HOW DOES SELF-REGULATION IMPACT STUDENT’S USE OF MATHEMATICAL STRATEGIES IN A REMEDIAL MATHEMATICS COURSE

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By

Michele Lynn Heron

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Dissertation written by

Michele Lynn Heron

B.S., Ohio Northern University, 1988

M.A.T., Bowling Green State University, 2002

Ph. D., Kent State University, 2010

Approved by

__________________________, Co-Director, Doctoral Dissertation Committee
Trish Koontz

__________________________, Co-Director, Doctoral Dissertation Committee
Anne Reynolds

__________________________, Member, Doctoral Dissertation Committee
Christopher Was

Accepted by

__________________________, Director, School of Teaching, Learning and Curriculum Studies
Alexa L. Sandmann

__________________________, Dean, College and Graduate School of Education, Health and Human Services
Daniel F. Mahony
HOW DOES SELF-REGULATION IMPACT STUDENT’S USE OF MATHEMATICAL STRATEGIES IN A REMEDIAL MATHEMATICS COURSE (163 pp.)

Co-Directors of Dissertation: Patricia Koontz, Ph.D.
Anne Reynolds, Ph.D.

In order to improve teaching strategies in a college level remedial mathematics course, this study seeks to investigate student perception while they attempt challenging mathematics tasks. The participants were 72 students enrolled in a mid-western university remedial mathematics course. A qualitative case study methodology was used to investigate the students’ experience. The On-Line Motivational Questionnaire (OMQ) was used to gather general information about the group and to help the researcher determine the five individuals to be interviewed. The findings of the study showed that students persisted less often, were less accurate in their solutions, and protected their well being when confronted with a challenging mathematics task. The mathematics tasks required the students to use proportional thinking and the study suggests that the participants were not as accurate when comparing ratios symbolically. The findings suggest that curriculum should include more problem solving strategies and should specifically focus on proportional reasoning.
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CHAPTER I
INTRODUCTION

Students have an opportunity to share their perceptions about their course experiences. At the end of each course the surveys are used to identify possible changes to improve the learning experience. Among other things, the students are asked to comment on their perception of their individual commitment levels, classroom environment, and teacher qualities. The comments provided by students are brief and lack detail; in general, the comments raise more questions than they answer. Unfortunately, there is no way to follow-up with the students because instructors receive the surveys the following semester. Using the summative evaluation as an instructional tool is challenging because of the timing both in the offering of the survey as well as in the turnaround time. The students in remedial math courses who respond limit their comments to what seem to be externally focused reasons for their success or failure. Data used to make decisions about instructional change require more than the elements offered by aforementioned surveys. Therefore, this author aims to use alternative ways to provide a more thoughtful look at students’ experiences during their remedial mathematics course.

The purpose of the study is to investigate the perceptions of students during their remedial mathematics course at the college level as a means for instructors to improve instruction. Richards (1996) stated that teachers can make changes to their classroom toward a more constructivist or inquiry type classroom as long as some assumptions are upheld. The assumptions include the need for communication between teacher and
student as well as between student and student. The primary role of teachers working from a constructivist platform is guiding students toward mathematical understanding. Being able to recognize the needs of all students and implementing the adjustments is a complex task but also a necessary condition so the teacher can pose challenging questions that encourage the student to negotiate mathematical meaning. The negotiation is a two-way interaction, in which student voices are necessary. Understanding how students perceive a mathematical problem and how students think about the problem informs teacher decision-making. The student perception includes the context of the problem, prior experience with mathematics, content knowledge, and affect. This perception may help or hinder task completion. How the student perceives the problem may depend on many factors relating to cognitive or metacognitive perception.

There is a great deal of research about what motivates students to learn (Bandura, 1986; Zimmerman & Kitsantas, 1997; Zimmerman, 2000; Deci & Ryan, 2000; Brophy, 2008). Studies about motivation have led to more specific conversations about self-regulation (SR) and self-regulated learners (SRL). Self-regulated learning is defined by Pintrich (2000), as “an active, constructive process whereby learners set goals for their learning and then attempt to monitor, regulate, and control their cognition, motivation and behavior, guided and constrained by their goals and the contextual features in the environment” (p. 453). By understanding how students self-regulate, there is an opportunity to understand how students actively participate in learning. This, in turn, becomes an instructional tool for teachers, who then may understand how students undertake the negotiation of meaning-making when problem solving.
(SR) has been studied extensively in the field of psychology but we are beginning to see other fields of study such as education, use the research to further understanding. Self-regulation is the degree that students are metacognitively, motivationally, and behaviorally active in their learning (Zimmerman & Martinez-Pons, 1986). More recently, Zimmerman (2000) stated that SR is a cycle of thoughts, feelings, and action feedback that is used to attain goals. The research relating to SR has suggested many relationships between individuals and their actions. For example, SR research suggests that students who are strong self-regulators also are high achievers (Zimmerman & Martinez-Pons, 1990; Pintrich, Roeser, & De Groot, 1994). Students who do not possess strong SR skills are less likely to be successful in academic settings (Zimmerman & Cleary, 2006). Research also has suggested that SR skills can be learned in a classroom setting (Pape, Bell & Yetkin, 2003; Schunk & Ertner, 2000; Perels, Gurtler, & Schmitz, 2005; Cleary & Zimmerman, 2004; Sawyer, Graham, & Harris, 1992). Research in SR evolved after the Pintrich & De Groot (1990) study suggested the need for domain-specific research. Research continues to evolve, as Boekaerts (2002) argues that domain-specific research is necessary but also that motivation changes during specific moments. Boekaerts (1992) suggests when students approach a task they are balancing two types of behaviors. The student is considering content knowledge or procedural knowledge and is also attempting to support his own well-being in the classroom. The theory of adaptive learning process (ALP) mediates the relationship between these two behaviors (Boekaerts, 2002). Students are motivated to complete a task for a variety of reasons but the reasons are mapped into two basic behaviors related either to the student’s
understanding of the content or into an attributional element. Attributions are perceived causes of outcomes or ways that individuals explain reasons for the occurrence of events (Weiner, 1986). For example, after a test is returned, students will explain low scores by stating the teacher did not explain the material or that the test was too long. These attributions are external causes making the individual believe he has no control. The student may claim he or she did not study enough or did not get enough sleep before the test. These are internal causes that the student does control. Students may choose to place the cause outside of their control to help preserve their well-being. As would be expected with studies relating to how humans learn, there are conflicting studies and multiple models used to interpret findings. The models differ because of the complexities of SR but the models consistently propose that students are actively engaged and that individuals are in control of their thoughts, feelings and actions (Boekaerts, 2002). This central frame is used throughout most studies in discussions providing a common thread. Although Pintrich and Zimmerman offer frameworks for understanding self-regulation, Boekaerts’ framework offers insight into the moment that students interact with the information, thus providing a real-time look at student perception. What happens in the specific moment when students must make cognitive and metacognitive decisions? Boekaerts (1992) argued that knowing a student’s motivational beliefs in general cannot inform a teacher about a student’s motivation in a given moment. What are the components that impact student decision-making within a specific task and how does this information support the teaching process? A teacher’s awareness of student thought can change according to Schorr and Lesh (2003). The study suggested that teachers made
transitions in their focus of the mathematics and on the level of student work (Schorr & Lesh, 2003). Having a better understanding of student perception within a given moment provides greater detail about factors affecting motivation and attribution to the teacher.

This study is interested in investigating student perception when posed with a challenging mathematical task for the purpose of informing instructional strategy. The mathematical task was chosen to create a cognitive overload. If the problems seem too familiar, students may not experience the moment when they have to decide whether their own personal well-being is threatened. Proportional reasoning is considered one of the most complex mathematical tasks and deemed essential for transitioning to higher-level mathematics courses (Lamon, 2007; Jitendra et al., 2009). Lamon (2007) defined proportional reasoning as the ability to supply “reasons in support of claims made about the structural relationships among four quantities” (p. 637). In order to investigate student perception the On-Line Motivational Questionnaire (OMQ) by Boekaerts (1992) was used. The OMQ will illuminate elements of self-regulation during a specific mathematical task. This information may be extended through interviews meant to highlight either mathematical or affective issues during the task. As with Schorr and Lesh (2003), the teachers will make adjustments to their questioning after listening to their students. In order to build a stronger instructional technique or curriculum the teacher/researcher must build a stronger connection with the students and the students’ thinking process. Sample questions might include: where in the task do students reach a stopping point? If students do not complete the task, what are the reasons for not completing? If the student does complete the task, what kinds of procedures are used in
the problem? Are these procedures different from what the teacher expected? How do these ideas and concerns become negotiated as part of the social learning structure of the class? To understand more about these questions an understanding of SR and social constructivism is necessary. Chapter two will elaborate in greater detail but a brief history and description are provided here.

**Social Constructivism**

One of the contributing factors to student response to a mathematical question is prior experience in mathematics courses. Classrooms where the teacher is the transmitter of ideas and the student is the recipient would be considered traditional (Battista, 1999). In this setting students are not necessarily expected to problem-solve or reason. They are expected to report back what they have been told by the teacher. When experiencing a mathematics classroom where students are provided the opportunity to demonstrate their own understanding of a problem, there may be a conflict for students whose mathematics backgrounds are traditional classrooms. These students would expect their mathematics class to be traditional. Experiencing classrooms where they are participants in their own learning may be an adjustment for these students. For this reason, it is expected that students may experience difficulty initially with the expectations of the course. However, the assumption that the student is an active participant in their learning is not a new theme. This is a general assumption in SR studies. Active participation also is an assumption in constructivist learning and the reform mathematics classroom (NCTM, 2000). The reform mathematics classroom incorporates high expectations while accommodating for difference; assessments are not just indicators of an end, but they
enhance student learning and support instructional decisions; learning is done with understanding; and teachers are always making improvements to create a challenging and supporting learning environment (NCTM, 2000). The constructivist learning theory is the dominant paradigm in mathematics education research because it places the student at the center of learning. Traditional classrooms as described by Battista (1999) are teacher-centered and have the general daily structure in which the class reviews homework from the previous night, the teacher provides the material for the next section, the students practice the material and homework is assigned. This traditional form of teaching mathematics does not effectively support problem solving and reasoning (Battista, 1999). The reform mathematics classroom holds a constructivist view of learning with the student at the center of the classroom and gives priority to process over product. Although correct solutions are valued, it is more important to know how students understand the mathematics and how students think about mathematics. Dewey (1938) expressed this well when he wrote

> What avail is it to win prescribed amounts of information about geography and history, to win ability to read and write, if in the process the individual loses his own soul: loses his appreciation of things worthwhile, of the values to which these things are relative; if he loses desire to apply what he has learned and, above all, loses the ability to extract meaning from his future experiences as they occur? (p. 49)

This statement encourages us to look at teacher practices and remember that learning is more than answering questions and has implications far removed from our
own classrooms. The current paradigm in mathematics education centers on constructivism. The basic tenets of constructivism are that students build knowledge individually and that students are active participants as learners. Social constructivism additionally contends that learning within a classroom is not an individual activity. There are “taken-as-shared” (Cobb & Bauersfeld, 1995) mathematical norms that each classroom develops. Students and teachers negotiate their mathematical meaning not just individually but also within the social setting (Cobb, 2000). How the students negotiate the lesson as provided by the teacher will depend on the norms established in the classroom. Are students expected to explain or describe their solutions? Do students explore their own line of inquiry, or do they follow the lead of the teacher? These questions are important to understanding how students interpret or negotiate the mathematics in the classroom and, in turn, how the teacher interprets the students’ meaning.

**Social Cognitive Theory of Self-Regulation**

The social cognitive theory of SR is a cyclical process and is described in three phases by Zimmerman (2000). The forethought phase relates to goal setting, the volitional phase relates to the process or action, and the self-reflection phase relates to the individual’s reflections on her goals and actions. SR occurs within the cognitive and metacognitive experiences of learning. Completing a mathematics problem has to do with the ability to understand the necessary strategies. Even if the strategies are understood, there is still recognition by the student that the problem has value or can help the student achieve goals. Understanding why a student does or does not complete an
activity is extremely complex and unlikely to be the same every time a problem is presented. Understanding self-efficacy, prior experience, attempts not to fail, value of the task, sense of belonging, or content knowledge are a few of the many decision factors faced by students each time a mathematics problem is posed to them. Because each problem will have different context, value, and strategy, understanding the reasons for student decision-making is very challenging. Success in a mathematics course is not simply dependent on content knowledge or affect but these ideas are interrelated (Pintrich et al., 1994). Studies of SR have shown several interesting results, and refinements in the frameworks for asking questions have allowed researchers to see a deeper, more thoughtful description of the student’s SR. The beginning studies on SR were primarily empirical in nature. Studies used self-reported Likert scale models to elicit student perceptions. However, understanding the complex elements of SR has required that studies include both qualitative and quantitative data to produce a more cohesive and comprehensive picture of SR (Schunk & Ertner, 2000). Most frameworks agree with four general assumptions of the SRL framework posed by Pintrich. The four assumptions are that the learner is active, the learner can regulate, the learner has criterion to compare and self-regulation connections of personal with contextual (Pintrich, 2000).

**Four Assumptions of Self-Regulation**

The four assumptions reflect what researchers consider as a baseline for students when learning. The first assumption states that the individual is active in his learning. When students are active in their learning, knowledge is constructed by the individuals and not simply transmitted by the teacher. This follows the constructivist paradigm that
teachers and students negotiate meaning within a social setting. The second assumption is that the learner regulates one’s own actions. Students are not just receiving information in a passive way. Instead they are making choices about how they will interact with the information. Quantitative studies have shown that SR does play an essential role in learning but there is now a call for studies to take a deeper look at the interaction between the student and learning (Pintrich, 2004). The information available to educators is extensive. Studies have shown that strong self-regulation is correlated to high achievers and motivation (Pintrich, 2004) as well as a greater likelihood of seeking help from peers (Zimmerman & Martinez-Pons, 1986). SR can increase persistence (Pokay & Blumenfeld 1990; Butler, 1987; Ames, 1992), achievement (Zimmerman & Martinez-Pons, 1990; Pintrich et al., 1994), and confidence (Ames, 1992). Given the relationships between these skills and SR, it is important to note that studies indicate that SR can be taught and learned: (Pape et al., 2003; Schunk & Ertner, 2000; Perels et al., 2005; Cleary & Zimmerman, 2004; Sawyer et al., 1992). Although it is not the intention of this study to teach SR skills, it is important to know the ways these skills potentially could support student learning. If students in the remedial mathematics courses are weak SR learners, investigating the kinds of skills that are strong or weak will help inform future instruction. These empirical studies show connections but raise other questions about why these connections were made. Early SR studies with interviews were asked specifically about the types of methods students employ when completing tasks. SR researchers have been clear in suggesting that studies involving SR and other motivational factors such as self-efficacy (SE) should not be generalized studies
(Bandura, 1986; Schunk, 1990; Pintrich, 2004; Pajares & Miller, 1995). The interview questions should not be just about the methods for self-regulating in general. The interview questions are in relation to the individual’s OMQ feedback, which also may include content-specific mathematical issues to address.

The third and fourth assumptions relate to the classroom setting suggesting that students have a way to compare their work and effort to others in the classroom along with their own personal experiences in and out of the classroom. The remedial mathematics classes in this study use a curriculum that studies a broad scope of content. The course includes a large portion of problem-solving elements that requires students to make connections to contextualized problems. The students are encouraged to work cooperatively and to discuss their ideas within the classroom with the teacher and with other students, creating a sense of community. This study will examine the kinds of SR skills students bring to the remedial mathematics course within the context of problem solving. Schoenfeld (1992) suggested that problem-solving should be linked to a common type of problems as a way to eliminate students’ misunderstandings and concerns about problem solving. Because problem-solving is a broad topic, this study will focus on problem solving related to proportional thinking because of the connection to higher-order mathematical thinking. The link between problem solving (PS) and SR is contextually important in the remedial mathematics courses. Understanding where students’ challenges occur will inform teacher decision-making. Studies have linked SR with PS already. The Perels et al. (2005) study illustrated that teaching both SR and PS strategies at the same time was more productive than teaching either individually. Other
studies suggested that PS skills and SR skills are both essential skills in learning mathematics (Fuchs et al., 2003). Suggestions for future studies include combining SR with context, using both quantitative and qualitative data, and including more studies focusing the reflection phase of SR (Pintrich, 2000). This research will focus on students’ interpretations, representations and reflections. This implies an understanding of domain-specific elements of mathematical learning that include computation, the deductive reasoning processes employed, and the rules and procedures students learn to execute. The framework suggested by Boekaerts (1992) involves the assumptions from Pintrich included with student thinking in a specific moment.

**Purpose of Study**

Negotiating meaning in a mathematics classroom is complex. The teacher brings her own perspective of mathematics along with experiences with learning and perceived understanding of how others learn. Students also bring prior experiences to the classroom.

When classroom time is limited, the opportunity to share thoughtful and meaningful discussions with all students is challenging. Often the classroom has a primary focus on content with student interest and student affect further down the list of priorities. Given that learning involves more than content knowledge, there should be opportunity to consider student perception during the actual task. What are the stumbling blocks or the door-opening moments for students? What are students thinking during these processes? The OMQ provides an initial view into the moment when students are working on a challenging task. It is the intention of this study to investigate student
perception of the task, and then follow up with interviews with a smaller group to focus on some of their challenges based on feedback from the OMQ. Are the challenges related to mathematical concepts, task strategies or with affective elements? This information then is used as formative assessment for the instructors of the remedial mathematics courses and ultimately improves teaching and learning of mathematics.

**Statement of the Problem**

In order to improve teaching strategies in the remedial mathematics courses, this study seeks to investigate student perception while attempting challenging mathematics tasks. Although there is feedback from the students work and student responses in class this information is limited in that there is no information about why a student made choices about completion, the amount of time needed to complete the task, or any of the other metacognitive responses that students had in the moment. Research in mathematics education is suggesting that both students and teachers should have a deeper understanding of the process by which students do their mathematics. This interpretation of student understanding of the problem can be accomplished superficially through questioning in class but there are more questions when students do not complete assignments. This study intends to investigate student interpretation of problem solving within the task itself. How does self-regulation impact student’s use of mathematical strategies in a remedial mathematics course? What indicators, if any, suggest that students are actively participating while attempting to complete the mathematics task? What criterion, if any, do students use while attempting the mathematics task? What mathematical strategies, if any, do students use to solve the mathematics tasks?
CHAPTER II
REVIEW OF THE LITERATURE

Mathematics education as a separate area of study is relatively young compared to other fields, such as psychology or mathematics. Mathematics education researchers borrow theoretical frameworks from other disciplines because of the shared content (Lerman, 2000). Researchers draw from the field of mathematics when focusing on content, they draw from the field of sociology when focusing on interactions within the classroom, or they draw from the field of psychology when focusing on individual experiences. Learning mathematics is a complex process, and therefore, requires more than one perspective. A brief historical perspective discusses influences from other fields on the newer field of mathematics education.

Mathematics Education

The Woods Hole Conference in 1959 may have been one of the first times that all areas of study gathered to begin sorting through the needs of a mathematics classroom. Mathematicians, general educators, and psychologists at the conference discussed the path for mathematics in the United States. Mathematics and science curriculum were under scrutiny because Russia had launched Sputnik, and there was a national sense that the United States was not developing enough mathematicians or scientists to keep up with other countries. The United States government created grants encouraging researchers to develop new curriculum for K-12 schools. The conference goals were to collaborately develop the next generation of mathematics curriculum for classrooms in the United States. The conference leader, Bruner (1960), followed the conference with a
book he authored summarizing the need for a change to mathematics curriculum but also about the importance of attitude.

To instill such attitudes by teaching requires something more than the mere presentation of fundamental ideas. Just what it takes to bring off such teaching is something on which a great deal of research is needed, but it would seem that an important ingredient is needed, but it would seem that an important ingredient is a sense of excitement about discovery – discovery of regularities of previously unrecognized relations and similarities between ideas, with a resulting sense of self-confidence in one’s abilities. (Bruner, 1960, p. 20)

The collection of individuals from various fields of study allowed questions to be answered immediately by experts on learning, motivation, mathematics and education. With the support of government grants these researchers set the field of mathematics education in motion. The resulting curriculum was known as New Math. A negative connotation is associated with the collection of curriculum used in the research projects, even though many of the projects showed positive learning results. The reasons for the perceived failure of the curriculum designs was centered on community concerns about the challenging level of mathematics that would be taught to younger children. Because the curriculum was so different from the traditional forms of mathematics curriculum parents in the community were accustomed to, there was a sense that parents could not help their own children understand the concepts. The material was age-appropriate but the result was a very different mathematics curriculum. A notable study lead by Davis (1990) stated that the discovery approaches discussed in the 1950s and 1960s were
successful because students learn more mathematics when they develop the mathematics themselves. Davis wrote that these ideas were possible in a classroom as long as the teacher believed that learning could take place with discovery. “When pedagogical practice is unsupported by appropriate theories of learning and theories of knowledge it is unlikely to flourish” (Davis, 1996, p. 5). Even though the research supported student learning the programs were considered ineffective. This is likely due to different reasons: all of the projects were lumped together; the goals of the projects were very different from the traditional practices, there was a lack of adequate theory for comparison; or the need to give more prominence to actual classroom episodes (Davis, 1990). Studies that had similar objectives and therefore should be considered together included the Madison project, the Illinois Arithmetic Project and the Science Curriculum Improvement Study project (Davis, 1990). These projects were not really new in the sense Colburn wrote books in the 1800s that introduced similar ideas. His induction process has been linked to the discovery process emphasized in the 1960s (Jones & Coxford, 1970). Mathematics education has moved between an emphasis on traditional and inquiry-based throughout the century without making a final determination yet. Presently, there is continued effort for curriculum development aligned with similar goals to those from the New Math studies. Mathematics curriculum development includes emphasis on discovery and inquiry as seen in the Connected Mathematics series for middle grades or the Integrated Mathematics Program for high school grades. Current researchers continue to build on the strengths of student-centered, discovery-based mathematics curriculum. Studies like Boaler’s study of two secondary schools in England demonstrate the reasons why change
in mathematics curriculum and instruction is necessary (Boaler, 2000). The study
examined differences in two schools’ approaches to mathematics instruction and
curriculum. The two schools that were examined by the study utilized different
mathematical curriculum and instruction. One school used the traditional form of
curriculum and instruction, with the teacher as the information provider and students
practicing independently. The other school focused on problem-solving, open-ended
questions; the interests of the students; and connections between mathematics learned in
school and mathematics experienced in the community. The study illustrated that
students from both schools had similar achievement scores on the national assessment
test but the students from the second school tested better at problem solving, they had
more interest in future study and could connect their classroom mathematics outside of
the classroom. The efforts of mathematics education are to continue demonstrating what
has been shown the mathematics classroom can be a place that shows mathematics is
valuable outside the classroom, where students think creatively, where students are
challenged to understand the meaning of the mathematics, and that mathematics is not for
a select few but rather for anyone (Davis, 1990; Boaler, 2000). The continued effort to
enact change is not a simple one or it would have been accomplished already. So in
keeping with Davis’ suggestion not to repeat the past there is the need to look at
classrooms with a theoretical lens that supports student learning and thinking about
mathematics.

Because the mathematics classroom is a combination of so many complex ideas,
it seems appropriate to consider the theories that best support student learning and critical
thinking. Historically, the field of mathematics education moves between two learning theories while building theories connected to specific content. Constructivist theories support the individual as a learner and not as a recipient of information. This keeps the learner as the central piece, with the main purpose being the exploration while the behaviorist approach keeps the curriculum as the central piece. Using constructivism as a central theory of learning keeps the individual in the center but does not disregard the curriculum. The individual has his or her own interests and experiences, which affect their interactions in a mathematics classroom so it is also important to consider the lens of the student experience. But understanding the student experience is not as simple as asking the teacher. Teacher’s perception of what students understand is often incorrect (Yusof & Tall, 1999). When students are asked to describe their perceptions of a given experience, these experiences are attached to emotions that have been built up for many years and may have little to do with the current understanding and more to do with prior experience. For this reason, questions related to student perception should be as specific as possible to provide information about the current experience.

The distinction of the theory of learning associated with mathematics education requires a discussion of the Math Wars still apparent in the field. Riley (1998) referred to the “two sides” of the mathematics curriculum debate. This is oversimplifying a complex and challenging discussion. According to Riley, the two sides consist of those who advocate a traditional mathematics curriculum and those who advocate a reform curriculum. Most would consider traditional curriculum as process-product with the teacher as the expert who provides direct instruction while the students receive the
information and practice until they understand. The curriculum is objective-oriented with clearly defined steps that must be followed in sequence. Reform curriculum will vary depending on the reform group but in general this curriculum consists of less direct instruction with more student interaction as a means either to discover the mathematics in their own way or through problem solving or experimentation. It is interesting that we wonder why mathematics curriculum has not shown consistent results or improvement through the years when the curriculum pendulum swings back and forth through the continuum of methodology and pedagogy. It seems prudent to discontinue the “Math Wars” debate in lieu of emphasizing research and curriculum development that supports student learning along with student interest in future learning. The development of lifelong learners is more important that the ability to pass a specific exam.

**Learning Mathematics**

What is it to learn mathematics? This is a complex question because there are different views of what it means to learn as well as what mathematics means. Lambdin and Walcott (2007) stated there have been six phases in mathematics education since the early 1900s. Drill and practice is typically related to Thorndike’s connectionism or a focus on basic skills rather than meaning (Kliebard, 2004). Meaningful arithmetic is linked to Brownell’s (1935) vision of mathematics education being a mixture of practice and a connection to contextualizing mathematics. The New Math phase has the focus of mathematics on the basic structures of mathematics and on building understanding through challenging topics that were spiraled through the curriculum. Back to basics is a return to the connectionism or drill and practice. This is replaced with a focus on
problem solving or learning through discovery. Currently we are considered to be in the accountability phase with a focus on testing. What is interesting about the historical perspective is the cyclical nature of the arguments. Although there has always been more than one view at any one time, one of the views tends to come to the forefront in any particular time frame. The 20th century began with the belief that learning mathematics was the same as practicing how to compute numbers. The content was handed down to the students and then practiced until consistency was shown. This was replaced with a more constructivist view that individuals must be able to develop their own knowledge. They can do so through meaning-making activities and discovery. When it appears that students don’t appear to know how to do basic computation the focus changes back to the connectionism, which, as shown earlier in the century is an effective way to get students to be able to answer problems but will not help them understand the deeper meanings and the contextual uses for the mathematics. So back to constructivism and a focus on problem solving and discovery learning the focus goes. Whether this is a result of the NCTM Standards (2000) being written as a list of items to learn, the next phase seems to be taking us back to the connectionism phases where students get the information from the teacher and practice the problems until they are able to pass state-created accountability tests. At what point do we stop swinging the pendulum from both sides of the Math War and settle on the idea that learning requires a little of each of these elements? What a person really knows and what really matters in mathematics education is an individual internalization of interpersonal practices and cultural knowledge (Lamon, 2003).
Current Trends

Let us examine the current trends in learning mathematics. The Math War has placed communities concerned about their school’s scoring against mathematics educators who see mathematics as more than a list of multiple-choice questions. Essentially the Math Wars are highlighting what has happened in mathematics education since the beginning of the 1900s. There still is a fight over whether students learn by being told and then practicing or if knowledge is built up individually by the student. These two sides are an oversimplification of the problem. The current paradigm for mathematics education is constructivism. We see constructivism in researchers like Piaget who is considered a radical constructivist because he believed that the individual built up their own knowledge (Flavell, 1963). In addition, he stated individuals do not discover an already existing reality; instead each individual builds his or her own reality. Learning is cyclic in nature where an individual has moments of equilibrium and disequilibrium. If an individual has her own understanding of an object and that understanding is somehow shaken, this disequilibrium leads the individual to assimilate or accommodate the idea. Assimilation is bringing in the idea so it fits with already understood schema, whereas accommodation requires that new schema and old schema have to change to reach a level of equilibrium. The process is ongoing as the schema become more challenging and abstract. When the individual works through this cycle using reflective abstraction the individual is reaching the highest level of thinking according to Piaget (Flavell, 1963).
Vygotsky (1978) was also known as a constructivist and shared much in common with Piaget, believed that knowledge is built up by the individual but that social and cultural context played a crucial part in this development. Vygotsky (1978) believed that children used language as a way to build their understanding. The challenge with constructivism within research is the necessary interpretations that have to be made by the researcher to understand what the individual has built up and how the individual is interpreting the schema.

**Sociomathematical Norms**

Current research has recognized this challenge and encourages researchers to be able to look at how individuals build their knowledge but also how sociomathematical norms play a role on these schemas as the individuals are working within a social context like a classroom. Cobb and Yackel (1996) described sociomathematical norms as a “taken-as-shared” understanding of mathematical ideas. The required depth of explanation required for describing the solution to a mathematics problem is a sociomathematical norm. The students and the teacher have to agree on the level of detail required. Classrooms have social norms relating to how individuals interact with one another. Sociomathematical norms also relate to how individuals interact but these interactions are specific to mathematics. To help students effectively reflect and problem-solve, the classroom community has to be reconsidered. Classrooms also should include time for children to think about mathematics concepts in their own way. Classrooms should encourage problem solving and communication as a means of
reflecting about concepts. In other words, a classroom should have a focus on the negotiation of meaning rather than a focus on imposing the meaning (Battista, 1999).

**Self-Regulation**

The process of regulating cognitive processes has been studied for several decades. Zimmerman (2000) defined self-regulation (SR) as the “self-generated thoughts, feelings, and actions that are planned and cyclically adapted based on the performance feedback to attain goals” (p. 13). The cycle described by Zimmerman (1986) includes three phases: a forethought phase, a monitoring phase, and a reflection phase. The study of SR has been developing since the 1970s and has included some version of this cycle throughout. There are four assumptions that hold for SR research. First, the individual is actively participating in the process. Second, the individual can regulate his or her own learning. Third, individuals have a criterion against which to compare their actions. Fourth, the individual regulates their cognition and behavior and eventual completion of a task (Pintrich, 2000).

Early SR research was based on attribution and self-efficacy (SE) studies. Attribution theorists contend that student perceptions of the causes of their academic successes and failures determine their expectancies for future performance (Weiner, 1986). There is empirical evidence suggesting a relationship between causal attributions and personal efficacy. Highly efficacious students believe performance outcomes to be personally controllable, so they tend to attribute failure to factors that they can change. Students with low SE attribute failure to uncontrollable factors, thereby increasing feelings of despair and helplessness (Zimmerman & Cleary, 2006). Self-efficacy (SE) is
the individual’s perception of his or her ability to complete a task (Bandura, 1986).
Studies on SE suggested it is associated with persistence (Bouffard – Bouchard, Parent, & Larivee, 1991; Schunk, 1981). Bandura (1986) suggested that studies relating to SE be domain-specific because motivation changes between domains. Individuals may be more confident in a chemistry class rather than a literature class. In the mathematics classroom, perception of mathematics ability has strong longitudinal effects on future efficacy beliefs and perceptions of ability to do mathematics (Meece, Wigfield, & Eccles, 1990). Understanding how students feel about their mathematical ability impacts the kinds of effort they will commit to a task but more importantly their beliefs impact their decision to take other mathematics courses or to continue learning. Self-efficacy is a perception of oneself and not a psychological trait so the transition to studies including self-regulation provided another line of research to determine connections psychological traits of individuals and classroom outcomes.

Metacognition

Metacognition in simple terms is how we think about our thinking. More specifically it is defined as the ability to monitor, evaluate, and make plans for one’s learning (Tobias & Everson, 2009). Because this is an internal structure the tools used to measure metacognition depend on self-reporting. For this reason, researchers also are interested in an individual’s ability to know what they know. How aware are individuals of their own ability? Self-reporting will raise a validity issue because individuals may not be candid or may not understand their own higher order processes. One tool used to report metacognition is the Knowledge Monitoring Assessment (KMA). The KMA
reflects how well students monitor their knowledge with an emphasis on accuracy between when they believe they know with what they know. When an individual is first presented with a challenging task, the individual makes a determination of his or her ability to do the task. This initial appraisal is then matched with the individual’s accuracy after completing the task. There are four possibilities with the KMA. For accurate knowledge monitoring, the individual will have a positive appraisal and an accurate solution or the individual will have a negative appraisal with an inaccurate solution. Accurate knowledge monitoring is an indication that the individual can make better determinations of his study needs when working on a specific idea. The presumption is that individuals with accurate knowledge monitoring will know when additional study time is necessary along with the kind of studying is required. Inaccurate knowledge monitors will have a positive appraisal with an inaccurate solution or a negative appraisal with an accurate solution. These individuals tend to over-estimate or under-estimate their abilities. As a result, they do not study when necessary or they study material that does not need to be studied. Those individuals who over-estimate their ability will be surprised when they do not do well on an exam. Sample comments from students will be that they knew the material before the test. Those students who under-estimate will comment that the test was difficult and that their score will be low but will end up doing well on the test or assignment. Learners who are accurate knowledge monitors will have more success filtering the vast amount of information provided in courses to focus on material they have not mastered.
**Self-Regulation Framework**

This section will highlight some of the findings within the SR framework that have created baselines for future research. This section also will highlight several researchers whose work has been continually developed throughout the past few decades.

Initial studies of SR focused on the individual phases described by Zimmerman (1986). Self-regulated learning (SRL) is based on the same theoretical base as Zimmerman but is more specific to student understanding. Research includes focus on goals or goal setting, the determination of SR can be learned, and self-talk as a learning strategy. Goals are described in a variety of ways in SRL, for example, goals are separated based on internal versus external influences. Mastery compares understanding to a personally determined criterion, and performance compares understanding to others’ standards. Ames and Archer (1980) suggested that mastery goals are linked to the attributional belief that effort leads to success, supporting an effort-outcome perception that is central to the attributional model of achievement. Attributional beliefs are described by Weiner (1986) as student perception of the causes for outcomes affects learning behavior. If a student believes the reason for failure is caused by something outside of their control, the student is more likely to give up (Weiner, 1986). Causal attributions are linked to self-efficacy and, therefore, to motivation, making attributions part of the student’s complex goal-setting phase. Butler (1987) showed that mastery goals increase based on the amount of time spent on learning tasks, persistence when problems are difficult, and most importantly, the quality of the student engagement in learning. The goals that students set will determine their approach to the second phase.
The reflection phase deals with the individual’s self-control and self-observation. The self-observation does not relate to the evaluation or judgment of task but rather to monitoring of the task process. Studies relating to this phase suggest that students can enhance learning if they use self-talk. Wong, Lawson, and Keeves (2002) ran a study of 47 high school freshmen in Australia. Students were given the mathematical beliefs questionnaire, assessed on prior geometry knowledge, and trained on self-talk procedures. The students were given problems to solve and were encouraged to talk out loud during the process. The researchers suggested that students who participated in the self-explanation during the problem solving had a learning advantage over the students who were not trained to self-explain (Wong et al., 2002). Creating opportunities for student success is important but what is most important about this strategy is its flexibility of use. The self-explanation is not content-specific allowing the student to use the skill in many areas. Self-talk also can help the teacher understand the thought process used by students. In a study of four freshman college students in Sweden, the researcher interviewed the students while working on challenging algebra problems. The interviews suggested that the students were not flexible thinkers. The students would try to use current content and methods to solve problems instead of using strategies involving elementary concepts (Lithner, 2000). These students believed that the method for solving was the most current method taught. The researcher uses the discussions with the students to understand the thought process while the student is attempting the problem. The teacher is then able to adjust instruction to demonstrate that more than current strategies can possibly be used.
The types of tasks and task strategies that students employ also is a part of the performance phase. Pape et al. (2003) demonstrated that tasks that are challenging and interesting enhance student effort and persistence. The researchers worked with a seventh-grade teacher and her classroom. The teacher was concerned about student effort during problem-solving settings. She made adjustments to the classroom norms: she developed challenging and interesting mathematical tasks; she implemented classroom norms encouraging mathematical discourse; she emphasized group work and support; she built SR theory into the curriculum; and she encouraged students to feel positively about their problem-solving efforts (Pape et al., 2003). The study suggested that a “critical” element for student learning was the meaningful mathematical tasks using multiple representations (Pape et al., 2003). Student participation in mathematical problem solving is impacted by content knowledge, strategy knowledge, prior experience, task value and interest. One of the goals of reform mathematics is considering the student when creating “real life” application. Application for the sake of creating word problems will not necessarily gain student interest or task value. Creating problems that are challenging yet of interest to the student is important.

Research in the field of SR has developed several instruments for data collection to meet the complexity of the studies. The Motivated Strategies for Learning Questionnaire (MSLQ) was designed by Pintrich, Smith, Garcia and McKeachie (1993) to determine connections among elements such as learning strategies, motivation, study management, value placement, self-efficacy, and affect. The MSLQ is like many of the empirical instruments for researching SRL. Students are self-reporting on their
perceptions using a Likert scale. The complexity of SR is illustrated through the use of 81 questions on the MSLQ. The complexity also is illustrated through the variety of instruments used in SRLL research. Zimmerman and Martinez-Pons (1988) utilized the Self-Regulated Learning Interview Schedule (SRLIS) which investigates student perception of the 14 self-regulated learning strategies linked to strong SR individuals. Zimmerman and Martinez-Pons (1986) point to fourteen strategies that are used by high achievers. The list includes: self-evaluation, organizing and transforming, goal-setting, seeking information, keeping records, environmental structuring, self-consequences, rehearsing and memorizing, seeking assistance, and reviewing. Boekaerts’ (1992) framework works from the same four assumptions as described by Pintrich (2000) but takes a task-specific view of self-regulation. Boekaerts’ Model of Adaptable Learning presumes that goals are interconnected. These goals may be based on an external component or internal components. Boekaerts’ (1992) model recognizes that students are continually deciding if their well-being is taken care of. Individuals will want to feel safe in the learning situation. Appraisals are a “nonstop evaluation process that results in emotions/action readiness of upcoming and ongoing learning activities” (Boekaerts & Niemivirta, 2000, p. 427). The Model of Adaptable Learning attempts to determine what metacognitive factors are interacting with the content of the task or the affective state of the individual. Research using this model suggests the necessity for task attraction for any meaningful action to occur (Boekaerts, Seegers & Vermeer, 1995). The questionnaire does not illicit specific responses as to reasons for the lack of task attraction, so a qualitative element is necessary for a stronger understanding of the student’s responses.
“Achievement effects are mediated by the self-regulatory activities that students engage to reach learning and performance goals” (Boekaerts & Corno, 2005, p 201). Boekaerts (1996) suggested research should not be related to hypothetical learning situations within a content domain; instead, the research should take into account the specific learning task. An example study by Seegers and Boekaerts (1993) exemplifies the use of the On-Line Motivational Questionnaire (OMQ) to investigate the interactions among motivation, task attraction, affect, and effort. Variables like attribution style, goal orientation or self-efficacy were suggested to influence how the individual feels about his or her ability to do the task or the personal relevance of the task. This study was conducted with sixth-graders using four mathematics problems as the task.

Research relating SR to context-specific content has been a focus of SR researchers since the Pintrich & De Groot (1990) study suggested that domain-specific knowledge is necessary to understand student SR. Researchers like De Corte (1995) recognize the need for more than the algorithmic approach to teaching mathematics. Within the mathematics domain research has demonstrated a connection between SR and problem-solving. Because studies suggest SR can be taught in an academic setting (Schunk & Zimmerman, 1998; Zimmerman & Martinez-Pons, 1990), domain-specific studies also exist. Perels et al. (2005) suggested that training both SR and problem-solving simultaneously is a stronger stance than teaching either skill independently. Problem-solving is an extremely broad topic, but awareness of how students solve and think about their task tells us how much effort will be made and how much learning may occur. Understanding that students are more likely to feel confident when working on
computational problems over word problems (Vermeer, Boekaerts, & Seegers, 2000) suggests that recognizing why students do not feel confident when problem solving is important. The use of multiple tools provides a deeper picture of the participants. Tools used to investigate SR in a mathematics classroom include the Goal Orientation Questionnaire (GOQ) developed by Seegers, Van Putten, and De Brabander (2002). The questionnaire has questions specific to the experience in a mathematics classroom. The GOQ also is used with the Self-Concept of Mathematics Ability Questionnaire developed by Bong & Skaalvik, (2003). This questionnaire measures cognitive aspects but was developed to work with the GOQ.

**Problem Solving**

How problem solving is perceived has been changing over the years, so the research about problem solving has been changing as well. While looking at the types of research related to problem solving, it becomes apparent that the definition of problem-solving has evolved over the years. While the definition has been evolving, so have the major issues, many of which still are works in progress. There seems to be a need for a clear definition of problem solving, attention to issues related to instruction, a better measurement tool to measure problem solving performance, and to work on issues relating to how problem-solving skills can be transferred (Lester, 1985). Lester also is concerned that the area of problem solving is receiving attention because other topics such as constructivism are pulling attention away. This may be true, but there also is the possibility that problem solving is part of other topics such as reform curriculum. The National Council of Teachers of Mathematics (NCTM, 2000) has included problem
solving as one of their process standards. As a result, problem solving is becoming part of the overall idea of reform curriculum. While looking through the history of problem solving, this researcher was interested in the kinds of research already available as well as how problem solving is defined and how problem solving is incorporated into classrooms.

Initially, problem solving was directly related to word problems but has evolved to include a classroom culture. The first major influential voice in problem solving appeared in 1945. Polya wrote the book *How to solve it*, in which he described a heuristic approach to problem-solving. Students are provided four steps for solving word problems. The steps are (a) “understand the problem,” (b) “see how the items connect,” (c) “carry out a plan,” and (d) “look back at the problem” (Polya, 1945, pp. 5-6). All steps are necessary to understand the problem. In the behavioristic setting of this time period, the steps would provide the general guidelines, but types of problems would have to be grouped so students could practice techniques. Understanding would be demonstrated by showing the appropriate technique for the appropriate type of problem. Multiple approaches and personal experience might play a role in the problem solving, but it would not be required. Depending on what is valued in a classroom, this may or may not be an important consideration. Textbooks used for college algebra still reference the same heuristic steps today.

The 1950s brought an emphasis of change to our educational system. This was the era of concern for our mathematics and science curriculum; in particular, the United States had lost the race to launch a satellite into orbit. This concern prompted an increase
in research funding by the government and, therefore, an increase in research. Before all
of this concern, studies tended toward process and product where the methodologies of a
classroom were studied to determine effectiveness (Schoenfeld, 1987). The 1950s
marked an interest in alternative methodologies, including a series of studies that most
remember as New Math (Davis & Maher, 1990). Given that most mathematics
instruction was based on behaviorism, instruction focused on careful sequencing in small
steps that were repeated often to gain proficiency. Because this instructional method was
not meeting the needs of schools, the focus of curriculum changed from primarily
procedural to conceptual. The New Math studies were assumed to be failures because
their ideas did not become common in our schools. As New Math has been studied it has
been noted that many of the programs were very successful but because there were many
different studies all under the same heading of New Math the successful were grouped
with the unsuccessful attempts (Davis & Maher, 1990). There seems to be a general lack
of trust that teaching conceptually can have a positive learning result. Instead the “back
to basics” drill and practice are more common. Part of what needs to take place in the
study of problem-solving is to be sure that studies are cohesive and have similar goals.
This time around, mathematics educators have the advantage of the National Council of
Teachers of Mathematics (NCTM), a group that ties the studies of mathematics together
by providing a general framework for curriculum. After the perception of failure of New
Math, mathematics education returned to the behavioristic model. However, it was too
late to go back entirely because the question of the adequacy of this form of instruction
was already suspect. So what are the alternatives?
The philosophers and psychologists during the 1950s were looking at learning strategies but not solely at learning strategies specific to mathematics education. Following the suggestion of Kilpatrick (1969), researchers began to focus not only on the methods of mathematics but also on the thought process while doing mathematics. Researchers were using ideas from psychologists and philosophers to relate what they noticed about how their students tried to make sense of mathematics. Cognitive researchers wanted to know what knowledge began with, the strategies the student already understood, and how these two ideas worked together (Schoenfeld, 1987). Researchers were beginning to see that students brought their own experiences to a problem and that it was necessary to determine how students utilized these experiences with their mathematics problem solving. As researchers began to notice more about the processes that the students were using, they began to compare how factors such as gender, class, affect, ethnography, and other social influences affect student process and student understanding (Kieran, 1994). This seemed to follow the general education research pattern into the 1980s and 1990s. In recent decades, research has included a more postmodern look at mathematics education. Studies include the more than how a student learns but why one student learns and another does not. Researchers bring ideas of learning and understanding together through constructivism, cognition and Piaget’s framework while including the social perspective of Vygotsky (Kieran, 1994).

One particular series of studies was cognitively based. The researchers were interested in developing an understanding of how young students approach subtraction problems. The researchers constructed a listing of the types of subtraction problems then
asked third-graders to solve the problems. The researchers also were interested in how the students’ teachers perceived the difficulty of the different types of problems. Teacher knowledge was a primary variable in a study by Carpenter, Fennema, Peterson, and Carey (1988). As it turns out, teachers believed that students would have difficulty with some problems because of the computation challenges but actually students used their own understanding of numbers and found alternative ways to solve the problems that did not require the challenging computations (Carpenter et al., 1988). This study suggests that the teacher’s beliefs impact how types of problems are approached. This study illustrates that teachers may be introducing strategies for solving problems that are not necessary. Students may already have their own means of solving the problems that fit with their current understanding of mathematics. This underscores the necessity for teachers and students to negotiate mathematical meaning and strategies together. The strategies should not be imposed on the students but rather seen as supporting student learning. Because the problems were defined into categories, comparisons could be made about how a student conceptualizes or models two problems that require addition. Is modeling always the same for addition or does it change when the wording changes? The researchers are interested in how the students modeled the problem. Did the students use manipulatives, drawings, or did they count on their fingers? They were interested in the perceptions the teachers may bring to the problem; do they know the method their students will use? The study used quantitative data and interviews to possibly help explain the data provided from the tests. The study noted several items of interest. Teachers had difficulty explaining the differences between the different types of
problems, and they tended to focus on syntactic instead of semantics when trying to
determine difficulty. Teachers tended to know the problems that would give their
students difficulty but were able to predict the strategy the student would use to solve the
problem about half the time. Their findings were that ‘teachers are reflective, thoughtful
individuals and that teaching is a complex, cognitively demanding process involving
problem solving and decision making” (Carpenter et al., 1988). In addition, the article
emphasized the importance of communication between the student and the teacher. The
conversation is not one directional but continual in both directions. The constructivist
theory of learning requires the same communication. To know where students thought
processes begin and how they bring information together helps the teacher know how to
ask the next question.

The next year, part of the group from above and others continued the study to
examine whether teachers who were asked to study research on student development of
problem-solving of addition and subtraction would have an effect on the student learning
or problem solving process (Carpenter, Fennema, Peterson, Chiang, & Loe, 1989).
Some teachers attended a workshop where they studied research on problem-solving
while other teachers were chosen to be part of the control group. None of the teachers
were given any prescribed instructional practices. The findings indicated that the
workshop group “encouraged students to use a variety of strategies and they listened to
processes the students used more than the control group” (Carpenter et al., 1989). As a
result of the years of working with teachers in the study, teacher belief changed. This is
fundamental because teachers tend to teach they way they were taught. The study
indicated that teachers were concerned about whether the focus on problem solving would take away from basic computation, and teachers also were concerned that the problems were not within the ability level of their students. The study showed the problems were within the level of the students and that the students in all classes regardless of which group did equally well at basic computation. The studies reveal the necessity for teachers and students to negotiate mathematical meaning together and also provides insight into how teacher belief can be changed when there is an understanding of student perception. Strategies for solving problems in mathematics are often predetermined, but this study shows that as students moved from one type of subtraction question to the next, strategies or models incorporated from previous problems were integrated. However, if the problem required additional information students could draw on other systems. This followed what Vygotsky had indicated with his zones of proximal development where a student’s use of systems may or may not be used depending on factors such as guidance from a peer or adult (Lesh & Harel, 2003). The researchers demonstrated that learning is difficult to fit into any one model. For some areas of study, specialized constructs need to be developed to understand the learning process.

Boaler (2002) conducted a study that encompasses ideas that reflect concerns of postmodernists as well as mathematicians. This three-year study focused on two schools in England whose approaches were distinctly different. One school followed a very traditional path and another followed a discovery-learning approach. The study followed students through grades that equate with grades eight, nine, and ten in the United States. The two schools were chosen because students from both schools used the same
traditional method with the same textbooks in the years prior to eighth grade. They were similar in socioeconomic levels and community make-up, with no significant differences between the scores of the student scores on a national exam given at the beginning of eighth grade (Boaler, 2002). This study is unusual for many reasons but primarily because of the extreme differences in philosophies of the two schools, Amber Hill and Phoenix Park. Amber Hill followed a traditional teaching methodology. Students were tracked based on mathematical ability, and teachers used textbooks as the central form for curriculum decision. Classrooms usually were set up in rows with teachers providing the lessons and students practicing individually at their desks. In contrast, Phoenix Park students were not tracked but worked in mixed-ability groups. Textbooks were rarely used, and students were given long-term projects for their subjects. There is some initial discussion provided by the teacher, and throughout the project the teacher may offer suggested topics to consider but otherwise the students work interdependently on the problems. The study compared student attitude, classroom interaction, teacher attitude, and student learning outcome. The study has many points of interest that cannot all be included in this paper, but I will point to several items of interest.

Where Amber Hill classes were structured and usually quiet, Phoenix Park classes tended to be noisy, with some students discussing their projects and others not. Students at Amber Hill did not feel they could apply their mathematics to the outside world, whereas the Phoenix Park students felt that mathematics was naturally part of their surroundings. Motivation and interest at Phoenix Park was stronger than at Amber Hill, according to interviews with students. Retention of information and conceptual
understanding of their mathematics was greater for Phoenix Park students than Amber Hill. Students in England are given a national test at the end of their tenth grade. Not all students take the test; if the school believes the student will not pass the test, the student will not attempt the test. Phoenix Park had 94% of their students attempt the test, and Amber Hill had 84% attempt the test. A larger percent of Phoenix Park students received passing grades. Amber Hill had 16% of their students not receive passing scores, and Phoenix Park had 7% not receive passing scores. Given this is only one year of data, it is interesting that Phoenix Park had more students attempt and fewer fail the exam. The study raised questions about the preference of girls to work in the Phoenix Park setting where assignments were less competitive and projects had an emphasis on connection. The study seemed to indicate that girls’ attitudes about their own mathematics ability were stronger in Phoenix Park. Only 1% of the Phoenix Park girls stated they were bad at mathematics, compared to 13% of the Amber Hill girls (who were part of the higher track of students).

What is exciting about the study is the chance to look at students’ perceptions of their own knowledge and also differences in conceptual or procedural knowledge. The general criticism of reform is that students given projects are not given sufficient time to practice necessary procedures in mathematics. However, what the study seems to suggest is that students who participate in problem-based classrooms are as successful, if not more successful, than students participating in traditional classes. What were they successful at? Motivation was higher, affect was greater, interdependence was greater, understanding of the purpose of mathematics was greater, and test scores were better. If
these are things that are valued then, yes, the Phoenix Park students were more successful. If what is valued is control, quiet classrooms, or individual practice, then Amber Hill was more successful. Boaler (2002) stated that Phoenix Park recognized that to prepare their students for the test, they had to stop doing projects and work on procedural lessons during the last half of the tenth-grade year. There could be many interpretations of this, but these interpretations will depend on the values of the person reading the study. This could be a comment on how projects have shortcomings and must be supplemented or it could be a comment on what the tests are valuing. Fleener (2002), a postmodern philosopher with a background in teaching mathematics, pointed to the current concerns with how well our students are prepared to compete in the world today and how significant changes are required of the current structures. But we should also be considering other models beyond the control-driven structure of our current curriculums. But we should also be wary of the curriculum suggestions of the past and consider what is to be valued. The New Math of the 1950s and 1960s offered a window from which to view the future. Many perceived this research a failure. These studies attempted to create curriculums that built student conceptual understanding. So why did these ideas not continue? It was suggested by Davis (1990) that these new forms of curriculum that encouraged student conceptual understanding were more difficult to test. Given the primary focus of behaviorism both in teacher training and in the classrooms, this idea would be difficult to adapt to. Careful consideration should be given to the kinds of reform that are presented to our classrooms as there are many traditions and values in question here.
**Ratio and Proportion**

The study is not about building mathematical knowledge specifically but discussions of self-regulation are domain-specific, which means the content has importance as well. This study chose to use proportion and ratio problems for several reasons. The mathematics community recognizes several key topics in mathematics considered to be critical to understanding problem solving and higher-level mathematics. According to Lamon (2007), the research related to rational numbers has been limited in the last decade and requires intensive studies to begin understanding the challenging intricacies involved in learning about rational numbers. This study may provide insight into ways to begin studying rational numbers with college students in remedial mathematics courses, although Lamon (2007) sees the need for longitudinal studies beginning with students in grades three, four, and five. The information gained from the study may provide insight into directions for supporting instruction.

Some of the common terms used in proportional studies include unitizing, norming, and partitioning. Unitizing is the measurement assigned to a given quantity (Lamon, 1993). Unitizing is used by individuals to describe quantities in a variety of ways. Individuals who can create multiple ways to describe a quantity or have more flexibility of thinking with ratio have stronger proportional reasoning (Lamon, 2007). An example of unitizing would be the ability to see multiple ways to group twenty-four cans of soda. The groups could be two 12-packs or two 6-packs. The decision about which group is more efficient is then another layer of monitoring used by the student to understand the ratio. Partitioning is the equal division of a single object or a set of
objects (Lamon, 2007). Partitioning is considered the first necessary skill for young
students when introduced to fractions. Norming is the process of redefining a system to a
fixed unit (Freudenthal, 1983). The standardization of a measurement might be the scale
factor on a map. Freudenthal (1983) suggested the relationship between standardizing
the diameter of the earth’s diameter to the head of a pin. The measure of the head of a
pin is 1 mm. This allows me to consider the distance of the earth to the sun as 10 m and
the diameter of the sun to be 10 cm. The standardization process can be helpful when
making comparisons with numbers or measurements that may be challenging or too
abstract to understand. Understanding how students perceive these comments within the
study may provide insight into their learning process.

Connections

Research in SR evolved after the Pintrich and De Groot (1990) study that
suggested the need for domain-specific research. An example study by Seegers and
Boekaerts (1993) exemplified the use of the OMQ to investigate the interactions among
motivation, task attraction, affect, and effort. Variables like attribution style, goal
orientation or self-efficacy were suggested to influence how the individual feels about
their ability to do the task or personal relevance. This study was conducted with sixth
graders using four mathematics problems as the task.

The On-Line Motivation Questionnaire (OMQ) designed by Boekaerts
(2002) was developed to measure students’ understanding and affect about tasks at the
moment the task is presented. This measurement is useful but is somewhat limited.
Recognizing that each student’s prior experiences are different and that these experiences
are reflected in their choices, additional interview questions may provide insight into student perception that the OMQ cannot provide.
CHAPTER III

METHODOLOGY

This chapter outlines the methodology for this study with two major sections. The first section is a discussion and rationale for use of the qualitative research approach, and the second section is a discussion of the procedures for the study.

Traditional research designs generally are found in one of two camps; quantitative or qualitative. Quantitative studies follow a positivist paradigm and tend to have a singular focus allowing the researcher to make inferences about that one element (Morse, 2003). Because quantitative designs follow a scientific approach with attempts to limit variation as much as possible by including random samples and choosing appropriate sample groups, the studies often can be generalized to other groups. For example, a study by Fenema, Franke, Carpenter and Carey (1993) compared teacher perception of problem difficulty with student performance. The quantitative study had a strong research design and was replicated in other studies, giving the study’s generalizability added strength. The study indicated that teacher perception of difficulty did not always align with student performance (Fenema et al., 1993). The researchers suggested that teacher perception was based on the adult form of solving the problem. Teachers determined the difficulty of a problem based on the number or types of steps that adults would use to solve the problem even though students used alternative strategies with success. The strategies used by the students to solve the problems deemed difficult by teachers were not considered traditional methods. These findings suggest that teachers may impose their own understanding without considering alternative methods seen by students.
Qualitative studies tend to follow a naturalistic paradigm. Researchers are less removed from the study. Studies typically are not of randomly selected groups but rather studies of groups with a central theme. The researcher gathers data using observation or interview techniques and then codes the data which is statistically analyzed. Because the subjects are placed in a more naturalistic setting, the researcher triangulates the codes with a variety of sources and because the researcher is less removed, qualitative studies can provide a vivid image of the setting and the interactions of the individuals. One of the weaknesses of qualitative studies is the use of small groups, making generalizability difficult. The Manouchehri and Goodman (2000) study illustrated that understanding differences among teacher approaches involves classroom observation, documentation, interview, and teacher self-reporting. Mathematics education research currently is working within a constructivist paradigm, recognizing that both the individuals who are participating in mathematics class along with the mathematics itself are complex. Constructivism is the learning philosophy that individuals build up their own knowledge (von Glasersfeld, 1990). The trends in mathematics education research suggest a more naturalistic setting as a way of combating the traditional view of learning mathematics. Traditionally, mathematics is seen as a list of algorithms or rules that are to be drilled and memorized by students (Lesh & Carmona, 2003). The questions associated with the number of correct answers can be answered with a quantitative study. However, the view that mathematics is a process instead of a product is changing how researchers ask questions and how studies are therefore designed. Questions in mathematics education are related to both solution and to thought process. How and why students come to
understand and complete problems is as important as a correct solution. Stroup & Wilensky (2000) contend that formal large scale assessments are counter to constructivist learning because the large scale assessments do not consider process.

Case-study research provides a method of gaining insight into a given setting by allowing the researcher to ask a “how” and “why” question (Merriam, 1988). The case study design gives significance to who is part of the study rather than methodology (Stake, 2000). The case gives the reader an opportunity to experience what the researcher is experiencing through the descriptive narration (Stake, 2000). The process is highlighted rather than the product. Merriam (1988) suggests the case study focus on specific phenomenon. This study focuses on the moment that students are confronted with a challenging problem. The student perception of that problem may be connected to many things but the focus will be on how students self-regulate and determine mathematical strategies employed.

Studies relating to self-regulation primarily have been quantitative in nature. The studies have suggested several important findings. Studies have focused on specific elements connecting problem-solving instructional strategies with self-regulation strategies to support student learning (Perels et al., 2005). Studies have connected motivational beliefs positively with self-regulation strategies, as in Pintrich et al. (1994). Studies clearly suggest that self-regulation is a major contributor to student success in classrooms. Researchers like Boekaerts have conducted studies focusing task-specific learning because self-regulation theorists agree that domain-specific studies are important. Boekaerts (2002) extended this one step further and placed the self-regulation
and motivation on specific tasks or content. The task-specific measurement tool used in Boekaerts’ studies was the On-Line Motivation Questionnaire (OMQ) which provides connections such as correlations between task-perceived relevance and learning intention (Boekaerts, 2002). The significance to this study is that the OMQ provides quantifiable data but does not provide specific feedback to the instructor about what kinds of tasks might be perceived as relevant to students. For this reason this study will investigate student perception with both the OMQ and interviews. The interviews will assist the instructor to receive feedback from the students about their SR and mathematical strategies.

Goldin (2000) pose3 a framework for task-based interviews to explore mathematical behaviors. The framework uses qualitative interview methods to scientifically investigate how students interact with mathematics. Lesh, Lovitts, and Kelly (2000) stated that research in mathematics education should include an engaged researcher who sees students as complex and self-organizing individuals. Once students are perceived as complex individuals, the measurement tools also must match this complexity. Understanding the whole experience will enhance the information available to the instructor as a means of improving the classroom experience.

Knowing the retention rates for students in the developmental courses is important but it does not provide feedback to instructors about how to make necessary adjustments to improve the teaching and learning of mathematics. Data about the number of students who do not answer specific questions on exams also is available but, again, does not provide information about the reasons for the incorrect answer. This study
explored the relationship between student attribution and persistence along with student content knowledge to provide feedback to the instructor. It is possible that student feedback will show that students do not self-regulate or self-monitor their actions during the course. In this case, instructors can include elements to improve student self-regulation along with mathematics content.

Students participating in remedial mathematics courses in a college setting are assigned these classes as a result of placement tests such as the COMPASS or ACT exams. The courses are a combination of traditional instruction paired with on-line practice offering immediate feedback of understanding. There is an assumption that the information provided to the student is then used by the student for future efforts. If students do not do well on an assignment it is left to the student to seek support from the instructor or tutors. This presumption that students are strong self-regulators is part of the question within this study. How does self-regulation impact student’s use of mathematical strategies in a remedial math course? What indicators, if any, suggest that students are actively participating while attempting to complete the mathematics task? What criterion, if any, do students use while attempting the mathematics task? What mathematical strategies, if any, do students use to solve the mathematics task?

**Research Methodology**

This research study was a qualitative observational case study (Bogdan & Biklen, 2006). The focus of the study is the experience in a college-level remedial mathematics course. The large number of students who failed the first two remedial mathematics course was a concern for the mathematics department. Several reasons were suggested
for the high failure rate, so the intent of this study was to investigate some of the possibilities. The study investigated student perception of their ability to complete tasks at the moment a challenging task was presented to them. The initial phase of the data collection was the OMQ. The OMQ was chosen because it surveyed student perception just before and just after students were given the mathematical tasks. This data then was used to help describe the whole group who participated in the study and also to help select the interview participants.

The data collection consisted of four stages. During the first stage, the participants read the five mathematical tasks but were asked not to attempt the tasks. After reading each of the tasks, the participants were to answer “yes” or “no” to the Knowledge Monitoring Assessment (KMA) questions of confidence in their ability to do the task. The initial stage asked students to read each problem and answer “yes” or “no” if they felt confident in completing the task. During the second stage, the participants completed the appraisal portion of the OMQ. The 19 Likert-scale questions asked students to describe their perception of the five tasks they read during the first stage. The purpose of the second stage was to provide the researcher with an initial participant self-reporting of mood, self-efficacy and intended effort. During the third stage, the participants attempted the five mathematical tasks. The problems were chosen for several reasons. The OMQ recognizes the significance of domain-specific research within self-regulation. The topic of ratio and proportion was chosen because the topics are taught within the remedial courses, making the material relevant to the students and because the topic is central to understanding higher-order thinking in mathematics.
Lamon, 2007). The five tasks were used in prior proportion studies. There are two general types of proportion problems. One type is the comparison of two ratios; when all four parts of the proportion are known. The other type is a proportion where three parts are known and the fourth part is unknown. Two of the five tasks on the OMQ compare ratios; one of these two tasks was iconic while the other was symbolic. Another two problems were part unknown type of problems. One of these tasks was a lengthy word problem - 89 words to be exact. The fifth task included the rate, miles per hour. The task is considered a challenging type of proportion problem because of the non-integer solution. That is, the factor required to compare the two ratios is non-integral. Cramer and Post (1993) suggest these proportion problems are more challenging. These problems were chosen to illuminate types of mathematical challenges students in remedial mathematics courses may have. There was no time limit imposed during the third stage. During the fourth stage, the participants were given the attribution portion of the OMQ. This section included ten Likert-scale questions, where participants report their perception of how well they completed the tasks and how they feel about the experience. An additional question was included at the end of the questionnaire asking participants to volunteer for the interview portion of the study. Participants moved through the stages at an individual rate. There was no expectation that all individuals will complete the stages at the same time. The stages were not timed, but the questionnaire was given during the class time with the expectation that other work would be completed giving the participants a self-imposed time limit.
The KMA reflects how well students monitor their knowledge with an emphasis on accuracy between when they believe they know with what they know. When an individual is first presented with a challenging task, the individual makes a determination of his or her ability to complete the task successfully. This initial appraisal then is matched with the individual’s accuracy after completing the task. The coefficient is a number between -1 and +1. When the score is closer to -1, then this means the participants are not accurate knowledge monitors. When the score is closer to +1, the participants are accurate knowledge monitors. The KMA portion of the study combines the information from stage one, when participants answer “yes” or “no” to their confidence in completing the task, with stage three when students attempt the task. An answer of “yes” is a positive appraisal and an answer of no is a negative appraisal. This appraisal then is matched with the accuracy of the task in stage three. An accurate knowledge monitor would be a positive appraisal with an accurate solution or a negative appraisal with an inaccurate solution. Students believe they can answer the question and then do so, or students believe they cannot answer the question and then do not do so. An inaccurate knowledge monitor would have a positive appraisal with an inaccurate solutions or a negative appraisal with an accurate solution. The inaccurate knowledge monitor either over-estimates or under-estimates his ability. As a result, the participant may study too much or not enough.

The data from the OMQ Likert scales was grouped according to prior OMQ studies. Questions were grouped for subjective competence that included self-efficacy and success perception; task appraisal that included perceived relevance, task attraction
and subjective competence; and intended effort. The OMQ Likert data was not collected for comparison but rather for possible use for comparison with future studies with students in the remedial mathematics courses. Additionally, the OMQ data was examined for possible questions to be used during the semi-structured interview portion of the study.

The sub-group of six participants purposefully selected for the interviews was chosen based on the following criteria. First, the participant volunteered to be interviewed. Second, the participant did not complete at least one of the tasks during stage three of the OMQ. Third, the participant had a low knowledge monitoring score. Low Hamman coefficients indicated that the participant had difficulty determining his or her ability to be accurate on tasks. The semi-structured interviews were audio-taped and transcribed. A video was taken of each of the interviewed participants’ paper so the researcher could match the written work with the participants’ verbal description. All five participants were interviewed once but when asked to return for a second interview only four returned. Data from one of the two who did not return was not sufficient to be used in the study, so this participant’s data was not used. This decision was based on the number of tasks the participants discussed in the interviews. The data provided by the OMQ and KMA provide descriptive information but were not able to answer the question of “why” in many cases. Those students who did not complete or attempt some of the mathematics tasks did not describe their specific reasons for not completing or attempting the tasks. The semi-structured interviews were used to provide a detailed description of the student reasoning while working on challenging mathematics tasks. The questions
used for interviews followed a specific protocol, but the questions varied during the interview when specific moments of interest appeared.

The analysis of the interviews involved coding the conversation using open coding (Strauss & Corbin, 1990). The codes were based on Zimmerman’s cycle of self-regulation (2000), the assumptions used by self-regulation researchers (Pintrich, 2000), and proportional thinking (Lamon, 2007). The interview codes were analyzed by an Associate Professor of Mathematics using the same definitions then discussed to minimize bias by the researcher. The topic of the professor’s doctorate study was mathematical beliefs and problem solving.

**Validity and Reliability**

Validity is necessary for the researcher to make inferences about groups similar to the group studied (Creswell & Clark, 2007). The quantitative data provided by the OMQ was used as a resource for developing questions in the qualitative portion of the study. Threats to validity for a qualitative study include: participant selection, sample size, and choice of measurement tool (Creswell & Plano Clark, 2007). To counter the validity issues with this type of study, the study used a measurement tool with proven validity, chose more participants for the quantitative portion than the qualitative, and chose participants for the qualitative portion based on significant results. The subscales for the OMQ included subjective-competence, success expectation, self-efficacy, task attraction, task relevance, and learning intention had Cronbach’s alphas ranging from 0.72 to 0.86. All of the scales were considered satisfactory. The OMQ was validated through several studies. The validity studies have shown that the OMQ has scale scores that have normal
distribution, except for Emotional State 1 and Emotional State 2 (Boekerts, 2002). The emotional states were questions of how the individual felt just before beginning the tasks (Emotional State 1) and just after the tasks were attempted (Emotional State 2). Other classroom environment factors could have impacted these scores. Boekaerts (2002) wrote that the reason for the skew is a perception by the students that unless the question was an exam question the task would not produce an intense negative emotion.

Closing

One of the intentions of the study was to share the findings with instructors of the remedial courses as Pandiscio (2002) suggested that teachers would adjust teaching methods if given opportunities to question their beliefs. Boekaerts developed the OMQ and permission was given to use the instrument in this study. The OMQ will be used to determine the degree to which students self-regulated when solving challenging mathematics questions and will also be used to investigate possible attributional factors that lead students to not persist. The information provided by the study will be used to enhance instructional strategies in the remedial courses.
CHAPTER IV
DATA

This chapter will include five sections describing the data collected in several formats. The first section will discuss metacognition data within the Knowledge Monitoring Assessment (KMA). The second section will discuss the four assumptions associated with self-regulation (SR). The third section will include specific information from the On-Line Motivational Questionnaire (OMQ) including appraisal, mathematical task and attribution data. The fourth section will discuss findings from the interviews. The fifth section will discuss the data as they relate to the research questions. The participants in the study were students in five sections of remedial mathematics chosen for convenience. The 72 individuals who agreed to participate in the study all completed a majority of the OMQ that included mathematical tasks selected for this study. The tables in this chapter show different totals for the number of participants. As a result of varying completeness of questionnaires, 62 individuals completed the entire questionnaire, including the KMA and the three OMQ sections. Sixty-nine individuals completed the three OMQ sections but did not complete the KMA. Because the three OMQ sections and the knowledge monitoring appraisal data are used for different purposes, the completed total number of completed questionnaires was used for each data collection.

The theoretical framework for this study is centered on Boekaerts’ (2002) model of adaptable learning. Boekaerts suggested that the moment students are given a task they are utilizing information from their own content knowledge, their attitudes about the task
or their beliefs about prior experiences along with the environment (Boekaerts et al., 1995). That is, a student may complete a series of tasks because there is no threat to his or her wellbeing and the task falls within his or her cognitive understanding. Reasons for not completing a task range from a lack of content knowledge, prior experiences, or mood at that moment. All theorists in the field do not study self-regulation within specific moments, but there are four assumptions researchers agree upon. The four assumptions are: the learner is active, the learner can regulate, the learner has criteria to compare and the learner makes SR connections of personal with contextual (Pintrich, 2000). Zimmerman (2000) suggested that all individuals self-regulate but that some individuals self-regulate at a higher level or at a level that produces expected outcomes. This study can presume that all participants were self-regulating, but one might ask how a student’s SR impact work with mathematical tasks in a remedial mathematics course. When considering student perception about learning mathematics, there is a plethora of variables to consider. The research must consider the personal and contextual elements of learning. Recognition that an individual’s interest and motivation varies within a contextual domain is reflected through the use of specific tasks on the OMQ. At the same time, the OMQ recognizes that the personal context or the mood of an individual also may affect learning. The OMQ is a tool used to measure Boekaerts’ (1992) theoretical perspective of SR. The mathematical tasks are labeled A, B, C, D and E. The tasks are provided below. The problems were used in studies by Lamon (1993).
A. Ellen, Jim, and Steve bought 3 helium-filled balloons and paid $2.00 for all three. They decided to go back and get enough balloons for all the students in their class. How much did they have to pay for 24 balloons.

B. Use the pictures below to answer the following question: who gets more pizza, the girls or the boys? Explain your answer.

![Pizza Illustration]

C. In a certain town, the demand for apartments was analyzed, and it was determined that to meet the community’s needs builders would be required to build apartments in the following way: Every time they build 3 one-bedroom apartments, they should build 4 two-bedroom apartments, and 1 three-bedroom apartment. Suppose a builder is planning to build a large apartment complex containing between 35 and 45 apartments. Exactly how many apartments should the contractor build to meet this regulation? How many one-bedroom, two-bedroom, and three-bedroom apartments will the apartment building contain?

D. Compare the ratios 3:5 and 9:19. Which ratio is larger? Explain your answer.

E. Frank is buying a new car that gets 40 miles per gallon. His current vehicle gets 30 miles per gallon. If we presume that Frank drives 14,000 miles in the next
year and the average price for a gallon of gas is $2.75 – how much will Frank save if he buys the new car?

**Metacognition**

Information about student metacognition also was included within the OMQ. Current research in metacognition was used to investigate a more specific element of SR. This study includes the question of whether students in the remedial mathematics classes monitor their knowledge effectively or not. Knowledge monitoring is the ability of an individual to know what he knows (Tobias & Everson, 2009). Students who do not knowledge monitor well may spend either too much time studying material they already understand or possibly do not study at all because it is believed the material is already understood when it is not. The Hamman coefficient, suggested as the measurement for KMA, uses the four possible outcomes and provides a score ranging from -1 to +1 to indicate knowledge monitoring strength. To determine the four possible outcomes the individual is asked to read the tasks before attempting them and provide a “yes” or “no” response to the question of his or her confidence in ability to complete the task. Then the individual was asked to attempt the task. The individual’s appraisal of ability compared with the accuracy of the solution provides the four possibilities. The possibilities and notation are provided in Table 1.
Table 1

Knowledge Monitoring Possible Scores

<table>
<thead>
<tr>
<th>Appraisal</th>
<th>Accuracy</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can complete</td>
<td>Correct</td>
<td>(+, +)</td>
</tr>
<tr>
<td>I can complete</td>
<td>Incorrect</td>
<td>(+, –)</td>
</tr>
<tr>
<td>I cannot complete</td>
<td>Correct</td>
<td>(–, +)</td>
</tr>
<tr>
<td>I cannot complete</td>
<td>Incorrect</td>
<td>(–, –)</td>
</tr>
</tbody>
</table>

Individuals who match their belief in ability to complete the task with actual ability are accurate knowledge monitors and their Hamman score will be between 0 and +1. The (+, +) and (–, –) columns of the table indicate an accurate knowledge monitor. This is not a measure of how well the material is understood but rather is an indication that the individual understands his or her own ability to do the task. Individuals who do not match their appraisal with accuracy are weak knowledge monitors and will have a score of -1 to 0. This study includes five tasks the possible scores are listed below in Table 2.

Table 2

Knowledge Monitoring Percents (n = 62)

<table>
<thead>
<tr>
<th># of problems</th>
<th>Possible Hamman Score</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>8</td>
<td>12.9%</td>
</tr>
<tr>
<td>4</td>
<td>0.6</td>
<td>22</td>
<td>35.5%</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>18</td>
<td>29.0%</td>
</tr>
<tr>
<td>2</td>
<td>-0.2</td>
<td>10</td>
<td>16.1%</td>
</tr>
<tr>
<td>1</td>
<td>-0.6</td>
<td>3</td>
<td>4.8%</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>1.6%</td>
</tr>
</tbody>
</table>

The table shows approximately 48% of the individuals who completed the survey were able to monitor their ability at least 4 out of 5 times. The Hamman score of 1
indicates the individual provided appraisals for the five tasks prior to completing and the appraisals matched the accuracy for all five tasks. Of the eight individuals five of the individuals had positive appraisal that matched their five accurate answers. One individual had one positive and four negative appraisals and was accurate about all five tasks. This means the individual did not feel confident about four of the tasks and was not able to complete those four tasks accurately. The score of one does not indicate the individual was able to complete the tasks correctly and provide appropriate explanations when requested, only that the individual was aware of which tasks he or she was able to complete.

The KMA was used to determine whether the participants in the study were monitoring their knowledge well. Do students recognize their ability to complete a task accurately prior to beginning the task? Table 2 shows the participants’ individual Hamman scores and the frequency for each of the possible scores. Prior research studies did not use a cut-off value to determine whether individuals were monitoring well or not well. Because this study is related to a specific topic and content area, the determination for a cut-off was a means of informing the reader and the instructors in the courses. The table shows 48% of the participants were able to monitor their knowledge of the tasks at least 4 out of 5 times. This was considered acceptable by the researcher. The topic is familiar and instructed within the remedial courses. The researcher also would consider monitoring correctly 0, 1 or 2 times out of 5 is not acceptable. Those monitoring correctly 3 out of 5 were determined to have an unacceptable monitoring ability. This was determined by the researcher and based on the expectation of the courses. These are
questions the participants would be expected to solve. The inability to monitor suggests that students may not be using their resources in the most efficient way.

The monitoring score was used to determine whether participants would be interesting interview candidates. The Hamman scores for the interviewees were not all high, but this is expected because poor monitoring was one of the factors for choosing interview participants. The interviewees KMA score, or Hamman scores are shown below. Two of the interviewees had a Hamman coefficient of -0.2 indicating the participant monitored accurately 2 out of 5 times. One interviewee had a score of +0.2 indicating the participant monitored accurately 3 out of 5 times while the other two interviewees had Hamman coefficients of +0.6, indicating they monitored accurately 4 out of 5 times. Because several of the interviewees did not monitor well, the interviews were able to investigate reasons why participants show either over-confidence or lack of confidence in their ability. The interviewees were then asked to share their process for completing the tasks.

Table 3 showed the detailed information about each of the four possible KMA categories. The data on Table 3 showed that a majority of the individuals showed positive appraisal for the problems. Table 3 also suggested the participants felt they would be able to complete the tasks except in the case of problem C. Problem C was the only one of the five tasks that showed more negative appraisals.
Table 3

Knowledge Monitoring Totals among Complete Questionnaires (n = 62)

<table>
<thead>
<tr>
<th>Problem</th>
<th>Accurate (+, +)</th>
<th>Inaccurate (+, -) &amp; (-, -)</th>
<th>Accurate (+, +) &amp; (-, -)</th>
<th>Inaccurate (+, -) &amp; (-, +)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>44</td>
<td>46 (74%)</td>
<td>16 (26%)</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>43</td>
<td>45 (73%)</td>
<td>17 (27%)</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>17</td>
<td>42 (68%)</td>
<td>20 (32%)</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>28</td>
<td>36 (58%)</td>
<td>26 (42%)</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>28</td>
<td>36 (58%)</td>
<td>26 (42%)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>160</td>
<td>205 (66%)</td>
<td>105 (34%)</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 shows the relationship between positive appraisal and negative appraisal. All Hamman coefficient scores are positive but more students were able to monitor their knowledge with problems A and B. During interviews, Marla stated that problem A, “…seemed like normal every day math”. Problem B was completed using a variety of rational number strategies including partitioning and measurement. The visual component available in the problem may have helped students use contextualized information to answer the question rather than proportional reasoning. Students were not as positive about problem D as they were about problem B. Table 4 shows that students were also less accurate with problem D than with problem B. The mathematical tasks are provided at the beginning of the chapter and in Appendix A.
Table 4

*Appraisal and Accuracy of five tasks*

<table>
<thead>
<tr>
<th>Problem</th>
<th>Positive appraisal</th>
<th>Negative appraisal</th>
<th>Accuracy percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>97%</td>
<td>3%</td>
<td>74%</td>
</tr>
<tr>
<td>B</td>
<td>95%</td>
<td>5%</td>
<td>71%</td>
</tr>
<tr>
<td>C</td>
<td>45%</td>
<td>55%</td>
<td>44%</td>
</tr>
<tr>
<td>D</td>
<td>77%</td>
<td>23%</td>
<td>54%</td>
</tr>
<tr>
<td>E</td>
<td>82%</td>
<td>18%</td>
<td>47%</td>
</tr>
</tbody>
</table>

The KMA appraisals are generally positive (except for problem C). Students being interviewed would continue working through problems even after stating a lack of confidence or a sense of how to begin. When I asked Ken about question B, he responded: “When I look at this…um…I don’t see how to equally make that work in my mind. Because if I third it here I have 9 pieces I still have 2 left over so that each girl gets one with a remainder with 2 pieces left over. That’s an equality thing to these three. But there are 2 times as many girls as there are boys. So. I wasn’t sure, and I can’t remember if I did answer this question. And I can’t remember what I was thinking or what I did.” Ken’s initial response was confusion and a lack of confidence. When asked what he would do today, Ken responded with the monitoring and reflection portion of his self-regulation cycle. “Today, it’s… I can’t figure out how to equally blend 7 into 3. And the other is taking these 3 pieces and coming up with 21 and giving each girl 3 pieces is the other thought process. But that seems complicated to me.”
He has a process, but it seems complicated and possibly not worth completing; however, he continues because he wants to be successful and wants to show what he knows. Consistently students will make attempts at problems to show they are trying to understand. There are times when students leave items blank or leave pictures or verbal cues behind that indicate confusion or lack of motivation to complete the task. There is no way to determine the reason for the lack of effort unless it is possible to discuss this with the student individually.

The problem in Figure 1 suggests the student might have a possible response. The reason for making the effort to tell the instructor the problem could be completed while not making an effort to actually begin is curious. The interview might have given the student the chance to explain the thought process.

Figure 1. Student effort

The question mark in Figure 2 could be a question of a strategy or a question of the meaning of key terms in the problem. The student made the effort to leave a symbolic statement, but additional information would be helpful to determine his thinking.
The response in Figure 3 provides some insight into the reason for the lack of attempt. What is interesting is the student understands the symbols as ratio but offers no detail for the instructor to determine whether this is really the case. Why leave the problem completely blank? Is the student preserving well-being by not providing a solution that is incorrect or does the student have absolutely no contextual connection to be made?

Interest in student feedback is a central part of the study. One of the shortcomings of the study is the request for students to volunteer to be interviewed on the day the OMQ was given. A majority of the individuals who volunteered to be interviewed did not leave blanks or similar statements on their questionnaires. With no information from the
student, there is no way to determine reasons and ultimately ways to support the students in the future. The interviews with students who left blanks would have allowed the students an opportunity to share their thinking. As shown in the following interaction, Dale was uncertain about the interpretation of problem A. He left the problem blank even though his proportional thinking was strong.

Teacher (T): So if …I mean if…Do you feel like there’s anything…I mean do have absolutely no idea or do you have some ideas and not sure about putting them down?

Dale: Well I’m not sure about putting them down. All right. OK, this one. If I take time I can answer it because they paid 3 dollars. They paid 20 dollars for 3 and they got 24 students in the class, so to guess balloons for all these students. They do not tell me how many balloons each student is getting. That is one of the things that was giving me problems because I don’t know how many balloons one student is getting, so I would just say 3 times 24 times 2.

T: Would you write that down. Write you’re mathematical process for me.

Dale: I would say 3 times 24. 24 times 3 is 72 then I would say 72 times 2 (Typing on the calculator).

During the interviews, another student, Ken, left a problem blank and was not able to make any connections to prior knowledge that would help him complete the problem.

Ken: Frank is looking at a car that gets 40 miles to the gallon. His current vehicle gets 30 miles per gallon. If we presume that Frank drives 14,000 miles in the next
year and the average price of gallon of gas is $2.75. How much will Frank save if he buys the new car? OK. He’s driving 14000 miles in a year. I don’t know that I read this. And right now I’m not sure how to what a plan of attack would be.

T: Any gut reaction?

Ken: Um no because I’m not sure where to go. I mean he’s buying another car obviously to save gas and I don’t know if this number relates to this number or this number. And the price to me is irrelevant. It’s just getting to the proportion you want.

T: You’re not sure how to set that up?

Ken: No

Both individuals left the problem blank but for very different reasons. The reasons for the blanks might provide insight for future instruction. Future studies should consider leaving the possibility of inviting these individuals on a more personal level. There should be opportunities to find out information about individuals who leave blanks, question marks, or statements about the students’ lack of work.

**Assumptions Related to Self-Regulation**

**First Assumption**

This study is investigating SR within a remedial mathematics course prompting the need to observe four assumptions made by researchers of SR. The assumptions were observed in the following ways. The first assumption states that students are active (Pintrich, 2000). To determine whether the learner is active during the task the participant is expected to demonstrate some form of mathematical reasoning. An
example of a participant’s reasoning can be seen in the work for problem B in Figure 4. The reasoning is evident in the drawings, symbols, and words. Any of these forms of representation are evidence of reasoning. The accuracy of the solution does not determine whether the participant actively pursues the solution rather, demonstration by the participant that he or she is pursuing the solution is considered active in SR.

![Figure 4. Active Self-Regulation](image)

Figure 4 illustrates the active pursuit of the solution that the boys have more pizza, even though the representations suggest the girls have more pizza. This problem offers additional reasoning for the use of interviews. When the representation within the problem suggests correct process and reasoning but an incorrect solution is obtained, there is a question of what the student was thinking.

Figure 5 shows reasoning through connection with current material. The content of the specific course for this student involved simultaneous equations, so the strategy was readily available to the student because it had been highlighted during the course. Again, an interview with the individual might highlight student thinking about variable use and meaning.
E. Frank is buying a new car that gets 40 miles per gallon. His current vehicle gets 30 miles per gallon. If we presume that Frank drives 14,000 miles in the next year and the average price for a gallon of gas is $2.75 – how much will Frank save if he buys the new car?

\[
\begin{align*}
40x + 30y &= 14,000 \\
x + y &= 2.75 \\
110 - 40y + 30y &= 14,000 \\
10x - 10y &= 14,000 \\
\frac{-110}{10} &= 13,000 \\
\frac{13,000}{10} &= 1,300
\end{align*}
\]

Figure 5. Active Self-Regulation

Figure 6 shows that reasoning may occur but that reflection about reasonableness of the answer does not. The mathematical error that occurs in the division of 1400 and 4 could be a result of forgetting that an adjustment to place value was used. The individual may have reflected on the reasonableness and added the additional 0 to 350 thinking that 3500 was more likely. This individual was not interviewed so the reason for the error remains unknown.

Figure 7 shows that students are willing to share more about their thinking than the mathematics. Reasoning can occur through explanation of answers or of possible
strategies or, as in this case, confusion with the meaning of the question. The explanation in Figure 7 also highlights that the student used criterion for comparison. This student is comparing this solution with possible students in the class. This is also a reflection of the comparison of their answer with solutions the teacher may be looking for.

Figure 7. Active Self-Regulation

Second Assumption

The second assumption states that individuals regulate (Pintrich, 2000). The presumption is that all individuals have the potential to monitor, control, and regulate. This study uses the specific framework from Zimmerman’s cycle of self-regulation (Zimmerman, 2000) to identify moments of regulation during the interviews. Tables 5 through 9 list a sequencing of goals, monitoring and reflection statements for each individual while discussing each of the five tasks on the OMQ. The tables reflect Zimmerman’s (2000) cyclical model of SR
Table 5

*Regulation During Interview for Problem A:*

<table>
<thead>
<tr>
<th></th>
<th>Goal</th>
<th>Monitor</th>
<th>Reflect</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ken</td>
<td>Well three of them cost $2. Another 8 batches of 3 would cost them 16 dollars</td>
<td>8 times $2 equals 16 at least.</td>
<td>I can take that number and divide it by the total number and come up with 8 times 2 to come up with 16.</td>
<td></td>
</tr>
<tr>
<td>Kevin</td>
<td>Well they paid $2 for three balloons.</td>
<td>So I divided 2 by 3 to get the price per balloon. Then they asked what it would cost for 24 of these balloons so I multiplied the .66 or the .67</td>
<td>I rounded up by 24 and that’s how I got my final answer.</td>
<td>Norming</td>
</tr>
<tr>
<td>Dale</td>
<td>When I look at this I think I’m going to have to set up the problem. If I can’t set up the problem then I can’t work it. If I take time I can answer it because they paid $3, they paid $20 for 3 and they got 24 students in the class. So to guess balloons for all these students. They do not tell me how many balloons each student is getting. That is one of the things that was giving me</td>
<td>I would say 3 times 24. 24 times 3 is 72 then I would say 72 times 2 (typing on the calculator). So 72… so I say that is how much they are going to spend.</td>
<td>So what I would do is I would divide 72 by 24 by my own idea then each would get 3.</td>
<td></td>
</tr>
</tbody>
</table>
problems because I don’t know how many balloons one student is getting. So I would just say 3 times 24 times 2.

Sarah
So these three kids bought balloons and paid $2. Uh, also how much did they have to pay for 24 balloons?

So I did like a cross product. So 3 balloons cost $2 should be equal to 24 balloons and we don’t know how much. And when we multiply 3x and 48. So 3x = 48 so I will find out that x = $16.

Cross-product algorithm

Marla
You have 3 balloons and they paid $2 for all three of them but there are 24 balloons.

I just divide that by 3 which is 8 and times 2 to get $16.

I feel pretty confident. Just seemed like an easy problem

Table 6

Regulation During Interview for Problem B:

<table>
<thead>
<tr>
<th>Goal</th>
<th>Reflect</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ken</td>
<td>7 girls and 3 boys and the boys each get one third of the</td>
<td>But there are 2 times as many girls as there are boys. So.</td>
</tr>
<tr>
<td></td>
<td>Because if I third it here, I have 9 pieces. I still have</td>
<td></td>
</tr>
</tbody>
</table>

Problem B self-regulation

Use the pictures below to answer the following question: Who gets more pizza, the girls or the boys? Explain your answer. (Lamon, 2007)
<table>
<thead>
<tr>
<th></th>
<th>pizza. At least that’s the way I look at it. The girls – counts to 7. When I look at this...um...I don’t see how to equally make that work in my mind.</th>
<th>2 left over so that each girl gets one with a remainder with 2 pieces left over. That’s an equality thing to these three.</th>
<th>I wasn’t sure.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ken 2nd</td>
<td>I can’t figure out how to equally blend 7 into 3. And the other is taking these 3 pieces and coming up with 21 and giving each girl 3 pieces is the other thought process. But that seems complicated to me.</td>
<td>Complicated in that you take each circle and cut it 7 times each.</td>
<td>So, um, my thought...and that’s the only logical solution so that each of them gets the equal third. Cuz they’re actually getting a third of everything.</td>
</tr>
<tr>
<td>Kevin</td>
<td>Visually I saw a fraction</td>
<td>There are 3 pizzas to 7 girls and 1 pizza to 3 boys. So I did that and found a decimal form that divided the 7 into the 3 and the 3 into the 1.</td>
<td>T: So you are comparing the decimals? Kevin: Yeah</td>
</tr>
<tr>
<td>Dale</td>
<td>The got, 1, 2, 3, 4, 5, 6, 7. 7 girls and they have 3. If I take this and go 1, 2, 3 and take this and go 1.2 3. I still got one big one left. So that’s only for 1 person.</td>
<td>So they can carry half, half, half, half, and this for this group here they only got 1.</td>
<td>So there’s no little bit half to give to them but I would say that girls would get more.</td>
</tr>
<tr>
<td>Sarah</td>
<td>So, I would say that I would take 3 pizzas and divide by 7 girls. For the boys, I divide 1 by</td>
<td>I get four tenths and ... and get three tenths.</td>
<td>So I would say that the girls get more pizza because they get four tenth for each girl and .4 is</td>
</tr>
</tbody>
</table>
Table 7

*Regulation During Interview for Problem C:*

<table>
<thead>
<tr>
<th><strong>Problem C self-regulation</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>In a certain town, the demand for apartments was analyzed, and it was determined that to meet the community’s needs builders would be required to build apartments in the following way: Every time they build 3 one-bedroom apartments, they should build 4 two-bedroom apartments, and 1 three-bedroom apartment. Suppose a builder is planning to build a large apartment complex containing between 35 and 45 apartments. Exactly how many apartments should the contractor build to meet this regulation? How many one-bedroom, two-bedroom, and three-bedroom apartments will the apartment building contain? (Lamon, 2007)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Goal</strong></th>
<th><strong>Monitor</strong></th>
<th><strong>Reflect</strong></th>
<th><strong>Comment</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ken</td>
<td>How many one-bedroom, two-bedroom, and three-bedroom apartments and I was thoroughly confused. So they’re both odd numbers. 35 and 45 are odd…45 is</td>
<td>To combine them totally saying it was one building instead of how it’s written it sounds like there’s two buildings so I came up with 80 apartments total and for every 3</td>
<td>T: Your confidence level with this one Ken: About 50-50. Because I don’t feel confident in my algebraic skills to break it down with an off the top-of-my-head algebraic problem. Teacher prompting Misinterpreting the problem</td>
</tr>
</tbody>
</table>
divisible by 3 and 9…35 isn’t…splitting them you have a fractional piece. I just guessed.

one-bedroom there should be 4…so I took the larger of the two and said that if there were 40 which is half of this (pointing to 80)…and then took the remaining half and said that there was probably another 10 for two bedroom and then the balance was 3 bedroom. So there were 30.

Kevin I scribbled out 2 here and flipping back and forth on what number I needed to end up with. From there I was going to break it out, so here’s 35 and 45 so to me the middle is 40. Now how do I get that for every 3 one-bedroom and 4 two-bedroom and I kind of wrote it that way,

The problem said 3 one-bedroom apartments. Every time they did 3 of these they had to build 4 two-bedroom apts. So I built a scale or a ladder there and then and one bedroom for every three they built. So now that I’m reading that – it might be different. I kind of built this and then the problem asked for a number between 35 and 45…and 45 is kind of right in the middle of those so I shot for 40. My first time through I got a 7 and I thinking well 42 because you

T: The 40 appeared because it’s between 35 and 45?
Kevin: Yes – that was to me between…to choose something in between.
T: So why not choose 42?
Kevin: It was just over the half way mark
can divide 7 into that and maybe you can come up with some numbers for that. So I said no, 42 isn’t in the middle so I chose 40. Then what I was going to do was take whatever number I came up with these three numbers into that 40 and somehow get the numbers out. So for every 3 one bedroom, you had to build 4 two-bedroom and 1 three-bedroom. 40 is in between 35 and 45. So 8. Now I can’t remember my thought process. So 8 divides into 40…5.

Sarah: I know, like, there should be some kind of formula. See I’m better with numbers. I don’t really like word problems. I started out counting, started feeling out. I didn’t know what I was doing. I was just thinking crazy. So it’s definitely a ratio so 3 to 4 to 1 and you have to I can do this like an unknown number like 3x, 4x and 1x and just add them so that would be like 8x. For some reason I think this number is because it can be divided by 8. So x = 5, 3 times 5… 15 one-bedroom, 4 times 5…20 two-bedroom and 1 I probably shouldn’t just pick the number just 40 because it can be divided by 8. For me it’s the only number that makes sense.

Proportional reasoning
| Marla                        | Something needed to be x, so I felt like taking that... so x would be equal to the three-bedroom apartment, yeah | I added them all up – I don’t know why I have two different numbers. Oh yeah, I did that with the 35 apartments and then the 45 apartments. Then I added those together, then I subtracted those things which gave me 11. Again. I don’t know if that’s right. | T: So you’re saying there are 11 three-bedroom apartments, there are 15 two-bedroom and 14 one-bedroom. Marla: yes T: And that’s because there are 4 more two-bedrooms than three-bedroom Marla: Oh it would be less than that wouldn’t it T: What do you mean by that? Marla: Well if you have 1 but then 4 minus 1 would be 3, 3 minus 1 would be 2. Am I just totally off the radar? |
| Dale                        | No data because Dale did not return for second interview | times 5... 5 three-bedroom. | Additive rather than multiplicative |
**Problem D self-regulation**

Compare the ratios 3: 5 and 9:19. Which ratio is larger? Explain your answer. (Lamon, 2007)

<table>
<thead>
<tr>
<th>Name</th>
<th>Goal</th>
<th>Monitor</th>
<th>Reflect</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ken</td>
<td>I’m really lost in ratioing. I’m thinking you’re looking…you’re comparing fractions.</td>
<td>Ken: I mean I’m just trying to take a circle and divide it so it is 5 equal pieces. I don’t know. Then shading those pieces versus a circle that is 19 pieces and then just taking 9 of those.</td>
<td>T: Do you feel there is any other process that might make you more confident? Ken: Not right at the moment…I can’t come up with anything that has been shown to me in math class that even brings that to light.</td>
<td>Teacher prompting</td>
</tr>
<tr>
<td>Kevin</td>
<td>Still trying to get a ratio. And I still ended up with fractions.</td>
<td>Over half and under half. I wrote it out a little different.</td>
<td>But I approached the same way.</td>
<td>Comparison to common fraction (1/2)</td>
</tr>
<tr>
<td>Dale</td>
<td>I didn’t understand, so I just guess. I look at this 3, 5 and 9, 19. I actually look at this. I don’t know why they have this stuff in the middle. (the : symbol)</td>
<td>But I just decide that because 3 and 5 right here and 9 and 19 are bigger than 3 and 5.</td>
<td>So this is how I came up with my answer.</td>
<td>Additive reasoning</td>
</tr>
<tr>
<td>Sarah</td>
<td>All the ratios are given in both.</td>
<td>3 to 5 and 9 over 19…I just divide it and I see what it come out to be</td>
<td>So 6/10 is bigger than 4/10.</td>
<td>Measurement</td>
</tr>
<tr>
<td>Marla</td>
<td>Which ratio is larger? I’ll just do that the same way I did the other one.</td>
<td>Let’s see, 1.6 and 2.11 so the 9 to 19 is bigger. I divided the 3 into the 5 and the 9</td>
<td>How do you know which number do you need to divide into what? But another way you</td>
<td>Inverted ratio</td>
</tr>
</tbody>
</table>
Marla continued: Maybe if you can find a number that the 5 and the 19 go into together or whatever and just multiply that. To make this 20 you have to multiply by 4 which is 12 which is bigger than 9 which means that your outcome here (3:5) is bigger than your outcome here (9:19).

T: Any of those ring truer for you
Marla: The first one
T: Why?
Marla: If you take the dots (referring to colon) so you’d say 9 out of 19 that would be your main number.

Table 9

*Regulation During Interview for Problem E:*

<table>
<thead>
<tr>
<th>Problem E self-regulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frank is buying a new car that gets 40 miles per gallon. His current vehicle gets 30 miles per gallon. If we presume that Frank drives 14,000 miles in the next year and the average price for a gallon of gas is $2.75 – how much will Frank save if he buys the new car? (Lamon, 2007)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Goal</th>
<th>Monitor</th>
<th>Reflect</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ken</td>
<td>And right now I’m not sure how to what a plan of attack would be. I mean he’s buying another car obviously to save gas and I don’t know if this number relates to this number or this number. And the price to me is irrelevant. It’s just No work</td>
<td>No work</td>
<td>No work</td>
</tr>
<tr>
<td>Kevin</td>
<td>Well..the new car gets 30 miles to the gallon his current car gets 40 miles to the gallon. I would work with both of those first, cuz you gotta find a difference.</td>
<td>14000 miles…so I would probably take the 40 divide it by 14000 to find how many, no, how many times 40 mpg fits in and that would give me the number of gallons of gas he uses so then I would divide the 14000 and the 40 then the 30 and the 14000 and that would give me the number of gallons he is actually using up or consuming. Then I would use the $2.75 times the number of gallons that he’s using per car. I would take those two answers and subtract them and that is how much he saves.</td>
<td>‘The more I thought about it I was a little more comfortable with it.</td>
</tr>
<tr>
<td>Dale</td>
<td>I took it twice now there are these ones on the test. I didn’t even try. If I can set up the problem then I can do the problem.</td>
<td>Because if you using one gallon per mile he’s gonna drive 14000 miles then I’m gonna multiply 14000 plus the one gallon times 2.75 per gallon and he’s gonna be driving 14000 miles for the whole year. So</td>
<td>T: So that’s how much he’s going to spend $38500 Dale: Yes T: OK. Does that seem like a lot of money? Dale: Something is wrong here T: Does it seem like a lot of money?</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Concern over miles per gallon rate</td>
</tr>
<tr>
<td>Name</td>
<td>Comment</td>
<td>Calculation</td>
<td>Result</td>
</tr>
<tr>
<td>-------</td>
<td>-------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Marla</td>
<td>I don’t know where to start – too many numbers. I’m really gonna have to do the math for both sides</td>
<td>If that one gets 40 mpg then multiply by 10 to get 400. Then 300 over here. 14000 divided by 400. that would be 35 tanks of gas. 14000 divided by 300 is 46.67. If the tank is 27.50 then I multiply that by 35 so that’s $962.50. then I subtract to get $320.92.</td>
<td></td>
</tr>
<tr>
<td>Sarah</td>
<td>We need to make two ratios for the new car and the current car.</td>
<td>So 2.75 over [pause] x then 30 mpg over 14000. So it’s going to be 30x = 38500. X is 1283.33</td>
<td>I’m not sure I set up the proportion right.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Third Assumption**

The third assumption of self-regulation states that individuals have criteria for comparing their ability (Pintrich, 2000). Participants use criteria in different ways. Comparisons are made to other approaches, to reasonable solutions, to the work and to the ability of other students or teachers, as well as to their own prior experience. When determining criteria in this study, an external reviewer provided a triangulation for comparison. The criteria used in the table are determined by reading the talk-aloud portion of the mathematical tasks. Criteria discussed outside of the mathematical task are
not included in the table as the criteria may have been a prompt from the interviewer. The talk-aloud portion of the problems were coded for possible criteria then compared with the external reviewer for agreement. The four criteria discussed in this study are: the comparison to the teacher’s process or answer (teacher); the comparison to their own process or answer (rhetorical); the comparison to prior experience or example (experience); and the lack of a comparison (non-specific criteria). All students who were interviewed expressed criteria with prior experience or prior mathematical examples. This experience ranged from general experience in other mathematics courses or with specific mathematical content. The example is an interaction with Ken while discussing Problem A.

T: Problem A you felt more confident because you could tell me another method.

Ken: Right. I was very confident of what I looked at. It was basic math, it wasn’t algebra. This is algebra. I’m uncertain how to check this to see who got more.

Four of the six interviewees asked questions during the interview reflecting an interest in knowing whether their thinking or process was correct. This criteria was labeled rhetorical because the questions were meant rhetorically during the interview. An example is shown with Marla during her explanation of problem B.

T: What do you mean by that?

Marla: Well if you have 1, but then 4 minus 1 would be 3. Three minus 1 would be 2. Am I totally off the radar?

Participants commented on the lack of criteria during reflections. This was seen when students were making guesses during the process. The comments could have been
noise to fill the air at the time but also could indicate a lack of mathematical strategy so the comments were of interest to the researcher. When Marla was discussing the appropriate method for solving problem D, she considered inverting the ratios. In the process, she discovered the solution was different depending on how the ratios were inverted. Her uncertainty led to the following interaction that illustrates non-specific criteria.

T: …What are you thinking?
Marla: I don’t even know.
T: Say out loud what doesn’t make sense.
Marla: How do you know which number you need to divide into what?

While reflecting on their process for the mathematical tasks the interviewees commented on the correctness of their answer. This is slightly different from the teacher as criteria. Questions were not directed at the teacher or interviewer but rather a rhetorical question was posed during reflection phase. An example from Dale is provided.

T: You multiplied 14,000 times 2.75?
Dale: Because if you are using one gallon per mile he’s gonna drive 14,000 miles.

 Then I’m gonna multiply 14,000 plus the one gallon times 2.75 per gallon and he’s gonna be driving 14,000 miles for the whole year so that’s why I just multiply this times this. But I don’t know if that’s right.
Table 10 shows the frequency of each type of criteria. The four criteria discussed in this study are: the comparison to the teacher’s process or answer (teacher); the comparison to their own process or answer (rhetorical); the comparison to prior experience or example (experience); and the lack of a comparison (non-specific). Table 10 shows a relatively equal distribution of types of criteria used by the students who were interviewed.

Table 10

*Types of criteria used by individuals during interview*

<table>
<thead>
<tr>
<th></th>
<th>Past Experience</th>
<th>Teacher</th>
<th>Rhetorical (Self comparison)</th>
<th>Non-Specific Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marla</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Dale</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Sarah</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Kevin</td>
<td></td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Ken</td>
<td>2</td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>11</strong></td>
<td><strong>8</strong></td>
<td><strong>10</strong></td>
<td><strong>5</strong></td>
</tr>
</tbody>
</table>

**Fourth Assumption**

The fourth assumption of self-regulation is that individuals “regulate their cognition, motivation, and behavior that mediate the relationships between the person, context, and eventual achievement” (Pintrich, 2000, p 453). When students described their thought process during the interviews participants were encouraged to share anything that entered their minds while working on the process. The participants’ descriptions centered on the mathematical process but also included experiences or
context from their own lives that helped the individual make sense of the problem. Marla compared her process for comparing the ratios in problem B with grocery shopping.

T: Any particular reason you know your answer is right?

Marla: …Again, maybe it comes from trying to decide when you’re at the grocery store what’s a better deal. Do you go for the 32 or do you get 2 of the 16?

T: So you relate it to something else?

Marla: Yeah.

All participants who were interviewed made comments relating to all four assumptions. Some participants referenced one assumption more than another, but the framework for self-regulation was evident during the interviews. Each of the assumptions was discussed above with tables to illustrate examples of each. The investigation provided opportunities for the interviewees to share elements from these assumptions through their original survey, their interview comments, and from work completed during the interview time. Each of the interviewees made connections to context, their past experience, and to content. The conversations during interviews illuminated more of the internal conversation students have while thinking about their mathematics. The comments during the interviews clarified some of the written work students used to answer the tasks on the OMQ.

**On-Line Motivational Questionnaire**

Boekaerts’ (1992) model for adaptable learners considers the complexity of motivation, interest, and preservation of well-being while students are completing tasks in a given domain. This study is interested in the individual’s perception of affect and
attribution while attempting mathematical tasks. Boekaerts’ (1992) model of adaptive learning measures domain-specific variables such as goal orientation and perception of ability. The model also measures the task-specific measures (Perceived Relevance, Perceived Task Attraction, and Subjective Competence). The domain-specific variables influence the task-specific variables which in turn determine effort (Seegers, Van Putten, & De Brabander, 2002). Since the study is not a comparative study, the data in Table 11 is available for future research and for a general picture of this group of students.

Table 11

Whole Group Appraisal Means

<table>
<thead>
<tr>
<th>Appraisal</th>
<th>Whole group mean (n = 69)</th>
<th>Interview group mean (n = 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective Competence</td>
<td>2.66</td>
<td>2.58</td>
</tr>
<tr>
<td>Self-efficacy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Success perception</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task Appraisal</td>
<td>2.66</td>
<td>2.70</td>
</tr>
<tr>
<td>Perceived Relevance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Task Attraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subjective Competence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intended Effort</td>
<td>3.02</td>
<td>3.42</td>
</tr>
</tbody>
</table>

There were three sections of data from the OMQ. The first section measured student appraisal of the task prior to beginning the task. The second section asked the students to attempt the tasks. The third section measured attributions of the students. Prior studies used data from the OMQ to compare groups, but this study was not comparative. The OMQ data were to be used to inform the instructors if items measured by the OMQ were unusual. The data was also used to support the qualitative portion of
the study. The information informed the interview protocol and identified factors for choosing participants for interview.

The data from the OMQ were to inform instructors by highlighting items from the OMQ with unusually high or low means. The means for the whole group tended to stay around the expected mean of 2.5. The appraisals measured items like self-efficacy and task attraction to determine whether students were confident in their ability and whether the problems seemed relevant enough to put effort into. The participants generally were confident in their ability to do the tasks, found the tasks relevant and planned to put effort into the tasks. The group, as a whole had scores around the expected mean with no unusually high or low scores. This prompted the need for a more specific measurement tool to look at elements within self-regulation, such as goal setting or help seeking in future studies. The OMQ also measured attribution. This information could inform the instructors about the reasons for perceived success or failure. Again, the scores tended toward the theoretical mean. The intention was to determine whether students would place the cause for their success or failure on elements like luck or on attributions where the student has no sense of control. The results suggested the opposite. The participants did not feel the cause for failure was luck. The participants seemed to suggest the problems were difficult and there was no experience with these types of problems. This could be interpreted to mean that the participants were not shown how to do these exact problems. Again, the lack of unusual data was a limitation of the study. A more specific measurement tool in addition to the OMQ should provide a stronger picture for future studies.
The participants who were interviewed also shared similar data as a sub-group. They were confident in their ability to do the tasks, found the tasks relevant and planned to put effort into the tasks. The attributions were not related to luck but rather suggested the tasks were difficult and they could have tried harder. The interviewees all shared one common element. At the end of the attribution section, the participants were to make a choice about whether they did well or not and then answer questions specific to their choice. All of the interviewees answered questions based on their choice that they had not done well on the tasks. All of the interviewees were not confident of their solutions at the end of the OMQ.

The information provided by the OMQ was used to determine questions for the interview protocol and to choose factors for the interviewees. The protocol asked students to describe their work on the mathematical tasks but additional questions were used in the protocol based on the general feedback of the OMQ. The protocol included questions about help-seeking, feelings about mathematics class, and specific learning tools like calculator use. This information did not inform the research questions but could inform the instructors for the courses. The interviewees were chosen based on persistence, monitoring, and by their responses on the OMQ tasks. The quantitative information allowed the researcher to find participants who might offer insight into the student experience.

The interviewees all believed they were not good mathematics students in high school but that their experience in college was more positive. There was no general reason for the improved experience, only that the material did not seem as difficult as
they remembered. The interviewees also discussed help-seeking. All of the participants stated that at some point, they had asked the teacher for help. Only one interviewee sought support through the tutoring center. Four of the interviewees looked for similar problems in the textbook or their notes prior to asking for help. One interviewee found Web sites with videos of teachers showing similar problems. All of the interviewees responded that having access to a calculator was not necessary or in some cases the calculator caused more confusion. This is a question that requires follow up because only one interviewee would be considered a traditionally aged college student. The other interviewees may not have had access to calculators while in high school.

Most SR studies use multiple tools because of the number of factors involved. Examples of measurement tools used in collaboration with the OMQ or MSLQ include The Goal Orientation Questionnaire (GOQ) developed by Seegers et al., (2002). The questionnaire has questions specific to the experience in a mathematics classroom. The GOQ also is used with the Self-Concept of Mathematics Ability Questionnaire developed by Bong and Skaalvik, (2003). This questionnaire measures cognitive aspects but developed to work with the GOQ.

Affect

Boekaerts (1995) suggested that the moment students are given a task they are utilizing information from their own content knowledge, their attitudes about the task or their beliefs about prior experiences along with the environment. The model of adaptable learning theorizes the interaction of these elements when individuals are presented with a task. The appraisals from individuals shape their actions. An individual who reads the
task and recognizes the task or can relate the task to prior experiences will have positive appraisals of the situation. The individual will consider the three components. The content is familiar and therefore possible to complete. If the individual believes the task is interesting and relevant and at the same time the individual does not feel threatened by attempting the task, then more effort is given to the task. Prior studies have suggested that interest in the task along with the self-perception of ability to complete the task determine willingness to exert resources to completing the task (Boekaerts, 1995).

Boekaerts (1995) also suggests that individuals will avoid doing tasks when they are afraid of losing a sense of well-being. By attributing a lack of effort to low cognitive ability or to an external cause such as the noise in the room, the student is released from feeling they are responsible for learning the material. In other words, students want to stay at a comfortable and balanced position in the classroom. The student wants to learn but is more willing to take on challenging tasks when the task is related to prior knowledge and the individual can make connections to prior positive experiences (Boekaerts, 1995).

Comments from the individuals who were interviewed showed the affective and attributional elements that Boekaerts (1995) refers to in her model. Marla expressed her feelings when asked about her confidence. Marla stated, “It’s like having a test. It’s anxiety.” She follows with a statement of “…wanting to do well.” Later in her interview she expressed concern about problem E stating “I hate this one.” This was her initial reaction even though she completed the problem correctly. She reflected after completing the problem by stating, “… some things just take time. They aren’t going to
be automatic.” She “finds it annoying” not to be able to complete a problem because she considers herself a problem-solver. Her affect was linked to wanting to find solutions and to do well. She placed a lot of significance on creating her own success. The comments from Marla were very different from Ken’s comments. Ken stated, “… word problems, they scare me” and “that problem scared the living crap out of me.” The reaction to the word problems was much stronger and much more negative. The reason for not completing tasks related to not knowing what the teacher wants. During the explanation of problem E he stated, “… I don’t know if this number relates to this number or this number. And the price to me is irrelevant. It’s just getting to the proportion you (meaning the teacher) want.” Ken is a non-traditional student who has come back to school later in his career and attributes the age differentiation and workload to his success in class. He stated when he walked into class “… everyone is half my age and they’ve had three months of off time. I’ve had 30 years of off time.” Ken’s comments suggested an external attribution for not completing the tasks or doing well in the course.

Mathematical Tasks

This study is not specifically about the process or problem-solving strategies used by college students in a remedial mathematics course. However, understanding student perception of the problems is integral to SR. The problems chosen for the tasks in the OMQ were selected because the topics are a part of the remedial mathematics courses, and proportional reasoning is seen as a stepping stone to higher-level mathematics reasoning (Lamon, 2007). The mathematical tasks were reviewed for insights into
student SR. The terminology, used for discussions within each proportional problem, is used for consistency with other proportional reasoning studies. Of the 69 participants, 25 individuals left a total of 35 blank tasks. Sixteen of the individuals left a single task blank, and only one individual left three items blank. Table 12 shows how many blanks were left for each task.

Table 12

*Number of blank tasks*

<table>
<thead>
<tr>
<th>Number of blank items</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>47</td>
<td>65.3%</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>22.2%</td>
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<tr>
<td>2</td>
<td>8</td>
<td>11.1%</td>
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<tr>
<td>3</td>
<td>1</td>
<td>1.4%</td>
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<tr>
<td>4</td>
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</tr>
<tr>
<td>5</td>
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<td>0%</td>
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</table>

Table 13

*Mathematical tasks left blank*

<table>
<thead>
<tr>
<th>Problem</th>
<th># left blank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>17</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
</tr>
<tr>
<td>E</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td>35</td>
</tr>
</tbody>
</table>

Data related to problems C and E appear unusual. The problems are provided below.
C. In a certain town, the demand for apartments was analyzed, and it was determined that to meet the community’s needs builders would be required to build apartments in the following way: Every time they build 3 one-bedroom apartments, they should build 4 two-bedroom apartments, and 1 three-bedroom apartment. Suppose a builder is planning to build a large apartment complex containing between 35 and 45 apartments. Exactly how many apartments should the contractor build to meet this regulation? How many one-bedroom, two-bedroom, and three-bedroom apartments will the apartment building contain?

E. Frank is buying a new car that gets 40 miles per gallon. His current vehicle gets 30 miles per gallon. If we presume that Frank drives 14,000 miles in the next year and the average price for a gallon of gas is $2.75 – how much will Frank save if he buys the new car?

Problems C and E were left blank most often and when completed, these problems were the most frequently missed. Problem C stands out as a very unique problem. Problem C contains the most words, had the greatest number of blank answers, and was the most frequently missed. Problems C and E are most frequently incorrect and also most frequently blank. The correct solutions for Problems C and E require multiple steps to complete. One possible solution for Problem C starts with connecting the ratios between the types of apartments with the total number of possible apartments to determine the factor in the ratios.

- Ratio of apartment types: 4:3:1
- Possible number of apartments 35 – 45
Assign a factor of k to determine the number of apartment complexes.

4k is the number of one-bedroom apartments, 3k is the number of two-bedroom apartments, and 1k is the number of three-bedroom apartments.

4k + 3k + 1k = 8k

This means the total number of apartments is a multiple of 8. The only multiple of 8 between 35 and 45 is 40.

8k = 40

k = 5

So there will be 20 one-bedroom apartments, 15 two-bedroom apartments, and 5 three-bedroom apartments.

Student work did not necessarily show all the steps or explain reasoning for the choice of 40 total apartments. Student work also utilized a counting method for the solution rather than proportional reasoning. The counting method still required multiple steps.

A possible solution for problem E would include determining the cost of gas for the two vehicles then subtracting to determine the savings. Again, the solution requires several steps. Problems C and E require several layers of thinking to complete the tasks. Even though both problems were most often left blank and answered incorrectly, problem C had more negative appraisals prior to beginning the problems.

The data from the other three problems did not offer any unique qualities. Possible solutions for problems B and D are similar and require the comparison of two ratios. The ratios may be represented as visual images, fractions, or decimal forms. A
possible solution for problem A would be a proportion. The problems were chosen from prior studies about proportional reasoning but also because the problem types, especially problems A, B and D, are common questions in ratio and proportion lessons. The difference in the layers of thinking or the number of words used to ask the questions were mentioned by some of the interviewees.

**Interviews**

The results from the tasks raised some questions that the interviews might answer. The interviews were coded for SR cycles and placed in table format. The tables are organized by task and then for each individual. Comments from interviewees were coded according to Zimmerman’s (2000) cycle of SR. These quotes then were analyzed for patterns along with the data collected from the OMQ. The discussion of mathematical tasks begins with the task that stands out for several reasons. Problem C has the greatest number of negative appraisals. Students were not confident in their ability to complete the task prior to beginning the tasks. Only 44% of the participants were able to answer the question correctly. Additionally, the problem was left blank more than any other problem. This problem stands out as a challenging problem for the students. Possible reasons for the difficulty could be the length of the problem. The problem consisted of more words, 89 words to be exact, than any other problem. Sarah and Marla made the following comments about problem C during the interview.

Sarah: “It’s kind of like too much because like too much information. Let’s say I see something like this it just like signs…its just numbers…and here you have to think and see the whole picture..
Marla: “There were too many numbers in that.”

Of the 35 blank tasks from all participants, almost half of the blank tasks, 17 to be exact, were from problem C. The problem had the most words and to complete the task students had to complete several steps. This was not a single-step task. During the interviews participants commented on not knowing the number of apartments that were to be built. A range of 35 to 45 was provided in the problem. The following examples illustrate how Ken chose the number of apartments.

Ken: “So they’re both odd numbers. 35 and 45 are odd…45 is divisible by 3 and 9… 35 isn’t… Splitting them, you have a fractional piece. I just guessed.

T: “40 is in between 35 and 45.”

The challenge involved with making the connection between the number of apartments and the necessary ratio of apartment types required persistence and reasoning skill. Ken commented about the need for a process.

Ken: “I don’t feel confident in my algebraic skills to break it down with an off the top-of-my-head algebraic problem.”

This raises questions for future studies and implications for teachers. If students are unable to see the whole process at the beginning of the problem, does this encourage students to stop? Problems C and E both required several steps to complete and both problems are left blank most often.

Problem A has positive appraisals and was most frequently answered correctly. The problem did not require several steps. Most of the interviewees discussed their strategy with ease. The problem seemed straightforward and corresponds with the
positive appraisals in the KMA data. Mathematical strategies varied on this problem. Strategies used to solve problem A were typically a cross-product process (Figure 8) and norming (Figure 9). The cross-product algorithm is a common approach but does not necessarily indicate proportional thinking (Lamon, 2007). Norming could indicate proportional reasoning, but problem A is an example of a problem where norming creates an inaccurate result. The lack of reflection to determine accuracy in Figure 9 illustrates a lack of reflection on the appropriateness of the solution.

![Figure 8. Norming](image)

![Figure 9. Lack of Reflection](image)

Figure 10 shows a counting strategy. This individual used a visual representation as a means of counting to find the solution. Lamon (2007) suggested this does not represent proportional reasoning because the individual does not show the factor or multiple required in the proportion.
Ken: “I was very confident of what I looked at. It was basic math. It wasn’t algebra.”

Dale: “Well. When I look at this, I think I’m going to have to set up the problem. If I can’t set up the problem then I can’t work it.”

Marla: “Um … it just seemed like an easy problem.” “Yeah – it just seemed like normal every day math.”

Dale did not attempt problem A. When asked why he left it blank, he stated he didn’t have a strategy for solving the problem but during the interview he was able to complete the task. His response showed confusion about the number of balloons each student would be receiving. He made the assumption that each student would receive three balloons and then completed the problem correctly based on this assumption. This shows that even when the student is capable of answering the task completely, confusion about specifics of the problem may cause the individual to stop. Self-regulation cycles are shown in Appendix A for all the problems. The SR comments are brief and concise
with very few comments about confusion or lack of confidence. The problem type is
common and requires a single-step strategy.

In problem B, students are asked to compare two ratios given the picture of seven
girls sharing three pizzas and three boys sharing one pizza. One possible solution used
by students in the study involved partitioning. Partitioning is the division of a whole into
equal parts. The boys’ single pizza is cut into three equal parts. To compare this
partition with the girls’ pizza the three pizzas for the girls are cut into three equal parts
too. Since each individual would be given a slice of pizza that is one third of a whole
pizza the girls have two slices left over to share, indicating the girls have more pizza.

Lamon (2007) questioned if this process illustrates proportional reasoning. Lamon
(2007) defined proportional reasoning as “…supplying reasons in support of claims made
about the structural relationships among four quantities, (say a, b, c, d) in a context
simultaneously involving covariance of quantities and invariance of ratios or products;
this would consist of the ability to discern a multiplicative relationship between two
quantities as well as the ability to extend the same relationship to other pairs of
quantities.” (pp. 637-638) Instead of proportional reasoning, a counting principle could
be used with this representation. Counting the number of slices that each boy and each
girl receives does not necessarily demonstrate an understanding of the relationship
involved in proportions. For this reason, students in the study may have a reasonable
solution but the solution may not indicate proportional reasoning. The OMQ results
suggested that students felt confident and demonstrated this through positive appraisals
and accuracy.
Problem B was a comparison of ratios with an iconic image of the ratio. Partitioning was used to explain the solution. An example of partitioning is shown in Figure 11. The idea of fair sharing was used to illustrate the girls had more pizza. Illustrating fair-share can be done by counting the number of pieces of pizza. This did not necessarily indicate students used proportional reasoning.

![Figure 11. Fair-Share](image)

Another strategy for problem B and also for problem D was a measurement strategy to compare the ratios when converted to decimal form. Figure 12 shows an example of measurement. Figure 13 illustrates how the measurement strategy can be inaccurate when a student does not have a strong understanding of ratios. This individual inverted the ratio resulting in an inaccurate solution.
Problem B was a comparison of two ratios. The ratios were provided as an image: three pizzas shared among seven girls and one pizza shared with three boys. The question asks who gets more pizza and to explain. Table 4 shows participants felt confident about the problem as the number of positive appraisals was 59. Participants also were accurate, with 74% of the participants able to answer and to explain the answer correctly. Only one individual left problem B blank. Approaches varied but converting the ratios to decimals was a common approach as was partitioning the pizzas into fair share slices.
The study did not intend to study specific mathematical strategies but the student work illustrates a naïve understanding of proportional thinking that should be discussed with the instructors. This will be addressed later in chapter 5.

Marla: {Goal} “So there’s 3 boys to 1 pizza and here there are 7 girls to 3 pizzas, and I just divided to get my solution.” {Monitor} “So there’s 3 boys to 1 pizza and here there are 7 girls to 3 pizzas and I just divided to get my solution.” {Reflect} “Cuz they have .42 a piece and they only have .33 continuing.”

Kevin: {Goal} “Visually I saw a fraction.” {Monitor} “There are 3 pizzas to 7 girls, and 1 pizza to 3 boys so I did that and found a decimal form that divided the 7 into the 3 and the 3 into the 1.” {Reflect} Teacher “So you are comparing the decimals” Kevin “Yeah”

Ken: {Goal} 7 girls and 3 boys and the boys each get one third of the pizza at least that’s the way I look at it. The girls count to 7. When I look at this … um … I don’t see how to equally make that work in my mind. {Monitor} “Because if I third it here I have 9 pieces I still have 2 left over so that each girl gets one with a remainder with 2 pieces left over. That’s an equality thing to these three.” {Reflect} “But there are 2 times as many girls as there are boys. So. I wasn’t sure, and I can’t remember if I did answer this question. And I can’t remember what I was thinking or what I did.”

The interviews provided detail about how students were self-regulating while working on the tasks. The detail provided some insight into how to support students from a SR standpoint. The mathematical task’s data provided additional information regarding
the mathematical thinking process. Self-regulation studies are domain-specific because of the differences in student perception within different domains (Pintrich & De Groot, 1990). The mathematics that students discuss in the interviews is also of importance. The next section highlights some of the mathematical strategies and interview statements used by students.

Problem B asks students to compare ratios of an iconic image of three pizzas for seven girls with one pizza for three boys. Several strategies can work here to determine who gets more pizza. Partitioning the pizza slices to compare the fair share is one possibility, or another would be to convert the ratios to decimal form. The conversion to decimal form was not consistent. The six individuals who were interviewed used different techniques. Four individuals wrote the ratio as 7:3 and divided 3 into 7. As a result the boys appear to have more pizza. This mathematical strategy neglects the concept of rate or unitizing per person. It may be worth changing the picture to see whether a majority of students write the ratio as the picture shows. When Sarah was asked about inverting the ratio, she noticed the comparison became 2.33 compared to 3.0, making the boys appear to have more. When asked what the 2.33 and 3 represented the response was confusion. When asked which strategy seemed more appropriate Sarah felt more confident with the 2.33 and 3.0 answers because of the “whole” numbers. This statement about decimals was a repeat finding in the interviews.

When looking at all participants in the study, the results from Problem B should be connected to Problem D because both problems require a comparison of ratios. Problem B shows the ratio in iconic form and problem D shows the ratio in symbolic
form. The results from problem D show that students were less confident, as this problem has the second highest negative appraisal number after problem C. Twenty-two percent of the participants had negative appraisal for problem D, but 54% of the participants were able to answer and explain correctly. Four individuals left the problem blank. Problems B and D ask students to compare two ratios but 74% were able to answer B, and only 54% could answer D. Thirty-eight individuals answered one of the problems correctly but not the other. Figure 14 is an example of a lack of explanation. Without mathematical reasoning, written explanation, or with limited explanation, the problems were marked incorrect.

D. Compare the ratios 3: 5 and 9:19. Which ratio is larger? Explain your answer.

3: 5 because it is larger.

Figure 14. Lack of explanation

Figure 15 shows another example of simplistic solution strategy. The solution is based on which of the numbers is larger instead of the multiplicative relationship between the numbers.

D. Compare the ratios 3: 5 and 9:19. Which ratio is larger? Explain your answer.

3: 5 and 9:19. 9:19 is larger. Higher numbers.

Figure 15. Overly simplistic explanation
The simplistic solution is compared with the solution for problem B. Work form problems B and D are shown from the same individual. Problem B is completed using a partitioning strategy and shown in Figure 16. The iconic images available offer context for answering the question, but when the iconic image is replaced with the symbolic version of ratios, as in problem D, the student has no proportional strategy. The difference in strategies, along with the difference in representations and explanations, has an implication to teacher. Chapter 5 will discuss implications for reasons that the representation of ratio using the colon in problem D may have been the cause for the difference in accuracy between problems B and D. Meyer (2001) discussed the transition from iconic to abstract representations, stating the intended purpose to create meaningful connections can be lost if the transition is made too quickly. Students were able to show understanding of comparing fractions with the iconic representation, but the transition to the abstract symbolic representation requires additional connections. This is an implication for instructional strategies.

B. Use the pictures below to answer the following question: who gets more pizza, the girls or the boys? Explain your answer.

![Figure 16. Iconic representation](image)

*Figure 16. Iconic representation*
Over half of the participants were unable to answer the ratio comparison problems. This comparison suggests importance to the instructors and should be addressed in the classroom. The confusion about notation and comparison of ratios or even the comparison of fractions may need to be addressed in the courses.

Problem D was counted incorrect when the answer was not explained. Eleven individuals stated the correct ratio but were not able to present an appropriate argument to justify their answer. Both problem D and E show students were overestimating their ability. The (+, −) column of Table 3 indicates problems D and E have the highest frequency of confident appraisal and inaccurate solution. Students felt they would be able to complete the tasks but were not accurate.

Problem E had the second highest number of blanks after problem C, and 47% of the students who attempted the problem were able to answer and explain correctly. The lack of accuracy, the tendency to leave both problems blank, and the need for more than one major step to complete each problem seem related. The interviews revealed a few possible reasons. Dale was immediately concerned about the miles per gallon. He explained, “… there are these ones on the test. I didn’t even try. If I can set up the problem then I can do the problem.” Regarding this problem, Marla said: “I don’t know where to start – too many numbers. I’m really gonna have to do the math for both sides.”

The results of the mathematical tasks suggest that problems that require more than one major step to complete were more often left blank or were attempted incorrectly. The results of the mathematical tasks also suggest the need to focus on multiple approaches to problem solving. Suggestions are offered in Chapter 5. There seems to be
several general comments that suggest the need to support student SR. Recognition of their own thinking may change their perception of ability. Three students commented during their interviews about answers with decimals creating a lack of confidence stating that they hated decimals.

Several participants referenced what seemed to be their own definition of mathematics. Their definition impacts the interactions and responsibilities in the classroom. Dale referenced a need to know the rules. He stated, “If I can’t set up the problem then I can’t work it.” Sarah comments: “Sometimes when I do math I don’t necessarily follow the rules.” Later in the interview Sarah stated, “I know, like, there should be some kind of formula.” Sarah seems to explicitly define mathematics by stating: “… it’s math, so most of the time you have the set of formulas and set theories. That’s what you’re supposed to do. Ken explained the teacher determines the rules when he stated: “… I don’t care that it’s wrong, it’s what you (the teacher) want.” All the statements about rules and the teacher expectation placed the teacher as the expert and the student as the rule-follower.

**Self-Regulation and Mathematical Tasks**

The results from the OMQ and the results from the mathematical tasks offer a picture of experience of students in college-level remedial mathematics. The investigation was framed with SR. The research questions investigate the assumptions of SR theory and investigate student self-regulation in a given moment while attempting a mathematical task. The findings above are now framed within the research questions for later discussion in Chapter 5 for findings and implications.
**First Research Question**

How does self-regulation impact student’s use of mathematical strategies in a remedial mathematics course? The investigation revolves around the connections found between self-regulation and mathematical strategy. The mathematical strategies are limited to the five tasks provided within the OMQ. Strategies were shared during the interviews. Strategies also are available from all participants’ OMQ task solutions.

Tables 2, 3 and 4 offer detail of student perception of whether they know what they think they know. The accurate knowledge monitoring columns, the (+,+), and (−,−) columns, together make up 205 of the possible 310 tasks appraised by all participants. Student metacognition is apparent in Table 2; the participants are able to monitor correctly about 66% of the time. During the interviews, Marla stated, “There were too many numbers in that.” Sarah stated, “It’s kind of like too much.” And Ken stated, “… I don’t feel confident in my algebraic skills to break it down with an off the top-of-my-head algebraic problem.”

The Seegers et al. (2002) study also found that when students had negative appraisal of their ability to complete a specific type of task then less effort was given to similar tasks in the future. Problem C had similar outcomes. Participants negatively appraised their ability to complete the task and this problem was most often left blank. Not only did participants not attempt problem C most often, participants were least accurate with the task.

Of interest is problem E which showed similar results with high numbers of blank tasks and low accuracy but was not similar with knowledge monitoring. Problem E
shows the largest number of overconfident appraisals if you refer to the (+, –) column of Table 3. The number is not unusual in the column but given the other similarities to problem C, why would students appraise their ability more positively on problem E but still have similar accuracy issues? Again, the interviews provided some insight. Ken and Dale had not completed the task on the original OMQ and continued having difficulty during the interview. Ken stated, “I have no idea.” Dale recognized problem E as a similar type from another standardized test and stated, “… there are these ones on the test. I didn’t even try.” However, Marla started by saying she “hated the problem … there are too many numbers but finished it … with “I guess some things just take time. They aren’t going to be automatic.”

Table 4 shows the relationship between the positive appraisal and intended effort. Students perceive their ability to complete problems A and B as being very high, and the accuracy reflects the confidence. The table shows an interesting connection between SR and mathematical strategy when comparing the accuracy of problems B and D. Both questions ask the participants to compare two ratios, but problem B is in iconic form and problem D is in symbolic form. More students were able to answer problem B (71%), the iconic form, accurately than problem D (54%). During interviews, the participants were asked if problems B and D were the same or different and to explain their reasoning. The interviews did not provide much insight because most of the participants, while talking through the problems, realized the process used to answer each question was essentially the same.
Second Research Question

What indicators, if any, suggest that students are actively participating while attempting to complete the mathematics task? This question was investigated through Zimmerman’s (2000) cyclic SR framework of goal setting, monitoring and reflection. Tables 5, 6, 7, 8 and 9 provide example SR cycles suggesting that the participants were active in their SR. The On-Line Motivational Questionnaire was used to investigate student perception of self-regulation at the moment a challenging mathematics problem was to be solved. Table 11 provides the group appraisal means from the OMQ but because this group is not being compared the information is a simple listing for future reference. Prior studies using the OMQ have shown that subjective competence and task appraisal mediate intended effort (Seegers et al., 2002).

Tables 5, 6, 7, 8, and 9 illustrate the students are regulating but it should be noted that second interviews were given to four of the six individuals as a way to provide additional information for these tables. The students were asked to talk through each of the problems and then were prompted at the end of each of the problems to describe their confidence. This prompt often provided the reflective portion of the cycle. This raises a question of how well students reflect without prompting. Reflection was apparent during the interviews even when not prompted. Students did not always express all of their thought processes during the interviews until prompted but reflection after completion of a challenging problem is essential. Studies relating self-reflection or self-talk during problem solving (Wong et al., 2002) or the use of heuristic processes that include the final step of determining accuracy could be used to support the students in these courses.
**Third Research Question**

What self-regulation criteria, if any, do students use while attempting the mathematics tasks? This question is specific to the moment when students are discussing their process for the mathematical tasks so the types of criteria were first analyzed by the researcher and a colleague individually then discussed to agree on types. The four major types of criteria found while students were attempting the tasks are discussed in greater detail earlier in the chapter with examples provided. Other forms of criteria were found outside the discussion of specific problems and may be of interest in future studies. The three criteria discussed in this study are: the comparison to the teacher’s process or answer (teacher); the comparison to their own process or answer (rhetorical); and the comparison to prior experience or example (experience). Table 10 shows a relatively equal distribution of types of criteria used by the students who were interviewed. In the process of describing their thought process, the reflection included statements that suggested an interest in wanting to know the answer to the question before determining their confidence. Marla explained her solution to problem C and finished by asking, “Am I just totally off the radar?”

This type of response along with responses from individuals who were matching experiences in the classroom with possible approaches to the problem without looking for their own experiences or their own context to support their thinking, relates to research in mathematics learning. The social context of the classroom and social negotiation of mathematical structures and strategies is the suggested environment that promotes the emergent perspective of constructivism. Cobb and Yackel (1996) discussed the
importance of recognizing that interactions in a mathematics classroom are experienced from both social and psychological theoretical perspectives. The roles defined by each individual also are developed through social norms defined in the classroom (Cobb & Yackel, 1996). While the request from Marla to know whether she was doing well may appear as a personal belief about who is the expert in the classroom. The comment also reflects the social norms of the classroom placing the teacher as the expert and the student as the novice.

Fourth Research Question

What mathematical strategies, if any, do students use to solve the mathematics tasks? Because studies involving SR and metacognition focus, at least in part, on the accuracy of the solution, this study also is interested in the types of mathematical strategies used by students during the OMQ. An accurate solution may be achieved through strong reasoning but if the intention of the course also is to improve student understanding of underlying concepts, then the process, strategy and reasoning should be part of the conversation. The five problems chosen for the OMQ mathematical tasks were taken from prior studies. Because Susan Lamon (2007) stated ratio and proportion topics to be a critical gateway to higher-order mathematical thinking, the study chose problems from her prior studies to be included. Student work showed similar examples to studies discussed by Lamon (2007). As shown earlier in the chapter reasoning through norming, unitizing, partitioning, and proportional reasoning were evident. Other forms of reasoning such as counting and cross-multiplication methods also were demonstrated. Many of these strategies produced accurate solutions, but further study is needed to
determine the level of proportional reasoning developed by students in these courses. If this topic is a gateway to higher order thinking then future success of the students depends on their flexibility of thought within this topic.

During the interviews, problems B and D originally were seen as similar types of problems with similar methods for solving. Most students wrote the ratios in decimal form to use for comparison. One of the conversations took an unexpected direction when the individual suggested that the numbers in the ratios should be inverted for division. The result changed the solution. When asked to explain which inverted form of the ratio then converted to decimal is the correct method, the students were unable to determine the correct method. This sequence of questions with the two individuals provided insight into the students’ reliance on procedure without understanding. Both students were very confident that converting to decimal form was an appropriate procedure, but neither could determine why inverting the ratios produced different results, and therefore, which result was appropriate. This unexpected set of questions also could provide insight into why problem B was answered accurately more often than problem D. The context clues provided by the iconic image in problem B helped, as suggested by Marla when she stated: “… this one had a cute little picture, so it made it easier. The three boys share the one pizza. Each boy receives one third of the pizza.” This method mimics the use of fractions. Students recognize the three equal parts of the whole pizza. Without the context in problem D there is no way to determine the whole. More importantly, the context with ratios is not always part to whole. Ratios can be represented part to part, part to whole, or whole to part. Seeing the ratio as a single unit is one of the necessary
components recognized by Lamon’s (2007) studies. Students in the courses could benefit from a more comprehensive study of ratio and proportion.

The question focuses on the mathematical strategies found within the solutions to the five mathematical tasks. Research relating to rational numbers, ratio and proportion had a stronger presence in previous decades but Lamon (2007) suggests the need to take on the topic again as she perceives these topics as the necessary gateways to higher order mathematical thinking. The five tasks were taken from earlier studies by Lamon (2007); not surprisingly the strategies used by the participants were similar to those used by individuals in earlier studies. The work shown on the OMQ mathematical tasks illustrates partitioning, unitizing, norming and proportional reasoning. Other forms of reasoning are also illustrated including counting and cross-product strategies. The strategies used by participants may have been used successfully in the past but may not support higher-order thinking necessary to answer more challenging questions. The next step is to determine how the information provided from the study informs instructors in the remedial mathematics courses.
CHAPTER V

FINDINGS

The purpose of the investigation was to add to the research concerning the complexities of learning in a mathematics classroom. Cobb (2007) suggested a focus on the practicality of the theoretical perspective and the future goals of student learning. Research in mathematics education still encompasses several theoretical frameworks. Cobb (2007) highlighted four perspectives in his discussion of philosophy in mathematics education. The perspectives he discussed included experimental psychology, cognitive psychology, socio-cultural theory, and distributed cognition. His purpose was not to claim one perspective over another but to point out strengths and weaknesses. This study seeks to investigate the perceptions of students during their remedial mathematics course at the college level as a means for the instructors to improve instruction. A mathematical disposition has been given central importance in learning in a mathematics classroom (De Corte, Verschaffel, & Op’t Eynde, 2000). Responsibility is greater for the student and teacher to achieve this. A teacher’s beliefs about student ability, instructional strategies and classroom norms are critical. Research suggests that teacher beliefs of what challenges students does not match student ability (Nathan & Koedinger, 2000). Instruction impacts student’s ability to understand mathematical tasks. Lamon (2001) suggested that instruction focused on reasoning without rules enables students to remember information longer and to know when to use the information (p. 162). The teacher establishes how mathematics is discussed in the classroom. Sohmer, Michaels, O’Connor and Resnick (2009) suggested that the exchange of mathematical ideas in the
classroom can be established as a communication norm. Changing the conversations so both the student and teacher are accountable for more than an answer can encourage greater content competence. The language of the classroom is an interaction of ideas with a common goal to understand the meaning of the concept. This study is therefore seen as the beginning of a long and necessary conversation between the teacher and student.

This study was an opportunity to investigate student self-perception and self-regulation (SR) as it relates to content knowledge or personal motivation within the subject domain. From an instructor’s perspective, feedback from students at the completion of a course can be limited. Because the evaluations are generalized for any course, the feedback does not provide specific information, for example, that could help the instructor understand why a student stops making the necessary effort to be successful; in the course. Interaction with individual students is possible if the students are present in class and will share. In order to improve teaching there is a need to understand each student’s current experience, along with prior experience, and content knowledge. The study was designed to investigate how a group of students in the remedial mathematics courses reacted to specific mathematical tasks. The On-Line Motivational Questionnaire (OMQ) was used to provide an initial picture of the individual. The student provided information about their affect, appraisal, mathematical skill, and their attribution about their mathematical tasks. The snapshot of self-perception provides insight at the moment a student is provided a mathematical task. The goal was to provide challenging mathematical tasks that would result in a lack of persistence. The
students who showed a lack of persistence then would be offered an opportunity to be interviewed to determine whether the lack of persistence was related to mathematical strategy or to SR or to a combination of both. The limitations section will discuss in greater detail that the students who did not persist in the tasks also did not volunteer to be interviewed. Of the six participants who were interviewed, two did not persist at a couple of the tasks. The insights from these individuals may provide support for teacher instruction, but future studies should include additional interviewees. The data included Hamman scores to discuss knowledge monitoring. The interview transcripts were used to demonstrate self-regulation occurring in the course while highlighting SR and mathematical strategies that are relevant to instructors. These findings are similar to findings from prior studies but also contribute to the research through interview information and subject choice.

Self-regulation connected the cognitive, metacognitive and domain-specific elements needed to look at mathematics learning. Boekaerts’ (1992) model of adaptable learning provided the jumping-off point. The OMQ provides a momentary look at an individual’s motivation and mathematical strategy. While in the process of working on a series of mathematical tasks the individual was asked to describe his or her perception of task relevance, intended effort, emotional state along with other scaled items.

**Summary of Findings**

The findings for the study are organized by findings related to SR and metacognition followed by findings related to mathematical strategies related to SR. The framework for self-regulation was taken from: the four assumptions used in SR research;
the self-regulation cycles as described by Zimmerman (2000); and Boekaerts model of adaptable learning. The following sections describe the findings suggested by the data from the study outlined in Chapter 3. In short, the study suggests that while students were attempting the five tasks within the OMQ they: were self-regulating; had accurate monitoring of their knowledge; and did not persist in attempts to complete multiple step word problems. Each of the findings is described in greater detail below.

**Metacognition**

A majority of the appraisal responses were positive, showing the participants were confident they could complete the tasks. The positive responses suggested participants knew what they thought they knew, or, in other words, the participants were aware of the types of tasks they could complete accurately. Participants were most accurate with problems A and B. Table 4 shows problem A with 97% positive appraisal and problem B with 95% positive appraisal prior to beginning the tasks. The confidence translates into the highest accuracy percentages as well. The accuracy for problem A was 74% and for problem B was 71%. These are common problems asked in the remedial mathematics courses, which may be the reason for the positive appraisal and accuracy. Problems C, D, and E offer different outcomes and therefore suggest implications to support student needs.

Knowledge monitoring studies emphasize the importance of understanding what one knows and does not know. Knowledge monitoring accuracy occurs when participants’ appraisal match their accuracy. This includes when individuals negatively appraise their knowledge prior to the task and then do not complete the task accurately.
This appears as the (–, –) column of Table 3. Students may be aware of their ability but this awareness also impacts effort to complete the task. More than half the participants, 55%, negatively appraised problem C. This lack of confidence could inhibit persistence or confidence to complete the task. Classrooms that emphasize autonomy, mathematical meaning, persistence, and learning encourage positive emotions from the participants (Kazemi & Stipek, 2002). The negative appraisals of problem C are an implication for instruction and curriculum. These implications are discussed later.

Knowledge monitoring of problems D and E also inform this study. The percentage of accurate knowledge monitors for the problems was 58%. The focus should be on the individuals who did not monitor correctly. Problems D and E had 42% of the participants not monitor correctly with a majority of this 42% overestimating their ability. The (+, −) column for these two problems had the highest values. Thirty-two percent of the participants over-estimated their ability in problem D, and 37% overestimated their ability in problem E. One third of the participants thought they could answer the problem correctly but did not. The over-estimation can cause students not to study material when actually the material is not understood. The result in classes would be heard as frustrated comments from students who say they knew the material but just did not do well on the test. The over-estimation of ability is another implication for instruction and will be discussed later in the chapter.

Self-Regulation

The study suggested that participants in the study self-regulate. Participants who volunteered to be interviewed in the study demonstrated the four assumptions made by
SR researchers. Participants were active, they regulated, they had criteria for comparison and personal mediated context (Pintrich, 2000). Tables 5 through 9 illustrate the SR cycle suggested by Zimmerman (2000). The tables provided the indicators suggesting the students were actively participating while attempting to complete the tasks. Table 10 shows the participants have criteria for comparison. The information suggests answers to two of the research questions intended to investigate whether students are self-regulating. According to the data, the students are regulating while attempting mathematical tasks.

There are additional measurement tools that could provide additional information about the students and should be considered in future studies.

During interviews, participants demonstrated their mathematical processes by thinking aloud while attempting the task. Student appraisals of ability to complete the tasks also followed a study by Seegers et al. (2002) study suggesting student appraisal of a task impacted their effort in similar tasks in the future. Problems A and problem C illustrated the point. A vast majority of the participants, 97%, appraised their ability to complete problem A positively. The task was perceived as a task that in previous setting students were successful doing. Participants were accurate with their solutions; 74% completed the problem successfully, and no one left the problem blank. The positive experiences prior to the current experience impacted current effort. Problem C shows the converse of this, but it still matched the Seegers et al. (2002) study. A negative prior experience produced less effort. More than half, 58%, of participants indicated a negative appraisal prior to completing the task. Only 44% answered the question
correctly and 24% of the participants left the problem blank. The negative appraisal matched with the lack of effort in this case.

This study suggested the participants self-regulate and that participants were metacognitively aware of their ability to complete the tasks. Additional findings related to the five tasks students attempted could be related. Participants completed the first two tasks accurately and with confident appraisals but the last three tasks were not completed. This seems to suggest that participants do not monitor well and do not self-regulate.

**Mathematics**

This section outlines the implication for teachers in the remedial mathematics program. The goal was to use information provided by the students to inform decisions in the classroom. The study suggests that students self-regulate and monitor their knowledge but the following section suggests how mathematics content and strategy may support students’ needs in the remedial mathematics courses. Appraisal, accuracy and knowledge monitoring for problems A and B were different from those of problems C, D, and E. Can this difference provide insight for the instructors in the remedial mathematics courses?

**OMQ tasks related to positive knowledge monitoring.** Students assigned positive appraisal to problems A and B and also were able to answer these tasks more accurately than any of the other tasks. This information could suggest the problems were not challenging for the participants. However, the strategies used to solve these two problems inform student thinking during the study. During interviews, Ken stated problems A and B were “normal”. Marla said the problems were “regular math”.

According to the interviews, the problems were “regular” or “normal” problems. The strategies used to solve the problems were standard algorithmic and fair-share strategies. The problems could be completed with one-step algorithms or fair-share explanations, neither of which required much cognitive load, so participants were confident and accurate. Problems C, D, and E required more than the simplistic strategies used in the first two problems. This additional cognitive load could be the reason students had negative appraisals for problem C and inaccurate solutions for problems C, D and E.

**OMQ tasks related to poor knowledge monitoring.** A closer look at problems C, D, and E along with statements from interviewed participants was used to develop findings related to the mathematical tasks. The OMQ data showed fewer than half of the participants were able to complete the three tasks accurately. The quantitative and qualitative data suggests student’s cognitive load is extensive. A lack of strategy availability and a lack of content knowledge limited student ability to complete the tasks.

Problem C had many unique quantitative elements. Students were not confident about their ability to complete the task. Seventeen participants, 25%, left problem C blank. Only 44% of the participants completed the problem correctly. Problem C was written using the most words, 97. Problem C also required multiple steps to complete. Interview statements indicated reasons participants were not confident or accurate. Marla stated problem C had too many words. She also attempted the problem using a technique used in class just prior to the OMQ without any proportional reasoning. Ken misunderstood the meaning of part of the problem. The interviews consistently suggested that the cognitive load for problem C was extensive. Participants did not have
a strategy, could not interpret the problem, and could not repeat thinking. This suggests a need for teachers to incorporate problem-solving or additional mathematical strategies into the curriculum. If a heuristic problem-solving process similar to Polya’s (1945) were used, instruction would include both a process for solving a variety of tasks that includes a follow up or reflection stage to determine reasonableness. The National Council of Teachers of Mathematics (NCTM, 2000) sees the purpose of problem solving as a means to adapt strategies while monitoring the process and reflecting on the problem. This adaptability is not evident from the students.

In terms of mathematics specifically, problems B and D raise another interesting element. Problem B had an iconic representation of ratios to compare and problem D had a symbolic representation of ratios to compare. The study suggested that a “critical” element for student learning was the meaningful mathematical tasks using multiple representations (Pape et al., 2003). Lamon’s (2001) study suggested the delay of procedural algorithms to solve problems involving fractions helped students build deeper understanding of the concepts. The study focused on the use of multiple representations, to compare and contrast flexibility within each of the representations. The result was better decision-making skills for choosing the appropriate representation for the given task. Problems B and D were similar types of problems, but each used different representations of the question. Flexible strategy use might have increased the accuracy of problem D. Problem B was appraised positively and answered correctly by 71% of the participants. However, problem D was appraised positively and answered correctly by only 54% of the participants. The process for answering the questions could play a part
in understanding why participants overestimated their ability to complete the task. During the interviews, the participant viewed problems B and D as the same. This could be a result of the participants using the measurement strategy of converting to decimal. The conversion to decimal form was a sufficient strategy. When two of the participants, Marla and Sarah, were asked to elaborate why they used this representation of the ratio and not the inverted form both participants become uncertain. This suggested a surface-level understanding of ratio. Even though the participants had used this process successfully in the past, neither was aware of why this process was appropriate rather than the inverted forms of the ratios. The implication is a need for greater depth of understanding for complex topics like proportional reasoning topics. Even simplistic strategies can be used by individuals but not understood entirely. NCTM (2000) agreed that understanding the facts of mathematics, the procedures along with the concepts provides a solid platform. The ability to connect new ideas with conceptual understanding allows the individual to be flexible when problem-solving (Schoenfeld, 1988).

Problem E was similar to problem C with a long list of quantitative data. Twelve participants, 17%, left problem E blank. Only 47% of the participants completed the problem correctly. Problem E had the second highest number of words. Problem E also required multiple steps to complete. The appraisals for problems C and E were different in three respects. Problem E was given primarily positive appraisals, students were initially confident they could complete the problem, and problem E used a rate of miles per gallon, a familiar context. Problem E had the highest number of participants in the
over-confident column (+,-) from Table 3. Interview statements provided insight for instructors. Marla stated the problem had “too many numbers” and that she “hated this one.” Dale said he “stopped when he saw the problem.” The problem reminded him of similar problems that had given him difficulty on a placement test. Ken stated he was “not sure where to go.” Both Marla and Dale completed the task with prompting during the interview, but Ken could not. While Marla had completed the problem accurately during the original OMQ, Dale had left the problem blank. The initial cognitive load seemed extensive. Ken’s reasoning for the incomplete problem was the lack of an appropriate strategy the teacher would want to see. The participants appraised their ability to complete the task positively but were not accurate. Marla stated, “… the problem just takes time.” The lack of accuracy, as with problem C, could relate to cognitive load. Students did not perceive a strategy after beginning the task. A possible suggestion for instructors is to include problem-solving as problem-solving instruction provides general steps for students to follow. In the same way prompting from the interviewer helped Dale complete the problem; the problem-solving steps provide prompts for the students. Again, the study suggests the need for instructors to incorporate problem-solving strategies into the course. One research question asks what mathematical strategies, if any, were used by students? The participants used a variety of strategies, some of which were elegant, some were efficient, and some were simplistic, while others were not helpful. The study highlights the limited strategies available to students when problems were more challenging. This only enhances the need for instructors to provide additional strategy support.
The addition of mathematical strategies is one implication from the study, but the limited number of interviewed participants suggests continued investigation with students in the remedial mathematics courses. Future investigations should consider the use of an improved research design that allows the teacher to make adjustments to the course through data-based decision-making. Cobb’s (2000) design experiment offers this design. Specific implications for teachers of the remedial mathematics courses are offered in the next section.

**Implications**

This study offers a deeper look at what students are thinking at the moment they are given a challenging mathematical task. Research suggests that teachers are not aware of their students’ abilities (Yusof & Tall, 1999). Making connections from what is seen through quantitative and qualitative data collection provides insight for the instructors of these courses as well as future direction for studies.

This study was an investigation to determine how SR impacted student perception in a mathematics class. Specifically, the researcher was interested in reasons for a lack of persistence with challenging problems. Prior studies have suggested that prior failures, motivation, and self-efficacy are possible factors in the lack of persistence. The study suggests that students are accurate knowledge monitors, even when uncertain of their ability to complete a task, and that students do self-regulate. Even though students regulate and monitor well, their accuracy with the tasks was inconsistent. The implication is that teacher intervention should include more than SR and knowledge monitoring support.
The implications for change relate to instructional strategies, curriculum, and policy change. The expectation for change may be slow but this section offers suggestions for where to begin. The implications relating to instructional strategy and curriculum are based on the lack of flexible strategy use and naïve representation.

Possible interventions for flexibility of mathematical strategy are part of implementation of problem-solving strategies and self-regulation strategies. Perels et al. (2005) study suggests that instruction that includes problem-solving and self-regulation skills at the same time improves student achievement. Problem-solving heuristics include the follow-up step of understanding your answer and reflecting on the reasonableness. Building problem-solving strategies supports the need for additional mathematical strategies while also supporting a reflective stage that encourages determining reasonableness of the solutions. The interviews suggested instructor prompting was necessary to hear reflections from the participants. These reflections could be introduced through a heuristic problem solving strategy similar to Polya’s problem-solving heuristic (1945). This intervention would be in written format.

Other studies have used self-talk strategies (Wong et al., 2002) with similar strategies but in a verbal format. Wong et al. (2002) ran a study of 47 high school freshmen in Australia. In this study, students were given a mathematical beliefs questionnaire, assessed on prior geometry knowledge, and trained on self-talk procedures. The students were given problems to solve and were encouraged to talk out loud during the process. The researchers suggested that students who participated in the self-explanation during the problem solving had a learning advantage over students who
were not trained to self-explain (Wong et al., 2002). Creating opportunities for student success is important but what is most important about this strategy is its flexibility of use. The self-explanation is not content-specific allowing the student to use this skill in many areas. Self-talk also can help the teacher understand the thought process of students. The addition of classroom discourse that promotes autonomy and problem solving requires appropriate levels of challenging problems. The remedial courses have a common curriculum that includes challenging mathematics problems meant to extend student thinking beyond the typical textbook questions. These problems provide a solid platform, but additional content is necessary given the representational issues related to problem D. Problem D is an example of material that instructors would presume individuals would have studied from middle school or high school. Research has suggested (Lamon, 2007) that teaching with a focus on procedure and results without understanding is more likely to be forgotten by students. If prior experiences have been procedural then a focus on critical concepts like proportional reasoning should be given greater emphasis in the remedial mathematics content.

Policy changes for the courses also are suggested based on the findings from this study. Creating an environment where students are encouraged to be autonomous, to share meaningfully with others, and to take risks is a complex task. These changes require professional development for the instructors and then freedom to make adjustments in the courses as students need. De Corte (1995) suggests four skills necessary for problem-solving environments: domain-specific knowledge, heuristic methods, metacognitive knowledge, and affective components (p. 37). These changes
also presume teachers see the need to change. Nathan and Koedinger (2000) report that teachers may have an “expert blind spot” (p. 209). This blind spot refers to teachers over-estimating students’ ability to interpret symbolic representation meaningfully. Encouraging the change may have to become a part of policy changes given that instructors may not know that their classroom procedures could be part of the reasons students are not successful in the courses.

**Limitations**

Limitations in the study begin with the sample size. The intended sample size was more than 100 because each of the courses has approximately 25 students enrolled. A convenience sample of five sections of the courses was chosen. Fewer than 100 students were in attendance the day the questionnaire was given. This is typical for these courses due to withdrawals or personal decisions to stop attending. Protocol for the questionnaire was based on the general enrollment of 25 students in each section. The intended sample size was to be more than 100 participants but because of attendance and withdrawals, 72 participants volunteered to complete the questionnaire. Due to incomplete questionnaires, the sample sizes to determine the Hamman coefficient and OMQ data were fewer than 72. Sixty-two questionnaires contained completed KMA information and 69 questionnaires contained completed information for the OMQ portion of the study. Informing educators and researchers was a goal of this research study, but the small sample size impacts the generalizability. The OMQ is a quantitative tool used in SR research. The tasks on the OMQ were not randomly placed for the participants, which may be a cause for the number of inaccurate and blank responses to problem E.
Therefore, results based on number of blank or incorrect solutions should be scrutinized carefully. The OMQ provided the snapshot of student perception when presented with a challenging mathematics problem; however, the use of the one-time sample may have limited the usefulness of the OMQ as a comparison tool. The addition of another measurement tool such as the Goal Orientation Questionnaire (GOQ) could be used to look at specific elements of SR such as goal setting (Seegers et al., 2002). The GOQ also is used with the Self-Concept of Mathematics Ability Questionnaire developed by Bong and Skaalvik, (2003). This questionnaire measures cognitive aspects but was developed to work with the GOQ.

The most profound limitation was a result of the process for selecting volunteers for the interviews. The purpose of the interviews was to examine student self-perception to a greater depth. Most importantly, the interviews were meant to be with individuals who did not persist at any one of the mathematical tasks on the OMQ. The interview protocol had participants volunteer to interview, but the participants who did not persist also were not willing to be interviewed. Thirty-two volunteered to be interviewed. Of these volunteers only seven were individuals who had left at least one task blank on the OMQ. Of the seven who left blanks only two responded to the invitation to be interviewed. This limited feedback related to reasons for a lack of persistence. The two individuals who volunteered to be interviewed and also had left tasks blank did provide insight into the study but it is clear that more data will be necessary to determine significant connections.
The method chosen for the study was meant to be an investigation, however, the method could be extended to include a design experiment as suggested by Cobb (2000). Cobb’s longitudinal design incorporates the same design included in this study with the addition of the teacher and researcher working together to make adaptations to the material and pedagogy as data is collected. The design experiment does not wait until the end of the semester to compare or contrast designs that may not be providing sufficient support for students. This fits well with self-regulated learning frameworks that require several layers of training for the teacher as well as with the students to create an environment where students are given opportunities to explore new regulation habits that provide positive outcomes. Presently, the design experiment would be limited in practice with the remedial mathematics courses. The courses have standardized assessments and instructional timelines. Instructional strategies vary with each instructor but content variation is not permitted. Changes within the courses also must occur beyond the level of instruction and curriculum to include policy changes.

**Future Efforts**

Additional questions were identified during the process of the study. Future efforts should consider alternative tasks to investigate the relationship between challenging problems and other factors such as reading comprehension, problem length, or tasks requiring multiple steps. Any of these changes should include challenging tasks. These changes also may inform knowledge monitoring or self-regulation in a new way.
Closing

Throughout my educational experiences, as a teacher and a student, I enter the classroom with the intention of answering a long list of questions. As the questions are answered, not surprisingly, a new and often longer list of questions remains. This defines this experience as well. The study began with questions that were too big to be answered in a dissertation, but as the questions became more defined the list of interesting additional questions surfaced. The experience of interviewing the participants began a dialogue that will continue for years.

The learning curve as a researcher was a steep one but all the experiences will act as platforms for future projects. I look forward to the next experiences and the interactions with the individuals.
APPENDIX A

MATHEMATICAL TASKS ON OMQ
Appendix A
Mathematical Tasks on OMQ

The following problems are from Lamon (2007).

A. Ellen, Jim, and Steve bought 3 helium-filled balloons and paid $2.00 for all three. They decided to go back and get enough balloons for all the students in their class. How much did they have to pay for 24 balloons?

B. Use the pictures below to answer the following question: who gets more pizza, the girls or the boys? Explain your answer.

C. In a certain town, the demand for apartments was analyzed, and it was determined that to meet the community’s needs builders would be required to build apartments in the following way: Every time they build 3 one-bedroom apartments, they should build 4 two-bedroom apartments, and 1 three-bedroom apartment. Suppose a builder is planning to build a large apartment complex containing between 35 and 45 apartments. Exactly how many apartments should the contractor build to meet this regulation? How many one-bedroom, two-bedroom, and three-bedroom apartments will the apartment building contain?

D. Compare the ratios 3: 5 and 9:19. Which ratio is larger? Explain your answer.

E. Frank is buying a new car that gets 40 miles per gallon. His current vehicle gets 30 miles per gallon. If we presume that Frank drives 14,000 miles in the next year and the average price for a gallon of gas is $2.75 – how much will Frank save if he buys the new car?
APPENDIX B

INTERVIEW PROTOCOL
Appendix B

Interview Protocol

First round of interviews:

A. Focus on the tasks that were not attempted and on tasks where participant did not monitor correctly.
B. Questions will vary depending on student response and possible SR or mathematical strategy paths.

Questions:

1. How do you feel about mathematics class?
2. Do you find a calculator is helpful?
3. How do you go for help when you need it?
4. Can you explain your process for this problem?
5. Did this problem seem familiar?
6. Specific to problems B and D: Do you believe problems B and D are the same or different – Explain.
7. Additional questions should be asked relating to mathematical strategy or thought process.

Second round of interviews:

1. Focus on cycle of self-regulation – students will describe their process for solving each of the problems even if they had already solved the problem correctly.
2. How do the participants feel about their process?
3. Encourage students to share the tiniest of detail – whatever crosses their mind while describing how to solve the problem.

4. Describe your thought process for this problem
   a. How confident are you with your solution (even if the solution is correct)
5. Do you have another way of solving this problem
6. Additional questions should be asked relating to SR cycle, mathematical strategy or thought process.
APPENDIX C

ON-LINE MOTIVATIONAL QUESTIONNAIRE
Appendix C

On-Line Motivational Questionnaire

Thank you for taking the time to complete the Motivation Questionnaire.

The purpose of the questionnaire is to investigate how students perceive their own ability while completing a mathematics task. Your accurate responses will help the instructor adjust teaching strategies to better support student learning.

General instructions:

A. Read the problems without attempting them.

B. Answer the first set of survey questions about your initial perceptions of the problems before you begin the problems.

C. Solve the problems.

D. Answer the second set of survey questions about your perceptions of the problems after you finish the work.
Read the problems but do not attempt to solve them yet.

Do you feel confident that you will be able to solve problem A?  yes___  no____
A.  Ellen, Jim, and Steve bought 3 helium-filled balloons and paid $2.00 for all three. They decided to go back and get enough balloons for all the students in their class. How much did they have to pay for 24 balloons?

Do you feel confident that you will be able to solve problem A?  yes___  no____

B.  Use the pictures below to answer the following question: who gets more pizza, the girls or the boys? Explain your answer.

Do you feel confident that you will be able to solve problem B?  yes___  no____

C.  In a certain town, the demand for apartments was analyzed, and it was determined that to meet the community’s needs builders would be required to build apartments in the following way: Every time they build 3 one-bedroom apartments, they should build 4 two-bedroom apartments, and 1 three-bedroom apartment. Suppose a builder is planning to build a large apartment complex containing between 35 and 45 apartments. Exactly how many apartments should the contractor build to meet this regulation? How many one-bedroom, two-bedroom, and three-bedroom apartments will the apartment building contain?

Do you feel confident that you will be able to solve problem C?  yes___  no____
D. Compare the ratios 3:5 and 9:19. Which ratio is larger? Explain your answer.

<table>
<thead>
<tr>
<th>Do you feel confident that you will be able to solve problem D?</th>
<th>yes</th>
<th>no</th>
</tr>
</thead>
</table>

E. Frank is buying a new car that gets 40 miles per gallon. His current vehicle gets 30 miles per gallon. If we presume that Frank drives 14,000 miles in the next year and the average price for a gallon of gas is $2.75 – how much will Frank save if he buys the new car?

<table>
<thead>
<tr>
<th>Do you feel confident that you will be able to solve problem E?</th>
<th>yes</th>
<th>no</th>
</tr>
</thead>
</table>
On-Line Motivation Questionnaire

Answer the 18 questions below before beginning the problems.

Name:____________________________  Course:________________________

Date of birth:______________  Gender:______________

Read every question and fill in exactly one circle that best describes you. DO NOT leave it blank.
1. Are you in the mood to start the task?
   - Not at all in the mood
   - Not much in the mood
   - In the mood
   - Very much in the mood

2. How good do you think you are at this type of task?
   - Not at all good
   - Not so good
   - Good
   - Very good

3. How useful do you consider the task?
   - Not at all useful
   - Not so useful
   - Useful
   - Very useful

4. How easy is this task for you?
   - Not at all easy
   - Not so easy
   - Easy
   - Very easy

5. How well do you expect to do on this task?
   - Not at all well
   - Not so well
   - Well
   - Very well

6. How enthusiastic are you about this task?
   - Not at all enthusiastic

7. How often can you succeed at this kind of task?
   - Hardly ever
   - Now and then
   - Regularly
   - Nearly always

8. How much attention do you plan to devote to this task?
   - Not much attention
   - Some attention
   - Much attention
   - Very much attention

9. How important do you find it to do well on this task?
   - Not important at all
   - Not so important
   - Important
   - Very important

10. How difficult do you find this task?
    - Not at all difficult
     - Not so difficult
     - Difficult
     - Very difficult

11. How pleasant do you find this task?
    - Not at all pleasant
     - Not so pleasant
     - Pleasant
     - Very pleasant
12. How much effort do you have to put into this task in order to get a passing grade?
- I don’t have to do my best at all
- I have to make a bit of an effort
- I have to try hard
- I have to try as hard as I can

13. My goal on this task is…
- To just get a pass
- To make sure I pass
- To make sure I perform well
- To make sure I perform very well

14. What sort of grade do you expect to get for this task?
- Below passing level
- Around passing level
- Above passing level
- Considerably above passing level

15. How much effort are you going to put into this task?
- Very little
- Some
- Much
- My very best

16. How good are you at this task in comparison to your classmates?
- Far below average
- Just below average
- Just above average
- Far above average

17. If you could take as much time as you want, how long would you spend on this task?
- I’d just begin and decide later how long to keep working
- I’d work until I think I’ve just made a passing grade
- I’d work until I’ve done a reasonably good job
- I’d work until I feel truly satisfied with myself
18. How do you feel right now, just before starting the task?

- a. Not at ease 0 0 0 0 at ease
- b. Not nervous nervous 0 0 0 0
- c. Not fine 0 0 0 0 fine
- d. Not worried worried 0 0 0 0
- e. Not confident confident 0 0 0 0
- f. Not annoyed annoyed 0 0 0 0

19a. What is the time? ______________
Answer the questions.

A. Ellen, Jim, and Steve bought 3 helium-filled balloons and paid $2.00 for all three. They decided to go back and get enough balloons for all the students in their class. How much did they have to pay for 24 balloons?

B. Use the pictures below to answer the following question: who gets more pizza, the girls or the boys? Explain your answer.

C. In a certain town, the demand for apartments was analyzed, and it was determined that to meet the community’s needs builders would be required to build apartments in the following way: Every time they build 3 one-bedroom apartments, they should build 4 two-bedroom apartments, and 1 three-bedroom apartment. Suppose a builder is planning to build a large apartment complex containing between 35 and 45 apartments. Exactly how many apartments should the contractor build to meet this regulation? How many one-bedroom, two-bedroom, and three-bedroom apartments will the apartment building contain?

D. Compare the ratios 3:5 and 9:19. Which ratio is larger? Explain your answer.

E. Frank is buying a new car that gets 40 miles per gallon. His current vehicle gets 30 miles per gallon. If we presume that Frank drives 14,000 miles in the next year and the average price for a gallon of gas is $2.75 – how much will Frank save if he buys the new car?

19b. What time is it? ____________________
1. How do you feel just after finishing the task?

   a. Not relieved
   b. Not at ease
   c. Not nervous
   d. Not satisfied
   e. Not fed up
   f. Not fine
   g. Not worried
   h. Not confident
   i. Not annoyed
   j. Not concerned

2. How carefully did you do the task? I paid...

   o Not much attention
   o Some attention
   o Much attention
   o Very much attention

3. How difficult did you find this task?

   o Not at all difficult
   o Not so difficult
   o Difficult
   o Very difficult

4. How long did you continue working on the task?

   o I started, but I didn’t continue very long
   o I worked until I thought I would just pass
   o I worked until I thought I’d done a reasonably good job
   o I worked until I was really satisfied with myself

5. What sort of grade do you expect to get for this task?

   o Below passing level
   o Around passing level
   o Above passing level
6. How much effort did you put into the task?
   - Very little
   - Some
   - Much
   - My very best

7. How well did you do this task?
   - Not at all well
   - Not so well
   - Well
   - Very well

8. How useful do you consider this kind of task?
   - Not at all useful
   - Not so useful
   - Useful
   - Very useful

SKIP TO question 10 if you DID NOT do well on the five tasks. If you did well on the five tasks answer question 9.

9. I did well on this task…
   a. Because I am good at this type of task
   b. Because I was in the mood to do the task
   c. Because I am lucky
   d. Because I did my best
   e. Because I found it an easy task
   f. Because I thought it was a pleasant task
   g. Because I knew how to handle the task
   h. Because I already knew a lot about the subject of this task

Answer question 10 if you did not do well on the tasks

10. I did not do well on this task…
a. Because I am not good at this type of task
b. Because I don’t like doing this type of task
c. Because I had bad luck
d. Because I didn’t do my best
e. Because I found it a difficult task
f. Because I thought it was an unpleasant task
g. Because I didn’t know how to handle the task
h. Because I hardly knew anything about the subject of the task
REFERENCES
REFERENCES


B. Greer (Eds.), *Theories of mathematical learning* (pp. 69-76). Mahwah, NJ: Lawrence Erlbaum.


