Fair Sharing of Costs and Revenue through Transfer Pricing in Supply Chains with Stochastic Demand

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Abstract

Supply chain coordination has attracted much attention lately. On the one hand, it helps create more streamlined business processes and enables cost reduction. On the other hand, supply chain members may become more concerned about their own individual profits than the overall supply chain profits. However, the question of how to divide total profits fairly among supply chain members still exists, especially in a stochastic demand market. Therefore, drawing from the accounting concept of transfer prices, this research will complement the research stream of fair division of supply chain profits among supply chain members and should help coordinate supply chains.

I first investigate fairness in supply chain profit division. Common principles of justice are used to show what conditions should be satisfied for a profit division to be fair. Thereafter, a value-sharing method based on the Shapley value from cooperative game theory (Shapley, 1953) is proposed to determine the transfer prices among supply chain members. The Shapley value solution is an appealing allocation rule since it takes into account the potential payoff from alternative options. Once reached, such an agreement could simplify the process of bargaining on transfer prices which embody fairness.

To illustrate the Shapley value solution and procedures, a two-echelon supply chain consisting of one supplier and two heterogeneous retailers is examined. Each company is an autonomous profit-generating entity. Each retailer faces a stochastic demand, and thus the combined demand of the supplier from the two retailers is also random. In addition to
their independent operations, each company has the option to conduct transactions for the same product from external markets. My goal is to figure out ideal transfer prices for products delivered among supply chain members. These transfer prices will achieve the suggested profit allocations among the three companies.

This research will contribute to the literature in several aspects: transfer pricing with stochastically varying demand; solution procedures and results for supply chain profit fair division; and supply chain coordination. My research suggests a procedure for setting fair transfer prices among supply chain members, which helps promote supply chain coordination.
Chapter 1 Introduction

Recently, supply chain management research has stressed the importance of coordination in decentralized supply chains\(^1\) (e.g., Lariviere, 1999; Corbett et al., 2004; Cachon & Lariviere, 2001; Gerchak & Wang, 2004; Bernstein & Federgruen, 2005; Cachon & Lariviere, 2005). Supply chain coordination helps create more streamlined business processes and enables cost reduction. It has been shown that supply chain members are not primarily concerned about the supply chain total profits, but about their individual profits (e.g., Cachon, 1999; Fransoo et al., 2001). It is critical for each supply chain member to achieve a certain level of profits to insure supply chain coordination. \textit{Supply chain coordination} is a series of activities which are taken by independently managed parties in a supply chain to act in the same way as operated if they were by one single decision maker. It often leads to global cost reduction, higher service levels, a reduction of the bullwhip effects\(^2\), better resource utilization and effective market demand responses. One critical question in coordinating a decentralized supply chain is how to price products shipped between supply chain members. The role of transfer prices as a coordinating mechanism has been explored for this problem (e.g., Yeom & Balachandran, 2000). \textit{Transfer prices} are the prices of products and services charged between separately running entities, including companies or divisions in multi-division firms. Despite the extensive literature, transfer pricing remains a complex problem in a stochastic demand market. Both revenue and costs are always jointly necessary for a comprehensive description and analysis of supply chain transfer pricing.

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\(^1\) In a decentralized supply chain, each supply chain member is operating independently as an autonomous unit to optimize its own objective.

\(^2\) The bullwhip effect refers to a phenomenon whereby the variance of orders to the supplier may be larger than that of sales to the buyer and the distortion increases as orders move upstream. (Lee, 2004).
The traditional study of transfer pricing focuses on profit maximization as the sole objective of a firm. However, fairness is a substantial issue when dividing the total supply chain profits. Kumar et al. (1995) demonstrate that fairness is a significant determinant of supply chain relationship quality through an empirical study. An unfair transfer price would hurt supply chain coordination if supply chain members feel they are not treated fairly in transactions. A well-accepted profit division scheme helps integrate supply chain members to form a seamless process. As indicated by Lee (2000), supply chain integration consists of three dimensions: information integration, coordination, and organizational linkage. Information integration denotes the sharing of information and knowledge within a supply chain, while organization linkage refers to the construction and maintenance of communication channels between supply chain members. Coordination helps to redeploy resources to the supply members after information and knowledge are shared. The division plan of ultimate profits provides an anticipated outcome for all supply chain members. Each supply chain member can assess whether their efforts are worthy or not. Therefore, the profit division scheme, including revenue and compatible costs, if any, is of interest to the managers of supply chain companies. The *revenue* from the sale of a product is the selling price multiplied by the number of units sold; this is the total income from sales. The *cost* is the sum of the fixed cost and variable cost; the former is incurred no matter what the output and the latter varies with the output. The *profit* is the difference between the revenue and the cost. In this dissertation, I simultaneously consider both revenue and costs, resulting from sales, productions, and transactions, for two retailers and one supplier. In the end, the total
profits of the system will be taken to be divided fairly among the two retailers and the supplier.

Transfer prices in transactions are very important to a firm’s investment decisions. Transfer pricing in supply chains has been explored with deterministic demands (e.g., Gjerdrum et al., 2002; Rosenthal, 2008). Yet, it is widely assumed in supply chain management literature that the demand of a supply chain is stochastic (Cachon, 2003). There is, however, a gap in the literature in that there is a paucity of studies on transfer pricing among supply chain members with stochastic demand. Furthermore, although fairness has attracted a lot of attention in research, the concept of fairness has not been well documented in the supply chain profit division area.

The objective of the research presented herein is to explore fair sharing of revenue and costs among supply chain members in a stochastic demand market. I evaluate the concept of fairness by three main principles: the principles of equity, utility and equality. A two-echelon supply chain will be examined consisting of one supplier and two heterogeneous retailers; each company is an autonomous profit-generating entity. Each retailer faces a stochastic demand, and thus the combined demand of the supplier from the two retailers is also random. In addition to their independent operations, each company has the option to conduct transactions for the same product with external companies.

In particular, I consider the role of transfer prices on supply chain coordination. Transfer price is a term that is generally used by divisions affiliated to the same organization to denote product/service unit price of the intra-company transactions (Vaysman, 1998). However, the terminology has also been extended to refer to payments among companies that are not affiliated with one organization, e.g., decentralized supply
chain members (e.g., Gjerdrum et al., 2002). Company or division managers negotiate appropriate transfer prices to trade products or services, and the transfer prices are set to maximize their profits. This practice is called transfer pricing. I propose a value-sharing method based on the Shapley value from cooperative game theory to determine transfer prices of delivered products among supply chain members. The Shapley value solution is an appealing allocation rule since it takes into account the potential payoff from alternative options. The Shapley value of each player is partially determined by how much the player can get by remaining independent from the grand coalition. The Shapley value assigns a higher payoff to players who contribute more to the grand coalition. Once reached, such an agreement could simplify the process of bargaining on transfer prices which embody fairness. Rosenthal (2008) calculates transfer prices in a vertically integrated supply chain, where supply chain members share technology and transaction costs. A limitation of Rosenthal’s work (2008) is his focus on deterministic demand. My research will extend his work by studying a supply chain facing stochastic demands and establish a fair division scheme of profits. The main contributions of the research presented herein consist of (1) the analysis of the transfer pricing with stochastically varying demand, (2) the efficiency of solution procedures and results for supply chain profit fair division, (3) the highlighting of transfer prices to supply chain coordination.

Cachon (2003) has demonstrated that revenue sharing contracts, buyback contracts, quantity-discount contracts, and quantity flexible contracts could coordinate supply chains. For any of these contracts a contract agreement is only achieved after the profit allocation scheme is accepted by all the members. Although the allocations of supply chain profits from contract coordination are completed after the realization of the
quantities demanded, agreements regarding the allocation scheme must be reached before the realization of demand. The goal of my dissertation is to figure out ideal transfer prices for products delivered between supply chain members. These transfer prices guarantee fair profit allocations among the supply chain members and determine the key aspects of a contract, which eventually coordinates supply chains.

The remainder of the dissertation is organized as follows. A literature review is presented on transfer pricing in multi-division companies and in supply chains, and profit division fairness in Section 2. In Section 3, the core and the Shapley value are introduced. Section 3 also includes a detail review of current research over supply chain management, using the Shapley value as a solution. A brief introduction of methodologies and numerical examples are presented in Section 4, followed by a summary and conclusion in Section 5.
Chapter 2 Literature Review

2.1 Intra-company transfer pricing in multidivisional companies

Intra-company transfer pricing in an organization has been widely examined (e.g., Manes & Verrecchia, 1982; Baldenius et al., 2004). An organization is a coalition of divisions with diverse interests and often different objectives, but profit optimization should be emphasized. If there are significant levels of intra-company transactions with inappropriate transfer prices, it could cause improper investment decisions and also demotivate division managers. Researchers have studied transfer pricing of product transactions between divisions. Hirshleifer (1956) claimed that, in a tax-free world, the intra-company transfer price should be equal to the marginal cost of the supplying division. The marginal cost method works when there is neither an external market for the same products nor an agreeable transfer price reached between the two divisions. These conditions for the marginal cost method are not always satisfied and the method is unlikely to hold in practice (Benke & Edwards, 1980). Manes and Verrecchia (1982) apply both Massachusetts formula (MF) and the Shapley value adjusted MF to determine intra-company transfer prices. MF is an equal-weighted average of the assets, payroll, and sales revenue, which is used by many states in the U.S. to allocate income to state jurisdictions. Wielenberg (2000) presents a transfer-pricing scheme in a divisionalized firm, simultaneously investigating investment and capacity decisions. Wettstein (1994) analyzes a resource allocation problem in an n-division firm. A mechanism is constructed to accomplish the profit-maximizing allocation. Meanwhile, transfer prices are defined as the outcome of a Nash equilibrium. One issue these papers missed is the discussion of the fairness of allocation: Will the suggested division scheme work if the divisions or
members care about fairness? If divisions care about fairness, for example, a Nash equilibrium might not be reached as expected in Wettstein (1994).

There is also a substantial literature on transfer pricing for multidivisional firms operating in different tax rate areas. In a tax-free world, or in unique tax rate areas, fair transfer pricing is necessary to correctly evaluate division performance. But in different tax rate areas, transfer prices have been found in the literature to be a tool for optimizing ultimate profits of the organization, as long as the transfer prices are legal. Copithorne (1971), Horst (1971), Samuelson (1982), Halperin and Srinidhi (1987), and Harris and Sansing (1998) have discussed how to set transfer prices in transactions given the government regulations and tax rates or tariffs in different areas or countries. The balance of government regulations and profit maximization is central to the total corporation performance. Baldenius et al. (2004) study a multinational firm with its divisions facing different income tax rates. For cost-based and market-based transfer pricing, it is suggested that a weighted average of the pre-tax marginal cost and the most favorable arm’s length price should be taken as the optimal intra-company transfer price. The arm’s length price is defined, according to the U.S. Internal Revenue Code of 1986, as “the amount which would have been charged in independent transactions with unrelated parties under the same or similar circumstances” (Abdallah, 1989). The arm’s length range is a range of reliable results from application of one of the following pricing methods: single method, selection of comparables, and comparables included in arm’s length range (Income Tax Regulations § 1.482-1(e)). Within the arm’s length range, the price that minimizes the firm’s total tax liability for a certain amount of transfer products is the most favorable arm’s length price.
For a multinational company (MNC), transfer pricing is also a tax optimization issue. Relative tax rates in the host or home countries have to be considered for transfer pricing by top management, which greatly affects the after-tax income of the company. Vidal and Goetschalckx (2001) present a mathematical programming model to maximize the after tax profits of a multinational corporation. The system considered is composed of internal suppliers, manufacturing plants, and distribution centers (DCs). In their model, transfer prices, the flow of the product and the allocation of transportation costs are taken as the decision variables. It is assumed that: transfer prices could be identified as centralized decisions; transfer price variables have a lower and an upper bound; transfer prices of one product depend on production cost; the customer demand is uncertain; all incomes of the suppliers, manufacturers, and DCs are not taxable until the shareholders get dividends; and only import duties are considered. The problem is set as a non-convex optimization problem with a linear objective function, a set of linear constraints, and a set of bilinear equalities. The problem is solved by successively fixing one set of variables and iterating the remaining Linear Programming (LP) for the other set. But the question of whether the outcome through this procedure is the global optimum is not answered. Another shortcoming in Vidal and Goetschalckx (2001) is that some of the assumptions in the model are rather strong.

Alles and Datar (1998) study a model of oligopolistic competition with two decentralized firms, each selling two products. The marketing manager and the CEO of each firm interact in a sequence as a Stackelberg game\(^3\), with the latter as the leader. The

\(^3\) The Stackelberg model (Osborne, 2004) can be applied to an industrial setting where only two firms are competing and one firm is positioned as the natural leader. To make the optimal decision, the natural leader has to consider the follower’s subsequent profit-maximization decisions. Suppose firm 1 is the leader who decides to produce a quantity, \( Q_1 \), and firm 2 is the follower who responds by selecting a quantity \( Q_2 \). The
CEO wants to determine the transfer price to maximize the organization’s profits, while the marketing manager would like to select the product price to maximize his division’s profits. The transfer price set by the CEO is the reported per unit manufacturing cost of this firm. The problem solved in this paper is to maximize the marketing department’s profits, rather than the firm’s total profits. To maximize profits, a cost-plus transfer price is finally chosen. The research results by Alles and Datar (1998) are consistent with existent survey evidence on transfer pricing. By examining the actions of the marketing manager and CEO of two decentralized firms, their research model is different from ours.

Villegas and Ouenniche (2008) examine a partially decentralized multi-division company, where each division makes decisions independently, and headquarters coordinate all the divisions. The headquarters would coordinate all the divisions, and those divisions who are not willing to work towards the corporate goals will suffer penalties including a restricted access to corporate resources. Corporate constraints and divisional constraints, or corporate goals and divisional goals may not be consistent. The research by Villegas and Ouenniche (2008) is to determine the transfer prices for the use of corporate resources to maximize the repatriated earnings and to derive managerial guidelines for the whole company. For a MNC, repatriated earnings refer to the earnings returning from foreign subsidiaries to their home country. One comprehensive unconstrained mathematical model is presented that incorporates a large number of the previous research factors into this type of supply chain problem: transfer prices, trade quantities and transportation cost allocations. One of its limitations is the absence of a

total quantity in the market is \( Q = Q_1 + Q_2 \). Given the quantity \( Q \) chosen by firm 1, presumably, firm 1 should expect that the follower will attempt to maximize profits with \( Q_2 \), which is a function of \( Q_1 \). Then firm 1 maximizes its profits by choosing a quantity, \( Q_1 \).
specific solution procedure for the model, which makes it difficult to apply the results to other contexts.

2.2 Inter-company transfer pricing in supply chains

Transfer prices may also refer to payments among supply chain companies. Gjerdrum et al. (2002) explore a two-echelon supply chain, which consists of two independent enterprises with symmetric information and facing deterministic demand. It is assumed that several transfer price levels are preset between the two enterprises before any decision making, and the selected transfer price is not necessarily the same as the market price. Transfer price levels are different unit product prices paid between the two companies. The objective is to maximize the profits of both enterprises by selecting a transfer price. The Nash bargaining solution is applied to obtain a fair profit division among players. The problem is formulated as a mixed integer non-linear programming (MINLP) model and the branch-and-bound algorithm is used to determine the most appropriate transfer price level. The supply chain profit maximization is set by a single level approach which maximizes the total supply chain profits without considering individual profits. It is demonstrated that the Nash approach could obtain a solution very close to the supply chain profit maximization. In their work, although hypothetically transfer price levels can be set before decision making, presetting transfer price levels is not practical. The assumption of deterministic demand is also not practical.

In the study by Rosenthal (2008), transfer prices are paid along a supply chain in multi-echelon corporations, and each corporation has the option to purchase or to sell intermediate goods in the supply chain or external markets. It is also assumed that each
division sells one unit of intermediate products to its downstream division, and the agreed unit prices are obtained from outside sources. Some transaction activities and technology of supply chain members are common and costs could be shared if two companies are united. The author set up a cooperative game among all divisions. When the supply chain behaves efficiently, which means the cost function of each division is subadditive, the author proved it a convex game. Being subadditive means that the value of a union of two coalitions is no more than the sum of the separate coalition values. The core of a convex game is nonempty (Shapley, 1971) and the Shapley value allocation lies on the centroid of the core. Thereafter, a procedure is presented to derive transfer prices between divisions using the Shapley value. The paper shows how to set a fair (in the Shapley sense), and acceptable solution to all supply chain members, who are vertically integrated with inputs and outputs at each level. One limitation in the paper is that the demand of the final product market is fixed.

Lakhal (2006) also studies the interactions between different firms. But the companies form a manufacturing network and are sharing risks of investment; the products pass through these companies with manufacturing collaboration. The objective is to maximize operating profits of the manufacturing network. His paper proposes a mathematical model to determine transfer prices between firms in the manufacturing network. One of the weaknesses is that the question of whether the transfer price is fair to all the players is not answered.

2.3 Fairness of profit division
It is common sense that fairness affects the behavior of a lot of people. In economics, many researchers have considered issues of fairness to be important in evaluating economic outcomes. Kahneman et al. (1986) conduct an empirical study to show that the customers’ feelings about the fairness of firms’ short-run pricing decisions restricts some firms from fully exploiting their monopoly power. Some empirical studies demonstrate that firms’ wage setting is greatly influenced by workers’ views about what constitutes a fair wage (e.g., Campbell & Kamlani, 1997). Fehr and Schmidt (1999) explore the role of fairness in competitive environments. They show that fairness considerations will impact market outcomes even in highly competitive environments. Bolton and Ockenfels (2000) develop a theory of equity (fairness), reciprocity, and competition to explain individual behavior in a wide class of games. Rabin (1993) adds fairness consideration into economic models and presents the concept of “fairness equilibrium”. He studies how people’s attitudes toward fairness affect their behavior and well-being and shows fairness matters. Santos-Pinto (2008) claims that “inequity aversion (players’ concern about fairness)” could explain many puzzling experimental findings in endogenous timing games.

The same principle that fairness affects people’s behavior is also applied to transfer pricing since negotiators care about relative profits on transfer pricing. This is consistent with the results of the experiment conducted by Luft and Libby (1997). But which system can be viewed as a fair one or how can wealth be distributed in a society fairly? There are a lot of theories of justice which stress how welfare and income should be fairly distributed among all members of a society (Rawls, 1971; Mill, 1993; Walzer, 1983;

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4 In an endogenous timing game, the order of players’ move is explicitly endogenously determined (Robson, 1990), rather than an exogenous specification of a simultaneous or a sequential move.
Deutsch, 1985). Ordinarily we consider that an allocation is *fair* if everyone receives what they are entitled to, in the sense that the benefits and burdens that are put on them because of their particular characteristics and circumstances. Previous studies have shown that when asked about beliefs concerning wealth distributive justice, people rarely refer to one single principle of distribution. As Zajac (1985) emphasizes, in popular discourse, fairness is a mixture of principles that no simple and concise formulation may be able to capture. Baumol (1986) believes that a variety of standards of fairness are evolving from one or both of the two basic criteria: the Pareto improvement criterion and the criterion of superfairness. The Pareto improvement criterion offers “as a test of the equitability of any proposal for change the universality of participation in the benefits of that change”. If everyone affected is expected to benefit, or at least not be harmed, then the change is a Pareto improvement. The Pareto improvement criterion, generally, is not the same as Pareto optimality. Obviously, a move to a Pareto optimum from an initial position, which was not optimal, is not necessarily a Pareto improvement—some persons may well be harmed by such a change. On the other hand, a Pareto improvement need not reach a Pareto optimum. While the criterion of superfairness indicates that a distribution is superfair only “if each class of participants prefers its own share to the share received by another group” (p. 15).

More generally, principles of equity, principles of utility, and principles of equality (or the egalitarian principles) are identified as the main principles of justice (Aalberg, 2003). Equity was one of the critical principles recognized by Rescher (1972) and Deutsch (1985). The principle of equity assumes that, to be a just distribution, welfare and outcomes should be distributed among individuals in proportion to their contributions.
Those who contribute more should be rewarded proportionally more than those who contribute less. Equity theory also assumes that individuals always want to maximize their own outcomes. The principle of utility assumes that a distribution should maximize the total utility (Mill, 1993). Utility is a measure of relative pleasure, satisfaction, desirability, or the realization of preferences. In other words, outcomes should be distributed in a way that the total welfare of all of the participants is maximized. Usually, equality or egalitarian requires equal shares in the distribution of outcomes (Greenberg & Cohen, 1982). However, the principle of equality does not necessarily imply identical outcome for each individual without regard to particular circumstances. The basic implication is that everyone should be rewarded as equally as possible, and hence the differences in rewards received by participants should be limited. The principle of equality (egalitarian) suggests that there should be a lower bound to make sure that everyone receives a decent minimum, and also suggests an upper bound that no one should be allowed to exceed.

Other concepts have also been used to evaluate fairness, e.g., the theory of justice by Rawls (1971). Rawls examines a number of definitions of fairness (such as maximizing total utility). Rawls (1971) believes that an unequal distribution of wealth and income is unfair unless it improves the receipts of the least favored person in society. The least favored person in a game in this dissertation refers to the person who has the lowest worth as a one-player coalition. A fair distribution of wealth and income does not need to assign equal portion to everyone, but it must be “to everyone’s advantage”. “Everyone’s advantage” could be interpreted as a combination of the principle of efficiency and the difference principle. The principle of efficiency, also known as Pareto efficiency,
signifies that no one will be better off without simultaneously making any one else worse off. The difference principle indicates that the difference in players’ payoffs is permissible only if the difference is to the advantage of those least favored. In other words, the inequality in distribution is justifiable only if changing it would make the least favored person even worse off. The unevenly distributed benefits to those most favored at least contribute to the gains of those least favored.

Eccles (1985) highlights that attention should be paid to the fairness of the transfer pricing policy and the resulting transfer pricing conflict as well. For example, if we take the marginal cost-based method, the transfer price is to be set as the marginal cost of the selling division. Suppose there is an external market for the products where the market price is reasonably higher than the marginal cost. The seller believes that he will get more if he sells in the market and desires a higher price, and does not perceive fairness in transactions with the original buyer. But the original buyer is happy with the transfer price. The desired prices of both parties are conflicting. Therefore, it is critical to figure out an acceptable transfer price for both of them.

It is reasonable that if a firm or division has the option to purchase from or sell to external markets, the firm or division will not be willing to pay more or accept less than the external market price. In Ghosh’s study (2000) of transfer price negotiation among trading divisions, perceived fairness of the transfer pricing is taken as a dependent variable, when he investigates the complementary effect of sourcing and compensation structure. But are the negotiators willing to act on these expectations? The experimental study by Kachelmeier and Towry (2002) reveals that how participants negotiate transfer prices determines the answer. Only when parties negotiate face-to-face with unrestricted
communication, fairness-based transfer pricing will come out. This process is very difficult and expensive to perform practically.
Chapter 3 Cooperative Games and Supply Chains

3.1 The core and the Shapley value

Both the core and the Shapley value are commonly used in cooperative games, where the players are allowed to form binding agreements. The core and the Shapley value are two alternative solution concepts to divide the total payoff among all players in cooperative games. The core of a game is measuring the stability of the solution concept, while the Shapley value selects one payoff vectors as a solution to the game. In a coalition game, a unique payoff vector can be reached through the Shapley value.

A cooperative game is a pair \((N; v)\), where \(N = \{1, 2, \ldots, n\}\) is the set of players, and \(v: 2^N \rightarrow \mathbb{R}\) is the characteristic function. \(n\) is the total number of players who may form a coalition. Any subset of \(N\), denoted by \(S\), is referred to as a coalition and \(v(S)\) is the worth of \(S\). For any coalition \(S \subseteq N\), the number of players in \(S\) is denoted by \(|S|\). A characteristic function \(v\), defined on all subsets of \(N\), is a real-valued function that shows the amount of payoff received by the players of the coalition \(S\). By definition, \(v(\emptyset) = 0\). For game \(v\), \(x_i(v)\) is an allocation function that is used to assign a payoff of \(x_i\) to player \(i\) in \(N\). Assuming that \(N\) represents the grand coalition, an allocation \(x_i\) is in the core of game \(v\) if and only

\[
\sum_{i \in N} x_i = v(N) \quad \text{and} \quad \sum_{i \in S} x_i \geq v(S)
\]

(1)

for any \(S \subseteq N\). A core allocation divides the total payoff of the grand coalition among all the players, and the sum of the payoffs to the players of each subcoalition \(S\) is no less than the payoff of the subcoalition \(S\). With an allocation in the core of a game, no player has an incentive to quit from the grand coalition \(N\) since leaving the grand coalition will
cause it to get less. If there exists an allocation scheme, which makes the players in $S$ obtain more than by the core allocation, then the sum of the total players’ payoff would exceed $v(N)$. It is logically incorrect. Thus, a core allocation is reminiscent of an equilibrium. However, the core of a cooperative game does not necessarily exist.

Consider an example with three players $N = \{1, 2, 3\}$ (Moulin, 2003). $v(S)$ is defined as the possible payoff of the players in the coalition $S$. Then let the payoffs be as follows: $v(1) = v(2) = v(3) = 0$, $v(12) = v(13) = v(23) = v(123) = 1$. First, we assume that the core of this game exists. We know the sum of the payoffs to all of the three players should be 1: $x_1 + x_2 + x_3 = 1$; since $x_1, x_2,$ and $x_3$ cannot be negative or zero, the payoff to Player 1, Player 2, and Player 3 are all between 0 and 1: $0 < x_1, x_2, x_3 < 1$. But Player 1 and Player 2 have incentives to leave the grand coalition $N$ together, since they could get 1 without Player 3. $x_1 + x_2 < 1 = v(12)$, which indicates that equation (1) is not satisfied and the core is empty.

So what conditions on game $v$ guarantee the core is not empty? One of the early answers is presented by Shapley (1971) by showing that for a convex game, the core is not empty. Suppose $S$ and $T$ are two finite subsets of $N$. A game $v$ is convex (Shapley, 1971) if its characteristic function satisfies

$$v(S) + v(T) \leq v(S \cup T) + v(S \cap T)$$

for any $S, T \in N$. A function which satisfies inequality (2) is defined as a supermodular function (Topkis, 1998). It has also been shown that the supermodularity of $v$ is equivalent to

$$v(T' \cup i) - v(T') \leq v(T \cup i) - v(T)$$

(3)
for any $T' \subseteq T \subseteq N$, and $i \notin T'$ or $T$, but $i \in N$ (Driessen, 1988). It implies that “the incentive for joining a coalition increases as the coalition grows”. In a convex game, the Shapley value is an element of the core (Shapley, 1971).

The Shapley value is an expected payoff to player $i$, $\phi_i(v)$, which is the only solution that satisfies the four axioms below (Shapley, 1953).

1) The first axiom is the symmetry axiom. This axiom illustrates that players would be assigned the same value if they are treated identically by the characteristic function, which means these players would contribute to any subcoalition in a symmetric way. For each player, only his contribution to the subcoalitions determines the players’ Shapley value allocation. 2) The second axiom, named the efficiency axiom, states that the sum of $\phi_i(v)$ over all players $i$ in any carrier $U$ is equal to $v(U)$. A carrier is any set $U \subseteq N$ with $v(S) = v(S \cap U)$ for all $S \subseteq U$. This axiom implies that the Shapley value assigns the total value of the coalition to the players in $N$. 3) The third axiom is the dummy axiom. If $v(S \cup i) = v(S)$ for all $S$, then player $i$ must get zero. 4) The last axiom, called the law of aggregation, requires that if we combine two games described by characteristic functions $v$ and $w$, then $\phi_i(v+w) = \phi_i(v) + \phi_i(w)$ for any player $i$ in $N$. This axiom is also known as the additivity axiom, which specifies how game values must be related to one another. It shows that the total Shapley value of two games $v$ and $w$, which are played at the same time, equals to the sum of the Shapley value when the player plays the game $v$ and $w$ at different times.

The Shapley value $\phi_i$ is defined by the following formula (Mas-colell et al., 1995):

$$\phi_i(v) = \frac{1}{n!} \sum_{\tau} [v(B(\tau, i) \cup i) - v(B(\tau, i))],$$
where $\phi_i(\nu)$ is the Shapley value for player $i$ in game $\nu$; $n$ is the maximum possible number of players in the coalition; $B(\tau,i)$ is the set of players who come into the coalition in ordering $\tau$ before player $i$. In particular, assuming all orderings are equally likely, the Shapley value can be further written as

$$\phi_i(\nu) = \sum_{S \subseteq N \setminus \{i\}} \frac{(s)!(n-s-1)!}{n!} \left[ \nu(S \cup i) - \nu(S) \right].$$  \hspace{1cm} (4)

In equation (4), $s$ is the number of players already joined in the coalition before player $i$. $\nu(S)$ is the worth of coalition $S$, $\nu(S \cup i)$ is the payoff after player $i$ has joined $S$, and the summation is over all subcoalitions $S$ in $N$. We can see that the term $[\nu(S \cup i) - \nu(S)]$ represents the contribution of the player $i$ joining $S$. However the fractional term, $\frac{(s)!(n-s-1)!}{n!}$, indicates the probability that a coalition $S$ is already formed before player $i$ joins $S$. Therefore, player $i$ will have a Shapley value that is the expected sum of all the incremental values resulting from the player joining $S$ weighted with the probabilities of the player joining the respective coalition. Payoffs to each player assigned by the Shapley value can also be viewed as the power of the corresponding player when negotiating in transactions (Winter, 2002).

Next I show one example to illustrate the interpretation of the Shapley value: the simple game with two players ($N = \{1,2\}$). There are just two orderings in the game, player 1 joining player 2 or vice versa. In the first ordering, player 2 contributes $\nu(2)$ and player 1 contributes $\nu(N) - \nu(2)$. In the second ordering, player 1 contributes $\nu(1)$ and player 2 contributes $\nu(N) - \nu(1)$. Then the average of the total contributions of each player is the Shapley value. Since my research model involves three players, next I
describe a game with just three players: player 1, player 2, and player 3. There are six orderings totally, and suppose the “worth” of player \( i \) is \( v(i) \):

<table>
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<th>Ordering</th>
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<th>Player 2</th>
<th>Player 3</th>
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<td>( v(123) - v(12) )</td>
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<td>(2,3,1)</td>
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<td>( v(2) )</td>
<td>( v(23) - v(2) )</td>
</tr>
<tr>
<td>(3,1,2)</td>
<td>( v(13) - v(3) )</td>
<td>( v(123) - v(13) )</td>
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<tr>
<td>(3,2,1)</td>
<td>( v(123) - v(23) )</td>
<td>( v(23) - v(3) )</td>
<td>( v(3) )</td>
</tr>
</tbody>
</table>

Table 1 the marginal contributions in a three-player game

Table 1 has enumerated all of the orderings about how the three players enter the coalition. If we add all marginal contribution of each player, it turns out to be the Shapley value. Take player 1 as an example.

The average of the marginal contributions of player 1

\[
= \frac{1}{6} \left[ v(1) + v(1) + (v(12) - v(2)) + (v(123) - v(23)) + (v(13) - v(3)) + (v(123) - v(23)) \right]
\]

\[
= \frac{1}{3} v(1) + \frac{1}{6} (v(12) - v(2)) + \frac{1}{6} (v(13) - v(3)) + \frac{1}{3} (v(123) - v(23)).
\]

Next consider the fairness of the Shapley value allocation scheme from more general perspectives. As mentioned, the Shapley value of a player is an average of marginal contributions of the player joining subcoalitions, which indicates fairness.
hand, we might check the three main principles of fairness. Recall the principle of equity assures that those who contribute more should be rewarded proportionally more than those who contribute less. In the Shapley value, \( [v(S \cup i) - v(S)] \) represents the contribution of the player \( i \) joining \( S \). Thus the assigned Shapley value is positively related to the marginal contribution, in a sense that those who contribute more when they join \( S \) would be assigned more than those who contribute less. According to Axiom 1), only a player’s contribution to the coalition determines his or her Shapley value. Thus, the Shapley value allocation reflects the principle of equity. The principle of utility requires that an applied distribution should maximize the total utility. According to the efficiency axiom of the Shapley value, the payoff of all of the members is maximized when the grand coalition is formed, while the Shapley value assigns the total value of the coalition to all the players in the game. Thus the principle of utility is satisfied.

The principle of equality or the egalitarian principle requires equal shares in the distribution of outcomes, although it does not necessarily imply identical outcome of each individual without regard to particular circumstances. Again, consider the most basic case of a two-player game with \( N = \{1, 2\} \), and the gain from the coalition is \( v(N) - v(1) - v(2) \). If we divide the gain from the coalition equally between the two players, the payoffs to player 1 and to player 2, denoted by \( \varphi_N(1) \) and \( \varphi_N(2) \), are

\[
\varphi_N(1) = v(1) + \frac{1}{2} (v(N) - v(1) - v(2)),
\]

(5)

and

\[
\varphi_N(2) = v(2) + \frac{1}{2} (v(N) - v(1) - v(2)),
\]

(6)
where \( \varphi_S(i) \) denotes the payoff of player \( i \) in the game whose players are all of the members in \( S \).

The payoffs of player 1 and 2 in (5) and (6) are just the same as the Shapley value, which can be acquired directly by applying formula (4) in a two-player game. It seems that the Shapley value follows the egalitarian principle with each player acquiring half of the total gain (Mas-coll et al., 1995). In the two-player game, \( \nu(1) = \varphi_{N\setminus\{2\}}(1) \) and \( \nu(2) = \varphi_{N\setminus\{1\}}(2) \), where \( N\setminus\{2\} \) and \( N\setminus\{1\} \) denote the subcoalition with all players but player 2 and player 1, respectively. Then we may rearrange equations (5) and (6) as

\[
\varphi_N(1) - \varphi_{N\setminus\{2\}}(1) = \varphi_N(2) - \varphi_{N\setminus\{1\}}(2),
\]

and

\[
\varphi_N(1) + \varphi_N(2) = \nu(N),
\]

respectively. One interpretation of equation (7) is that the contribution differences are preserved: what player 1 gets out of the presence of player 2 is the same as player 2 gets out of the presence of player 1 (Mas-coll et al., 1995). Next we expand on the discussion in Mas-coll et al. (1995), and consider a game with more than two players. Given a subset \( T \) of \( N \) \( (T \subset N) \), then to follow the egalitarian principle, for every game with \( T \) and players \( j, k \in T \), contribution differences are preserved in a manner similar to the two player case:

\[
\varphi_T(j) - \varphi_{T\setminus\{k\}}(j) = \varphi_T(k) - \varphi_{T\setminus\{j\}}(k), \quad \text{for all } j, k \in T \subset N,
\]

and

\[
\sum_i \varphi_T(i) = \nu(T), \quad \text{for all } T \subset N.
\]
Expression (9) determines \( \varphi_T(j) \) uniquely. Next we can proceed inductively. Suppose that we have defined \( \varphi_T(i) \) for all \( T \subset N \), \( i \in T \). Note that (9) allows us to express every \( \varphi_N(i) \) as a function of \( \varphi_N(1) \):

\[
\varphi_N(i) = \varphi_N(1) + \varphi_{N\setminus\{1\}}(i) - \varphi_{N\setminus\{i\}}(1), \quad \text{for all } i \neq 1, \text{ and}
\]

\[
\sum_i \varphi_N(i) = v(N).
\]

Recalling \( n = |N| \), we have

\[
\varphi_N(2) = \varphi_N(1) + \varphi_{N\setminus\{1\}}(2) - \varphi_{N\setminus\{2\}}(1),
\]

\[
\varphi_N(3) = \varphi_N(1) + \varphi_{N\setminus\{1\}}(3) - \varphi_{N\setminus\{3\}}(1),
\]

\[
\vdots
\]

\[
\varphi_N(n) = \varphi_N(1) + \varphi_{N\setminus\{1\}}(n) - \varphi_{N\setminus\{n\}}(1),
\]

and

\[
v(N) = \varphi_N(1) + \varphi_N(2) + \ldots.
\]

After adding both sides up, we obtain

\[
v(N) = n\varphi_N(1) + \varphi_{N\setminus\{1\}}(2) + \varphi_{N\setminus\{1\}}(3) + \ldots - \varphi_{N\setminus\{2\}}(1) - \varphi_{N\setminus\{3\}}(1) - \ldots = n\varphi_N(1) - \sum_{i=1}^{n-1} \varphi_{N\setminus\{i\}}(1).
\]

Therefore,

\[
\varphi_N(1) = \frac{1}{n} \left[ v(N) - \sum_{i=1}^{n-1} \varphi_{N\setminus\{i\}}(1) + \sum_{i=1}^{n-1} \varphi_{N\setminus\{i\}}(1) \right]
\]

\[
= \frac{1}{n} \left[ v(N) - v(N \setminus \{1\}) + \sum_{i=1}^{n-1} \varphi_{N\setminus\{i\}}(1) \right].
\]

\[(10)\]
and for any player \( i, i \in N \),

\[
\varphi_N(i) = \frac{1}{n} \left[ v(N) - \sum_{k \neq i} \varphi_{N \setminus \{i\}}(k) + \sum_{k \neq i} \varphi_{N \setminus \{k\}}(i) \right]
\]

\[
= \frac{1}{n} \left[ v(N) - v(N \setminus \{i\}) + \sum_{k \neq i} \varphi_{N \setminus \{k\}}(i) \right].
\]  

(11)

Therefore, an allocation scheme provided in (11) embodies the egalitarian principle. On the other hand, the Shapley value turns out to be the same as in (11) (Mas-colell et al., 1995):

\[
\phi(v) = \frac{1}{n} \left[ v(N) - v(N \setminus \{i\}) + \sum_{k \neq i} \phi_{N \setminus \{k\}}(i) \right].
\]  

(12)

Therefore, the Shapley value is consistent with the allocation scheme suggested by the egalitarian principle as in expression (11). Thus the principle of equality or egalitarian is satisfied.

I would like to use one example (Table 2) to show whether the Shapley value scheme is fair from the perspective of Rawls (1971): a fair division scheme of profits has to follow the principle of efficiency and the difference principle. Since the maximum profits could be only acquired by the grand coalition and all of the profits are shared by members (see Axiom 2), no one could get more without hurting anyone else. So the Shapley value guarantees Pareto efficiency. In other words, the principle of efficiency is satisfied. Next, examine whether the allocation suggested by the Shapley value satisfies the difference principle. The difference principle requires that if the legitimate expectations of the most
advantaged were less, the payoff of the least advantaged would also be less. However, the

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Table 2 Lowering the most favored player’s payoffs in a three-player game

conception of the difference principle is not defined mathematically. While there may be
general proof for the difference principle, I only provide a numerical example in this article.

In the game with player $N = \{1, 2, ..., n\}$, if the expected “worth” in the grand coalition
to player $i$ is reduced, it is reasonable to assume that the incentive and contribution of
player $i$ to the grand coalition are both decreased. Meanwhile we might assume the
payoffs of all of the coalitions, except the grand coalition, keep constant, since those
coalitions do not change. In Table 2, simulations are applied in EXCEL for games with 3
players. Player 1 is the most favored player and player 3 is the least favored one. In the
beginning, the payoff of player 1 is 10, that of player 2 is 6, and that of player 3 is 3. The
worth of the subcoalition of player 1 and 2 is 20; the worth of the subcoalition with
player 1 and player 3 is 18; the worth of the subcoalition with player 2 and player 3 is 12.
Initially, the assigned payoff of player 1 (the Shapley value of player 1) is 14.17 and the assigned payoff of player 3 is 6.67. If the expected payoff of player 1 is reduced, his/her incentive and contribution to the grand coalition will decrease, and we may assume the payoff of the grand coalition decreases, say 0.1. Meanwhile, the payoffs of isolated players keep constant since they are known parameters and the worth of those subcoalitions are also known parameters. Accordingly, we can see the assigned payoff of player 3 is also less. In Table 2, the yellow column (Column I) shows the assigned payoffs (the Shapley value) of the most favored player and the blue column (Column K) shows the assigned payoffs (the Shapley value) of the least favored player. We show that with a decreased Shapley value of player $i$ (the most favored player), the Shapley value assigned to player $j$ (the least favored player) is lowered. The results can also be drawn from equation (12) directly. The conclusion is consistent with the statement of the difference principle: if we lower the payoff of the most favored member, the Shapley value of the least favored member will also be less.

Therefore, in this example, the Shapley value allocation satisfies both the principle of efficiency and the difference principle and it is a just allocation scheme.

Actually, it is still not clear mathematically that which player is the most favored person and which player is the least favored person in a game, since the identification of players can only be made after the Shapley value allocation is finished. I would like to take it as a future research question. Answers to this question will be supportive to the proof of whether the Shapley value satisfies the difference principle.

3.2 The application of the Shapley value in supply chain management
Lately, the Shapley value has been used in a few supply chain management (SCM) papers as a solution methodology. For example, Bartholdi and Kemahlioğlu-Ziya (2005) use the Shapley value to allocate supply chain savings which result from inventory pooling for all of the parties in a supply chain. By analyzing the inventory pooling among two retailers and one supplier, they find that the allocations suggested by the Shapley value are acceptable and coordinate supply chains.

Granot and Sošić (2003) present a three-stage model of a decentralized distribution system consisting of multiple retailers (see Figure 1). In the first stage, each retailer places an independent order, $X$, before the random demand is realized; in the second stage, after demand realization $D$, a retailer determines the amount of residual supply, $H$, or residual demand, $E$, to share with other retailers; in the third stage, an additional profit, $R(X,D,H,E)$, is allocated among retailers, and this profit results from residual inventories transshipped. These additional profits are referred to as residual

![Figure 1 A supply chain interaction model with $n$ retailers](image)
profits. Let $r_i$, $c_i$, and $v_i$ denote the retail price, cost, and salvage value for retailer $i$, respectively. $t_{ij}$ denotes unit shipping cost from retailer $i$ to retailer $j$.

Suppose $R_s^*(X,D,H,E)$ is the maximal residual profits in the second stage for retailers $S (S \subseteq N)$, with a shipping pattern $Y$, then

$$R_s^*(X,D,H,E) := \max_Y \sum_{i,j \in S} (r_i - v_i - t_{ij})Y_{ij}$$

s.t. $$\sum_{j \in S} Y_{ij} \leq H_i, \quad i \in S$$

$$\sum_{j \in S} Y_{ij} \leq E_j, \quad j \in S$$

$$Y_{ij} \geq 0 \quad i, j \in S$$

The paper attempts to find a fair allocation scheme for the maximal residual profits, $R_n^*(X,D,H,E)$, among all the retailers. The characteristic function in this game $\nu(S)$ is constructed using the potential residual profits that the subcoalition $S$ could obtain. Then, the payoff of the grand coalition, $\nu(N)$, equals to $R_n^*(X,D,H,E)$. The Shapley value induces an allocation that does not result in a decrease in the total residual profits, but the allocation is not a core allocation. Therefore, during the third stage, the players do not have benefit incentives to form subcoalitions and it is possible that retailers will not share any of its leftover inventories/demands with other retailers, which will cause a zero residual profit. Thus an enforcement mechanism, e.g. a contract, is necessary to ensure the maximum residual profits.

In the following study by Sošić (2006), the Shapley value is again used in the third stage to distribute profits from residual inventory transshipment among retailers. However, this time retailers are farsighted. A ‘farsighted’ retailer will consider both his
actions and how other retailers would react to his actions. Formula (4) (Section 3.1) is applied to get the Shapley value of each retailer in different scenarios. One scenario, in a game with three retailers, is that the residual supply for retailer 3 to be shared is at least the sum of the residual demand of retailer 1 and retailer 2. When no alliance is formed, or the alliance composed of retailer 1 and retailer 2, each player receives a zero residual profit. The Shapley value of retailer 1 is

\[
\phi_1 = \frac{0!}{3!} \left( v(\emptyset \cup 1) - v(\emptyset) \right) + \frac{1!}{3!} \left( v(2 \cup 1) - v(2) \right) \\
+ \frac{1!}{3!} \left( v(3 \cup 1) - v(3) \right) + \frac{2!}{3!} \left( v(23 \cup 1) - v(23) \right) \\
= \frac{E_1}{2}.
\]

In the same way, the Shapley values allocated to retailer 2 and retailer 3 are \( \frac{E_2}{2} \) and \( \frac{E_1 + E_2}{2} \), respectively. Since the Shapley value allocation is not a core of the game and the grand coalition of myopic retailers is not stable, (myopic retailers are those who do not consider reactions of other players to their actions), the solution concepts of largest consistent set (LCS), (see Chwe, 1994), and equilibrium process of coalition formulation (EPCF), (see Konishi & Ray, 2003), are used to check the farsighted stability of the grand coalition. They prove that the grand coalition is a farsighted stable outcome for a transshipment game whether or not the retailers have identical unit residual profits. For farsighted retailers, no enforcement mechanism is needed to keep their commitment, and the profits from residual inventory transshipment are always maximized under the Shapley value allocation. In their study, the assumption of random demand that each
retailer faces is consistent with our study. But Sošić (2006) examines the transshipment problem only among multiple retailers, while we study a supply chain consisting of both retailers and supplier and explore profit allocation scheme more than residual profits.

Robinson (1993) reexamines a cooperative game initially presented by Gerchak and Gupta (1991). He uses the Shapley value to allocate a fair cost sharing among a number of retailers. These retailers have one common central supplier, and their inventory model follows the one constructed by Gerchak and Gupta (1991). Gerchak and Gupta consider a continuous-review \((Q, r)\) single-period inventory model with full back ordering, where \((Q, r)\) represents “order \(Q\) items when inventory level reaches \(r\)”. Furthermore, \(N = \{1, 2, ..., n\}\) denotes the set of retailers with common ownership. Each retailer faces random demands \(D_i\) over one time unit with an expected demand \(E(D_i)\). The lead time is assumed to be the same for each retailer. During the lead time, the random demands \(X_i\) are assumed to follow a distribution function \(G_i\) with a density function \(g_i\) and expected value \(E(X_i)\). Suppose each retailer has a common order setup cost \(B\), unit holding cost per unit time \(h\) and unit shortage cost \(\eta\). An expected cost per unit time of retailer \(i\) is

\[
C_i(Q_i, r_i) = B[E(D_i)]/Q_i + h[Q_i/2 + r_i - E(X_i)] + \eta E(D_i)R_i(r_i)/Q_i \tag{13}
\]

Where \(R_i(r_i)\) is the expected shortage amount per unit time and \(R_i(r_i) = E[(X_i - r_i)^+] = \int_{r_i}^{\infty} (x - r_i)g_i(x)dx\). \([r_i - E(X_i)]\) is the expected safety stock for store \(i\).

Now set the first partial derivatives of \(C_i(Q_i, r_i)\) with respect to each argument equal to zero to minimize the cost \(C_i(Q_i, r_i)\). The optimal solutions of \((Q, r)\) are:
In addition, Gerchak and Gupta (1991) prove that centralization of the inventory system is the best option if the demand of the central system is the sum of each individual retailer’s expected demand. They also show that the cost functions are subadditive. However, only the retailers who join a subcoalition are ensured to be better off, and some retailers may be worse off in the centralized system.

Robinson (1993) then presents a cost allocation scheme based on the Shapley value that makes every retailer better off. With \( N \) players, the cost function of the subcoalition \( S \) is \( C(S) = C(Q^*_S, r^*_S) \) where \( S \subseteq N \). Let \( \nu(S) \) represent savings (benefits) by centralizing a group of retailers (a subcoalition):

\[
\nu(S) = \sum_{i \in S} C(Q^*_i, r^*_i) - C(Q^*_S, r^*_S)
\]

To ensure that no one is worse off after a new retailer joins, the conditions of core membership are satisfied for each subcoalition. The core of a concave cost game is non-empty and the Shapley value is one of the core allocations. The Shapley value is then introduced as an allocation that ensures every retailer benefit from centralization.

A critical implicit assumption of the inventory model by Gerchak and Gupta (1991) is that the performance of each retailer is identical. This assumption may not always be realistic in most cases, and the model can be adjusted by assigning different parameter values to each retailer. Furthermore, the retailers make their decisions independently and all of the shortage demand is backordered. It is highly possible that customers of one retailer, if finding one product stock out, will seek products from another retailer if the
two retailers are in one district. Robinson (1993) considers the allocation of costs among only retailers. My work extends it to a two-echelon model, including a supplier and retailers.

Additionally, the Shapley value has been used to divide profits of supply chains in a couple of papers. Manes and Verrecchia (1982) apply both the Massachusetts formula (MF) (defined in Section 2.1) and the Shapley value adjusted MF to set intra-company transfer prices. One limitation of this model is that it only studies a constant product flow; a second shortcoming of this model is that it does not explain whether the presented allocation is fair to all of the parties.

In the existing literature, the closest research to my work is done by Rosenthal (2008), where a cooperative game is set up among all divisions in a vertically integrated supply chain and a procedure is provided to obtain transfer prices between divisions using the Shapley value. Unlike my model, demand is deterministic. In my model, the retailers face a random demand and order quantity decisions are made by the retailers.

It is assumed that each division has the option of transacting with outside markets or with other divisions. It is also assumed that transactions among divisions are involved with high costs; the technology and transactions activities are compatible among divisions; and the transactions between divisions could save some fixed costs compared with the case when they trade the same products with outside markets. Cost sharing contributes to subadditive cost functions where two divisions’ shared cost is less than the sum of the two divisions’ individual costs.

In Rosenthal (2008)’s model, these parameters are known: the supply chain revenue from the outside market, the product purchasing costs of each division, the market price
for the intermediate goods, fixed operating costs and fixed cost shared among each subset of these divisions. A division sells a certain amount of products to its downstream division and the profits are allocated among all upstream divisions in the supply chain. With subadditive coalition cost functions, the game between divisions is shown to be a convex game, where the core of the game exists and the Shapley value of the game is the centroid of the core. First, a system of equalities about the revenue of every subcoalition is created. Next calculate the Shapley value for all divisions with respect to the transfer prices between divisions. Since the Shapley value lies in the core of game, further are

Figure 2  A vertically integrated supply chain with Division A, B and C

reached according to the rule that the payoff of any subcoalition is no less than the sum of individual payoffs. Finally, a system of inequalities is reached with respect to the desired transfer prices, which are the unfinished product prices that allocate the profits in a well-accepted way.
We can know more about the procedure through mathematical details of the following example from Rosenthal (2008): three divisions, \( A \), \( B \), and \( C \), are integrated in a supply chain (Figure 2). The cost functions are subadditive. Suppose one unit of intermediate goods is transferred from upstream divisions to downstream divisions. We know acquisition cost \( c_i \) from outside, fixed cost \( f_i \) for each division, fixed cost \( f_s \) shared among each subset \( S \) of these divisions, outside market price \( p_i' \), transfer price \( p_i \), \( i = (A, B, C) \) : \( c_A = 25 \), \( c_B = 10 \), \( c_C = 20 \), \( f_A = 20 \), \( f_B = 12 \), \( f_C = 30 \), \( f_{AB} = 28 \), \( f_{AC} = 49 \), \( f_{BC} = 41 \), \( f_{ABC} = 57 \), 
\[
p_A' = 50, \quad p_B' = 75, \quad p_C' = 135.
\]
Then the profits per unit of the subcoalitions \( (A, B, C, AB, AC, BC \text{and} ABC) \) are as follows:
\[
\begin{align*}
\pi_A &= p_A - (c_A + f_A) = p_A - 45, \\
\pi_B &= p_B - (p_A + c_B + f_B) = p_B - p_A - 22, \\
\pi_C &= p_C' - p_B - c_C - f_C = 135 - 20 - 30 - p_B = 85 - p_B, \\
\pi_{AB} &= p_B - c_A - c_B - f_{AB} = p_B - 25 - 10 - 28 = p_B - 63, \\
\pi_{AC} &= p_C - c_A - c_C - f_{AC} + p_A - p_B = 135 - 45 - 49 + p_A - p_B = 41 + p_A - p_B, \\
\pi_{BC} &= p_C - c_B - c_C - f_{BC} - p_A = 135 - 30 - 41 - p_A = 64 - p_A, \\
\pi_{ABC} &= p_C - c_A - c_B - c_C - f_{ABC} = 135 - 25 - 10 - 20 - 57 = 23.
\end{align*}
\]
Equation (4) is then used to get the Shapley value of each division: \( \varphi_A = p_A - 42.83 \), \( \varphi_B = p_B - p_A - 19.83 \) and \( \varphi_C = 85.67 - p_B \).

Because the Shapley value lies in the core of the game, it is expected that each division would get more than when they keep isolated from the coalition; the Shapley value of any
two divisions is no less than the sum of their payoffs if they are binding but staying isolated from the third one. Thus,

\[ \phi_A \geq p_A' - c_A - f_A = 5, \]
\[ \phi_B \geq p_B' - c_B - f_B - p_A' = 3, \]
\[ \phi_C \geq p_C' - c_B - f_B - p_B' = 10. \]
\[ \phi_A + \phi_B \geq p_B' - c_A - c_B - f_{AB} = 75 - 25 - 10 - 28 = 12, \]
\[ \phi_B + \phi_C \geq p_C' - c_B - c_C - f_{BC} = 135 - 10 - 20 - 41 - 50 = 14, \]
\[ \phi_A + \phi_C \geq p_C' - p_B' - c_C + (p_A' - c_A) - f_{AC} = 135 - 75 - 20 - (50 - 25) - 49 = 16. \]

Therefore, the final constraints (Figure 3) are reached as a system of inequalities:

\[ 47.83 \leq p_A \leq 51.83; \quad 74.67 \leq p_B \leq 75.67; \quad 22.83 \leq p_B - p_A \leq 26.83. \]

Figure 3  The Shapley value solution of Rosenthal (2008)

One limitation of this model, as mentioned, is that it does not consider demand. The study could be extended if letting the retailers face stochastic demands. This author does not aim to explain what would occur from the decision maker’s selection of different
transfer prices within the range, wherein each set of values provides stable outcomes of the game.
Chapter 4 The Models

4.1 Model assumptions

The purpose of the methodology section is to develop a sequence of mathematical models to compute fair transfer prices among supply chain members. It is assumed that supply chain members form a grand coalition to maximize total supply chain profits in a specific period of time. (Refer to 4.2 for the notation of all parameters.)

In this dissertation, I investigate a decentralized supply chain with one supplier and two heterogeneous retailers (denoted by \( R_0, R_1, \) and \( R_2 \), respectively) as in Figure 4. The two retailers are in different markets. It is assumed that the demands of the two markets are independent. In the game of \( R_0, R_1, \) and \( R_2 \), the characteristic function \( \nu(S) \), where \( \nu : 2^N \rightarrow \mathbb{R} \), is defined as the worth of the coalition \( S \). Complete information is further assumed: each player knows the other’s cost information - including purchase cost, holding cost, shortage cost, fixed cost (e.g., technology and transactions cost), etc, and revenue information. In the grand coalition, the profit generated from sales by the two retailers is to be allocated among the three players. Since the objective is to divide the total profits of the grand coalition among the three players, I will only prove that the grand coalition is stable. Not as in the literature of coalition structure (Owen, 1977; Hart, S. and Kurz, M.,

\[ \text{Supplier } R_0 \]

\[ \text{Retailer 1 } R_1 \quad \text{Retailer 2 } R_2 \]

Figure 4 The supply chain model with one supplier and two retailers
I do not study whether all of the coalitions in the game will form or not. The “worth”s of these coalitions are calculated for the application of the Shapley value if these coalitions form. Let $R_i$’s demand be normally distributed with density function $f_1(d)$, $R_2$’s demand be normally distributed with density function $f_2(d)$, and $f_1(d) \neq f_2(d)$. The two retailers remain independent. The sum of the two random variables of demands is still normal. The normal demand function is widely used in the literature (e.g., Gerchak & Gupta, 1991; Cachon, 2003).

Both retailers order a certain amount of products from the supplier to fulfill their own stochastic demand. It is assumed that if the two retailers form a coalition, they will share a common warehouse and the common unit holding cost is less than either individual unit holding cost ($h < h_i, i = 1, 2$). The retail price $p_i$ is determined by each retailer. The supplier sells a common product to two retailers at a wholesale price $p_0$ with $c_0 < p_0 < p_i$, whereby $i = 1, 2$, and $c_0$ is the manufacturing cost per unit of $R_0$. It is interesting to figure out how much the transfer price should be if the retailers and the supplier form an alliance. Transfer prices are the wholesale prices when all three players cooperate. In this dissertation, a wholesale price denotes the market price charged by the supplier to a retailer without any agreement between them. The transfer price herein is the unit payment between the supplier and a retailer within a coalition. It should be made clear that with a cost saving coalition ($B_G < B_0 + B_{12} < B_0 + B_1 + B_2$), the transfer price is very likely different from the wholesale price to indicate a relatively fair profit allocation. The supplier has enough capacity to satisfy orders from the retailers --- no capacity constraints. The firms are in one country, without tax rate difference. Following the
assumption by Rosenthal (2008), transactions between the supplier and a retailer could save some fixed cost as compared with the case without any agreement. So do holding costs. Cost functions are accordingly subadditive.

The whole model development will be stated as in Figure 5.
Figure 5  The Flowchart of the Methodology

The Shapley value Formula

\[ \phi(v) = \sum_{S \subseteq \pi(0)} \frac{|S|!(|M|-|S|-1)!}{|M|!} [\pi(S \cup i) - \pi(S)] \]
4.2 Notations

*Parameters:*

\( f_i(d) \): Demand density function of retailer \( i \) \((i = 1,2)\)

\( D_i \): Demand of retailer \( i \) \((i = 1,2)\) (units)

\( Z \): Sum of the demands to the two retailers (units)

\( g(z) \): Probability density function of the sum of two retailers’ demand

\( B_i \): Fixed cost of retailer \( i \) \((i = 1,2)\) per order (\$/order)

\( h_i \): Unit holding cost of retailer \( i \) \((i = 1,2)\) (\$/unit)

\( h \): Unit holding cost of retailers if the two retailers form a coalition (\$/unit)

\( \eta_i \): Unit shortage cost of retailer \( i \) \((i = 1,2)\) (\$/unit), where \( \eta_i > h_i \)

\( p_0 \): The wholesale price of \( R_0 \) (\$/unit)

\( p_1 \): Retail price per unit product of \( R_1 \) (\$/unit), where \( p_1 > p_0 \)

\( p_2 \): Retail price per unit product of \( R_2 \) (\$/unit), where \( p_2 > p_0 \)

\( B_0 \): Fixed cost of \( R_0 \) per order from \( R_1 \) or \( R_2 \) (\$/order)

\( c_0 \): Manufacturing cost per unit of \( R_0 \) (\$/unit), where \( c_0 < p_0 \)

\( B_{01} \): Fixed cost per order if \( R_0 \) and \( R_1 \) make a coalition (\$/order)

\( B_{02} \): Fixed cost per order if \( R_0 \) and \( R_2 \) make a coalition (\$/order)

\( B_{12} \): Fixed cost per order if \( R_1 \) and \( R_2 \) make a coalition (\$/order)

\( B_{G} \): Fixed cost per order of the grand coalition (\$/order)

\( B_{G}^i \): Fixed cost of player \( i \) per order in the grand coalition \((i = 0,1,2)\) (\$/order)
**Variables:**

- $\pi_i$: Profit function of player $i$ ($i = 0, 1, 2$)
- $\pi_{0i}$: Profit function of $R_0$ resulting from transactions with $R_i$
- $\pi_{0j}$: Profit function of $R_0$ resulting from transactions with $R_2$
- $\pi_{ij}$: Total profits of the subcoalition composed of $R_i$ and $R_j$ ($i, j = 0, 1, 2; i \neq j$)
- $\pi_G$: Total profits of the grand coalition
- $\pi^G_i$: Player $i$’s expected profit if the grand coalition forms but before transfer prices are paid

- $q_i$: Order quantity of retailer $i$ ($i = 1, 2$) (units)
- $q^G_i$: Order quantity of retailer $i$ if the grand coalition forms
- $TP_i^G$: The transfer price between $R_0$ and $R_i$ when $R_0$, $R_1$, and $R_2$ form a coalition. $TP_i > 0$ denotes a net payment from retailer $i$ to the supplier, while $TP_i < 0$ denotes an opposite payment from the supplier to retailer $i$ ( /unit)

4.3 Model development

In this three-player game, there are seven possible coalitions, $S \in \{\{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$. The analysis proceeds as follows. First, we determine coalition $S$’s worth, $\nu(S)$. In this supply chain, $\nu(S)$ is simply the sum of the coalition members’ expected profits, $\pi_S$. Such profit levels are well-defined as shown in the following subsections. Note that for a coalition consisting of one player, the coalition’s worth is equal to this player’s profit when all three players remain independent.
Second, we calculate the Shapley value using equation (4).

Finally, we use retailer \( i \)'s Shapley Value to calculate the transfer price paid to the supplier by retailer \( i \). Indeed, to determine the transfer prices paid by the two retailers, we assume that the grand coalition forms and that each player earns a payoff equal to its Shapley Value from game \( (3; v) \). More specifically, for \( j = 0, 1, 2 \), let \( \pi_j^G \) denote player \( j \)'s expected profit if the grand coalition forms but before transfer prices are paid, where

\[
\sum_{j=0}^{3} \pi_j^G = \pi_G.
\]

Then retailer \( i \)'s transfer price is defined as \( TP_i = \frac{\pi_i^G}{q_i^G} \), where \( q_i^G \) is retailer \( i \)'s optimal order quantity if the grand coalition forms \( (i = 1, 2) \).

Calculating transfer prices requires an additional step because \( \pi_i^G \) is not well-defined. Indeed, to characterize \( \pi_i^G \), the fixed cost of the grand coalition must be allocated to the three players. For consistency, we assume that the Shapley value is used as the fixed cost allocation rule. Specifically, we assume that each player’s share of the fixed cost in the grand coalition is the Shapley Value of a cooperative game. In this game, each coalition’s worth is the fixed cost it would incur if that coalition were to form\(^5\).

For the sake of clarity, we now show that in a fully coordinated supply chain consisting of three players (i.e., one that plays the cooperative game \( (3; v) \) and uses the transfer prices defined above), each player does earn its Shapley Value as expected payoff. It is obvious from the definition of transfer prices that retailer \( i \)'s expected payoff is equal to \( \phi_i(v) \). Meanwhile, the amount transferred to the supplier via transfer prices is

\(^5\) Formally, characterizing the fixed cost allocation requires defining a game \( (3; w) \), where \( w(S) = B_S \) is the given total fixed cost coalition \( S \) would incur if it were to form. We assume that the fixed cost allocated to player \( i \) is equal to his Shapley Value \( \phi_i(w) \).
\[(TP_1(q_1^G)) + (TP_2(q_2^G)) = \pi_1^G - \phi_1 + \pi_2^G - \phi_2.\]

Furthermore, by definition of the Shapley Value, \(\phi_0 = \pi_G - \phi_1 - \phi_2\) and by definition of \(\pi_i^G, \pi_G = \pi_0^G + \pi_1^G + \pi_2^G.\) Therefore,

\[(TP_1(q_1^G)) + (TP_2(q_2^G)) = \pi_G - \pi_0^G = (\pi_G - \phi_0) = \phi_0 - \pi_0^G.\]

But since in the grand coalition, the supplier’s only source of revenue is obtained from transfer prices, it is clear that \(\pi_0^G = -c_0(q_1^G + q_2^G) - B_0^G,\) where \(B_0^G\) is the fixed cost allocated to the supplier. Hence,

\[(TP_1(q_1^G)) + (TP_2(q_2^G)) = \phi_0 + c_0(q_1^G + q_2^G) + B_0^G,\]

so that

\[(TP_1(q_1^G)) + (TP_2(q_2^G)) - c_0(q_1^G + q_2^G) - B_0^G = \phi_0.\]

Therefore, the supplier’s expected profit, which is on the left-hand side of the above equation, is also equal to its Shapley Value from game \((3; v)\).

4.3.1 The case of one-member subcoalitions

Here, no coalition is formed among the three players. In this case, each retailer chooses his order quantity to maximize his own profits and the supplier would deliver the amount ordered by the retailers. \(R_1\) orders a quantity of products based on his demand function, and resells the products at price \(p_1\). So does \(R_2\) but with a price \(p_2\). \(R_0\) would deliver the products with a quantity ordered by the retailers for a wholesale price \(p_0\), since he has enough capacity to meet the demand from the retailers. Each retailer faces a newsvendor problem.
To calculate the worth $v(R_i), i = 0, 1, 2$, for $R_0$, $R_1$, and $R_2$, a retailer’s optimal order quantity $q_i^N, i = 1, 2$ is to be determined. $v(R_i)$ is the expected profit functions of $R_0$, $R_1$, and $R_2$, respectively. Use $\pi_i (i = 0, 1, 2)$ to denote the profit functions of $R_0$, $R_1$, and $R_2$, therefore,

$$\text{Max } \pi_i(q_i) = E[p_i \min(q_i, D_i) - \eta_i ([D_i - q_i]^+) - h_i ([q_i - D_i]^+) - B_i - p_0 q_i]$$

(14)

where $i = 1, 2$,

and

$$[D_i - q_i]^+ = \begin{cases} D_i - q_i, & \text{if } (D_i - q_i) \geq 0 \\ 0, & \text{if } (D_i - q_i) < 0 \end{cases}.$$ 

Suppose the optimal order quantities of $R_i (i = 1, 2)$ are $q_1^N$ and $q_2^N$, respectively. Then the worth of $R_i$ is

$$v(R_i) = \pi_i(q_i^N).$$

With the optimal order quantity, $q_i^N$, the total payment of $R_i$ to $R_0$ is $p_0 q_i^N$. The resulting order fixed cost of $R_0$ is $B_0$. Let the profit of $R_0$ resulting from $R_i$ be $\pi_{0i}$, then $\pi_{0i} = p_0 q_i^N - B_0$. For a transaction of $R_0$ with $R_2$, the total payment of $R_2$ to $R_0$ is $p_0 q_2^N$ with an optimal order quantity, $q_2^N$, and the resulting order fixed cost of $R_0$ is $B_0$ as well. Let the profit of $R_0$ resulting from $R_2$ be $\pi_{02}$, then $\pi_{02} = p_0 q_2^N - B_0$. Hence the revenue of $R_0$ is $p_0 q_1^N + p_0 q_2^N$ and the cost of $R_0$ is $B_0 + B_0 + c_0 q_1^N + c_0 q_2^N$. Accordingly, the profit functions of $R_0$ resulting from transactions with $R_1$ and $R_2$, respectively, are

---

6 The evidence of the existence and uniqueness of $q_i^N$ and $q_2^N$ is shown later on in Sec. 4.3.4.
\[ \pi_0 = p_0 q_1^N - c_0 q_1^N - B_0, \] and
\[ \pi_0 = p_0 q_2^N - c_0 q_2^N - B_0. \]

So the profit functions of \( R_0 \) is the sum of profits from transactions with the two retailers
\[ v(R_0) = \pi_0 = \pi_0 + \pi_0 = p_0 (q_1^N + q_2^N) - c_0 (q_1^N + q_2^N) - 2B_0. \] (15)

4.3.2 The case of two-member subcoalitions

There are three scenarios if two of the players are cooperating.

Scenaro 1: \( R_1 \) forms a coalition with \( R_0 \), with \( R_2 \) staying isolated.

In this scenario, \( R_1 \) and \( R_0 \) would like to maximize their total profits. The subcoalition revenue is \( p_1 \min(q_1, D_1) \). The total fixed cost is now \( B_{01} \), with the assumption of fixed cost saving, \( B_{01} < B_0 + B_1 \). The manufacturing cost of the order quantity \( q_1 \) is \( c_0 q_1 \). Then
\[ \text{Max } \pi_{01}(q_1) = E \left[ p_1 \min(q_1, D_1) - \eta_1 \left( [D_1 - q_1]^+ \right) - h_1 \left( [q_1 - D_1]^+ \right) - B_{01} - c_0 q_1 \right]. \] (16)

Let \( q_1^* \) denote the order quantity when \( \pi_{01}(q_1) \) is maximized\(^7\). The payoff of \( R_2 \) is the same as in Section 4.3.1, since \( R_2 \) orders to optimize his own profits. Hence, the worth of the subcoalition composed of \( R_1 \) and \( R_0 \), \( v(R_0 R_1) \), is
\[ v(R_0 R_1) = \pi_{01}(q_1^*) + \pi_0. \]

Except the payoff of the subcoalition \( R_0 \) and \( R_1 \), \( R_0 \) would get some profits from transactions with \( R_2 \), even without any subcoalition between \( R_0 \) and \( R_2 \).

\(^7\) The evidence of the existence and uniqueness of \( q_1^* \) is shown later on in Sec. 4.3.4.
**Scenario 2:** $R_2$ forms a coalition with $R_0$, with $R_i$ staying isolated.

Similarly, we would like to maximize the expected profit function of the subcoalition of $R_0$ and $R_2$ by

$$\text{Max } \pi_{02}(q_2) = E\left[p_2 \min(q_2, D_2) - \eta_2 \left(\left[D_2 - q_2\right]^+\right) - h_2 \left(\left[q_2 - D_2\right]^+\right) - B_{02} - c_0 q_2\right]. \quad (17)$$

Let $q_2^\ast$ denote the order quantity when $\pi_{02}(q_2)$ is maximized\(^8\). The payoff of $R_1$ is the same as in Section 4.3.1, since $R_1$ orders to optimize his own profits. Hence the worth of the subcoalition composed of $R_2$ and $R_0$ is

$$\nu(R_0R_2) = \pi_{02}(q_2^\ast) + \pi_{01}.$$  

**Scenario 3:** A subcoalition composed of $R_1$ and $R_2$, with $R_0$ staying isolated.

If $R_1$ and $R_2$ form a subcoalition, we assume they will place an order together and share one common warehouse. The fixed ordering cost is reasonably assumed to be less than the sum of their individual fixed costs: $B_{12} < B_1 + B_2$. The common unit holding cost is reduced and less than their individual unit holding costs: $h < \min(h_1, h_2)$.

$$\text{Max } \pi_{12}(q_1, q_2) = E\left[\sum_{i=1,2} p_i \min(q_i, D_i) - \sum_{i=1,2} \eta_i \left(\left[D_i - q_i\right]^+\right) - h \left(\left[q_i + q_2\right] - \left(D_1 + D_2\right)\right)^+\right]$$

$$- B_{12} - p_0 (q_1 + q_2). \quad (18)$$

Let $q_1^\ast$ and $q_2^\ast$ denote the order quantity when $\pi_{12}(q_1, q_2)$ is maximized\(^9\). The worth of the subcoalition, $\nu(R_1R_2)$, is

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\(^8\) The evidence of the existence and uniqueness of $q_2^\ast$ is shown later on in Sec. 4.3.4.

\(^9\)
\( \nu(R_1, R_2) = \pi_{12}(q_1^R, q_2^R). \)

4.3.3 The case of the grand coalition

In the grand coalition, the two retailers would place an order together to the supplier and share one common warehouse. Their common unit holding cost is still \( h \). The order quantity is the sum of their individual orders but would optimize the payoff of the grand coalition. Recall we assume that the total fixed cost of the coalition is less than the sum of their individual fixed costs: \( B_G < B_0 + B_{12} < B_0 + B_1 + B_2 \). The worth of the grand coalition can be constructed as in Formula (19), which is also the expected profit function of the grand coalition. To get the optimal order quantity of retailer \( i (i = 1, 2) q_i^G \), we would like to maximize the expected profit function of the grand coalition:

\[
\begin{aligned}
\text{Max } \pi_G(q_1, q_2) &= E \left[ \sum_{i=1,2} p_i \min(q_i, D_i) - \sum_{i=1,2} \eta_i \left( [D_i - q_i]^+ \right) - h \left( [q_1 + q_2 - (D_1 + D_2)]^+ \right) \right] \\
&\quad - B_G - c_0 (q_1 + q_2). 
\end{aligned}
\]  

(19)

The worth of the grand coalition, \( \nu(R_0, R_1, R_2) \), is

\[ \nu(R_0, R_1, R_2) = \pi_G(q_1^G, q_2^G). \]

4.3.4 Solution of the mathematical models

Formula (14), (16), and (17) are all typical newsvendor problems (Nahmias, 2004).

Regarding formula (14)

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9 The evidence of the existence and uniqueness of \( q_1^R \) and \( q_2^R \) is shown later on in Sec. 4.3.4.

10 The evidence of the existence and uniqueness of \( q_i^G \) is shown later on in Sec. 4.3.4.
\[ \pi_i = E \left[ p_i \min(q_i, D_i) - \eta_i \left( [D_i - q_i]^- \right) - h_i \left( [q_i - D_i]^- \right) - B_i - p_o q_i \right]. \]

It follows for \( R_i \) that

\[ \pi_i = E \left[ p_i \min(q_i, D_i) - \eta_i \left( [D_i - q_i]^- \right) - h_i \left( [q_i - D_i]^- \right) - B_i - p_o q_i \right] \]

\[ = p_i \left( \int_0^{q_i} x f_i(x) dx + \int_{q_i}^{\infty} f_i(x) dx \right) - \eta_i \int_{q_i}^{\infty} (x - q_i) f_i(x) dx - h_i \int_0^{q_i} (q_i - x) f_i(x) dx - B_i - p_o q_i. \]

The derivatives of \( \pi_i \) can be obtained by Leibniz formula:

\[ \frac{d\pi_i}{dq_i} = p_i \left( q_i f_i(q_i) - q_i f_i(q_i) + \int_{q_i}^{\infty} f_i(x) dx \right) - \eta_i \left( -(q_i - q_i) f_i(q_i) - \int_{q_i}^{\infty} f_i(x) dx \right) \]

\[ - h_i \left( (q_i - q_i) f_i(q_i) + \int_{q_i}^{\infty} f_i(x) dx \right) - p_o \]

\[ = p_i \left( \int_{q_i}^{\infty} f_i(x) dx \right) + \eta_i \int_{q_i}^{\infty} f_i(x) dx - h_i \int_0^{q_i} f_i(x) dx - p_o \]

\[ \frac{d\pi_i}{dq_i} = p_i (1 - F_i(q_i)) + \eta_i (1 - F_i(q_i)) - h_i F_i(q_i) - p_o, \quad \text{and} \]

\[ \frac{d^2 \pi_i}{dq_i^2} = (-h_i - \eta_i - p_i) f_i(q_i). \]  

(20)

Since \( f_i(q_i) > 0 \), the second derivative \((-h_i - \eta_i - p_i) f_i(q_i) < 0\). Thereafter, the optimal order quantity \( q_i^N \) is obtained when

\[ \frac{d\pi_i}{dq_i} = p_i (1 - F_i(q_i)) + \eta_i (1 - F_i(q_i)) - h_i F_i(q_i) - p_o = 0. \]

Rearranging terms gives

\[ F_i(q_i^N) = \frac{p_i - p_o + \eta_i}{p_i + h_i + \eta_i}. \]  

(22)
Formula (22) gives the rule to acquire the optimal order quantity \( q_1^N \): an order quantity need to be found where the demand cumulative distribution function \( F_i(\cdot) \) equals the fraction on the right hand side. Since \( p_1 > p_0 > 0 \) and \( \eta_i > 0 \), then \( p_1 - p_0 + \eta_i > 0 \) and 
\[
\frac{p_1 - p_0 + \eta_i}{p_1 + h_i + \eta_i} > 0 \;; \text{ since } \frac{p_1 - p_0 + \eta_i}{p_1 + h_i + \eta_i} < \frac{p_1 + \eta_i}{p_1 + h_i + \eta_i} \leq 1.
\]
Then there must exist one \( q_1^N \), which satisfies (22). In addition, with the monotonicity of a probability cumulative function \( F_i(\cdot) \), the optimal order quantity \( q_1^N \) is unique.

Similarly, see the 1st and 2nd derivatives for \( R_2 \):
\[
\frac{d\pi_2}{dq_2} = p_2(1 - F_2(q_2)) + \eta_2(1 - F_2(q_2)) - h_2 F_2(q_2) - p_0; \tag{23}
\]
\[
\frac{d^2\pi_2}{dq_2^2} = (-h_2 - \eta_2 - p_2)f_2(q_2). \tag{24}
\]
With a non-positive 2nd derivative, the optimal order quantity \( q_2^N \) is obtained when 
\[
\frac{d\pi_2}{dq_2} = p_2(1 - F_2(q_2)) + \eta_2(1 - F_2(q_2)) - h_2 F_2(q_2) - p_0 = 0.
\]
Rearranging terms gives 
\[
F_2(q_2^N) = \frac{p_2 - p_0 + \eta_2}{p_2 + h_2 + \eta_2}. \tag{25}
\]
Again, Formula (25) gives the rule to acquire the optimal order quantity \( q_2^N \) when \( R_2 \) stays outside any coalition. Since \( p_2 > p_0 > 0 \) and \( \eta_2 > 0 \), then \( p_2 - p_0 + \eta_2 > 0 \) and 
\[
\frac{p_2 - p_0 + \eta_2}{p_2 + h_2 + \eta_2} > 0 \;; \text{ since } \frac{p_2 - p_0 + \eta_2}{p_2 + h_2 + \eta_2} < \frac{p_2 + \eta_2}{p_2 + h_2 + \eta_2} \leq 1.
\]
there must exist one \( q_2^N \), which satisfies (25). In addition, with the monotonicity of a probability cumulative function \( F_2(\cdot) \), the optimal order quantity \( q_2^N \) is unique. \( q_2^N \) is reached where the demand cumulative function equals to the fraction \( \frac{p_2 - p_0 + \eta_2}{p_2 + h_2 + \eta_2} \).

Therefore, the profit functions of \( R_1, R_2, \) and \( R_0 \) are as follows

\[
\pi_1 = E \left[ p_1 \min(q_i^N, D_i) - \eta_i \left( [D_i - q_i^N]^+ \right) - h_i \left( [q_i^N - D_i]^+ \right) - B_i - p_0 q_i^N \right]; \\
\pi_2 = E \left[ p_2 \min(q_2^N, D_2) - \eta_2 \left( [D_2 - q_2^N]^+ \right) - h_2 \left( [q_2^N - D_2]^+ \right) - B_i - p_0 q_2^N \right]; \\
\pi_0 = p_0 (q_1^N + q_2^N) - c_0 (q_1^N + q_2^N) - 2B_0.
\]

The same actions are taken for Formula (16) to determine the value of \( q_1 \) that maximizes the profit of \( R_1 \) and \( R_0 \), \( \nu(R_0, R_1) \):

\[
\pi_{01} = E \left[ p_1 \min(q_i, D_i) - \eta_i \left( [D_i - q_i]^+ \right) - h_i \left( [q_i - D_i]^+ \right) - B_{01} - c_0 q_i \right].
\]

\[
\frac{d\pi_{01}}{dq_i} = p_1 (1 - F_i(q_i)) + \eta_i (1 - F_i(q_i)) - h_i F_i(q_i) - c_0 \; ; \\
\frac{d^2\pi_{01}}{dq_i^2} = (-h_i - p_i - \eta_i) f_i(q_i) .
\]

With a non-positive 2\(^{nd}\) derivative, the optimal order quantity \( q_1^S \) can be reached if

\[
\frac{d\pi_{01}}{dq_i} = p_1 (1 - F_i(q_i)) + \eta_i (1 - F_i(q_i)) - h_i F_i(q_i) - c_0 = 0 .
\]

Rearranging terms gives

\[
F_i(q_i^S) = \frac{p_1 - c_0 + \eta_i}{p_1 + h_i + \eta_i} .
\]
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$q_i^s$ determined in formula (31) would be used as the optimal order quantity when just $R_i$ and $R_0$ form a subcoalition. Since $p_i > p_0 > c_0 > 0$ and $\eta_i > 0$, then $p_i - c_0 + \eta_i > 0$ and

\[
\frac{p_i - c_0 + \eta_i}{p_i + h_i + \eta_i} > 0; \quad \text{since} \quad p_i - c_0 + \eta_i < p_i + \eta_i < p_i + h_i + \eta_i, \quad \frac{p_2 - p_0 + \eta_2}{p_2 + h_2 + \eta_2} < 1. \quad \text{Then there must exist one } q_i^s, \text{ which satisfies (31). In addition, with the monotonicity of a probability cumulative function } F_i(\cdot), \text{ the optimal order quantity } q_i^s \text{ is unique. } q_i^s \text{ is reached where the demand cumulative function equals to the fraction } \frac{p_i - c_0 + \eta_i}{p_i + h_i + \eta_i}.

Therefore, the maximum of the subcoalition profit functions of $R_0$ and $R_i$ is

\[
\pi_{0i} = E \left[ p_i \min(q_i^s, D_i) - \eta_i \left( [D_i - q_i^s]^+ \right) - h_i \left( [q_i^s - D_i]^+ \right) - B_{0i} - c_0 q_i^s \right]. \quad (32)
\]

Similarly for Formula (17), to determine the value of $q_2$ that maximizes the profit of $R_2$ and $R_0$, $\nu(R_0, R_2)$:

\[
\pi_{02} = E \left[ p_2 \min(q_2, D_2) - \eta_2 \left( [D_2 - q_2]^+ \right) - h_2 \left( [q_2 - D_2]^+ \right) - B_{02} - c_0 q_2 \right].
\]

\[
\frac{d\pi_{02}}{dq_2} = p_2 (1 - F_1(q_2)) + \eta_2 (1 - F_2(q_2)) - h_2 F_2(q_2) - c_0; \quad (33)
\]

\[
\frac{d^2\pi_{02}}{dq_2^2} = (-\eta_2 - h_2 - p_2) f_2(q_2). \quad (34)
\]

Again, with a non-positive second derivative, the optimal order quantity $q_2^s$ is reached by

\[
\frac{d\pi_{02}}{dq_2} = p_2 (1 - F_2(q_2)) + \eta_2 (1 - F_2(q_2)) - h_2 F_2(q_2) - c_0 = 0.
\]

Rearranging terms gives
\[ F_2(q_2^*) = \frac{p_2 - c_0 + \eta_2}{p_2 + h_2 + \eta_2}. \] (35)

\( q_2^* \) determined in formula (35) would be used as the optimal order quantity when just \( R_2 \) and \( R_0 \) form a subcoalition. Since \( p_2 > p_0 > c_0 > 0 \) and \( \eta_2 > 0 \), then \( p_2 - p_0 + \eta_2 > 0 \) and \( \frac{p_2 - c_0 + \eta_2}{p_2 + h_2 + \eta_2} > 0 \); since \( p_2 - c_0 + \eta_2 < p_2 + \eta_2 < p_2 + h_2 + \eta_2 \), \( \frac{p_2 - c_0 + \eta_2}{p_2 + h_2 + \eta_2} < 1 \). Then there must exist one \( q_2^* \), which satisfies (35). In addition, with the monotonicity of a probability cumulative function \( F_2() \), the optimal order quantity \( q_2^* \) is unique. \( q_2^* \) is reached where the demand cumulative function equals to the fraction \( \frac{p_2 - c_0 + \eta_2}{p_2 + h_2 + \eta_2} \).

Therefore, the maximum of the subcoalition profit functions of \( R_0 \) and \( R_2 \) is

\[ \pi_{02} = E\left[ p_2 \min(q_2^*, D_2) - \eta_2 \left( [D_2 - q_2^*]^+ \right) - h_2 \left( [q_2^* - D_2]^+ \right) - B_{02} - c_0 q_2^* \right]. \] (36)

Formula (18) and (19) are not typical newsvendor problems, each with two variables. However, the value of \( q_1 \) and \( q_2 \) can be determined simultaneously that maximize the payoff of the subcoalition consisting of \( R_1 \) and \( R_2 \), \( v(R_1 R_2) \):

\[ \pi_{12} = E\left[ \sum_{i=1,2} p_i \min(q_i, D_i) - \sum_{i=1,2} \eta_i \left( [D_i - q_i]^+ \right) - h \left( [(q_i + q_2) - (D_i + D_2)]^+ \right) \right] - B_{12} - p_0 (q_1 + q_2). \]

Since demands are following normal distribution functions, the sum of two normal random variable, \( Z \), is still following a normal distribution, \( g(z) \). Here, \( E[Z] = E[D_1] + E[D_2] \) and \( Var(Z) = Var(D_1) + Var(D_2) \).

\[ \frac{\partial \pi_{12}}{\partial q_1} = p_1 (1 - F_1(q_1)) + \eta_1 (1 - F_1(q_1)) - hG(q_1 + q_2) - p_0; \] (37)
and
\[ \frac{\partial \pi_{12}}{\partial q_2} = p_2(1 - F_2(q_2)) + \eta_2(1 - F_2(q_2)) - hG(q_1 + q_2) - p_0. \] (38)

\[ \frac{\partial^2 \pi_{12}}{\partial q_1^2} = -(p_1 + \eta_1)f_1(q_1) - hg(q_1 + q_2); \] (39)

\[ \frac{\partial^2 \pi_{12}}{\partial q_2^2} = -(p_2 + \eta_2)f_2(q_2) - hg(q_1 + q_2); \] (40)

\[ \frac{\partial^2 \pi_{12}}{\partial q_1 \partial q_2} = -hg(q_1 + q_2). \] (41)

Since
\[ \frac{\partial^2 \pi_{12}}{\partial q_1^2} \frac{\partial^2 \pi_{12}}{\partial q_2^2} - \left( \frac{\partial^2 \pi_{12}}{\partial q_1 \partial q_2} \right)^2 \]
\[ = (- (p_1 + \eta_1)f_1(q_1) - hg(q_1 + q_2))(- (p_2 + \eta_2)f_2(q_2) - hg(q_1 + q_2)) - (-hg(q_1 + q_2))^2 \]
\[ = ((p_1 + \eta_1)f_1(q_1) + hg(q_1 + q_2))((p_2 + \eta_2)f_2(q_2) + hg(q_1 + q_2)) - (hg(q_1 + q_2))^2 \]
\[ = [(p_1 + \eta_1)f_1(q_1)][(p_2 + \eta_2)f_2(q_2)] + h(g(q_1 + q_2))\left[(p_1 + \eta_1)f_1(q_1) + (p_2 + \eta_2)f_2(q_2)\right] \]
\[ > 0, \] (42)

and
\[ \frac{\partial^2 \pi_{12}}{\partial q_1^2} = -(p_1 + \eta_1)f_1(q_1) - h(g(q_1 + q_2)) < 0, \] (43)

The optimal order quantity \( q_1^R \) of \( R_1 \) and \( q_2^R \) of \( R_2 \) can be obtained if
\[ \frac{\partial \pi_{12}}{\partial q_1} = p_1(1 - F_1(q_1)) + \eta_1(1 - F_1(q_1)) - hG(q_1 + q_2) - p_0 = 0, \]

and
\[ \frac{\partial \pi_{12}}{\partial q_2} = p_2 (1 - F_2(q_2)) + \eta_2 (1 - F_2(q_2)) - hG(q_i + q_2) - p_0 = 0. \]

Rearranging terms gives

\[ (p_1 + \eta_1) (1 - F_1(q_i^R)) - hG(q_i^R + q_2^R) = p_0, \quad (44) \]

and

\[ (p_2 + \eta_2) (1 - F_2(q_2^R)) - hG(q_i^R + q_2^R) = p_0. \quad (45) \]

$q_i^R$ and $q_2^R$ determined by equations (44) and (45) simultaneously are used as the optimal order quantity of $R_i$ and $R_2$ when they form a subcoalition. Next, I will show the existence and uniqueness of $q_i^R$ and $q_2^R$. Because $0 \leq G(\cdot) \leq 1$, $(p_1 + \eta_1) (1 - F_1(q_i^R)) \geq p_0$ and $(p_1 + \eta_1) (1 - F_1(q_i^R)) \leq p_0 + h$. With $p_i > p_0$ and $\eta_i > h$, $F_i(q_i^R) < 1$ and $F_i(q_i^R) > 0$. Similarly, $F_2(q_2^R) < 1$ and $F_2(q_2^R) > 0$. Thus, there must be at least one $q_i^R$ and $q_2^R$ satisfy the equation system (44) and (45). Meanwhile, since $(p_1 + \eta_1) (1 - F_1(q_i^R)) = (p_2 + \eta_2) (1 - F_2(q_2^R))$, $F_i(q_i^R)$ and $F_2(q_2^R)$ have a positive linear relationship. With the increasing monotonicity of cumulative probability functions, $F_i(\cdot)$ and $F_2(\cdot)$, $q_i^R$ and $q_2^R$ have a monotonically increasing relationship. Furthermore, the cumulative probability function $G(\cdot)$ is also a monotonically increasing function. Since equations (44) and (45) can be further written as

\[ p_i + \eta_i - p_0 = (p_1 + \eta_1) F_i(q_i^R) + hG(q_i^R + q_2^R) \]

and

\[ p_2 + \eta_2 - p_0 = (p_2 + \eta_2) F_2(q_2^R) + hG(q_i^R + q_2^R), \]

there is at most one set of solutions that could satisfy the equation system (44) and (45). Suppose there is one set of solutions $q_i$ and $q_2$, no matter increasing $q_i$ and $q_2$ or
decreasing $q_1$ and $q_2$ will not make the previous two equations balance any more. So far, the existence and uniqueness of the solutions to the equation system (44) and (45) have been proved.

Therefore, the maximum of the subcoalition ($R_1$ and $R_2$) profit functions is:

$$\pi_{12} = E \left[ \sum_{i=1,2} p_i \min(q_i^R, D_i) - \sum_{i=1,2} \eta_i \left( \left[ D_i - q_i^R \right]^+ \right) - h \left( \left[ (q_i^R + q_i^R) - (D_i + D_2) \right]^+ \right) \right] - B_{12} - p_0 (q_1^R + q_2^R).$$  \hspace{1cm} (46)

Similarly, formula (19) is also a maximizing problem with two variables. Recall

$$\pi_G = E \left[ \sum_{i=1,2} p_i \min(q_i, D_i) - \sum_{i=1,2} \eta_i \left( \left[ D_i - q_i \right]^+ \right) - h \left( \left[ (q_i + q_2 - (D_i + D_2)) \right]^+ \right) \right] - B_G - c_0 (q_1 + q_2)$$

Taking the first partial derivates with respect to $q_1$ and $q_2$:

$$\frac{\partial \pi_G}{\partial q_1} = p_1 (1 - F_1(q_1)) + \eta_1 (1 - F_1(q_1)) - hG(q_1 + q_2) - c_0,$$ \hspace{1cm} (47)

and

$$\frac{\partial \pi_G}{\partial q_2} = p_2 (1 - F_2(q_2)) + \eta_2 (1 - F_2(q_2)) - hG(q_1 + q_2) - c_0.$$ \hspace{1cm} (48)

$$\frac{\partial^2 \pi_G}{\partial q_1^2} = -(p_1 + \eta_1) f_1(q_1) - hg(q_1 + q_2),$$ \hspace{1cm} (49)

$$\frac{\partial^2 \pi_G}{\partial q_2^2} = -(p_2 + \eta_2) f_2(q_2) - hg(q_1 + q_2),$$ \hspace{1cm} (50)

and

$$\frac{\partial^2 \pi_G}{\partial q_1 \partial q_2} = -hg(q_1 + q_2).$$ \hspace{1cm} (51)

Since
\[
\frac{\partial^2 \pi_G}{\partial q_1^2} \frac{\partial^2 \pi_G}{\partial q_2^2} - \left( \frac{\partial^2 \pi_G}{\partial q_1 \partial q_2} \right)^2 \\
= \left( - (p_1 + \eta_1 f_1(q_1) - h g(q_1 + q_2)) \right) \left( - (p_2 + \eta_2 f_2(q_2) - h g(q_1 + q_2)) \right) - \left( - h g(q_1 + q_2) \right)^2 \\
= \left[ (p_1 + \eta_1 f_1(q_1)) \right] \left[ (p_2 + \eta_2 f_2(q_2)) + h g(q_1 + q_2) \right] \left[ (p_1 + \eta_1 f_1(q_1)) + (p_2 + \eta_2 f_2(q_2)) \right] \\
> 0 ,
\]

(52)

and

\[
\frac{\partial^2 \pi_G}{\partial q_1^2} = -(p_1 + \eta_1 f_1(q_1)) - h(g(q_1 + q_2)) < 0 ,
\]

(53)

The optimal order quantity \( q_1^G \) of \( R_1 \) and \( q_2^G \) of \( R_2 \) can be obtained when the two first derivatives equal 0.

\[
\frac{\partial \pi_G}{\partial q_1} = p_1 (1 - F_1(q_1)) + \eta_1 (1 - F_1(q_1)) - hG(q_1 + q_2) - c_0 = 0 ,
\]

and

\[
\frac{\partial \pi_G}{\partial q_2} = p_2 (1 - F_2(q_2)) + \eta_2 (1 - F_2(q_2)) - hG(q_1 + q_2) - c_0 = 0 .
\]

Rearranging terms gives

\[
(p_1 + \eta_1) (1 - F_1(q_1^G)) - hG(q_1^G + q_2^G) = c_0 ,
\]

(54)

and

\[
(p_2 + \eta_2) (1 - F_2(q_2^G)) - hG(q_1^G + q_2^G) = c_0 .
\]

(55)

\( q_1^G \) and \( q_2^G \) determined by equations (54) and (55) would be used as the optimal order quantity of \( R_1 \) and \( R_2 \) in the grand coalition. Because \( 0 \leq G(\cdot) \leq 1 \),

\[
(p_1 + \eta_1) (1 - F_1(q_1^G)) \geq c_0 \quad \text{and} \quad (p_1 + \eta_1) (1 - F_1(q_1^G)) \leq c_0 + h .
\]

With \( p_1 > c_0 \) and \( \eta_1 > h \),
\(F_i(q_i^G) < 1\) and \(F_i(q_i^G) > 0\). Similarly, \(F_2(q_2^G) < 1\) and \(F_2(q_2^G) > 0\). Thus, there must be at least one \(q_i^G\) and \(q_z^G\) satisfy the equation system (54) and (55). Meanwhile, since
\[
(p_1 + \eta)(1 - F_i(q_i^G)) = (p_2 + \eta)(1 - F_2(q_2^G)),
\]
\(F_i(q_i^G)\) and \(F_2(q_2^G)\) have a positive linear relationship. With the increasing monotonicity of cumulative probability functions, \(F_i(\cdot)\) and \(F_2(\cdot)\), \(q_i^G\) and \(q_2^G\) have a monotonically increasing relationship. Furthermore, the cumulative probability function \(G(\cdot)\) is also a monotonically increasing function. Since equations (54) and (55) can be further written as
\[
p_1 + \eta_1 - c_0 = (p_1 + \eta_1)F_i(q_i^G) + hG(q_i^G + q_z^G) \quad \text{and}
\]
\[
p_2 + \eta_2 - c_0 = (p_2 + \eta_2)F_2(q_2^G) + hG(q_i^G + q_z^G),
\]
there is at most one set of solutions that could satisfy the equation system (54) and (55). No matter increasing \(q_i\) and \(q_z\) or decreasing \(q_i\) and \(q_z\) will not make the previous two equations balance any more. So far, the existence and uniqueness of the solutions to the equation system (54) and (55) have been proved.

Therefore, the maximum of the grand coalition profit functions is:
\[
\pi_G = E \left[ \sum_{i=1,2} p_i \min(q_i^G, D_i) - \sum_{i=1,2} \eta_i \left( \left[ D_i - q_i^G \right]^+ \right) - h \left( \left[ q_i^G + q_z^G - (D_i + D_z) \right]^+ \right) \right] - B_G - c_0 (q_i^G + q_z^G) .
\]

Accordingly, retailer 1 and retailer 2’s expected profits after the grand coalition forms but before transfer prices are paid, respectively, are
\[
\pi_1^G = E \left[ p_i \min(q_i^G, D_1) - \eta_i \left( \left[ D_i - q_i^G \right]^+ \right) - h \left( \left[ q_i^G - D_i \right]^+ \right) \right] - B_i^G , \quad \text{and}
\]
\[ \pi_2^G = E \left[ p_2 \min(q_2^G, D_2) - \eta_2 \left( \left[ D_2 - q_2^G \right]^+ \right) - h \left( \left[ q_2^G - D_2 \right]^+ \right) \right] - B_2^G. \]

4.3.5 The transfer pricing problem

Given the revenue that the supply chain generates from sales by \( R_1 \) and \( R_2 \), given the fixed costs for all of the members and subcoalitions, and given the holding rates and shortage rates of \( R_1 \) and \( R_2 \), I will set transfer prices between the supplier and each retailer to allocate the costs and revenue in a fair way. With the assumption of subadditive cost function, the game by \( R_0 \), \( R_1 \) and \( R_2 \) is convex with a non-empty core (Rosenthal, 2008). Hence there exists at least one solution that satisfies the conditions of the core. The Shapley value allocation is the centroid of the core set in a convex game (Shapley, 1971).

The objective is to set fair transfer prices between the supplier and each retailer. The core allocation insures that the total payoff of the grand coalition is split among all the players, and the sum of the payoffs to the players of each coalition \( S \) is not less than the payoff of the coalition \( S \). With an allocation in the core of a game, no player can get more without lowering other’s payoff. The Shapley value is the unique payoff division that divides the payoff of the grand coalition and satisfies the four mentioned axioms. By aggregating the contribution of players in their various opportunities, the Shapley value could further measure the bargaining power of players in a game (Winter, 2002).

The Shapley value solution is thus used as our profit division rule to set fair transfer prices. Using formula (4), the Shapley values of \( R_0 \), \( R_1 \), and \( R_2 \) can be obtained as follows, respectively:
\[
\phi_1 = \frac{0!(3-0-1)!}{3!}(v(\emptyset \cup R_1) - v(\emptyset)) + \frac{1!(3-1-1)!}{3!}(v(R_1, R_2) - v(R_2)) \\
\hspace{2cm} + \frac{1!(3-1-1)!}{3!}(v(R_1R_2) - v(R_1)) + \frac{2!(3-2-1)!}{3!}(v(R_1R_2R_2) - v(R_1R_2)) \\
= \frac{1}{3}v(R_1) + \frac{1}{6}(v(R_1R_2) - v(R_2)) + \frac{1}{6}(v(R_1R_2) - v(R_2)) + \frac{1}{3}(v(R_1R_2R_2) - v(R_1R_2)).
\]

Therefore,
\[
\phi_1 = \frac{1}{3}\pi_1 + \frac{1}{6}(\pi_{12} - \pi_2) + \frac{1}{6}(\pi_{01} + \pi_{02} - \pi_0) + \frac{1}{3}(\pi_G - \pi_{01} - \pi_{02}). \tag{57}
\]

\[
\phi_2 = \frac{0!(3-0-1)!}{3!}(v(\emptyset \cup R_2) - v(\emptyset)) + \frac{1!(3-1-1)!}{3!}(v(R_2, R_1) - v(R_1)) \\
\hspace{2cm} + \frac{1!(3-1-1)!}{3!}(v(R_2R_2) - v(R_2)) + \frac{2!(3-2-1)!}{3!}(v(R_2R_2R_2) - v(R_2R_2)) \\
= \frac{1}{3}v(R_2) + \frac{1}{6}(v(R_2R_1) - v(R_1)) + \frac{1}{6}(v(R_2R_1) - v(R_1)) + \frac{1}{3}(v(R_2R_2R_2) - v(R_2R_2)).
\]

Therefore,
\[
\phi_2 = \frac{1}{3}\pi_2 + \frac{1}{6}(\pi_{12} - \pi_1) + \frac{1}{6}(\pi_{02} + \pi_{01} - \pi_0) + \frac{1}{3}(\pi_G - \pi_{01} - \pi_{02}). \tag{58}
\]

\[
\phi_0 = \frac{0!(3-0-1)!}{3!}(v(\emptyset \cup R_0) - v(\emptyset)) + \frac{1!(3-1-1)!}{3!}(v(R_0, R_1) - v(R_1)) \\
\hspace{2cm} + \frac{1!(3-1-1)!}{3!}(v(R_0R_2) - v(R_2)) + \frac{2!(3-2-1)!}{3!}(v(R_0R_2R_2) - v(R_2R_2)) \\
= \frac{1}{3}v(R_0) + \frac{1}{6}(v(R_0R_1) - v(R_1)) + \frac{1}{6}(v(R_0R_2) - v(R_2)) + \frac{1}{3}(v(R_0R_2R_2) - v(R_2R_2)) \\
= \frac{1}{3}\pi_0 + \frac{1}{6}(\pi_{01} + \pi_{02} - \pi_1) + \frac{1}{6}(\pi_{02} + \pi_{01} - \pi_2) + \frac{1}{3}(\pi_G - \pi_{02}).
\]

Therefore,
\[ \phi_0 = \frac{1}{3} \pi_0 + \frac{1}{6} (\pi_{01} + \pi_{02} - \pi_1) + \frac{1}{6} (\pi_{02} + \pi_{03} - \pi_2) + \frac{1}{3} (\pi_G - \pi_{12}). \]  

(59)

In formula (57), (58) and (59), the value of \( \pi_1, \pi_2, \pi_0, \pi_{01}, \pi_{02}, \pi_{12}, \) and \( \pi_G \) have been already reached in Section 4.3.4. It is easy to verify that \( \phi_0 + \phi_1 + \phi_2 = \pi_G \).

In addition, in the grand coalition, \( R_1 \) sells \( q_i^G \) at price \( p_1 \) and \( R_2 \) sells \( q_2^G \) at price \( p_2 \).

The following steps are to allocate the fixed cost of the grand coalition among the three players by the Shapley value. Given \( B_0, B_1, B_2, B_{01}, B_{02}, \) and \( B_{012} \), the fixed cost of the grand coalition is allocated by formula (4):

\[
B_i^G = \frac{0!(3-0-1)!}{3!} (B_1 - \nu(\emptyset)) + \frac{1!(3-1-1)!}{3!} (B_{12} - B_2) + \frac{1!(3-1-1)!}{3!} (B_{01} - B_0) \\
+ \frac{2!(3-2-1)!}{3!} (B_{012} - B_{02}) \\
= \frac{1}{3} B_1 + \frac{1}{6} (B_{12} - B_2) + \frac{1}{6} (B_{01} - B_0) + \frac{1}{3} (B_{012} - B_{02}).
\]

Similarly, the fixed cost assigned to retailer 2 and the supplier can be obtained:

\[
B_2^G = \frac{1}{3} B_2 + \frac{1}{6} (B_{12} - B_1) + \frac{1}{6} (B_{02} - B_0) + \frac{1}{3} (B_{012} - B_{01}), \text{ and}
\]

\[
B_0^G = \frac{1}{3} B_0 + \frac{1}{6} (B_{01} - B_1) + \frac{1}{6} (B_{02} - B_2) + \frac{1}{3} (B_{012} - B_{12}).
\]

Now the transfer price between \( R_0 \) and \( R_1, TP_1 \),

\[
TP_1 = \frac{\pi_i^G - \phi_i}{q_i^G}, \text{ and}
\]

\[
(TP_1)(q_i^G) = E\left[p_1 \min(q_i^G, D_i) - \eta_i \left([q_i^G - D_i]^+\right) - h\left([q_i^G - D_i]^+\right)\right] - B_i^G - \phi_i. \tag{60}
\]

Similarly, we get the transfer price, \( TP_2 \), between \( R_0 \) and \( R_2 \).
\[ TP_2 = \frac{\pi^G_2 - \phi_2}{q^G_2}, \text{ and} \]

\[ (TP_2)(q^G_2) = E\left[p_2 \min(q^G_2, D_2) - \eta_2 \left([D_2 - q^G_2]_+ - h([q^G_2 - D_2]_+)\right) - B^G_2 - \phi_2 \right]. \quad (61) \]

The suggested transfer prices divide the whole revenue of the grand coalition among the supplier and the two retailers in a fair way.

Remark: Formula (60) and (61) do not involve \( \phi_0 \), but only \( \phi_1 \) and \( \phi_2 \), because \( \phi_0, \phi_1, \) and \( \phi_2 \) have an underlying relation: \( \phi_0 + \phi_1 + \phi_2 = \pi_G. \)

4.4 Numerical examples

The first set of numerical example was designed to show the whole methodology procedure associated with transfer pricing assuming normal demand distribution functions. For each subcoalition of three players, the profit functions can be obtained following the suggested solution in Sec. 4.3. With the profit functions, the Shapley value formula (4) is then used to reach the ideal transfer prices between retailers and the supplier.

Demands to retailer 1 and retailer 2 are both normally distributed, defined as \( D_1 \) and \( D_2 \), with \( D_1 \sim N(1000,200) \) and \( D_2 \sim N(800,150) \). \( D_1 + D_2 \) is also a normal random variable, with \( D_1 + D_2 \sim N(1800,250) \). Other parameters are assigned as follows:

\[
B_1 = $100; \quad B_2 = $80; \quad h_1 = $.6/\text{unit}; \quad h_2 = $.8/\text{unit}; \quad \eta_1 = $1.2; \quad \eta_2 = $1; \quad p_1 = $10; \\
p_2 = $11; \quad c_0 = $2; \quad p_0 = $5; \quad B_0 = $150; \quad B_{01} = $120; \quad B_{02} = $100; \quad B_{12} = $80; \quad B_G = $125; \\
h = $.3/\text{unit}.
\]

One-member coalition
\[
F_1(q_1^N) = .5254; \quad q_1^N = 1012.75; \quad F_1(0) = 0
\]
\[
F_2(q_2^N) = .5469; \quad q_2^N = 817.67; \quad F_2(0) = 0
\]

**Retailer 1**

\[
\pi_i = E\left(p_i \min(q_i^N, D_i) - \eta_i \left([D_i - q_i^N]_+\right) - h_i \left([q_i^N - D_i]_+\right) - B_i - p_i q_i^N\right)
\]
\[
= p_i \left(\int_{0}^{q_i} x f_i(x)dx + \int_{q_i}^{\infty} q_i f_i(x)dx\right) - \eta_i \int_{0}^{q_i} (x - q_i) f_i(x)dx - h_i \int_{0}^{q_i} (q_i - x) f_i(x)dx - B_i - p_i q_i
\]

Since

Calculating each term of the right hand side of

\[
\int_{0}^{q_i} x f_i(x)dx = \int_{0}^{q_i} x \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx
\]
\[
= -\sigma^2 \int_{0}^{q_i} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx + \mu (F_i(q_i) - F_i(0))
\]
\[
= -\frac{\sigma}{\sqrt{2\pi}} e^{-\frac{(q_i-\mu)^2}{2\sigma^2}} + \mu (F_i(q_i) - F_i(0))
\]
\[
= \frac{200}{\sqrt{2\pi}} \left( e^{-\frac{1000}{2\sigma^2}} - e^{-\frac{(1012.75-1000)}{2\sigma^2}} \right) + 1000 (0.5254 - 0) = \frac{200}{\sqrt{2\pi}} (0 - 99797) + 525.4
\]
\[
= (79.788)(-99797) + 525.4 = -79.626 + 525.4 = 445.8
\]

\[
\int_{q_i}^{\infty} q_i f_i(x)dx = q_i (1-F_i(q_i)) = 1012.75*(1-.5254) = 480.7
\]

\[
\int_{0}^{q_i} (x - q_i) f_i(x)dx = \int_{0}^{q_i} x f_i(x)dx - \int_{q_i}^{q_i} q_i f_i(x)dx = \int_{0}^{q_i} x \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx - \int_{q_i}^{q_i} q_i f_i(x)dx
\]
\[
= \left[ -\frac{\sigma}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \right]_{q_i}^{+\infty} + \mu (1-F_i(q_i)) = -480.7 = \left[ \frac{\sigma}{\sqrt{2\pi}} e^{-\frac{(q_i-\mu)^2}{2\sigma^2}} - 0 \right] + \mu (1-F_i(q_i)) - 480.7
\]
\[
\int_{q_1}^{q_2} (q_1 - x)f_1(x)dx = \int_{0}^{q_1} q_1f_1(x)dx - \int_{0}^{q_2} x f_1(x)dx = q_1(F_1(q_1) - F_1(0)) - 445.8 = 1012.75 \times (0.5254 - 0) - 445.8 = 86.3
\]

Therefore,
\[
\pi_1 = p_1(445.8 + 480.7) - \eta_1(73.53) - h_1(86.3) - B_1 - p_0(1012.75) = 10 \times (445.8 + 480.7) - 1.2 \times (73.53) - 0.6 \times (86.3) - 100 - 5 \times 10^{1.75} = 3961.2
\]

\(v(1) = 3961.2\)

**Retailer 2**

\[
\pi_2 = E \left( p_2 \min(q_2^N, D_2) - \eta_2 \left( \left[ D_2 - q_2^N \right]^+ \right) - h_2 \left( \left[ q_2^N - D_2 \right]^+ \right) - B_2 - p_0 q_2^N \right)
\]
\[
= p_2 \left( \int_{0}^{q_2} q_2f_2(x)dx + \int_{q_2}^{+\infty} q_2f_2(x)dx \right) - \eta_2 \int_{q_2}^{+\infty} (x - q_2)f_2(x)dx - h_2 \int_{0}^{q_2} (q_2 - x)f_2(x)dx - B_2 - p_0 q_2
\]

Since
\[
\int_{0}^{q_2} q_2f_2(x)dx = \frac{\sigma}{\sqrt{2\pi}} \left( e^{\frac{-\mu^2}{2}} - e^{\frac{-(q_2 - \mu)^2}{2\sigma^2}} \right) + \mu \left( F_2(q_2) - F_2(0) \right)
\]
\[
= (59.84) \times (0.993086) + 800 \times (0.546875 - 0) = 378.1
\]

\[
\int_{q_2}^{+\infty} q_2f_2(x)dx = q_2(1 - F_2(q_2)) = 817.67 \times (1 - 0.546875) = 370.51
\]

\[
\int_{q_2}^{+\infty} (x - q_2)f_2(x)dx = \left[ - \frac{\sigma}{\sqrt{2\pi}} e^{\frac{-(x - \mu)^2}{2\sigma^2}} \right]_{q_2}^{+\infty} + \mu(1 - F_2(q_2)) - \int_{q_2}^{+\infty} q_2f_2(x)dx
\]
\[
= (59.84 \times 0.99309) + 800 \times (1 - 0.546875) - 370.51 = 51.42
\]

\[
\int_{0}^{q_2} (q_2 - x)f_2(x)dx = \int_{0}^{q_2} q_2f_2(x)dx - \int_{0}^{q_2} x f_2(x)dx = q_2(F_2(q_2) - F_2(0)) - 378.1 = 817.67 \times (0.546875 - 0) - 378.1 = 69.1
\]
Therefore,
\[
\pi_2 = p_2 (378.1 + 370.5) - \eta_2 (51.42) - h_2 (69.1) - B_2 - p_0 q_2 = 11*(378.1 + 370.5) - 1*51.42 - 0.8*69.1 - 80 - 5*817.67 = 3959.6
\]

\(\nu(2) = 3959.6\)

**The Supplier**

\[
\pi_0 = p_0 (q_1^N + q_2^N) - c_0 (q_1^N + q_2^N) - 2B_0 \\
= 5* (1012.75 + 817.67) - 2* (1012.75 + 817.67) - 2*150 = 5191.3
\]

\[\pi_0 = 5*1012.75 - 2*1012.75 - 150 = 2888.3\]

\[\pi_0 = 5*817.67 - 2*817.67 - 150 = 2303\]

\[\nu(0) = 5191.3\]

**Two-member coalition**

\[F_1(q_1^s) = .7797; \quad q_1^s = 1154.21; \quad F_1(0) = 0; \quad \sigma_1 = 200\]

\[F_2(q_2^s) = .7813; \quad q_2^s = 916.46; \quad F_2(0) = 0; \quad \sigma_2 = 150\]

**Retailer 1 & the supplier**

\[
\pi_{01} = E \left( p_1 \min(q_1, D_1) - \eta_1 \left( [D_1 - q_1]_+ \right) - h_1 \left( [q_1 - D_1]_+ \right) - B_{01} - c_0 q_1 \right)
\]

\[
= p_1 \left( \int_0^{q_1} x f_1(x) dx + \int_{q_1}^{\infty} q_1 f_1(x) dx \right) - \eta_1 \int_{q_1}^{\infty} (x - q_1) f_1(x) dx - h_1 \int_0^{q_1} (q_1 - x) f_1(x) dx - B_{01} - c_0 q_1
\]

Since

\[
\int_0^{q_1} x f_1(x) dx = \int_0^{q_1} x \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \int_0^{q_1} (x - \mu) \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx + \mu \int_0^{q_1} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx
\]
\[
\begin{align*}
-\sigma^2 & \int_0^\infty \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} d\frac{-(x-\mu)^2}{2\sigma^2} + \mu(F_1(q_1) - F_1(0)) \\
&= -\frac{\sigma}{\sqrt{2\pi}} \left| q_1 \right| + \mu(F_1(q_1) - F_1(0)) = -\frac{\sigma}{\sqrt{2\pi}} \left( e^{2\sigma^2} - e^{-\frac{(q_1-\mu)^2}{2\sigma^2}} \right) + \mu(F_1(q_1) - F_1(0)) \\
&= 200 \left( \frac{-1000^2}{e^{2(200)^2}} - e^{-\frac{(1145.21-1000)^2}{2(200)^2}} \right) + 1000 (0.7797 - 0) = 200 \sqrt{2\pi} (0.7429) + 779.7 \\
&= (79.788) (-.7429) + 779.7 = 720.43
\end{align*}
\]

\[
\int_{q_2}^{q_1} q_1 f_1(x) dx = q_1 (1 - F_1(q_1)) = 1154.21 \times (1 - 0.77966) = 254.32
\]

\[
\begin{align*}
\int_{q_2}^{q_1} (x - q_1) f_1(x) dx &= \int_{q_2}^{q_1} x f_1(x) dx - \int_{q_2}^{q_1} q_1 f_1(x) dx \\
&= \int_{q_2}^{q_1} x \left( \frac{1}{\sigma \sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}} \right) dx - \int_{q_2}^{q_1} q_1 f_1(x) dx \\
&= \left[ -\frac{\sigma}{\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}} \right]_{q_2}^{q_1} + \mu (1 - F_1(q_1)) \right] - 254.32 = \left[ \frac{\sigma}{\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma^2}} \right]_{q_1}^{q_2} = \mu (1 - F_1(q_1)) - 254.32 \\
&= 79.788 \times .7429 + 1000 \times (1 - 0.77966) - 254.32 = 25.29
\end{align*}
\]

\[
\int_{0}^{q_1} (q_1 - x) f_1(x) dx = \int_{0}^{q_1} q_1 f_1(x) dx - \int_{0}^{q_1} x f_1(x) dx = q_1 (F_1(q_1) - F_1(0)) - 720.43 \\
= 1154.21 \times (0.77966) - 720.43 = 179.46
\]

Therefore,
\[
\pi_{01} = p_1 \times (720.43 + 254.32) - \eta_1 (25.29) - h_1 (179.46) - B_{01} - c_0 q_1 \\
= 10 \times (720.43+254.32) -1.2 \times 25.29 - .6 \times 179.46 - 120 - 2 \times 1154.21 = 7181.06
\]

Profits of \( R_1 \) resulting from the transactions with \( R_2 \) are
\[
= 5 \times (817.67) - 2 \times 817.67 - 150 = 2303.01
\]

\[
\nu(01) = 7181.06 + 2303.01 = 9484.07
\]

Retailer 2 and the supplier
\[ \pi_{02} = E\left(p_2 \min(q_2, D_2) - \eta_2 \left([D_2 - q_2]_+\right) - h_2 \left([q_2 - D_2]_+\right) - B_{02} - c_0 q_2 \right) \]
\[ = p_2 \left(\int_{q_2}^{0} x f_2(x)dx + \int_{q_2}^{+\infty} q_2 f_2(x)dx\right) - \eta_2 \left(\int_{q_2}^{+\infty} (x - q_2) f_2(x)dx - h_2 \int_{0}^{q_2} (q_2 - x) f_2(x)dx\right) - B_{02} - c_0 q_2 \]
\[ \int_{0}^{q_2} x f_2(x)dx = -\frac{\sigma}{\sqrt{2\pi}} \left(e^{\frac{-\mu^2}{2\sigma^2}} - e^{\frac{-\left(q_2 - \mu\right)^2}{2\sigma^2}}\right) + \mu \left(F_2(q_2) - F_2(0)\right) \]
\[ = (59.841)\times(-.73978) + 800 \times(.7813) = 580.77 \]
\[ \int_{q_2}^{+\infty} q_2 f_2(x)dx = q_2 \left(1 - F_2(q_2)\right) = 916.46 \times (1 - .7813) = 200.43 \]
\[ \int_{q_2}^{+\infty} (x - q_2) f_2(x)dx = \left[\int_{q_2}^{+\infty} -\frac{\sigma}{\sqrt{2\pi}} e^{\frac{-\left(x - \mu\right)^2}{2\sigma^2}} dx\right]_{q_2}^{+\infty} + \mu \left(1 - F_2(q_2)\right) - \int_{q_2}^{+\infty} q_2 f_2(x)dx \]
\[ = (-59.8413)\times(-.73978) + 800 \times (1 - .7813) - 200.43 = 18.8 \]
\[ \int_{0}^{q_2} (q_2 - x) f_2(x)dx = \int_{0}^{q_2} q_2 f_2(x)dx - \int_{0}^{q_2} x f_2(x)dx = q_2 \left(F_2(q_2) - F_2(0)\right) - 580.77 \]
\[ = 916.46 \times (.7813) - 580.77 = 135.26 \]

Therefore,
\[ \pi_{02} = p_2 \left(\int_{0}^{q_2} x f_2(x)dx + \int_{q_2}^{+\infty} q_2 f_2(x)dx\right) - \eta_2 \left(\int_{q_2}^{+\infty} (x - q_2) f_2(x)dx - h_2 \int_{0}^{q_2} (q_2 - x) f_2(x)dx\right) - B_{02} - c_0 q_2 \]
\[ = 11 \times (580.77 + 200.43) - \eta_2 (18.8) - h_2 (135.26) - B_{02} - c_0 q_2 \]
\[ = 6533.27 \]

Profits of \( R_0 \) resulting from the transactions with \( R_i \) are
\[ \pi_0 = 5 \times 1012.75 - 2 \times 1012.75 - 150 = 2887.35 \]
\[ \nu(02) = 6533.27 + 2887.35 = 9420.62 \]
\[ q_1^R = 1019.21; \quad q_2^R = 826.09; \quad q_1^R + q_2^R = 1845.3 \]

\[ F_1(q_1^R) = .5383; \quad F_2(q_2^R) = .5691 \]

\[ \pi_{12} = E \left( \sum_{i=1,2} p_i \min(q_i, D_i) - \sum_{i=1,2} \eta_i \left( [D_i - q_i]^+ - h((q_i + q_2) - (D_i + D_2))^{-} \right) \right) - B_{12} - p_0(q_1 + q_2) \]

\[ = p_1 \left( \int_{q_1}^{\infty} x f_1(x) dx + \int_{q_1}^{+\infty} q_1 f_1(x) dx \right) - \eta_1 \int_{q_1}^{+\infty} (x - q_1) f_1(x) dx + \quad(\int_{q_2}^{+\infty} x f_2(x) dx + \int_{q_2}^{+\infty} q_1 f_2(x) dx) \]

\[ - \eta_2 \int_{q_2}^{+\infty} (x - q_2) f_2(x) dx - h \int_{0}^{q_2} (q_1 + q_2 - x) g(x) dx - B_{12} - p_0(q_1 + q_2) \]

\[ \int_{0}^{q_1} x f_1(x) dx = -\frac{\sigma}{\sqrt{2\pi}} e^{-\frac{-(q_1-\mu)^2}{2\sigma^2}} + \mu (F_1(q_1) - F_1(0)) \]

\[ = (79.788) \times (0.9954) + 1000 \times (0.5383 - 0) = 458.88 \]

\[ \int_{q_1}^{+\infty} q_1 f_1(x) dx = q_1 (1 - F_1(q_1)) = 1019.21 \times (1 - 0.5383) = 470.57 \]

\[ \int_{q_1}^{+\infty} (x - q_1) f_1(x) dx = \int_{q_1}^{q_1} x \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{-(x-\mu)^2}{2\sigma^2}} dx - \int_{q_1}^{+\infty} q_1 f_1(x) dx = \quad\left[ -\frac{\sigma}{\sqrt{2\pi}} e^{-\frac{-(x-\mu)^2}{2\sigma^2}} \right]_{q_1}^{+\infty} + \mu (1 - F_1(q_1)) \]

\[ - \int_{q_1}^{+\infty} q_1 f_1(x) dx = \quad\left[ \frac{\sigma}{\sqrt{2\pi}} e^{-\frac{-(q_1-\mu)^2}{2\sigma^2}} \right]_{0}^{q_1} + \mu (1 - F_1(q_1)) - 458.88 \]

\[ = (79.788) \times (0.99598) + 1000 \times (0.5383 - 458.88) = 82.24 \]

\[ \int_{0}^{q_2} x f_2(x) dx = -\frac{\sigma}{\sqrt{2\pi}} e^{-\frac{-(x-\mu)^2}{2\sigma^2}} + \mu (F_2(q_2) - F_2(0)) \]

\[ = (59.841) \times (-0.98499) + 800 \times (0.5691 - 0) = 396.34 \]

\[ \int_{q_2}^{+\infty} q_2 f_2(x) dx = q_2 (1 - F_2(q_2)) = 826.09 \times (1 - 0.5691) = 355.96 \]

\[ \int_{q_2}^{+\infty} (x - q_2) f_2(x) dx = \int_{q_2}^{+\infty} x \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{-(x-\mu)^2}{2\sigma^2}} dx - \int_{q_2}^{+\infty} q_2 f_2(x) dx \]

\[ = (59.841) \times (-0.98499) + 800 \times (0.5691 - 0) = 396.34 \]
\[
\begin{align*}
&= \left[-\frac{\sigma}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \right]_{q_2}^{+\infty} + \mu(1-F_2(q_2)) - \int_{q_2}^{+\infty} q_2 f_2(x) dx \\
&= \frac{\sigma}{\sqrt{2\pi}} e^{-\frac{(q_1-q_2)^2}{2\sigma^2}} + \mu(1-F_2(q_2)) - 355.96 = (59.841) \cdot 0.9499 + 800(1-0.5691) - 355.96 = 47.7
\end{align*}
\]

\[
\int_{0}^{q_1+q_2} (q_1 + q_2 - x)g(x)dx = (q_1 + q_2) \int_{0}^{q_1+q_2} g(x)dx - \int_{0}^{q_1+q_2} xg(x)dx = (q_1 + q_2)
\]

\[
(G(q_1 + q_2) - G(0)) - \int_{0}^{q_1+q_2} xg(x)dx = (q_1 + q_2)(G(q_1 + q_2) - G(0)) - \frac{\sigma}{\sqrt{2\pi}}
\]

\[
* \left(e^{\frac{-\mu^2}{2\sigma^2}} - e^{\frac{-\eta_1+q_2}{2\sigma^2}} \right) - \mu(G(q_1 + q_2) - G(0))
\]

\[
= 1945.3 \cdot 0.57189 - 99.7356 \cdot 0.97143 - 1800 \cdot 0.57189 = 179.98
\]

Therefore,

\[
\begin{align*}
\pi_{12} &= p_1(458.88 + 470.57) - \eta_1(82.24) + p_2(396.34 + 355.96) - \eta_2(47.7) - h(179.98) \\
-B_{12} &= p_0(q_1 + q_2) \\
&= 10 \cdot (458.88 + 470.57) - 1.2 \cdot 82.24 + 11 \cdot (396.34 + 355.96) - 1 \cdot 47.7 - 3 \cdot 179.98 - 80 - 5 \cdot 1845.3 \\
&= 8062.92
\end{align*}
\]

\[v(12) = 8062.92\]

The grand coalition

\[
\begin{align*}
q_1^G &= 1166.72; \quad F_1(q_1^G) = 0.7978 \\
q_2^G &= 932.36; \quad F_2(q_2^G) = 0.8113 \\
q_1^G + q_2^G &= 2099.08; \quad G(q_1^G + q_2^G) = 0.8842
\end{align*}
\]

\[
\begin{align*}
\pi_G &= E \left( \sum_{i=1,2} p_i \min(q_i^G, D_i) - \sum_{i=1,2} \eta_i \left( [D_i - q_i^G]^+ - h([q_i^G + q_i^G - (D_i + D_i)]^+) \right) \right) \\
-B_G - c_0(q_1^G + q_2^G) &= p_1 \left( \int_{0}^{q_1} xf_1(x)dx + \int_{q_1}^{+\infty} q_1 f_1(x)dx \right) - \eta_1 \int_{q_1}^{+\infty} (x - q_1) f_1(x)dx \\
&+ p_2 \left( \int_{0}^{q_2} xf_2(x)dx + \int_{q_2}^{+\infty} q_2 f_2(x)dx \right) - \eta_2 \int_{q_2}^{+\infty} (x - q_2) f_2(x)dx - h \int_{0}^{q_1+q_2} (q_1 + q_2 - x) g(x)dx \\
-B_G - c_0(q_1 + q_2)
\end{align*}
\]
\[
\int_0^{q_1} xf_1(x)dx = \frac{\sigma}{\sqrt{2\pi}} \left( \frac{-\mu^2}{e^{2\sigma^2}} - e^{\frac{-(q_1-\mu)^2}{2\sigma^2}} \right) + \mu \left( F_1(q_1) - F_1(0) \right) = (79.788)*(-.7065) + 1000*(.7978) = 741.43
\]

\[
\int_{q_1}^{+\infty} q_1 f_1(x)dx = q_1(1-F_1(q_1)) = 1166.72*(1-.7978) = 235.91
\]

\[
\int_{q_1}^{+\infty} (x-q_1) f_1(x)dx = \int_{q_1}^{+\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx - \int_{q_1}^{+\infty} q_1 f_1(x)dx = \left[ -\frac{\sigma}{\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \right]_{q_1}^{+\infty} + \mu(1-F_1(q_1)) - \int_{q_1}^{+\infty} q_1 f_1(x)dx = \left[ \frac{\sigma}{\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} - 0 \right] + \mu(1-F_1(q_1))
\]

\[
-235.91 = (79.788)*(.7065) + 1000*(1-.7978) - 235.91 = 22.66
\]

\[
\int_0^{q_2} xf_2(x)dx = \frac{\sigma}{\sqrt{2\pi}} \left( \frac{-\mu^2}{e^{2\sigma^2}} - e^{\frac{-(q_2-\mu)^2}{2\sigma^2}} \right) + \mu \left( F_2(q_2) - F_2(0) \right) = (59.841)*(-.67752) + 800*(.8113) = 608.5
\]

\[
\int_{q_2}^{+\infty} q_2 f_2(x)dx = q_2(1-F_2(q_2)) = 932.36*(1-.8113) = 175.94
\]

\[
\int_{q_2}^{+\infty} (x-q_2) f_2(x)dx = \int_{q_2}^{+\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx - \int_{q_2}^{+\infty} q_2 f_2(x)dx = \left[ -\frac{\sigma}{\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \right]_{q_2}^{+\infty} + \mu(1-F_2(q_2)) - \int_{q_2}^{+\infty} q_2 f_2(x)dx = \left[ \frac{\sigma}{\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} + \mu(1-F_2(q_2)) \right]
\]

\[
-175.94 = (59.841)*(.67752) + 800*(1-.8113) - 175.94 = 15.56
\]

\[
\int_0^{q_1+q_2} (q_1 + q_2 - x)g(x)dx = (q_1 + q_2) \int_0^{q_1+q_2} g(x)dx - \int_0^{q_1+q_2} xg(x)dx = (q_1 + q_2) \left( G(q_1 + q_2) - G(0) \right) - \int_0^{q_1+q_2} xg(x)dx = (q_1 + q_2) \left( G(q_1 + q_2) - G(0) \right) - \frac{\sigma}{\sqrt{2\pi}} \left( \frac{-\mu^2}{e^{2\sigma^2}} - e^{\frac{-(q_1+q_2-\mu)^2}{2\sigma^2}} \right) - \mu \left( G(q_1 + q_2) - G(0) \right)
\]

\[
= 2099.08*.8842-(99.74)*(-.4889)-1800*.8842 = 313.21
\]
Therefore,
\[ \pi_G = p_1(741.43 + 235.91) - \eta_1(22.66) + p_2(608.5 + 175.94) - \eta_2(15.56) - h(313.21) \]
\[ -B_c - c_0(q_1 + q_2) = 10*[741.43+235.91]-1.2*22.66+11*[608.5+175.94]-1*15.56-3*313.21-125-2*2099.08 = 13942.4 \]

\[ \nu(012) = 13065.6 \]

**To summarize:**

\[ \nu(1) = 3961.2, \nu(2) = 3959.6, \nu(0) = 5191.3 \]

\[ \nu(01) = 9487.1, \nu(02) = 9420.6, \nu(12) = 8062.9 \]

\[ \nu(012) = 13942.4 \]

*Use formula (4) to compute the Shapley value of each member:*

\[ \phi_1 = \frac{1}{3} v(R_1) + \frac{1}{6} (v(R_1R_2) - v(R_2)) + \frac{1}{6} (v(R_1R_0) - v(R_0)) + \frac{1}{3} (v(R_0R_1R_2) - v(R_0R_2)) \]
\[ = \frac{1}{3} \times 3961.2 + \frac{1}{6} \times (8062.9 - 3959.6) + \frac{1}{6} \times (9487.1 - 5191.3) + \frac{1}{3} \times (13942.4 - 9420.6) \]
\[ = 4243 \]

\[ \phi_2 = \frac{1}{3} v(R_2) + \frac{1}{6} (v(R_1R_2) - v(R_1)) + \frac{1}{6} (v(R_2R_0) - v(R_0)) + \frac{1}{3} (v(R_0R_1R_2) - v(R_0R_1)) \]
\[ = \frac{1}{3} \times 3959.6 + \frac{1}{6} \times (8062.9 - 3961.2) + \frac{1}{6} \times (9420.6 - 5191.3) + \frac{1}{3} \times (13942.4 - 9487.1) \]
\[ = 4208 \]

\[ \phi_0 = \frac{1}{3} v(R_0) + \frac{1}{6} (v(R_0R_1) - v(R_1)) + \frac{1}{6} (v(R_0R_2) - v(R_2)) + \frac{1}{3} (v(R_0R_1R_2) - v(R_1R_2)) \]
\[ = \frac{1}{3} \times 5101.3 + \frac{1}{6} \times (9487.1 - 3961.2) + \frac{1}{6} \times (9420.6 - 3959.6) + \frac{1}{3} \times (13942.4 - 8062.9) \]
\[ = 5491 \]

The percentage of profit increases for each member:

\[ R_i: \frac{\phi_i - \nu(1)}{\nu(1)} \times 100% = \frac{4243 - 3961}{3961} \times 100% = 7.1\% \]
\[ R_2 : \frac{\phi_2 - v(2)}{v(2)} \times 100\% = \frac{4208 - 3960}{3960} \times 100\% = 6.3\% \]

\[ R_0 : \frac{\phi_0 - v(0)}{v(0)} \times 100\% = \frac{5491 - 5101}{5101} \times 100\% = 7.6\% \]

**Transfer pricing:**

First, I use formula (4) again to calculate the allocation of fixed costs to each member in the grand coalition

\[ B_1^G = 36.7 \]
\[ B_2^G = 16.7 \]
\[ B_0^G = 71.7 \]

With the known parameters:
\[ q_1^G = 1166.72; \ F_1(q_1^G) = .7978 \]
\[ q_2^G = 932.36; \ F_2(q_2^G) = .8113 \]

\[ (TP_1)(q_1^G) = p_i \min(q_i^G, D_i) - \eta_i \left( [D_i - q_i^G]^+ \right) - h \left( [q_i^G - D_i]^+ \right) - B_i^G - \phi_i \]

\[ = p_i \left( \int_0^{q_i^G} xf_i(x)dx + \int_{q_i^G}^{\infty} q_i f_i(x)dx \right) - \eta_i \int_{q_i^G}^{\infty} (x - q_i) f_i(x)dx 
- h \int_0^{q_i^G} (q_i - x) f_i(x)dx - B_i^G - \phi_i \]

From the grand coalition calculation, we know
\[ \int_0^{q_i^G} xf_i(x)dx = 741.43 \]
\[ \int_{q_i^G}^{\infty} q_i f_i(x)dx = 235.91 \]
\[ \int_{q_i^G}^{\infty} (x - q_i) f_i(x)dx = 22.66 \]

\[ \int_0^{q_i^G} (q_i - x) f_i(x)dx = \int_0^{q_i^G} q_i f_i(x)dx - \int_0^{q_i^G} xf_i(x)dx = q_i \int_0^{q_i^G} f_i(x)dx - 741.43 
= 1166.72 \times .7978 - 741.43 = 189 \]

\[ (TP_1)(q_1^G) = p_i(741.43 + 235.91) - \eta_i(22.66) - h(189) - B_i^G - \phi_i 
= 10*(741.43+235.91) - 1.2*(22.66)-.3*(189)-36.7-4243 
= 5409.81 \]

\[ TP_1 = \frac{5409.81}{q_1^G} = 4.65 \]
\[ (TP_2)(q^G_2) = E \left[ p_2 \min(q^G_2, D_2) - \eta_2 \left( [D_2 - q^G_2] - h \left( [q^G_2 - D_2] \right) \right) \right] - B_2^G - \phi_2 \]
\[ = p_2 \left( \int_{0}^{q_2} xf_2(x)dx + \int_{q_2}^{\infty} q_2 f_2(x)dx \right) - \eta_2 \int_{q_2}^{\infty} (x - q_2) f_2(x)dx \]
\[ - h \int_{0}^{q_2} (q_2 - x) f_2(x)dx - B_2^G - \phi_2 \]

From the grand coalition calculation, we know
\[ \int_{0}^{q_2} xf_2(x)dx = 608.5 \]
\[ \int_{q_2}^{\infty} q_2 f_2(x)dx = 175.94 \]
\[ \int_{q_2}^{\infty} (x - q_2) f_2(x)dx = 15.56 \]

\[ \int_{0}^{q_2} (q_2 - x) f_2(x)dx = \int_{0}^{q_2} q_2 f_2(x)dx - \int_{0}^{q_2} xf_2(x)dx = 932.36 \times 0.8113 - 608.5 = 147.92 \]

\[ (TP_2)(q^G_2) = p_2 (608.5 + 175.94) - \eta_2 (15.56) - h(147.92) - B_2^G - \phi_2 \]
\[ = 11 \times (608.5 + 175.94) - 1 \times (15.56) - 0.3 \times (147.92) - 16.7 - 4208 \]
\[ = 4344.2 \]

\[ TP_2 = \frac{4344.2}{q^G_2} = 4.66 \]

The suggested transfer prices between the two retailers and the supplier are 4.65 and 4.66, respectively. Given the wholesale price, 5, the two prices must be acceptable to both sides of transactions.

Actually, only non-negative values of demand can be observed. So in the second numerical example, left truncated normal distributions are used to model demand functions of both retailer 1 and retailer 2. The concept of a truncated distribution can be found in a variety of production processes (e.g., Cho & Govindaluri, 2002; Phillips & Cho, 2000). Demands to retailer 1 and retailer 2 are now both assumed to be truncated.
normal distribution functions, defined as $T_1$ and $T_2$, which are left-truncated from $D_1 \sim N(1000,200)$ and $D_2 \sim N(800,150)$ on the point $L = 0$, respectively, by

$$
t_1(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\frac{f_1(x)}{\int_0^{+\infty} f_1(x)dx} & \text{if } x \geq 0
\end{cases}
$$

$$
t_2(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\frac{f_2(x)}{\int_0^{+\infty} f_2(x)dx} & \text{if } x \geq 0
\end{cases}
$$

Other parameters are assigned the same as in the previous example: $B_1 = 100\$; $B_2 = 80\$; $h_1 = .6\$/unit; $h_2 = .8\$/unit; $\eta_1 = 1.2\$; $\eta_2 = 1\$; $p_1 = 10\$; $p_2 = 11\$; $c_0 = 2\$; $p_o = 5\$; $B_0 = 150\$; $B_{01} = 120\$; $B_{02} = 100\$; $B_{12} = 80\$; $B_o = 125\$; $h = .3\$/unit. With the known probability density functions $f_1(x)$ and $f_2(x)$, the left-truncated normal density function at zero can be reached as follows:

$$
t_1(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\frac{f_1(x)}{0.99999973} & \text{if } x \geq 0
\end{cases}
$$

$$
t_2(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\frac{f_2(x)}{0.99999995} & \text{if } x \geq 0
\end{cases}, \text{ respectively.}
$$

$D_1 + D_2$ is also a normal random variable, with $D_1 + D_2 \sim N(1800, \sqrt{200^2 + 150^2})$. With the means of $D_1$ and $D_2$ far away from 0, their truncated functions $t_1$ and $t_2$ are very close to $D_1$ and $D_2$. The sum of demands, $T$, is also very close to the function which is left-truncated on the point of 0 from $D_1 + D_2$. Evidence can be reached by simulation.
with EXCEL (5000 sample data for the sum of \( t_1 \) and \( t_2 \) and 5000 sample data for the left-truncated function from \( D_1 + D_2 \)). After conducting an F-test \(( \alpha = 0.05)\) over the two samples, we can see that the variances of the two samples are equal. Then we conduct a two sample t-test \(( \alpha = 0.05)\) assuming equal variance, and with a p-value 0.78 \((> \alpha)\), we cannot reject the null hypothesis that the mean difference is zero. Next both of the two samples are fitted to a normal distribution by Monte Carlo Simulation (Software package: Arena), and the Kolmogorov-Smirnov test \(( p \text{ value } > .15)\) shows that both of the two samples are following a normal distribution. Statistically, we say that the two samples are following the same normal distribution with equal mean and variance. A summarized table of the differences between the two functions is given to show their closeness (see Table 3). Therefore in the following numerical example, we use the left-truncated function from \( D_1 + D_2 \) to approximate the demand function, \( T \).

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Variance</th>
<th>Max difference between CDF and ( N(1800,250) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of the two truncated normal functions</td>
<td>1800</td>
<td>62500</td>
<td>0.00959 ((\text{K-S test}))</td>
</tr>
<tr>
<td>Truncated function of the sum of the two normal functions</td>
<td>1800</td>
<td>61504</td>
<td>0.00842 ((\text{K-S test}))</td>
</tr>
<tr>
<td>Difference Percentage</td>
<td>0</td>
<td>1.59</td>
<td>12.2</td>
</tr>
</tbody>
</table>

**Table 3** Differences between \( T \) and the truncated function from \( D_1 + D_2 \)

The corresponding left-truncated normal density function of the sum of the demands is
\[ t(x) = \begin{cases} 
0 & \text{if } x < 0 \\
g(x) \cdot \frac{1}{0.999999999997} & \text{if } x \geq 0
\end{cases} \]

Referring to the table of cumulative area of the left-truncated standard normal distribution (Khasawneh et al., 2005), the numerical example can be feasibly conducted. But in this specific example,

\[ t_1(x) = \begin{cases} 
0 & \text{if } x < 0 \\
f_1(x) \cdot \frac{1}{0.99999973} & \text{if } x \geq 0
\end{cases} \approx \begin{cases} 
0 & \text{if } x < 0 \\
f_1(x) & \text{if } x \geq 0
\end{cases} ; \]

\[ t_2(x) = \begin{cases} 
0 & \text{if } x < 0 \\
f_2(x) \cdot \frac{1}{0.9999995} & \text{if } x \geq 0
\end{cases} \approx \begin{cases} 
0 & \text{if } x < 0 \\
f_2(x) & \text{if } x \geq 0
\end{cases} ; \]

\[ t(x) = \begin{cases} 
0 & \text{if } x < 0 \\
g(x) \cdot \frac{1}{0.999999999997} & \text{if } x \geq 0
\end{cases} \approx \begin{cases} 
0 & \text{if } x < 0 \\
g(x) & \text{if } x \geq 0
\end{cases} . \]

Therefore, in this second numerical example, the whole procedure to calculate the transfer prices are the same as in the first example (keep all of the other parameters constant). Since the means of the demand functions are far away from zero, the approximation action does not affect the outcomes too much. So even if left truncated normal distributions are used to model demand functions of both retailer 1 and retailer 2, we can still obtain acceptable transfer prices using the suggested procedures.
Chapter 5 Discussion and Summary

Even though the literature on transfer pricing is quite extensive, not much research focuses on supply chain transfer pricing and very little research work has been done on transfer pricing with stochastic demands. In this dissertation, I studied a supply chain with one supplier and two retailers, where order quantities are considered as decision variables. These decisions take into account production costs, holding costs, shortage costs, sales revenue, etc. The research discussed herein contributes to the supply chain transfer pricing literature by relaxing the assumption of deterministic demand. Furthermore, the concept of fairness is not well documented in the area of supply chain profit division. In this study, I calculate transfer prices among supply chain members with one supplier and two retailers in different markets. Commonly accepted theories of justice are used to operationalize the concept of fairness. The Shapley value solution is explored to obtain a fair sharing of revenues and costs. First, I assume that the demands of each retailer are stochastic. Second, a cooperative game with three players in a supply chain is constructed for which the Shapley value is applied to allocate the total profits of the grand coalition to the three players. The maximized expected profit of each coalition is figured out by choosing the order quantity of each retailer. For the two-retailer coalition, it is assumed that the retailers will share a common warehouse and unit holding cost is reduced. Cost reduction makes the game convex, thus the Shapley value is the centroid of the core. Finally, the transfer prices may be determined according to their individual expected allocated profits suggested by the Shapley value. The whole procedure is illustrated by a numerical example. To be more practical, the demand functions of retailers, initially two normal distributions, are replaced with their left-
truncated normal distribution at point zero, respectively. Since the means of the two distribution functions are far away from zero, the whole procedure turns out to be workable in the case of truncated normal distribution functions as well. My dissertation has shed light on the difficulties in achieving supply chain coordination when retailers face stochastic demand.

The analysis has examined how a decentralized supply chain can coordinate to divide the global supply chain profits fairly. The presented procedure can be well applied to supply chain coordination practice. Even when there are multiple suppliers and retailers, my analysis provides a procedure to follow for supply chain profit division. My results draw a clear profit picture for those companies in a cooperative game. Cooperation in turn yields significant cost reductions for supply chain members. I emphasize that cost saving by cooperation is a means toward global profit maximization, since it raises the allocated profit of each player.

From a managerial perspective, the transfer prices obtained by my procedure would give an acceptable suggestion when supply chain members are bargaining on contract agreements. A supply chain contract has been shown to be an effective tool to coordinate supply chains, but how to reach supply chain contract agreements still needs more attention. Since supply chain members are more concerned about their individual profits, the agreements on transfer prices are critical for all members. The suggested transfer prices would help them skip the bargaining process, and reach the final stage directly. Stochastic demand functions are widely assumed in supply chain contract research. Given this, the transfer prices obtained by my procedure have the advantage of incorporating practical market demand into a model which embodies fairness in profit division.
Furthermore, a supply chain contract is difficult to perform if the supply chain members are independent firms. Sometimes legal actions are necessary to ensure performance of a successfully negotiated contract, which results in huge costs and hurts supply chain relationships. However, fairly settled contracts are more likely to be executed by supply chain members. In other words, the elimination of negotiation and execution trouble brings about administration cost savings for repeated transactions and contributes to harmonious supply chain relationships.

Distributive decisions involve a variety of factors, and a fair allocation scheme may not always be accepted. The profit division suggested by the Shapley value may not be accepted by more powerful players since they have the power to earn more. Power can be broadly defined as the ability of one negotiator to influence other negotiators’ decisions (Bacharach & Lawler, 1981; Kelley & Thibaut, 1978). The power of a player would affect the player’s behavior. For example, powerful players might demand more and concede less (De Dreu & Van Kleef, 2004), and have a higher tendency to use threats and bluffs to get their way (Lawler, 1992). Powerful players are also less likely to care and think about the less powerful ones than vice versa (Keltner et al., 2003). It could be true that since powerful players have more resources, they can act at will without causing serious negative outcomes to themselves. For instance, as the most profitable car giant, Toyota has the power and intent to earn more benefits when working with its suppliers. Many American suppliers are willing to supply to Toyota, and they even celebrated when they first received orders from Toyota. Toyota outsources a high percentage of its manufacturing costs (around 70 to 80 percent). Effective outsourcing can help manufacturing companies improve quality service, delivery performance, and flexibility
to meet changing business conditions and customer satisfaction, while lowering costs due to economies of scales. It is well known that suppliers contribute greatly to Toyota’s innovation and success. If those suppliers are not treated fairly in profit division, their incentives to conduct cost-cutting activities or other joint improvement activities are very likely to decrease. Yet Toyota, the player with more power, might benefit from acting self-interested in the short term. However, since Toyota has such great reliance on suppliers, a long term relationships with those suppliers is necessary for its success. Therefore, although Toyota has the power to negotiate self-interestedly, its goal of long term profit maximization should not be at the expense of its suppliers. Fairness of profit division is one of the most critical factors for a long-term relationship that involves trust and mutual well-being. Future research can focus more on power in profit allocation: how to measure and assign power among players with certain assumptions.

A natural extension of my model would be to consider profit division among multi-stage supply chain members with coordination. Multi-stage supply chains are widely studied for delivery performance (Guiffrida, 2009), inventory management (Khouja, 2003), profit division in a constant demand market (Rosenthal, 2008), etc. This multi-stage extension in a stochastic market would bring further realism as it would determine the profit division scheme for a long term relationship, very likely with repeated transactions. My proposed procedure might be ideal for transfer prices for transactions among multi-stage supply chain members.

Another extension of my research concerns the possibility of retailer competition in the demand market. For certain industries, it seems plausible that the competition would affect the expected sales revenue, especially when one retailer is out of stock. One would
expect that competitive interaction in the demand market will have an impact on the desired choice of acceptable transfer prices.

As indicated by Ronchi et al. (2007), supply chain coordination is much less frequently adopted than expected. This may be due to firm interaction problems, such as lack of mutual trust, conflict of risk and benefit sharing allocation, etc. My model improves supply chain coordination by providing procedures to set fair transfer prices among individual supply chain members. I believe that the class of problems in transfer pricing for supply chain transactions represents a promising area for future research and practice.
References:


