INTENSE, ULTRASHORT PULSE, VECTOR WAVE PROPAGATION IN OPTICAL FIBERS

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INTENSE, ULTRASHORT PULSE, VECTOR WAVE

PROPAGATION IN OPTICAL FIBERS

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ABSTRACT

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The planned research is initially motivated by experiments on twisted fiber to examine the polarization of the output pulses. The initial polarization launched into the fiber evolves to a new final state that asymptotically moves to one of two opposite circular polarizations. The initial research was to program the vector wave equations of one and coupled solitons in a twisted fiber including the additional nonlinear terms stimulated Raman scattering and self-steepening. The high-twist fiber eliminates small linear birefringence at the expense of introducing circular birefringence manifested in the group velocities. The vector equations are naturally written in the circular polarization basis. To verify the numerical results, I made a sojourn to INAOE in Puebla, Mexico to run an experiment and compare the results. The numerical compare extremely well with the experimental results. For one soliton, the output polarization of the twisted fiber follows the input with high fluctuations. However, for the coupled soliton input, when the
input polarization is close to linear, we observe a very abrupt polarization switch from nearly negative circular, \(-45^\circ\) to nearly positive circular, \(45^\circ\) over a very narrow range of the input ellipticities.

The literature is full of simulations of super-continuum generation using scalar wave equations, but we have not seen any report on the polarization of the output Supercontinuum light. Again, motivated by the experiments on polarization evolution in optical fibers we wanted to study the vector wave equations at higher incident powers to discover what the polarization state of the output waves are in an extreme nonlinear situation.
ACKNOWLEDGEMENTS

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<th>Description</th>
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<tbody>
<tr>
<td>CW</td>
<td>Continuous Wave</td>
</tr>
<tr>
<td>SVEA</td>
<td>Slowly Varying Envelope Approximation</td>
</tr>
<tr>
<td>NLSE</td>
<td>Nonlinear Schrodinger Equation</td>
</tr>
<tr>
<td>GVD</td>
<td>Group Velocity Dispersion</td>
</tr>
<tr>
<td>CNLSE</td>
<td>Coupled Nonlinear Schrodinger Equations</td>
</tr>
<tr>
<td>PMF</td>
<td>Polarization Maintaining Fiber</td>
</tr>
<tr>
<td>SPM</td>
<td>Self-phase Modulation</td>
</tr>
<tr>
<td>XPM</td>
<td>Cross-phase Modulation</td>
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<tr>
<td>TOD</td>
<td>Third Order Dispersion</td>
</tr>
<tr>
<td>FWM</td>
<td>Four-wave Mixing</td>
</tr>
<tr>
<td>SRS</td>
<td>Stimulated Raman Scattering</td>
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<tr>
<td>CBF</td>
<td>Circular Birefringent Fiber</td>
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<tr>
<td>QWP</td>
<td>Quarter-wave Plate</td>
</tr>
<tr>
<td>EDF</td>
<td>Erbium-Doped Fiber</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>EDFA</td>
<td>Erbium-Doped Fiber Amplifier</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>FM</td>
<td>Faraday Mirror</td>
</tr>
<tr>
<td>OSA</td>
<td>Optical Spectrum Analyzer</td>
</tr>
<tr>
<td>SMF</td>
<td>Single Mode Fiber</td>
</tr>
<tr>
<td>PC</td>
<td>Polarization Controller</td>
</tr>
<tr>
<td>PBS</td>
<td>Polarization Beam Splitter</td>
</tr>
<tr>
<td>SC</td>
<td>Supercontinuum</td>
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<tr>
<td>PCF</td>
<td>Photonic Crystal Fiber</td>
</tr>
<tr>
<td>WDM</td>
<td>Wavelength Division Multiplexing</td>
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<tr>
<td>GNLSE</td>
<td>Generalized Nonlinear Schrodinger Equation</td>
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<td>SSFM</td>
<td>Split-Step Fourier Method</td>
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CHAPTER 1

INTRODUCTION

1.1 Motivation

Linear birefringence of standard optical fibers is small and random, which leads to fluctuations in the polarization and additional dispersion for long-haul fiber communications systems. To understand the effect of this phenomenon the propagation of solitons is treated as a vector beam instead of a scalar one. One way to eliminate the random linear birefringence is by twisting the fiber to average the effects of linear birefringence. As a consequence the fiber twist produces high circular birefringence, which changes the polarization of a soliton to circular polarization [1]. In [2] and [3], the polarization of a propagated pulse in a twisted fiber was studied experimentally with an experimental setup shown in Figure 1.1. The seed source is a distributed feedback (DFB) laser that uses pulsed operation. The pulses are amplified in an erbium-doped fiber amplifier (EDFA) and the polarization is fixed by a fiber polarizer and a fiber quarter-wave retarder (QWR). The fiber under test is 500 m of twisted fiber. After the twisted fiber the polarization is changed from circular to linear polarization and separated by a fiber polarization beam splitter (PBS). Both circular polarizations are measured by the same detector by putting a delay line in one arm after the PBS.
When a circularly polarized beam is passed through a twisted fiber, the output has elliptical polarization, which can be decomposed into orthogonal circular polarized components. However, when untwisted fiber was used, the output polarization has random fluctuations. Figure 1.2 and Figure 1.3 show the input versus output polarization ellipticity at a wavelength of 1580 nm for twisted and untwisted fibers, respectively. Dashed line in Figure 1.2 represents input ellipticity and the output ellipticity is indicated by a solid line passing through the data points. The data in the figures clearly demonstrate an unusual nonlinear optical response in a twisted fiber; using input pulses with sufficient energy output polarization asymptotic evolves to a plateau value state depending on the input polarization. Twisted fibers are a type of circular birefringence fiber (CBF). Their linear polarization isotropy makes them useful for nonlinear optical applications [4], unlike standard optical fibers where the nonlinear polarization evolution of a pulse can also be affected by residual birefringence. When the input polarization is approximately circularly polarized, i.e. the polarization controller QWR1 has a principal axis is set at 45° or 135° from the linear polarization axis, the output is also circularly polarized. However, the output polarization tends to the same value over a wide range of QWR angles corresponding to elliptically polarized input pulses.
Our research is designed primarily to understand the experimental results presented in the two figures, **Figure 1.2** and **Figure 1.3**. Numerical simulations were performed using a set of dynamical equations for the vector fields; the effect of fiber twist was also
incorporated into the equations. Our simulations indicate a higher degree of polarization for the output pulse; it is close to circular polarization and there is an abrupt change between the elliptical polarizations as they angle crosses the linear polarization axis, i.e. between a tendency for negative and positive helicity polarizations.

Initially, I wanted to model and verify the results presented in Figure 1.2 above. However, we did not get good agreement between the theoretical and experimental results, and Dr. Haus thought a visit to the experimental group would clear up the issues. It turns out that a long collaboration existed with the group was the Instituto Nacional de Astrofisica, Optica y Electronica, INAOE in Tonantzintla, Mexico and so began a two month sojourn to a new land. There I had the opportunity to meet the group of Drs. Baldemar Ibarra Escamilla and Evgeni Kuzin and to participate in experiments. Since we are already working with fairly high input fields, we also wanted to explore Supercontinuum generation starting from the same set of dynamical equations, but with higher energy per pulse to further broaden the output spectrum. In our studies we measure the relationship between the input and output polarization and the spectrum for energetic pulses propagating through a twisted fiber. The pulse energy is large enough so that a large nonlinear optical response is observed during propagation in the fiber.

### 1.2 Nonlinear Schrödinger Equation in a Fiber

The pulse shape as it propagates through an optical fiber can be deformed by linear dispersion and its spectrum and shape are affected by nonlinear characteristics of the fiber [5]. The group velocity dispersion in a fiber occurs because the pulse’s group velocity is a wavelength dependent, and the self-phase modulation nonlinearity produces an intensity dependent phase velocity. If the fiber nonlinearity and group velocity
dispersion were designed in a way that they cancel each other, a soliton pulse can be formed. Solitons have the property of propagating a very long distance with no distortion. The propagation of optical waves can be governed and described by Maxwell’s equations, which defined as

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}, \quad (1.1) \]

\[ \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}_f, \quad (1.2) \]

\[ \nabla \cdot \mathbf{D} = \rho_f, \quad \tag{1.3} \]

\[ \nabla \cdot \mathbf{B} = 0. \quad (1.4) \]

Where \( \mathbf{E} \) and \( \mathbf{H} \) are the electric and magnetic fields, respectively; \( \mu_0 \) is the vacuum permeability, and \( \mathbf{D} \) and \( \mathbf{B} \) are the electric and magnetic flux density. Since the fiber does not have free charges, we assume that the current density \( \mathbf{J}_f \) and charge density \( \rho_f \) are zero. Electric field \( \mathbf{E} \) affects the electric flux density \( \mathbf{D} \) which can be defined as

\[ \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \quad (1.5) \]

where the induced polarization \( \mathbf{P} \) is

\[ \mathbf{P} = \varepsilon_0 \left[ \chi^{(1)} \mathbf{E} + \chi^{(2)} : \mathbf{E} \mathbf{E} + \chi^{(3)} : \mathbf{E} \mathbf{E} \mathbf{E} + \cdots \right]. \quad (1.6) \]

The first term dominates over other terms and contributes to the linear polarization, \( \mathbf{P}_L \), and the dielectric constant. The second term with tensor \( \chi^{(2)} \) vanishes in a medium like glass since it is a centrosymmetric medium. Third term is nonzero and is the leading contribution to nonlinear polarization, \( \mathbf{P}_{NL} \), in an optical fiber. Maxwell’s equations can be manipulated to derive the vector wave equation in the form (derived in Appendix A)

\[ \nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}_L}{\partial t^2} + \mu_0 \frac{\partial^2 \mathbf{P}_{NL}}{\partial t^2}. \quad (1.7) \]

A monochromatic field is expressed in complex form as
\[ E(r, t) = \hat{e} \tilde{E}(r, \omega) e^{-i\omega t}, \] (1.8)

where \(\omega\) is the signal frequency and \(\hat{e}\) is the field polarization direction. To study the evolution of the pulse in a fiber, it is easier to transfer the field from the time domain to the frequency domain. So, we deal with \(\tilde{E}\) as a Fourier transform of \(E\), defined as

\[ \tilde{E}(r, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} E(r, t) e^{i\omega t} \, dt. \] (1.9)

The scalar wave equation (1.7) can be written as

\[ \nabla^2 \tilde{E} + \varepsilon (x, y, \omega) k_0^2 \tilde{E} = -\omega^2 \mu_0 \tilde{P}_{NL}, \] (1.10)

where \(\varepsilon (x, y, \omega) = 1 + \chi^{(1)}(x, y, \omega)\) is the dielectric function, \(k_0^2 = \omega^2 / c^2\) is the free space wave number, and \(\tilde{P}_{NL}\) is the Fourier transform of the nonlinear polarization. Since here we are not dealing with a continuous wave, CW, but a signal with an envelope, pulse form, we can write the electric field as an angular frequency-dependent function is expanded around the central angular frequency, \(\omega_0\)

\[ \tilde{E}(r, \omega - \omega_0) = F(x, y) \tilde{A}(z, \omega - \omega_0) e^{i\beta_0 z}, \] (1.11)

where \(F(x, y)\) is the transverse field profile, which describes the shape of the fiber mode at angular frequency \(\omega_0\). \(\tilde{A}\) is the envelope of the signal, and the exponential factor governs the spatial phase shift with \(\beta_0\) the propagation constant, whose value will be determined below. The scalar wave equation (1.10) can be solved by the assumed solution, equation (1.11) as

\[ \tilde{A} \frac{\partial^2 F}{\partial x^2} e^{i\beta_0 z} + \tilde{A} \frac{\partial^2 F}{\partial y^2} e^{i\beta_0 z} + F \left[ \frac{\partial^2 \tilde{A}}{\partial z^2} e^{i\beta_0 z} + 2i\beta_0 \frac{\partial \tilde{A}}{\partial z} e^{i\beta_0 z} - \beta_0^2 \tilde{A} e^{i\beta_0 z} \right] + \varepsilon (x, y, \omega) k_0^2 \tilde{A} F e^{i\beta_0 z} = -\omega^2 \mu_0 \tilde{P}_{NL}. \] (1.12)

, the fiber mode function, \(F\), at the central frequency \(\omega_0\) satisfies the wave equation in the following form \((k_0^2 = \omega_0^2 / c^2\) below)
\[ \nabla^2 F + \left[ \varepsilon(x, y, \omega_0)k_0^2 - \beta_0^2 \right]F = 0, \quad (1.13) \]

where a constant envelope \( \tilde{A} \) is an amplitude for the fiber mode with the transverse spatial function \( F \). The solution of equation (1.13) for optical fiber geometry can be described as propagation modes in the optical fiber and the dispersive eigenvalues are the propagation constant \( \beta_0 \). We define the frequency dependent propagation constant \( \beta(\omega)^2 = \varepsilon(\omega)k_0^2 \), which can be expanded about the central frequency. Solving the equation (1.12) for the envelope \( \tilde{A} \) leads to

\[
2i\beta_0 \frac{\partial A}{\partial z} + F \frac{\partial^2 A}{\partial z^2} + \left[ \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \beta(\omega)^2 F \right] \tilde{A} - \beta_0^2 \tilde{A} F = -\omega^2 \mu_0 \tilde{P}_{NL}. \quad (1.14)
\]

Assuming the slowly varying envelope approximation (SVEA), the \( \frac{\partial^2 A}{\partial z^2} \) can be neglected. By the fiber mode function, equation (1.13), \( \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \varepsilon(\omega_0)k_0^2 F = \beta_0^2 F \), the envelope equation (1.14) can be written as

\[
F \left\{ 2i\beta_0 \frac{\partial A}{\partial z} + \left[ \beta(\omega)^2 - \beta_0^2 \right] \tilde{A} \right\} = -\omega^2 \mu_0 \tilde{P}_{NL}. \quad (1.15)
\]

Equation (1.15) leads to a wave equation in a form of (derivation in Appendix B)

\[
\frac{\partial A}{\partial z} + \frac{1}{\beta_1} \frac{\partial A}{\partial t} + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A = i\gamma |A|^2 A. \quad (1.16)
\]

This is a simple, scalar form of a propagating wave in an optical fiber which called nonlinear Schrodinger equation (NLSE), where \( A \) is the slowly varying field amplitude. The pulse envelope propagates in a group velocity \( v_g \) where \( v_g = \frac{1}{\beta_1} \), \( \beta_2 \) describes the pulse envelope spreading during propagation and is called the group velocity dispersion GVD and \( \alpha \) represents the fiber loss. The right-hand side is responsible for the fiber nonlinearity with a coefficient \( \gamma \). Depending on the input pulse width \( T_0 \) and peak
power $P_0$, dispersion or nonlinearity may dominate. The importance of each contribution can be determined by the natural length scales of each term. The dispersion and nonlinear lengths are defined as:

$$L_D = \frac{r_0^2}{|\beta_2'|}, \quad L_{NL} = \frac{1}{yP_0},$$

(1.17)

respectively. Specifically, the dispersion length, $L_D$, is the distance over which an initial Gaussian pulse’s width would broaden by $\sqrt{2}$ and the nonlinear length, $L_{NL}$, is the distance over which the phase of the wave changes by 1 radian due to the nonlinearity.

If $L_D \sim L$, and $L_{NL} \gg L$, where $L$ is the fiber length, then fiber dispersion will dominate.

On the other hand, if $L_{NL} \sim L$, and $L_D \gg L$, the nonlinearity will dominate the wave evolution. The interesting case where the two lengths are in the same order of the fiber length that leads to the formation of fundamental and higher order solitons. A fundamental soliton propagates without changing shape and higher-order solitons periodically undulate.

### 1.3 Vector Wave, Coupled Mode Equations

A simple derived form of the coupled nonlinear Schrödinger equations (CNLSE) assumes that the polarization state of the incident light is changing while the pulse travels through the optical fiber [2][6]. In practice, fibers are randomly birefringent with a degree of $B_m = |n_x - n_y|$, where $n_x$ and $n_y$ are the refractive indices of the orthogonal polarized fiber modes. In case of manufacturing a polarization maintaining fibers (PMF), inner stress or core shaping can be designed in a way that produces a constant birefringence over the fiber in order to overcome random birefringence. Another way to solve the problem of random birefringence is to twist the fiber and thus average the linear
birefringence during propagation. However, twisting the fiber produces a circular birefringence [7], which introduces another dynamical effect. Circular birefringence decreases the sensitivity of the fiber to the inner and outer conditions. Evolution of the two linear polarization components propagating in a fiber satisfy the following set of equations

\[
\frac{\partial A_x}{\partial z} + \beta_{1x} \frac{\partial A_x}{\partial t} + i\frac{\beta_2}{2} \frac{\partial^2 A_x}{\partial t^2} + \frac{\alpha}{2} A_x = i\gamma \left(|A_x|^2 + \frac{2}{3} |A_y|^2\right) A_x + i\frac{\gamma}{3} A_x^* A_y^2 e^{-2i\Delta \beta z},
\]

(1.18)

\[
\frac{\partial A_y}{\partial z} + \beta_{1y} \frac{\partial A_y}{\partial t} + i\frac{\beta_2}{2} \frac{\partial^2 A_y}{\partial t^2} + \frac{\alpha}{2} A_y = i\gamma \left(|A_y|^2 + \frac{2}{3} |A_x|^2\right) A_y + i\frac{\gamma}{3} A_y^* A_x^2 e^{2i\Delta \beta z}.
\]

(1.19)

where $\beta_{1x}$ and $\beta_{1y}$ are the inverse group velocities for the two polarization components in $ps/km$. $\beta_2$ is the second order dispersion in $ps^2/km$ which can be negative or positive. If the wavelength $\lambda$ is in the normal dispersion regime where it is smaller than the zero-dispersion wavelength $\lambda_D$, $\beta_2$ is positive. However, if the wavelength $\lambda$ is in the anomalous dispersion regime, i.e. longer than the zero dispersion wavelength $\lambda_D$, $\beta_2$ is negative and an optical soliton can be formed [6]. The optical soliton is a stable nonlinear envelope wave that is a balance between dispersion and nonlinearity. In the case where the nonlinearity is small, $\beta_2$ dominates to broaden the pulse while propagating through the fiber. $\alpha$ is the fiber loss with units $1/km$. $\gamma$ is the fiber nonlinearity in $(W.km)^{-1}$.

Equations (1.18) and (1.19) can be written in a circularly polarized wave basis by using

\[
A_+ = \frac{A_x e^{i\Delta \beta z/2} + i A_y e^{-i\Delta \beta z/2}}{\sqrt{2}},
\]

(1.20)

\[
A_- = \frac{A_x e^{i\Delta \beta z/2} - i A_y e^{-i\Delta \beta z/2}}{\sqrt{2}},
\]

(1.21)

where $A_+$ and $A_-$ are the right and left circularly polarized components, respectively.

Substituting equations (1.20) and (1.21) in equations (1.18) and (1.19) and assuming the
case of a very low birefringent fiber, $\beta_{1x} \approx \beta_{1y} = \beta_1$ gives (derivation in Appendix C without the twist phase term)

$$
\frac{\partial A_+}{\partial z} + \frac{i\Delta \beta}{2} A_- + \beta_1 \frac{\partial A_+}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_+}{\partial t^2} + \frac{\alpha}{2} A_+ = \frac{i^2}{3} \gamma (|A_+|^2 + 2|A_-|^2) A_+ ,
$$

(1.22)

$$
\frac{\partial A_-}{\partial z} - \frac{i\Delta \beta}{2} A_+ - \beta_1 \frac{\partial A_-}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_-}{\partial t^2} + \frac{\alpha}{2} A_- = \frac{i^2}{3} \gamma (|A_-|^2 + 2|A_+|^2) A_- .
$$

(1.23)

$\Delta \beta$ describes the effect of the linear birefringence of the fiber and defined as:

$$
\Delta \beta = \frac{2\pi(n_x-n_y)}{\lambda}.
$$

(1.24)

$\beta_1$ is the inverse of the velocity of the optical pulse which called the group velocity $v_g$.

First and second terms on the right side are the self-phase modulation SPM and cross-phase modulation XPM, respectively.

In case of ultrashort pulses, the additional nonlinear terms should be included that significantly affect the pulse’s evolution. A brief description of the effects of each of the high order parameters will be described in the next sections.

### 1.4 Additional Linear and Nonlinear Effects

#### 1.4.1 Third Order Dispersion

When the wavelength of the pulse is at or very close to the zero dispersion wavelength, $\lambda \approx \lambda_D$ where the waveguide and material dispersions cancel each other out, the second order dispersion $\beta_2 \approx 0$. At this case, third order dispersion $\beta_3$ dominates and has a sufficient effect to the GVD[6], which results in pulse broadening. Third order dispersion, TOD, leads to a soliton break up, which is more severe for higher order solitons. Higher order solitons mainly consist of multiple fundamental solitons traveling
with similar group velocities in the fiber. The presence of TOD changes the relative velocities, which leads to a pulse break-up of the pulse [8].

1.4.2 Stimulated Raman Scattering

Elastic light scattering defines the situation where no energy transferred from one field to another during the propagation through the medium [6]. However, for inelastic scattering phenomena that occur due to nonlinear affects controlled by the $\chi^{(3)}$ response in equation (1.6), such as, four-wave mixing (FWM) and stimulated Raman scattering (SRS), a part of the optical wave energy will be transferred to different frequencies. More details on the effects of (SRS) on the evolution of an optical pulse are in supercontinuum generation section. FWM and SRS have a dominate effect over other nonlinear terms. When a continuous frequency shift for a soliton pulse was observed in [9], it becomes an important to study the effects of higher order nonlinearities such as FWM and SRS since they promote large shifts. Frequency broadening of the pulse spectrum happens because the SRS does a high to low-frequency energy transfer, producing what is called Stokes waves. In the SRS, the energy transfer occurs when the pump power exceeds the threshold value which defined by [10]

$$P_p^{th} = 16 \frac{A\alpha}{g_R}$$

(1.25)

where $A$ in the mode area in $cm^2$, the absorption coefficient $\alpha$ has units $cm^{-1}$, and the fiber peak gain $g_R$ in $cm/W$. The energy transfer is linear with pulse propagation, this leads to a linear red-frequency shift. One issue of transferring energy from one mode to another is that it limits the efficiency of multichannel systems. SRS has a similar effect of TOD on higher order solitons. SRS affects a higher order soliton and produces a number
of fundamental solitons by changing their relative velocities [11]. This leads to what is called soliton fission or soliton break up.

### 1.4.3 Self-Steepening

A self-steepening is a process where the pulse shape changes while traveling through a fiber. This occurs when the refractive index of the fiber is an intensity dependent [12]. Having an intensity-dependent refractive index leads to pulse wings to be propagated faster than the peak when the dispersion is not present. This process creates an optical shock. Even though the presence of dispersion will reduce the steepening of the wings and makes them smooth [6], the pulse center will travel slower and be delayed and shifted to the trailing wing. Delay can be approximated as \( \tau = s z \), where \( s \) is the steepening coefficient and \( z \) is the propagation distance. As for other higher nonlinear terms, self-steepening breaks up high-order solitons into \( N \) fundamental solitons causing soliton fission.

After giving a brief explanation of high-order nonlinear terms, we should include their effects in propagation equations (1.22) and (1.23) without the fiber twist phase included

\[
\frac{\partial A_+}{\partial z} + \frac{i \Delta \beta}{2} A_- + \beta_1 \frac{\partial A_+}{\partial t} + \frac{i \beta_2}{2} \frac{\partial^2 A_+}{\partial t^2} - \frac{\beta_3}{6} \frac{\partial^3 A_+}{\partial t^3} + \frac{\alpha}{2} A_+ = R_+ + is \frac{\partial}{\partial t}(R_+), \tag{1.26}
\]

\[
\frac{\partial A_-}{\partial z} - \frac{i \Delta \beta}{2} A_+ - \beta_1 \frac{\partial A_-}{\partial t} + \frac{i \beta_2}{2} \frac{\partial^2 A_-}{\partial t^2} - \frac{\beta_3}{6} \frac{\partial^3 A_-}{\partial t^3} + \frac{\alpha}{2} A_- = R_- + is \frac{\partial}{\partial t}(R_-). \tag{1.27}
\]

\( \beta_3 \) governs the third order dispersion effect, \( s = 1/\omega_0 \) is the self-steepening coefficient and the nonlinear terms \( R_-, R_+ \) are
\[ R_{\pm} = R_{\pm} + \frac{2i}{3} \gamma \left( \left( |A_{\pm}|^2 + 2|A_{\mp}|^2 \right) A_{\pm} \right), \]  

(1.28)

Where \( R_{\pm} \) governs the SRS effect with Raman characteristic time of \( \tau_R \). In many research reports and papers in literature, total power was used to calculate the SRS effects [13][14][15]. In our calculations, we take into our consideration the difference between the orthogonal and parallel Raman gain coefficients with the ratio between them \( \alpha_R = 0.3 \) [16]. In the linear basis, the SRS effects on the two components of polarizations are described as:

\[ R_{x,y} = -\frac{i\gamma R}{2} \left\{ \partial t \left( |A_{x,y}|^2 + \alpha_R |A_{y,x}|^2 \right) A_{x,y} \right\}. \]  

(1.29)

Converting to circular basis with the same process we did for Eqs. (1.21) and (1.22) gives:

\[ R_{\pm} = -\frac{i\gamma R}{2} \left\{ \frac{1+\alpha_R}{2} \partial t(|A_+|^2 + |A_-|^2)A_{\pm} + (1 - \alpha_R )\partial t[Re(A_+A_-^*)]A_{\mp} \right\}. \]  

(1.30)

In Section 2.4, we show the difference between using the total power for SRS effect (\( \alpha_R = 1 \)) and considering the ratio between orthogonal and parallel Raman gain coefficients (\( \alpha_R = 0.3 \)).

### 1.5 Conclusion

To study the properties and characteristics of the polarization, we have to deal with a wave as a vector instead of a scalar. In this chapter, we derive the nonlinear Schrödinger equation starting with the Maxwell’s equations. In Section 1.3, the vector wave is shown and how it gets transferred from linear polarized form to a circular polarized form as presented in Eqs. (1.22) and (1.23). Some of higher order effects are presented in Sec. 1.4, as third order dispersion, stimulated Raman scattering SRS, and self-steepening. Stimulated Raman scattering, is a nonlinear effect where the propagating pulse produces a
high to low-frequency energy transfer, called the Stokes wave, which results in a red-frequency shift in the propagated wave. The third effect is called self-steepening, which happen when the refractive index is an intensity dependent parameter. In this case the wings of the pulse, which have lower intensity will travel faster the center that can cause asymmetric reshaping of the wave to form an optical shock.
CHAPTER 2

PROPAGATION IN TWISTED FIBERS

2.1 Introduction

In this chapter, we study the impact of fiber twisting on the polarization of the output pulse. We have experimental results on pulse propagation in twisted fibers that we can directly compare with the numerical simulations. As mentioned briefly in Section 1.3, twisting the fiber can be used to average the linear birefringence during propagation. However, twisting the fiber produces a circular birefringence [7], which produces another dynamical effect that is accounted for in our equations. In Section 0 and 2.4, we discuss numerically and experimentally the nonlinear polarization dynamics for single and coupled solitons, respectively. Evolution of two polarization components traveling in twisted fibers described by the two equations

\[
\frac{\partial A_+}{\partial z} + \frac{i\Delta \kappa}{2} A_+ + \frac{i\Delta \beta}{2} A_- + \beta_1 \frac{\partial A_+}{\partial t} + \frac{i\beta_2 A_+}{2} \frac{\partial^2 A_+}{\partial t^2} - \frac{\beta_3 A_+}{6} \frac{\partial^3 A_+}{\partial t^3} + \frac{\alpha}{2} A_+ = R_+ + i\frac{s}{\partial t}(R_+), \tag{2.1}
\]

\[
\frac{\partial A_-}{\partial z} - \frac{i\Delta \kappa}{2} A_- - \frac{i\Delta \beta}{2} A_+ - \beta_1 \frac{\partial A_-}{\partial t} + \frac{i\beta_2 A_-}{2} \frac{\partial^2 A_-}{\partial t^2} - \frac{\beta_3 A_-}{6} \frac{\partial^3 A_-}{\partial t^3} + \frac{\alpha}{2} A_- = R_- + i\frac{s}{\partial t}(R_-). \tag{2.2}
\]

The parameter \(\Delta \kappa\) describes the effect of the circular birefringence produced by twisting the fiber and defined as:

\[
\Delta \kappa = \frac{g}{2} \tau, \tag{2.3}
\]
where $g$ is measured from the elastooptic tensor for the silica and found to be in the range from 0.13 to 0.16, and $\tau$ is the twist rate in radians/m. The $\beta_1$ is related to the inverse group velocity difference between the two circular polarizations; in other words it accounts for the walk-off between two polarized field pulses due to group velocity differences arising from fiber twist and can be calculated using the relation [17]

$$\beta_1 = \tau \frac{\lambda^2}{2\pi c} \frac{dg}{d\lambda},$$

where $\lambda$ is the wavelength of the input signal and $c$ is the speed of light in free space.

With using the results of [18] at wavelength of 1.55 $\mu$m, $\frac{dg}{d\lambda} = 7.2 \times 10^{-3}$ $\mu$m$^{-1}$.

### 2.2 Experimental Setup

The experimental setup used to measure the pulse properties propagated in a twisted fiber and the fiber ring laser used as a source are illustrated in Figure 2.1 (a) and (b), respectively.
The passively mode-locked ring cavity laser in Figure 2.1(b) uses a 10-m length of twisted fiber and a double pass erbium-doped fiber amplifier (EDFA) with a Faraday rotator followed by a Faraday mirror at one end, which makes the polarization ellipticity in the cavity defined and stable[19]. Twisting the 10-m fiber in the cavity is to eliminate the residual linear birefringence by producing higher value of the circular birefringence. After propagation through the 60-cm EDFA, high birefringence produced, hence we use
the Faraday rotator FR followed by the Faraday mirror FM. Beam goes through the FR, it rotates 45 degrees, reflected by the FM, then rotates a 45 degrees, thus a beam will be rotated 90 degrees before passing the EDFA through the way back. The second pass through the EDFA produces birefringence on the pulse that rotated by 90 degrees, which eliminate the effect of the birefringence that produced in the first pass. As a result, the pulse enters the EDFA has the same ellipticity as the pulse comes out of it. The mode-locking in the cavity initiated by the modulator which also used here as a polarizer. After the mode-locking starts, the radio frequency RF generator is switched off because it causes some noise. Ellipticity and azimuth angle of the pulse can be adjusted by the variable retarder; these are critical parameters that determine the nonlinear polarization rotation rate. Initially, we measured the ellipticity and azimuth angles and determined the region where mode-locking occurs; a map of the results were generated, as shown in Figure 2.2. The shaded area shows the parameter region where the mode-locking occurs. After appropriate adjustment of the ellipticity and azimuth was fixed we could change the pulse generation mode of the laser, i.e. single or paired solitons, by changing the pump power. The laser output is measured and monitored with an Optical Spectrum Analyzer (OSA) and an autocorrelator. The mode-locked laser output pulses were amplified by a 3 m double pass EDFA, see Figure 2.1(a). The polarization, measured by an in-line polarimeter, is controlled by the polarization controller PC. The pulses are coupled into a 500 m fiber twisted with a rate of 6 turns/m. At the output, a Quarter Wave Plate (QWP) and a polarization beam splitter (PBS) separates the left and right circular components of the pulses, which can be separately displayed on the oscilloscope by using a fiber delay line in one arm.
Figure 2.2 A map of the azimuth and ellipticity angles that show where the mode-locking occurs.
2.3 Numerical and Experimental Results of Single Soliton Input.

In this section, we compare numerical and experimental results for the polarization evolution of single soliton in twisted fibers. The numerical input pulse is shown on Figure 2.3. The pulse width is $17\,\text{ps}$ with a peak power of $75\,\text{W}$.

![Input pulse shape for our numerical calculations. Noise is added to the amplitude.](image1)

![Experimental single soliton pulse spectrum. The peaks correspond to dispersive continuum radiation that is shed from the pulse.](image2)
**Figure 2.4** shows the experimental single soliton spectrum with a 6.6 nm bandwidth and strong dispersive wave peaks. Input pulse amplified by propagating through a double pass 3m of EDFA. Amplification factor of the EDFA controlled by its current. Initially, we reduce the current to the minimum level where the input pulse propagate into the 500 m with no amplification. Since the nonlinearity of the fiber is intensity dependent, a propagating pulse with low power is not highly affected by the fiber nonlinearity and the polarization ellipticity does not change, too. Polarization ellipticity is calculated by:

\[
\rho = \tan^{-1}\left(\frac{\sqrt{P_+} - \sqrt{P_-}}{\sqrt{P_+} + \sqrt{P_-}}\right),
\]  

(2.5)

where \(P_+\) and \(P_-\) are the powers of the two polarization components which separated after the polarization beam splitter PMS. To check the appropriate azimuth angle, we fix the input ellipticity at linear, 0°, and measure the output ellipticity at different azimuth angles, see **Figure 2.5**. The dependence of the output ellipticity on the input is represented on **Figure 2.6**. The output ellipticity follows the input ellipticity except at the tails, where the input is close to circular polarization. This results from imperfection of the photonic components in the system. For example, the PBS ports have a leakage 4% in one port due to the finite extinction coefficient. A circular polarization input becomes a linear polarization after the QWP, which supposed to propagate through one arm of the PMS. However, we measure about 96% in one arm and 4% in the other one. This error is responsible for dropping the output ellipticity from 45° to 35° in our measurements.
Figure 2.5 Dependence of the output ellipticity on the input azimuth angle with an ellipticity of $0^\circ$.

Figure 2.6 Experimental dependence of the output ellipticity on the input at low input power.
After the measured results were consistent with expectations based on linear fiber optics at low power, we increased the output power of the EDFA to study the nonlinear fiber optic effects on the pulse polarization. Figure 2.7 shows the output spectra for the single soliton input. The spectrum has a noise-like shape around the pump wavelength and a single-soliton-shape pulse appears with wavelength shifted by 40 to 80 nm depending on the input power and the input ellipticity. The experimentally measured dependence of the soliton wavelength shift on the input ellipticity is demonstrated in Figure 2.8. Equations (2.1) and (2.2) are used with parameters of \(\beta_1 = 0.173 \text{ps/km}, \beta_2 = -25 \text{ps}^2/\text{km}, \beta_3 = 0.02 \text{ps}^3/\text{km}, \gamma = 1.5/\text{W.km}, \tau_R = 3 \text{fs}, \alpha_R = 0.3, \ g = 0.16, \tau = 2\pi \times 6 \text{rad/m}.

![Figure 2.7](image-url)

Figure 2.7 Output spectra for single-soliton case with a linear, elliptical, and circular input polarization; red – right circular input polarization, green – right elliptical polarization, blue – linear polarization.
With using the CNLSE, Eqs. (2.1) and (2.2), we study the evolution of the two components of the field $A_+$ and $A_-$, Eq. (2.4)(2.5) is used the calculate the output ellipticity at different input ellipticities. Figure 2.9 (a) and (b) show experimental and numerical dependence of the output ellipticity on the input ellipticity with a single soliton input. With some fluctuations, the output ellipticity follows the input ellipticity.
Figure 2.9 Experiment (a) and simulation (b) results for the dependence of the output ellipticity on the input ellipticity for a single soliton input.
The numerical simulation results of the time evolution of $A_+$ and $A_-$ are plotted in Figure 2.10 (a). The initial Gaussian shaped pulse breaks up, at first mainly due to four-wave mixing terms creating self- and cross-phase modulation. The initial pulse increases in amplitude and breaks up into individual pulses first due to the strong nonlinear focusing effect. As a consequence of the nonlinear focusing effect stimulated Raman scattering becomes stronger, which is evident in the final spectrum, which has more energy at red-shifted wavelengths, see Figure 2.7. A close-up view of the time evolution in Figure 2.10 (b) reveals the complexity of the pulse evolution with multiple soliton-like pulses and a broad background.
Figure 2.10 (a) Energy evolution of $A_+$ and $A_-$ as they propagate through the fiber with a linearly polarized input. (b) Close-up at $d=500$ m.
2.4 Numerical and Experimental Results for Two Solitons Input.

In this section, we show the pulse and the ellipticity evolution in a twisted fiber for a coupled soliton pulse. By changing the pump power in the fiber ring laser shown on Figure 2.1 (b), we are able to generate a coupled soliton pulse. The separation between solitons in the paired soliton, as determined by the autocorrelator measurements, is 14 ps as shown on Figure 2.11, which corresponds well to the modulation frequency of the spectrum observed in Figure 2.12. The experimental and numerically simulated output spectra with different input polarizations of a coupled pulse are shown on Figure 2.13 and Figure 2.14, respectively.

![Autocorrelator output of coupled solitons](image.png)
Figure 2.12 Experimental pair-Soliton optical spectrum.

Figure 2.13 Experimental output power spectra for pair-soliton case with a linear, elliptical, and circular input polarization; red – right circular input polarization, green – right elliptical polarization, blue – linear polarization.
When the paired-pulse is launched into the twisted fiber, the spectrum has a broad, plateau structure with a bandwidth exceeding 100 nm depending on the input polarization and input power. We measured and calculated the output polarization ellipticity at different wavelengths within the plateau region. Figure 2.15(a) and (b) show the dependence of the output ellipticity on the input ellipticity at 1585 nm (blue) and 1580 nm (red) in experiment and simulation, respectively. We observe a very abrupt polarization switch from nearly left to nearly right circular polarization when the input polarization is close to linear. Output circular polarized pulse propagate through the QWP, which placed at 45° to the principal axes of the fiber, hence a linear polarized pulse in produced. Then, it propagates through the polarization beam splitter PBS, which
split the two components into two ways. So, producing a perfect circular polarized pulse at the output of the 500m twisted fiber should give one of the pulse’s components with 0 intensity. Output experimental amplitudes of the two components, $A_+$ and $A_-$ at different input ellipticity are shown on Figure 2.16. We see that at any input ellipticity, one of the components is very small comparing to the other one. In the simulation, the coupled pulses do not collide with one another; however, after propagating some distance, they break up and walk-off one another; the intense pulses interact with one another as they propagate and create the polarization change through stimulated Raman scattering effects. We found through simulations that when the pair pulses were separated in larger delay, the interaction between the pair pulses is delayed and the polarization evolution occurs over larger propagation distance.

From Figure 2.15, we see how the energy transfers from one field’s component to another, as mentioned in Section 1.4.2. At any input polarization, transferring energy from one component to another one produces a nearly circular polarized output, no matter the input polarization. From simulation, Table 2.1 presents the output intensity at wavelength 1585 nm of the two components $A_+$ and $A_-$ and the phase difference between them, $\delta = \delta_y - \delta_x$ at different input ellipticity. The output at input ellipticity $-33^\circ, 6^\circ,$ and $1.5^\circ$ are shown on Figure 2.17.
Figure 2.15 Output ellipticity versus input ellipticity for the pulse pairs at the laser output. The polarization state is reported for two wavelengths: 1580 nm (red) and 1585 nm (blue). (a) Experiment and (b) simulation results.
Figure 2.16 Experimental output intensity of $A_+$ and $A_-$ at input ellipticity (a) $44^\circ$, (b) $20^\circ$, (c) $0^\circ$, (d) $-20^\circ$. 
Table 2.1 Output intensity of the two components $A_+$ and $A_-$ and the phase difference between them at different input ellipticity. Simulation results.

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Figure 2.17 Output at different input ellipticity. Simulation results.

**Figure 2.18** (a) shows the evolution of the two components of the field, $A_+$ and $A_-$ at different sections in the fiber with a linearly polarized input. A close-up view of the time domain evolution is shown on **Figure 2.18** (b). We notice the energy transfer from $A_+$ to $A_-$ as they propagate. Initially, pulse breaks up into its individual pulses due to the nonlinear effects of the fiber. Pulse breaking up resulting in an amplitude increasing, hence the effect of stimulated Raman scattering increases and more red-wavelength shift occurs. **Figure 2.19** shows the evolution of the two components of the field, $A_+$ and $A_-$ at different sections in the fiber with a circular polarized input.
Figure 2.18 (a) Numerical simulations results for the evolution of $A_+$ and $A_-$ as they propagate through the fiber with a linearly polarized input. (b) expanded scale at $d=500$ m.
Figure 2.19 Numerical simulation results for the evolution of $A_+$ and $A_-$ as they propagate through the fiber with a circular polarized input.

**Figure 2.20** (a) and (b) show the overall distribution for $A_+$ and $A_-$ components as they propagate through the twisted fiber with linear and circular polarized inputs, respectively. To show the abrupt change from negative to positive circular polarization, many shots have been taken at linear polarized input for different wavelengths, see **Figure 2.21**. We clearly notice the discontinuous change in polarization evolution around the input polarization with $0^\circ$ ellipticity. At wavelength 1582 nm **Figure 2.21** (a) the output ellipticity changes between $28.3^\circ$ and $-30.2^\circ$ in average.
Figure 2.20 Numerical simulation showing the mean intensity evolution for each circular polarization with propagation distance in a twisted fiber with (a) linear polarization and (b) circular polarized input.
Figure 2.21 Experimental output ellipticity for single shots at wavelength 1582 nm (a) and 1600 nm (b) with a linear polarized input.
2.4.1 Analytical Results for Propagation Through Untwisted Fiber

In our experiments and numerical simulations we observed how twisting the fiber can eliminate the linear random birefringence. Twisting the fiber causing the energy of one component field to be transferred to another one, which eventually results in producing a circular ellipticity by using Eq. (2.5). The twist factor, \( \tau \) in Eqs. (2.1) and (2.2), set to be 0 and the output ellipticity is calculated. The dependence of the output ellipticity on the input ellipticity is shown on Figure 2.22. Clearly, one can see how twisting the fiber can maintain the polarization to be nearly circular by comparing Figure 2.15 and Figure 2.22.

![Figure 2.22 Output ellipticity versus input ellipticity for the pulse pairs propagating through untwisted fiber. The polarization state is reported for two wavelengths: 1580 nm (red) and 1585 nm (blue).](image-url)
2.4.2 Raman Gain Contribution to the Polarization

As mentioned in Equation (1.29), we do not use the total power to calculate the effects of the stimulated Raman scattering. However, we take into our consideration the difference between the orthogonal and parallel Raman gain coefficients with the ratio between them as $\alpha_R = 0.3[16]$. In many researches and papers in literature, total power was used to calculate the SRS effects [13-15]. If we use the total power by taking $\alpha_R = 1$, the output ellipticity dependence on the input for coupled solitons input at wavelengths of 1580 nm and 1585 nm are shown on Figure 2.23. The behavior is completely deferent than the analytical and experimental results shown on Figure 2.15.

![Graph showing analytical output ellipticity versus input ellipticity for the pulse pairs at the laser output. The polarization state is reported for two wavelengths: 1580 nm (red) and 1585 nm (blue). Total power used to calculate the SRS with $\alpha_R=1$.](graph.png)
2.5 Conclusion

Regular fibers, untwisted ones do not preserve the polarization of propagated wave through them. This caused by many reasons. Fibers which have a perfect circular core and cladding have two equal refractive indexes of the principal axes, $n_x, n_y$. Since the core is not a very accurate, $n_x \neq n_y$, which make the propagation constants $\beta_x$ and $\beta_y$ not equal. One way to average the residual linear birefringence is twisting the fiber. Twisting the fiber also produces a circular birefringence which larger than the linear birefringence, at least one order of magnitude. We study the polarization dynamic for one and coupled-soliton pulse experimentally and numerically. The numerical results compare extremely well with the experimental results. For the single soliton pulse, the output spectrum has a soliton-like shape shifted by 40 to 80 nm to longer wavelength. Shift results from the stimulated Raman scattering and its value depends on the input ellipticity and the input power. The output ellipticity follows the input ellipticity with high fluctuations that occurs because of the presence of the fiber nonlinearity. For the coupled-soliton input case, the output spectrum has a broad and flat structure that exceed 100 nm. The output ellipticity was measured and calculated at different wavelengths at this flat area and very similar results have gotten. The output polarization has a very abrupt change from nearly left circular $-45^\circ$ to nearly right circular $+45^\circ$ within a very narrow range of input ellipticities.

Pump waves produce two types of stoke waves, a parallel Stokes to the pump which give a parallel Raman gain and perpendicular Stokes to the pump which gives a perpendicular Raman gain. We take into our consideration the ratio between these two gains. Some researches in the literature assume that two gain values are the same, $\alpha_R = 1$, and we numerically proved that this assumption is not valid.
Polarization evolution of a coupled-soliton case in untwisted fiber has been studied numerically. The results show that the polarization is random and cannot be preserved through the propagation because of the linear birefringence of the fiber.
CHAPTER 3

SUPERCONTINUUM GENERATION

3.1 Introduction

One of the peculiar features of nonlinear wave propagation is the generation of an optical spectrum with a wide range of frequencies; thus, the name supercontinuum. In nonlinear optics, different nonlinear effects, such as stimulated Raman scattering and self-phase modulation occur at the same time causing a broadening in the propagated pulse, often spanning over 500 nm. The supercontinuum (SC) phenomenon generates a wide range of frequencies in the output spectrum from a narrow band input pulse [20]. In the supercontinuum generation experiments, a photonic crystal fiber PCF may be specially designed to tightly confine the mode field to a small cross-sectional area thus increasing the effective nonlinearity.

Figure 3.1 shows a cross section of the PCF that used in experiment in [20] and [21].

Figure 3.1 (Left) shows a cross section of the PCF, and (Right) shows the central details.
PCFs are constructed with air holes surrounding the core where the wave propagates by the total internal reflection process. The PCF that used for supercontinuum generation has a property of high air-fill fraction which gives strong confinement and nonlinearity[20]. PCF’s also have a wide spectral range for the operation of the single mode[14]. Supercontinuum generation is attractive for different applications, such as spectroscopy using a tunable femtosecond laser source. Moreover, it has been proposed as an alternative source for a dense wavelength division multiplexing WDM optical communications system where it emits a wide range of wavelengths at the same time[6].

3.2 Scalar Waves

The generalized nonlinear Schrödinger equation GNLSE of $A(z,T)$ is

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2} - \sum_{m=2}^{M} \left( \frac{i^{m+1}}{m!} \beta_m \frac{\partial^m A}{\partial T^m} \right) =$$

$$i\gamma \left( 1 + i\tau_{\text{shock}} \frac{\partial}{\partial T} \right) \left( A(z, t) \int_{-\infty}^{+\infty} R(t') |A(z, t - t')|^2 dt' + i\Gamma_R(z,t) \right),$$

(3.1)

where the left side of the equation represents the linear effects on propagation with a loss with coefficient of $\alpha$ and dispersion with coefficients of $\beta_m$ that found from Taylor expansion. The right side represents the nonlinear effects with a coefficient $\gamma = \omega_0 n_2(\omega_0)/c A_{\text{eff}}(\omega_0)$, where $n_2(\omega_0)$ is the nonlinear refractive index and $A_{\text{eff}}(\omega_0)$ is the mode effective area. The $\Gamma_R$ represents Raman noise and it will be neglected in the simulation. The time derivative defines the dispersion of the nonlinearity which usually comes with effects such as the optical shock with time scale of $\tau_{\text{shock}} = \tau_0 = 1/\omega_0$, which will be considered here. A corrected form of $\tau_{\text{shock}}$ has been derived by Blow and
Wood where the frequency dependent of \( A_{\text{eff}}(\omega_0) \) and \( n_2(\omega_0) \) were included. The corrected form defined as [22]

\[
\tau_{\text{shock}} = \tau_0 - \left[ \frac{1}{n_{\text{eff}}(\omega)} \frac{dn_{\text{eff}}(\omega)}{d\omega} \right]_{\omega_0} - \left[ \frac{1}{A_{\text{eff}}(\omega)} \frac{dA_{\text{eff}}(\omega)}{d\omega} \right]_{\omega_0} .
\]

(3.2)

The dependence of the refractive index on frequency is very small compare to the change of the mode area with the frequency, so, the second term can be ignored. Since the mode decreases with the increasing of the frequency, third term will be negative, which means that the effective mode area will increase the shock time coefficient \( \tau_{\text{shock}} \). The response function \( R(T') \) includes the electronic and Raman effects on the nonlinearity by

\[
R(T) = (1 - f_R)\delta(t) + f_R h_R(t).
\]

(3.3)

Where \( f_R \) shows the effect of the Raman response to the nonlinearity of the function. \( h_R \) represents the Raman gain which usually measured experimentally. However, Blow and Wood derived a useful form in [22] to approximate the contribution of \( h_R \) by

\[
h_R = \frac{\tau_1^2 + \tau_2^2}{\tau_1 \tau_2} \sin \left( \frac{t}{\tau_1} \right) \exp \left( - \frac{t}{\tau_2} \right).
\]

(3.4)

Where \( \tau_1 \) and \( \tau_2 \) are parameters that can be chosen to get best approximation for the Raman gain response. They have been chosen in [22] to be \( \tau_1 = 12.2 \) fs and \( \tau_2 = 32 \) fs.

In order to solve the GNLSE, equation (3.1), we separate the linear \( L \) and nonlinear \( NL \) terms and use the Split-Step Fourier Method (SSFM). Equation (3.1) can be written as

\[
\frac{\partial A}{\partial z} = (L + NL)A(z, T).
\]

(3.5)

Where \( L \) and \( NL \) defined as

\[
L = \frac{\alpha}{2} - \sum_{m=2}^{M} \left( \frac{\beta_m}{m!} \frac{\partial^{m+1} A}{\partial T^m} \right),
\]

(3.6)
\[ NL = i\gamma \left( 1 + \frac{i}{\omega_0} \frac{\partial}{\partial t} \right) \left( A(z, t) \int_{-\infty}^{+\infty} R(t') |A(z, t - t')|^2 dt' \right) \]

\[ = i\gamma(1 - f_R) \left[ |A|^2 + \frac{i}{\omega_0} \frac{\partial}{\partial t} (A|A|^2) \right] + i\gamma f_R \left[ \int_{-\infty}^{+\infty} h_R |A|^2 \, dt + \right. \]

\[ \left. \frac{i}{\omega_0} \frac{\partial}{\partial t} \left( A \int_{-\infty}^{+\infty} h_R |A|^2 \, dt \right) \right]. \]

The evolution of an optical pulse in a segment of a fiber from \( z \) to \( z + h \) can be approximately presented by the split-step Fourier method as

\[ A(z + h, T) = \exp \left( \frac{h}{2} L \right) \exp \left( \int_{z}^{z+h} NL(z) \, dz \right) \exp \left( \frac{h}{2} L \right) A(z, T). \quad (3.7) \]

**3.3 Vector Waves**

To study the polarization properties in SC generation, we deal with the vector wave components and the GNLS can be written as[14]

\[ \frac{\partial}{\partial z} A_\pm = \left( \frac{i\Delta\beta_0}{2} - \frac{\Delta\beta_1}{2} \frac{\partial}{\partial t} \right) A_\mp + \sum_{m=2}^{\infty} \left( \frac{i^{m+1}}{m!} \beta_m \frac{\partial^m}{\partial t^m} A_\pm \right) - \frac{\alpha}{2} A_\pm + \]

\[ i\gamma \left( 1 + \frac{i}{\omega_0} \frac{\partial}{\partial t} \right) \left[ (1 - f_R) \left( \frac{2}{3} |A_\pm|^2 + \frac{4}{3} |A_\mp|^2 \right) A_\pm + f_R A_\pm \int_{-\infty}^{t} h_R(t - t') \left( |A_\pm|^2 + |A_\mp|^2 \right) dt' \right] \quad (3.9) \]

Where \( A_+ \) and \( A_- \) are the two components of the field. \( \Delta\beta_0 = \beta_x - \beta_y = B\omega_0/c \) is the fiber birefringent and \( B \) assumed to be \( 10^{-5} \) [14], and \( \Delta\beta_1 \) is the group velocity mismatch between the slow and fast axes. The linearly polarized components of the field can be described as

\[ A_x = A_+ + A_- \frac{\alpha}{\sqrt{2}} e^{-i\Delta\beta_0 z/2}, \quad (3.10) \]

\[ A_y = A_+ - A_- \frac{i}{\sqrt{2}} e^{i\Delta\beta_0 z/2}. \quad (3.11) \]
3.4 Simulation Results

The literature apparently does not report Supercontinuum generation using vector wave equations. We want to study the vector properties of Supercontinuum generation. Our preliminary studies are illustrated here. Two input field are identical and set as $A_± = \sqrt{P} \text{sech}(\frac{T}{t_0})$, where $P = 10 kW$, and $t_0 = 28.4 fs$. Chromatic dispersion coefficients are

$\beta_2 = -11.830 \times 10^{-3} \frac{ps^2}{m}, \beta_3 = 8.1038 \times 10^{-5} \frac{ps^3}{m}, \beta_4 = -9.5205 \times 10^{-8} \frac{ps^4}{m}, \beta_5 =$

$2.0737 \times 10^{-10} \frac{ps^5}{m}, \beta_6 = -5.3943 \times 10^{-13} \frac{ps^6}{m}, \beta_7 = 1.3486 \times 10^{-15} \frac{ps^7}{m}, \beta_8 =$

$-2.5495 \times 10^{-18} \frac{ps^8}{m}, \beta_9 = 3.0524 \times 10^{-21} \frac{ps^9}{m}, \beta_{10} = 1.7140 \times 10^{-24} \frac{ps^{10}}{m}$. At wavelength $\lambda = 835 nm$, nonlinear coefficient $\gamma = 0.11 \ (Wm)^{-1}, f_R = 0.18$ and fiber loss, $\alpha$ is negligible since we are simulating a short PCF with a length of 6 cm. First, we apply a scalar case with an input field $A = \sqrt{P} \text{sech}(\frac{T}{t_0})$ in the simulation in order to have a clear comparison with the vector case. Figure 3.2 and Figure 3.3 show the input and output spectra of a scalar and vector waves, respectively, with parameters mentioned above and the vector wave is polarized with a 45° ellipticity. The vector wave generates a broader SC spectrum than the scalar one with a range over 700 nm compared to about 600 nm for the scalar wave.
Figure 3.2 Input and output spectra with a scalar wave input.

Figure 3.3 Input and output spectra with a polarized vector wave input at 45°.

For the vector wave propagation, Figure 3.4 shows the temporal, spectral, and ellipticity evolutions of an input with an ellipticity of 10° and power of 4 kW. Four snapshots of the pulse inside the fiber are shown at 0.5 cm, 1 cm, 3 cm and 6 cm. The ellipticity defined as

\[ e(\lambda) = \tan^{-1} \left( \frac{\sqrt{A_+} - \sqrt{A_-}}{\sqrt{A_+} + \sqrt{A_-}} \right). \]  

(3.12)
Following the temporal evolution, both vector components get compressed as they propagated, then they produce additional pulses by fission of the input pulse. The input breaks-up into multiple pulses after a 1 cm propagation distance; the shift of the power spectrum to longer wavelength is attributed to intrapulse Raman scattering. Spectrum evolution shows that a broadband SC for more than an octave was achieved after 6 cm propagation distance. The spectrum broadens because of the self-phase modulation, cross-phase modulation and the Raman scattering. Ellipticity goes through very complicated evolutions as the wave propagates through the PCF.

Figure 3.4 Temporal, spectral, and ellipticity evolution of input with an input ellipticity of 10° and input power of 4 kW.
3.4.1 Input Polarization Effect on the Output Polarization

Ellipticity evolution of a propagated pulse in the PCF depends on the input ellipticity. To avert the polarization fluctuation issue in the SC generation process input pulse is launched with a polarization along one of the principal axes of the PCF; in this case the linearly polarized pulse can propagate through the fiber with no change in the ellipticity. **Figure 3.5** shows the temporal, spectral, and polarization output of the wave for a polarized input at 1°, 20°, 30°, and -44°. Our simulations show that the output ellipticity is more stable when the input polarization is almost a linearly or circularly polarized. However, as the polarization at each wavelength shows large fluctuations in value covering the entire range of ellipticities in the spectrum.

![Figure 3.5 Temporal, spectral, and ellipticity output for a polarized input at (a) 1°, (b) 20°, (c) 30°, and (d) -44°.](image-url)
3.4.2 Input Power Effect on the Output Polarization

From the results of the previous sub-section we further studied the effect of input power $P$ on the spectrum and the polarization of SC wave. At this point, noise is introduced in the simulation, which is a very important factor that causes large pulse-to-pulse fluctuations. So, 10 simulations were done with a white noise to study the effect of the input power on the spectrum and the polarization. As the input power increases, the bandwidth of the output SC wave increases, as well. However, the output ellipticity exhibits larger fluctuations and develops complex correlations as the input pulse power increases. Higher power produces higher modulation polarization instability gain [23], which causes greater fluctuations in the output ellipticity. Output data for a wave vector input with ellipticity of 2° and different input powers are shown on Figure 3.6. It demonstrates how increasing the power causes broader spectra and higher ellipticity randomness in the pulse spectrum. The ellipticity fluctuations are more or less symmetric around the input wave’s carrier wavelength, which suggests that four-wave mixing is largely responsible as the mechanism driving the large fluctuations.
3.4.3 Effect of Twisting the PCF on the Ellipticity

In this section, we study the effects of twisting the PCF on the output ellipticity. Different twist rates at PCF length of 6 cm and 1 m were studied. Figure 3.7 shows the output ellipticity at different twist rates of a 6 cm PCF with an input power of 4 kW and ellipticity of 10°. For such a short fiber, it is possible to generate a supercontinuum, but we do not notice any dependence of the output ellipticity on the fiber twist rate. For that reason, a simulation for a PCF fiber of 1 m length has been studied. Figure 3.8 shows the output ellipticity at different twist rate. Applying circular birefringence on a photonic crystal fiber by twisting it would not stabilize the ellipticity. This may happen because of the structure of the fiber. Unstable ellipticity could cause an issue when the SC used as a source for an
application that need a constant polarization. In [14], it was suggested to use a polarizer at the fiber output to solve this problem. Another way to resolve this issue is to have a linear or circular polarized input, where they propagate with much less fluctuations in the ellipticity, see Figure 3.5. Latter method is recommended to avoid using a broadband polarizer.

Figure 3.7 Spectrum and output ellipticity at twist rate of (a) 0 (b) 500 (c) 1000 turns/m for a 6 cm PCF with an input power of 4 kW and 10° ellipticity.
3.5 Conclusion

In this chapter, we discussed the difference between the scalar and vector wave in generating a supercontinuum. We study how the output ellipticity of the SC depends on the input ellipticity and conclude that the output ellipticity could be stable if the input was pumped into one of the principal axes. We then made an initial examination of the relation between the input power and the output ellipticity to discover how increasing the power would broaden spectrum and affect the ellipticity. We found that as the power increases, the ellipticity spectrum becomes less stable. We also found out that twisting the PCF would
not settle the ellipticity to a constant value. Although a very high twist regime was not studied.
CHAPTER 4  
SUMMARY AND FUTURE WORK  

In this dissertation, we examined the polarization dynamics in a twisted fiber. To study the properties and characteristics of the polarization, we have to deal with a wave as a vector instead of a scalar. In Chapter 1, we derive the nonlinear Schrödinger equation starting with the Maxwell’s equations. Eq. (1.11) was used as a solution form for the scalar wave. First, we solve for the linear part by solving for the transverse field $F(x,y)$ and derive an equation of motion for the signal’s envelope function, $\tilde{A}$. Then, we solve for the nonlinear part which governs the nonlinear polarization $\tilde{P}_{NL}$. In Section 1.3, the vector wave equation is presented and we showed how to transform it from the linear polarization basis to a circular polarization basis, as written in Eqs. (1.22) and (1.23). Some of higher order perturbation effects are discuss in Sec. 1.4; those include third-order dispersion, stimulated Raman scattering SRS, and self-steepening contributions. Third-order dispersion can be dominant at or near the zero dispersion wavelength and it leads to pulse broadening and eventually pulse break up. Stimulated Raman scattering, SRS is a third-order nonlinear effect that causes the propagating pulse to shift its carrier frequency to longer wavelengths, i.e. a red-frequency shift. SRS produces Stokes wave by having an energy transfer from high to low-frequencies. Third effect is the self-steepening, which is a correction to the pulse propagation that includes the intensity dependence of the refractive index. In this
case the center of the pulse will deform its shape and overtake the low intensity wings of the pulse that causes an optical shock wave.

Regular fibers, untwisted ones do not preserve the polarization of propagated wave through them. There are many reasons for this depolarization effect. Ideally, fibers with a perfect circular core and cladding have two equal refractive indexes of the principal axes, $n_x, n_y$. However, in real fibers the core’s circularity is not a perfect so that the principal indices are not degenerate, i.e. $n_x \neq n_y$; this makes the propagation constants $\beta_x$ and $\beta_y$ unequal.

So, in a single mode fiber with two degenerate modes $HE_{11x}$ and $HE_{11y}$ phase velocity will not be the same which described as:

$$\nu_{f,x} = \frac{2 \pi f}{\beta_x},$$  \quad (4.1)  

$$\nu_{f,y} = \frac{2 \pi f}{\beta_y}. $$  \quad (4.2)

The mode phase shift, which determine the output polarization state is:

$$\Delta \phi = (\bar{\beta}_x - \bar{\beta}_y) L. $$  \quad (4.3)

Where $\bar{\beta}_x$ and $\bar{\beta}_y$ are the average propagation constants in the two principal axes, $x$ and $y$, and $L$ is the fiber length. Linear birefringence can be produced by other internal and external causes. For instance, non-homogeneity of the cladding density results in non-uniform pressure on the core, $P_x \neq P_y$, and external stress on the fiber, fiber’s bending and the temperature cause linear birefringence in the fiber.
One way to average the residual linear birefringence is twisting the fiber. Twisting the fiber also produces a circular birefringence which larger than the linear birefringence, at least one order of magnitude. Circular birefringence depends on the twist rate and defined as:

$$\Delta \phi = g L \tau.$$  \hfill (4.4)

The twist rate is the torsion angle $\delta$ per unit length $l$, see Figure 4.1.

![Figure 4.1 A twisted fiber with torsion angle $\delta$.](image)

The phase shift of the right and left circular polarization defined as

$$\phi^+ = \frac{2 \pi}{\lambda} L \left( n_0 + \frac{\Delta n_c}{2} \right).$$ \hfill (4.5)

$$\phi^- = \frac{2 \pi}{\lambda} L \left( n_0 - \frac{\Delta n_c}{2} \right).$$ \hfill (4.6)

Comparing the phase difference $\Delta \phi = \phi^+ - \phi^-$, from Eqs. (4.5) and (4.6) with Eq. (4.4) gives

$$\Delta n_c = \frac{\lambda}{2 \pi} g \tau.$$ \hfill (4.7)

Where $\Delta n_c$ is the difference between the refractive indexes of the right and left circular polarized waves which equal to $1.488 \times 10^{-6}$ at wavelength of 1.55 $\mu m$ and twist rate 6 $turns/m$. We study the polarization dynamics for one and coupled-soliton pulse experimentally and numerically. The numerical results compare extremely well with the experimental results. For the single soliton pulse, the output spectrum has a noise like
spectrum around the pump wavelength and a soliton-like shape shifted by 40 to 80 nm to longer wavelength. Shift results from the stimulated Raman scattering and its value depends on the input ellipticity and the input power. The ellipticity has been measured at the shifted soliton wavelength by adjusting the monochromator and changing the input ellipticity. The output ellipticity follows the input ellipticity with high fluctuations that occurs because of the presence of the fiber nonlinearity. For the coupled-soliton input case, the output spectrum has a broad and flat (plateau) structure whose spectral width exceeds 100 nm. The output ellipticity was measured and calculated at different wavelengths over the plateau area and very similar results were found. The output polarization has a very abrupt change from nearly left circular $-45^\circ$ to nearly right circular $+45^\circ$ within a very short range of input ellipticities. Experimentally, some parts can be enhanced or changed to improve the results. For example, the polarization beam splitter’s finite extinction is responsible for the output ellipticity’s values which falls short of measuring circular polarization at the output.

Raman gain can have two values, $g_{//}$ where the Stoke waves are parallel to the pump and $g_\perp$ where the Stoke waves are perpendicular to the pump. In the simulation, we take into our consideration the ratio $\alpha_R = g_\perp/g_{//}$ which depends in the frequency between the Stokes and the pump. For short frequency, the ratio is close to 0.3. Some researches in the literature assume that two gain values are the same, $\alpha_R = 1$, and we numerically proved that this assumption is not valid. Polarization evolution of coupled-soliton case in untwisted fiber has been numerically studied. The results show that the polarization is random and cannot be preserved through the propagation because of the linear birefringence of the fiber.
The mode-locked laser that used as pulse source generates a stable single and coupled-soliton pulses with a fixed pulse width. It is recommended to study the polarization dynamics in a twisted fiber for single and paired-pulses with a controlled pulse-width. Another case to study in the future, is the polarization of different types of input pulses.

We numerically study the polarization evolution with supercontinuum generation by a 6 cm photonic crystal fiber. Vector wave is used to study the polarization dynamics. We study the impact of the input ellipticity on the output ellipticity. Results show that the polarization behavior is very complicated and unstable unless with a linear or circular polarized inputs. Effect of the input power on the output polarization is presented. Higher power generates broader spectrum and produces a more complicated polarization, as well. This can be a result from the high nonlinearity of the fiber.

A circular birefringent fiber is created by twisting a standard fiber at different twist rates and the polarization evolution is studied. We note that twisting a photonic crystal fiber (PCF) is not expected to affect the output polarization state. A test was made on 1 m length of PCF and no change was observed. It may be that the structure of the PCF suppresses the twist effect and a much higher twist rate may be required to observe a twist related effect. Measuring the ellipticity of a twisted photonic crystal fiber experimentally in the future will be a good way to study super-continuum generation and polarization in the twisted PCF.
REFERENCES


APPENDICES

APPENDIX A

SVEA Wave Propagation Equation

The vector wave equation is derived following the usual process using Maxwell’s equations, as shown below

\[ \nabla \times \nabla \times \mathbf{E} = -\nabla \times \left[ \mu_0 \frac{\partial \mathbf{H}}{\partial t} \right] = -\mu_0 \frac{\partial}{\partial t} [\nabla \times \mathbf{H}] = -\mu_0 \frac{\partial}{\partial t} \left[ \frac{\partial \mathbf{D}}{\partial t} \right] \\
= -\mu_0 \frac{\partial^2}{\partial t^2} [\varepsilon_0 \mathbf{E} + \mathbf{P}] = -\mu_0 \frac{\partial^2}{\partial t^2} (\varepsilon_0 \mathbf{E}) - \mu_0 \frac{\partial^2}{\partial t^2} \mathbf{P}, \quad (A.1) \]

So, using the vector operator identity we have

\[ \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \mu_0 \left[ \frac{\partial^2 P_L}{\partial t^2} + \frac{\partial^2 P_{NL}}{\partial t^2} \right], \quad (A.2) \]

With assuming no charge density, \( \rho_f = 0 \), \( (\nabla \cdot \mathbf{E}) = 0 \) and equation (A.2) reduced to

\[ \nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} = \mu_0 \frac{\partial^2 P_L}{\partial t^2} + \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}. \quad (A.3) \]
APPENDIX B

Scalar Nonlinear Schrodinger Equation

In the wave equation (1.15), we assume that $\beta \approx \beta_0$, so

$$\beta^2 - \beta_0^2 = (\beta - \beta_0)(\beta + \beta_0) \approx 2\beta_0(\beta - \beta_0). \quad (B.1)$$

So, equation (1.15), can be written as

$$F \left\{ 2i\beta_0 \frac{\partial A}{\partial z} + 2\beta_0 (\beta - \beta_0)A \right\} = -\omega^2 \mu_0 \tilde{P}_{NL}(w). \quad (B.2)$$

We can convert the right side of equation (B.2), the nonlinear part of the equation to the time domain and considering the average nonlinear polarization which gives

$$\frac{\int_{-\infty}^{\infty} F F^* \, dx \, dy}{\int_{-\infty}^{\infty} |F|^2 \, dx \, dy} \left\{ \frac{\partial A}{\partial z} - i(\beta - \beta_0)A \right\} = -\frac{\omega_0^2 \mu_0 \int_{-\infty}^{\infty} \tilde{P}_{NL}(t) F^* \, dx \, dy}{2i \beta_0 \int_{-\infty}^{\infty} |F|^2 \, dx \, dy}. \quad (B.3)$$

The nonlinear polarization define as $\tilde{P}_{NL}(t) = \varepsilon_0 \chi^{(3)} |E|^2 E$. We can rewrite the equation (B.3) as

$$\frac{\partial A}{\partial z} - i(\beta - \beta_0)A = i \frac{\omega_0^2 \mu_0 \varepsilon_0}{2 \beta_0} \left[ \frac{\int_{-\infty}^{\infty} \chi^{(3)} |F|^2 |A|^2 F A^* \, dx \, dy}{\int_{-\infty}^{\infty} |F|^2 \, dx \, dy} \right]. \quad (B.4)$$

The wave number $\beta_0(\omega)$ can be approximated as $\beta_0(\omega) = n(\omega_0)\omega_0/c$ and the third order susceptibility $\chi^{(3)} = \frac{\varepsilon_{NL}}{|E|^2} = \frac{2n \Delta n}{|E|^2}$, where $\Delta n = \bar{n}_2 |E|^2 + \frac{i\alpha}{2k_0}$. These leads to
\[
\frac{\partial A}{\partial z} - i(\beta - \beta_0)\tilde{A} = i \frac{w_0^2 \mu_0 \varepsilon_0 c}{2n(\omega_0)\omega_0} |A|^2 A \left[ 2n(\bar{n}_2 |E|^2 + i\alpha) \frac{\int_{-\infty}^{\infty} |F|^4 dx dy}{|E|^2 \int_{-\infty}^{\infty} |F|^2 dx dy} \right].
\] (B.5)

Where \( \Delta n \) is a small perturbation results from the nonlinearity, \( \bar{n}_2 \) is the nonlinear index coefficient and \( \alpha \) is the absorption coefficient. With \( \mu_0 \varepsilon_0 = 1/c^2 \), and \( k_0 = \omega_0/c \), it becomes

\[
\frac{\partial A}{\partial z} - i(\beta - \beta_0)\tilde{A} = i k_0 |A|^2 A \left[ \bar{n}_2 \left[ \frac{\int_{-\infty}^{\infty} |F|^4 dx dy}{\int_{-\infty}^{\infty} |F|^2 dx dy} + \frac{i\alpha}{2k_0|A|^2} \right] \right].
\] (B.6)

Since there are a band of frequencies, \( \beta(\omega) \) is equal to \( \beta_0 \) for \( \omega = \omega_0 \), the Taylor expansion of \( \beta \) around \( \omega_0 \) yields

\[
\beta(\omega) = \beta_0 + (\omega - \omega_0) \frac{\partial \beta}{\partial \omega}|_{\omega_0} + \frac{(\omega - \omega_0)^2}{2} \frac{\partial^2 \beta}{\partial \omega^2}|_{\omega_0} + \cdots.
\] (B.7)

We denote \( \frac{\partial \beta}{\partial \omega} \) as \( \beta_1 \) and \( \frac{\partial^2 \beta}{\partial \omega^2} \) as \( \beta_2 \). Equation (B.6) can be described as

\[
\frac{\partial A}{\partial z} - i \left[ (\omega - \omega_0)\beta_1 + \frac{(\omega - \omega_0)^2}{2} \beta_2 + \cdots \right] \tilde{A} = i \gamma(\omega)|A|^2 A - \frac{i\alpha}{2} A,
\] (B.8)

where the nonlinearity coefficient \( \gamma(\omega) \) defined as

\[
\gamma(\omega) = \bar{n}_2 k_0 \frac{\int_{-\infty}^{\infty} |F|^4 dx dy}{\int_{-\infty}^{\infty} |F|^2 dx dy}.
\] (B.9)

Now we may return to the time domain for the left side of the equation (B.8) by taking the inverse Fourier transform, IFT as

\[
A(z, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{A}(z, \omega - \omega_0) e^{-i(\omega - \omega_0)t} d\omega.
\] (B.10)

Taking the IFT for (B.8) gives

\[
\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A = i \gamma |A|^2 A.
\] (B.11)
APPENDIX C

Vector Nonlinear Amplitude Equations

The vector wave equations in the linear polarization basis are

\[
\frac{\partial A_x}{\partial z} + \beta_{1x} \frac{\partial A_x}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_x}{\partial t^2} + \frac{\alpha}{2} A_x = i\gamma \left( |A_x|^2 + \frac{2}{3} |A_y|^2 \right) A_x + \frac{i\gamma}{3} A_x^* A_y^2 e^{-2i\Delta \beta z}
\]

(C.1)

\[
\frac{\partial A_y}{\partial z} + \beta_{1y} \frac{\partial A_y}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_y}{\partial t^2} + \frac{\alpha}{2} A_y = i\gamma \left( |A_y|^2 + \frac{2}{3} |A_x|^2 \right) A_y + \frac{i\gamma}{3} A_y^* A_x^2 e^{2i\Delta \beta z}
\]

(C.2)

Equations (C.1) and (C.2) can be written in a circularly polarized wave basis by using

\[
A_+ = \frac{A_x e^{i\Delta \beta z/2} + iA_y e^{-i\Delta \beta z/2}}{\sqrt{2}},
\]

(C.3)

\[
A_- = \frac{A_x e^{i\Delta \beta z/2} - iA_y e^{-i\Delta \beta z/2}}{\sqrt{2}}.
\]

(C.4)

So,

\[
A_x = \frac{A_+ + A_-}{\sqrt{2}} e^{-i\Delta \beta z/2},
\]

(C.5)
\[ A_y = \frac{-iA_+ + iA_-}{\sqrt{2}} e^{i\Delta \beta z/2}. \] (C.6)

\[ |A_x|^2 = \frac{|A_+|^2 + A_+ A_- + A_+^* A_- + |A_-|^2}{2}. \] (C.7)

\[ |A_y|^2 = \frac{|A_+|^2 - A_+ A_- - A_+^* A_- + |A_-|^2}{2}. \] (C.8)

\[ A_x^* A_y^2 = \left( \frac{A_+ + A_-}{\sqrt{2}} \right) e^{i\Delta \beta z/2} \left( \frac{-|A_+|^2 + 2A_+ A_- - A_-^2}{2} \right) e^{i\Delta \beta z} = \]

\[ e^{3i\Delta \beta z/2} \left( \frac{|A_+|^2 + 2|A_+|^2 A_- - A_+ A_-^2 - 2|A_-|^2 A_+ - |A_-|^2 A_-}{2\sqrt{2}} \right). \] (C.9)

\[ A_y^* A_x^2 = \left( \frac{iA_+ - iA_-}{\sqrt{2}} \right) e^{-i\Delta \beta z/2} \left( \frac{A_+^2 + 2A_+ A_- + A_-^2}{2} \right) e^{-i\Delta \beta z} = \]

\[ e^{-3i\Delta \beta z/2} \left( \frac{|A_+|^2 + 2|A_+|^2 A_- + |A_-|^2 A_+ - |A_-|^2 A_-}{2\sqrt{2}} \right). \] (C.10)

Equation (C.1) and (C.2) become

\[
\begin{align*}
\frac{\partial A_+}{\partial z} + \frac{\partial A_-}{\partial z} - \frac{i\Delta \beta}{2} (A_+ + A_-) + & \beta_1 \frac{\partial A_+}{\partial t} + \beta_1 \frac{\partial A_-}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_+}{\partial t^2} + \frac{i\beta_2}{2} \frac{\partial^2 A_-}{\partial t^2} \\
+ & \frac{\alpha}{2} A_+ + \frac{\alpha}{2} A_- = i\gamma \left( \frac{5|A_+|^2 + A_+ A_- + A_+^* A_- + 5|A_-|^2}{6} \right) (A_+ + A_-) + \\
& \frac{i\gamma}{3} e^{\frac{3i\Delta \beta z}{2}} \left( \frac{-|A_+|^2 A_+ + 2|A_+|^2 A_- - A_+ A_-^2 - A_+^2 A_-^* - 2|A_-|^2 A_+ - |A_-|^2 A_-}{2\sqrt{2}} \right) \left( \frac{\sqrt{2}}{e^{i\Delta \beta z}} \right) e^{-2i\Delta \beta z}.
\end{align*}
\] (C.11)
\[-\frac{\partial A_+}{\partial z} + \frac{\partial A_-}{\partial z} - \frac{i\Delta \beta}{2} (A_+ - A_-) - \beta_1 \frac{\partial A_+}{\partial t} + \beta_1 \frac{\partial A_-}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 A_+}{\partial t^2} + \frac{i\beta_2}{2} \frac{\partial^2 A_-}{\partial t^2} \]  

\[-\frac{\alpha}{2} A_+ + \frac{\alpha}{2} A_- = i\gamma \left( \frac{-5|A_+|^2 + A_+ A_- + A_+^* A_-^* - 5|A_-|^2}{6} \right) (A_+ - A_-) + \]  

\[i\gamma \frac{e^{-3i\Delta \beta z}}{2} \left( |A_+|^2 A_+ + 2|A_+|^2 A_- + i A_+^* A_-^* - 2i |A_-|^2 A_+ - i |A_-|^2 A_- \right) \left( \frac{\sqrt{2}}{ie^{-\frac{\Delta \beta z}{2}}} \right) e^{2i\Delta \beta z}. \]

Subtract Equation (C.12) from (C.11), and divide by 2 gives

\[\frac{\partial A_+}{\partial z} - \frac{i\Delta \beta}{2} A_- + \beta_1 \frac{\partial A_+}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_+}{\partial t^2} + \frac{\alpha}{2} A_+ = \frac{2i}{3} \gamma (|A_+|^2 + 2|A_-|^2) A_+, \]  

Adding Equation (C.11) to (C.11), and divide by 2 gives

\[\frac{\partial A_-}{\partial z} - \frac{i\Delta \beta}{2} A_+ + \beta_1 \frac{\partial A_-}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_-}{\partial t^2} + \frac{\alpha}{2} A_- = \frac{2i}{3} \gamma (|A_-|^2 + 2|A_+|^2) A_-., \]