The Benefits of Cooperative Learning in the Secondary Math Classroom

Rick Weaver

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Dr. Suzanne McFarland, Advisor

Dr. Suzanne McFarland, Coordinator
Master of Arts in Education Program
Chair, Division of Education
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# Table of Contents

## Chapter I
- Introduction .......................................................... 1
- Statement of Problem ............................................... 1
- Justification ............................................................ 1
- Definitions of Terms .................................................. 2
- Limitation and Appropriate Use of Results .................. 2

## Chapter II
- Review of Related Literature .................................... 3

## Chapter III
- Methods and Procedures .......................................... 12
- Participants ............................................................ 12
- Treatment/Intervention ............................................. 12
- Instruments/Protocols .............................................. 14
- Procedures ............................................................. 16
- Timeline ................................................................. 18
- Data Analysis .......................................................... 18

## Chapter IV
- Results ..................................................................... 20

## Chapter V
- Discussion ............................................................... 38
- Meaning of Findings ................................................ 38
- Recommendations .................................................... 40
Summary ............................................................................................................. 40

Conclusion ......................................................................................................... 41

Reference List .................................................................................................... 42

Appendices

A .......................................................................................................................... 43

B .......................................................................................................................... 44

C .......................................................................................................................... 46

D .......................................................................................................................... 48

E .......................................................................................................................... 50

F .......................................................................................................................... 52
CHAPTER 1: INTRODUCTION

Introduction

As a secondary teacher it was the researchers ambition to implement cooperative learning into the secondary math classroom. Lecture based learning was no longer effective in reaching every student. The researcher wanted to discover the benefits of cooperative learning and incorporate it into the secondary math classroom.

Statement of the Problem

The purpose of this project was to discover the benefits of implementing cooperative learning strategies in the secondary math classroom. The research questions that guided the project were:

1. How did the professional literature define cooperative learning?
2. What strategies did the professional literature suggest to implement into the classroom?
3. What did the professional literature indicate were the benefits of cooperative learning (CL)?
4. What are the benefits of incorporating CL in the secondary math classroom?

Justification

The researcher chose cooperative learning in the secondary math classroom as an alternative to the traditional lecture based learning method. The researcher had discovered that lecturing was no longer an effective way to reach every student. The researcher wanted to find another approach that was effective in involving every student in the classroom. It was the desire of the researcher that this project would benefit other secondary math teachers.
Definition of Terms

**Benefits** = Something that promotes or enhances the students well being.

**Cooperative Learning** = The instructional use of small groups through which students work together to maximize their own and each other’s learning.

**Motivate** = To provide with an incentive

**Secondary Math Student** = Any student that is in grades 9 – 12

**Strategies** = A plan of action intended to accomplish a specific goal.

Limitations and Appropriate Use of Results

There were several limitations that affected the results of this project. The class was of medium size with about 18-20 students. The class was 95% Caucasian students. The researcher had a limited amount of time as the project was done during a 9 week period. These limitations may have influenced the results and therefore, may not be applicable to all classes.
CHAPTER II: REVIEW OF LITERATURE

The purpose of this project was to discover the benefits of implementing cooperative learning strategies in a secondary math classroom. The research questions that guided the project were:

1. How did the professional literature define cooperative learning?
2. What cooperative learning strategies did the professional literature suggest to implement into the classroom?
3. What did the professional literature indicate were the benefits of cooperative learning (CL)?
4. What are the benefits of incorporating CL in the secondary math classroom?

How did the Professional Literature Define Cooperative Learning?

In order to answer question one, a review of literature was conducted. Panitz (1999) defined cooperative learning (CL) as students working in groups to accomplish a task successfully. The author stated that this included all learning strategies in which students worked in groups to accomplish specific learning objectives.

When CL was implemented in the classroom, teachers needed to understand how to group the students. Watson and Marshall stated (as cited in Baer, 2002) that CL was most often associated with heterogeneous grouping. The authors also stated that “heterogeneous grouping of students in CL is so commonly accepted that it is often included as part of the definition of CL.” (292). The authors also discussed that homogeneous groups were also possible. Tobias explained (as cited in Baer, 2002) that elementary schools commonly used heterogeneous grouping where secondary schools used more homogeneous grouping. Loveless and Tobias discussed (as cited in Baer,
2002) that grouping students based on past achievement was considered a controversial practice even though high achieving students generally achieved more. The problem with grouping based on achievement was that low achieving students had varied successes (Baer, 2002).

Nastasi (1991) reported that there were three types of CL groups, which included team learning, expert groups, and collaborative task completion. She stated that in team learning, the students were directed to learn assigned material and also assist team members in learning the material for later recall. However, she stated that the expert group approach allowed students to be responsible for the teaching. An expert student taught the material to the other students in the group. Finally, Nastasi indicated that when students were given an assignment such as a worksheet, students were put in a group and completed the assignment in a cooperative manner. This approach was called collaborative task completion.

Baloche (1998) stated that the teacher should assign specific roles to each student in the group. The author stated that there were nine different roles that could be assigned. The first role was the checker and his/her job was to make sure that everyone understood the material. The second role was the scout and he/she was responsible for asking help of other groups. The third role was the timekeeper and he/she needed to keep the group on task. The fourth role was the active listener and it required that the student repeat what had been said. The fifth role was the questioner and he/she needed to ask questions and get opinions from the group members. Baloche continued with the sixth role, which was the summarizer, and he/she needed to pull everything together so the presentation went smoothly. The seventh role was the encourager and he/she had to make sure that all
participants were enthused about the material. The eighth role was the materials manager and he/she needed to collect all the material for the group. Finally, the last role was the reader and he/she needed to read and present the material to the group.

In order for cooperative learning to be effective two goals needed to be met. Goal one was for someone to assume the leadership role, and goal two was for every student to participate equally (Gettinger and McManus, 2001). Furthermore, Johnson and Johnson (as cited in Nastasi, 1991) remarked that these goals were met when students had mutual goals, developed a division of labor, and had specific roles for each member of the group.

Cooperative Learning was a learning strategy where students worked together to master material that was originally presented by the teacher (Slavin, as cited in McManus and Gettinger, 2001). CL had three types of groups. The groups included team learning, expert groups, and collaborative task completion (Nastasi, 1991). In addition, there were also nine different roles that the teacher should assign (Baloche, 1998). Baloche commented that these nine roles were checker, scout, timekeeper, active listener, questioner, summarizer, encourager, materials manager, and reader. Finally, CL had two goals that needed to be met. Gettinger and McManus (2001) stated that the two goals were that someone became the group leader and every student participated equally.

What Cooperative Learning Strategies did the Professional Literature Suggest to Implement into the Classroom?

In order to answer question two, a review of literature was conducted. The selection of which cooperative strategy to use should be based on its usefulness towards the topic (Putnam, 1997). Furthermore, the professional literature suggested several

The first cooperative learning strategy that Johnson, Johnson, and Holubec (1994) suggested was the jigsaw method. The authors indicated that the jigsaw method was divided into 5 parts. The first part was to divide the class into cooperative groups. Each member in the group was responsible for one part of the assignment. The second part was called the preparation pairs. Every member that was assigned a specific part met with the other members of the other groups to discuss the material. The authors pointed out that they have two goals in this part. They want to become an expert on their material and they needed to plan how to teach the material to the other members of the group. The third part was the practice pairs. The members got together with other members to discuss the best ways to teach the material. The final step as discussed by Johnson, Johnson, and Holubec was to get back into their cooperative groups and accomplish two important tasks, the first was to teach their area of expertise and the second was to learn the material being taught from the other members.

The second cooperative learning strategy that the literature suggested was the think-pair-share method. Putnam (1997) stated that the teacher gave the students a topic to think about to themselves. Then the students were paired up with another student to discuss the material. Finally, the students shared their ideas with the class.

The third cooperative learning strategy that Putnam (1997) suggested was the numbered heads together. The author pointed out that the class was divided into groups, and the teacher asked each group a question. Next the students consulted with everyone
in the group to make sure they knew the answer, and then the teacher called upon one student to answer the question (Putnam, 1997).

The fourth cooperative learning strategy that Putnam (1997) suggested was the pair's check. The author explained that first the student's worked in pairs within a group of four. Then the students work on problems and they alternated, one solved the problem while the other students coached them. Finally, after every two problems, the students checked with the other group to see if they had the same answers (Putnam, 1997).

The final strategy, as suggested by Bol and Whicker (1997), was known as the student teams – achievement divisions (STAD). Slavin (as cited in Bol and Whicker, 1997) remarked that this strategy had group competition along with cooperation within the groups. Bol and Whicker (1997) explained that every member of the group was required to learn the material since point accumulation depended on each individual group member's performance. The authors pointed out that the team score was based on each individual's improvement on quiz scores.

The literature suggested five cooperative learning strategies to implement into a secondary math classroom. The cooperative strategies were the jigsaw method, the think-pair-share method, the numbered heads together, the pair's check, and the student teams – achievement divisions (STAD). Putnam (1997) stated that cooperative learning strategies helped students achieve success and maximized their participation.

**What did the Professional Literature Indicate were the Benefits of Cooperative Learning?**

In order to answer question three, a review of literature was conducted. In chapter one, benefits of cooperative learning were defined as something that promoted or
enhanced the students well being. The professional literature that was reviewed discussed several benefits of cooperative learning in a secondary math classroom.

The first benefit of cooperative learning that was discussed by Johnson and Johnson (as cited in Panitz, 1999) was the enhancement of the student’s self-esteem. The authors stated that when students helped each other to achieve their goal, it raised the performance level of each group member. In addition, Panitz (1997) indicated that when groups take ownership they came together as one to achieve the desired goal. The author remarked that this was very important for students who had a history of failing. In addition, Johnson and Johnson (as cited in Gettinger and McManus, 2001) indicated that students felt liked and supported by students when cooperative learning was implemented in their classroom. Nastasi (1991) also reported that cooperative learning increased personal support in the classroom and decreased feelings of alienation. Furthermore, Slavin (as cited in Gettinger and McManus, 2001) added that when students feel liked by their peers and have success in the classroom their self-esteem will rise.

The second benefit of cooperative learning that was discussed by Panitz (1997) was the reduction of classroom anxiety. The author pointed out that in a traditional classroom when a teacher called on a student and the student got the answer wrong, the student felt singled out. Panitz then stated that during cooperative learning groups the whole group was responsible for the answer and thus eliminated one student from feeling singled out. When a mistake was made it became a teaching moment instead of singling out an individual student for a mistake (Panitz, 1999). When classroom anxiety was lessoned, Dishon and O’Leary (as cited in Gettinger and McManus, 2001) reported that
students were more likely to share information and ideas, ask questions, follow directions, and stay on task.

The third benefit of cooperative learning was the development of positive attitudes between students and teachers (Panitz, 1999). Panitz indicated that participation levels increased and involvement between the teacher and the cooperative groups became personal. In addition, Panitz also stated that teachers became more familiar with students' behaviors because they were given more time to explain their thoughts and actions within the cooperative group setting. Cooperative learning promoted more active involvement in learning, and interactions between students and teachers increased (Nastasi, 1991). Nastasi pointed out that cooperative learning improved student's attitudes towards math, it improved their attitude about their teacher, it improved their learning in all areas, and most importantly it improved how they thought about learning.

The fourth benefit of cooperative learning as indicated by Panitz (1999) was that it sets high expectations for both the students and the teacher. The author reported that when students were responsible for their learning as well as other students learning their expectation level of learning was increased. According to Nastasi (1999) the use of CL raised expectations the more times cooperative learning was used in the classroom. Students were encouraged to engage in conversation, which was more likely to enhance motivation and attitudes toward learning. When motivation and attitudes toward learning were changed, student's grades began to increase (Putnam, 1997). Baloche (1998) stated that in order to raise the expectation level of a classroom the teacher needed to empower all students by giving them responsibilities within the cooperative group. The author also remarked that by creating instruction that required discussion and was not easily solved,
all types of students would learn. Finally, she explained that if the teacher assigned easy tasks, which required little discussion, then some students would be held back from learning.

The final benefit of cooperative learning was the development of social interaction skills by the students (Panitz, 1999). Johnson, Johnson, and Holubec (as cited in Panitz, 1997) remarked that social skills were needed in order to work cooperatively. Cooperative learning encouraged students to verbalize their thoughts, which in turn improved their achievement (Gettinger and McManus, 2001). In addition, Gettinger and McManus (2001) explained that since students were required to discuss class material, provide support, give feedback, and encourage other group members, social skills were developed. Furthermore, Webb (as cited in Gettinger and McManus, 2001) reported that two verbal processes occurred. The two processes were giving help and receiving help. When a student gave help it could have been an elaborate explanation or simply the answer. The author indicated that when students were giving directions from the teacher that they didn’t understand, usually other students could explain it in words that made more sense to the student. The students also learned to work together, resolved conflicts among the group, and learned to respect the opinions of others in the group (Bol and Whicker, 1997).

The literature discussed five benefits of cooperative learning in a secondary math classroom. The benefits were increased self-esteem, the reduction of classroom anxiety, the development of positive attitudes between students and teachers, increased expectation levels, and the development of social interaction skills (Baloche, Bol and Whicker, Gettinger and McManus, Nastasi, and Panitz, 1998, 1997, 2001, 1991, 1997).
Conclusion

A review of literature was conducted in order to define cooperative learning, identify what cooperative learning strategies were suggested to implement in a secondary math classroom, and discover the benefits of cooperative learning. The literature defined cooperative learning as group work where students worked together to understand the material presented by the teacher (Johnson and Johnson, 1990). In addition, the literature suggested that cooperative learning had two goals. The first goal was for someone to become the leader and the second goal was for every student to participate in the group (Gettinger and McManus, 2001). Next, the literature suggested five strategies to implement into a secondary math classroom. The cooperative learning strategies were the jigsaw method, think-pair-share-method, numbered heads together, pair’s check, and student teams – achievement divisions (STAD) (Bol and Whicker, Johnson, Johnson, and Holubec, and Putnam, 1997, 1994, 1997). Finally, the literature listed 5 major benefits of implementing cooperative learning in a secondary classroom. The first benefit was the enhancement of student’s self-esteem. The second benefit was the reduction of anxiety in the classroom. The third benefit was the development of positive attitudes between students and teachers. The fourth benefit was that cooperative learning set high standards for both the students and the teacher. The final benefit of cooperative learning was the development of social skills by the students (Gettinger and McManus, Nastasi, and Panitz, 2001, 1999, 1999).
CHAPTER III: METHODS AND PROCEDURES

The purpose of this project was to discover the benefits of implementing cooperative learning strategies in the secondary math classroom. The research questions that guided the project were:

5. How did the professional literature define cooperative learning?
6. What strategies did the professional literature suggest to implement into the classroom?
7. What did the professional literature indicate were the benefits of cooperative learning (CL)?
8. What are the benefits of incorporating CL in the secondary math classroom?

Participants

Sixty-one secondary high school students in grades 9-12 participated in the study. The students were placed into the algebra classes by the guidance office based on their schedule availability. The school was located in the Midwest and was a small rural community. The study included 29 males and 32 girls.

Intervention

The intervention was the implementation of cooperative groups in the secondary math classroom. The cooperative group work took place twice a week. The members of the group were assigned specific roles, and members had to successfully fulfill the requirements of their roles. The group members were randomly chosen and the members of each group changed during the study. The students were responsible for completing the assigned task in their groups and presenting it to the class. Some of the assignments included problem solving, OGT writing prompts, and vocabulary. The students were
taught chapters 2-5 using several cooperative learning strategies. The strategies included the jigsaw method, think-pair-share, numbered heads together, pair's check, and students teams-achievement divisions (STAD). The researcher used cooperative learning strategies for two of the algebra classes. The other class was used to compare results in order to determine what benefits the other two classes gained for the cooperative learning strategies.

The first cooperative learning strategy that was implemented into the secondary math classroom was the think-pair-share method. Students were asked to take out a piece of paper and answer three questions about rational numbers. The students were then paired up with another student to discuss their answers. They were instructed to discuss the answers for five minutes and decide between the two of them what the right answers for each question were. When the students were done with the discussion, the class went over the answers together. The questions were used to help teach lesson 2-1 on adding and subtracting rational numbers.

The second cooperative learning strategy that was implemented was the numbered heads together method. The students were put into groups of four and then split into pairs within each group. They were given a worksheet on probability and odds to work on with their partner. The worksheet was divided into 3 sections and the students were instructed to stop after each section and check their work with the other group. While they were checking their answers they were instructed to discuss how they came up with their answers and figure out which group had the right answer. When they were completed with the first section they were instructed to do the same procedure for the other two sections.
The third cooperative learning strategy that was implemented was the jigsaw method. The class was split into four different groups. Each group was in charge of learning a section of the lesson. Also, each member was given a number from one to four. The four sections of the lesson were graphing reflections, translations, dilations, and rotations. After each group had mastered its section, it was paired up with members of another group. Each member was then in charge of teaching the others his/her section of the lesson. When everyone had finished they had all learned to graph reflections, translations, dilations, and rotations.

The final cooperative learning strategy that was implemented was students teams-achievement divisions (STAD). Each student in the class was given a numerical ranking with one being the smartest students. The rankings were on a scale of 1-4. The students never saw these rankings. The class was then divided into even groups for the duration of Chapter 5. One student in each group was assigned a specific role. The roles included team leader, task manager, timekeeper, and note taker. The students worked in these groups 2-3 times a week. It was the responsibility of each group member to make sure that everyone in the group understood each lesson. At the end of the chapter, the students took the chapter test and the group with the highest average received bonus points.

*Instruments / Protocols*

The data collected for this study was gathered through an attitude survey, which was given prior to the implementation of cooperative learning and following the use of cooperative learning. The survey was administered to determine if attitudes about math improved after the use of cooperative learning took place in the classroom.
Attitude Survey

The students were asked to fill out a survey about how they felt about math. The survey was given to the students at the beginning of the school year. The survey was then given to the students at the end of the study to see if their views about math had changed. The survey contained 5 questions that probed the students about their attitude towards math. The survey also attempted to look at student's successes in math and their attitudes towards math. See Appendix A for a copy of the survey.

The first question on the survey asked the students what was their favorite subject. The student had six choices. The choices included math, health, history, foreign language, science, and gym.

The second question asked the students if they liked math. The students had a yes, no choice.

The third question asked the students how math could be more enjoyable? Students had five choices. The choices included rewards, do nothing special, group work, less homework, and no assigned seats.

The fourth question asked the students if they had success in math. The students had a yes, no choice.

The final question asked the students if they had anxiety towards their math class. The students had a yes, no choice.

Chapter 1 Test

The students were instructed using the lecture-based method for chapter 1. Chapter 1 was review and was used as a measuring tool to determine where the students
were mathematically. The content included writing algebraic expressions, using the
distributive property, and simplifying expressions. Since the content was review there
was no cooperative learning involved. See Appendix B for a copy of Chapter 1 test.

Chapter Tests

Each chapter test contained 25 questions and each question was worth four points.
The test was provided from the book, as the researcher did not generate the test. The
researcher graded each problem in the same manner. If the student did not attempt the
problem or provided work that wasn’t remotely close to the solution, the student received
no points. If the student demonstrated some knowledge of solving the problem, but failed
to solve it correctly, they were given 1 or 2 points depending on how much of the
problem they solved correctly. If the problem was solved almost completely right but the
student failed in solving one minor part, then the student received 3 points. Finally, if the
student solved the problem correctly, the student received 4 points.

Procedures

The first step in starting the study was that the researcher met with the principal to
discuss the implementation of cooperative learning in the math classroom. After
receiving permission to go forward with the study, the researcher examined the classes
and decided which classes to choose for the study.

Attitude Survey

The students of all three classes were asked to fill out the survey during the first
twenty minutes of class. The students were informed why they were filling out the
survey and why they needed to answer it honestly. When the students were finished with
the survey they were instructed to turn it over and wait for the teacher to come around
and pick it up. When all the surveys were collected the teacher put them in a folder to compare them with the second survey at a later time.

Chapter 1 Test

During the teaching of Chapter 1, there was no cooperative learning. Every lesson included the teacher lecturing while the students took notes. At the end of every lesson,

The teacher assigned specific problems for homework. The next day the class would go over the answers to the homework and then the teacher would teach the next lesson. This continued throughout Chapter 1.

Chapter Tests

The researcher administered the tests for Chapters 2-5 in the same manner. Each class was given review problems the day before the test. The classes were grouped together to work on the review. Each group was responsible for going over the review problems and making sure every student in the group understood each problem. The researcher would walk around to each group and ask students questions regarding the review problems to make sure that every student understood the problems.

The following day when the test was given, students were required to hand in their review sheet prior to taking the test. Each student was then instructed to put all their books away and clear their desks. The only things that were permitted for the test were a calculator, a pencil, and a scratch sheet of paper. The students were given the whole period to take the test.
After Chapter 5 was completed, every student took the Chapter 5 test separately. The group's grades were averaged and the group with the highest average received 5 bonus points on their chapter 5 test.

**Timeline**

The researcher talked to the principal to obtain permission during the last week of August. The students began working in cooperative groups at the beginning of September. They continued to work with cooperative learning strategies at least once a week and sometimes twice a week until the end of November. The students were then asked to complete a follow up survey to the first one that they took before the cooperative learning took place. The follow up survey was given to the students during the first week of January.

**Data Analysis**

*Attitude Surveys*

The researcher analyzed the data obtained from the attitude survey. The researcher used the same procedure for analyzing both attitude surveys. The survey was broken down into 5 questions and each question was analyzed separately. The researcher also analyzed each question for each class separately.

For question one, the student's answers were tallied to determine their favorite subjects. Question two was tallied by computing the percentage of yes, no answers. For question three, the student's answers were tallied to determine how math could be more enjoyable. Question four was tallied by computing the percentage of yes, no answers. Finally, question five was also tallied by computing the percentage of yes, no answers.
Chapter I Test

The researcher also analyzed the data obtained from test #1. The researcher analyzed each class separately in the same way. The researcher analyzed the average test score, the highest score, and the lowest score for each class. The researcher then calculated the grades by percentages for each class separately.

Chapter Tests

The researcher also analyzed the data obtained from the other chapter tests. The researcher analyzed each class separately but analyzed each class the same way. The researcher analyzed the average test score, the highest score, and the lowest score for each class. The researcher then calculated the grades by percentages for each class separately.

Conclusion

Chapter 3 explained the procedures and methods that were used to research question #4. Research was done to discover the benefits of cooperative learning in the secondary math classroom. The chapter took an in-depth look at the participants, treatment/intervention, instruments/protocol, procedures, timeline, and data analysis. The results were discussed in Chapter 4.
Chapter IV: Results

The purpose of this project was to determine the benefits of implementing cooperative learning strategies in a secondary math classroom. The research questions that guided the project were:

9. How did the professional literature define cooperative learning?

10. What cooperative learning strategies did the professional literature suggest to implement into the classroom?

11. What did the professional literature indicate were the benefits of cooperative learning (CL)?

12. What are the benefits of incorporating CL in the secondary math classroom?

Chapter III described the methods and procedures that were utilized to gather the data for chapter IV. The data was obtained from 2 sources. The first source was an attitude survey, which was given at the beginning and the end of the project. The second resource was the chapter tests. The students took five chapter tests that were used to obtain the results.

Pre and Post Attitude Survey

The survey contained five questions that the students were asked to answer. The survey was designed to determine students' attitudes about math as compared to other school subjects. Two of the questions were open ended and the students could answer them any way, and three of the questions were yes, no answers.

Question 1

The first question asked each student to state his or her favorite subject. Their choices were math, health, history, English, Spanish, science, and gym.
Three classes participated in the study. Period 2 had 15 students participate in the survey and 40% listed history as their favorite subject. Health was 2\textsuperscript{nd} with 21%, followed by math, Spanish, and science at 13%. Gym and English received 0% from period 2 students. Period 4 had 16 students participate in the survey and math was the favorite subject for 32% of the students. Science was listed by 25% of the students, while 19% chose history. Health, Spanish, English, and gym were listed by 6% of the students. Period 7 had 16 students participate in the survey and 37% chose history as their favorite subject. Nineteen percent listed science as their favorite, while 13% of the students listed Spanish and gym. Math, health, and English were listed by 6% of the students.

The students generated 13 different responses through participation in the second administration of the post attitude survey. During the first attitude survey they only generated seven different responses. The choices that they listed were math, health, art, English, history, science, business, yearbook, survival skills, psychology, French, gym, and none. Math and history received 25% of Period 2's vote as their favorite classes. Health, art, and business each gained 12% of the votes, while English and science received 7% of the votes. In the Period 4 class 22% of the students claimed they didn’t have a favorite class. Sixteen percent of the students felt that math was their favorite subject. Gym, history, art, and business claimed 11% of the students’ votes as their favorite classes, while health, Spanish, and science received 6% of the students’ votes. Thirty-three percent of the Period 7 students felt that history was their favorite subject. Math was the 2\textsuperscript{nd} highest total with 22% of the votes. Science and survival skills each had 10% of the students listing them as their favorite subjects, while yearbook, business, psychology, French, and English had 5% of the votes.
The data showed that most students’ favorite classes changed after the study was completed with math gaining from the first survey to the second.

**Question 2**

The second question asked the students if they liked math. The students responded by simply answering yes or no. Fifty three percent of the students in Period 2 reported that they liked math, while 47% didn’t. Period 4 had 78% that liked math and only 22% that didn’t. Finally, 59% of the students in 7th period responded that they liked math, with 41% responded with a no.

The student’s opinion of math changed according to the second survey. Sixty three percent of the students in Period 2 liked math, while 37% said they didn’t like math.
In the Period 4 class 72% of the students reported liking math while 28% didn’t, and in the Period 7 class 63% of the students liked math and 37% of the students didn’t like math. The percentage of students who liked math increased in Periods 2 and 7 but declined slightly in Period 4. Figure 2 shows the percentage of each classes yes vote for survey I and II.

![Bar Chart](image)

**Figure 2.** Comparison of students who liked math before and after the survey.

The data shows that two of the three classes liked math more after the survey was completed.

**Question 3**

The third question asked the students how math could be more enjoyable. The answers were categorized into 5 choices. The choices were rewards, do nothing, more
group work, less homework, and no assigned seats. Thirty eight percent of the students in Period 2 believed that more group work or less homework would make math more enjoyable. The rest of the class was evenly divided between rewards, do nothing, and no assigned seats, each of which received 8% of the class. Period 4 also believed that more group work or less homework would lead to a more enjoyable math class. They both earned 28% of the class. Rewards and do nothing had 22% of Period 4 thinking that would make math more enjoyable. Zero percent of the class thought that no assigned seats would make a difference. Thirty three percent of Period 7 believed that more group work or doing nothing was the secret to making math more enjoyable. Twenty eight percent of the class believed that less homework would do the trick and 6% thought that rewards were the way to go. Zero percent of the class thought that not having assigned seats would make math more enjoyable.

The eleven suggestions for making math more enjoyable from the second administration of the survey were group work, rewards, less homework, nothing, more challenges, have math at a different period, play games, have a different book, have a different routine, make math easier, and finally, do more individual work. Forty four percent of the students in Period 2 thought that doing more group work would make math more enjoyable. Twenty five percent of the students thought that there was nothing that could be done to make math more enjoyable. Nineteen percent of the students chose less homework, while 6% thought rewards or more challenges would help. Twenty eight percent of the students in Period 4 thought more group work was the answer with 22% of them saying that nothing could be done. Less homework was selected by 18% of the students, while 11% wanted a different routine. The other two suggestions were to have
math class during a different period or use a different book, each of these were chosen by 6% of the students. Thirty seven percent of the students in Period 7 felt that less homework was the key to making math more enjoyable. Thirty two percent of the students thought that math was too hard and it needed to be made easier to enjoy it. Twenty one percent of the students wanted more group work, while 10% wanted more individual work. Figure 3 displays the common responses from the first survey and the second one.

![Figure 3](image)

*Figure 3. Comparison of the five common choices for how to make math more enjoyable before and after the use of cooperative learning.*

The data shows that the opinions of the students changed from the beginning of the study and after the study. The only thing that stayed the same was that both surveys
showed that students thought that group work was the best way to make math more enjoyable.

*Question 4*

The fourth question asked the students if they had success in math. The students were to respond with a yes or no answer. Sixty percent of the students in Period 2 responded that they felt they had success in math, while 40% felt that they didn’t. In Period 4, 67% of the students answered yes, while 33% answered no. Finally, in the Period 7, 47% of the students felt that they were successful in math and 53% felt that they weren’t.

For the post survey 94% of the students in Period 2 felt that they had success in math, while only 6% felt that they didn’t. The average was a little lower in Period 4 with 72% of the students feeling that they were successful in math and 28% believing they weren’t. The percentage dropped even more in Period 7 with 58% of the students feeling they had success and 42% felt that they didn’t have success in math. Figure 4 compares the percentage of yes answers for both surveys by class.
Figure 4. Comparison of the percentage of yes responses on “Did you have success in math?” for each class before and after the study.

The data suggests that students felt they had more success in math after the study was completed. Each class had a higher percentage in the post-survey than they did in the pre-survey with Period 2 showing the most change.

Question 5

The fifth question asked the students if they had anxiety towards math. They were asked to respond by answering either yes or no. Seventy three percent of the students in Period 2 felt that they had no anxiety towards math, while 27% answered that they did have anxiety towards math. Eighty-three percent of the Period 4 students felt that they had no anxiety towards math, while 17% felt that they did have anxiety towards
math. In the Period 7 class, 71% of the students answered no to having anxiety towards math, while 29% answered yes.

The post-survey showed that 75% of the students in Period 2 reported having no anxiety towards math, while 25% of the students felt they did have anxiety. The percentage was much higher in Period 4 with 94% of the students responding that they felt no anxiety towards math. Six percent of the students claimed they did feel anxiety. Seventy nine percent of the students in Period 7 felt no anxiety and 21% said they did have anxiety towards math. Figure 5 compares the results of the no responses to the question "Do you have anxiety towards your math class?" for both surveys.

![Bar chart comparing no responses to math anxiety for Period 2, Period 4, and Period 7 on both surveys.](image)

**Figure 5.** Comparison of the percentages of no responses to the question "Do you have anxiety towards your math class?" for each class on the pre and post surveys.
The data suggests that the students felt less anxiety towards math after the study was completed. Every class had a higher percentage of no responses in the second survey.

Chapter 1 Test

Chapter 1 test was given after each lesson in the chapter was taught. The lecture method was used to teach Chapter 1. Chapter 1 was a review of Pre-Algebra. The researcher used the Chapter 1 test as a measuring stick to determine where each student’s mathematical skill level was. The average score for Period 2 was an 84%. The highest score was a 98% and the lowest score was a 68%. Seven students received A’s, 5 students received B’s, 4 students received C’s, and 1 student got a D. The average score for Period 4 was an 85%. The highest score was a 97% and the lowest score was a 65%. Seven students in the class received A’s, 8 students received B’s, 4 received C’s, and 1 student got a D. The average score for Period 7 on the chapter 1 test was a 79%. The highest score was a 96% and the lowest score was a 54%. Five of the students received A’s, 3 students got B’s, 7 of the students received C’s, 2 got D’s and only 1 student failed the test. Figure 6 shows the percentages of each class’s average score on the Chapter 1 test.
The data suggested that Period 2 and 4 class members had a good understanding of the review material and period seven lacked in some basic skills.

**Chapter Tests**

Chapters 2-5 were taught using a variety of cooperative learning strategies. The researcher wanted to see what benefits occurred from using cooperative learning. The researcher used cooperative learning for Periods 2 and 7, and maintained the lecture-based method for Period 4.
Figure 7. Comparison of the average score on the Chapter 2 test for the three classes.

The data suggested that Period 2 benefited more from the cooperative learning strategies than Period 7 did. It also suggested that Period 4 and Period 7 are achieving at about the same level in spite of different instructional strategies.

Chapter 3

Chapter 3 test was given after each lesson in the chapter was taught and the students had a review day. The average score for Period 2 was a 69%. The highest score was an 88% and the lowest score was a 35%. Zero of the seventeen students received A’s for Chapter 3. Seven of the students received B’s, 6 of the students received C’s, 1 student got a D, and 4 students failed the test. The average score for Period 4 was a 71%. The highest score was an 87% and the lowest score was a 0%. Nobody in period 4
received an A on the test. Ten of the students received B’s, 4 students received C’s, 4 students got D’s, and 3 students failed the test. The average score for Period 7 was a 56%. The highest score was a 90% and the lowest score was a 0%. One of the students received an A on the test. Six of the students received B’s, 3 of them received C’s, 1 student got a D, and 8 students failed the test. Figure 8 shows the average percentages from each class period for the Chapter 3 tests.

![Chapter 3 Average Scores Graph](image)

**Figure 8.** Comparison of average scores for the three classes on the Chapter 3 test.

The data suggested that Period 4 scored better without the use of cooperative learning. The data also suggested that Period 2 is still performing at an acceptable level. Period 7 is still not performing at an acceptable level, even with the use of cooperative learning being implemented.
Chapter 4

Chapter 4 test was given after each lesson in the chapter was taught and the students had a review day. The average score for Period 2 was an 88%. The highest score was a 100% and the lowest score was a 61%. Nine of the students received A’s on the test. Five of the students received B’s, 1 student received a D, and 1 student failed the test. The average score for Period 4 was also an 88%. The highest score was a 99% and the lowest score was a 79%. Ten of the students received A’s, 10 students also received B’s, and 2 students got C’s. The average score for Period 7 was a 79.6%. The highest score was a 97% and the lowest score was a 65%. Seven of the eighteen students received A’s. Six of the students received B’s, 4 of the students received C’s, and 1 student got a D. Figure 9 displays the average percentage for each class’s Chapter 4 test scores.
The data suggested that periods 2 and 4 are performing at an exceptional rate. Period 2 was still being taught the chapters through cooperative learning strategies and Period 4 was still being taught by the lecture-based method. Period 7 was still performing at a level that was lower than the other two classes.

Chapter 5

Chapter 5 test was given after each lesson in the chapter was taught and the students had a review day. The students were assigned to a cooperative group that they worked with each day. The average for Period 2 was a 76.6%. The highest score was a 99% and the lowest score was a 5%. Four of the sixteen students received A’s on the test. Five of the students received B’s, 3 of the students received C’s, 3 students got D’s,
and 1 student failed the test. The class average on the Chapter 5 test for Period 4 was a 65.5%. The highest score was a 97% and the lowest score was a 0%. Two of the twenty-two students received A’s on the test. Nine of the students received B’s, 2 students received C’s, 3 students got D’s, and 6 students failed the test. The average for Period 7 was a 65%. The highest score was a 106% and the lowest score was a 36%. Two of the eighteen students received A’s on the test. Four of the students received B’s, 3 students received C’s, 2 students got D’s, and 7 students failed the test. Figure 10 shows the percentages of each class score for the Chapter 5 tests.

Figure 10. Comparison of the average score for the three classes after the chapter 5 test.
The data suggested that Period 2 is learning through cooperative learning better than Periods 4 and 7. The data also suggests that Period 7 is still achieving at a low level in spite of the cooperative learning.

Conclusion

Chapter 4 took an in-depth look at the results of both attitudinal survey’s and the five chapter tests. First, the chapter compared both attitudinal surveys to see if cooperative learning changed students’ opinions about math. Next, the chapter compared the three classes studied using the Chapter 1 test. Finally, Chapter 4 displayed the results of the other chapter tests and compared each of the three classes. The following chapter will discuss the meaning of findings, a summary, and recommendations to further enhance the study.
Chapter V: Discussion

The purpose of this project was to determine the benefits of implementing cooperative learning strategies in a secondary math classroom. The research questions that guided the project were:

13. How did the professional literature define cooperative learning?

14. What cooperative learning strategies did the professional literature suggest to implement into the classroom?

15. What did the professional literature indicate were the benefits of cooperative learning (CL)?

16. What are the benefits of incorporating CL in the secondary math classroom?

Chapter III described the methods and procedures that were utilized to gather the data for chapter IV. The data was obtained from 2 sources. The first source was an attitude survey, which was given at the beginning and the end of the project. The second resource was the chapter tests. The students took five chapter tests that were used to obtain the results.

*Meaning of Findings*

The finding that the percentage of students who liked math increased in Periods 2 and 7 but decreased for Period 4 supports the fact that students enjoy cooperative learning strategies over the traditional lecture based method. In this study, Periods 2 and 7 were introduced to cooperative learning strategies while Period 4 had the same routine everyday. They went over the homework, took notes over the next lesson, and worked on their homework quietly. The students complained to the researcher numerous times that
the class was boring. The students in Periods 2 and 7 commented numerous times to the researcher that this was the most fun they had ever had in a math class.

The finding that the percentage of students who felt they had success in math increased in every class doesn't support the fact that cooperative learning led to the increased success rate. Even though the percentages increased more for Periods 2 and 7 than it did for Period 4, all three classes still increased. The finding shows that the lecture based method can still be an effective means for instruction.

The finding that Period 2 had an average score of an 84% on the chapter 1 test, Period 4 had an average score of 85%, and period 7 had an average score of 79% showed that the classes didn’t start on an even level. Periods 2 and 4 had an advantage over Period 7 since they had more students who had a higher level of math knowledge. This higher level of knowledge continued to show as each chapter test was taken.

The finding that Period 2 had an average score of 77% on the Chapter 5 test, Period 4 had an average score of 65.5% and period 7 had an average score of 65% showed that Period 2 benefited the most from the cooperative learning and Period 7 still continued to have the lowest average score. The researcher felt that Period 4 was falling behind the other classes since they weren’t having cooperative learning implemented into their lessons. For Chapter 5 all three classes were instructed using cooperative learning. The class averages for Period 4 and 7 weren’t what the researcher had hoped for but it is important to know that last year when Chapter 5 was taught using the lecture based method, 80% of the students failed the test and the researcher had to reteach Chapter 5.
Summary

The purpose of this project was to determine the benefits of cooperative learning in the secondary math classroom. The students were given a pre and post attitudinal survey to see if cooperative learning changed their views of math. The surveys were given at the beginning of the project and at the end of the project. Two of the classes that participated in the project were instructed using cooperative learning strategies while the other class was instructed using the lecture based method. The researcher used cooperative learning strategies in the classroom at least two times a week. The researcher used the think – pair – share method, the numbered heads together method, the jigsaw method; the pairs check method, and the students teams achievement division method. The students were evaluated at the end of each chapter by taking the chapter test. The results of both surveys and the chapter tests suggest that the students did benefit from the implementation of cooperative learning.

Recommendations

The researcher concluded that in order for this project to be more beneficial there are several recommendations that need to be made. First, the study needs to span a longer period of time. This particular project was done over a 3-month period. In order for the results to have more meaning, the study should last at least the whole school year. Second, the project should be done throughout the whole math department in this particular school. The study would have more meaning if every teacher in the building used various instructional methods and the findings were compared. Finally, the study needs to be conducted throughout many school systems. This particular study took place
in a rural community where sixty-one students participated. For this study to have any meaning thousands of students need to participate with several different backgrounds.

Conclusion

This chapter explained the findings from Chapter 4. The researcher concluded that cooperative learning benefited the classes that were involved. The students who were exposed to cooperative learning seemed to score higher on the tests and felt better about math after the study. The researcher felt that some recommended changes needed to take place in order to make the project more meaningful.
REFERENCES


Appendix A

Algebra Survey I

Name____________________

1.) What is your favorite subject? Explain.

2.) Do you like math? Explain.

3.) How could math be more enjoyable?

4.) Do you have success in math? Explain.

5.) Do you have anxiety towards your math class? Explain.
Chapter 1 Test, Form 2C

For Questions 1 and 2, write an algebraic expression for each verbal expression.

1. the sum of the square of a number and 34
2. the product of 5 and twice a number
3. Write a verbal expression for $4n^3 + 6$.
5. Evaluate $3w + (8 - v)t$ if $w = 4$, $v = 5$ and $t = 2$.
6. Find the solution of $5b - 13 = 22$ if the replacement set is {5, 6, 7, 8, 9}.
7. Solve $\frac{6 + 3^2(4)}{7 - 1} = y$.
8. Find the solution set for $2(6 - x) < 10$ if the replacement set is {0, 1, 2, 3, 4}.

For Questions 9 and 10, name the property used in each equation. Then find the value of $n$.

9. $5 + 0 = n$
10. $7 + (4 + 6) = 7 + n$

11. Evaluate $4(5 \cdot 1 + 20)$. Name the property used in each step.

12. Rewrite $3(14 - 5)$ using the Distributive Property. Then simplify.

Simplify each expression.

13. $15w - 6w + 14w^2$
14. $7(2y + 1) + 3y$

For Questions 15 and 16, evaluate each expression.

15. $32 + 5 + 8 + 15$
16. $\frac{1}{3} \cdot 4 \cdot 9 \cdot \frac{1}{2}$

17. Identify the hypothesis and conclusion of the following statement.
   *I will attend football practice on Monday.*

18. Find a counterexample for the following statement.
   *If the sum of two numbers is odd, then the two numbers are odd numbers.*
19. The line graph shows the number of students per computer in U.S. public schools. Explain how the graph can be fixed so it is not misleading.

![Graph of Students per Computer in U.S. Public Schools]

Source: World Almanac

Use the table that shows the percent of students enrolled in private schools.

20. Between what two consecutive school years did the percent change the most?

<table>
<thead>
<tr>
<th>School Year</th>
<th>Percent Enrolled</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959-60</td>
<td>16.1</td>
</tr>
<tr>
<td>1969-70</td>
<td>12.1</td>
</tr>
<tr>
<td>1979-80</td>
<td>12.0</td>
</tr>
<tr>
<td>1989-90</td>
<td>11.7</td>
</tr>
<tr>
<td>1999-00</td>
<td>11.3</td>
</tr>
</tbody>
</table>

Source: World Almanac

21. Describe the trend in enrollment in private schools since 1959.

Use the graph that shows temperature as a function of time.

22. Identify the independent and dependent variables.

23. Name the ordered pair at point C and explain what it represents.

For Questions 24 and 25, use the table that shows 2001 airmail letter rates to Greenland.

24. Write the data as a set of ordered pairs.

<table>
<thead>
<tr>
<th>Weight (oz)</th>
<th>Rate ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>4.80</td>
</tr>
<tr>
<td>6.0</td>
<td>4.80</td>
</tr>
<tr>
<td>7.0</td>
<td>5.60</td>
</tr>
<tr>
<td>8.0</td>
<td>6.40</td>
</tr>
</tbody>
</table>

Source: World Almanac

25. Draw a graph that shows the relationship between the weight of a letter sent airmail and the total cost.

Bonus Use grouping symbols, exponents, and symbols for addition, subtraction, multiplication, and division with the digits 1, 9, 8, and 7 (in that order) to form expressions that will yield each value.

a. 6  

b. 7  

c. 9
Chapter 2 Test, Form 3

1. Name the coordinates of the points graphed on the number line.

For Questions 2 and 3, graph each set of numbers.

2. \{..., -5, -3, -1, 1, 3, 5, ...\}

3. \{integers less than -2 or greater than 3\}

4. Evaluate \( |2 + b| - a \) if \( a = \frac{2}{3} \) and \( b = -\frac{1}{9} \).

5. Find the sum of \(-3\frac{1}{6}\) and \(1\frac{2}{3}\).

6. The highest point of elevation in the state of California is 14,494 feet at Mount Whitney. The lowest point is -282 feet at Death Valley. What is the difference in elevation of these two points?

7. Find \(-45 + (-91) + 23\).

8. Find \((\frac{-3}{7}) - (\frac{-6}{5})\).

9. A bank in Los Angeles, California is a 40-story building. The average height of each story is 12.9 feet. How tall is the bank?

Evaluate each expression if \(w = 12\), \(x = -7\), \(y = \frac{3}{4}\), and \(z = -\frac{5}{6}\).

10. \(4x^2 - 3w\)

11. \(xyz - 5yz\)

For Questions 12 and 13, simplify each expression.

12. \(-\frac{6(7 - 11)}{5(-13 + 12)}\)

13. \(\frac{24x - 42y}{6}\)

14. Evaluate \(\frac{(a + b)}{c}\) if \(a = -6.3\), \(b = 20.7\), and \(c = 4.5\).
Use the list that shows the number of silver medals won by 15 countries participating in the 1998 Winter Olympic Games. Source: World Almanac

9, 10, 6, 5, 5, 3, 4, 4, 1, 6, 1, 6, 2, 1, 1

15. Make a line plot of the data.


Use the list that shows the height in meters of the winning high jump for women in the Summer Olympic Games from 1936 to 2000. Source: World Almanac

1.60, 1.68, 1.67, 1.76, 1.85, 1.90, 1.82, 1.92, 1.93, 1.97, 2.02, 2.03, 2.02, 2.05, 2.01

17. Make a stem-and-leaf plot of the data.

18. Which measure of central tendency best describes the data? Explain.

For Questions 19-21, a state lottery game uses ping pong balls numbered 0-39. Balls with the numbers 20, 8, 7, and 1 have already been selected in the weekly drawing.

19. Find the probability of drawing a ball with a number less than 10 as the fifth ball.

20. Find the odds of drawing a ball with a number between 10 and 30 as the fifth ball.

21. Find the probability of drawing a ball with the number 42 as the fifth ball.

22. The probability that an event will occur is 30%. What are the odds that the event will not occur?

23. Find \( \pm \sqrt{\frac{16}{625}} \).

24. Name the set or sets of numbers to which the real number \( \sqrt[4]{36} \) belongs.

25. Write \(-\frac{2}{9}, -\frac{2}{11}, -0.2, -\frac{1}{\sqrt{16}}, -1\) in order from least to greatest.

Bonus Simplify \( \frac{-2(4.9 + 8)}{2.5(6.4 - 4)} \).
For Questions 1-6, solve each equation.

1. \( n + 39 = 12 \)
2. \( w + (-8) = -21 \)
3. \( -6n = 16 \)
4. \( -13 = \frac{n}{4} \)
5. \( \frac{3}{4}h = \frac{-45}{52} \)
6. \( -\frac{a}{6} + 7 = -14 \)

7. If \( x - 5 = 12 \), what is the value of \( x - 9 \)?

8. Translate the following into an equation.
   A number \( x \) is decreased by 45. The result is then divided by 12. Then 20 is added to this new result to give a final result of five times the difference of 32 and the number \( x \).

For Questions 9 and 10, write an equation for each problem. Then solve the equation.

9. Three-fifths of what number equals one?

10. The product of 2 more than a number and 10 is 36 more than 8 times the number. What is the number?

11. Translate the following equation into a verbal sentence.
    \( 5(2x + 3y) = y^2 - 2x^3 \)

12. Solve the following problem by working backward.
    Shyam invested money in the stock market. In the first year, his stock increased 20%. He paid his stock broker $300 and then lost $450. He withdrew $500, and then his remaining investment doubled. Shyam's investment is now worth $7100. How much was Shyam's original investment?

13. Use cross products to determine whether the pair of ratios \( \frac{42}{48} \) and \( \frac{63}{72} \) form a proportion. Write yes or no.

14. Solve the proportion \( \frac{t + 4}{t - 2} = \frac{1}{4} \).

15. A blueprint for a house states that 2 inches represents 8 feet. If the width of a window is 2.5 inches on the blueprint, what is the width of the actual window?
Chapter 3 Test, Form 3 (continued)

For Questions 16-18, solve each equation.

16. $6 - 2y = 7y + 13$

17. $3x - 5(x - 6) = 2(10 - x) + 10$

18. $5(7 - a) - 3(a + 4) - 4 = 4(a - 3) + 7$

19. Solve $ax - n = r$ for $x$.

20. Solve $\frac{4x + t}{r} = s$ for $x$.

21. State whether the percent of change is a percent of increase or a percent of decrease. Then find the percent of change.
   
   original: 75, new: 84

22. A jacket costs $75.00 retail. A warehouse outlet discounts the price by 20%. If the sales tax is 6%, find the final price.

23. Calvin invested $7500 for one year, part at 12% annual interest and the rest at 10% annual interest. His total interest for the year was $890. How much money did he invest at 12%?

24. Two airplanes leave the Atlanta airport at the same time, traveling in opposite directions. One plane travels 30 miles per hour faster than the other. After 3 hours, the planes are 3150 miles apart. What is the rate of each plane?

25. PHYSICS A ball is thrown straight up at an initial velocity of 53 feet per second. In the first 1.5 seconds, it travels 42 feet. The formula $s = \left(\frac{u + v}{2}\right)t$ represents the vertical distance $s$ that an object travels in $t$ seconds, where $u$ represents the initial velocity of the object and $v$ represents the velocity of the object at the end of $t$ seconds. Find the velocity of the ball at the end of 1.5 seconds.

Bonus Paloma Rey drove to work on Wednesday at 40 miles per hour and arrived one minute late. She left home at the same time on Thursday, drove 45 miles per hour, and arrived one minute early. How far does Ms. Rey drive to work?
Determine whether each relation is a function.

16. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

17. 

For Questions 18 and 19, if \( g(x) = x^2 - 2x + 4 \), find each value.

18. \( g(-5) \)

19. \( g(t - 1) \)

20. Determine whether the sequence 7, 11, 15, 19, ... is an arithmetic sequence. If so, state the common difference.

21. Find the next three terms of the arithmetic sequence -25, -22, -19, -16, ...

22. Write an equation for the \( n \)th term of the sequence 3, 12, 21, 30, ... Then graph the first five terms of the sequence.

23. Find the next two items in the pattern.

24. Use the table below that shows the amount of gasoline a car consumes for different distances driven.

<table>
<thead>
<tr>
<th>Distance (mi)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gasoline (gal)</td>
<td>0.05</td>
<td>0.10</td>
<td>0.15</td>
<td>0.20</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Write an equation in function notation for the relationship between distance and gasoline used.

25. Joseph wants to reduce a rectangular photograph that is 3 inches wide and 5 inches tall by a scale factor of 2.5 so it will fit on his report. What will be the dimensions of the new photograph? Use a coordinate system to draw a photograph of the original rectangle and the reduced photograph. Place one corner of each photograph at the origin and for each photograph write the coordinates of the other three vertices.

Bonus Graph the points \((4, -2), (4, 3), (-2, 3)\). Find a fourth point that completes a rectangle with the given three points, then graph the rectangle on the same coordinate plane.
Write the ordered pair for each point shown at the right. Name the quadrant in which the point is located.

1. M
2. N
3. P

Plot each point on a coordinate plane.
4. A(−3, 0)
5. B(2, −2)
6. C(1, 3)

For Questions 7 and 8, determine whether each transformation is a reflection, translation, dilation, or rotation.

7. 
8. 

9. Express the relation {(8, 7), (−2, 0), (1, 0), (4, −5), (−1, 3)} as a mapping. Then determine the domain and range.

10. Express the relation shown in the graph as a set of ordered pairs. Then write the inverse of the relation.

11. Find the solution set for 3x − y = 7, given the replacement set {(2, 1), (−3, −16), (5, 8), (4, −19)}.

12. Solve y = −2x − 1 if the domain is {−3, −2, −1, 0, 1}. Graph the solution set.

For Questions 13 and 14, determine whether each equation is a linear equation. If so, write the equation in standard form.

13. \( \frac{1}{x} + \frac{1}{y} = \frac{2}{3} \)

14. 4x = 2y

15. Graph the equation x + 4y = 0.
Chapter 5 Test, Form 2C

For Questions 1–3, find the slope of the line passing through each pair of points. If the slope is undefined, write “undefined.”

1. (2, 5) and (3, 6) 1. _______
2. (-1, 3) and (6, 3) 2. _______
3. (6, -4) and (-3, 7) 3. _______

4. Find the value of r so that the line through (-4, 8) and (r, -6) has a slope of \( \frac{2}{3} \). 4. _______

5. In 1968, vehicle emission standards allowed 6.3 hydrocarbons released per mile driven. By 1980, the standards allowed only 0.41 hydrocarbons per mile driven. What was the rate of change from 1968 to 1980? 5. _______

6. Graph \( y = -\frac{1}{2}x \). 6.

7. If a shark can swim 27 miles in 9 hours, how many miles will it swim in 12 hours? 7. _______

8. Write a linear equation in slope-intercept form to model the situation: A telephone company charges $28.75 per month plus 10¢ a minute for long-distance calls. 8. _______

9. Write an equation in standard form of the line that passes through (7, -3) and has a y-intercept of 2. 9. _______

10. Write the slope-intercept form of an equation for the line graphed at the right. 10. _______

11. Graph the line with y-intercept 3 and slope \(-\frac{3}{4}\). 11. _______
12. Write an equation in slope-intercept form for the line that passes through \((-1, -2)\) and \((3, 4)\).

13. Write an equation in standard form for the line whose slope is undefined and passes through \((-6, 4)\).

14. Write an equation in point-slope form for the line that has slope \(\frac{1}{3}\) and passes through \((-2, 8)\).

15. Write the standard form of the equation \(y + 4 = \frac{12}{7}(x - 1)\).

16. Write the slope-intercept form of the equation \(y - 2 = 3(x - 4)\).

17. Write the slope-intercept form of the equation of the line parallel to the graph of \(2x + y = 5\) that passes through \((0, 1)\).

18. Write the slope-intercept form of the equation of the line perpendicular to the graph of \(y = -\frac{3}{2}x - 7\) that passes through \((3, -2)\).

For Questions 19 and 20, use the data in the table.

<table>
<thead>
<tr>
<th>Time Spent Studying (min)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score Received (percent)</td>
<td>53</td>
<td>67</td>
<td>78</td>
<td>87</td>
<td>95</td>
</tr>
</tbody>
</table>

19. Make a scatter plot relating time spent studying to the score received.

20. Write the slope-intercept form of the equation for a line of fit for the data. Use your equation to predict a student’s score if the student spent 35 minutes studying.

Bonus In a certain lake, a 1-year-old bluegill fish is 3 inches long, while a 4-year-old bluegill fish is 6.6 inches long. Assuming the growth rate can be approximated by a linear equation, write an equation in slope-intercept form for the length \(l\) of a bluegill fish in inches after \(t\) years. Then use the equation to determine the age of a 9-inch bluegill.