, “The rate that the planes are going and the final distance that the two planes wanted to be at.” Raul’s explanation of the term “rate” was “rate means basically how fast it is going. In this case it is the measurement of hours.” Raul named
the units of rate as “300 miles per hour. Miles per hour means how many miles you will cover in a given hour. That is how you determine the speed.” When I asked Raul to compare the terms “rate” and “speed”, Raul replied, “I guess they are the same because the only way you would use rate would be in terms of speed. If you are talking about a motion, they would be the same. If you are talking about finances, they are different.” Raul also named the units of speed as “kilometers per hour”. When asked what the unknown was, Raul’s answer was, “The amount of hours it takes to get to that distance.” The participant identified attributes of a motion as “forward or background”. When asked about attributes of motion discussed in the problem, Raul replied, “When something is moving, the speed at which it is moving and how long it takes to get there. [They are] rate, time, speed, direction.” The participant identified the formula connecting the attributes of a motion as “In this case it would be 300 mph, so it would be the speed times the time equals the distance.” When I asked Raul to explain what the arrows on his picture stand for, he answered, “The arrows are the two planes going in two different directions and as they go, this one is going faster than the other, so the hours could have been different to get to 1000, the speeds given.” Raul was also asked if he visualized the story told in the problem or rather focused on calculations. His replied was, “The formula would have helped me a lot and I would have just plugged the numbers in. But if I don’t have the formula, I would try to visualize it first. I think about these two planes because they are going in different directions, but if they were going in the same direction, I would just think about the numbers.” When asked about relevance of this problem to real
life, Raul replied, “Not to my life right now, but it could be if I was actually traveling, or even driving.”

Figure 29. Raul’s solution of motion problem I

**General description of Raul’s experience solving motion problem I.** Raul restated the problem properly and was able to identify the given and the unknown as well. When Raul was asked about the units of rate, he didn’t only identify them properly but in addition stated that it is a distance covered within an hour. Raul recalled two attributes of a motion-- rate and time, but he had difficulty recollecting distance. After Raul was reminded about the distance, he recalled the proper formula for it. Raul attempted to solve the problem arithmetically using the trial and error method. His answer was incorrect given that he modeled the problem as two independent planes flying 1000 miles each instead of 1000 miles being the total distance. The participant did not introduce any variable and/or an equation. The graphical representation consisted of two arrows representing two planes. When asked about visualizing the problem, Raul stated that he
would try to apply a formula first, and if this approach would not work, then he would visualize the problem. By making that statement, Raul admitted that he would prefer routine substitution rather than modeling the problem.

Observation and particular description of Raul’s experience solving mixture problem I. Raul began with silent reading and proceeded with writing notes. He then stated, “I think the answer is 32 ounces but I might be wrong.” Raul restated the problem and explained the way he obtained the answer as, “First I need to know how many ounces equal times amount of ounces. It’s a 50% alcohol solution, must be mixed with the 80 ounces. I definitely had more trouble with this one.” When asked about the meaning of 50% solution, the participant stated, “It’s half. In this case, it was half of 80 ounces, or half of the total. It means that 50% of it is alcohol, and the other 50% is other, or not alcohol.” When asked to describe 40 milligrams of 50% alcohol solution, Raul replied that half the bottle is alcohol. When asked to clarify the answer, Raul added that it is “20 mg and the rest is something else.” When asked to identify the contents of the twenty percent alcohol solution, Raul replied, “It means that twenty of it is alcohol, and 80% is something else.” After that Raul was asked to determine the total amount and estimate the possible concentration of the mixture of ten ounces of 50% solution and fifteen ounces of 80% alcohol solution. He properly responded that the total amount would be 25 ounces, that is, the sum of the original amounts of the ingredients and the concentration would be approximately 65%. I then asked Raul why in his opinion the given problem would cause difficulties to solve. He replied, “I think it was just that it was not clear how
to put together the percentages. The fact that you need to come up with the ounces and it only gives you one instance of 80 ounces. I got more confused with this one.” When doing calculations, the participant had not been using a calculator, so I asked the reason for it. Raul answered, “If you type the wrong numbers into it, it’s going to give you the wrong answer. I trust it [a calculator] but at times it becomes a number that is easily miscalculated, negative 237 + negative 413. It’s easy to forget to carry a one or something.” Raul also stated that the problem is relevant to real life, but failed to come up with the examples. The participant admitted that he works as a bartender at the present time.

![Image](image.png)

**Figure 30.** Raul’s solution of mixture problem I

**General description of Raul’s experience solving mixture problem I.** Raul began with reading the text silently and then taking to writing the notes shortly after. He was not articulate during the process. Raul admitted that this problem [mixture] is more
difficult that the motion one. The participant attempted to solve the given problem arithmetically but failed to obtain correct answer. At the same time, when I asked him to describe the contents of 40 ounces of 50% alcohol solution, Raul properly replied that there was twenty ounces of alcohol in it. When solving the given problem, Raul got confused by the fact that he needed to work with the amounts of two solutions and their concentrations at the same time. He made all his calculations manually saying that using a calculator may lead to a mistake if a wrong number was pressed into it.

Kate

Observation and particular description of Kate’s experience with math education. Kate is a 52-year-old student who holds a Bachelors Degree in Psychology. She has worked as a social worker for a county and now she is seeking a degree in nursing. During the interview Kate appeared to be neither emotional, nor articulate. She was answering the researcher’s questions in a monotone, unanimated voice but at the same time she was being polite and thoughtful. When asked about her earliest math experience, Kate replied, “I have always liked math and I have always excelled at it.” Kate revealed that she liked math all the way through high school. She also complemented her math teachers saying, “They were good [math teachers] because they were very strict”. The problems began when Kate was taking calculus in college. She stated, “I liked it [mathematics]. I did not like calculus only. It was so hard for me, all those integrals that I don’t understand.” When asked about her current math class
experience, Kate replied, “I was nervous at the beginning because I was out of school for a long time, and to know math you have to do it really every day. But I am fine now. And my professor is very good and patient”. When asked about solving word problems, Kate said, “Yes [about remembering solving word problems], and I had a lot of difficulty with them because I did not know.” Kate didn’t disclose any information about her family and/or friends helping with math education.

**General description of Kate’s experience with math education.** Kate seemed to have mostly positive experiences with mathematics education through her elementary, middle, and high school years. Her recollection of the math teachers she has had was positive as well. Kate’s troubles began when she needed to take Calculus in order to earn a Bachelors Degree. She did remember solving word problems and stated that it was always difficult. Kate didn’t provide any information about her family’s involvement with her schooling. Her attitude toward the present math class and her instructor is very positive. Kate admitted feeling content and confident in that math class.

**Observation and particular description of Kate’s experience solving motion problem II.** Kate seemed to be calm and confident when beginning to solve the problem. She announced that she was done shortly after she began reading it. Kate restated the problem as, “There are two vehicles, one is a car, and the other one is a bus going in opposite directions. The car is traveling toward the second city at 50 miles per hour, and the bus is leaving the second city going toward the first one at 55 miles per hour. The distance between them is 315 miles. And the question they want to know is
how long it will take them [the vehicles] to meet. And so what I did I made the chart of rate, time, and distance because it is what I was taught in class.” When asked if she would be able to solve the problem without the chart, Kate replied, “No, it would be very hard for me to do it without the chart. The chart helps me to understand better because it organizes things, and I know from class that if those two vehicles are moving in opposite directions, then you add the two equations together, and I would put them together from the chart.” When asked again why she would add the expression “when the vehicles were moving in opposite directions,” Kate replied, “Because it is the equation. I don’t really know. The formula says that if they are going in two different directions, then you have to add.” Kate also added that she would have subtracted if “they [the vehicles] are moving in the same direction because they overlap; one vehicle overtakes the other one.” When asked about the reason for subtraction, Kate replied, “I don’t know. I just know the formula and know the situation, I just plug it in.” The participant identified the values of 50 and 55 as “how fast the vehicles are going.” When asked if it would make any difference if she would begin the chart with the slower vehicle, Kate replied, “I don’t think it would because you are adding. I don’t know how it work if we would be subtracting.” Kate stated that her variable x stands for “unknown time because they are asking me how long it would take for them to meet. And since I don’t know how long it takes, then I just mark it as x.” When asked about attributes of a motion, Kate replied, “If it is constantly going or stops somewhere, time, what time it leaves, or if it leaves earlier, or later, west, east, the direction, hours, minutes, how long it takes when they are a
certain distance apart, rate.” When asked about the formula that connects rate, time, and
distance, Kate replied, “There is a formula that will help you to get distance when you
multiply rate and time.” She identified units of distance as “miles, kilograms, oh no, not
kilograms, meters, kilometers.” Kate explained the meaning of the product of 55 and the
variable x as, “That is rate times the time. It gives you distance. And we have to solve for
x.” She explained the meaning of 50x as “it is for the slower vehicle. Rate times time is
the distance. And if I add these two together, it gives me the total distance.” Kate
disclosed that when solving the problem she “concentrated on numbers, calculations, and
on the chart to make it easier because otherwise I know rate times time equals distance
and would try to solve for that, but then I would not know if I am supposed to add or
subtract those two equations together or what.”

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<table>
<thead>
<tr>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
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<tbody>
<tr>
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<td>x</td>
<td>55x</td>
</tr>
<tr>
<td>50</td>
<td>x</td>
<td>50x</td>
</tr>
</tbody>
</table>
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\[55x + 50x = 315\]
\[105x = 315\]
\[x = 3\text{hrs}\]

Figure 31. Kate’s solution of motion problem II

**General description of Kate’s experience solving motion problem II.** Kate
appeared calm and content at the beginning of the interview. When asked to solve the
problem, the participant created the chart and wrote an equation. She admitted that she was shown the chart in her algebra class and would have difficulty solving the problem without it. Kate was able to correctly set up and solve an equation. Nevertheless, when asked why she added the terms of the equation (the products of rate and time), Kate replied that she followed the formula shown in class. She was quite open about just plugging in the given values without comprehending the situation. Kate was able to restate the problem, indentify the given and the unknown, and formulate the distance formula without visible difficulties. When answering my questions, she appeared confident and focused.

Observation and particular description of Kate’s experience solving mixture problem II. Kate began with silent reading and proceeded with making a chart. When asked the reason for the chart, Kate replied that she was shown it in her Algebra class. She identified the meaning of the term strong as “strong percentage, chemical. I guess one of them is strongest, so I put that down. One of them is weaker. And then the mixture is here (pointing to the chart).” When asked to clarify her expression “larger,” the participant replied, “A higher percentage of whatever they are asking me. It is a higher percentage of that chemical or whatever.” Kate properly identified the units of the value twenty as percent and the units of the value four as ounces. She identified the meaning of the word “total” in the chart as “The twenty is the percentage amount; the percentage times the amount gives me the total volume whatever the item is. So I multiplied the higher percentage amount by four ounces, and I came out with 80 ounces.” When asked
to identify the meaning of these 80 ounces, Kate replied while rereading the problem, “In this case, it is 80 ounces of alcohol solution.” When asked about the meaning of the product of fifteen percent and the x, Kate replied, “We don’t know how much we need to combine with the 80 ounces in order to get what we need, so I put x for the amount. And we multiply to get the volume, the total amount of alcohol solution that I need.” When asked the reason for adding two products together, Kate responded, “Because they are asking me how much I need to get seventeen percent, so I don’t even know if I did this right. Yes, because it is going to give me the total amount that I need. I am solving for x.” Kate described the meaning of the equation as “If I multiply, if I have a certain item, if I have four ounces of twenty percent alcohol, to get the total amount of that item, I have to multiply the twenty times four. And I know that two of them added together are going to give me the total amount of seventeen percent solution, but since I don’t know how much percent of that second solution I need, then I need to find out what it is, so x is representing the unknown.” Kate described the meaning of the expression “20% alcohol solution” as “how strong this solution is” and then failed to provide more information about it. At the end of the interview, Kate admitted that the problem is relevant to real life and that mixing of ingredients is being done in chemical science.
Figure 32. Kate’s solution of mixture problem II

**General description of Kate’s experience solving mixture problem II.** Kate seemed quite confident during the interview. She identified the given and the unknown without visible difficulties. Nevertheless, when she was asked to identify the meaning of the product of the amount of a solution by its concentration, Kate replied that it would be a total amount and then failed to clarify that expression. She solved the problem by setting up and solving an algebraic equation. At the same time, the participant failed to explain the meaning of her equation in spite of the fact that her equation and its solution were correct. When asked later to calculate the amount and to approximate the
concentration of the mixture of two solutions, Kate incorrectly added both the amounts of
the ingredients and their concentrations.

**Martin**

**Observation and particular description of Martin’s experience with math education.** Martin is 26-year-old student taking classes in order to get a degree in Culinary Arts. He graduated from a high school where he took Algebra and Geometry. Martin served in the US Army for a few years after that. One of his duties was being an air traffic controller. Martin was one of the first students joining the study. He seemed to be very enthusiastic about sharing his experience, open to the conversation, and had a very positive attitude toward learning. Martin recalled his earliest math experience as, “I remember kindergarten with the jellybeans. You would have to do addition like if you have two jellybeans and you have five more, how many jellybeans is that? Yes, [I was good at mathematics], I think so. At least, I was ok with it”. When talking about his post-arithmetic experience, Martin was the only participant who believed that it was entirely his teachers’ fault that he wasn’t successful in his math classes. Martin stated, “When it started to go bad, the math, it was in a middle school in a Prealgebra class because I hated my math teacher. He would not explain anything at all. He just would turn to the board to solve a problem, and then say that it is how you do it and nothing else.” Martin also added, “In high school, I had one teacher who was good. She explained every individual step. In her class I did ok. But then I had a choice of doing geometry, and because I was
in arts at the same time, I took geometry instead of continuing with algebra.” In addition, Martin admitted that algebraic concepts caused additional difficulties as well. He said, “For me it is easier with just numbers. Sometimes variables are harder. Numbers would be easier than variables. The problem with two parts of an equation, sometimes I forget what to do. Like when you have a subtraction, and there is a positive or a negative sign, that is the part I occasionally have troubles with because instead of being an actual number, it is a letter. And in my head a letter cannot be a number”. When asked about his present experience with mathematics, Martin revealed, “Last semester I did a math course that is Prealgebra. That teacher actually was the best math teacher I have ever had. She was phenomenal. She was like my geometry teacher explaining each individual step and why each step should be done this way and why if you do it differently, it is wrong.” He described his attitude toward mathematics now as, “I do not particularly like it or dislike it. It is just there.” When asked if he believed that mathematics is relevant to real life, Martin replied, “Oh, of course [mathematics is relevant to real life]. Examples would be going to a grocery store. I have x dollars, and I have to get everything I can get with only x dollars. And math is going to be more helpful when I start my cooking to do constant conversions between recipes, depending on how many people want something.” When asked if he remembered solving word problems, Martin replied that, “We used to have them all the time in high school, in Prealgebra and Geometry. They were difficult. They are relevant to everyday activities. For example, word problems with graphs are the most
relevant, also when some person goes to a store and checks how much this is and that is.”

There was no evidence of doing any informal math activities with family/friends.

**General description of Martin’s experience with math education.** Martin appeared eager to share his experience and seemed content throughout the interview. He expressed the feeling of being comfortable with math in elementary school until he reached his middle school pre-algebra class. The participant believed that his attitude toward his math education changed because his math teacher simply showed how to complete a problem without explaining all the necessary steps involved. Even now, working with variables caused Martin to experience some difficulties. Martin didn’t recall doing any informal math activities with his family, but remembered helping his father do taxes and doing flashcards with his mother to assist with his school homework. Martin believed that mathematics is relative to real life and sited cooking as an example. He also stated that word problems are relevant to real life but admitted that they were difficult to solve. Martin’s attitude toward mathematics was neither positive nor negative. The participant said that it [mathematics] is just there.

**Observation and particular description of Martin’s experience solving motion problem I.** Martin informed the researcher that he didn’t like using a calculator right before he was asked to solve the first problem. Martin restated the problem as follows: “Two planes are at the same point and going opposite ways; for example, one goes to England, and the other one to California.” After that, the participant made a graphical representation of the problem showing two planes moving in opposite
directions. When asked what was given, Martin responded, “Objects, rates, and how fast they are traveling. Ok, again, two planes, their speed of travel, 300 miles the faster one and 200 miles per hour for the second one.” Martin stated that he identified speed by the units given–miles per hour. He also added, “And there is a distance they are needed to travel–1000 miles.” When asked what we were looking for, Martin replied, “We are looking for how many hours until the planes are 1000 miles apart.” When Martin was asked about the attributes of motion and the connection between them, his reply was, “Connection between speed, distance and time? I do not think so [that the attributes are connected]. I mean distance and speed combined, but separately they are not.” Martin admitted not knowing any synonyms for the term “speed”. Martin identified the unknown as “The time it takes for two planes to be 1000 miles apart.” When asked if enough information is given to solve the problem, he replied, “Yes.” Martin stated that he would not be able to solve the problem differently. When asked if he had used a variable, Martin replied, “Yes, x hours.” He also explained his approach to solving the problem as follows, “There are two planes, so I showed two planes and wrote down their speeds. Since we have to find the hours, so it is x hours. Oh, I forgot a zero here. They are supposed to be 1000 miles apart. I was thinking something but it was not right. That is why I crossed it out. I was thinking that 300 divided by 200, but then I thought it would not come out right. Then I thought of doing how long it would take plane one to be 1000 miles away. This is how I got 3.333. Then I decided to do how long it would take for plane two to be 1000 miles away. I got five hours by division. Then I got 5 divided by
3.333 and got 1.5002 hours.” When asked to elaborate on the division, Martin said, “Because I measured so for the total 1000 miles for each individual plane and then divided both of them because they both are the same distance at the same time, so I divided to get the answer.” Martin admitted that usually he wouldn’t draw pictures, but he did draw one since he was asked to do so. Martin disclosed driving on a regular basis. When asked if the problem was relevant to real life, Martin replied, “Yea, when I was in the army, I was an air traffic controller, so you have to know this stuff, and otherwise they would not let you do the stuff.”

\[
\begin{align*}
2 \text{ mph} & \times 1000 \\
300 & \div 200 \\
& = 1.5002 \\
& = 3.333 \\
\end{align*}
\]

\[
\begin{align*}
3.33 & \text{ hr first plane} \\
5 \text{ hr second plane} \\
1.50 & \text{ sec hours to reach 1000 miles apart}
\end{align*}
\]

\[300 \text{ miles west, 300 miles east, 500 miles south, 1000 miles north} \]

*Figure 33. Martin’s solution of motion problem I*
General description of Martin’s experience solving motion problem I. Martin was able to adequately restate the problem and identify the given and the unknown information. Nevertheless, he became visibly confused when I asked him about the connection between distance, rate, and time. Martin was not able to state the formula \( D=RT \), but still used the formula intuitively when calculating the time needed by each plane to cover 1000 miles. Martin failed to see the fact the two planes are moving concurrently and he obtained two separate time intervals instead. He then obtained the quotient of the hours. No clear explanation of the quotient was given. Martin admitted that he made the graphical representation only because I asked him. Martin shared that he was an air controller while being in the army thus claiming that he has knowledge of the situation. In spite of some confusion when asked about the formula, the participant seemed quite self-confident.

Observation and particular description of Martin’s experience solving mixture problem I. Martin began with silent reading and continued by saying, “Yes, I have seen something like this in a biology course. We had similar problems last semester. So, there is 50% solution and twenty percent. There is \( x \) ounces to make 40%. So, there is twenty over 80, which is two over eight, which is one over four. And then 50% solution is one half, and then to make a 40% alcohol solution. I do not know.” When asked if this problem is relevant to real life, Martin responded, “Yes, it is, but I am not exactly sure how to do it.” When I asked Martin what he attempted to find dividing twenty by 80, he replied, “The amount of alcohol in the ounces, the fraction of the alcohol in the ounces. It
must be mixed with twenty ounces of the alcohol. I was trying to find the alcohol content. It is one quarter alcohol. And then I did the same thing to another solution. 50\% is of course half alcohol. So, by putting together one quarter and one half we have to get 40\%.” After that Martin was asked to determine the total amount and estimate the possible concentration of the mixture of two liters of 20\% solution and five liters of 60\% alcohol solution. He properly responded that the total amount would be seven liters, that is the sum of the original amounts of the ingredients and the concentration would be approximately 80\%, that is the sum of the given percentages.

![Figure 34. Martin’s solution of mixture problem I](image)

**Figure 34.** Martin’s solution of mixture problem I

**General description of Martin’s experience solving mixture problem I.**

Martin was not able to restate the problem adequately. He confused different concentrations and the amounts. Martin attempted to solve the problem arithmetically by dividing the percentages. No clear explanation of the division was given. Martin refused to use a calculator, proudly stating that he would prefer doing calculations mentally. The
participant employed an arithmetic approach to the problem, but failed to obtain the correct answer. No variables, equation(s) were introduced. No pictures, diagrams etc. were used either. Martin admitted though that this problem is relevant to real life.

Observation and particular description of Martin’s experience solving motion problem II. As with the first interview, the participant appeared eager to show his knowledge and looked self-confident. Martin began the solution process with reading the problem silently and proceeded with writing notes. He completed his solution shortly. When asked to explain his approach, Martin replied, “I did my boxes. There are six boxes. There is time times rate equals distance. The top column has the fastest vehicle, which is 55 miles per hour. The time is unknown, so it is x. We multiply across to get 55x. The second one is in the second column, so it is 50. And the x is unknown, so it is 50x. The total distance is 315. Then you add it down and it equals 55x plus 50x equals 315. We combine the xs, and the time is three hours.” Martin disclosed that the chart was shown in class. When asked if he would be able to solve the problem without the chart, Martin replied, “I don’t know. I think it is easier for me because I can see how it all goes.” He also stated that he began the chart with the faster vehicle because “it is how she [the teacher] showed us. The faster is on the top and then the slower”. When asked to explain the reason he multiplied 55 by the unknown, the participant responded, “Because the formula says that rate times time equals distance.” He stated that the units of the value of 55 are miles per hour, and the units of the x are hours. When asked what the units of the product of 55 and the x are, Martin replied, “Hours also, total hours”.
about the meaning of the product of 55 and the x, Martin replied, “55 miles per hour times the time to reach whatever, time maybe, or total distance.” Martin identified the meaning of the product of 50 and x as “total distance.” When asked why he added these two products, Martin replied, “Because the combined distance between two cities was 315, so you have to add to equal 315. You have one car going to one city, and the other car going to the other city, and they meet somewhere in the middle. They leave at the same time, so you add it to get 315 miles.” Per my request Martin made a picture presenting the situation that would require the products of rates and times to be subtracted. He complied and stated, “This car is going here and that car is going here (drawing). Here I would subtract the larger time from the smaller.” Martin added that the problem is relevant to real life and made the following example: “If you have one person going one way and the other person going towards them, and you want to meet somewhere in between, you can figure it out.”

![Figure 35. Martin’s solution of motion problem II](image-url)
General description of Martin’s experience solving motion problem II.

Martin solved the problem using the chart shown in class and obtained a correct answer algebraically. He was able restate the problem and to give clear explanations of the terms of the equation. Nevertheless, sometimes he would confuse the units of rate and time and seemed to have difficulties explaining the meaning of the equation. When asked the reason for particular operations, Martin stated that it was done according to his teacher’s directions. As during the first interview, Martin made the picture only when was asked to do so.

Observation and particular description of Martin’s experience solving mixture problem II. Martin read the problem silently and immediately began writing notes. He set up a chart and stated that the reason for the chart was, “it organizes all the numbers and makes it easier to see the numbers before you write the equation.” He also stated that the chart was shown in his math class. When asked if he would be able to solve the problem without the chart, Martin replied, “No, I do not think so. I might solve the motion problem without the chart, but not the mixture one.” When asked to explain his thinking process, Martin responded, “The first column is the strong mixture or higher concentration, which was twenty percent. And you have to convert to a decimal, so it is .20. And it is total of four ounces, so it goes in the middle; and the total multiply across--.20 times four ounces to get the percentage. The next one you have the weaker mixture. It is .15, total ounces is unknown, so .15x. The mixture is .17 and x plus four because you have four ounces of the original and the unknown smaller one. It goes 17 (x + 4). So, it
becomes twenty times four plus fifteen $x$ equals seventeen $x$ plus 68. You solve it and get six ounces.” When asked to identify the meaning of the product of the twenty percent and four ounces, Martin replied, “[It is] concentration.” When asked to identify the meaning of the product of the fifteen percent and $x$ ounces, the participant replied, “Total amount with the percentage. I do not know how to explain. It equals the total percentage of the concentration—the fifteen percent.” Martin described the meaning of the number twenty as “It is because you have to take it out of the decimal. It is where the twenty came from.” When asked to explain the meaning of the sum of four ounces and the $x$, he replied, “It is four ounces of twenty percent solution known and unknown for the 15%, so the total ounces is $x$ plus four.” When asked to approximate the concentration of the mixture of the 50% orange juice solution and 40% juice solution, Martin replied, “It depends on how many liters, between 42% and 48%.” He also added that the concentration cannot be lower than 40% and higher than 50%.

![Figure 36. Martin’s solution of mixture problem II](image-url)
General description of Martin’s experience solving mixture problem II.

Martin began solving the mixture problem using a chart. He commented that it corresponded to the teacher’s directions in class. He also admitted that he would not be able to solve the problem without it. As before, he would read and solve the problem in silence. It took Martin just a few minutes to obtain the correct answer. Martin was able to set up a proper equation but failed to logically explain the meanings of the terms and the meaning of the equation. At the same time, when he was asked to identify the contents of the 50% alcohol solution, the participant answered the question correctly. Martin didn’t produce any graphical representations.

Nell

Observation and particular description of Nell’s experience with math education. Nell is a 30-year-old student who is a Licensed Practical Nurse. She graduated from a vocational high school and the last math class she took was Business Math. Nell appeared smiling, articulate, and talkative during the interview. She returned to school to get her Associate Degree in Nursing. When asked about her earliest math experience, Nell recalled, “My earliest memory was probably my third grade. I remember I got 98% on a math test. I do not know why I remember that but I remember getting it and my teacher said because I did that I can go be a hall monitor or something. So, I got like a prize for doing well in math.” Nell’s tone of voice changed a bit when she began talking about her post-arithmetic experience. It became quite tense and apprehensive.
Nell stated that her middle school mathematics experience was rather negative. She attributed her lack of success to a few different factors by saying, “And I don’t think it was very hard in elementary school, but then when I got into middle school, I was not paying nearly the same amount of attention to math that I should have been. My mind was on boys or something. I remember taking Prealgebra in probably ninth grade and I did ok in that class. I probably got B; it was not terribly difficult. And then when I got into tenth grade, it was my first Algebra class. I remember I had this teacher who put problems on overhead projector and had very monotone voice. He would turn the lights down in the class every day to use the projector, and I felt lost and confused. He didn’t give us the opportunity to ask questions or maybe I was embarrassed to ask questions. For the first week, I have tried. And after that, I told myself that I am not going to try to do this anymore. The guy I had before him [the teacher] was very engaging. Maybe if I would have a different teacher, I would have done better. From one side I blamed the teacher, but from another side, I did not have the drive.” Nell also admitted that mathematics was “overwhelming” at that time. When talking about her present class, Nell disclosed that she felt quite content. She said, “And where I am now, I have a drive because I have a goal. And so it does not matter what teacher it is or circumstances are. I know that I have to do it. Well, honestly, if I could have not taken math class, I wouldn’t. I am still apprehensive but with better attitude.” Nell also disclosed that her family was neither supportive in her math studies when she was young, nor would help her with math homework. There was no evidence of any informal mathematical activity in Nell’s life.
When asked if mathematics is relevant to everyday life, Nell replied, “Yes [math is relevant to real life]. Especially these word problems about the distance and the time, because my sister lives in North Carolina and she drives slower than I do. So, we have to calculate the time when we should meet without waiting for another person.” She described her experience with solving word problems as follows: “My first reaction [to word problems] is hesitation, because my biggest problem is …ok, I get the information and can picture it in my head, but how to take it from words and put into a problem? It is difficult. And so I wish that I just can look at a problem and take this number and that number and divide this number and that number and get the answer. But I don’t feel there yet.”

**General description of Nell’s experience with math education.** Nell appeared to be affirmative toward her being in college in general and determined to succeed in her math class in particular. Nell disclosed that her elementary school experience with arithmetic was quite positive and she had satisfactory success in that class. Nevertheless, the participant admitted that her attitude changed in the middle and high school algebra environment as the result of poor teaching and the lack of her own interest and effort. She believed that mathematics as well as word problems are related to everyday life experience, but admitted that she associated them with her academic experience at school rather than any other circumstances. Nell’s attitude toward her present math class seemed to be quite positive since, as she admitted, having a goal now of getting a degree made her work hard in class and, consequently, be more successful than before. She also
admitted that, if possible, she would not have taken mathematics, but since it was a requirement, she intended to do her best and have an open mind toward the experience.

**Observation and particular description of Nell’s experience solving motion problem II.** Nell began with reading the problem aloud and immediately started looking confused and uncertain. She said, “Two cities, oh, I don’t like this kind.” Nell explained the reason for such an attitude as, “Because I don’t have my cheat sheet [the note card allowed on the exam]. Two cities are 315 miles apart. A car leaves one of the cities traveling toward the second city at 50 miles per hour. At the same time a bus leaves the second city bound for the first city at 55 miles per hour. How long would it take for them to meet? So, the total travel time is 315 miles. Now I know I need to make an equation, but I don’t know how to set my equation up. I can see it happening in my head, but I don’t know what I need to do to make it work.” Nell identified the attributes of a motion as “miles per hour, how fast we are going, speed or distance, wait a minute; isn’t speed times time equals distance?” When asked to state the formula connecting attributes of motion, Nell said, “Speed times time equals distance.” She also came up with “how fast” as the synonym of “speed” and “miles per hour” being the units of it. Nell identified the units of distance as “it can be anything, can be miles, and can be kilomiles, no, kilometers, feet.” She stated that there was enough information to solve the problem, but admitted that even though she had seen similar problems in class, she still didn’t know how to solve the one given during the interview.
General description of Nell’s experience solving motion problem II. When asked to solve a motion problem, the participant immediately reacted negatively saying that she couldn’t solve it without her notes and did not like this type of problems in general. She was able to restate the problem clearly and made an adequate, though incomplete picture to illustrate it. She also was able to define and identify the attributes of a motion and their units. Nevertheless, she was not able to solve the problem either arithmetically or algebraically.

Observation and particular description of Nell’s experience solving mixture problem II. Nell started with reading the problem aloud and proceeded with restating it as follows: “Basically you are going to mix two solutions. One has so much alcohol. The other has so much alcohol. And together you want them to get 17% alcohol solution.” The participant seemed to be in a positive mood at the beginning of the interview and it didn’t change as the meeting progressed. Nell stated that the problem is relevant to real life and made the following example: “Nursing, yes, how much of something is going to add to how much of something else is going to equal a percent of something else. Well, I
think of Cool-aid for my kids. How much sugar are you going to put in—one cup of sugar or two cups of sugar? And if you mix two different amounts, how much sugar is it going to make? Do I want to do it [solving mixture problems] every day? No, I don’t want to think about it.” When asked if the problem looked familiar, Nell replied, “In class–yes. Outside of class on purpose--no.” She also added that when shopping she would not check the labels of the drinks she buys. Nell then proceeded by saying, “I feel like it was months ago when we did this. I don’t know what to do next. Well, I know you have to add them together. I am thinking I have to add these two together to get the 17% alcohol solution.” When asked to explain the meaning of the expression twenty percent alcohol, Nell replied, “It means that there is twenty percent alcohol, twenty parts alcohol to 80 parts water. And this one means there is fifteen parts alcohol to 85 parts water. And this means your total ounces. This is [x] what we don’t know. We don’t know how many ounces of this is going to equal this.” Nell identified her unknown x as “x is the percent we don’t know of fifteen percent to make the seventeen percent.” Nell added then that the units of this x are ounces. She identified the given as follows: “We know that there is four ounces of twenty percent solution; we know that the total amount we are looking for is seventeen percent solution. And we know that there is fifteen percent of a certain solution. We are looking for the number of ounces of the fifteen percent solution. Now I just don’t know what I have to do with all this information and how to set this problem up.” Nell seemed to give up on the problem, but when I asked her to reconsider, she began working on her notes again. When asked to comment on her equation, the
participant replied, “I know it is going to be 4x. This is going to be x plus 4. X plus four is the number of ounces of the fifteen percent solution; x plus the four ounces of the twenty percent solution.” When asked if she recalled solving percent problems before, Nell answered no. When working on her chart, Nell commented, “I have to change the percent to a decimal. So I figure that you have to add that twenty percent and the fifteen percent together to get a seventeen percent solution. So, I multiplied twenty. I did this one wrong. This should have been fifteen times, no, twenty times four. Twenty times four, two hundred times four, eight hundred. I want to subtract my x, minus one fifty plus twenty x, minus 680 x equals six ounces.” Nell explained the meaning of her equation as “My equation says that I have a fifteen percent solution multiplied by the amount of that fifteen percent solution and added to the twenty percent of four ounces multiplied by the percent of those four ounces. And then together those equal the amount multiplied by the percent of the final solution.” When asked to identify the meaning of the product of twenty percent by four ounces, Nell responded, “Two hundred, well, you have to change it into a decimal and then back because of the fifteen percent. I don’t know how to say it.” When asked why she multiplied the concentration and the amount of the solution and the meaning of the product, Nell responded, “I am not really sure why you multiply them together. I know why they equal that [the right side of the equation] and you have to add them together. [The terms are being added] because you have got one type of the solution added to another type of solution to equal a certain solution.” When asked about the units of the product of the concentration and the amount, she said that it would be ounces.
When asked if she used all information given in the problem, Nell responded uncertainly, “Yes, well, not necessarily.” When asked if there was information that was not used, she said, “Yes, alcohol”. When I then asked Nell to approximate the concentration of the mixture of a 50% and 70% solutions, she replied, “It has to be somewhere in the middle. I would guess 60.”

\begin{equation}
\begin{align*}
150x + 200 \cdot 4 &= 170(x + 4) \\
150x + 800 &= 170x + 680 \\
-150x &= 680 \\
680 &= 680 \\
130 &= \frac{20x}{20} \\
602 &= x
\end{align*}
\end{equation}

\[ \text{Figure 38. Nell’s solution of mixture problem II} \]

**General description of Nell’s experience solving mixture problem II.** Nell admitted that the given problem is relevant to real life; nevertheless, she revealed that she was not interested in thinking about it on an everyday basis. Soon after the first attempt to
solve the problem, Nell announced that she didn’t know how to proceed; however, after some deliberation she made the second attempt. Nell was able to set up and solve the proper equation, but she failed to explain the meaning of the individual terms as well as the meaning of the whole equation. She was confused with the terms such as alcohol solution and alcohol. When asked about the meaning of the product of the concentration of a solution by its amount, Nell responded that she didn’t know. The participant didn’t make any attempts to produce a graphic representation, so I asked her to do so. Nell then produced quite a coherent picture of two solutions being combined into one. She properly identified the given and the unknown quantities as well. Nell also appropriately estimated the concentration of the mixture of two solutions.

**Tammy**

**Observation and particular description of Tammy’s experience with math education.** Tammy is a 29-year-old student. She holds an Associate’s Degree in Culinary Arts. Tammy graduated from a high school where she took Algebra and Geometry. During the interview she appeared to be a quiet, soft spoken individual with a positive attitude toward the conversation. When asked about her earliest math experience, Tammy recalled, “In elementary school, I remember addition, subtraction, multiplication, how to divide and working my way up till I got to the middle school.” The participant then revealed that she has had difficulties in her post-arithmetic years. Tammy supposed that her lack of effort was the reason she was not as successful as she wished. She
admitted, “In a high school, I was in a kind of advanced math. I remember I was not good at it. Going to a math class sometimes makes me feel uncomfortable because I know it is the subject that I am not good at it. Because I don’t think I really focused on it, I don’t believe that I put myself into it. I don’t think I was so bad; I just didn’t try harder and studied more. Instead of realizing it now when I am in college, I should have known it earlier.” Tammy described her present attitude toward math as, “I have got to the point where now if I go into the situation with more positive attitudes that I do better. When I started here, I was in Basic Math, which was last semester. The teacher that I had was awesome. She was the best teacher I have ever had and so having her just opened the door for me. When I got to the next math, it is difficult, but I just know to have an open mind and I can get through this. So, I just have to know that no matter what the obstacles are, whether it is math or something else, I can get through it. I am real comfortable now.” Tammy’s recollection of solving word problems was, “Yes, I remember solving word problems and they were hard. Maybe it is difficult for me to understand the wording.” Tammy also added, “Yes, they are [word problems are relevant to real life]. Some word problems teach you how to do things you would not necessarily think about, like when you go shopping. Once you take a math class, you know how to figure out tax or if something is on sale you know how much you are supposed to pay for it.” She described her math activities at home as, “Not leisure [math activities]; it was math I was supposed to do as far as education.”
General description of Tammy’s experience with mathematics education.

Tammy appeared to be positive about being interviewed. She recalled her placement in advanced math class when being in middle/high school, but did not think that it was justified, since she was not good at math. Tammy attributed her limited math skills to the lack of her effort and concentration. Her attitude toward the math class she is enrolled in this semester is very optimistic. She pointed out that the change of attitude is due to her last semester’s math teacher. Tammy recalled that growing up her math exposure was only about doing her homework. No games, puzzles, or any other informal mathematics experience was part of her upbringing. She recalled solving word problems in middle/high school, but admitted that they were hard for her. Tammy ascribed the cause of her difficulties with word problems to the confusing wording and her inability to figure out what was given and what needed to be determined.

Observation and particular description of Tammy’s experience solving motion problem I. Tammy began solving the problem by silent reading. At the same time, she seemed to start looking uncertain and puzzled. When asked to restate the problem, she said, “Two planes leave the airport at the same time; they are going in opposite directions. One plane is going 300 miles per hour faster; the slower plane is going 200 miles per hour. And it wants to know how many hours will the planes be 1000 miles apart.” Tammy identified the given as “They leave airport at the same time. They are going in two different directions. One plane is faster than another. It is 100 miles faster than the other one.” I asked Tammy what 200 and 300 stand for, and the reply was,
“Miles per hour.” When asked about attributes or characteristics of a motion, she first didn’t reply; and when asked again what can be measured during an object moving, Tammy replied uncertainly, “The steps and how long, time”. Tammy clarified “steps” as “spend or a ruler.” She also said uncertainly that the units of spend are “kilo something, meters or miles.” When asked what attribute of a motion can be measured using miles, the participant seemed very confused and uncertain; it took her a while to respond, “The distance”. When asked what attributes of a motion are represented by 200 and 300, she replied, “They are the distance between each other, how far they are--rates maybe.” She explained the term “rate” as “it is the amount of the faster plane or the amount of the slower plane. I am not exactly sure.” Tammy acknowledged that when she drives, she determines her rate using a speedometer. She also came up with “speed” as synonym for rate. When asked about the unknown, Tammy said, “It wants to know how many hours apart it would be till they are 1000 miles apart. I think we are supposed to convert these. Instead of miles it should be hours.” No clear explanation of the conversion was given. When asked about the formula connecting the attributes of a motion, Tammy replied, “I have to multiply them. I am pretty sure there is [a connection], but I don’t know it.” The participant stated that the problem is relevant to real life and gave the following example: “Because cars on a road go slower than I do sometimes and I am a little bit ahead of them.”
General description of Tammy’s experience solving motion problem I.

Tammy restated the problem without visual difficulties. She properly identified the given and the unknown. Nevertheless, Tammy had visible difficulties recognizing attributes of motion and couldn’t recall the formula that connects them. She attempted to solve the problem by setting up a proportion that consisted of the rates of the planes and the given distance. Tammy didn’t provide any comprehensive explanation of the approach. All the time during the process of solving the problem Tammy looked puzzled. She used x as the unknown but didn’t provide what the variable stands for. Tammy did set up an equation, but it was not the correct one. No graphical representations were done. The participant admitted that these word problems are relevant to real life though.

Observation and particular description of Tammy’s experience solving mixture problem I. Tammy began with reading the problem silently and proceeded by restating it as “They want 50% solution alcohol and you have to do 80 ounces and twenty percent alcohol solution to make 40% alcohol solution. I have no idea.” When asked to
identify “50 % solution,” Tammy responded, “It is half of the solution, half of alcohol solution.” Tammy was not able to identify the other half of the solution. I then asked her to calculate the total amount and to estimate the concentration of the mixture of ten ounces of the 50% solution and five ounces of the 80 % solution. Tammy replied that the total amount would be fifteen ounces and the possible concentration would be 130 %. The participant admitted that she was not sure if the original mixture problem is relevant to real life. I also asked Tammy to calculate an amount of alcohol in twenty ounces of the 50 % alcohol solution, but Tammy failed to provide a logical answer.

![Figure 40. Tammy’s solution of mixture problem I](image)

General description of Tammy’s experience solving mixture problem I. After reading the problem silently, Tammy was not able to restate it correctly. She didn’t identify the given and unknown either. When asked about the meaning of the percent of a solution, Tammy was not able to either explain or indentify percentage and concentration of a solution properly. No graphic representation was provided. No variables or equations were used either. When asked about a mixture of two solutions, the participant added the amounts of ingredients to calculate the total amount and added the given percentages to obtain the concentration of the mixture as well. Tammy admitted that she was not sure if the mixture problem is relevant to real life though.
Observation and particular description of Tammy’s experience solving motion problem II. Tammy appeared to be open toward the conversation but at the same time quite unsure in her mathematical ability to solve word problems. She seemed comfortable during the interview; however her tone of voice very quiet. Tammy began the solution process with reading the problem silently and taking notes. As soon as she read the problem, she began acting exceedingly shy. Tammy restated the motion problem as follows: “It is a car leaving from one city going to the second city, and both cities are 315 miles apart. So, the car is going 50 miles per hour, going toward the city. The bus is coming the opposite way to the city the car is leaving going 55 miles per hour. They want to know how long it will take for them to meet.” Tammy disclosed that she was taught solving this type of word problem in her algebra class and stated uncertainly that the problem is relevant to real life. She made a picture illustrating the story only upon the request. When asked to show where the distance of 315 miles was on the picture, Tammy replied, “The distance, I cannot see it; it is how far apart.” When I then asked Tammy to recall the attributes of motion involved in the problem, she replied, “They are driving, distance, time.” When asked about another attribute, Tammy said, “I don’t know.” When asked to identify the given information, the participant replied, “How many miles apart, how fast they are going, the speed.” When I asked Tammy to recall the formula that connects these attributes, she responded, “Yea [they are connected]. I think there is one, but I don’t know it.” Tammy explained her second attempt to solve the problem as “Yes, I know the distance; I know how fast they are going.” When asked about units of the
attributes of a motion, the participant admitted that she was not familiar with the term units. After my explanation, she hesitantly identified the units of distance as “like a yard, minutes maybe, I don’t know.” Tammy also admitted driving on a regular basis.

![Image of Tammy's solution of motion problem II]

**Figure 41.** Tammy’s solution of motion problem II

**General description of Tammy’s experience solving motion problem II.**

Tammy was able to restate the problem and identify the given. She attempted to set up a proportion to solve the problem. Tammy didn’t provide a clear reason for using the proportion. She uncertainly identified the attributes of a motion and suggested that they are connected, but failed to come up with a formula connecting them. Tammy was quite confused when discussing the units of distance and rate.

**Observation and particular description Tammy’s experience solving mixture problem II.** Tammy began with the silent reading of the problem. Almost immediately
she announced, “I don’t know this one; I don’t know.” When I then asked her to restate the problem, Tammy replied, “They want to know how many ounces of the alcohol solution must be mixed with four ounces of another alcohol solution to get seventeen percent of the solution.” Tammy then disclosed that she did similar problems in class and described her attitude toward solving these problems as “I don’t like them. These problems always were giving me a problem.” Since the participant made no attempt to draw a picture, I asked her to do so. Tammy described her picture as “How many ounces of a fifteen percent alcohol solution must be mixed with four ounces of a twenty percent alcohol solution to make up a seventeen percent alcohol solution.” When asked about the number of solutions being mixed together, the participant answered hesitantly, “Three, two, three, no, we get one.” Tammy presented the way she approached the problem as “I took the four ounces that you need of the twenty percent solution and four point two zero and then what we don’t know is how much we need to make the seventeen percent solution, right? So, I put x for the seventeen percent solution on the bottom.” Since the statement made by Tammy consisted of two equal ratios, I asked her to identify the statement. Tammy replied, “I don’t remember. I want to say it, but I don’t know if it is right; it is a linear equation.” I then told Tammy that it would be a proportion and also asked her why she used the proportion. Tammy replied, “Because there is something missing and we are trying to find it.” When asked if all the given information was used, Tammy replied, “No, I have not. I left the fifteen out. It is needed because you have to mix this one [fifteen percent] with four ounces of that one [twenty percent] to get that
one.” After working on the problem for a while, Tammy stated, “No, I cannot figure this out.” When asked to explain the term concentration, she failed to provide a logical answer. When asked to identify the contents of twenty ounces of 50% alcohol solution, she replied, “It means 50% alcohol in there, in the solution.” Tammy then identified the rest of the solution as “It is twenty ounces, 50% of alcohol.” When asked again about the rest of the solution, Tammy replied that it is water. When asked to calculate 50% of twenty ounces, Tammy replied indecisively, “I don’t know. You would multiply I think. Point fifty times twenty, ten.” Tammy also identified 50% as half of a hundred. Tammy also approximated the concentration and calculated the amount of the mixture of twenty ounces of 50% alcohol solution and thirty ounces of 70% alcohol solution as 50 ounces and 120% which is the sum of 50% and 70%. She identified the meaning of the ratio of four and twenty hundredth in her solution as “I thought that I would divide but it is multiplication because I was not thinking like that. I was thinking about a proportion to make it easy to solve, to find out how much solution you would need to make the seventeen percent.” When asked again to try to solve the problem, Tammy responded, “I just don’t know. You can probably use multiplication to solve it, but I don’t know how.”
Figure 42. Tammy’s solution of mixture problem II

General description of Tammy’s experience solving mixture problem II.

Shortly after Tammy read the problem, her facial expression became quite upset and she announced that she didn’t remember how to solve the problem. When asked to restate the problem, Tammy complied with visible difficulty and her statement was not quite logical. She proceeded with stating that we are looking for the amount of the fifteen percent solution to make a seventeen percent solution. Tammy attempted to set up an equation that was a proportion. When asked to explain the reason for using the proportion, she replied that we use it [the proportion] since there is an unknown quantity and it [the proportion] would be easy to use.
Sarah

Observation and particular description of Sarah’s experience with math education. Sarah is a 19-year-old student who recently graduated from high school. She has not had a work experience and began taking college courses right after high school graduation. Sarah emerged rather shy at the beginning of the interview but began feeling more confident as the interview progressed. When asked what she recalled about her earliest math experience, Sarah replied, “Probably at school, probably in the first grade or something. I remember teacher trying to teach us about tens or twenties. When I first started math when I was a little kid, I did not develop an attitude, but when I got older and saw a bunch of numbers I would like blank out.” Sarah viewed the struggling with algebraic concepts being the reason she was having difficulties in algebra classes. She stated, “Yea [I would get nervous], when it would be more complex; I really did not like math. Like in high school a combination of numbers and variables would make me nervous. When I was in eighth grade, there were teachers who were very challenging, so I would not go to their classes; I went to another class, so it was slower.” When asked about her attitude toward the present math class. Sarah replied, “Now it is [the Beginning Algebra] pretty easy so far. Now it is fine.” She recalled dealing with word problems as, “I remember in elementary school some people would look at them [word problems] and be afraid. But me… they do not intimidate me too much. I just look at the numbers and write down the equation. Sometimes when the equation is hard, then it is hard for me.” Sarah also recalled that her father would help her with the math homework when it was
needed. She didn’t recall doing any informal mathematical activity when she was younger.

**General description of Sarah’s experience with mathematics education.**
Sarah recalled having a neutral attitude toward mathematics in elementary school, but later, during her middle and high school years, mathematics would make her feel uncomfortable. Sarah believed that the reason for the change in her attitude was the difficulty she was having dealing with algebraic concepts in general and variables in particular. Sarah also stated that she feels quite content and confident in her present algebra class since this material is quite easy for her. The participant acknowledged that solving word problems doesn’t intimidate her unless the problem and its equation are challenging. She also recalled that her father was the one helping her with school work if she needed it. Sarah appeared quite content and confident throughout the interview.

**Observation and particular description of Sarah’s experience solving motion problem I.** Sarah began with reading the problem silently. After that, she started writing her thoughts. She announced shortly, “Ok, I finished. I think it depends on the velocity. It says they are flying in opposite directions, so, it depends on the velocity.” When asked what was given, Sarah replied, “Velocity, yes, it says that the rate is 300. Should I tell you the answer? I think that after ten hours.” Sarah explained her thinking process as follows: “First, it was a little confusing, so I was not sure how to go about solving it. I put down plane one 300, and plane two 200.” When I asked Sarah to explain what “300” stands for, she replied, “It is miles per hour.” When asked about characteristics of a
motion, she replied, “The direction, speed, time, rate, distance, acceleration.” She identified “speed” and “rate” as “two different things. Rate is distance multiplied by time? I forgot.” She also added, “Speed is like a constant, constant speed. I forgot. I took physics in 12th grade.” When asked about the unknown, Sarah stated, “We are looking for when the distance between them is 1000.” Sarah identified the connection between distance, rate, and time as “Yes, there is [connection], because distance times time, no, distance divided by time, right, distance divided by time equals rate.” When I asked Sarah why she multiplied 300 and three, she replied, “It is 900…I was trying to get 1000… to see how long would it take.” When asked why she multiplied 300 by four, Sarah replied, “I was trying to see when they will be 1000 apart. I was trying to figure out how to write an equation, but did not know how to. So, I was just playing around.” When asked why she multiplied each number by ten, Sarah replied, “1000 miles, so how many hours is there before they reach 1000 miles, so I multiplied by 10. Then each would have traveled 3000 miles and 2000 miles.” She also added that 1000 is the difference between 3000 and 2000, so the answer is ten hours.” When asked if she believed it is the right answer, Sarah said, “I am not sure because of the whole physics aspect of this. I was only looking at the numbers.” I asked Sarah if when solving word problems, she would visualize the story told and/or concentrate on numbers and calculations. Sarah replied, “No, [I was] only thinking about numbers given.” When asked about relevancy of the problem to real life, she replied, “It sounds like there are lots of internal factors that need to be monitored, like temperature, to make sure it is exactly accurate.” Sarah revealed
that she has flown in an airplane a number of times, but added that when flying she would not think about mathematics.

**Figure 43.** Sarah’s solution of motion problem I

**General description of Sarah’s experience solving motion problem I.** Sarah was able to restate the problem and to properly identify the given and the unknown. Then she proceeded with a “trial and error” technique using ten as a testing number. She determined by multiplying the rate of each plane by ten that one plane would fly 2000 miles away and the other plane would fly 3000 miles. Sarah then calculated that the difference between these two numbers is 1000, that is, the given distance. Her conclusion was that the answer is ten hours. She used the distance formula when she multiplied the rate of each plane by time [ten hours] and later stated the correct formula as well. When
solving the problem, Sarah failed to recognize the fact that the total distance between the planes is the sum of the individual distances, not the difference. Her explanation of why she subtracted the distances was quite vague. The participant used neither a variable nor an equation. Sarah’s comment about the relevancy of the problem to real life was not clear. Sarah did not use a calculator.

**Observation and particular description of Sarah’s experience solving mixture problem I.** After reading the problem aloud, Sarah asked, “Is it supposed to make sense?” I then asked Sarah when in her opinion people would need/want to mix two solutions in real life. She responded, “Nurses need to do this all the time. I just don’t know if there is enough information [is given to solve the problem].” When asked about the number of solutions mentioned in the problem, Sarah said that it would be three. She also indicated that two of them would be mixed together in order to obtain the third one. When asked about the total amount of the mixture consisting of three liters of one solution and five liters of another one, she replied that would be eight, the sum of five and three. When she was asked to approximate the possible concentration of the solution comprised of 50 % alcohol solution and 80 % alcohol solution, she replied, “130%.” The participant identified percent as “one out of 100” and “50% alcohol solution” as “half of it is alcohol.” When asked about the nature of the rest of the solution, she said, “The rest is probably water.” Sarah identified the twenty percent alcohol solution as “80 % of water.” When Sarah attempted to solve the given problem again, she introduced the variable $x$ as “ounces” and added that she was looking for “how many ounces of 50 %
solution have to be mixed with 50 times x must be plus 80 times point twenty equals point forty.” Sarah presented her approach as follows: “80 times point twenty equals sixteen. Point fifty times x plus sixteen equals point forty. Then you do minus sixteen and minus zero six five. [It is] fifteen point sixty. I am not sure if I am doing this right. [it is] 50 x plus sixteen; then divide by point fifty. The answer is about 41. I am not sure it makes sense. [The answer is] 41 ounces.” When asked why it might not make sense, Sarah replied, “Because 41 over 80 equals 40 % alcohol solution. Does it equal 40 over 100?”

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\frac{x}{50} + \frac{80}{80} = \frac{x}{40} \quad \frac{41}{80} = \frac{410}{100}
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.50x + 80 \cdot .20 = .40
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.50x + 16 = .40
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percent solution”, she presented coherent answers showing her grasp of the concept of concentration. Sarah properly identified the amount of the mixture of two solutions as the sum of the ingredients’ amounts, but at the same time added the given concentrations in order to determine the concentration of the mixture. When Sarah decided to try solving the problem again, she properly represented the variable, x, as the amount of the 50% solution and attempted to set up an equation. Although her equation was not correct (the amount of the final solution was missing), it did consist of the proper terms representing the products of the given concentrations and the corresponding amounts. Sarah obtained the answer to her equation and intuitively decided that her answer didn’t make sense. No coherent explanation of the opinion was given. No pictures, diagrams, etc. were used.

Observation and particular description of Sarah’s experience solving motion problem II. Sarah began with silent reading and proceeded by drawing a chart and writing the notes. She stated that the reason for using the chart was “because of the class” and asked if her solution was correct. Sarah also acknowledged that 55 miles per hour represented the rate of the car and the x was used to represent the unknown time. When asked about the meaning of the product of 55 and the x, Sarah replied, “The rate times the time for both of them [equals] the distance [in] miles per hour.” When asked to explain the meaning of the product of 50 and the variable x, she said, “It is the rate times the time [equals] distance of the bus.” When asked why she added the products, Sarah replied, “Because of the opposite direction. If they are going in the same direction, then you subtract. If they are going in opposite directions, it means they are equal, you add.” Sarah
described the picture she made as follows: “They [the vehicles] meet; they are in the same place. They are coming from different directions. It is the distance they took together. But we don’t know the time to get there. The cities are 315 miles apart, so after the x hours they are going to meet.” Sarah also identified the units of the value 315 as “miles” and the units of the products of 55x and 50x as “it is hours.”

**Figure 45.** Sarah’s solution of motion problem II

**General description of Sarah’s experience solving motion problem II.** During the second interview, Sarah seemed as pleasant and content as during the first one. She even appeared enjoying the challenge of solving word problems. When asked to solve the motion problem, Sarah got the correct answer within a few minutes. She solved the problem algebraically without using a calculator for arithmetic operations. Sarah used a
chart shown in class. When asked to explain the meanings of the terms of the equation, Sarah appeared hesitant, and first replied that it is a product. Only at some point later, Sarah uncertainly identified one of the products as the distance covered by the bus. The picture produced showed two vehicles coming toward each other. When asked to connect the picture and the products of rate and time of the vehicles, Sarah failed to produce a clear answer. She also got confused when asked about the units of the products. Her answer was that it is hours.

Observation and particular description of Sarah’s experience solving mixture problem II. Sarah began the solution process with silent reading and shortly after she announced that she solved the problem. The participant used a chart, and when asked why she did it this way, Sarah replied, “It is how we were taught in class.” When asked if she could do it without the chart, Sarah replied, “Technically, I could. I could just make my own thing. Like 20 times four, but it is just easier.” When I asked Sarah about the meaning of the product of twenty percent and four ounces, she replied, “It is 80, twenty percent, twenty times four ounces, the total of the stronger solution.” When asked to clarify the expression “the total of” Sarah added, “The total of alcohol solution, like the alcohol with water, the total solution.” She identified the meaning of the product of 15 and the variable x as “the 15 is the alcohol part and the x, we are not sure.” When I then asked Sarah to explain the meaning of the variable, she replied, “X is the alcohol. We don’t know how many ounces of alcohol.” When asked about the meaning of the sum of four and the x on the right side of her equation, Sarah replied, “[It is] the total amount
of alcohol.” When asked about the units of the quantity, she got confused and asked me to clarify the term units. After additional explanation, Sarah said, “Two and ten.” In addition, Sarah identified the units of the unknown quantity x as well as the sum of the x and four as ounces. Sarah identified the value of seventeen as “the percentage of the mixture, the amount of alcohol per 100 units.” When asked about the strong and the weak solutions, she replied, “It is a higher percentage, stronger mixture of alcohol.” When asked to approximate the concentration of the mixture of twenty percent juice drink and 40% juice drink, Sarah replied it would be the sum of twenty and 40 that is 60%. When asked to explain the meaning of her equation, Sarah responded, “It says that the strong mixture of alcohol solution plus the weak mixture of alcohol solution equals the mixture.” When asked again about the meaning of the products of fifteen percent and x and the twenty percent and four ounces, she replied, “The strong and the weak, the percentage of alcohol.”

![Figure 46. Sarah’s solution of mixture problem II](image)
General description of Sarah’s experience solving mixture problem II. When asked to solve the mixture problem, Sarah came up with the solution after a few minutes of starting reading the problem. Her approach was setting up and solving an equation. The equation was set up properly and the answer she obtained was correct. Nevertheless, when Sarah was asked to explain the meaning of the product of an amount of a solution and its concentration (four ounces and twenty percent), she replied that it was a total amount of the solution. At this point in the interview, Sarah seemed to become hesitant and apprehensive. Sarah set up a chart and admitted that it was how she was taught in class. She was able to properly identify the given and the unknown information in the problem. When asked why she added the x and four ounces, Sarah identified the sum as the total amount of the alcohol. Her answers pointed out to the participant’s misunderstanding of the concepts of alcohol, concentration of a solution, and the alcohol solution. In addition, the participant was not able to identify the units of the amount properly; the term ‘units’ was perplexing to her. At the same time, when she was asked to define a concentration of the solution, she replied correctly. When Sarah was asked to approximate the concentration of the mixture of two solutions, she incorrectly added the two concentration values. The participant didn’t produce any graphical representations.

Sal

Observation and particular description of Sal’s experience with math education. Sal is a 46-year-old student who is returning to school after a number of
years working in fast-food restaurants and machine shops. He is seeking an Information Technology degree. Sal seemed calm at the beginning of the interview and eager to talk. His appearance was rather disorganized but at the same time he looked prepared and immediately put paper, pencil, and a calculator on the desk looking ready to work. Sal sounded quite enthusiastic about getting a degree in IT but worried about his capability to handle the math requirements. When asked about his earliest experience with math education, Sal replied, “I remember that when I was at the grade school level, math was one of my best subjects.” The participant then disclosed that the situation with the math success changed to the worse after elementary school. Sal attributed his difficulties with mathematics to frequent school change and complexity of algebraic concepts. He stated, “Then [after elementary school] I had problems with algebra. My father was in the military, so I switched schools a lot, so it always screwed up my classes. Then I came up here for ninth grade and finished my classes in California. This moving around really screwed my classes, even math. Once I got into Geometry, it started to get really hard for me and then some of the word problems like Pythagorean Theorem were very difficult. I just remember when given an example that x times y equals c for a reason, I need hard numbers, so it is when I started failing because I was not getting the equations.” When asked about his present attitude toward mathematics and the class, Sal replied, “Now it is very important that I do understand everything. I feel that when I don’t understand I have to ask questions. And if I ask a question and the professor is willing to answer it, it is ok. I do not feel nervous in class.” When I asked Sal if he remembers solving word problems,
Sal replied, “Yes, and they were a little bit more difficult.” Sal acknowledged mathematics and word problems to be relevant to everyday life and made the following examples: “Adding, subtraction, figuring out percentages, how much you are paying for your house, percentage you have to put aside to pay your taxes, loans and stuff.” The participant didn’t disclose any information about his family’s involvement with his schooling.

**General description of Sal’s experience with mathematics education.** Sal recalled his elementary (grade) school years as his earliest math experience. He stated that mathematics was his best subject at that point. As time went on, his family was obligated to move a lot since his father was in the military. This was one of the reasons that Sal began having difficulties in post-elementary mathematics at school. He also stated that algebraic concepts involving variables and geometry were difficult for him to grasp. Sal remembered solving word problems before and admitted that it was a difficult process for him. As of right now, Sal admitted feeling comfortable in his math class, and sounded excited to understand the material and is determined to obtain the degree.

**Observation and particular description of Sal’s experience solving motion problem I.** Sal began by reading the problem silently. He reported shortly, “Ok, the fast plane will take three point thirty three hours to arrive at the location and the slower plane will reach 1000 miles in five hours.” When asked to restate the problem and to identify the given and the unknown, Sal replied, “What is given is the miles. There are two planes. One can fly 300 miles per hour and the slower one can fly 200 miles per hour.
The top miles [are given]. They both have to be equal 1000 miles. We are looking for how long it would take each plane to reach 1000 miles. I think what you asked me before was how many hours it will take for two planes to equal 1000 miles.” He identified 1000 miles as “that would be the end goal” and added, “Ok, now I looked at the problem in a different way. They are flying in opposite directions, so you are trying to find out between the one going faster and the one going slower how much time it will allow before they reach 1000 miles apart, not from both of them from zero point.” He presented the thinking process as, “It is 1000 miles apart. And one plane is traveling 200; one plane is traveling 300. That is 500 an hour each and then I believe now the answer is two hours.” When I asked Sal to explain how the answer was obtained, he replied, “Once I read the problem correctly and then I saw that one plane travels at 300 miles in an hour, so they both are still hourly. And the slower plane travels at 200, so first I added 300 to the 200 to get 500.” When asked why he added the two rates, Sal responded, “Because they are going in two different directions and you are trying to find the end.” Sal added that the planes will be 1000 miles away from each other in two hours. When asked to identify “1000 miles”, Sal replied that it is “a linear number, a line graph”. Sal identified attributes of motion as “How fast it is going, rate or speed, angle of descent, time and miles”. Sal described “the miles” as “I am only thinking of the line graph where there is a point in the middle and the one goes this way and the other in the opposite way. The ending result is 1000 miles.” When asked about the formula he used to calculate the answer, Sal replied, “One plane travels 300 miles in this direction in one hour. The other
plane is slower, traveling at 200 miles the opposite way. The 1000 miles was the end result, so then I added the 300 to the 200 going the opposite way. I did it because my first calculation gave me a solid number. I did division because I want to know how many times 500 goes into 1000, which is my final answer.” Sal was asked again about the division and his reply was, “I am not sure. Division sounded the way it would be because first you added the fast plane and the slower plane. We are starting at zero, so one plane went one way and the other plane went the other way. We are finding how many miles are between the both of them. When I did it wrong, I was calculating each plane. Then I realized that one plane was 300 and it moved 600 and if I add the other one 200 and 200. It came up 500 and 500. Once you know that, you can divide 500 into the 1000. That is the answer.” When asked if he remembered solving similar problems before, Sal replied that he was not sure. He admitted that the problem is relevant to real life and presented the following explanation: “Well, we watch the gas in our car. So, we see how many miles we get on the tank of gas and I was keeping records of it. When I took geometry back in high school, we did triangles and stuff and how hard you would hit the ball so it bounces and stuff.”
General description of Sal’s experience solving word problem I. Sal obtained the answer to the problem very shortly and without hesitation. He first viewed the motion of two planes as a motion of two independent vehicles rather than simultaneous as stated in the problem. Later on Sal reread the problem and was able to obtain the correct answer. Sal solved the motion problem arithmetically. No variable and/or equation were used. In spite of the fact that he obtained the correct answer by dividing distance by rate, Sal was not able to state the formula connecting the three attributes of a motion.

Observation and particular description of Sal’s experience solving mixture problem I. Sal began with reading the problem aloud. He said, “How many ounces of 50% solution must be mixed with 80 ounces of twenty percent solution. I am not sure I am doing this right. I am guessing 200 ounces, but I am not sure with this one.” When asked to rephrase the problem, Sal replied, “How many ounces, ounces are the solution, 50% alcohol, and I am thinking of that like a fraction which would be 50 over a 100, and
then we have a given of 80 ounces twenty percent alcohol. So, this is a fraction twenty
over 100 alcohol solution, but I don’t know if I got 40 % as a solution too.” When asked
about the meaning of the expression “50% alcohol solution,” Sal replied, “It means half.”
When I asked Sal to elaborate on the answer, Sal responded that half would be alcohol
and “it doesn’t say what the rest is, a lesser solution of alcohol maybe.” When asked to
identify the components of twenty ounces of the 50% alcohol solution, Sal replied, “It has
50 % alcohol and another unknown solution.” He also added that there are ten ounces of
alcohol in it. Sal’s examples of real life application of different mixtures were,
“Household, cleaning, when you are mixing drinks at a bar, may be quarts of oil for your
car.” When I then asked Sal to calculate the total amount and to estimate the
concentration of the mixture of twenty percent of 50% alcohol and ten ounces of 30%
alcohol solution, he replied that it would be 30 ounces of 80 % solution since both
answers are the sums of the initial ingredients. When asked to explain his solution of the
original mixture problem, Sal replied, “I was thinking that back in high school the teacher
drilled that all goes below the line, so I always have it as a fraction because here it says
‘how many ounces of 50 %’, so I would put it below the line.” Sal identified the
numerator of the fraction as “It is 200. I was using it for how many ounces”. When asked
for the meaning of the quotient of 200 and 50, Sal replied, “That would be forty ounces
over here.” He described the second fraction as “That stands for 80 ounces of another
fluid that is twenty percent alcohol. So, then I am supposed to cross-multiply and then
divide. You don’t have this answer, so, you multiply these two together and then you divide the twenty.”

![Figure 48. Sal’s solution of the mixture problem I.](image)

**General description of Sal’s experience solving the mixture problem I.** When asked to solve the problem, Sal got visibly confused and kept rereading the problem. The solution did not come as easily to him as with the motion problem. He admitted that he was not sure how to solve it. Sal didn’t rephrase the given problem correctly and was not able to determine the given and the unknown coherently. When asked to identify the components of the 50% alcohol solution, he didn’t answer properly and confused the terms “alcohol” and “alcohol solution.” Sal attempted to solve the problem using a proportion but was not able to explain the thinking process. In addition, when asked to combine two solutions, the participant mistakenly added both and the amounts and the concentration percent.

**Observation and particular description of Sal’s experience solving motion problem II.** Sal began by reading the problem aloud. He proceeded by writing notes and restating the problem as follows: “Two cities, 315 miles. Car leaves one of the cities
traveling toward another city at 50 miles per hour. At the same time a bus left the second city traveling at 55 miles an hour. How long will it take, ok, 50 miles? Here it is. I probably didn’t do it right, but after three hours they will meet. Do you want the distance? The bus is going this way 55 and after three hours it will be 165. That one is going 50 miles that way, and after three hours it will be 150 miles. When I take three hours both of them, it comes out total of 315.” After obtaining the correct answer, Sal stated, “But there’s got to be an easier way to do it than how I did it. I am going to talk to you about it after the interview.” Sal then added that he used all the information given and that the problem was relevant to real life. When asked to recall the attributes of a motion that were portrayed in the problem, Sal replied, “Rate times time times distance.” Sal proceeded by writing it down and repeating to himself, “Rate times time times distance”. When asked to identify the meaning of the product of all three characteristics, Sal replied, “Speed”. When asked to identify the units of rate, Sal responded, “How fast we are going.” When asked if he understood the term units, Sal replied, “Yes, per hour.” He identified units of time as hours and units of distance as miles. When asked if he could solve the problem differently, Sal responded, “Actually I am thinking that if you have rate times time it will equal distance, not times distance. I just did it a different way up here by drawing a little diagram and I have my total distance and how fast each car is going in opposite directions.” Sal also admitted solving similar problems in class and recalled that in class he was shown an algebraic approach to solve the problem. Sal also disclosed that he forgot it as soon as he left the classroom.
General description of Sal’s experience solving motion problem II. Sal was able to restate the problem and to indentify the given and the unknown without visible difficulties. He then solved the problem arithmetically using the picture he made. The participant admitted that he believed there is an easier way to solve the problem, but stated that he didn’t know how to. Sal appeared quite content during most of the interview. However, he started looking a bit upset admitting that he forgot an algebraic approach to the solution. Sal made a coherent but incomplete picture illustrating the situation. When asked about the attributes of motion, he presented the distance formula as a product of rate, time, and distance at first. Sal was able to provide the correct formula at some point later in the interview. There is no apparent evidence that the participant understood the meaning of the term units since his answers about it were not clear.

Figure 49. Sal’s solution of notion problem II
Observation and particular description of Sal’s solution of mixture problem

II. Sal began with reading the problem silently. He proceeded with restating it as “How many ounces of fifteen percent solution must be mixed with four ounces. So, I have two sets of fifteen, twenty, seventeen, total, four ounces equals. Now, I am not getting this one. I have the numbers but I am not sure how to put them into a formula.” When I then asked Sal if the problem was relevant to real life, Sal replied, “Sure, it can be with old lawnmower. You have to put gas and mix oil with it. Cars have the same thing. Also, when you have to bake a cake, you have to know how much oil to put in, how many eggs.” Sal identified the contents of twenty ounces of 50% alcohol solution as 50% alcohol. When asked to further elaborate on the question, Sal added, “Half is alcohol, twenty ounces of alcohol solution. The other half is water or something else. Half of twenty ounces is ten ounces.” He also added that ten ounces would be water and ten ounces would be the alcohol. When I then asked Sal if these ten ounces would be pure alcohol or the alcohol solution, Sal replied, “Alcohol solution because of the water mixing with it.” When asked to approximate the concentration of the mixture of a 50% and 70% alcohol solutions, Sal replied, “Let me say 120.” When asked if a similar problem was discussed in class, Sal stated, “Probably, but I forgot it.” The participant didn’t attempt to produce a graphical representation, so I asked him to make a picture illustrating the problem. While doing the picture, Sal stated, “I only got one number I really know. I set up my little jars with twenty percent. This one I like because this one gives me all the information. It says four ounces and four ounces equals twenty percent
alcohol. I got four ounces; it is twenty percent alcohol, and it means the rest is 80% water. So, four ounces, this is almost like a triangle problem. Because it is just like this (drawing a triangle)--80 percent up here, twenty percent over here, and four ounces are over there. It is all equals 100. Well, I have to find out how many ounces over here. I got the base alcohol is fifteen percent, so I can figure out this is 85% water. Now I should be able to find ounces, right? I have fifteen percent alcohol. So, would this be five point six ounces? Because I divided 85 by fifteen to get my ounces it comes out five point six and a lot of sixes.” When asked about his final answer, Sal replied, “It should be adding the ounces up. It would be nine point six.” When asked if he could solve the problem algebraically, Sal replied that he forgot how to do it. When asked about the value and the meaning of the product of twenty percent and four ounces, Sal replied, “Eighty percent water. Because there is twenty percent solution makes a whole 100 percent, then the other half has to be 80 percent water.” Sal also mentioned that it was easier for him to solve a motion problem.

Figure 50. Sal’s solution of mixture problem II
General description of Sal’s experience solving mixture problem II. Sal arrived for the interview in a very positive attitude, but quickly became quite upset by his inability to solve the mixture problem. There is no apparent evidence that he understood the given and the unknown since Sal’s responses were not clear. When I asked Sal to identify the contents of twenty ounces of 50% alcohol solution, he failed to provide a clear answer talking interchangeably about alcohol and the alcohol solution. When asked to approximate the concentration of the mixture of two solutions, Sal incorrectly added the given concentrations. The participant used no variable. He set up no equation either. Sal attempted to solve the problem arithmetically, but his thinking was not logical. In his solution Sal found a ratio of eighty five percent to fifteen percent but failed to explain convincingly the reason for the step. The participant didn’t attempt to produce any graphical representations, so he was asked to make a picture. Sal’s picture represented a triangle with three values: twenty percent, eighty percent, and four ounces as the vertices. His description of the picture was not apparent.

Nick

Observation and particular description of Nick’s experience with math education. Nick is a 50-year-old full time student who retired after serving twenty years in the US Army. Now he is seeking a degree in nursing. During the interview, Nick seemed to be quite talkative and eager to share his thoughts. He seemed to be in a good mood and was excited about being in school again. At the same time, the participant was
clearly having difficulty focusing and staying on task. When asked about his school years prior to the army, Nick admitted that he didn’t remember much except dropping out shortly before graduation and then earning his GED right before the military service. He stated, “The most recent time for me is when I was in university and I took the math course there. I don’t know about school. The experience I had at the university was that math came in tricks. Now I know that I missed something very important in that. It was not just problem solving, it was the understanding and applications. I had lots of social problems. And now I am in high school and I got distracted by other things. I totally lost my focus. I was bringing home good grade report cards because my father would give me money to do well, and I got very good at making Bs out of Ds etc. Nick described his present attitude toward mathematics education as, “And now, at a later stage in my life, I recognize that there is something incredible going on here and I can make a connection to the knowledge and an education. I have not had it before. This [mathematics] is about everyday life; this is about the world that we experience around us every day. Even I appreciate art, I always have, but I understand that there is science in everything. There are principles in place with geometry and all the other disciplines that go into, just putting a building in place. I always have been fascinated by this kind of stuff- how nature has geometry in it. It is an equation going into these incredible possibilities of the multidirectional thing fighting for the sun.” Nick didn’t make any comments about his family and/or any informal math activities in his life when he was younger.
General description of Nick’s experience with mathematics education. At the beginning of the interview Nick admitted that he had no recollection of his math experience prior to his university years except the fact that he was neither feeling comfortable in math class nor successful in it. Nick’s explanation of his difficulties in math class was based on his lack of understanding algebraic concepts, lack of interest in studying in general, and social problems. Nick sounded upset when stating that he didn’t recall anything prior to his university years. When asked about his attitude toward being again a student, he appeared excited and eager to learn mathematics. He has also mentioned his strong belief that mathematics is everything around us.

Observation and particular description of Nick’s experience solving motion problem I. When asked to solve the problem, Nick stated, “I am thinking about rate, time, and distance, and I know a particular way of laying this out, where you can put all of the information, how we have been taught and it will always solve the problem given that you have a correct variable and correct information based on the words in here. I got to piece this out in my head like the handout that I have. I am going to draw this box, ok, and I know that it has three columns and three [rows] the opposite direction, so it has nine boxes. I know that there is a faster time and there is a slower time, and then we have--I think in this case there is no variable here because then it would be a mixture problem. So, we have time, distance, and, I believe, that is total. I am going to read this: two planes leave the same airport at the same time flying in opposite directions, so right away I am picturing this as a center point and two lines going away. The rest of the information is
they are flying in opposite directions. They leave at the same time, and it is important.

Rate of the faster plane is 300 miles per hour, so, right away I am going to plug it in here–300 mph. The rate of the slower plane is 200 mph; we are going to write for the slower plane. The question is in how many hours the planes will be 1000 miles apart.

Because you have two different rates, it is not going to be the same for each, but we are looking at 1000 miles.” When asked to identify the unknown, Nick replied, “Right now I am thinking for--here I am going to have a little difficulty. I am saying that in my mind there is an x and x minus or x plus depending on how it is going. I have an equation that would give us this piece of information: A plus B is going to equal to C. The sum is going to be equal to this number. If I just say x is the faster. I can say that it is going to take the slower plane longer to get to the 1000 miles than the faster one. I know the answer is five but I am being stuck now here. Let me do an equation: x, I do know this is going to be x. The faster plane will be the x minus. The slower plane is x. And there is going to be 1000 miles. This is such a simple problem but I am stuck.” When asked to recall the formula that connects three attributes of a motion, Nick replied, “Ok, that is easy. And it is probably where I should have gone because time times the distance equals the total amount. Time and distance, so 300 miles per hour, this is the speed and the rate of time, and then you have the distance which is going to be a 1000.” When asked about the formula again, Nick added, “Time times distance equals total amount of time. If miles are 5,280 feet, this is 300 of those, we would multiply that. Now we have zero (doing calculations), six, eight, this is a lot of feet. So, 1000 feet means in one hour we cover
these many. Now we divide by 60 which give us feet per minute. 1,584,000 divided by 60 equals 26,300 feet per minute. I think I have to divide.” When asked to explain the meaning of his term “total,” Nick responded, “Total amount of time the plane flew to 1000 feet.”

![Diagram of Nick's solution of motion problem](image)

**Figure 51.** Nick’s solution of motion problem I

**General description of Nick’s experience solving motion problem I.** When solving the problem, Nick stated that it was easy and he just did it in class. He recalled using a chart and a variable in class. Nick identified attributes of motion and their units but failed to recall the proper formula for them. He also stated properly what was given and what is unknown. It seemed that Nick was concentrating on rote memorization to
follow the procedure shown in class instead of thinking about the concept of motion. He knew that he was expected to use $x$ as the unknown but was unable to use it as an algebraic representation of the time he was looking for. In addition, he envisioned the situation as motion of one vehicle at a time, thus failing to see a simultaneous motion of two vehicles. When he attempted to solve the problem arithmetically using the correct formula, he began doing unnecessary unit conversions and got lost.

**Observation and particular description of Nick’s experience solving mixture problem II.** After reading the problem aloud, Nick stated, “It is about mixture. Here I am more confident [than with motion problem]. This is the teaching aid (drawing a chart). The largest amount is on the top. What are we looking for here? I have lost my entire formula for this.” At this point the participant began looking uncertain and confused. When asked the meaning of the expression “50% alcohol solution” Nick responded, “That is a solution which means it is a mixture. Now, half of it is alcohol and the other is something else, water or something.” Nick stated that the problem was relevant to real life and presented cooking as an example of mixing solutions. Nick also approximated the concentration and calculated the amount of the mixture of two solutions--twenty ounces of a 50% alcohol solution and fifteen ounces of a 70% alcohol solution--as “The whole volume is 35 ounces. I would add those two [the percentages] and come up with something in the middle, which is 60%, but it is roughly. I am sure there is a math principle behind it.” Nick then identified the contents of twenty ounces of the 50% alcohol solution as “It is always going to be 50%, but alcohol is the amount, so
each ounce is 50. Well, it [the alcohol] is half of the twenty ounces, so, ten ounces.”

When Nick stated that $x$ is the representation of the unknown, I asked him about the expression representing the mixture of four ounces and $x$ ounces. Nick replied that it would be four plus the $x$. When asked to try to solve the problem again, Nick responded, “This is the simplest thing and I am kind of frustrated because when I look at my graph, I have done it repetitiously. It is confusing me. I am really having a hard time here.”

**Figure 52.** Nick’s solution of mixture problem II

**General description of Nick’s experience solving mixture problem II.** While reading the mixture problem, Nick stated that he was more confident with this problem than with the motion one. He stated that a 50% alcohol solution means half alcohol and the rest is water or something else. Nick attempted to set up the chart introduced in his math class, but failed to complete it and set up and solve an equation. No graphical representation had been done by the participant. When Nick was offered to find the
amount of alcohol in twenty ounces of 50% solution, he got the correct answer arithmetically without visible difficulty. When Nick was offered an additional problem about two solutions mixed together, he was able to estimate the concentration of the mixture properly. When asked to attempt to solve the given problem again, Nick tried to restate it but failed to do it coherently. He attempted to solve the problem neither algebraically nor arithmetically.

Lora

Observation and particular description of Lora’s experience with math education. Lora is a 20-year-old high school graduate. She works as a store clerk and is currently seeking a degree in social work. Lora appeared very talkative and articulate during the interview. She presented an extremely positive outlook at herself, her family, and life in general. Lora recalled her earliest math experience as follows: “My memory begins in elementary school. I do not remember the first grade. Second grade I do remember. Second grade I had Mrs. S. who was probably my favorite teacher to this day. She definitely taught me the basics of math, concepts of addition, subtraction, and then multiplying and dividing. I really do not remember struggling with math and I was an average student. Then I remember math in seventh and eighth grades. There were some problems and my mom kind of helped with the process and I got it. I was home-schooled for awhile and before entering high school I took a placement test. I got put in Algebra 1A during my first year and then Algebra 1B my second year, the sophomore, because I
was not technically at the level to be put in Algebra 1. It literally looked like a foreign language to me. Then in my junior year I got into Geometry. It was so hard! We also had a professor who was very bland. So, I believe it depends on the teachers’ approach and how they teach you. Our Geometry professor was not good at it, so this class was difficult. I was in Precalculus during my senior year. In Precalculus it was extremely hard I would say, but again the professor was pretty bland like my geometry teacher. I got a C in that class. I have never had a traumatizing experience in a math class. It just took longer for me to understand it than for others. And algebra was always my favorite. Yes, in high school it made me nervous. But the class I am in now is not nervous at all.” When asked to talk more about her college math experience, the participant disclosed, “I took Intro to Logic class which is a form of math. There were no numbers, only rules you have to know, so I took that class. The professor was great. At first I was completely lost, but he set up study groups and all of a sudden I remember one day it just clicked, just clicked. So, now I am older and more mature, and I look at math differently than when I was in high school.” When asked to talk about her vision of mathematics now, Lora stated, “Present tense I do think that math is very relevant to real life. When I was back in high school I did not see any relevance at all. In high school I was obsessed with art, with more social things, and I did not see how math really could have equated with all of that.” Lora acknowledged that her mom would help with her school work when it was needed. There were no math games or puzzles though. When asked about her experience solving word problems, Lora replied that she remembered it and “it was very easy like
multiplying and adding. But I got to a number \( a \) [a variable] being equal to this decimal number and \( b \) [another variable] equals this fraction number, or a square root of that number, and that it was difficult.” When I asked Lora again if solving word problems would make her feel uncomfortable, the participant replied, “No, I would not say that. They [word problems] would make me hesitant at first, because I would have to think what angle or what would I do first. You have to figure what equals what and then plug in different numbers.”

**General description of Lora’s experience with mathematics education.** Lora appeared very content and comfortable throughout the interview. She disclosed that her attitude toward elementary school arithmetic was very positive and she also complemented her math teachers as using good techniques to explain new material. Lora also reported she has experienced difficulties learning Algebra and Geometry during her middle and high school years. She attributed the obstacles to the complexities of the subjects as well as to poor teaching. As of right now, Lora stated that she felt very comfortable and confident in her math class and believed that she is good at mathematics. The participant remembered solving word problems before and stated that even the process of solving them didn’t make her uncomfortable; it was not simple. She also made a comment that working just with numbers was easy, but working with unknowns was not.

**Observation and particular description of Lora’s experience solving motion problem I.** Lora began with reading the problem aloud clearly and articulately. Then,
after thinking for a while, she said, “Here I got the answer, but I did not write down the
equation properly. It is four hours. I just thought of it logically in my head and did the
math. What I did was the rate of the faster plane is going 300 miles per hour; the slower
plane is going 200 miles per hour. So this is 1000 miles apart. They start at the same
point and are going in complete opposite directions. I think of it as being the same spot,
but totally going opposite directions, so they are kind of connecting with one another or
more so, and going with a different pace. So it is two hours and equals 600. And I did 200
miles at the time of two hours which is 400. So, this is 600 for the faster plane and 400
for the other plane. I added them together and it would be 1000. From that I just thought
it makes sense since it is pretty even playing with numbers. So, I added two plus two, and
it is four hours.” Lora identified rate as “Rate? I am not quite sure. Probably, rate is a
pace you are going with, steady pace or a constant pace, steady pace or constant motion.”
When Lora was asked about attributes of motion, she first asked me to clarify the
question and then replied, “You definitely can measure the distance, from point A to
point B. You can measure possibly the inconsistency; for example you have to stop at the
red light or the stop sign.” When I asked Lora to explain the meaning of the value of two
in her calculations, she replied, “It represents the amount of time, the amount of hours
that you are constantly moving at the rate of 300 miles per hour.” The participant
explained that she performed the multiplication as “the equivalence of time and rate.”
When asked to explain the meaning of 600, Lora replied, “It is an actual amount of time.
I am not sure.” Lora presented the formula that connects the attributes of motion as D =
RT. The participant identified the given and unknown as “We are looking for distance apart. What is given is the rate. They did not give me any time at all. Oh, they did give me the time because it says ‘per hour’. What I have to figure out is the number of time, but it is separate from miles per hour. So, I am not quite sure how it is.” She explained her answer as “It represents the hours. Oh, it is the time. So, they gave me the rate; they did not give me the hours, so, we are looking for the hours.” The participant also stated that she wouldn’t be able to solve the problem differently. When asked if she had seen similar problems before, Lora replied, “Oh, yea. But they were a little bit harder than this one. And I remember that it is definitely a formula for this on; there is a formula for everything. Our teacher would make us to use the formula to solve it, which makes it very logical and ties it together. It should be an equation.”

![Figure 53. Lora’s solution of motion problem I](image)

**General description of Lora’s experience solving motion problem I.** The participant seemed to be very confident during the process of solving the problem. She derived the correct answer by trial and error method shortly after reading the problem aloud. When asked about the attributes of motion, Lora was not sure about the answer,
but after getting some clarification, replied that it would be distance, number of stops, direction of the motion, and time. Lora stated the formula \( D=RT \) correctly but failed to identify the proper units of the variables in it. She also stated that the slower plane would arrive at the final destination later that the faster one, thus revealing the lack of understanding the situation. Lora didn’t use any variable or equations. She didn’t produce any picture or diagram either.

**Observation and particular description of Lora’s experience solving mixture problem I.** Lora began by reading the problem aloud and proceeded by restating it as follows: “How many ounces of 50% alcohol, we want to find the ounces, must be mixed with 80 ounces, so they are giving us 80 ounces, and it is a problem with equation, 80 ounces of 20% alcohol solution to make 40%. What we want to find is 40% alcohol. So, how many ounces of 50% alcohol solution must be mixed with 80 ounces.” When she was asked if she saw similar problems before, Lora replied, “I definitely remember seeing this but there is some type of connection that I have not seen yet.” She also stated that the problem is relevant to real life and that people would mix different solutions if they are forensic scientists or need to color their hair or cook. When asked to identify the term “50% alcohol solution,” Lora replied, “The whole bottle of alcohol solution, so it is 50% of that possibly. I equated 50 with half, so it will be another 50 that we are not using.” When asked about the nature of the other half, she stated, “Yes, it is also an alcohol. So, there is 50% alcohol we are using and there is 50% alcohol we are not using.” When asked to identify “20% alcohol solution”, the participant replied, “Twenty
percent is only out of one hundred percent total. It is one fourth; no, it is not one fourth. So, out of one hundred you take away twenty percent, so you still have eighty percent of the bottle left.”

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<th>50% alcohol</th>
<th>Ounces</th>
<th>80 ounces</th>
<th>20% alcohol</th>
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*Figure 54. Lora’s solution of mixture problem I*

**General description of Lora’s experience when solving mixture problem I.**

The participant read the problem aloud and restated it coherently. Nevertheless, she admitted right away that she was confused and didn’t know how to solve it. When Lora was asked to explain the meaning of “50 % solution”, she replied that half of the solution would be used now and the rest later, thus presenting the lack of comprehension of the concept of concentration of a solution. Lora didn’t attempt to solve the problem either arithmetically or algebraically. No picture/diagram was used.

**Observation and particular description of Lora’s experience solving motion problem II.** As during the first interview, Lora seemed to be quite confident in her mathematical ability and was open toward the interview. She began with reading the problem silently, drawing a chart, and writing notes. When asked to restate the problem, she responded, “So, there are two different cities; they are 315 miles apart approximately.
From one of the cities a car is traveling to the other and is going exactly in the direction of the other city which is in a 315 mile radius. And it is traveling 50 miles per hour at the opposite city. A bus is traveling to the opposite city where the car is coming from and traveling at 55 miles per hour. I did not get to read the bottom part but I think we have to say how much time it takes both of them to meet one another till they cross paths. I think I remember the formula for this one. It is going to take them three hours until they meet.”

When asked where she was taught to use the chart, Lora replied, “I saw the chart in class this semester. Our teacher basically introduced it to us to break down distance apart from each other like word problems.” Lora identified the value of 55 as “the miles per hour the bus is traveling, the exact miles that you are traveling, like the average amount of miles that you are traveling in one hour.” When asked to describe the attributes of a motion, she said, “Oh, we are measuring its distance, we are measuring steady distance, how far it goes. We are measuring the pace it goes with.” Lora also added that “pace” is similar to the rate. When asked to explain the meaning of the variable used, Lora replied, “X is the amount of hours and we measure time using hours.” Lora identified the connection between the attributes of motion as “I think they say distance equals time over pace.”

When asked about the units of distance, the participant replied, “From point A to point B.” After some explanations, she added, “Oh, units, we measure it by miles.” When asked about the units of rate or pace, she said, “It is the average.” When asked to explain the meaning of the product of 55 and the x, Lora replied, “It stands for the amount of time the bus takes to reach the car.” When asked about the meaning of the product of 50 and
the x, she responded, “That stands for the pace that car is going times the amount of time it takes to meet up with the bus.” Lora explained her equation as “The total amount of time it would take. I am adding them [the products of 50x and 55x] together because it is going to equal the total amount between both the car and the bus.” When she was asked to clarify the term “total amount”, Lora replied, “Total amount of hours. Even though one is going less comparing to the other one, it is basically they are going at their own pace. With them going with their own pace, how long till they meet, it is both of them added together, contributing to meeting one another. After learning what I have learned, it is going to be equal to 315 no matter what because it is the total amount of distance between the two, so there is no way that when they are added to each other; it is not equal to 315. I decided whatever my equation is going to be, it is going to equal 315 and then I had to figure what is missing. It is time. The reason I put the addition symbol is because we want to add the times together and both equal 315 miles. We need time for both of them, so I just put x near 55 miles times that and 50 miles times x.” Lora identified the meaning of the product of 55 and the x as “We were definitely told in class to put 55 times x. I think the reason behind it is because we multiply something; just duplicate the number. You get a total and you divide by the total amount.” Lora also stated that she has used all the information given and added that the problem can be solved differently as follows: “Instead of doing an equation, you can use common sense to try to figure it out. But I am not quite sure how it will work with this, so I don’t know. I think it is the best, most accurate way to solve it using an equation.”
General description of Lora’s experience solving motion problem II. After reading the problem silently, Lora proceeded with setting up a chart and an equation. She stated that she learned to do both in her current algebra class. Lora restated the problem coherently and identified the given and the unknown. She also set up a correct equation and solved it without visible difficulties. The picture was coherent as well presenting two vehicles moving in opposite directions and the initial distance between them. However, Lora kept getting confused when she was asked about the meaning of the equation she set up and the meanings of the terms of the equation, therefore failing to identify the products of the rates and the time as the distances covered by individual vehicles. Her explanation of the equation was that she completed the problem the way she was shown in class.
Observation and particular description of Lora’s experience solving mixture problem II. Lora began with silently reading and writing notes. She then stated, “I think I set it up properly. The only thing why the problem makes me nervous [is] I forgot how she [the teacher] writes down the equation. I forgot the exact arrangement of the numbers. But I think I got the whole [chart] set up properly.” When asked to restate the problem, Lora replied, “You have a combination of fifteen percent solution of alcohol. Well, you have two solutions, the fifteen percent and the twenty percent. The twenty percent solution is four ounces. I cannot remember; it is some kind of substance. So, with those two given, we have to identify how many ounces I need to, or how many ounces actually are in the seventeen percent of alcohol solution. So, you are given one already which is the twenty percent. You already know, it is four ounces. The fifteen percent, you are not given. I think you only need one [solution] to figure it out. How many ounces are in the seventeen percent?” When asked to explain the expression “fifteen percent alcohol solution,” Lora replied, “I am guessing it is total out of a 100. Fifteen percent is fifteen percent of the total amount of solution. It has fifteen percent alcohol in it.” When then asked to describe the rest of the solution, she replied, “It is unknown. The 85 % of it is unknown. It is not identified.” When asked to identify the ingredients of twenty ounces of a 50% alcohol solution, Lora said, “Fifty percent is alcohol and that it ten ounces then. Ten ounces is half of the total amount of solution, 50% of it, half of it; half of it has alcoholic solution in it, half of twenty ounces.” When asked about the units of the values of fifteen, twenty, and seventeen, she replied, “It is the largest amount, the medium, and
the smallest.” When asked again if she knew the term units, Lora said no. After some explanation Lora identified the units of four and the unknown x as ounces. Lora identified the variable used as “The unknown amount of alcoholic solution.” When asked to specify the solution, the participant didn’t give an answer. Lora didn’t attempt to provide a graphic representation, so I asked her to make a picture illustrating the problem. Lora first drew three containers and identified them properly, but at the end she combined all of them into a whole and labeled it as 100%. She described her picture as “What I drew is the total 100%; it is all a big circle. Inside the circle is the fifteen percent of alcohol solution in its own little bubble. The unknown is the ounces we have to add. Ounces we are adding to it are four ounces of twenty percent solution. But we still have to figure out what ounces it takes to get to the seventeen percent alcohol solution.” She identified 100% as “the amount of overall alcoholic solution.” The participant also stated that she learned the chart in math class and that enough information was given to solve the problem. When asked about the meaning of the value of 80 that is the product of twenty and four, Lora responded, “I am not sure. I multiply the twenty times four. That is the total amount of twenty percent solution and the ounces combined.” When asked again to solve the problem, Lora replied, “I feel bad, but I cannot remember the equation format. I don’t think I will be able to remember it from the top of my head.” After a while she added, “It is an estimated number. I think it is three and three-fourth ounces alcoholic solution. I didn’t do an equation. I did in my mind. There are five twenties in a hundred. There are four ounces in each five sets of twenty percent, so there are five twenties in a
hundred, so there are five fours also in a hundred because twenty basically equals four ounces. So, four times five got me twenty ounces total. Then I took the seventeen and this is where I just estimated the number because seventeen is not an even number; even is twenty. It is right in the middle; it is not fifteen percent, and it is not twenty percent. I know it is at least three ounces, so it is three and three-fourths.”

Figure 56. Lora’s solution of the mixture problem II

**General description of Lora’s experience solving mixture problem II.** Lora disclosed that her attitude toward solving word problems had changed to a positive after learning the process in her algebra class. Nevertheless, Lora had visible difficulty restating the problem coherently and identifying the given and the unknown. When answering my questions, she would often say, “I guess.” When asked about the concept of a percentage, the participant properly replied that it is part of 100. When asked about the ingredients of four ounces of the twenty percent alcohol solution, the participant
became visibly hesitant and failed to provide a clear answer. She kept confusing the terms solution, alcohol solution, and alcohol. Lora attempted to set up and use the chart shown in math class, but failed to do it properly saying that she couldn’t remember. Lora also attempted to set up an equation, but couldn’t do it either. She tried to estimate the answer to the problem, but her explanation of the procedure was not logical.

Tom

Observation and particular description of Tom’s experience solving motion problem I. Tom began by reading it silently and writing notes. He then described his thinking process as follows: “Yes, I know it is ten hours. I did it in my head. The way I did it is the rate of one plane is 300 miles per hour, the rate of another plane is 200 miles per hour, and the difference is 100. The question is how many hours it would take to get 1000 miles. So, I divided 1000 miles by 100 and got 10.” When asked to explain the term “rate”, Tom’s reply was “It is speed.” He added that “rate” and “speed” both represent “how fast you are traveling”. Tom identified the given as “the speed of the other plane. They are flying in opposite directions. They are leaving the same airport and how many hours it would take to be 1000 miles apart.” When asked what we are looking for, Tom replied, “The hours, the miles per hour, the rate, the time.” When asked to explain why he subtracted the rates of the planes, Tom responded, “Because that is the difference between them. And it is looking for how far away it would be. Well, I subtracted the
difference and I divided by a thousand because they both travel with this equal distance all the time. It is why I divided by one hundred.”

Figure 57. Tom’s solution of motion problem I.

General description of Tom’s experience solving motion problem I. Tom appeared to be relaxed and content during our conversation. It looked as though he did not have a negative attitude toward either mathematics or word problem solving. Tom properly identified the given and the unknown. He obtained a solution to the problem shortly after he finished reading it. Tom seemed to understand the attributes of motion like rate, time, and distance, but failed to see the simultaneous motion of two planes in opposite directions. The participant attempted to solve the problem arithmetically without
using a variable and or an equation. Tom intuitively divided the given distance by the rate to obtain the value of time but did not state the formula. Somehow Tom decided to subtract the given rates instead of adding them, so the answer obtained was incorrect. He added the picture of two planes only when I asked him to do so.

**Observation and particular description of Tom’s experience solving mixture problem I.** After reading the problem silently, Tom restated the problem as follows: “I need 40% alcohol solution. I have 20% and 50%. 50% needs to be mixed with 80 ounces of 20%.” After that Tom indicated that he didn’t know how to proceed and that he was ready for my questions. When asked about the meaning of 50% solution, Tom responded, “It is diluted by half” and the ingredients are “possibly water and alcohol.” Tom identified twenty percent alcohol solution as “it is eighty and twenty; eighty is alcohol.” When I asked Tom to determine the total amount and approximate concentration of the mixture of ten ounces of 50% solution and five ounces of 80% solution, he responded that the amount “would be fifteen [ounces] and the concentration would be “about 65%”. When asked if enough information was given to solve the original problem, Tom admitted that he didn’t know. When asked to determine the amount of alcohol in twenty ounces of 30 % alcohol solution, Tom responded, “Is 30% alcohol and the rest is the solution? It would be 6.” When asked how he obtained the answer, Tom replied, “I took 30% of twenty which is six and I did it in my head. I took ten percent of twenty and then multiplied by three.” Tom added that he has not seen similar problems before and he didn’t believe that it is relevant to everyday life.
General description of Tom’s experience solving mixture problem I. When asked to solve the second problem, Tom seemed to restate it without visible difficulty and attempted to solve it right away. He properly identified an alcohol solution as a mixture of alcohol and water and presented the 50% solution as half of each. At the same time he presented the 20% solution as 80% alcohol and twenty percent water. Tom was not able to solve the mixture problem either arithmetically or algebraically. No picture, chart, or graph was done. At the same time he properly estimated the possible concentration of the mixture of 50% solution and 80% solution as about 65%. The participant did all the calculations mentally.

Observation and particular description of Tom’s experience solving motion problem II. Tom began with reading the problem aloud and proceeded with writing notes. He stated shortly after, “Two cities are 315 miles apart, ok, three hours.” When asked to restate the problem, Tom replied, “Two cities are 315 miles apart. A car leaves
one of the cities and is traveling toward the second city at 50 miles an hour. At the same
time a bus leaves the second city and traveling at 55 miles per hour. How long will it take
for them to meet?” He identified the unknown and the given as “Unknown is the time and
what was given is the distance and rate.” When asked to recall the formula that connects
attributes of motion, Tom replied, “Rate equals distance over time or something like
that.” When replying to the question, Tom appeared quite uncertain though. He then
added, “I think rate equals distance over time but I am not sure. I used this
rate/time/distance table.” The participant disclosed that he was shown the table in his
math class. When asked about units of rate, Tom replied uncertainly, “50 and 55 miles
per hour.” After that Tom admitted that he was not sure what the term units stand for.
After some clarification, he replied, “50 units and 55 units.” He also identified the units
of distance as “three hundred and fifteen.” When asked if he could have solved the
problem without the chart, Tom failed to provide a clear answer. When I then asked Tom
for the reason he added the values of 55 and 50, he responded, “Because they are moving
toward each other and covering this distance. So, I am adding the distance.” According to
Tom’s notes, the last statement was that 105 are equal to 315. When asked to clarify the
statement, Tom replied, “I would divide 105 by 315 and I would get three.” He also
added that the units of this value three would be “three hours because 105 is miles per
hour.” When asked to make a picture illustrating the problem, Tom replied, “I am not an
artist”. After some deliberation, he silently drew the picture. Tom also added that there is
enough information given to solve the problem and that he has seen similar problems in
his math class. When asked if the problem is relevant to real life, he stated, “Oh, yes. People drive all the time.” When then asked about his attitude toward solving word problems, Tom replied, “I dissect every word practically because from high school they are the worst. And my teacher taught me critical words.”

Figure 59. Tom’s solution of motion problem II

**General description of Tom’s experience solving motion problem II.** Tom restated the problem without visible difficulty and identified the given and the unknown properly. He also attempted to make a picture when was asked. His picture seemed to be coherent but incomplete. Tom obtained the correct answer shortly after reading the problem. He solved the problem applying a combination of arithmetic and algebraic approaches by introducing a variable correctly but without setting a correct equation.
Tom was not able to identify units of the attributes of a motion correctly. He set up a chart similar to the one shown in class but failed to present the use of it rationally.

**Observation and particular description of Tom’s experience solving mixture problem II.** Tom began with reading the problem aloud. Immediately after reading, he became visibly upset and confused and stated, “I will try. I hate word problems.” When asked about the reason, Tom replied, “To me it is just more difficult. It is easier to just see the numbers or the letters and to go from there.” When asked for the reason students are being asked to solve word problems in a math class, Tom replied, “Because there are problems that need to be solved.” He admitted in a few moments that word problems actually reflect real life situations. Tom proceeded by restating the problem as follows: “How many ounces of the fifteen percent, so x, I know what you are looking for, how many ounces of fifteen percent solution should be added to, I must say I don’t know. I am trying to figure this out. No, I cannot. I am trying to combine the two to make seventeen percent.” Tom identified the given information as “the four ounces of twenty percent and x ounces of fifteen percent” and the unknown as “x ounces of fifteen percent combined with the other to make seventeen percent.” Tom admitted solving similar problems in class and added, “Yes, a long time ago, maybe one month; it feels like a long time ago. We were doing a chart. I remember the motion problems, rate times time. No, I cannot remember how we set up the chart.” When asked if he could solve the problem without the chart, Tom said no. The participant didn’t make any attempt to make a graphical representation, so I asked him to do so. Tom then made a picture presenting three objects
containing something inside and labeled them according to the given percentages. He described his picture as follows: “So, if these are filled, at fifteen percent and twenty percent, they equal seventeen percent. How many ounces of fifteen percent must be mixed with four ounces, four ounces of this is x, have to equal that.” Tom explained the meaning of the expression fifteen percent alcohol solution as “the dilute or how strong it is. The other is 85 percent, water or whatever, the basis.” When asked again to identify the nature of the fifteen percent alcohol solution, the participant said, “It is the alcohol.” When asked about the twenty percent alcohol solution, Tom responded, “I am mixing four ounces of that with the fifteen percent.” When I then asked Tom to identify the contents of four ounces of twenty percent concentration alcohol solution, he replied, “Eighty. There is twenty percent, so twenty of it is alcohol. The rest would be eighty.” Tom also added that these four ounces are not a pure alcohol. When asked to calculate the amount of alcohol in four ounces of the twenty percent alcohol solution, Tom replied hesitantly, “It would be twenty to eighty. It is still twenty percent, water, alcohol. No, I am really drawing a blank.” When I asked Tom to determine the expression representing the total amount of x ounces of one solution and four ounces of another, he replied, “Seventeen percent.” Tom explained his reasoning as follows: “It would be six ounces. I am doing it in my head. If you take fifteen percent and twenty percent and just divide it by two you get seventeen and a half percent, no, four ounces of twenty percent. You are going to need little more ounces since it is not as strong as fifteen percent.” When asked to approximate the concentration of the mixture of 50% and 70% concentration solutions,
Tom replied, “It would be 60 because there is a difference of twenty. It is how I would do it in my head. It is the sum divided by two in my opinion. I don’t know.” When asked about the amount of the mixture of twenty ounces and 30 ounces of two different solutions, Tom replied, “It would be 25, no, 50.” When then asked if there was a difference between the concentration of a solution and its amount, Tom said yes and added, “Ounces are pounds, percent goes to 100.” When asked again about the amount of the mixture of x ounces and four ounces together, he replied, “It would be four x.” In addition, Tom stated that the problem is relevant to real life if it is related to what one does.

![Figure 60. Tom’s solution of mixture problem II](image)

**General description of Tom’s experience solving mixture problem II.** Tom became instantly upset after looking at the problem and then stated that it is difficult for him to solve. The participant nevertheless was able to restate the problem and to identify
the given and the unknown. Tom also produced a picture representing the problem upon my request. The picture was logical but incomplete lacking the addition sign between the initial components. When asked a number of questions about the nature of a concentration of a solution and the amount of pure alcohol in it, Tom became even more frustrated. He was confusing terms such as solution, total amount, and percentage. He attempted to reproduce a chart shown in class but admitted that he didn’t remembered it. There was no clear evidence that Tom comprehended the concept of concentration since his answers were sporadic and indistinct. He attempted to set up an equation and even managed to construct the proper right sight of the equation, but failed to produce the left side as the product of the percentage and the amount of the mixture. He estimated the answer as six ounces, but his explanation of it was not clear. Tom also failed to recognize that the expression for the mixture of x ounces of a solution and four ounces of another solution would be the sum of the x and four. At the same time he properly estimated the concentration of the mixture of 50% and 70% solutions as approximately 60%.

Cross-Case General Descriptions of Participants’ Experiences

Mathematics Education

This section offers the cross-case analysis of the participants’ experiences with mathematics education. Tables 7- 8 and Figures 61-63 (Appendix C) captured the synthesis of the participants’ experiences.
Almost all the participants mentioned their elementary school mathematics classroom as the earliest experience with math education. And for most of them, this experience was a pleasant one consisting of feelings of success and content studying mathematics. One participant revealed that he didn’t remember anything prior to his university years, and the other one mentioned that his elementary school years were not as pleasant, noting his lack of attention and the interest being the reason for it. Almost all the participants gave a lot of credit to their mathematics teachers throughout their school years beginning with elementary school. The qualities praised in math teachers were patience, concerns, and caring toward students. The participants mentioned the following teaching methods as beneficial ones: manipulatives (jellybeans and blocks) and memory aids (flashcards). There was no single participant who mentioned anything negative about their first mathematics teacher(s). Most of the participants mentioned family involvement with the mathematics studies. This involvement included general support for studying and some help with school math homework. At the same time, the participants stated that they didn’t recall being engaged in any games, puzzles, or other informal activities related to mathematics and/or learning in general.

The participants’ attitude changed during their middle school years. The participants admitted losing interest toward school in general and mathematics in particular as the result of various reasons. The reasons for change included the following: increased interest in social life, increased challenge to complete mathematics, and various domestic circumstances such as moving with the parents’ jobs. Only in two instances, the
participants blamed their teachers as the cause of their failure. The difficulties in the mathematics classroom mentioned were dealing with variables rather than with numbers and lack of understanding of the content such as solving equations and word problems. One of the crucial factors of success or failure according to the participants was the teachers’ attitudes toward their students and the subject.

There is a great deal of attitude change toward studying in general and mathematics in particular happening at this point in the participants’ lives. All of them have mentioned their determination to succeed at school and in their math study, willingness to work hard in order to reach their goals and earn a degree. In terms of mathematics study, a number of participants stated their determination to understand the subject, not just to pass it. This “aha” moment is based on the realization that mathematics is really contained within the life around us.

When asked about solving word problems, all the participants admitted that solving word problems is difficult and uncomfortable. Only two students stated that they didn’t like it, but were not intimidated by it. The causes mentioned were dealing with variables and reading comprehension. All the participants admitted that the word problems offered in the math classes throughout their school years are relevant to real life.
Motion Problem I

This section offers the cross-case analysis of the participants’ experiences with solving the motion problem I. Table 9 (Appendix D) captured the synthesis of the participants’ experiences.

Most participants began solving the problem with silent reading. All but two of the participants were able to rephrase the problem meaningfully. Nevertheless, the participants had mixed experiences when identifying the unknown and the given. Some of them were able to identify the information rather quickly and some of them would deliberate with themselves about the information. During the process, the terminology and the units were not always used correctly. Even though the term “rate” was given in the text, a few participants had trouble identifying it. There was no clear evidence that the participants have a clear understanding of what the term “units” stands for and the connection between the term and the corresponding numbers. Participants, for the most part, attempted to apply the distance formula intuitively. They were not able to apply the formula directly. Some participants had difficulties with identifying the attributes of the motion. Other participants could not state the formula D=RT. Some could not do either. This caused almost all participants apprehension and anxiety. Only a few participants introduced a variable into the process, but then failed to use the variable correctly in an equation. Some of these students would not set up an equation at all. There were a few students who were able to obtain the correct answer arithmetically, but only one student was able to do so algebraically. This one student, however, could not explain the set-up
of the equation. Half of the participants considered the motion as two independent vehicles and did not consider the relationship between the two moving objects. All participants acknowledged that the problem they were solving was relevant to real life. All participants revealed that they drive. Students realized the authenticity of the problem presented; however, they could not execute the proper problem solving technique to correctly solve the problem. Some participants lacked not only the arithmetic and/or algebraic skills necessary, but also the reading comprehension skills. At this level of mathematics, the students were, for the most part, not ready to tackle this motion problem. They lacked the ability to model the situation of the problem and the physical attributes of it. Only three students initially presented a graphical representation of the situation. Other students lacked the problem solving technique to visually represent the situation. These students only drew a picture when asked by the researcher. The pictures presented were not complete. The students simply had difficulty translating real world, authentic activities to the mathematics classroom.

**Mixture Problem I**

This section offers the cross-case data analyses of the participants’ experiences with solving the mixture problem I. Table 10 and Figures 67 & 68 (Appendix E) describe the synthesis of the participants’ experiences.

When solving the mixture problem, almost all participants began with silent reading. When asked to restate the problem, only half of them were able to restate it
coherently and identify the given and the unknown. No participants but one, Sarah, were able to introduce a variable, set up an equation and obtain a correct answer. At the same time, when asked to solve a one-step problem about the amount of alcohol in a given amount of solution and given its concentration, half of the participants were able to solve it by multiplying the percentage by the amount of the solution. The rest of the participants stated that 50% percent means half, but were not able to coherently define the ingredients of such solution. Almost of the participants agreed that the given problem is relevant to real life and were able to come up with coherent examples. The entire participant group was asked to calculate the total amount and to approximate the possible concentration of the mixture of two given solutions. All of them were able to calculate the total amount properly saying it is the sum of the original amounts, but only two participants approximated the resultant concentration as the average of the given ones. The rest of the participants added the percentages as well. These students didn’t express any concern about the consequential percentage being above 100%.

Motion Problem II

This section offers the cross-case data analysis of the participants’ experiences with solving the motion problem II. Table 11 and Figures 69 & 70 (Appendix F) captured the synthesis of the participants’ experiences.

Similar to the first interview, most of the participants began solving the problem with silent reading. Only two of them then read it aloud. After the students were taught to
solve this type of word problem in class, all of the participants were able to paraphrase
the problem correctly and to identify the unknown and the given. At the same time, some
of them were able to identify the information rather quickly and some of them would
deliberate with themselves about the information. There is clear evidence that some
participants still have difficulties with units of the attributes of motion. Most of them
began identifying them correctly and improved their terminology as the interview
proceeded. All of the participants but four applied the distance formula correctly at this
time. Two participants used it incorrectly and two students did not use it at all. All of the
participants but four introduced a variable and attempted to solve the motion problem
algebraically. Half of the participants then proceeded with setting up and solving a proper
equation. Two students had incorrect equations and six of the participants did not use an
equation at all. Three participants ended up without any answer to the problem. Half of
the participants attempted to use a picture. These pictures were incomplete though. The
participants presented a picture only upon the researcher’s request. Seven participants
obtained a correct answer algebraically and the rest of them obtained a correct answer
arithmetically. Out of all the participants who obtained the correct answer, only three
were able to explain the meaning of the equation and/or arithmetic expression logically.
The rest of the participants when asked to explain the steps replied that it was how they
were shown the procedure in class. As during the first interview, all the participants
admitted that they drive on a regular basis and claimed the realism of the problem.
Mixture Problem II

This section offers the cross-case data analysis of the participants’ experiences solving the mixture problem II. Table 12 and Figures 71-72 (Appendix G) depict the synthesis of the participants’ experiences.

When solving the mixture problem after learning the process in class, almost all participants approached it with a positive attitude. Half of them began the process with silent reading and the other half read the problem out loud. When asked to restate the problem, all but five participants were able to restate it accurately and identify the given and the unknown. All participants but one introduced a variable, but out of these, seven participants did not use the variable properly. All participants but five were able to set up a proper equation and then solve it correctly. Nevertheless, none of the participants was able to explain the meanings of the separate terms of the equation and/or the whole equation. When asked for the reason for multiplication of the concentration by the amount, the participants would respond “don’t know,” “it was shown in class,” or “I don’t follow your question.”

When asked additional questions about finding contents of twenty ounces of 50% alcohol solution, half of the participants were able to determine that half of it, ten ounces, is alcohol. Still, all of them when answering additional questions were confusing the terms alcohol, alcohol solution, total, and total amount. Not a single participant among those who used the proper concentration formula explained the meaning of the product of the amount of a solution by its concentration as the amount of pure substance (alcohol in
Almost all of the participants agreed that the given problem was relevant to real life and were able to come up with coherent examples. The entire participant group was asked to calculate the total amount and to approximate the possible concentration of the mixture of two given solutions. As during the first interview, all of them were able to calculate the total amount properly saying it is the sum of the original amounts. Only six participants, however, approximated the resultant concentration as the average of the given ones. The rest of the participants added the percentages as well. Again, as before, the participants didn’t express any concern about the consequential percentage being above 100%. While attempting to solve the problem, not a single participant made an effort to draw a picture. Nine participants were asked to draw a picture, and only three participants presented complete images consisting of two containers added together and resulting with the third one and proper labeling. All the participants seemed to be in a good mood and had a positive attitude toward the meeting at the beginning of the second interview, and almost all of them but two got frustrated and confused when solving the mixture problem.

**Findings of the Study**

The purpose of the study is to understand and describe the adult students’ learning experience while solving mathematical word problems. Ultimately, this could lead to understanding the cognitive meanings of the algebraic thinking of adult learners. I intended to look for the following: (a) attitudes and beliefs of adult students toward
learning mathematics and mathematical problem solving, (b) the mathematical content knowledge adult learners assess when solving word problems, (c) approaches solving word problems used by adult learners, (d) the adult students’ symbolic language and the dynamics of reasoning and thinking. My interpretive commentaries below are based on analyzing of the collected data.

Attitudes and Beliefs of Adult Students Regarding Mathematics Education and Word Problem Solving

For most of the participants, their earliest experience with mathematics education was their elementary school. This experience was a pleasant one and consisted of feeling successful and content when studying mathematics. The participants’ attitudes changed during their middle school years. The participants admitted losing interest toward school in general and mathematics in particular, for various reasons. The reasons for this change in attitude included the following: an increased interest in social life, increased challenge to focus on mathematics class as studies became more complex with each grade and various domestic circumstances such as moving with the parents’ jobs.

In terms of the socialization process, as children grew older, their focus on socializing seemed to get prioritized over their concentration on their studies. The difficulties in the mathematics classroom mentioned were a result of dealing with variables rather than with numbers, and abstract concepts rather than concrete ones. This led to the lack of understanding how to solve equations and word problems. In reference
to changing circumstances in the children’s lives, it seemed that big life changes would almost always have an effect on the learning process. As young students were forced into adjusting to new schools and different schedules, their ability to concentrate on mathematics would decrease.

Almost all the participants gave a lot of credit to their mathematics teachers throughout their school years beginning with elementary school. The qualities praised in math teachers were patience, concern, and caring toward students. Only two participants blamed their teachers as the cause of their failure. Only half of the participants mentioned that their family was involved and encouraging with their mathematics studies, and just one of these students discussed participating in informal mathematics activities at home. From understanding how the upbringing and social surroundings of the participants changed their experience with mathematics education, it is clear those different environments at school and at home affected the individual’s attitude about mathematics education.

Interestingly, it seemed that a shift in attitude towards mathematics education took place several times throughout the lives of the participants in this study: the one already noted, the shift between elementary education and middle school, and then again in their young adulthood in their decisions to attend community college. All of the participants mentioned their determination to succeed at school and in their math study, willingness to work hard in order to reach their goals and earn a degree. For all the participants, the decision to return to school was intentional. In terms of mathematics
study, a number of participants stated their determination to understand the subject, not just to pass it, but to get a deeper understanding of the subject and begin applying this to real-life situations.

When asked about solving word problems, all the participants admitted that solving word problems is difficult and uncomfortable. Only two students stated that they didn’t like it, but were not intimidated by it. The causes of difficulties mentioned were dealing with variables and reading comprehension. All the participants admitted that the word problems offered in the math classes throughout their school years are relevant to real life.

**Mathematical Content Knowledge and Strategies Solving Motion Problems Prior to Classroom Lesson**

I would like to summarize this part of the study by using the Schoenfeld’s terms *resources, heuristics, control, and belief systems*. The adult students who participated in the study seemed to be lacking the *resources* necessary to comprehend the attributes of motion and the connection between them. The participants at times were able to intuitively use the distance formula, but were unable to state it and/or its components and/or the units involved. At this point, the participants were not inclined to either use a variable or solve the problem algebraically. There is a definite preference for arithmetic reasoning since all of the participants but one attempted to solve the problem arithmetically. In addition, mathematics is contingent on the use of clear and explicit
terminology (Schoenfeld, 1985, p.56). The participants of the study didn’t show the
possession of the mathematics terminology related to the concept of motion. Talking
about heuristics, some participants of the study hadn’t had a plan to carry out. The
students who did still wouldn’t draw figure/pictures and wouldn’t restate the problem
and/or identify the given and the unknown unless asked to do so. Not a single participant
worked the given problem backwards. The students who did obtain the answer would not
check it at the end. Only a few participants introduced a variable into the process, but
then failed to use the variable correctly in an equation. Some of these students would not
set up an equation at all. All of the participants of the study admitted the fact that they
drive on a regular basis; nevertheless, the concept of motion and its characteristics
seemed unclear to them. This revealed that the participants presented no use of applying
real-life cognitive skills to mathematics word problems.

Concerning the control issue, the few students who did attempt to draw a picture
would not always provide a comprehensive image that captured the full picture. The
students who did apply the distance formula failed to recognize the motion described in
the problem as a simultaneous motion of two vehicles and did not consider the
relationship between the two cars. All the participants realized the authenticity of the
problem presented; however, they could not execute the proper problem solving
technique to correctly solve the problem. At this level of mathematics the students were,
for the most part, not ready to tackle this motion problem and simply had difficulty
transferring every day, real-world activities to the mathematics classroom. This reveals
that the participants did not attempt to apply real-life skills. At this point, participants became quite apprehensive and anxious while attempting to solve the problem although they were initially very enthusiastic about being in a mathematics classroom again.

**Mathematical Content Knowledge and Strategies Solving Mixture Problems Prior to Classroom Lesson**

When solving the mixture problem during the first interview, most of the participants stated that they would not know how to solve it without obtaining any additional knowledge. This reveals that they didn’t have any prior experience or couldn’t access the right information from their memory in order to solve the problem. When asked to restate the problem, only half of them were able to restate it comprehensively and identify the given and the unknown. Only one participant was able to introduce a variable, to set up an equation, and to obtain the correct answer. When asked questions about the information provided in the problem, the participants revealed the insufficient resources by not knowing the meaning of the concentration and the product of the amount of solution by its concentration. Most of the participants got confused when asked about the unit as well. At the same time, when asked to solve a concrete problem about a 50% alcohol solution, half of the participants were able to determine that 50% percent means half, but were not able to define the ingredients of such solution. Almost all of the participants agreed that the given problem is relevant to real life and were able to come up with proper examples. The entire participant group was asked to calculate the total
amount and to approximate the possible concentration of the mixture of two given solutions. All of them were able to calculate the total amount properly saying it is the sum of the original amounts, but only two participants approximated the resultant concentration as the average of the given ones. The rest of the participants added the percentages as well. These students didn’t express any concern about the consequential percentage being above 100%. There was terminology misunderstanding again. The participants were confusing the terms concentration, alcoholic solution, alcohol, and total amount.

At this point, the participants failed to identify the process needed to acquire a solution. They didn’t attempt to introduce a variable and/or set up an equation. As with the motion problem, the participants didn’t attempt to draw a picture or use any other graphical representation. In addition, most of the participants seemed to get overwhelmed and upset while attempting to solve the problem and/or answer my questions about the percent and concentration. Amongst the participants, there was a nurse, a bartender, several chefs and stay home moms, all jobs that could seemingly find real-life circumstances that could potentially relate to the problem. However the participants failed to apply any real-life critical thinking skills in a mathematics classroom.
Mathematical Content Knowledge and Strategies Solving Motion Problems after the Classroom Lesson

When solving the motion problem during the second interview, most of the participants began to make better use of their resources. All of the participants were able to restate the problem and identify the unknown and the given accurately. At the same time, a few participants still had difficulties with the attributes of motion, their units, and the distance formula. All of the participants but four applied the distance formula correctly. These participants were then successful at computing the time for a single vehicle. All of the participants but four introduced a variable and attempted to solve the motion problem algebraically. Half of the participants were able to obtain the correct answer algebraically. There is an apparent increase in the level of cognitive thinking used at this point since half of the participants set up and solved a proper equation. Most of the participants used a chart presented in class. Nevertheless, only three participants were able to explain the meaning of the equation set as the sum of two distances being equal to the total distance. This revealed that the participants knew the type of problem, followed the example outlined, yet failed to show their comprehension of the matter. The rest of the participants when asked to explain the steps replied that it was how they were shown the procedure in class. One participant did set up and solve an equation properly but at the same time, failed to state or write the distance formula. When talking about the heuristics and control, only a few participants attempted to use a picture to visualize the problem. Additionally, nobody tried to estimate and/or check the result after completion.
Half of the participants were asked by me to draw a picture. Half of the resulting pictures were incomplete, showing only the arrows representing the motion of two vehicles without the total distance and/or showing the distance given as a dot. Half of the participants solved the problem arithmetically thus failing to learn the algebraic approach to solving word problems in class. In addition, some of the participants still failed to find the meaning of the simultaneous motion from the context of the problem. No multiple representations had been used during the solution process.

As during the first interview, all the participants admitted that they drive on a regular basis and claimed that the given problem was relevant to a real-life situation.

**Mathematical Content Knowledge and Strategies Solving Mixture Problems after the Classroom Lesson**

There is a change in attitude observed during the solution of mixture problem after learning in class. Participants seemed to have an open mind and positive attitude approaching the problem but almost all became apprehensive and uncertain during the solution process. Two students ended up almost crying.

When asked to restate the problem, all but five participants were able to restate it logically and identify the given and the unknown. There is a definite increase in the resources utilized since all participants but one introduced a variable when attempting to solve the problem. Still, only seven participants used it properly and were able to set up a correct equation and solve it. A number of participants did correctly present the left side
of the equation as the sum of the amounts of pure alcohol in each original solution, but failed to recognize that the right side is the product of the amount of the final mixture by its concentration. The latest observation leads to the conclusion that the notion of equivalence is difficult for the adult students involved in the study to grasp.

A number of participants introduced a variable but failed to use it appropriately. At the same time, none of the participants were able to explain either the meanings of the separate terms of the equation as amounts of pure alcohol in each solution or the meaning of the whole equation. When asked for the reason for multiplication of the concentration by the amount, the participants would respond “don’t know,” “it was shown in class,” or “I don’t follow your question.” The students did learn the only strategy to solving the problem was to set up and solve the equation, but they still had difficulties executing it. The study also revealed that there is a lack of comprehension of such concepts as concentration, solution, and amount. There was no evidence that any participant clearly understood the meaning of the product of the amount of the solution by its concentration as the amount of pure substance. It seemed that the students involved in the study didn’t build the equation by understanding the process. Instead, they memorized the procedure and mimicked a previous example. When asked additional questions about finding the contents of twenty ounces of the 50% alcohol solution, half of the participants were able to determine that half of it, ten ounces, is alcohol. At the same time, the concrete example of the twenty percent solution would cause confusion, thus revealing that the students failed to execute any analogy.
In terms of heuristics and control, the study revealed that even after the topic was taught in class, the participants would not draw pictures to help with visualization of the problem unless asked by me, thus demonstrating a reluctance to use visual representations, preferring letter-symbolic ones. Nine participants were asked to make a picture, and only three participants presented comprehensive images consisting of two containers added together and resulting with a third one and proper labeling. The adult learners would not estimate the answer and/or execute analogies and would not restate the given problem unless asked to do so. Most of the students remembered that the example in class was solved using a chart. Consequently, they tried to memorize the chart without clear understanding of the components. Even most of the participants restated the problem logically thus showing their understanding of the situation described, half of them were unable to derive the plan of the solution and execute it properly. There is no evidence that the participants went through a critical analysis of the protocol necessary to either solve the problem or to provide an explanation of their work. Some participants would easily give up if they were not able to solve the problem quickly. In most cases, the strategies used were algebraic. This tells me that in most cases an algebraic approach was most retained from the participants’ classroom experience. While attempting to solve the problem, no numerical substitution or guessing had been attempted in order to arrive at the conclusion.

During the second interview, most of the participants agreed that the given problem is relevant to real life and were able to come up with appropriate examples. The
entire participant group was asked to calculate the total amount and to approximate the possible concentration of the mixture of two given solutions. As during the first interview, all of them were able to calculate the total amount properly saying it is the sum of the original amounts. Only six of these participants, however, approximated the resultant concentration as an average of the given ones. The rest of the participants added the percentages as well. As before, the participants didn’t express any concern about the consequential percentage being above 100%. It seems that there is no critical thinking that connects this problem to a real-life experience, a tactic that would typically jumpstart the process to resolve the problem. No construction, or meaning, was brought from domains like physics or chemistry, as participants only channeled their knowledge of algebra. Real-life experience such as professional careers (chefs and nurses were among participants in the study) were not referenced either to help with the solution process. Ultimately, there was a significant lack of utilizing outside knowledge or life experience in resolving the problem.
CHAPTER V

DISCUSSION OF FINDINGS AND IMPLICATIONS

This chapter begins with a summary of the findings relative to the research questions. This chapter also situates the findings within existing research. The chapter ends with the implications of the study’s findings for practice and research.

The purpose of the study is to understand and describe the experiences adult learners have while solving mathematical word problems. The research questions posed to accomplish this were

1. What attitudes and beliefs do adult students hold regarding mathematics education in general and word problem solving in particular?

2. What mathematical content knowledge do adult learners access when solving word problems?
   a) How is this knowledge used? How is it chosen?
   b) Why does the solution evolve the way it does?

3. What strategies do adult learners use to solve word problems?
   a) What formal approaches are employed?
   b) What informal approaches are employed?
   c) What are the adult students’ symbolic language and the dynamics of reasoning/thinking?
RQ1: The Attitudes and Beliefs of Adult Students Regarding Mathematics Education

Final Synthesis

Findings for the first research question revealed a relationship between participants’ cognition and affect. The attitudes, feelings and beliefs that the participants in the study hold toward mathematics and problem solving are an integral part of their mathematics learning experience. In addition, this relationship is rather dynamic, since there is a pattern in adult students’ attitudes toward mathematics education that involves their experiences in adolescence and how those experiences transcend into their secondary education. Contentment and feelings of confidence while in an elementary school arithmetic environment were replaced by discontent and lack of success in a secondary school algebraic environment. This feeling was ultimately replaced by determination in the college algebraic environment. This newfound commitment to academic success is often based on the students’ need to improve their economic situation and the realization that the mathematics courses are the hoops they must jump through to obtain the degree and/or the life style they are striving for.

Situating Findings with the Current Research

Adult students who participated in my study revealed that all the educational experiences in their past as well as in the present are associated with their emotional changes such as attitude toward schooling, socialization with peers and family, and
maturation. Almost all the participants mentioned their elementary school mathematics classroom as the earliest experience with math education. This period of time is portrayed with feelings of success and contentment when studying mathematics. As time progressed, they revised these experiences when continuing their education. The students reported losing interest to study in general and in mathematics in particular once reaching the post algebra period in middle and high school. This period is also aligned with the feelings of failure and cognitive difficulties connected to the algebra concepts that changed from concrete to abstract. In addition, the participants of the study revealed that all negative dispositions toward mathematics education that originated in their past did alter the way they approach math education as adults. At this point, even as the participants are determined to stay in school and strive for a better life, they still feel insecure about their mathematical abilities. In particular, most of them have difficulties with such concepts as variables, solving equations, and word problems. These findings seemed to agree with the number of studies concerning socio-psychological issues of adult students learning mathematics. Swindell (1995), Kasworm (1990), and Gal (2000) reported on the indirect deficiencies of adult learners such as negative prior educational experiences, lack of motivation and inadequate study skills. Many (e.g., Agar & Knopfmacher, 1995; Davis, 1996; Dhamma & Colwell, 2000; Gal, 2000; Ginsburg, Manly, & Schmitt, 2006; McLeod, 1992; Taylor, 1995) have argued that how well a numeracy situation is managed depends not only on the knowledge of mathematical rules and operations and linguistic skills, but also on the students’ beliefs, attitudes,
metacognitive habits and skills, self-concept, and feelings about the situation. When interviewing adult students who were taking developmental algebra, Safford (2000) reported that they expressed negative feelings toward math education such as fear of asking questions, phobia for math in general, lack of understanding leading to the fear of the subject, fear of failure, negative attitude and lack of knowledge of an instructor. The findings of the study also agree with Carter and Yackel (1989) and Evans (2000) who stated that affect in general and mathematical anxiety in particular were considered as relatively established characteristics of an adult learner, characteristics which have an ongoing effect on mathematical thinking, performance, and participation in mathematics courses. Ultimately, most of the participants admitted that their fear of mathematics was an obstacle to achieving their goals at some point or another in their life. In addition, when studying research on adults learning mathematics, Kasworm (1990) reported that chronological age is not the key variable in the process of learning mathematics. He stated that rather life experience, previous education, sociocultural context and attitude are more important. The age of the participants of the study ranged from eighteen years old to fifty. The data of my study reported that the participants’ attitudes and beliefs toward math education were similar without any correlation to their age.

As I presented above, the findings of my study align with the current research on the connection of emotions and cognitive experiences of adult learners. At the same time, the uniqueness of the study is in revealing that there is the clear pattern in the connection. As a refinement to the current research, my study suggests that positive emotions
connected to an elementary school/arithmetic period, and then hesitation/confusion/negative emotions/avoidance connected with a post-arithmetic/algebraic period and later an intentional decision to go back to school deemed as a norm for the students I surveyed.

**RQ1: Adult Students’ Attitudes toward Solving Word Problems**

**Final Synthesis**

The first research question also includes the topic of adult students’ attitude and beliefs toward solving mathematical word problems. The major finding of the study concerning the research question was that the participants expressed having difficulties with algebraic concepts in general and word problems in particular.

**Situating Finding with the Current Research**

The finding of the study about the adult students’ attitude toward solving mathematical word problems corresponds to the findings expressed in a number of research projects (Cummins, 1991; De Corte & Verschaffel, 1988; Greeno and Heller, 1983; Greer, 1993; Moyer, Sowder, Threadgill & Moyer, 1984; Nesher, 1982; Okamoto, 1996; Pape, 2004; Riley & Fuson, 1992; Vergnaud, 1982) that pointed to the fact that students, children and adults, have difficulties solving word problems. There are a number of difficulties that students encounter when attempting to solve word problems. Some of these difficulties such as phrasing of the problem, understanding the abstract
language of algebra, and reading comprehension, are among the ones mentioned by the participants of my study. Since this theme is directly linked to the second and the third research questions of the study, more information on the solving of the word problems will be given in the subsequent sections.

**Implications for Affective Domain (RQ1)**

I would like to begin the discussion of the implication of my study with the words of Evans (2000) who claims

“In much of the research on the use of mathematics by adults, the situation has not changed much from that described in psychology by Vygotsky three generations ago. There is still little or no explicit acknowledgment of the importance of the affective – feelings of anxiety, frustration, pleasure, and/or satisfaction which attend the learning of mathematics and the solution of numerate problems” (p.108).

Consequently, the current research states that one of the fundamental goals of adult mathematics education is not only to deal with purely cognitive issues, but also with students’ dispositions and beliefs toward mathematics education. To work on the latter, mathematics educators should present quantitative reasoning as a practical means to approach life’s challenges as a gate opener, instead of a gatekeeper (Benn, 1997; Gal, Ginsburg, & Schau, 1997). They also should be particularly sensitive to their students’ previous experiences of mathematics and their cultural and social approaches to
quantitative situations (Coben, 2000). As I have stated above, the uniqueness of this study is in revealing the pattern of adult students’ affective domain within quite an extended period of time. Consequently, the implication of the study’s findings is to help adult learners with understanding the pattern and with supporting the present determination to succeed in this often difficult but crucial endeavor.

The participants of the study revealed that they have negative attitudes toward mathematics and problem solving. In particular, these negative connotations were noted when students were enrolled in pre-algebra and algebra courses. This corresponded to his or her adolescent years. In addition, the participants revealed that their attitudes and emotions negatively affected their learning outcomes. They did not put forth much effort or very much time. Therefore, there is a great importance and necessity for instructors in changing these attitudes and beliefs in order to assure learning. In order to proactively encourage a positive mathematics attitude amongst adult students, the results of my study imply that there is a set of knowledge and techniques that instructors should employ. Since these recommendations are beyond the scope of the specific outcomes of the course, instructors will need to be carefully motivated to embrace them. It would not be sufficient to simply present this information within department meetings, faculty colloquiums, or other small-scale faculty development opportunities (especially if this is a directive of the administration). Instead, these venues may be used to provide the initial motivation based on research. Following that, a program of activities or workshops, designed by master teachers who have already embraced these techniques, can be offered
over the course of a semester or academic year. Instructors may be motivated by the efficacy of these concepts as research suggests. Additionally, through completion of this program, instructors may earn a designation as a Specialist of Math and Affective Domain to legitimize the comprehensiveness of the training and motivate the instructor with special recognition. The program would be designed based on the specific needs of the school rather than a one-size-fits-all approach. For example, a single-campus school may design a two-week workshop comprised of evening meetings throughout the semester, but a multi-campus school may choose to offer an intensive one-week retreat between semesters. Whatever the logistical design, the program should be motivated by other instructors and should include the following:

- **Conversations with students** – There is tremendous value in holding a conversation with each student regarding their attitudes and previous math experience during the first week of class. Such conversations may take place as a group discussion or as an individual talk. Some instructors may be a bit intimidated as well with this verbal discussion. In this case, a written questionnaire could be completed with relaxing classical music playing in the background. The questionnaires could be done anonymously if the instructor chooses. Instructors should urge their students to be honest when describing their feelings, even if feelings are negative. In a follow-up group discussion, instructors should take the time to write a few key words from each student on the board and then ask the group to discuss. This can foster a support system and a true sense of
belongingness for students so they can understand that many others share their struggle with learning math.

- **Emotional and social development of students** - Instructors need to be aware of the fact that adult students (as the data of this study revealed) have a great deal of difficulty learning math during the prealgebra-algebra period. The participants explained this phenomenon as having no aptitude toward mathematics. At the same time, the data collected revealed that this period was marked by active socializing and losing interest in schooling in general and mathematics in particular. Consequently, when discussing the prealgebra-algebra period, it is vital to explain to the students that the adolescent years (when algebra is usually taught) are filled with many changes such as maturation, shifting social demands, and conflicting role demands. These impactful changes may lead to positive or negative outcomes upon the overall educational experience. It is also imperative to ask students to reflect on their behavior and attitude toward school in their adolescence. Perhaps this can prompt them to consider how their behavior and attitudes rather than the lack of ability produced the negativity toward math.

- **Growth mindset** – Students must learn that their basic intelligence is malleable and they can expand it with work; and instructors need to facilitate this. Since intelligence is malleable, humans are capable of learning and mastering new things at any time. To reinforce the scientific validity of the message, the video should be shown that discusses how the brain, and hence intelligence, is capable
of growing making connections throughout life (Andersen, 2007; Aronson, Fried, & Good, 2002; Blackwell, Trzesniewski, & Dweck, 2007; Dweck, 2010; PERTS, 2011; Stigler, 2012). Such information would help students feel more confident so that they can become smarter and achieve more.

- **Concepts versus procedures** - Assignments which deal with mathematical concepts in addition to the procedures being taught in class have farther-reaching effects. For example, career projects could be assigned. Interviews could be conducted and the use of mathematics in these careers could be explored. Another example of such an assignment can be an “Art Museum Project”. The students are sent to an Art Museum to explore mathematical concepts in artwork. After that they do the class presentation and write in his or her math journal about the experience. Another suggested project is to ask students to bring in and present an object in class that is relevant to their understanding of mathematics. All of these projects explore mathematical concepts and will help students to realize mathematics in the world around them. The realization that mathematics is relevant to them will improve a students’ attitude toward learning mathematics (Langer-Osuna, 2011; Lobato & Siebert, 2002; Palm, 2008; Plus Magazine, 2014; SCALE Institute, 2009; Science Education Resource Center at Carleton University, 2014).

- **Creating a comfortable learning environment** - When appropriate, instructors may use educational/mathematical games in class to alleviate the students’
negative feelings about mathematics and put them at ease. Games, when used properly promote students being actively engaged with the mathematical concepts being reviewed. The games also promote a comfortable and nurturing environment (Plus Magazine, 2014; SCALE Institute, 2009).

- **Scaffolding and assessments as tools to teach problem-solving** - When teaching solving word problems, instructors can divide the problem into single tasks, because students are apprehensive to solve problems that require multiple steps. With single task assessments, the students are provided with the needed reinforcements and encouragements (Alter, 2012; Jonassen, 2003; Reusser, 2000).

**RQ2: Content Knowledge Adult Learners Access Solving Word Problems**

**Final Synthesis**

The major finding of the study related to the second research question is that the majority of the participants in the study were not ready to approach the traditional word problems offered in beginning algebra courses. My findings give evidence that one of the core reasons for students’ lack of readiness is the absence of necessary cognitive resources/previous knowledge (Schoenfeld, 1985). In addition, the majority of the participants displayed no transfer of learning between the classroom and everyday activities.
Situating Findings with the Current Research

The findings of the study directly related to the mathematical resources of adult students correlate with the findings expressed by Lave (1988), Saxe (1991), Schliemann and Acioly (1989), and Wedege (1998). These researchers reported the apparent contradiction demonstrated by adults having difficulties with mathematics in formal educational settings, but performing competently in everyday quantitative situations. My findings also concur with those of Schoenfeld (1985) who argued that the “naïve physics” presents difficulties for students and that the students often follow their often incorrect intuitions/preconceptions about physical situations that are interfering with the learning of the formal principles. Moreover, the study findings support the notion of Lester (1994), Passmore (2003), and Schoenfeld (1997) who argued that besides training in the strategies of problem solving, students also need training in the resources and control of the problem solving process. The lack of cognitive resources (such as the deficiency of knowledge of basic science including the formula $D = RT$ and the concept of concentrations) and control strategies (such as metacognition) prevented further success for the participants. The findings of my study revealed that while the participants acquired the knowledge of everyday concrete, authentic mathematics such as remembering that in order to obtain a time needed for traveling one needs to divide the value of miles by the value of rate and/or that 50% solution means half, and 80% solution has more of something than the 50% one, neither the scientific knowing nor the critical one has taken place yet. When the participants were presented with a situation of two
vehicles moving simultaneously and/or two solutions being combined, they were not able to solve the problem. In addition, the findings of the study about the adult students lacking the resources for solving real life word problems can be connected to the concept of *commonsense* presented by Colleran, O’Donoghue, and Murphy (2002). The authors affirmed that usual, everyday commonsense provides a confident basis on which adult learners began quantitative problem-solving processes. *Commonsense* is the basis for forming the *scientific knowing* which is then employed when an individual engages a new situation and the mental processes move from the concrete to the abstract, and the *critical knowing* which enables learners to solve quantitative problems. The finding of my study seemed to support the argument by Colleran, O’Donoghue, and Murphy (2002) which stated that the critical knowing cannot be achieved without the scientific one, and the latter cannot be achieved without *commonsense*. The participants in my study lacked the resources as many participants did in other studies. This research project, however, is truly unique because the participants showed weakness in solving word problems in the specific conceptual areas asked – motion and concentration. The concepts of motion and concentration are authentic to real life. The participants do drive and they do work with mixtures such as cooking and shopping in their everyday life. Some of the participants were nurses and chefs. One participant used to be an air traffic controller. However, they were not able to transfer their knowledge to complete the word problems successfully. These two concepts are the basis for the majority of the word problems presented in
Beginning Algebra courses. The data extrapolated from my research is pertinent to the teaching of these courses.

**RQ2: The Evolution of the Answer**

**Final Synthesis**

The data collected relevant to the evolution of the answer revealed that most of the participants failed to obtain a reasonable answer when solving the word problems during the first interview. In addition, the participants who were able to solve the given problems by an equation during the second interview were not able to explain their method of solving the problem or why they chose the method. Solving the offered word problems by an equation was the only algebraic method used by the participants. The fact that the participants were not able to explain their method of solving the problems and/or why they chose the method demonstrated that no situational level of understanding took place. According to Hung (2000), the situational level of understanding is one of the vital characteristics of a successful problem solver and comprises the ability to comprehend the inner relationships in the problem statements and the solution process. This third level of understanding is founded on the first level, the symbol level, and the second one, the problem level.
Situating Findings with the Current Research

As the examples above reveal, no situational level of understanding took place. Hung (2000) stated that achieving only the first level—the symbol level of understanding—and, sometimes, the second one—the problem level is typical for children solving word problems. The distinctiveness of my study is in supporting that this argument is true for adult learners as well. The participants of the study seemed to be deficient of the comprehension of the physical meaning of the word problems, thus failing to achieve the problem level of understanding. Consequently, they failed to solve and/or comprehend the problems, thus failing to reach the situational level of understanding.

The findings of the study also seemed to correspond to the notion by Ginsburg, et al. (2006), Nunes, Schliemann, & Carraher (1993), Verschaffel, de Corte & Lazure (1994), and Greer (1993, 1997) who reported that extensive experience with traditional school mathematics word problems develops in young students a strong inclination to exclude real world knowledge and reasonable thought from their solution process. Verschaffel and de Corte (1997) also highlighted the role of being familiar with “the game of” school word problems—puzzle like tasks—as a main factor of the ability to deal successfully with word problems. The exclusive findings of my study pointed out that the adult students, similar to the widespread tendency of children, also answer school mathematics word problems irrespective of the reality of the situations described by the text of these problems. In addition, they derive the answer by following a procedure rather than truly understanding a problem.
It is imperative to mention that Schoenfeld (1992) argued that one of most common beliefs of students of all ages taking mathematics classes is that the mathematics learned in school has little or nothing to do with the real world. Even though the participants of my study all stated that the word problems in general and the ones offered in the study were related to real life, they were unable to see the connections between what they know of their own worlds to model any of it with mathematics.

**RQ3: Approaches for Solving Word Problems and the Reasoning and Thinking when Solving Word Problems**

**Final Synthesis**

The data collected revealed that prior to any formal instruction in dealing with solving these types of problems, students used informal/arithmetic strategies in solving the problems. After formal instructions, some but not all (N=8) students attempted to use the algebraic methods/solving by equation they were taught in class. The rest of the students still attempted to solve the problems arithmetically. The participants were able to restate the problems logically; however, the participants experienced difficulties in using formal mathematical tools in the representation of the problems or the solutions and/or comprehending the relationships between the situation presented and the representations (symbolic and/or pictorial) to solve the problems correctly. The participants who attempted to employ the representations used letter-symbolic representations, such as variables and equations, over graphical, such as pictures and graphs. The students who
used letter-symbolic representations and graphical ones failed to make a connection between the representations. Some of the participants were not able to model the problems correctly. A model is a specific structure of some kind that symbolizes the features of an object, a situation, or a class or situations or phenomena (Goldin, 2008, p.184). From the cognitive prospective, mathematical problem solving processes begin with reading of the problem’s text and proceed with forming an object-based or mental model constructed on prior knowledge and the text elements (Nathan et al., 1992). Verschaffel, Greer, and Corte (2000) proposed that mathematics has two faces, “one being its role in describing aspects of the world, and the other the construction of abstract structures; modeling forms the link between these two aspects” (p. 125). The starting point of a modeling process is an actual situation under analysis. The first stage is the construction of mental model mediated by external representations, reflecting the main variables in the situation and relations between them. At the next stage, mathematical equations are set up that reflect the model. After that the equations are worked out and interpreted. As the data revealed, there was no evidence that the participants were able to analyze the situation given, construct a mental model, and set up and solve an equation. If, at times, the proper equation was set up, it was not interpreted properly afterwards. In summary, the participants of the study were not successful in their mathematical modeling.
Situating Findings with the Current Research

The findings of the study relevant to the methods used by students when solving word problems and their thinking processes seemed to correspond to the notion expressed by Shoenfeld (1985, 1992), Lester (1994), Lesh (1982), and Silver (1982) who argued that the lack of mastery of heuristics and the control issues (Shoenfeld, 1985) including metacognition would lead to failure when solving mathematical problems in general. The participants of the study seemed to have difficulties with choosing and pursuing the constructive approach, the ability to understand and recover from wrong choices, and the ability to maintain an internal and/or external dialogue by either asking themselves questions or asking me. The current research reports on the lack of heuristics and control mastery leading to the lack of success in solving mathematics problems in general. The distinction of this study’s findings is in reporting a similar outcome when solving mathematical word problems by adult learners in particular.

The findings of the study also seemed to support the notion expressed by Doerr & English (2003), English (2006), Goldin (2003; 2008), Lesh & Doerr (2003), Janvier (1987), Kaput (2007), Lesh & Zawojewski (2007), Pape (2004), Thompson & Yoon (2007), Kintsch & Greeno (1985), and Vergnaud (1998) which argued that the lack of engagement in modeling activities is one of the major reasons for the deficiency of success when solving mathematical problems. The data collected suggested that the participants were not actively involved in modeling activities since they did not present any clear indication of seeing the problem(s) as combination of a story told, graphical and
numerical representations used, and the answer obtained. The results presented in the current research are based on the mathematical problem solving by young students. The uniqueness of my study is in reporting that the modeling activities for adult students solving word problems are similar to children’s.

Another fundamental issue related to this and the previous research is the representations used by adult students when solving word problems. A representation is a configuration that represents something else in some way (Goldin, 2008). There are external and internal representational systems for mathematics. Some external systems include formulas, algebraic concepts, and theorems. Internal systems include such aspects as language, personal constructs, visual and spatial imagery, problem solving heuristics, and affects (Goldin, 2008; Kaput, 1994). Internal representations and abstraction are closely related mental constructs. Each type of representation articulates different meanings of mathematical concepts. In addition, a fluent and flexible use of multiple representations of “structurally the same” mathematical concept is associated with deep conceptual understanding (Dreyfus & Eisenberg, 1996, p. 268).

The findings of the study seemed to support the argument presented by Cifarelli (1993), Goldin (2008), Kieran (2007), Radford (2004), Heyd-Matzuyanim and Sfard (2012) and Sfard and Linchevski (1994) who stated that no effective learning in general and problem solving in particular takes place when young students don’t make inferences between internal representations and external representations. This study also confirms current research with the consistent difficulties the participants had with deriving
meanings from an algebraic structure itself, involving the letter-symbolic and graphical
and the meanings from the problem context. At the same time, my study is revealing that
not only young students but adult students as well neither have had strong external
representations constructed previously, such as knowledge of the concept of motion and
/or concentration, nor had strong internal ones, such as inability to draw on the intuitive
ideas to help solve word problems. The participating adult students had trouble
visualizing the stories, converting them into pictures, translating between the text and
algebraic symbols and creating quantitative representations such as equations and charts.
In addition, the participants’ weakness toward pictorial representations might have added
to creating a barrier for the development of conceptual understanding of the problem
solving process. Furthermore, the findings of my study show that adult students, as well
as children, experience difficulties with aligning the semantics and syntactic of ordinary
language with the structure of formal mathematical reasoning.

Sfard (1991) argued that the same representation or mathematical concept may
sometimes be interpreted as process and at other times as object. In other words, the
representation or concepts may be conceived both operationally and structurally (p. 193).
She also argued that these two different ways to see mathematical constructs are
Sfard and Linchevski (1994), and Heyd-Matzuyanim and Sfard (2012) stated that the
ability to capture the structural feature is not easy to achieve. This ability is crucial for the
development of algebraic reasoning and successful problem solving. The researchers
stated that high school students more often than not cannot cope with problems which do not yield to the standard algorithm. Similarly, the findings of my study revealed that if at times the participants were able to set and solve a correct equation, they were not able to explain either the concept of motion of two vehicles or the mixture concept logically, thus revealing that the crucial junctions in the development of algebraic thinking where a transition from one level to another takes place are missing.

The findings of the study also seemed to support the argument by Nunes, Schliemann, & Carraher (1993) who stated that the transfer of knowledge progresses from informal mathematics knowledge towards the formal knowledge rather than the reverse process during the problem solving activity. When asked to solve the motion problem during the first interview, the participants who solved it did so informally by using an arithmetic approach. Also, they did it orally. Even after the algebraic approach was taught in their algebra class, half of the participants solved it based on their intuition rather than on the formal approach taught. Thus, the data collected revealed that despite spending a good deal of time studying algebraic concepts and techniques to apply to word problems solving, the participants were not able to combine the intuitive/informal ideas with formal procedures. Furthermore, the participants’ approaches were often based on incorrect informal assumptions rather than correct formal knowledge of the concepts of motion and concentration. The participants were unsure of the formula D=RT and at times used it incorrectly. They also failed to calculate the amount of an agent as the product of the concentration and the amount of the solution. If the Nunes, Schliemann, &
Carracher (1993) argument was for mathematical problem solving activity in general, the present research finding uniquely supports the argument that the notion is true for word problem solving in particular as well.

According to the current research, the factors detailing the difficulties students encounter in word problem solving are (a) the type of problem situation, (b) the exact phrasing of the problem, (c) the particular numbers used, (d) the age and instructional background of the students, (e) the linguistic knowledge of a student, (f) comprehending abstract language of mathematics (Cummins, 1991; De Corte & Verschaffel, 1988; Greeno & Heller, 1983; Greer, 1993; Nesher, 1982; Okamoto, 1996; Pape, 2004; Riley & Fuson, 1992; Sfard, 2013; Vergnaud, 1982). These studies focused on middle and high school students taking algebra. Even though in my study I didn’t focus on each individual category presented above, the data collected seemed to support the argument that the type of a word problem (motion versus mixture) has an influence on the process of the word problem solving by adult students as well. As the data collected revealed, the majority of the participants admitted that solving motion type word problems is a less difficult task than solving concentration type word problems.

The findings of this study also support the argument made by Caldwell and Goldin (1979), Nathan & Koedinger (2000), Bednarz & Janvier (1996) and Riley et al. (1983) that concrete verbal (arithmetic, result-unknown) problems are substantially less difficult than abstract (algebraic, start unknown) ones. Bednarz & Janvier (1996), Stacey & MacGregor (2000), Ginsburg, Manly, & Schmitt (2006), and Herskovics & Linchevski
(1994) also stated that for some students difficulties in the word problem solving process are caused and/or accompanied by a refusal to operate on the unknown and, consequently, dealing with the symbolism used to present the relationships in the equation. The results of my research agree with the statements of the relevant literature by revealing that even though some participants were able to solve a single-step arithmetic problem asking for an amount of agent in a solution, they were not able to solve an algebraic, start-unknown problem about two solutions mixed together; thus stating that for the adult learners as well as for children algebraic approaches present more difficulties than arithmetic ones since such change requires a new way of thinking.

**Implications for Instruction (RQ2 and RQ3)**

The findings of my study indicate that students taught in a traditional mathematics classroom undervalue, or altogether omit, personal real world knowledge and reasonable thought from their solution process. Based upon these findings, the following are recommendations for teaching mathematics to adult learners:

- **Connecting to real-life experiences** – Facilitation of this connection occurs by teaching students to transfer their personal real-life experiences into solving mathematical word problems by (a) gathering the information about students’ educational and occupational background at the beginning of the semester, and (b) creating mathematical word problems that reflect (based upon) the concepts/real-life situations described by the students. Such problems will be drawing students’
attention to realistic modeling (Depaepe, De Corte, & Verschaffel, 2010; Palm, 2008; Vamvakoussi, Van Dooren, & Verschaffel, 2012).

- **Learning in context** - The teaching of word problems currently offered in mathematics classroom should be supplemented by an integrated approach when motion problems are taught in physics class and/or lab and concentration (mixture) problems are taught in chemistry class and/or lab. Geometry and measurement word problems should be taught using actual measuring devises and/or graphing software. Thus, students then would not implicitly learn to follow the directions given in class without actual comprehension, but would learn modeling using real or pseudo-real situations (such as visual objects, physical models, motion sensors and/or technology) that stimulate algebraic thinking. Using such authentic modeling would help students to transfer the internal/informal knowledge to the external/abstract knowledge and bring sense to the formal abstract notations used in algebraic problem solving. In addition, this modeling would provide more informal learning experience to ensure the movement to more abstract understanding of the concepts taught.

- **Adult-oriented math** – Expand the types of word problems offered in class to include ones based on the activities the adult learners participate in everyday life outside of their professional activities including but not limited to: (a) purchase and/or consumption of food and other necessities, (b) involvement and/or rising children and taking care of elderly, and (c) paying for rent/mortgage/car.
• **Maintaining a balanced focus** – When teaching solving word problems, incorporate a balance of generality and situational features; thus enabling students to anticipate the similarities across situations, and differences respectively. That is, social language needs to be connected to academic language.

• **Language and literacy** - Teaching solving word problems should incorporate the concept of terminology since there is a critical problem between the language we use and mathematical language. Students should be encouraged to translate mathematical concepts into their own words. However, as understanding occurs, precise terminology should be a goal. For example, in the study, the participants have difficulties understanding and applying such terms as concentration, percentage, attributes of motion, and amount. Instructors should model proper terminology in class while at the same time constantly assessing the students’ comprehension. There can be written assignments in which terminology will need to be included. The knowledge of terminology should be part of the assessment process. Nevertheless, the terminology and the sentence structures used in the word problems should be no more difficult that the ones used in the real life situation. (Sfard, A. (2013).

• **Specific and authentic contexts** - The information given in the word problems should be specific, not general; the data and the values must be realistic in the sense of identical to the corresponding numbers in real life situations (Depaepe, De Corte, & Verschaffel, 2010).
• **Teaching students to analyze** - The instructor should promote the development of the metacognition skills needed for word problem solving. They should teach the learners how to analyze the problems and their components, establish inter-relations between the given and the unknown, estimate and analyze the validity of the answers obtained, and analyze similarities and differences between situations. For example, when teaching motion word problems, learners should be taught to identify the given attributes of motion, the unknown, and the formula needed to find the unknown. In addition, learners should be presented with situations of singular motion, independent motion, and dependent motion. Special attention should be paid to the texts used. If the textbooks chosen do not provide the opportunity to do this, then there is a need to reconsider the texts used in the courses.

• **Understanding the reasonableness of answers** - Students should be encouraged to make sense of any solutions they offer to problems. That is, they are able to say why the answer makes sense given the original question/problem.

• **Collaborative learning** - Instructors should carefully develop collaborative learning activities so the students would be put in a situation that requires asking/answering questions related to the problem (Science Education Resource Center at Carleton University, 2014; PERTS, 2011).

• **Use of multiple representations** - The use of representations such as diagrams, tables, graphs, and equations may increase the chances of success by ensuring the
movement to more abstract understanding. Since good representational ability is critical to the development of solutions to many real-life problems solving, the step of creating the representations should be graded separately. Furthermore, students should be requested to provide different types of representations such as pictures, diagrams, charts to earn full credit. It is pertinent to remember that diagrams are powerful strategies to use since they help to unpack the structure of the problem, present the connection(s) between the objects given, and/or abstract concepts concrete. In case that drawing such diagram becomes a stumbling block, the students need to be provided them at the beginning. The students should be asked to present a diagram that has at least two characteristics: (a) image only, where the information from the problem is depicted, and (b) organizer, where the data or information depicted to help understand the problem. Instructors can model drawing his or her interpretations of the word problems during a lecture. The assessment can be designed to incorporate this strategy. Practical suggestions can be made during a professional development workshop where best practices are discussed. An instructor should bear in mind the courses students will be taking and choose strategies that are portable to those courses (Carnegie Mellon University Eberly Center, 2013; Heyd-Matzuyanim, & Sfard, 2012; Van Garderen, 2007).

- **Developing writing skills** - Quantitative writing assignments should be a mandatory part of the word problem solution process. Quantitative writing is the
written explanation of a quantitative analysis, thus it facilitates learners’ understanding of the problem’s structure and concepts rather than simply applying formulae. These assignments ask students to produce a claim with supportive reason and evidence rather than “the answer”. The students should be asked to write the following: (a) Restate the problem in your own words, (b) Identify the given the unknown, (c) Hypothesize: I have… I want to find…If I… then…, and (d) Estimate the answer. The quantitative writing assignment should be a separately graded part of the overall assessment (Science Education Resource Center at Carleton University, 2013).

- **Learning through assessment** - Instructors should consider increasing the quantity of assessments of a topic of word problem solving since more learning occurs through testing (Association for Psychological Science, 2012; Belluck, 2011).

- **Professional development** - Continued professional development is necessary to understand various teaching pedagogies. Since the teaching and learning philosophies of educators greatly impact the problem solving capabilities of our students, teaching pedagogy and styles are crucial in adult education. By understanding multiple philosophical approaches, instructors can better appreciate and recognize a variety of learner characteristics and teaching methods. One of these pedagogical approaches is the Constructivist approach that puts emphasis on socio-cultural and cognitive perspectives, consideration of students’ goals, values
and beliefs. In addition, instructors must stay current with adult mathematics learning research and best practices. This can be obtained through further education and by attending workshops and conferences (Carnegie Mellon University Eberly Center, 2013; Science Education Resource Center at Carleton University, 2014).

**Implications for Future Research**

Based on what I found relevant to adult students solving word problems, I believe that in-depth studying of the informal ways adult students solve algebraic word problems would enhance the effort of math educators to help students make progress toward making sense of motion, mixture, and other related word problems. In my study, I observed the participants attempting to solve the motion problem using an informal arithmetic approach. Nevertheless, the variety of the informal ways to solve word problems was outside of the scope of my research; therefore, additional research in the area might bring more light into the discussion. In addition, more research is needed in the area of adult students using single and multi-representations including binary quantitative representations such as charts, pictures, and graphs. In addition, there is a notion expressed in the relevant literature that creating communities of practice would benefit adult students’ success in a mathematics classroom. This notion is based on the Vygotsky’s zone of proximal development and scaffolding and states that knowledge is not only contextually but socially situated. Consequently, I would also suggest the topic
of collaborative word problem solving by adult students for future research. Another issue that plays an important role in word problem solving is reading comprehension of mathematical texts/symbols/data. So the interrelationship between mathematical word problems and reading comprehension of adult learners would be an important addition to the topic. Yerushalmy (2000) and Schwartz and Hershkowitz (1999) when studying the long term impact of a graphing technology and function-based algebra on problem-solving abilities on middle school students stated that both approaches enhance the level of success. Therefore similar research based on the adult learners would undeniably be of great value to the field.

In addition, I would recommend future research on the affective domain of students in different educational settings, such as various four-year schools and trade schools. I would also recommend studying the affective domain of solving word problems outside of the formal school setting. Another recommended research would be a large scale quantitative research. These studies might yield different and/or additional data.

**Conclusion**

This study has provided me with a great opportunity to carefully examine the experiences of adult students solving mathematical word problems. In the end, I was supplied with a great deal of information about adult learners’ mathematical thinking when solving word problems and the reasons the learners experience so much difficulty during the process. The study’s conclusion was that adult students experience difficulties
when solving algebraic word problems partially as the result of the deficiency in their
cognitive resources as well as their beliefs, attitudes, and experiences of mathematics.
Solving mathematical problems is one of the fundamental standards of math education
(NCTM Standards, 1989; Common Core State Standards, 2010); therefore, I believe any
research that is able to aid our students to become successful is vital. Without any doubt,
there is much yet to be learned, but I believe that the results of this study do add to the
overall research in the area. The findings of this study and the future research on the topic
can provide teachers with the necessary information for improving the current curriculum
and implementing the mathematical modeling integral to the Common Core.
Furthermore, this study will promote and encourage more pertinent research in the field
of adult mathematics education. This research is greatly needed to provide our society
with a better prepared workforce. The findings of the study can improve our students’
level of success in math classes, thus leading to the boost of their self-esteem and
willingness to persist in their educational endeavors. The field of adult mathematical
education is an enormous endeavor which has many diverse and unique characteristics.
One unique characteristic of the mathematics classroom is the great amount of anxiety
caused by negative prior experiences that the students exhibit. For many of them,
previous mathematics education at school was based on a memorization of processes.
This rote memorization was not successful. Other societal and educational structures also
play a significant role in adult education. These structures include methods of instruction,
cultural background, informal mathematical knowledge and cognitive resources. In the
light of the above, I hope that my study can make valuable insights to pedagogical approaches to teaching adults mathematics and to the research in the field.
APPENDIXES
APPENDIX A

SAMPLE INTERVIEW
Appendix A

Sample Interview

R = Researcher

P = Participant

R: Can you please tell me everything you remember about your previous math experience? I am interested in your experience prior to coming to college.

P: I guess, in elementary school I was pretty average in math, but as a kid I was very easily distracted. And it was always where I would get an understanding of 70 or 75 percent of whatever we were covering. And because I was daydreaming or talking, or drawing pictures, I would miss the part of bringing it all together. And so, you know, when it was time to do the homework, I would do it, or my mother would make me do it, and when she wasn’t looking, I simply would not do it. And I would go to school the next day and try to disappear in the classroom, so teacher would not call on me.

R: Was your mom able to help you with math homework if you would ask?

P: No, if it was simple problems… my mother was very impatient…I can’t say she wouldn’t have helped me, but I wouldn’t ask her.

R: I would say that she has stressed the importance of the school work.

P: Yes, she would compare my stepfather’s annual salary to her cousin’s annual salary. My stepfather was a sanitation worker for the city and my mother’s cousin was the valedictorian of her class and was working somewhere in the educational...
system. That always made an impression on me. In order to succeed you have to, you know, you have to go to school.

R: What can you recall about your math teachers then?

P: My math teachers were some of the best teachers that I have ever met. It was never them. It was always me. I think that …you see I went to school in the predominantly black community. And the teachers were genuinely concerned and wanted their students to learn. But there were classes where I was the class clown, I wouldn’t do the work, was disruptive in class and the teachers would give up on me. As I look back, they were truly good.

R: Do you remember the last math class that you took at school?

P: In high school? The last one I remember was in tenth grade. It was business math class.

R: Have you taken algebra or geometry?

P: No. They gave me the choice…the first day of school…the students had to go and see the guidance counselor so the guidance counselor would help them to map out the curriculum. I remember being in line to see the guidance counselor, and it was a girl in front of me, and the guidance counselor comes out with a pen and pad and asked the student: “what do you want to do when you graduate high school? What kind of work do you want to do?” And the girl in front of me said she wanted to work in a department store. And that is what I said when it was my turn. And so they sent me to Business Math, English, Social Studies, and Black
history. When I was asked what I wanted to do, that is when I shot myself in a foot. I was not in reality. So, school had nothing to do with my reality.

R: How long since you graduated from high school?

P: I didn’t graduate, I dropped out of school. I was eighteen years old, in the eleventh grade and I knew for me to continue to the twelfth grade, I would have a full schedule of classes to make up, because I didn’t do anything in the eleventh grade. So, I dropped out and joined the marine core. And I got my GED while in the Marine Core. And I didn’t score very high on it.

R: How many years have you been in the military?

P: Four

R: Have you done any math studies since then?

P: This is my third time doing Beginning Algebra. I am fairly sure that I could’ve passed both times, but both times I dropped out. I don’t know if I have gotten “A” or “B”, but I think I would have scored high enough to move on.

R: How do you feel being in math class now?

P: It makes me kind of nervous.

R: Why does it make you nervous?

P: First of all, I don’t test very well. As we cover something in class, I can pretty much rap my brain around it. But by the time I leave the classroom, I have forgotten a lot. And then in the evening, when I am at home doing my homework, I forget a lot. But I don’t cheat. I work out a problem, I come up with my answer
and then I check it with the back of the book. It takes me a very long time to do it. So, I check my answer with the back of the book, and if I am wrong, I go through the problem again. But still math makes me nervous.

R: What do you think makes a person a good problem solver?

P: I don’t know. I think that talents that we have are gifts, and some people have been given the gift of math, and they go into the trades of the employment areas that this country needs to become a civilization. We need mathematicians to build bridges, to build infrastructure, houses that will not fall, and these things require math.

R: What kind of degree do you have in mind?

P: Music therapy.

R: Now I am going to ask you to solve two word problems. Please read the first problem and try to solve it. Please show all your steps. Do you have a calculator?

P: Yes

R: You are welcome to use it if needed. Notes, graph, and formulas - all are welcome. Do you drive?

P: Yes

R: Do you work at the present time?

P: No

R: What kind of work have you been doing after the discharge from the military?
P: I had so many jobs since I left the marines, I lost count. The ones that were pretty good were with the County of Commissioners as a stock clerk. I worked at Control Data Corporation; I worked for the East Ohio Gas company as meter reader; I worked for the Department of Defense in the finance and accounting.

R: Was the latter job related to working with numbers?

P: No

R: Ok, thank you. Now please try to solve this problem

P: (Reading the first problem quietly for four minutes before starting writing the ideas down) Ok, I probably didn’t write it out right. It is six hours.

R: Please explain how you got the answer.

P: I have learned in Arithmetic and Prealgebra that distance equals rate times time.
So, the first plane …I wrote the distance…distance they gave me – 1000 miles – equals 300 miles per hour divided by 60 which is an hour. I got 18000, which is a ridiculous number to me.

R: What are the units of 18000?

P: Break it down in minutes… you divide that 18000 by 60

R: What are you thinking about?

P: Ok, I got a hundred hours here. It takes 60 minutes to take an hour. I got a hundred something and so I am subtracting 60 from that.

R: What is given in the problem?

P: How fast the planes are going and how many miles apart.
R: What are we looking for?

P: How many miles apart they will be …after flying…at a certain rate. How many hours would it take?

R: So, what are we looking for? What do we measure using hours?

P: Seconds, minutes

R: What concept do we measure in hours?

P: Time

R: Do you remember the formula that connects distance, rate, and time?

P: Yes

R: Have you used the formula in your solution?

P: Yes

R: Where have you used it?

P: 300 miles per hour is the rate, the hour is the time, and the distance is the 1000 miles apart. Then I did the same for the second plane and then I subtracted it. Obviously, the first plane will reach the certain point faster than the second plane will. You want to know when they are 1000 miles apart, so I subtracted the information of the second plane from the information of the first plane.

R: And you got six hours, correct?

P: Actually, I got an hour and 40 minutes. Because when I subtract 200, that is how long it will take the second plane; from 300 it is how long it will take the first
one, I got 100. One hour is 60 minutes; I subtracted that from 100 and it gives me
40 minutes left over, an hour and 40 minutes.

R: Do you think this problem is relevant to real life?
P: Oh, yea

R: Do you remember solving similar problems before?
P: No

R: Do you remember flying in an airplane?
P: Yes.

R: Now we are going to look at the second problem. Can you please tell me
everything you can about the solution beginning with restating the problem

P: You are trying to mix 50% alcohol with 20% alcohol to make 40 proof.

R: When in real life would people need to mix together two solutions?
P: To create a chemical, a reaction, a cause and an effect, to mix ingredients to do
something with it… because the properties of two or more ingredients are
needed to do something that they want.

R: When you multiplied .20 and 16, what were you looking for?
P: Sixteen ounces make a pound, so that is the only thing I know that to measure
by ounces. Sixteen may not be percentages. The percentages were given.

R: What is .20?
P: .20 is 20%.

R: You also multiplied .5 by .16. Please explain the meaning of the product.
P: Point fifty is 50%. And 50% multiplied by .16 is 8.00. 20% multiplied by .16 is 3.00. I subtracted 3.00 from 8.00.

R: You got three and twenty hundredth, what does it stand for?

P: Ounces…percentage…I don’t know what it represents.

R: What about the second number: eight?

P: Same thing, I really have no idea what it represents.

R: What is your final answer?

P: Nineteen point twenty.

R: And what is the meaning of the value?

P: I would say it is ounces but I was going to multiply those 19.20 by .80 or 80 ounces.

R: So, what is the final answer?

P: Thirty five hundreds thirty six

R: And what are the units of the number?

P: Ounces

R: When you put two solutions together, 20 ounces of one, and 30 ounces of another one, what is the final amount or the amount of the mixture?

P: Fifty

R: If you mix together two solutions: 50% solution and 70 % alcohol solution, what is the possible concentration of the mixture?

P: 120%
R: Do you think this problem is relevant to real life?

P: Sure

R: What does 50% alcohol solution mean?

P: Half is alcohol, half is water.
APPENDIX B

LETTER OF CONSENT
Appendix B

Letter of Consent

Informed Consent to Participate in a Research Study

*Study Title:* Investigating the Adult Learners’ Experience when Solving Mathematical Word Problems

**Principal Investigator:** Ellen Brook

You are being invited to participate in a research study. This consent form will provide you with information on the research project, what you will need to do, and the associated risks and benefits of the research. Your participation is voluntary. Please read this form carefully. It is important that you ask questions and fully understand the research in order to make an informed decision. You will receive a copy of this document to take with you.

**Purpose:** The purpose of the study is to understand and describe the experiences adult learners have while solving mathematical word problems in order to

- better understand the deep and different sociocultural and cognitive meanings of algebraic thinking of adult learners when attempting to solve word problems in order to provide teachers with new approaches to teach algebra;
- provide some insights about the way of introducing and structuring the topic;
- rethink the role of word problems in teaching algebra.

**Procedures:** You are enrolled in Beginning Algebra course for the Spring 2012 semester. All students enrolled in this section of the course will be invited to participate. I intend to
conduct interviews and observations before and after the formal teaching of solving word problems in the chapter on Linear Equations (approximately in January and March). You will be offered to solve a mathematical word problem and then asked questions about your approach. In addition, I will collect all artifacts (pictures, diagrams, graphs, etc.) produced by the students while attempting word problem solving. You are being asked for your permission to being audio and video taped and for collection of these artifacts which will then be analyzed for this investigation. Subsequent interviews are possible to clear up the data obtained by previous interviewing.

I am planning to design the observation and interviews to take about one to one-and-a half hour period. In every observation and interview, optional objects would be provided for external representation, such as paper, pencils, markers, and a hand calculator.

**Audio and Video Recording and Photography**

In addition, you are being asked for your permission to being audio and video taped during the observations and interviews. The tapes will be kept in the locked cabinet in my office after being analyzed for about 5 years. You have an option of seeing or hearing the tapes prior to their use.

**Benefits**

- Since all the word problems for this study belong to the mathematical curriculum, you, as a student, may benefit from the opportunity to reflect on their reasoning process as they work on solving these problems;

- As a participant, you also may benefit from knowing that you experience could inform future teaching practice or curriculum decisions in adult mathematics education.

**Risks and Discomforts**

- Only my former students and other instructors’ students will be invited to participate in the research in order to reduce any concern to participate in the study and to avoid the conflict of interests;

- The results of the study will be shared with the participants to ensure reciprocity between the participants and myself as a researcher.

- The participants will be notified before they engage in the research that they can withdraw at any moment;
• The participants will be notified that they have the rights to ask questions, obtain a copy of the results, and have their privacy respected;

There are no other foreseeable risks for the adult participants in the project.

**Privacy and Confidentiality**
To protect your privacy and anonymity, pseudonyms will be used in material that will be published or publicly displayed. I will further disguise, alter, or remove identifiers that could reveal your identity. In addition, the name and location of the community college and other identifiers will be protected. After the completion of the project, data, once analyzed, will be kept in my office for a number of years (up to 5 years).

**Compensation**
If you participate in the research, you will be offered $20 Tri-C bookstore gift card as well as once-a-week math tutoring before or after your math class.

**Voluntary Participation**
Taking part in this research study is entirely up to you. You may choose not to participate or you may discontinue your participation at any time without penalty or loss of benefits to which you are otherwise entitled. You will be informed of any new, relevant information that may affect your health, welfare, or willingness to continue your study participation. In addition, please be informed that participation or nonparticipation will not affect your math course grade.

**Contact Information**
If you want to know more about this research project, please call me at (216)-987-2318 or my supervising faculty, Dr. Michael Mikusa at mmikusa@kent.edu, Dr. Joanne Caniglia, at jcanigl1@kent.edu, or Dr. Susan V. Iverson at siverson@kent.edu. The project has been approved by Kent State University. If you have questions about Kent State University’s rules for research, please call Dr. John West, Vice President of Research, Division of Research and Graduate Studies (Tel. 330.672.2704).

**Consent Statement and Signature**
I have read this consent form and have had the opportunity to have my questions answered to my satisfaction. I voluntarily agree to participate in this study. I understand that a copy of this consent will be provided to me for future reference.

________________________________________  __________________
Participant Signature                      Date

**Audio/Video tape Consent Form**
I agree to be audio/videotaped during my participation in this study.

________________________________________________________________________________________
Participant Signature                          Date

I have been told that I have the right to listen to the recording of the interview before it is used. I have decided that I:

____want to listen to the recording         ____do not want to listen to the recording

Sign now below if you do not want to listen to the recording. If you want to listen to the recording, you will be asked to sign after listening to them.

Ellen Brook  may / may not  (circle one) use the audio-tapes/video tapes made of me. The original tapes or copies may be used for:

____this research project ______publication ______presentation at professional meetings

________________________________________________________________________________________
Participant Signature                          Date

Address:
APPENDIX C

CROSS-CASE DATA ANALYSIS OF PARTICIPANTS’ EXPERIENCE WITH MATH EDUCATION
Appendix C

Cross-Case Data Analysis of Participants’ Experience with Math Education

Table 8. *Cross-Case Data Analysis of Participants’ Experience with Math Education*

<table>
<thead>
<tr>
<th>Participant’s Name</th>
<th>Earliest Arithmetic Experiences</th>
<th>Post Arithmetic Experiences</th>
<th>Present Math Education Experience</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mina</td>
<td>AM, SA, TG</td>
<td>ANE, SDA, SM, TP</td>
<td>AM, ARL, SG, TG</td>
</tr>
<tr>
<td>John</td>
<td>AM, BED, SA, TG</td>
<td>ANE, BED, SDA, SM</td>
<td>ANE, SM, TG, ARL</td>
</tr>
<tr>
<td>Ron</td>
<td>AC, TG</td>
<td>ANE, ANR, TP</td>
<td>SDA, ARL</td>
</tr>
<tr>
<td>Jana</td>
<td>AC, SA</td>
<td>AD, BNI, SDA</td>
<td>AC, AM, ARL, AS, TG</td>
</tr>
<tr>
<td>Ken</td>
<td>AM, SG, TG</td>
<td>AM, ANRL, SA</td>
<td>AC, AM, ARL, AS</td>
</tr>
<tr>
<td>Liz</td>
<td>AC, SG, TG</td>
<td>ANRL, BNI, SA</td>
<td>AC, ARL, AS</td>
</tr>
<tr>
<td>Rocky</td>
<td>AC, SG, TG</td>
<td>ANRL, SDA</td>
<td>AC, AM, ARL, SG</td>
</tr>
<tr>
<td>Raul</td>
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<td>AC, BNI, TG</td>
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<td>Kate</td>
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<td>AC, SA</td>
<td>AC, TG</td>
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<tr>
<td>Nell</td>
<td>AC, SA, TG</td>
<td>ANE, ANRL, BNI, TP</td>
<td>ARL, AS, AM, TG</td>
</tr>
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<td>Martin</td>
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<td>SDA, TP</td>
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<td>Tammy</td>
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<td>AM, AS, TG</td>
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<tr>
<td></td>
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<td>AC, SG, TG</td>
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<td>------</td>
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</tr>
<tr>
<td>Sarah</td>
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<td>ANE, SDA</td>
<td>AC, SG, TG</td>
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<td>Sal</td>
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<tr>
<td>Nick</td>
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<td>ANRL, BNI, SDA</td>
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<tr>
<td>Lora</td>
<td>AC, SA, TG</td>
<td>ANE, ANRL, SDA, TP</td>
<td>AC, ARL, TG</td>
</tr>
</tbody>
</table>

The following three figures (bar graphs) depict the participants’ attitudes toward their math education. The first graph represents the frequency of times students describe their attitudes and beliefs towards mathematics. The second graph represents his or her measurement of success at various stages of their schooling. The third graph represents the participants’ perception of their math teachers. The three different colors represent three periods of time—elementary/arithmetic, middle/high school/post arithmetic and the present time (college)/algebraic.
Figure 61. Participants' Experiences with Math Education Data: Attitude Toward Math Education
Figure 62. Participants' Experience with Math Education: Level of Success
Figure 63. Participants’ Experience with Math Education: Teacher/Teaching Perception and Student Behavior toward Math
Table 9. Cross-Case Data Analysis of Participants’ Experience with Solving Word Problems, Family Involvement, and Participation in Formal/Informal Mathematical Activities

<table>
<thead>
<tr>
<th>Participant’s Name</th>
<th>Solving Word Problems</th>
<th>Family Involvement</th>
<th>Formal/Informal Math Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mina</td>
<td>WPU</td>
<td>HS</td>
<td>MF</td>
</tr>
<tr>
<td>John</td>
<td>WPN</td>
<td>HS</td>
<td>MF</td>
</tr>
<tr>
<td>Ron</td>
<td>WPU</td>
<td>HS</td>
<td>MF</td>
</tr>
<tr>
<td>Jana</td>
<td>WPN</td>
<td>HS</td>
<td>MF</td>
</tr>
<tr>
<td>Ken</td>
<td>WPN</td>
<td>HS</td>
<td>MF, MI</td>
</tr>
<tr>
<td>Liz</td>
<td>WPU</td>
<td>HN</td>
<td>MF</td>
</tr>
<tr>
<td>Rocky</td>
<td>WPU</td>
<td>HS</td>
<td>MF</td>
</tr>
<tr>
<td>Raul</td>
<td>WPC</td>
<td>HS</td>
<td>MF</td>
</tr>
<tr>
<td>Kate</td>
<td>WPU</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>WPU</td>
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Figure 64. Participants' Experience with Math Education: Solving Word Problems, Family Involvement, and Participation in Mathematical Activities
APPENDIX D

CROSS-CASE DATA ANALYSIS OF THE PARTICIPANTS’ EXPERIENCES

SOLVING MOTION PROBLEM I
## Appendix D

### Cross-Case Data Analysis of the Participants’ Experience Solving Motion Problem I

**Table 10. Cross-Case Data Analysis of the Participants’ Experience Solving Motion Problem I**

<table>
<thead>
<tr>
<th>Name</th>
<th>Understanding the Problem</th>
<th>D = RT Formula</th>
<th>Use of Variable</th>
<th>Use of Equation</th>
<th>Graphical Representation</th>
<th>Correct Answer</th>
<th>Use of Calculator</th>
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<tbody>
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<td>HAI</td>
<td>RC</td>
</tr>
</tbody>
</table>
The following two figures (bar graphs) depict the participant’s experience when solving the motion problem during the first interview. The first graph represents the frequency of times students understood and rephrased the problem correctly as well as the resources used (such as the distance formula). The second graph represents the frequency of times students used heuristics when solving the first motion problem.

Figure 65. Participants' Experience with Solving Motion Problem I: Resources
Figure 66. Participants' Experience with Motion Problem I: Heuristics
APPENDIX E
CROSS-CASE DATA ANALYSIS OF THE PARTICIPANTS’ EXPERIENCES
SOLVING MIXTURE PROBLEM I
Appendix E

Table 11. Cross-Case Data Analysis of the Participants’ Experiences Solving Mixture Problem I

<table>
<thead>
<tr>
<th>Name</th>
<th>Understanding the Problem</th>
<th>Understanding Concept of Concentration</th>
<th>Use of Variable</th>
<th>Use of Equation</th>
<th>Graphical Representation</th>
<th>Correct Answer</th>
<th>Use of a Calculator</th>
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</tbody>
</table>

The following two figures (bar graphs) depict the participant’s experience when solving the mixture problem during the first interview. The first graph represents the frequency of times students understood and rephrased the problem correctly as well as the resources used (such as the formula used to calculate the amount of the agent as a product of the percentage and the total amount of the solution). The second graph...
represents the frequency of times students used heuristics when solving the first mixture problem.

Figure 67. Participants' Experiences Solving Mixture Problem I: Resources
Figure 68. Participants’ Experiences Solving Mixture Problem II: Heuristics
APPENDIX F
CROSS-CASE DATA ANALYSIS OF THE PARTICIPANTS: EXPERIENCES
SOLVING MOTION PROBLEM II
Appendix F

Cross-Case Data Analysis of the Participants’ Experiences Solving Motion Problem II

Table 12. Cross-Case Data Analysis of the Participants’ Experiences Solving Motion Problem II

<table>
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<th>Name</th>
<th>Understanding the Problem</th>
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359
The following two figures (bar graphs) depict the participant’s experience solving the motion problem during the second interview. The first graph represents the frequency of times students understood and rephrased the problem correctly as well as the resources used. The second graph represents the frequency of times students used heuristics when solving the second motion problem.
Figure 69. Participants’ Experiences Solving Motion Problem II: Resources
Figure 70. Participants’ Experiences Solving Motion Problem II: Heuristic
APPENDIX G

CROSS CASE DATA ANALYSIS OF THE PARTICIPANTS’ EXPERIENCES

SOLVING MIXTURE PROBLEM II
Appendix G

Cross-Case Data Analysis of the Participants’ Experiences Solving Mixture Problem

II

Table 13. Cross-Case Data Analysis of the Participants’ Experience Solving Mixture Problem II

<table>
<thead>
<tr>
<th>Name</th>
<th>Understanding the Problem</th>
<th>Understanding the Concept of Concentration</th>
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<th>Use of Equation</th>
<th>Graphical Representation</th>
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</table>
The following two figures (bar graphs) depict the participants’ experiences solving mixture problem during the second interview. The first graph represents the frequency of times students understood and rephrased the problem correctly as well as the resources used (such as the formula to calculate the amount of the agent as a product of the percentage and the amount of the solution). The second graph represents the frequency of times students used heuristics when solving the second mixture problem.

Figure 71. Participants’ Experiences Solving Mixture Problem II: Resources
Figure 72. Participants’ Experiences Solving Mixture Problem II: Heuristic
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REFERENCES


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