SPATIALLY TARGETED ACTIVATION OF A SMP

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The Degree of
Master of Science in Aerospace Engineering

By
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# SPATIALLY TARGETED ACTIVATION OF A SMP

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ABSTRACT

SPATIALLY TARGETED ACTIVATION OF A SMP

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The purpose of the research is to identify the mechanical properties of a skin, an infilled honeycomb, for morphing aircraft and investigate ways to optimize the geometry of the honeycomb. The first phase of the project included identifying the material properties of the honeycomb and SMP. The next phase included researching available models and developing an appropriate means to optimize the unit cell, this included experimental models, computational models, and various analytical models. Existing analytical models worked well for the infilled honeycomb at low infill modulus but when the infill modulus comes close to the honeycomb modulus a rule of mixtures is a better approximation of the composite modulus. The analytical equation also showed good correlation in plane in the primary orthogonal axes but poor correlation in shear. The sensitivity studies with the analytical equations showed no local minima leaving the constraints and objective function as the leading factor in any optimized solution. The research lays the ground work for future design of morphing aircraft skins.
Dedicated to my friends and family who encouraged me to finish my thesis.
ACKNOWLEDGEMENTS

My special thanks are in order to committee members and advisors, James Joo, Greg Reich, Rich Beblo, and Aaron Altman who gave me advice on my masters project and helped me finish this thesis. I would also like to thanks Brian Smyers and Nate DeLeon who were instrumental in setting up and preparing the lab for the my experiments. Finally I would like to thank the Professors and staff at the University of Dayton for everything from teaching me classes to reserving my conference room for my defense.
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NOMENCLATURE

$A$ – area of panel (15 x 20 in.)
$C$ – constraint constant matrix
$D$ – degree of freedom
$E_{xc}$ – composite Young’s modulus in the $x$ direction
$E_{yc}$ – composite Young’s modulus in the $y$ direction
$E_H$ – honeycomb’s Young’s modulus
$E_I$ – SMP Infill Young’s modulus
$G$ – shear modulus
$I$ – cell wall’s mass moment of inertia
$K$ – Stiffness Matrix
$N$ – number of cells
$P$ – pressure
$Q$ – constraint constant matrix
$R$ – reaction force
$T_g$ – glass transition temperature
$W$ – weight of the entire panel
$a$ – horizontal cell wall length
$c$ – cell depth
$b$ – panel length
$d$ – cell wall thickness
$l$ – cell wall length
$m$ – width of the cell panel
\( \nu \) – Poisson ratio

\( \nu_c \) – honeycomb composite Poisson ratio

\( \nu_i \) – infill Poisson ratio

\( w \) – applied out of plane deflection

\( x_o \) – \( l \cos \theta \)

\( y_o \) – \( l \sin \theta \)

\( \theta \) – cell angle

\( \alpha \) – ratio of length to width

\( \delta \) – out of plane panel deflection

\( \lambda \) – Lagrange multiplier

\( \rho_i \) – density of the infill

\( \rho_h \) – density of the honeycomb
### ACRONYMS

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<tr>
<td>2D</td>
<td>2 Dimensional</td>
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<tr>
<td>AFRL</td>
<td>Air Force Research Laboratory</td>
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<td>AR</td>
<td>Aspect Ratio</td>
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<tr>
<td>DA</td>
<td>Decylamine</td>
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<tr>
<td>DARPA</td>
<td>Defense Advance Research Project Agency</td>
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<tr>
<td>DIC</td>
<td>Digital Image Correlation</td>
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<td>DMA</td>
<td>Dynamic Mechanical Analysis</td>
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<td>DOF</td>
<td>Degrees of Freedom</td>
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<td>NGDE</td>
<td>Neopentyl Glycol Diglycidyl Ether</td>
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<td>N-MAS</td>
<td>Next Generation Morphing Aircraft Requirements</td>
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<tr>
<td>PBC</td>
<td>Periodic Boundary Conditions</td>
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<td>REF</td>
<td>Reference</td>
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<tr>
<td>RTV</td>
<td>Silicone Rubber</td>
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<td>SMP</td>
<td>Shape Memory Polymer</td>
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<td>UDRI</td>
<td>University of Dayton Research Institute</td>
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CHAPTER 1
INTRODUCTION

Interest in reconfigurable skin systems has increased due to the attention of morphing aircraft. Reconfigurable skin system can allow drastic changes in planform shape and area while keeping aerodynamic integrity. Morphing aircraft can then be optimized for specific mission profiles.

For instance, a high camber wing at low speeds will produce more lift efficiently while a low camber wing, at high speeds, reduces drag and allows for better maneuverability such as a fighter. An airliner has a dihedral wing to increase the aircraft’s stability. Variable sweep will allow an aircraft to perform better at higher speed by sweeping the wing and increase the efficiency at lower speeds with an un-swept wing. By changing the wing’s span an aircraft can benefit by better cruise/dash performance and roll control (Bubert, Kothera, & Wereley, 2008).

The aerodynamic performance of a single aircraft can be improved by tailoring the morphing for specific mission segments as seen in Andersen et al. (Andersen, 2007). Andersen showed a variety of mission segments in which a different configuration optimizes the aerodynamic performance such as dash, cruise, climb, loiter, etc. The constraints for the current design will be based on the constraints used in Andersen’s model. Currently, designs for wing shape change employ wings that can change their area by as much as 100%. Some of these applications suggest that materials with low in-plane stiffness and relatively high out-of-plane stiffness may be required (Joo, Reich, & Westfall, 2009).
The current goal is to create a monolithic skin that will not create wrinkling when deformed and still be able to withstand strain more than traditional materials while undergoing aerodynamic loads. To address this issue cellular structures were examined. Cellular structures provide high out-of-plane stiffness and low in-plane stiffness. By filling the cells with a polymer, a monolithic skin with higher out-of-plane properties will be created.

1.1 Morphing Aircraft Requirements

Defense Advance Research Project Agency (DARPA) published a set of requirements for Next Generation Morphing Aircraft Structures (N-MAS). Asheghiani et al. designed a morphing skin using the N-MAS requirements in their design seen below: (Asheghian, Reich, Enke, & Kudva, 2010)

- Skin panel weight < 4.8 kg/m2
- Maximum out-of-plane deflection < 2.54 mm with a load of 19152 Pa
- Nominal panel size = 38.1 cm by 50.8 cm
- Total shear angle change of 45 degrees
- No wrinkling of the skin even in extreme morphed configurations

Olympio et al. looked at a honeycomb design for a morphing wing. In addition to the above N-MAS requirements he included no local buckling of the cellular substructure under a load of 19152 Pa (Olympio K. Y., 2010)

1.2 Current State of the Art

Many proposed designs looked at multi-layer skins in an attempt to meet the requirements. Bubert et al. looked at large area change by creating a passive elastomeric matrix composite. The matrix composite consisted of a composite top sheet and a honeycomb structure, both of which exhibited near zero in-plane Poisson ratio. The focus was on the composite arrangements and substructure’s configurations. The skin demonstrated a 100% uniaxial strain with a 100% increase in surface area while exhibiting a deflection of 1.2 mm out-of-plane with a
9576 Pa load (Bubert, Kothera, & Wereley, 2008). Bubert did not look at shear and the out-of-plane deflection it only went up to less than half of the required 19152 Pa load.

Murray et al. looked at using a flexible matrix composite for one-dimensional morphing to increase span-, chord-, or camber-change. The flexible matrix composite is composed of stiff fibers aligned in a soft matrix creating relatively low actuation cost but remains rigid in orthogonal directions. The authors found that aligning the fibers orthogonal to the morphing direction does not impede the modulus or actuation cost and allows high strain. With a low matrix modulus the out-of-plane loads cause high deformation unless pretension is applied to the skin. By aligning the fibers in the pretension direction, the chances of rupture are minimized and out of plane displacements are reduced (Murray G. G., 2010).

Olympio et al. looked at flexible skin for shear wing morphing. By tensioning a low modulus material over a honeycomb structure the skin is able to limit out-of-plane displacements and reduce wrinkling in the skin. The design of the cellular structure included a strain relief feature to reduce the peak strain and actuation cost. Elliptical, Gaussian, or Cosine shaped curved strands were found to reduce the actuation cost by 30% over straight strands. The peak strains reduced to 1.2% from 3.3% of the straight strands; however, at high morphing angles of the strands near the edges caused wrinkling to occur (Olympio K. Y., 2010).

Joo et al. showed a design of a passive, engineered composite skin for morphing aircraft applications using a two-step design process. The first step in the process was to determine bulk material properties for the skin and the layout of attachments between the skin and underlying substructure. This resulted in a distribution of bulk properties across the skin. The second step utilized these property values as constraints to match the found bulk property in a material optimization in order to determine the layout of a unit cell. For the proof of the concept, a 2D engineered skin was designed using the proposed two-step process (Joo, Reich, & Westfall, 2009).
1.3 Project Outline

Cellular structures have been investigated by researchers for morphing applications. Most of these studies were limited to open cell or elastomer covered designs. [3-6] Cellular structures are compliant structures with relatively low in-plane stiffness, while the out-of-plane stiffness depends on the thickness of the structure. Low stiffness is obtained by the empty space within each cell, which provides space for the nearby thin structures to deform. However, when the space is filled or sandwiched between plates, the typical arrangement for honeycomb sandwich structures, it becomes stiff. This means the stiffness of the cellular composite is determined by the deformation of each cell, which can be controlled by filling cells with different materials. When the infill modulus is low, the skin will act like a mechanism and the modulus may vary in different directions. When the infill is hard the skin will become rigid. (Bubert, Kothera, & Wereley, 2008), (Murray G. G., 2010) (Andersen, 2007)

The system concept proposed is to use a honeycomb structure filled with a shape memory polymer (SMP) as seen in Figure 1. SMP is a variable stiffness material that will change stiffness when activated with stimuli such as light, heat, and chemicals. Using SMP as the infill in this application allows for the skin to display varying degrees of stiffness depending on the temperature.
Here the cells in blue are stiff (cold) while the cell in red is soft (hot). By specifically heating individual cells the global material properties can be tailored to the specific needs of the aircraft. When all the cells are cool the SMP is hard and the composite acts like a normal material. When all of the cells are hot, the material becomes soft and easily strained in different directions. More importantly, when cells are heated in patterns, the global stiffness of the material can be tailored. Some sample patterns are shown in Figure 2.
The all stiff skin would act like a normal skin and the all flexible minimizes the force required to strain the skin in any direction. The $0^\circ/90^\circ$ and the $-45^\circ/45^\circ$ samples would be used for stretching and shearing the skin respectively. With further investigation additional patterns can be made with the heating arrangements such as large honeycombs or auxetic honeycombs which would be mission specific.

The overall design of the Spatially Targeted Activation of a Reconfigurable Skin is a multi-step process looking at both the System Concept and Heating Concept as shown in Figure 3.

![Figure 3: Project Overview](image)

In order to fully explore the design space a model was created analytically, experimentally, and computationally. The first step is identifying the system concept and what the system is trying to achieve. Then an investigation into the material properties and its characteristics was performed. Using the material characteristics an analytical model was
developed. FEA was performed on the model to verify whether the analytical model correctly captured the behavior of the system. Finally the model was validated through experiments.

In conjunction with afore mentioned steps, heating methods for the system was developed but was not the focus of this thesis and is only shown for reference. An optimization of the honeycomb cell was performed based on the analytical model and is in APPENDIX C.

The system concept focuses on the material characteristics and validation of the models and methods needed to accurately capture the behavior of the infilled composite. The heating concept focuses on how to activate the SMP to the hot state. The heating concept was separated because multiple types of SMP activation can be used and the goal of this thesis is to characterize the honeycomb composite. Figure 4 shows the focus of this thesis.

Figure 4: Road Map: Material Characterization
This was accomplished by characterizing the SMP properties and the honeycomb properties, then combining the data gathered to create an analytical model validated through experimental work and FEA modeling. The material and honeycomb characterization focused on understanding the mechanical properties of the honeycomb and SMP. Both were researched and modelled and then experimented on. The SMP experiments provided the modulus and transition temperature along with a better understanding of how the SMP work. The honeycomb experiments were performed to get a better understand of how the mechanism works. The composite modeling looked at the analytical equations to be used in the final optimization. The honeycomb experiments and FEA modeling was used to validate the analytical model. Finally, an optimization on the composite itself was run to find the idealized cell shape which is located in APPENDIX C.

The hypothesis: the properties of a Spatially Targeted Activation of SMP is a feasible design for morphing aircraft as defined by the N-MAS requirements. In investigating the above hypothesis and exploring the design space of a filled honeycomb skin with spatially prescribable stiffness properties, an accurate analytical model predicting the skin’s properties given different materials and honeycomb configurations is required. To validate such a model, experimental characterization of representative empty honeycomb, infill, and filled honeycomb at various angles, temperatures, and boundary conditions is necessary.
CHAPTER 2
SHAPE MEMORY POLYMER

The first step is to characterize the material properties of the Shape Memory Polymer (SMP) and to verify through experimentation that the information from the literature review is accurate. Mechanical and thermal properties of the SMP significantly effects the overall properties of the composite. This section, Material Characterization of the Shape Memory Polymer, is the second step in the design process as illustrated in Figure 5.

Figure 5: Road Map: Material Characterization
The SMP was the fundamental method that allowed for spatially prescribing stiffness throughout the system. The experiments validates the fabrication process of the SMP. The hot and cold modulus will be used in the analytical equation and FEA modeling for the hard and soft states. The transition temperature will be used when running the composite experiments to know how to heat the samples.

2.1 Current State of the Art

2.1.1 SMP Characteristics

SMP is a class of polymers that will change shape when an external stimulus is applied. SMP can exhibit the shape memory behavior from variety of methods but the focus in this thesis will be on thermal activation.

SMP possess many advantages including excellent processability, light weight, and highly flexibility. The shape memory (SM) cycle for a thermo-responsive SMP consists of 4 steps: deformation, cooling, fixing, and recovery phases which are seen in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Deformation</th>
<th>Deform the SMP to a constant strain and hold above the glass transition temperature, $T_g$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Cooling</td>
<td>Cool the SMP below the glass transition temperature, $T_g$.</td>
</tr>
<tr>
<td>3</td>
<td>Fixing</td>
<td>Release the SMP and it will hold its shape.</td>
</tr>
<tr>
<td>4</td>
<td>Recovery</td>
<td>Heat the SMP above the glass transition temperature, $T_g$ and it recovery to its original shape.</td>
</tr>
<tr>
<td>5</td>
<td>Repeat</td>
<td>Return to step 1</td>
</tr>
</tbody>
</table>

The SM cycle described in Table 1 assumes the polymer has already been set in its permanent shape (usually during the curing process).

The stress-strain-temperature curve of the SM cycle is shown in Figure 6.
Figure 6: Stress-Strain-Temperature Curve of the SM Cycle (Lendlein, 2007)

Figure 6 shows that as the SMP is deformed at a high temperature the stress increases. At step 2 the temperature is reduced. In step 3 the applied stress is released. In step 4 the applied stress drops back to zero, and finally heating up the sample again causes the SMP to return it its original shape.

2.1.2 GM SMP

The General Motors (GM) SMP used in this study was based upon Xie et al.’s work because the recipe is commercially available, ingredients were readily available, and the transition temperature was tailorable (Xie & Rousseau, 2009). The main two chemicals are EPON 826 and Jeffamine D230. Neopentyl glycol diglycidyl ether (NGDE) and Decylamine (DA) can be added to change the transition temperature and modulus of the SMP. The Table 2 shows various recipes available for the GM SMP and Figure 7 Dynamic Mechanical Analysis (DMA) curves of the Reference (REF) GM SMP and NGDE SMPs.
Table 2: GM Formulation and Properties

<table>
<thead>
<tr>
<th>Sample</th>
<th>EPON 826 (mol)</th>
<th>D230 (mol)</th>
<th>DA (mol)</th>
<th>NGDE (mol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>REF</td>
<td>0.02</td>
<td>0.01</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>DA1</td>
<td>0.02</td>
<td>0.0075</td>
<td>0.005</td>
<td>-</td>
</tr>
<tr>
<td>DA2</td>
<td>0.02</td>
<td>0.005</td>
<td>0.05</td>
<td>-</td>
</tr>
<tr>
<td>DA3</td>
<td>0.02</td>
<td>0.0025</td>
<td>0.015</td>
<td>-</td>
</tr>
<tr>
<td>DA4</td>
<td>0.02</td>
<td>0</td>
<td>0.02</td>
<td>-</td>
</tr>
<tr>
<td>NGDE1</td>
<td>0.015</td>
<td>0.01</td>
<td>-</td>
<td>0.0005</td>
</tr>
<tr>
<td>NGDE2</td>
<td>0.01</td>
<td>0.01</td>
<td>-</td>
<td>0.01</td>
</tr>
<tr>
<td>NGDE3</td>
<td>0.005</td>
<td>0.01</td>
<td>-</td>
<td>0.015</td>
</tr>
<tr>
<td>NGDE4</td>
<td>0</td>
<td>0.01</td>
<td>-</td>
<td>0.02</td>
</tr>
</tbody>
</table>

The DA samples were left out because the focus was only on the REF SMP as this was the SMP used in the experimentation. The transition temperature is 89°C.

2.2 Experimentation

Multiple tests were run with the SMP to determine its tensile modulus, compressive modulus, transition temperature, and to detect any significant effects of aging.
2.2.1 SMP Test Setup

2.2.1.1 Tension Test

2.2.1.1.1 Tension Sample Preparation

When creating SMP a few methods were tried based on previous experience. The goal of the manufacturing was to create a square sample that could then be machined into dog bone specimens to run tensile tests on them. Multiple processes were tried and Table 3 shows the summary of the attempts and their results.

<table>
<thead>
<tr>
<th>SMP Mold Attempted</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandwich glass plates with RTV sealant and cured on its side</td>
<td>Too many bubbles, SMP was stuck to the glass plates, and mold leaked</td>
</tr>
<tr>
<td>Sandwich glass plates with RTV sealant, mold release, and cured on its side</td>
<td>Too many bubbles, SMP was stuck to the glass plates, and mold leaked</td>
</tr>
<tr>
<td>Sandwich glass plates with RTV sealant, Teflon fabric, and cured standing up</td>
<td>Teflon fabric wrinkled and caused variations in the thickness of the sample</td>
</tr>
<tr>
<td>Open mold, with RTV to create boundaries and Teflon fabric as the base</td>
<td>Teflon fabric wrinkled and caused variations in the thickness of the sample</td>
</tr>
<tr>
<td>Silicon mold</td>
<td>Silicone was too porous and it created bubbles in the SMP</td>
</tr>
<tr>
<td>Teflon sheet machined to have a square mold and through holes with plugs to easily release the sample</td>
<td>Good Sample</td>
</tr>
</tbody>
</table>

The curing cycle uses a three step 1 hour at 100°C thermal cure and 1/½ hour at 150°C post cure process.

Once the SMP cured dog bone samples were cut out using a CNC router as seen in Figure 8, the samples were weighed and the cross-sectional area was measured in three locations as shown in Figure 8 and the average was taken. The samples gage length of about 1.8 in., depth of 0.025” and width of 0.16 in. Reflective tape was attached in the middle of the sample to allow the laser to read the strain with a nominal gage length of 2 in. with sample sized per ASTM D638
2.2.1.1.2 Tension Test Equipment

The specimens were tested in tension on a MTS QTest /1L Elite controller with a Rebew 20D 3187-104 load cell (max error -0.91%). The strain was measured by two methods: MTS software and a EIR Laser extensometer model LE-01 (non-linearity ±0.0001 in., Resolution 0.0001 in.). A Type K Thermocouple was attached to the back side of the sample. A B&K Precision 720 K-Type Thermocouple Reader measured the temperature. A Custom thermochamber was created out of Fiberfrax© and a heat pipe connected to shop compressed air was used to heat the chamber. The pressure of the air was not measured but was varied as needed to increase the air flow. A Powerstate Variable Autotransformer was used to supply electricity to the Omega “T” Type Air Process Heater heat pipe. The samples were griped with mechanical screw grips. The test was performed per ASTM D638 standards.
2.2.1.1.3 Tensile Test Procedure

The tensile test was run on the same specimen at varying degrees in order to determine its $T_g$. Below is the tensile test procedure:

1. Mount the sample to the grips and sandwich double sided sandpaper to help prevent slipping.
2. Attach the thermocouple and close the thermal chamber.
3. Heat the sample past the desired temperature for the test and re-tighten the grips
4. Heat/Cool the sample back to the desired temperature range
5. Adjust the crosshead displacement to zero out the load cell, straightening the SMP sample
6. Setup up the laser and zero the displacement
7. Run the test at a constant strain of 12.7 mm/min up to 1% strain and then return the sample to zero strain.
It is important to re-tighten the sample after it is heated up otherwise it will slip when it becomes soft and rubber-like. Table 4 shows the test matrix that was used.

<table>
<thead>
<tr>
<th>Number of Runs</th>
<th>Thermo-Chamber Temperature</th>
<th>Precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room Temperature</td>
<td>21°C</td>
<td>±3°C</td>
</tr>
<tr>
<td>Every 10-15°C</td>
<td>35°C 50°C 65°C 75°C 90°C 105°C</td>
<td>±5°C</td>
</tr>
<tr>
<td>Max Temp</td>
<td>115°C</td>
<td>+ 5°C</td>
</tr>
</tbody>
</table>

The test matrix shown in Table 4 was used to calculate the modulus of the test specimens. From the data gathered a stress strain curve was created so a modulus can be calculated for each sample at each temperature. The equation for calculating the Young’s modulus can be seen in equation (1).

\[ E = \frac{\sigma}{\varepsilon} \] (1)

The modulus can then be graphed w.r.t. temperature, and by curve fitting a MATLAB© modified Heaviside function to the data, a \( T_g \) can be obtained. The \( T_g \) is found by calculating the temperature at the inflection point of the curve show in equation (2). This will show the transition temperature of the SMP.

\[ H = (E_{UL} - E_{LL}) \left( 1 - \left( 1 + e^{-2S(T-T_g)} \right)^{-1} \right) \] (2)

Where \( E_{UL} \) is the upper modulus, \( E_{LL} \) is the lower modulus, \( S \) is the slope of the curve at \( T_g \), and \( T_g \) is the transition temperature.

An aging study was conducted on the tension test samples. All the samples were created on the same day and from the same batch of SMP mixture. The samples were weighed each week to measure any evaporation of chemicals. Then, beginning day one, samples were tested on a weekly basis to check for changes in the modulus, weight, and \( T_g \). Each sample was weighed on the day of the test, and the room’s humidity and the ambient temperature were recorded.
2.2.1.2 Compression Test Specimen

2.2.1.2.1 Compression Sample Preparation

The SMP compression test sample was processed the same as the SMP tension test samples in 2.2.1.1. The difference was in the mold used. Figure 10 shows the hollow cylinder used to create the compression test samples.

![Compression Test Sample Preparation](image)

The cylinder was sprayed with mold release and the hose clips were tightened. A Teflon® plug was used to stopper one end and the mold was closed. It was then filled with SMP and allowed to cure vertically. The sample was removed from the mold and it was machined to uniform thickness with a lathe. The nominal size was 0.5 in. diameter and a height of 1 in. Five samples were produced.

2.2.1.2.2 Compression Test Equipment

The compression test equipment included the same test frame and controller as the SMP tensile test equipment, 2.2.1.1.2. The attachment for the test frame is a metal plate on which the compression sample sits. The top extension arm has a sanded down bolt that was used to evenly compress the sample.
2.2.1.2.3 Compression Test Procedure

Below is the compression test procedure where each sample is run past the $T_g$ value.

1. Mount the sample
2. Attach the thermocouple, move crosshead head to just above the sample, and close the thermal chamber.
3. Heat the sample to 95°C
4. Zero out the load cell and displacement
5. Run the test at a constant strain of 12.7 mm/min to 15% strain.

Calculation the modulus of the sample is the same as section 2.2.1.1.

2.2.2 SMP Test Results

2.2.2.1 SMP Tension Results

The tension tests and aging study were conducted because the SMP uses a resin based polymer chemicals that evaporate over time. The two-step curing process with different temperatures uses the first temperature to set the polymer and the second to evaporate any leftover chemicals. Additional evaporation could occur over time and this study is to address if that happens and what effects it has the modulus. Some of the samples created for the SMP tension test were used to run this study.

Figure 11 shows the stress-strain curve of a single sample at 130°C.
Some samples slipped and if so the modulus was calculated from the linear portion which was linear with an $R^2$ value of at least 0.98. This was done for each specimen at every temperature described in the test matrix shown back in Table 4.

The transition temperature was analyzed by taking the moduli and its corresponding temperature for an individual sample and running it through modified Heaviside function. The modified Heaviside function is a step function that provides the upper limit and lower limit of the Young’s Modulus $E_{UL}$ and $E_{LL}$, Transition Temperature, and slope of the curve ($S$) as seen in equation (4) and then graphed in Figure 12

$$H = (E_{UL} - E_{LL}) \left( 1 - \left( 1 + e^{-2S(T - T_g)} \right)^{-1} \right)$$

(3)

The slope curve, $S$, is significant for characterizing SMPs because it shows how quickly the SMP transitions from the hard state to the soft state. For the purpose of this study the temperature range of hot state and cold state is important for the full composite experiments.

![Figure 12: SMP Modulus Vs. Temperature With a Modified-Heaviside Curve Fit With Error Bars](image)

From the data collected, no discernible reduction in the modulus was calculated as time progressed and after 5 weeks the testing stopped. Also, the weight of the individual sample did not change as the time progressed.
Figure 13 shows multiple data sets overlaid on top of one another.

![Graph showing SMP Modulus Versus Temperature for 5 Weeks With a Modified-Heaviside Curve Fit With Error Bars]

The list of \(T_g\) and modulus for 5 weeks can be seen in Table 5 and graphed in Figure 14 and Figure 15.

### Table 5: List of \(T_g\) for the Neat SMP

<table>
<thead>
<tr>
<th>Sample</th>
<th>(T_g) (°F)</th>
<th>(E_{UL}) (MPa)</th>
<th>(E_{LL}) (MPa)</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>79.5</td>
<td>906</td>
<td>15</td>
<td>0.15</td>
</tr>
<tr>
<td>2</td>
<td>77.0</td>
<td>1071</td>
<td>16</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>77.0</td>
<td>948</td>
<td>16</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>77.0</td>
<td>948</td>
<td>16</td>
<td>0.1</td>
</tr>
<tr>
<td>5</td>
<td>77.3</td>
<td>965</td>
<td>17</td>
<td>0.1</td>
</tr>
<tr>
<td>Average</td>
<td>77.6</td>
<td>968</td>
<td>16</td>
<td>0.13</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1.1</td>
<td>62</td>
<td>0.1</td>
<td>0.02</td>
</tr>
</tbody>
</table>
The published value by Xie et al. was 89°C but the data clearly shows a lower $T_g$. As the table shows, the $T_g$ of the SMP was not affected by aging after 5 weeks. The variation in the $T_g$ could be due to the exact temperature of the SMP verse the thermocouple measured temperature. The temperature can differ because of poor adhesion to the sample, or because the hot air comes in at a slightly different angle and heats the thermocouple before it heats the sample. Also, the
sample was assumed to be uniform temperature and did soak in the desired environment for an extended period of time, but heat variations could still occur throughout the sample.

In addition modulus in the hard state through experimentation was shown to be 968 MPa compared to the published value of about 1250 MPa. Also the aging study showed no decrease in sample weight and no change in the modulus of the SMP. Differences could occur in the accuracy of the measurement of the cross section or the accuracy of the tensile machines.

2.2.2.2 SMP Compression Results

The compression test was conducted to look for any differences in the compression modulus verses the tension modulus. Differences could affect the final reconfigurable skin due to the area of the cell undergoing compression then tension at different points in the strain.

Figure 16 shows the stress strain curve of a single sample.

![Stress Strain Curve](image)

**Figure 16: SMP Soft Compression Stress Strain Curve**

The point of initial contact occurred as the crosshead initially engaged the sample. The linear section was found by using the MATLAB smooth function to find the most linear portion of the data.

The results for the five tests are shown in Table 6.
Table 6: SMP Soft Compression Test Results

<table>
<thead>
<tr>
<th>Test</th>
<th>Pa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.318E+07</td>
</tr>
<tr>
<td>2</td>
<td>1.508E+07</td>
</tr>
<tr>
<td>3</td>
<td>1.434E+07</td>
</tr>
<tr>
<td>4</td>
<td>1.349E+07</td>
</tr>
<tr>
<td>5</td>
<td>1.315E+07</td>
</tr>
<tr>
<td>Average</td>
<td>1.385E+07</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.411E+05</td>
</tr>
</tbody>
</table>

The variation in the results could be due to stress concentrations caused by milling out the specimen from the mold or an SMP being poorly mixed in the manufacturing process.

2.3 SMP Discussion

The tension tests showed a $10^\circ$C difference in the $T_g$ but for the purposes of the composite experiments knowing the temperature of the soft state and hard state is sufficient. The hard and soft state moduli had an 18% and a 6% percent difference versus the published values respectively. The hard and soft moduli will be used in the FEA modeling and analytical model. The SMP compression and shear modulus was not found in literature. The compression tests showed 13.4% difference from the tensile tests.

The hard and soft Young’s moduli will be used in the FEA modeling and analytical model. The differences in the moduli was never fully investigated. It is important to note that the FEA shows parts of the infill undergoing compression while the analytical model assumes the infill undergoes compression. The other material constant of isotropic material, the Poisson ratio used for analytical and FEA modeling was assumed to be 0.5 which is a common polymer Poisson ratio. The $T_g$ was used in the honeycomb composite experiments to know what temperature to heat the composite to.
CHAPTER 3
HONEYCOMB

An experimentally validated analytical model is required to achieve the overall goal of exploring the design space of an infilled honeycomb with local spatially controllable stiffness. To this purpose, experimentally characterizing the constituent materials of the composite, including SMP in Chapter 2 and the empty honeycomb presented here in Chapter 3, is the next step in the process. The characterization of the honeycomb properties is the second step in the design process which is illustrated in Figure 17. The background on honeycomb structures and equations to model them will be presented. In addition, the experiments were run in the tensile case to compare to the analytical model.

Figure 17: Road Map: Honeycomb Characterization
The experiments and analytical modeling of the honeycomb was used to better understand how the final composite will perform. Honeycomb acts like a mechanism and has different moduli in the X and Y direction. Similar trends in the X and Y honeycomb moduli are expected in the analytical and experimental models.

3.1 Current State of the Art

3.1.1 Honeycomb Manufacturing

The manufacturing of honeycomb is an important aspect of the project as it dictates the shape and size of the honeycomb cells. There are several methods to produce honeycomb but only one of them will be focused on here. Figure 18 shows the dimensions that will be used when describing honeycombs.

![Figure 18: Honeycomb Dimensions](image)
To start, thin sheets materials are used. The width of the sheet will determine the depth of the honeycomb cell, \( c \), and the thickness of the material will determine the thickness of the honeycomb walls, \( d \). The thin sheets are then layered together and glued or welded at constant intervals. Then the next layer is welded in an offset of exactly half the length of the previous layer to the next sheet with every two layers having the same segment of the sheet attached as seen in Figure 19.

![Figure 19: Manufacturing of a Honeycomb Structure: The Left Picture Shows the Flat Sheets Attached Together by the Red With the Distance Exaggerated. The Right Picture Shows the Same Five Sheets Pulled Apart.](image)

The attached sections of the sheets will become the \( a \) walls and the unattached segments are the \( l \) walls. This is why in honeycomb structures \( a \) beams are twice as thick as the \( l \) beam. The length of the welded segment will dictate the length of the \( a \) beam and the distance between each weld will dictate the length of the \( l \) beam. At this point the honeycomb looks like a simple layer of sheets. To create the honeycomb shape, the top sheet and bottom sheet of the stack are evenly strained causing the \( l \) walls to rotate by bending at the joint. This results in \( l \) walls of the honeycomb sheet rotating at an angle \( \theta \) creating two sets of parallel sides. The result can be seen on the right side of Figure 19. The \( a \) beams of the honeycomb sheet stay parallel to each other and do not rotate. Some of the problems with this method of manufacturing are the residual stresses that occur while straining the sheets apart, the geometry of the corner of the honeycomb are hard to capture accurately in models, and the attachment between the different layers of honeycomb either through welding, gluing or some other means is difficult to model. Any error
in the process such as straining, misalignment, and poor attachment can cause the honeycomb to have stress concentrations. (Composites, Honeycomb Attributes and Properties, 1999)

3.1.2 Analytical Model

3.1.2.1 Olympio

Olympio et al. looked at an empty honeycomb model. He assumed the cell walls could be modeled as shear deformable beam-rod elements and that boundary effects were negligible (Kingnide Olympio, 2010) (Olympio K. R., 2009) (Murray G. G., 2009). His model was based off of Gibson and Ashby’s work (Gibson & Ashby, 1997). They calculated the Young’s modulus in the x and y direction, the Poisson ratio in the XY and YX directions, the Shear modulus in all three principal directions, and the max strain in the x direction. Figure 20 shows the dimension of the unit cell and coordinate axis definition.

![Figure 20: Olympio Model Cell Dimensions (Olympio K. R., 2009)](image)

The equivalent modulus of the honeycomb assumes perfect honeycomb, perfect attachment where the honeycomb layers are attached, and uniform angles, \( \theta \), at the joints.
The first equation is the Young’s modulus in the X direction of the honeycomb as can be seen in equation (4).

\[
\frac{E_x}{E} = \frac{\beta^3 \cos \theta}{(\alpha + \sin \theta) \sin^2 \theta} \times \frac{1}{1 + (\kappa + \cot^2 \theta)\beta^2}
\]  

(4)

Next is the Young’s modulus in the Y direction shown in equation (5).

\[
\frac{E_y}{E} = \frac{\beta^3 (\alpha + \sin \theta)}{\cos^3 \theta} \times \frac{1}{1 + (\kappa + \tan^2 \theta + 2\alpha/(\eta \cos^2 \theta))\beta^2}
\]  

(5)

Where \( \beta = t_l/l \) (thickness to-length ratio of inclined wall), \( \alpha = h/l \) (cell aspect ratio), \( \kappa \) is the coefficient of shear deformation of the beam, typically \( \kappa = 2.4 + 1.5v \), and \( \eta = t_v/t_l \) (vertical wall to inclined wall thickness ratio).

The behavior of the honeycomb modulus in the X and Y direction will be important in the analysis of the reconfigurable skin. Figure 21 shows the modulus in relationship to \( \beta \) which is the ratio of thickness to length of the inclined wall.

![Figure 21: Empty Honeycomb Ratio of Young’s Moduli in the X Direction Verse the Cell Angle for Three Beta Values.](image)

This shows that as the honeycomb walls become thinner in respect to its width the equivalent modulus decreases, which is expected. This graph also shows that as the cell angle...
decreases there is a spike in the modulus around 0°. Another interesting aspect of this graph is that as \( \beta \) decreases the honeycomb starts acting more nonlinearly in the region around 0°, with a sharp increase at 0°.

Another important feature of the honeycomb is the shear modulus. For the analysis only small strains are considered and the equations representing the shear modulus are presented in equations (6) and (7).

\[
G_{xy} = \frac{\cos^2 \theta}{(\alpha + \sin \theta) \sin \theta} \times \frac{1 + (\kappa - 1)\beta^2}{1 + (\kappa + \cot^2 \theta)\beta^2} \quad (6)
\]

\[
G_{yx} = \frac{\sin \theta (\alpha + \sin \theta)}{\cos^2 \theta} \times \frac{1 + (\kappa - 1)\beta^2}{1 + (\kappa + \tan^2 \theta + 2\alpha/\eta \cos^2 \theta)\beta^2} \quad (7)
\]

The shear modulus of honeycomb cell divided by the honeycomb modulus is shown in Figure 22.

![Figure 22: Empty Honeycomb Ratio of Shear Moduli to the Shear Angle in the XY for Different \( \eta \)](image)

29
The shear modulus increases as the cell angle increases from 0°. Below 0° the cell wall starts to overlap with the η and α values show as they would be non-physical. It should also be noted that as the aspect ratio increases the modulus of the honeycomb reduces. (Olympio K. R., 2009)

The analytical solution for honeycomb has been widely developed and verified with FEA analysis. Some of the equations pertaining to this thesis include the ratio of the Young’s modulus of the material to that of a standard honeycomb, ratio of the shear modulus, and the Poisson ratio of the honeycomb. The benefit of creating a ratio is it allows the analysis to focus on the honeycomb geometry.

3.1.2.2 Masters and Evans

Masters and Evans based their model on a two-dimensional honeycomb and considered a single cell. (Masters & Evans, 1996) The cell walls were considered to flex as a cantilever beam. The result was two equations, (8) and (9), when multiplied by the material modulus of the honeycomb, gives the resulting modulus of an empty honeycomb.

\[
E_x = \frac{(a + x_o)}{y_o \left( \frac{y_o^2 l}{d^3} + \frac{x_o^2}{d l} + \frac{a}{d} \right)} E_h \tag{8}
\]

\[
E_y = \frac{y_o}{(a + x_o) \left( \frac{x_o^2 l}{d^3} + \frac{y_o^3}{d l} \right)} E_h \tag{9}
\]

Where the variables used are defined by Figure 23
3.1.2.3 El-Sayed

El-Sayed based his model off of the strain energy of a unit cell. El-Sayed assumed that the cell walls that bent had the inflexion at the midpoint and his equations can be seen in (10) and (11). (El-Sayed F., 1976) The

\[ E_x = \frac{(a + y_o \cot \theta) \sin \theta}{cy_o^2 \left( \frac{y_o^2}{12I} + \frac{\cos^2 \theta}{A} \right)} E_h \]  \hspace{1cm} (10)

\[ E_y = \frac{\sin \theta}{c(a + y_o \cot \theta) \left( \frac{y_o^2 \cot^2 \theta}{12I} + \frac{\sin^2 \theta}{A} \right)} E_h \] \hspace{1cm} (11)

Where the variables used are define by Figure 23

3.2 Experimentation

Tensile tests were run on the empty honeycomb to gain a better understanding of how the honeycomb contributes to the overall reconfigurable skin performance. The experimentation was necessary because the honeycomb composite model will need to be verified by testing and the behavior of an infilled honeycomb is unknown.
3.2.1 Honeycomb Test Setup

The honeycomb tensile testing was based upon ASTM C 363/C 363M – 09 Standard for Test method for Node Tensile Strength of Honeycomb Core Materials which states the standards for tensile testing bond strength of honeycomb core materials.

3.2.1.1 Honeycomb Sample Preparation

The honeycomb was ordered from McMaster Carr ©. The dimensions of the honeycomb were found by taking an average of the honeycomb cells at 5 locations. Measurements were taken using calipers and the average can be seen in Table 7.

Table 7: Experimental Honeycomb Properties

<table>
<thead>
<tr>
<th>$a$</th>
<th>$l$</th>
<th>$d$</th>
<th>$c$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall Length (mm)</td>
<td>Wall Length (mm)</td>
<td>Thickness (mm)</td>
<td>Depth (mm)</td>
<td>Angle (deg)</td>
</tr>
<tr>
<td>5.24</td>
<td>9.28</td>
<td>0.44</td>
<td>6.44</td>
<td>45.96</td>
</tr>
</tbody>
</table>

The honeycomb was cut into 130 x 260 mm segments with scissors using a template to ensure repeatability. A total of 16 test samples were cut, 8 for the Y direction and 8 for the X direction.

3.2.1.2 Honeycomb Test Equipment

The empty honeycomb tensile tests used the MTS QTest for the SMP tensile tests. Custom test grips were made out of aluminum and shown in Figure 24.
The dimensions of the mount are shown in Figure 24. One of the key things to note is the bottom slit has a rubber stopper to prevent the bolts from sliding. Without the stopper the bolts had a tendency to shift during the test and provide uneven strain on the sample.

3.2.1.2.1 Honeycomb Testing Procedure

The honeycomb testing procedure is as follows:

1. Mount the test specimen at the first complete row of honeycomb cells by sliding bolts through the open honeycomb cells and let it hang. If residual stress in the manufacturing of the honeycomb causes the bottom of the sample to not align with the test mount, procure a new sample.

2. Lower the mount so the last row of honeycomb cells is lined up with the bottom mount and slide the bolts through it so that it hangs. The rubber stopper keeps the screws in place and does not allow them to move.
3. Raise the crosshead so the last row of honeycomb cells is just about to touch the bolts and then zero the load cell.

4. Run the test at 12.7 mm/min and run until the sample breaks.

3.2.2 Honeycomb Results

3.2.2.1 Honeycomb X Tensile Results

Figure 25 shows the stress verses strain of test data from an empty honeycomb X tensile test.

Two of the key features identified in Figure 25, point of full contact and the linear section are significant. The point of full contact is not identified as point of contact because every sample had some residual stress. Sometimes one cell may be in contact while the other cells may not be. The sample would straighten out in the first few seconds of the test. The linear portion only occurs after all the cells have engaged the bolts and for a short period the honeycomb exhibits a linear response for 0.01 strains, after which the slope of the curve starts to increase.

Table 8 shows the honeycomb modulus in the X direction for all 8 tests, the average, and the standard deviation.
Table 8: Honeycomb X Tension Modulus

<table>
<thead>
<tr>
<th>Test</th>
<th>Pa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.03E+04</td>
</tr>
<tr>
<td>2</td>
<td>5.98E+04</td>
</tr>
<tr>
<td>3</td>
<td>5.60E+04</td>
</tr>
<tr>
<td>4</td>
<td>5.09E+04</td>
</tr>
<tr>
<td>5</td>
<td>8.69E+04</td>
</tr>
<tr>
<td>6</td>
<td>6.01E+04</td>
</tr>
<tr>
<td>7</td>
<td>6.60E+04</td>
</tr>
<tr>
<td>8</td>
<td>2.38E+04</td>
</tr>
<tr>
<td>Average</td>
<td>5.55E+04</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>5.80E+04</td>
</tr>
</tbody>
</table>

One major source of error was the residual strain in the honeycomb preventing it from lining up with the test grips uniformly. In extreme cases, when the honeycomb hung from the top grip the honeycomb sheet curved so none of the lower cells could have bolts placed through them. While that particular sample was thrown out, this did happen to lesser degrees with most of the samples. Another source of error was the variation in the honeycomb. As mentioned earlier, the dimensions of the honeycomb were an average of the entire sheet. However, some samples ended up with another row of cells for the same 230 mm. A third source of error could have been from the pre-strain that occurs in the honeycomb walls during manufacturing.

3.2.2.2 Honeycomb Y Tensile Results

Figure 26 shows the stress verses strain test data from honeycomb Y tensile test.
As with the honeycomb X tension test, two of the key features identified in Figure 25, the point of full contact (as noted visually during testing) and the linear section, are significant. It is important to note that the linear section spans 0.03 strains and the y axis is half that of Figure 25.

Table 9 shows the honeycomb modulus in the Y direction for all 8 tests, the average, and the standard deviation.

**Table 9: Honeycomb Y Tension Modulus**

<table>
<thead>
<tr>
<th>Test</th>
<th>Pa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.68E+04</td>
</tr>
<tr>
<td>2</td>
<td>2.23E+04</td>
</tr>
<tr>
<td>3</td>
<td>1.82E+04</td>
</tr>
<tr>
<td>4</td>
<td>2.34E+04</td>
</tr>
<tr>
<td>5</td>
<td>1.70E+04</td>
</tr>
<tr>
<td>6</td>
<td>1.90E+04</td>
</tr>
<tr>
<td>7</td>
<td>2.13E+04</td>
</tr>
<tr>
<td>8</td>
<td>2.26E+04</td>
</tr>
<tr>
<td>Average</td>
<td>2.01E+04</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2.64E+03</td>
</tr>
</tbody>
</table>

Same errors can occur in the Y direction as the X direction.
3.3 Honeycomb Discussion

Figure 27 shows the data from the both the X and Y honeycomb tensile test graphically displayed with the 95% confident interval.

![Figure 27: X and Y Honeycomb Tension Moduli](image)

Both the x and y directions have two data points outside of the 95% confidence interval, 3 and 5, and 1 and 5, respectively. The fact that 2 of the data points are outside of the confidence intervals show the data is erratic and can only be used for rough estimates of the actual modulus. The honeycomb has a noticeably lower modulus in the Y direction which will be seen later in the in composite moduli. Also as the stress-strain curves show, the moduli is nonlinear due to the changes in the cell angle as it strains which again is a factor of it acting like a mechanism.

An experimentally validated analytical model is required to achieve the overall goal of exploring the design space of an infilled honeycomb with local spatially controllable stiffness. In Chapter 2 the SMP was characterized and in Chapter 3 the honeycomb characterized. Next step will be to create the composite experimental model.
The experimental characterization of the honeycomb composite presented in this section will use the SMP modulus and $T_g$ found in chapter 2 and use the same honeycomb characterized in chapter 3. The road map, shown in Figure 49, shows the focus of the design process at this step. The basics of the reconfigurable skin are presented in this section is the experimental characterization.

Figure 28: Road Map: Composite Characterization
Investigating the utility of an infilled honeycomb morphing skin via a trade space study/design tool requires the analytical model in Chapter 6 to verified experimentally and validated via FEA modeling and experimental results in Chapter 5.

The experiment was the basis by which the analytical results and/or trends are validated and understood. These two models along with the FEA model helps explore the design space. In addition, the experimentation sample preparation helped with understanding how the honeycomb composite could be made and difficulties in producing it.

4.1 Test Setup

4.1.1 Sample Preparation

The reconfigurable skin test specimens were created using a modified process of the SMP shear samples. The modified process was used because it kept the SMP from de-bonding from the honeycomb structure which occurred if the entire sample was made at once. The same honeycomb was used as in section in 3.2.1.

Once the honeycomb was cut to size it was soaked in a nitric acid bath at 23.5% concentration for 10 minutes. The honeycomb samples were then rinsed and dried by placing them in the oven at 100°C for about 20 minutes or until all the droplets evaporated. This was done because if a little of the acid was left on it caused a reaction with the SMP during the curing process.

The three step pouring process is detailed in Table 10.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Step</th>
<th>Cure Time at 100°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Add First Layer</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Add Second Layer</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Add Final Layer</td>
<td>3</td>
</tr>
</tbody>
</table>

The first layer is a thin film that was spread on the bottom of the mold with the honeycomb placed on top of it. Then it was cured in the oven for an hour. The second layer
filled every other row of honeycomb cells and again cured for an hour. The final layer filled the rest of the cells. The final step, curing the sample for 3 hours at 100°C, has the total amount of energy of proposed curing cycle of the SMP but at a lower peak temperature.

4.1.1.1 Tension Test Specimens

The first step was to create the tensile test specimen Teflon© mold with through holes seen in Figure 29. The tensile test specimens were 12.7 x 25.4 cm and a sample is shown in Figure 29.

This particular specimen had a few issues with it. When pouring the SMP, it is important to make sure the specimen is level, every cell is evenly filled, and the top layer is smooth. Problems usually occurred because the SMP starts becoming highly viscous the longer it sits out making it difficult to pour. The thickness of the honeycomb was about 6.44 mm but after the SMP was filled the average thickness was 7 ±1 mm.
Once the sample is removed from the mold, a slight slope in the SMP could be seen at the edges. This was sanded down so it could be fixed to the test fixture. For the hard tensile test a 2.54 cm x 12.7 cm x 0.3175 cm aluminum plate was epoxied to the top and bottom of the sample. The plate helped distribute the loads when the grips clamped down on it.

4.1.2 Testing Equipment

4.1.2.1 Soft Tensile Test

Figure 30 shows the test equipment used in the soft tensile test in the X and Y direction. The test was based off of ASTM C363 which is ASTM standard for testing honeycomb since no standard currently exists for testing infilled honeycomb composite.

Figure 30: Test Setup of the Soft Tensile Test. Left Picture Has the Enclosed Thermal Chamber. The Right Picture Has the Front Cover Removed. 1 Is the Heat Pipe Used to Control the Temperature, 2 Are the Thermal Couples, 3 Is the Load Cell, 4 Is the Power Source for the Heater, and 5 Is the Piece of Metal Used to Attach the Specimen to the Grips.

Figure 30 has the same setup as 2.2.1.1 except for a change of grips.
The soft tensile test procedure is listed below:

1. Mount the samples to the L-bracket at the top and bottom of the sample
2. Attach the thermocouple on the side across from the heat pipe.
3. Close the thermal chamber and start the heater.
4. Open the thermal chamber and at 110°C and retighten the grips
5. Close the thermal chamber and reheat the test specimen back up to 110°C.
6. The sample will expand due to the heat; adjust the grip displacement to zero out the load cell, straightening the test specimen
7. Run the test at a constant strain of 12.7 mm/min up until it breaks and collect the data with the MTS software.

4.1.2.2 Hard Tensile Tests

The hard tensile test was setup up on a MTS load frame [3A]. The strain was obtained by using the crosshead displacement measurement. The specimen was attached with 3” grips that were hydraulically actuated. A problem occurred when gripping the specimen. Too high of a grip strength would crush the specimen causing it to crack and too low of a pressure would allow the specimen to slip. The tensile test was run at 12.7 mm/min until the sample breaks.

4.2 Tensile Experimental Test Results

The stress strain curves of the soft and hard tensile tests are shown in Figure 31 and Figure 33 respectively. The linear section of the data either occurred initially or after the test fixture slipped a little bit before the loss joints were under enough tension.

The Young’s modulus was calculated in the same manner as section 2.2.1.1.

The modulus was calculated over the linear section marked by the blue line. The black points are the selection of the data used. Sometimes the sample slipped as in the on the right figure in Figure 31 and the modulus was taken to be only the linear portion after the slip occurred.

Figure 31 shows one sample of the soft tensile test in the X and Y directions.
Figure 31: Soft Tensile Stress Strain Curve. Note That the Two Graphs Have Different Axes.

The black data points show how the modulus was calculated with the points showing the raw data. In the Y soft tensile case the specimen slipped initially and this is depicted.

Figure 32 shows a soft Y tensile test specimen that broke, with the crack depicted by the arrows.

Figure 32: Broken Soft Y Tensile Hot

The crack initiated slowly and then rapidly spread across the sample. The crack started at the side (where the hot air used to heat the thermal chamber entered) and propagated across 90%
of the specimen. The initiation of the crack is thought to be caused by the thermal stresses applied in the area that is locally heated greater than the surrounding area. Cracks only went across the infill on the two edges of the test specimen. The crack followed the honeycomb where the infill de-bonded from the honeycomb walls. Along these segments, the cracks didn’t always go all the way through the thickness which is believed to be a result of the thin layer of the infill that was sometimes above and below the honeycomb itself. When the sample broke across the honeycomb beams, the beam in some areas broke while in others it stayed intact.

All four of the Y hot tensile test specimens cracked. Only a few of the X tensile specimens showed signs of a crack starting at the side where the hot air entered the thermal chamber. This is believed to be a result of the way the honeycomb is manufactured. The thin sheets of metal are layered and bonded together, so when the honeycomb is pulled in the Y direction, the $a$ beams already have a weak point and that combined with the poor bonding of the infill to the honeycomb help propagate the crack across the honeycomb.

Figure 33 shows the X and Y hard tensile test stress strain curves.

![Figure 33: Hard Tensile Stress Strain Curve. Note That the X Axis is Different for Each Plot.](image)
In both of the samples, initial slip is shown to have occurred which is thought to be a result of the grip pressure and the slip in the grip going away. The black data points show the area where the modulus was calculated.

A total of 3 of the 4 X hard tensile test samples broke when tested. Figure 34 shows an X hard tensile test specimen where a break occurred along the ridge of the honeycomb.

![Figure 34: X Hard Tensile Sample Failure](image)

When the tensile test broke it occurred in a fraction of a second with a load crack. Real time crack propagation analysis couldn’t be done since it happened so swiftly. Note that at the top side of the picture the crack propagated across a couple rows and at one point it went through the infill. The rest of the break followed closely to the adhesion of the honeycomb to the infill. Some samples had cracks where the metal plate was epoxied to it. While the breaks looked similar to Figure 34, those pictures weren’t shown because they were likely due to a stress
concentration that occurred due to the attachment of the grips to the test samples. In some samples when the grips clamped down, an audible crack could be heard.

All four of the Y hard tensile samples broke, Figure 35 shows one such test sample.

![Figure 35: Y Hard Tensile Sample Failure](image)

Like the X hard test sample, the break occurred nearly instantaneously with a loud booming noise. A notable difference between the X and Y tensile soft tests is that the honeycomb de-bonded from itself down the a beam and then proceeded to break across the infill of the honeycomb. The breaks that occurred along through the infill are thought to be a result of stress concentrations. The stress concentrations occur at the joints because the beams try to bend but are resisted by the infill.

In a few locations the crack propagated along the l beams but this is thought to be a result of poor bonding in those locations.

A DIC analysis was conducted on one hard tensile test in order to look for strain concentrations. A smaller, 5” sample had to be used because the load frame that had the DIC system setup had a height restriction. The result of the DIC of the hard X direction can be seen in
Figure 36, note that the sample is rotated so what appears as the Y direction in the figure is really measuring the X direction of the honeycomb.

The image show areas of high strain as a percentage to the rest of the model.

The result of the DIC of the hard Y direction can be seen in Figure 37.
Again the specific strain is not being calculated but the figure clearly shows regions of higher stress around the honeycomb structure.

The results for the soft and hard tensile tests can be seen in Table 11.
The composite effective Young’s modulus in the X and Y directions for the hard (cold) state are nearly the same due to the high stiffness. The high stiffness restrained the beams from bending causing the skin to behave similarly to that of an isotropic material. In comparison, for the soft (hot) tensile tests, the X and Y modulus differ by a factor of 2.32. In this case the honeycomb is able to act as mechanism and so the effect of the honeycomb structure dominates the strength of skin. The honeycomb beams bent at the corners and influenced the modulus to a much greater affect. Like the empty honeycomb results the X tensile has a higher modulus than the Y tensile.

Using the modulus and $T_g$ from Chapter 2 and the trends of the X and Y modulus from Chapter 3, the experimental composite model provided means by which the analytical properties of the composite will be validated in Chapter 6. The next step is to investigate the FEA model to make sure the assumptions used in the analytical equation are valid.
CHAPTER 5
COMPOSITE FEA

The experimental study of the honeycomb composite in Chapter 4 along with the FEA model in this chapter will be used to verify the analytical model presented in Chapter 6. The roadmap, shown in Figure 49, shows the focus of the design process at this step. The basics of the reconfigurable skin are presented in this section including the analytical model of the composite, experiment characterization, the FEA modeling of the skin, and a first level cell optimization.

Figure 38: Road Map: Composite Characterization
Investigating the utility of an infilled honeycomb morphing skin via a trade space study/design tool requires an accurate analytical model; which is validated via FEA modeling and experimental results. The SMP modulus from Chapter 2 and the honeycomb properties from Chapter 3 were used for input to the FEA model presented in this Chapter.

5.1 Overview and Assumptions

A finite element analysis was run on a unit cell model in ABAQUS v6.10 to compare to experimental and analytical results. The model was based upon homogenization theory which states a single periodic cell can be used to extrapolate the material properties if proper periodic boundary conditions are used. This was chosen because it reduced the size of the model and allowed the material properties to be extrapolated to a sheet of any size and not just a specific case. Most importantly the FEA model will show how the honeycomb cell behaves and verify whether modeling it with thin beam theory assumptions is valid.

The homogenization theory assumes that an individual honeycomb cell is significantly smaller than the resulting sheet. The individual cell in this model was the repeating unit as shown on the right hand side of Figure 53. The analytical model showed that keeping the same ratio of the honeycomb geometries: wall length to thickness, but changing the actual size of the cell didn’t change the resulting modulus of the composite.

A key feature in the homogenization method is the periodic boundary conditions (PBC). Simply stated, whatever happens on one side of the cell happens on the opposite side including displacement and stress distribution.

5.1.1 PBC Constraints

The finite element method can deal with constraints in a few different manners. The constraints include applying a load, forced displacement, and linear constraints (aka tying one node to another). One common method is applying Lagrange multipliers, it is easy to implement with commercial FEA software as it does not require access to the source code. Cook et al (Cook,
writes a linear constraint equation that couples Degrees of Freedom (DOF) \( \{D\} \) in the form of equation (12):

\[
[\mathbf{C}]\{D\} = \{Q\} \tag{12}
\]

where \([\mathbf{C}]\) and \(\{Q\}\) are constants. This equation is rewritten to make it equal to zero as seen in (13).

\[
[\mathbf{C}]\{D\} - \{Q\} = \{0\} \tag{13}
\]

Equation (13) is then multiplied by the row vector \(\{\lambda\}^T\) that contains as many Lagrange multipliers as there are constraints. The potential energy equation that the finite element method solves can then be written in the form of equation (14).

\[
\Pi_p = \frac{1}{2} \{D\}^T[\mathbf{K}]{\{D\}} - \{D\}^T\{R\} + \{\lambda\}^T([\mathbf{C}]\{D\} - \{Q\}) \tag{14}
\]

where \(\Pi_p\) is the potential energy, \([\mathbf{K}]\) is the stiffness matrix and \(\{R\}\) is the reaction force. Since the term on the left hand side of equation (14) is equal to zero, it can be added to the equation. The next step is to make \(\Pi_p\) stationary by taking the partial of equation (13) with respect to the DOF and the Lagrange multipliers as seen in (15), (16), and (17).

\[
\frac{\partial \Pi_p}{\partial \mathbf{D}} = 0 \quad \frac{\partial \Pi_p}{\partial \lambda} = 0 \tag{15}
\]

\[
\frac{\partial \Pi_p}{\partial D} = [\mathbf{K}]\{D\} - \{R\} + \{\lambda\}^T[\mathbf{C}] \tag{16}
\]

\[
\frac{\partial \Pi_p}{\partial \lambda} = ([\mathbf{C}]\{D\} - \{Q\}) = 0 \tag{17}
\]

The result is two equations that can be written in matrix form, seen in equation (18).

\[
\begin{bmatrix}
[\mathbf{C}] & {\mathbf{C}}^T \\
\mathbf{C}^T & 0
\end{bmatrix} \begin{bmatrix}
\{D\} \\
\{\lambda\}
\end{bmatrix} = \begin{bmatrix}
\{R\} \\
\{Q\}
\end{bmatrix} \tag{18}
\]

By solving this equation the reaction force and Lagrange multipliers can be found.

5.1.2 Three Node Example of PBC

Figure 39 shows a 1D example using the previously described method to find the reaction forces and displacements of the nodes. A set displacement of 0.1 is applied and stiffness, \(\mathbf{K}\), of

52
each element is 1. $u_1$, $u_2$, and $u_3$ are nodes on the part and $u_4$ is a reference node that is not connected to the system.

![Figure 39: Simple 3 Node System, (u1-u3) With a Reference Node, (u4) Using Periodic Boundary Conditions With Stiffness, K, in Each Element](image)

The constraints on the problem can be seen in equations (19), (20), and (21). Equation (19) represents the total applied displacement of the system. Equation (20) fixes the displacement of the system so no translation of the system occurs. Equation (21) is the application of the PBC using linear constraint which states the motion of $u_3$ is equal to the motion of $u_1$ plus $u_4$.

$$Q = u_4 = 0.1$$ (19)

$$u_2 = 0$$ (20)

$$u_3 = u_1 + u_4$$ (21)

Node $u_2$ is fixed to eliminate free body translation of the model in the X direction, equation (20), the net displacement is enforced by equation (19), and the PBC shown in equation (21) forces the system to displace equally at both node 1 and 3. It doesn’t matter which node (1, 2, or 3) on the system is fixed at zero, only that one node is (some commercial programs won’t allow any node to be chosen because it thinks the system is over constrained) . All the fixed node does is remove rigid body motion of the part. In theory this doesn’t have to be applied because node $u_1$ and $u_3$ move relative to each other but in order to remove the rigid motion of the
assembly, this boundary condition is necessary. If this boundary condition was left out then \( F_2 \) would become 0 and the system would still be able to be solved. In this model node \( u_2 \) was chosen because otherwise the model would be a trivial 1D line problem.

The global stiffness matrix of the assembly, not including the reference node, is shown in equation (22).

\[
\begin{bmatrix}
  K & -K & 0 \\
  -K & 2K & -K \\
  0 & -K & K \\
\end{bmatrix}
\begin{bmatrix}
  u_1 \\
  u_2 \\
  u_3 \\
\end{bmatrix}
= \begin{bmatrix}
  F_1 \\
  F_2 \\
  F_3 \\
\end{bmatrix}
\tag{22}
\]

The PBC constraint can be rewritten in matrix form by using equation (13) to form equation (23).

\[
\begin{bmatrix}
  1 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
  u_1 \\
  u_2 \\
  u_3 \\
\end{bmatrix}
- \begin{bmatrix}
  u_4 \\
\end{bmatrix}
= \begin{bmatrix}
  0 \\
\end{bmatrix}
\tag{23}
\]

Then equation (23) can be used to rewrite the global stiffness matrix to include the constraints as seen in equation (24).

\[
\begin{bmatrix}
  K & -K & 0 & 1 \\
  -K & 2K & -K & 0 \\
  0 & -K & K & -1 \\
  1 & 0 & -1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
  u_1 \\
  u_2 \\
  u_3 \\
  \lambda \\
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  F_2 \\
  0 \\
  \lambda \\
\end{bmatrix}
\tag{24}
\]

If \( K \) is 1 and the boundary conditions are plugged back into the equation, equation (25) is produced.

\[
\begin{bmatrix}
  1 & -1 & 0 & 1 \\
  -1 & 2 & -1 & 0 \\
  0 & -1 & 1 & -1 \\
  1 & 0 & -1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
  u_2 \\
  u_3 \\
  \lambda \\
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  F_2 \\
  0 \\
  0.1 \\
\end{bmatrix}
\tag{25}
\]

Then the system of equations produces the following results, seen in equation (26), (27), (28), and (29).

\[
\lambda = -0.5 
\tag{26}
\]

\[
u_1 = 0.05 
\tag{27}
\]

\[
u_3 = -0.05 
\tag{28}
\]

\[
F_2 = 0 
\tag{29}
\]
Node 1 and 3 displace the same distance away from node 2, 0.5, producing a zero reaction force on node 2. Also note that the tensile force needed to displace the system 0.1 apart is -0.5.

5.1.3 Finding the Effective Poisson Ratio with PBC

A method to prescribe PBC to a perfect square of an isotropic material with a Poisson ratio of 0.33 that is in tension in the x direction is described below. First, three reference nodes are chosen: 1 and 2 corresponding to the x, and y faces have to be created. Then constrain the X faces with reference node 1 and Y faces with reference node 2. Reference node 1 is given a positive displacement of 1 while reference node 2 is prescribed a negative displacement of 0.33. The result is tension in the X direction and compression in the Y directions with zero reaction forces on reference node 2.

When dealing with an anisotropic material where the Poisson ratio is unknown a different method is needed. No heuristic method to find the Poisson ratio was found in the literature so one was developed. Below is the series of steps used to find the Poisson ratio of an anisotropic material.

Below is the heuristic that was developed to find Poisson ratio of a material in tension:

1. Assume Poisson ratio for the transverse directions
   - A good starting point is found by running a test case with no constraints in the transverse direction and start with that Poisson ratio
2. Find the resulting reaction force in the transverse direction
3. Reduce/Increase the displacement to minimize the reaction force
4. Repeat until reaction force equals zero
5. Once the final reaction force equals zero a model with the correct Poisson ratio is found.

The advantage of using this method is that it is simple and always works. The major drawback is that it takes a while to get to the answer especially when the model is very large. There are a couple of improvements that can be used to increase the speed of this method. One
way is to run the model with a smaller mesh to speed the process up and then use a refined mesh to narrow in on the exact value. Another improvement for finding a good starting point is to run the model without the periodic boundary conditions and use the resultant Poisson ratio as the initial guess. Another noticeable improvement is to plot the reference force in the transverse direction against the displacement of the corresponding reference node.

5.1.4 Creating PBC in ABAQUS

Applying PBC to an ABAQUS model was accomplished through linear constraints. Linear constraints tie the displacement from a node on one side to the displacement of the node on the corresponding side as shown by Cook et al. The equation for this is shown in equation (30).

\[ u_a - u_b = 0 \]  

(30)

Three problems arose: how to enforce a displacement, how to recover the reaction force, and finally how to deal with the corner nodes.

By changing the equation (30) so it does not equal 0, a net displacement between the sides can be created. This was accomplished through the creation of a reference node adding a third term to the equation seen in equation (31).

\[ u_a - u_b = u_{\text{Ref}} \]  

(31)

ABAQUS linear constraint equations have to be equal to zero which transforms equation (31) to (32).

\[ u_a - u_b - u_{\text{Ref}} = 0 \]  

(32)

The reference node was created in ABAQUS by adding another part instance that only contained a reference node. By tying these three sets of nodes together a net displacement can be found from one side to the other.

If all the nodes on one side and their respective DOF are tied to the opposing side with the same reference node the cumulative reaction force from the entire side can be recovered.
The third problem with applying linear constraint equations in ABAQUS is constraining the corners. The nodes at the corner all move simultaneously as seen in Figure 40.

\[ D = C_4 = A_4 \]  

Equation (33) shows the relative displacement of the D corner and how it moves exactly the same both the A and C corners.

\[ C = B_4 = D_3 \]  

Equation (34) shows the relative displacement of the C corner and how it moves with both the B and D corners.

It can be seen that the same thing happens for the A and B corners. Therefore all corner nodes move together in a shear. If the unit cell undergoes tension/compression then the nodes on the opposite face move further/closer to each other but in the transverse direction they have the same displacements. When implementing the linear constraint equations it is important to only tie the nodes together once.

5.2 FEA Setup

FEA analysis was run on a unit cell of the experimental honeycomb with a soft infill and hard infill. Test cases were also run for shear in the XY and YX directions, tensile X and Y
directions, and a case study was performed with varying the shear deflection to see how it impacted the results as seen in APPENDIX C.

The model FEA analysis was conducted in ABAQUS. The model was based upon the experiment. The material properties are shown in Table 12. FEA model dimensions are shown in Table 12 and Figure 41.

Table 12: FEA Model Dimensions and Material Properties

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total X Length</td>
<td>23.55525 mm</td>
</tr>
<tr>
<td>Total Y Length</td>
<td>13.89597 mm</td>
</tr>
<tr>
<td>( a )</td>
<td>5.327 mm</td>
</tr>
<tr>
<td>( c )</td>
<td>1 mm</td>
</tr>
<tr>
<td>( d )</td>
<td>0.044 mm</td>
</tr>
<tr>
<td>( l )</td>
<td>9.28 mm</td>
</tr>
<tr>
<td>( \theta )</td>
<td>45.964(^\circ)</td>
</tr>
<tr>
<td>SMP (hard)</td>
<td>968 MPa</td>
</tr>
<tr>
<td>SMP (soft)</td>
<td>1.6 MPa</td>
</tr>
<tr>
<td>Aluminum</td>
<td>69 GPa</td>
</tr>
</tbody>
</table>

Figure 41: FEA Honeycomb Model
In this way the results of the experimental work can be compared to the FEA. The FEA modeled the hard state and soft state for the two shear directions, XY and YX, and the two tensile directions, the X and Y shown in the test matrix in Table 13.

<table>
<thead>
<tr>
<th>Test</th>
<th>Hard</th>
<th>Soft</th>
</tr>
</thead>
<tbody>
<tr>
<td>XY Shear (0.5°)</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>YX Shear (0.5°)</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>X Tensile (1%)</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Y Tensile (1%)</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

The model was meshed with 2.3 million elements seen in Figure 42.

The FEA tensile test was conducted to find the equivalent Young’s modulus of the honeycomb. Four test cases run for tensile X hard and soft and tensile Y hard and soft. A strain of 1% was set for each loading. The resulting displacement of each side can be seen in Table 14. Table 14 shows the original length of the honeycomb sides, the state of the SMP, the amount of strain, the resulting strain, and the final lengths of the unit cell. Poisson’s ratio was determined.
by the method described in section 5.1.3 and those values, indicated in Table 14, were used to determine the X and Y modulus.

Table 14: Tension Displacements

<table>
<thead>
<tr>
<th>Original Length</th>
<th>State</th>
<th>Strain</th>
<th>Poisson</th>
<th>Δx</th>
<th>Δy</th>
<th>Final x Length</th>
<th>Final y Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>X 23.5523</td>
<td>Hot</td>
<td>0.01</td>
<td>1.61</td>
<td>0.235525</td>
<td>-0.21623652</td>
<td>23.788025</td>
<td>13.21460348</td>
</tr>
<tr>
<td></td>
<td>Cold</td>
<td>0.01</td>
<td>0.6085</td>
<td>0.235525</td>
<td>-0.08172666</td>
<td>23.788025</td>
<td>13.34911334</td>
</tr>
<tr>
<td>Y 13.43084</td>
<td>Hot</td>
<td>0.01</td>
<td>0.59</td>
<td>-0.13895975</td>
<td>0.1343084</td>
<td>23.41354025</td>
<td>13.5651484</td>
</tr>
<tr>
<td></td>
<td>Cold</td>
<td>0.01</td>
<td>0.4745</td>
<td>-0.11175661</td>
<td>0.1343084</td>
<td>23.44074339</td>
<td>13.5651484</td>
</tr>
</tbody>
</table>

The model was subject to six boundary conditions. The first two boundary conditions prevent translation and rotation of the model in equation (35). The next four boundary conditions applied the type of loading. For the tensile case the reference node that corresponds to the X faces and Y faces respectively are zero in the transverse direction and equal to the strain shown in Table 14. These boundary conditions are shown in equations (36), (37), and (38).

\[
u_{x_{Top\ Left\ Corner}} = \nu_{y_{Top\ Left\ Corner}} = 0 \tag{35}\]

\[
u_x = \Delta x \tag{36}\]

\[
u_{y_{REF1}} = \nu_{x_{REF2}} = 0 \tag{37}\]

\[
u_{y_{REF2}} = \Delta y \tag{38}\]

5.3 FEA Results

The FEA results for the tensile and shear test cases are shown in Table 15.

Table 15: Results of the FEA Tensile Tests

<table>
<thead>
<tr>
<th>V</th>
<th>Shear</th>
<th>Tensile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>YX Soft</td>
<td>XY Soft</td>
</tr>
<tr>
<td>Modulus</td>
<td>1.77E+07</td>
<td>1.77E+07</td>
</tr>
</tbody>
</table>

\(\nu\) is the Poisson ratio of the system. The difference of the soft state Poisson ratio versus the hard state is large. This is due to the skin acting less like a mechanism in the hard state because the Young’s modulus of the infill is similar to that of the honeycomb.
The modulus of the Y tension is lower than the X tension for both the hard and soft conditions. This again can be attributed to the geometry of the cell due to the initial angle \( \theta \).

The large difference in the soft and hard state modulus is also attributed to the skin acting like a mechanism in the soft state and as single material in the hard state due to the similarities of the materials’ modulus.

The FEA tensile contour plots showed similar stress concentrations for the X and Y directions for both the soft and hard starts, therefore only the X soft tensile case will be shown.

Figure 43, Figure 44, and Figure 45 shows von Mises, S11, and S22 contour plot of the soft X tension.

![Figure 43: FEA Tensile Contour Plots of Soft X Tension With a Focus on the Von Mises Stress in the Infill.](image)
The S11 stress contributes to the high stress in the infill. This is expected since the cell is undergoing tension in the 1-direction. The stress above and below the a beam is low due to most of the load being carried by the beam. It is important to note though that the magnitude of the S22 stress is high around the l beams, this is due to the honeycomb acting like a mechanism and
compressing. The search for the correct Poisson ratio requires that the net force on the top and bottom of the figure equals zero but the internal stresses do not.

Figure 46 shows the FEA contour plot of the soft X (1) tension with a focus on the axial stress in the top left $l$ beam while Figure 47 shows a plot of the axial stress along a line through the top left $l$ beams at the top, middle, and bottom of the beam plotted against their location along the beam.

Figure 46: FEA Tensile in the $l$ Direction With a Soft Infill in the Axial Direction of the Top Left $l$ Beam. A Focus Is Shown on the Top Left $l$ Beam With Units in MPa
The entire beam is in tension but the S shaped curve clearly shows multiple bending at the middle and end of the beams. The area of high stress is at the corners where nonlinear deformation occurs. Since the FEA model is linear it does not correctly model the stresses at the ends of the beam.

Figure 48 shows the FEA contour plot of the soft X tension with stresses plotted along the axis of the α beams with a focus on the stresses in the beam. A plot of the stresses along the top, middle, and bottom path of the beam is not shown here because the data was too noisy due to the mesh not being fine enough along the unit cell and stress concentrations that occur at either end of the beam.
The figure shows an increase in stress at the corner with a 2 MPa variation from the end of the beam to the center. The slight peak in the center of the beam is thought to be caused by the infill which has a lower stress near the center causing the beam to carry the stress instead. At either end of the beam the stress concentrations are due to the nonlinear plastic deformation that the FEA model does not capture.

The tensile modulus has a large variation between the soft and cold states in the X and Y directions. The X soft modulus was 2.8x greater than the Y soft modulus. The reasons for this can be attributed to the fact that honeycomb acts like a mechanism and have a large impact on the overall modulus. The larger value in the X direction was due to a large amount of the load carried by the $a$ beams which has a high modulus while the Y direction has a lower modulus because the $l$ beams bend causing the unit cell to elongate with a lower force.

The hard tensile modulus was very similar in the X and Y direction. This is due to the unit cell acting less like a mechanism and more like a composite skin when the infill is in the hard state. Consequently it is expected to see similar moduli in both directions. As the figures show the beams still bend causing the Y direction modulus to be lower than the X direction modulus.
The focus on the FEA was understand the cell behavior specifically mechanics of the cell. Moving forward the Soft Tensile modulus in the Y direction of 2.04E+7 Pa will be used to look at the analytical model. The Soft tensile modulus in the X direction was is likely an error and not used to verify the analytical model. The Hard X and Y tensile modulus was 1.46E+9 Pa and 1.33E+9 respectively and they were used to verify the analytical model.
CHAPTER 6

COMPOSITE ANALYTICS

An accurate analytical model for the design space of a filled honeycomb skin with spatially prescribably stiffness properties given different materials and honeycomb configurations, presented in this chapter, requires experimental characterization of representative empty honeycomb, infill, and filled honeycomb at various angles, temperatures, and boundary conditions shown in Chapters 1-5. The road map, shown in Figure 49, shows the focus of the design process at this step.

Figure 49: Road Map: Composite Characterization
The material characterization of the SMP presented in Chapter 2 provides the inputs for the Young’s modulus and glass transition temperature as the same SMP is used. The characterization of honeycomb and its anisotropic behavior was seen in Chapter 3. Chapter 4 presented the experimental results of a honeycomb composite that will validate the analytical model presented in this Chapter and the same honeycomb properties will be used as inputs to the analytical model. The FEA model in Chapter 5 provides an understanding of how the cell behaviors under tension and provides verification of the assumptions used in the analytical model.

6.1 El-Sayed

El-Sayed et al (El-Sayed F., 1976) developed equations for a honeycomb with a low modulus infill. El-Sayed assumed that the deformations remained small and the sides of the hexagon remain straight due to the infill. Other implied assumptions were thin beam theory which states the aspect ratio of the beam has to be 8 or greater. The honeycomb as mentioned in section 3.2.1.1 is perfectly elastic. The honeycomb is completely uniform. Assuming the deformations remain small, thin beam theory holds true, and allow for Castigliano’s theory to be applied which is what El-Sayed used to derive his models. Castigliano’s Theory states “If an elastic system is supported so that rigid-body displacements of the system are prevented, and if certain concentrated forces of magnitudes, \( F_1, F_2, \ldots F_p \) act on the system, in addition to distributed loads and thermal strains, the displacement component of \( q_i \) of the point of application of the force \( F_i, \) is determined by the equation (39).” (Boresi, 2003)

\[
q_i = \frac{\partial U}{\partial F_i}, \, i = 1,2, \ldots p
\]

(39)

Where \( U \) is the strain energy. The assumptions of perfect adhesion and uniform honeycomb simplify the model. In order to relate the strain distributions to the applied forces, the Potential Energy Method was used for analysis.
El Sayed’s model uses the assumption that the deformations remain relatively small, neglects axial deformation of the honeycomb walls, and the sides of the infill remain straight. These assumptions as will be shown in section 5.2 don’t always hold true and at large deformations these will be a large source of error. Figure 50 shows the unit cell used for creating the model and the additional variables used in the following equations, where $a, l, d,$ and $\theta$ are the horizontal wall length, slanted wall length, wall thickness, and cell angle. $x_o$ and $y_o$ are a function of the cell geometry and are $l \cos \theta$ and $l \sin \theta$ respectively. A dimension not shown but important to the analysis is the cell depth, $c$.

![Figure 50: Honeycomb Dimensions and Unit Cell](image_url)

El-Sayed looks at the strain energy in a strip, $d_y$, of a infill as shown in Figure 51.
Then by integrating over the whole element, the strain energy is obtained. The potential energy of the system can then be found by subtracting the work of the external forces from the strain energy.

El-Sayed assumed the sides of the honeycomb walls remain straight and the deformations are small. For a given load and displacement the modulus in the principal directions, as defined as the X and Y axis in Figure 50, can be obtained as shown in Equations (40) and (41).

\[ E_{xc} = B_H E_H + B_I E_I \]  \hspace{1cm} (40)
\[ E_{yc} = D_H E_H + D_I E_I \]  \hspace{1cm} (41)

Where the coefficients to calculate the composite modulus in the X direction are shown in equations (42) and (43).

\[ B_H = \frac{12I(a + x_o)}{c y_o^2 I \left( y_o + \frac{12I \cos^2 \theta}{y_o A} + \frac{6al}{y_o A} \right)} \]  \hspace{1cm} (42)

\[ B_I = \frac{(a + x_o) [K]}{2y_o(1 - v^2) l \left( \frac{y_o}{x_o} \right)^2} \]  \hspace{1cm} (43)

The coefficients to calculate the composite modulus in the Y direction are seen in equations (44) and (45).
\[ D_H = \frac{12Iy_o}{c(a + x_o)(lx_o^2 + \frac{12Iy_o\sin\theta}{A})} \] (44)

\[ D_I = \frac{y_o[K]}{2(a + x_o)(1 - v_i^2)} \] (45)

In both of these equations \( K \) is defined by equation (46).

\[
[K] = \left[ \left( \frac{y_o}{x_o} \right)^3 - \frac{a}{x_o} \left( \frac{y_o}{x_o} \right)^3 + \frac{1}{2} \left( \frac{a}{x_o} \right)^2 \left( \frac{y_o}{x_o} \right)^3 \ln \left( 1 + \frac{2x_o}{a} \right) + \frac{x_o}{y_o} + \frac{a}{y_o} + 2v_i \frac{y_o}{x_o} \right]
\] (46)

\( I \) is the second moment of area of the beam which is a function of the beam width. \( A \) is the cross-sectional area of the beam. \( E_H \) is the elastic modulus of the honeycomb material presented in 3.2.1.1 and \( E_I \) is the elastic modulus of the infill found experimentally in Chapter 2. \( v_i \) is the Poisson ratio of the infill which was assumed to be 0.5 for polymers. This model is limited to deformations of about 30% strain before the model starts behaving nonlinearly and for the composites where the infill is three plus orders of magnitude below that of the honeycomb. El-Sayed compared his model to an experimental test with cell geometries of \( l = 7.3 \text{ mm}, d = 0.1 \text{ mm}, a = 7.3 \text{ mm}, \theta = 60^\circ \) as seen in Figure 52.
The infill, Flexane 30, had a modulus of 1.08 MPa and the aluminum honeycomb had a modulus of 70.3 GPa. El-Sayed plotted the load versus the percent strain. The experimental test used a honeycomb that was 3 cells wide by 20 cells long.

6.1.1 Murray’s Investigation of El-Sayed’s Model

Murray et al (Murray G. G., 2009) looked at polymer-filled honeycombs for structural damping. The goal of the research was to examine if high-modulus and high damping could be obtained with this design. Murray looked at El-Sayed’s equations and compared the moduli in the principle directions and the strain energy of the cell to an FEA analysis. Figure 53 shows the unit cell he used and his applied boundary conditions.

Figure 52: Load/Displacement Relationship for Experimental and Analytical Filled Honeycomb Filled (Left) and Unfilled (Right) (El-Sayed A. J., 1979)
Murray used ANSYS© and modeled the honeycomb walls with (BEAM3) beam elements and the infill with (SHELL63) shell elements. The image on the right of Figure 53 shows the unit cell boundary conditions parallel to the loading direction highlighted in green and they are constrained so they move the same amount perpendicular to the loading direction. The bottom of the model is constrained in the y-direction only, and the top the displacement is uniform across the width of the unit cell. Also the corners of the honeycomb marked by a star are constrained to have zero rotation about the z-axis.

The results of Murray’s investigation into the accuracy of the infill are reproduced in Figure 54, which used Aluminum alloy 2024-T4 for the honeycomb with a modulus of 70 GPa.
The predicted honeycomb modulus was accurate for a honeycomb with a modulus of about three orders of magnitude greater than the infill modulus, but the error increased with increasing infill. This was due to two of the assumptions El-Sayed made: 1) the elastic bending of the honeycomb walls was not influenced by the filler, and 2) the bending of the walls was not used in the calculations of the strain in the filler. The strain energy calculations of the model were off by a significant margin. What is important to note is that El-Sayed’s equations capture the general trends of different honeycomb configurations even if it isn’t quite accurate.

6.2 Puttmann et al. Model

Puttmann et al (Puttmann, Beblo, Joo, Reich, & Smyers, 2012) derived an equation that predicted the deflection of a flat plate.
The bending moments of a plate are based upon equations (47) and (48).

\[
M_{bcv} = 0.009wb^2(1 + 2\alpha^2 - \alpha^4) + \frac{v_i wb^2}{8(3 + 4\alpha^4)}, \quad (47)
\]

\[
M_{acv} = \frac{wb^2}{8(3 + 4\alpha^4)} + 0.009v_i wb^2(1 + 2\alpha^2 - \alpha^4), \quad (48)
\]

Where \( M_{bcv} \) and \( M_{acv} \) is the bending moment, \( w \) is the pressure applied to the panel, \( v_i \) is the Poisson ratio of the equivalent honeycomb composite (this case assumed 0.5), \( b \) is the width of the panel, and \( \alpha \) is the ratio of the width to the length, \( m \), of the panel as seen in equation (49).

\[
\alpha = \frac{b}{m}, \quad (49)
\]

These moments are used in Castigliano’s theorem for deflections, resulting in equation (50). If \( E_{xc} \) and \( E_{yc} \) are equal, the maximum deflection of the plate calculated by equation (50) differs from that by flat plate theory of an isotropic material by less than 2%.

\[
\delta = \frac{2m^2 b^2}{5c^3(m + b)^2} \left[ \frac{M_{bcv}(3m + b)}{E_{cy}} + \frac{M_{acv}(m + 3b)}{E_{cx}} \right], \quad (50)
\]

6.3 Composite Discussion

The analytical results of the soft tensile of the analytical modeling, FEA modeling, and experimentation are shown in Figure 75.
The first thing to note is that the FEA soft tensile in the X direction is significantly lower than either the analytical or composite testing. The focus on the FEA was understand the cell behavior specifically mechanics of the cell. While FEA results are in the x direction are likely in error the FEA analysis does significant beam bending as seen in 5.3, which the analytical equation is based upon. The lower Y modulus versus the X modulus has the same trend in the composite experiment and analytical model like the honeycomb experiments showed in Chapter 3. The percent difference of for the Y and X soft moduli was 34% and 12% respectively. Even with the high percent difference of the Y soft modulus the analytical model shows similar trends with the composite experiments.

The analytical results of the soft tensile of the analytical modeling, FEA modeling, and experimentation are shown in Figure 56.
The analytical model shows a much larger difference between the X and Y model than the FEA or the experiments. If the FEA model is looked at again, beam bending plays a less significant role in cell behavior. This would indicate that the analytical model based on beam bending is not as accurate as Figure 56 clearly shows. What is not addressed directly in this thesis was a rule of mixture equation which more closely models the composite experimental results with a Modulus of 1.63E+9 PA in the X and Y direction.

When comparing the design to the N-MAS requirements it fails in most categories. A panel of 38.1 x 50.8 cm deflecting only 2.54 mm would weigh 8.3 kg or 48 kg/ m² compared to the requirement of 4.8 kg/m². In order to reduce the overall weight partially filled cells or cells layered with a light foam core could significantly reduce the weight while still maintain structural rigidity and a smooth surface.
CHAPTER 7

CONCLUSIONS

This thesis investigated the design space of the Spatially Targeted Activation of SMP with experimental, computational, and analytical models as defined by the N-MAS requirements. The SMP and honeycomb selection allowed for tailorable properties and cheap quick experiments but some of the characteristics were undesirable. The models created were able to closely correlate the in plane modulus of the honeycomb composite. The optimization in APPENDIX C revealed the design space and the necessary constraints needed to further pursue this research. The next steps will be to refine the current work and research the next steps into creating the morphing skin system.

The material selection proved to be a viable option to investigate the composite skin’s properties. The SMP selected could be tailored to different modulus and transition temperatures. Unfortunately the hard state was very brittle and when failure occurred, it happened quickly across the entire specimen. The SMP in the soft state showed promising characteristics in tension and compression.

The honeycomb used for the experiment was a commercially available option. This provided a quick and cheap way to test multiple samples. The manufacturing process for the honeycomb was unknown, but from difficulties with creating uniform test specimens it can be surmised that residual stresses were created during the manufacturing of the honeycomb. The design space study revealed that the geometry of the honeycomb significantly impacts its performance. Correcting for inconsistencies in the honeycomb cell and reducing the residual
stress that occurs during the manufacturing process, the experimental results accuracy and consistency can be increased.

The experimental tests were conducted with commercially available honeycomb and therefore were not intended to meet the N-MAS requirements. The honeycomb skin developed, showed promising results in the experimental tests. First, it tied all the research together and showed that the computational model and analytical models had the correct trends. It also verified that the theory behind Spatially Targeted Activation of SMP is plausible but needs more work. The skin’s weight ultimately ended up being significantly too heavy for the N-MAS requirements. The breaks in the samples during testing were generally severe and further work needs to be done to address this issue.

The analytical model of SMP and empty honeycomb was well documented in the literature but the composite structural model of an infilled honeycomb had limited capabilities, specifically in its ability to accurately show the modulus of the material when the honeycomb walls aspect ratio becomes small and the Thin Beam Theory used to create the model no longer applies. The analytical model showed similar trends to both the experimental and computational models.

The computational model agreed with experimental results and the analytical model in the tension test. The computational model showed areas of high stress in the corners in both the Aluminum and SMP. The computational model and experimental results had a difference of 26% but this was thought to be experimental error and differences between the geometry of the physical model and the analytical model. As the analytical model shows small differences in cell wall thickness, beam length, and cell angle, θ, can cause significant differences in results and the experimental honeycomb sometimes had large variations between honeycomb cells.

The following areas need further research before the morphing skin design is a viable option for aircraft skin and meets the N-MAS requirements. First, a better selection of SMP and honeycomb material that will give an increase in performance and consistent results. Second, the
computational, analytical, and experimental models need to increase accuracy and robustness. Third, an improved optimization study done after a better definition of the constraints. Fourth, control methods and algorithms to optimally heat the SMP in a consistent manner with the desired motions of the skin. Finally, a number of items not addressed in this thesis need to be investigated, including heating configurations, the optimization of heating configurations, the ability to control heating pattern, abnormal cell geometries, and mixed cell geometry.

The presented work of an experimentally and FEA validated analytical model predicting the response of a filled honeycomb was found to be a viable concept for morphing aircraft skins and has been proven to be a valuable resource for future exploration of morphing skins.


*43rd Annual Structures, Structural Dynamics, and Materials Conference.* Denver, Colorado: AIAA.


APPENDIX A

Composite Shear Experiment

An accurate analytical model for the design space of a filled honeycomb skin with spatially prescribably stiffness properties given different materials and honeycomb configurations presented in Chapter 4 focused on the Young Modulus of the material in the x and y directions. The shear modulus of the honeycomb is experimentally obtained in this appendix.

A.1 Shear Test Specimen

A.1.1 Sample Preparation

The first step is to create the shear test specimens is to mill out the Teflon© molds. The Teflon© mold was cut to 12.7 cm x 12.7 cm. The mold was prepared with a release agent to allow the sample to easily be removed after it was cured. The sample was then poured and cured as in 2.2.1.1. Figure 57 shows the size of the sample even though this sample has a honeycomb embedded in it.
The holes were drilled with a CNC router and the mold was made without corners to allow the shear test fixture to rotate at the four corners.

Additional post-processing was done to smooth the edges down with sand paper and then a speckled decal applied to be used for the Digital Image Correlation (DIC) shown in Figure 58. DIC is an optical method of tracking strain by measuring changes from one set of pictures to the next in a set time interval. This is usually done on a speckled pattern.
A.1.2 Shear Test Equipment

Both the hard and soft shear test used the same test bed. The soft shear test included the same instruments to control and measure the temperature as in 2.2.2.1. Figure 59 shows the shear test setup.
The shear specimens were tested on an MTS 858 load frame at WPAFB by me. The extension arm, pivot, mounting bracket and custom shear test fixture is described below. The thermal chamber was made out of ¾” plywood insulated by Compression-Resistant Polyimide Insulation. The Sony XCDU100 cameras with Schneider C-mount lens and lights were focused on the specimen. ARAMIS®, DIC software, was used to analyze the pictures and to measure the shear angle. What is not shown in the picture is the Omega “T” Type Air Process heater AHP-5051 heat pipe that enters in the rear. The temperature was measured with thermocouples placed on the back side of the test specimen and read with a thermocouple reader. Double-sided sandpaper is layered between the test sample and test fixture to prevent the sample from slipping. A closer look at the test fixture is shown in Figure 60.
The aluminum block was fixed to the test frame with O-Clamps. The rigid steel L brackets were then affixed to aluminum block. The pins going through the steel L bracket allow three of the four linkages to rotate while keeping one side vertical. The linkages housed the test sample by clamping down on them with bolts on all four sides. The extension arm displaces vertically and is pinned at the top to the load frame so it is able to rotate and apply axial force through the rod.

A.1.3 Shear Test Procedure

The shear testing procedure for both the hard and soft shear test is described below. The only difference is with the hard test no heating or thermal chamber was used.

1. Calibrate the cameras used for DIC. Make sure that the deviation is below 0.031 pixels. Note this was done every time the cameras or lighting was adjusted.
2. Mount the shear test specimens to the 4 bar linkage. Place pieces of sandpaper between the test specimen and the sample. Tighten the screws around the sample to make sure it is clamped down.

3. Insert the test specimen into the custom test frame and attach the pins at the four corners. Note that the extension arm is not connected to the test frame yet.

4. Raise the crosshead of the load frame so the pin at the top can easily slide in, zero the load cell, and then slide the pin in.

5. Heat the thermal chamber up to 110°C. Account for any thermal expansion by raising or lowering the crosshead. (Not included in the hard test, skip to step 6)

6. Take 2 sets of pictures before the machine start to get a baseline for the DIC.

7. Run the test at 12.7 mm/min until the test specimen breaks. Take pictures while the test is running. The camera software is not linked to the test frame and was started manually. Pictures were taken every 4 seconds.

The use of DIC allowed for an average shear angle across the entire specimen to be measured. This simplified the calculation of the shear modulus. The equation to calculate the shear modulus is shown in equation where $\gamma$ is the average shear angle and $\tau$ is the shear stress (51).

$$G = \frac{\tau}{\gamma}$$  \hspace{1cm} (51)

With pictures taken every four seconds, the data from the load frame was correlated with the time pictures. The shear stress force required to shear the specimen, $F$, over the area of the side $A$ as seen in equation (52).

$$\tau = \frac{F}{A}$$  \hspace{1cm} (52)

where $A$ is the cross-sectional area of the side of the specimen that is deflected.
A.2 SMP Shear Results

The stress strain curve for each of the shear cases: Soft neat SMP are presented in Figure 61.

![Graph showing stress strain curve]

Figure 61: Neat Shear Stress Strain Curve, on the Left is Soft SMP and on the Right is the Hard SMP. Note that Both the X and Y Axis Are Different on Each Graph.

In the soft test case the fixture did not slip and the modulus was taken over the entire length of the specimen until it broke. The data points at 0.05-0.07 rad occurred after the sample broke. On the hard test case the fixture slipped and the initial few degrees were not taken into account.

Table 16 shows the modulus of the hard and soft neat SMP tests.

<table>
<thead>
<tr>
<th></th>
<th>Neat SMP Shear Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hard</td>
<td>1.269E+09 Pa</td>
</tr>
<tr>
<td>Soft</td>
<td>1.061E+06 Pa</td>
</tr>
</tbody>
</table>

Figure 62 show the broken soft shear test.
The soft sample broke at very low strains. At about 0.05 rad, the sample bowed out and ripped in a circular pattern as seen in Figure 62 over a period of about 8-12 seconds, slower than the hard shear crack propagation. The initial crack propagation immediately dropped the load on the test frame.

The shear modulus for polymer with a poison ratio of 0.3 is expected to be 1/3 of the Young’s modulus. The hard shear modulus 300% higher than expected and soft shear modulus was 20% lower than expected.
A.3 Composite Shear Experimental Test Results

The stress-strain curves of the soft and hard shear tests where based upon the shear data from the DIC and the loads in the MTS load frame [2B]. The Young’s modulus was calculated by equation

\[ G = \frac{\sigma}{\varphi} \]  

Where \( \varphi \) is the shear angle from the DIC data, \( \sigma \) is the stress, and \( \varepsilon \) is the strain. Stress is calculated by equation (54)

\[ \sigma = \frac{F}{A} \]  

Where \( F \) is the force measured from the load frame and \( A \) is the initial cross-sectional area of the specimen which was calculated by taking the average thickness measured at all four corners and the midpoint of the sides prior to installation into the grips.

The stress strain curve for the of soft shear cases XY and YX are presented in Figure 63.

![Figure 63: XY and XY Soft Shear Stress Strain Curve. Note That the Two Figures Have Different Y Axis.](image)

As the picture Figure 63 shows, after the linear region the honeycomb starts acting nonlinearly. The points at which samples broke were not captured in the data collection. The first few
fractions of a degree was not included in the calculations of the modulus as it was assumed to be slippage in the test setup.

None of the samples failed at the 1 radian shear that all the specimens were run to. On one sample this test was run longer to see the failure, Figure 64 shows a broken YX soft shear sample.

Figure 64: YX Soft Shear Sample Broken

The crack occurred slowly across the entire diagonal of the honeycomb with no noticeable start or finish. The honeycomb infill stays nearly intact throughout the entire sample and the honeycomb de-bonds from the SMP without cracking going through the infill itself. The honeycomb α beams de-bonded from itself instead of the actual infill cracking. The breaks occurred corner to corner because at the shear angle at which it broke, a large tension force occurs from corner to corner while a large compression force occurs in the other direction causing the SMP to bulge out.
In addition to a break going diagonally across the honeycomb structure, another break occurred at the top left corner of the figure where the material bulged out, going in the perpendicular direction. The breaks that occur around the edges are thought to be a result of stress concentrations from the mounting of the specimen.

The result of the DIC of the hard YX direction can be seen in Figure 65.

![Figure 65: DIC of Hard YX Shear Test Percent Strain in the Y Direction](image)

Again the relative strain is shown as a percentage as the figure clearly shows regions of higher stress. Unlike the tension cases no discernable honeycomb pattern can be found. The areas
of high strain and low strain go across honeycomb cells and don’t have a defined impact as in the tension cases.

The stress strain curve for the each of the hard shear cases: hard XY and YX are presented in Figure 66.

![Figure 66: XY and YX Hard Shear Stress Strain Curve in the Left and Right Figure Respectively](image)

The section of the points where the blue line is drawn through was used to calculate the modulus. The first few fractions of a degree was not included in the calculations of the modulus as it was assumed to be slippage in the test setup. The graph on the left broke which is why the extra data points around 0.001 and 0.0045 radians on the left figure and the extra data point around 0.005 radians on the right figure.

When the samples did break they looked like Figure 67.
The crack seemed to occur instantaneously and it was followed by a loud bang. Most of the cracks occurred along the honeycomb structure itself. This indicates that the adhesion of the infill was imperfect. In the center of the picture cracks have occurred across the infill. The crack still mimicked the path of the honeycomb beams. It is believed that the cracks that started at the adhesion of the infill and honeycomb occurred first. Also this is probably a result of air bubbles that sometimes were trapped in the SMP during the manufacturing process despite degassing the SMP first. Additional cracks occurred where the specimen was mounted to the test frame, this was result of the mountings’ fixture tightening.

The results for the soft and hard shear tests are shown in Table 17.
Table 17: Experimental Shear Test Results, XY, YX, Plane and Soft and Hard

<table>
<thead>
<tr>
<th></th>
<th>Cold XY</th>
<th>Cold YX</th>
<th>Hot XY</th>
<th>Hot YX</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.087E+09</td>
<td>1.027E+09</td>
<td>1.693E+07</td>
<td>1.375E+07</td>
</tr>
<tr>
<td>2</td>
<td>1.300E+09</td>
<td>1.171E+09</td>
<td>1.242E+07</td>
<td>1.527E+07</td>
</tr>
<tr>
<td>3</td>
<td>1.154E+09</td>
<td>1.236E+09</td>
<td>1.135E+07</td>
<td>1.238E+07</td>
</tr>
<tr>
<td>4</td>
<td>1.234E+09</td>
<td>1.085E+09</td>
<td>1.493E+07</td>
<td>1.072E+07</td>
</tr>
<tr>
<td>Average</td>
<td>1.194E+09</td>
<td>1.130E+09</td>
<td>1.391E+07</td>
<td>1.303E+07</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>9.289E+07</td>
<td>9.229E+07</td>
<td>2.512E+06</td>
<td>1.940E+06</td>
</tr>
</tbody>
</table>

Note the modulus for XY and YX in both the hard and soft cases are very similar. The confidence intervals overlap so the two directions in the hard and soft case are statistically the same.

The large variations in the samples contribute to the differences in samples moduli such as the manufacturing process as mentioned in 4.1.1. The variation in the thickness due to overfill was thought to be a large contributing factor in the hard shear tests. Despite each sample having different thicknesses the stress calculations don’t take into account the lack of the honeycomb going through the entire thickness. The variation in the cell geometries and the residual stress of the honeycomb has a larger contributing factor for the soft shear tests because in the soft state the beams bend so the variations will affect the results to a much greater degree. Another contribution to the variation in the test samples are stress concentrations that may or may not occur during the honeycomb manufacture process and the preparation of the samples.

For the hard shear tests, the largest contributing factor to the modulus would be the ratio of infill to the honeycomb. If the honeycomb makes up a larger percentage of the composite, the modulus of the composite would change. The variation in the honeycomb is thought to have little affect since the composite acts less like a mechanism and more like single material.

For the soft shear tests the variations in the honeycomb geometry would be a larger factor since the low infill modulus allows the honeycomb to deform and act like a mechanism. Looking at an increased cell angle, means larger residual stresses will occur in the honeycomb due to the manufacturing process. The beams would bend differently which wouldn’t be seen in the hard state. Other ways the mechanisms of the honeycomb could change would be to change the aspect
ratio of the honeycomb beams. Smaller aspect ratios would stiffen the beam again affecting the bending of the beams and therefore the mechanism of the honeycomb.
APPENDIX B

COMPOSITE FEA SHEAR MODEL

An accurate analytical model for the design space of a filled honeycomb skin with spatially prescribable stiffness properties given different materials and honeycomb configurations presented in Chapter 5 focused on the Young Modulus of the material in the x and y directions. The shear modulus of the honeycomb is validated appendix through FEA.

Four shear cases were run to match the modulus of the experimental data with the FEA data. Again four shear test cases ran were, XY hard and soft and YX hard and soft. The boundary conditions of the shear model compresses the opposing faces and offsets the faces in the transverse direction as shown in equation (55), (56), and (57) for the XY shear and equations (55), (58), and (59), for YX respectively. The transverse faces remain unconstrained (apart from the PBC) and equal distance apart. The imposed BC is shown in Table 18 where $\Delta x$ and $\Delta y$ correspond to the faces undergoing shear.

\[
\begin{align*}
    u_{xTopLeftCorner} &= u_{yTopLeftCorner} = 0 \\
    u_{xREF1} &= \Delta x & u_{yREF1} &= \Delta y & \text{XY Shear} \\
    u_{yREF2} &= u_{xREF2} = 0 \\
    u_{xREF2} &= \Delta x & u_{yREF2} &= \Delta y & \text{YX Shear} \\
    u_{yREF1} &= u_{xREF1} = 0
\end{align*}
\]
Table 18: XY and YX Shear Displacements

<table>
<thead>
<tr>
<th></th>
<th>deg</th>
<th>rad</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>XY</td>
<td>0.5</td>
<td>0.008726646</td>
<td>0.000896912</td>
<td>0.205555725</td>
</tr>
<tr>
<td>YX</td>
<td>0.5</td>
<td>0.008726646</td>
<td>0.117204702</td>
<td>0.000511405</td>
</tr>
</tbody>
</table>

B.1 Composite FEA Shear

Figure 68, Figure 69, Figure 70, and Figure 71 show the FEA contour plot of the YX soft shear with a focus on the infill of the von Mises, S11, S22, and S12 stresses.

Figure 68: YX Soft Shear with a Focus on the Von Mises Stress in the Infill.

Figure 69: YX Soft Shear with a Focus on the S11 Stress in the Infill.
Comparing Figure 68 to Figure 71 the infill directly above and below the $\alpha$ beam carries a higher stress due to the shear stress that dominates the stress in the infill. The shear stress around the $l$ beams is carried by the honeycomb beams. By reducing length of the $l$ beam the shear force necessary to deform the honeycomb would reduce. Also increasing the length of the $l$ beams would reduce the bending stress and therefore reduce the force needed to deform the unit cell.
Figure 72 shows the FEA contour plot of the YX soft shear test condition of the top left \( l \) beam with a focus on the axial stresses along the beam. Figure 73 shows the same test conditions but a line plot of the axial stress through the top, middle, and bottom of the \( l \) beam plotted against their location along the beam test.

**Figure 72: YX Soft Shear Mises of the Entire Honeycomb**

**Figure 73: YX Soft Shear Axial Strain Along the Top Left Beam**
The large absolute stresses at either end are most likely due to stress concentrations that occur at the beam joints. The constant tension through the center line of the beam signifies that the beam is in tension in addition to bending.

Figure 74 shows the FEA contour plot of the YX soft shear test condition of the top left $a$ beam with a focus on the axial stresses along the beam while Figure 75 shows a plot of the axial stress along a line through the top left $l$ beams at the top, middle, and bottom of the beam plotted against their location along the beam.

![Figure 74: YX Soft Shear S11 with the Coordinate System Y Axis Along the Bottom Left Slanted Beam to Focus on the Stresses in the Beam.](image)

![Figure 75: YX Soft Shear Axial Strain Along the Top Right Beam](image)
The stress distribution seen in the α beam is similar to that of the l beam. The major difference is the center of the beam is not higher and the magnitude of stress due to bending is 3 times greater.

Very little difference was seen between the hard states and soft states, consequently they were not shown. The bending was not as pronounced but the honeycomb still displayed the characteristics in the hard versus the soft state.

The first thing to note from the model is that the XY and YX shear test cases were the same for the hard and for the soft test cases. This is due to the symmetry in the unit cell. It is assumed that in other directions (parallel/perpendicular) this would be different.

An analytical equation was not developed but the FEA and experimental results are summarized in Figure 76 and Figure 77.

![Figure 76: Summary of the Soft Shear FEA and Experimental Results](image)
The first thing to note is the results for the XY and YX shear moduli are identical for the hard and for the soft state are identical. The difference between the XY/YX soft and hard shear is 6% and 5% respectively. The difference is small and could be an error in experiment.
Using the analytical model for the design space of a filled honeycomb skin with spatially
prescribably stiffness properties given presented in Chapter 6, this appendix looks at an
optimization that tailors the modulus of the honeycomb by changing the geometry of the unit cell.

The honeycomb unit cell geometry was optimized to minimize the Young’s modulus in
the $x$ and $y$ direction. This will ultimately reduce the actuation energy needed to morph the
composite skin. The panel of interest was based on the Next Gen – Morphing Aircraft Structure
(N-MAS) Phase II design requirements with dimensions of .508 x .381 m, constrained to deflect
2.5 mm in the middle of the plate when a 19,152 N/m$^2$ load is applied. The infill SMP has a
modulus of 1.6e9 Pa, which is only modeled in its soft state, and the honeycomb a modulus
2.1e11 Pa.

The optimization will highlight some key features of the analytical model,
specifically, the large contribution the thickness, cell angle, and horizontal wall length, $d$, $\theta$, and $l$
respectively because those terms are cubed.

C.1 Design Variables

The design variables are the horizontal wall length, $a$, slanted wall length, $l$, wall
thickness $d$, and cell angle, $\theta$. The depth of the cell was not included because the minimum
weight was always an active constraint, and the depth could be solved for directly. Consequently
the weight will not show up as a constraint but instead be imbedded in Equations (40), (41), and
(50) (repeated below).
C.2 Objective Function

The objective of the design is to minimize Equation (60) because this will give the lowest actuation cost in both the $x$ and $y$ direction

$$E_{cx} + E_{cy}$$ (60)

C.3 Constraints

The honeycomb panel was limited to a 2.5 mm deflection, seen in Equation (61).

$$\delta \leq 2.5$$ (61)

The El-Sayed equation for equivalent moduli (40) and (41) is accurate near 74 degrees, so the cell angle was limited to a range around 74 degrees. The cell angle is also constrained to make sure the final skin would still be able to strain nearly the same distance in both the $x$ and $y$ directions resulting in Equation (62). The N-MAS requirements of shear angle of 45 degrees was taken into consideration and used as an estimate when deciding this constraint.

$$55 \leq \theta \leq 75$$ (62)

The cell wall thickness, $d$, had a lower bound constraint of 0.2 mm which is the lowest manufactured honeycomb cell wall width, Equation (63)

$$0.2 \leq d$$ (63)

The upper bound of the horizontal wall and slanted wall is constrained, seen in equation (64) with units of mm.

$$l, a \leq 100$$ (64)

The weight of the panel is initially constrained by using the depth of the cell as a design variable. When the weight constraint is consistently active the cell depth, $c$ can be solved for and so it is removed from the design variables. The lowest weight that attained convergence was 8.3 kg. This weight significantly violates the MAS requirements, but because this study focused on exploring the design space, this value is used.
\[ \delta = \frac{2m^2b^2}{5c^3(m+b)^3} \left[ \frac{M_{bcv}(3m+b)}{E_{cy}} + \frac{M_{acv}(m+3b)}{E_{cx}} \right] \]  

(50)

The weight of the system is derived by computing the total number of cells in a panel, seen in Equation (65).

\[ N = \frac{A}{2y_0(a + x_o)} \]  

(65)

Here A is the area of the entire panel. The area of the infill and of the honeycomb structure is then computed as seen in Equations (66) and (67)

\[ A_i = 2(y_o - d)(x_o - d) + 2((a - d)(y_o - d)) \]  

(66)

\[ A_H = 2y_o(a + x_o) - A_i \]  

(67)

Finally, knowing the area of the cells times the respective density of the infill and honeycomb structure the weight can be calculated for a given cell depth, Equation (68).

\[ W = (A_i \rho_i + A_H \rho_H) cN \]  

(68)

C.4 Results

The optimization was run with MATLAB© Optimtool. The function will accept upper and lower bounds for the design variables and nonlinear equalities. Optimtool selects the ‘best’ algorithm to use which was a mixture of the sequential quadratic programming, Quasi-Newton, and line search algorithms.

A sensitivity study was conducted looking at the effects of the aspect ratio (AR) of the slanted wall, \( l \), and horizontal wall length \( a \), versus the wall thickness, \( d \), as seen in Figure 78 and Figure 79.
Figure 78: Young's Modulus in the y (Bottom) and x (Top) Direction of the Optimal Cell as a Function of Slanted Wall Length, \( l \), Versus the Wall Thickness, \( d \).
Figure 79: Young's Modulus in the y (Bottom) and x (Top) Direction of the Optimal Cell as a Function of Horizontal Wall Length, $a$, Versus the Wall Thickness, $d$.

$E_{xc}$ and $E_{yc}$ are both significantly dependent on the thickness of the cell wall. As the thickness increases the modulus increases. In the case of the slanted wall length, $l$, as the AR ratio increases the $E_{xc}$ decreases but $E_{yc}$ sees an increase in the modulus with an increase in the slanted wall length independent of the AR. The opposite is true for the horizontal wall length, $a$, in which $E_{yc}$ decreases with an increase in the AR and $E_{xc}$ is independent of the AR. Looking at the analytical model, the thickness of the cell is cubed, since it is based upon the thin beam theory, which clearly the most significant contribution to the modulus.
In addition as the honeycomb experiment and the honeycomb composite experimentation and FEA model all show a lower a modulus in the Y direction than the X direction. The same can be lower modulus in the Y direction can be seen in the rest of the sensitivity studies.

The second sensitivity study looked at the horizontal wall length, \( l \) versus the slanted wall length \( a \), and how it effects \( E_{yc} \) and \( E_{xc} \) seen in Figure 80.
Figure 80: Young's Modulus in the x Direction, $E_{xc}$ (Top) and y Direction, $E_{yc}$ (Bottom) of the Optimal Cell as a Function of the Slanted Wall Length, $l$, and Cell Angle, $\theta$.

Both $E_{xc}$ and $E_{yc}$ show a decrease in the modulus as the slanted wall length, $l$, decreases.

The horizontal wall length, $a$, sees a near linear increase in $E_{xc}$ as the length increases and a near
linear decrease in $E_{yc}$ as the length increases. $l$ is also cubed in the analytical model in the $x_o$ and $y_o$ term and it has a greater effect on the modulus than the $a$ beam.

The final sensitivity study looked at $E_{xc}$ and $E_{yc}$. Comparing the horizontal and slanted wall length to the cell angle, $\theta$ in Figure 81 and Figure 82.
Figure 81: Young's Modulus in the y (Bottom) and x (Top) Direction of the Optimal Cell as a Function of Horizontal Wall Length, $a$, Versus the Cell Angle, $\theta$. 
The cell angle, $\theta$, causes $E_{xc}$ to decrease and $E_{yc}$ to increase which is the opposite effect of the horizontal wall length, $a$.

The sensitivity study illustrates a few interesting features. First, the single largest contributing factor to the overall moduli was the wall thickness which increased the modulus by 4 orders of magnitude with a range of 0-10 mm. Second, there is no local minima in the graphs.
This means that any optimized geometry will be limited by imposed geometric constraints making the determination of the geometric constraints extremely important.

The optimization was started at various starting points ranging with each variable varied throughout their domains. The optimization ended up producing different $l$, $d$, and $a$ depending on initial conditions but $\theta$ consistently is limited by the upper bound constraint. The deflection was always an active constraint which is expected. More interesting is that the ratio of $a:l$ and $a:d$ were constant for each solution at 0.3 and 26.4. Presented in Table 19, are two sets of results that have a ratio of 0.3 and 26.4 respectively. Note that the moduli are the same, the deflection was limited to 2.5 mm, and $\theta$ was 75°.

<table>
<thead>
<tr>
<th>$E_xe$</th>
<th>$E_ye$</th>
<th>$\delta$</th>
<th>$\theta$</th>
<th>$l$</th>
<th>$d$</th>
<th>$a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pa</td>
<td>Pa</td>
<td>mm</td>
<td>deg</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
</tr>
<tr>
<td>1.55E+9</td>
<td>1.36E+9</td>
<td>2.5</td>
<td>75</td>
<td>16.19</td>
<td>2.01</td>
<td>52.14</td>
</tr>
<tr>
<td>1.55E+9</td>
<td>1.36E+9</td>
<td>2.5</td>
<td>75</td>
<td>1.619</td>
<td>.201</td>
<td>5.28</td>
</tr>
</tbody>
</table>

As stated early the weight of the cell was selected to be 8.3 kg by running a series of iterations. So while the weight isn’t a regular constraint, it was a limiting factor in the design of the cell.