SUPPRESSION OF MOIRÉ PATTERNS IN DIGITAL HOLOGRAPHY

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SUPPRESSION OF MOIRÉ PATTERNS IN DIGITAL HOLOGRAPHY

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ABSTRACT

SUPPRESSION OF MOIRÉ PATTERNS IN DIGITAL HOLOGRAPHY

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Digital holography, which utilizes semiconductor sensors rather than sensitized films as the recording medium, is a very attractive and convenient approach in today’s holography technology. On one hand, coherent interference holograms usually contain high spatial frequency fringes as part of the phase information. On the other hand, the spatial resolution of digital holography is restricted by the limitations of imaging devices, such as the pixel size. The Moiré effect, a pattern deformation effect emerging from image undersampling, occurs and vitiates the quality of the reconstructed image. In this work, the formation of Moiré patterns, which is the result of Moiré effect, is systematically studied in both Fresnel and Fraunhofer regions. Quantitative descriptions of Moiré patterns are established for both plane wave source and point source holography. Experimental results show that the locations of Moiré patterns can be predicted with high accuracy. Meanwhile, both theoretical analysis and experiments demonstrate that ultra-short pulse illumination
can be utilized to eliminate Moiré patterns efficiently. Moreover, the formation of Moiré patterns in imaging systems under the influence of aberrations is studied. Quantitative descriptions are also established. The potential application of Moiré patterns in the determination of aberration for imaging systems is also proposed.
Dedicated to my parents, friends, 

and the memorable two years at the University of Dayton.
ACKNOWLEDGMENTS

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<th>Description</th>
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<tr>
<td>1D</td>
<td>1-dimensional</td>
</tr>
<tr>
<td>2D</td>
<td>2-dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>3-dimensional</td>
</tr>
<tr>
<td>ANDi</td>
<td>all-normal-dispersion</td>
</tr>
<tr>
<td>CCD</td>
<td>charge-coupled device</td>
</tr>
<tr>
<td>CW</td>
<td>continuous wave</td>
</tr>
<tr>
<td>DH</td>
<td>digital holography</td>
</tr>
<tr>
<td>DIH</td>
<td>digital in-line holography</td>
</tr>
<tr>
<td>GVD</td>
<td>group-velocity dispersion</td>
</tr>
<tr>
<td>HWP</td>
<td>half-wave plate</td>
</tr>
<tr>
<td>MOS</td>
<td>metal-oxide-semiconductor</td>
</tr>
<tr>
<td>MO-SF</td>
<td>microscope objective-spatial filter</td>
</tr>
<tr>
<td>NPE</td>
<td>nonlinear polarization evolution</td>
</tr>
<tr>
<td>PBS</td>
<td>polarizing beamsplitter</td>
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<tr>
<td>QWP</td>
<td>quarter-wave plate</td>
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<tr>
<td>SMF</td>
<td>single mode fiber</td>
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WDM wavelength-division multiplexer
LIST OF SYMBOLS

$A$  

amplitude of optical field

$d$  

diameter of fiber

$E$  

optical field

$f$  

focal length

$f_s$  

sampling rate

$f_c$  

highest frequency in the signal

$g_{PSF}$  

point spread function

$h$  

spatial impulse response for propagation

$I$  

intensity distribution

$k_0$  

propagation constant

$k_x, k_y$  

spatial angular frequencies along $x$ and $y$ directions

$p$  

pixel size

$R_1$  

radius of circular block

$s$  

fringe spacing

$T$  

pulse duration

$T_{fringe\_space}$  

space between the interference fringes
<table>
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$w_{040}$</td>
<td>spherical aberration coefficient</td>
</tr>
<tr>
<td>$x_{\text{Moiré pattern}}$</td>
<td>position of Moiré pattern</td>
</tr>
<tr>
<td>$z$</td>
<td>recording distance</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>reconstructed optical field</td>
</tr>
<tr>
<td>$\theta$</td>
<td>angle of inclination</td>
</tr>
<tr>
<td>$\rho$</td>
<td>distance between origin and a point on observation</td>
</tr>
<tr>
<td>Plane</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>spherical aberration</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>oscillation phase</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>central wavelength of illuminating beam</td>
</tr>
<tr>
<td>$\psi$</td>
<td>phase of optical field</td>
</tr>
<tr>
<td>$\omega$</td>
<td>instantaneous (temporal) angular frequency</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>central frequency of illumination beam</td>
</tr>
<tr>
<td>$\nabla^2$</td>
<td>Laplacian operator</td>
</tr>
<tr>
<td>$\mathcal{F}$</td>
<td>Fourier transform operator</td>
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CHAPTER 1
INTRODUCTION

The topic of this thesis is concerned with the study of two methods to suppress the unwanted Moiré patterns in digital holography (DH). The suppression of Moiré patterns in digital hologram using pulsed illumination during recording is first investigated in details in the thesis. A second method to suppress Moiré patterns by exploiting the aberration in imaging systems is proposed.

1.1 Organization of the Thesis

The thesis starts with the introduction to the research area in Chapter 1. The development of digital (in-line) holography (DIH) is described. Furthermore, a summary of the harmful Moiré effects during the recording and reconstruction processes is presented.

In Chapter 2, the theoretical background of this thesis is introduced, including the formation of the Moiré patterns on the recording charge-coupled device (CCD) array, the foundations of DH, and the operational principle of the all-normal-dispersion (ANDi) femtosecond fiber laser.
In Chapter 3, the suppression of the Moiré patterns in digital holograms using pulsed illumination is discussed. A one-dimensional (1D) object such as a long optical fiber and a 1D recording geometry is initially considered. First, the far-field approximation is introduced to the recording geometry to simplify the results and locate the positions of the Moiré patterns. Simulation results based on the expression with specific functions are demonstrated. The analysis is extended to the general case where the far-field approximation relaxed. Both simulations and experimental results are presented in this Chapter, which is followed up with the property of angular dependence of the number of Moiré fringes. Finally, the analysis is extended to a 2-dimensional (2D) object, e.g., a small circular block.

In Chapter 4, aberration and its effects on the Moiré patterns in imaging systems are considered. A comparison between a single lens and a confocal lens system is presented with and without aberrations. In order to facilitate the mathematical analysis with aberration, only spherical aberrations are considered in the thesis.

Chapter 5 concludes the thesis, along with proposals for future work.

1.2 Introduction

The concept of holography, known as recording the entire optical field with both the amplitude and the phase information, was first introduced by Gabor [1–3]. In traditional holography, photographic plates or films are usually used to record the interference pattern between the wave scattered or reflected by an object and the reference wave. The entire
information of an object can be reconstructed successfully by illuminating with the same reference beam [1, 4, 5]. Instead of recording the hologram through the recording medium such as a film, in DH, a CCD camera is utilized to record the intensity distribution of the interference pattern between two waves electronically. The 3-dimensional (3D) information of the object is reconstructed numerically using the standard diffraction formula [2].

DH, especially in the in-line configuration, is useful for a variety of applications including biomedicine, biophotonics [6–9], nanophotonics and industrial metrologies. DH has shown excellent performance in achieving micron level 3D images with relatively low cost, high speed recording and high resolution hologram [10], compared to off-axis methods [11–13]. However, unwanted Moiré patterns are observed during in-line DH recording and its reconstruction due to the interaction of the fringe patterns and the periodic structure of detectors on CCD devices [14, 15]. In order to suppress these harmful Moiré patterns, a spatially incoherent source has been used [16, 17]. Alternatively, pulsed laser illumination has been demonstrated as an efficient means to eliminate the Moiré effects [18, 19].

In this thesis, the intensity and location of the Moiré patterns are systematically studied and well predicted in both Fresnel and Fraunhofer regions. It is demonstrated that the Moiré patterns can be effectively eliminated by utilizing pulsed laser illumination, for both 1D and 2D objects and recording geometries. Additionally, the aberration and its effects on the position of Moiré patterns in both single lens and confocal lens imaging systems are
investigated. For the first time to the best of our knowledge, it is shown that Moiré patterns may not exist in single-lens systems for certain ranges of values of the spherical aberration. The potential application of Moiré patterns in the aberration determination of imaging systems is proposed, based on our analysis.
CHAPTER 2
THEORETICAL BACKGROUND

In this Chapter, a brief background Moiré effects is first presented. These effects are known to occur during the recording and reconstruction of digital holograms, especially during CW illumination of the object. The basic principles of holography and digital holography, including recording and reconstruction, are discussed. Since it has been shown that Moiré effects can be eliminated using pulsed illumination of the object, the basic principles of operation of a convenient laser source for use in our experiments, viz., the all-normal-dispersion (ANDi) fiber laser, which can switch between CW and pulsed operation, are discussed.

2.1 Principle of Moiré Effects from a CCD Camera

In order to give a complete explanation about the Moiré effects in a digital recording device, the structure and working principle of CCD are presented at the beginning of this Section. Thereafter, the Nyquist theorem as well as the undersampling phenomenon in the CCD camera are illustrated.
2.1.1 The CCD camera

There are different kinds of technology generally used for light detection, such as consumer and professional digital cameras and active pixel sensors [20,21]. As a solid-state imaging device, the semiconductor pixels of a CCD which are p-doped metal-oxide-semiconductor (MOS) capacitors collect electrical charges in response to the incident photons. Specifically, MOS capacitors and buried channels are created by ion implantation [22]. When image acquisition begins, the CCD converts the integrated charges into a digital value. A CCD is an integrated circuit and made out of a photoactive region and a transmission region. The photoactive region (pixels) are light sensitive elements with a two dimensional capacitor array etched on a silicon surface. When the CCD captures an image, each capacitor of the photoactive region accumulates charges proportional to the light intensity (the photoelectric effect) and transfers to its neighbor. A charge amplifier converts the charge to a sequence of voltages and turns it into a digital copy of the image falling on regularly-spaced CCD cells.

2.1.2 The Nyquist theorem

According to the Nyquist theorem, when a continuous function is sampled to a discrete sequence, the sampling rate should be at least twice the maximum frequency contained in the signal to avoid harmful aliasing, i.e.,

\[ f_s \geq 2f_c. \]  \hspace{1cm} (2.1)
\( f_s \) is the sampling rate and \( f_c \) is the highest frequency in the signal [23]. When sampling with a frequency lower than the Nyquist rate, aliasing arises when a signal is captured and reconstructed improperly with an insufficient sampling rate. For instance, reconstruction from the samples may be adversely affected by the aliasing without knowing the original signal as shown in Fig. 2.1 below. The black curve shows the original signal. Due to the low sampling frequency, the signal can only be reconstructed to the red curve with loss of information based on the limited sample points as shown in the figure. Some methods to prevent aliasing can be adopted, such as a higher sampling rate or pre-filtering to form band limited signal. In this thesis, an effective method to remove the unwanted aliasing (Moiré patterns) is proposed, by employing ultra-short pulse illumination.

![Figure 2.1: Under-sampling and its effect on signal reconstruction.](image)

2.1.3 Moiré patterns from a CCD camera

The phenomenon of Moiré patterns is a visualization effect, which can be usually observed by superposing two Ronchi gratings with equal opaque and transparent
regions [24]. In fact, the Moiré phenomenon is very common, and shows up in our daily lives when two transparent objects with repetitive patterns overlap. In general, these Moiré patterns are obtained as a pattern of light and dark lines and shifts when we move the two overlapping objects.

Due to the regular 2D repetitive structure of charged-coupled cell arrays, a CCD camera itself plays the role of one of periodic pattern. When it is used to observe another periodic pattern, the continuous signal is sampled by those discrete pixels of the CCD camera. Then Moiré patterns appear and are captured by the CCD camera itself [15]. The interference between two periodic patterns, such as lines or stripes with periodic structures, is recorded by the CCD camera. According to the Nyquist theorem, under-sampling (Moiré pattern) happens when

\[ p < 2T_{\text{fringe,space}}, \]  

\( T_{\text{fringe,space}} \) represents the space between the interference fringes and \( p \) is the pixel size of the CCD camera. As shown in Fig. 2.2, Moiré patterns are observed when two periodic patterns are overlapped.
Figure 2.2: Two examples of Moiré patterns, shown in the sequence of Figures (a-c) and (d-f). (a) Periodic structure of charged-coupled cell arrays with pixels size $p = 6.7\mu m$. (b) Repetitive fringes with angle $\theta = 2^\circ$. (c) Superposition result from (a) and (b). (d) Periodic structure of charged-coupled cell arrays with $p = 6.7\mu m$. (e) Repetitive fringes with angle $\theta = 5^\circ$. (f) Superposition result from (d) and (e).

The lattices shown in Figs. 2.2(a) and (d) represent the repetitive structure of a CCD camera with pixel size $p = 6.7\mu m$. Fringe patterns in Figs. 2.2(b) and (d) have been plotted with different values of $\theta$ where $\theta$ is the inclination angle between the fringe and the vertical axis. The space between the fringe patterns is chosen to vary from two times of the pixel size on the left and then decreasing to the pixel size on the right. By superposing the CCD structure with fringe patterns in Figs. 2.2(c) and (d), the production of Moiré fringes can be observed. It is interesting to note that the apparent Moiré pattern in Fig. 2.2(b) is a result of sampling of the computer itself.

From Figs. 2.2(c) and (f), the angular dependence of the Moiré patterns is readily observed. The fringe density in the Moiré patterns $\theta$ increases with and the shape
changes. This phenomenon is tested and verified experimentally and reported later in this thesis.

If the period of fringes on the CCD camera $T_{\text{fringe space}}$ is a function of the $x$ or $y$ coordinates, such as the diffraction pattern of a fiber as will be explored later, Moiré patterns will appear if

$$T_{\text{fringe space}} = p.$$

(2.3)

It is also possible to predict the central position of the Moiré fringe patterns by solving the above equation.

### 2.2 Digital Holography

This Section starts with the theoretical background of the Fresnel and Fraunhofer diffraction in DH. Furthermore, the structure of digital in-line holography is presented and the numerical reconstruction by the convolution approach is introduced.

#### 2.2.1 Fresnel and Fraunhofer diffraction

When an optical wave is incident on an object whose dimensions are in the order of a few wavelengths, the intensity pattern behind the object may comprise dark and bright regions instead of its geometrical shadow. According to Huygen’s principle, every point on a wavefront can be considered as a new source point of a secondary spherical disturbance. The phenomenon of diffraction, which is a consequence of the superposition of all these secondary waves, plays an important role in the recording and reconstruction of holograms.
Assuming $E(x,y;z)$ is the optical field at an arbitrary point, it obeys the Helmholtz equation

$$\nabla^2 E(x,y;z) + k_0^2 E(x,y;z) = 0,$$  \hspace{1cm} (2.4)

$\nabla^2$ is Laplacian operator, $k_0$ is the propagation constant, $x$ and $y$ are transverse coordinates and $z$ is the propagation distance. The Fourier transform of Eq. (2.4) is given by

$$\frac{d^2 \tilde{E}(k_x,k_y;z)}{d^2 z} + k_0^2 [1 - (k_x/k_0)^2 - (k_y/k_0)^2] \tilde{E}(k_x,k_y;z) = 0.$$  \hspace{1cm} (2.5)

In Eq. (2.5), $\tilde{E}(k_x,k_y;z)$ is the Fourier transform of $E(x,y,z)$ and expressed as

$$\tilde{E}(k_x,k_y;z) = \mathfrak{F}[E(x,y;z)] = \iint_{-\infty}^{+\infty} E(x,y;z) \exp(j(k_x x + k_y y))
dx dy,$$  \hspace{1cm} (2.6)

$k_x$ and $k_y$ are the spatial angular frequencies. The optical field after propagating a distance $z$ can be found by solving Eq. (2.5) to yield

$$\tilde{E}(k_x,k_y;z) = \tilde{E}(k_x,k_y;0) \exp \left\{ -j k_0 \sqrt{1 - (k_x/k_0)^2 - (k_y/k_0)^2} z \right\}$$

$$= \tilde{E}(k_x,k_y;0) g_{PSF}(k_x,k_y;z).$$  \hspace{1cm} (2.7)

The equation above leads to the well-known Fresnel-Kirchhoff or Rayleigh-Sommerfeld integral [1,2]. Upon applying the inverse Fourier transform on both sides, the optical field can be expressed in spatial coordinates as [25]

$$E(x,y;z) = E(x,y;0) \ast g_{PSF}(x,y;z),$$  \hspace{1cm} (2.8)

where we have,

$$g_{PSF}(x,y;z) = \mathfrak{F}^{-1}\{\tilde{g}_{PSF}(x,y;z)\}$$
\[
\begin{align*}
&= \mathcal{Z}^{-1}\left\{ \exp\left\{ -jk_0 \sqrt{1 - \left( \frac{k_x}{k_0} \right)^2 - \left( \frac{k_y}{k_0} \right)^2} z \right\} \right\} \\
&= jk_0 \left( \frac{z}{\rho} \right) \left( 1 + \frac{1}{jk_0 \rho} \right) \exp\left( -jk_0 \rho \right) \\
&\approx jk_0 \frac{\exp\left( -jk_0 \rho \right)}{2\pi \rho}, \\
\end{align*}
\]

with \( \rho = \sqrt{x^2 + y^2 + z^2} \). In simplifying from the third line of Eq. (2.9) to the last line, it is assumed that the distance between the object and the observation point is usually much larger than the optical wavelength so that \( k_0 \gg 1/\rho \) and \( z \approx \rho \). Thus the simple expression as shown in the last line in Eq. (2.9) can be obtained by substituting these approximations. \( g_{PSF}(x, y; z) \) is the point spread function which represents the wave field diverging from a point source at a distance \( z \).

In Fig. 2.3, the \((\xi, \eta)\) plane is assumed to be the object plane and it is assumed that the object is illuminated with a plane wave traveling along the \( z \) direction. The Huygens-Fresnel principle can be rewritten to calculate the complex wave field at the plane \((x, y)\) as

\[
E(x, y; z) = \frac{jk_0}{2\pi r_{01}} \int_{\text{Aperture area}} E(\xi, \eta; 0) \exp(-jk_0 r_{01}) d\xi d\eta. \quad (2.10)
\]

Figure 2.3: Diffraction geometry [1].
In order to introduce a more simple expression for the Huygens-Fresnel principle, an approximation based on the binomial expansion of square root is applied and given by

$$r_{01} = \sqrt{(x - \xi)^2 + (y - \eta)^2 + z^2} \approx z [1 + \frac{1}{2} \left(\frac{x - \xi}{z}\right)^2 + \left(\frac{y - \eta}{z}\right)^2].$$

(2.11)

Therefore the Eq. (2.10) can be rewritten in the form

$$E(x, y) = \frac{jk_0}{2\pi z} \iint E(\xi, \eta) \exp\left[-jk_0 \frac{(x - \xi)^2 + (y - \eta)^2}{2z}\right] d\xi d\eta$$

$$= \frac{jk_0}{2\pi z} \exp\left[-jk_0 \frac{(x^2 + y^2)}{2z}\right] \iint E(\xi, \eta) \exp\left[-jk_0 \frac{(\xi^2 + \eta^2)}{2z}\right]$$

$$\exp\left[jk_0 \frac{(x\xi + y\eta)}{2z}\right] d\xi d\eta$$

$$= \iint E(\xi, \eta) h(x - \xi, y - \eta) d\xi d\eta,$$

(2.12)

where

$$h(x, y) = \frac{jk_0}{2\pi z} \exp\left[-jk_0 \frac{x^2 + y^2}{2z}\right],$$

(2.13)

In writing the above equations, it has been assumed that the distance in the denominator of Eq. (2.9) can be replaced by $z$.

Equation (2.12) can be further simplified using the Fraunhofer approximation which is given by

$$z \gg \frac{k_0 (\xi^2 + \eta^2)_{\max}}{2}.$$ 

(2.14)

When the approximation is satisfied, the quadratic phase term $\exp\left[-jk_0 \frac{(\xi^2 + \eta^2)}{2z}\right]$ in Eq. (2.12) becomes unity. With the stronger Fraunhofer approximation, the complex optical field at the $(x, y)$ plane now becomes

$$E(x, y) = \frac{jk_0}{2\pi z} \iint E(\xi, \eta) \exp\left[-jk_0 \frac{(x - \xi)^2 + (y - \eta)^2}{2z}\right] d\xi d\eta$$
\[
\frac{jk_0}{2\pi z} \exp \left[ -jk_0 \frac{(x^2+y^2)}{2z} \right] \iint E(\xi, \eta) \exp \left[ jk_0 \frac{(x\xi+y\eta)}{2z} \right] d\xi d\eta. \quad (2.15)
\]

Thus in the region of Fraunhofer diffraction, the expression of the observed field above is simply proportional to the scaled Fourier transform of the optical field distribution at the object plane \((\xi, \eta)\).

### 2.2.2 Digital in-line holography (DIH)

The theory of holography is based on the interference and diffraction of waves, one from the object and one from the reference. A set-up to record holograms usually consists of a light source, lenses and a recording device, e.g. recording medium.

The direct quantity which can be measured by recording medium is the intensity. Considering the interference between two monochromatic waves, the complex amplitudes for each individual can be written as [1]:

\[
E_o(x, y, z) = A_o(x, y) \exp[-j\psi_o(x, y)], \quad (2.16)
\]

\[
E_{ref}(x, y, z) = A_{ref}(x, y) \exp[-j\psi_{ref}(x, y)], \quad (2.17)
\]

\(A_o(x, y)\) is the amplitude of the optical filed from the object on the recording medium and \(\psi_o(x, y)\) represents the phase. Similarly, \(A_{ref}(x, y)\) and \(\psi_{ref}(x, y)\) are amplitude and phase of the reference beam, respectively. When the object and reference waves interfere on the recording material, the intensity of the interference pattern depends on the individual intensities and the phase difference between them, as [2]

\[
I(x, y) = |E_o(x, y) + E_{ref}(x, y)|^2
\]

\[
= [E_o(x, y) + E_{ref}(x, y)][E_o(x, y) + E_{ref}(x, y)]^* 
\]
\[ = I_o(x, y) + I_{\text{ref}}(x, y) + 2\sqrt{I_o(x, y)I_{\text{ref}}(x, y)}\cos[\psi_o(x, y) - \psi_{\text{ref}}(x, y)] \] . (2.18)

Holograms can be on-axis or off-axis. Off-axis [26,27] holography has played an important role in optical metrology system and three dimensional visualization [28,29]. Although off-axis holography can readily eliminate the twin-image problem associated with in-line holography, the resolution limitation of current CCD sensors prevents faithful recording of the hologram if the angle between the object wave and the reference is larger than a few degrees. Furthermore, the restricted reconstructed image results the inefficient use of the CCD pixels in the off-axis setups [30].

The above drawbacks of off-axis digital holography can be overcome using in-line holography which has a simplified recording geometry but suffers from the twin-image problem. The application of in-line setup are holography of transparent (phase) objects such as biological samples like cells or holography of small particles [31,32]. An alternative to in-line holography is phase-shifting digital holography [32].

In DIH, the hologram is again recorded by a CCD camera. Based on the two-dimensional hologram, the three dimensional structure of the original object can be reconstructed numerically through a computer. Not only the intensity distribution, but also the phase distribution can be obtained from the digital hologram. Figure 2.4 shows the fundamental schematic of a DIH system. A (semi-transparent or small) object is placed at a distance \( z \) in front of the CCD. The reference wave and the diffracted wave from the object are coaxial and propagate to the CCD normally. The interference fringe
patterns corresponding to the superposition of the two beams are stored by the CCD camera and reconstructed numerically by using a computer.

![Diagram of in-line setup.](image)

**Figure 2.4: Diagram of in-line setup.**

### 2.2.3 Numerical reconstruction by the convolution approach

By illuminating the hologram with the same reference wave (also called the reading beam), the original object can be reconstructed successfully. In a simple mathematical model, the original object information can be found by [25]

\[
I(x, y)E_{ref}^*(x, y) = |E_o(x, y) + E_{ref}(x, y)|^2E_{ref}^*(x, y)
\]

\[
= I_o(x, y)E_{ref}^*(x, y) + I_{ref}(x, y)E_{ref}^*(x, y)
+ E_o(x, y)E_{ref}^2(x, y) + I_{ref}(x, y)E_o^*(x, y). \tag{2.19}
\]

From Eq. (2.19), it is seen that by multiplying the conjugate of the reference beam, the phase contribution of the reference beam is removed. The term \( I_{ref}(x, y)E_o^*(x, y) \) with the real image information is obtained.

In contrast to analog holography, which reconstructs the object image by physically illuminating the recording medium with the reference beam, the propagation of light
beyond the illuminated hologram is numerically simulated in digital holography to reconstruct the complex field of the object based on the data measured via a CCD camera in DH, as shown in Fig. 2.5.

![Figure 2.5: (a) Hologram recording, (b) reconstruction with $E_{ref}$, (c) reconstruction with $E_{ref}^*$ [25].](image)

The theoretical background for numerical phase and amplitude reconstruction of an object is scalar diffraction theory. The reconstruction methods for Fresnel holograms through Fresnel-transform diffraction formula and the convolution-type diffraction integral have been reported, considering different diffraction properties of the system configurations [33].

![Figure 2.6: Coordinate system for DH reconstruction [25].](image)
The numerical reconstruction method named “convolution approach” has been first demonstrated by Demetrakopoulos and Mittra, where the reconstruction of holograms is achieved numerically by using the convolution theorem \[34\]. The optical field could be reconstructed by illuminating the hologram with \( E^*_R \) and then propagated a distance \( d \), which can be expressed with standard notation in DH \[2\]

\[
\Gamma(\xi, \eta) = \iint h(x, y)E^*_R(x, y)g_{PSF}(\xi, \eta, x, y)dxdy, \tag{2.20}
\]

where

\[
g_{PSF}(\xi, \eta, x, y) = \frac{e^{-j k_0 \sqrt{(x-\xi)^2+(y-\eta)^2+d^2}}}{\lambda \sqrt{(x-\xi)^2+(y-\eta)^2+d^2}}. \tag{2.21}
\]

This linear system is space-invariant. Therefore the impulse response \( g_{PSF}(\xi, \eta, x, y) \) depending on the distance \( x - \xi \) and \( y - \eta \) is rewritten as

\[
g_{PSF}(\xi, \eta, x, y) = g_{PSF}(x - \xi, y - \eta). \tag{2.22}
\]

From the results above, it follows

\[
\Gamma(\xi, \eta) = \iint h(x, y)E^*_R(x, y)g_{PSF}(\xi, \eta, x, y)dxdy
\]

\[
= \iint h(x, y)E^*_R(x, y)\frac{e^{-j k_0 \sqrt{\xi^2+\eta^2+d^2}}}{\lambda \sqrt{\xi^2+\eta^2+d^2}}dxdy
\]

\[
= (hE^*_R) \ast g_{PSF}
\]

\[
= \mathbb{F}^{-1}_{x,y}\{\mathbb{F}_{x,y}(hE^*_R) \cdot \tilde{g}_{PSF}\}. \tag{2.23}
\]

The convolution theorem indicates that Fourier transform of the convolution is the product of the individual Fourier transforms. Thus the reconstruction field of the real image \( \Gamma(\xi, \eta) \) can be found simply by taking the inverse Fourier transform of the product of the Fourier transform of \( h \cdot E^*_R \) and the Fourier transform of \( g_{PSF} \).
The numerically discretized form of the impulse response function is represented by [2]

$$g_{PSF}(k, l) = \frac{\exp(-jk_0 \sqrt{(k - \frac{N}{2})^2 \Delta x^2 + (l - \frac{N}{2})^2 \Delta y^2 + d^2})}{\lambda \sqrt{(k - \frac{N}{2})^2 + (l - \frac{N}{2})^2 + d^2}}. \quad (2.24)$$

The Fourier transform of the impulse response function can be expressed in the form [2]

$$\tilde{g}_{PSF}(m, n) = \exp[-jk_0d \sqrt{1 - \frac{\lambda^2(n + \frac{N^2\Delta x^2}{2d\lambda})}{N^2\Delta x^2} - \frac{\lambda^2(m + \frac{N^2\Delta y^2}{2d\lambda})}{N^2\Delta y^2}}]. \quad (2.25)$$

$\Delta x$ and $\Delta y$ are the pixel sizes of the detector where the hologram image is recorded and $\Delta \xi$ and $\Delta \eta$ represent the resolution of the reconstructed object image. When the image is reconstructed by the convolution approach, they should satisfy

$$\Delta \xi = \Delta x, \quad \Delta \eta = \Delta y. \quad (2.26)$$

By applying the Fast Fourier Transform (FFT) algorithm, the discrete Fresnel transform formula for reconstruction can be calculated effectively.

### 2.3 All-normal-dispersion (ANDi) Femtosecond Fiber Laser

In this thesis, the effect of CW and pulsed illumination in the recording of holograms is examined particularly in connection with the elimination of Moiré patterns. A convenient means of achieving this is a mode-locked fiber laser, which can readily be switched between CW and pulsed modes. Among a variety of fiber laser designs, an ANDI femtosecond laser is used in the experiment. A brief description of the ANDi laser is presented here for the benefit of readers.

In modern femtosecond lasers, group-velocity dispersion (GVD) and nonlinear effects need to be controlled to generate ultrashort femtosecond pulses. In typical mode-locked
lasers, GVD is controlled by applying prism pairs [35], diffraction grating pairs [36], and chirped mirrors [37]. Pulse propagation in the presence of a gain, nonlinear phase shift and dispersion is modeled by the modified nonlinear Schrödinger equation [40].

$$- j \frac{\partial A}{\partial z} = \frac{j}{2} \alpha A + \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} - \gamma |A|^2 A. \quad (2.27)$$

As shown in equation (2.27), the nonlinear effects and normal GVD ($\beta_2 > 0$) have same sign of the phase. Therefore, it was strongly believed that anomalous GVD is necessary to generate femtosecond pulses from mode-locked lasers [38,39]. In conventional laser designs, intracavity anomalous GVD has been ubiquitous.

Pulses as short as 55 fs with normal GVD have been generated by Buckley et al by introducing a frequency filter to stabilize the mode locked Yb-doped fiber laser [41]. Surprising, it was also possible to generate femtosecond pulses without anomalous GVD in the cavity. Such laser was referred as an ANDi fiber laser which generated femtosecond pulses [42].

The schematic of the laser is shown in Fig. 2.7. All components in the ring laser are normally dispersive. Since no bulky grating pair is necessary, an ANDi laser offers advantages in practical applications. A 980 nm pump laser is coupled into the laser cavity through a wavelength-division multiplexer (WDM). Then the multiplexer is spliced with a 3 m long single mode fiber (SMF), a 20 cm Yb-doped gain fiber and a 1 m SMF in a proper order.
In ANDi fiber laser, combined effect of a half-waveplate (HWP), two quarter-waveplates (QWP) and a polarizing beamsplitter (PBS) induces nonlinear polarization evolution (NPE) which serves as a saturable absorber. A spectral filter centered at 1030\text{nm} is put in the laser cavity. It is placed after the beam splitter, which serves as an output port, to achieve the shortest and highest energy pulse. At the same time, a 4\text{um} core diameter gain fiber induces substantial accumulation of nonlinear phase shift for the spectral broadening. By rotating waveplates in the cavity, the laser can be mode-locked and generates pulse trains. The repetition rate measured by a fast detector is 45MHz. The ANDi fiber laser provides a convenient way to switch between CW and pulsed operation by simply fine tuning waveplates in the laser cavity. The net GVD of the laser is high normal GVD to produce highly chirped pulses. The chirped pulses can be dechirped by a grating pair out of the laser cavity. The pulse duration after dechirping can be measured an autocorrelation system. For the holography experiment, an ANDi laser available in the laboratory was used. The laser output spectrum and the
autocorrelation for femtosecond pulses from the ANDi fiber laser are shown in Figs. 2.8(a) and (b). The dechirped pulse duration was 245 fs.

Figure 2.8: Output of the ANDi fiber laser: (a) spectrum, (b) interferometric autocorrelation.

2.4 Conclusion

In this Chapter, the theoretical background about the Moiré effects in the recording and reconstruction processes of digital holography has been introduced. Hologram recording and reconstruction methods in digital holography have been discussed. In light of the fact that both CW and pulsed illumination are required for the experimental part of the work, the operation of the ANDi fiber laser has been summarized. In the next Chapter, the formation and suppression of Moiré patterns is discussed in detail.
CHAPTER 3
SUPPRESSION OF MOIRÉ PATTERNS IN PULSED DIGITAL HOLOGRAM

In the last Chapter, the theoretical background of Moiré effects for the CCD camera has been discussed. Hologram construction and reconstruction methods for the Gabor type setup have been summarized. The ability of ANDi laser to switch between ultra-short pulse and CW has been illustrated. In this Chapter, the pulsed digital hologram is demonstrated as an efficient way to avoid Moiré patterns effects in the cases of 1D as well as 2D, using illustrative numerical simulations and careful experiments.

3.1 Suppression of Moiré Patterns for a One Dimensional Object

The experimental setup utilized in this part is shown in Fig. 3.1. A high quality beam from an ANDi laser is used as the illumination source. The beam is first focused and spatially filtered by a microscope objective-spatial filter (MO-SF) combination and then collimated by a convex lens L1 of focal length $f$. Next, the collimated beam illuminates a bare optical fiber which works as a 1D object. The width of this fiber, $d$, is much
smaller than the beam size. After a propagation distance $z$, the diffraction pattern is captured by a CCD array with pixel size $p$.

![Figure 3.1: Schematic of the fiber hologram recorded with plane wave illumination. MO-SF, microscope objective-spatial filter; L1, collimating lens; $z$, the distance between the fiber and CCD.](image)

### 3.1.1 The Fraunhofer region

Since the height of the fiber is much larger than its width, the problem can be regarded as one-dimensional. The fiber can be considered to be a uniformly blocking object of width $d$. When illuminated by a collimated beam, the optical field immediately behind the object can therefore be approximated as $1 - \text{rect} \left( \frac{x}{d} \right)$. The field after a distance of propagation $z$ is

$$E(x, z) = \left[1 - \text{rect} \left( \frac{x}{d} \right) \right] \otimes h(x, z) = 1 - \text{rect} \left( \frac{x}{d} \right) \otimes h(x, z) = 1 - E_{\text{diff}}(x, z), \ (3.1)$$

where $E_{\text{diff}}(x, z)$ is the Fresnel diffracted field from rectangular aperture. This can be calculated as

$$E_{\text{diff}}(x, z) = \text{rect} \left( \frac{x}{d} \right) \otimes h(x, z)$$

$$= \mathcal{F}^{-1} \{ \tilde{E}_0(k_x) H(k_x) \}$$
\[
\frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{E}_0(k_x) e^{jk_0x/2k_0} e^{-jk_0x} dk_x \\
= \frac{1}{2\pi} e^{-jk_0x^2/2z} \int_{-\infty}^{+\infty} \tilde{E}_0(k_x) e^{jk_0z} e^{jk_0x} e^{-jk_0} dk_x. \tag{3.2}
\]

In Eq. (3.2), \( \tilde{E}_0(k_x) \) is the Fourier transform of \( \text{rect}(\frac{x}{d}) \) and expressed as
\[
\tilde{E}_0(k_x) = \Im \left[ \text{rect}(\frac{x}{d}) \right] = \frac{d}{2\pi} \sin(c(\frac{dk}{2x})). \tag{3.3}
\]

Using the relation [43]
\[
\int_0^{\infty} g(\alpha) e^{k\alpha} d\alpha \approx \sqrt{-\frac{\pi}{2f''(\alpha_0)}} g(\alpha_0) e^{k\alpha_0} \frac{e^{k\alpha_0}}{\sqrt{k}} \tag{3.4}
\]
for large \( k \), where \( \alpha_0 \) is the saddle point defined by \( f'(\alpha_0) = 0 \) and applying to Eq. (3.2) by writing
\[
k f(k_x) = j \frac{z}{2k_0} (k_x - \frac{x}{z} k_0)^2, \tag{3.5a}
\]
or equivalently,
\[
k = j \frac{z}{2k_0}, f(k_x) = (k_x - \frac{x}{z} k_0)^2, \tag{3.5b}
\]
the saddle point can be obtained from setting
\[
f'(k_x) = 2(k_x - \frac{x}{z} k_0) = 0, \tag{3.6}
\]
which implies
\[
k_x = \frac{x}{z} k_0. \tag{3.7}
\]

Also, at this location, \( f''(k_x) = 2 \). Thus, the optical field in the Fraunhofer (or far-field) regime can be written as
\[
E_{diff}(x, z) = \frac{1}{2\pi} e^{-jk_0x^2/2z} \int_{-\infty}^{+\infty} \tilde{E}_0(k_x) e^{j\frac{z}{2k_0}(k_x - \frac{x}{z} k_0)^2} dk_x \\
= \frac{e^{-jk_0x^2/2z}}{2\pi} \sqrt{-\frac{\pi}{4}} \tilde{E}_0(k_x) \frac{1}{\sqrt{j \frac{z}{2k_0}}}. 
\]

25
\[
\begin{align*}
\frac{1}{\sqrt{2\pi}} e^{-j k_0 x^2 / 2z} \bar{E}_0(k_x) \\
= \frac{d}{\sqrt{\lambda z}} \exp(j \frac{\pi x^2}{\lambda z} - \frac{\pi}{4}) \text{sinc}(\frac{x d}{\lambda z}) \\
= h'(x, z) F(x, z),
\end{align*}
\] (3.8)

where \(k_0 = 2\pi / \lambda\) and

\[
h'(x, z) = \frac{1}{\sqrt{\lambda z}} \exp(j \frac{\pi x^2}{\lambda z} - \frac{\pi}{4}),
\] (3.9)

\[
F(x, z) = d \text{sinc}(\frac{x d}{\lambda z}).
\] (3.10)

Comparing Eq. (3.10) with the general form without far-field approximation (which will be derived in the following Chapter), the advantage with respect to the Fraunhofer far-field approximation method is that the intensity distribution is expressible as a specific function \(F(x, z)\), which is a sinc function.

As discussed in Chapter 2, a CCD camera is only capable of recording the intensity distribution rather than the complex amplitude. Without worrying about the scaling constants, the intensity distribution recorded by the camera at a distance \(z\) can be derived as

\[
I_z(x, z) = E(x, z) \cdot E^*(x, z) = 1 - [h'(x, z) + h'^*(x, z)] F(x, z) + |F(x, z)|^2,
\] (3.11)

also

\[
[h'(x, z) + h'^*(x, z)] F(x, z) = \frac{2d}{\sqrt{\lambda z}} \cos(\frac{\pi x^2}{\lambda z} - \frac{\pi}{4}) \text{sinc}(\frac{d x}{\lambda z}).
\] (3.12)

As evident from Eq. (3.12), a new chirp function, the \(\cos(\frac{\pi x^2}{\lambda z} - \frac{\pi}{4})\) term, is introduced into the intensity distribution, which modulates the diffraction patterns with dark and bright interference fringes. The phase information of the diffraction field are
preserved by these interference fringes. The oscillation phase \( \Phi(x, z) \) and the instantaneous angular frequency \( \omega(x, z) \) of the chirp function are given by, respectively,

\[
\Phi(x, z) = \frac{\pi x^2}{\lambda z} - \frac{\pi}{4}, \quad \omega(x, z) = \frac{d \Phi}{dx} = \frac{2\pi x}{\lambda z}.
\] (3.13)

Therefore, the fringe spacing can be simply calculated by

\[
s(x, z) = \frac{2\pi}{\omega(x, z)} = \frac{\lambda z}{x}.
\] (3.14)

Which also means that the period of the fringes \( s(x, z) \) is directly proportional to the recording distance \( z \) and inversely proportional to the transverse \( x \) coordinate.

Based on Eqs. (3.11) and (3.12), the variation of the intensity as a function of \( x \) is plotted in Fig. 3.2(a). The corresponding two-dimensional brightness plot of the diffraction pattern is displayed in Fig. 3.2(b).
Figure 3.2: Simulation results of (a) variation of intensity as a function of \(x\) and (b) the brightness plot displayed in two dimensions. \(d = 40\mu m, \lambda = 1030nm, z = 10mm\).

In the experiment, the intensity distribution is sampled during recording by the CCD camera. The sampling quality of the recording process depends on the pixel size of the CCD camera. However, when the pixel size \(q\) of the CCD camera is higher than half of the fringe spacing, the camera starts to record aliasing (which is also called Moiré patterns), instead of the actual intensity distribution. The center of the Moiré pattern is located where the fringe spacing is equal to the pixel size of CCD, which can be solved as

\[
x_{\text{moire pattern}} = \frac{\lambda z}{p}.
\]
The sampling of a CCD device can be modeled by a convolution with a rect function and followed by a multiplication with a comb function, whose period equals to the width of the rect function. The variation of the intensity which is sampled by the CCD camera as a function of $x$ is plotted in Fig. 3.3(a). Comparing with Fig. 3.2(a), the Moiré effect can be observed on each side as non-expected low spatial frequency regions, which are marked in Fig. 3.3(a). The corresponding two-dimensional brightness plot of the diffraction pattern is displayed in Fig. 3.3(b) with observable Moiré fringes on both sides.

Figure 3.3: Simulation results of (a) variation of intensity sampled by the CCD camera as a function of $x$ and (b) the brightness plot displayed in two dimensions. $d = 40\mu m$, $\lambda = 1030nm$, $z = 10mm$. The arrows show the location(s) of the Moiré patterns.
So far, all of our derivations and simulation analysis are established under the far-field approximation (i.e. $\pi d^2/(2\lambda z) << 1$). However, results from far-field approximation are not perfectly suitable for arbitrary recording distances because of the restrictive condition above. For instance, due to the very nature of diffraction, far-field distributions have larger size and lower intensity than the near field or intermediate field patterns. Thus it is unsuitable for small sensor size or higher sensor illumination thresholds. Also, the far-field approximation cannot be applicable if the size of the object is large or the propagating distance is short. Thus, a more practical analysis is necessary to explore hologram recording in the general case, as discussed in the next Section.

### 3.1.2 The Fresnel region

The location of the Moiré pattern determined for the Fraunhofer diffraction case is now re-evaluated for the Fresnel region. As will be shown, the same conclusion can be derived even without the far-field approximation. The optical field at the plane of the CCD camera is obtained in the same way as before, viz.,

$$E(x, z) = [1 - \text{rect}(\frac{x}{d})] \otimes h(x, z) = 1 - \text{rect}(\frac{x}{d}) \otimes h(x, z). \quad (3.16)$$

We define

$$E_{\text{diff}}(x, z) = \text{rect}(\frac{x}{d}) \otimes h(x, z)$$

$$= \int_{-\infty}^{+\infty} \text{rect}(\frac{u}{d}) e^{-j\frac{\lambda_0}{2}(x-u)^2/2z} du$$

$$= e^{-j\frac{\lambda_0}{2}x^2} \int_{-\infty}^{+\infty} \text{rect}(\frac{u}{d}) e^{-j\frac{\lambda_0}{2}(u^2-2ux)} du$$
In Eq. (3.17),

\[
G(x, z) = \int_{-\infty}^{+\infty} \text{rect} \left( \frac{u}{d} \right) e^{-j k_0 \left( \frac{u^2 - 2ux}{2z} \right)} du, \tag{3.18}
\]

\[
\text{rect} \left( \frac{x}{d} \right) = \begin{cases} 
1, & \text{for } -\frac{d}{2} \leq x \leq \frac{d}{2} \\
0, & \text{elsewhere},
\end{cases} \tag{3.19}
\]

\[
h(x, z) = \exp \left( -\frac{j k_0 x^2}{2z} \right). \tag{3.20}
\]

Thus, the intensity distribution recorded by the camera at a distance \(z\) from the fiber can be expressed as

\[
I_z(x, z) = E(x, z) \cdot E^*(x, z)
\]

\[
= 1 - \left[ E_{\text{diff}}(x, z) + E_{\text{diff}}^*(x, z) \right] + |E_{\text{diff}}(x, z)|^2
\]

\[
= 1 - \left[ G(x, z)h(x, z) + G^*(x, z)h^*(x, z) \right] + |G(x, z)|^2
\]

\[
= 1 - 2\text{Re}[E_{\text{diff}}(x, z)] + |E_{\text{diff}}(x, z)|^2. \tag{3.21}
\]

In the absence of far-field approximation, \(G(x, z)\) is an integral as shown in Eq. (3.18). However, it cannot be further simplified analytically.

For the analysis of aliasing, only the fringe spacing is of interest. Now, any periodic function has the same period as its derivative. This implies that the period of \(\text{Re}[E_{\text{diff}}(x, z)]\) is equivalent to the period of \(\text{Re}[E'_{\text{diff}}(x, z)]\). Now,

\[
\text{Re}[E'_{\text{diff}}(x, z)] = \text{Re} \left[ \frac{\partial E_{\text{diff}}(x, z)}{\partial x} \right]
\]

\[
= \text{Re} \left[ \int_{-\frac{d}{2}}^{\frac{d}{2}} \left( -\frac{j k_0}{z} \right) (x - u)e^{-j k_0 \left( \frac{(x-u)^2}{2z} \right)} du \right]
\]
Re \[ -e^{-\frac{jk_0}{2z}(x-d_z)^2} + e^{-\frac{jk_0}{2z}(x+d_z)^2} \] 
\[ = -2\sin\left[ \frac{\pi}{\lambda z} \left( x^2 + \frac{d_z^2}{4} \right) \right] \sin \left( \frac{\pi}{\lambda z} dx \right). \]  
(3.22)

Assuming that the term \( \sin\left( \frac{\pi}{\lambda z} dx \right) \) is much less oscillating than the function \( \sin\left[ \frac{\pi}{\lambda z} (x^2 + \frac{d_z^2}{4}) \right] \), particularly for small \( z \), the fringe spacing can be found through the oscillation phase \( \Phi(x, z) \) and the instantaneous angular frequency \( \omega(x, z) \) of the chirp function which are given as

\[ \Phi(x, z) = \frac{\pi}{\lambda z} \left( x^2 + \frac{d_z^2}{4} \right), \quad \omega(x, z) = \frac{d\Phi(x,z)}{dx} = \frac{2\pi x}{\lambda z}. \]  
(3.23)

The resulting space between interference fringes is described by

\[ s(x, z) = \frac{2\pi}{\omega(x, z)} = \frac{\lambda z}{x}, \]  
(3.24)

which yields the position of the Moiré patterns as

\[ x_{\text{Moiré pattern}} = \frac{\lambda z}{p}, \]  
(3.25)

identical to Eq. (3.14) derived for the Fraunhofer case.

It is reiterated that in the case without far-field approximation, the intensity distribution of the diffraction pattern cannot be analytically determined. However, the propagation of the optical field and the intensity distribution can be simulated by numerical method (which will be shown in Section 3.1.4). In the next Section, the method to suppress the unwanted Moiré patterns by illuminating with ultra-short pulses is demonstrated.

### 3.1.3 Ultra-short pulse illumination

As illustrated by Nicolas et al. [18], the harmful Moiré patterns can be smoothened by ultra-short pulse illumination. In our case, femtosecond pulse trains can be generated by
switching ANDi laser into the mode-locked mode. The temporal function of a Gaussian pulse can be written as

\[ u(t) = \exp(-j\omega_0 t - \frac{t^2}{T^2}). \tag{3.26} \]

By applying Fourier transformation in time, the spectrum can be shown to be

\[ U(\omega) = T\sqrt{\pi}\exp[-\frac{T^2}{4} (\omega - \omega_0)^2]. \tag{3.27} \]

\( T \) denotes the pulse width of the laser, and \( \omega_0 \) is the central frequency of the illumination beam. Applying the relation \( \omega = \frac{2\pi c}{\lambda} = k_0 c \) (technically \( k_0 \) is no longer a constant) the expression of intensity distribution recorded by the CCD as a function of \( \omega \) is given by

\[ I_z(x, \omega) = E(x, \omega) \cdot E^*(x, \omega) = 1 - [G(x, \omega)h(x, \omega) + G^*(x, \omega)h^*(x, \omega)] + |G(x, \omega)|^2. \tag{3.28} \]

It has been shown by Nicolas et al. [18] that the intensity distribution can be considered as the contribution of each spectral component, according to

\[ I_{z,pulse}(x) = C \int_{-\infty}^{+\infty} I_z(x, \omega)|U(\omega)|^2 d\omega, \tag{3.29} \]

where \( C \) is a normalization coefficient.

\[ I_{z,pulse}(x) = \int_{-\infty}^{+\infty} \exp[-\frac{T^2}{2} (\omega - \omega_0)^2]d \omega - \int_{-\infty}^{+\infty} G(x, \omega)h(x, \omega)\exp[-\frac{T^2}{2} (\omega - \omega_0)^2]d \omega. \tag{3.30} \]

When all integrals are calculated [19], the intensity distribution recorded by the CCD with short-pulse illumination is expressible as

\[ I_{z, Pulse}(x) = 1 - [G(x)h(x) + G^*(x)h^*(x)]W(x, z, T) + |G(x)|^2. \tag{3.31} \]

with
\[ W(x, z, T) = \exp\left(-\frac{x^4}{8c^2z^2T^2}\right). \quad (3.32) \]

Figure 3.4: Plot of \( W(x, z, T) \) for \( z = 10 \text{mm}, T = 247 \text{fs} \).

By comparing Eq. (3.31) with the intensity distribution derived earlier in Eq. (3.21) for CW illumination, it is clear that the chirp function \( [G(x)h(x) + G^*(x)h^*(x)] \) is multiplied with a super-Gaussian function \( W(x, z, T) \), which filters the high frequency efficiently. The comparison of intensity distributions by CW and pulsed laser illumination is plotted in Fig 3.5. A fiber with diameter \( d = 40 \mu \text{m} \) is assumed as the object and located at \( z = 10 \text{mm} \) from the CCD camera. The central wavelength of the laser is \( \lambda = 1030 \text{nm} \). The simulation results of the intensity distribution sampled by the CCD camera are shown in Fig. 3.5 (a) for CW illumination and Fig. 3.5 (b) for pulse illumination with 245fs duration. It shows clearly that the Moiré effects are removed by the ultra-short pulse illumination successfully.
Figure 3.5: The intensity distribution of the diffraction pattern sampled by the CCD camera (a) with CW illumination, (b) with short-pulse illumination. $d = 40 \mu m$, $\lambda = 1030 nm$, $T = 247 fs$. 
By combining Eq. (3.21) and Eq. (3.31), the intensity distribution of the diffraction patterns from an optical fiber illuminated by a plane wave at a distance $z$ can be given in a general form

$$I_{z,\text{Pulse}}(x) = 1 - [G(x)h(x) + G^*(x)h^*(x)]W(x, z, T) + |G(x)|^2,$$  \hspace{1cm} (3.33)

where,

$$W(x, z, T) = \begin{cases} 
1, & \text{for CW}, \\
\exp\left[-\frac{x^4}{8c^2z^2T^2}\right], & \text{for pulse}.
\end{cases} \hspace{1cm} (3.34)$$

### 3.1.4 Simulation and experimental results for the Fresnel hologram and digital reconstruction of a fiber

Our experimental setup comprises a collimated beam of light of wavelength $\lambda = 1030\text{nm}$ from an ANDi femtosecond fiber laser illuminating an optical fiber with diameter $d = 245\mu m$ (Rayleigh range 50$\text{mm}$) and a recording CCD camera of pixel size $p = 6.7\mu m$. The recording distance is $z = 15\text{mm}$. By rotating the wave plates (saturable absorber) in the ANDi laser cavity, it is possible to switch from CW operation to ultra-short pulse generation with pulse duration $T = 247fs$ and a repetition rate of 45$\text{MHz}$. 

The simulation results of the intensity distribution at the position of CCD with CW illumination are shown in Fig. 3.6(a). The digital reconstruction using the Fresnel diffraction formula is shown in Fig. 3.6(b) as well. As expected, the unwanted Moiré patterns are observed in both the recorded hologram and reconstruction results under CW
illumination. Similar results for ultra-short pulse illumination are shown in Figs. 3.6(c) and (d), respectively. As clearly evident, the Moiré fringes are eliminated with the ultra-short pulse illumination. The patterns close to the fiber is generated due to the shrinkage of the image to fit the paper. The process of adjusting the exposure is applied to make the patterns more obvious by playing with color bar.

Figure 3.6: Numerical results of (a) intensity distribution and (b) digital reconstructions of a fiber with CW illumination, $\lambda = 1030\text{nm}$. The results with ultra-short pulse illumination, $T = 247\text{fs}$ and $\lambda = 1030\text{nm}$ are shown in (c), (d), respectively.

The corresponding experimental results are shown in Fig. 3.7. The ultra-short pulse plays the role of a filter to smooth the harmful aliasing in both of the hologram recording and reconstruction results. As clearly evident, the experimental results agree well with the simulation results.
3.2 Suppression of Moiré Patterns for a Two Dimensional Object

The simulation (and experimental) results above are performed based on a 1D diffraction model of a \textit{rect} function whose position continually changes along the vertical axis. In this Section, the suppression of the Moiré patterns for a two dimensional object with ultra-short pulse illumination is investigated.

3.2.1 Mathematical background

A two dimensional object (e.g., a circular block with a radius $R_1 = 40 \mu m$) is illuminated by a plane wave. The amplitude transmittance for a circular aperture is given by

$$t(r) = \text{circ} \left( \frac{r}{R_1} \right),$$

(3.35)
where $r = \sqrt{x^2 + y^2}$ is the radial coordinate in the plane of the aperture, which has a Fourier transform [1]

$$B\{\text{circ} \left( \frac{r}{R_1} \right) \} = \pi \omega^2 \frac{J_1(2\pi R_1 \rho)}{\pi R_1 \rho},$$  \hspace{1cm} (3.36)

where the substitution $\rho = r/\lambda z$ is required to transform to spatial coordinates in the far field for the Fraunhofer diffraction pattern. The optical field of the diffraction pattern of a circular block at the recording plane after a propagation distance $z$ can be obtained as

$$E(x, y, z) = \left[ 1 - \text{circ} \left( \frac{r}{R_1} \right) \right] \otimes h(x, y, z).$$ \hspace{1cm} (3.37)

As before, under far-field approximation, the intensity distribution in the Fraunhofer diffraction pattern is found by just extending Eq. (3.21) to two dimensional coordinates to yield,

$$I_{\text{circular block}}(x, y, z) = 1 - [h(x, y, z) + h^*(x, y, z)]F(x, y, z) + |F(x, y, z)|^2,$$ \hspace{1cm} (3.38)

$$h'(x, y, z) = \frac{1}{\lambda z} \exp j \left( \frac{\pi(x^2 + y^2)}{\lambda z} - \frac{\pi}{2} \right),$$ \hspace{1cm} (3.39)

$$F(x, y, z) = \pi \omega^2 \frac{J_1(2\pi R_1 \rho)}{\pi R_1 \rho} = 2\pi R_1^2 \frac{J_1 \left( \frac{k_0 R_1 \sqrt{x^2 + y^2}}{z} \right)}{k_0 \frac{R_1 \sqrt{x^2 + y^2}}{z}}.$$ \hspace{1cm} (3.40)

When the circular block is illuminated by ultra-short pulses, it can be proved that the intensity distribution of the diffraction pattern can be achieved by simply expanding the super-Gaussian function to the two dimensional form as

$$W(x, y, z, T) = \exp \left[ - \frac{(x^4 + y^4)}{8c^2 z^2 T^2} \right].$$ \hspace{1cm} (3.41)

In general, the intensity distribution of the diffraction of a two dimensional object can be expressed as
\[ I_z(x, y, z) = 1 - \left[ G(x, y, z) h(x, y, z) + G^*(x, y, z) h^*(x, y, z) \right] W(x, y, z, T) + |G(x, y, z)|^2, \quad (3.42) \]

and

\[ W(x, y, z, T) = \begin{cases} 1, \quad &\text{for CW}, \\ \exp \left[ -\frac{(x^4+y^4)}{8c^2z^2T^2} \right], &\text{for pulse}. \end{cases} \quad (3.43) \]

### 3.2.2 Simulation results

Simulation results of the recorded hologram along with Moiré patterns during CW illumination of a circular object with a radius \( R_1 = 40\mu m \) and with illumination wavelength \( \lambda = 1030nm \) are shown in Fig. 3.8(a). Like the case of the 1D fiber, the Moiré patterns resulting from under-sampling of the CCD camera with CW illumination can be suppressed significantly by using ultra-short pulses, as shown in Fig. 3.8(b).

![Simulated intensity distributions of hologram of a circular block](image)

Figure 3.8: Simulated intensity distributions of hologram of a circular block (a) with CW illumination, (b) with ultra-short pulse illumination, \( T = 247fs \).
3.3 Conclusion

In this Chapter, the harmful Moiré fringes due to the under-sampling of the CCD camera have been discussed quantitatively in both Fraunhofer and Fresnel regimes. It is shown that the Moiré patterns, which appear during recording and reconstruction for CW illumination are successfully eliminated by applying ultra-short pulse illumination. In Chapter 4, the Moiré effects in different imaging systems are evaluated and compared numerically and experimentally.
CHAPTER 4
ABERRATIONS AND THEIR EFFECTS ON MOIRÉ PATTERNS IN IMAGING SYSTEMS

In the last Chapter, the suppression of Moiré patterns with ultra-short pulses illumination has been demonstrated. For some holographic applications, an imaging system is desirable and preferable, such as holographic microscopy. In this Chapter, the performance of the Moiré effects using spherical illumination is compared with the plane wave illumination. The effect of aberration on the spatial frequency bandwidth of a recorded hologram is investigated in a single lens imaging system and a confocal lens imaging system, respectively. It is shown that a right amount of aberration also mitigates aliasing.

4.1 Theoretical Analysis without Aberration

In this Section, the basic imaging systems used in holography, viz., single lens imaging system and confocal imaging system are first introduced. Their similarity with holography using a point source and plane wave (or collimated) source, respectively are discussed.
4.1.1 Spherical wave illumination

Other than a plane wave, a point source which leads to a spherical wavefront is also widely used as the illumination source in holography. In Fig. 4.1, lens L1 is used to produce a collimated beam from the point source and focused by L2 to finally evolve into a diverging spherical wave beyond the back focal point of L2. The object (optical fiber) is placed at a distance $f$ behind the back focal point of L2. The hologram of the fiber is recorded at a distance $z$ by the CCD camera.

Figure 4.1: Schematic of the fiber hologram recorded with spherical wave illumination. MO-SF is a microscope objective – spatial filter combination to generate a point source; L1, L2 form a 4f system to image the point source at a distance $f$ beyond L2. $z$ is the distance between fiber and CCD.

The optical fields of the object beam and reference beam reaching the plane of the CCD camera can be shown, respectively, as

$$E_o(x, y, z) = [e^{-jk_0(x^2+y^2)/2f} \text{rect}(\frac{x}{d})] \otimes h(x, y; z)$$

$$= e^{-\frac{jk_0(x^2+y^2)}{2z}} \int_{-\infty}^{+\infty} e^{jk_0\left[y'^2\left(-\frac{1}{2f} - \frac{1}{2z}\right) + \frac{2yy'}{2z}\right]} dy'$$
\[ \int_{-\infty}^{+\infty} e^{jk_0(x^2+y^2)/2z} e^{jRy^2} \cdot CK(x), \quad (4.1) \]

\[ E_{\text{ref}}(x, y, z) = e^{-jk_0(x^2+y^2)/2z} \otimes h(x, y; z) \]

\[ = e^{-jk_0(x^2+y^2)/2z} e^{jRy^2} e^{jRx^2} C^2, \quad (4.2) \]

hence

\[ K(x, z) = \int_{-\frac{d}{2}}^{\frac{d}{2}} e^{jk_0(x'^2+\frac{2yx'}{z})} \cdot d' \cdot dx'. \quad (4.3) \]

Here, \( C \) is a constant number and \( R = \frac{k_0d'}{2z^2}, \) \( d' = \frac{fz}{f+z}. \) As mentioned earlier in the first Chapter, the reference beam and the object beam interferes with each other. The CCD records the intensity distribution of the interference pattern of the fiber at a distance \( z, \)

which is given by

\[ I(x, z) = |E_{\text{ref}}(x, y, z) - E_o(x, y, z)|^2 \]

\[ = 1 - C^3 [K(x, z)e^{-jRx^2} + K^*(x, z)e^{jRx^2}] + C^2 |K(x, z)|^2 \]

\[ = 1 - 2C^3 \cdot \text{Re}[E_k(x, z)] + C^2 |K(x, z)|^2, \quad (4.4) \]

it follows that

\[ E_k(x, z) = K(x, z)e^{-jRx^2} = \int_{-\frac{d}{2}}^{\frac{d}{2}} e^{jk_0\left(-\frac{1}{2d'}(x'-\frac{fx}{f+z})^2}\right) dx'. \quad (4.5) \]

From Eq. (4.4), the intensity distribution of the diffraction pattern is only related to \( x \)
coordinate, which also means that, even though the spherical wave introduces a phase term in \( y \) direction, there is no interference between the object wave and reference wave in this direction.
The fringe spacing of the interference patterns is not obvious through the integral shown in Eq. (4.5). Fortunately, the fringe spacing of the interference patterns can be calculated by finding the period of the chirp function \( \text{Re}[E_k(x, z)] \), which is also equal to the period of the function \( \text{Re}[E'_k(x, z)] \), which is, the derivative of \( \text{Re}[E_k(x, z)] \). This can be expressed as

\[
\text{Re}[E'_k(x, z)] = \text{Re}[\frac{\partial E_k(x, z)}{\partial x}]
\]

\[
= \text{Re}\left[\int_{-\frac{d}{2}}^{\frac{d}{2}} -j k_0 \frac{1}{d'} (x' - \frac{fx}{f+z}) e^{j k_0 \left(-\frac{1}{2d'}(x' - \frac{fx}{f+z})^2\right)} dx'\right]
\]

\[
= \text{Re}\left[e^{-\frac{j k_0 d}{2d'} \left(-\frac{fx}{f+z}\right)^2} - e^{-\frac{j k_0 d}{2d'} \left(-\frac{fx}{f+z}\right)^2}\right]
\]

\[
= \cos\left[\frac{k_0}{2d'} \left(\frac{fx}{f+z} - \frac{d}{z}\right)^2\right] - \cos\left[\frac{k_0}{2d'} \left(\frac{fx}{f+z} + \frac{d}{2}\right)^2\right]
\]

\[
= 2\sin\left(\frac{\pi}{ld'} \left(\left(\frac{f}{f+z}\right)^2 x^2 + \frac{d^2}{4}\right)\right) \sin\left(\frac{\pi}{ld'} dx\right). \quad (4.6)
\]

As noted in Chapter 3, the term \( \sin\left(\frac{\pi}{ld'} dx\right) \) is much less oscillatory than the term \( \sin\left(\frac{\pi}{ld'} d' x\right) \).

The way to determine the fringe spacing is the same as that discussed in the previous Chapter. The oscillation phase \( \Phi(x, z) \) and the instantaneous angular frequency \( \omega(x, z) \) of the chirp function can be found as

\[
\Phi(x, z) = \frac{\pi}{ld'} \left[\left(\frac{f}{f+z}\right)^2 x^2 + \frac{d^2}{4}\right], \quad \omega(x, z) = \frac{d\Phi(x, z)}{dx} = \frac{k_0 x}{d' z^2}, \quad (4.7)
\]

hence

\[
s(x, z) = \frac{2\pi}{\omega(x, z)} = \frac{\lambda z^2}{d' x}. \quad (4.8)
\]
Thus, the fringe spacing of the diffraction patterns can be predicted by the chirp term
\[ C^3(K(x)e^{-jRx^2} + K^*(x)e^{jRx^2}), \]
and is \( s(x,z) = \frac{\lambda z^2}{d'} \). Due to the limitation of pixel size of the CCD, under-sampling occurs when the period of the fringes equals to two times of the pixel size. Hence, the Moiré pattern starts from the position \( x_0 = \frac{\lambda z(f+z)}{2fp} \). The central position of the Moiré pattern can be determined by
\[
x_{\text{Moiré pattern}} = \frac{\lambda z}{p} + \frac{\lambda z^2}{fp}.
\] (4.9)

The experimental results of the intensity distribution recording by the CCD camera are shown in Fig. 4.2. As we can see from Fig. 4.2(a), the Moiré fringes are visible under CW illumination and removed by ultra-short pulse illumination in Fig. 4.2(b). In the experiment, the fiber is illuminated by a spherical wave. The focal length of L2 is \( f = 5\text{cm} \).

![Figure 4.2](image)

Figure 4.2 Recording hologram of the fiber with spherical wave illumination (a) with CW illumination, (b) with ultra-short pulse illumination. The region within the box is the region of interest where Moiré patterns appear for CW illumination.
4.1.2 Comparison between spherical wave illumination and plane wave illumination

By solving Eq. (3.25) and Eq. (4.9) with different values of the recording distance, the positions of Moiré patterns variation for spherical illumination and plane wave illumination are plotted in Fig. 4.3, respectively, which are distinguished by blue and red curves. The corresponding experimental results, which are shown in the same plot, agree well with the theoretical results.

Due to an additional term $\frac{\lambda z^2}{f_p}$ in Eq. (4.9), the position of the Moiré patterns is shifted by a distance $\frac{\lambda z^2}{f_p}$ with spherical wave illumination as shown in Fig. 4.3.

![Figure 4.3: The position of Moiré patterns as a function of the recording distance. The results are calculated from Eq. (3.25) for red curve (plane wave) and Eq. (4.9) for blue curve (spherical wave). $f = 50mm$, $d = 245\mu m$, $p = 6.7\mu m$.](image)

4.1.3 Single lens imaging system

In this case, the fiber is imaged with a single lens system. The experimental setup is shown in Fig. 4.4. The same fiber is placed at a distance $2f$ in front of L2 in order to
achieve a same size image at the image plane. Then the CCD camera records the

diffraction patterns at a distance \( z \) after the image plane.

Figure 4.4: Schematic of the fiber hologram recorded in a single lens system. The object
distance and image distance for lens L2 are \( 2f \); \( f \), the focal length of lens L2; L1, the
collimated lens; \( z \), the distance between fiber and CCD.

To determine the intensity distribution of the interference pattern, the point spread
function from the object plane to the image plane is obtained first:

\[
g_{PSF}(x, y; x_0, y_0) = -e^{-j k_0 \frac{x_0^2 + y_0^2}{d_0}} \cdot \delta(x + x_0, y + y_0). \quad (4.10)
\]

Based on the point spread function, the optical field due to the fiber at the image plane can
be written as

\[
E_{image, plane} = \int -rect \left( \frac{x_0}{d} \right) e^{-j k_0 \frac{x_0^2 + y_0^2}{d_0}} \cdot \delta(x + x_0, y + y_0) \, dx_0 \, dy_0 \\
= -rect \left( \frac{-x}{d} \right) \cdot e^{-j k_0 \frac{x^2 + y^2}{d_0}}. \quad (4.11)
\]

After propagating distance \( z \), the optical fields of the object beam and reference beam at
the plane of CCD can be expressed as, respectively,

\[
E_o(x, y, z) = \iint rect \left( \frac{x'}{d} \right) \cdot e^{-j k_0 \frac{(x')^2 + (y')^2}{d_0}} \cdot e^{-j k_0 \frac{(x-x')^2 + (y-y')^2}{2z}} \, dx' \, dy' \\
= e^{-j k_0 \frac{x^2 + y^2}{2z}} e^{j R y^2} CK(x), \quad (4.12)
\]
\[ E_{\text{ref}}(x, y, z) = e^{-j k_0 \frac{(x^2 + y^2)}{2f}} \otimes h(x, y, z) \]
\[ = \iint e^{-j k_0 \frac{x'^2 + y'^2}{2f}} \cdot e^{-j k_0 \frac{(x-x')^2 + (y-y')^2}{2z}} \, dx' \, dy' \]
\[ = e^{-j k_0 (x^2 + y^2)/2z} e^{j R_y^2} e^{j R_x^2} C^2. \quad (4.13) \]

Again, C is a constant, \( R = \frac{k_0 d'}{2z^2} \), \( d' = \frac{fz}{f+z} \), and
\[ K(x, z) = \int_{-\frac{d}{2}}^{\frac{d}{2}} e^{j k_0 (x'^2 \left( -\frac{1}{2d'} \right) + \frac{xy}{z})} \, dx'. \quad (4.14) \]

Finally, the intensity distribution at the plane of the CCD can be calculated as
\[ I(x, z) = \left| E_{\text{ref}}(x, y, z) - E_o(x, y, z) \right|^2 \]
\[ = 1 - C^3 [K(x, z) e^{-j R x^2} + K^*(x, z) e^{j R x^2}] + C^2 |K(x, z)|^2 \]
\[ = 1 - 2C^3 \cdot \text{Re}[E_k(x, z)] + C^2 |K(x, z)|^2. \quad (4.15) \]

As a result, the fringe spacing can be determined by finding the phase and its derivative:
\[ \Phi(x, z) = \frac{\pi}{\lambda d'} \left[ \left( \frac{f}{f+z} \right)^2 x^2 + \frac{d^2}{4} \right], \quad \omega(x, z) = \frac{d\Phi(x, z)}{dx} = \frac{k_0 x}{d'z^2}, \quad (4.16) \]
so that
\[ s(x, z) = \frac{2\pi}{\omega(x, z)} = \frac{\lambda z^2}{d'x}. \quad (4.17) \]

Moreover, the position of Moiré patterns can be expressed by
\[ x_{\text{Moiré pattern}} = \frac{\lambda z}{p} + \frac{\lambda z^2}{fp}. \quad (4.18) \]

Upon comparing Eq. (4.4) with Eq. (4.15), there is no differences for the intensity distribution between the spherical wave laser beam illumination and the image system with a single lens. Also the two expressions to determine the position of Moiré patterns for the two systems, Eq. (4.9) and Eq. (4.18), are comparable.
As shown earlier, Moiré patterns can be observed when the fiber is illuminated by the spherical wave experimentally. The same experimental phenomenon for the single lens system is expected at first.

Surprisingly, however, when the fiber is imaged by an optical lens (the same lens used in the Section 4.1.1) in the experiment, no Moiré patterns can be observed at any location behind the lens. The Moiré fringes cannot be observed for both CW and ultra-short pulse illumination, as shown in Fig. 4.5. The low frequency fringes are actually the result of the inference effects from the optical components in the laser cavity or the protecting glass of the CCD camera. These fringes, which are observed in most hologram recordings, have no impact on the reconstruction results.

Figure 4.5: Recorded hologram of the fiber in a single lens image system (a) with CW illumination, (b) with ultra-short pulse illumination.
The conflict between the mathematical derivations and the experimental results motivates us to explore more. In the next Section, similar analysis and experiments with a con-focal lens system are investigated.

4.1.4 The confocal lens imaging system

The fiber is illuminated with a collimated plane wave and imaged by a $4f$ lens system as shown in Fig. 4.6. The same fiber is placed at the front focal plane of lens L2. L2 and L3 share the same focal length $f$. The distance between L2 and L3 is $2f$. The image plane of the lens system is the back focal plane of lens L3. Obviously, the image of the fiber is formed at the image plane without magnification for this $4f$ imaging system.

![Figure 4.6: Schematic of the recording of the fiber hologram in a confocal lens system.](image)

Figure 4.6: Schematic of the recording of the fiber hologram in a confocal lens system. $f$ is the focal length of lens L2, L3; L1 is the collimated lens; $z$ is the distance between fiber and CCD.

First, the point spread function from the object plane to the image plane can be written as

$$g_{PSF}(x, y; x_0, y_0) = \delta(x + x_0, y + y_0). \quad (4.19)$$

The optical field at the image plane can be calculated by
\[ E_{\text{imageplane}}(x) = \iint [1 - \text{rect}(\frac{x-x_0}{d})] \delta(x + x_0, y + y_0) dx_0 dy_0 = 1 - \text{rect}(\frac{x}{d}). \] (4.20)

From the equation above, the optical field at the image plane is the same as the optical field at the object plane. This also means that, when a fiber imaged by a confocal lens system, the diffraction pattern at the observing distance \( z \) from the image plane is equivalent to the situation when illuminating the fiber with the plane wave directly. Finally, the complex optical field at the CCD plane is the same as Eq. (3.21).

By performing calculations similar to that in Chapter 3, the period of the interference fringes can be shown to be

\[ s(x, z) = \frac{2\pi}{\omega} = \frac{\lambda z}{x}. \] (4.21)

The expression for the position of the Moiré pattern is given by

\[ x_{\text{Moiré pattern}} = \frac{\lambda z}{p}. \] (4.22)

The experimental recordings of the intensity distribution of the diffraction patterns for a fiber in a confocal lens system using CW and pulsed illumination are shown in Fig. 4.7. The Moiré fringes are visible in Fig. 4.7(a) with CW illumination and suppressed by the pulse illumination efficiently from Fig. 4.7(b).
4.2 Aberration and its Effects on Moiré Patterns in Imaging Systems

In the previous derivations, the optical aberrations of the image lens have been ignored for simplicity. In this Section, these derivations are redone by considering an aberration term in order to achieve more accurate expressions for the intensity distribution of the diffraction patterns.

4.2.1 The single lens imaging system with aberration

The aberration of a lens can be defined in terms of a phase function $\phi(x, y)$. Then the point spread function from the object plane to the image plane is rewritten as

$$g_{PSF}(x, y; x_0, y_0) = \frac{k_0^2}{4\pi^2d_0d_i} e^{-jk_0\frac{x_0^2+y_0^2}{2d_0}} \cdot e^{-jk_0\frac{x^2+y^2}{2d_i}} \cdot \iint e^{ik_0\left[x\frac{x_0}{d_0}+y\frac{y_0}{d_0}\right]} \cdot \left[ e^{ik_0\left[x\frac{x}{d_0}+y\frac{y}{d_i}\right]} \right]$$
In order to simplify the derivation with the complicated integral, only the \( x \) direction is considered here

\[
g_{PSF}(x, x_0) = -\frac{k_0^2}{4\pi^2 d_0 d_i} e^{-jk_0\frac{x^2}{2d_0}} e^{-jk_0\frac{x'}{2d_i}} \int e^{j k_0 \left[ x' \left( \frac{x_0}{d_0} + \frac{x}{d_i} \right) \right]} \phi(x') dx'
\]

which implies

\[
P(x, x_0) = -\frac{k_0^2}{4\pi^2 d_0 d_i} e^{-jk_0\frac{(x-x_0)(x+x_0)}{2d_0}} \int e^{j k_0 \left[ x' \left( \frac{x_0}{d_0} + \frac{x}{d_i} \right) \right]} \phi(x') dx'
\]

Since the object distance is equal to the image distance, there is no magnification for this imaging system. An approximate result can be obtained by assuming \( (x - x_0) \ll x \), so that

\[
P(x, x_0) = -\frac{k_0^2}{4\pi^2 d_0 d_i} \int e^{j k_0 \left[ x' \left( \frac{x_0}{d_0} + \frac{x}{d_i} \right) \right]} \phi(x') dx'.
\]

By examining the input-output relations for the linear imaging, it turns out to be a space invariant linear system. For such a system, Eq. (4.26) can be rewritten with the substitution

\[
P(x_1, x_0) = P(x_1 - x_0) = P(x_1 + x_0),
\]

where a mirror symmetry of the object is assumed for simplicity. Then the optical field of the object beam after propagating distance \( z \) from the image plane can be found as

\[
E_o(x, z) = \iint \text{rect} \left( \frac{X_0}{d} \right) P(x_1, x_0) e^{-jk_0 \frac{x'^2}{2d_i}} e^{-jk_0 \frac{(x-x_1)^2}{2z}} dx_0 dx_1
\]
\[ = e^{-jk_0(x_0^2 - d'x_0^2) \int \frac{x_0}{d} \int \text{rect} \left( \frac{x_0}{d} \right) P(x_1, x_0) e^{-jk_0 \frac{(x_1 - x_0)^2}{2d'}} dx_0 dx_1} \]

\[ = -e^{-jk_0(x_0^2 - d'x_0^2) \int \frac{x_0}{d} \int \text{rect} \left( \frac{x_0}{d} \right) P_0(x_1 - x_0) e^{-jk_0 \frac{(x_1 - x_0)^2}{2d'}} dx_0 dx_1}. \quad (4.28) \]

Under far-field approximation, by applying variable substitution \( x_0' = x_1 - x_0 \), Eq. (4.28) can be revised as

\[ E_o(x, z) = e^{-jk_0(x_0^2 - d'x_0^2) \int \frac{x_0}{d} \int \text{rect} \left( \frac{x_0}{d} \right) P(x_0') e^{jk_0 \frac{x_0 x_0'}{z}} dx_0' dx_1}. \quad (4.29) \]

Substituting \( x_1' = x_1 - x_0' \), \( E_o(x, z) \) is given by

\[ E_o(x, z) = e^{-jk_0(x_0^2 - d'x_0^2) \int \frac{x_0'}{d} \int \text{rect} \left( \frac{x_0'}{d} \right) P(x_0') e^{jk_0 \frac{x_0 + x_0'}{z}} dx_0' dx_1'} \]

\[ = e^{-jk_0(x_0^2 - d'x_0^2) \text{sinc}(\frac{k_0 x}{zd})} \int e^{jk_0 \frac{x_0 x_0'}{z}} P(x_0') dx_0'. \quad (4.30) \]

Accordingly, our objective is now to calculate the integral \( \int e^{jk_0 \frac{x_0 x_0'}{z}} P(x_0') dx_0'. \)

Due to the small size of fiber, we can assume \( x_0 \approx 0 \). \( P(x_0') \) can be expressed as

\[ P(x_0') = \int_{-\infty}^{+\infty} e^{ik_0 \frac{x_0 x_0'}{d_i}} \cdot \phi(x') dx'. \quad (4.31) \]

Furthermore, the integral is given by

\[ \int_{-\infty}^{+\infty} e^{jk_0 \frac{x_0 x_0'}{z} P(x_0')} dx_0' = \int_{-\infty}^{+\infty} e^{jk_0 \frac{x_0 x_0'}{z} P(x_0')} e^{jk_0 \frac{x_0 x_0'}{d_i}} \phi(x') dx' \]

\[ = \int_{-\infty}^{+\infty} \left[ \int_{-\infty}^{+\infty} e^{jk_0 \frac{x_0 x_0'}{d_i + z} P(x_0')} dx_0' \right] \phi(x') dx' \]

\[ = \int_{-\infty}^{+\infty} \delta \left[ k_0 \left( \frac{x_0'}{d_i} + \frac{x_0'}{z} \right) \right] \phi(x') dx' \]

\[ = \phi\left( -\frac{d_i}{z} \right). \quad (4.32) \]
\( \phi(x) \) is defined to be the aberration term of optical lens. For lens, there are actually different kinds of aberrations which are categorized into two classes: monochromatic and chromatic. Chromatic aberrations have no influence for our narrow wavelength band holographic system. Second order monochromatic aberrations, such as tilt and defocus, are irrelevant since the recording plane does not fall on the image plane. Fourth order monochromatic aberrations, also known as Seidel aberrations, however, have the most crucial impact on the quality of the recorded patterns because they can change the wave front from spherical to aspherical. For our small sized object, spherical aberration which is usually labeled as \( W_{040} \) term of Seidel aberrations plays the most important role in distorting the final hologram. Here, only the spherical aberration in the \( x \) direction with \( \phi(x) = e^{jW_{040}x^4} \) is considered for simplicity, then

\[
\int_{-\infty}^{+\infty} e^{j\frac{k_0 x x_0}{z}} P(x_0') \, dx_0' = \phi(-\frac{d}{z} x) = e^{jW_{040}(-\frac{d}{z})^4 x^4} . \tag{4.33}
\]

Substituting Eq. (4.33) in Eq. (4.30), the optical field of the object beam is given by

\[
E_o(x, z) = e^{-j\frac{k_0(x_0^2)}{2z}} \int \int \text{rect} \left( \frac{x_1'}{d} \right) P(x_0') e^{j\frac{k_0(x_1' + x_0^2)}{z}} \, dx_0' \, dx_1'
\]

\[
= e^{-j\frac{k_0(x_0^2)}{2z}} \sin\left( \frac{k_0 x}{zd} \right) \int_{-\infty}^{+\infty} e^{j\frac{k_0 x x_0'}{z}} P(x_0') \, dx_0'
\]

\[
= e^{-j\frac{k_0(x_0^2)}{2z}} \sin\left( \frac{k_0 x}{zd} \right) e^{jW_{040}(-\frac{d}{z})^4 x^4} . \tag{4.34}
\]

In a similar way, the optical field of the reference beam after propagating distance \( z \) from the image plane is

\[
E_{\text{ref}}(x, z) = \int \int P(x_1, x_0) e^{-j\frac{k_0(x_1^2)}{2z}} e^{-j\frac{k_0(x-x_1)^2}{2z}} \, dx_0 \, dx_1
\]
\[ e^{-jk_0(\frac{x^2}{2z} - \frac{d'x^2}{2z^2})} \int\int P(x_1, x_0) e^{-j\frac{x_1}{2d'}(\frac{d'}{2})^2} dx_0 dx_1, \quad (4.35) \]

where

\[ \frac{1}{d'} = \frac{1}{f} + \frac{1}{z}. \quad (4.36) \]

Note that for a space-invariant system, by substitution Eq. (4.27) into Eq. (4.35), the corresponding expression for \( E_{\text{ref}}(x, z) \) is

\[ E_{\text{ref}}(x, z) = e^{-jk_0\frac{x^2}{2z} - \frac{d'x^2}{2z^2}} \int\int P(x_1, x_0) e^{-j\frac{x_1}{2d'}(\frac{d'}{2})^2} dx_0 dx_1 \]

\[ = e^{-jk_0\frac{x^2}{2z} - \frac{d'x^2}{2z^2}} \int\int P(x_1 - x_0) e^{-j\frac{x_1}{2d'}(\frac{d'}{2})^2} dx_0 dx_1 \]

\[ = e^{-jk_0\frac{x^2}{2z} - \frac{d'x^2}{2z^2}} \int_{-\infty}^{+\infty} \left[ \int_{-\infty}^{+\infty} P(x_1 - x_0) dx_0 \right] e^{-j\frac{x_1}{2d'}(\frac{d'}{2})^2} dx_1. \quad (4.37) \]

Since the point spread function for the image system needs to be finite, as a part of the point spread function, \( P(x_1 - x_0) \) has to be equal to 0 at infinity. So we have

\[ \frac{d}{dx_1} \left[ \int_{-\infty}^{+\infty} P(x_1 - x_0) dx_0 \right] = \int_{-\infty}^{+\infty} P'(x_1 - x_0) dx_0 \]

\[ = - \int_{-\infty}^{+\infty} P(x_1 - x_0) = 0. \quad (4.38) \]

In this way, \( \int_{-\infty}^{+\infty} P(x_1 - x_0) dx_0 \) can be proved to be a constant based on Eq. (4.38).

Now by combining Eq. (4.37) and Eq. (4.38), \( E_{\text{ref}}(x, z) \) is expressed as

\[ E_{\text{ref}}(x, z) = C_2 e^{-jk_0\frac{x^2}{2z} - \frac{d'x^2}{2z^2}}. \quad (4.39) \]

\( C_2 \) is a constant.

Finally, the intensity distribution recorded by the CCD camera is given by

\[ I(x, z) = \left| E_{\text{ref}}(x, z) - E_o(x, z) \right|^2 \]

\[ = C_2^2 - C_2 \text{sinc} \left( \frac{k_0\alpha}{zd} \right) \left[ e^{-jk_0\frac{d'x^2}{2z^2}} e^{jW_{640}(\frac{d_1}{\lambda})} x^4 + e^{jW_{640}(\frac{d_1}{\lambda})} x^4 \right] \]
\[ + \text{sinc}^2\left(\frac{k_0x}{zd}\right) \]
\[ = C_2^2 - 2C_2 \text{sinc}\left(\frac{k_0x}{zd}\right)\cos[k_0 \frac{d'x^2}{2z^2} - W_{040} \left(\frac{d_1}{z}\right)^4 x^4] + \text{sinc}^2\left(\frac{k_0x}{zd}\right) \quad (4.40) \]

As mentioned earlier, the spacing between the interference fringes is decided by the chirp function \(\cos[k_0 \frac{d'x^2}{2z^2} - W_{040} \left(\frac{d_1}{z}\right)^4 x^4]\), therefore
\[ \omega(x) = k_0 \frac{d'x}{z^2} - 4W_{040} \left(\frac{d_1}{z}\right)^4 x^3, \quad (4.41) \]
and
\[ s(x) = \frac{2\pi}{\omega(x)} = \frac{2\pi}{k_0 \frac{d'x}{z^2} - 4W_{040} \left(\frac{d_1}{z}\right)^4 x^3}. \quad (4.42) \]

Since the fringe spacing \(s(x)\) is a function of the \(x\) coordinate and the spherical aberration of the optical lens, the Moiré pattern shows up at different positions for the image systems with different spherical aberrations. Due to the limitation of pixel size of the CCD, under-sampling starts from the position where the fringe spacing is equal to twice the pixel size. Also the center of the Moiré pattern is located at the position where the fringe spacing is the same as the pixel size of the CCD camera. The position of the Moiré patterns \(x_{\text{Moiré pattern}}\) is found by solving the equation
\[ s(x) = p. \quad (4.43) \]
where \(p = 6.7\mu m\) is the pixel size of CCD camera. This equation can be solved using MATLAB, as shown in the last Section.

In the following subsection, the effect of aberration in a confocal lens imaging system is investigated.
4.2.2 The confocal lens imaging system with aberration

The point spread function from the object plane to the image plane needs to be revised in the presence of aberration, and is given by

\[
g_{PSF}(x, x_0; y, y_0) = \left\{ e^{j k_0 \frac{(x+x_0)^2+(y+y_0)^2}{2f}} \phi(x,y) \right\} * h(x,y,f)
\]

\[
= \iint e^{j k_0 \frac{(x'+x_0)^2+(y'+y_0)^2}{2f}} \phi(x',y') e^{-j k_0 \frac{(x-x')^2+(y-y')^2}{2f}} dxdy'. \tag{4.44}
\]

Note that, only considering \( x \) component, the \( g_{PSF} \) reduces to

\[
g_{PSF}(x, x_0) = e^{j k_0 \frac{x_0^2-x^2}{2f}} \int_{-\infty}^{+\infty} \phi(x') e^{j k_0 \frac{(xx'+x_0x')}{f}} dx'.
\]

\[
= e^{j k_0 \frac{x_0-x+x}{f}} \int_{-\infty}^{+\infty} \phi(x') e^{j k_0 \frac{(xx'+x_0x')}{f}} dx'. \tag{4.45}
\]

For this con-focal lens system, since the magnification is unity, the approximation \((x - x_0) \ll x\) can be applied to Eq. (4.45), then the point spread function can be rewritten as

\[
g_{PSF}(x, x_0) = \int_{-\infty}^{+\infty} \phi(x') e^{j k_0 \frac{(x+x_0)x'}{f}} dx'. \tag{4.46}
\]

The optical field of the object beam reaching the CCD camera can be expressed as

\[
E_0(x,z) = \iint \text{rect} \left( \frac{x_0}{d} \right) g_{PSF}(x_1, x_0) e^{-j k_0 \frac{(x-x_0)^2}{2z}} dx_0 dx_1. \tag{4.47}
\]

Variable substitution can be applied to such a space invariant linear system, which means that

\[
g_{PSF}(x, x_0) = g_{PSF}(x - x_0) = g_{PSF}(x + x_0). \tag{4.48}
\]

Based on the point spread function, the optical field of the object beam and the reference beam at a distance \( z \) beyond the image plane can be expressed, respectively, as
\[
E_0(x, z) = \iint \text{rect} \left(\frac{x_0}{d}\right) g_{PSF}(x_1, x_0) e^{-j k_0 \frac{(x-x_1)^2}{2z}} dx_0 dx_1 \\
= \iint \text{rect} \left(\frac{x_0}{d}\right) g_{PSF}(x_1 - x_0) e^{-j k_0 \frac{(x_1-x)^2}{2z}} dx_0 dx_1, \quad (4.49)
\]

\[
E_{\text{ref}}(x, z) = \iint g_{PSF}(x_1, x_0) e^{-j k_0 \frac{(x_1-x)^2}{2z}} dx_0 dx_1. \quad (4.50)
\]

Comparing with the single lens system, \( E_0(x, z) \) and \( E_{\text{ref}}(x, z) \) can be obtained for this confocal system by simply letting \( d_i = f \) and \( d' = z \) in Eq. (4.28) and Eq. (4.35).

The approximations applied in Section 4.2.1 are also suitable for the confocal lens system, which also means that the intensity distribution of the diffraction pattern at the front plane of CCD can be obtained from derivations similar to that in Section 4.2.1. The optical field of the object beam and reference beam can therefore be expressed as

\[
E_0(x, z) = e^{-j k_0 \frac{x^2}{2z}} \sin c \left(\frac{k_0 x}{zd}\right) e^{j W_{040} \left(\frac{f}{z}\right)^4 x^4},
\]

\[
E_{\text{ref}}(x, z) = C_2, \quad (4.52)
\]

Hence the intensity distribution of the diffraction pattern can be expressed as

\[
I(x, z) = \left| E_{\text{ref}}(x, z) - E_0(x, z) \right|^2
\]

\[
= C_2^2 - 2C_2 \sin c \left(\frac{k_0 x}{zd}\right) \cos \left[k_0 \frac{x^2}{2z} - W_{040} \left(\frac{f}{z}\right)^4 x^4 \right] + \sin c^2 \left(\frac{k_0 x}{zd}\right). \quad (4.53)
\]

The fringe spacing is given by

\[
s(x) = \frac{2\pi}{k_0 \frac{x}{z^4} W_{040} \left(\frac{f}{z}\right)^4 x^3}. \quad (4.54)
\]

The position of the Moiré pattern \( x_{\text{Moiré pattern}} \) can be predicted by solving the equation

\[
s(x) = p. \quad (4.55)
\]

Again, \( p = 6.7 \mu m \) is the pixel size of the CCD camera.
4.2.3 Comparison between the single lens and confocal lens systems in presence of aberrations

For both the single lens and confocal lens systems, the effect of the aberration on Moiré patterns has been derived. By solving Eq. (4.43) and Eq. (4.55) with different values of the aberration coefficient, the variation of the position of the Moiré patterns with the amount of spherical aberration in single lens system and confocal lens system are plotted in Fig. 4.8.

![Figure 4.8: Position of Moiré patterns as a function of the spherical aberration coefficient. The results are calculated from Eq. (4.42) for red curve (single lens system) and Eq. (4.54) for blue curve (confocal lens system).](image)

Figure 4.8: Position of Moiré patterns as a function of the spherical aberration coefficient. The results are calculated from Eq. (4.42) for red curve (single lens system) and Eq. (4.54) for blue curve (confocal lens system). $f = 50\text{mm, } d = 245\mu\text{m, } z = 10\text{mm, exit pupil diameter, } 2.5\text{mm}$.

The horizontal and vertical axes in Fig. 4.8 represent the spherical aberration coefficient ($W_{040}$) and the position of Moiré patterns, respectively. The single lens system and confocal lens system are distinguished by red and blue lines in the plot. It is
clear that the variation of the spherical aberration term $W_{040}$ has much larger effect on the position of the Moiré pattern in the single lens system as compared to the confocal system. In the case of the single lens system, the position of Moiré pattern shows more sensitivity to the aberration compared to the confocal lens system. In other words, the single lens imaging system has a more significant change of the position of the Moiré pattern for a given aberration term $W_{040}$ compared to the confocal system. Furthermore, the position of the Moiré pattern is relatively invariant in the confocal lens system, unless a relatively large aberration exists.

It is worthwhile to note that the same lenses are used in both image systems in our case. Comparing Eq. (4.42) and Eq. (4.54), the expression for the fringe period in the single lens system has an extra multiplier ($2^4$) in the $x^3$ term, resulting a shift or disappearance of the Moiré pattern as long as the spherical aberration is large enough. This successfully explains the clear observation of Moiré patterns in the confocal lens system, as well as the absence of Moiré patterns in the single lens system.

4.3 Conclusion

Expressions for the position of the Moiré patterns has been derived for both spherical wave illumination and plane wave illumination. According to numerical results, spherical illumination actually results in a displacement of the Moiré patterns, which has been proved by the experimental results. Secondly, it has been shown that the spherical aberration term plays a non-negligible role in the analysis of Moiré effects for both single
lens and confocal lens systems. For imaging systems, the position of Moiré patterns also depends on the spherical aberration term. However, the single lens system turns out to be more sensitive to the aberration term than a confocal lens system. With the same aberration coefficient $W_{040}$, the displacement of the Moiré patterns for a single lens system is more than the confocal lens system, which may be responsible for the disappearance of the Moiré patterns with larger aberration coefficients during digital recording of the holograms.
CHAPTER 5
CONCLUSIONS

The harmful effects of Moiré patterns during digital recording and reconstruction are analyzed for both cases of 1D object (a fiber) and 2D object (a circular block). Theoretical analysis in the absence of far-field approximation is carried out and numerical results are calculated. Experimental results for a 1D object are shown as well, and are consistent with theoretical predictions. It is shown that the unwanted Moiré effects are successfully eliminated by using ultra-short pulse illumination during the recording process not only in simulations but also in experiment.

For the first time to the best of our knowledge, it is shown that spherical aberrations play a non-trivial role in the analysis of Moiré effects. By including the spherical aberration term it is shown that the location of the Moiré patterns critically depends on the spherical aberration in both single lens and confocal lens system. The theoretical analysis shows that the single lens system is more sensitive to aberrations compared to the confocal lens system. It is shown that Moiré patterns may not be recorded for large values of the aberration. In fact, instead of smoothing the Moiré patterns by using ultra-short pulse
illumination, introducing a certain amount of aberration may be a novel method to eliminate the unwanted Moiré patterns in digital holography. A technique on determining the amount of aberration in an optical system by locating the position of the Moiré pattern is possible based on the results of the present work and will be pursued in the near future.
BIBLIOGRAPHY


