MODELING ANISOPLANATIC EFFECTS
FROM ATMOSPHERIC TURBULENCE
ACROSS SLANTED OPTICAL PATHS IN IMAGERY

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MODELING ANISOPLANATIC EFFECTS FROM ATMOSPHERIC TURBULENCE
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ABSTRACT

MODELING ANISOPLANATIC EFFECTS FROM ATMOSPHERIC TURBULENCE ACROSS SLANTED OPTICAL PATHS IN IMAGERY

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When viewing objects over long distances, atmospheric turbulence introduces significant aberrations in imagery from optics with large apertures. We present a model for simulating turbulent effects in imagery using a technique similar to Bos and Roggemann’s model \[1\]. This simulation will support efforts in developing innovative turbulence mitigation techniques and replacing expensive flight tests. The technique implements the commonly used split-step beam propagation method with phase screens optimally placed along the optical path. This method is used to supply a turbulence distorted point spread function (PSF) along the unique, optical path from the object to the camera aperture for each pixel of an image. The image is then distorted by scaling and summing each PSF with the appropriate surrounding area of the corresponding pixel for new pixel values. Very large phase screens have been integrated into the simulation to account for low spatial frequencies and wind speed in video. Additionally, a modified version of Schmidt’s method \[2\] is implemented for estimating statistics for the individual phase screens in the model and for angle spectrum propagation through free space. The proposed model has the capability of simulating over horizontal or slanted paths using the Huffnagel Valley turbulence profile. For verification purposes,
analysis of average simulated PSFs for short and long exposures and angle of arrival were compared to theoretical results. Further analysis of simulated error statistics were carried out against varying elevation in the atmosphere.
For My Family and Friends
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CHAPTER I

INTRODUCTION

A major issue facing modern electro-optical (EO) and infrared (IR) imaging systems that take measurements along optical paths is atmospheric turbulence. Historically in imaging, astronomy has been the main area in implementing methods to reduce the effects of turbulence in imagery from telescopes. As camera technology has progressed and spatial resolution improved for longer distances, this problem of turbulence has surfaced in long distance imaging as well; therefore, many methods of modeling and mitigating atmospheric turbulence originally developed for astronomy have been retrofitted for long distance imaging. The research accomplished in this work focuses specifically on the process of modeling turbulent effects in imagery. The importance of simulation is directly related to mitigation. Mitigation techniques often require expensive data collections to gather perturbed imagery for development. These expenditures can be reduced by implementing simulated methods rather than the costly, time-consuming alternatives. Additionally, simulations provide a truth image which can be used to determine the effectiveness of the mitigated imagery.

Understanding how EO/IR radiation travels long distances from an object to the camera’s aperture is key in reproducing the effects of the atmosphere. Light, when traveling through the atmosphere, is affected by both free space propagation and random phase aberrations due to turbulence. To determine the diffraction resulting from propagation and turbulence, we have implemented the split-step wave propagation (SSWP) method common to other recent simulation models [1, 3, 4].
Ideally, imagery will have manageable turbulence aberrations within the scene. Common statistical approximations for estimating these distortions include Kolmogorov, Tatarskii and von Kármán [5]. However, these assumptions are only valid for instances with light turbulence where the camera’s viewing angle is within the isoplanatic angle, the largest field angle over which turbulence effects are approximately the same. For cases in which the camera viewing angle is greater than the isoplanatic angle, the imaging system is anisoplanatic. This issue of anisoplanatism becomes unavoidable for wide field-of-view (FOV) imaging systems where the optical path distance is typically very long and the isoplanatic angle is as small as a few pixels in measurement. There are several recent techniques that account for anisoplanatic effects in numerical simulation models. Lachinova et al. developed a unique method for incoherent anisoplanatic imaging using the brightness function by ray tracing through Kolmogorov phase screens to determine the perturbed intensity distribution [6]. Chaolan et al. proposed a model for anisoplanatic imaging for astronomical purposes by using Zernike Polynomials and superposition to determine phase distortion from atmospheric turbulence [7]. Underwood and Voelz integrated anisoplanatism into their model by using SSWP to collect single intensity realizations while iterating the process for an accurate average estimate of the perturbed image [8]. Bos and Roggemann addressed anisoplanatism over a horizontal optical path by splitting the simulation into blocks smaller than the isoplanatic angle and then using SSWP through Kolmogorov phase screens to find PSFs representing the distortion from the turbulence [1].

The proposed method is similar to Bos and Roggemann’s for simulated anisoplanatic imaging over horizontal optical paths. Schmidt’s techniques [2] for point source creation, phase screen generation, and angle spectrum propagation (ASP) are applied to calculate a grid of PSFs that represent the desired phase distortions for each pixel in an image. Each of these PSFs is scaled by the corresponding pixel intensity and summed to determine the new perturbed image containing the anisoplanatic effects from atmospheric turbulence. For simulating video, the region of interest
(ROI) for each frame is shifted through large rectangular phase screens to simulate temporal effects of turbulence frame to frame. Additionally, the model has been modified to accommodate slanted paths by adjusting the parameter values of the system to incorporate the Huffnagel-Valley turbulence profile (HVTP) [9].

The rest of the paper is organized as follows. Chapter 2 provides theoretical background for the various techniques used including the propagation method, sampling criteria and atmospheric turbulence parameters. In Chapter 3 the numerical model is detailed from generating phase screens to creating simulated video. Chapter 4 provides the results of simulation verification by comparing long/short exposure PSFs and Angle of Arrival (AoA) measurements to theoretical calculations. Additional analysis is performed by examining error statistics from turbulence over varying degrees of elevation in the simulation. Chapter 5 addresses conclusions and recommendations for future work.
CHAPTER II
THEORETICAL BACKGROUND

In any simulation the results produced from the model should match closely to expected theoretical results. The transition from real, continuous processes such as spherical propagation and atmospheric turbulence must be as seamless as possible. A comparison of diagrams for the realistic and simulated systems is shown in Figure 2.1.

Both systems demonstrate how a PSF is determined from a single point source affected by turbulence aberrations. In the theoretical system the point source (i.e. a laser beam) travels through the atmosphere that contains continuously varying temperatures and pressures. These fluctuations change the index of refraction adversely affecting the point source as it passes through the bulk turbulence until it is captured by the camera. In the simulated system, the PSF is found by manipulating the wavefront of a point source as it propagates through phase screens to the aperture. This wavefront is defined in phaser notation as

\[ u(x, y) = m(x, y)e^{i\phi(x, y)} \]  

where \( m(x, y) \) is the magnitude and \( \phi(x, y) \) is the phase. This form allows each point in the propagation grid to have direction and amplitude. By implementing this concept, a spherical point source can be applied in the simulation. The point source provides the initial wavefront, \( u_0(x, y) \). The phase and magnitude are affected as the wavefront propagates along the optical path. The
Figure 2.1: Comparison of theoretical and simulated systems: (a) The theoretical representation of the realistic process, (b) The simulated representation of turbulence model.

Phase screens are defined as transmission functions in the form of $t(x, y) = e^{i\phi(x, y)}$ and applied as $u_{n+1}(x, y) = u_n(x, y)t_n(x, y)$. Once the wavefront arrives at the aperture, it is collimated to a plane wave, masked according to the size of the aperture and propagated the remaining distance to the focal plane with a Fourier transform.

This PSF generation process will be explained in greater detail in Section 3.2.3. First, there are three major components of the simulation that are represented discretely when transitioning.

1 Image by Dr. Russell Hardie, University of Dayton, used here with permission.
from the theoretical model: the point source, propagation and turbulence. It is very important to understand these alterations from real world functions to computer simulated estimates to ensure accuracy in the simulation. Each of these components are detailed in the following sections.

2.1 Point Source

The theoretical representation of a point source is a dirac delta function. However, this depiction of a point source is unrealistic in the discrete domain given its Fourier spectrum contains infinite spatial bandwidth. Therefore, a point source that produces a band-limited spectrum is generally used for discrete cases. For the proposed simulation, each point source travels through free space from the object to the camera aperture using a form of Fresnel propagation. The area of the spectrum from each point source must appropriately cover the size of the camera aperture for accurate results. With this constraint in mind we create our point sources to match the spectrum properties we desire in the aperture plane after propagation.

Schmidt’s method of generating point sources \([2]\) is implemented here. Using Fresnel diffraction to estimate the wavefront of the propagated point source at the aperture, the spectrum is band-limited by windowing the wavefront,

\[
U(r_2) = \frac{e^{ik\Delta z}}{i\lambda\Delta z} W(r_2) e^{i\frac{k}{2\Delta z}|r_2-r_c|^2}, \quad (2.2)
\]

where \(r_2 = (x_2, y_2)\) is the observation plane coordinates, \(r_c = (x_c, y_c)\) is the location of the point source in the source plane and \(W(r_2 - r_c)\) is the window function. Additional optical parameters include \(k\) as the wavenumber, \(\lambda\) as the optical wavelength and \(\Delta z\) as the propagation distance. The goal is to have a uniform distribution across the aperture once the point source is propagated over the distance \(\Delta z\). With this in mind, the ideal window should be either a rect or circ function in order to cover the circular shaped aperture. The window in this case is a \(\text{rect}(x, y)\). By taking an inverse
Fourier transform and simplifying, the appropriate form for the point source can be found to be

\[ U_{pt}(r_1) = A e^{-i \frac{k}{2\Delta z} r_1^2} e^{i \frac{k}{2\Delta z} r_1^2} e^{-i \frac{k}{\Delta z} r_c \cdot r_1} \times \left( \frac{d}{\lambda \Delta z} \right)^2 \text{sinc} \left( \frac{d(x_1 - x_c)}{\lambda \Delta z} \right) \text{sinc} \left( \frac{d(y_1 - y_c)}{\lambda \Delta z} \right) \]  

(2.3)

where \( r_1 = (x_1, y_1) \) is the source plane coordinates and \( d \) is the width of the rect function. For even better results, a Gaussian term is implemented to mitigate some of the aliasing produced by the process. The resulting source signal is called the sinc-Gaussian point source. Examples of the point source and the wavefront after propagation to the aperture are shown in Figure 2.2.

Figure 2.2: Point Source Examples: (a) Point source generated at the object, (b) Propagated wavefront at aperture, (c) 1-D profile of propagated wavefront at aperture
Figures 2.2.a and 2.2.b show the relative intensity of the wavefront from the point source as it is generated and propagated across a distance of 7000 meters. The parameters for the example simulation can be seen in Appendix A. The ringing effects seen in Fig. 2.2.c after propagation are a side effect from band limiting the spectrum. The point source is generated to provide a rectangular spectrum in the pupil plane with a length that is four times the size of the aperture diameter. This ensures that there is a uniform surface covering the aperture after propagation, even with the ringing effects.

2.2 Propagation

There are many different methods of free-space propagation that can be found in literature [10]; however, since we implement phase screens in our model, the split-step propagation method proves an ideal option. As Bos and Roggemann describe, placing a single phase screen at the object or aperture plane results in poor representation of diffraction or atmospheric effects respectively [1]. Following this observation, the simulation propagates \( n + 1 \) times where \( n \) is the number of phase screens. The next two subsections describe the sampling constraint needed for avoiding aliasing and the applied method for propagation.

2.2.1 Sampling Constraint

Sampling in itself is often a tricky method that can be very complex with many constraints to avoid aliasing. For our simulation, we chose the critical sampling constraint (CSC) derived by Voelz for Fresnel propagators [11]. Starting with the definition of the Fresnel transfer function,

\[
H(f_X, f_Y) = e^{jkz} \exp[-j \pi \lambda z (f_X^2 + f_Y^2)],
\]

(2.4)

Voelz extracts the phase term to isolate the frequency part of the transfer function,

\[
\phi_H(f_X, f_Y) = \pi \lambda z (f_X^2 + f_Y^2),
\]

(2.5)
where \( z \) is the total propagation distance and \((f_X, f_Y)\) are the frequency domain coordinates. Working exclusively in the x-direction, we wish to determine an ideal constraint for the sampling frequency interval \( \Delta f_X \). The criterion Voelz utilizes for verification purposes is

\[
\Delta f_X \left| \frac{\partial \phi_H}{\partial f_X} \right|_{\text{max}} \leq \pi
\]  

(2.6)

where \( \frac{\partial \phi_H}{\partial f_X} \) is the phase slope. The previous statement requires the maximum change in absolute phase to be less than or equal to a cycle of \( \pi \) between any two adjacent samples to prevent aliasing. Simplified, the expression becomes the desired criterion in the frequency domain

\[
\Delta f_X \leq \frac{1}{\lambda z 2 |f_{X \text{max}}|}.
\]  

(2.7)

The corresponding criterion for the spatial domain can easily be determined by substituting in \( \Delta f_X = \frac{1}{D_p} \) with \( D_p \) as the side length of the grid size and \( |f_{X \text{max}}| = \frac{1}{2\Delta x} \) resulting in

\[
\Delta x \leq \frac{\lambda z}{D_p}.
\]  

(2.8)

Specifically, the CSC is

\[
\Delta x = \frac{\lambda z}{D_p} = \sqrt{\frac{\lambda z}{N}}.
\]  

(2.9)

where \( N \) is the number of samples in the propagation grid. Equation (2.9) provides the ideal sample spacing to avoid oversampling/undersampling and demonstrates the closest estimate to analytical results [11]. Using this criteria, the relationship between \( D_p \) and \( N \) can be observed in Figure 2.3 using the parameters from Appendix A.

To accurately cover the scope of the simulation, the size of pupil plane width must be greater than the side length of the rectangular wavefront of the point source after it propagates to the camera aperture. Remember that the length of the propagated rectangular wavefront is set to four times the diameter of the aperture when the point source is generated. For the example parameters, this constraint is 0.8136 (m). Therefore, the number of samples chosen must satisfy this requirement. In
MATLAB, the common Fast Fourier Transform (FFT) function which we use for propagation runs more efficiently with grid sizes to the power of 2. Thus, the ideal set of parameters is determined as a sample number of 256 which gives a pupil plane width of 0.9699 (m).

2.2.2 Fresnel Angular Spectrum Propagation

The two most common methods of propagation are Fresnel and Fraunhofer. Fraunhofer is an approximation of Fresnel that is valid for long propagation distances [10]. The total distance \( z \) of our simulation is usually long enough to be valid for Fraunhofer approximation. However, implementing split-step propagation separates the propagation into smaller distances \( \Delta z \) that cause...
Fraunhofer to be invalid in most cases. Therefore, Fresnel propagation must be used for experimentation. Note that we also chose to propagate spherically rather than planar since a point source naturally propagates in a spherical manner [10].

The proposed simulation uses the angle spectrum form of Fresnel propagation since it enables the use of MATLAB’s FFT function for more efficient computation. While there are many methods for ASP, we’ve implemented a slightly modified version of Schmidt’s method [2] for our model.

To start, the convolution form of the Fresnel diffraction integral,

\[ U(x_2, y_2) = U(x_1, y_1) \otimes \left[ \frac{e^{ik\Delta z}}{i\lambda\Delta z} \right] e^{i\frac{k}{2\lambda\Delta z}(x_2^2 + y_2^2)} \]  \hspace{1cm} (2.10)

is rewritten using special notation,

\[ U(r_2) = F^{-1}[r_2, f_1] \{ H(f_1) (F[f_1, r_1] \{ U(r_1) \}) \} \] \hspace{1cm} (2.11)

Each of these parameters are defined as follows:

\[ \mathcal{F}[r, f] \{ U(r) \} = \int_{-\infty}^{\infty} U(r) e^{-i2\pi f \cdot r} dr \] \hspace{1cm} (2.12)

is the Fourier transform integral,

\[ \mathcal{F}^{-1}[f, r] \{ U(f) \} = \int_{-\infty}^{\infty} U(f) e^{i2\pi f \cdot r} df \] \hspace{1cm} (2.13)

is the inverse Fourier transform integral,

\[ H(f_1) = e^{ik\Delta z} e^{-i\pi \lambda \Delta z(f_2^2 + f_{y1}^2)} \] \hspace{1cm} (2.14)

is the transfer function of free-space propagation, and \( U \) is the wavefront under propagation. In his method, Schmidt derives a version of the ASP that implements a scaling factor \( m \) so that he can choose differing sample spacings \( \delta_1 \) and \( \delta_2 \) at the source and observation planes respectively. This is done to avoid aliasing after propagation. His modified expression for ASP after simplification is

\[ U(r_2) = Q \left[ \frac{m \pm 1}{m \Delta z}, r_2 \right] \mathcal{F}^{-1} \left[ f_1, \mp \frac{r_2}{m} \right] Q_2 \left[ \pm \frac{\Delta z}{m}, f_1 \right] \mathcal{F}[r_1, f_1] Q \left[ \frac{1 \pm m}{\Delta z}, r_1 \right] \left( \mp \frac{1}{m} \right) \{ U(r_1) \} \] \hspace{1cm} (2.15)
where
\[ Q[c, r]\{ U(r) \} = e^{i \frac{k}{2} c |r|^2} U(r) \] (2.16)

and
\[ Q_2[d, r]\{ U(r) \} = e^{i \pi^2 \frac{2d}{k} |r|^2} U(r) \] (2.17)

are the quadratic phase factors. The proposed simulation avoids this scaling factor by keeping the sample spacings consistent during each propagation. Aliasing is not an issue provided that the CSC is accounted for. The sample spacings are then set to \( \delta_1 = \delta_2 \) letting \( m = 1 \) which modifies equation (2.15) to be
\[ U(r_2) = Q[0, r_2]\{ \mathcal{F}^{-1}[f_1, r_2]\{ Q_2[-\Delta z, f_1]\{ \mathcal{F}[r_1, f_1]\{ Q[0, r_1]\{ U(r_1) \} \} \} \} \} \]
\[ = \mathcal{F}^{-1}[f_1, r_2]\{ Q_2[-\Delta z, f_1]\{ \mathcal{F}[r_1, f_1]\{ U(r_1) \} \} \} . \] (2.18)

As shown in equation (2.18), the propagation is simply performed by multiplying the wavefront with a quadratic phase term in the Fourier domain.

### 2.3 Atmospheric Turbulence

The effects of turbulence in imagery have been studied for many years covering a wide array of theories and techniques. Most of the literature traces back to three sources of origin: Kolmogorov, Tatarskii and Fried. Kolmogorov [12] established a statistical model for atmospheric turbulence while Tatarskii [10] and Fried [13] extended Kolmogorov’s work into the field of optics. The theories and mathematical concepts developed from their work are vastly varied. However, there are several key parameters that should be addressed for better understanding of the proposed model.

Atmospheric turbulence originates from the current of air motion caused by the heating and cooling of the Earth. This temperature change causes pockets of air that grow in size until they break up into smaller pockets again and the process repeats. These pockets of air are randomly distributed with each containing specific temperatures. This is important to note because the light that imagery captures is affected by the index of refraction of the medium it travels through. Since
index of refraction is affected by temperature, the index of refraction of the medium will be a random distribution as well. When the volume over which the index of refraction is uniform, the pockets of air are referred to as eddies. The eddies’ size and position can be spatially represented using the Power Spectral Density (PSD) of the random index of refraction [14].

In the proposed model, this PSD is calculated and used to characterize the phase screens. There are several different forms of the PSD which originate from Kolmogorov theory. We use the Modified von Kármán (MVK) PSD [14].

\[
S_{\phi}^{\text{MVK}}(\rho) = \frac{0.0333 e^{-\rho^2/\rho_m^2}}{(\rho^2 + \rho_0^2)^{11/6}} \text{C}_n^2, \tag{2.19}
\]

where \(\rho_0 = \frac{1}{L_0}\) and \(\rho_m = \frac{5.92}{2\pi l_0}\). The MVK PSD equation is in terms of the radial spatial frequency, \(\rho = \sqrt{u^2 + v^2}\). The parameters \(l_0\) and \(L_0\) are defined as the inner and outer scales, respectively.

The figure below demonstrates how eddies are represented in the atmosphere in terms of inner and outer scales [5].

The outer scale defines the largest possible size of an eddy before it breaks down into smaller eddies. The inner scale gives the smallest possible size of an eddy before it dissipates into energy. When compared to other PSDs, the MVK proves to be more flexible with varying experiments. This is due to the MVK having no constraints in spatial frequencies for the PSD to be valid [5]. The most important parameter in the MVK PSD is the Refractive Index Structure Parameter, \(\text{C}_n^2\). The \(\text{C}_n^2\) characterizes the degree of variation of refractive index over a given region. This key parameter influences all of the turbulence statistics derived for the model.

Note that \(\text{C}_n^2\) is actually constant as presented in equation 2.4. This holds true when the region of interest is over a horizontal path. When altitude is introduced, the \(\text{C}_n^2\) varies with respect to height, \(h\), above ground. There are several profiles that have been established to estimate \(\text{C}_n^2(h)\).
The most commonly used profile is the HVTP \cite{9},

\[ C_n^2(h) = 0.00594 \left( \frac{\nu}{27} \right)^2 \left( 10^{-5} h \right)^{10} e^{-\frac{h}{1000}} + 2.7 \times 10^{-16} e^{-\frac{h}{1500}} + A e^{-\frac{h}{1000}}, \]  

(2.20)

which we have implemented in the simulation. The parameter \( A \) represents the turbulence strength at ground level while \( \nu \) is the high altitude wind speed. The most commonly used HVTP is the HV5/7 which sets \( A = 1.7 \times 10^{-14} \text{ m}^{-2/3} \) and \( \nu = 21 \text{ m/s} \).

One of several different ways of representing turbulence in simulations is through using phase screens. The proposed model splits the simulation into layers of turbulence along the scope of the experiment. Each layer of turbulence volume is replaced by an equivalent thin phase screen. The thickness of the phase screen must be much less than the propagation distance to be considered thin.
Phase screens are a single instance of random phase perturbations of the atmosphere in a given plane.

We will go further into detail of generating phase screens in Sect. 3.1. First, it’s important to address several key parameters that characterize atmospheric turbulence in optics. Perhaps the most important for the simulation is the atmospheric coherence diameter, $r_0$

$$r_0 = \left[ 0.423k^2 \int_{z=0}^{z=L} C_n^2(z) \left( \frac{z}{L} \right)^{5/3} dz \right]^{-3/5}$$

with $L$ as the total distance of the optical path.

The term $r_0$, also known as Fried’s parameter, is the diameter (meters) of the circular area where the root mean square (RMS) wavefront aberrations from atmospheric affects equals 1 radian. It acts as a “knee” in the optical resolution curve for a camera aperture affected by turbulence. If $r_0$ is less than the camera’s aperture diameter, resolution is affected more by turbulence. For $r_0$ greater than the aperture size, optical diffraction is a greater influence in resolution.

Another important parameter in defining atmospheric turbulence in an optical system is log-amplitude variance, $\sigma^2 \chi$

$$\sigma^2 \chi = 0.563k^{7/6}L^{5/6} \int_{z=0}^{z=L} C_n^2(z) \left( \frac{z}{L} \right)^{5/6} \left( 1 - \frac{z}{L} \right)^{5/6} dz.$$ 

The parameter $\sigma^2 \chi$ captures the variance in the amplitude of the wavefront due to the atmosphere. It is used as a reliable measure of the strength of scintillation in optics.

The final turbulence parameter to be addressed is the isoplanatic angle, $\theta_0$

$$\theta_0 = \left[ 2.91k^2L^{5/3} \int_{z=0}^{z=L} C_n^2(z) \left( 1 - \frac{z}{L} \right)^{5/3} dz \right]^{-3/5}.$$ 

The term $\theta_0$ is defined as the largest field angle (radians) over which turbulence effects are approximately the same. Viewing angles larger than the isoplanatic angle will have anisoplanatic issues. While the isoplanatic angle is not only useful for generation of the phase screens, it is also
important to note its influence in modeling anisoplanatic effects. The proposed model creates PSFs for each pixel in order to stay within the scope of the isoplanatic angle, so that turbulence effects are correctly represented. This process is explained in further detail in Section 3.2.2.
CHAPTER III

NUMERICAL SIMULATION

In this chapter, we present the observation model. There are many steps we take to generate video containing anisoplanatic atmospheric effects from turbulence:

1. Create sinc-Gaussian based point source. Ensure that the point source is generated such that a uniform distribution covers the scope of the pupil after propagation (Section 2.1).

2. Generate large MVK phase screens by filtering random white Gaussian noise with the MVK PSD (Section 3.1.1). Locations for phase screens along the optical path are optimally determined using Wallner's method (Section 3.1.2). Individual phase screen $r_{ij}$s, used for calculating the MVK PSDs, are derived by finding the optimal solution to an under-determined system of equations (Section 3.1.3). The phase screens’ size should encompass the scope of the entire anisoplanatic process including wind speed for video (Section 3.2.1).

3. Adjust phase screen size for a single frame process. This includes phase shifting to the appropriate frame’s ROI (Section 3.2.1), cropping to minimum phase screen size (Section 3.2.2) and zeroing data in phase screens outside the scope of the anisoplanatic technique (Section 3.2.2).
4. For each unique PSF, use ASP to propagate the point source from a grid of points at the object through the appropriate portions of the phase screens to the camera aperture, apply a pupil mask to the resulting perturbed wavefront and FFT to get the PSF (Section 3.2.3).

5. Repeat step 4 for every pixel in the image (or every \( n \)th pixel if pixel skip is enabled; see Section 3.2.2 for “pixel skip” description) to determine PSFs.

6. Compute a weighted sum with each PSF and the area of the corresponding pixel to determine new intensity values for each pixel in the final perturbed image. Save the new perturbed image and apply wind speed as a phase shift in the Fourier domain to set a new ROI of the large phase screens for the next frame in the video (Section 3.2.2).

7. Repeat steps 3-6 for each frame until full video is compiled.

In the following sections we will cover the finer details for these steps in the numerical model.

### 3.1 Phase Screen Generation

To start, we must setup several important components of the model. These include generating the point source described in Section 2.1, setting up mesh grids with the sample spacing conforming to the CSC listed in Section 2.2.1, and generating the phase screens using the turbulence parameters introduced in Section 2.3. There are many moving parts in generating and manipulating phase screens. The flowchart in Figure 3.1 demonstrates the process for designing phase screens.

In the following paragraphs we will address our methods of optimizing phase screen locations, deriving individual phase screen statistics, and creating the phase screens using the MVK PSDs.

#### 3.1.1 PSD Filtering Approach

The steps taken in designing a single phase screen should first be addressed. As discussed in Section 2.3, the MVK PSD is used in the simulation for characterizing the phase screens. We
implement Schmidt’s modified version \([2]\) of equation (2.19) for Fried’s parameter to get

\[
S_{\varphi}^{\text{MVK}}(\rho) = \frac{0.49 e^{-\rho^2/\rho_0^2}}{r_0^{5/3} \left( \rho^2 + \rho_0^2 \right)^{11/6}}.
\]  

(3.1)

This version is easier to implement since \(r_0\)’s are calculated for the MVK PSD of each phase screen along the path (described in further detail in Section 3.1.3). An example MVK PSD is shown in Figure 3.2 using the parameters from Appendix A.

The proposed method is slightly different than the traditional phase screen generation method using FFTs \([2]\). Using the CSC sample spacing \(\Delta x\), the sampling frequency of the phase screen is defined as \(f_s = 1/\Delta x\) cycles/meter. The discrete (or sampled) version of \(S_{\varphi}^{\text{MVK}}(\rho)\) is then easily found using equation (3.1). The random draws using the Gaussian distribution are generated with unit variance and scaled by \(f_s\) to ensure the samples are consistent with a MVK PSD value of unity.
within the simulation. These random samples are then filtered with the square root of the MVK PSD to generate a random distribution of samples from the MVK PSD pattern. The result is the random phase, \( \phi(x, y) \), used to create the phase screens. The phase screens are setup to act like thin lenses with a transmission function of \( t(x, y) = Ae^{j\phi(x, y)} \). This makes it simple to apply the phase fluctuations to the wavefront as it’s propagated by simply scaling with the transmission functions of the phase screens. Note that we ignore amplitude fluctuations throughout the simulation, setting \( A = 1 \), while focusing only on the phase fluctuations in the manipulated wavefront. Figure 3.3 provides a representation of the phase, \( \phi(x, y) \), for a MVK phase screen using the MVK PSD shown in Figure 3.2.
3.1.2 Determining Optimal Phase Screen Locations

Locations of phase screens are critical for a simulation along a slanted path. For the horizontal path case, it is not as important since the stagnant elevation provides a constant $C_n^2$ when using the HVTP. When altitude is introduced, $C_n^2$ varies respectively with height as shown in Figure 3.4 and described in Section 2.3.

Since turbulence does not vary linearly with altitude, it is not ideal to place the phase screens at equidistant locations. For this matter, Wallner’s technique [15] is implemented for optimally placing phase screens along an optical path. Wallner provides two conditions for optimum layer and section
Here is the text converted into a natural plain text representation:

Figure 3.4: Huffnagel Valley Turbulence Profile

heights:

\[ \int_{h_i}^{H_i} C_n^2(h)(h - h_i)^{2/3} \, dh = \int_{H_{i-1}}^{h_i} C_n^2(h)(h_i - h)^{2/3} \, dh \]  

(3.2)

and

\[ H_i = \frac{h_i + h_{i+1}}{2} \]  

(3.3)

for \( i = 1, 2, \ldots, n \) with \( n \) number of phase screens. Layer locations are defined as \( H_i \) and sections are located at \( h_i \). Since the simulation splits the scope of process into layers, we can easily apply these conditions to find optimal phase screen locations. By setting initial layer and phase screen locations, the conditions can be used to find the remaining locations. The first layer, \( H_1 \), will always be zero. However, \( h_1 \) must be chosen to provide a final layer boundary at the distance of the experiment. Therefore, a binary recursive search is used to find the \( h_1 \) guess that places the final layer boundary.
at the propagation distance, $\Delta z$. Once the height locations for each phase screen are determined, simple geometry is used to determine the corresponding locations, $z_i$, of the phase screens along the optical path.

### 3.1.3 Finding Individual Phase Screen Statistics

Finding statistics of each phase screen in a simulation is often referred to as an art rather than a science. While there are many methods in prescribed literature, the proposed simulation implements a traditional method common to astronomical imaging scenarios. This method generates phase screen statistics that represent the bulk turbulence parameters introduced in Section 2.3. In this case, the statistic we choose for generating each of the phase screens is Fried’s parameter, $r_{0_i}$, for $i = 1, 2, \ldots n$.

For the proposed method, we follow a modified version of Schmidt’s process for determining each of the $r_{0_i}$s [2]. The isoplanatic angle has been added as another constraint to achieve better estimation of the $r_{0_i}$s. The constraining equations from the global turbulence parameters must first be determined. The global turbulence equations introduced in the theoretical Section 2.3 are discretized to statistics at each phase screen, $n$:

\[
r_0 = \left[ 0.423k^2 \sum_{i=1}^{n} C_{n_i}^2 \left( \frac{z_i}{L} \right)^{5/3} \Delta z \right]^{-3/5}, \quad (3.4)
\]

\[
\sigma_x^2 = 0.563k^{7/6}L^{5/6} \sum_{i=1}^{n} C_{n_i}^4 \left( \frac{z_i}{L} \right)^{5/6} \left( 1 - \frac{z_i}{L} \right)^{5/6} \Delta z, \quad (3.5)
\]

and

\[
\theta_0 = \left[ 2.91k^2L^{5/3} \sum_{i=1}^{n} C_{n_i}^2 \left( 1 - \frac{z_i}{L} \right)^{5/3} \Delta z \right]^{-3/5}. \quad (3.6)
\]

The term $\Delta z$ represents the thickness of the extended turbulence from the $i$th phase screen while $z_i$ are the locations of the phase screens gathered from the method described in Section 3.1.2. Using the definition [2]

\[
r_{0_i}^{-5/3} = 0.423k^2C_{n_i}^2 \Delta z, \quad (3.7)
\]
these equations are then adjusted to be defined in terms of $r_0$:

\[
\[ r_0 = \left[ \sum_{i=1}^{n} r_0_i - \frac{5}{3} \left( \frac{z_i}{L} \right)^{5/3} \right]^{-3/5}, \tag{3.8} \]
\]

\[
\sigma^2 = 1.33 k^{-5/6} L^{5/6} \sum_{i=1}^{n} r_0_i - \frac{5}{3} \left( \frac{z_i}{L} \right)^{5/6} \left( 1 - \frac{z_i}{L} \right)^{5/6}, \tag{3.9} \]

and

\[
\theta_0 = \left[ 6.8794 L^{5/3} \sum_{i=1}^{n} r_0_i - \frac{5}{3} \left( 1 - \frac{z_i}{L} \right)^{5/3} \right]^{-3/5}. \tag{3.10} \]

With the constraints now defined, a minimization technique using MATLAB’s `fmincon()` is applied using the under determined system of equations to find the $r_0_i$s that provide global statistics close to theoretical. Figure 3.5 shows the comparison theoretical global parameters with the discretized bulk parameters calculated from the array of $r_0_i$s.

The statistics were plotted against varying degrees of elevation. The results include both techniques of positioning phase screens using optimized locations and spacing the phase screens equidistantly. The figures indicate that optimizing locations provides better overall estimates than equidistantly spacing the locations. Note that the equal spacing method does work when many additional phase screens are implemented, but this causes the simulation to become computationally inefficient when propagation is implemented.

### 3.2 Anisoplanatic Process

The overall process for including anisoplanatic effects in imagery includes many moving parts. It’s best to begin with the overarching design for creating video with anisoplanatic turbulence effects and then move to describing the remaining processes in further detail. The flowchart in Figure 3.6 shows the method for implementing anisoplanatism in video.
3.2.1 Implementing Wind Speed for Temporal Correlation

One of the inputs that has not been explained yet is the object grid. This grid indicates where each of the point sources are located in the object plane. Since a PSF is generated for each pixel, the number of samples for the grid must be equivalent to the number of samples for the image. Geometrical optics are used to determine the sample spacing of the object grid related to the Nyquist sample spacing provided in the focal plane.
We have demonstrated up to this point what phase screens are and how they are generated. When utilizing the phase screens, spatial and temporal warping from turbulence in video must be correlated. Temporal correlation is implemented in the simulation by creating large rectangled phase screens. The length of the screens must be large enough to encompass a single frame process which will be discussed at length later. The width is determined by the video time, \( t \) (sec), the number of frames per second, \( n_{fps} \), and the wind speed, \( v_w \) (m/sec). Using the sample spacing \( \Delta x \) from the CSC, the wind speed in pixels is defined as

\[
\upsilon_p = \frac{v_w}{\Delta x n_{fps}}. \tag{3.11}
\]

Note that positive values of \( \upsilon \) can be input to replicate West to East wind motion while negative values indicate the opposite effect. The number of samples for the width of the phase screens is then found to be

\[
N_{XPS} = \lceil N_{YPS} + |\upsilon_p| n_{fps} t \rceil. \tag{3.12}
\]

The notation \( \lceil \rceil \) is mathematically used to indicate rounding up to the nearest integer.

Using these prescribed metrics, each of the phase screens is generated as large rectangles with sample sizes \( N_{XPS} \times N_{YPS} \) and spacing \( \Delta x \). For each frame in the video, the appropriate ROI of
the rectangled phase screens is cropped in the size of $N_{ypS} \times N_{ypS}$. This ROI is determined using a phase shift in the Fourier domain,

$$\phi_n(x, y) = \mathcal{F}^{-1} \{ e^{-j2\pi v_p \Delta x (n-1)u} \mathcal{F}\{\phi_0(x, y)\} \}. \quad (3.13)$$

The term $\phi_0(x, y)$ is the phase for the first frame and $\phi_n(x, y)$ is the phase for the $n$th frame.

### 3.2.2 Single Frame Process

A flowchart detailing the single frame anisoplanatic design is shown in Figure 3.7.

There are several final adjustments to be made to the phase screens before they can be implemented into the generating PSFs process. Note again that the simulation must demonstrate correlated spatial and temporal warping from turbulence. The temporal correlation is accounted for by implementing wind speed into the video process. Spatial correlation is represented through the technique used for integrating anisoplanatic effects in the simulation. To help explain this technique, Figures 3.8.a-c show an example of manipulating three phase screens for applying anisoplanatism.
Figure 3.8: Phase Screen Manipulation for Anisoplanatic Imaging Technique: (a) Determine phase screen sizes needed for scope of simulation, (b) Zero out unused information in phase screens for efficient computation, (c) Crop ROI of phase screens along the optical path from \((i,j)\) to the pupil plane.

As shown in Figure 3.8a, the minimum phase screen size is the size of the object grid plus the pupil plane width. This width can be found using Equation (2.9) for the CSC with \(D_p\) as the pupil plane size. The parameter \(N_{Y_{PS}}\) mentioned earlier must provide at least the minimum phase screen size for the anisoplanatic technique to work correctly. Note that it is preferred to use a larger number
of samples for $N_{\text{PS}}$ to initially create the phase screens and then crop to the minimum phase screen size. This gives a better representation of low spatial frequencies when generating the phase screens. Also, it is important to clarify that the phase screens in Figure 3.8 are equidistantly spaced for explanation purposes only. The phase screens are optimally located using method described in Section 3.1.2.

Once the phase screens are at minimum phase screen size, the unused information is zeroed out (Figure 3.8b). The ROIs for the anisoplanatic technique can be geometrically determined by locating the maximum extent of the simulation when propagating from each corner of the object grid. The phase screens do not actually change in size during this process.

Figure 3.8c presents the most important visual of the group. For each unique path from location $(i, j)$ in the object grid to the center of the pupil plane, a set of locations is found where the optical path crosses through each phase screen. These locations are used as the centers of the ROIs for cropping smaller phase screens used during propagation. The size of these smaller phase screens is equivalent to the pupil plane width. With the appropriate portions of phase screens extracted, the PSF for the unique $(i, j)$ path can be generated. Note that this technique will cause nearby pixels to have overlapping portions of the smaller phase screens used for propagation. This is where the spatial correlation in turbulence is found in the anisoplanatic technique as demonstrated in Figure 3.9.

After the PSF for the $(i, j)$ path is generated, it must be interpolated down to the Nyquist sample spacing in the focal plane and be peak normalized before being applied. Additionally, scintillation is removed since amplitude fluctuations are ignored during the propagation process. Remember that this is because $A = 1$ when the transmission functions of the phase screens are created.

This process is repeated for every point in the object grid until a stack of PSFs is stored for application to the image. After interpolation, only a small area of the PSF is non-zero, so the center
of it can be cropped to conserve storage space. Each PSF represents unique warping effects from propagating through the phase screens for every pixel in the image. To apply the PSF effects to each pixel of the image, a back projection is performed. For example, if each PSF is 35x35, the PSF is scaled and summed with the 35x35 area of intensity values from the image with the corresponding pixel as the center. The result of the partial convolution is the new pixel value for the perturbed image.

For the proposed model, the total processing time has been decreased by skipping every $n$ points in the object grid. This respectively generates PSFs every $n$ pixels. The remaining PSFs are estimated using bilinear interpolation and then applied in the same manner as before.

### 3.2.3 PSF Generation

The process for generating PSFs is demonstrated in the flowchart shown in Figure 3.10. The two inputs are the cropped phase screens from the method shown in Figure 3.8c and the point source.

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3 Image by Dr. Russell Hardie, University of Dayton, used here with permission
described in section 2.1. Since scintillation is ignored, every optical path from an object grid point \((i, j)\) to the center of the aperture is treated as a completely horizontal propagation. This concept is visualized in Figure 3.11.

By treating every propagation from the object to the aperture as horizontal, the complex anisoplanatic technique becomes the simple process shown in Figure 2.1b repeated for every pixel (or every \(n\) pixels for faster speed). A generated spherical wave point source propagates through each phase screen. An example of this process affecting the wavefront can be viewed in Figure 3.12 using parameters from Appendix A.

Propagation is performed using ASP defined in section 2.2.2. Each of the phase screens is represented as transmission functions in the form of \(t(x, y) = e^{j\phi(x,y)}\) and calculated using the various methods prescribed in multiple previous sections. The wavefront \(u(x, y)\) is perturbed by scaling by the phase factors from ASP in the Fourier domain and multiplying with the transmission
Figure 3.11: Demonstration of ignoring tilt in anisoplanatic method: (a) Line of sight propagation scope, (b) Horizontal propagation scope after removing tilt.  

function for each phase screen at each step along the optical path. After the final propagation to the aperture, the perturbed spherical wavefront is collimated to transform the spherical wave into an incident plane wave. This allows for Fourier optics to be implemented when the wavefront is propagated the remaining distance to the focal plane as in a camera imaging system [10]. After collimation, an aperture mask is applied to the incident wavefront. The PSF is finally obtained by taking a Fourier transform of the intensity of the masked wavefront. An example PSF with turbulence effects is shown in Figure 3.13 using parameters provided in Appendix A.

Image by Dr. Russell Hardie, University of Dayton, used here with permission.
Figure 3.12: Demonstration of Wavefront Propagating through Phase Screens: (a) Wavefront at first phase screen, (b) Wavefront at second phase screen, (c) Wavefront at third phase screen, (d) Wavefront at aperture

The characteristics of the PSF demonstrate the expected, turbulence effects. Careful observation shows that the perturbed center of the PSF is moved indicating warping effects. The PSF also shows scattering effects as energy spreads outwards from the center of the plot.
Figure 3.13: Example PSF with Turbulence Effects: (a) Top-down view of PSF, (b) 3D plot of PSF
CHAPTER IV

RESULTS

The experimental parameters used to test the slanted path capability of the model can be viewed in Appendix A. There are several metrics used for verifying and testing the simulation. Theoretical long exposure (LE) and short exposure (SE) PSFs are compared to averaged realizations of simulated PSFs. In addition, the theoretical RMS AoA is calculated in terms of pixel tilt displacement and compared to simulated results. For observational purposes error statistics between many realizations of the true and perturbed images were also averaged and compared over varying experiments.

The parameters listed in Appendix A are adopted from actual experiments used for optical tests. The model has been tested over varying degrees of slant path from $0^\circ$ horizontal along the ground to $90^\circ$ vertical through the atmosphere. In this case the object on the ground remains stationary while the camera changes position along varying slant paths. The only thing that changes in these circumstances is the height; the propagation distance stays the same throughout the testing. The specific angles chosen for this experiment are $2^\circ, 4^\circ, 6^\circ, 8^\circ, 10^\circ, 20^\circ, 30^\circ$ and $90^\circ$. There are more angles closer to horizontal since the paths will have more turbulence when closer to the ground. There are also fewer angles above $30^\circ$ since the simulation becomes diffraction limited around $25^\circ$ where turbulence is less of a factor. The horizontal $0^\circ$ case, left out of this experiment, contains very severe turbulence over a long distance causing the model to perform worse when simulated.
with the given parameters. This could be improved with additional phase screens which will be demonstrated as well. However, simulation parameters are kept consistent in order to compare performance metrics over varying degrees of altitude.

4.1 Validation

Theoretical SE and LE PSFs are calculated by multiplying the Atmospheric Transfer Function (ATF) with the diffraction limited Optical Transfer Function (OTF) and applying an inverse Fourier transform to the product. The ATF is defined as

\[ H_{\text{atm}}(\rho) = \exp\left\{-6.88\left(\frac{\rho D}{2\rho_c r_0}\right)^{5/3}\left[1 - \alpha\left(\frac{\rho}{2\rho_c}\right)^{1/3}\right]\right\} \]  \hspace{1cm} (4.1)

where \(\alpha\) is 0 for LE with no tilt correction and 1 for SE with no scintillation. The parameter \(\rho_c\) is the diffraction cutoff frequency and \(\rho\) is the radial spatial frequency as previously defined. The OTF is listed as

\[ H_{\text{dif}}(\rho) = \frac{2\pi}{\cos^{-1}\left(\frac{\rho}{2\rho_c}\right)} - \frac{\rho}{2\rho_c} \sqrt{1 - \left(\frac{\rho}{2\rho_c}\right)^2} \]  \hspace{1cm} (4.2)

for \(\rho \leq \rho_c\).

PSFs for each realization are determined by implementing the PSF generation process detailed in section 3.2.2. Average SE PSFs are found by removing tilt and averaging 1000 realizations of PSFs. Average LE PSFs were determined by averaging 1000 realizations of PSFs without removing tilt. The resulting PSFs over varying degrees of elevation are shown in Figures 4.1-4.4.

The plots demonstrate good agreement between theoretical and simulated PSFs even in the more severe turbulence cases. As described in section 2.3, the Fried parameter determines the degree of influence of turbulence in imagery relative to the camera’s aperture. In this case, the aperture size is 0.2101 (m). Referring to Figure 3.5, \(r_0\) becomes greater than the aperture around 25° indicating the simulation becomes more diffraction limited than turbulence limited. The PSFs are consistent with
Figure 4.1: Comparison of SE PSFs across Lower Angles of Elevation

this deduction since there is little to no variation between the simulated and diffraction PSFs after 20\(^\circ\).

Another method of validation is comparing the RMS AoA for simulated and theoretical results. Theoretical RMS AoA can be calculated using [17]

\[
\psi_{RMS} = \sqrt{2.91D^{-1/3} \int_{z=0}^{z=L} C_n^2(z) \left(\frac{z}{L}\right)^{5/3} dz}.
\]  

Equation (4.3) is in units of radians. Using geometrical optics, the result can be converted to pixel displacement for easier comparison to simulated data. Simulated RMS AoA is calculated over 1000
realizations by averaging the tilt that is removed from the SE PSFs for each case of elevation. The comparison of theory and simulation demonstrate favorable agreement as shown in Figure 4.5. Note that for the PSFs there is still some degree of error between simulation and theory in lower angles of elevation. This can be attributed to inaccurately representing more severe turbulence with an inadequate number of phase screens. If the amount of phase screens implemented is increased, the outcomes prove to be more favorable. An analysis of improvements in tripling phase screen number at 2° can be seen in Figure 4.6.

Figure 4.2: Comparison of SE PSFs across Higher Angles of Elevation
The SE PSF barely changes while the LE PSF shows more agreement with the theoretical. The trade-off with improved performance is the requirement of more computational time. Further analysis of the optimal number of phase screens can be viewed in [1].

4.2 Studying Error Statistics

With the model validated against theoretical means, attention can be given to the effects of turbulence in imagery. The image used for experimentation is the camera man image provided
by MATLAB (Fig 4.7). Single realizations of perturbed images using the prescribed numerical simulation with the parameters given in Appendix A are shown in Figures 4.8 and 4.9.

As expected, the perturbed images demonstrate varying effects of turbulence depending on the slant path case. Lower angles demonstrate severe warping and blurring due to turbulence while higher angles again indicate diffraction as a limiting effect. However, it is difficult to see the warping effects in imagery in higher angles with single frames. Our simulated videos demonstrate warping effects even at 90°.
Additional metrics are used to analyze the error produced in imagery across different slant paths. The mean squared error (MSE), mean absolute error (MAE), signal-to-noise ratio (SNR) and peak signal-to-noise ratio (PSNR) are all common measurements of error implemented in image analysis. These metrics are calculated for 50 independent realizations and then averaged to obtain the representative error along each slant path in the experiment (Figure 4.10).

The error statistics point to the expected effects resulting from turbulence. The MSE and MAE plots show the decrease in error as the angle of elevation increases. PSNR and SNR results indicate better performance as the path crosses through lower turbulence at higher angles of elevation. These
results are all consistent with the intuitive understanding of stronger turbulence effects closer to ground level.
Figure 4.7: True Image Used for Experimentation
Figure 4.8: Images Distorted from Atmospheric Turbulence across Lower Angles of Elevation
Figure 4.9: Images Distorted from Atmospheric Turbulence across Higher Angles of Elevation
Figure 4.10: Error Metrics between Perturbed and True Images across Varying Degrees of Elevation: (a) Mean Absolute Error, (b) Mean Squared Error, (c) Peak Signal-to-Noise Ratio and (d) Signal-to-Noise Ratio
A model for simulating atmospheric turbulence with anisoplantic effects for imaging across slanted paths has been described and tested. Anisoplanatic effects from turbulence are accounted for by generating a representative PSF for each pixel in an image. Each PSF is generated by creating a spherical wave, sinc-Gaussian point source, propagating through phase screens to the aperture, collimating to an incident plane wave, cropping the aperture mask and taking an FFT to propagate to the focal plane. Scintillation is removed from the PSFs since amplitude fluctuations from turbulence are ignored. The resulting stack of PSFs are each scaled and summed with the corresponding area of the pixel for the perturbed image pixel values. Perturbed videos are simulated by creating large, rectangular phase screens and shifting to new ROIs to repeat the single image process for each frame. Phase screens are created using the MVK PSD filtering method. Phase screen locations are determined using a method of optimizing the layers each phase screen represents with recursive binary using several constraints. Individual phase screen statistics are accurately estimated by implementing a minimization technique with the bulk turbulence parameters as constraints. Results show the validity of the simulation by comparing the averages of many independent realizations of SE PSFs, LE PSFs and RMS AoA to theoretical calculations over varying degrees of elevation. Additionally, perturbed images and error statistics from the simulation demonstrate realistic effects from atmospheric turbulence.
While proposed methods of verifying the simulation were successful, other metrics such as structure functions and differential tilt variance [18] will also be implemented to further validate the model. Additional studies will also be performed to observe functionality over varying wavelengths in the optical spectrum as well as possibly including amplitude effects in future versions of the simulation. Plans are currently in place for implementing the model to aid in developing turbulence mitigation algorithms in the near future.
BIBLIOGRAPHY


APPENDIX A

EXAMPLE SIMULATION PARAMETERS

Table 1.1: Simulation Parameters

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