FOURIER MULTISPECTRAL IMAGING
IN THE SHORTWAVE INFRARED

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By
Matthew David Howard

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FOURIER MULTISPECTRAL IMAGING IN THE SHORTWAVE INFRARED

Name: Howard, Matthew David

APPROVED BY:

Keigo Hirakawa, Ph.D.
Advisor Committee Chairman
Associate Professor, Electrical and Computer Engineering

Andrew Sarangan, Ph.D.
Committee Member
Professor, Electro-Optics

Kenneth Barnard, Ph.D.
Committee Member
Principal Engineer, Air Force Research Laboratory

Robert J. Wilkens, Ph.D., P.E.
Associate Dean for Research and Innovation
Professor
School of Engineering

Eddy M. Rojas, Ph.D., M.A., P.E.
Dean, School of Engineering
ABSTRACT

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Name: Howard, Matthew David
University of Dayton

Advisor: Dr. Keigo Hirakawa

Most multispectral systems try to measure incoming spectra with narrow-band filters at specific central wavelengths which are chosen based upon the application. A shortwave infrared (SWIR) multispectral system using concepts from Fourier multispectral imaging was designed with approximately sinusoidal spectral transmission filters. The filters were implemented as single cavity thin film resonators of varying thicknesses. The prototype system was evaluated using narrow-band spectra with single peaks, narrow-band spectra with multiple peaks, and broadband spectra with atmospheric absorption characteristics. The results show that this technique preserves spectral peaks and absorption bands even though the filters are not perfectly sinusoidal. Additionally, preliminary detection results were evaluated using a synthetic scene.
For Frank
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CHAPTER I
INTRODUCTION

Spectral imaging has been a field of interest as far back as the development of three color imaging in the 1850’s. Since then, much progress has been made, especially after the introduction of the digital camera in the 1970’s. Modern spectral imaging devices extend the traditional three visible color channels to tens or even hundreds of channels used to measure spectral information anywhere from the ultraviolet to thermal bands and beyond. By measuring the intensity of light at different wavelengths, it is possible to detect anomalous materials in a scene and even identify specific materials, something that would not be possible with a panchromatic sensor. Such devices commonly find uses in military, agriculture, mining, and other application areas.

The two modalities of capturing many spectral channels of an incident light field are hyperspectral and multispectral. The main distinction between the two is that hyperspectral captures at an arbitrarily higher spectral resolution than multispectral. Typically multispectral systems are implemented with a discrete set of spectral filters while hyperspectral systems generally aim to produce a fine resolution representation of the incoming spectrum, often through use of dispersive optical elements that separate the incoming spectrum’s wavelengths spatially. As an example comparison, a narrow-band hyperspectral system may capture 200 wavelength channels with 10 nm spectral bandwidth while a narrow-band multispectral system captures only 9 channels with 100 nm spectral bandwidth. If no assumptions are made about the signal other than that its spectral range is

1
2000 nm, the hyperspectral system will do a considerably better job of representing the spectrum of the incident light field since the multispectral bands’ spectral range is not wide enough. Additionally, even if the spectral bandwidths were changed to cover a large enough range, the multispectral system can only represent up to a 9 dimensional spectrum while the hyperspectral can represent up to a 200 dimensional spectrum. Hence, the multispectral measurements can be considered a lossy compressed version of the hyperspectral measurements, where the compression takes place in hardware instead of digitally after the measurements have been made. Since the lossiness of the compression may negatively impact applications such as material detection and identification, the choice for the filter shape in the multispectral system dictates the performance of the system and must be considered carefully.

This work will focus on the spectrum of light in the shortwave infrared (SWIR), a region where many materials have distinct spectral shapes which makes material detection and identification possible. Other spectral ranges with distinct material spectral features such as longwave infrared generally require cryo-cooled sensors while SWIR sensors work around room temperature, making a compact SWIR system very practical. Compared to shorter wavelengths, SWIR penetrates haze and fog much better as well as being less affected by distortions due to atmospheric turbulence [2].

The goal of this work is to develop a novel multispectral imaging system in the SWIR range that preserves the spectral content of the signal without increasing the number of spectral measurements. This is accomplished by leveraging the recent adaptation of Fourier spectrometry from hyperspectral to multispectral imaging found in [3] to design filters that compress the incoming spectral signal better than conventional narrow-band filters. The prior work demonstrated the filter-based Fourier spectrometry in the very near infrared (VNIR) range using filters with approximately sinusoidal spectral transmission. This thesis extends the work to the SWIR range; the filter design technique and prototyping effort as well as some preliminary detection results are described.
CHAPTER II
BACKGROUND

2.1 Existing Imaging Spectrometry Techniques

Referred to as a “data cube,” a hyperspectral image consists of two spatial dimensions and one spectral dimension. Since such spatial-spectral data is of higher dimensionality than a common detector array with only two spatial dimensions, designers must make a trade-off to capture all three dimensions. Existing imaging techniques can be categorized into two broad categories—scanning systems that require time-multiplexed images to construct the full data cube and snapshot systems which capture all three dimensions at once with a trade-off between how much spatial and spectral content can be captured. In the following section, degradations caused by scene changes during collection (motion, illumination, etc.) in a time-multiplexed system will be referred to as ”change artifacts.” For a more complete review of existing spectrometry techniques, refer to [1].

2.1.1 Scanning Systems

Scanning spectrometers are common outside of lab settings due to their ability to measure full resolution data cubes. In many applications, the scene does not rapidly change with time and temporal artifacts do not present a significant problem. The hardware is relatively easy to fabricate, resulting in many readily available commercial products. Several common system architectures are discussed below.
**Point Scanning Spectrometer**

In a point scanning spectrometer, the light from a single point in space is dispersed with a diffraction grating across a line of detector elements, causing each detector to measure a different wavelength. This configuration can be used in conjunction with two galvo mirrors that scan over the two spatial dimensions to form a complete data cube. The advantage of such a scheme is a fast readout rate but with high risk of change artifacts due to the large number of time-multiplexed measurements required. Such systems have largely been rendered obsolete by line scanning spectrometers which take advantage of large format detector arrays.

**Line Scanning Spectrometer**

Also called pushbroom or whiskbroom spectrometers, a line scanning spectrometer uses a slit aperture with a dispersive element to collect one spatial dimension and the spectral dimension at the same time. In order to form the other spatial dimension, the scene must be scanned across the aperture; this can be accomplished by moving the scene across a stationary imaging system as on a conveyor belt, by moving the system across the scene as on an aircraft, or with a single galvo mirror that sweeps the imaging system’s field of view across a stationary scene. The advantage of a line scanning spectrometer is that the full spectral dimension is collected over one of the spatial dimensions which eliminates change artifacts in the direction of the aperture slit; high risk of change artifacts will still exist in the direction of the scan.

**Tunable Filter Camera**

A tunable filter camera consists of a large format detector array with tunable spectral filters placed in its optical path. Unlike the point and line scanning spectrometers, the system measures the two spatial dimensions at once and time multiplexes the spectral content. The spectral filters can be implemented with a variable filter such as a mechanical filter wheel, or an electronically tunable
filter such as an adjustable Fabry-Perot etalon [4, 5], a tunable liquid-crystal filter [6], or an acoustooptic tunable filter [7]. The filter switching times can range from on the order of microseconds to seconds; the faster the switching time, the smaller the risk of change artifacts appearing. The advantage of tunable filters is that spatial content of the scene will be preserved in each time-multiplexed image, and that there are no moving parts in electronically tunable systems; however, the spectral content of the data cube can be distorted by the change artifacts.

**Imaging Fourier Transform Spectrometer**

![Michelson interferometer using a cube beamsplitter.](image)

Similar to using tunable filters, an imaging Fourier transform spectrometer takes time-multiplexed spectral measurements to form the spectral dimension of the data cube. The scanned element in a Fourier transform spectrometer is one mirror of a Michelson interferometer that is varied to collect slices at multiple optical path difference (OPD) values [8]. A visualization of a Michelson interferometer is provided in Figure 2.1.
By varying the OPD, which is equivalent to time-delays one branch of the beam’s path, the
time coherence of the light can be measured at the detector plane. The spectrogram made of many of
these measurements can then be Fourier transformed to attain the spectrum represented with respect
to spatial units, the more commonly used representation.

Another more recent implementation is to vary the OPD values with moving birefringent prisms
[9]. The advantage of a Fourier approach is higher light throughput and robustness to noise [10].
These systems require some post-processing to reconstruct the signal and can be sensitive to vibra-
tions or temperature changes.

**Computed Tomography (CT) Imaging Spectrometer**

In a CT imaging spectrometer, the three dimensional data cube is reconstructed by taking mul-
tiple slices from different angles, accomplished through the use of a rotating direct vision prism
which allows one wavelength to pass undeviated while dispersing the others [11]. The data cube
reconstruction is computed using the multiplicative algebraic reconstruction technique, an iterative
method. This step requires a large amount of computation and will have change artifacts present.
Additionally, the detector is not used efficiently compared to other techniques due to large areas on
the detector array with no information during each measurement.

**2.1.2 Snapshot Systems**

Existing techniques for snapshot imaging spectroscopy have been developed since as far back as
the 1930’s. In certain configurations, the capture rate can be fast enough to allow the possibility of
video frame rates. These techniques eliminate the change artifacts caused by the time multiplexing
of scanning systems; however, many of the techniques suffer from complexity of hardware, sensitiv-
ities to alignment, and difficulties in calibration. Most of these techniques have poor signal strength
due to elements reflecting away light or inefficiencies in gratings. Additionally, some require high computational complexity for post-processing.

**Integral Field Spectrometry**

![Figure 2.2: Visualization of Bowen image slicer. A circular image is sliced into strips and realigned into a strip. (Taken from [1].) ](image)

In astronomy, the most popular approaches to snapshot spectroscopy are integral field systems where each measurement consists of an integration over a region of the object. The first such technique emerged using mirrors, beginning with the Bowen image slicer which was modified into a version using a prism-coupled plane parallel plate that is less sensitive to alignment and vibration [12]. The image is sliced and realigned into a slit, as shown in Figure 2.2, so that it can be coupled with a conventional slit spectrometer as described in 2.1.1. Although the field of view in the slit direction is reduced compared to a standard line scanning spectrometer, this hardware configuration has the advantage of concurrently measuring spectral information for scene points along the direction perpendicular to the slit. Similar devices have been created using fiber bundles [13] and lenslet arrays [14] to rearrange the image into a slit.
Multispectral Beamsplitting

Beamsplitting into spectral channels allows the two spatial dimensions to be fully measured while the spectral dimension is sparsely measured. Typical setups split the incoming light into 3-6 spectral channels, each measured by a different detector array [15]. There is a limit to the number of times that the beam can be split due to light loss. A setup can be made with tilted thin-film filters operating in double pass to image all channels onto a single detector array as shown in Figure 2.3; however, this reduces the field of view and has increased transmission loss due to the double pass configuration [16].

CT Imaging Spectrometer

The snapshot CT imaging spectrometer is a dispersive spectrometer where the spatial and spectral information is allowed to mix at the detector. Specifically, it uses a 2D dispersion pattern to cause the mixing to vary at different locations on the detector array, which allows computational uncombination in post-processing [17]. This technique suffers from high computational complexity as well as difficulty in manufacturing the 2D dispersive element.
Multi-aperture Filtered Camera

The multi-aperture filtered camera is similar to multispectral beamsplitting. Multiple detectors image different spectrally filtered versions the scene so that the spatial dimensions are sampled fully and the light spectrum is sparsely measured. Different filters are placed in front of each detector or a lenslet array is used to create subimages which can be viewed through different filters on a single detector array [18, 19]. This setup suffers from drawbacks similar to the multispectral beamsplitting approach.

Tunable Echelle Imager

A tunable echelle imager adds a tunable Febry-Perot etalon before the diffraction grating of a dispersive spectrometer to reduce the spectral content so that subimages are created around each peak of the etalon transmission [20]. This encoding keeps spatial and spectral features from mixing on the detector array, allowing the three dimensional data cube to be captured when the slit is made wider; however, the spectral resolution is limited to the where the peaks of the etalon transmission occur and the spatial extent must be limited so that no mixing occurs on the detector array. The balance can be adjusted to measure more spatial and less spectral content by reducing the etalon length or increasing it to measure more spectral and less spatial.

Spectrally Resolving Detector Arrays

Spectrally resolving detector arrays work at the pixel level by placing different filters at each pixel location on a detector array. This is analogous to the Bayer color filter arrays used in conventional color cameras [21]. In multispectral imaging, the color filter array is replaced by a spectral filter array, which employs many more filters to increase spectral resolution as shown in Figure 2.4. These systems are robust to vibrations and temperature changes but at an increased risk of aliasing.
due to the spatial multiplexing that reduces the sampling rate of each spectral channel. Additionally, manufacturing pixel-sized filters is difficult.

**Image-Replicating Imaging Spectrometer**

In an image-replicating imaging spectrometer, the incoming image is split into multiple subimages by placing Wallaston beamsplitting polarizers between thick waveplate retarders [22]. By splitting the beam multiple times, up to 8 spectral channels have been measured at the same time on a single detector array [23]. Hardware limitations theoretically prevent more than 16 channels being measured [24].

**Coded Aperture Snapshot Spectral Imager (CASSI)**

CASSI uses a compressive sensing approach to combine spatial and spectral content in such a way that it can be undone in post-processing. It is a generalization of a coded aperture spectrometer which places a binary encoded mask in the optical path that is typically encoded in a
pseudo-Hadamard pattern where the negative values have been replaced with zeros due to transmission amplitudes being between zero and one [25, 26]. The light is then passed through a standard dispersive spectrometer which will smear the spatial and spectral information; however, since columns of the mask form an orthogonalizable binary code, this process can be undone computationally. For CASSI, a the binary mask is randomly generated. The spatial-spectral projection onto the detector array is modulated by the code of the binary mask in such a way that every wavelength experiences a shifted modulation code. If the underlying signal has a sparse representation in a linearly transformed domain, a compressive sensing algorithm can be used to estimate the datacube. Reconstruction requires linear programming, an iterative algorithm with non-negligible complexity.

2.2 Detection in Spectral Images

Spectral images tend to be difficult for human observers to comprehend due to limitations on what humans are able to perceive with their own eyes. Computers, however, are able to operate on spectral images like high dimensional vector spaces which allows the creation of automatic detection algorithms. Two fundamental detection statistics in the spectral imaging community are the Spectral Angle Mapper (SAM) and the Adaptive Coherence Estimator (ACE) statistics.

2.2.1 Spectral Angle Mapper

The SAM statistic, directly measures similarity between two vectors based off the angle between them. This generally does not perform acceptably when the data contains noisy measurements, but its simplicity makes it desirable for a quick evaluation. The SAM statistic is defined as:

\[
SAM = \frac{\psi^T y}{\sqrt{(\psi^T \psi)(y^T y)}},
\]

where \( \psi \) is the target signal and \( y \) is the signal under test. Geometrically this can be interpreted as \( SAM = \cos(\phi) \), where \( \phi \) is the angle between the signals \( \psi \) and \( y \).
2.2.2 Adaptive Coherence Estimator

The ACE statistic [27], a detection statistic originally developed for radar detection applications, has found use in discrete detection problems such as hyperspectral or multispectral detection. The statistic was proposed for detecting a target signal in noise where the signal covariance and signal level are both unknown. The ACE statistic is defined as:

\[
ACE = \frac{|\psi^T R^{-1} y|^2}{(\psi^T R^{-1} \psi)(y^T R^{-1} y)},
\]

(2.2)

where \( \psi \) is the target signal, \( y \) is the signal under test, and \( R \) is the estimated covariance structure of \( y \). \( R \) is estimated from training data, or frequently in the case of hyperspectral imaging, scene data. Since the image has not been classified, under the assumption that all pixels come from the same distribution, \( R \) can be estimated by using each pixel as a sample point. The scene is assumed to be largely background material, making the covariance calculated approximately that of the background even though some target pixels were included. Ideally, the covariance structure should be calculated for the background and target separately which would be used to whiten their respective vectors; however, since the target locations are unknown, the target covariance is assumed to be the same as the background.

A geometric interpretation of the ACE statistic is that it represents \( \cos^2(\phi) \) where \( \phi \) is the angle made between the whitened signals \( R^{-1/2} \psi \) and \( R^{-1/2} y \). Due to the whitening step, the ACE statistic is invariant to non-singular linear transforms; this allows the ACE statistic to be calculated on multispectral measurements without first reconstructing a wavelength spectrum, a fact that will be utilized in Section 5.4.
CHAPTER III
FILTER DESIGN

3.1 Review of Fourier Multispectral Imaging

The filter design process in this work is inspired by [28], which outlines a process for sampling the light spectrum in the OPD domain. Let us first review the key contributions of that work.

A measurement proportional to the photon radiance of a spectral signal $x(\lambda), \lambda \in [\lambda_{\text{min}}, \lambda_{\text{max}}]$ measured at the detector located at $(n_0, n_1)$ can be written as

$$m(n_0, n_1) = \langle x(n_0, n_1, \lambda), r(n_0, n_1, \lambda) \rangle = \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} x(n_0, n_1, \lambda)r(n_0, n_1, \lambda)d\lambda, \quad (3.3)$$

where $r(n_0, n_1, \lambda)$ is a spectral response function of the system. Because interference in thin film filters is periodic with respect to wavenumber instead of wavelength, consider an equivalent equation through change of variables,

$$m(n_0, n_1) = \langle x(n_0, n_1, \sigma), r(n_0, n_1, \sigma) \rangle = \int_{\sigma_{\text{min}}}^{\sigma_{\text{max}}} \frac{x(n_0, n_1, 1/\sigma)}{\sigma^2}r(n_0, n_1, 1/\sigma)d\sigma. \quad (3.4)$$

In the case of a Michelson interferometer as in Figure 2.1, the systems response functions can be written as $r_\psi(\sigma) = \cos(2\pi\psi\sigma)$. This produces an interferogram which can be Fourier transformed to obtain the wavenumber spectrum. Since $\psi$ can be stepped in arbitrarily small amounts, $m(\psi)$ is a continuous function and the spectrum can be reconstructed using:

$$\hat{x}(n_0, n_1, \sigma) = \int_{-\infty}^{\infty} m_\psi(n_0, n_1)e^{2\pi i \psi \sigma}d\psi. \quad (3.5)$$
$m_\psi(n_0, n_1)$ is an auto-correlation, an even function, which allows the above to be rewritten as:

$$\hat{x}(n_0, n_1, \sigma) = 2 \int_0^\infty m_\psi(n_0, n_1) \cos(2\pi \psi \sigma) d\psi.$$  \hspace{1cm} (3.6)

A multispectral imaging system is composed of $K$ discrete system response functions $r_k, k \in [0, 1, \ldots, K-1]$. In Fourier multispectral imaging (FMSI), the response functions are chosen to be $r_k = \cos(2\pi k \zeta \sigma)$ where $\zeta$ defines the sampling rate of the OPD’s. This can be thought of as taking the inverse cosine transform of the spectrum and then sampling it $K$ times at a sampling rate $\zeta$. Since imaging detectors have finite cutoff wavelengths, a value for $\zeta$ exists that will not introduce spectral aliasing in the reconstruction.

The spectrum can be reconstructed via Equation (3.5). This can be rewritten in a summation due to the discrete sampling of the measurements,

$$\hat{x}(n_0, n_1, \sigma) = 2 \zeta \sum_{k=0}^{\infty} m_k(n_0, n_1) \cos(2\pi k \zeta \sigma).$$  \hspace{1cm} (3.7)

Since only the first $K$ values of $m_k$ are non-zero, the above can be written as an ideal low pass,

$$\hat{x}(n_0, n_1, \sigma) = 2 \zeta \sum_{k=0}^{K-1} m_k(n_0, n_1) \cos(2\pi k \zeta \sigma),$$  \hspace{1cm} (3.8)

which can be visualized as multiplying $m_k$ by a brick wall filter in Equation (3.7). Since the reconstructed spectrum will be an ideal low pass version of the true spectrum, it is equivalent to the true spectrum convolved with a sinc function whose width is controlled by $K$. The peaks and troughs of the true spectrum will be preserved due to the symmetry of the sinc function. Since the cosine transform typically compacts signal energy, choosing cosines as the system response functions will preserve the signal $x(n_0, n_1, \lambda)$ more efficiently than many other choices. The downside to cosine system response functions is that they are difficult to realize in hardware and must be approximated. The following section provides a way to understand the effects of choosing arbitrary response functions on signal reconstruction as well as a method for reconstructing the spectrum.
3.2 FMSI with Imperfect Filters

It is worth noting that in practice \( r_k(n_0, n_1, \lambda) \) as outlined in Section 3.1 is comprised of not only physical filters, such as interference filters placed in the optical path, but also implicit filters such as the quantum efficiency of the detector. Nevertheless, it is believed that the benefits of FMSI are retained even when the response functions deviated from perfect sinusoids by small amounts. We verify this claim experimentally in Chapter 4.3, but an alternative approach to (3.7) is required to reconstruct the spectrum in a practical system. Below we develop such a framework.

Assuming that multiple measurements can be made at a single point and dropping the spatial dependence for convenience, we can write the system in matrix form:

\[
\begin{bmatrix}
    \langle r_1(\lambda), x(\lambda) \rangle \\
    \langle r_2(\lambda), x(\lambda) \rangle \\
    \vdots \\
    \langle r_N(\lambda), x(\lambda) \rangle
\end{bmatrix}
= \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_N \end{bmatrix},
\]

(3.9)

Assuming that the system response functions \( r_k \) and photon radiance \( x \) can be represented by some finite duration discrete sampling, the inner product can be split into a matrix multiplication:

\[
\Delta\lambda \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix} x = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_N \end{bmatrix},
\]

(3.10)

where the response functions are row vectors, and the light spectrum \( x \) is a column vector. \( \Delta\lambda \) is a scalar introduced during the approximation of the inner product integral by a sum. The scaling factor \( \Delta\lambda \) is dropped in the following equations for convenience.

Since in general the functions \( r_1, r_2, ..., r_N \) are not orthogonal to one another, consider transforming these functions to an orthonormal basis \( u_1, u_2, ..., u_N \) via the singular value decomposition,

\[
\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix} = VS \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix},
\]

(3.11)
The matrix of the left singular vectors, $V$, is a $k \times k$ matrix that represents a rotation in the space of $U^T$, and $S$ is a $k \times k$ diagonal matrix containing the singular values which scales the vectors of $U^T$ before rotation.

Making the substitution into Equation (3.10), we obtain a forward model for the spectral imaging system,

$$
\begin{bmatrix}
m_1 \\
m_2 \\
\vdots \\
m_N 
\end{bmatrix} = V S U^T x
$$

(3.12)

From the forward model in this form, we can see that our best estimate for $x$ is given by

$$
\hat{x} = U S^{-1} V^T \begin{bmatrix}
m_1 \\
m_2 \\
\vdots \\
m_N 
\end{bmatrix}
$$

Further, we recognize that $\hat{x}$ is contained in the column space of $U$. The vectors of $U$ should be chosen in such a way that the projection of $x$ onto the lower dimensional space, $U$, is close to $x$. A good choice of vectors would be the principal components of possible spectra since these would diagonalize the covariance structure; these would be difficult to realize in hardware and design would be dependent on training data.

This establishes a starting point for filter design where the basis functions in $U$ are chosen so that an optimal estimate of $x$ is contained in their column space. This process can be used to carry out the reconstruction in (3.7) in a discrete sense by setting the vectors in $U$ as the cosine terms, $V = I$, and $S = I 2\zeta$; care must be taken that the vectors $U$ remain orthogonal since they will now be cosines that do not extend infinitely in $\sigma$. 

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CHAPTER IV
SYSTEM PROTOTYPING

In order to verify performance of the multispectral system using approximately sinusoidal response functions as outlined in Chapter 2.2.2, a prototype was implemented as a tunable filter camera using a filter wheel with 8 thin film interference filters and a space with no filter. The intention for future work is implementation as a spectrally resolving detector array.

4.1 Filter Fabrication

The implementation of the designed filters was accomplished by creating Fabry-Perot resonators. The wavenumber transmission dependence can be approximated by:

\[ T(\sigma) = \frac{1}{1 + F \sin^2(2\pi \sigma n \xi)} = \frac{2}{2 + F - F \cos(4\pi \sigma n \xi)} \]

where \( n \) is the refractive index of the material, \( \xi \) is the thickness of the etalon, and \( F \) is the finesse of the cavity which increases as \( n \) increases. As \( F \) reduces, \( T(\sigma) \) approaches a sinusoidal shape but has smaller modulation depth on the \( \cos \) term. A scenario with a fixed optical path difference is shown in Figure 4.1 where the finesse of the cavity is varied; it can be seen that the smaller finesse values produce more sinusoidal shapes. Since the modulation is the signal we are actually interested in, we desire a higher modulation while tolerating the deviation from pure sinusoidal shape. A low modulation will result in all the system response functions being significantly correlated due to the
mean offset of the filter transmission functions; at the limit of a very small modulation, each will measure approximately the panchromatic image.

Silicon was chosen as the film material due to its high index of refraction and transparency in the SWIR range. This provides one of the highest finesse values that we could attain while also reducing the thickness of the filter needed to generate specific periodicity.

The silicon films were RF sputtered onto 1 inch diameter quartz glass substrates with thicknesses approximately in multiples of 110 nm which provides a sampling rate \( \zeta \), as outlined in Section 3.1, of around 450 nm. This ensures that there will not be aliasing in the reconstruction. The filter transmission functions were measured using an Agilent Cary UV-Vis-NIR spectrometer with step size of 1 nm.
Some drawbacks to using Si include absorption at shorter wavelengths, as evident in the measured spectra in Figure 4.2, and poor control of material purity. It is believed that oxygen in the chamber, although in very small quantities, oxidized some of the Si forming a mixture with a lower index of refraction which will change the OPD of the cavity. Since there is little to no control over the residual oxygen in the chamber, the mixture changed each time and produced very different film material properties. This was compensated for by making the transmission peaks align at the short wavelength end which causes the film thickness difference to change between consecutive filters.

Other materials could be used that might have more reliable material properties, but Si has the significant benefit of a high index of refraction. This allows the films to be thinner which helps avoid film stress and other physical problems. More exotic deposition techniques could be attempted to purge the residual oxygen in order to improve the Si deposition.
4.2 Prototype Hardware

Figure 4.3: Filter wheel with filters installed (camera and lens not shown). The 50 mm lens is stopped to approximately $f/2$

The 8 filters were placed in front of a Sensors Unlimited Micro-SWIR 320CSX InGaAs camera with a 50 mm StingRay lens as shown in Figure 4.3 (camera removed). The lens was stopped down to a 1 inch aperture to accommodate the size of the filter substrates. The quantum efficiency of the camera was measured after the manufacturer’s non-uniformity correction (NUC) for each pixel. A monochromator was used to generate input signals from 850 nm to 1750 nm which were measured by a spectrometer for reference. A dark current offset was calculated per pixel which was used to correct all subsequent measurements; this was done by closing the shutter on the monochromator. It was assumed that the manufacturer’s NUC corrected to a linear response to incoming light but failed to fully correct the dark current offset; a full calibration could be completed to address any residual non-linearity by producing signals of various intensity.

The sensor’s average quantum efficiency has sharp cutoffs around 950 nm and 1700 nm as can be seen in Figure 4.4. Because this spectral range does not span more than an octave, no cutoff filters were required as were in [28]. Although the average quantum efficiency is shown here, each
pixel response is determined by the specific quantum efficiency at that pixel during the spectral reconstruction.

4.3 Theoretical Performance

The first step considered in system evaluation was to invert the filter design process to determine the performance that could be expected from the system we actually built. The filter transmission functions were placed in a matrix and decomposed using the SVD. It was found that the condition number, the ratio of the largest singular value to the smallest, of this matrix was 22.5, considerably higher than what was initially expected. A larger condition number corresponds to an amplification of measurement errors which can produce large errors in the reconstruction. Since the condition number was larger than expected, filter transmission functions were removed from the matrix one at a time, and the condition number was recomputed to analyze the contribution of each. When the filter transmission function for the slot with no filter was removed, the condition number was

![Graph showing the average response of detectors in 320CSX camera.](image)

Figure 4.4: Average response of detectors in 320CSX camera
Figure 4.5: SVD of spectral filter transmission without detector response included

reduced the most to 13.3. This indicates that the panchromatic image with no detector response considered is significantly correlated with the other measurements. The rows of the orthonormal matrix \( U \) for both cases were inspected and are shown in Figure 4.5; the first 8 vectors of the case with 9 transmission functions are nearly identical to those in the case with 8. It was concluded that the filter transmission function corresponding to open slot does not contribute heavily to the vectors corresponding to the 8 largest singular values.

Next, the spectral response functions \( r_k \), obtained by multiplying the thin film filter transmissions by the average quantum efficiency of the detector array, shown in Figure 4.6, were analyzed. The SVD was computed again with the condition number for the 9 response function case rising to 101, 8 times higher than the condition number of 12.7 found in the 8 function case. The error amplification is exacerbated when the response functions include the quantum efficiency of the detector. The first 8 vectors of the orthonormal bases are still roughly the same as shown in Figure 4.7. The reason for the increased condition number is believed to be that the panchromatic response
Figure 4.6: Average system response functions with spectral filter transmission and detector response included

is highly correlated with the other response functions; this causes the 9 measurements to approximately reside in an 8 dimensional subspace, which will be highly unstable when inverting to a 9 dimensional space. As a result of this, measurement errors can result in considerably large variation in the signal reconstruction when 9 measurements are used; with 8 measurements, the boost in error is significantly smaller.
Figure 4.7: SVD of system response functions with detector response included
CHAPTER V
SYSTEM EVALUATION

For all spectra depicted in the following sections, the values were normalized to remove constant multipliers due to integration time, pixel size, and electron conversion rate; all spectra are proportional to photon radiance. False color images of the reconstructions are provided to visualize variation in the reconstruction of uniform objects. The false color conversion was done by taking the reconstructed spectra and assigning red, green, and blue values based on the integration over
the filter transmission functions shown in Figure 5.1. These curves were chosen to roughly align with the transmission bands of the atmosphere, leading to the uneven spacing of the peaks. The negative values in the reconstructions caused by Gibbs phenomenon were included in the integral even though they do not have physical meaning. This was done so that no non-linear processes such as clipping or absolute values were introduced in the display process.

5.1 Narrow-band Input Evaluation

Evaluation of the prototype began with generating narrow-band spectra with known center wavelengths and attempting to reconstruct the signal while preserving the location of the spectral peak. Input spectra were generated using a monochromator with central wavelengths of 1100, 1500, 1650, and 950 nm; the first two are well within the range of response of the InGaAs detector, while the latter two are at the boundaries of the detector sensitivity. As can be seen in Figures 5.2-5.4, 8 measurements localized the peaks of the input spectra better than 9 measurements; however, as shown in Figure 5.5, 9 measurements performed better when the peak was close to the short wavelength cutoff of the detector.

By examining the bases in Figure 4.7, one can recognize that the extra vector consists largely of information at the short wavelengths that is not represented well in the other 8 vectors; this reasonably explains the increased performance by 9 measurements in the 950 nm reconstruction. Even though the spectral transmission functions are not purely sinusoidal, the benefit of Fourier multispectral imaging to localize peaks is still visible.
Figure 5.2: 1100 nm input and reconstructed signals
Figure 5.3: 1500 nm input and reconstructed signals
Figure 5.4: 1650 nm input and reconstructed signals
Figure 5.5: 950 nm input and reconstructed signals
5.2 Multiple Narrow-band Input Evaluation

Spectra consisting of narrow-band peaks at multiple wavelengths were then generated using Oriel spectral calibration lamps which emit at specific spectral lines. Figure 5.6 shows the results for a Kr lamp and Figure 5.7 shows results for a Hg(Ar) lamp. One problem should be noted in the images in Figures 5.6 and 5.7 where there are concentric circles of varying color. This is caused by rings of varying radii that occur for each filter measure as shown in Figure 5.8. If the rings were the same for every filter, there would be no color shift, but there would be an intensity fluctuation. We can write this in a mathematical way by defining $\hat{m}$ as the “true” measurements that should be seen without the interference and $F$ as a location specific fluctuation matrix that scales each measurement based upon the interference pattern seen. Let the measurements seen be $m = F\hat{m}$. For only an intensity fluctuation to occur, $F$ must be equal to $\alpha I$, where $\alpha$ is a constant scalar; this will simply scale $\hat{m}$ without changing its direction. If $F$ is a diagonal matrix with different elements on the diagonal, a vector rotation will take place in addition to scaling which results in a color shift.
Figure 5.6: Kr gas lamp input and reconstructed signals
Figure 5.7: Hg(Ar) gas lamp input and reconstructed signals
Figure 5.8: Hg(Ar) gas lamp images
The rings are believed to be caused by the extended coherence length of the light creating a cavity within the substrate which will cause Newton’s rings to form at the image plane due to the curve induced by film stress. The curve of each filter was measured using a Zygo interferometer and has been summarized in Table ?? by the peak to valley change of the filters. The height change is given in waves based on the wavelength of 632 nm that the interferometer uses. Since the measurement was only of the front film surface, the large absorption at 632 nm in the film was not an issue. Each filter surface is clearly spherical which will produce the rings seen; an example surface profile found from the interferometer is shown in Figure 5.9. The ring structure only appears when illuminating by a source with narrow spectral features such as these calibration gas lamps or florescent room lighting; standard use cases will not be affected by this degradation.

Table 5.1: Peak to valley change of filters given in waves based off 632 nm wavelength.

<table>
<thead>
<tr>
<th>Filter Number</th>
<th>Peak to Valley Change (@ 632nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5084 wv</td>
</tr>
<tr>
<td>2</td>
<td>1.169 wv</td>
</tr>
<tr>
<td>3</td>
<td>1.042 wv</td>
</tr>
<tr>
<td>4</td>
<td>0.9343 wv</td>
</tr>
<tr>
<td>5</td>
<td>1.170 wv</td>
</tr>
<tr>
<td>6</td>
<td>1.120 wv</td>
</tr>
<tr>
<td>7</td>
<td>1.291 wv</td>
</tr>
<tr>
<td>8</td>
<td>1.120 wv</td>
</tr>
</tbody>
</table>
5.3 Broad-band Outdoor Evaluation

For the final evaluation, the system was taken outside on a day with low cloud cover to ensure temporally consistent illumination. Images were captured of 6 different materials illuminated by sunlight: a canvas drop cloth, a blue plastic drop cloth, purple insulation foam board, styrofoam, a 99% reflective spectralon panel, and natural ground. First, reconstructions of the signals were completed in the same manner as previous evaluations. The results are displayed in Figures 5.10-5.15 with the reference spectrum measured in situ with an ASD Field Spec Pro spectrometer. When using 8 measurements, the reconstruction produces reasonable reconstructions of the spectra. In the 9 measurement case, sometimes the reconstruction does well as in Figure 5.11b, but largely it is not reliable. Additionally, there is a large variation in the reconstruction for 9 measurements as can be visualized in the false color images. We conclude that the 8 measurement case is a more accurate
representation of the incoming light spectrum and that it will more consistently produce the same “color” reconstruction.
Figure 5.10: Canvas drop cloth input and reconstructed signals
Figure 5.11: Blue plastic drop cloth input and reconstructed signals
Figure 5.12: Purple insulation foam board input and reconstructed signals
Figure 5.13: Styrofoam input and reconstructed signals
Figure 5.14: Spectralon input and reconstructed signals
Figure 5.15: Natural ground input and reconstructed signals
5.4 Detection Evaluation

As a surrogate for collecting a scene consisting of target materials and a large portion of background, measurements from the 5 non-grass materials in Section 5.3 were synthetically appended to the image of natural ground; a representative image is displayed in Figure 5.16. This allowed for a preliminary evaluation of the system’s ability to do material detection. Three scenarios were generated corresponding to scenes containing 25% target pixels, 10% target pixels and 1% target pixels. These values were arbitrarily chosen but are believed to be representative of typical scenes. Receiver operability characteristic (ROC) curves using the ACE statistic for these scenarios are displayed in Figures 5.17-5.19. The same reference was used for every scenario where it was taken as the average measurement of 51200 pixels from each material. As can be seen in Figure 5.20, some materials are confused with others before they are confused with the background. The improvement as the number of target pixels is decreased is believed to be caused largely by the system’s ability to separate target pixels from the background very well but lack of ability to separate targets from one another. Additionally, when there are a large number of target pixels, estimating the covariance
structure of the background also becomes an issue because the assumption that there are relatively few targets is violated.

In comparison to a hyperspectral system, the detection rate is poor; however, there is no consideration in these detection results for the spatial information that the multispectral system can potentially collect concurrently. These detection results can be considered a lower bound for the detection performance of the proposed multispectral system since they treat only the spectral information. Future data collected from a real scene will be used to verify the lower bound and attempt to utilize spatial information.
Figure 5.18: ROC curves for 10% target pixel synthetic scene

(a) Canvas Drop Cloth   (b) Blue Drop Cloth   (c) Purple Foam
(d) Spectralon         (e) Styrofoam

Figure 5.19: ROC curves for 1% target pixel synthetic scene

(a) Canvas Drop Cloth   (b) Blue Drop Cloth   (c) Purple Foam
(d) Spectralon         (e) Styrofoam
Figure 5.20: Image of ACE statistic using styrofoam as the reference (rightmost slice)
CHAPTER VI
CONCLUSION

In this work, we have created a prototype multispectral SWIR system using Fourier multispectral imaging as a starting point. The system was evaluated based upon performance localizing peaks of narrow-band spectra and ability to represent broadband spectra with and without the typical atmospheric absorption present. The spectra were reconstructed by inverting an underdetermined system of equations that approximate the forward model of the imaging system. Preliminary detection results were produced by creating a synthetic scene with embedded targets. The prototype presented was able to localize peaks of various narrow-band spectra well and was able to reasonably represent broadband spectra with and without absorption. The preliminary detection results presented are promising, considering no spatial information was utilized.

Future work will include collecting a full scene along side a scanning slit spectrometer for benchmarking. The detection method could be improved by including spatial information. Additionally, the full scene can be used for preliminary spatial analysis for a spectrally resolving detector array version of the prototype presented here.

A future iteration of this system could benefit from filter transmission functions designed with consideration of the detector response and manufacturability. Additionally, filter transmission shapes
could be designed based upon knowledge of incoming signals since, especially for solar illumination with absorption bands, similar signals are frequently seen; this work was intended to be a signal agnostic design.

The implementation of the filters in this work could be improved by stabilizing the material properties of the deposited Si, possibly by purging the chamber with an inert gas to help remove additional oxygen. Additionally, films could be deposited on the front and rear surfaces to reduce the curvature of the filters induced by film stress; this could be accomplished by placing an anti-reflective coating on the rear surface of the substrate.


OE/Aerospace Sensing and Dual Use Photonics. International Society for Optics and 

Auxiliary Instrumentation for Large Telescopes, 1972, pp. 175–183.


[16] W. E. Orryn, D. A. Basiji, P. Morrissey, T. George, B. Hall, C. Zimmerman, and D. Perry, 


fabry-perot spectrometer for astronomical imaging.” Astronomy and Astrophysics Supplement 


dimensions,” in Biomedical Optics 2003. International Society for Optics and Photonics, 
2003, pp. 46–54.

[23] A. Gorman, D. W. Fletcher-Holmes, and A. R. Harvey, “Generalization of the lyot filter and 
its application to snapshot spectral imaging,” Optics express, vol. 18, no. 6, pp. 5602–5608, 
2010.


