INCOHERENT IMAGING IN THE PRESENCE OF ATMOSPHERIC TURBULENCE AND REFRACTIVITY

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INCOHERENT IMAGING IN THE PRESENCE OF ATMOSPHERIC TURBULENCE AND REFRACTIVITY

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ABSTRACT

INCOHERENT IMAGE PROPAGATION IN PRESENCE OF ATMOSPHERIC TURBULENCE AND REFRACTIVITY

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Atmospheric turbulence, associated with its refractive-index inhomogeneities (refractivity), may severely affect long range incoherent images formation. Example of this impact includes image blurring, motion, warping and anisotropic geometrical distortions. Currently, the effects of turbulence and refractivity on image formation are considered as being mutually independent and analysed separately using the Fresnel diffraction (wave-optics) and geometrical optics (ray tracing) approaches, respectively. Such independent treatment of turbulence and refractivity effects have certain limitations. Atmospheric refractivity may result in significant deviations of optical wave propagation direction. This effect is commonly referred as the ray bending which, in turn, may lead to a change in turbulence characteristics such as the refractive index structure parameter $C_n^2$ that is commonly considered as a function of altitude $h$ above the ground. Correspondingly, optical wave refraction, especially in extended-range imaging scenarios, could affect the turbulence-induced optical aberrations.
In this work, we analyze the incoherent image formation in atmosphere in the presence of both atmospheric turbulence and refractivity using numerical simulations based on the brightness function (BF) technique.

Using the BF technique, the incoherent imaging system modulation transfer function (MTF) estimation is performed via direct numerical analysis of visibility of sine-test patterns of different spatial frequencies. The test patterns are assumed to be imaged through a volume medium with turbulence and refractivity-induced refractive index inhomogeneities. The major effects observed in numerical simulations, include the spatial frequency shift between frequency of a sine-test object and its image, and spatial non-uniformity of the sine-pattern image distortion which is referred to as the refractivity-induced image anisoplanatism. Both effects depend on the location and strength of the localized refractive index structure with respect to the imaging (wave propagation) geometry. The MTFs corresponding to distributed (volume) turbulence with and without atmospheric refractivity are also compared.

Next, the joint impact of atmospheric turbulence and inverse temperature layer (ITL) on optical mirage formation is analyzed. The dependency of both desert- (superior) and ocean-type (inferior) mirage image formation on ITL characteristics (temperature inversion and location of the ITL) have been studied. The impact of atmospheric turbulence strength on mirage image qualities is also analyzed.

Finally, a numerical analysis is conducted to study the impact of localized refractive index inhomogeneities on image quality. It is shown that image quality strongly depends on atmospheric turbulence strength and locations along the optical path. To characterize this impact, two metrics are proposed and developed to measure the image quality as a function
of turbulence strength and location. The impact of inverse temperature layer on the
developed image quality metrics is also studied.
Dedicated to my wife, sisters and parents,
for all the support that made this journey possible
ACKNOWLEDGEMENTS

First of all, I would like to specially thank my advisor Dr. Mikhail Vorontsov, for his wonderful guidance, vast knowledge, advice, and continuous encouragement and support throughout this work. Without him this would have been impossible. I would also like to thank Dr. Svetlana Lachinova for giving her valuable time for advising and paper, presentation editing. I must thank Dr. Shiyi Wang for his encouragement and tips which lead me through a wonderful start in my graduate life.

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Last but not least, I would specially thank to my wife, sisters and parents for being my strength and stay behind me all the time.
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<tr>
<td>2D</td>
<td>Two-dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>Three-dimensional</td>
</tr>
<tr>
<td>BF</td>
<td>Brightness function</td>
</tr>
<tr>
<td>IQM</td>
<td>Image quality metric</td>
</tr>
<tr>
<td>ITL</td>
<td>Inverse temperature layer</td>
</tr>
<tr>
<td>MC</td>
<td>Monte-Carlo</td>
</tr>
<tr>
<td>MCF</td>
<td>Mutual correlation function</td>
</tr>
<tr>
<td>MTF</td>
<td>Modulation transfer function</td>
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<td>MUSA76</td>
<td>Modified USA 1976</td>
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<tr>
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<tr>
<td>$\alpha$</td>
<td>Temperature gradient in troposphere</td>
</tr>
<tr>
<td>$A$</td>
<td>Optical field</td>
</tr>
<tr>
<td>$A_D$</td>
<td>Constant for</td>
</tr>
<tr>
<td>$b_{\text{img}}$</td>
<td>Image size</td>
</tr>
<tr>
<td>$b_{\text{obj}}$</td>
<td>Object size</td>
</tr>
<tr>
<td>$B$</td>
<td>Brightness function</td>
</tr>
<tr>
<td>$B_P$</td>
<td>Fourier transform of overlapping function</td>
</tr>
<tr>
<td>$C_n^2$</td>
<td>Refractive index structure parameter</td>
</tr>
<tr>
<td>$D$</td>
<td>Lens diameter</td>
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<tr>
<td>$D_n$</td>
<td>Structure function</td>
</tr>
<tr>
<td>$D/r_0$</td>
<td>Ratio to characterize turbulence</td>
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<tr>
<td>$f_{\text{cut}}$</td>
<td>Cutoff frequency in vacuum</td>
</tr>
<tr>
<td>$f_{\text{img}}$</td>
<td>Image spatial frequency</td>
</tr>
<tr>
<td>$f_{\text{obj}}$</td>
<td>Object spatial frequency</td>
</tr>
<tr>
<td>$\hat{f}_{\text{obj}}$</td>
<td>Normalized object spatial frequency</td>
</tr>
<tr>
<td>$F$</td>
<td>Lens focal length</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>$h$</td>
<td>Height above ground</td>
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</table>
\( h_{\text{img}} \)
Lens elevation

\( h_{\text{obj}} \)
Object elevation

\( h_{\text{ITL}} \)
ITL center elevation

\( H^d_I \)
MTF in vacuum

\( \langle H_I \rangle_{\text{LE}} \)
Long-exposure MTF

\( l \)
Object intensity distribution

\( l_{\text{img}} \)
Image plane intensity distribution

\( I_{\text{LE}}^{E} \)
Long-exposure image intensity distribution

\( J \)
Image quality metric symbol

\( k \)
Wave number

\( l_0 \)
Inner scale

\( L_0 \)
Outer scale

\( L_i \)
Image distance

\( L_s \)
Object distance

\( m \)
Molecular mass of ideal air

\( M \)
Magnification factor

\( M_{\text{turb}} \)
Number of turbulence screens

\( n \)
Atmosphere refractive index field

\( n_0 \)
Constant part of atmosphere refractive index

\( n_{\text{refr}} \)
Refractivity component

\( n_{\text{turb}} \)
Turbulence component

\( N_{\text{lens}} \)
Numerical grid in pupil-plane
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</tr>
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<td>Pressure and pupil function</td>
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<tr>
<td>$P_0$</td>
<td>Pressure on the ground</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>Three dimensional coordinates</td>
</tr>
<tr>
<td>$r$</td>
<td>Two dimensional coordinates</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Fried parameter</td>
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<tr>
<td>$R$</td>
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<td>$\mathbf{R}$</td>
<td>BF trajectory coordinate</td>
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<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature profile</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Temperature on the ground</td>
</tr>
<tr>
<td>$w_{ITL}$</td>
<td>ITL diffuse parameter</td>
</tr>
<tr>
<td>$W$</td>
<td>Wind speed</td>
</tr>
<tr>
<td>$z$</td>
<td>Propagation distance</td>
</tr>
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<td>$\tau_{turb}$</td>
<td>Turbulence correlation time</td>
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<tr>
<td>$\Delta T$</td>
<td>Temperature inversion</td>
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<tr>
<td>$\rho$</td>
<td>Distance between two points</td>
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<tr>
<td>$\Gamma$</td>
<td>Mutual correlation function</td>
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<tr>
<td>$\Gamma_P$</td>
<td>Aperture overlapping function</td>
</tr>
<tr>
<td>$\mathbf{k}$</td>
<td>BF trajectory vector</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>BF trajectory angular vector</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Delta function</td>
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<tr>
<td>$\Delta$</td>
<td>Object displacement from ITL</td>
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<th>Symbol</th>
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<tr>
<td>$\Delta f$</td>
<td>Frequency shift</td>
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<td>$\Delta z$</td>
<td>Distance between phase screens</td>
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<td>$\lambda$</td>
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CHAPTER 1
INTRODUCTION AND OBJECTIVES

Long-distance incoherent imaging in the Earth’s atmosphere can be strongly influenced by refractive index spatial inhomogeneities resulting from complicated dynamics of air masses [1-7]. Under the commonly used simplifications in atmospheric optics, these inhomogeneities can be represented as a sum of two major components that are associated with atmospheric refractivity and optical turbulence.

Fig. 1. Example of incoherent image in presence of (a) atmospheric refractivity, (b) turbulence and (c) both atmospheric refractivity and turbulence.
Electro-magnetic energy coming from a distant source and reaching the observer inside the atmosphere will not propagate along a straight line but in a curved path as shown in Fig. 1.1(a). This is called atmospheric refractivity which is seen in a number of optical (UV to LWIR) phenomena. Mirage and looming are two well-known visible light examples.

In addition to atmospheric refractivity effect, there are small scale, random changes of refractive index present associated with turbulence. This will have the effect of random refraction and is responsible for the scintillation of stars, called twinkling, and image distortion as shown in Fig. 1.1(b). The joint effort of atmospheric refractivity and turbulence are included in the image of the world which we see every day. One example of incoherent images formed in the presence of both effects is shown in Fig. 1.1(c).

Considerable efforts have been made to the study of incoherent visible and near-IR image formation in atmosphere. Much progress regarding numerical computational technique development on image propagation has been made in the past. Conventional Monte Carlo (MC) techniques have been used to perform numerical simulation of anisoplanatic imaging through the atmosphere. This has been done based on fast Fourier transform wave propagation computations with the atmospheric inhomogenieties modeled by a set of thin phase screens [7]. While the MC technique is enough to deal with small point-like light sources, it has difficulties dealing with imaging of extended objects due to extremely time consuming computational algorithms. For image propagation through atmosphere with strong refractive gradients, a framework of geometrical optics in terms of optical ray tracing connecting transmitter and receiver planes is commonly involved [8]. This simplified explanation is quite limited since it doesn’t take into account important information such as diffraction on the imaging system aperture and the presence of
atmospheric turbulence induced refractive index fluctuations along the propagation path. Correspondingly, the conventional approach doesn’t provide adequate tools for predictive numerical simulations of incoherent image formation in atmospheric conditions that are characterized by the presence of both refractivity and turbulence.

This dissertation includes four main contributions: further develop the existing brightness function technique to take both atmospheric turbulence and refractivity into account for incoherent imaging simulation; apply the brightness function (BF) technique for the analysis of imaging system modulation transfer function (MTF) in the presence of both atmospheric turbulence and refractivity; study the desert-type and ocean-type mirage phenomenon; characterize the impact of atmospheric turbulence strength and locations on image qualities by developing novel image quality metrics which are sensitive to different types of image degradations.

The next section (chapter 2) covers background then chapter 3 covers research experiments and computational demonstrating the three major contributions of this project. Chapter 4 presents conclusions and potential future research.

A number of conference publications and one journal paper have resulted from the work thus far. These are listed below:

2. Z. Yang, S. L. Lachinova, M. A. Vorontsov and D. A. LeMaster. “Image quality characterization in highly anisoplanatic conditions.” Proc. SPIE Long-
range imaging. International Society for Optics and Photonics, 2017, pp. 10204-12.


CHAPTER 2
BACKGROUND

2.1 Refractive index inhomogeneity in Earth’s atmosphere

Under the commonly used atmospheric optics simplifications, for optical wavelength of $\lambda$, the refractive index field $n(\lambda, \mathbf{r}, t)$ can be represented as a sum of three major components: a non-deviated constant component $n_0$, a continuous changing component associated with atmospheric systematic refractivity effect $n_{\text{refr}}(\lambda, \mathbf{r}, t)$ and a random fluctuation component in both space and time associate with turbulence effect $n_{\text{turb}}(\lambda, \mathbf{r}, t)$. The expression for this atmosphere refractive index model is expressed as below:

$$n(\lambda, \mathbf{r}, t) = n_0 + n_{\text{refr}}(\lambda, \mathbf{r}, t) + n_{\text{turb}}(\lambda, \mathbf{r}, t),$$

(2.1)

where $\mathbf{r} = \{x, y, z\}$ is the coordinate vector and $t$ is the time. The refractivity term $n_{\text{refr}}(\mathbf{r})$ describes time-average, large-scale deviations in the refractive index field from the undistorted value $n_0$, while the term $n_{\text{turb}}(\mathbf{r}, t)$ represents turbulence-induced random rapidly refractive index fluctuations (eddies). The characteristic correlation time $\tau_{\text{turb}}$ is around $10^{-3} – 10^{-2}$ s and spatial scales belonging the inertial subrange (inner $l_0$ and outer $L_0$ scales) extending from a few mm to several meters [9]. Contrary to the optical turbulence dynamics, noticeable changes in atmospheric refractivity occur over a
timeframe of several hours. Correspondingly for typical observation times (from seconds to less than hour), the refractivity term in Eq. (2.1) can be considered as stationary.

2.2 Atmospheric refractivity models

The real dynamics of atmospheric refractivity is complicated and inconvenient for the implementation in scientific researches. It is necessary to have a functional description of refractive index variance over space which are both suitable for numerical computational application and flexible enough to describe the characteristics of the atmosphere. In this section below, the atmosphere model used in this work will be introduced which is based on the U.S. standard atmosphere of 1976 (US1976).

2.2.1 Modified US 1976 standard atmosphere model (MUSA76)

Numerous research work have been done to predict the complicated atmosphere refractive index field distribution with a simple analytical equation. In general, atmosphere refractive index is strongly depends on the density of air which is high near the earth’s surface due to the gravitational pull of the planet. Weather and geographic location also play an important role in the density of the air at a given altitude. In earlier research work, the deviation of the averaged atmosphere refractive index of dry air \( n(\lambda, \mathbf{r}, t) \) from its free-space value \( n_0 = 1 \) is estimated to be dependent on temperature \( T(\mathbf{r}, t) \), pressure \( P(\mathbf{r}, t) \) and wavelength \( \lambda \). In this model, the dependence on height \( h \) above sea level is assumed and the time dependent as well as the impact of water vapor on refractive index variations are ignored, i.e. \( n(\lambda, \mathbf{r}, t) = n(\lambda, \mathbf{r}) = n(\lambda, h) \). The analytical expression for refractivity term in Eq. (2.1) can be presented as below for dry air [8, 10]:

\[
\]
\[ n_{\text{refr}}(\lambda, h) - 1 = A_D(\lambda) \frac{P(h)}{T(h)}, \quad (2.2) \]

where \( A_D(\lambda) \) is the reduced refractivity coefficient for air given in hPa\(^{-1}\)K. Several different forms for \( A_D(\lambda) \) have been developed which all agree at the level of 0.1% in visible range and Edlin’s Sellemeir [11] form was used in this work with \( \lambda \) given in micrometer:

\[
A_D(\lambda) = 10^{-8} \left[ 8342.13 + \frac{2406030}{130 - 1/\lambda^2} + \frac{15997}{38.9 - 1/\lambda^2} \right] \frac{288.15}{1013.25}. \tag{2.3}
\]

If air is considered to be an ideal gas with molecular mass \( m \) in gram, then the temperature and pressure are related to each other by the differential equation [8, 10]:

\[
\frac{dP}{dh} = -\frac{mg}{R} \frac{P(h)}{T(h)}, \tag{2.4}
\]

where \( R \) is Boltzmann’s constant and \( g \) is the acceleration of gravity. The density of air is most sensitive to changes in air temperature, so most atmospheric models are based on the air temperature profile with altitude. Once the temperature profile is known, the pressure profile can be found using Eq. (2.4), and the refractive index profile can then be calculated.

Atmospheric temperature profiles can be based on deterministic models, measured data, or even meteorological prediction models. We adopt the US1976 atmosphere temperature profile in this work, modified in the troposphere to allow for a free choice of the temperature \( T_0 \) at ground or sea level. The thus modified US1976 atmosphere has been
Fig. 2. The dependence of temperature profiles on elevation are plotted in (a), refractivity profiles in (b) and refractive index gradient profiles in (c). MUSA76 is the modified US 1976 atmosphere model, sea-level temperature $T_0 = 288.15\, \text{K}$; Desert type ITL represent desert-type inverse temperature layer model with $T_0 = 288.15\, \text{K}, h_{ITL} = 4\, \text{m}, w_{ITL} = 0.5\, \text{m}$ and $\Delta T = -2\, \text{K}$; and Ocean type ITL stands for ocean-type inverse temperature layer model with $T_0 = 288.15\, \text{K}, h_{ITL} = 45\, \text{m}, w_{ITL} = 4\, \text{m}$ and $\Delta T = 5\, \text{K}$.
denoted as MUSA76 [8, 10]. This temperature profile is characterized by piecewise constant temperature gradients \((\alpha = -6.5 \text{ K/km} \text{ in troposphere})\) with an expression as below:

\[
T(h) = T_0 + \alpha h.
\]  

(2.5)

Substituting it into Eq. (2.4) and solving for the air pressure, obtains the pressure profile as a function of altitude, which is given by

\[
P(h) = \frac{P_0}{T_0} \left( \frac{T_0}{T_0 + \alpha h} \right)^{\frac{mg}{R \alpha}},
\]

(2.6)

where \(P_0\) is the adjustable sea-level pressure, \(R = 8314.472 \text{ J/kmol K}^{-1}\) is the universal gas constant, \(g = 9.8 \text{ m/s}^2\) is the acceleration of gravity. With the knowledge of both the pressure and temperature profile, the refractivity profile can be found using Eq. (2.2):

\[
n(\lambda, h) = 1 + A_\nu(\lambda) \frac{P_0}{T_0} \left( \frac{T_0}{T_0 + \alpha h} \right)^{\frac{mg}{R \alpha}}.
\]

(2.7)

The corresponding profiles of the MUSA76 temperature profile for \(T_0 = 288.15 \text{ K}\), refractivity \(n(\lambda, h) - 1\) and the refractive index gradient are plotted in Fig. 2.1 (a), (b) and (c) correspondingly, labeled as MUSA76. A mono-decreasing of temperature and refractive index are observed as the elevation increase.

Ray tracing for the MUSA76 atmosphere is shown in Fig. 2.2(a). The observer’s height is 15 m. Ray trajectories over 50 km for different initial angles are presented for the height above the ground below 100m. The ray trajectories do not cross for this case.

This refractivity induced by the standard MUSA76 model can’t describe other phenomenon like optical mirages. These phenomenon are resulted from large scale highly
spatially localized refractive index structures, such as inverse temperature layers which will be discussed in the following section.

### 2.2.2 Inverse temperature layer (ITL) atmospheric model

One well studied analytical temperature profile created to study the formation of optical mirage phenomena is the inverse temperature layer (ITL) atmosphere model. This model is analytically described by appending a warm or cold layer to the MUSA76 atmosphere, for which we use an analytical form, borrowed from the theory of the electron gas, where it is known as the Fermi distribution:

\[
T(h) = T_0 + \alpha h - \Delta T + \frac{\Delta T}{1 + \exp[-(h - h_{ITL})/w_{ITL}]},
\]

(2.8)

Here \( h_{ITL} \) is the height of the isotherm about which the added temperature profile is centered, \( \Delta T \) is the temperature jump across the inversion, and the diffuseness parameter \( w_{ITL} \) determines the width of the jump.

Depending on the sign of temperature variation, one can distinguish the desert (\( \Delta T < 0 \)) and ocean (\( \Delta T > 0 \)) ITL types. The corresponding temperature profiles, refractivity profiles, and refractive index profiles versus elevation for both desert-type and ocean type inverse temperature layer atmospheric models are plotted in Fig. 2.1(a), (b) and (c), labeled as desert-type ITL and ocean-type ITL, respectively. A strong positive refractive index gradient in the ITL center for desert-type ITL model is observed in Fig. 2.1(c); while for the ocean-type ITL model, a negative gradient is observed in the ITL.

The effect of desert-type ITL on the transformation of ray tracing is shown in Fig. 2.2(b). While the initial angle of rays are within certain range, the ray trajectories are
Fig. 2.2 Ray tracing for (a) MUSA76 atmosphere, (b) desert-type ITL atmosphere and (c) ocean-type ITL atmosphere. The observer’s height is at 15 m, and all the parameters are defined the same as in the example shown in Fig. 2.1.

bended upwards when passing through the inversion and lead to ray crossing. The corresponding image is mirrored below the true image and lead to desert-type mirage effect. Consider the ray tracing in ocean-type ITL atmosphere shown in Fig. 2.2(c). Rays, initial angles within certain range, are bended downwards as passing through the inversion. This
lead to an additional mirrored image on top of the true image and corresponding to ocean-type mirage phenomena.

2.3 Kolmogorov atmospheric turbulence model

In this section, the Kolmogorov atmospheric turbulence model is reviewed and discussed. Atmospheric turbulence produces small fluctuations in the index of refraction which comes from randomly shaped pockets of air with relatively uniform temperature and are called “turbulence eddies”. The refractive index fluctuations convert to phase distortion of the wavefront as light passes through a turbulence.

Since turbulence is a random process phenomenon, it can only be described statistically. In many atmospheric optics applications, large-scale turbulences whose size significantly exceeds an optical system’s aperture size or the propagating beam size have a relatively small influence on optical system performance. The parameter associated with the largest optical turbulence size is known as the turbulence outer scale \( L_0 \). To exclude large-scale refractive index inhomogeneities Kolmogorov and Obukhov proposed to use statistical characteristic known as the refractive index structure function which is defined as \([12, 13]\):

\[
D_n(\mathbf{r}_1, \mathbf{r}_2) = \left\langle [n_{\text{turb}}(\mathbf{r}_1) - n_{\text{turb}}(\mathbf{r}_2)]^2 \right\rangle. \tag{2.9}
\]

Kolmogorov’s theory was presented in the terms of a set of hypotheses and one of them is that there is a subrange in the separation distance for which the refractive index deviations \( n_{\text{turb}}(\mathbf{r}) \) can be considered as a locally uniform and isotropic random field \([12-14]\). The parameter associate with the smallest scale refractive index inhomogeneities is
known as inner scale \( \ell_0 \) and the interval bounded by the outer and inner scale is called inertial subrange [15].

For locally uniform and isotropic turbulence the dependence of \( D_n(\vec{r}_i, \vec{r}_j) \) was found by Kolmogorov and Obukhov [13, 16, 17]. They demonstrate that for the points \( \vec{r}_i \) and \( \vec{r}_j \) for which the difference vector modulus \( \rho = |\vec{r}_i - \vec{r}_j| \) belongs to the inertial subrange, the structure function in Eq. (2.9) satisfies the two-thirds power law

\[
D_n(\rho) = C_n^2 \rho^{2/3}, \quad l_0 < \rho \leq L_0, \tag{2.10}
\]

where the proportionality constant \( C_n^2 \), called the refractive index structure parameter is a function of atmospheric pressure, temperature, height above ground, observation time, and geographical locations. The refractive index structure function \( C_n^2 \) plays a fundamental role in atmospheric optics by being present in practically all expressions that describe optical wave propagation in the atmosphere. The turbulence associated with this two-thirds power dependence is often referred to as Kolmogorov turbulence. There are other atmospheric turbulence models like Andrews, Tatarskii and Von Karman, while in this dissertation, the classic Kolmogorov turbulence model is used.

There are several phenomenological models that describe the altitude profile of the structure parameter [18-22]. The most commonly used models for inland daytime conditions are the Gracheva-Guivich and Hufnagel-Valley (HV) turbulence profile models [19, 22]. The advantage of HV model is that it represents the smooth, analytical dependence of \( C_n^2 \) on the altitude and contains two “free” parameters that can be adjusted to account for turbulence strength near the ground [the parameter \( C_n^2(0) \)], and the high-altitude wind speed \( W \):

\[
\]
\[ C_n^2(h) = C_n^2(0) \exp(-h/10^2) + 5.94 \times 10^{-53} h^{10} (W/27)^2 \exp(-h/10^3) + 2.7 \times 10^{-16} \exp(-h/1500). \] (2.11)

Commonly used values for these two parameters are \( C_n^2(0) = 1.7 \times 10^{-14} \text{ m}^{-2/3} \) and \( W = 21 \text{ m/s} \), the so-called HV-5/7 model since it is associated with a fried radius of 5 cm and has a isoplanatic angle of 7 \( \mu \text{rad} \) at the wavelength of 5 \( \mu \text{m} \) for zenith path. Figure 2.3 shows Hufnagel-Valley turbulence profiles calculated for the two different values of both \( C_n^2(0) \) and \( W \). As can be seen in Fig. 2.3(a) the ground-turbulence level value of \( C_n^2(0) \) has no effect on \( C_n^2 \) values above height of \( h > 1 \text{ km} \). On the contrary, variation of the wind speed parameter \( W \) impacts \( C_n^2 \) but only at altitude above 6-7 km.

![Fig. 2.3 Atmospheric turbulence structure parameter \( C_n^2 \) altitude profiles corresponding to the Hufnagel-Valley model for a fixed wind speed parameter of \( W = 21 \text{ m/s} \) in (a), with the solid line corresponding to \( C_n^2(0) = 1.7 \times 10^{-14} \text{ m}^{-2/3} \) and dashed line to \( C_n^2(0) = 1.7 \times 10^{-13} \text{ m}^{-2/3} \); and for \( C_n^2(0) = 1.7 \times 10^{-13} \text{ m}^{-2/3} \) in (b) where the solid line corresponding to \( W = 21 \text{ m/s} \) and the dashed line to \( W = 51 \text{ m/s} \).](image)

Another important parameter that characterize the strength of atmospheric turbulence is the Fried coherence length \( r_0 \) [23]. For a plane wave propagating along the
direction of the zenith angle \( \theta_z \), \( r_0 \) is given by the following expression which accounts for the vertical structure of \( C_n^2(h) \):

\[
 r_0 = 1.68 \left\{ k^2 \int_0^L C_n^2[h(z)] dz \right\}^{-3/5}, \tag{2.12}
\]

where the height above ground \( h(z) = h_0 + z \cos(\theta_z) \) is a function of the propagation distance \( z \), zenith angle \( \theta_z \) and height above the ground at the beginning of the propagation path \( h_0 \). The ratio \( D/r_0 \) of the optical system diameter \( D \) to the Fried coherence length \( r_0 \) is a commonly used parameter for characterizing the impact of turbulence on optical system performance.

2.4 Mathematical and numerical techniques for analysis of image propagation in atmosphere

2.4.1 Basic mathematic background of incoherent imaging

Consider the conversional optical imaging system as depicted in Fig. 2.4. The imaging system is represented by a thin lens of diameter \( D \) and focal length \( F \). We assume the coordinate system with original at the lens center \( z = 0 \) and the optical axis \( (z-axis) \) passing through the object plane at \( z = L_o \). An extended object is located at the object plane orientated orthogonal to the optical axis and the imaging sensor is located at the imaging plane \( z = -L_i \). The distance can be obtained from the lens formula: \( 1/F = 1/L_o + 1/L_i \). Denote the complex field at the image plane as \( A(r,-L_i,t) \) and the complex field at the lens pupil plane as \( A(r,0,t) \), where \( r = \{x,y\} \) is the transverse coordinate vector and \( t \) is the time.
Using the lens transform and the Fresnel diffraction integral, propagate from lens pupil plane, the optical field at the image plane is given by

\[ A(r, -L_i, t) = \frac{ik}{2\pi L_i} \exp(-ikL_i) \int_{-\infty}^{\infty} P(r') A(r', 0, t) \exp \left[ \frac{ik}{2F} |r'|^2 - \frac{ik}{2L_i} |r - r'|^2 \right] d^2r' \]  

(2.13)

where \( P(r) \) is the imaging lens pupil aperture stepwise function defined as unity inside the aperture area and as zero otherwise, \( F \) is the focal length of the lens, and \( k = \frac{2\pi}{\lambda} \) is the wavenumber. By taking into account the lens formula \( 1/F = 1/L_i + 1/L_s \), Eq. (2.13) can be represented as

\[ A(r, -L_i, t) = \frac{ik}{2\pi L_i} \exp(-ikL_i - \frac{ikr^2}{2L_i}) \int_{-\infty}^{\infty} P(r') A(r', 0, t) \exp \left[ \frac{ik}{2L_s} |r'|^2 + \frac{ik}{L_i} rr' \right] d^2r' \]  

(2.14)

Assume that an imaging photo-senor integration time \( \tau_{ph} \) is smaller with respect to the characteristics time \( \tau_{at} \) of the atmospheric turbulence induced refractive index change (frozen turbulence model), but significantly exceeds either the coherence time \( \tau_c \) of the optical wave that illuminates the object or the object surface roughness update time \( \tau_s \).
(incoherent illumination), so that $\tau_c, \tau_s \ll \tau_{ph} < \tau_{aw}$. Under this assumption, the intensity distribution in the image plane registered by the sensor is given by (the proportional constant is insignificant and neglected here)

$$I_{\text{img}}(\mathbf{r}, t) \equiv \left\langle A(\mathbf{r}, -L, t) A^*(\mathbf{r}, -L, t) \right\rangle_s = \int \int P(\mathbf{r'}) P(\mathbf{r}*) \left\langle A(\mathbf{r'}, 0, t) A^*(\mathbf{r}^*, 0, t) \right\rangle_s \times \exp \left[ \frac{ik}{2L_s} (|\mathbf{r}|^2 - |\mathbf{r}^*|^2) + \frac{ik}{L_i} \mathbf{r}(\mathbf{r'} - \mathbf{r}^*) \right] d^2\mathbf{r'}d^2\mathbf{r}^*. \quad (2.15)$$

The incoherent short exposure imaging corresponding to averaging over ensemble of random realizations of object surface roughness $\left\langle \cdot \right\rangle_s$ under “frozen” refractive index inhomogeneities. To numerically solve for the image intensity, several technologies have been developed including Monte Carlo method and brightness function technique.

### 2.4.2 Brightness function approach

The basic idea of the brightness function approach is to perform averaging of the instantaneous image-plane intensities over ensemble of random target surface roughness realizations analytically [24-26]. For the following calculations, this target surface roughness is assumed delta correlated and corresponding to incoherent imaging in this case. Start from the analytical representation of incoherent short-exposure image expressed in Eq. (2.15), we will introduce the sum and the difference coordinates [25],

$$\mathbf{R} \equiv (\mathbf{r}' + \mathbf{r}^*)/2, \quad \mathbf{p} \equiv \mathbf{r}' - \mathbf{r}^*. \quad (2.16)$$

and using the relation

$$|\mathbf{r}'|^2 - |\mathbf{r}^*|^2 = 2\mathbf{R}\mathbf{p}. \quad (2.17)$$

Then intensity in the image plane can be written as
\[ I_{img}(r,t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(R+\rho/2)P(R-\rho/2) \Gamma(\rho,R,0,t) \exp \left[ ik \left( \frac{R}{L_s} + \frac{r}{L_i} \right) \rho \right] d^3 \rho d^3 R, \quad (2.18) \]

where we use lens formula \(1/F = 1/L_i + 1/L_s\). The mutual correlation function (MCF) \(\Gamma(\rho,R,0,t)\) defined at the lens pupil plane \(z=0\) is given as

\[ \Gamma(\rho,R,0,t) = \left\{ A(R+\rho/2,0,t)A'(R-\rho/2,0,t) \right\}_s. \quad (2.19) \]

Based on Eq. (2.18), short exposure (“frozen turbulence”) incoherent image can be obtained through the computation of mutual coherent function at lens pupil plane. To achieve this goal, two problems need to be solved: compute MCF on pupil-plane; solve the 4D integrals.

By introducing a vector \(\kappa = k(R/L_s + r/L_i)\), the integral in Eq. (2.18) can be expressed in the form

\[ I_{img}(r,t) = \int_{-\infty}^{\infty} d^3 R \int_{-\infty}^{\infty} \Gamma_p(\rho,R) \Gamma(\rho,R,0,t) \exp[i\kappa \rho] d^3 \rho, \quad (2.20) \]

where \(\Gamma_p(\rho,R) \equiv P(R+\rho/2)P(R-\rho/2)\) is the aperture overlapping function. Using the Fourier transform convolution theorem, with the accuracy to a constant of proportionality, the integral in Eq. (2.20) can be represented as

\[ I_{img}(r,t) = \int_{-\infty}^{\infty} d^3 R \int_{-\infty}^{\infty} B_p(\kappa-\kappa',R)B(\kappa',R,0,t) d^3 \kappa'. \quad (2.21) \]

where \(B_p(\kappa,R)\) and \(B(\kappa,R,0,t)\) are the Fourier transform of functions \(\Gamma_p(\rho,R)\) and \(\Gamma(\rho,R,0,t)\) over the difference coordinate \(\rho\) so that [24, 25]

\[ B_p(\kappa,R) = \frac{1}{(2\pi)^2} \int \Gamma_p(\rho,R) \exp(i\kappa \rho) d^3 \rho, \quad (2.22) \]
\[ B(\kappa, \mathbf{R}, 0, t) = \frac{1}{(2\pi)^2} \int \Gamma(\mathbf{p}, \mathbf{R}, 0, t) \exp(i \kappa \mathbf{p}) d^2 \mathbf{p} . \] (2.23)

Function \( B(\kappa, \mathbf{R}, 0, t) \) is the brightness function at the lens pupil-plane \( z = 0 \), defined as the Fourier transform of the mutual coherent function \( \Gamma(\mathbf{p}, \mathbf{R}, z, t) = \langle A(\mathbf{R} + \mathbf{p}/2, z, t) A^*(\mathbf{R} - \mathbf{p}/2, z, t) \rangle_s \) over the difference coordinate.

For a lens with infinity large aperture \( P(\mathbf{r}) = \text{const} \), the Fourier transform of the aperture overlapping function \( B_p(\kappa, \mathbf{R}) = \delta(\kappa) \). Then Eq. (2.21) can be written as the 2-D integration of brightness function on pupil-plane:

\[ I_{\text{img}}(\mathbf{r}, t) = \int B(\kappa, \mathbf{R}, 0, t) d^2 \mathbf{R} . \] (2.24)

For a lens with Gaussian aperture function \( P(\mathbf{r}) = \exp(-r^2/2a^2) \), then the overlapping function \( \Gamma_p(\mathbf{p}, \mathbf{R}) = \exp\left[-(\mathbf{R}/a)^2\right]\exp\left[-(\mathbf{p}/2a)^2\right] \) and the corresponding Fourier transform of overlapping function over difference coordinate is \( B_p(\kappa, \mathbf{R}) = \exp\left[-(\mathbf{R}/a)^2\right]\exp\left(-a^2\kappa^2\right) \). Substituting to Eq. (2.21), we obtain the image intensity from Gaussian aperture is

\[ I_{\text{img}}(\mathbf{r}, t) = \int e^{-\mathbf{r}^2/(4a^2)} d^2 \mathbf{R} \int e^{-\kappa^2/(4a^2)} B(\kappa', \mathbf{R}, 0, t) d^2 \kappa' . \] (2.25)

For the case of a lens with step-wise circular aperture of diameter \( D \), \( P(\mathbf{r}) = 1 \) when \(|\mathbf{r}| \leq D/2 \) and \( P(\mathbf{r}) = 0 \) otherwise, the problem is more complicated and can only be solved numerically. Based on Eq. (2.22), by fixing coordinate \( \mathbf{R} = (R_x, R_y) \) on pupil-plane, the overlapping function \( \Gamma_p(\mathbf{p}, \mathbf{R}) \) is a 2-D function of \( \mathbf{p} \) and correspondingly \( B_p(\kappa, \mathbf{R}) \) is a function of \( \kappa \). To solve for \( \Gamma_p(\mathbf{p}, \mathbf{R}) \), it comes to the following set of equations
\[(R_x + \frac{\rho_x}{2})^2 + (R_y + \frac{\rho_y}{2})^2 \leq (\frac{D}{2})^2, \quad (R_x - \frac{\rho_x}{2})^2 + (R_y - \frac{\rho_y}{2})^2 \leq (\frac{D}{2})^2.\] Then \(B_p(\kappa, R)\) is the Fourier transform of the solution of the above \(\Gamma_p(\rho, R)\) for the fixed coordinate of \(R\).

Several examples of overlapping function \(\Gamma_p(\rho, R)\) and the corresponding \(B_p(\kappa, R)\) are shown in Fig. 2.5 for different coordinates \(R\). Image intensity distribution described in Eq. (2.21) is obtained by integrating the convolution \(\int_{-\infty}^{\infty} B_p(\kappa - \kappa', R) B(\kappa', R, 0, t) d^3\kappa'\) over pupil-plane coordinate \(R\).

![Fig. 2.5 Aperture overlapping function \(\Gamma_p(\rho, R)\) and the corresponding Fourier transform \(B_p(\kappa, R)\) for different pupil-plane coordinates \(R\) are shown in the first and second row, respectively.](image)

Equation (2.21) established the relationship between the image intensity and the brightness function at the pupil-plane. The further relationship between the pupil-plane brightness function and the intensity \(I(r, L_s, t)\) at the object plane is needed for a full solution. This by computing groups of trajectories corresponding to constant BF values, which depart from points \(R\) at the lens aperture plane \(z = 0\) at different angular vectors \(\theta\) belonging to a cone \(|\theta| \leq \theta_0\), and reach the object plane \(z = L_s\), as described in Ref.[24, 25].

The transport equation for the brightness function from object plane to receiver is given by...
\[
\frac{\partial B(\kappa, R, z, t)}{\partial z} + k^{-1} \nabla_R B(\kappa, R, z, t) + k \nabla_R n(R, z, t) \nabla_\kappa B(\kappa, R, z, t) = 0, \tag{2.26}
\]

where \(\nabla_R\) and \(\nabla_\kappa\) are the directional gradients and \(n(R, z, t)\) is the refractive index field along the propagation path. For the case of a spatially incoherent extended object with the intensity distribution \(I(r, L, t)\) describing the object shape with respect to the receiver aperture, the boundary condition for the brightness function at the object plane \(z = L\) is written as [24]

\[
B(\kappa_L, R_L, L, t) = c I(R_L, L, t), \tag{2.27}
\]

where \(R_L = R(z = L)\), \(\kappa_L = \kappa(z = L)\) and \(c\) is the proportionality coefficient insignificant for our analysis. By introducing the angular vector \(\theta = \{\theta_x, \theta_y\}\) and representing \(\kappa\) as \(\{k_x, k_y\} = k \{\theta_x, \theta_y\}\), the vector \(\kappa\) can be associated with the wavevector that enters plane \(z\) at the angle \(\theta\), then \(\theta = R/L_z + r/L_i\). If \(\theta_L = \theta(z = L)\), Eq. (2.27) can be rewritten as

\[
B(\theta_L, R_L, L, t) = c I(R_L, L, t). \tag{2.28}
\]

The above expression indicates that within the receiver field of view (FOV), the angular component \(B(\theta_L, R_L, L, t)\) of the field intensity \(I(r, L, t)\) scattered off the object plane is independent of the angle of \(\theta_L\).

Based on Ref. [24, 25] that the transport equation in Eq. (2.73) can be reduced to a system of first order ordinary differential equations describing the brightness function characteristics (trajectories) \(R\) and \(\theta\) corresponding to brightness function constant values:

\[
\frac{dR(z, t)}{dz} = -\theta(z, t), \tag{2.29a}
\]
\[
\frac{d\theta(z,t)}{dz} = -\nabla_R \left[ n_0 + n_{\text{refr}}(R, z, t) + n_{\text{turb}}(R, z, t) \right]. \tag{2.29b}
\]

The first equation defines \( \theta \) as a vector tangent to the trajectory, the second equation describes evolution of the tangent vector along the optical axis Oz caused by the inhomogeneities in the propagation medium refractive index [both refractivity term \( n_{\text{refr}}(R, z, t) \) and turbulence term \( n_{\text{turb}}(R, z, t) \)]. For a single trajectory, coordinates \( R \) and \( R_0 \equiv R(z=0) \) along with angular vector \( \theta_L \) and \( \theta_0 \equiv \theta(z=0) \) define mapping between the object and pupil-plane. For a single trajectory, the following mapping condition follows from the definition of the brightness function trajectory:

\[
B(\theta_L, R_L, L_s, t) = B(\theta_0, R_0, 0, t). \tag{2.30}
\]

Equations (2.28)-(2.30) build the relationship between the brightness function at the pupil-plane and the intensity at the object plane. Combined with Eq. (2.21), they provide the desired coupling of the intensities at the image and object planes.

As follows from the above theoretical analysis, the brightness function at the receiver plane can be obtained by numerically solving for the differential equations in Eq. (2.29) with the intensity distribution at the object plane used for computation of the brightness function boundary condition expressed in Eq. (2.28). This corresponds to a tracing of the brightness function trajectory from the object plane to the pupil plane. However, as pointed out in Ref. [24], there is an advantage from computational point of view to solve the ray equations in the reverse direction, that is, from the pupil plane to the object plane. This is equivalent to a sign change of the variable \( z \) in Eq. (2.29).

In the numerical simulations, the associated turbulence refractive index term \( n_{\text{turb}}(R, z, t) \) in Eq. [2.29(b)] was represented by a set of \( M_{\text{turb}} \) statistically independent
random thin refractive index screens equidistantly distributed between the test object and imaging lens aperture, as shown in Fig. 2.6. Using the thin-screen representation of the turbulence-induced refractive index fluctuations, for Eq. [2.29(b)] we obtain:

\[
\frac{d\theta(z,t)}{dz} = \nabla_R \left[ n_0 + n_{\text{refr}}(R, z, t) \right] + \sum_{m=1}^{M} \delta(z - z_m) \nabla_R n^{(m)}_{\text{turb}}(R, t). \tag{2.31}
\]

Here \( \delta(z - z_m) \) is Delta function at \( z_m = m\Delta z \), and

\[
n^{(m)}_{\text{turb}}(R) = \int_{z_m - \Delta z/2}^{z_m + \Delta z/2} n_{\text{turb}}(R, z) \, dz, \quad m = 1, \ldots, M_{\text{turb}}, \tag{2.32}
\]

is the 2D function describing \( m \)th thin refractive index screen and \( \Delta z = L/M_{\text{turb}} \).

Numerical integration of the set of Eqs (2.29a), (2.31), and (2.32) was performed using \( N_{\text{lens}} \) groups of BF trajectories at equidistant 2D grid of points \( \{ R_i^0 \} \) inside the lens aperture area \( (l = 1, \ldots, N_{\text{lens}}) \). The group of the BF trajectories at each point \( R_i^0 \) is defined by the set of angular vectors \( \theta_{l,k}^0 = R_l^0 / L_s + r_k^i / L_i \) corresponding to image plane points \( \{ r_k \} \), \( k = 1, \ldots, N_{\text{img}} \). Numerical integration of the set of Eqs (2.29a), (2.31), and (2.32) allows to define BF values \( B(\theta, R, z = 0) \) at the lens pupil plane at the grid points \( \{ R_i^0, \theta_{l,k}^0 \} \) and hence, using Eq. (2.21), compute the image-plane intensity distribution \( I_{\text{img}}(r_k) \) at the numerical grid corresponding to the image plane points \( \{ r_k \} \). A detailed description of the BF imaging computation technique is given in [25, 27].

The representation of 3D refractive index field by a set of 2D thin screens corresponds to the conventional split-step-operator technique used in computational atmospheric wave-optics [28, 29]. Note that in the BF imaging approach considered, the refractivity \( n_{\text{refr}}(R, z) \) and turbulence \( n_{\text{turb}}(R, z) \) refractive index components in Eq.
(2.29b) are treated differently. Contrary to \( n_{\text{turb}}(\mathbf{R}, z) \), the refractivity-induced 3D refractive index field \( n_{\text{turb}}(\mathbf{R}, z) \) does not need to be approximated by thin refractivity screens. To provide better numerical simulations accuracy, BF trajectories between subsequent turbulence screens \{ \( n_{\text{ref}}^{(m)}(\mathbf{R}) \) \} in Eq. (2.31) are obtained by direct computation of function \( n_{\text{ref}}(\mathbf{R}, z) \) and its gradient based on the chosen temperature profile model described in Section 2.2.

Fig. 2.6 Schematics of anisoplanatic image computation using brightness function trajectories for layered atmospheric turbulence model and assume no refractivity effect.

The schematic of anisoplanatic image computational algorithm using brightness function trajectories are portrayed in Fig. 2.6. For simplicity, we depict three phase screens to simulate the atmospheric turbulence and assume that the refractive index distribution between phase screens is optically homogeneous (vacuum), where the brightness function trajectory represents a straight line.

In this algorithm as shown in Fig. 2.6, the image plane is divided into different independent spatial points \{ \( \mathbf{r}_i \) \}, the computation include the following steps:
(a) At the pupil-plane, fix a starting point for BF trajectories $\mathbf{R}_0^j$, $j=1,\ldots,N_{\text{lens}}$, and generate a groups of trajectories with the initial angular vectors (slopes) 
\[ \{\theta_0^{ij}=\mathbf{R}_0^j/L_s+r_j/L_z\} \]
associate with a group of image plane coordinates \(\{\mathbf{r}_i\}\), 
\[ i=1,\ldots,N_{\text{img}}. \]

(b) Trace the $N_{\text{img}}$ trajectories with different angular vector \(\{\theta_0^{ij}\}\) from the start point to the first phase screen by integrating the differential equations of (2.29a), (2.31) to the first turbulence screen with the refractivity term in Eq. (2.31) defined as constant for this case. The trajectories reach the phase screen at points of 
\[ \{\mathbf{R}_{zi}^{ij}=\mathbf{R}^{ij}(z=z_i)\} \]
and with angular vectors \(\{\theta_{zi}^{ij}=\theta^{ij}(z=z_i)\}\).

(c) Pass trajectories through the turbulence screen $n_{\text{lamb}}^{(1)}(\mathbf{R},t)$ which can result in a change of the trajectory slopes [Eq. (2.31)] by the angular vector 
\[ \Delta \theta_{zi}^{ij} = \nabla_{\mathbf{R}} n_{\text{lamb}}^{(1)}(\mathbf{R}_{zi}^{ij},t). \]
The angular vectors of BF trajectories after the first phase screen are therefore given by \(\{\theta_{zi}^{ij}+\Delta \theta_{zi}^{ij}\}\).

(d) Continue trajectories to the object plane by repeating steps (b)-(c) at the coordinates of 
\[ \{\mathbf{R}_L^{ij}=\mathbf{R}^{ij}(z=L_s)\} \]
with angular vectors \(\{\theta_L^{ij}=\theta^{ij}(z=L_s)\}\).

(e) Compute the BF values for each trajectory by applying the boundary condition given in Eq. (2.28). Then the BF values at the pupil plane for a fixed $\mathbf{R}_0^j$ and a group of$\{\mathbf{r}_i\}$ are obtained as 
\[ B(\theta_0^{ij}=\mathbf{R}_0^j/L_s+r_j/L_z,\mathbf{R}_0^j,0,t)=I(\mathbf{R}_L^{ij},L_s,t) \].
(f) Numerically compute the aperture overlapping function $\Gamma_p(\rho, R_0^j)$ (a function of difference coordinate) with the fixed pupil-plane point $R_0^j$. Apply Fourier transform Eq. (2.22) to get $B_p(0, R_0^j)$. 

(g) Compute the convolution of $B_p(0, R_0^j)$ with $B(0, R_0^j, 0, t)$ over $\theta$ to calculate the image component (sub-image) for lens point of $R_0^j$ and accounts for the aperture diffraction effect. Examples of image intensity components for different lens points are shown in Fig. 2.7 (center column).

(h) Repeat steps (a) – (g) for all $N_{\text{lens}}$ start points on pupil-plane and compute all the image components associated with each $\{R_0^j\}$.

(i) Sum all the image components $\{B_p(0, R_0^j) \ast B(0, R_0^j, 0, t)\}$ computed in previous step to solve for image plane intensity distribution $I_{\text{img}}(r, t)$ given in Eq. (2.21). Example of calculated image are shown in the right-hand side of Fig. 2.7.

Fig. 2.7 Left-hand image is the schematic illustration of discretized pupil-plane coordinates $R_0^j$. The middle column is the image components calculated from each single lens points and the right-hand side is the calculated image plane intensity distribution by summation of all image components.
For the more general case that when the medium between turbulence screens is not homogeneous, exist refractive index gradient for example, the corresponding schematic for incoherent image modeling using BF technique is portrayed in Fig. 2.8. Compared with the schematic shown in Fig. 2.6, the difference is that the brightness function trajectories are not straight lines between turbulence screens and refractivity effects need to be accounted in Step (b) to calculate the associated evolution of angular vector and trajectory coordinates at each location with Eqs. (2.29a) and (2.31).

Fig. 2.8 Schematics of anisoplanatic image computation using brightness function trajectories for layered atmospheric turbulence model and refractivity effect exist between phase screens.

As mentioned above, in this BF approach, atmospheric turbulence effect is accounted by passing BF trajectories through turbulence screens and result in an update of trajectory slopes; the atmospheric refractivity effects are taken into account as trajectories are traced between turbulence screens with the medium refractive index field defined as \( n_{refr}(\mathbf{R}, z) \). The image obtained for a single atmospheric realization using Eq. (2.21) is referred to as the short-exposure incoherent image \( \{I_{\text{img}}^{(j)}(\mathbf{r})\} \), \( j = 1, \ldots, N_{at} \). Long-exposure images are obtained by statistically averaging the short-exposure images over the ensemble of large number of uncorrelated random realizations of the turbulence-induced refractive
index inhomogeneities, \( I_{\text{img}}^{LE}(r) = \left\langle I_{\text{img}}^{(j)}(r) \right\rangle_{N_{\omega}} \). Also in this dissertation, we consider incoherent anisoplanatic imaging systems. Here “incoherent” means that the field of the reflected light out of different point (pixel) of the object is uncorrelated; “anisoplanatic” means that optical field from different point of the object pass through different atmosphere.
CHAPTER 3
INCOHERENT ANISOPLANATIC IMAGING IN PRESENCE OF
ATMOSPHERIC TURBULENCE AND REFRACTIVITY

Prepared with the BF technique, here in this chapter, we will introduce the research projects we have accomplished with this technique and present the numerical simulation results. Three projects have been conducted which include the analysis of imaging system MTF in atmosphere, optical mirage formation in presence of turbulence and the characterization of image quality in localized atmosphere conditions. All three projects are associated with an incoherent imaging system operating in the atmosphere with the presence of both atmospheric turbulence and refractivity.

First, a general schematic of an incoherent imaging system as shown in Fig. 3.1 is considered for all numerical modeling of incoherent image formation through an optically inhomogeneous medium. An incoherently illuminated extended object of size \( b_{obj} \) is located a distance \( h_{obj} \) above the ground in the atmosphere. In the numerical simulations, the imaging system was represented by a thin lens of diameter \( D \) and focal length \( F \). It was assumed that the lens is located at an elevation of \( h_{img} \) above the ground a distance \( L \) from the test-object. We consider the coordinate system with origins at the lens center and optical axis (z-axis) orthogonal to the test-object. In this coordinate system, the test-object image
is formed at the plane \( z = -L_{\text{img}} \), where the distance \( L_{\text{img}} \) can be obtained from the lens formula: \( 1/F = 1/L + 1/L_{\text{img}} \).

The optically inhomogeneous medium is characterized by continuously distributed atmospheric turbulence and refractivity induced by an inverse temperature layer (ITL). The atmospheric turbulence is assumed to obey the Kolmogorov power spectrum and turbulence strength is characterized by the ratio of \( D/r_0 \). All the transverse to the optical axis variables and parameters, including object size \( b_{\text{obj}} \), Fried parameter \( r_0 \), are normalized by the receiving aperture radius \( a_0 = D/2 \). The longitudinal variables and parameters, such as \( L \) and \( F \), are normalized by the diffraction length \( L_{\text{diff}} = ka_0^2/2 \).

![Fig. 3. 1 A schematic illustrating optical imaging through an optically inhomogeneous medium including atmospheric turbulence and refractivity caused by an inverse temperature layer (ITL).](image)

In numerical simulations, split-operator technique was used to represent the volume turbulence with a set of \( M_{\text{turb}} \) statistically independent random thin turbulence screens equidistantly distributed between the test object and imaging lens aperture, as discussed in Section 2.3. The refractivity-induced 3D refractive index field \( n_{\text{refr}} (\mathbf{R}, z) \) does not need to
be approximated by thin refractivity screens and direct computation of function $n_{ref}(R, z)$ and its gradient based on the temperature profile model defined in Section 2.2 was applied.

### 3.1 Imaging system modulation transfer function in the presence of atmospheric turbulence and refractivity

It is well-known that the presence of atmospheric turbulence may severely affect spatial resolution of imaging systems [7, 23]. The turbulence impact on imaging system resolution is commonly evaluated in terms of modulation transfer function (MTF) that describes visibility of sine-type test patterns of various spatial frequencies which are observed with an imaging system [30-32]. Optical refractivity is another atmospheric effect the imaging system performance and MTF depend on [8, 33]. Currently, in analysis of imaging systems both these effects are considered independently, and analysed using correspondingly the Fresnel diffraction (wave-optics) [9] and geometrical optics (ray tracing) [34] approaches.

Such separate treatment of turbulence and refractivity has certain limitations. Indeed, atmospheric refractivity may result in significant deviation in the altitude $h$ of optical wave propagation trajectory – the effect commonly referred to as the ray bending – and hence impact the atmospheric turbulence characteristics that are dependent on altitude $h$ above the ground, such as the refractive index structure parameter $C_n^2$. Correspondingly, optical wave refraction, especially for extended-range imaging scenarios, could also affect the turbulence-induced optical aberrations and hence the imaging system MTF.

In the framework of the MUSA76 model, the relatively slowly temperature change with height $h$ increase results in a smooth refractive index spatial modulation inside the air.
volume, which is essential for image formation along the propagation path from an object to the imaging system aperture. The atmospheric refractivity impact on optical wave propagation can be accounted by solely considering tip- and tile-type wavefront phase aberrations that are slowly changing along the propagation path and only result in vertical displacement of the long-exposure (atmospheric-averaged) image. This refractivity type does not lead to geometrical distortions of the long-exposure images of the sine-type test patterns used for the MTF analysis.

This may be not true in the presence of spatially localized refractive index structures, such as those caused by inverse temperature layers (ITL) frequently observed in the atmosphere. The ITL-induced refractive index structures may cause highly anisotropic large-scale aberrations of optical waves propagating near or through such refractive index inhomogeneities. These large-scale, quasi-stationary (on the scale of the turbulence change) wavefront phase aberrations could result in spatially nonuniform, dependent on the propagation trajectory (anisoplanatic) distortions of the long-exposure images that directly affect the imaging system MTF.

In this section, we analyze these coupled turbulence and refractivity induced effects by considering an MTF of an incoherent, monochromatic imaging system using numerical simulations based on the brightness function (BF) technique. The MTF estimation is performed via direct numerical analysis of visibility of sine-type test patterns of different spatial frequencies. The test patterns are assumed to be imaged through a volume medium with turbulence- and refractivity-induced refractive index inhomogeneities as described in Chapter 2. The turbulence effects are described in the framework of the classical
Kolmogorov turbulence model. The atmospheric refractivity is simulated using a combined MUSA76 and ITL temperature profile model.

![Fig. 3.2 A schematic illustrations of the imaging system geometry for MTF analysis. The inserted horizontally orientated grey-scale sine-pattern in top-left is an example of test-object used in the following numerical simulations. Atmospheric turbulence is represented by a set of thin refractive index screens, and the ITL is illustrated by the colour gradient.](image)

Schematic illustration of an imaging system used for the MTF analysis is presented in Fig. 3.2. A square test object with horizontally oriented sine-type grey-scale test pattern is applied for the numerical calculations. The test object’s reflectivity is described by the normalized function

\[ R_{obj}(\mathbf{r}) = W(\mathbf{r}) \left[ 1 + \sin \left( 2\pi f_{obj} \cdot y \right) \right], \]

where \( \mathbf{r} = \{x, y\} \) is the coordinate vector in the object plane, and \( W(\mathbf{r}) \) and \( f_{obj} \) are correspondingly the step-wise function that is equal to unity inside the square area of size \( b_0 \times b_0 \) and zero otherwise, and the sine object spatial frequency. An example of a sine-type test object with \( b_0 = 100 \text{ cm} \) and \( f_{obj} = 0.12 \text{ cm}^{-1} \) which was used in the numerical simulations is inserted in top-left in Fig. 3.2. Assuming the imaging system represented by a lens of aperture diameter \( D = 30 \text{ cm} \) and focal length \( F = 1.2 \text{ m} \). The square grid for image is \( N_{img} = n_{img} \times n_{img} = 400 \times 400 \) and \( N_{lens} = n_i \times n_i = 20 \times 20 \) for lens pupil-plane. The imaging lens is located at an elevation of \( h_{img} = 4.0 \text{ m} \) above the ground and a distance \( L = 10 \text{ km} \) from
the test-object. In the analysis we used the normalized spatial frequency \( \hat{f}_{\text{obj}} = f_{\text{obj}} / f_{\text{cut}} \), where \( f_{\text{cut}} = D / (\lambda L) = 0.3 \text{ cm}^{-1} \) is the MTF cut-off frequency for propagation in vacuum [7].

### 3.1.1 MTF in presence of atmospheric turbulence

**Pupil-plane turbulence: numerical model evaluation**

Consider first the impact of solely atmospheric turbulence on incoherent imaging system MTF. For a diffraction limited imaging system, the analytical expression of modulation transfer function (MTF) as a function of normalized object spatial frequency \( \hat{f}_{\text{obj}} \) is given as [7, 30]:

\[
H_i^d(\hat{f}_{\text{obj}}) = \frac{2}{\pi} \arccos(\hat{f}_{\text{obj}}) - \hat{f}_{\text{obj}} \sqrt{1 - \hat{f}_{\text{obj}}^2}. \tag{3.1}
\]

In the presence of Kolmogorov turbulence layer modelled by the pupil-plane turbulence screen (pupil-plane turbulence model), the long-exposure MTF of the imaging system is given by [7, 32]:

\[
\left< H_i(\hat{f}_{\text{obj}}) \right>_{\text{LE}} = H_i^d(\hat{f}_{\text{obj}}) \exp \left[ -3.44 \left( \frac{D}{r_0} \right)^{5/3} \hat{f}_{\text{obj}} \right]. \tag{3.2}
\]

A comparison of theoretical and simulation results of the imaging system MTF in free space as well as in the atmospheric turbulence is conducted to verify the validation of applying the BF technique for incoherent imaging system performance analysis.

Numerical calculations of long-exposure imaging system MTF in homogeneous medium and in presence of atmospheric turbulence modeled by pupil-plane turbulence screen are performed and the comparison of simulation and the theoretical expression are presented in Fig. 3.3(a). The solid lines are theoretical expressions and the circles are the corresponding numerical calculations for three cases: diffraction-limited MTF, long-exposure MTF expressed by Eq. (3.2) with the
Fig. 3.3 (a) Comparison of theoretical (solid curves) and numerical simulation (circles) results of imaging system MTF as a function of the normalized spatial frequency $\hat{f}_{obj}$. The three pair of curves are diffraction-limited MTF, long-exposure MTF in pupil-plane turbulence with turbulence strength characterized by the ratio $D/r_0=1$ and $D/r_0=10$. The comparison of numerical calculated long-exposure MTF in the presence of pupil-plane turbulence and distributed turbulence model are plotted in (b) $D/r_0=1$ and (c) $D/r_0=10$. Circles and dots denote MTF values obtained in pupil-plane and distributed turbulence, respectively. Inserted images in both (b) and (c) are example calculations of images corresponding to identical sine patterns with normalized spatial frequency of $\hat{f}_{obj}=0.1$ in presence of pupil-plane and distributed turbulence models. The number of atmospheric turbulence realizations for long-exposure image calculation is $N_{at}=500$, $N_{lens}=20$ in BF technique.
turbulence strength defined as \( D / r_0 = 1 \) and \( D / r_0 = 10 \). Good agreement between simulation results and theoretical expressions is observed in Fig. 3.3(a) which indicates the validation of applying brightness function technique approach for MTF based incoherent imaging system performance analysis.

*Distributed atmospheric turbulence*

Consider now long-exposure imaging system MTF in the presence of the distributed atmospheric turbulence. In this case, the continuous atmospheric turbulence is modeled by a set of \( M_{\text{turb}} = 10 \) equidistantly located phase screens along the propagation path. Assuming the same turbulence strengths defined above, a comparison of numerical simulation results of long-exposure MTFs in presence of either pupil-plane (circles) or distributed turbulence (dots) are shown in Fig. 3.3(b) (weak turbulence \( D / r_0 = 1 \)) and Fig. 3.3(c) (stronger turbulence \( D / r_0 = 10 \)). Based on these two figures, in the whole frequency range, the MTF values calculated in distributed turbulence are higher than those calculated in pupil-plane turbulence. It is more straightforward if we compare the visibilities of images corresponding to identical object calculated in different turbulence models [the inserted images in both Figs. 3.3(b) and 3.3(c)]. As expected, higher spatial resolution is observed for imaging system operating in distributed turbulence than those in pupil-plane turbulence.

### 3.1.2 MTF in presence of inverse temperature layer

Then consider the impact of solely atmospheric refractivity on incoherent imaging system MTF. Assume the same imaging system illustrated in Fig. 3.2 with test-object elevated at height \( h_{obj} \) above the ground, represents a sine-type pattern of spatial frequency \( f_{obj} \). The ITL-induced refractivity layer of width \( w_{\text{ITL}} = 1.0 \) m and temperature inversion
$\Delta T$ was located at the elevation $h_{ITL} = 8.0\text{ m}$ above the ground. The distance between the object and the ITL centre $\Delta = h_{obj} - h_{ITL}$ was varied by considering the object displacement along the vertical axis $y$.

![Example calculations of images in presence of desert-type ITL for different $\Delta$ values.](image)

Fig. 3.4 Example calculations of images in presence of desert-type ITL for different $\Delta$ values. The test-objects are horizontally and vertically orientated sine-patterns with normalized spatial frequency $\hat{f}_{obj} = 0.4$ for the first and second rows, respectively. The columns are the corresponding images for different $\Delta$ values and parameters for ITL models are defined as follows: $w_{ITL} = 1.0\text{ m}$, $h_{ITL} = 8.0\text{ m}$, $\Delta T = -0.5\text{ K}$.

Examples of the image-plane intensity distributions $I_{img}(r)$ computed using the described above BF technique in the desert-type ITL for different values of $\Delta$ are presented Fig. 3.4. Images in the first and second rows are corresponding to test-object shown in the top-left of Fig. 3.2 (horizontally orientated sine-pattern) and the same above object with vertically oriented sine-patterns, respectively. In both cases, a reduction of image size along the $y$-axis, compared to the associated object, is observed. Also an increase of the image sine-pattern frequency can be seen in the first row while the vertically oriented sine-patterns in the second row exhibit no frequency changes. To characterize
these image size and spatial frequency changes, we introduce parameters of image size magnification (or demagnification) factor $M = \frac{b_{\text{img}}}{b_0}$ and relative frequency deviation between image and object $\Delta f = \left( \frac{f_{\text{img}}}{f_{\text{obj}}} - 1 \right)$, where $b_{\text{img}}$ is image size along y-axis, $f_{\text{img}}$ is image spatial frequency. Since the variation of spatial frequency across each image is small (less than 5 percent), the presented $f_{\text{img}}$ are corresponding to the averaged image spatial frequency. For this case, the minimum $M$ and maximum $\Delta f$ is observed at $\Delta = 0$ m as seen in the bottom of the first row images. The deviation of image spatial frequency in respect to the object can introduce problems for imaging system MTF characterization which is based on the assumption that test-object and image frequency preserves. Besides, comparing the first and second row images, this frequency shift is anisotropic and depends on the orientation of sine-patterns which is also not expected in imaging system MTF analysis.

Fig. 3.5 Example calculations of images in presence of ocean-type ITL for different $\Delta$ values. The test-objects are horizontally and vertically orientated sine-patterns with normalized spatial frequency $\hat{f}_{\text{obj}} = 0.4$ for the first and second rows, respectively. The columns are the corresponding images for different $\Delta$ and parameters for ITL models are defined as follows: $w_{\text{ITL}} = 1.0$ m, $h_{\text{ITL}} = 8.0$ m, $\Delta T = 0.5$ K.
Similar calculations as above for the ocean-type ITL case are performed and presented in Fig. 3.5. For the horizontally orientated sine-pattern shown in the first row, based on the calculated $M$ and $\Delta f$ values displayed in the bottom of each image, the computed image patterns size increase first and then decrease along $y$-axis with the further increase of $\Delta$. On the other hand, the image spatial frequency changes in the opposite way - decrease first and then increase. For the vertically orientated sine-pattern images, still no change of the image spatial frequency observed.

| $|\Delta T| = 0.1\, \text{K}$ | $|\Delta T| = 0.2\, \text{K}$ | $|\Delta T| = 0.3\, \text{K}$ | $|\Delta T| = 0.4\, \text{K}$ | $|\Delta T| = 0.5\, \text{K}$ |
|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $M=0.90, \Delta f=0.11$ | $M=0.82, \Delta f=0.21$ | $M=0.75, \Delta f=0.33$ | $M=0.69, \Delta f=0.47$ | $M=0.65, \Delta f=0.53$ |
| $M=1.12, \Delta f=0.10$ | $M=1.28, \Delta f=0.21$ | $M=1.50, \Delta f=0.33$ | $M=1.76, \Delta f=0.43$ | $M=1.98, \Delta f=0.50$ |

Fig. 3.6 Example calculations of images in presence of desert- and ocean-type ITLs for different temperature inversion values $\Delta T$ are shown in the first and second rows, respectively. The test-object is horizontally oriented sine-pattern with normalized spatial frequency $\hat{f}_{obj} = 0.4$. The columns are the corresponding images for different $|\Delta T|$: $\Delta T < 0$ for the desert- and $\Delta T > 0$ for the ocean-type ITLs. Other parameters for ITL models are defined as follows: $w_{ITL} = 1.0\, \text{m}$, $h_{ITL} = 8.0\, \text{m}$, and the object-ITL relative position $\Delta = -1\, \text{m}$.

Another example of images computed for different temperature inversion $|\Delta T|$ in presence of both desert- ($\Delta T < 0$) and ocean-type ($\Delta T > 0$) ITLs are presented in Fig. 3.6. For the desert-type ITL case, shown in the first row, image size gradually decreases as the temperature inversion increases, while in the ocean-type ITL case, on the contrary, larger
temperature inversion is associated with greater image size magnification, as can be seen in the second row of Fig. 3.6. Considering the change of image spatial frequencies, the increase of temperature inversion will lead to further modulation of image frequency to higher or lower values with respect to the object in desert- or ocean-type ITLs, respectively.

The above three examples highlight the impact of refractive index structures, in this case refractivity induced by ITLs, on incoherent image formation. Such refractive index inhomogeneities can result in highly spatially non-uniform (vertical and horizontal directions) distortions of the images. Similar phenomena of atmospheric refractivity resulted highly anisotropic distortion were observed in several time-lapse imagery experiments [35-37].

Qualitative analysis of how image size and spatial frequency changes along with the variation of object-ITL relative position Δ and ITL temperature inversion |ΔT| are presented in above examples. The quantitative analysis of these dependencies for both desert- and ocean-type ITLs are presented in Fig. 3.7. Here we consider only the object with horizontally orientated sine-patterns. The dependency of magnification factor \( M \) and relative spatial frequency shift \( \Delta f \) on \( \Delta \) are shown in Figs. 3.7(a) and 3.7(c), respectively. For the desert-type ITL case, \( M \) is always less than 1 (demagnification) and \( \Delta f \) is positive (“blue-shift”), the minimum value of \( M \) and maximum value of \( \Delta f \) is observed as the object is placed in the centre of the ITL (\( \Delta = 0 \) m). For the ocean-type ITL, the transition from \( M > 1 \) (magnification), \( \Delta f < 0 \) (“red-shift”) to \( M < 1 \), \( \Delta f > 0 \) is observed at \( \sim \Delta = 1 \) m.

For the other case, the dependency of \( M \) and \( \Delta f \) on temperature inversion are shown in Figs. 3.7(b) and 3.7(d), respectively. As the temperature inversion increases, \( M \)
monotonically decreases and $\Delta f$ increases for the desert-type ITL cases; while $M$ increases and $\Delta f$ decreases for the ocean-type ITL case.

Fig. 3.7 The dependencies of the image magnification factor $M$ and the relative image spatial frequency deviation $\Delta f$ on the relative object-ITL positions $\Delta$ are shown in (a), (c); and on the temperature inversion absolute values $|\Delta T|$ are presented in (b), (d), respectively, for both desert- and ocean-type ITLs. Parameters for ITL models are defined as $w_{ITL} = 1.0 \text{ m}$, $h_{ITL} = 8.0 \text{ m}$, $\Delta T = -0.5 \text{ K}$ for desert- and $\Delta T = 0.5 \text{ K}$ for ocean-type ITLs in (a), (c) and the relative position parameter is fixed at $\Delta = -1 \text{ m}$ in (b), (d).

The above examples illustrated the effects of image size and spatial frequency change due to the presence of atmospheric refractivity induced by ITLs. But in reality, refractivity can also cause the change of image pattern visibility as shown in Fig. 3.8. Figure 3.8 (a) is the free-space image calculated from the same horizontally oriented sine-pattern used in above examples, and 3.8(b), 3.8(c) are the corresponding images calculated
in the desert- and ocean-type ITLs, respectively. The right-hand-side of each figure shows the cross-section along the $y$-axis. Compared with the image in free-space, image pattern in Fig. 3.8(b) present with higher spatial frequency and lower visibility while the pattern in Fig. 3.8(c) exhibit lower spatial frequency and higher visibility. Since imaging system MTF is directly measured based on image pattern visibilities, the presence of atmospheric refractivity solely can impact imaging system MTF even in the absence of turbulence. Also this impact depends on the imaging path geometry and ITL characteristics.

Fig. 3.8 (a) is free-space image of the horizontally orientated sine-pattern test-object of spatial frequency $\tilde{f}_{obj} = 0.4$ and (b), (c) are images calculated for the desert- and the ocean-type ITLs with $\Delta = -1$ m, respectively. The right-hand-side of each image are the corresponding cross sections. Parameters for ITL models are: $w_{ITL} = 1.0$ m, $h_{ITL} = 8.0$ m, $\Delta T = -0.5$ K for desert- and $\Delta T = 0.5$ K for ocean-type ITLs.

High resolution imagery is very important in fields of study that demands accurate feedback information provided by optical systems. This triggered the rapid development of image processing techniques which are aimed at achieving high spatial resolution from low-resolution images through image post-processing [38, 39]. Besides image post-processing techniques, high resolution images directly from optical system is also important. Our study indicate that the image spatial frequency “red-shift” due to the presence of an ocean-type ITL is one possible solution to this problem. This technique benefits from refractive gradient originating from the ocean-type ITL which shifts indistinguishable high spatial frequency components to lower spatial frequency.
components which become resolvable with optical imaging systems. Based on the analysis in previous section this technique works when the object and optical imaging system are located on the same side of the ITL which can be referred as the “red-shit” condition.

Fig. 3.9 An example of super resolution image formation in presence of an ocean-type ITL. (a) is the test-object and (b) is the calculated image in free space; (c),(d),(e) and (f) are images through the atmosphere in presence of the ocean-type ITL with $\Delta = -2, -1, 0$ and 1m respectively.

Here we discuss one more example about the ocean-type ITL assistant image resolution enhancement. The object is a square array as shown in Fig. 3.9(a) and the image obtained in free space with the optical imaging system is in Fig. 3.9(b) which exhibits low resolution between the neighboring squares in both horizontal and vertical directions. With the same imaging system discussed above, we recalculate the image in ocean-type ITL presented atmosphere as displayed in Figs. 3.9(c)-3.9(f) with $\Delta$ values varied from -2m to 1m with a step of 1m (move object). When the “red-shift” condition is satisfied as in Figs. 3.9(c) and 3.9(d), both images exhibit resolution enhancement in vertical direction when compared with the image in free space. While, if the “red-shift” condition is not satisfied, a decrease of the image resolution is noticed in Fig. 3.9(f). Another
point that is worth noticing is the similarity of the image resolution in the horizontal direction for all the calculated images. This can be explained by the fact that this ocean-type ITL model is only applied vertically (ideal numerical model) and will have no effect horizontally.

3.1.3 MTF in presence of atmospheric turbulence and desert-type inverse temperature layer

Now consider the joint impacts of atmospheric turbulence and refractivity on the incoherent imaging system MTF. The analysis was carried out by comparing the MTFs of imaging systems operating in three atmospheric conditions: atmospheric turbulence only, refractivity only, and both turbulence and refractivity. For atmospheric turbulence, the HV-5/7 model was used for computing the \( C_n^2(h) \) profile along the path and the turbulence effect was modelled by a set of 10 turbulence screens following the Kolmogorov power spectrum. The refractivity effect was described by the desert-type ITL model defined in above section with the temperature inversion \( \Delta T = -0.5 \, \text{K} \) and the object-ITL relative position fixed at \( \Delta = -1 \, \text{m} \).

As discussed in Section 3.1.2, images calculated in the presence of refractivity is exhibit broadband frequencies and the defined image mean spatial frequency \( f_{\text{img}} \) is shifted from the object spatial frequency \( f_{\text{obj}} \). This spatial frequency deviation can cause problem in imaging system MTF calculation and for the comparison of the results with no refractivity in present. Here we ignored the frequency change of the image and consider the imaging system MTFs associated with the object spatial frequencies \( f_{\text{obj}} \). Also for the case with atmospheric refractivity in presented, the MTF is defined as the image averaged
Fig. 3.10 A comparison of imaging system MTF in three different atmospheric conditions: turbulence only, refractivity only, and both turbulence and refractivity. Distributed turbulence model is assumed, and refractivity effect is described by the desert-type ITL with the parameters defined as $w_{ITL} = 1.0 \text{ m}$, $\Delta T = -0.5 \text{ K}$, and $h_c = 8.0 \text{ m}$; $\Delta = -1.0 \text{ m}$ is fixed for all cases. The turbulence strength is characterized by the ratio of $D/r_0 = 1$ in (a) and $D/r_0 = 10$ in (b).

Contrast: $\text{MTF} = \left( \langle I_{\text{max}} \rangle - \langle I_{\text{min}} \rangle \right) / \left( \langle I_{\text{max}} \rangle + \langle I_{\text{min}} \rangle \right)$, where $\langle I_{\text{max}} \rangle$ and $\langle I_{\text{min}} \rangle$ are the averaged local maximum and minimum intensities of the image.

For all three atmospheric conditions, the dependencies of the imaging system MTFs on the normalized object spatial frequencies $\hat{f}_{\text{obj}}$ are plotted in Figs 3.10(a) and 3.10(b) for different turbulence strengths $D/r_0 = 1$ and $D/r_0 = 10$, respectively. Note that the MTF for the condition of refractivity only is the same in both figures. For the case with turbulence existed, the MTFs are computed based on visibility of long-exposure (averaged over 500 of statistically independent thin turbulence screen sets) images.

Based on Figs 3.10(a) and 3.10(b), at each normalized spatial frequency $\hat{f}_{\text{obj}}$, MTF values calculated in presence of both turbulence and refractivity are lower than those calculated with only one component. Also, both atmospheric-turbulence- and desert-type-ITL-induced refractivity effects solely can result in imaging system resolution decline. The
presence of atmospheric turbulence could lead to image quality degradation by giving rise to the long-exposure image blur [7, 30-32] and stronger turbulence strength is associated with lower image qualities. While the desert-type-ITL-induced atmospheric refractivity can have impact on the imaging system resolution by introducing spatially anisotropic distortions to the images (decrease of image size and increase of image spatial frequency) and result in a decrease of image pattern visibilities. By combining these two effects, in this case, a further decline of the imaging system resolution is observed in both Figs 3.10(a) and 3.10(b).

3.2 Optical mirage formation in presence of atmospheric turbulence

The study of images transmitted through a refracting atmosphere has a history that goes back to 17th century. Phenomenon that can occur including shortening or lengthening of the normal horizon, and seeing multiple elevated images of a single source which is known as optical-mirage.

Optical-mirages are commonly studied and analyzed in framework of geometrical optics in terms of optical rays bending occurring in propagation through an atmospheric layer with strong refractive index gradient. This simplified explanation is quite limited since it doesn’t take into account such important factor as diffraction for image formation which results from imaging system aperture and presence of atmospheric turbulence-induced refractive index inhomogeneity along the propagation path from object to imaging system aperture. These factors, neglected in the geometrical optics approach, may significantly impact mirage-image formation [26]. Correspondingly, the conventional approach doesn’t provide adequate tools for predictive numerical simulations of mirage-
image as well as for analysis of more general problems of image formation in atmospheric conditions that are characterized by the presence of both strong refractivity and turbulence. As introduced in Section 2.4, for its powerful capability, the recently developed brightness function technique is applied in this section to address this challenging problem of analysis of image formation in presence of both turbulence and refractivity.

### 3.2.1 Experimental observation of ocean-type mirage phenomenon

A set of experiments were carried out in the San Diego bay area in 2014. In these experiments, optical-mirage phenomena of distant ship targets over the ocean were observed and recorded under different atmospheric conditions. A few examples of mirage images are shown in Fig. 3.11.

Figure 3.11(a) and 3.11(b) are two frames from the same video which recorded the mirage phenomena at different times (different atmospheric conditions). Two virtual images (one inverted image and one erect image) on top of the ordinary image are observed in both figures while the separations between ordinary and virtual images changes for different conditions. Fig. 3.11(c) shows the separation of all three images under different observation conditions and Fig. 3.11(d) shows the bend up of the sea level. Besides the mirage effects resulted from large scale refractive index gradient, atmospheric turbulence resulted image degradations are also observed in all figures.
Fig. 3.11 Examples of experimental observed mirage images of ships in San Diego Bay.

3.2.2 Mirage image formation in presence of ocean-type inverse temperature layer and atmospheric turbulence

For the numerical calculations, the imaging system lens diameter $D=30\text{cm}$ and focal length $F=1.2\text{m}$ is elevated at $h_{\text{img}}=18\text{m}$. Object (the ship) height is defined as $b_o=25\text{m}$ with grid points of $N_{\text{img}}=n_{\text{img}}\times n_{\text{img}}=1024\times1024$ and located a distance $L=20\text{km}$ away from the imaging system.
Fig. 3.12 Example of short-exposure mirage image calculations in ocean-type ITL through turbulence. The ITL model parameters are defined as $h_{ITL} = 30 \text{ m}$, $w_{ITL} = 2 \text{ m}$. (a) is the scene for imaging (object); (b), (c), and (d) are numerically calculated mirage images as the ITL parameter $\Delta T = 2 \text{ K}$, 3K, and 4K, respectively. The turbulence strength is defined as $D/r_0 = 5$ for all cases.

First we have studied the dependence of mirage image formation on ITL strength. The scene we are trying to image is shown in Fig 3.12(a) which is a transport ship in the ocean. Examples of calculated images of the object for different temperature inversion $\Delta T$ in the ocean-type ITL are shown in Fig. 3.12(b) to 3.12(d). Atmospheric turbulence strength is $D/r_0 = 5$ for all the calculations. The ITL center elevation $h_{ITL}$ is above the object (transport ship) and as the temperature inversion increases, as can be seen in Fig. 3.12, the mirage virtual images gradually shows up, the separation between two virtual images also increases.
Fig. 3.13 Examples calculation of short-exposure mirage images at different ITL center elevations. (a)-(d) are the corresponding mirage images when the ITL center elevation changes from $h_{\text{ITL}} = 28\,\text{m}$, $30\,\text{m}$, $32\,\text{m}$, and $34\,\text{m}$. The parameters for ocean-type ITL are defined as $\Delta T = 4\,\text{K}$, $w_{\text{ITL}} = 2\,\text{m}$ and the atmospheric turbulence strength $D/r_0 = 5$.

The next step is to study how ITL center elevation will affect the mirage-image formation. The same object is used and ITL width and temperature inversion is fixed, as the ITL center position $h_{\text{ITL}}$ changes from $28\,\text{m}$ to $34\,\text{m}$ with a step of $2\,\text{m}$, the corresponding calculated images are shown in Fig. 3.13(a) to 3.13(d). As we can see, virtual images gradually disappear as the ITL moves away from the object (ship) and eventually no mirage phenomenon can be observed as the distance between object and ITL center is large enough. As the ITL is moved far away from the object, the impact from the ITL is too weak to generate mirage effect.
Fig. 3.14 Dependence of mirage images quality on atmospheric turbulence strength $D/r_0$. Short-exposure mirage images for (a) $D/r_0 = 5$, (b) $D/r_0 = 10$, (c) $D/r_0 = 20$, and (d) $D/r_0 = 30$. The ITL parameters are defined as $\Delta T = 4K$, $h_{ITL} = 30$ m, $w_{ITL} = 2$ m.

Atmospheric turbulence also plays an important role in image formation. As shown in Fig. 3.14, examples of computed images in ocean-type ITL for different atmospheric turbulence strength is exhibited. By comparing the short-exposure images shown in Fig. 3.14(a) - 3.14(d), one can conclude that stronger turbulence can resulted in larger image distortions and associate with lower image qualities.

### 3.2.3 Mirage image formation in presence of desert-type inverse temperature layer and atmospheric turbulence

Similar phenomenon of mirage image formation in desert-type ITL is studied in this section. In this case the imaging lens with identical parameters as described above is used
and is elevated at $h_{\text{img}} = 15\text{m}$ above the ground. For this case, the object is a square area with a watch tower stand in the desert as shown in Fig. 3.15(a). The height of the tower is $b_{\text{obj}} = 25\text{m}$ and located a distance $L = 20\text{km}$ away from the imaging lens.

Fig. 3.15 Example of short-exposure mirage image calculations in desert-type ITL through turbulence. The ITL model parameters are defined as $h_{\text{ITL}} = 5\text{m}$, $w_{\text{ITL}} = 0.5\text{m}$. (a) is the scene for imaging (object); (b), (c), and (d) are numerically calculated mirage images as the ITL parameter $\Delta T = -1.0\text{K}$, $-1.5\text{K}$, and $-2.0\text{K}$, respectively. The turbulence strength is defined as $D/r_{\theta} = 5$ for all the cases.

Examples of image-pane intensity distribution computed for the desert-type ITL using the BF technique shown in Fig. 3.15(b)-3.15(d) for different values of the temperature inversion. Correspondingly from Fig. 3.15 (b) to 3.15(d), $\Delta T = -1.0\text{K}$, $-1.5\text{K}$ and $-2.0\text{K}$. Based on these images, as the temperature inversion increase, the impacted area of this ITL increases and as a result the part of tower that displayed in the inverted virtual image increased and the separation between the ordinary and inverted
image decreases. For the above calculations, atmospheric turbulence strength is a constant and is defined as \( D/r_0 = 5 \).

Fig. 3.16 Examples calculation of short-exposure mirage images at different ITL center elevations. The ITL parameters \( \Delta T = -2.0 \text{K} \), \( w_{\text{ITL}} = 0.5 \text{m} \), (a)-(d) are the corresponding computed images when the ITL elevation center changes from \( h_{\text{ITL}} = 3.0 \text{m} \), \( 5.0 \text{m} \), \( 7.0 \text{m} \) and \( 9.0 \text{m} \). The atmospheric turbulence strength are defined as \( D/r_0 = 5 \) in all cases.

Then we will discuss how ITL location will affect the mirage image formation. With the same imaging geometry as above, the calculated images for different ITL center elevations are shown in Fig. 3.16. For this case, the strength of the ITL is fixed while the impacted area of the ITL to the object changes. As can be seen in Fig. 3.16, as the ITL center elevation increase, the portion of object that will have mirage effect changes and in this case, the joint location of the inverted image and the ordinary image move upwards.
Fig. 3.17 Examples calculation of short-exposure mirage images at turbulence strength. The ITL parameters $\Delta T = -2.0K$, $h_{ITL} = 5\text{m}$, $w_{ITL} = 0.5\text{m}$, (a)-(d) are the corresponding mirage images for the atmospheric turbulence strength $D/r_0 = 5$, $D/r_0 = 10$, $D/r_0 = 20$, and $D/r_0 = 30$, respectively.

Examples of the object images obtained in desert-type ITL with the increase of atmospheric turbulence strength are presented in Fig. 3.17. In this case, the ITL parameters are fixed and the turbulence strength increases from $D/r_0 = 5$, $D/r_0 = 10$, $D/r_0 = 20$, to $D/r_0 = 30$ for the calculated images in Fig. 3.17(a) to 3.17(d), respectively. As can be seen from these images, poor image quality resulted from the increased image distortions are observed as the turbulence strength increases.
3.3 Image quality characterization in highly anisoplanatic conditions

Image quality metrics (IQMs) that are commonly used in astronomical applications for performance evaluation of adaptive optics systems (e.g. sharpness functions) were developed under assumption of isoplanatic or quasi-isoplanatic imaging conditions [40-42]. Under these conditions, atmospheric turbulence causes the image blur. While for the case highly anisoplanatic imaging conditions typically happened in long-range image observation in the atmosphere, another type of turbulence-induced image distortions characterized by the presence of geometric image warping is commonly observed. Besides the atmospheric turbulence, the presence of atmospheric reactivity may result in additional anisotropic distortions to the image as already discussed in Section 3.1.

To characterize the quality of anisoplanatic images qualities, both image blur and geometrical distortions need to be accounted. Since the existed image quality metrics (IQMs) (e.g. sharpness functions) are not sensitive to geometrical image distortions, new image quality metrics that are sensitive to both the image blur and warping is needed.

In this section, we present two new IQMs which can potentially be used for characterization of incoherent anisoplanatic images obtained in the presence of a spatially localized atmospheric turbulence and ITL along the imaging path. The analysis is performed by means of numerical simulations based on the BF technique.

For the anisoplanatic image quality characterization, considering the scenario of incoherent imaging in presence of localized atmospheric refractive index structures as shown in Fig. 3.18. An imaging system (telescope) placed on the ground images an extended object (resolution chart) located a distance $L$ from it. In the optical path from the telescope to the target, there exist localized atmospheric turbulence layers and ITL
Here for simplification, single atmospheric turbulence layer is considered.

![Schematic of incoherent imaging in presence of localized refractive index structures.](image)

Fig. 3.18 Schematic of incoherent imaging in presence of localized refractive index structures.

In the numerical simulation, the imaging system is represented with a thin lens of diameter $D = 30$ cm and focal length $F = 120$ cm. The Air Force resolution chart of size $b_0 = 10$ m is applied as the object and is placed a distance $L=10$ km from the lens. Atmospheric turbulence is represented by a single thin phase screen located a distance of $\Delta z$ from pupil-plane which can change from 0 to $L$ along the path.

### 3.3.1 Anisoplantic image characterization in atmospheric turbulence

First let’s consider the impact from atmospheric turbulence solely. Example calculations of incoherent images of resolution chart for different turbulence layer locations along the path are shown in Fig. 3.19. Based on these images, as the distance between turbulence screen and lens increases, the image sharpness increases as expected. Besides image blurring, geometrical image distortion can also be observed from these images and the strength of this geometrical distortion also depends on the turbulence location. Since
both image blur and geometric distortion existed in each image, to characterize the image quality accurately, the developed image quality metrics need to be sensitive to both image blur and geometric distortions. The developed IQMs are tested by applying for the characterization of anisoplanatic images obtained in the presence of spatially localized turbulence layer along the image path.

![Fig. 3.19 Examples of calculated short-exposure images for different turbulence layer locations. From (a) to (d), images are calculated with the turbulence layer located at $\Delta z = 0.2L$, $0.4L$, $0.6L$, and $0.8L$, respectively.](image)

*Image sharpness metrics for anisoplanatic image quality characterization*

Consider first the existing image sharpness metrics. With the imaging system described above, images for different atmospheric turbulence strength and locations are calculated. For different turbulence location and strength, three commonly used image sharpness metrics [43] shown in Table 1 are calculated based on the calculated intensity distributions on image plane $I(r)$. Here $n$ is the number of uncorrelated random turbulence realizations, and $J(t_n)$ is the metric value in $n$th turbulence realization.
Table 1. Commonly used image sharpness metrics and definitions

<table>
<thead>
<tr>
<th>#</th>
<th>Metric description</th>
<th>Definition</th>
<th>Ensemble-average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Intensity-squared</td>
<td>$J_2 = \int l^2(r) , dr$</td>
<td>$\langle J_2(t_n) \rangle_n$</td>
</tr>
<tr>
<td>2</td>
<td>Gradient-based</td>
<td>$J_v = \int</td>
<td>\nabla I(r)</td>
</tr>
<tr>
<td>3</td>
<td>Squared gradient-based</td>
<td>$J_{v^2} = \int</td>
<td>\nabla I(r)</td>
</tr>
</tbody>
</table>

By averaging over $n = 500$ realizations, the dependency of all three image sharpness metrics on turbulence strength and locations are plotted in Fig. 3.20. Each curve plots the associated image sharpness metric for different turbulence layer location $\Delta z$ with the turbulence strength defined beside it. The solid curves from top to bottom represent the three corresponding metric (#1, #2 and #3) at the turbulence strength of $D/r_0 = 20$. Then the two curves on the top of the figure (two dashed curve) are metric #1 at different turbulence strengths of $D/r_0 = 10$ and $D/r_0 = 15$.

Fig. 3.20 The plot of image sharpness metrics for images calculated with turbulence layer located at different locations $\Delta z$ and for different turbulence strength. Three solid curves from top to bottom represent the metrics of #1, #2 and #3 for turbulence strength $D/r_0 = 20$, respectively. The top two dashed curves represent image sharpness metrics #1 when turbulence strength $D/r_0 = 10$, and $D/r_0 = 15$, respectively. X-axis is the location of turbulence layer along the propagation path and y-axis is the image sharpness metric values.

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As can be seen from Fig. 3.20, image sharpness metric #3 is the most sensitive to turbulence layer location changes which corresponding to a change of values from 0 to 1. The first three curves show that stronger turbulence corresponding to lower sharpness. All three image sharpness metrics have a monotonic dependency on turbulence layer locations: metrics value increase as turbulence layer moved closer to target plane. This simple relationship can only be applied for the characterization of image blur while is not enough to account for the presence of other geometrical distortions which also have impact on the image quality. In the following, additional metrics which are sensitive to both image blur and geometrical distortion will be presented.

*Inter-frame correlation based image quality metric*

Correlation between image frames can be applied for the determination of image deformation from each other and the strength of this deformation is reflected by the correlation coefficient. Since geometrical distortions is also one type of image deformation, the inter-frame correlation coefficient can be potentially served as one of the image quality metrics. Considering that image quality metric sensitivity can be increased with edge-images, the edge-image based inter-frame correlation coefficient is applied.

The computation of this metric includes the following steps and an example calculation of inter-frame correlation map from original frame of images is shown in Fig. 3.21:

1. Fix the turbulence layer location along the propagation path;
2. Calculate and save the corresponding image $I_{img}(r,t_n)$ for each turbulence realization and repeat for 500 realizations;
3. Apply Sobel edge detection to all saved images and calculate the edge image of them \( \nabla I_{\text{img}}(r,t_n) \);

4. Convolve edge images from Step 3 with Gaussian kernel \( G(r) = \exp(-r^2 / \sigma^2) \)
   where \( \sigma \) defines the Gaussian width to get \( M_{\nu}(r,t_n) = \nabla I_{\text{img}}(r,t_n) \ast G(r) \);

5. Calculate the correlation between neighboring frames of smoothed edge images from Step 4 and get 499 inter-frame correlation maps \( C_{\nu}(r,t_n,t_{n+1}) = \text{Corr}[M_{\nu}(r,t_n),M_{\nu}(r,t_{n+1})] \);

6. Calculate the auto-correlation of all smoothed edge images from Step 4 and get 500 auto-correlation maps \( C_{\nu}(r,t_n,t_n) \);

7. Find the maximum values from all correlation maps calculated in Step 5 and compute the average value \( \langle \max C_{\nu}(r,t_n,t_{n+1}) \rangle_n \) and apply the same calculation for auto-correlation maps and get \( \langle \max C_{\nu}(r,t_n,t_n) \rangle_n \);

8. The metric is defined as the ratio of these two values \( C = \frac{\langle \max C_{\nu}(r,t_n,t_{n+1}) \rangle_n}{\langle \max C_{\nu}(r,t_n,t_n) \rangle_n} \);
9. Change turbulence layer location along the path and repeat Steps 2 to 8 to calculate metric values at each position.

Analyzed in the above examples, the amount of image geometrical distortion resulted from turbulence phase screen depends on its location and strength. Also the increase of geometrical distortion can result in a decrease of image inter-frame correlation and lead to decline of metric values. The dependence of this metric on turbulence layer locations and strength is plotted in Fig. 3.22. From top to bottom, the three curves represent the metric values for different turbulence locations at the turbulence strength of \( \frac{D}{r_0} = 10 \), 15 and 20, respectively.

![Correlation coefficient graph](image)

Fig. 3.22 Three curves from top to bottom represent the dependence of inter-frame correlation based metric on turbulence layer locations for turbulence strength is \( \frac{D}{r_0} = 10 \), \( \frac{D}{r_0} = 15 \), and \( \frac{D}{r_0} = 20 \) respectively. The x-axis is the location of turbulence layer along the propagation path and y-axis is the metrics value.

From these plots, there is a local minimum of the metric values for each turbulence strength for the turbulence layer located around \( \Delta z = 0.2L \). This indicate that based on this
image quality metric, a minimum image quality which takes into account both image blur and geometric distortion is found near the pupil-plane. Since large-scale image distortion (image motion) resulted from atmospheric turbulence is expected to be more significant when the turbulence layer is located closer to the pupil-plane, it can be captured by this image quality metric.

*Intensity Scintillation Based Image Quality Metric*

Another image quality metric is developed by considering that geometrical distortion can result in image intensity fluctuation in each pixel. A good representation of the intensity fluctuation is the intensity scintillation index which is the obtained by normalizing the intensity variance by the averaged intensity at each pixel. Thus, the intensity scintillation index is sensitive to both geometrical distortion and image blur. By computing the image intensity scintillation over all pixels, a scintillation index map can be obtained and a statistical representation of this map can be served as another image quality metric. For this case, the mean scintillation index over the image map is adopted.

\[
S_t(r) = \frac{\left\langle I_{\text{img}}^2(r, t) \rightangle - \left(\left\langle I_{\text{img}}(r, t)\right\rangle\right)^2}{\left\langle I_{\text{img}}(r, t)\right\rangle^2}
\]

*Fig. 3.23* Example calculation of intensity scintillation index map from original frames of images for different turbulence realizations.
The computation of this metric includes the following steps and an example calculation of inter-frame correlation map from original frame of images is shown in Fig. 3.23:

1. Fix the turbulence layer location along the propagation path;
2. Calculate and save the corresponding image $I_{\text{img}}(r, t_n)$ for each turbulence realization and repeat for 500 realizations;
3. Convolve images from Step 2 with Gaussian kernel $G(r) = \exp(-r^2 / a^2)$ where $a$ defines the Gaussian width to get $M(r, t_n) = I_{\text{img}}(r, t_n) * G(r)$;
4. Calculate the intensity scintillation index for each pixel by using the 500 smoothed image from Step 3 and get the intensity scintillation index map $S_I(r) = \left( \frac{I^2_{\text{img}}(r, t_n) - \langle I_{\text{img}}(r, t_n) \rangle^2}{\langle I_{\text{img}}(r, t_n) \rangle^2} \right)$, only for those pixels with non-zero averaged intensity;
5. Change turbulence layer location and repeat Steps 2 to 4 and calculate all intensity scintillation index maps for each location;
6. Calculate the mean value of each intensity scintillation index map $\bar{S}_I(r)$

Four example calculations of intensity scintillation index map for turbulence layer located at $\Delta z = 0.2L$, $\Delta z = 0.4L$, $\Delta z = 0.6L$ and $\Delta z = 0.8L$ are shown in Fig. 3.24. As can be seen from these images, the number of pixels with scintillation index decreases while the peak value increased as the turbulence layer moved to the target plane.
Fig. 3.24 Example of image scintillation index map for turbulence layer located at (a) $\Delta z = 0.2L$; (b) $\Delta z = 0.4L$, (c) $\Delta z = 0.6L$ and (d) $\Delta z = 0.8L$.

The metric values for different turbulence layer locations at different turbulence strengths are plotted in Fig. 3.25. The metrics values increase first then decrease as the turbulence layer moving closer to the target plane. A local maximum metric value is observed around $\Delta z = 0.6L$ (near target plane) for all three curves which indicate that this metric is sensitivity to small-scale geometric image distortion (image wrapping).
Fig. 3.25 The dependency of mean intensity scintillation index of images on turbulence strength and locations. The three curves are correspondingly for different turbulence strength of $D/r_0 = 10$, 15 and 20.

3.3.2 Anisoplantic image characterization in presence of both atmospheric turbulence and ITL

With the above discussed newly developed image quality metrics, here we consider the impact of atmospheric refractivity induced by ITL on the performance of these metrics. The analysis is carried out by comparing the image quality metrics for different turbulence layer locations with and without ITL-induced refractivity. The refractivity effect is described by the same desert-type ITL model defined in above sections with the parameters defined as $h_{ITL} = 8.0 \text{ m}$ and $w_{ITL} = 1.0 \text{ m}$.

Examples of the image-plane intensity distribution computed in presence of atmospheric turbulence layer with and without desert-type ITL are shown in Fig. 3.26. The turbulence layer is located on pupil-plane $\Delta z = 0$ with a turbulence strength of $D/r_0 = 10$. As already discussed in Section 3.1, the presence of ITL can lead to a modulation of the
image size and spatial frequency as can be seen in Fig. 3.26(b) and 3.26(c) with the
temperature inversion $\Delta T = -0.5K$ and $\Delta T = -1.0K$, respectively. Due to the presence of
this desert-type ITL, the image size shrink and can bring change to the performance of
IQMs.

![Fig. 3.26 Numerical simulated images of the resolution chart for pupil-plane turbulence with the strength of $D/r_0 = 10$ under the condition of (a) no ITL; (b) desert-type ITL with $\Delta T = -0.5K$; (c) desert-type ITL with $\Delta T = -1.0K$.]

The qualitative analysis of the change of IQM values depends on the presence of
ITL is shown in Fig. 3.17. In this plot, we compared the IQMs under the atmosphere
conditions with only atmospheric turbulence ($D/r_0 = 20$) and with both turbulence and
desert-type ITL of different temperature inversion. As can be seen in Fig. 3.27(a), the inter-
frame correlation coefficient increase due to the presence of desert-type ITL and for larger
temperature inversion, the metric values increase more. Since the presence of desert-type
ITL lead to a shrink of the image size which will make the image deformation less visible
and can lead to an increase of the inter-frame correlation. While even with the impact from
ITL-induced refractivity, this metric is still sensitive to both image blur and geometric
distortions since they still have the local minimum around $\Delta z = 0.2L$.

For the mean intensity scintillation index metric shown in Fig. 3.27(b), the presence
of the desert-type ITL resulted in a decrease of the metric values and stronger temperature
inversion can lead to lower intensity scintillation index. For this case, the presence of ITL can result in a non-sensitivity to the geometric distortion. As seen from the two black curves in fig. 3.27(b) which represent the mean scintillation index metric for the presence of ITLs, the local maximum smoothed out around \( \Delta z = 0.6L \) which indicate that the presence of refractivity can lead to sensitivity decrease of this IQM to geometrical distortion.

![Fig. 3.27 Numerical simulated images of the resolution chart for pupil-plane turbulence with the strength of \( D/r_0 = 10 \) under the condition of (a) no ITL; (b) desert-type ITL with \( \Delta T = -0.5 \text{ K} \); (c) desert-type ITL with \( \Delta T = -1.0 \text{ K} \).](image-url)
CHAPTER 4
CONCLUSION AND FUTURE WORK

4.1 Conclusion

In this dissertation, we have further developed the brightness function technique to be able to account for both atmospheric turbulence and refractivity. In this BF approach, the atmospheric turbulence effect is accounted for with thin turbulence screens. The refractivity effect is accounted for without the approximation of thin refractivity screens, direct computation of refractive index distribution is applied for better numerical simulation accuracy.

The BF technique is also applied for the numerical analysis of incoherent imaging in presence of atmospheric turbulence and refractivity. Several research projects have been conducted and the numerical simulation results have been presented. Specifically, in Section 3.1, we have analyzed impacts of atmospheric refractivity and turbulence on the incoherent monochromatic imaging system MTF using numerical simulations based on the brightness function technique. Atmospheric refractivity was represented by highly spatially localized refractive index structures that result from inverse temperature layers (ITLs) of both desert and ocean type. Atmospheric turbulence effects were modelled using a conventional wave-optics approach based on the split-step operator technique with a set of statistically independent Kolmogorov phase screens located along.
the imaging path. The split-step operator technique was generalized to describe the impact of both turbulence and refractivity in numerical simulations of incoherent imaging systems. An extensive set of numerical simulations was performed for selected imaging system parameters, propagation geometries, and ITL and turbulence characteristics. The analysis shows that ITL-induced refractivity that is located in the vicinity of an air volume that is essential for image formation creates highly spatially anisotropic low-order spatially distributed phase aberrations, which may significantly impact imaging system performance. The numerical analysis demonstrates that these spatially distributed phase aberrations (spatially distributed lensing effect) may result in a significant highly anisotropic change in the image size, leading to either image elongation or shrinkage in the direction orthogonal to the ITL.

For the case of sine-type test objects used for analysis of the imaging system modulation transfer function (MTF), these image distortions result in a spatial frequency shift in the observed images of horizontally oriented sine-patterns. In turn, this frequency shift also affects visibility of the sine-pattern in the direction orthogonal to the ITF, and hence the imaging system MTF becomes highly anisotropic.

Numerical simulations show that the joint effects of atmospheric turbulence and ITF-induced refractivity cause the MTF to be a non-linear function dependent on imaging system characteristics, propagation path geometry, and atmospheric turbulence and ITL-induced refractivity parameters.

The results achieved demonstrate challenges for conventional MTF-based performance evaluation of imaging systems that operate in the presence of ITL-type refractive structures located in the vicinity of an air volume that is essential for image
formation. At the same time, the described effects of ITL-induced image distortions open new opportunities for image-based atmospheric refractivity sensing.

In Section 3.2, the application of BF for the study of optical mirage formation in turbulence was investigated. The formation of mirage images is strongly dependent on the characteristics of the ITL, specifically, the temperature inversion and relative position of the ITL to the object. For both desert- and ocean-type mirages, numerical calculations have been performed and demonstrate that the mirage phenomenon shows up as the temperature inversion increases to a certain value depending on the imaging geometry. Also the relative position of the ITL from the object and determine the portion of images that will have mirage phenomena and it is shown that as the ITL becomes far away from the object, no mirage phenomena can be observed. Additionally, the impact of atmospheric turbulence strength on mirage image qualities have been studied.

In Section 3.3, the dependence of image quality on atmospheric turbulence strength and turbulence-screen locations was explored. It is shown that for anisoplanatic images, the presence of atmospheric turbulence can introduce both image blur and geometric distortions to the image and this is modulated by both the turbulence strength and locations. To characterize these anisoplanatic image, we have developed new IQMs which are sensitive to both image blur and geometrical distortion. In contrary to existing image sharpness metrics which have monotonic dependence on turbulence layer locations, the developed IQMs exhibit local minima or maxima as turbulence layer location changes depends on the sensitivity of the metric to large- or scale geometrical distortions. Also, we have studied the impact of ITL-induced refractivity on metric performance which shows that the presence of ITL can decrease the sensitivity of both IQMs to geometrical
distortions. One potential application of these IQMs is for real-time image restoration especially when strong geometrical distortion exists. Another application could be atmospheric turbulence sensing. With the combination of both IQMs, it is possible to calculate the location and strength of the turbulence layer.

4.2 Future Work

In this section, the continuation of some parts of this dissertation is described as future work. The work of the imaging system MTF analysis can be extended in another direction. Since the presence of refractivity can lead to a modification to the image size and spatial frequency and eventually have impact on image visibilities, it is possible to utilize these results and apply them for image-based refractivity sensing.

The work of IQM development for anisoplantic image characterization can be further extended. We can apply these metrics for image restoration and compare the results with other existed IQMs. Furthermore algorithms are needed to finalize the work of image quality metrics based atmospheric turbulence strength calculation and turbulence layer localization. A computational algorithm which can take different image quality metrics values as input and calculate turbulence strength and turbulence layer locations as outputs is still needed.
REFERENCES


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